

## Article

# Theoretical Foundation of the Relationship between Three Definitions of Effective Density and Particle Size

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**Abstract:** Effective density ( $\rho_e$ ) is universally used in atmospheric science as an alternative measure of the density ( $\rho$ ) of aerosol particles, and its definitions can be expressed in terms of the particle mass ( $m_p$ ),  $\rho$ , mobility diameter ( $D_m$ ), vacuum aerodynamic diameter ( $D_{va}$ ), and dynamic shape factor ( $\chi$ ), as  $\rho_e^I = 6m_p/(\pi \cdot D_m^3)$ ,  $\rho_e^{II} = \rho/\chi$ , and  $\rho_e^{III} = D_{va}/D_m$ . However, the theoretical foundation of these three definitions of  $\rho_e$  is still poorly understood before their application. Here, we explore the relationship between  $\rho_e$  and aerosol size through theoretical calculation. This study finds, for the first time, that  $\rho_e^I$  and  $\rho_e^{III}$  inherently decrease with increasing size for aspherical particles with a fixed  $\rho$  and  $\chi$ . We further elucidate that these inherent decreasing tendencies are governed by  $\chi$ , and the ratio of the Cunningham Slip Correction Factor of the volume-equivalent diameter to that of the mobility diameter ( $C_c(D_{ve})/C_c(D_m)$ ), but not by  $\rho$ . Taking the variable  $\chi$  into consideration, the relationships of  $\rho_e^I$  and  $\rho_e^{III}$  to particle size become more complicated, which suggests that the values of  $\rho_e^I$  and  $\rho_e^{III}$  have little indication of the size-resolved physicochemical properties of particles. On the contrary,  $\rho_e^{II}$  is independent on size for fixed  $\chi$  and  $\rho$ , which indicates that the change in  $\rho_e^{II}$  with size can better indicate the change in morphology and the transformation of the chemical compositions of particles. Our new insights into the essence of three  $\rho_e$ s provide an accurate and crucial theoretical foundation for their application.

**Keywords:** effective density; dynamic shape factor; chemical composition; size



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## 1. Introduction

Atmospheric particles play an important role in air quality, human health, and global climate change, which strongly depend on their chemical and physical properties [1,2]. Effective density ( $\rho_e$ ), one of the physical quantities, has been widely adopted in the characterization of the properties of aerosols as an alternative measure of density ( $\rho$ ) [3,4].  $\rho_e$  can serve as a link between the important characteristics of aerosol particles, such as volume-equivalent diameter ( $D_{ve}$ ) and aerodynamic diameter ( $D_a$ ) (presented in Appendix A—Table A1) [5], and as a tracer for new particle formation [6,7] and the atmospheric ageing processes [8]. Moreover,  $\rho_e$  can also provide an insight into particle morphology [9].

Due to differences in measurement methods, three effective densities are defined in atmospheric science, which are systematically reviewed in the work of DeCarlo, et al. [10]. The first definition ( $\rho_e^I$ ) describes  $\rho_e$  as the ratio of the particle mass ( $m_p$ ) to the apparent

volume, calculated assuming a spherical particle with a diameter equal to the measured mobility diameter ( $D_m$ ) (presented in Appendix A—Table A1) [10]:

$$\rho_e^I = \frac{6m_p}{\pi D_m^3} \quad (1)$$

where  $m_p$  is equal to  $1/6 \pi \cdot \rho \cdot D_{ve}^3$ .  $D_m$  is related to  $D_{ve}$ , as shown in Equation (2):

$$\frac{D_m}{C_c(D_m)} = \frac{D_{ve}}{C_c(D_{ve})} \chi \quad (2)$$

where  $\chi$  is the dynamic shape factor and  $C_c(D)$  represents the Cunningham Slip Correction Factor, which is calculated by Equation (3):

$$C_c(D) = 1 + \frac{\lambda}{D} \left( A + B \cdot \exp\left(\frac{C \cdot D}{\lambda}\right) \right) \quad (3)$$

where  $\lambda$  is the mean free path of the gas molecules, and  $A$ ,  $B$  and  $C$  are empirically determined constants specific to the analysis system. Substituting Equation (2) into Equation (1) results in the final form of  $\rho_e^I$ , as shown in Equation (4):

$$\rho_e^I = \frac{\rho}{\chi^3} \cdot \left( \frac{C_c(D_{ve})}{C_c(D_m)} \right)^3 \quad (4)$$

