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SIMULATED SCHEDULE DELAY MITIGATION VIA FLOAT ALLOCATION

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Abstract: As delays in construction project schedules are widely documented, new approaches to manage this problem are needed. Previous studies have explored float as an inherent ability of schedules to absorb delays. However, all ignore the critical path; some recommendations as to float ownership directly contradict the inherently unfair 'first come first serve' principle; and none have derived testable methods to fairly allocate float to multiple participants. This study therefore employs the mathematical analogy of decision-making among a group of unequally sized individuals to explicitly allocate project float a priori to those most vulnerable – the critical path. While intuition might indicate allocation proportional to activity duration, or perhaps equal shares, it is demonstrated that neither is truly fair and a mathematical compromise can be found. The performance of this approach is tested quantitatively by simulating a case example of project schedules with and without such protection. Delays are modeled as probabilistic events that affect activities. It is found that even a relatively small amount of project float allocated along the critical path provides significant delay mitigation.

1 INTRODUCTION

Construction project schedules in many countries unfortunately are frequently inundated with delays (Gündüz *et al.* 2013). *Delays* in this research are defined in a strictly temporal view, i.e. negative changes of start and/or finish of activities between the as-planned and as-updated project schedules. What makes delays particularly perilous is the fact that they are not an isolated local phenomenon within schedules, but may initiate a cascading set of ripple effects in later activities that depend on the earlier delayed one. Whatever the diverse causes of such delays are, which may stem from one or several *risk factors* that are materializing (González *et al.* 2013), is less relevant to this discussion than the unfortunate outcome, that it becomes likely that the contractually agreed turnover date will be met, i.e. the entire project will be late.

Contractual effects of late completion typically are that the contractor is required to reimburse the owner for consequential cost (liquidated damages) of postponed occupancy of the new facility, which often leads to further side-effects, e.g. protracted court cases over liability that may damage the reputation and lose future clients. The central question remains not one of identifying and treating symptoms, but of curing the cause: *What inherent defenses does a project schedule have against incurring delays*? Such capabilities could take two forms, either preventive (lowering the probability of a delay occurring) – akin to inoculation; or reactive (gaining better means of alleviating a delay that occurs) – akin to therapeutics in medicine.

To begin answering this question, this research will first review existing theory of project management regarding conceptual limitations of traditional scheduling and opportunities for protecting against delays, then explore potential theory in decision-making from which inspiration for such resilience will be gleaned.

2 LITERATURE REVIEW

2.1 Scheduling Assumptions

Construction schedules are traditionally conceived and handled as networks, which mathematically are acyclic 'graphs' (Chen *et al.* 2001) that consist of nodes (activities) and arcs (links) between them, whose direction is dictated by the passing of time. Exploiting a rather simple and repetitive structure of relations of temporal data among activities led to the so-called *critical path method* (Kelley and Walker 1959) with which dates are sequentially calculated: Within activities, adding the duration to its start gives its finish ($S + D = F$). And between activities, all successors must finish as a condition for the successor to start ($max\{F_{\text{pred}}\} = S_{\text{succ}}$). This forward-looking calculation assumes the best case that all activities start as early as possible at their *earliest* (E) dates. Its inverse, the backward-looking calculation assumes the worst case that they start as late as possible at *latest* (L) dates, so as to just not yet delay the project finish itself.

Comparing these two scenarios gives information for each activity on *how much it can be delayed* without impacting (a) any successor (free float, $FF_{\text{pred}} = min\{ES_{\text{succ}}\} - EF_{\text{pred}}$), or 'only' (b) the project finish (total float, $TF = LF - EF = LS - ES$). Thus *float* is defined by an assumed delay. Due to the aforementioned vital importance of the project finish for contractual liability purposes, this research will focus on the total float. Float in the literature has been presented as the opposite of *criticality*, which has been defined as having "zero total float" (de la Garza *et al.* 2007, p. 836), and inspired the name for this scheduling technique. A subset of activities within any network schedule forms a continuous path – the eponymous critical path – between the first and last activity, which just like the overall sequential schedule may branch and merge.

