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Role of Uncertainty in Life-Cycle Design of Concrete Structures

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ABSTRACT: This paper investigates the time-variant uncertainty effects on the life-cycle safety, robustness and redundancy of deteriorating concrete structures. The investigated performance indicators are evaluated based on a general methodology for concrete structures subjected to diffusive attacks from external aggressive agents. The effects of the uncertainty associated with the random variables involved are quantified and compared by means of proper time-variant importance factors. The application to a reinforced concrete frame under different corrosion damage scenarios shows that the uncertainty effects vary over time with different trends for structural safety, robustness and redundancy.

1. INTRODUCTION

In the classical approach to structural design, the level of performance and functionality is usually specified with reference to structural safety and serviceability and uncertainty is taken into account by means of semi-probabilistic approaches where only a few parameters are considered as random variables. For reinforced concrete structures, random variables are generally associated with material properties, such as concrete and steel strengths, and nominal values are assumed for other mechanical and geometrical parameters (CEN-EN 1992-1-1 2004).

For undamaged concrete structures, this approach is calibrated by conservative codes and is proven to be effective for practical purposes. However, the effects of aging and deterioration processes due to aggressive chemical attacks and other physical damage mechanisms can lead over time to unsatisfactory structural performance (Ellingwood 2005). As a consequence, for deteriorating structures the required level of safety and serviceability should be ensured not only at the initial time, but over the expected service life (Frangopol and Ellingwood 2010, Frangopol 2011, Biondini and Frangopol 2014b). For this purpose, a proper calibration of life-cycle design procedures must consider the time variation of the uncertainty effects associated with the design parameters. In addition, a more comprehensive description of the lifetime structural performance is necessary when aging and deterioration are considered. This is achieved by means of time-variant structural performance indicators, such as robustness and redundancy (Frangopol and Curley 1987, Biondini and Restelli 2008, Biondini 2009, Ghosn *et al.* 2010, Okasha and Frangopol 2010, Decò *et al.* 2011, Saydam and Frangopol 2011, Frangopol and Zhu 2012, Biondini and Frangopol 2014a).

The uncertainty effects on structural safety and serviceability of deteriorating concrete structures have been investigated elsewhere (Biondini *et al.* 2006b, 2008). In this paper, the investigation is extended to compare the time-variant uncertainty effects on the life-cycle structural performance in terms of safety, robustness and redundancy. These indicators are evaluated based on a general methodology for concrete structures subjected to diffusive attacks from external aggressive agents (Biondini *et al.*

2004, 2006a, Biondini and Frangopol 2008). The effects of the uncertainty associated with each random variable are quantified and compared by means of time-variant importance factors computed by regression analyses.

The proposed approach is illustrated through the application to a reinforced concrete frame under different corrosion damage scenarios. The results show that the uncertainty effects and the relative importance of each random variable may vary over time with different trends for structural safety, robustness and redundancy. This further emphasizes the importance of a life-cycle design approach based on multiple performance indicators to take properly into account the time-variant uncertainty effects associated with deterioration processes.

2. DAMAGE MODELING

In concrete structures damage is often induced by diffusion of aggressive agents, such as sulfates and chlorides, which may involve deterioration of concrete and corrosion of reinforcement (CEB 1992).

2.1. Local Damage

The main effect of the corrosion process is a reduction of the reinforcing steel bar area. Such percentage can be effectively described by means of a dimensionless damage index $\delta_s=\delta_s(t)$ which provides a direct measure of damage within the range [0,1]. The corrosion process causes also a reduction of steel ductility (Apostolopoulos and Papadakis 2008). Moreover, the formation of oxidation products may lead to propagation of longitudinal cracks and concrete cover spalling (Vidal *et al*. 2004). These effects can be modeled as a function of the damage index δ_s as proposed in Biondini and Vergani (2014). However, this paper will focus on the effects of corrosion in terms of mass loss of the steel bars. To this aim, a deterioration process with no damage of concrete and uniform corrosion of steel bars is considered (Biondini and Frangopol 2013).