The second definition ( $\rho_e^{II}$ ) is the ratio of  $\rho$  to  $\chi$  [11]:

$$\rho_e^{II} = \frac{\rho}{\chi} \quad (5)$$

The third definition ( $\rho_e^{III}$ ) is the ratio of vacuum aerodynamic diameter ( $D_{va}$ ) (presented in Appendix A) and  $D_m$ :

$$\rho_e^{III} = \frac{D_{va}}{D_m} \rho_0 \quad (6)$$

where  $\rho_0$  represents the standard density of 1.0 g/cm<sup>3</sup> [10].  $D_{va}$  also depends on  $D_{ve}$ :

$$D_{va} = \frac{\rho}{\rho_0} \frac{D_{ve}}{\chi} \quad (7)$$

Combining the Equations (2), (6), and (7) obtains the final form of  $\rho_e^{III}$ , as shown in Equation (8) [12].

$$\rho_e^{III} = \frac{C_c(D_{ve})}{\chi^2 \cdot C_c(D_m)} \rho \quad (8)$$

For spherical particles, three  $\rho_e$ s are equal to their  $\rho$ . For aspherical particles, the three definitions of  $\rho_e$  should not yield the same numerical values, because they capture slightly different particle properties [10].

Based on the three definitions of effective density, copious studies have measured the values of aerosol effective density using different methods [13]. Hitherto,  $\rho_e^I$  is the most widely used definition in atmospheric science, because it can be measured using a variety of simple methods [13], such as the setup of a differential mobility analyzer (DMA), centrifugal particle mass analyzer (CPMA), condensation particle counter (CPC) [14], and DMA single-particle soot photometer (SP2) [15]. Although  $\rho_e^{III}$  can be measured using the tandem setup of a DMA—mass spectrometer [16] and the parallel setup of a scanning mobility particle spectrometer (SMPS) and aerosol mass spectrometer (AMS) [17], there are not many studies using the values of this definition to characterize aerosol properties [18–20]. Worse than  $\rho_e^{III}$ , until recently, there was no method to measure the value of  $\rho_e^{II}$  of an aspherical particle. However, Peng et al. filled this gap by developing a new method

of combining an aerodynamic aerosol classifier (AAC) with single-particle aerosol mass spectrometry (SPAMS) [21]. By using the established methods, previous studies measured the size-resolved  $\rho_e$  to indicate the change in morphology and/or the transformation of the chemical compositions of particles. Some studies reported  $\rho_e$  decreasing with an increase in particle size [22–27], while the other studies found  $\rho_e$  to be independent of [28] or to increase [7] with particle size. These studies ascribed such discrepancies among the particles with different sizes to different voids [24,26],  $\chi$  [23,25], morphology [4], chemical composition [8], and/or atmospheric processes [29,30]. However, the inherent relationship between the three definitions of  $\rho_e$  and the particle size were not understood before their application in these studies.

In this study, we focus on the theoretical calculation of  $\rho_e$  to probe the inherent relationship between  $\rho_e$  and the particle size for the three definitions of  $\rho_e$ , and to explore the factors resulting in their inherent relationship.

## 2. Materials and Methods

Table 1 presents the set values of  $\rho$ ,  $\chi$ , and  $D_m$  for Y and Z particles, which were used as surrogate particles in this study.  $\rho_e^I$ ,  $\rho_e^{II}$  and  $\rho_e^{III}$  for these particles, with a  $D_m$  of 40 nm, 80 nm, 150 nm, 250 nm, 350 nm, 450 nm, and 550 nm, were calculated using the values of  $\rho$ ,  $\chi$ , and  $C_c(D)$ . Although Zieger, et al. [31] found that the  $\chi$  of NaCl depends on particle size, which suggests that  $\chi$  varies with particle size, we first assumed particles with a fixed  $\chi$  value in the calculation, to facilitate probing the essential relationship between the three  $\rho_e$ s and particle size. In this study, the effect of variable  $\chi$  on the relationship between  $\rho_e$  and size is discussed.

**Table 1.** Information ( $\rho$ ,  $\chi$ , and  $D_m$ ) for calculating the three  $\rho_e$  for particles of Y and Z.

