3 THERMODYNAMICS

Contents

Recall from physics that **kinetic energy** relates to the motion of objects, while **potential energy** relates to the attraction between objects. These energies can apply on the **macroscale** — to large-scale objects consisting of many molecules. They can also apply on the **microscale** — to individual molecules, atoms, and subatomic particles. Energy on the microscale is known as **internal energy**, and a portion of internal energy is what we call **heat**.

Energy can change forms between kinetic, potential, and other energy types. It can also change scale. The conversion between microscale and macroscale energies was studied extensively during the industrial revolution to design better engines. This study is called **thermodynamics**.

The field of thermodynamics also applies to the atmosphere. The microscale energy of heat can cause the macroscale motions we call winds. Microscale attractions enable water-vapor molecules to condense into macroscale cloud drops and rain.

In this chapter, we will investigate the interplay between internal energy and macroscale effects in the atmosphere. First, focus on internal energy.

INTERNAL ENERGY

Definitions

In thermodynamics, **internal energy** consists of the sum of <u>microscopic</u> (molecular scale) kinetic and potential energy.

Microscopic <u>kinetic</u> energies include random movement (translation) of molecules, molecular vibration and rotation, electron motion and spin, and nuclear spin. The sum of these kinetic energies is called **sensible energy**, which we humans can sense (i.e., feel) and measure as **temperature**.

Microscopic potential energy is associated with forces that bind masses together. It takes energy to pull two masses apart and break their bonds. This is analogous to increasing the microscopic potential energy of the system. When the two masses snap back together, their microscopic potential energy is released back into other energy forms. Three forms of binding energy are:

- **latent** — bonds between molecules
- **chemical** — bonds between atoms
- **nuclear** — sub-atomic bonds

We will ignore chemical reactions and nuclear explosions here.

Latent energy is associated with **phase change** (solid, liquid, gas). In solids, the molecules are bound closely together in a somewhat rigid lattice. In liquids, molecules can more easily move relative to each other, but are still held close together. In gases, the molecules are further apart and have much weaker bonds.

For example, starting with cold ice (in lower left corner of Fig. 3.1), adding energy causes the temperature to increase (a sensible effect), but only up to the melting point (0°C at standard sea-level pressure). Further addition of energy forces bonds of the solid lattice to stretch, enabling more fluid movement of the molecules. This is melting, a latent effect that occurs with no temperature change. After all the ice has melted, if you add more energy then the liquid warms (a sensible effect), but only up to the boiling point (100°C). Subsequent addition of energy forces further stretching of the molecular bonds to allow freer movement of the molecules; namely, evaporation (a latent effect) with no temperature change. After all the liquid has vaporized, any more energy added increases the water-vapor temperature (a sensible effect).

Fig. 3.1 can also be traversed in the opposite direction by removing internal energy. Starting with hot water vapor, the sequence is cooling of the vapor, condensation, cooling of the liquid, freezing, and finally cooling of the ice.

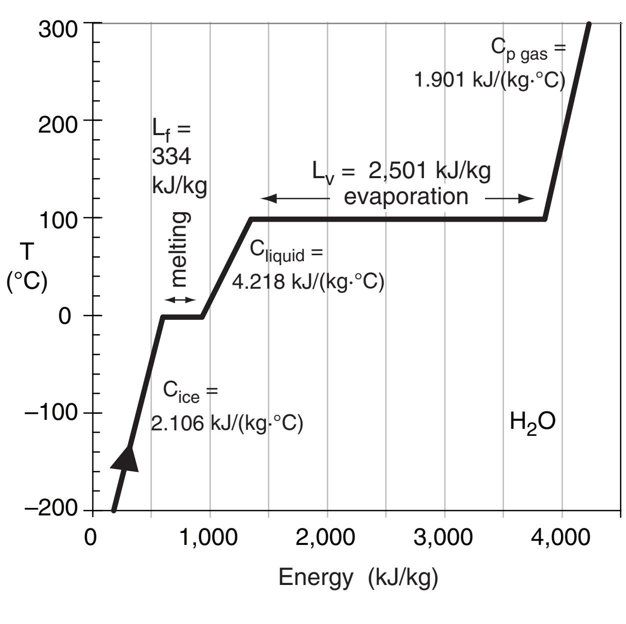


Figure 3.1
Sensible and latent energy for water.

Possession and Transfer of Energy

We cannot ignore the connection between the microscale and the macroscale. A macroscale object such as a cannon ball consists of billions of microscale molecules and atoms, each possessing internal energy. Summing over the mass of all the molecules in the cannon ball gives us the total internal energy (sensible + latent energy) that the cannon ball **possesses**.

Thermal energy transferred to or from the macroscale object can increase or decrease the internal energy it possesses. This is analogous to your bank account, where money transferred (deposited or withdrawn) causes an increase or decrease to the total funds you possess.

Transfer of Heat

Define the **transfer** of **thermal energy** as Δq. It has energy units (J kg^{-1}). In this text, we will refer to Δq as **heat transferred**, although in engineering texts it is just called heat.

Latent Energy Possessed

Define the **latent heat** Q_E as the latent energy (J) possessed by the total mass m of all the molecules in an object.

But usually we are more interested in the change of possessed latent heat ΔQ_E associated with some process that changes the phase of Δm kilograms of material, such as phase change of water:

$$\Delta Q_E = L \cdot \Delta m_{water} \tag{3.1}$$

For example, if we transfer Δq amount of thermal energy into water that is already at 100°C at sea-level pressure, then we can anticipate that the amount of water evaporated will be given by: $\Delta m_{water} = \Delta q \cdot m_{water} / L_v$. [A sample application is on page 86.]

Different materials have different strengths of bonds, so they have different constants of proportionality L (called the **latent heat factor**) between ΔQ_E and Δm. For water (H_2O), those latent heat factors are given in Table 3-1.

Sensible Energy Possessed

The sensible energy (J) possessed by the total mass m of all the molecules in an object (such as air molecules contained in a finite volume) is $m_{air} \cdot C_v \cdot T$, where T is absolute temperature of the air. The constant of proportionality C_v is called the **specific heat at constant volume**. Its value depends on the material. For dry air, $C_{vd\ air}$ = 717 J kg^{-1} K^{-1}.

Again, we are interested more in the change of sensible energy possessed. We might suspect that it should be proportional to the change in temperature ΔT, but there is a complication that is best approached using the First Law of Thermodynamics.

FIRST LAW OF THERMODYNAMICS

Definition

Let Δq (J kg^{-1}) be the amount of thermal energy you add to a stationary mass m of air. Some of this energy warms the air (i.e., the internal energy increases). But as air warms, its volume expands by amount ΔV and pushes against the surrounding atmosphere (which to good approximation is pushing back with constant pressure P). Thus, a portion of the thermal energy input does not go into warming the air, but goes into macroscopic movement.

To illustrate, consider a column of air having base of area A and height d. Resting on top of this column is more air, the weight of which causes pressure P at the top of the column. Suppose the volume ($V = A{\cdot}d$) expansion is all in the vertical, so that $\Delta V = A{\cdot}\Delta d$. For the column top to rise, it must counteract the downward pressure force from the air above; namely, the top of the column must push up with pressure P as it moves distance Δd.

Recall that **work** W is force times distance ($W = F{\cdot}\Delta d$). Also, pressure is force per unit area ($P = F/A$) Thus, the work done on the atmosphere by the expanding column is $W = F{\cdot}\Delta d = P{\cdot}A{\cdot}\Delta d = P{\cdot}\Delta V$.

The **First Law of Thermodynamics** says that energy is conserved, thus the thermal energy input must equal the sum of warming (a microscopic effect) and work done per unit mass (a macroscopic effect):

$$\Delta q = C_v{\cdot}\Delta T + P{\cdot}(\Delta V/m) \tag{3.2a}$$

Apply to the Atmosphere

But $P{\cdot}(\Delta V/m) = \Delta(P{\cdot}V/m) - V{\cdot}\Delta P/m$ (using the product rule of calculus). Also, $P{\cdot}V/m = P/\rho = \Re{\cdot}T$ from the ideal gas law, where $\rho = m/V$ is air density and $\Re$ is the gas constant. Thus, $\Delta(P{\cdot}V/m) = \Delta(\Re{\cdot}T) = \Re{\cdot}\Delta T$ because $\Re$ is constant. Using this in eq. (3.2a) gives:

$$\Delta q = C_v{\cdot}\Delta T + \Re{\cdot}\Delta T - \Delta P/\rho \tag{3.2b}$$

By definition for an ideal gas: $C_v + \Re \equiv C_p$, where C_p is the **specific heat of air at constant pressure**. The INFO box on the next page gives:

$$C_p = C_{p\ humid\ air} \approx C_{pd} \cdot (1 + 1.84{\cdot}r) \tag{3.3}$$

where C_{pd} is the dry-air specific heat at constant pressure, and the water-vapor mixing ratio r has units ($g_{water\ vapor}/g_{dry\ air}$). See the Water Vapor chapter for more details on humidity.

Table 3-1. Latent heat factors L for water (H_2O), for the phase-change processes indicated.

Process Name & Direction	L (J kg^{-1})	Process Name & Direction
	vapor	
evaporation ↑	$L_v = 2.5\text{x}10^6$	↓ condensation
	liquid	
melting ↑	$L_f = 3.34\text{x}10^5$	↓ freezing (fusion)
	solid	
	vapor	
sublimation ↑	$L_d = 2.83\text{x}10^6$	↓ deposition
	solid	
↑ requires transfer of thermal energy Δq TO water from the surrounding air.		↓ requires transfer of thermal energy Δq FROM water to the surrounding air.

Sample Application

Suppose 3 kg of water as vapor condenses to liquid. What is the value of latent heat transferred to the air?

Find the Answer

Given: $L_v = 2.5\text{x}10^6$ J·kg^{-1}, $m_{vapor} = 3$ kg.
Find: ΔQ_E = ? J

Apply eq. (3.1):
$\Delta Q_E = (2.5\text{x}10^6\ \text{J·kg}^{-1}) \cdot (3\ \text{kg}) = \underline{7{,}500\ \text{kJ}}$

Check: Physics & units are reasonable.
Exposition: Three liters is a small quantity of water (equivalent to 3 mm depth in a bathtub). Yet it represents a large quantity of latent heat.

Sample Application

Find the specific heat at constant pressure for humid air holding 10 g of water vapor per kg of dry air.

Find the Answer

Given: $r = (10\ g_{vapor}) / (1000\ g_{dry\ air}) = 0.01$ g/g
Find: C_p = ? J·kg^{-1}·K^{-1}

Apply eq. (3.3):
$C_p = (1004\ \text{J·kg}^{-1}\text{·K}^{-1}) \cdot [\ 1 + (1.84{\cdot}(0.01\ \text{g/g}))\ ]$
$= \underline{1022.5\ \text{J·kg}^{-1}\text{·K}^{-1}}$

Check: Units OK. Magnitude reasonable.
Exposition: Even a modest amount of water vapor can cause a significant increase in specific heat. See Chapter 4 for typical ranges of r in the atmosphere.

INFO • Specific Heat Cp for Air

The specific heat at constant pressure C_p for air is the average of the specific heats for its constituents, weighted by their relative abundance:

$$m_T \cdot C_p = m_d \cdot C_{pd} + m_v \cdot C_{pv} \tag{3I.1}$$

where $m_T = m_d + m_v$ is the total mass of air (as a sum of mass of dry air m_d and water vapor m_v), and C_{pd} and C_{pv} are the specific heats for dry air and water vapor, respectively.

Define a mixing ratio r of water vapor as $r = m_v / m_d$. Typically, r is of order 0.01 g/g. Then eq. (3I.1) becomes:

$$C_p = (1 - r) \cdot C_{pd} \cdot [1 + r \cdot C_{pv}/C_{pd}] \tag{3I.2}$$

Given: C_{pd} = 1004 J·kg^{-1}·K^{-1} at 0°C for dry air, and C_{pv} = 1850 J·kg^{-1}·K^{-1} for water vapor, eq. (3I.2) becomes

$$C_p \approx C_{pd} \cdot [1 + 1.84 \cdot r] \tag{3I.3}$$

Even for dry air, the specific heat varies slightly with temperature, as shown in the figure below:

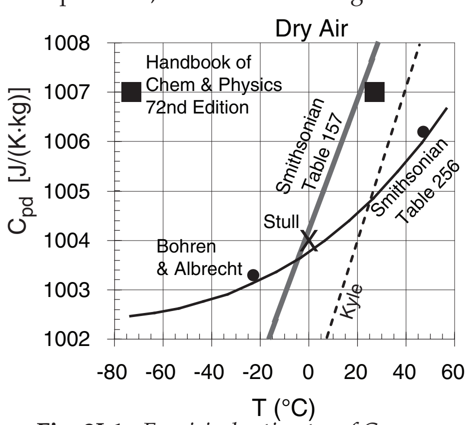


Fig. 3I.1. *Empirical estimates of* C_{pd}.

In this book, we will use C_{pd} = 1004 J·kg^{-1}·K^{-1} , and will approximate it as being constant.

Sample Application

What heat transfer is needed to cause 3 kg of dry air to cool by 10°C?

Find the Answer

Given: C_{pd} = 1004. J·kg^{-1}·K^{-1}, m_{air} = 3 kg , ΔT = –10°C
Find: ΔQ_H = ? J

Apply eq. (3.4b):

$$\Delta Q_H = m_{air} \cdot C_{pd} \cdot \Delta T$$
$$= (3\text{ kg}) \cdot (1004.\text{ J·kg}^{-1}\text{·K}^{-1}) \cdot (-10°\text{C}) = \underline{\mathbf{-30.12\ kJ}}$$

Check: Physics & units are reasonable.
Exposition: On a hot day, this is the energy your air conditioner must extract from the air in your car.

Appendix B lists some specific heats; e.g.:
$C_{pd\ air}$ = 1004 J·kg^{-1}·K^{-1} for dry air at const. pressure,
$C_{vd\ air}$ = 717 J kg^{-1} K^{-1} for dry air at const. volume,
C_{liq} ≈ 4217.6 J·kg^{-1}·K^{-1} for liquid water at 0°C,
C_{ice} ≈ 2106 J·kg^{-1}·K^{-1} for ice at 0°C,
C_{pv} = 1850 J·kg^{-1}·K^{-1} for pure water vapor at 0°C.

You can combine the first two terms on the right of eq. (3.2b) to give a form of the **First Law of Thermodynamics** that is easier to use for the atmosphere:

$$\underset{\textit{heat transferred}}{\Delta q} = \underset{\textit{enthalpy change}}{C_p \cdot \Delta T} - \Delta P/\rho \tag{3.2c}$$

Enthalpy vs. Sensible Heat

From our derivation of eq. (3.2c) we saw that the first term on the right includes both the microscopic effect of a temperature change (internal energy or heat possessed) <u>and</u> the macroscopic effect of that same temperature change. Hence, we cannot call that term "heat possessed" — instead we need a new name.

To this end, define **enthalpy** as $h = C_p \cdot T$ with units J kg^{-1}. The corresponding **enthalpy change** is:

$$\Delta h = C_p \cdot \Delta T \tag{3.4a}$$

which is the first term on the right side of eq. (3.4). It is a characteristic <u>possessed</u> by the air.

By tradition, meteorologists often use the word **sensible heat** in place of the word enthalpy. This can be confusing because of the overloading of the word "heat". Here is a table that might help you keep these definitions straight:

Table 3-2. Heat terminology.

Quant. (J/kg)	Character-istic	Terminology	
		Meteorology	Engineering
Δq	transferred	heat transferred	heat
$C_p \cdot \Delta T$	possessed	sensible heat	enthalpy

With this in mind, the **First Law of Thermo** per unit air mass can be annotated as follows:

$$\underset{\textit{heat transferred}}{\Delta q} = \underset{\textit{sensible heat}}{C_p \cdot \Delta T} - \Delta P/\rho \qquad \bullet(3.2d)$$

This form is useful in meteorology. When rising air parcels experience a decrease in surrounding atmospheric pressure, the last term is non-zero.

The change of **sensible-heat** (ΔQ_H) possessed by air mass m_{air} changing its temperature by ΔT is thus:

$$\Delta Q_H = m_{air} \cdot C_p \cdot \Delta T \tag{3.4b}$$

INFO • Cp vs. Cv

C_v — Specific Heat at Constant Volume

Consider a sealed box of fixed volume V filled with air, as sketched in Fig. 3I.2a below. The number of air molecules (idealized by the little spheres) can't change, so the air density ρ is constant. Suppose that initially, the air temperature T_o is cool, as represented by the short arrows denoting the movement of each molecule in box 3I.2a. When each molecule bounces off a wall of the container, it imparts a small force. The sum of forces from all molecules that bounce off a wall of area A results in an air pressure P_o.

If you add Δq thermal energy to air in the box, the temperature rises to T_2 (represented by longer arrows in Fig. 3I.2b). Also, when each molecule bounces off a wall, it imparts a greater force because it is moving faster. Thus, the pressure P_2 will be larger. This is expected from the ideal gas law under constant density, for which

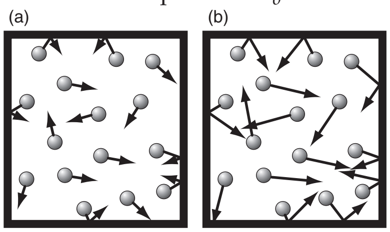


Fig. 3I.2. *Molecules in a fixed volume.*

$$P_o/T_o = P_2/T_2 = \text{constant} = \rho\cdot\Re \qquad (3I.4)$$

Different materials warm by different amounts when you add heat. If you know how much thermal energy Δq you added per kilogram of material, and you measure the resulting increase in temperature $T_2 - T_o$, then you can empirically determine the **specific heat at constant volume:**

$$C_v = \Delta q/(T_2 - T_o) \qquad (3I.5)$$

which is about $C_v = 717 \text{ J·kg}^{-1}\text{·K}^{-1}$ for dry air.

C_p — Specific Heat at Constant Pressure

For a different scenario, consider a box (Fig. 3I.3c) with a frictionless moveable piston at the top. The weight of the stationary piston is balanced by the pressure of the gas trapped below it. If you add Δq thermal energy to the air, the molecules will move faster (Fig. 3I.3d), and exert greater pressure against the piston and against the other walls of the chamber. But the weight of the piston hasn't changed, so the increased pressure of the gas causes the piston to rise.

But when any molecule bounces off the piston and helps move it, the molecule loses some of its microscopic kinetic energy. (An analogy is when a billiard ball bounces off an empty cardboard box sitting in the middle of the billiard table. The box moves a bit when hit by the ball, and the ball returns more slowly.) The result is that the gas temperature T_1 in Fig. 3I.3e is not as warm as in Figs. 3I.2b or 3I.3d, but is warmer than the initial temperature; namely, $T_o < T_1 < T_2$.

(continues in next column)

(continuation of INFO on Cp vs. Cv)

The molecules spread out within the larger volume in Fig. 3I.3e. Thus, air density ρ decreases, causing fewer molecules near the piston to push against it. The combined effects of decreasing density and temperature cause the air pressure to decrease as the piston rises. Eventually the piston stops rising at the point where the air pressure balances the piston weight, as shown in Fig. 3I.3e.