The corrosion rate of steel depends on the concentration of the aggressive agent (Bertolini *et al.* 2004). Based on available information (Pastore and Pedeferri 1994), the damage index $\delta_{sm}=\delta_s(\mathbf{x}_m,t)$ of the m^{th} reinforcing steel bar located at point $\mathbf{x}_m=(y_m,z_m)$ over a member cross-section is related at each time instant t to the diffusion process by assuming a linear relationship between the rate of damage and the mass concentration $C=C(\mathbf{x}_m,t)$ of the aggressive agent (Biondini *et al.* 2004):

$$\frac{\partial \delta_s(\mathbf{x}_m,t)}{\partial t} = \frac{C(\mathbf{x}_m,t)}{C_s \Delta t_s}, \quad t \geq t_{0m} \tag{1}$$

where C_s is the value of constant concentration which would lead to a complete damage of the steel bar over the time interval Δt_s, $t_{0m} = \min\{t \mid C(\mathbf{x}_m,t) \geq C_{cr}\}$ is the corrosion initiation time and C_{cr} is a critical threshold of concentration.

The space and time distributions of concentration $C=C(\mathbf{x},t)$ are affected by the diffusivity D of concrete (Glicksman 2000). An accurate numerical solution of the Fick's differential equations, predicting (a) the diffusive flux to the concentration under the assumption of steady state and (b) how diffusion causes the concentration to change with time, can be achieved by means of cellular automata (Biondini *et al.* 2004, 2006a).

2.2. Global Damage

The *local* damage index δ_{sm} provides a time-variant measure of the corrosion damage of the m^{th} steel bar. A *global* measure of steel damage $\Delta_s=\Delta_s(t)$ may be derived from δ_{sm} by a weighted average over the cross-section (Biondini 2009):

$$\Delta_s(t) = \frac{\sum_m w_{sm}(t)\delta_{sm}(t)A_{sm}}{\sum_m w_{sm}(t)A_{sm}} \tag{2}$$

where A_{sm} is the area of the m^{th} steel bar and $w_{sm}=w_{sm}(t)$ is a suitable weight function. Same weights $w_{sm}(t)=w_0$ can be adopted if there are no bars playing a specific role in the damage process. It is worth noting that this cross-sectional formulation can be extended at the structural level by integration over all members of the system.

3. PERFORMANCE INDICATORS

3.1. Failure Loads

For concrete frame structures, limit states of interest are the occurrence of the first local failure of a critical cross-section, that represents a warning for initiation of damage propagation, and the global collapse of the structural system. Denoting $\lambda \geq 0$ a scalar multiplier of the live loads, these limit states can be identified by the limit load multipliers λ_1 and λ_c associated to reaching of first local failure and structural collapse, respectively. Since structural performance deteriorates over time, the functions $\lambda_1=\lambda_1(t)$ and $\lambda_c=\lambda_c(t)$ need to be evaluated by time-variant structural analyses taking into account the effects of damage (Biondini *et al.* 2004, 2006a). By assuming that shear failures are avoided by a proper capacity design, the limit load multipliers λ_1 and λ_c can be computed under the hypotheses of linear elastic behavior up to first local failure, and perfect plasticity at structural collapse, respectively (Biondini and Frangopol 2008).

3.2. Safety Factor

The main objective of structural design is to ensure an adequate level of safety against the limit state of collapse. A structure is safe if the design load multiplier $\lambda^*=\lambda^*(t)$ is no larger than the collapse value $\lambda_c=\lambda_c(t)$. The safety criterion can be expressed as follows:

$$\Theta(\lambda^*, \lambda_c) = \frac{\lambda_c(t)}{\lambda^*(t)} \geq 1 \qquad (3)$$

where $\Theta(\lambda^*, \lambda_c)$ is a safety factor.

3.3. Robustness Factor

Structural robustness measures the ability of the system to suffer an amount of damage not disproportionate with respect to the causes of the damage itself (Ellingwood 2006). A robustness measure is obtained by comparing the system performance in the original state, in which the structure is fully intact, and in a perturbed state, in which a damage scenario is applied (Frangopol and Curley 1987, Biondini and Restelli 2008). The ratio of the collapse load multiplier $\lambda_c=\lambda_c(t)$ to its initial value $\lambda_{c0}=\lambda_c(0)$ is assumed as time-variant performance index within the range [0;1]:

$$\rho(t) = \frac{\lambda_c(t)}{\lambda_{c0}} \qquad (4)$$

The performance index $\rho=\rho(t)$ is hence compared with the global damage index $\Delta\equiv\Delta_s=\Delta_s(t)$ representing the amount of steel corrosion over the structure within the range [0;1]. The following robustness criterion has been proposed in Biondini (2009):

$$R(\rho, \Delta) = \rho(t)^\alpha + \Delta(t)^\alpha \geq 1 \qquad (5)$$

where $R=R(\rho, \Delta)$ is a robustness factor, and α is a shape parameter of the boundary $R=R(\rho, \Delta)=1$. A value $\alpha=1$ indicates a proportionality between loss of performance and damage. The structural system is robust when $R\geq1$, and not robust otherwise ($R<1$).

