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Localization Parameters for Two Interacting Particles in Disordered Two-Dimensional Finite Lattices

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Abstract: I study spreading of two interacting hardcore bosons in disordered two-dimensional finite lattices from an initial occupation of two adjacent sites. The parameters related to the spreading of the particles provide an insight on the effect of interaction. I find that the presence of interaction makes the particles less localized than the non-interacting ones within the range of disorder strength $W \leq 4$ and interaction strength $V \leq 4$. If the interaction strength is higher, then particles localize more. A transition with changes in the character of dominant correlations is found at critical disorder strengths for each chosen strength of interaction. The nature of correlations between the particles as nearest neighbours becomes dominant beyond these disorder strengths.

Keywords: interaction; disordered system; two dimension; localization

1. Introduction

The study of Anderson [1] on quantum particles in disordered one-dimensional lattices had initiated the study on the phenomena of localization more than sixty years ago. Since then, a plethora of theoretical and experimental studies have been performed to understand more about this phenomena. Scaling theory [2] had predicted a localization–delocalization transition for three-dimensional disordered systems and absence of it in lower dimensional disordered systems. The extension of the study of localization for the last few decades for the interacting particles has opened up the highly researched area of many body localization. The work of Fleishman and Anderson [3], Finkelstein [4], Giamarchi and Schulz [5], Basko et al. [6] and Gornyi et al. [7] are considered the most prominent ones in this field. The study of such localization has now been made possible experimentally also with developments in optical lattices. Recently, Schreiber et al. [8] have performed such studies and made observations on the effect of interaction on localization from an imbalance of density distribution between odd and even sites of a 1D disordered lattice. Other such experiments [9,10] have also studied a similar effect of interaction on localization properties of many particles.

The numerical studies on single particle localization are realized from finding the spread of a particle from an initial location on a disordered lattice. The dynamics of the spread, points to the signature of a diffusive, sub-diffusive or localized state [11]. A similar approach can be taken for studying the localization phenomena in the presence of interaction. The interplay of disorder and interaction in localizing the particles can be observed from the spread of two interacting particles in a lattice from an initial location. In this article, the localization behaviour of two hardcore bosons has been studied from the spread of the particles over a finite two-dimensional disordered lattice from an initial occupation at adjacent sites. The case of such localization of two interacting particles in one-dimensional lattices has been investigated thoroughly before [11–17] since the 1990s. These studies were motivated by the observation of persistent currents in 1D wires [18–20]. The two-dimensional disordered systems are of interest due to its proximity to localization–delocalization transition predicted by Abrahams et al. [2]. However, the scaling theory predicts an absence of such transition. The presence of interaction in making such a transition has been predicted by experimental studies of Kravchenko et al. [21,22] and subsequent theoretical studies [23–26].

The numerical study of localization of two interacting particles in large 2D disordered lattices can be performed with Green's functions. The exact calculations for Green's functions in a disordered system can be performed with the method of full diagonalization, which becomes inefficient with increasing lattice size. In a recently developed algorithm [27,28], based on recursive calculation of two-particle Green's functions, such calculations have been made more efficient to perform for larger lattice systems. However, certain approximations has to be employed which has been justified in detail in a separate work [28]. These calculations not only helps in gaining understanding on the effect of interaction on localization, but also provide information on the correlations of the particles. These correlations between particles calculated from two-particle Green's functions reveal further details on the phases of localization in the disordered systems.

For the calculation of localization parameters, one of the prescribed approaches is to calculate a two-particle Green's propagator from the center of the lattice to boundaries [29] with combination of scaling arguments [30]. This approach has been implemented in few of the previous studies [23,24]. However, computational difficulty prohibits such calculations for large system sizes and often entails significant finite size effects.

In this article, the inverse participation ratio (IPR) is calculated as a localization parameter. The calculation of IPR involves the computation of all Green's propagators for a given interaction strength and a strength of disorder of the lattice. The IPR can be taken as a macroscopic parameter dependent on the distribution of particles and is less susceptible to finite size effects. The IPR numbers are expected to provide an understanding on the length scale of localization of two interacting particles in finite 2D disordered systems. Additional informations on the localization are obtained from a parameter of total correlations between the particles calculated from two-particle Green's functions. The correlation parameters provide more information on the structure under the density distribution of the particles in localized systems. In this study, the IPR and correlation parameters are calculated for a broad range of disorder strengths of the lattices and interaction strengths between the particles. This broad range of calculations help in gaining further insights into the range of expected length scales of localization. With the additional information on correlations, separate phases can be identified between localized density distributions in finite disordered 2D systems.

