Seismic data processing with curvelets: a multiscale and nonlinear approach

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joint work with Deli Wang, Gilles Hennenfent and Peyman Moghaddam

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Motivation

Exploit *two* aspects of curvelets, namely their

- parsimoniousness
- invariance under certain operators

Formulate

- *non-adaptive* wavefield reconstruction algorithms
- *data-adaptive* matching algorithms

Applications

- *nonlinear* sampling theory for wavefields
- *nonlinear* migration-amplitude recovery
- *nonlinear* primary-multiple separation

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Approach

Employ parsimoniousness by sparsity promotion.

Exploit behavior of certain operators in phase space

- diagonalization <=> curvelet domain *scaling*
- smoothness <=> *structure* of phase space

Combine *parsimoniousness* with *structure* in phase space

- *diagonal* approximation operators
- *stable* amplitude recovery
- improved *adaptive* separation

Migration-amplitude recovery methods are based on

- diagonal approximation of Pseudo's
- estimate *scaling* from a *reference* vector and demigrated-migrated *reference* vector
  - Illumination-based normalization (Rickett '02)
  - Amplitude corrections (Guitton '04)
  - Amplitude scaling (Symes '07)

Primary-multiple separation methods are based on

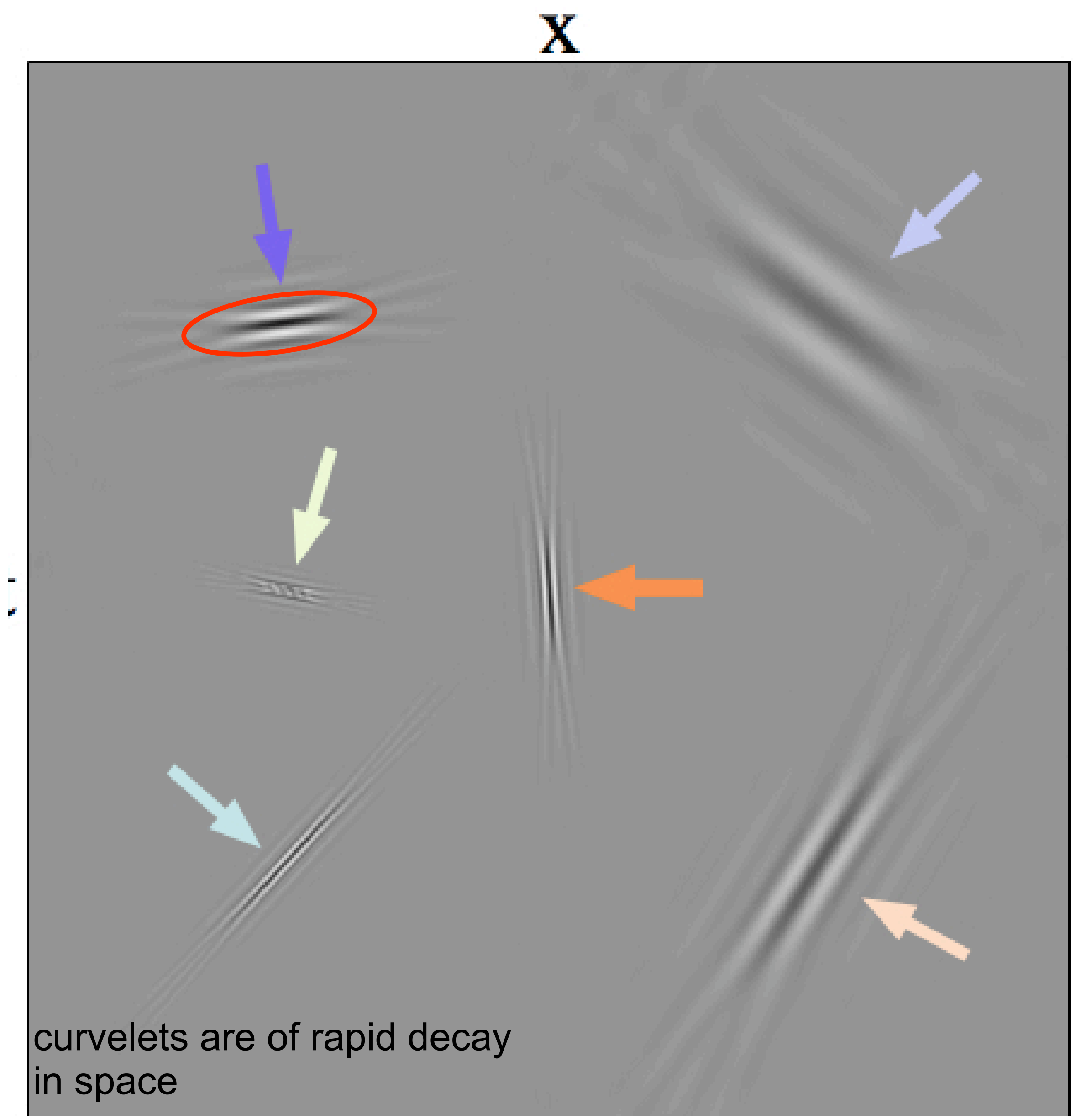
- diagonal approximation in the Fourier domain
- estimate *scaling* from mismatch pred. multiples & data
  - adaptive subtraction (Verschuur and Berkhout '97)