Hidden assumptions also lie in the inputs to such analysis, notably links and durations. Links (or schedule 'logic') are primarily determined based on technical reasons per laws of physics, but may also be modified for administrative reasons or even to artificially generate opportunities for success or failure (Zack 1992). This research assumes that links are fixed. Durations are determined based on work quantity as taken off plan drawings and specifications, combined with the crew choice and their unit productivity. However, one must consider that forecasting durations is always loaded with uncertainty. While studies have explored distributions to provide realistic durations for probabilistic scheduling (Khamooshi and Cioffi 2013), project managers in the construction industry have not been oblivious to the need for realistic duration estimates as opposed to overly optimistic ones. In practice, how well it is accomplished depends on the expertise of individual project managers, who determine 'raw' durations (from productivity data) and empirically extend them to 'realistic' ones by including a contingency (Barraza 2011). *Contingency* refers to both time or cost (Xie *et al.* 2012), but here it means additional time. This research assumes that raw durations are known to a user of this new approach, here presumed to be the general contractor. Studies have explored how contingency aids in company performance (Deng and Smyth 2013). It "has been used in project planning to cover activities delays, oversights, and unknowns and as a cushion for possible time-estimating errors" (Barraza 2011, p. 259). Since time contingency is essentially float, it will be referred to as such for clarity. The nature, quantity, and most beneficial distribution of such contingency, or float, will be explored next.

2.2 Criticality Paradox

Whereas risk has been explored in detail, with many studies listing risk factors and how to rank or weigh them, often using survey methodologies (e.g. Tran and Molenaar 2012), its realization in form of delays has been studied primarily to create numerous competing or even contradictory approaches to distribute liability between owner and contractor (Yang and Kao 2012) after excusable delays have been forgiven, i.e. assigning blame, and pondering the vexing issue of concurrent delays (Ibbs *et al.* 2011). However, *a posteriori* analysis, while unfortunately necessary, is not even reactive, but merely inculpative. In other words, delay liability appears has been the focus rather than delay mitigation or, even better, avoidance.

Rephrasing the definition of *criticality* as 'an inability to absorb delays' reveals that it is based upon an assumption itself, that activities in network schedules will necessarily have unequal capabilities to handle delays. However, schedulers control the manner in which activities are linked in network schedules (Zack 1992), so that criticality is essentially an artifact of the schedule itself. Criticality as it is currently defined for critical path method is not an inherent quality of the activity based on its nature or risk, although this fact is often hidden, because long or difficult activities often actually turn out to be part of the critical path.

Artificially dividing activities into being critical or not by whether or not they have total float in fact creates a paradox (Thompson and Lucko 2012, p. 488): “By definition, noncritical subcontractors have float, but the critical ones have none, even though they need it most.” Such two classes, of which critical activities are *designed* to be on the verge of failure, run counter to what should be the optimum goal of a scheduler: *Creating a schedule that attempts to minimize the overall risk level (or probability of local delays affecting any successor) of its activities by giving each the amount of float that it actually needs.* In other words, this approach will advocate for a schedule that does not contain a ‘critical path’ in the traditional definition.

2.3 Float Ownership

Although a notion of overcoming the well-established concept of a critical path may sound ambitious, it is not out of line with project management in practice: Project managers *already* attempt to set contingency by considering which of the various activities ‘needs more time’. However, this is performed empirically by “subjective management of the project time contingency” (Barraza 2011, p. 260), influenced by various factors, including the contract type that is used (Smith and Bohn 1999), but lacking any rigorous scientific foundation. Thus realism and quality (in terms of predictive ability) of a schedule depend to a large degree on such an individual’s expertise. Quantifying and formalizing this wisdom is the purpose of this research.