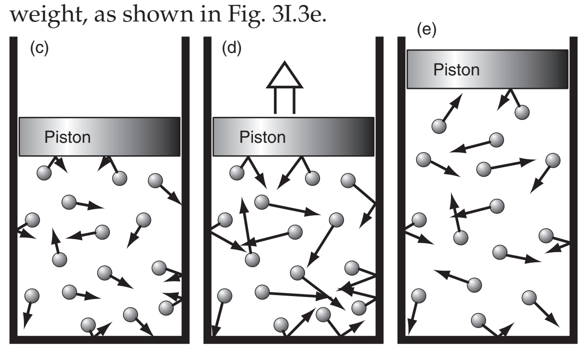


Fig. 3I.3. *Molecules in a constant-pressure chamber.*

Hence, this is an isobaric process (determined by the weight of the piston in this contrived example). The ideal gas law for constant pressure says:

$$\rho_o \cdot T_o = \rho_1 \cdot T_1 = \text{constant} = P/\Re \qquad (3I.6)$$

If you know how much thermal energy Δq you added per kilogram of material, and you measure the resulting increase in temperature $T_1 - T_o$, then you can empirically determine the **specific heat at constant pressure:**

$$C_p = \Delta q/(T_1 - T_o) \qquad (3I.7)$$

which is about $C_p = 1004 \text{ J·kg}^{-1}\text{·K}^{-1}$ for dry air.

C_p vs. C_v

Thus, in a constant pressure situation, the addition of Δq thermal energy results in less warming [$(T_1 - T_o) < (T_2 - T_o)$] than at constant volume. The reason is that, for constant pressure, some of the random microscopic kinetic energy of the molecules is converted into the macroscale **work** of expanding the air and moving the piston up against the pull of gravity. Such conservation of energy is partly described by the First Law of Thermodynamics.

In the atmosphere, the pressure at any altitude is determined by the weight of all the air molecules above that altitude (namely, the "piston" is all the air molecules above). If you add a small amount of thermal energy to air molecules at that one altitude, then you haven't significantly affected the number of molecules above, hence the pressure is constant. Thus, for most atmospheric situations it is appropriate to use C_p, not C_v, when forecasting temperature changes associated with the transfer of thermal energy into or from an air parcel.

FRAMEWORKS

The First Law of Thermodynamics is a powerful tool that we can apply to different frameworks.

Lagrangian vs. Eulerian

One framework, called **Eulerian**, is fixed relative to a position on the Earth's surface. Thus, if there is a wind, then the air blows through this framework, so we need to be concerned about what heat is carried in by the wind. Weather forecasts for specific points on a map utilize this framework.

Another framework is called **Lagrangian**. It moves with the air — its position is constantly changing. This is handy for investigating what happens to air as it rises or sinks.

Eulerian and Lagrangian frameworks can be used for a variety of budget equations:
• Heat Budget (First Law of Thermo)
• Momentum Budget (Newton's Second Law)
• Moisture Budget (conservation of water),
We will use these frameworks in other chapters too.

Air Parcels

Sometimes a large cluster of air molecules will move together through the atmosphere, as if they were enclosed by a hypothetical balloon about the diameter of two city blocks. We can use a Lagrangian framework that moves with this cluster or "blob", in order to study changes of its temperature, momentum, and moisture.

When these air blobs move through the atmosphere, myriad eddies (swirls of turbulent motion) tend to mix some of the outside air with the air just inside the blob (such as the mixing you see in smoke rising from a campfire). Thus, warmer or colder air could be added to (**entrained** in through the sides of) the blob, and some air from inside could be lost (**detrained**) to the surrounding atmosphere. Also, in the real atmosphere, atmospheric radiation can heat or cool the air blob. These processes complicate the thermodynamic study of real air blobs.

But to gain some insight into the thermodynamics of air, we can imagine a simplified situation where radiative effects are relatively small, and where the turbulent entrainment/detrainment happens only in the outer portions of the air blob, leaving an inner core somewhat protected. This is indeed observed in the real atmosphere. So consider the protected inner core (about the diameter of a city block) as an **air parcel**.

Whenever you see discussions regarding air parcels, you should immediately associated them with Lagrangian frameworks. This is the case for the next section.

HEAT BUDGET OF AN UNSATURATED AIR PARCEL

In this chapter, we consider a special case: unsaturated air parcels, for which no liquid or solid water is involved. The word "**dry**" is used to imply that phase changes of water are not considered. Nonetheless, the air CAN contain water vapor. In the next chapter we include the effects of saturation and possible phase changes in a "**moist**" analysis.

Lagrangian Form of the First Law of Thermo

The pressure of an air parcel usually equals that of its surrounding environment, which decreases exponentially with height. Thus, the last term of eq. (3.2d) will be non-zero for a rising or sinking air parcel as its pressure changes to match the pressure of its environment. But the pressure change with height was given by the hydrostatic equation in Chapter 1: $\Delta P/\rho = -\,|g|\cdot\Delta z$. We can use this to rewrite the First Law of Thermo in the Lagrangian framework of a moving air parcel:

$$\Delta T = -\left(\frac{|g|}{C_p}\right)\cdot\Delta z + \frac{\Delta q}{C_p} \qquad \bullet(3.5)$$

Sample Application

A 5 kg air parcel of initial temperature 5°C rises 1 km and thermally loses 15 kJ of energy due to IR radiation. What is the final temperature of the parcel?

Find the Answer

Given: ΔQ = –15,000 J, m_{air} = 5 kg, Δz = 1000 m ,
Find: T = ? K

Convert from energy to energy/mass:
$\Delta q = \Delta Q/m_{air}$ = (–15000 J)/(5 kg) = –3000 J kg^{-1} .
With lack of humidity info, assume dry air.
C_p = 1004. J·kg^{-1}·K^{-1} (= units m^2 s^{-2} K^{-1})
Apply eq. (3.5):
ΔT = –[(9.8 m s^{-2})/ (1004 m^2 s^{-2} K^{-1})]·(1000m) + [(–3000 J kg^{-1}) / (1004. J·kg^{-1}·K^{-1})]
= (– 9.76 – 3.00) K = –12.76 °C

Check: Physics & units are reasonable.
Exposition: Because we are working with a temperature difference, recall that 1 K of temperature difference = 1°C of temperature difference. Hence, we could replace the Kelvin units with °C. The net result is that the rising air parcel cools due to both IR radiative cooling and work done on the atmosphere as the parcel rises.

Various processes can cause heat transfer (Δq). The sun could heat the air, or IR radiation could cool the air. Water vapor could condense and release its latent heat back into sensible heat. Exothermic chemical reactions or radioactive decay could occur among air pollutants carried within the parcel. Internal turbulence could dissipate into heat. Molecular conduction in the air is very weak, but turbulence could mix warmer or cooler air into the air parcel. Other processes such as convection and advection (Fig. 3.2) do not change the parcel's temperature, but can move the air parcel along with the heat that it possesses.

Eq. (3.5) represents a **heat budget**. Namely, parcel temperature (which indicates heat possessed) is conserved unless it moves to a different height (where the pressure is different) or if heat is transferred to or from it. Thus, eq. (3.5) and the other First Law of Thermo eqs. (3.2) are also known as **heat conservation** equations.

Lapse-rate Definition

Define the **lapse rate**, Γ, as the amount of temperature <u>decrease</u> with altitude:

$$\Gamma = -\frac{T_2 - T_1}{z_2 - z_1} = -\frac{\Delta T}{\Delta z} \tag{3.6}$$

Note that the lapse rate is the negative of the **vertical temperature gradient** $\Delta T/\Delta z$.

We separately consider the lapse rates inside the air parcel, and in the surrounding environment outside. Inside the air parcel, all the processes illustrated in Fig. 3.2 could apply, causing the parcel's temperature to change with changing altitude. The resulting $\Delta T/\Delta z$ (times –1) defines a **process lapse rate** (Fig. 3.3).

Outside the air parcel, assume the environmental air is relatively stationary. This is the ambient environment through which the air parcel moves. But this environment could have different temperatures at different altitudes, allowing us to define an **environmental lapse rate** (Fig. 3.3). By sampling the ambient air at different heights using weather instruments such as radiosondes (weather balloons) and then plotting T vs. z as a graph, the result is an **environmental sounding** or **vertical temperature profile** of the environment. The environmental sounding changes as the weather evolves, but this is usually slow relative to parcel processes. Thus, the environment is often approximated as being unchanging (i.e., static).

The temperature difference (Fig. 3.3) between the parcel and its environment is crucial for determining parcel buoyancy and storm development. This is our motivation for examining both lapse rates.

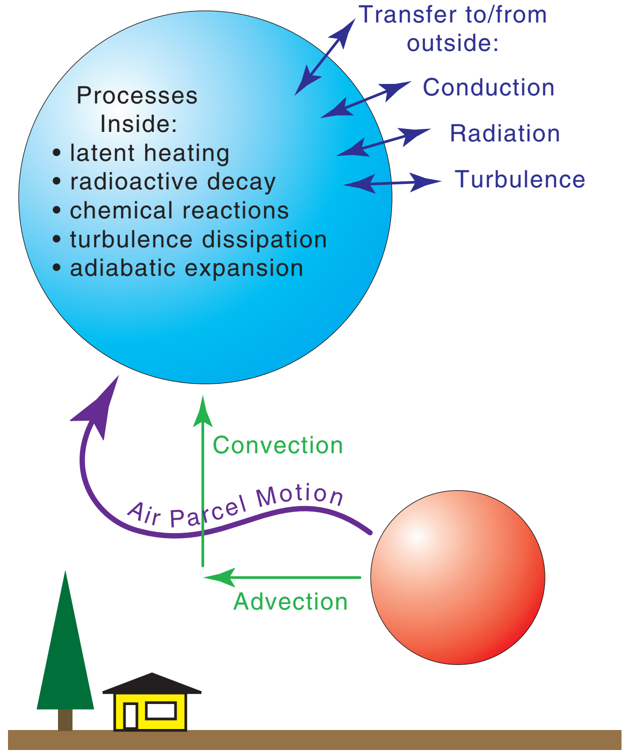


Figure 3.2
Internal and external processes affecting air-parcel temperature.

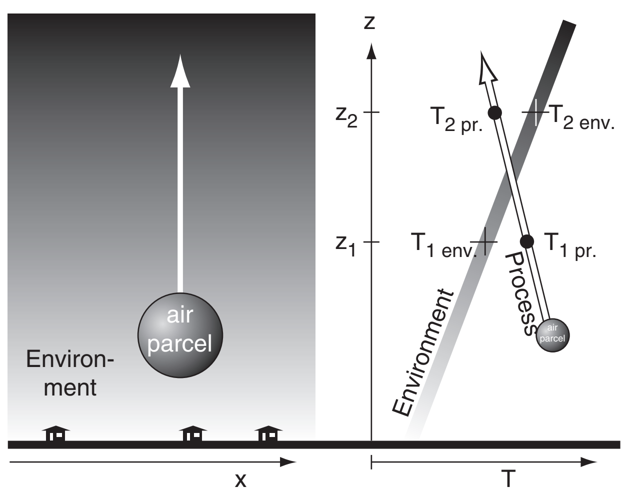


Figure 3.3
Left: Sketch of a physical situation, showing an air parcel moving through an environment. In the environment, darker colors indicate warmer air. Right: Temperature profiles for the environment and the air parcel. The environmental air is not moving. In it, air at height z_1 has temperature $T_{1\ env}$, and air at z_2 has $T_{2\ env}$. The air parcel has an initially warm temperature, but its temperature changes as it rises: becoming $T_{1\ pr}$ at height z_1, and later becoming $T_{2\ pr}$ at height z_2. In the <u>environment</u> of this example, temperature <u>in</u>creases as height increases, implying a <u>negative</u> <u>environmental</u> lapse rate. However, the air <u>parcel's</u> temperature <u>decreases</u> with height, implying a <u>positive</u> <u>parcel</u> lapse rate.

Sample Application

Find the lapse rate in the troposphere for a standard atmosphere.

Find the Answer

Given: Std. Atmos. Table 1-5 in Chapter 1, , where
T= –56.5°C at z = 11 km, and T = +15°C at z = 0 km.
Find: Γ = ? °C km^{-1}
Apply eq. (3.6): $\Gamma = -(-56.5 - 15°C) / (11-0 \text{ km})$
$= \underline{\mathbf{+6.5\ °C\ km}}^{-1}$

Check: Positive Γ, because T decreases with z.
Exposition: This is the environmental lapse rate of the troposphere. It indicates a static background state.

HIGHER MATH • Adiabatic Lapse Rate in Pressure Coordinates

Start with the First Law of Thermodynamics (eq. 3.2d), but written more precisely using virtual temperature T_v to account for arbitrary concentrations of water vapor in the air. Set Δq = 0 because adiabatic means no heat transfer:

$$\mathrm{d}P = \rho \cdot C_p \cdot \mathrm{d}T_v$$

Use the ideal gas law $\rho = P / (\Re_d \cdot T_v)$ to eliminate ρ:

$$\mathrm{d}P = \frac{P \cdot C_p \cdot \mathrm{d}T_v}{\Re_d \cdot T_v}$$

Group temperature & pressure terms on opposite sides of the eq.:

$$\frac{\mathrm{d}P}{P} = \frac{C_p}{\Re_d} \cdot \frac{\mathrm{d}T_v}{T_v}$$

Integrate from starting (P_1, T_{v1}) to ending (P_2, T_{v2}):

$$\int_{P_1}^{P_2} \frac{\mathrm{d}P}{P} = \frac{C_p}{\Re_d} \cdot \int_{T_{v1}}^{T_{v2}} \frac{\mathrm{d}T_v}{T_v}$$

assuming $C_p/\Re_d$ is somewhat constant. The integral is:

$$\ln(P)\Big|_{P_1}^{P_2} = (C_p / \Re_d) \cdot \ln(T_v)\Big|_{T_{v1}}^{T_{v2}}$$

Insert limits of integration. Also: ln(a) –ln(b) = ln(a/b). Thus:

$$\ln\left(\frac{P_2}{P_1}\right) = (C_p / \Re_d) \cdot \ln\left(\frac{T_{v2}}{T_{v1}}\right)$$

Multiply both sides by $\Re_d / C_p$:

$$(\Re_d / C_p) \cdot \ln\left(\frac{P_2}{P_1}\right) = \ln\left(\frac{T_{v2}}{T_{v1}}\right)$$

Use the relationship: $a \cdot \ln(b) = \ln(b^a)$:

$$\ln\left[\left(\frac{P_2}{P_1}\right)^{\Re_d / C_p}\right] = \ln\left(\frac{T_{v2}}{T_{v1}}\right)$$

The anti-log of the equation ($e^{LHS} = e^{RHS}$) yields:

$$\boxed{\left(\frac{P_2}{P_1}\right)^{\Re_d / C_p} = \frac{T_{v2}}{T_{v1}}} \qquad (3.10)$$

Dry Adiabatic Lapse Rate

The word **adiabatic** means zero heat transfer (Δq = 0). For the protected inner core of air parcels, this means no thermal energy entering or leaving the air parcel from outside (Fig. 3.2). Nonetheless, internal processes are allowed.

For the special case of humid air with no liquid water or ice carried with the parcel (and no water phase changes; hence, a "dry" process), eq. (3.5) gives:

$$\frac{\Delta T}{\Delta z} = -\left(\frac{|g|}{C_p}\right) = -9.8 \text{ K km}^{-1} \qquad \bullet(3.7)$$

Recalling that the lapse rate is the negative of the vertical temperature gradient, we can define a **"dry" adiabatic lapse rate** Γ_d as:

$$\Gamma_d = 9.8 \text{ K km}^{-1} = 9.8 \text{ °C km}^{-1} \qquad (3.8)$$

(Degrees K and °C are interchangeable in this equation for this process lapse rate, because they represent a temperature change with height.)

The HIGHER MATH box at left shows how this dry adiabatic lapse rate can be expressed as a function of pressure P:

$$\frac{\Delta T}{T} = \frac{\Re_d}{C_p} \cdot \left(\frac{\Delta P}{P}\right) \qquad (3.9)$$

or

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\Re_d / C_p} \qquad \bullet(3.10)$$

where $\Re_d/C_p$ = 0.28571 (dimensionless) for dry air, and where temperatures are in Kelvin.

Sample Application

An air parcel with initial (z, P, T) = (100m, 100 kPa, 20°C) rises adiabatically to (z, P) = (1950 m, 80 kPa). Find its new T, & compare eqs. (3.7) & (3.10).

Find the Answer

Given: P_1 = 100 kPa, P_2 = 80 kPa, T_1 = 20°C = 293K
z_1 = 100 m, z_2 = 1950 m
Find: T_2 = ? °C

First, apply eq. (3.7), which is a function of z:
$T_2 = T_1 + (\Delta z)\cdot(-\Gamma_d)$ = 20°C – (1950–100m)·(0.0098°C/m)
= 20°C – 18.1°C = **1.9°C**.

Compare with eq. (3.10), which is a function of P:
T_2 = (293K) · [(80kPa)/(100kPa)]$^{0.28571}$
T_2 = 293K · 0.9382 = 274.9 K = **1.9°C**

Check: Both equations give the same answer, so either equation would have been sufficient by itself.

Potential-temperature Definition

When an air parcel rises/sinks "dry" adiabatically into regions of lower/higher pressure, its temperature changes due to work done by/on the parcel, even though no thermal energy has been removed/added. Define a new temperature variable called the **potential temperature** θ that is proportional to the sensible heat contained in the parcel, but which is unaffected by work done by/on the parcel.

Namely, potential temperature is constant for an adiabatic process (i.e., $\Delta q = 0$) such as air-parcel ascent. Thus, we can use it as a **conserved variable**. θ can increase/decrease when sensible heat is added/removed. Such **diabatic** (non-adiabatic) heat transfer processes include turbulent mixing, condensation, and radiative heating (i.e., $\Delta q \neq 0$).

Knowing the air temperature T at altitude z, you can calculate the value of potential temperature θ from:

$$\theta(z) = T(z) + \Gamma_d \cdot z \qquad \bullet(3.11)$$

The units (K or °C) of $\theta(z)$ are the same as the units of $T(z)$. There is no standard for z, so some people use height above mean sea level (MSL), while others use height above local ground level (AGL).

If, instead, you know air temperature T at pressure-level P, then you can find the value of θ from:

$$\theta = T \cdot \left(\frac{P_o}{P}\right)^{\Re_d / C_p} \qquad \bullet(3.12)$$

where $\Re_d/C_p$ = 0.28571 (dimensionless) and where temperatures must be in Kelvin. A reference pressure of P_o = 100 kPa is often used, although some people use the local surface pressure instead. In this book we will assume that the surface pressure equals the reference pressure of P_o = 100 kPa and will use z = 0 at that surface, unless stated otherwise.

Both eqs. (3.11) and (3.12) show that $\theta = T$ at $z = 0$ or at $P = P_o$. Thus θ is the actual temperature that an air parcel potentially has if lowered to the reference level adiabatically.

