5th International/11th Construction Specialty Conference
5e International/11e Conférence spécialisée sur la construction

Vancouver, British Columbia
June 8 to June 10, 2015 / *8 juin au 10 juin 2015*

MODELING SUBWAY RISK ASSESSMENT USING FUZZY LOGIC

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Abstract: According to the Canadian Urban Transit Association (CUTA 2012), 140 Billion CAD is required to maintain, rehabilitate, and replace subway infrastructure between the years 2010 and 2014. However, transit authorities are faced by a fund scarcity problem which is hindering them from addressing all the network rehabilitation requirements in an efficient manner. The solution according to the 2013 America's infrastructure report card is to adopt a comprehensive asset management system to maximize investments. This research develops a risk assessment model for subway stations. Probability of failure of different subway elements are developed using Weibull reliability curves. Consequences of failure are measured against three predefined attributes these are financial, operational, and social impacts of failure. Finally, a criticality index measures the respective station criticality derived from its particular size, location in proximity to different attraction types, and, nature of use. A qualitative approach with the help of expert judgment is adopted to integrate the indices using the Fuzzy Analytic Network Process with application to Fuzzy Preference Programming. The three models are integrated into a fuzzy rule based risk index model to compute element and station expected risk index. The output of the model is a comprehensive risk index that can be used to prioritize elements across stations for rehabilitation. The model is verified through an actual case study comparing elements across six stations and computing probability of failure, consequence of failure, criticality and the risk index. This paper illustrates the general framework of the proposed methodology which will help decision makers prioritize stations and elements across stations for rehabilitation based upon their risk index.

1 INTRODUCTION

Subway systems failure is associated with consequences like multiple fatalities or injuries, partial or complete loss of service, major traffic disruptions, and, different socio-economic effects. A subway network is composed of diverse components and systems operating simultaneously to deliver the required service. The component diversity causes a level of complexity which complicates the process of assessing and maintaining the network at the desired level of service. In addition, the problem of fund scarcity faced by most public authorities converts it into a tough task. According to (Semaan 2011), “Société de Transport de Montréal” estimated a required amount of 5.1 Billion CAD for maintenance of its subway system for the next ten years. Different systems operating in a subway network compete for rehabilitation priorities while having various consequences of failure and multiple failure modes which turns the prioritization process into a tough task. Moreover, elements operating in a subway network pose diverse rehabilitation and maintenance needs based on their role in the network hierarchy and operation. The current method used for prioritizing subway stations is visual inspection. Hastak and Baim (2001) stated that in the subway stations context, inspections are used to identify the needed assessment for rehabilitation work. However, since no federal or state regulatory is used for inspections; the development and the implementation of the inspection standards in mainly the transit management responsibility

(Russel et al. 1997). This research aims at developing a risk assessment model on a network level based upon the visual inspection reports subway structural elements. This is a four-phased model in which sub-models for measuring the components of a risk equation, namely probability and consequence of failure, are first developed. The paper proceeds to suggest an addition to the classical risk equation to be better suited for the case of subway networks. The risk equation components are then combined using the fuzzy inference system.

The following section presents a background for the current practices adopted in addition to the available researches. The background is followed by the methodology section in which the developed models are explained in details. Finally, a case study is presented to demonstrate how the model works and its validation.

1.1 Background

1.1.1 Subway Assessment Efforts

The literature demonstrates research and industrial efforts to assess the condition of subway stations and rank stations for maintenance and rehabilitation. California transit authority developed an evaluation system for stations and ranked them on a scale from excellent to poor based on predefined criteria combined using a weighted average technique (Abu-Mallouh 1999). Whereas, Metropolitan Transit Authority of New York Transit developed a ranking system for condition assessment by assigning points to different functional factors (Abu-Mallouh 1999). London Transit developed the Key Performance Indicator to evaluate the performance of stations from its customers' point of view using a direct evaluation of customer satisfaction through surveys and interviews. The Paris Rapid Transit Authority worked on developing a selection procedure for stations in need of rehabilitation, the model used a seven functional criteria selection procedure.

