Reliability Analysis of Water Distribution Networks Using Minimum Cut Set Approach
(Collaborative project funded by Qatar Foundation)
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Alaa Alhawari
Water Distribution Networks

Introduction

Background

Model Development

Model Implementation

Conclusion

Water Distribution Network (WDN)

Source

Pipes

Hydrants

Valves

Pumps

Nodes/ Demand nodes
Water Distribution Networks

- 719630 km of water pipes in Canada
- 15.4% of linear assets rated “fair” to “very poor”
- 14.4% of non linear assets rated “fair” to “very poor”
- $25.9 billion estimated for rehabilitation

(Canadian Infrastructure Report Card, 2012)
Importance of Reliability

- Main task of WDN
  - Pressure
  - Availability
  - Water Quality

- Reliability
  - Planning
  - Operation

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- Conclusion
Objectives

Mechanical Reliability

Component Reliability
- Pipes
- Valves
- Hydrants

Segment Reliability
Collection of components

Network Reliability
Collection of segments

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Reliability Assessment Methods

Connectivity/ Topological
- Shamsi (1990)
- Quimpo and Shamsi (1991)
- Ostfeld (2004)
- Yannopoulos and Spiliotis (2013)

Hydraulic
- Xu and Goulter (1999)
- Shinstine et al. (2002)
- Zhuang et al. (2011)

Entropy as a reliability surrogate
- Prasad and Tanyimboh (2008)
- Tanyimboh et al. (2011)
- Gheisi and Naser (2014)
Reliability Assessment Methods

Tung (1985) discussed six techniques

- Conditional Probability Approach
- Tie Set Analysis
- Cut Set Method
- Connection Matrix Method
- Event Tree Technique
- Fault Tree Analysis
Network Reliability Flowchart

Start

Compute failure rate of all components

Compute component reliability

Compute segment reliability and failure probability of all segments

Perform cut set analysis for all nodes

Compute network reliability based on cut sets

Stop
### Failure Rate ($\lambda_t$)

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**Failure Rate ($\lambda_t$)**

\[ \lambda_{Pipe} = 6 \times 10^{-6}X^2 + 0.0004X + 0.0026 \]

- $X$ is Age of pipe

**Component Failure Rate**

\[ \lambda_{Component} = \frac{N_f}{\text{Length of Segment}} \]

- $N_f$ is Number of failures per year
Reliability Assessment

Component Reliability

- $R_c = e^{-\lambda t}$
- $R_c$ is Component Reliability

Segment Reliability

- $R_{seg} = \sum_{i=1}^{n} R_c w_i$
- $R_{seg}$ is Segment Reliability
- Relative Weight ($w_i$) = Weight of Components / Sum of Weights of all Components

Component Weights (Salman A., 2011)

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe</td>
<td>38%</td>
</tr>
<tr>
<td>Hydrant</td>
<td>31%</td>
</tr>
<tr>
<td>Isolation Valve</td>
<td>28%</td>
</tr>
<tr>
<td>Control Valve</td>
<td>3%</td>
</tr>
</tbody>
</table>
Identification of Minimum Cut Sets

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Path Matrix

First order cut sets

- Find all possible paths from source node to destination node.
- Create path matrix

- From the path matrix, check if any column is non zero.
- Any non zero column is a first order cut set.

Second order cut sets

- Combine any two columns representing segments in a path matrix and check if their addition creates a non zero column.
- The resultant non zero column of combination of segments is second order cut set.
Identification of Minimum Cut Sets

Path Matrix

\[
P = \begin{bmatrix}
A & B & C \\
1 & 1 & 0 \\
1 & 0 & 1
\end{bmatrix}
\]

\{A-B and A-C\}
Identification of Minimum Cut Sets

Second order cut sets

\[ P = \begin{bmatrix} A+B & B+C & C+A \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \]