A **virtual potential temperature** θ_v for humid air having water-vapor mixing ratio r but containing no solid or liquid water is defined as:

$$\theta_v = \theta \cdot [1 + (a \cdot r)] \qquad (3.13)$$

where a = 0.61 $g_{air}/g_{water\ vapor}$. If the air contains ice crystals, cloud drops, or rain drops, then virtual potential temperature is given by:

$$\theta_v = \theta \cdot [1 + (a \cdot r) - r_L - r_I] \qquad \bullet(3.14)$$

where r_L is the liquid-water mixing ratio and r_I = ice mixing ratio. Mixing ratio is described in the Wa-

Sample Application

Find θ for air of T = 15°C at z = 750 m?

Find the Answer

Given: T = 15°C, z = 750 m,
Find: θ = ? °C

Apply eq. (3.11), and assume no ice or rain drops.
θ = 15°C + (9.8 °C km^{-1})· (0.75 km) = **22.35 °C**

Check: Physics & units are reasonable.
Exposition: Notice that potential temperatures are warmer than actual air temperatures for $z > 0$.

Sample Application

Find θ for air at P = 70 kPa with T = 10°C?

Find the Answer

Given: P = 70 kPa , T = 10°C = 283 K, P_o = 100 kPa
Find: θ = ? °C

Apply eq. (3.12): θ = (283 K) · [(100 kPa)/(70 kPa)]$^{0.28571}$
θ = 313.6 K = **40.4 °C**

Check: Physics OK. θ is always greater than the actual T, for P smaller than the reference pressure.

Sample Application

What is the virtual potential temperature of air having potential temperature 15°C, mixing ratio 0.008 $g_{water\ vapor}/g_{air}$, and liquid water mixing ratio of: a) 0 ; b) 0.006 $g_{liq.water}/g_{air}$?

Find the Answer

Given: θ =15°C = 288 K, r = 0.008 $g_{water\ vapor}/g_{air}$,
a) r_L = 0 ; b) r_L =0.006 $g_{liq.water}/g_{air}$.
Find: θ_v = ? °C
Abbreviate "water vapor" with "wv" here.

a) Apply eq (3.13): θ_v = (288K)·
[1 + (0.61 g_{air}/g_{wv}) · (0.008 g_{wv}/g_{air})] .
Thus, θ_v = 289.4 K. Or subtract 273 to get Celsius: θ_v = **16.4 °C**.

a) Apply eq (3.14): θ_v = (288K)·
[1 + (0.61 g_{air}/g_{wv}) · (0.008 g_{wv}/g_{air}) – 0.006 $g_{liq.}/g_{air}$] = 287.7 K.
Subtract 273 to get Celsius: θ_v = **14.7°C**.

Check: Physics and units are reasonable.
Exposition: When no liquid water is present, virtual pot. temperatures are always warmer than potential temperatures, because water vapor is lighter than air.

However, liquid water is heavier than air, and has the opposite effect. This is called **liquid-water loading**, and makes the air act as if it were colder.

Sample Application

Given air at P = 70 kPa with T = –1°C, which is either (a) unsaturated, or (b) has $r_s = 5\ g_{water\ vapor}/kg_{air}$ and $r_L = 2\ g_{liq\ water}/kg_{air}$. Find θ_v.

Find the Answer

Given: (a) P = 70 kPa , T = –1°C = 272K,
(b) $r_s = 5\ g_{water\ vapor}/kg_{air} = 0.005\ g_{wv}/g_{air}$
$r_L = 2\ g_{liq\ water}/kg_{air} = 0.002\ g_{liq}/g_{air}$
Find: θ_v = ? °C

(a) Apply eq. (3.12) to find the potential temperature
$\theta = (272K) \cdot [(100\ kPa)/(70\ kPa)]^{0.28571} = 301\ K$
= **<u>28°C</u>** .

(b) Apply eq. (3.15):
$\theta_v = (301\ K) \cdot [\ 1 + (0.61\ g_{air}/g_{wv}) \cdot (0.005\ g_{wv}/g_{air}) - (0.002)\ ] = 301K \cdot (1.001) = \mathbf{\underline{301.3\ K}}$
$\theta_v = 301.3\ K - 273\ K = \mathbf{\underline{28.3°C}}$

Check: Physics & units are reasonable.
Exposition: For this example, there was little effect of the vapor and liquid water. However, for situations with greater water vapor or liquid water, the virtual potential temperature can differ by a few degrees, which can be important for estimating thunderstorm intensity.

ter Vapor chapter; it is the ratio of grams of water per gram of air. The θ and θ_v values in the previous three equations must be in units of Kelvin.

An advantage of θ_v is that it can be used to calculate the buoyancy of air parcels that contain water — useful for anticipating storm characteristics. θ_v is constant only when there is no phase changes and no heat transfer; namely, no latent or sensible heat is absorbed or released.

For air that is rising within clouds, with water vapor condensing, it is usually the case that the air is saturated (= 100% relative humidity; see the Water Vapor chapter for details). As a result, the water-vapor mixing ratio r can be replaced with r_s , the saturation mixing ratio.

$$\theta_v = \theta \cdot [1 + (a \cdot r_s) - r_L] \qquad \bullet(3.15)$$

where $a = 0.61\ g_{air}/g_{water\ vapor}$, as before.

However, there are other situations where the air is NOT saturated, but contains liquid water. An example is the non-cloudy air under a cloud base, through which rain is falling at its terminal velocity. For this case, eq (3.14) should be used with an unsaturated value of water-vapor mixing ratio. This situation occurs often, and can be responsible for damaging downbursts of air (see the Thunderstorm chapters).

Why use potential temperature? Because it makes it easier to compare the temperatures of air parcels at two different heights — important for determining if air will buoyantly rise to create thunderstorms. For example, suppose air parcel A has temperature T_A = 20°C at z = 0, while air parcel B has T_B = 15°C at z = 1 km. Parcel A is warmer than parcel B.

Does that mean that parcel A is buoyant (warmer and wants to rise) relative to parcel B? The answer is <u>no</u>, because when parcel B is moved dry adiabatically to the altitude of parcel A, then parcel B is 5°C warmer than parcel A due to adiabatic warming. In fact, you can move parcels A and B to any common altitude, and after considering their adiabatic warming or cooling, parcel B will always be 5°C warmer than parcel A.

The easiest way to summarize this effect is with potential temperature. Using eq. (3.11), we find that θ_A = 20°C and θ_B = 25°C approximately. θ_A and θ_B keep their values (because θ is a conserved variable) no matter to what common altitude you move them, thus θ_B is always 5 °C warmer than θ_A in this illustration.

Another application for potential temperature is to label lines on a thermodynamic diagram, such as described next.

Intro to Thermo Diagrams

Convection is a vertical circulation associated with "warm air rising" and "cold air sinking". Meteorologists forecast the **deep convection** of thunderstorms and their hazards, or the **shallow convection** of thermals that disperse air pollutants.

The phrase "warm air rising" relates to the temperature difference ΔT between an air parcel and its surrounding environment. Air-parcel-temperature variation with altitude can be anticipated using heat- and water-conservation relationships. However, the surrounding environmental temperature profile can have a somewhat arbitrary shape that can be measured by a sounding balloon, but which is not easily described by analytical equations. So it can be difficult to mathematically describe ΔT vs. altitude.

Instead, graphical solutions can be used to estimate buoyancy and convection. We call these graphs "**thermodynamic diagrams**". In this book, I will abbreviate the name as "**thermo diagram**".

The diagram is set up so that higher in the diagram corresponds to higher in the atmosphere. In the real atmosphere, pressure decreases approximately logarithmically with increasing altitude, so we often use pressure P along the y-axis as a surrogate for altitude. Along the x-axis is air temperature T. The thin green lines in Fig. 3.4 show the (P, T) basis for a thermo diagram as a semi-log graph.

We can use eq. (3.10) to solve for the "dry" adiabatic temperature change experienced by rising air parcels. These are plotted as the thick orange diagonal lines in Fig. 3.4 for a variety of starting temperatures at P = 100 kPa. These "**dry adiabat**" lines (also known as **isentropes**), are labeled with θ because potential temperature is conserved for adiabatic processes.

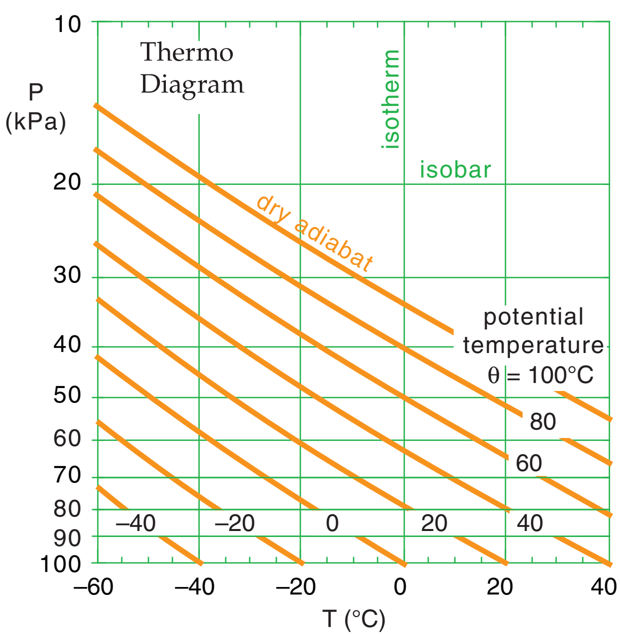


Figure 3.4 **(at left)**
Simplified ***thermo diagram****, showing isotherms (vertical green lines of constant temperature) and isobars (horizontal green lines of constant pressure). Isobars are plotted logarithmically, but with the scale reversed so that the highest pressure is at the bottom of the graph, corresponding to the bottom of the atmosphere. Dry adiabats are thick orange lines, showing the temperature variation of air parcels rising or sinking adiabatically.*

INFO • Create Your Own Thermo Diagram

One of the advantages of thermo diagrams is that you do NOT need to calculate adiabatic temperature changes, because they are already calculated and plotted for you for a variety of different starting temperatures. If the starting temperature you need is not already plotted, you can mentally interpolate between the drawn lines as you raise or lower air parcels.

However, it is a useful exercise to see how such a thermo diagram can be created with a tool as simple as a computer spreadsheet.

The green (or dark-grey) items in the spreadsheet below were typed directly as numbers or words. You can follow along on your own spreadsheet. (You don't need to use the same colors — black is OK.)

The orange (or light-grey) numbers were calculated by entering a formula (eq. 3.10) into the bottom leftmost orange cell, and then "filling up" and "filling right" that equation into the other orange cells. But before you fill up and right, be sure to use the dollar sign "$" as shown below. It holds the column ID constant if it appears in front of the ID letter, or holds the row constant if in front of the ID number. Here is the equation for the bottom left orange cell (B12):

= ((B$13+273) * ($A12/A13)^0.28571) - 273

	A	B	C	D
1	Create Your Own Thermo Diagram			
2				
3	P(kPa)	T(degC)	T(degC)	T(degC)
4	10	–152.3	–131.6	–110.9
5	20	–125.9	–100.6	–75.4
6	30	–107.8	–79.5	–51.1
7	40	–93.7	–62.9	–32.1
8	50	–81.9	–49.0	–16.2
9	60	–71.6	–37.1	–2.5
10	70	–62.6	–26.4	9.7
11	80	–54.4	–16.9	20.7
12	90	-46.9	–8.1	30.7
13	100	–40	0	40

Different spreadsheet versions have different ways to create graphs. Select the cells that I outlined with the dark blue rectangle. Click on the Graph button, select the "XY scatter", and then select the option that draws straight line segments without data points.

Under the Chart, Source Data menu, select Series. Then manually switch the columns for the X and Y data for each series — this does an axis switch. On the graph, click on the vertical axis to get the Format Axis dialog box, and select the Scale tab. Check the Logarithmic scale box, and the Values in Reverse Order box. A bit more tidying up will yield a graph with 3 curves similar to Fig. 3.4. Try adding more curves.

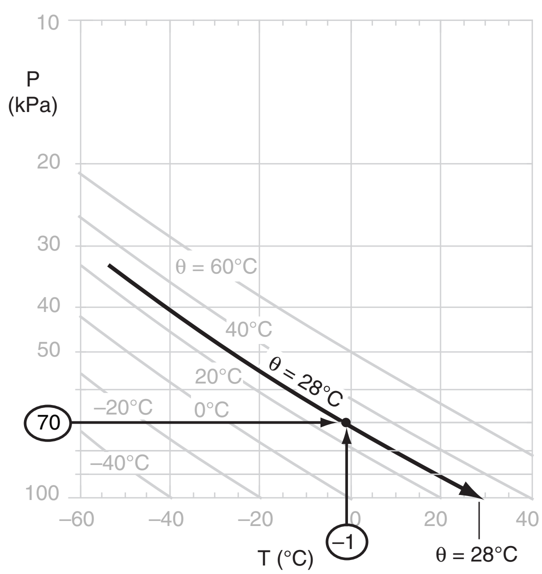
Sample Application

Given air at P = 70 kPa with T = –1°C. Find θ using the thermo diagram of Fig. 3.4.

Find the Answer

Given: P = 70 kPa , T = –1°C
Find: θ = ? °C

First, use the thermo diagram to find where the 70 kPa isobar and the –1°C isotherm intersect. (Since the –1°C isotherm wasn't drawn on this diagram, we must mentally interpolate between the lines that are drawn.) The adiabat that passes through this intersection point indicates the potential temperature (again, we must interpolate between the adiabats that are drawn). By extending this adiabat down to the reference pressure of 100 kPa, we can read off the temperature 28°C, which corresponds to a potential temperature of θ = <u>28°C</u> .



Check: Physics and units are reasonable.
Exposition: This exercise is the same as part (a) of the previous exercise, for which we calculated θ = 301 K = 28°C. Yes, the answers agree.

The advantage of using an existing printed thermo diagram is that we can draw a few lines and quickly find the answer without doing any calculations. So it can make our lives easier, once we learn how to use it.

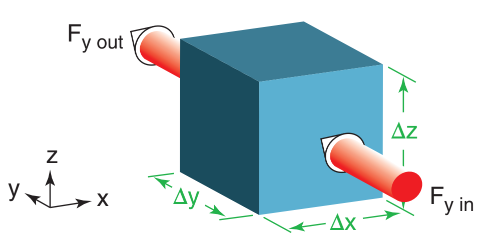


Figure 3.5
If heat flux $F_{y\ in}$ exceeds $F_{y\ out}$, then: (1) heat is deposited in the cube of air, making it hotter, and (2) ΔF/Δy is negative for this case. Similar fluxes can occur across the other faces.

If you know the initial (P, T) of the air parcel, then plot it as a point on the thermo diagram. Move parallel to the orange lines to the final pressure altitude. At that final point, read down vertically to find the parcel's final temperature.

HEAT BUDGET AT A FIXED LOCATION

Eulerian Form of the First Law of Thermo

Picture a cube of air at a fixed location relative to the ground (i.e., an Eulerian framework). By being fixed, the cube experiences only small, slow changes in pressure. As a result, the pressure-change term in the First Law of Thermo (eq. 3.2d) can usually be neglected. What remains is an equation that says thermal energy transferred (Δq) per unit mass causes temperature change: $\Delta T = \Delta q / C_p$.

Dividing this equation by time interval Δt gives a forecast equation for temperature: $\Delta T/\Delta t = (1/C_p)\cdot\Delta q/\Delta t$. A heat flux $\mathbb{F}$ (J m^{-2} s^{-1}, or W m^{-2}) into the volume could increase the temperature, but a heat flux out the other side could decrease the temperature. Thus, with both inflow and outflow of heat, net thermal energy will be transferred into the cube of air if the heat flux <u>de</u>creases with distance s across the cube: $\Delta q/\Delta t = -(1/\rho)\cdot\Delta\mathbb{F}/\Delta s$. The inverse density factor appears because Δq is energy <u>per unit mass</u>.

Heat flux convergence such as this causes warming, while **heat flux divergence** causes cooling. This **flux gradient** (change with flux across a distance) could happen in any of the three Cartesian directions. Thus, the temperature forecast equation becomes:

$$\frac{\Delta T}{\Delta t} = -\frac{1}{\rho \cdot C_p}\left[\frac{\Delta \mathbb{F}_x}{\Delta x} + \frac{\Delta \mathbb{F}_y}{\Delta y} + \frac{\Delta \mathbb{F}_z}{\Delta z}\right] + \frac{\Delta S_o}{C_p \cdot \Delta t} \tag{3.16}$$

where, for example, $\Delta\mathbb{F}_y/\Delta y$ is the change in northward-moving flux $\mathbb{F}_y$ across a north-south distance Δy (Fig. 3.5). Additional heat sources can occur inside the cube at rate $\Delta S_o/\Delta t$ (J kg^{-1} s^{-1}) such as when water vapor already inside the cube condenses into liquid and releases latent heat. The equation above is the Eulerian **heat-budget equation**, also sometimes called a **heat conservation** or **heat balance** equation.

Recall from Chapter 2 that we can define a kinematic flux by $F = \mathbb{F}/(\rho\cdot C_p)$ in units of K m s^{-1} (equivalent to °C m s^{-1}). Thus, eq. (3.16) becomes:

$$\frac{\Delta T}{\Delta t} = -\left[\frac{\Delta F_x}{\Delta x} + \frac{\Delta F_y}{\Delta y} + \frac{\Delta F_z}{\Delta z}\right] + \frac{\Delta S_o}{C_p \cdot \Delta t} \qquad \bullet(3.17)$$

We can also reframe this heat budget in terms of potential temperature, because with no movement of the cube of air itself, then $\Delta T = \Delta\theta$.

$$\frac{\Delta\theta}{\Delta t} = -\left[\frac{\Delta F_x}{\Delta x} + \frac{\Delta F_y}{\Delta y} + \frac{\Delta F_z}{\Delta z}\right] + \frac{\Delta S_o}{C_p \cdot \Delta t} \qquad \bullet(3.18)$$

You may have wondered why, in the previous figure, a $\Delta F_y/\Delta y$ was negative, even though heat was deposited into the cube. The reason is that for gradients, the difference-direction of the denominator must be the same direction as the numerator; e.g.:

$$\frac{\Delta F_y}{\Delta y} = \frac{F_{y\ northside} - F_{y\ southside}}{y_{northside} - y_{southside}} \qquad (3.19)$$

Similar care must be taken for gradients in the x and z directions.