If float can mitigate delays by being consumed, then it becomes a *commodity* with potential value to the project participants (de la Garza *et al.* 1991). A question naturally arises, *who owns float*? In practice the ‘first come, first serve’ approach of using float appears to prevail (Pasiphol 1994), which underlines the urgent need of a mathematical approach that is derived from a testable hypothesis. A multitude of studies in the literature has discussed float ownership, which continues to be contentious and unresolved. Among the arguments were that float should belong to the owner, who pays for a project, or the contractor, who is responsible for its schedule (Al-Gahtani 2006), and suggestions to share or split it (Prateapusanond 2003), but without any details on how it could be fairly and equitably accomplished. Splitting this Gordian knot once again can be achieved by rephrasing the initial question to overcome its implicit assumption: *Who among the project participants should be allocated float, and how much*? The dichotomy of owner and general contractor is not central to resolving this question, because in many cases the individual activities are performed by specialty subcontractors. As Al-Gahtani and Mohan (2007 p. 33) supported, float should be allocated by “linking the ownership with the party who carries the risk”. Subcontractors, being productive agents who perform the schedule activities, will therefore be the focus of this research.

2.4 Fair Allocation

The final step in the logical argument pertains to the question of *how much*? This is fundamentally an allocation problem, where float takes on the role of a valuable but limited commodity and subcontractors are competing parties who each desire to receive float to protect their individual activities against delays. Since such activities are very different, it is necessary to establish a quantitative measure of need for float (rather than binary criticality) by which the allocation can proceed to achieve a fair and equitable result.

3 RESEARCH METHODOLOGY

This research employs an analogy-adapting methodology to create a functioning mathematical approach and test its hypothesis. An unexpected intriguing analogy is found in the area of social choice (Thompson and Lucko 2011), which will be described in the following section. Different from previous work by these authors that first presented the idea; this research will compare three different possible measures of float allocation and visualize their respective performances regarding the goal of a fair and equitable allocation.

The quantitative measure of need for float will use the activity duration, i.e. it is simplifyingly assumed that long activities are at a higher risk of incurring delays. While this is certainly true, characterizing the risk of delays by duration alone in practice likely is less realistic. It is envisioned that a weighted combination of e.g. duration, cost, and any other quantitative measure of relevant risk factors may be used, which will be investigated under future research. It is assumed for the new approach that initially all durations are raw durations and that a regular calculation per the critical path method has been completed to obtain the raw project finish and a list of activities that are deemed critical. The difference between calculated raw project finish and the known contract turnover date is called contract float (*CF*), which is "post-CPM duration float that relates to the overall completion of the project" (Thompson and Lucko 2011, p. CN-002-7). While this approach may superficially resemble *critical chain project management* (CCPM) from the manufacturing industry, which intentionally accumulates all intermediate buffers into a single large one at the project finish (Steyn 2000), important differences exist, which counteract the simplistic assumptions of CCPM:

- CCPM does not explicitly distinguish raw from realistic durations, but simplistically assumes a generic 50% reduction of the planned duration to determine "the likely duration" (Zhao *et al.* 2010, p. 1056), which questionably assumes that on average all subcontractors inflate their estimates by as much;
- CCPM radically removes all 'buffers' from a schedule, even from the currently non-critical activities, into a single cumulative 'project buffer' (Rand 2000) at the project end, thus changing schedule dates;
- CCPM artificially and wilfully generates *severe criticality* across a schedule, because it assumes that the subcontractors will only work timely if they have the impression that they work with zero flexibility;
- CCPM over-ambitiously seeks to minimize project duration (Herroelen and Leus 2001), but does not focus on reducing the project risk by increasing the consistency of achieving a realistic duration;
- "CCPM does not assign safety time to each individual activity" (Ma *et al.* 2014, p. 1), which is inverse to the philosophy new approach, and prevents determining any mathematically fair float ownership;
- CCPM reinserts 'feeding' buffers into non-critical paths (Rand 2000), changing schedule dates *again*;
- CCPM continues to ignore the critical path, whose risk has increased when all 'safety' was removed.