Subways domain was shyly researched in academia with only a handful models assessing subway stations. Abu-Mallouh (1999) developed a model to optimize the number of stations accommodated within a given capital program for full and partial rehabilitation. Semaan (2009) developed a condition assessment model to diagnose specific subway stations and assess their conditions using an index (0-10). In a corresponding effort, (Farran 2006) developed a model to address life cycle costing for a single infrastructure element with probabilistic and condition rating approach for condition state. And finally, (Semaan 2011) developed a model to evaluate structural performance of different components in a subway network using performance curves for components and the entire network using reliability-based cumulative Weibull function.

It is noted that the reported transit management practices adopted a qualitative functional perspective to inspect and prioritize subway stations for rehabilitation. On the other hand, the academia focused mainly on structural quantitative models through condition assessment and deterioration models. While these two perspectives of assessment are vital; none of the reported literature integrates the functional and structural aspects of a subway station into a single model.

1.2 Methodology

The developed methodology aims at combining structural and functional perspectives of a subway network into a single risk assessment model. The structural integrity is assessed through a probability of failure sub-model whereas the functional perspective is assessed through the consequence of failure and criticality index sub-models. The output of the three sub-models is then integrated into a risk index model using 30 rules extracted from experts' knowledge. This section starts by presenting the network hierarchy used through the analysis and proceeds with the sub-models and model development.

1.2.1 Subway Hierarchy

A generic subway network hierarchy is presented in Figure 1. A typical subway line is composed of a number of station buildings. They operate by means of their composing systems such as electrical, mechanical, security and communication, and, structural. This research focuses only on the operational risk failure derived from the structural systems in a network. Therefore, the structural system is identified as a composition of stations, tunnels and auxiliary structures. These are composed of the elements located at the lowest level of the hierarchy. This hierarchy will be the basis of calculations through model development and its associated sub-models.

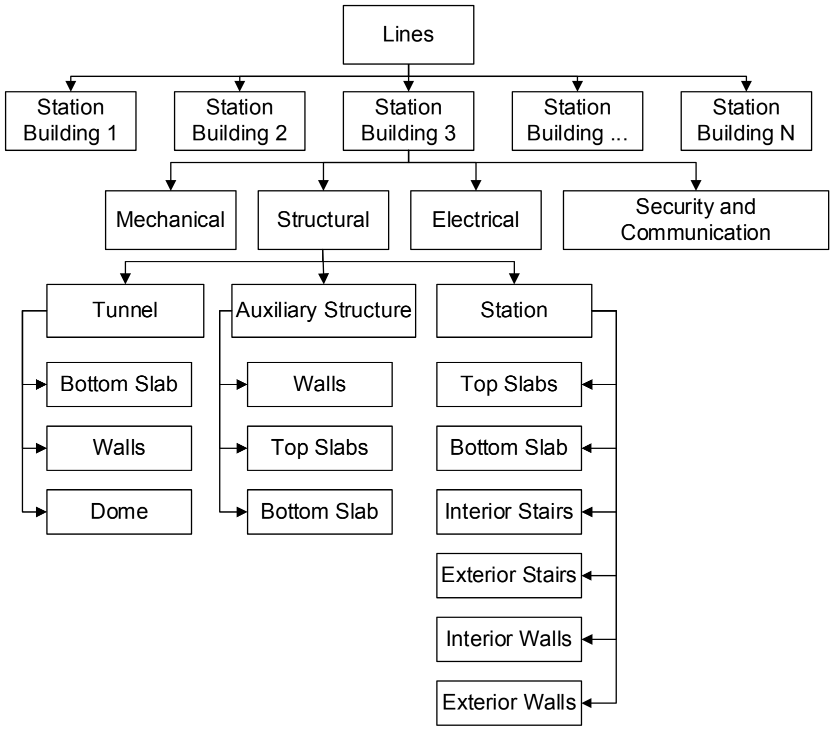

Figure 1: A Generic subway network hierarchy

1.2.2 Probability of Failure Sub-Model

The Probability of Failure (PoF) sub-model builds upon the performance model developed by (Semaan 2011). Semaan (2011) used reliability-based cumulative Weibull function to evaluate the structural performance of different components in a subway network and develop performance curves for subway components and the entire network. Reliability-based cumulative Weibull function takes a probabilistic approach that yields a reliability index, which is the inverse of the PoF. Therefore, PoF can be estimated as the inverse of the reliability and is shown in Equation [1]

[1] $\mathrm{PoF}_f = 1 - R(t) = 1 - e^{-\left(\frac{t-\alpha}{\tau}\right)^{\delta}}$

Where,
R (T) = Reliability, t = Time, δ = deterioration parameter, α = location parameter, τ= scale parameter, δ= and e = exponential.