Not only do we need to consider fluxes in each direction in eqs. (3.16 to 3.18), but for any one direction there might be more than one physical process causing fluxes. The other processes that we will discuss next are **conduction** (*cond*), **advection** (*adv*), **radiation** (*rad*), and **turbulence** (*turb*):

$$\frac{\Delta F_x}{\Delta x} = \left.\frac{\Delta F_x}{\Delta x}\right|_{adv} + \left.\frac{\Delta F_x}{\Delta x}\right|_{cond} + \left.\frac{\Delta F_x}{\Delta x}\right|_{turb} + \left.\frac{\Delta F_x}{\Delta x}\right|_{rad} \qquad (3.20)$$

$$\frac{\Delta F_y}{\Delta y} = \left.\frac{\Delta F_y}{\Delta y}\right|_{adv} + \left.\frac{\Delta F_y}{\Delta y}\right|_{cond} + \left.\frac{\Delta F_y}{\Delta y}\right|_{turb} + \left.\frac{\Delta F_y}{\Delta y}\right|_{rad} \qquad (3.21)$$

$$\frac{\Delta F_z}{\Delta z} = \left.\frac{\Delta F_z}{\Delta z}\right|_{adv} + \left.\frac{\Delta F_z}{\Delta z}\right|_{cond} + \left.\frac{\Delta F_z}{\Delta z}\right|_{turb} + \left.\frac{\Delta F_z}{\Delta z}\right|_{rad} \qquad (3.22)$$

In addition to describing these fluxes, we will estimate typical contributions of latent heating as a body source (ΔS_o), allowing us to simplify the full heat budget equation in an Eulerian framework.

Advection of Heat

The AMS *Glossary of Meteorology* (2000) defines advection as transport of an atmospheric property by the mass motion of the air (i.e., by the wind). **Temperature advection** transports heat. Faster winds blowing hotter air causes greater **advective heat flux**:

$$F_{x\ adv} = U \cdot T \qquad (3.23)$$

$$F_{y\ adv} = V \cdot T \qquad (3.24)$$

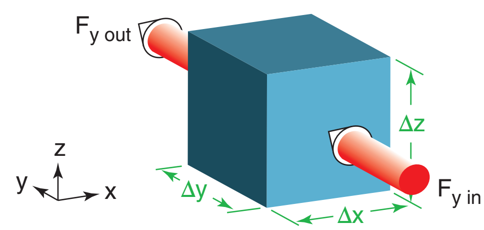
$$F_{z\ adv} = W \cdot T \qquad (3.25)$$

Sample Application

In the figure below, suppose that the incoming heat flux from the south is 5 W m^{-2}, and the outgoing on the north face of the cube is 7 W m^{-2}. (a) Convert these fluxes to kinematic units. (b) What is the value of the kinematic flux gradient? (c) Calculate the warming rate of air in the cube, assuming the cube has zero humidity and is at a fixed altitude where air density is 1 kg m^{-3}. The cube of air is 10 m on each side.

Find the Answer

Given: $\mathbb{F}_{y\ in}$= 5 W·m^{-2} , $\mathbb{F}_{y\ out}$ = 7 W·m^{-2}, Δy = 10 m
ρ = 1.0 kg m^{-3} ,
Find: a) $F_{x\ right}$ = ? K·m s^{-1}, $F_{x\ left}$ = ? K·m s^{-1}
b) $\Delta F_y/\Delta y$ = ? c) $\Delta T/\Delta t$ = ? K s^{-1}
From Appendix B: C_p = 1004 J·kg^{-1}·K^{-1}
Also, don't forget that 1 W = 1 J s^{-1} .
Diagram:



a) Apply eq. (2.11): $F = \mathbb{F} / (\rho \cdot C_p)$

$F_{y\ in}$ = (5 J·s^{-1}·m^{-2}) / [(1 kg m^{-3}) · (1004 J·kg^{-1}·K^{-1}))]
= **<u>4.98x10^{-3}</u>** **<u>K·m·s^{-1}</u>** .

$F_{y\ out}$ = (7 W·m^{-2}) / [(1 kg m^{-3}) · (1004 J·kg^{-1}·K^{-1}))]
= **<u>6.97x10^{-3}</u>** **<u>K·m·s^{-1}</u>** .

b) Recall from Chapter 1 that the direction of y is such that y increases toward the north. If we pick the south side as the origin of our coordinate system, then $y_{south\text{-}side}$ = 0 and $y_{north\text{-}side}$ = 10 m. Thus, the kinematic flux gradient (eq. 3.19) is

$$\frac{\Delta F_y}{\Delta y} = \frac{\left[(6.97\times10^{-3}) - (4.98\times10^{-3})\right](\mathrm{K\cdot m\cdot s^{-1}})}{[10-0]\ (\mathrm{m})}$$

= **<u>1.99x10^{-4}</u>** **<u>K·s^{-1}</u>**.

Putting this into eq. (3.21) and then that eq. into eq. (3.17) yields:

$\Delta T/\Delta t$ = **<u>– 1.99x10^{-4}</u>** **<u>K·s^{-1}</u>**.

Check: Physics & units are reasonable.
Exposition: The cube does not get warmer, it gets colder at a rate of about 0.72°C/hour. The reason is that more heat is leaving than entering, which gave a positive value for the flux gradient.

What happens if either of the two fluxes are negative? That means that heat is flowing from north to south. So the sign is critical in helping us determine the movement and convergence of heat.

Sample Application
The cube of air from Fig. 3.5 has T = 12°C along its south side, but smoothly increases in temperature to 15°C on the north side. This 100 km square cube is advecting toward the north at 25 km/hour. What warming rate at a fixed thermometer can be attributed to temperature advection?

Find the Answer
Given: V = 25 km h^{-1}, ΔT = 15 – 12°C = 3°C,
Δy = 100 km
Find: $\Delta T/\Delta t$ = ? °C h^{-1} due to advection

Apply eq. (3.27) in eq. (3.21), and apply that in eq. (3.17); namely, $\Delta T/\Delta t = -V \cdot (\Delta T/\Delta y)$
= – (25 km h^{-1}) · [3°C / 100 km] = **<u>– 0.75 °C h</u>**$^{-1}$.

Check: Physics and units are reasonable
Exposition: Note that the horizontal temperature gradient is positive (T increases as y increases) and V is positive (south wind), yet this causes negative temperature change (cooling). We call this **cold-air advection**, because colder air is blowing in.

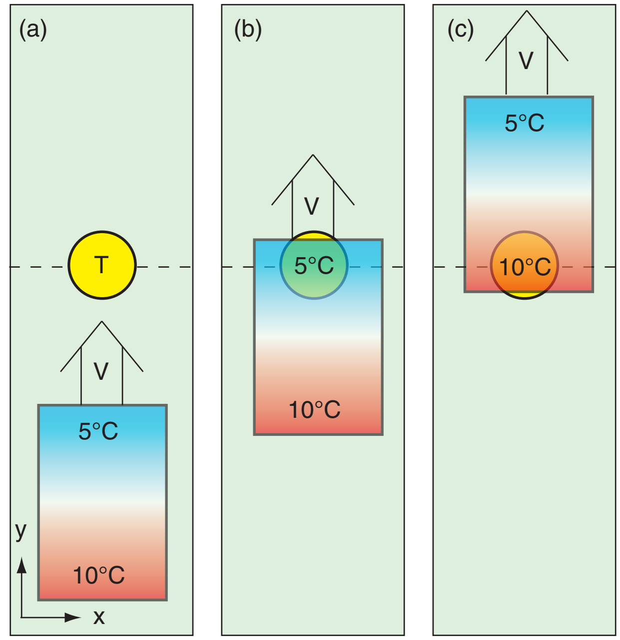


Figure 3.6
Top view of a grass field (green) with a fixed thermometer (T; yellow). Air with temperature gradient is advected north.

Sample Application
Given Fig. 3.6b, except assume that higher in the figure corresponds to higher in the atmosphere (i.e., replace y with z). Suppose that the 5°C air is at a relative altitude that is 500 m higher than that of the 10°C air. If the updraft is 500 m/(10 hours), what is the temperature at the thermometer after 10 hours?

Find the Answer
Given: Δz = 500 m, $T_{initial}$ = 5°C, W = 500 m/(10 h),
$\Delta T/\Delta z$ = (5–10°C)/(500 m) = –0.01°C/m
Find: T_{final} = ? °C after Δt = 10 h.

Looking at Fig. 3.6c, one might guess that the final air temperature should be 10°C. But Fig. 3.6c does not apply to <u>vertical</u> advection, because there is the added process of adiabatic expansion of the rising air.

The air that is initially 10°C in Fig. 3.6b will adiabatically cool 9.8°C/km of rise. Here, it rises only 0.5 km in the 10 h, so it cools 9.8°C/2 = 4.9°C. Its final temperature is 10°C – 4.9°C = **<u>5.1°C</u>**.

Check: Physics & units reasonable.
Exposition: The equations give the same result. Using eq. (3.28, 3.21 & 3.17): $\Delta T/\Delta t = -W \cdot (\Delta T/\Delta z + \Gamma_d)$.

Since we need to apply this over Δt = 10 h, multiply both sides by Δt: $\Delta T = -W \cdot \Delta t \cdot (\Delta T/\Delta z + \Gamma_d)$.
ΔT = – (500m/10h) · (10h) · (–0.01°C/m + 0.0098°C/m)
= –500m · (–0.0002°C/m) = + 0.1°C.
This 0.1°C warming added to the initial temperature of 5°C gives the final temperature = **<u>5.1°C</u>**.

Updrafts also cause heat transport, where buoyant updrafts are called **convection** while non-buoyant updrafts are called **advection**.

To illustrate temperature advection, consider a rectangular air parcel that is colder in the north and warmer in the south (Fig. 3.6). Namely, the temperature gradient $\Delta T/\Delta y$ = negative in this example. A south wind (V = positive) blows the air north toward a thermometer mounted on a stationary weather station. First the cold air reaches the thermometer (Fig. 3.6b). Later, the warm air blows over the thermometer (Fig. 3.6c). So the thermometer experiences warming with time ($\Delta T/\Delta t$ = positive) due to advection. Thus, it is not the advective flux $F_{x\ adv}$, but the gradient of advective flux ($\Delta F_{x\ adv}/\Delta y$) that causes a temperature change.

Although Fig. 3.6 illustrates only horizontal advection in one direction, we need to consider advective effects in all directions, including vertical. For a mean wind with nearly uniform speed:

$$\frac{\Delta F_{x\ adv}}{\Delta x} = \frac{U \cdot (T_{east} - T_{west})}{x_{east} - x_{west}} = U \cdot \frac{\Delta T}{\Delta x} \tag{3.26}$$

$$\frac{\Delta F_{y\ adv}}{\Delta y} = \frac{V \cdot (T_{north} - T_{south})}{y_{north} - y_{south}} = V \cdot \frac{\Delta T}{\Delta y} \tag{3.27}$$

$$\frac{\Delta F_{z\ adv}}{\Delta z} = W \cdot \left[\frac{\Delta T}{\Delta z} + \Gamma_d \right] \tag{3.28}$$

Rising air cools at the dry adiabatic lapse rate of Γ_d = 9.8 °C km^{-1} . Since temperature of a rising air parcel is not conserved, this lapse-rate term must be added to the temperature gradient in the vertical advection equation. This same factor (with no sign changes) works for descending air too.

We can combine eqs. (3.26 - 3.28) with eq. (3.11) to express advection in terms of potential temperature θ:

$$\frac{\Delta F_{x\ adv}}{\Delta x} = U \cdot \frac{\Delta \theta}{\Delta x} \qquad \bullet(3.29)$$

$$\frac{\Delta F_{y\ adv}}{\Delta y} = V \cdot \frac{\Delta \theta}{\Delta y} \qquad \bullet(3.30)$$

$$\frac{\Delta F_{z\ adv}}{\Delta z} = W \cdot \frac{\Delta \theta}{\Delta z} \qquad \bullet(3.31)$$

Molecular Conduction & Surface Fluxes

Molecular heat conduction is caused by microscopic-scale vibrations and movement of air molecules transferring some of their microscopic kinetic energy to adjacent molecules. Conduction is what gets heat from the solid soil surface or liquid ocean surface into the air. It also conducts surface heat further underground. Winds are not needed for conduction.

Vertical heat flux due to molecular conduction is:

$$\mathbb{F}_{z\ cond} = -k \cdot \frac{\Delta T}{\Delta z} \qquad (3.32)$$

where k is the molecular conductivity, which depends on the material doing the conducting. The molecular conductivity of air is k = 2.53x10^{-2} W·m^{-1}·K^{-1} at sea-level under standard conditions.

The molecular conductivity for air is small, and vertical temperature gradients are also small in most of the atmosphere, so a good approximation is:

$$\frac{\Delta F_{x\ cond}}{\Delta x} \approx \frac{\Delta F_{y\ cond}}{\Delta y} \approx \frac{\Delta F_{z\ cond}}{\Delta z} \approx 0 \qquad (3.33)$$

But near the ground, large vertical temperature gradients frequently occur in the bottom several mm of the atmosphere (Fig. 3.7). If you have ever walked barefoot on a black asphalt parking lot or road on a hot summer day, you know that the surface temperatures can be burning hot to the touch (hotter than 50°C) even though the air temperatures at the height of your ankles can be 30°C or cooler. This large temperature gradient compensates for the small molecular conductivity of air, to create important vertical heat fluxes at the surface.

Sample Application

The potential temperature of the air increases 5°C per 100 km distance east. If an east wind of 20 m s^{-1} is blowing, find the advective flux gradient, and the temperature change associated with this advection.

Find the Answer

Given: $\Delta\theta/\Delta x$ = 5°C/100 km = 5x10^{-5} °C m^{-1}
U = – 20 m s^{-1} (an east wind comes <u>from</u> the east)
Find: $\Delta F/\Delta y$ = ? °C s^{-1}, and $\Delta T/\Delta t$ = ? °C s^{-1}

Apply eq. (3.29): $\Delta F/\Delta x$ = (– 20 m s^{-1})·(5x10^{-5} °C m^{-1})
= **<u>–0.001 °C s^{-1}</u>**
Apply eq. (3.17) neglecting all other terms:
$\Delta T/\Delta t$ = – $\Delta F/\Delta x$ = – (–0.001°C s^{-1}) = **<u>+0.001 °C s^{-1}</u>**

Check: Physics reasonable. Sign appropriate, because we expect warming as the warm air is blown toward us from the east in this example.
Exposition: $\Delta T/\Delta t$ = 3.6°C h^{-1}, a rapid warming rate.

Sample Application

Suppose the temperature decreases from 50°C at the Earth's surface to 30°C at 5 mm above ground, as in Fig. 3.7. What is the vertical molecular heat flux?

Find the Answer

Given: ΔT = –20 °C, Δz = 0.005 m
k = 2.53x10^{-2} W·m^{-1}·K^{-1}
Find: $\mathbb{F}_{z\ cond}$ = ? W·m^{-2}

Apply eq. (3.32) :
$\mathbb{F}_{z\ cond}$ = –(2.53x10^{-2} W·m^{-1}·K^{-1}) · [–20°K/(0.005 m)]
= 101.2 W·m^{-2}

Check: Physics and units are reasonable.
Exposition: Although this is a fairly large heat flux into the bottom of the atmosphere, other processes described next (turbulence) can spread this heat over a layer of air roughly 1 km deep.

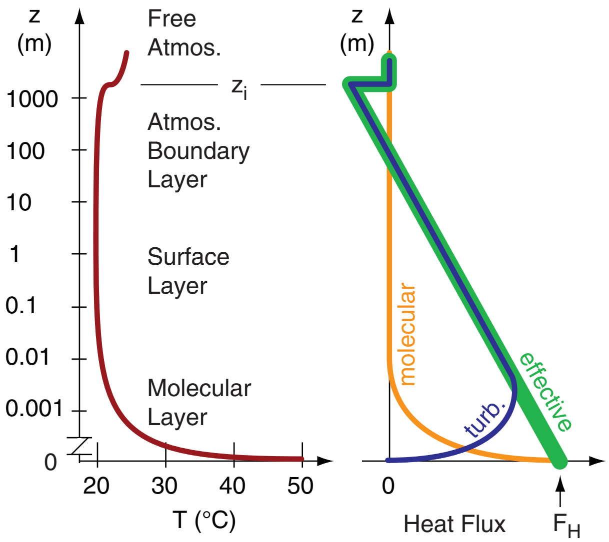


Figure 3.7
Relationship between temperature gradients and heat fluxes.

The bottom layer of the atmosphere that feels the influence of the earth's surface (i.e., the bottom boundary of the atmosphere) is known as the **atmospheric boundary layer** (ABL). This 1 to 2 km thick layer is often turbulent, meaning it has irregular gusts and whorls of motion. Meteorologists have devised an **effective turbulent heat flux** that is the sum of molecular and turbulent heat fluxes (Fig. 3.7), where turbulence is described in the next section. At the surface this effective flux is entirely due to molecular conduction, and above about 5 mm altitude the effective flux is mostly due to turbulence.

Instead of using eq. (3.32) to calculate molecular surface heat fluxes, most meteorologists approximate the effective surface turbulent heat flux, F_H, using what are called **bulk-transfer relationships**.

For windy conditions where most of the turbulence is caused by wind shear (change of wind speed or direction with altitude), you can use:

$$F_H = C_H \cdot M \cdot (\theta_{sfc} - \theta_{air}) \qquad \bullet(3.34)$$

or

$$F_H \cong C_H \cdot M \cdot (T_{sfc} - T_{air}) \qquad \bullet(3.35)$$

where (T_{sfc}, θ_{sfc}) are the temperature and potential temperature at the top few molecules (the skin) of the earth's surface, (T_{air} , θ_{air}) are the corresponding values in the air at 2 m above ground, and the wind speed at altitude 10 m is M. The empirical coefficient C_H is called the **bulk heat-transfer coefficient**. It is dimensionless, and varies from about $2\text{x}10^{-3}$ over smooth lakes or salt flats to about $2\text{x}10^{-2}$ for a rougher surface such as a forest. F_H is a kinematic flux.

For calm sunny conditions, turbulence is created by thermals of warm air rising due to their buoyancy. The resulting convective circulations cause so much stirring of the air that the ABL becomes a well **mixed layer** (ML). For this situation, you can use:

$$F_H = b_H \cdot w_B \cdot (\theta_{sfc} - \theta_{ML}) \qquad \bullet(3.36)$$

or

$$F_H = a_H \cdot w_* \cdot (\theta_{sfc} - \theta_{ML}) \qquad \bullet(3.37)$$

where a_H = 0.0063, is a dimensionless empirical **mixed-layer transport coefficient**, and b_H = $5\text{x}10^{-4}$ is called a **convective transport coefficient**. θ_{ML} is the mid-mixed-layer potential temperature (at height 500 m for a ML that is 1 km thick).

The w_B factor in eq. (3.36) is called the **buoyancy velocity scale** (m s^{-1}):

$$w_B = \left[\frac{|g| \cdot z_i}{T_{v\,ML}} \cdot (\theta_{v\,sfc} - \theta_{v\,ML}) \right]^{1/2} \qquad \bullet(3.38)$$

for a ML of depth z_i , and using gravitational acceleration $|g|$ = 9.8 m s^{-2}. ($\theta_{v\,sfc}$, $\theta_{v\,ML}$) are virtual

Sample Application

The wind is blowing at 10 m s^{-1} at height 10 m AGL. The 2 m air temperature is 15°C but the surface skin temperature is 30°C. What is the effective surface kinematic heat flux? Assume a surface of medium roughness having C_H = 0.01 .