2. Method

The Hamiltonian of this study contains terms for nearest neighbor hopping and nearest neighbor interaction for two hardcore bosons in a disordered 2D lattice:

$$H = \sum_{ij} \epsilon_{ij} a_{ij}^{\dagger} a_{ij} + \sum_{ij} t(a_{ij}^{\dagger} a_{i+1j} + a_{ij}^{\dagger} a_{ij+1}) + V \sum_{ij} (n_{ij} n_{i+1j} + n_{ij} n_{ij+1}). \tag{1}$$

Here (i, j) are the site indices on two axes of the lattice. The onsite energy ϵ_{ij} is chosen randomly from a uniform box distribution $[-\frac{W}{2}, \frac{W}{2}]$ of width W, which defines the disorder strength. The onsite energy (ϵ_{ij}) and disorder strength (W) is measured in the units of hopping term (t). The hopping parameter t is taken as a unit and constant for any edges between two sites on the lattice to simplify the calculations. The $a_{ij}^{\dagger}$ (a_{ij}) operators are creation (annihilation) operators for a particle at site (i, j) of the 2D lattice. The computational algorithm involved in the calculations is explained in detail in a separate article [28]. The Green's elements for a given disorder strength from an initial occupation of particles at adjacent sites in the middle of the lattice are calculated at very large times ($\tau = 1000$) using the algorithm. The lattice was considered to have 20 sites per dimension. To make the calculations more efficient, an approximation of a fixed maximum relative distance (r) was applied with $r = 5$ [28]. This parameter limits the number of Green's functions involved in the calculation. Any Green's functions with relative distances between the particles larger than r is not included in the calculation. This doesn't limit the location of the particles on the lattice. The assumption that those Green's functions with a larger r is negligible is justified in article [28]. The Green's elements were thus computed at sufficiently large times than that is required for the spreading of the particles to the lattice boundaries:

$$G(i,j,\tau) = \sum_{\omega} e^{-i\omega\tau} G(i,j,\omega). \tag{2}$$

Once all such Green's elements were found propagated from two particles occupying adjacent sites at the center of the disordered 2D lattice, that is, for every site indices (i,j) with $|i-j| \leq r$ for two particles, the joint density distribution (ρ), density distribution (ϱ) and inverse participation ratio ($\mathcal{I}$) were calculated for each realization of disorder, which were averaged afterwards over many realizations:

$$\rho(i,j,\tau) = |G(i,j,\tau)|^2, \tag{3}$$

$$\varrho(i,\tau) = \frac{1}{2}\sum_{j\neq i} \rho(i,j,\tau), \tag{4}$$

$$\mathcal{I} = \frac{\sum_i \varrho(i,\tau)^2}{\sum_i \varrho(i,\tau)}. \tag{5}$$

The inverse participation ratio (I) describes a density distribution on lattices qualitatively by measuring the inverse of a number that roughly describes how many sites have participated in that distribution. The lesser the I, the less localized is the distribution of the density. For a measure of total correlation (ζ) with respect to the distance between particles, the minimum step distance between the particles on the lattice (r_s, hamming distance) was taken as a measure of distance and the correlation elements were summed based on such distance irrespective of the locations of the particles on the lattice. This distance r_s is the most general form of distance on any general graph when hopping has no directional dependence. The measure of correlation indicates weights for different relative distances between two particles in the joint density distribution. The relative distance r_s is a scalar and doesn't depend on the location of the two particles over the lattice:

$$\zeta(r_s) = \sum_{|i-j|=r_s} \rho(i,j,\tau). \tag{6}$$

This parameter then reflects the total correlation within the joint density distribution in the disordered 2D lattice for each r_s. This measurement cannot be calculated directly from single particle density distribution. However, the two-particle Green's functions provide a direct insight on such correlations.

3. Results

The results of the calculations for medium-sized finite disordered 2D lattices are shown in Figure 1. The calculations involved combinations of disorder (W) and interaction strengths (V) within the range $1 \leq W \leq 12$ and $0 \leq V \leq 8$. The IPR numbers were found to be ranging from 0.004 to 0.04 within this broad range of disorder and interaction strengths. Higher I represents more localization of the particles.