Different system configuration requires different calculations for PoF values. The series-parallel reliability technique (Hillier and Lieberman 1972) entitles that any system is composed of components outlined in parallel, in series, or, in a combination of both. A system in parallel is a redundant system where components work simultaneously; hence, it can operate even if one of its components fails. This is the logic used to calculate the different PoF values. A subway network is composed of lines, stations, and auxiliary structures, the PoF is calculated for each system based on the configuration shown in Figure 2.

<u>*Station System (STA)*</u>: In a subway station system, the slab and stairs are redundant systems and can be considered as a parallel system. The wall system is a series system in which if any wall "fails" to perform, the whole station becomes unsafe, and thus does not perform. PoF of a station system can be computed using equation [2]

[2] P_{STA} = 1- [(1- $\prod_{i=1}^{n} P_{STEi} P_{STIi}$)*(1- $\prod_{i=1}^{n} P_{SEi} P_{SIi}$)*(1- $\prod_{i=1}^{n} (1-P_{WIi})(1-P_{WEi})$)]

Where,

P_{STAj} = Probability of station j failure, P_{STE} = Probability of exterior stairs failure, P_{STI}= Probability of interior stairs failure, P_{SE}= Probability of external slab failure, P_{SI}= Probability of internal slab failure, P_{WI}= Probability of internal wall failure, P_{WE}= Probability of external wall failure, and, i=1, 2 … n = station floor.

<u>*Tunnel System (TUN):*</u> A tunnel system operates in series in which it fails if any of its components fail, therefore, PoF values are calculated using equation [3]

[3] P_{TUN} =1 - $(1-P_D)*(1-P_w)*(1-P_s)$

Where;

P_{TUN} = Probability of tunnel failure, P_D = Probability of Dome failure, P_w = Probability of wall failure, P_s = Probability of slab failure.

<u>*Auxiliary structures System (AS):*</u> These systems operate in series in which it fails if any of its components fail, therefore, PoF are calculated using equation [4]

[4] $P_{Aux\ St}$ = 1 – $(1-P_w)$ (1- $P_{TS}*P_{BS}$)

Where;

$P_{Aux\ St}$ = Probability of auxiliary structure failure, P_W = Probability of walls failure, P_{TS} = Probability of top slab failure, and, P_{BS} = Probability of bottom slab failure.

<u>*A Line System:*</u> is composed of all stations, tunnel, and auxiliary structure systems operating on the line. These systems together operate in series whereas; the composition of each system operates in parallel. The stations systems are redundant system, they operate in parallel and will fail to operate when all stations in a line fail. Likewise, a line failure occurs when all tunnels on the line fail to operate. Same applies for the auxiliary structure, operating is parallel in a line systems. On the other hand, the three systems operate in series. If any of the systems fails entirely that means the subway line is in a failure status and can no more function effectively. The line hierarchy is shown in Figure 2 (a) and is computed using equation [5]

[5] $P_{line\ i} = 1 - [(1 - \prod_{n}^{i=1} P_{STA_i})^{*}(1 - \prod_{n}^{i=1} P_{TUN_i})^{*}(1 - \prod_{n}^{i=1} P_{AUX_i})]$

Where;

P_{line} = Probability of line failure, P_{STA} = Probability of station failure, P_{TUN} = Probability of tunnel failure, $P_{Aux\ St}$ = Probability of auxiliary structure failure, and i=1, 2 … n = number of systems in a line.

Subway Network; a subway network is composed of all the lines operating in the network. It can be concluded that the lines in a network operate in parallel. Hence, the network only fails when all the lines operating in the network fail. This can be computed using equation [6] and concluded from Figure 2 (b).

[6] $P_{Net} = \prod_{n}^{i=1} P_{Linei}$

Where;

P_{Net} = Probability of network failure, P_{Linei}= Probability of line failure.