Particles	$\rho$ (g/cm <sup>3</sup> )	$\chi$	$D_m$ (nm)
Y	1.0	2.00	40, 80, 150, 250, 350, 450, and 550
	1.4		
	1.8		
	2.2		
	2.6		
	3.0		
	3.4		
Z	1.80	1.05	40, 80, 150, 250, 350, 450, and 550
		1.10	
		1.20	
		1.60	
		2.00	
		2.50	

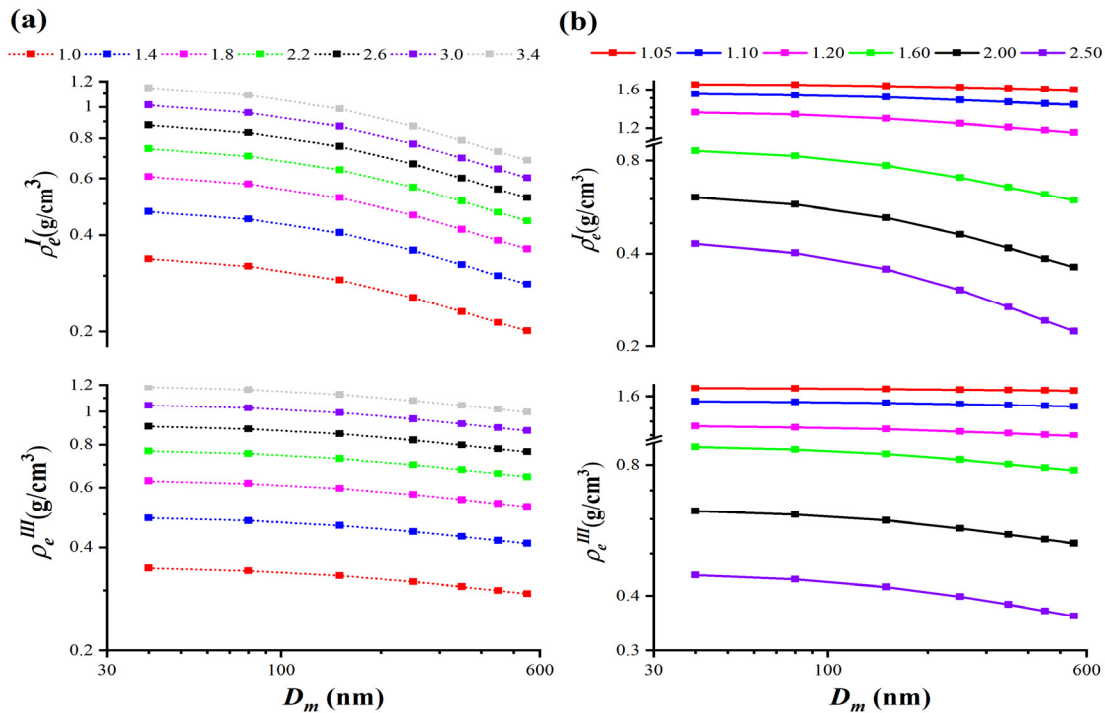
## 3. Results and Discussions

### 3.1. The Decrease in $\rho_e^I$ and $\rho_e^{III}$ with Particle Size for Aspherical Particles

Figure 1a,b presents the values of  $\rho_e^I$  and  $\rho_e^{III}$  for the Y and Z particles, respectively, which show that  $\rho_e^I$  and  $\rho_e^{III}$  decrease as the size increases for aspherical particles with fixed  $\rho$  and  $\chi$ . This result highlights, for the first time, the correlation of  $\rho_e^I$  and  $\rho_e^{III}$  with particle size through theoretical analysis. However, the inherent dependency of  $\rho_e^I$  and  $\rho_e^{III}$  on particle size was not considered in previous studies [17,20,32–36]. This omission raises concerns regarding the measurement technologies for  $\rho_e^I$  and  $\rho_e^{III}$  and the results of the morphology, voids, and atmospheric processes of the particles.

We take a closer look at the studies that adopted  $\rho_e^I$  and  $\rho_e^{III}$ . Previous studies established the methods of  $\rho_e^I$  through the measurement of  $D_m$  and  $D_a$ , and of  $\rho_e^{III}$  through the measurement of  $D_m$  and  $D_{va}$ .  $\rho_e^I$  and  $\rho_e^{III}$  were determined by fitting the size distributions of  $D_m$  and  $D_a$  in a narrow-overlap size range [32–35] or using the peak diameters of  $D_m$  and  $D_{va}$  [17,20], respectively. If the studies applied these methods to measure the size distributions of the particles with a  $\rho$  of 1.8 g/cm<sup>3</sup> and a  $\chi$  of 2.5, it would result in one value

to characterize  $\rho_e^I$  and  $\rho_e^{III}$ . However, this is inconsistent with the theoretical calculation in this study, which finds that the effective density of these particles ranges from 0.43 g/cm<sup>3</sup> at 40 nm to 0.22 g/cm<sup>3</sup> at 550 nm for  $\rho_e^I$ , and from 0.45 g/cm<sup>3</sup> at 40 nm to 0.36 g/cm<sup>3</sup> at 550 nm for  $\rho_e^{III}$ , respectively (Figure 1b). This suggests that using specific values for  $\rho_e^I$  and  $\rho_e^{III}$  to represent the whole measured size range needs to be reevaluated in future studies.