3.4. Redundancy Factor

Structural redundancy denotes the ability of the system to redistribute among its members the load which can no longer be sustained by some other damaged members after the occurrence of a local failure (Frangopol and Curley 1987). The load redistribution after the first local failure up to collapse depends on the difference between the limit load multipliers $\lambda_c=\lambda_c(t)$ and $\lambda_1=\lambda_1(t)$. The following quantity is assumed as time-variant measure of redundancy in the range [0;1] (Biondini and Frangopol 2014a):

$$\Lambda(\lambda_1, \lambda_c) = \frac{\lambda_c(t) - \lambda_1(t)}{\lambda_c(t)} \qquad (6)$$

The redundancy factor $\Lambda=\Lambda(t)$ is zero when there is no reserve of load capacity after the first failure ($\lambda_1=\lambda_c$), and tends to unity when the first failure load capacity is negligible with respect to the collapse load capacity ($\lambda_1<<\lambda_c$).

4. ROLE OF UNCERTAINTY

4.1. Probabilistic Model

The probabilistic model is formulated at cross-sectional level by assuming as random variables the strength of both concrete f_c and steel f_{sy}, the coordinates (y_p, z_p) of each nodal point p of the member cross-section, the coordinates (y_m, z_m) and diameter $\varnothing_m$ of each steel bar m, the diffusivity coefficient D, and the steel damage rate $q_s=(C_s\Delta t_s)^{-1}$. Nominal values are considered as mean values. The random variables are assumed uncorrelated with the probabilistic distributions and standard deviation values listed in Table 1 (Biondini and Frangopol 2013).

Table 1. Probabilistic model (nom = nominal value).

Random Variable ($t = 0$)	Distribution	St. Dev.
Concrete strength, f_c	Lognormal	5 MPa
Steel strength, f_{sy}	Lognormal	30 MPa
Coordinates of nodes, (y_p, z_p)	Normal	5 mm
Coordinates of bars, (y_m, z_m)	Normal	5 mm
Diameter of bars, $\varnothing_m$	Normal (*)	$0.10\varnothing_{m,nom}$
Diffusivity, D	Normal (*)	$0.10\ D_{nom}$
Steel damage rate, $q_s=(C_s\Delta t_s)^{-1}$	Normal (*)	$0.30\ q_{s,nom}$

(*) Truncated distributions with non negative outcomes.

4.2. Importance Factor

Let X denote a parameter of the problem, and Y a performance indicator. In order to investigate the time-variant effects of the uncertainty associated with each explanatory variable $X=X(t)$ on each response variable $Y=Y(t)$, the following standard variates are introduced (Biondini *et al.* 2008):

$$\xi(t) = \frac{|X(t) - \mu_X(t)|}{\sigma_X(t)} \tag{7}$$

$$\eta(t) = \frac{|Y(t) - \mu_Y(t)|}{\sigma_Y(t)} \tag{8}$$

where μ_X, σ_X, and μ_Y, σ_Y, are the time-variant mean value and standard deviation of the random variables X and Y, respectively. Based on a data sample of X and $Y=Y(X)$, a set of time-variant least squares linear regression can be performed:

$$\eta(t) = \alpha_{XY}(t)\xi(t) + \alpha_{0,XY}(t) \tag{9}$$

The regression coefficient $\alpha_{XY}=\alpha_{XY}(t)$ provide a time-dependent measure of the sensitivity of the response variable Y with respect to the explanatory variable X. This measure is weighted by means of the correlation coefficient $\rho_{XY}=\rho_{XY}(t)$ to account for the degree of linearity of the model $Y=Y(X)$:

$$I_{XY}(t) = \alpha_{XY}(t)\rho_{XY}(t) \tag{10}$$

where $I_{XY}=I_{XY}(t)$ is an importance factor of the time-variant uncertainty effects related to X and Y.

5. APPLICATION

5.1. Case study

The shear-type reinforced concrete frame shown in Figure 1 is considered (Biondini and Frangopol 2013, 2014a). The frame is subjected to a dead load q=32 kN/m applied on the beam and an horizontal load λF acting at top of the columns, with λ^*=1 and F=100 kN. The material strengths are f_c=40 MPa for concrete and f_{sy}=500 MPa for steel. Two exposure scenarios are studied, with columns exposed (I) on the outermost side only and (II) on the four sides, with concentration C_0.