We are interested in a formulation that

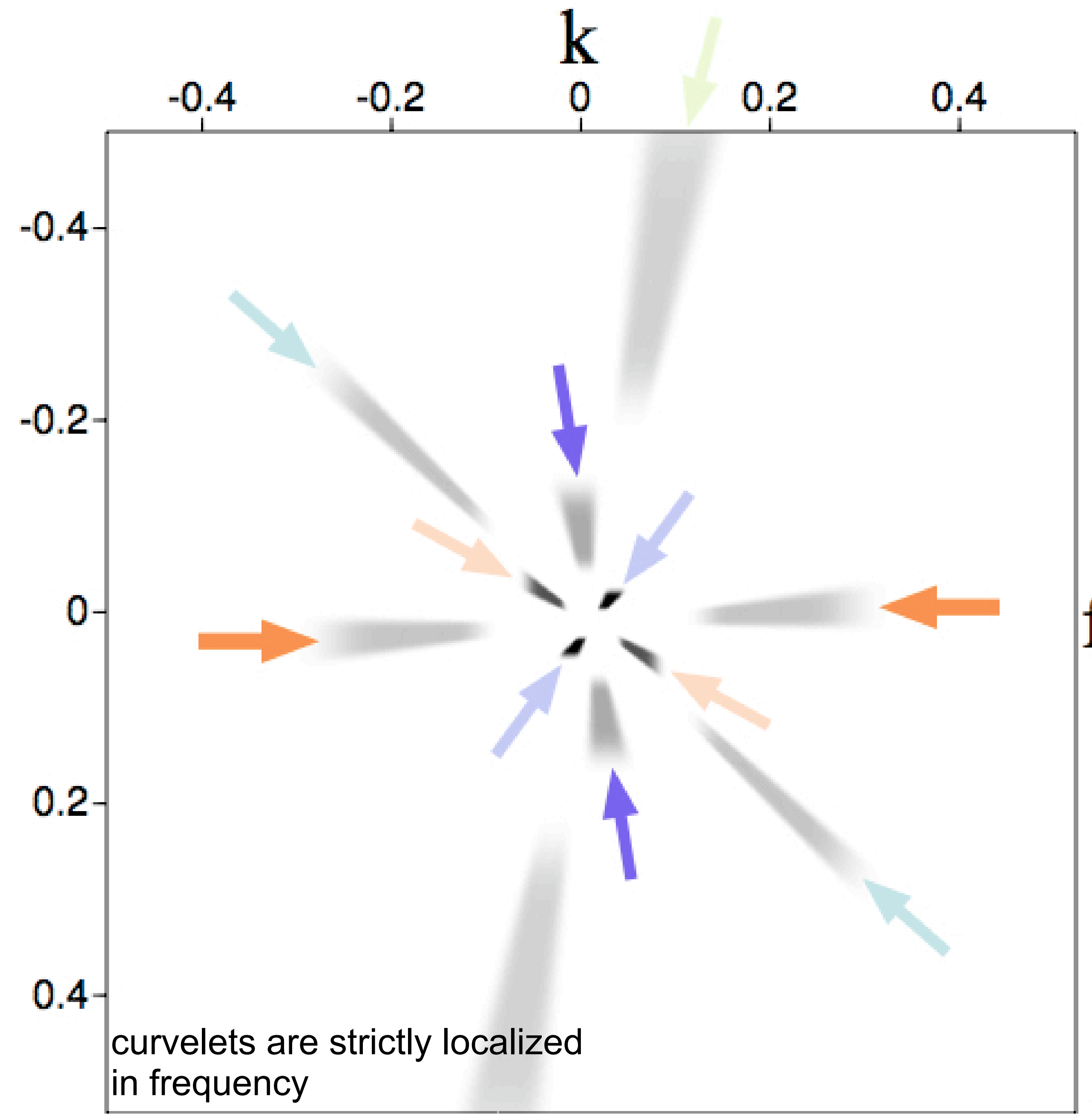
- estimates the scaling with smoothness control
- prevents overfitting
- allows for conflicting dips
- incorporates curvelet-domain sparsity promotion