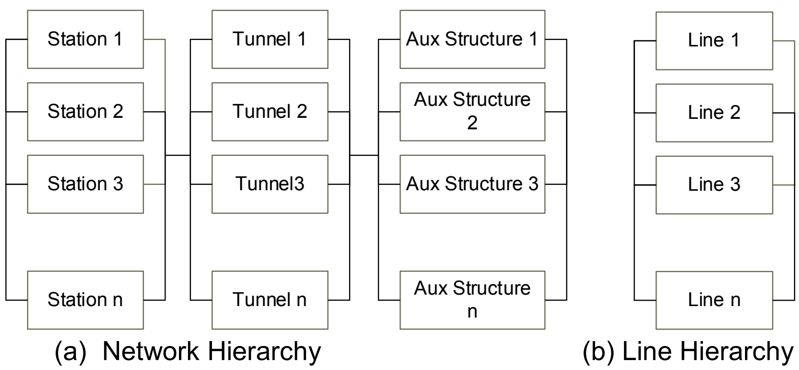

Figure 2: Schematic diagrams for network and line hierarchy.

1.2.3 Consequences of Failure Sub-Model

A generic risk management system should identify PoF and Consequences of Failure (CoF) to be combined later to produce a representative risk index. A formal review of failure consequences diverts attention away from maintenance tasks having little or no effects and focuses on maintenance tasks that are more effective. This ensures the maintenance spending is optimized and guarantees the inherent reliability of equipment is enhanced (Gonzalez et al. 2006). Indirect impacts of failure of a subway station include, but are not limited to, service disruption, passenger delay, loss of reputation, loss of revenue in addition to other socio-economic impacts reflected as the extent to which the failure affects adjacent services and customers benefiting from the service and the ease of providing an alternative service. However, only a fraction of the expected CoF can be monetized whereas most of the expected indirect CoF are difficult to monetize and measure (Muhlbauer and W Kent. 2004). One way to overcome the difficulty inherent in these calculations is measuring CoF using indices, which facilitates comparing between expected CoF and highlights areas of higher failure impacts.

This research determined factors affecting CoF calculations in terms of tangible and intangible impacts using the Triple Bottom line approach. This revealed a wide spectrum of consequences occurring at element and station levels. A station is composed of a number of elements operating simultaneously; based on the location of the element and its nature, the element failure might cause total, partial, or no station closure. This suggests CoF are element-dependent, Figure 3 outlines the CoF model. Based on literature and expert opinion, CoF are broadly grouped into financial, social, and, operational impacts of failure. It is noted that some factors could fall under two different perspectives simultaneously.

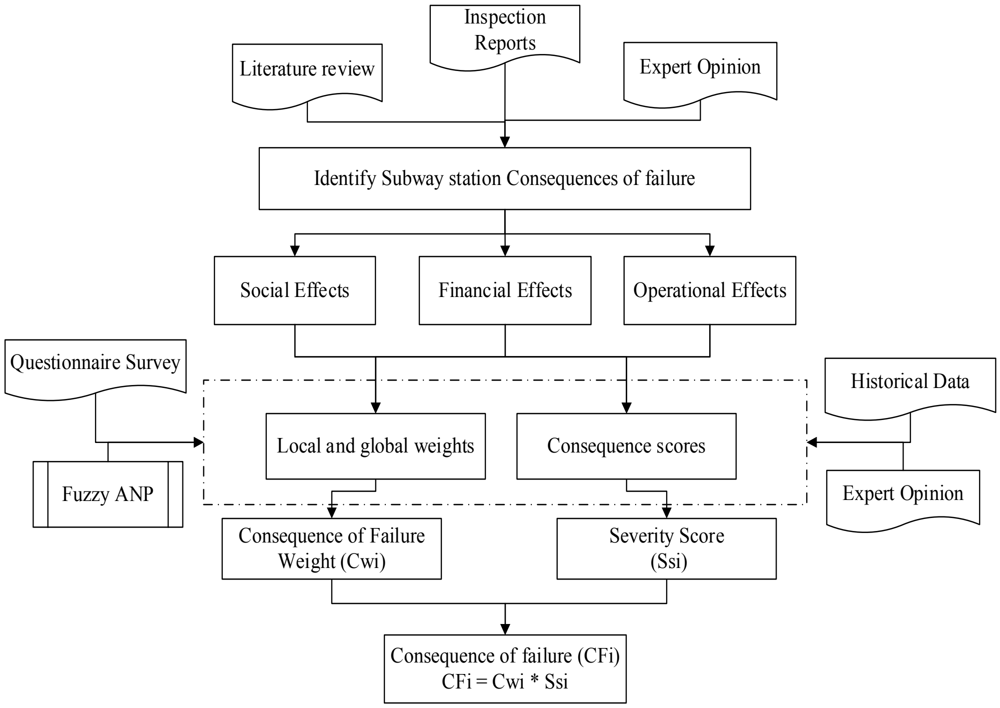

Figure 3: Consequences of failure model outline.