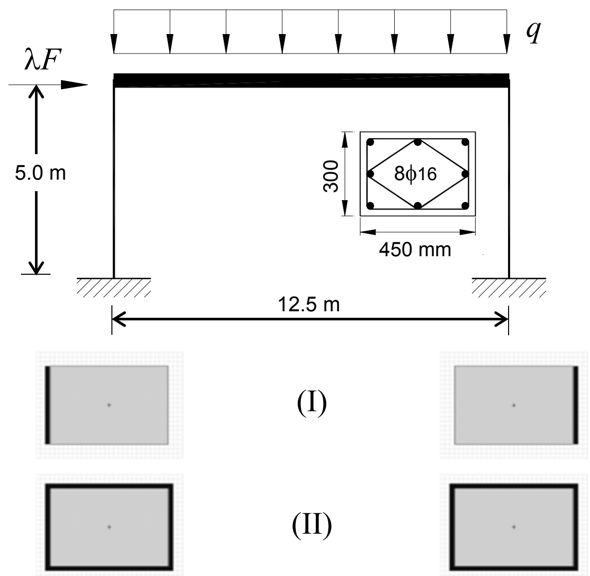


Figure 1. Reinforced concrete frame. Geometry, structural scheme, loading condition and exposure scenarios: (I) columns with exposure on one side; (II) columns with exposure on four sides.

5.2. Performance Indicators

The lifetime structural performance of the frame is investigated in terms of safety, robustness and redundancy by assuming a diffusivity coefficient $D = 10^{-11}$ m^2/sec and a severe corrosion process with C_s=C_0, Δt_s=50 years and C_{cr}=0. Figure 2 shows the evolution over a 50-year lifetime of the probabilistic parameters of the safety factor Θ, robustness factor R and redundancy factor Λ computed by Monte Carlo simulation for the two investigated exposure scenarios.

The results show that, as expected, safety deteriorates over time. Moreover, case (II) with full exposure is the worst damage scenario in terms of safety. Despite this trend, the frame is robust over the lifetime and case (I) with localized corrosion is the worst damage scenario in terms of robustness. For redundancy, a significant increase is obtained over time for case (I). However, the beneficial effects of damage are reduced for case (II), which is the worst damage scenario also in terms of redundancy.

In all cases, the uncertainty effects increase over time with damage. It is also interesting to note that the probability density functions (PDFs) of safety and robustness remain centered over the lifetime around mean values that are close to the nominal values. On the contrary, for redundancy the PDFs are characterized over the lifetime by mean values sensibly higher than the nominal values. This is due to the effects of randomness that, for the cases studied, emphasize the reserve of load capacity after the first local failure and lead, in this way, to an increase on average of the lifetime redundancy.

Therefore, safety, robustness and redundancy over time may exhibit opposite trends depending on the structural system and exposure scenario.