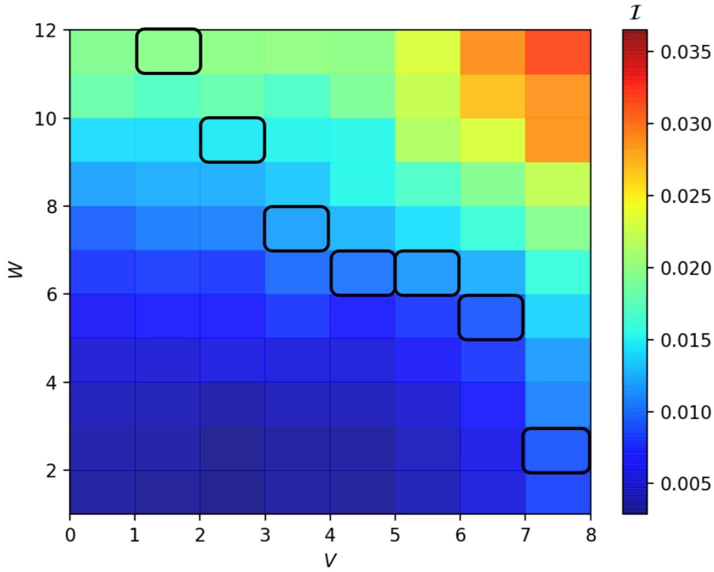

Figure 1. Two-dimensional disorder-interaction diagram with black squares showing the regions where nature of correlations change. The inverse participation ratios plotted are averaged over 320 realizations of disorder.

Figure 1 shows similar parameters of localization with an increase in interaction up to $V \leq 4$, when compared to $V = 0$ case, for the range of disorder strength $1 \leq W \leq 4$. This range can be termed as weak interaction and weak disorder region, where the interacting particles have smaller IPR and lesser localization compared to non-interacting ones. This is made clearly visible in Figure 2 with inverse participation numbers for fixed disorder cross sections plotted as a function of interaction strength.

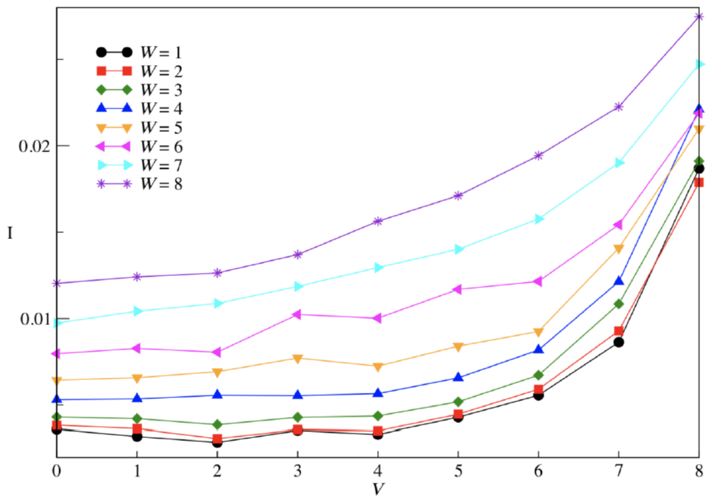

Figure 2. Cross sections from Figure 1 on disorder axis. It shows a decrease in IPR from that of non-interacting particles within the range $1 \leq W \leq 4$ and $0 \leq V \leq 4$.

Beyond the weak-disorder-weak-interaction regime, enhancement in localization is observed in the presence of interaction.

The marked squares on Figure 1 indicates a change in the correlations between the particles in their localization. The measure of total correlation $\zeta(r_s)$ for $r_s = 2$ is larger than that of $r_s = 1$ towards the lower IPR region of these squares. $\zeta(1)$ becomes larger than $\zeta(2)$ when either disorder or interaction is increased beyond these marked squares. The following Figure 3 exhibits one of such squares for $V = 4$, where it can be observed that $\zeta(2) > \zeta(1)$ beyond $W = 6$. This change in the underlined correlations between the particles in their localization can be observable for measurements involving two-particle correlations. This change also signifies not only a transition from low IPR to high IPR regions, but also a type of underlined phase of the particles which cannot be observed from single particle density measurements.