The curvelet transform

2-D curvelets



x-t

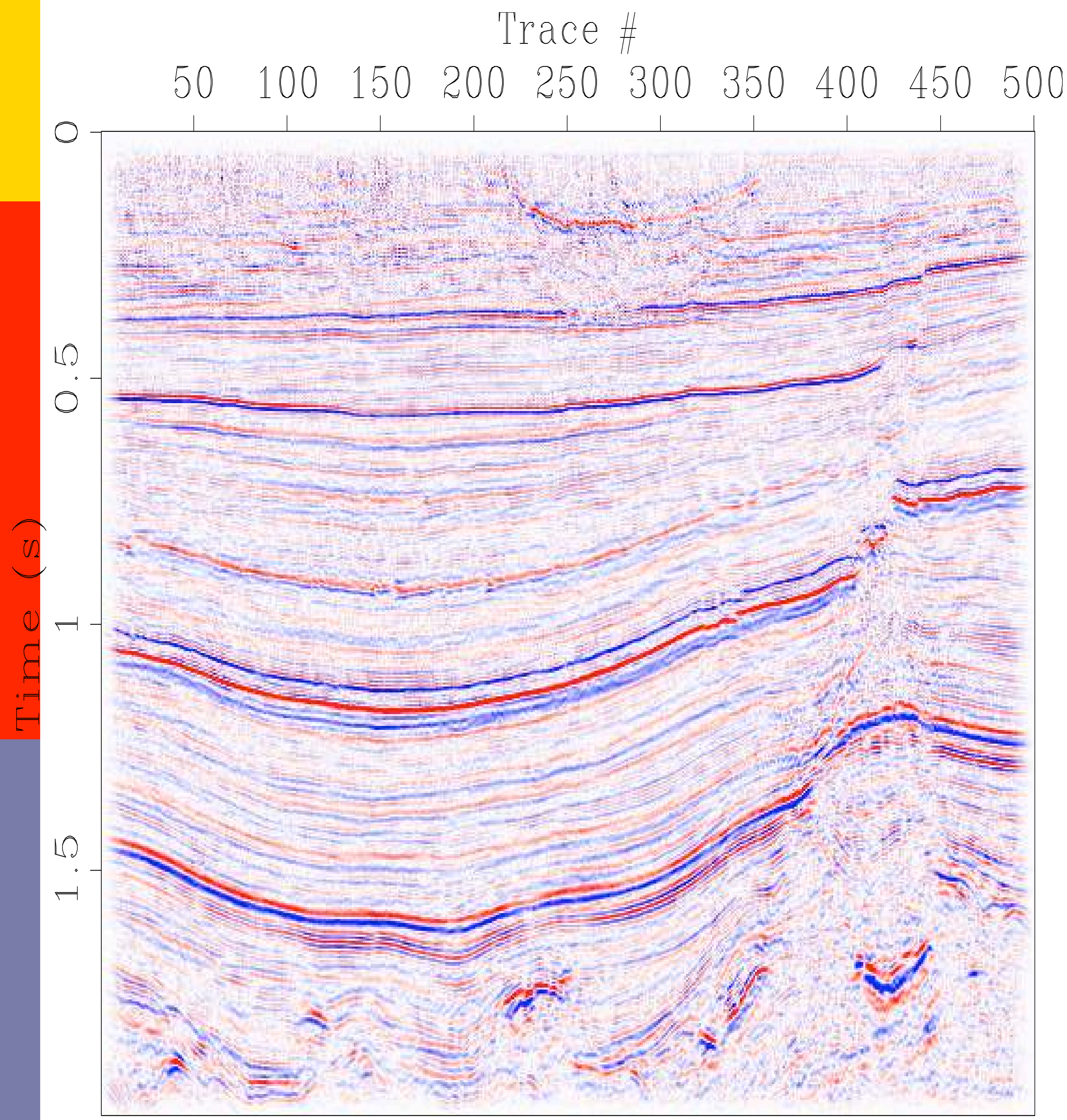


