Phase transitions in exploration seismology: statistical mechanics meets information theory

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joint work with
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Research interests

• Develop techniques to obtain higher quality images from (incomplete) data $\iff$ seismic imaging of transitions

• Characterization of reflectors $\iff$ estimation of singularity orders of imaged reflectors

• Understand physical processes that generate singular transitions in the earth $\iff$ Percolation phenomena

• Singularity-preserved upscaling
Today’s topics

Phase diagrams in the recovery of seismic data from incomplete measurements

• old ideas in geophysics reincarnated in the new field of “compressive sampling”

• describes regions that favor recovery

Phase diagrams in the description of seismic reflectors

• mixture models with critical points $\leftrightarrow$ reflectors

• a first step towards singularity-preserved upscaling
Phase-transition behavior in compressive sampling

joint work with Gilles Hennenfent

“Non-parametric seismic data recovery with curvelet frames” in revision for GJI
Data

nominal spatial sampling ~ 112.5m
CRSI

Spatial sampling ~ 12.5m
Consider $n$-random time samples from a signal with $k$-sparse Fourier spectrum, i.e.

$$signal \rightarrow y = Ax_0$$

with $A \in \mathbb{C}^{n \times N}$ the time restricted inverse Fourier transform.

The signal

$$f_0 = F^H x_0$$

with the $k$-non-zero spectrum can exactly be recovered.
Solve

\[
P_1 : \begin{cases}
\tilde{x} = \arg \min_{x \in \mathbb{R}^N} \| x \|_1 = \sum_{i=1}^{N} |x_i| & \text{s.t.} & y = Ax \\
\tilde{f} = F^H \tilde{x}.
\end{cases}
\]

When a traveler reaches a fork in the road, the $l_1$-norm tells him to take either one way or the other, but the $l_2$-norm instructs him to head off into the bushes. [Claerbout and Muir, 1973]

Recovery for Gaussian matrices when [Donoho and Tanner ‘06]

\[ n = k \times 2 \log(N/k) \]

For arbitrary measurement sparsity bases [Candes, Romberg & Tao ‘06]

\[ n = \mu^2 k \times \log N \]

for certain conditions on the matrix and sampling ....
Phase diagrams
11 solver [from Donoho et al ‘06]

In the white region

\[ \hat{x} = \text{arg min}_x \|x\|_1 \quad \text{subject to} \quad Ax = y \]

recovers exactly.
Phase transition

Has a second-order phase transition at an oversampling of 5

- transition becomes sharper for $n \to \infty$
- conceptual but unexplored ‘link’ with percolation theory
- $k$-neighborhoodness of polytopes undergoes a phase transition
2-D curvelets

- Curvelets are of rapid decay in space.
- Curvelets are strictly localized in frequency.

Oscillatory in one direction and smooth in the others!
3-D curvelets

Curvelets live in wedges in the 3 D Fourier plane...
Model

spatial sampling: 12.5 m
Data

20% traces remaining

avg. spatial sampling: 62.5 m
Interpolated result using CRSI

SNR = 9.26 dB

spatial sampling: 12.5 m
Difference
(no minimum velocity constraint)

SNR = 9.26 dB

spatial sampling: 12.5 m
New paradigm

Traditional data collection & compression paradigm
- ‘over emphasis’ on data collection
- extract essential features
- throw away the rest ....

New paradigm compression during sampling
- project onto measurements that breaks aliases
- recover with sparsity promotion

Exploration seismology
- ‘random’ sampling of seismic wavefields [Hennenfent & F.J.H ‘06]
- compressive wavefield extrapolation where eigenfunctions of the Helmholtz operator are used as the measurement basis [Lin & F.J.H ‘07]
Characterizing singularities

joint work Mohammad Maysami

“Seismic reflector characterization by a multiscale detection-estimation method” ‘07
Problem

• Delineate the stratigraphy from seismic images.
• Parameterize seismic transitions
  • beyond simplistic reflector models
  • consistent with observed intermittent behavior of sedimentary records
• Estimate the parameters from seismic images:
  • location
  • singularity order
  • instantaneous phase
Singularity characterization through waveforms
*[F.J.H ’98, ’00, ’03, ‘07]*

- generalization of zero- & first-order discontinuities
- measures wigglyness / # oscillations / sharpness
Singularity characterization through waveforms

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- generalization of zero- & first-order discontinuities
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Approach

[Wakin et al ‘05–‘07, M&H ‘07]

- Use a detection-estimation technique
  - multiscale detection => segmentation
  - multiscale Newton technique to estimate the parameterization
- Overlay the image with the parameterization
Seismic trace