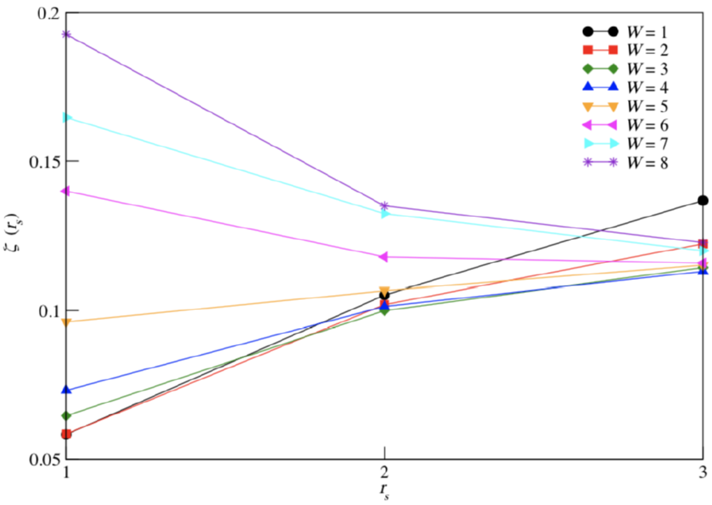

Figure 3. Total correlation of the two particles as a function of the interparticle (Hamming) distance. The correlations are summed for each such distance irrespective of their location at lattice for an interaction strength $V = 4$ with disorder strength ranging from $1 \leq W \leq 8$.

The density distribution of one such point from Figure 1 ($W = 1, V = 4$) is shown in Figure 4. As the distribution shows, after averaging over 320 realizations, the final distribution appears to be localized. This point from Figure 1 has a low IPR compared to other points of the diagram. However, the density distribution appears to be localized for a finite sized 2D disordered lattice.

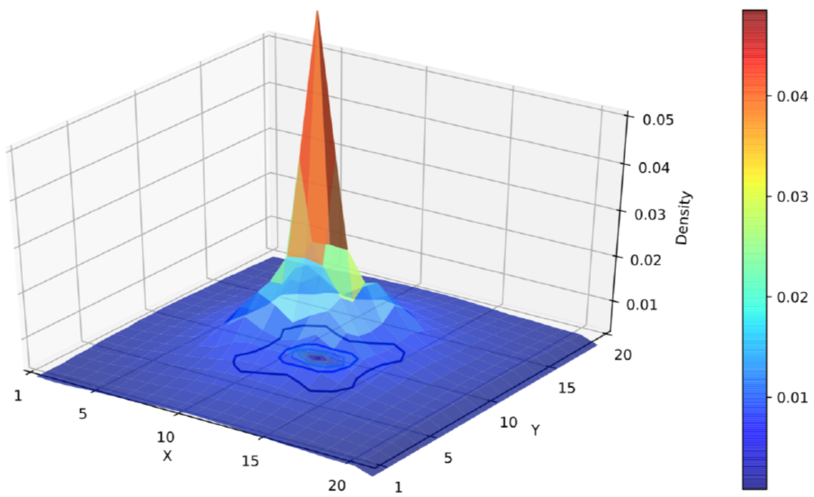

Figure 4. Density distribution of two particles on the two-dimensional lattice for $V = 4$ and $W = 1$, averaged over 320 realizations of disorder.

The scaling analysis of this specific point ($W = 1, V = 4$) from Figure 1 is described in Figure 5, which indicates a localized state as the IPR approaches a constancy for larger system sizes. However, a conclusion on the absence of delocalization for the model under consideration at this weak-disorder-weak-interaction regime may not be drawn as the calculations involved approximations which has been mentioned previously and the finite size of the lattice plays a role. Any such conclusion will require analysis of the full problem for large system sizes.