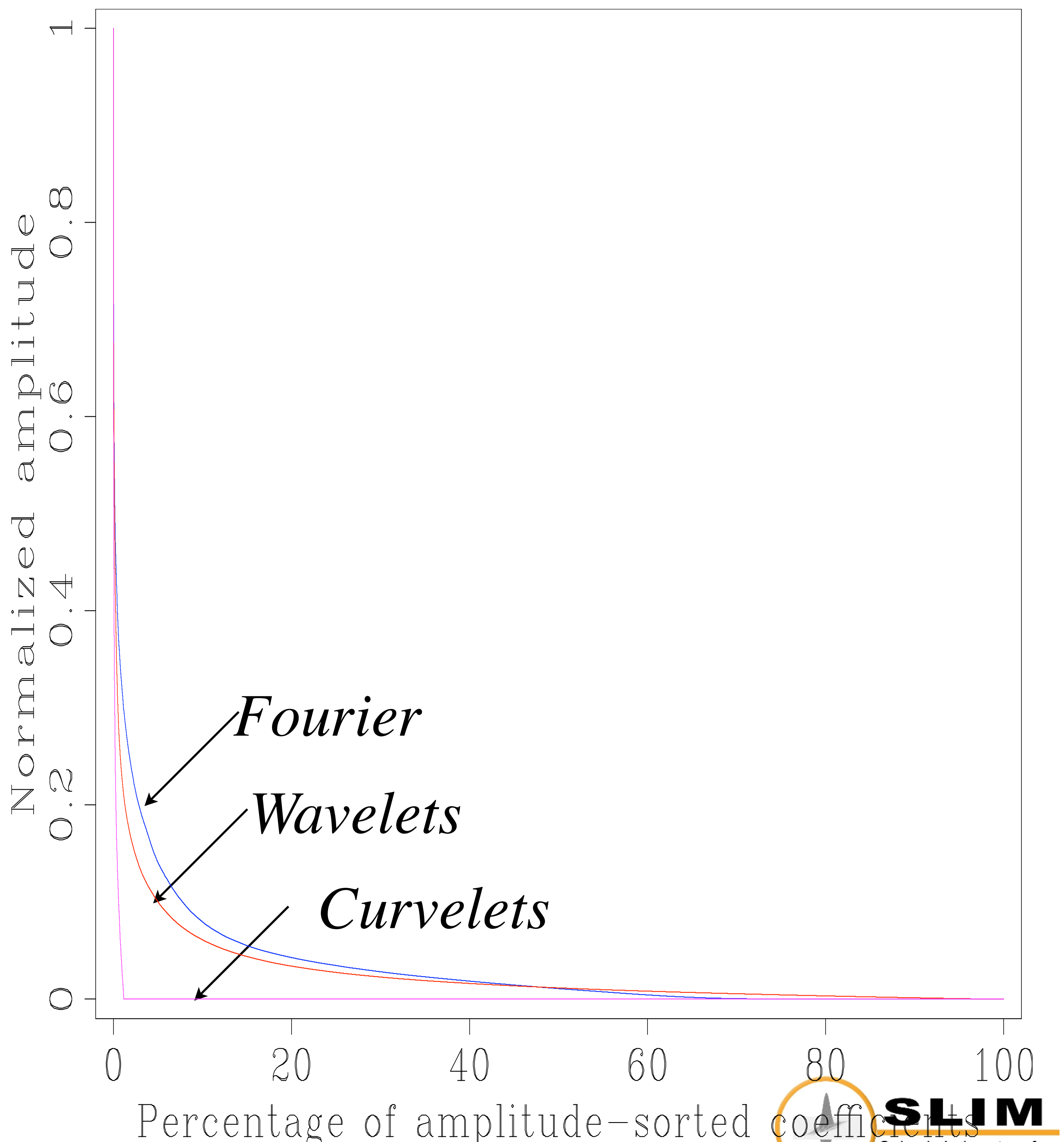
f-k

Oscillatory in one direction and smooth in the others!