Besides such conceptual discrepancies, neither CCPM nor similar buffering approaches that were purely simulation-based (Gurgun *et al.* 2013, Barraza 2011) and essentially functioned by "taking off contingency buffers from individual activities and pooling them", then "putting the resized or newly introduced buffer between activities" (Park and Peña-Mora 2004, p. 630), provided a rigorous scientific foundation for fair and equitable apportionment of buffers, or rather *CF*. They can therefore be considered to be rudimentary attempts of solving a related ambition, but differed from the analogy-based approach that is presented in this research by turning away from, instead of improving, the vital backbone of a project – its critical path.

Note that this methodology and its assumptions and intended outcome – a project schedule 'without' any critical path – are not as radical as they may initially appear. Project managers already will 'inflate' the raw durations to what they consider realistic ones, but they do so in an individual and empirical manner. In the best case, this research will thus formalize and codify the best professional practice by providing it with a quantitative scientific foundation whose entire predictive and preventative approach can be validated.

3.1.1 Research Analogy

Arrow (1964) distinguished decision-making approaches into being voting (which is used in democratic systems to reach political decisions), and marketplace mechanisms (which are used in capitalist systems to reach economic decisions). Under these approaches, persons or groups engage in decision-making in several ways; e.g. dictatorially or unilaterally, forming a coalition to compose a majority of similar-minded individuals, or collective bargaining (hierarchical negotiation). Such approaches are often refined by extra rules and constraints, e.g. veto power – an ability to overrule a decision, per its Latin meaning "*I forbid.*"

Regardless of its detailed approach, any type of decision-making process seeks to reach a consensus by systematically aggregating many individual views. A rich body of research has examined decision-making under the heading of *social choice*; within it problems like fair division (of desirable assets or undesirable chores) and voting models (for parliamentary processes) have been explored mathematically. These offer analogies for the internal structure and operation of network schedule systems. Stated briefly, participants in such system (here: project) are voters (here: subcontractors), who jointly generate a decision (here: to expend float or not) by their actions (here: timely completion or delay) (Thompson and Lucko 2011). This conceptual analogy warrants investigation, especially exploring the applicability of their tenets to network schedule systems and project management in general for the problem at hand; namely float allocation.

A fundamental question emerges when participants are not of equal size (e.g. countries in the European Union), yet each of them should receive a fair weight within a decision-making process. To solve this vexing problem, two related mathematical voting models have emerged; the Penrose (1946) square root law and the Banzhaf (1965) power index. The former was inspired by the founding of the United Nations. It addressed problems of 'resolute' blocs that may dominate decisions and small nations, whose view either is irrelevant or who may become the tiebreaker. In other words, a truly fair balance is necessary to ensure that participants of small size are neither never nor always influencing the decision. Transferred newly to project management, Penrose (1946, p. 53) serves as inspiration for the new float allocation:

> *In general, the power of the individual vote*[r] *can be measured by the amount by which his chance of being on the winning side exceeds one half. The power, thus defined, is the same as half the likelihood of a situation in which an individual vote can be decisive – that is to say, a situation in which the remaining votes are equally divided upon the issue at stake. The general formula for the probability of equal division of* n *random votes, where* n *is an even number, approaches* $\sqrt{2/n\pi}$ *when* n *is large. It follows that the power of the individual vote*[r] *is inversely proportional to the square root of the number of people in the committee.*

He distinguished voting weight (which is a simple factor) from voting power (how often a participant sways a decision). Of course, all participants should receive equal power, regardless of their size. For countries with population *n*, a balance between the two unfair extremes of *'one vote per country'* (proportional to n^0, which gives small ones too much power) versus *'one vote per person'* (proportional to n^1, which gives small ones too little power) is required for a fair and equitable decision-making process (Pöppe 2007). He suggested that the newly formed General Assembly should allocate votes based on the square root ($n^{0.5}$) of country population. This somewhat counterintuitive principle that using direct proportionality to number of voters in each country is not the fairest possible approach remains a central insight of these studies.