Find the Answer

Given: C_H = 0.01 , M = 10 m s^{-1} at z = 10 m
T_{sfc} = 30°C , T_{air} = 15°C at z = 2 m
Find: F_H = ? K·m·s^{-1}

Apply eq. (3.35):
F_H = (1x10^{-2})·(10 m s^{-1})·(30–15°C) = **<u>1.5 °C·m·s</u>**$^{-1}$

Check: Physics and units are reasonable.
Exposition: Recall that the relationship between dynamic and kinematic heat flux is $\mathbb{F}_H = \rho \cdot C_p \cdot F_H$. Thus, the dynamic heat flux is $\mathbb{F}_H \approx$ (1.2 kg m^{-3})·(1004 J kg^{-1} K^{-1})·(**<u>1.5 K·m·s</u>**$^{-1}$) = 1807. W m^{-2}. This is an exceptionally large surface heat flux — larger than the average solar irradiance of 1366 W·m^{-2}. But such a heat flux could occur where cool air is advecting over a very hot surface.

potential temperatures of the surface skin and in the mid-mixed layer, and T_v is the absolute virtual temperature (Kelvins) in the mid mixed layer. Typical updraft speeds in thermals are of order $0.02 \cdot w_B$. To good approximation, the denominator in eq. (3.38) can be approximated by $\theta_{v\,ML}$ (also in units of K).

Another convective velocity scale w_* is called the **Deardorff velocity**:

$$w_* = \left[\frac{|g| \cdot z_i}{T_v} \cdot F_{Hsfc} \right]^{1/3} \qquad \bullet(3.39)$$

for a surface kinematic heat flux of $F_{Hsfc} = F_H$. Often the Deardorff velocity is of order 1 to 2 m·s^{-1}, and the relationship between the two velocity scales is $w_* \approx 0.08 \cdot w_B$.

Later in the chapter, in the section on the Bowen ratio, you will see other formulas you can use to estimate F_H. Bulk transfer relationships can be used for other scalar fluxes at the surface including the moisture flux. For this case, replace temperature or potential-temperature differences with humidity differences between the surface skin and the mixed layer.

Sample Application

What is the value of F_H on a sunny day with no winds? Assume z_i = 3 km, no clouds, dry air, θ_{ML} = 290 K, and θ_{sfc} = 320 K.

Find the Answer

Given: θ_{sfc} = 320 K, θ_{ML} = 290 K, z_i = 3 km,
Find: $F_{z\ eff.sfc.}$ = ? K·m·s^{-1}

If the air is dry, then: $\theta_v = \theta$ (from eq. 3.13).
Apply eqs. (3.38) and (3.36):

$$w_B = \left[\frac{|9.8\text{m} \cdot \text{s}^{-2}| \cdot 3000\text{m}}{290\text{K}} \cdot (320\text{K} - 290\text{K}) \right]^{1/2}$$

$= (3041\ \text{m}^2\text{·s}^{-2})^{1/2} = 55.1$ m·s^{-1}

$F_H = (5\text{x}10^{-4}) \cdot (55.1\ \text{m·s}^{-1}) \cdot (320\ \text{K} - 290\ \text{K})$
$= \underline{\mathbf{0.83\ K·m·s^{-1}}}$

Check: Physics & units are reasonable.
Exposition: Notice how the temperature difference between the surface and the air enters both in the eq. for w_B and again for F_H. Thus, greater differences drive greater surface heat flux.

Atmospheric Turbulence

Superimposed on the average wind are somewhat-random faster and slower gusts. This **turbulence** is caused by **eddies** in the air that are constantly being created, changing, and dying. They exist as a superposition of many different size swirls (3 mm to 3 km). One eddy might move a cold blob of air out of any fixed Eulerian region, but another eddy might move air that is warmer into that same region. Although we don't try to forecast the heat transported by each individual eddy (an overwhelming task), we instead try to estimate the net heat flux caused by all the eddies. Namely, we resort to a statistical description of the effects of turbulence.

Turbulence in the air is analogous to turbulence in your teacup when you stir it. Namely, turbulence tends to blend all the ingredients into a uniform homogenous mixture. In the atmosphere, the mixing homogenizes individual variables such as potential temperature, humidity, and momentum (wind). The mixing rate depends on the strength of the turbulence, which can vary in space and time. We will focus on mixing of heat (potential temperature) here.

Fair Weather (no thunderstorms)

In a turbulent atmospheric boundary layer (ABL), daytime turbulence caused by convective thermals can transport heat from the sun-warmed Earth's surface and can distribute it more-or-less evenly through the ABL depth. The resulting turbulent heat fluxes decrease linearly with height was shown

Sample Application

Given an effective surface kinematic heat flux of 0.67 K·m·s^{-1}, find the Deardorff velocity for a dry, 1 km thick boundary layer of temperature 25°C

Find the Answer

Given: F_H = 0.67 K·m·s^{-1}, z_i = 1 km = 1000 m,
$T_v = T$ (because dry) = 25°C = 298 K.
Find: w_* = ? m s^{-1}

Apply eq. (3.39):
$w_* = [(9.8\ \text{m·s}^{-2}) \cdot (1000\text{m}) \cdot (0.67\ \text{K·m·s}^{-1})/(298\text{K})]^{1/3}$
$= \underline{2.8}$ m s^{-1}

Check: Physics & units are reasonable.
Exposition: Over land on hot sunny days, warm buoyant thermals often rise with a speed of the same order of magnitude as the Deardorff velocity.

Sample Application
Given the Sample Application at the top of the previous page, what is the value for vertical flux divergence for this calm, sunny ABL?

Find the Answer
Given: $F_H = 0.83$ K·m·s^{-1}, $z_i = 3000$ m
Find: $\Delta F_{z\ turb}/\Delta z = ?$ (K s^{-1})

Apply eq. (3.41):

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{-1.2 \cdot F_H}{z_i}$$

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{-1.2 \cdot (0.83\ \mathrm{K \cdot m/s})}{3000\mathrm{m}}$$

$$= \underline{\mathbf{-0.000332\ K{\cdot}s^{-1}}}$$

Check: Physics and units are reasonable.
Exposition: Recall from eq. (3.17) that a negative vertical gradient gives a positive warming with time — appropriate for a sunny day. The amount of warming is about 1.2°C/h. You might experience this warming rate over 10 hours on a hot sunny day.

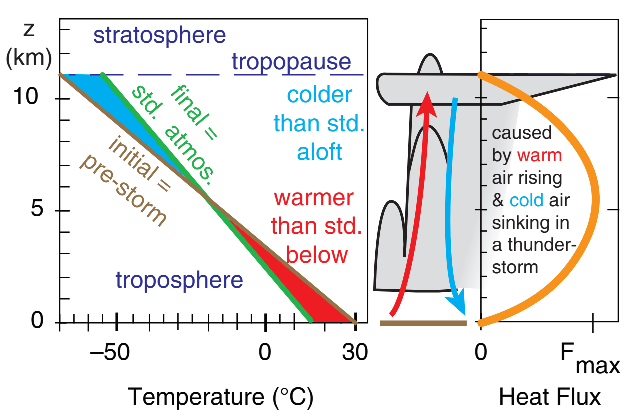


Figure 3.8
If a deep layer of cold air lies above a deep layer of warm air, such as in a pre-thunderstorm environment, then the air is statically unstable. This instability creates a thunderstorm, which not only causes overturning of tropospheric air, but also mixes the air. The final result can differ from storm to storm, but here we assume that the storm dies when the atmosphere has been mixed to the standard (std.)-atmosphere lapse rate of 6.5°C/km.

by the thick green line in Fig. 3.7. This line has a value at the bottom of the ABL as given by the effective surface flux ($F_{z\ bottom} = F_H$), and at the top has a value of ($F_{z\ top} \approx -0.2 \cdot F_H$) on less windy days. Thus, the flux-divergence term for turbulence (during sunny fair weather, within domain $0 < z < z_i$) is:

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{F_{z\ top} - F_{z\ bottom}}{z_i} \tag{3.40}$$

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{-1.2 \cdot F_H}{z_i} \tag{3.41}$$

for an ABL depth z_i of 0.2 to 3 km.

When no storm clouds are present, the air at $z > z_i$ is often not turbulent during daytime:

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx 0 \quad \text{above ABL top; for fair weather} \tag{3.42}$$

During clear nights of fair weather, turbulence can be very small over most of the lower 3 km of troposphere, except in the very lowest 100 m where wind shears can still create occasional turbulence.

Stormy Weather

Sometimes horizontal advection can move warm air under colder air. This makes the atmosphere statically unstable, allowing thunderstorms to form. These storms try to undo the instability by overturning the air — allowing the warm air to rise and cold air to sink. But the result is so violently turbulent that much mixing also takes place. The end result can sometimes be an atmosphere with a vertical gradient close to that of the standard atmosphere, as was discussed in Chapter 1. Namely, the atmosphere experiences **moist convective adjustment**, to adjust the initial less-stable lapse rate to one that is more stable.

The **standard atmospheric lapse rate** ($\Gamma_{sa} = -\Delta T/\Delta z$) is 6.5 K km^{-1}. Suppose that the initial lapse rate before the thunderstorm forms is Γ_{ps} ($= -\Delta T/\Delta z$). The amount of heat flux that is required to move the warm air up and cold air down during a storm lifetime of Δt (≈1 h) is:

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{z_T}{\Delta t} \cdot \left[\Gamma_{ps} - \Gamma_{sa}\right] \cdot \left(\frac{1}{2} - \frac{z}{z_T}\right) \tag{3.43}$$

where the troposphere depth is z_T (≈11 km). An initially unstable environment gives a positive value for the factor enclosed by square brackets.

Because thunderstorm motions do not penetrate below ground, and assuming no flux above the top of the storm, then the vertical turbulent heat flux must be zero at both the top and bottom of the troposphere, as was sketched in Fig. 3.8. The parabolic shape of the heat-flux curve has a maximum value of:

$$F_{\max} = z_T{}^2 \cdot \left[\Gamma_{ps} - \Gamma_{sa} \right] / (8 \cdot \Delta t) \qquad (3.44)$$

The thunderstorm also affects the heat budget via warming at all thunderstorm altitudes where condensation exceeds evaporation. Cooling at the thunderstorm top can be caused by IR radiation from the anvil cloud up into space. These heating and cooling effects should be added to the heat redistribution (heat moved from the bottom to the top of the storm) caused by turbulence.

So far, we focused on vertical flux gradients and the associated heating or cooling. Turbulence can also mix air horizontally, but the net horizontal heat transport is often negligibly small for both fair and stormy weather, because background temperature changes so gradually with distance in the horizontal. Thus, at all locations, a reasonable approximation is:

$$\frac{\Delta F_{x\ turb}}{\Delta x} \approx \frac{\Delta F_{y\ turb}}{\Delta y} \approx 0 \qquad (3.45)$$

Also, at locations with no turbulence there cannot be turbulent heat transport.

Solar and IR Radiation

Divide this topic into short-wave (solar) and long-wave (IR) radiation. Clear air is mostly transparent to solar radiation. Thus, the amount of short-wave radiation entering an air volume nearly equals the amount leaving. No flux gradient means that, to good approximation, you can neglect the direct solar heating of the air. However, sunlight is absorbed at the Earth's surface, which causes surface heat fluxes as already discussed. Sunlight is also absorbed in clouds or smoke, which can cause warming.

IR radiation is more complex, because air strongly absorbs a large portion of IR radiation flowing into a fixed volume, and re-radiates IR radiation outward in all directions. Radiation emission is related to T^4, according to the Stefan-Boltzmann law. In horizontal directions having weak temperature gradients, radiative flux divergence is negligibly small:

$$\frac{\Delta F_{x\ rad}}{\Delta x} \approx \frac{\Delta F_{y\ rad}}{\Delta y} \approx 0 \qquad (3.46)$$

But in the vertical, recall that temperature decreases with increasing altitude. Hence, more radiation would be lost upward from warmer air in the lower troposphere than is returned downward from the colder air aloft, which causes net cooling.

$$\frac{\Delta F_{z\ rad}}{\Delta z} \approx 0.1 \text{ to } 0.2 \text{ (K/h)} \qquad (3.47)$$

Sample Application

Suppose a pre-storm environment has a lapse rate of 9 °C km^{-1}. a) What is the maximum value of vertical heat flux near the middle of the troposphere during a storm lifetime? b) Calculate the vertical flux gradient at 1 km altitude due to the storm.

Find the Answer

Given: Γ_{ps} = 9 K km^{-1},
Find: (a) F_{max} = ? K·m s^{-1} (b) $\Delta F_{z\ turb}/\Delta z$ = ? (K s^{-1})
Assume: Γ_{sa} = 6.5 K km^{-1}, lifetime = Δt = 1 h = 3600 s, z_T = 11 km,

(a) Apply eq. (3.44):
F_{max} =(11,000m)·(11km)·[(9–6.5)(K km^{-1})]/[8·(3600s)]
= **<u>10.5 K m s</u>**$^{-1}$

(b) Apply eq. (3.43):

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{z_T}{\Delta t} \cdot \left[\Gamma_{ps} - \Gamma_{sa} \right] \cdot \left(\frac{1}{2} - \frac{z}{z_T} \right)$$

$$\frac{\Delta F_{z\ turb}}{\Delta z} \approx \frac{11\text{km}}{3600\text{s}} \cdot \left[(9 - 6.5) \frac{\text{K}}{\text{km}} \right] \cdot \left(\frac{1}{2} - \frac{1\text{km}}{11\text{km}} \right)$$

$\Delta F_{z\ turb}/\Delta z$ = **<u>0.0031 K s</u>**$^{-1}$

Check: Physics and units are reasonable.
Exposition: The magnitude of the max heat flux due to thunderstorms is much greater than the heat flux due to thermals in fair weather. Thunderstorms move large amounts of heat upward in the troposphere.

Based on Fig. 3.8, we would anticipate that storm turbulence should cool the bottom half of the stormy atmosphere. Indeed, the minus sign in eq. (3.17) combined with the positive sign of answer (b) above gives the expected cooling, not heating.

A SCIENTIFIC PERSPECTIVE • Expert vs. Novice

Expert scientists and engineers often solve problems, organize knowledge, and perceive structure differently than students and other novices.

Problem Solving	Novice	Expert
... is ...	a recall task	a process
... begins with ...	hunt for "the equation"	qualitative analysis
... uses classification based on ...	surface features	deep structure
...tools include ...	"the equation"	graphs, limits, diagrams, conservation laws, units, ...

Organizing Knowledge	Novice	Expert
Memory recall is ...	piecemeal	effortless retrieval of relevant collected facts
Reasoning by ...	jumping to hasty, unfounded conclusions	fast mental scan through a chain of possibilities
Conflicting data, ideas & conclusions are...	not recognized	recognized, pointing to need for more info
Related ideas are...	memorized as separate facts	integrated into a coherent big picture

Structure Perception	Novice	Expert
Cues about the structure are ...	missed	recognized and trigger new lines of thought
Disparate instances are...	separately classified based on surface features	recognized as having the same underlying structure
Tasks are performed...	before thinking about the organization	after data is organized to find structure
Theories that don't agree with data ...	are used without revision	identify ideas ripe for revision

(Paraphrased from Wendy Adams, Carl Wieman, Dan Schwartz, & Kathleen Harper.)

Internal Sources such as Latent Heat

Suppose $\Delta m_{condensing}$ kilograms of water vapor inside the storm condenses into liquid droplets and does not re-evaporate. It would release $L_v \cdot \Delta m_{condensing}$ Joules of latent heat. If this heating is spread vertically through the whole thunderstorm (a gross simplification) of air mass m_{air}, then the heating is:

$$\frac{\Delta So}{C_p \cdot \Delta t} = \frac{L_v}{C_p} \cdot \frac{\Delta m_{condensing}}{m_{air} \cdot \Delta t} \tag{3.48}$$

Because this warming does not require a heat flux across the storm boundaries, we define it as a "source term" that is internal to the thunderstorm. An opposite case of existing suspended cloud droplets that evaporate would yield the same equation, but with opposite sign as indicates net cooling.

In a real thunderstorm, some of the water vapor that initially condensed into cloud droplets can later evaporate. But any precipitation reaching the ground represents condensation that did not re-evaporate. Hence, we can use **rainfall rate** (RR) to estimate the internal latent heating rate:

$$\frac{\Delta So}{C_p \cdot \Delta t} = \frac{L_v}{C_p} \cdot \frac{\rho_{liq}}{\rho_{air}} \cdot \frac{RR}{z_{Trop}} \tag{3.49}$$

where the storm is assumed to fill a column of tropospheric air of depth z_{Trop}, liquid-water density is ρ_{liq} = 1000 kg·m^{-3}, latent-heat to specific heat ratio is L_v/C_p = 2500 K·kg$_{air}$·kg$_{liq}^{-1}$, and column-averaged air density is ρ_{air} = 0.689 kg·m^{-3} for z_{Trop} = 11 km.

Combining some of the values in eq. (3.49) gives:

$$\frac{\Delta So}{C_p \cdot \Delta t} = a \cdot RR \tag{3.50}$$

where a = 0.33 K (mm of rain)$^{-1}$, and RR has units [(mm of rain) s^{-1}]. Divide by 3600 for RR in mm h^{-1}.

Simplified Eulerian Net Heat Budget

You can insert the flux-gradient approximations from the previous subsections into eqs. (3.17 or 3.18) for the first law of thermo. Although the result looks complicated, you can simplify it by assuming the following are negligible within a fixed air volume: (1) vertical temperature advection by the mean wind; (2) horizontal turbulent heat transport; (3) molecular conduction; (4) short-wave heating of the air; (5) constant IR cooling.

You then get the following approximate Eulerian net heat-budget equation:

$$\left.\frac{\Delta T}{\Delta t}\right|_{x,y,z} = -\underbrace{\left[U\cdot\frac{\Delta T}{\Delta x} + V\cdot\frac{\Delta T}{\Delta y}\right]}_{advection} \underbrace{-\,0.1\frac{\text{K}}{\text{h}}}_{radiation}$$

$$\underbrace{-\frac{\Delta F_{z\ turb}(\theta)}{\Delta z}}_{turbulence} \underbrace{+\frac{L_v}{C_p}\cdot\frac{\Delta m_{condensing}}{m_{air}\cdot\Delta t}}_{latent\ heat} \qquad \bullet(3.51)$$

Later in this book you will see similar budget equations for other variables such as water vapor or momentum. In the turbulence term above, the (θ) indicates that this term is for heat flux divergence. Any of the terms on the right-hand side can be zero if the process it represents (advection, radiation, turbulence, condensation) is not active.

The net heat budget is important because you can use it to forecast air temperature at any altitude. Or, if you already know how the air temperature changes with time, you can use the net heat budget to see which processes are most important in causing this change.

The net heat budget applies to a volume of air having a finite mass. For the special case of the Earth's surface (infinitesimally thin; having no mass), you can write a simplified heat budget, as described next.

HEAT BUDGET AT EARTH'S SURFACE

So far, you examined the heat budget for a volume of air, where the volume was fixed (Eulerian) or moving (Lagrangian). Net imbalances of heat flux caused warming or cooling of air in the volume.