Obey *parabolic* scaling relation $\mathrm{length} \approx \mathrm{width}^2$

Coefficients Amplitude Decay In Transform Domains





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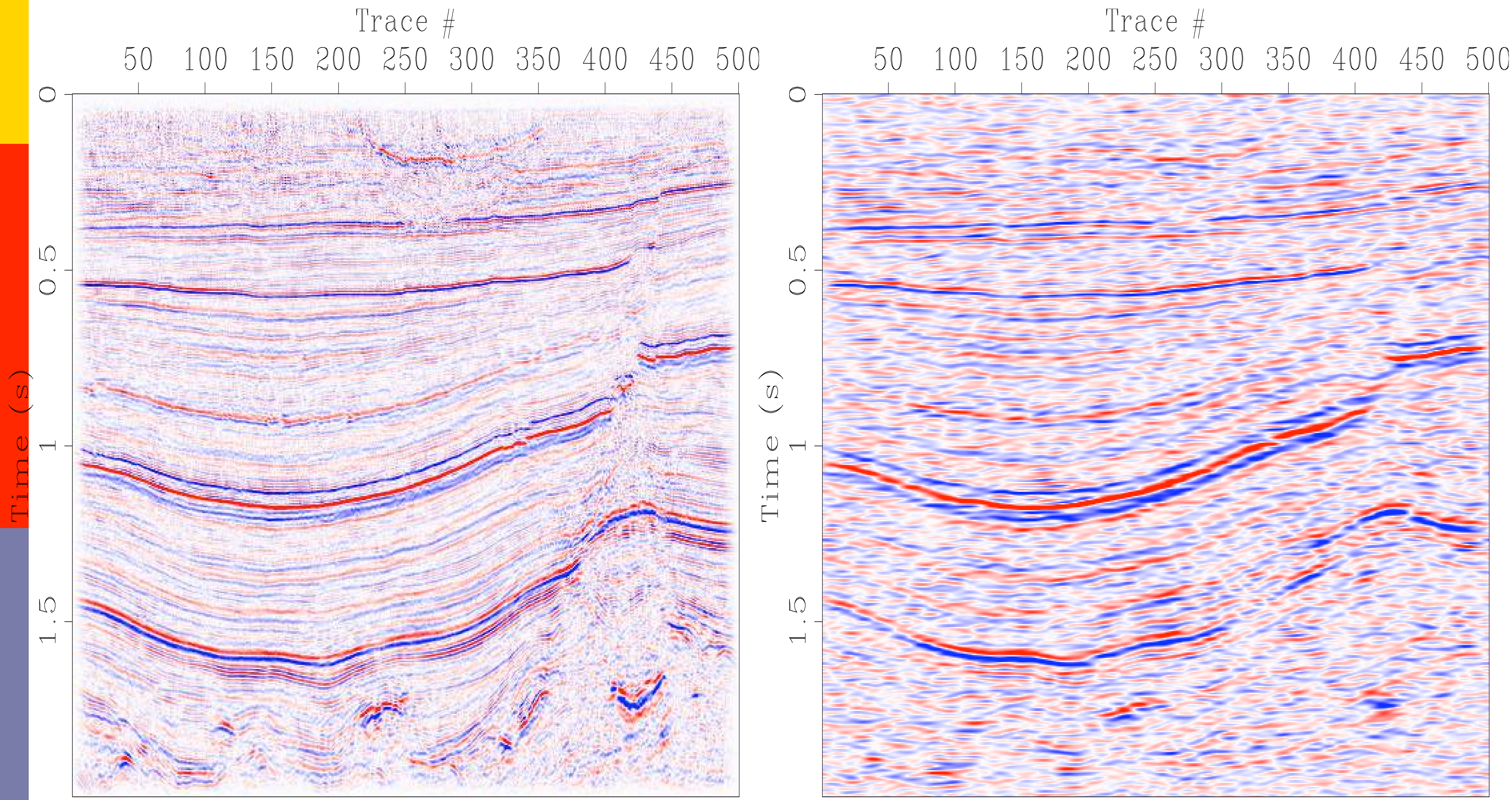
Partial Reconstruction
Fourier (1% largest coefficients)