Transferring this conceptual analogy to network schedules systems, political systems in representative democracies with two or more levels (federations like e.g. the United Nations, European Union, or United States) that make binary decisions provide the analogy, which correlates with critical subcontractors, who individually decide to consume their allocated float or not, and collaboratively build a project on time or not. Just as voters cast votes in an election (a decision forum), so subcontractors participate in a project with delay mitigation by deciding on the expenditure of float. "[V]oting typifies a possible determinant for the analogous behavior to those participating in large construction projects; and a potential mitigation approach to network schedule uncertainties and their impacts" (Thompson and Lucko 2011, p. CN-002-4). A question remains of whether $n^{0.5}$ is the most appropriate value, i.e. a pure Penrose approach to float allocation, or if applying this analogy to construction projects warrants another value between n^0 and n^1.

3.1.2 Research Hypothesis

It is hypothesized that allocating float to the activities on the critical path based on the square root of the measure of float need leads to less need to project float than an allocation based on order zero or one.

4 SIMULATION CALCULATIONS

4.1 Conceptual Approach

To efficiently perform the simulation calculations for the case example and derive comparisons, the entire new scheduling approach has been implemented in commercial computer software. Coding has followed a modular approach: Inputs are provided as a file that contains an activity list with name identifier for each activity as well as duration (distributions with its respective shape parameters) and point-to-point relations between the predecessors and successors that compose a network schedule. The number of repetitions of the simulation (i.e. runs) are specified separately. Output is written to another file and post-processed for analysis and interpretation. During each run the probability distribution of every activity is randomly sampled to determine its respective duration instance for one particular realization of the entire project.

Repeated simulation of a model that contains individual probabilistic variables to observe the probabilistic behavior of the entire system is known as Monte Carlo method (Chantaravarapan *et al.* 2004). It is often employed if the system (here a network schedule) is sufficiently complex to preclude a direct prediction of how exactly its elements will interact and cannot be solved by direct mathematical calculation. A distant conceptual relation of the new approach of this research exists to the so-called *criticality index* (Tang *et al.* 2013, p. 3238), which "is defined as the probability of an activity becoming critical". However, several fundamental differences must be understood to contrast the criticality index with the new float allocation:

- A criticality index does not pre-allocate any contract float, neither to critical nor to any other activities;
- A criticality index is based on duration distributions for realistic durations, not raw ones, even though the former are more difficult to determine than the latter, which can rely directly on productivity data;
- A criticality index is performed within the calculated project duration, oblivious of its contract deadline;
- A criticality index analysis, using a Monte Carlo approach, tracks whether or not the critical path will pass through a particular activity within a single randomized realization of a given network schedule;
- A criticality index analysis is randomized across a single variable, the duration from the distributions;
- A criticality index analysis generates a single quantitative values to indicate a form of 'meta-criticality';
- The float allocation is randomized across two variables, durations and also increasing available *CF*;
- The float allocation generates a curve for each activity that indicates at what total *CF* that is available to all critical activities within a schedule said activity ceases to exceed its own allocated portion of *CF*.

For continuity, the case example is selected from Thompson and Lucko (2011). The left half of Table 1 lists its input with 15 activities that initially amount to a 72-day-long project from mobilization (Mob.) to turnover (T/O) per a CPM calculation of raw durations. Seven bold activities are deemed critical and will be the ones that receive float from the three allocation mechanisms. For simplicity, links have no leads or lags and no link types other than finish-to-start are used; these items will be explored in future research. In the subsequent simulation it is assumed that the duration distribution of activities has an asymmetric triangular shape with a minimum at 90% of the planned duration per Table 1 and a maximum of 125% to reflect that fact that activities rarely take shorter than planned, but often longer (Thompson 2012). Other probability distributions, e.g. beta (Fente *et al.* 2000), will be examined within the future research as well.