**Figure 1.** (a) The size variations of  $\rho_e^I$  and  $\rho_e^{III}$  for the Y particles. The change in color from red to grey represents the increase in density from 1.0 to 3.4 g/cm<sup>3</sup>; (b) The size variations of  $\rho_e^I$  and  $\rho_e^{III}$  for the Z particles. The change in color from red to purple represents the increase in dynamic shape factor from 1.05 to 2.50.

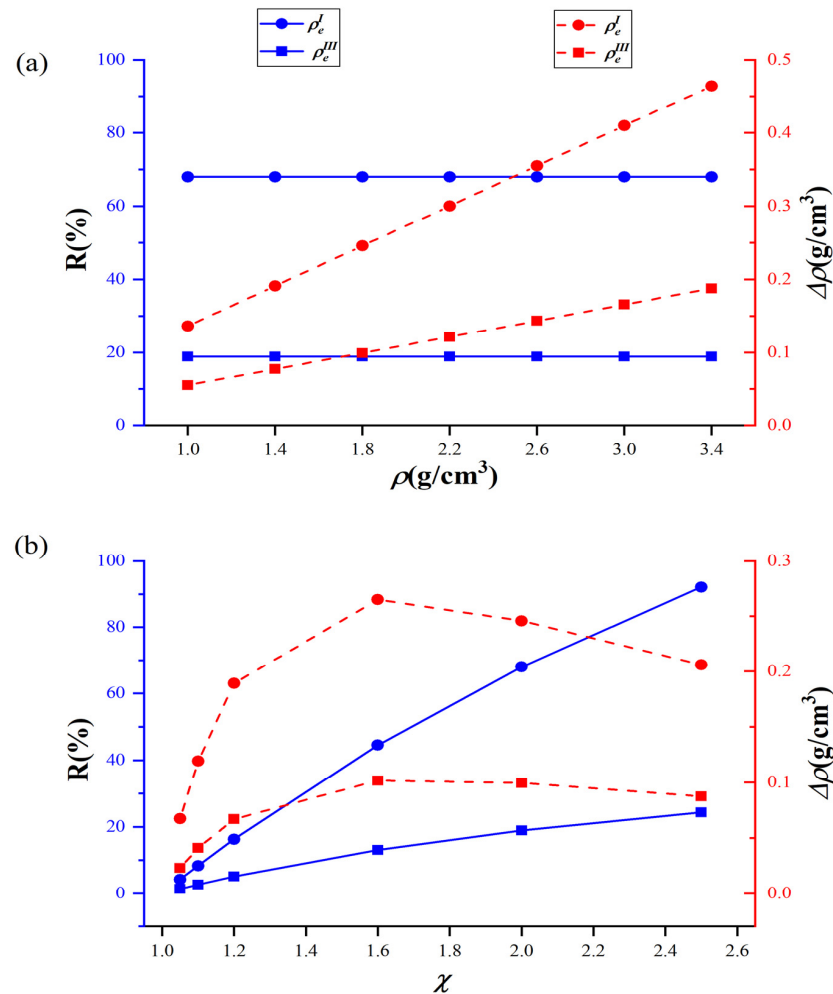
Park, et al. [22] found, for the first time, that the  $\rho_e^I$  of diesel particles decreases as particle size increases. Since then, dozens of studies have shown that the  $\rho_e^I$  and  $\rho_e^{III}$  of the primary particles, such as particles from vehicles [36,37] and biomass burning [19,38–40], exhibited a similar trend with particle size. These studies attributed this phenomenon to the increasing  $\chi$  [23,25] and more voids [24,26] with a large  $D_m$ . However, we obtained the size-resolved  $\rho_e^I$  and  $\rho_e^{III}$  for soot with fixed  $\chi$  and found that the effective density still decreases with the increasing particle size, even without the increase in the voids. Therefore, it is somewhat arbitrary to conclude that larger particles have a larger  $\chi$  and more voids based only on the measurements of  $\rho_e^I$  and  $\rho_e^{III}$  with different particle sizes.