The defined impacts of failure along different categories are interdependent; hence, the effect of a single impact cannot be measured independently without considering how other impacts affect and are affected by its occurrence. Therefore, Fuzzy Analytical Network Process (FANP) is selected to obtain relative weights of these factors. FANP addresses the interdependency inherent in the relation between these factors and accounts for the uncertainty caused by using expert opinions. The reader is referred to (Abouhamad and Zayed 2013a) for further model details. For each subway element, the consequence of failure index ($\mathrm{CoF_i}$) is computed using equation [7]

[7] $\mathrm{CoF_i} = \mathrm{CW_i} * \mathrm{Ss_i}$

Where;

$\mathrm{CoF_i}$= Consequence of Failure Index, $\mathrm{CW_i}$ = Criteria weight obtained using questionnaire survey and FANP, $\mathrm{Ss_i}$ = Severity score calculated from network data and inspection reports, i= elements operating per station.

Financial impacts are twofold; repair/replacement cost defined as the direct cost of repair or replacement and loss of revenue defined as the profit loss due to service interruption. Operational impacts is measured in terms of ease of providing alternative and time to repair. The ease of providing alternative is measured by means of available bus stops and reroutes in case of no service whereas, the time to repair is the time required to return the component to a full functioning state. The social impacts are measured by user traffic frequency, maximum allowable interruptions per year per station and the degree of service interruption whether partial, total or no interruption at all.

1.2.4 Criticality Index Model

This research introduces criticality for the scope of subway networks as the Criticality index. The subway network breakdown structure is assessed differently, the element is selected such that its criticality level is dominant and diverse enough to prevail over other network components. Consequently, subway stations are selected to be the focus of criticality analysis. Systems and subsystems share the same major role of delivering the service; however, their criticality is derived from their respective locations in stations that vary in criticality according to several factors. From this discussion, the concept of criticality propagation is introduced; criticality level propagates upwards and downwards in a hierarchy of a subway network such that they acquire the same criticality level as stations where they operate. Similarly, a line criticality is computed as the sum of criticality indices of stations existing on this line. For interconnecting systems such as tunnels and auxiliary structures, C_R is computed as the higher index of the two corresponding stations through which this system connects.

Factors contributing to station C_R are identified through historical data, expert opinion and by consulting current structure and map of several subway networks. Station criticality is a complex decision based on different attributes defined as; number of lines, number of levels, station use whether end or intermodal, and station proximity to different attraction locations. C_R factors defining a station differ in significance, thus, a weight component is introduced to the C_R equation to accommodate the subjective variability in attributes weight. Attribute scores are computed based upon the network under examination and individual station information. Station criticality is defined in terms of three main factors and seven sub factors or attributes. Amongst attributes identified, the station location is the most diverse. For further details about this model, the reader is referred to (Abouhamad and Zayed 2013b). Station criticality attributes are strongly connected, hence, cause and effects loops flow between them. Therefore, FANP with application to Fuzzy Preference Programming is used to compute the attributes weight. The Criticality Index model is outlined in Figure 4.