The following approach was taken for simulation. The first module performs the forward pass, backward pass, and float calculations of traditional CPM, which identifies critical activities. Afterward, it replaced the fixed planned durations with aforesaid triangular distribution and performs numerous repetitions (here 1,000) in a Monte Carlo type simulation of the now probabilistic schedule. Data analysis was performed on the list of seven critical activities only. Their actual durations as sampled from the distribution were compared per Equation 1 with the float that had been allocated, called distributed float *DF* (Thompson and Lucko 2011) and is a portion of the contract float *CF* itself that is available to the entire critical path.

[1] Actual duration – (planned duration + distributed float) = Float overrun

Besides the actual duration of each activity, *CF* was the second variable in the simulation. It was varied in integer increments between *CF* = {1 to 40} days. Efficiently, it was not necessary to re-run the simulation to count all of the individual cases of float overrun (or have sufficient float, which is of course desirable), or even to repeat it 30 times, once for each scenario of *CF*. Rather, only 'actual duration' in Equation 1 is randomized. 'Planned duration' is fixed and known from Table 1. And 'distributed float' depends only on the mechanism of float allocation, planned duration, and *CF*, but not actual duration. The reason for this uncoupling is that the entire approach is designed to function *a priori*, before actual durations are known.

Rounding presented an important challenge for float allocation, because it directly influenced how exactly the limited valuable *CF* became *DF* for each critical activity. It was rounded to integer days and any non-allocated single day of float was not explicitly assigned, but was left to 'first-come, first-serve'. However, activities were rounded up so that each activity could only start at an integer time, to reflect the fact that if a subcontractor finishes before the end of a workday in construction practice, the next one will not arrive for merely a few hours, but on the morning of the next regular workday as Bashford *et al.* (2007) implied.

Table 1: Case Example Input and Float Allocation Output

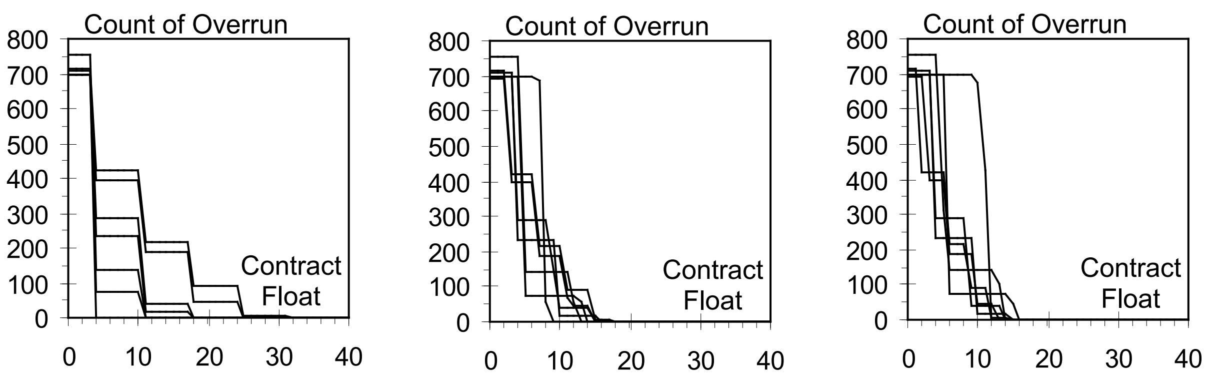
Traditional CPM Calculations									Duration$^{0.0}$				Duration$^{0.5}$				Duration$^{1.0}$			
Act.	Dur.	Succ.	*ES*	*LS*	*EF*	*LF*	*TF*	*FF*	*L*	*R*	*m*	*s*	*L*	*R*	*m*	*s*	*L*	*R*	*m*	*s*
Mob.	7	A, B, E	0	0	7	7	0	0	3	11	4.80	3.85	4	13	5.87	3.52	4	14	6.34	3.82
A	19	D, I, J	7	13	26	32	6	0	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
B	10	C	7	7	17	17	0	0	3	18	5.98	4.53	3	16	5.62	3.65	3	15	5.60	3.79
C	6	D, F, J	17	17	23	23	0	0	3	11	4.24	3.05	4	14	5.43	3.32	5	16	6.52	3.64
D	18	L	26	33	44	51	7	7	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
E	15	F, G	7	8	22	23	1	0	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
F	17	H, I, K	23	23	40	40	0	0	3	32	9.83	7.38	2	19	6.15	4.19	2	16	5.48	3.60
G	16	H, I, K	22	24	38	40	2	2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
H	6	M	40	53	46	59	13	0	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
I	11	L	40	40	51	51	0	0	3	18	6.79	5.21	3	15	6.06	3.72	3	14	5.85	3.72
J	19	L	26	32	45	51	6	6	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
K	15	T/O	40	54	55	69	14	14	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
L	18	T/O	51	51	69	69	0	0	3	32	10.69	8.36	2	18	6.60	4.75	1	15	5.17	4.04
M	10	T/O	46	59	56	69	13	13	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
T/O	3	N/A	69	69	72	72	0	0	3	4	3.50	0.00	6	9	7.56	0.37	9	12	11.07	0.69
Grand Average Across Activities									3.0	18.0	6.55	4.63	3.4	14.9	6.19	3.36	3.9	14.6	6.58	3.33