CWT

Modulus of CWT & MMLs

Scale index

Location
Singularity map
Estimated alpha

Offset [m]
2.5
2.6
2.7
2.8
2.9
3.0
3.1
3.2

Time [s]

Estimated alpha
-6.4
-5.6
-4.8
-4.0
-3.2
-2.4
-1.6
-0.8
0.0

Courtesy
CGG Veritas
Observations

- Stratigraphy is detected
- Parameterization provides information on the lithology
  - evidence of changes in exponents along clinoforms
- Method suffers from curvature in the imaged reflectors
- Extension to higher dimensions necessary
- Model that explains different types of transitions
- A step beyond the zero- & first-order discontinuities
Modeling seismic singularities

Joint work with Yves Bernabe (MIT)

“Seismic singularities at upper-mantle phase transitions: a site percolation model” GJI ‘04
Problem

Earth subsurface is highly heterogeneous

- sedimentary crust, upper-mantle transition zone & core-mantle boundary

Smooth relation volume fractions and the transport properties.

Homogenization/equivalent medium (EM) theory *smoothes* the singularities during *upscaling*

- relatively easy for *volumetric* properties (density)
- *notoriously difficult* for *transport* properties (velocity)

Q: How to preserve singularities in effective properties?
Our approach

Include connectivity in models for the effective properties of bi-compositional mixtures $\leftrightarrow$ SWITCH

Start with binary mixtures, e.g.

- sand-shale
- gas-hydrate, Opal-Opal CT
- upper-mantle mineralogy

Studied two cases:

- elastic properties upper mantle (H & B ’04)
- fluid-flow properties synthetic rock (B & H ’04)
Mixture laws for binary mixtures

Elastic case:

• Controlled by \textit{connectivity} of stiffest phase

Fluid-flow case:

• Controlled by \textit{connectivity} of high-conductivity phase

Note: Stiff phase = low porosity, low conductivity phase

No obvious link elastic-fluid flow properties ...
Singularity modeling
binary mixtures

LP  olivine

HP  β-spinel

Varying composition binary mixture

Site percolation

random process

$p_c < 0.59$

$p_c = 0.59$

$p_c > 0.59$

elastic properties

volume fraction

HS+

HS−
Singularity modeling
binary mixtures

HP  β-spinel

LP  olivine

Site percolation

random process

p_c < 0.59
p_c = 0.59
p_c > 0.59

Varying composition binary mixture

elastic properties

volume fraction

HS+

HS−
Singularity modeling
binary mixtures

LP  blue  olivine

Varying composition binary mixture

Site percolation
random process

HP  red  β-spinel

elastic properties

volume fraction

HS+
HS−
Mixing model

Homogeneous mixing (e.g., solid solution) of two phases (LP weak and HP strong) can only produce gradually varying elastic properties.

If Heterogeneous (e.g. emerging random macroscopic inclusions) mixing, then a singularity in the elastic properties must arise at the depth where the strong, HP phase becomes connected (observed in binary alloys).
Site-percolation model

Assume volume fractions $p$ and $q = 1 - p$, are linear functions of depth $z$.

At a critical depth $z_c$, which corresponds to the percolation threshold $p_c = p(z_c)$, an "infinite", connected HP cluster is formed.

For $z \geq z_c$

- not all HP inclusions belong to the infinite cluster.
- isolated HP inclusions can still be found, embedded in the remaining LP material and forming with it a mixture ($M$).
Site-percolation model

Above $z_c$ we have a weak LP matrix containing randomly distributed, non-percolating, strong HP inclusions.

Below $z_c$, a strong HP skeleton is intertwined with the weaker, mixed material $M$. 
Site-percolation model

Volume fraction $p^*$ of HP material that belongs to the infinite cluster

- is zero for $p < p_c$ (i.e., above $z_c$)
- has a power-law dependence on $(p - p_c)$ for $p \geq p_c$.

Hence, $p^*$ is given by:

$$p^* = p \left( \frac{p - p_c}{1 - p_c} \right) ^ \beta$$
Site-percolation model

Mixed $M$ is given by $q^* = (1 - p^*)$.

For $M$, we need the volume fractions of its LP and HP parts,

$$q_M = (1 - p)/(1 - p) + (p - p^*)$$

$$p_M = (1 - q_M),$$

yielding

$$p_M = 1 - q \frac{q}{1 - p \left( \frac{p-p_c}{1-p_c} \right)^\beta}$$
Site-percolation model

Binary mixture:

- **Strong** when its strong component is connected.
- **Weak otherwise.**

Assume *locally isotropic*

- Bin. mixtures bounded by Hashin-Shtrikman (HS).
- *Upper* HS bound when strong component connects, the *lower* one applies otherwise.