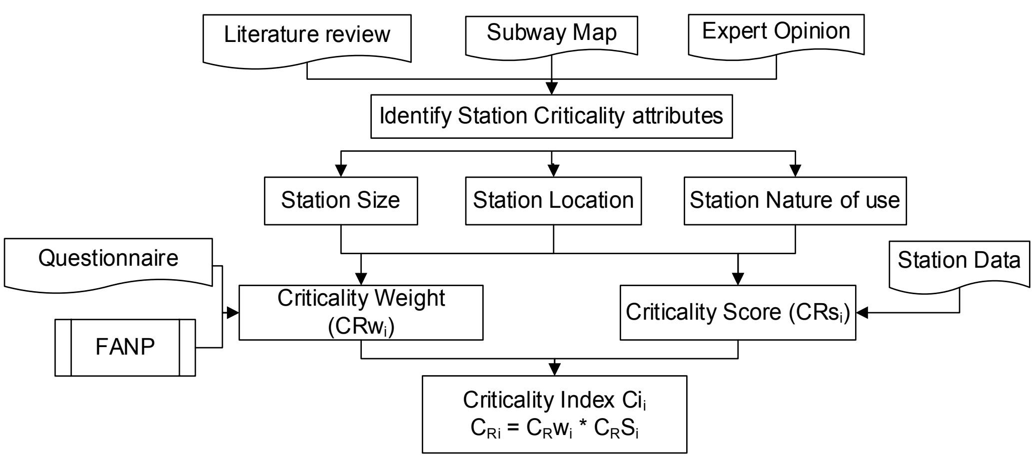

Figure 4: Criticality Index model outline.

Criticality Index per station (C_R) is computed using equation [8]

[8] $C_R = \sum_{i=1}^{n} C_R wi * C_R s$

Where; C_R= Criticality Index per station, $C_R w_i$= Criticality attributes weights calculated using questionnaire

surveys and FANP, $C_R s$ = Criticality scores calculated using current network data, and i=1,2, …,n, n= criticality attributes

1.2.5 Risk Index Model

Risk by definition is a combination of PoF and the severity of adverse effects (Lowrance 1967). When studying the risk level, it should be noted that elements with similar PoF might show wide variation in terms of consequences of failure and vice versa. In addition, critical elements with high consequences of failure usually compose a smaller portion of the overall network. Accordingly, focusing only on these elements would result in an unbalanced management practices since unexpected failures may occur in less-critical elements, which constitute the majority of the network. Furthermore, a comprehensive risk assessment should consider the relative importance of different components and systems of a subway network. A criticality index is introduced to measure the relative importance and consider it in the risk index development. Consequently, a new term is added to the risk equation, named as the criticality index

($\boldsymbol{C_R}$). Several methods exist to compute the risk index value, ranging from simple straightforward multiplication to more sophisticated computation of risk matrix.

The Fuzzy Rule Based (FRB) technique was selected to compute the risk index in this research. This method permits users to integrate their experience into the decision support system through using "if-then" rules. Fuzzy sets allow for a more precise presentation of element's membership particularly when it is difficult to determine the boundary of the set as crisp values. An FRB consists of a set of if-then rules defined over fuzzy sets (Masulli et al. 2007). The rules are usually created using "expert knowledge" (Castillo et al. 2008). The relationship between different fuzzy variables is represented by if-then rules of the form "If antecedent…… Then Consequent". In cases where the antecedent has more than one part, the fuzzy operator is applied to obtain one number representing the consequence for the antecedents of that rule. This is the number used afterwards to obtain the output function. The Mamdani fuzzy inference system (Mamdani and Assilian 1975) uses the min-max composition as defined in equation [9]

[9] $\mu_{C_K}(Z) = \max\left[min\left[\mu_{A_K}\left(input\ (x)\right), \mu_{B_K}\left(input\ (y)\right)\right]\right] k$

Where;

μ_{C_K} ,μ_{A_K}, μ_{B_K} are the membership functions for output "z" for rule "k",X and y are inputs.

Whereas in our case, the antecedent and the consequent are fuzzy propositions. The proposed model is performed using MATLAB® fuzzy logic toolbox. Mamdani algorithm based on experts' knowledge is used to construct the rule base. The model combines PoF, CoF, and C_R expressed as triangular membership functions. The min-max composition is used whereas the defuzzification was done using the Centre of Area method. The fuzzy risk equation solves equation [10] and is shown in equation [11]

[10] Risk Index = Probability of Failure * Consequence of Failure * Criticality Index

[11] Ri: IF PoF is Xi and CoF is Yi and C_R is Zi then Risk is Li

Where, i= 1, 2, 3 ….k , Xi. Yi, Zi, and Li are linguistic constants as defined in model, k = number of rules

The threshold for risk values are set based on the maximum allowable PoF and CoF values. This eliminates the major drawback of a risk matrix in differentiating between the two extreme cases of high PoF with low CoF and vice versa. It also ensures the highest priority is given to elements with most emerging rehabilitation need whether derived from high PoF or high CoF. Based upon feedback from experts, CoF is categorized into three levels based upon the combined effect of failure on financial, social, and operational levels. Criticality serves to define stations into normal stations with moderate importance and critical stations with higher criticality. All data incorporated in the risk index calculations is reserved for a detailed analysis of each station. The membership functions were selected based on literature review and unstructured interview with subway experts. A set of 30 rules (5 rules for PoF, 3 for CoF and 2 for C_R) is generated to develop the Risk Index.