4.2 Results and Interpretation

Float overrun was plotted as the response variable in comparison to the *CF* as the explanatory variable. Figure 1 shows side-by-side the three different mechanisms of float allocation: Equal shares (planned duration$^{0.0}$), square root of duration (planned duration$^{0.5}$), and directly proportional (planned duration$^{1.0}$). The number of occurrences of float overrun was counted for each *CF* scenario and for every activity. Said counts were of course different under the three mechanisms. Shapes of the curves in Figure 1 represent the efficacy of each mechanism to protect the critical path against delays. For low *CF* the critical activities have not yet received sufficient *DF* to be individually protected. This explains the 'plateau' of their curves in the left half of the diagrams: While on this plateau, activities will consume more float than they own, or have not even received any *DF* yet. The latter is due to the fact that only integer days of *CF* become *DF*; and until the *CF* exceeds the number of critical activities, critical activities exist that have an allocation of zero. Next, activities experience a more or less steep drop-off or 'cliff', as float shows its powerful effect in absorbing any delays locally. If all activities will remain within their *DF*, the entire project will not exceed its planned raw duration plus *CF*. For each activity, a relatively narrow range exists, where *DF* changes from insufficient (on average) to sufficient (on average). After this cliff, a lower plateau is reached in the right half of the diagrams, where activities have excess *DF* and the project contains more *CF* than is needed.

To capture the cliff behaviour, which is of vital interest for this analysis, these parameters were calculated from the data and listed in Table 1: Left and right cutoff *L* and *R* (the right edge of upper plateau and the left edge of lower plateau), and within this range a weighted mean *m* and weighted standard deviation *s* on the horizontal axis, which indicated where and how narrow the cliff was. For example, if an activity has a result of $y = \{10, 10, 1, 0, 0, 0\}$ counts of overrun for $x = \{0, 1, 2, 3, 4, 5\}$ days of *CF*, then $L = 1$; $R = 3$; $m = [9 \cdot (2 - 1) + 1 \cdot (3 - 2)] / (9 + 1) = 1.6$ (weighted by drops Δy within their integer ranges of x); and $s = \sqrt{\left[9 \cdot (1.5 - 1.6)^2 + 1 \cdot (2.5 - 1.6)^2\right] / \left[10 \cdot ((2 - 1)/2)\right]} = 0.42$ (average weighted distance of the integer range midpoints to m). Note that both m and s are only calculated across a cliff itself, not the plateaus besides it.