 Bulk modulus $K$ of the co-existence region above $z_c$ is given by the *lower* HS bound:

$$K = K_{LP} \left(1 + \frac{p(K_{HP} - K_{LP})}{q(K_{HP} - K_{LP})a_{LP} + K_{LP}}\right)$$
Below $z_c$ we must switch to the higher HS bound:

$$K = K_{HP} \left( 1 + \frac{q^*(K_M - K_{HP})}{p^*(K_M - K_{HP})a_{HP} + K_{HP}} \right)$$

Since the HP inclusions in M are isolated, $K_M$ is calculated using the lower HS bound:

$$K_M = K_{LP} \left( 1 + \frac{p_M(K_{HP} - K_{LP})}{q_M(K_{HP} - K_{LP})a_{LP} + K_{LP}} \right)$$
Site-percolation model

Major consequence of this model is that it predicts:

• a $\beta$-order, cusp-like singularity in the elastic moduli as the critical depth $z_c$ is approached from below (instead of a first-/zero-order discontinuity).

• Singularities that persist for vanishing contrasts.

• Density that does not behave singularly.
Elastic contrasts between LP and HP are small:

• Nearly coincident HS bounds.

• Excessively small contrasts.

Discard isotropy assumption:

• Horizontally-oriented oblate ellipsoidal inclusions which coalesce below \( z_c \) into long, vertical dendrites, leaving prolate M inclusions between them.

• Transversely isotropic structure.
Site-percolation model

Near normal incidence, $V_p$ and $V_s$ approach limiting values as the aspect ratio is goes to zero.

Same as replacing lower and higher HS bounds by Reuss and Voigt averages:

\[
K = \left( \frac{q}{K_{LP}} + \frac{p}{K_{HP}} \right)^{-1}
\]

(for $z < z_c$)

\[
K = q \cdot K_M + p \cdot K_{HP}
\]

(for $z \geq z_c$)

\[
K_M = \left( \frac{q_M}{K_{LP}} + \frac{p_M}{K_{HP}} \right)^{-1}
\]
Singularity model
Singularity model
upper-mantle transitions
Modeled data vs seismic

(a) CAN 410

(b) CAN 660

(c) Modelled

(d) Data

Depth km
Singularity-preserved upscaling

Joint work with Yves Bernabe (MIT)
The problem

• Equivalent medium based upscaling washes out the singularities

• Reflection seismology lives by virtue of singularities in the elastic moduli (transport properties)

• Propose a singularity preserving upscaling method:
  • upscales the lithology rather than the velocities

• Singularities can be due to sharp changes in composition or due to the switch ...
Switch vs no switch

![Graph showing velocity from lithology](attachment://graph.png)

- **Equivalent medium**
- **Percolation model**
Switch vs no switch

![Graph showing the comparison between switch and no switch scenarios. The y-axis represents velocity, and the x-axis represents volume fraction of shale. The graph includes two lines: Equivalent medium (dotted blue) and Percolation model (dashed red). A singularity point is indicated.](image_url)
Lithological upscaling

Upscaling of the lithology (volume fractions)
EM-upscaled reflectivity

Reflectivity for the Equivalen medium model

lost singularities
Perco.-upscaled reflectivity

Reflectivity for the Percolation model

preserved singularities
Relation to fluid flow
&
open problems

joint work with Yves Bernabe (MIT)
Sedimentary crust

What does this model buy us? Some insight in

- the **complexity** of transitions
- the creation of a **singularity** for smooth varying **composition**, e.g. when **clay lenses connect** ...
- the **morphology** at transitions
- linking **elastic** and **fluid** properties remains a **challenge** ....
Fluid flow

Connectivity of the high conductive phase

measured

- difficult to model
- difficult to measure

modelled

[B&H ‘04]
Steady-state flow equation:

\[
\frac{\partial}{\partial x} \left( \frac{k}{\mu} \frac{\partial P_p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{k}{\mu} \frac{\partial P_p}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{k}{\mu} \frac{\partial P_p}{\partial z} \right) = 0
\]
Sand-clay model
according HB model [HB ‘04]

velocities <=> permeability
Elastic versus Fluid

- **singularities in elastic** properties are **small**

- **singularities in fluid** properties are **large**

- **waves** are way more **sensitive** to singularities than **diffusion** driven **fluid** flow

- models for fluid and elastic transport are not integrated

- incorporate in Biot?
Conclusions

• Multiscale compressible signal representations that exploit higher-dim. geometry are indispensable for acquiring accurate information on the imaged waveforms.

• Imaged waveforms carry information on the fine structure of the reflectors.

• Percolation model provides an interesting perspective
  • on linking the micro-connectivity to singularities detected by waves
  • on providing an upscaling that preserves features = singularities that matter ....
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