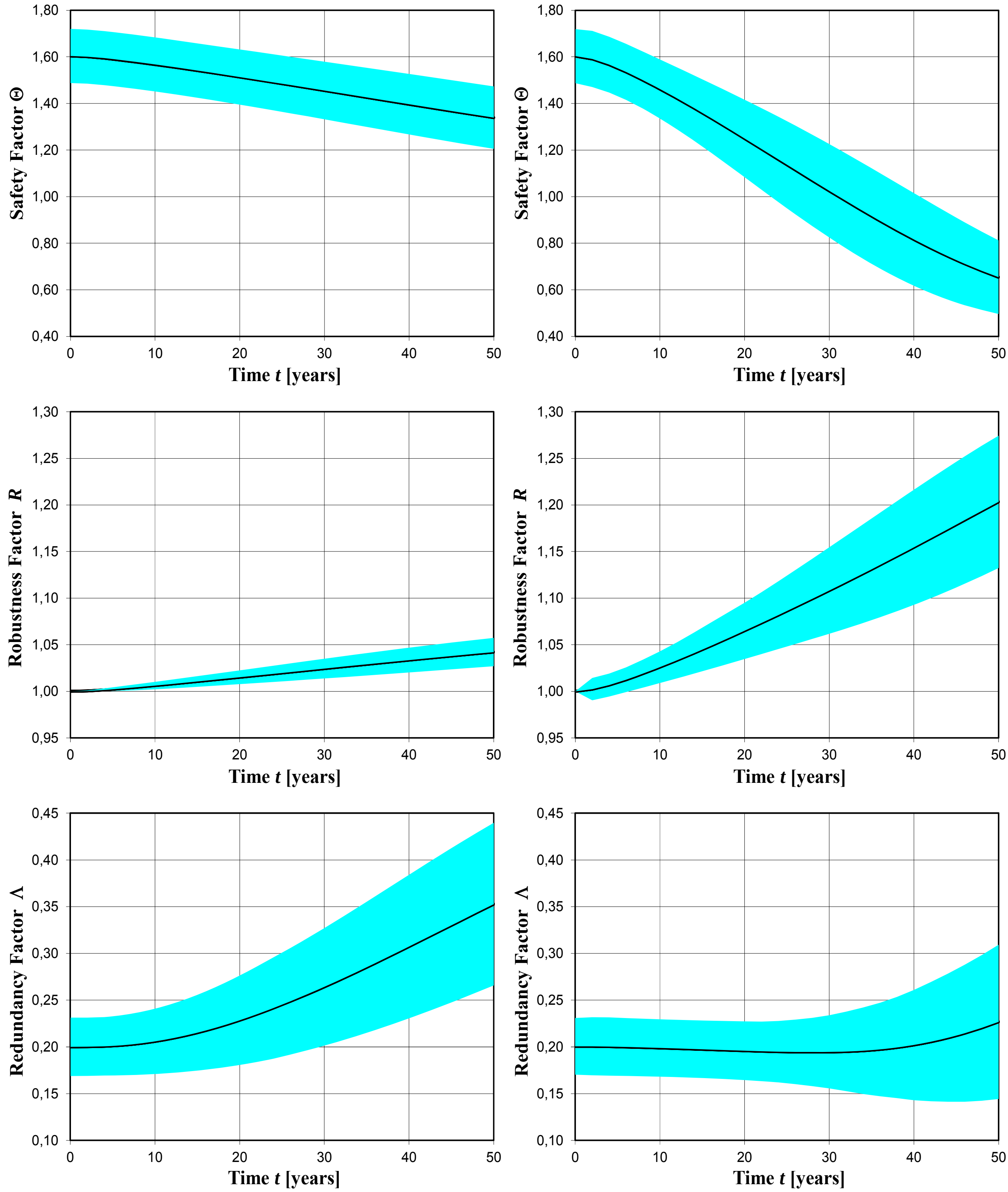
5.3. Uncertainty Effects

The role of the uncertainty effects on the life-cycle performance of the frame system is also investigated. Figure 3 shows the time evolution of the importance factors of the random variables which mainly affect safety, robustness and redundancy for the two investigated exposure scenarios. For the sake of synthesis, the importance factors associated with the area A_s of the steel bars are computed with reference to the mean values of the corresponding standard variates over the whole cross-section.

The safety factor Θ of the undamaged frame mainly depends on the steel strength f_{sy} and steel bar areas A_s. Such dependency quickly decreases over time, and the steel damage rate q_s becomes the more important parameter after about 40 years for case (I) and 10 years for case (II). However, the importance of this parameter tends to decrease when a severe cumulated damage occurs, as it happens for case (II) after about 35 years. A similar dependency on the steel damage rate q_s is found for the robustness factor R, which is mainly related to this variable only. Finally, the redundancy factor Λ of the undamaged frame mainly depends on the steel bar areas A_s. As for the case of safety, this dependency is reduced with damage and over time the importance of the steel damage rate q_s quickly increases. However, an opposite trend is observed for the two exposure scenarios, since the primary role of q_s is reached earlier for case (I) than for case (II), respectively after about 15 years and 25 years. Again, the importance of this parameter decreases under the severe cumulated damage occurring in case (II).