The values of  $\rho_e^I$  and  $\rho_e^{III}$  are generally used to indicate the aging process of particles in the atmosphere [8,41–43]. The atmospheric aging processes would change the size of the particles and, consequently, influence  $\rho_e^I$  and  $\rho_e^{III}$ , based on our theoretical calculation. Therefore, the change in  $\rho_e^I$  and  $\rho_e^{III}$  due to aging processes might be biased if the essence of  $\rho_e^I$  and  $\rho_e^{III}$  with particle size is not considered.

### 3.2. The Factors That Cause the Decrease in $\rho_e^I$ and $\rho_e^{III}$ with Particle Size

According to Equations (4) and (8),  $\rho_e^I$  and  $\rho_e^{III}$  are a function of  $\rho$ ,  $\chi$ , and  $C_c(D_{ve})/C_c(D_m)$ , so the three factors were evaluated for the decrease in  $\rho_e^I$  and  $\rho_e^{III}$  with particle size.  $\rho$  was evaluated by Y particles with fixed  $\chi$  but with the variable  $\rho$ . Figure 1a shows the variations of  $\rho_e^I$  and  $\rho_e^{III}$  of the Y particles. The values of  $\rho_e^I$  and  $\rho_e^{III}$  decrease monotonically with  $D_m$ , and  $\rho_e^I$  decreases more rapidly than  $\rho_e^{III}$ . Correspondingly, Figure 2a shows the  $\Delta\rho$  ( $\rho_{e,40nm} - \rho_{e,550nm}$ ) and R ( $(\rho_{e,40nm} - \rho_{e,550nm})/\rho_{e,550nm}$ ) of  $\rho_e^I$  and  $\rho_e^{III}$  of the Y particles as

a function of  $\rho$ . Obviously,  $\Delta\rho$  is proportional to  $\rho$  and the slope of  $\rho_e^I$  is greater than that of  $\rho_e^{III}$ , which is a result of the larger coefficient between  $\rho_e^I$  and  $\rho$  ( $(C_c(D_{ve})/C_c(D_m))^3/\chi^3$ ) (as shown in Equation (4)) than between  $\rho_e^{III}$  and  $\rho$  ( $C_c(D_{ve})/C_c(D_m)/\chi^2$ ) (as shown in Equation (8)). In particular,  $\rho_e^I$  and  $\rho_e^{III}$  have constant R ( $(\rho_{e,40nm} - \rho_{e,550nm})/\rho_{e,550nm}$ ) values of 68.0% and 18.9%, respectively. These findings lead to the conclusion that  $\rho$  determines the values of  $\rho_e^I$  and  $\rho_e^{III}$ , but does not affect the relationship of  $\rho_e^I$  and  $\rho_e^{III}$  with the particle size.

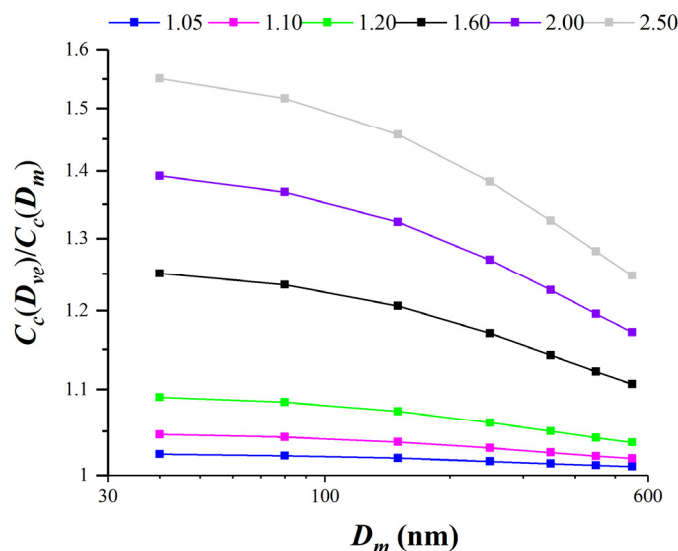


**Figure 2.** (a)  $\Delta\rho$  ( $=\rho_{e,40nm} - \rho_{e,550nm}$ ) and  $R$  ( $=\Delta\rho/\rho_{e,550nm}$ ) of  $\rho_e^I$  and  $\rho_e^{III}$  for the Y particles with different  $\rho$ ; (b)  $\Delta\rho$  ( $=\rho_{e,40nm} - \rho_{e,550nm}$ ) and  $R$  ( $=\Delta\rho/\rho_{e,550nm}$ ) of  $\rho_e^I$  and  $\rho_e^{III}$  for the Z particles with different  $\chi$ . Red and blue lines represent  $\Delta\rho$  and  $R$ , respectively.