But what happens at the Earth's surface, which is infinitesimally thin and thus has zero volume? No heat can be stored in this layer. Hence, the sum of all incoming and outgoing heat fluxes must exactly balance. The net flux at the surface must be zero.

Surface Heat-flux Balance

Recall that fluxes are defined to be positive for heat moving upward, regardless of whether these fluxes are in the soil or the atmosphere.

Relevant fluxes at the surface include:

$\mathbb{F}^*$ = net radiation between sfc. & atmos. (Chapter 2)

$\mathbb{F}_H$ = effective surface turbulent heat flux (the **sensible <u>H</u>eat flux**)

$\mathbb{F}_E$ = effective surface **latent heat flux** caused by <u>E</u>vaporation or condensation (dew formation)

Sample Application

Suppose a thunderstorm rains at rate 4 mm h^{-1}. What is the average heating rate in the troposphere?

Find the Answer

Given: RR = 4 mm·h^{-1}.

Find: $\Delta S_o/(C_p\cdot\Delta t)$ = ? K·h^{-1}

Apply eq. (3.50): $\Delta S_o/(C_p\cdot\Delta t)$ = 0.33 (K mm^{-1})·(4 mm h^{-1}) = <u>**1.32 K·h**</u>$^{-1}$

Check: Physics and units are reasonable.

Exposition: For fixed Eulerian volumes losing liquid water as precipitation, this heating rate is significant.

Sample Application

For a fixed Eulerian volume, what temperature increase occurs in 2 h if $\Delta m_{cond}/m_{air}$ = 1 g$_{water}$ kg$_{air}^{-1}$, $F_{H\ sfc}$ = 0.25 K·m·s^{-1} into a 1 km thick boundary layer, U = 0, V = 10 m s^{-1}, and $\Delta T/\Delta y$ = –2°C/100 km. Hint, approximate $L_v/C_p \approx$ 2.5 K (g$_{water}$ kg$_{air}^{-1}$)$^{-1}$.

Find the Answer

Given: (see above)

Find: ΔT = ? °C over a 2 hour period

For each term in eq. (3.51), multiply by Δt:

$$\text{Lat.Heat\_Source}\cdot\Delta t = \left(2.5\frac{\text{K}\cdot\text{kg}_{\text{air}}}{\text{g}_{\text{water}}}\right)\cdot\left(1\frac{\text{g}_{\text{water}}}{\text{kg}_{\text{air}}}\right) = \underline{\mathbf{+2.5°C}}$$

$$\text{Turb}\cdot\Delta t = -\frac{-1.2\cdot(0.25\text{K}\cdot\text{m/s})}{1000\text{m}}\cdot(7200\text{s}) = \underline{\mathbf{+2.16°C}}$$

$$\text{Adv}\cdot\Delta t = -\left[(10\text{m/s})\cdot\left(\frac{-2°\text{C}}{100000\text{m}}\right)\right]\cdot(7200\text{s}) = \underline{\mathbf{+1.44°C}}$$

$$\text{Rad}\cdot\Delta t = \left(-0.1\frac{\text{K}}{\text{h}}\right)\cdot(2\text{h}) = \underline{\mathbf{-0.2°C}}$$

Combining all the terms gives:

ΔT = (Latent + Turb + Adv + Rad)

= (2.5 + 2.16 + 1.44 – 0.2)°C = <u>**5.9 °C**</u> over 2 hours.

Check: Physics and units are reasonable.

Exposition: For this contrived example, all the terms (except advection in the x direction) were important. Many of these terms can be estimated by looking at weather maps. For example, cloudy conditions might shade the sun during daytime and reduce the surface heat flux. These same clouds can trap IR radiation, causing the net radiative loss to be near zero below cloud base. But if there are no clouds (i.e., no condensation) and no falling precipitation that evaporates on the way down, then the latent-heating term would be zero.

So there is no fixed answer for the Eulerian heat budget — it varies as the weather varies.

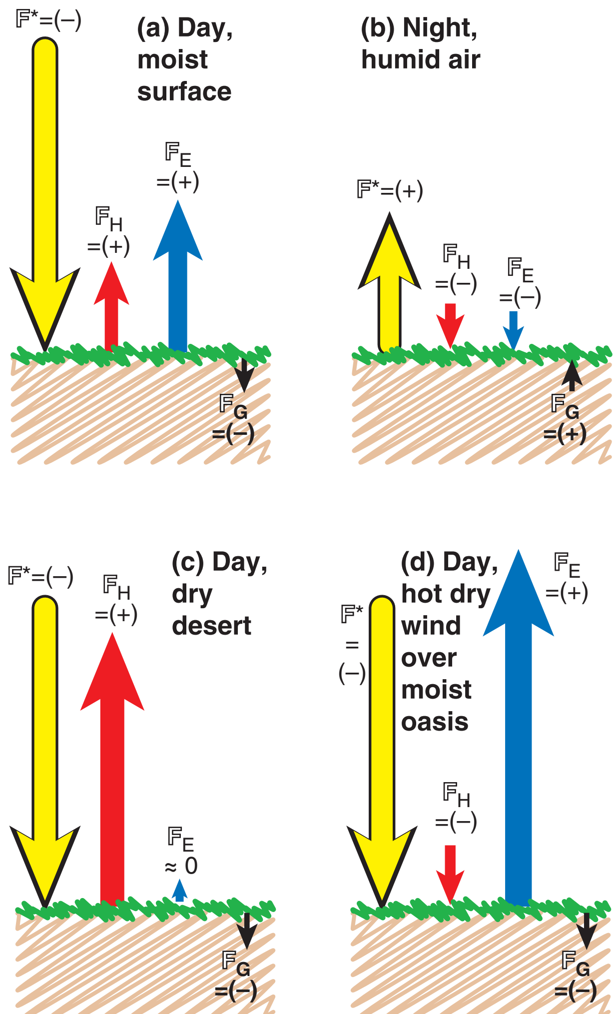


Figure 3.9
Illustration of signs and magnitudes of surface fluxes for various conditions. $\mathbb{F}$ = net radiative flux, $\mathbb{F}_H$ = sensible heat flux, $\mathbb{F}_E$ = latent heat flux, $\mathbb{F}_G$ = conductive heat flux into the ground.*

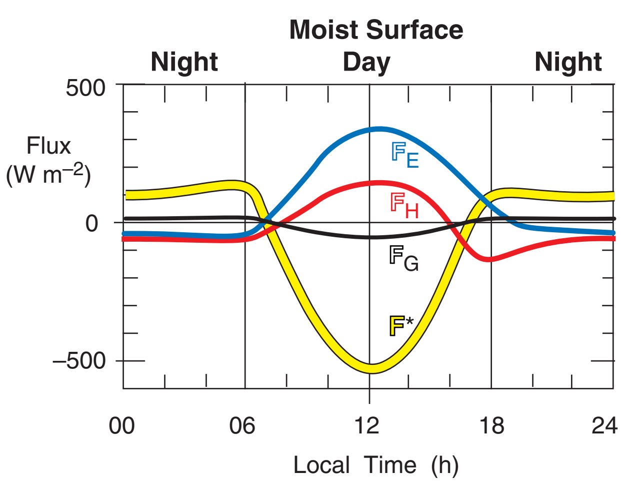


Figure 3.10
Daily variation of terms in the surface heat balance for a moist surface with humid air. Day and night correspond to (a) & (b) of the previous figure.

$\mathbb{F}_G$ = **molecular heat conduction** to/from deeper below the surface (e.g., Ground, oceans).

The surface balance for dynamic heat-fluxes (in units of W m^{-2}) is:

$$0 = \mathbb{F}^* + \mathbb{F}_H + \mathbb{F}_E - \mathbb{F}_G \qquad \bullet(3.52)$$

If you divide by $\rho_{air}{\cdot}C_p$ to get the balance in kinematic form (in units of K m s^{-1}), the result is:

$$0 = F^* + F_H + F_E - F_G \qquad \bullet(3.53)$$

The first 3 terms on the right are fluxes between the surface and the air above. The last term is between the surface and the Earth below (hence the – sign).

Examples of these fluxes and their signs are sketched in Fig. 3.9 for different surfaces and for day vs. night. For an irrigated lawn or crop, the typical **diurnal cycle** (daily evolution) of surface fluxes is sketched in Fig. 3.10. Essentially, net radiation F^* is an external forcing that drives the other fluxes.

A crude, first-order approximation for dynamic heat flux down into the soil is

$$\mathbb{F}_G \approx X \cdot \mathbb{F}^* \qquad (3.54)$$

with a corresponding kinematic heat flux of:

$$F_G \approx X \cdot F^* \qquad (3.55)$$

with factor X = (0.1, 0.5) for (daytime, nighttime).

There are different options for estimating the other terms in eqs. (3.52 or 3.53). For effective surface sensible heat flux you can use the bulk-transfer relationships already discussed (eqs. 3.34 to 3.37). For latent heat flux at the surface, similar bulk-transfer equations will be given in the Water-Vapor chapter. Another option for estimating latent and sensible heat fluxes at the surface is to utilize the Bowen-ratio, described next.

The Bowen Ratio

Define a **Bowen ratio**, B, as surface sensible-heat flux divided by surface latent-heat flux:

$$B = \frac{\mathbb{F}_H}{\mathbb{F}_E} = \frac{F_H}{F_E} \qquad (3.56)$$

Typical values are: 10 for arid locations, 5 for semi-arid locations, 0.5 over drier savanna, 0.2 over moist farmland, and 0.1 over oceans and lakes.

In the atmospheric **surface layer** (the bottom 10 to 25 m of the troposphere), surface effective sensible heat flux depends on $\Delta\theta/\Delta z$ — the potential-temperature gradient. Namely, $F_H = -K_H{\cdot}\Delta\theta/\Delta z$, where K_H is an **eddy diffusivity** for heat (see the Atmos. Boundary Layer chapter), z is height above ground, and the negative sign says that the heat flux flows down the local gradient (from hot toward cold air).

An analogous expression for effective surface moisture flux is $F_E = -K_E \cdot \Delta r/\Delta z$, where mixing ratio r is defined in the next chapter as mass of water vapor contained in each kg of dry air. If you approximate the eddy diffusivity for moisture, K_E, as equaling that for heat and if the vertical gradients are measured across the same air layer Δz, then you can write the Bowen ratio as:

$$B = \gamma \cdot \frac{\Delta\theta}{\Delta r} \tag{3.57}$$

for a **psychrometric constant** defined as $\gamma = C_p/L_v = 0.4\ (g_{water\ vapor}/kg_{air}) \cdot K^{-1}$.

Eq. (3.57) is appealing to use in field work because the difficult-to-measure fluxes have been replaced by easy-to-measure mean-temperature and humidity differences. Namely, if you erect a short tower in the surface layer and deploy **thermometers** at two different heights and mount **hygrometers** (for measuring humidity) at the same two heights (Fig. 3.11), then you can compute B. Don't forget to convert the temperature difference to potential-temperature difference: $\Delta\theta = T_2 - T_1 + (0.0098\ K\ m^{-1}) \cdot (z_2 - z_1)$.

With a bit of algebra you can combine eqs. (3.57, 3.56, 3.54, and 3.52) to yield effective surface sensible heat flux in dynamic units (W m^{-2}) as a function of net radiation:

$$\mathbb{F}_H = \frac{-0.9 \cdot \mathbb{F}^*}{\dfrac{\Delta r}{\gamma \cdot \Delta\theta} + 1} \tag{3.58}$$

or kinematic units (K m s^{-1}):

$$F_H = \frac{-0.9 \cdot F^*}{\dfrac{\Delta r}{\gamma \cdot \Delta\theta} + 1} \tag{3.59}$$

A bit more algebra yields the latent heat flux (W m^{-2}) caused by movement of water vapor to or from the surface:

$$\mathbb{F}_E = \frac{-0.9 \cdot \mathbb{F}^*}{\dfrac{\gamma \cdot \Delta\theta}{\Delta r} + 1} \tag{3.60}$$

or in kinematic units (K m s^{-1}):

$$F_E = \frac{-0.9 \cdot F^*}{\dfrac{\gamma \cdot \Delta\theta}{\Delta r} + 1} \tag{3.61}$$

The next chapter shows how to convert latent heat-flux values into waver-vapor fluxes.

If you have found sensible heat flux from eqs. (3.58 or 3.59), then the latent heat flux is easily found from:

$$\mathbb{F}_E = -0.9 \cdot \mathbb{F}^* - \mathbb{F}_H \tag{3.62}$$

or

$$F_E = -0.9 \cdot F^* - F_H \tag{3.63}$$

Sample Application

If the net radiation is –800 W·m^{-2} at the surface over a desert, then find sensible, latent, and ground fluxes.

Find the Answer

Given: $\mathbb{F}^* = -800\ W \cdot m^{-2}$
$B = 10$ for arid regions
Find: $\mathbb{F}_H$, $\mathbb{F}_E$ and $\mathbb{F}_G$ = ? W·m^{-2}

Because negative $\mathbb{F}^*$ implies daytime, use $X = 0.1$ in eq. (3.54): $\mathbb{F}_G = 0.1 \cdot \mathbb{F}^* = 0.1 \cdot (-800\ W \cdot m^{-2}) =$ **–80 W·m^{-2}**

Eqs. (3.52 & 3.56) can be manipulated to give:
$\mathbb{F}_E = (\mathbb{F}_G - \mathbb{F}^*) / (1 + B)$
$\mathbb{F}_H = B \cdot (\mathbb{F}_G - \mathbb{F}^*) / (1 + B)$
Thus,
$\mathbb{F}_E = (-80 + 800\ W \cdot m^{-2}) / (1 + 10)$
= **65.5 W·m^{-2}**

$\mathbb{F}_H = 10 \cdot (-80 + 800\ W \cdot m^{-2}) / (1 + 10)$
= **654.5 W·m^{-2}**

Check: Physics and units are reasonable. Also, we should confirm that the result gives a balanced energy budget. So apply eq. (3.52):

$0 = \mathbb{F}^* + \mathbb{F}_H + \mathbb{F}_E - \mathbb{F}_G$???
$0 = -800 + 654.5 + 65.5 + 80\ W \cdot m^{-2}$ True.

Exposition: Although typical values for the Bowen ratio were given on the previous page, the actual value for any given type of surface depends on so many factors that it is virtually useless when trying to use the Bowen ratio method to predict surface fluxes. However, the field-measurement approach shown in the figure below and in eqs. (3.58 - 3.63) does not require an a-priori Bowen ratio estimate. Hence, this field approach is quite accurate for measuring surface fluxes, except near sunrise and sunset.

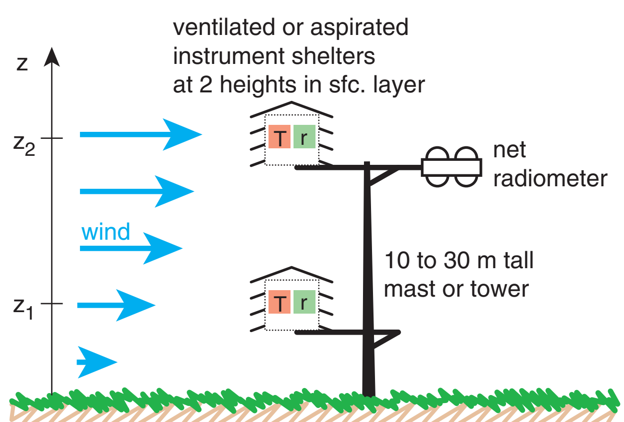


Figure 3.11
Field set-up for getting surface effective sensible and latent heat fluxes using the Bowen-ratio method. (T, r) are (thermometers, hygrometers) that are shielded from sunlight using ventilated instrument shelters. The net radiometer measures $\mathbb{F}^*$.

Sample Application

A Bowen-ratio field site observes the following:

index	z (m)	T (°C)	r (g_{vapor}/kg_{air})
2	15	16	7
1	1	20	12

with, $\mathbb{F}^* = -650\ W \cdot m^{-2}$. Find all surface fluxes.

Find the Answer

Given: info above.
Find: surface dynamic fluxes (W·m^{-2}) $\mathbb{F}_E$, $\mathbb{F}_H$, $\mathbb{F}_G$ = ?

First step is to find $\Delta\theta$:

$$\Delta\theta = T_2 - T_1 + (0.0098\ K\ m^{-1})\cdot(z_2 - z_1)$$
$$= 16\ K - 20\ K + (0.0098\ K\ m^{-1})\cdot(15m - 1m)$$
$$= -4\ K + 0.137\ K = -3.86\ K$$

Apply eq. (3.58)

$$\mathbb{F}_H = \frac{-0.9\cdot(-650 W\cdot m^{-2})}{\dfrac{(-5 g_{vap}/kg_{air})}{[0.4(g_{vap}/kg_{air})\cdot K^{-1}]\cdot(-3.86K)} + 1}$$

$\mathbb{F}_H$ = **<u>138 W·m</u>$^{-2}$**

Next, apply eq. (3.62):

$$\mathbb{F}_E = -0.9\cdot\mathbb{F}^* - \mathbb{F}_H$$
$$= -0.9\cdot(-650\ W\cdot m^{-2}) - 138.\ W\cdot m^{-2}$$
$$= \underline{\mathbf{447\ \ W\cdot m^{-2}}}$$

Finally, apply eq. (3.54): $\mathbb{F}_G = 0.1\cdot\mathbb{F}^*$ = **<u>–65 W m</u>$^{-2}$** .

Check: Physics & units are reasonable. Also, all the flux terms sum to zero, verifying the balance.

Exposition: The resulting Bowen ratio is B = 138/447 = 0.31, which suggests the site is irrigated farmland.

Sample Application

If wind speed is 30 km h^{-1} and actual air temperature is –25°C, find the wind-chill index.

Find the Answer

Given: M = 30 km h^{-1}, T_{air} = –25°C,
Find: $T_{wind\ chill}$ = ? °C.

Apply eq. (3.64a):

$$T_{wind\ chill} = [0.62\cdot(-25°C) + 13.1°C] + [0.51\cdot(-25°C) - 14.6°C]\cdot\left(\frac{30 km/h}{4.8 km/h}\right)^{0.16}$$

$$= [-2.4°C] + [-27.4°C]\cdot(1.34) = \underline{\mathbf{-39.1°C}}$$

Check: Physics and units are reasonable. Agrees with a value interpolated from Table 3-1.

Exposition: To keep warm, consider making a fire by burning pages of this book. The book is easier to replace than your fingers, toes, ears, or nose.

APPARENT TEMPERATURE INDICES

Warm-blooded (**homeothermic**) animals including humans generate heat internally via metabolism of the food we eat with the oxygen we breath. But we also rely on heat transfer with the environment to help maintain an internal core temperature of about 37°C (= 98.6°F). (Our skin is normally cooler — about 33.9°C = 93°F). Heat transfer occurs both via sensible heat fluxes (temperature difference between air and our skin or lungs) and latent heat fluxes (evaporation of moisture from our lungs and of perspiration from our skin).