1.3 Case Study and Model Implementation

An actual case study was conducted on a sub network in Montreal metro to validate the model and proof its robustness. Montreal subway is one of the oldest networks in North America, with 68 stations spreading on four lines and covering the north, east, and centre of the Island of Montreal. Six stations (SEG 1 to SEG 6) on three different lines are analysed in the model with one station being the interconnecting station. SEG 1 to SEG 3 fall on the same line given the name Line A, SEG 4 and SEG5 both fall on the second line B. SEG 6 falls on line C whereas, SEG 2 is the interconnecting station for the three lines. Stations were selected from literature review (Semaan 2011) and based upon availability of inspection reports for different indices calculations.

1.3.1 Sub-Models output

PoF is calculated using year 2014 as the base for calculations. The subway system hierarchy together with the equations presented earlier were used to compute PoF values for elements at the lowest level of the hierarchy then aggregated upwards to compute the integrated PoF values for stations, tunnels and auxiliary structures, identified as a segment (SEG).Sample output PoF values are illustrated in Figure 5.

A questionnaire survey was launched to gather the required data for CoF and Criticality models' development. The questionnaire conducted pairwise comparisons between attributes, sub-attributes and goals for each of the two sub-models. It also contained open ended questions for experts to provide their opinion on model development and suggest any required modifications. The output of the questionnaires are local and global weights for attributes in CoF and C_R models. Further details about the resultant weights can be found in (Abouhamad and Zayed 2014). FANP calculations are done using MATLAB® software and FPP as a prioritization tool.

Scores for CoF attributes were obtained from literature review (Farran, 2006) and current information of Montreal subway. Sample calculations for CoF index are seen in Figure 5. As stated earlier, CoF are calculated for elements at the lower level of the hierarchy then aggregated upwards. It is evident that CoF are highly affected by PoF value for each system since all the factors accounted for in the model are directly proportional with PoF value. Calculations for C_R were done for the entire Montréal subway network (68 stations). Two stations with maximum and minimum criticality levels were set as thresholds for normalizing the index for the six stations under study. Criticality index is defined as the functional role a station plays and thus is calculated on stations level. A tunnel criticality index is taken as the higher value for the two connecting stations, while auxiliary structures acquire the criticality index of corresponding subway station. This explains the constant C_R value per segment as seen in Figure 5. Unlike PoF and Cof where values are upwards aggregated, for an element level analysis, C_R values for a given element are the same as the station where it operates.

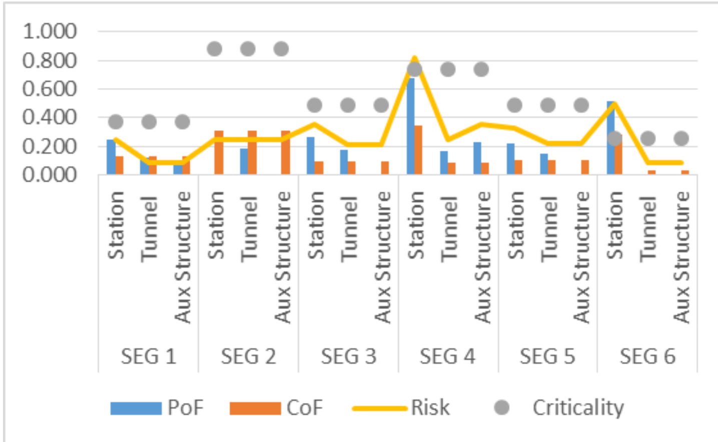

Figure 5: Integrated Risk Model Outputs

1.3.2 Risk Index Model

The output of the preceding three submodels is used as the input for the risk index model. Using rules derived from experts as seen in Figure 5, each submodel is assigned a value based on the degree to which it belongs to its membership function. The risk Index is calculated on the elements level where PoF and CoF are calculated. The series-parallel technique is then used to aggregate the risk index to higher levels of the hierarchy based upon the scope of analysis. Figure 5 demostrates the expected risk index calculated per elements and aggregated upwards per segments to the second level of the network hierarchy. Figure 6 shows a sample rules configuration used to develop the model. Figure 7 illustrate the resultant risk surface.