To explore the effects of  $\chi$  on the decrease in  $\rho_e^I$  and  $\rho_e^{III}$  with particle size, the  $\rho_e^I$  and  $\rho_e^{III}$  of the series of Z particles with a fixed  $\rho$  but the variable  $\chi$  are calculated and presented in Figure 1b. For particles with one value of  $D_m$ ,  $\rho_e^I$  and  $\rho_e^{III}$  decrease with increasing  $\chi$ . For  $\chi > 1.00$ ,  $\rho_e^I$  and  $\rho_e^{III}$  decrease with increasing  $D_m$ . Figure 2b shows  $\Delta\rho$  ( $\rho_{e,40nm} - \rho_{e,550nm}$ ) and  $R$  ( $\Delta\rho/\rho_{e,550nm}$ ) of  $\rho_e^I$  and  $\rho_e^{III}$  of the Z particles.  $\Delta\rho$  increases from 0.07 to 0.26 g/cm³ for  $\rho_e^I$  and from 0.02 to 0.10 g/cm³ for  $\rho_e^{III}$ , as  $\chi$  increases from 1.05 to 1.60. Then,  $\Delta\rho$  decreases from 0.26 g/cm³ to 0.21 g/cm³ for  $\rho_e^I$  and from 0.10 g/cm³ to 0.09 g/cm³ for  $\rho_e^{III}$ , as  $\chi$  increases from 1.60 to 2.50. Additionally,  $R$  increases from 4.2% to 92.1% for  $\rho_e^I$ , and increases from 1.4% to 24.3% for  $\rho_e^{III}$ , as  $\chi$  increases from 1.05 to 2.50. This implies that  $\chi$  plays a key role in the values of  $\rho_e^I$  and  $\rho_e^{III}$  and their dependence on particle size.

$\rho_e^I$  and  $\rho_e^{III}$  for the aspherical particles with fixed  $\rho$  and  $\chi$ , however, still depend on  $D_m$ , suggesting that  $C_c(D_{ve})/C_c(D_m)$  is responsible for the dependency. Figure 3 shows the relationship between  $C_c(D_{ve})/C_c(D_m)$  and particle size for the Z particles with a  $\rho$  of

1.80 g/cm<sup>3</sup> and a  $\chi$  ranging from 1.05 to 2.50. It can be seen that  $C_c(D_{ve})/C_c(D_m)$  is greater than 1 in the size range of 40–550 nm, and decreases slightly from 1.02 to 1.01 (0.9%), 1.05 to 1.02 (2.9%), and 1.09 to 1.04 (4.8%) for the particles with a  $\chi$  of 1.05, 1.10, and 1.20, respectively. The values of  $C_c(D_{ve})/C_c(D_m)$  have an evident downward trend, decreasing from 1.25 to 1.11 (12.6%), 1.39 to 1.17 (18.8%), and 1.55 to 1.25 (24.0%) for the particles with a  $\chi$  of 1.60, 2.00 and 2.50, respectively. Obviously, when  $\chi$  is larger,  $C_c(D_{ve})/C_c(D_m)$  decreases more rapidly with  $D_m$ .



**Figure 3.** The relationship between the  $C_c(D_{ve})/C_c(D_m)$  and  $D_m$  for the Z particles with fixed  $\rho$  of 1.80 g/cm<sup>3</sup> and different  $\chi$  values of 1.05, 1.10, 1.20, 1.60, 2.00, and 2.50, which are represented by different colors.

In this study, through the exploitation of the final forms of  $\rho_e^I$  and  $\rho_e^{III}$  (i.e., Equations (4) and (8)), we discover that  $\rho_e^I$  is proportional to  $(C_c(D_{ve})/C_c(D_m))^3$ , and  $\rho_e^{III}$  is proportional to  $C_c(D_{ve})/C_c(D_m)$ .  $C_c(D_{ve})/C_c(D_m)$  decreases as  $D_m$  increases (Figure 3), and therefore,  $\rho_e^I$  and  $\rho_e^{III}$  also definitely decrease as  $D_m$  increases. The role of  $\chi$  on  $C_c(D_{ve})/C_c(D_m)$  also explains why  $\chi$  determines the downward trends of  $\rho_e^I$  and  $\rho_e^{III}$  (Figure 2b). Furthermore, the cube value of  $C_c(D_{ve})/C_c(D_m)$  is greater than itself because it is greater than 1, which inevitably causes a faster downward trend of  $\rho_e^I$  with  $D_m$  than that of  $\rho_e^{III}$ . Generally, the values of  $\chi$  and  $C_c(D_{ve})$  are unknown when using effective density to substitute for density; therefore, the downward trends of  $\rho_e^I$  and  $\rho_e^{III}$  cannot be corrected in their application.