\{A, B\}, \{B, C\} and \{C, A\}
Reliability Assessment

Network Reliability

- Probability of Failure of Segments
  \[ Q = 1 - e^{-\lambda t} \]

- Identification of Minimum Cut Sets
  - Path Matrix
  - First Order Cut Sets
  - Second Order Cut Sets

- Mechanical Reliability based on Minimum Cut Sets
  \[ Q(MC_i) = \prod_{j=1}^{n} Q_j \]
  \[ R_N = 1 - \sum_{i=1}^{M} Q(MC_i) \]
Hypothetical Network

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<table>
<thead>
<tr>
<th>Seg.</th>
<th>Comp.</th>
<th>No. of Failures</th>
<th>Age X(yrs)</th>
<th>Seg. length(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>I.Valve 1</td>
<td>5</td>
<td>N.A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pipe</td>
<td>3</td>
<td>8.4</td>
<td>400</td>
</tr>
<tr>
<td>A</td>
<td>I.Valve 2</td>
<td>5</td>
<td>N.A</td>
<td></td>
</tr>
</tbody>
</table>
## Results

<table>
<thead>
<tr>
<th>Seg.</th>
<th>Comp.</th>
<th>No. of Failures</th>
<th>Age X(yrs)</th>
<th>Seg. length(m)</th>
<th>Failure rate ((\lambda t)) (Breaks/m)</th>
<th>Comp. reliability (R_c)</th>
<th>Weight</th>
<th>Relative weight (w_i)</th>
<th>Seg. reliability (R_{seg})</th>
<th>Probability of failure (Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>I.Valve 1</td>
<td>5</td>
<td>N.A</td>
<td>400</td>
<td>0.0125</td>
<td>0.9876</td>
<td>0.28</td>
<td>0.2979</td>
<td>0.990</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>Pipe</td>
<td>3</td>
<td>8.4</td>
<td></td>
<td>0.0064</td>
<td>0.9936</td>
<td>0.38</td>
<td>0.4043</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>I.Valve 2</td>
<td>5</td>
<td>N.A</td>
<td></td>
<td>0.0125</td>
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<td>0.28</td>
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</tr>
</tbody>
</table>

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### Results

#### Order of cut sets

<table>
<thead>
<tr>
<th>List of cut sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

#### Segment Seg. Reliability Probability of failure

<table>
<thead>
<tr>
<th>Segment</th>
<th>Seg. Reliability</th>
<th>Probability of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seg A.</td>
<td>0.989575987</td>
<td>0.010424013</td>
</tr>
<tr>
<td>Seg B.</td>
<td>0.986965692</td>
<td>0.013034308</td>
</tr>
<tr>
<td>Seg C.</td>
<td>0.994601779</td>
<td>0.005398221</td>
</tr>
<tr>
<td>Seg D.</td>
<td>0.988621382</td>
<td>0.011378618</td>
</tr>
<tr>
<td>Seg E.</td>
<td>0.993797096</td>
<td>0.006202904</td>
</tr>
<tr>
<td>Seg F.</td>
<td>0.985742613</td>
<td>0.014257387</td>
</tr>
<tr>
<td>Seg G.</td>
<td>0.982337837</td>
<td>0.017662163</td>
</tr>
<tr>
<td>Seg H.</td>
<td>0.981748611</td>
<td>0.018251389</td>
</tr>
</tbody>
</table>

\[
R_N = 1 - Q_N = 1 - \sum_{i=1}^{M} Q(MC_i)
\]

\[
Q_N = Q(MC_1) + Q(MC_2) + Q(MC_3) = 0.0107
\]

\[\therefore R_N = 1 - Q_N = 0.9893\]
Conclusion

- Requires very detailed historic break data of all the components including pipes
- Failure rate of pipes is based only on age
  - More parameters leads to more realistic predictions
  - Research should be extended to predict the failure rate of components other than pipe
- Assessed reliability assuming exponential distribution
  - More effective model is needed to assess reliability
Thank you

Questions?