The ranges of L and R and weighted grand averages and standard deviations across all critical provided a quantitative comparison of the efficacy of the three mechanisms. The equal shares (planned duration$^{0.0}$) has a wide range {3.0 to 18.0} within which all activities exhibit their drop-off behaviour, a weighted mean of 6.55, and a weighted standard deviation of 4.3, which indicates a merely gradual saturation of critical activities with their urgently needed *DF*. This approach is not recommended, due to being oblivious to the relative risk of activities, which here is represented by the proxy of duration. More interesting is comparing square root (planned duration$^{0.5}$) versus proportional shares (planned duration$^{1.0}$) approaches of Figure 1. The former has a narrower average range {3.4 to 14.9} of width 11.5 for all critical activities per Table 1; the latter is even slightly narrower {3.9 to 14.6) of width 10.7. Weighted means and standard deviations give a more distinct view: The former has a weighted mean of 6.19 compared to 6.58 of the latter. Clearly the square root model on average provides all critical activities earlier with the *DF* that is needed to satisfy their need. In fact, m is almost identical for (a) and (c) in Figure 1, which supports the hypothesis that (b) is the balance approach between these extremes. Finally, weighted standard deviations compare nearly identical at 3.36 versus 3.33 per Table 1. Both are much lower than s = 4.63 for (a). Testing for statistical significance at an alpha-level of 0.05 finds the difference between means not significant; however, this is most likely due to the very small case example with only seven different activities. More experimentation is therefore needed to substantiate the positive indications of the square root approach for float allocation.



(a) Equal Shares (b) Square Root (c) Proportional Shares

Figure 1: Case Example Performance for Three Mechanisms of Float Allocation

5 CONCLUSIONS AND RECOMMENDATIONS

This research has presented a vision for overcoming the long-standing and deeply entrenched paradigm within construction project scheduling by uncovering its various implicit assumptions. They have included focusing excessively on dissecting risk factors on the one hand and delay liability on the other hand; not explicitly distinguishing raw from realistic durations and basing criticality assessment on the latter, not the former; and artificially creating a paradoxical two-class society of critical versus non-critical activities that actively contradicts any desire to minimize the overall risk level. It is thus understandable that previous approaches to float ownership failed to solve the true underlying issue – minimizing risk by mitigating delays. This research stipulated that all critical activities should receive float, not by directly inserting it from its location between calculated and contract finish to being distributed along the critical path within a network schedule itself, but rather by providing an unambiguous approach to calculate optimum portions of ownership. This new approach is based on the premise that a directly proportional allocation to ‘size’ of the critical subcontractors’ contributions to the project would not fulfill the overall goal of being fair and equitable. Rather, three cases have been tested for numerical comparison; an allocation based on equal shares (n^0), the square root of size – or risk – ($n^{0.5}$), or proportional shares (n^1). Initial results are positive, but not statistically conclusive between the square root approach or proportional shares of float allocation. More experimentation with larger and more realistic examples of network schedules is recommended.

This research aids in fulfilling the unrealized vision of "Total Float Traded as Commodity" (de la Garza *et al.* 1991, p. 716). Only that it will not be the *TF* of non-critical activities, but *CF*, and that it will be designed to benefit critical activities. And that it will require having completed a pre-allocation of *CF*. Combined with the creative analogy from voting mathematics, which provides the seminal inspiration for how to achieve a provable fairness between large and small subcontractors who compete for *CF*, a market of opportunities can emerge. More research is necessary on how to determine dollar value, which should be listed in the contract to give all subcontractors certainty. Future research should investigate how the emphasis that is placed on measures of 'size' – whether it is duration, cost, risk (depending on type and nature of activity) – can be weighted and converted into a single numerical metric (Thompson and Lucko 2011). The current approach has merely assumed that a numeric value of risk (or size) is available as input. Closely related to this, it should also be examined how the potentially different subjective monetary valuations of *CF* by said contractors can be aggregated. Further analogies from areas of knowledge besides construction project management should be parsed to identify proven concepts that can handle subjective valuation.

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