The station in SEG4 has the highest risk index. This is the complied effect of a high PoF and C_R values despite somehow moderate expected CoF. It is noticed that the tunnel and auxiliary structure in the same segment share the same C_R level yet their risk index is low. This is clearly due to the low probability of operational failure of the two elements derived from low PoF and C_R. This resultant risk value is only available through a fuzzy risk index where the C_R affects the risk index only in case of an existent risk value. The station in SEG 6 comes next with an expected risk index of 0.5. This risk index is mainly affected by the moderately high PoF in spite of low CoF and C_R values. This also is attributed to the fuzzy risk model which triggers the expected risk index value based on interrelated criteria just like a human expert. The risk index for the remainder elements is considered within acceptable range since they all have low combinations of PoF, CoF and C_R values.

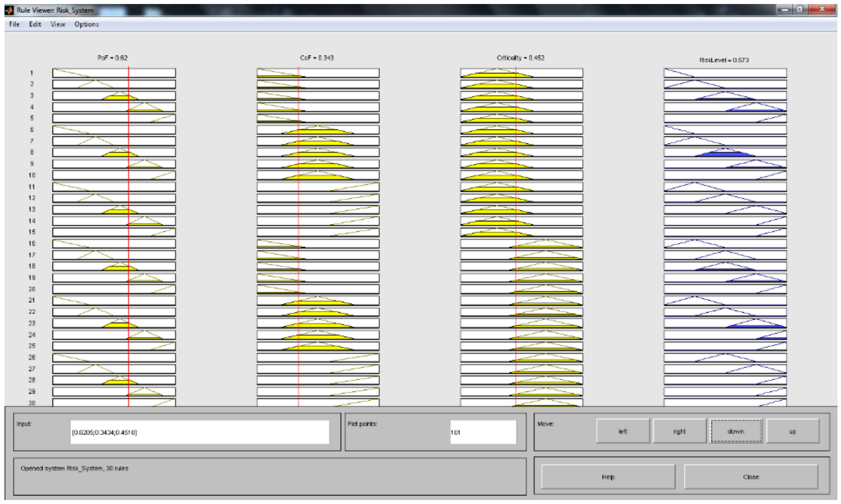

Figure 6: Sample Rules configuration for the Risk Index Model

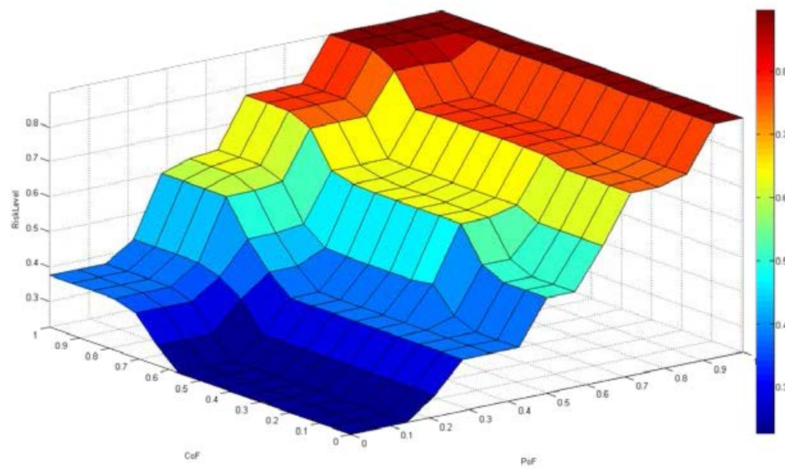

Figure 7: The resultant risk surface

1.4 Conclusion

This paper presented an overview of a network level risk assessment model. The risk index is developed using three sub-models and the fuzzy inference engine to incorporate experts' knowledge. The illustrated case study verifies the model can be used for ranking element in a subway hierarchy based upon its risk index. Furthermore, the sub-models can be used as standalone models for ranking stations for rehabilitations based on one perspective (Structural or operational) only rather than the combined risk effect. The use of fuzzy logic facilitates incorporating experts' knowledge into the model to account for the data scarcity and problem complexity. In addition, it overcomes the shortcomings of the conventional risk assessment techniques. The presented model is novel and expected to be of great benefits to academia, since the topic is poorly researched as well as industry, since the model is easy to use and straight forward.

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