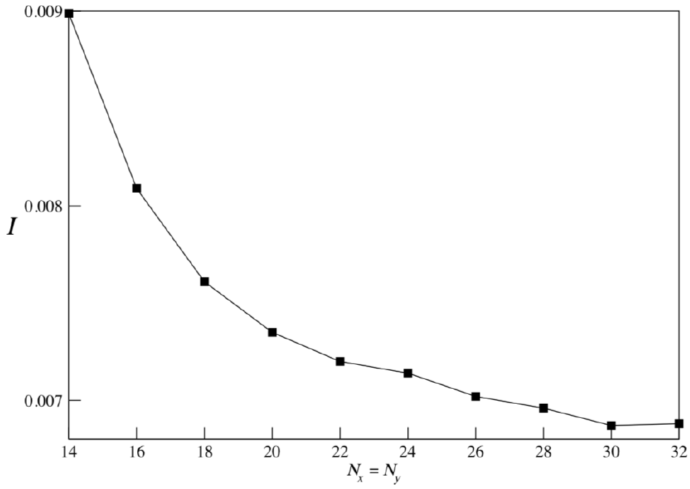

Figure 5. Scaling of $\mathcal{I}$ with the lattice size for $V = 4$ and $W = 1$, averaged over 50 realizations of disorder.

The effect of increasing localization, when both interaction and disorder is strong, can be interpreted as the effect of particles becoming correlated with lesser relative distance in the underlined dynamics and disorder enhancement of such correlations. The correlations with lesser inter-particle distance imply increased binding. However, from Figure 1, the change of such correlations signified with the black rectangles on the figure doesn't have a direct relation with inverse participation ratio numbers. The changes in different types of correlations in the underlined distribution add to differences between localized distributions.

Accompanied with the understanding on correlations within the localized distribution of particles, one can attempt to draw separate phases of localizations. A 3D distribution of the same Figure 1 is shown in Figure 6. The contours drawn on the lower surface of Figure 6 indicate such differences between localized phases. While the contours with IPR ≤ 0.01 reveal a centricity towards the origin, the contours with IPR ≥ 0.02 have centricity away from the origin on the diagram. This may be based on the correlations between the particles that can be compared from Figure 1. This region with IPR ≤ 0.01 shows a possibility of delocalized behaviour that may be corroborated with other works [23,24]. However, this study does not find any sign of delocalized behaviour, which may be a result of finite size of the systems under consideration combined with the approximations involved in the calculations.

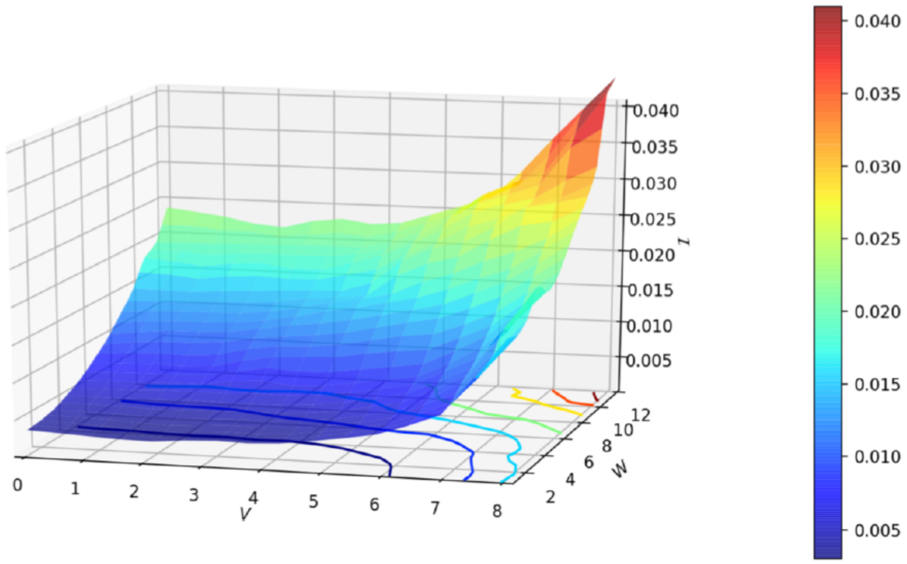

Figure 6. Three-dimensional disorder-interaction diagram. The inverse participation ratios plotted are averaged over 320 realizations of disorder.

4. Conclusions

This study has calculated localization parameters for disordered 2D lattice systems with higher accuracy. The calculations not only provide an understanding on the length scales on density distribution of the interacting particles, but also present an understanding on the correlations between the particles in their localized distribution. The inverse participation ratios calculated for a vast range of disorder and interaction parameters reveal the expected range of localization length under the effects of both interaction and disorder in disordered finite 2D systems. Although a localization–delocalization transition is not found in this study, it doesn't exclude such possibilities. Understanding of the correlations suggests different phases of localization within the broad ranges of disorder and interaction strengths.

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