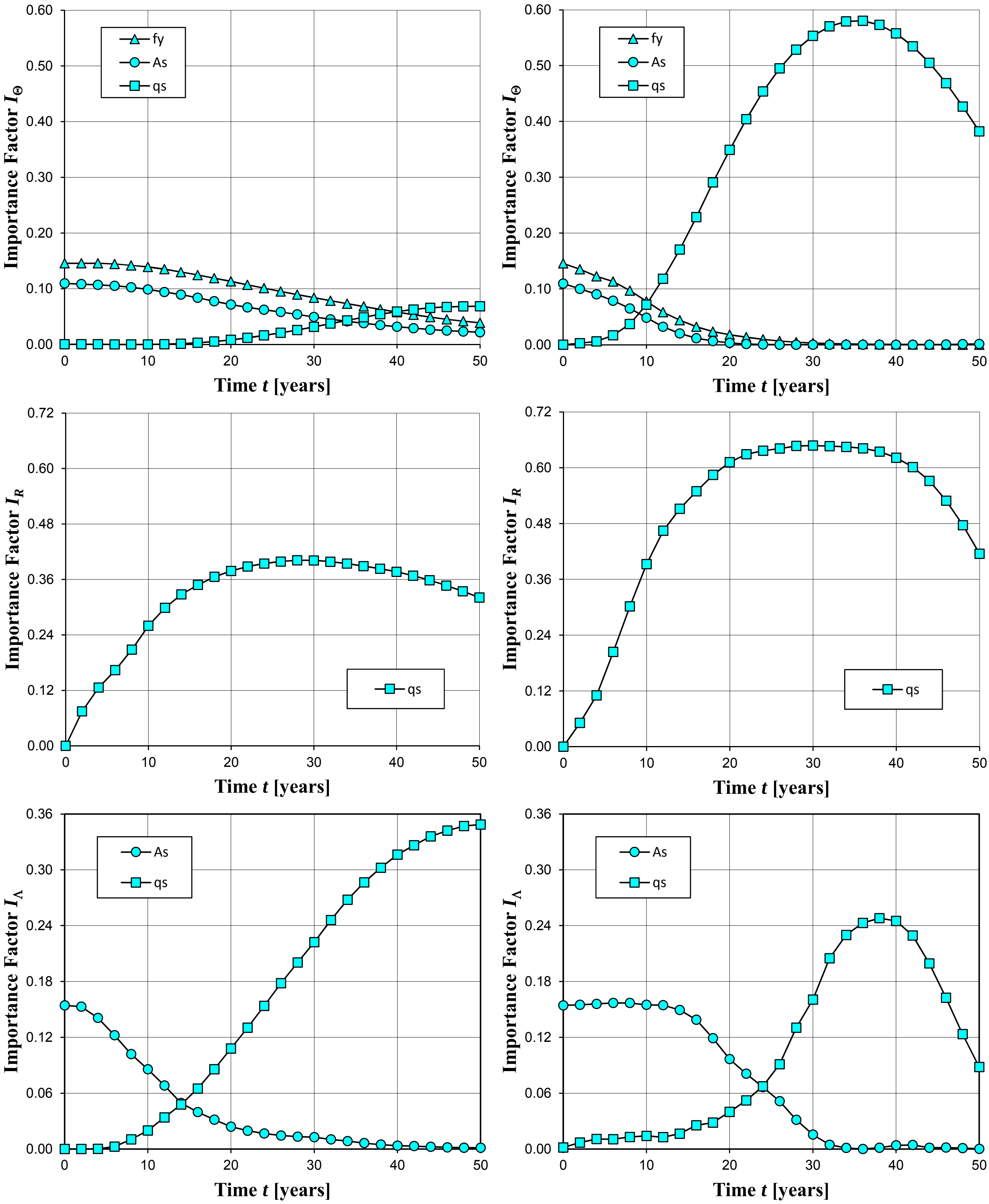
6. CONCLUSIONS

The lifetime safety, robustness and redundancy of concrete structures in aggressive environment have been investigated. The results of this study showed that the importance of the uncertainty effects associated with the design parameters may significantly vary over time, with different trends depending on the performance indicators. Therefore, the classical approach in which the main role in concrete design is played by the uncertainty associated with the material strengths needs to be reconsidered to account for the time-variant uncertainty effects associated to damage. Despite the necessity to extend the investigation by including the effects of time-variant loadings and to improve the regression models, it has been shown that the proposed importance factors can be used effectively to capture the time-variant role played by the uncertainty effects.



(a) *(b)*

Figure 2. Time evolution of the probabilistic parameters of safety factor Θ, robustness factor R and redundancy factor Λ for (a) scenario (I) with exposure on one side, and (b) scenario (II) with exposure on four sides. The shaded region is bounded by the mean plus/minus one standard deviation.



(a) *(b)*

Figure 3. Time evolution of the importance factor of the random variables affecting safety, robustness and redundancy for (a) scenario (I) with exposure on one side, and (b) scenario (II) with exposure on four sides.

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