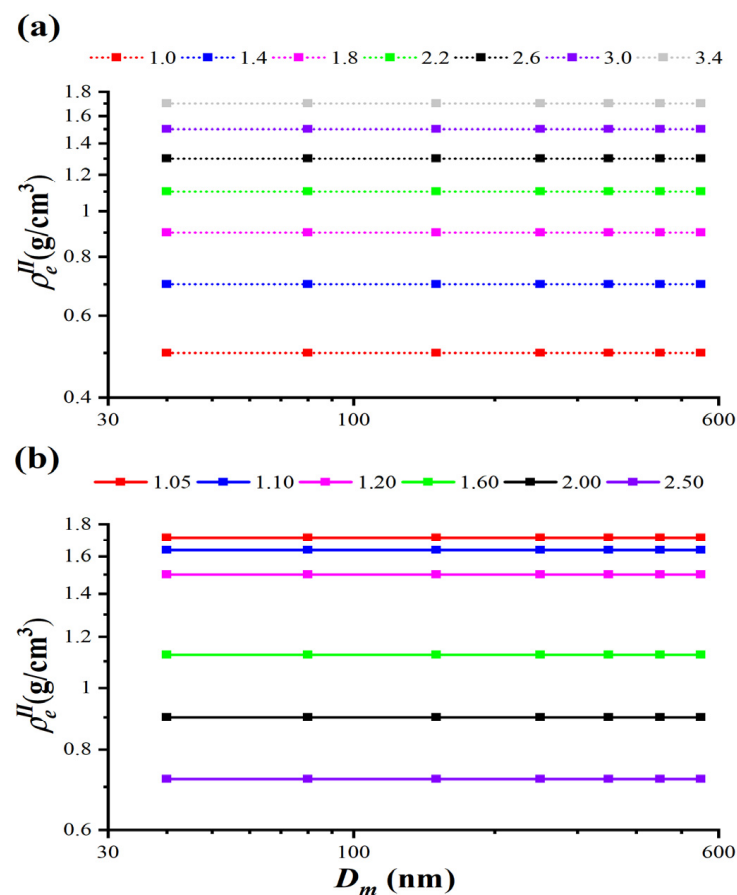
$\rho_e^I$  and  $\rho_e^{III}$  inherently decrease with the increasing size of aspherical particles, indicating that aerosols of one substance have a series of values of  $\rho_e^I$  and  $\rho_e^{III}$ , because aerosols have a wide size distribution. For example, particles with a  $\rho$  of 1.80 g/cm<sup>3</sup> and  $\chi$  of 2.50 obtain  $\rho_e^I$  and  $\rho_e^{III}$  values of 0.43 g/cm<sup>3</sup> and 0.45 g/cm<sup>3</sup> at 40 nm, and of 0.22 g/cm<sup>3</sup> and 0.36 g/cm<sup>3</sup> at 550 nm, respectively. A series of values makes the physical quantities of  $\rho_e^I$  and  $\rho_e^{III}$  undistinguishable and blurs their physical meaning, which is totally inconsistent with the values of density.

### 3.3. The Independent Relationship between $\rho_e^{II}$ and Particle Size

Figure 4 presents the  $\rho_e^{II}$  of Y and Z particles, showing that  $\rho_e^{II}$  only has one value in the entire range of sizes and that it is independent of particle size. This independent tendency is consistent with the characteristics of  $\rho$ . Therefore, it is more reasonable to use  $\rho_e^{II}$  as the alternative measure of density. The relationship between  $\rho_e$  and size is generally used to indicate the change in morphology and/or the transformation of the chemical compositions of particles. According to the new understanding of the relationship between the three effective densities and particle size from this study, Table 2 presents the relationship between the trends of effective density with size, and that of  $\chi$  and  $\rho$  with size.



