Reliability Analysis of Water Distribution Networks Using Minimum Cut Set Approach
(Collaborative project funded by Qatar Foundation)

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Water Distribution Networks

Water Distribution Network (WDN)

Source

Pipes

Hydrants

Valves

Pumps

Nodes/ Demand nodes

Demand Node

Pump

Hydrant

Valve

Pipe

Source

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Water Distribution Networks

- 719630 km of water pipes in Canada

- 15.4% of linear assets rated "fair" to "very poor"

- 14.4% of non linear assets rated "fair" to "very poor"

- $25.9 billion estimated for rehabilitation

(Canadian Infrastructure Report Card, 2012)

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Importance of Reliability

Reliability

Planning

Operation

$$$

Main task of WDN

Pressure ✓

Availability ✓

Water Quality ✓

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Types of Reliability

Reliability

Mechanical Reliability

Hydraulic Reliability

Water Quality Reliability

Pipes

Valves

Hydrants

Pumps

Pressure Head

Demand Variation

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Objectives

Introduction

Background

Model Development

Model Implementation

Conclusion

Mechanical Reliability

Component Reliability
- Pipes
- Valves
- Hydrants

Segment Reliability

Collection of components

Network Reliability

Collection of segments

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Reliability Assessment Methods

Connectivity/ Topological

- Shamsi (1990)
- Quimpo and Shamsi (1991)
- Ostfeld (2004)
- Yannopoulos and Spiliotis (2013)

Hydraulic

- Xu and Goulter (1999)
- Shinstine et al. (2002)
- Zhuang et al. (2011)

Entropy as a reliability surrogate

- Prasad and Tanyimboh (2008)
- Tanyimboh et al. (2011)
- Gheisi and Naser (2014)

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Reliability Assessment Methods

Tung (1985) discussed six techniques

- Conditional Probability Approach

- Tie Set Analysis

- Cut Set Method

- Connection Matrix Method

- Event Tree Technique

- Fault Tree Analysis

Minimum Cut Set

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Network Reliability Flowchart

Start

Compute failure rate of all components

Compute component reliability

Compute segment reliability and failure probability of all segments

Perform cut set analysis for all nodes

Compute network reliability based on cut sets

Stop

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Reliability Assessment

Failure Rate (λt)

$$\lambda_{Pipe} = 6 \times 10^{-6} X^2 + 0.0004X + 0.0026$$

X is Age of pipe

$$\lambda_{Component} = \frac{N_f}{Length\ of\ Segment}$$

N_f is Number of failures per year

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Reliability Assessment

Component Reliability

- $R_c = e^{-\lambda t}$
- R_c is Component Reliability

Segment Reliability

- $R_{Seg} = \sum_{i=1}^{n} R_c w_i$
- R_{seg} is Segment Reliability
- $Relative\ Weight\ (w_i) = $
$$\frac{Weight\ of\ Components}{Sum\ of\ Weights\ of\ all\ Components}$$

Component Weights (Salman A., 2011)	
Pipe	38%
Hydrant	31%
Isolation Valve	28%
Control Valve	3%

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Identification of Minimum Cut Sets

Path Matrix

- Find all possible paths from source node to destination node.
- Create path matrix

First order cut sets

- From the path matrix, check if any column is non zero.
- Any non zero column is a first order cut set.

Second order cut sets

- Combine any two columns representing segments in a path matrix and check if their addition creates a non zero column.
- The resultant non zero column of combination of segments is second order cut set.

Identification of Minimum Cut Sets

Path Matrix

$\{\text{A-B and A-C}\}$

$$P = \begin{bmatrix} A & B & C \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

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Identification of Minimum Cut Sets

Second order cut sets

$\{A, B\}, \{B, C\}$ and $\{C, A\}$

$$P = \begin{bmatrix} A+B & B+C & C+A \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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Reliability Assessment

Network Reliability

Probability of Failure of Segments

$Q = 1 - e^{-\lambda t}$

Identification of Minimum Cut Sets

Path Matrix

First Order Cut Sets

Second Order Cut Sets

Mechanical Reliability based on Minimum Cut Sets

$Q(MC_i) = \prod_{j=1}^{n} Q_j$

$R_N = 1 - \sum_{i=1}^{M} Q(MC_i)$

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Hypothetical Network

Demand Nodes

Segments

Source Node

Seg.	Comp.	No. of Failures	Age X(yrs)	Seg. length(m)
A	I.Valve 1	5	N.A	
	Pipe	3	8.4	400
	I.Valve 2	5	N.A	

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Results

Seg.	Comp.	No. of Failures	Age X(yrs)	Seg. length(m)	Failure rate (λt) (Breaks/m)	Comp. reliability R_c	Weight	Relative weight w_i	Seg. reliability R_{seg}	Probability of failure Q
	I.Valve 1	5	N.A		0.0125	0.9876	0.28	0.2979		
A	Pipe	3	8.4	400	0.0064	0.9936	0.38	0.4043	0.990	**0.010**
	I.Valve 2	5	N.A		0.0125	0.9876	0.28	0.2979		

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Results

Segment	Seg. Reliability	Probability of failure $Q = 1 - R_{seg}$
Seg A.	0.989575987	0.010424013
Seg B.	0.986965692	0.013034308
Seg C.	0.994601779	0.005398221
Seg D.	0.988621382	0.011378618
Seg E.	0.993797096	0.006202904
Seg F.	0.985742613	0.014257387
Seg G.	0.982337837	0.017662163
Seg H.	0.981748611	0.018251389

Order of cut sets	List of cut sets
1	{A}
2	{B, C}, {B, F}, {D, G}, {D, H}, {C, F} and {G, H}
3	{B, D, E}, {B, E, G}, {B, E, H}, {C, D, E}, {C, E, H}, {C, F, G}, {D, E, F}, {E, F, H}, and {E, F, G}

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$$R_N = 1 - Q_N = 1 - \sum_{i=1}^{M} Q(MC_i)$$

$$Q_N = Q(MC_1) + Q(MC_2) + Q(MC_3) = 0.0107$$

$$\therefore R_N = 1 - Q_N = \mathbf{0.9893}$$

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Conclusion

- Requires very detailed historic break data of all the components including pipes

- Failure rate of pipes is based only on age

  - More parameters leads to more realistic predictions

  - Research should be extended to predict the failure rate of components other than pipe

- Assessed reliability assuming exponential distribution

  - More effective model is needed to assess reliability

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