The temperature we "feel" on our skin depends on the air temperature and wind speed (as they both control the bulk heat transfer between our skin and the environment) and on humidity (is it affects how rapidly perspiration evaporates to cool us).

Define a reference state as being a person walking at speed M_o = 4.8 km h^{-1} through calm, moderately dry air. The <u>actual</u> air temperature is defined to be the temperature we "feel" for this <u>reference</u> state.

The **apparent temperature** is the temperature of a reference state that feels the same as it does for non-reference conditions. For example, faster winds in winter make the temperature feel colder (wind chill) than the actual air temperature, while higher humidities in summer make the air feel warmer (humidex or heat index).

Wind-Chill Temperature

The **wind-chill temperature index** is a measure of how cold the air feels to your exposed face. The official formula, as revised in 2001 by the USA and Canada, for wind chill in °C is:

$$T_{wind\ chill} = \left(a\cdot T_{air} + T_1\right) + \left(b\cdot T_{air} - T_2\right)\cdot\left(\frac{M}{M_o}\right)^{0.16} \quad \text{for } M > M_o \quad (3.64a)$$

and

$$T_{wind\ chill} = T_{air} \quad \text{for } M \le M_o \quad (3.64b)$$

where a = 0.62, b = 0.51, T_1 = 13.1°C, and T_2 = 14.6°C. M is the wind speed measured at the official anemometer height of 10 m. For $M < M_o$, the wind chill equals the actual air temperature. This index applies to non-rainy air.

Fig. 3.12 and Table 3-3 show that faster winds and colder temperatures make us "feel" colder. The data used to create eq. (3.64) was from volunteers in Canada who sat in refrigerated wind tunnels, wearing warm coats with only their face exposed.

At wind chills colder than –27°C, exposed skin can freeze in 10 to 30 minutes. At wind chills colder than –48°C: WARNING, exposed skin freezes in 2 to 5 min. At wind chills colder than –55°C: DANGER, exposed skin freezes in less than 2 minutes. In this danger zone is an increased risk of **frostbite** (fingers, toes, ears and nose numb or white), and **hypothermia** (drop in core body temperature).

Humidex and Heat Index

On hot days you feel warmer than the actual air temperature when the air is more humid, but you feel cooler when the air is drier due to evaporation of your perspiration. In extremely humid cases the air is so uncomfortable that there is the danger of heat stress. Two **apparent temperatures** that indicate this are **humidex** and **heat index**.

The set of equations below approximates Steadman's temperature-humidity index of sultriness (i.e., a **heat index**):

$$T_{heat\ index}(°C) = T_R + [T - T_R] \cdot \left(\frac{RH \cdot e_s}{100 \cdot e_R} \right)^p \tag{3.65a}$$

where e_R = 1.6 kPa is reference vapor pressure, and

$$T_R(°C) = 0.8841 \cdot T + (0.19°C) \tag{3.65b}$$

$$p = (0.0196°C^{-1}) \cdot T + 0.9031 \tag{3.65c}$$

$$e_s(\text{kPa}) = 0.611 \cdot \exp\left[5423 \left(\frac{1}{273.15} - \frac{1}{(T + 273.15)} \right) \right] \tag{3.65d}$$

The two input variables are T (dry bulb temperature in °C), and RH (the relative humidity, ranging from 0 for dry air to 100 for saturated air). Also, T_R (°C), and p are parameters, and e_s is the saturation vapor pressure, discussed in the Water Vapor chapter. Eqs. (3.65) assume that you are wearing a normal amount of clothing for warm weather, are in the shade or indoors, and a gentle breeze is blowing.

The dividing line between feeling warmer vs. feeling cooler is highlighted with the bold, underlined heat-index temperatures in Table 3-4.

In Canada, a **humidex** is defined as

$$T_{humidex}(°C) = T(°C) + a \cdot (e - b) \tag{3.66a}$$

where T is air temperature, a = 5.555 (°C kPa^{-1}), b = 1 kPa, and

$$e(\text{kPa}) = 0.611 \cdot \exp\left[5418 \left(\frac{1}{273.16} - \frac{1}{[T_d(°C) + 273.16]} \right) \right] \tag{3.66b}$$

Table 3-3. The wind-chill-index temperature (°C).

Wind Speed km·h^{-1}	Wind Speed m·s^{-1}	Actual Air Temperature (°C) –40	–30	–20	–10	0	10
60	**16.7**	–64	–50	–36	–23	–9	5
50	**13.9**	–63	–49	–35	–22	–8	6
40	**11.0**	–61	–48	–34	–21	–7	6
30	**8.3**	–58	–46	–33	–20	–6	7
20	**5.6**	–56	–43	–31	–18	–5	8
10	**2.8**	–51	–39	–27	–15	–3	9
0	**0**	–40	–30	–20	–10	0	10

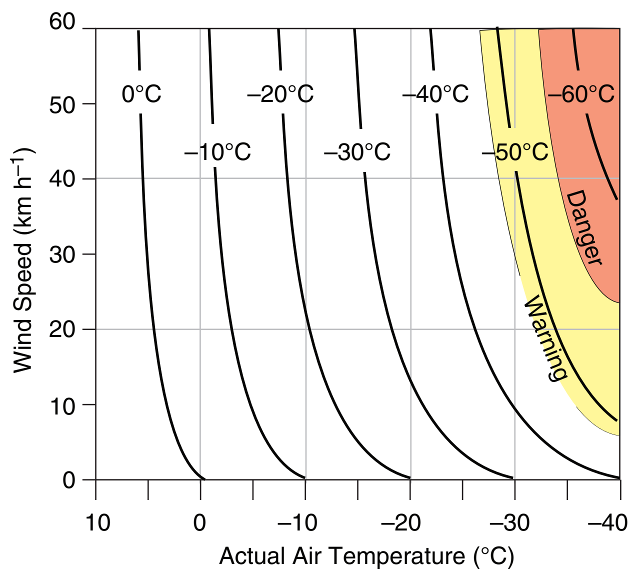


Figure 3.12
For any wind speed M and actual air temperature T read the wind-chill temperature index (°C) from the curves in this graph.

Table 3-4. Heat-index apparent temperature (°C).

Rel. Hum. (%)	Actual Air Temperature (°C) 20	25	30	35	40	45	50
100	21	29	41	61			
90	21	29	39	57			
80	21	28	37	52			
70	20	27	35	48			
60	20	26	34	45	62		
50	**19**	25	32	41	55		
40	19	**24**	30	38	49	66	
30	19	24	**29**	36	44	56	
20	18	23	28	**33**	40	48	59
10	18	23	27	32	**37**	**42**	**48**
0	18	22	27	31	36	40	44

Sample Application

Use the equations to find the heat index and humidex for an air temperature of 38°C and a relative humidity of 75% (which corresponds to a dew-point temperature of about 33°C).

Find the Answer

Given: $T = 38°C$, $RH = 75\%$, $T_d = 33°C$
Find: $T_{heat\ index} = ?\ °C$, $T_{humidex} = ?\ °C$

For heat index, use eqs. (3.65):

$T_R = 0.8841 \cdot (38) + 0.19 = 33.8°C$ (3.65b)
$p = 0.0196 \cdot (38) + 0.9031 = 1.65$ (3.65c)
$e_s = 0.611 \cdot \exp[5423 \cdot (\{1/273.15\} - \{1/(38+273.15)\})]$
$= 6.9$ kPa (3.65d)
$T_{heat\ index} = 33.8 + [38-33.8] \cdot (0.75 \cdot 6.9/1.6)^{1.65}$
$=$ **<u>62.9°C</u>** (3.65a)

For humidex, use eqs. (3.66):

$e = 0.611 \cdot \exp[5418 \cdot (\{1/273.16\} - \{1/(33+273.16)\})]$
$= 5.18$ kPa (3.66b)
$T_{humidex} = 38 + 5.555 \cdot (5.18-1) =$ **<u>61.2°C</u>** (3.66a)

Check: Units are reasonable. Values agree with extrapolation of Tables 3-4 and 3-5.

Exposition: These values are in the danger zone, meaning that people are likely to suffer heat stroke. The humidex and heat index values are nearly equal for this case.

Table 3-5. **Humidex** apparent temperature (°C)

T_d (°C)	Actual Air Temperature T (°C) 20	25	30	35	40	45	50
50							118
45						96	101
40					77	82	87
35				62	67	72	77
30			49	54	59	64	69
25		37	42	47	52	57	62
20	28	33	38	43	48	53	58
15	24	29	34	39	44	49	54
10	21	26	31	36	41	46	51
5	**<u>19</u>**	**<u>24</u>**	**<u>29</u>**	**<u>34</u>**	**<u>39</u>**	**<u>44</u>**	**<u>49</u>**
0	18	23	28	33	38	43	48
–5	17	22	27	32	37	42	47
–10	16	21	26	31	36	41	46

T_d is dew-point temperature, a humidity variable discussed in the Water Vapor chapter.

Humidex is also an indicator of summer discomfort due to heat and humidity (Table 3-3). Values above 40°C are uncomfortable, and values above 45°C are dangerous. **Heat stroke** is likely for humidex ≥ 54°C. This table also shows that for dry air ($T_d \le 5°C$) the air feels cooler than the actual air temperature.

TEMPERATURE SENSORS

Temperature sensors are generically called **thermometers**. Anything that changes with temperature can be used to measure temperature. Many materials expand when warm, so the size of the material can be calibrated into a temperature. Classical **liquid-in-glass thermometers** use either mercury or a dyed alcohol or glycol fluid that can expand from a reservoir or **bulb** up into a narrow tube.

House **thermostats** (temperature controls) often use a **bimetalic strip**, where two different metals are sandwiched together, and their different expansion rates with temperature causes the metal to bend as the temperature changes. Car thermostats use a wax that expands against a valve to redirect engine coolant to the radiator when hot. Some one-time use thermometers use wax that melts onto a piece of paper at a known temperature, changing is color.

Many electronic devices change with temperature, such as resistance of a wire, capacitance of a capacitor, or behavior of various transistors (**thermistors**). These changes can be measured electronically and displayed. **Thermocouples** (such as made by a junction between copper and constantan wires, where constantan is an alloy of roughly 60% copper and 40% nickel) generate a small amount of electricity that increases with temperature. **Liquid crystals** change their orientation with temperature, and can be designed to display temperature.

Sonic thermometers measure the speed of sound through air between closely placed transmitters and receivers of sound. Radio Acoustic Sounder Systems (**RASS**) transmit a loud pulse of sound upward from the ground, and then infer temperature vs. height via the speed that the sound wave propagates upward, as measured by a radio or microwave profiler.

Warmer objects emit more radiation, particularly in the infrared wavelengths. An **infrared thermometer** measures the intensity of these emissions to infer the temperature. Satellite remote sensors also detect emissions from the air upward into space, from which temperature profiles can be calculated (see the Satellites & Radar chapter).

Even thick layers of the atmosphere expand when they become warmer, allowing the **thickness** between two different atmospheric pressure levels to indicate average temperature in the layer.

REVIEW

Three types of heat budgets were covered in this chapter. All depend on the flow rate of energy per unit area (J m^{-2} s^{-1}) into or out of a region. This energy flow is called a flux, where the units above are usually rewritten in their equivalent form (W m^{-2}).

1) One type is a heat balance at the surface of the Earth. The surface has zero thickness — hence no air volume and no mass that can store or release heat. Thus, the input fluxes must exactly balance the output fluxes. Sunlight and IR radiation (see the Radiation chapter) must be balanced by the sum of conduction to/from the ground and effective turbulent fluxes of sensible and latent heat between the surface and the air.

2) Another type is an Eulerian budget for a fixed volume of air. If more heat enters than leaves, then the air temperature must increase (i.e., heat is stored in the volume). Processes that can move heat are advection, turbulence, and radiation. At the Earth's surface, an effective turbulent flux is defined that includes both the turbulent and conductive contributions. Also, heat can be released within the volume if water vapor condenses or radionuclides decay.

3) The third type is a Lagrangian budget that follows a mass of air (called an air parcel) as it rises or sinks through the surrounding environment. This is trickier because the parcel temperature can change even without moving heat into it via fluxes, and even without having water vapor evaporate or condense within it. This adiabatic temperature change is caused by work done on or by the parcel as it responds to the changing pressure as it moves vertically in the atmosphere. For unsaturated (non-cloudy) air, temperature of a rising air parcel decreases at the adiabatic lapse rate of 9.8°C km^{-1}. This process is critical for understanding turbulence, clouds, and storms, as we will cover in later chapters.

The actual air temperature can be measured by various thermometers. Humans feel the combined effects of actual air temperature, wind, and humidity as an apparent air temperature.

HOMEWORK EXERCISES

Broaden Knowledge & Comprehension

B1(§). For an upper-air weather station near you (or for a site specified by your instructor), get recent observation data of T vs. z or T vs. P from the internet, and plot the result on a copy of the thermodynamic diagram from this chapter.

B2. For an upper-air weather station near you (or for a site specified by your instructor), get an already-plotted recent sounding from the internet. Find the background isotherm and isobar lines, and compare their arrangement to the diagram (Fig. 3.4) in this chapter. We will learn more about other thermo-diagram formats in the Atmospheric Stability chapter.

B3. Use the internet to acquire temperatures at your town and also at a town about 100 km downwind of you. Also get the wind speeds in both towns and take an average. Use this average speed to calculate the contribution of advection to the local heating in the air between those two towns.

B4. Use the internet to acquire a weather map or other weather report that shows the observed near-surface air temperature just before sunrise at your location (or at another location specified by your instructor). For the same location, find a map or report of the temperature in mid afternoon. From these two observations, calculate the rate of temperature change over that time period. Also, qualitatively describe which terms in the Eulerian heat budget might be largest. (Hint: if windy, then perhaps advection is important. If clear skies, then heat transfer from the solar-heated ground might be important. Access other weather maps as needed to determine which physical process is most important for the temperature change.)

B5. Use the internet to acquire a local weather map of apparent temperature, such as wind-chill in winter or heat index (or humidex) in summer. If the map covers your location, compare how the air feels to you vs. the apparent temperature on the map.

B6. Use the internet to acquire images of 4 different types of temperature sensors (not 4 models of the same type of sensor).

Apply

A1. Find the change in sensible heat (enthalpy) (J) possessed by 3 kg of air that warms by __°C.
a. 1 b. 2 c. 3 d. 4 e. 5 f. 6 g. 7
h. 8 i. 9 j. 10 k. 11 m. 12

A2. Find the specific heat C_p of humid air having water-vapor mixing ratio ($g_{vapor}/g_{dry\ air}$) of:
a. 0.010 b. 0.012 c. 0.014 d. 0.016 e. 0.018
f. 0.020 h. 0.022 i. 0.024 j. 0.026 k. 0.028
m. 0.030

A3. Find the change in latent heat (J) for condensation of ___ kg of water vapor.
a. 0.2 b. 0.4 c. 0.6 d. 0.8 e. 1.0 f. 1.2 g. 1.4
h. 1.6 i. 1.8 j. 2.0 k. 2.2 m. 2.4

A4. Find the temperature change (°C) of air given the following values of heat transfer and pressure change, assuming air density of 1.2 kg m^{-3}.

	Δq (J kg^{-1})	ΔP (kPa)
a.	500	5
b.	1000	5
c.	1500	5
d.	2000	5
e.	2500	5
f.	3000	5
g.	500	10
h.	1000	10
i.	1500	10
j.	2000	10
k.	2500	10
m.	3000	10

A5. Find the change in temperature (°C) if an air parcel rises the following distances while experiencing the heat transfer values given below.

	Δq (J kg^{-1})	Δz (km)
a.	500	0.5
b.	1000	0.5
c.	1500	0.5
d.	2000	0.5
e.	2500	0.5
f.	3000	0.5
g.	500	1
h.	1000	1
i.	1500	1
j.	2000	1
k.	2500	1
m.	3000	1

A6. Given the following temperature change ΔT (°C) across a height difference of Δz = 4 km, find the lapse rate (°C km^{-1}):
a. 2 b. 5 c. 10 d. 20 e. 30 f. 40 g. 50
h. –2 i. –5 j. –10 k. –20 m. –30

A7. Find the final temperature (°C) of an air parcel with the following initial temperature and height change, for an adiabatic process.

	$T_{initial}$ (°C)	Δz (km)
a.	15	0.5
b.	15	–1.0
c.	15	1.5
d.	15	–2.0
e.	15	2.5
f.	15	–3.0
g.	5	0.5
h.	5	–1.0
i.	5	1.5
j.	5	–2.0
k.	5	2.5
m.	5	–3.0

A8. Using the equations (not using the thermo diagram), find the final temperature (°C) of dry air at a final pressure, if it starts with the initial temperature and pressure as given. (Assume adiabatic.)

	$T_{initial}$ (°C)	$P_{initial}$ (kPa)	P_{final}(kPa)
a.	5	100	80
b.	5	100	50
c.	5	80	50
d.	5	80	100
e.	0	60	80
f.	0	60	50
g.	0	80	40
h.	0	80	100
i.	–15	90	80
j.	–15	90	50
k.	–15	70	50
m.	–15	70	100

A9. Same as previous question, but use the thermo diagram Fig. 3.4.

A10. Given air with temperature and altitude as listed below, use formulas (not thermo diagrams) to calculate the potential temperature. Show all steps in your calculations.

	z (m)	T (°C)
a.	400	30
b.	800	20
c.	1,100	10
d.	1,500	5
e.	2,000	0
f.	6,000	–50
g.	10,000	–90
h.	–30	35
i.	700	3
j.	1,300	–5
k.	400	5
m.	2,000	–20

A11. Same as the previous exercise, but find the virtual potential temperature for humid air. Use a water-vapor mixing ratio of 0.01 $g_{vapor}/g_{dry\ air}$ if the air temperature is above freezing, and use 0.0015 $g_{vapor}/g_{dry\ air}$ if air temperature is below freezing. Assume the air contains no ice or liquid water.

A12. Given air with temperature and pressure as listed below, use formulas (not thermo diagrams) to calculate the potential temperature. Show all steps in your calculations.

	P (kPa)	T (°C)
a.	90	30
b.	80	20
c.	110	10
d.	70	5
e.	85	0
f.	40	–45
g.	20	–90
h.	105	35
i.	75	3
j.	60	–5
k.	65	5
m.	50	–20

A13. Same as previous exercise, but use the thermo diagram Fig. 3.4.

A14. Instead of equations, use the Fig 3.4 to find the actual air temperature (°C) given:

	P(kPa)	θ (°C)
a.	100	30
b.	80	30
c.	60	30
d.	90	10
e.	70	10
f.	50	10
g.	80	–10
h.	50	–10
i.	20	50

A15(§). Use a spreadsheet to calculate and plot a thermo diagram similar to Fig. 3.4 but with: isotherm grid lines every 10°C, and dry adiabats for every 10°C from –50°C to 80°C.

A16. Find the rate of temperature change (°C h^{-1}) in an Eulerian coordinate system with no internal heat source, given the kinematic flux divergence values below. Assume $\Delta x = \Delta y = \Delta z = 1$ km.