When  $\rho_e^I$  and  $\rho_e^{III}$  decrease with the increasing size of aspherical particles, they may be determined by the increase or invariability of  $\chi$  for particles with a fixed  $\rho$ , and the decrease or the invariability of  $\rho$  for particles with a fixed  $\chi$ . Even for the same physical quantity (e.g.,  $\chi$ ,  $\rho$ ), the change could result in the same trend of  $\rho_e^I$  and  $\rho_e^{III}$  with the particle size. In addition, it is worth noting that  $\chi$  may change with particle size irregularly, [31] which will render the relationship of  $\rho_e^I$  and  $\rho_e^{III}$  with size more complicated. The above discussion suggests that the size-resolved  $\rho_e^I$  and  $\rho_e^{III}$  actually have no ability to indicate the relationship of  $\chi$  and  $\rho$  with size. On the contrary, the increase, invariability, and decrease in  $\rho_e^{II}$  with the increasing size of aspherical particles specifically indicate the decrease in  $\chi$  and/or the increase in  $\rho$ , the invariability of  $\chi$  and/or  $\rho$ , and the increase in  $\chi$  and/or the decrease in  $\rho$ , respectively. This comparison implies that  $\rho_e^{II}$  is a better indicator for the change in morphology and the transformation of the chemical compositions of particles. Therefore, it is recommended to apply  $\rho_e^{II}$  as an alternative measure of  $\rho$  in studies involving particle sizes in atmospheric science.



**Figure 4.** (a) The size-resolved  $\rho_e^{II}$  for the Y particles with  $\rho$  from 1.0 to 3.4 g/cm<sup>3</sup>, which are represented by the change in line color from red to grey; (b) The size-resolved  $\rho_e^{II}$  for the Z particles with a  $\chi$  from 1.05 to 2.50, which are represented by the change in line color from red to purple.

**Table 2.** The trends of  $\rho_e^I$ ,  $\rho_e^{II}$ , and  $\rho_e^{III}$  with size of aspherical particles as  $\chi$  (a) and  $\rho$  (b) change.

The Trend of				
(a) Fixed value of $\rho$	$\chi$	$\rho_e^I$	$\rho_e^{II}$	$\rho_e^{III}$
	Increasing	Decreasing	Decreasing	Decreasing
	Invariant	Decreasing	Invariant	Decreasing

Table 2. Cont.

	Decreasing	Invariant or increasing	Increasing	Invariant or increasing
(b) Fixed value of $\chi$	$\rho$	$\rho_e^I$	$\rho_e^{II}$	$\rho_e^{III}$
	Increasing	Invariant or increasing	Increasing	Invariant or increasing
	Invariant	Decreasing	Invariant	Decreasing
	Decreasing	Decreasing	Decreasing	Decreasing

#### 4. Conclusions

This study forms a comprehensive theoretical analysis for the three definitions of effective density.  $\rho_e^{II}$  is found to be independent of particle size, while  $\rho_e^I$  and  $\rho_e^{III}$  decrease as the size of aspherical particles increases; these are determined by  $\chi$  and  $C_c(D_{ve})/C_c(D_m)$ , but not  $\rho$ , which suggests that the relationship between size and the definitions of  $\rho_e^I$  and  $\rho_e^{III}$  gives little indication of the size-resolved physicochemical properties of the particle. Compared to  $\rho_e^I$  and  $\rho_e^{III}$ , the values of  $\rho_e^{II}$  are better for indicating the change in morphology and the transformation of chemical compositions; thus, the definition of  $\rho_e^{II}$  is recommended as a more proper alternative measure of  $\rho$  in studies involving particle sizes in atmospheric science. These results lay a sound theoretical foundation for the three effective densities, which will help with their accurate application in the future studies.

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#### Appendix A

Table A1. Introductions of the diameters used in this paper.

Symbol	Definition	Derivation	Example of Measurement Instrument
$D_a$	Aerodynamic diameter is defined as the diameter of a sphere with standard density that settles at the same terminal velocity as the particle of interest.	$D_a = D_{ve} \sqrt{\frac{\rho_p C_c(D_{ve})}{\chi \rho_0 C_c(D_a)}}$	Aerodynamic Aerosol Classifier (AAC)



Table A1. Cont.

$D_{va}$	In the free-molecular regime, the aerodynamic diameter is called the vacuum aerodynamic diameter.	$D_{va} = \frac{\rho_p}{\rho_0} \frac{D_{ve}}{\chi}$	Single-Particle Aerosol Mass Spectrometry (SPAMS)
$D_m$	Mobility diameter is defined as the diameter of a sphere with the same migration velocity in a constant electric field as the particle of interest.	$\frac{D_m}{C_c(D_m)} = \frac{D_{ve}}{C_c(D_{ve})} \chi$	Differential Mobility Analyzer (DMA)
$D_{ve}$	Volume-equivalent diameter is defined as the diameter of a spherical particle of the same volume as the particle under consideration.	$D_{ve} = \sqrt[3]{\frac{6m_p}{\pi\rho_p}}$	AAC-SPAMS

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