	ΔF_x (K·m s^{-1})	ΔF_y (K·m s^{-1})	ΔF_z (K·m s^{-1})
a.	1	2	3
b.	1	2	–3
c.	1	–2	3
d.	1	–2	–3
e.	–1	2	3
f.	–1	2	–3
g.	–1	–2	3
h.	–1	–2	–3

A17. Given the wind and temperature gradient, find the value of the kinematic advective flux gradient (°C h^{-1}).

	V (m s^{-1})	$\Delta T/\Delta y$ (°C 100 km)
a.	5	–2
b.	5	2
c.	10	–5
d.	10	5
e.	–5	–2
f.	–5	2
g.	–10	–5
h.	–10	5

A18. Given the wind and temperature gradient, find the value of the kinematic advective flux gradient (°C h^{-1}).

	W (m s^{-1})	$\Delta T/\Delta z$ (°C km^{-1})
a.	5	–2
b.	5	2
c.	10	–5
d.	10	–10
e.	–5	–2
f.	–5	2
g.	–10	–5
h.	–10	–10

A19. Find the value of the conductive flux $\mathbb{F}_{z\ cond}$ (W m^{-2}) given a change of absolute temperature with height ($T_2 - T_1$ = value below) across a distance ($z_2 - z_1$ = 1 m):

a. –1 b. –2 c. –3 d. –4 e. –5 f. –6 g. –7
h. 1 i. 2 j. 3 k. 4 m. 5 n. 6 o. 7

A20. Find the effective surface turbulent heat flux (°C·m s^{-1}) over a forest for wind speed of 10 m s^{-1}, air temperature of 20°C, and surface temperature (°C) of

a. 21 b. 22 c. 23 d. 24 e. 25 f. 26 g. 27
h. 19 i. 18 j. 17 k. 16 m. 15 n. 14 o. 13

A21. Find the effective kinematic heat flux at the surface on a calm day, for a buoyant velocity scale of 50 m s^{-1}, a mixed-layer potential temperature of 25°C, and with a surface potential temperature (°C) of:

a. 26 b. 28 c. 30 d. 32 e. 34 f. 36 g. 38
h. 40 i. 42 j. 44 k. 46 m. 48 n. 50

A22. Find the effective kinematic heat flux at the surface on a calm day, for a Deardorff velocity of 2

m s^{-1}, a mixed-layer potential temperature of 24°C, and with a surface potential temperature (°C) of:
a. 26 b. 28 c. 30 d. 32 e. 34 f. 36 g. 38
h. 40 i. 42 j. 44 k. 46 m. 48 n. 50

A23. For dry air, find the buoyancy velocity scale, given a mixed-layer potential temperature of 25°C, a mixed-layer depth of 1.5 km, and with a surface potential temperature (°C) of:
a. 27 b. 30 c. 33 d. 36
e. 40 f. 43 g. 46 h. 50

A24. For dry air, find the Deardorff velocity w_* for an effective kinematic heat flux at the surface of 0.2 K·m s^{-1}, air temperature of 30°C, and mixed-layer depth (km) of:
a. 0.4 b. 0.6 c. 0.8 d. 1.0
e. 1.2 f. 1.4 g. 1.6 h. 1.8

A25. Find the value of vertical divergence of kinematic heat flux, if the flux at the top of a 200 m thick air layer is 0.10 K·m s^{-1}, and flux (K·m s^{-1}) at the bottom is:
a. 0.2 b. 0.18 c. 0.16 d. 0.14
e. 0.12 f. 0.10 g. 0.08 h. 0.06

A26. Given values of effective surface heat flux and boundary-layer depth for daytime during fair weather, what is the value of the turbulent-flux vertical gradient?

	F_H (K·m·s^{-1})	z_i (km)
a.	0.25	2.0
b.	0.15	1.5
c.	0.1	1.0
d.	0.03	0.3
e.	0.08	0.3
f.	0.12	0.8
g.	0.15	1.0
h.	0.25	1.5

A27. Given a pre-storm environment where the temperature varies linearly from 25°C at the Earth's surface to –60°C at 11 km (tropopause). What is the value of the vertical gradient of turbulent flux (K s^{-1}) for an altitude (km) of:
a. 0.1 b. 0.5 c. 1 d. 1.5 e. 2 f. 2.5 g. 3
h. 3.5 i. 4 j. 5 k. 6 m. 7 n. 8 o. 11

A28. Find the mid-tropospheric maximum value of heat flux (K·m s^{-1}) for a stormy atmosphere, where the troposphere is 11 km thick, and the air temperature at the top of the troposphere equals the air temperature of a standard atmosphere. But the air temperature (°C) at the ground is:
a. 16 b. 17 c. 18 d. 19 e. 20 f. 21 g. 22
h. 23 i. 24 j. 25 k. 26 m. 27 n. 28 o. 29

A29. Find the latent-heating rate (°C h^{-1}) averaged over the troposphere for a thunderstorm when the rainfall rate (mm h^{-1}) is:
a. 0.5 b. 1 c. 1.5 d. 2 e. 2.5 f. 3 g. 3.5
h. 4 i. 4.5 j. 5 k. 5.5 m. 6 n. 6.5 o. 7

A30. Given below the net radiative flux (W m^{-2}) reaching the surface, find the sum of sensible and latent heat fluxes (W m^{-2}) at the surface. (Hint: determine if it is day or night by the sign of the radiative flux.)
a. –600 b. –550 c. –500 d. –450 e. –400
f. –350 g. –300 h. –250 i. –200 j. –150
k. –100 m. –50 n. 50 o. 100 p. 150

A31. Same as the previous problem, but estimate the values of the sensible and latent heat fluxes (W m^{-2}) assuming a Bowen ratio of:
(1) 0.2 (2) 5.0

A32. Suppose you mounted instruments on a tower to observe temperature T and mixing ratio r at two heights in the surface layer (bottom 25 m of atmosphere) as given below. If a net radiation of –500 W m^{-2} was also measured at that site, then estimate the values of effective surface values of sensible heat flux and latent heat flux.

index	z(m)	T(°C)	r (g_{vap}/kg_{air})
2	10	T_2	10
1	2	20	15

where T_2 (°C) is:
a. 13.5 b. 13 c. 12.5 d. 12 e. 11.5 f. 11
g. 10.5 h. 10 i. 9.5 j. 9 k. 8.5 m. 8

A33. Not only can a stationary person feel wind chill when the wind blows, but a moving person in a calm wind can also feel wind chill, because most important is the speed of the air relative to the speed of the body. If you move at the speed given below through calm air of temperature given below, then you would feel a wind chill of what apparent temperature? Given: M (m s^{-1}), T (°C) .
a. 5, 5 b. 10, 5 c. 15, 5 d. 20, 5 e. 25, 5
f. 30, –10 g. 25, –10 h. 20, –10 i. 15, –10 j. 10, –10

A34(§). Modify eqs. (3.64) to use input and output temperatures in Fahrenheit and wind speeds in miles per hour. Calculate sufficient values to plot a graph similar to Fig 3.12 but in these new units.

A35. Find the heat index apparent temperature (°C) for an actual air temperature of 33°C and a relative humidity (%) of:
a. 5 b. 10 c. 20 d. 30 e. 40 f. 50 g. 60
h. 70 i. 75 j. 80 k. 85 m. 90 n. 90

A36. Find the humidex apparent air temperature (°C) for an actual air temperature of 33°C and a dew-point temperature (°C) of:
a. 32.5 b. 32 c. 31 d. 30 e. 29 f. 28 g. 27
h. 26 i. 25 j. 23 k. 20 m. 15 n. 10 o. 5

Evaluate & Analyze

E1. Assume that 1 kg of liquid water initially at 15°C is in an insulated container. Then you add 1 kg of ice into the container. The ice melts and the liquid water becomes colder. Eventually a final equilibrium is reached. Describe what you end up with at this final equilibrium?

E2. Explain in your own words why the units for specific heat C_p (J·kg^{-1}·K^{-1}) are slightly different than the units for the latent heat factor L (J·kg^{-1}). (Hint: read the INFO box on Internal Energy.)

E3. Explain in your own words why the magnitude of C_p should be larger than the magnitude of C_v. (Hint: read the INFO box on C_p vs. C_v).

E4. Consider the INFO box on C_p vs. C_v, with Fig. 3I.3c representing an initial state at equilibrium. Suppose you add some weight to the piston in Fig (c) causing the piston to become lower to reach a new equilibrium, but no thermal energy is added ($\Delta q = 0$). Describe what would happen to: (a) the molecules on average, (b) the gas temperature in the cylinder, (c) the air density in the cylinder, and (d) the air pressure in the cylinder.

E5. For the First Law of Thermodynamics (eq. 3.4d) which term(s) is are zero for a process that is:
a. adiabatic b. isothermal c. isobaric

E6. Start with eq. (3.4) and use algebra to derive equation (3.5). What did you need to assume to do this derivation? Does the result have any limitations?

E7. For Fig. 3.2, speculate on other processes not listed that might affect the air-parcel temperature.

E8. Using Fig. 3.3, explain in your own words the difference between a process lapse rate and an environmental lapse rate. Can both exist with different values at the same height? Why?

E9. Eq. (3.7) tells us that temperature of an adiabatically rising air parcel will decrease linearly with increasing height. In your own words, explain why you would NOT expect the same process to cause temperature to decrease linearly with decreasing pressure.

E10. If an air parcel rises isothermally (namely, heat is added or subtracted to maintain constant temperature), then what would happen to the potential temperature of the air parcel as it rises?

E11. Chinook winds (also known as foehn winds) consist of air descending down the lee slope of a mountain and then continuing some distance across the neighboring valley or plain. Why are Chinook winds usually warm when they reach the valley? (Hint: consider adiabatic descent of an air parcel.)

E12. In the definition of virtual potential temperature, why do liquid water drops and ice crystals cause the air to act heavier (i.e., colder virtual potential temperature), even though these particles are falling through the air?

E13. First make a photocopy of Fig. 3.4, so that you can keep the original Thermo Diagram clean.
a) On the copy, plot the vertical temperature profile for a standard atmosphere, as defined in Chapter 1. Suppose that this standard profile represents background environmental air.
b) On this same diagram, plat a point representing an air parcel at (P, T) = (100 kPa, 15°C). If you adiabatically lift this parcel to 50 kPa, what is its new temperature?
c) Is the parcel temperature a 50 kPa warmer or colder than the environment at that same pressure?

E14(§). For a standard atmosphere (see Chapt. 1), calculate potential temperature θ at z = 0, 2, 4, 6, 8, 10 km altitudes. Plot θ along the bottom axis and z along the vertical axis.

E15(§). Thermo diagrams often have many different types of lines superimposed. For example, on the background T vs. log-P diagram of Fig. 3.4 is plotted just one type of line: the dry adiabats. Instead of these adiabats, start with the same background of a T vs. log-P diagram, but instead draw lines connecting points of equal height (called **contour** lines). To calculate these lines, use the hypsometric equation from chapter 1 to solve for P vs. (z, T). Do this for the z = 2, 4, 6, 8, 10 km contours, where for any one height, plug in different values of T to find the corresponding values of P that define the contour.

E16. For advection to be a positive contribution (i.e., causing heating) and for wind that is in a positive coordinate direction, explain why the corresponding temperature gradient must be negative.

E17. Suppose that mild air (20°C at 10 m altitude) rests on top of a warm ocean (26°C at the surface), causing **convection** (vertical overturning of the air). If there is no mean horizontal wind, then the effective heat flux at the surface has what value? Assume a mixed layer that is 1200 m thick with average thermodynamic state of $r = 0.01$ g_{vapor}/g_{air} and $\theta =$ 15°C.

E18. Light travels faster in warm air than in cold. Use this info, along with Fig. 3.7, to explain why **inferior mirages** (reflections of the sky) are visible on hot surfaces such as asphalt roads. (Hint: Consider a wave front that is moving mostly horizontally, but also slightly downward at a small angle relative to the road surface, and track the forward movement of each part of this wave front — an optics method known as Huygens' Principle. See details in the atmospheric Optics chapter.)

E19. Under what conditions would eqs. (3.34 - 3.35) be expected to fail? Why?

E20. Use eqs. (3.37) and (3.39) to solve for the heat flux as a function of the temperature difference.

E21. In Fig. 3.8, the heat flux is greatest at the height where there is no change in the vertical temperature profile from before to after a storm. Why should that be the case?

E22. How fast does air temperature change if only if the only thermodynamic process that was active was direct IR cooling?

E23. In a thunderstorm, the amount of water condensation in the troposphere is often much greater than the amount of rain reaching the ground. Why is that, and how might it affect the heat budget averaged over the whole thunderstorm depth?

E24. Eq. (3.51) has what limitations?

E25. Comment on the relative strengths of advective vs. latent heating in an Eulerian system, given $V =$ 5 m s^{-1}, $\Delta T/\Delta y = -5$°C/1000km, and 1 g/kg of water condenses every 5 minutes.

E26. Create a figures similar to Fig. 3.9, but for:
a) daytime over a white concrete road,
b) nighttime black asphalt road.

E27. It is sometimes said that conductive heat flux into the ground is a response to radiative forcings at the surface. Is that statement compatible with the crude parameterization presented in this book for flux into the ground? Explain.

E28. What is the initial rate of change of average mixed-layer air temperature with horizontal distance downwind if the air is initially 5 °C colder than the water, given that the air blows over the water at speed 15 m s^{-1}? Consider entrainment into the top of the mixed layer, but neglect other heating or cooling processes.

E29. Can the parameterizations (eqs. 3.58 - 3.61) actually give a balanced heat budget? For what types of situations are these parameterizations valid?

E30. (§). Suppose that we used the heat transfer eq. (3.35) as a basis for deriving wind chill. The result might be a different wind-chill relationship:

$$T_{wind\ chill} = T_s + \left(T_{air} - T_s\right) \cdot \left[b + a \cdot \left(\frac{M + M_o}{M_o} \right)^{0.16} \right] + T_c \tag{3.67}$$

where T_s = 34.6°C is an effective skin temperature, and where, $a = 0.5$, $b = 0.62$, T_c = 4.2°C, and M_o = 4.8 km h^{-1}. Plot this equation as a graph similar to Fig. 3.12, and comment on the difference between the formula above and the actual wind-chill formula.

E31. Notice in Fig. 3.12 that the curves bend the most for slow wind speeds. Why might you expect this to be the case?

Synthesize

S1. Describe the change to the ocean if condensation caused cooling and evaporation caused heating of the air. Assume dry air above the ocean.

S2. Suppose that zero latent heat was associated with the phase changes of water. Describe the possible changes to climate and weather, if any?

S3. Describe the change to the atmosphere if rising air parcels became warmer adiabatically while sinking ones became cooler.

S4. Suppose that for each 1 km rise of an air parcel, the parcel mixes with an equal mass of surrounding environmental air. How would the process lapse rate for this rising air parcel be different (if at all) from the lapse rate of an adiabatically rising air parcel (having no mixing).

S5. Macro thermodynamics (the kind we've used in this chapter) considers the statistical state of a large collection of molecules that frequently collide with each other, and how they interact on average with their surroundings. Can this same macro thermodynamics be used in the exosphere, where individual air molecules are very far apart (i.e., have a large mean-free path) and rarely interact? Why? Also, explain if how heat budgets can be used in the exosphere.

S6. Could there be situations where environmental and process lapse rates are equal? If so, give some examples.

S7. Suppose that the virtual potential temperature was not affected by the amount of solid or liquid water in the air. How would weather and climate change, if at all?

S8. The background of the thermo diagram of Fig. 3.4 is an orthogonal grid, where the isotherms are plotted perpendicular to the isobars. Suppose you were to devise a new thermo diagram with the dry adiabats perpendicular to the isobars. On such a diagram, how would the isotherms be drawn? To answer this, draw a sketch of this new diagram, showing the isobars, adiabats, and isotherms. (Do this as a conceptual exercise, not by solving equations to get numbers.)

S9. Describe changes to Earth's surface heat balance if the geological crust was 1 km thick aluminum (an excellent conductor of heat) covering the whole Earth.

S10. Suppose you were on a train moving in a straight line at constant speed. You make measurements of the surrounding environmental air as the train moves down the track.

a) If the environmental air was calm, do you think your measurements are Eulerian, Lagrangian, or neither? Explain.

b) If the environmental air was moving in any arbitrary speed or direction, do you think your measurements are Eulerian, Lagrangian or neither? Explain.

c) Try to create a heat budget equation that works in the framework, given your constant speed of translation of M_o.

S11. Describe how atmospheric structure, climate, and weather would change if the troposphere were completely transparent to all IR radiation, but was mostly opaque to solar radiation.

S12. Describe how errors in surface sensible and latent heat flux estimates would increase as the temperature and humidity differences between the two measurement levels approached zero.

S13. The wind-chill concept shows how it feels colder when it is winder. For situations where the wind chill is much colder than the actual air temperature, to what temperature will an automobile engine cool after it is turned off? Why? (Assume the car is parked outside and is exposed to the wind.)

Sample Application

[This sample applies to eqs. 3.1 and 3.3, but was put here on the last page of the chapter because there was no room for it earlier in the chapter.]

How much dew must condense on the sides of a can of soda for it to warm the soda from 1°C to 16°C?

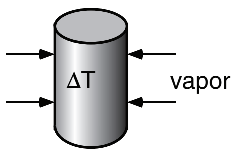
Hints: Neglect the heat capacity of the metal can. The density of liquid water is 1000 $kg \cdot m^{-3}$. Assume the density of soda equals that of pure water. Assume the volume of a can is 354 ml (milliliters), where 1 l = 10^{-3} m^3.

Find the Answer

Given: ρ_{water} = 1000 $kg \cdot m^{-3}$.
C_{liq} = 4200 $J \cdot kg^{-1} \cdot K^{-1}$
Volume (Vol) in Can = 354 ml
L_{cond} = + 2.5×10^6 $J \cdot kg^{-1}$
ΔT = 15 K

Find: Volume of Condensate

Sketch:



Equate the latent heat release by condensing water vapor (eq. 3.3) with the sensible heat gained by fluid in the can (eq. 3.1)

$$\Delta Q_E = \Delta Q_H$$

$$\rho_{condensate} \cdot (\Delta Vol\ of\ Condensate) \cdot L_{cond} = \rho_{soda} \cdot (Vol\ of\ Can) \cdot C_{liq} \cdot \Delta T$$

Assume the density of condensate and soda are equal, so they cancel. The equation can then be solved for *ΔVolume of Condensate*.

$\Delta Volume\ of\ Condensate = (Vol\ of\ Can) \cdot C_{liq} \cdot \Delta T\ L_{cond}$
= (354 ml)·(4200 $J \cdot kg^{-1} \cdot K^{-1}$)·(15 K) ($2.5 \times 10^6$ $J \cdot kg^{-1}$)
= **8.92 ml**

Check: Units OK. Sketch OK. Physics OK.

Exposition: Latent heats are so large that an amount of water equivalent to only 2.5% of the can volume needs to condense on the outside to warm the can by 15°C. Thus, to keep your can cool, insulate the outside to prevent dew from condensing.