SNR = 2.1 dB

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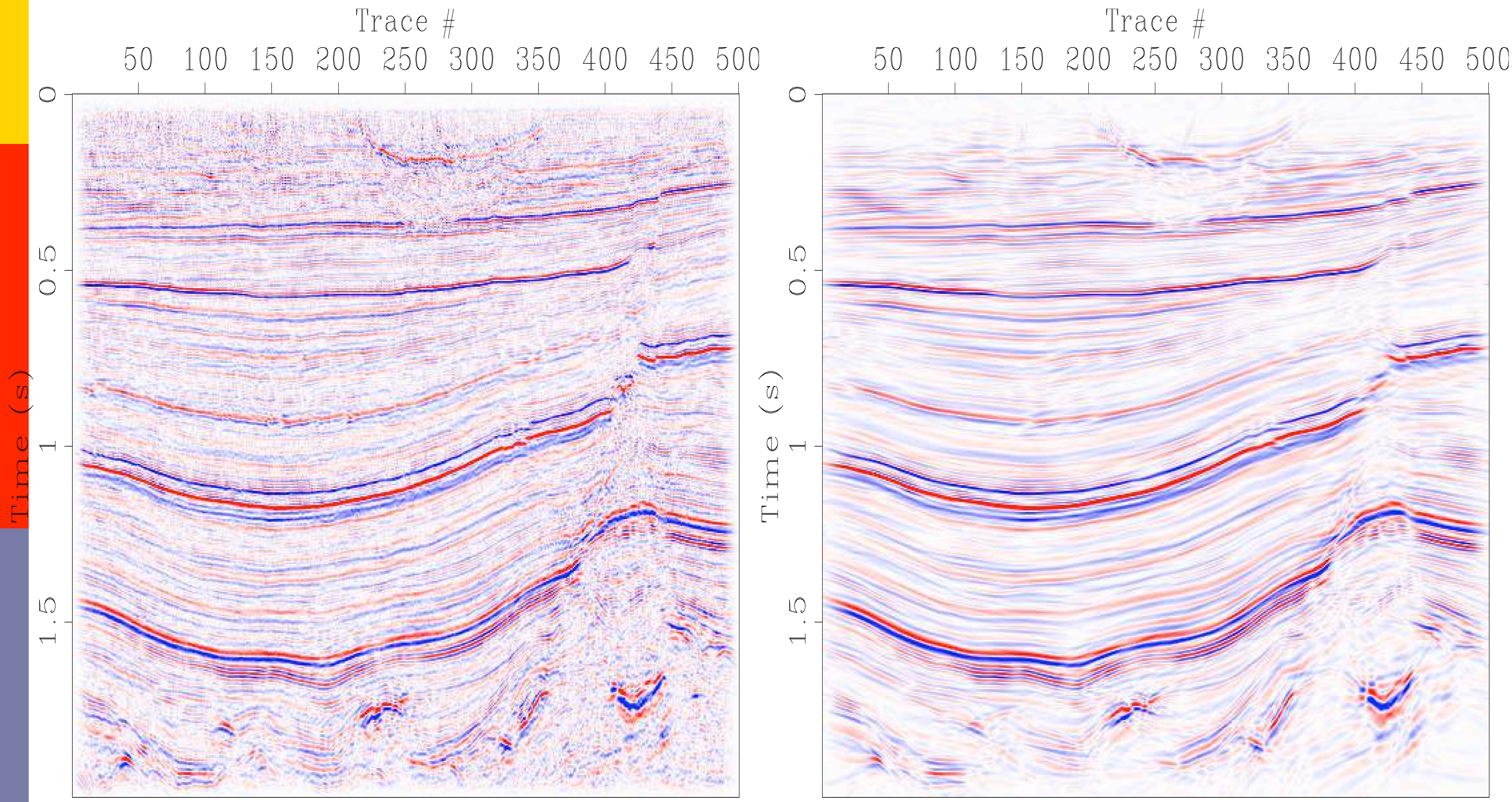
Partial Reconstruction
Curvelets (1% largest coefficients)



SNR = 6.0 dB

Non-adaptive curvelet-domain sparsity promotion

Linear quadratic (lsqr):

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_2 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon$$

- **model Gaussian**

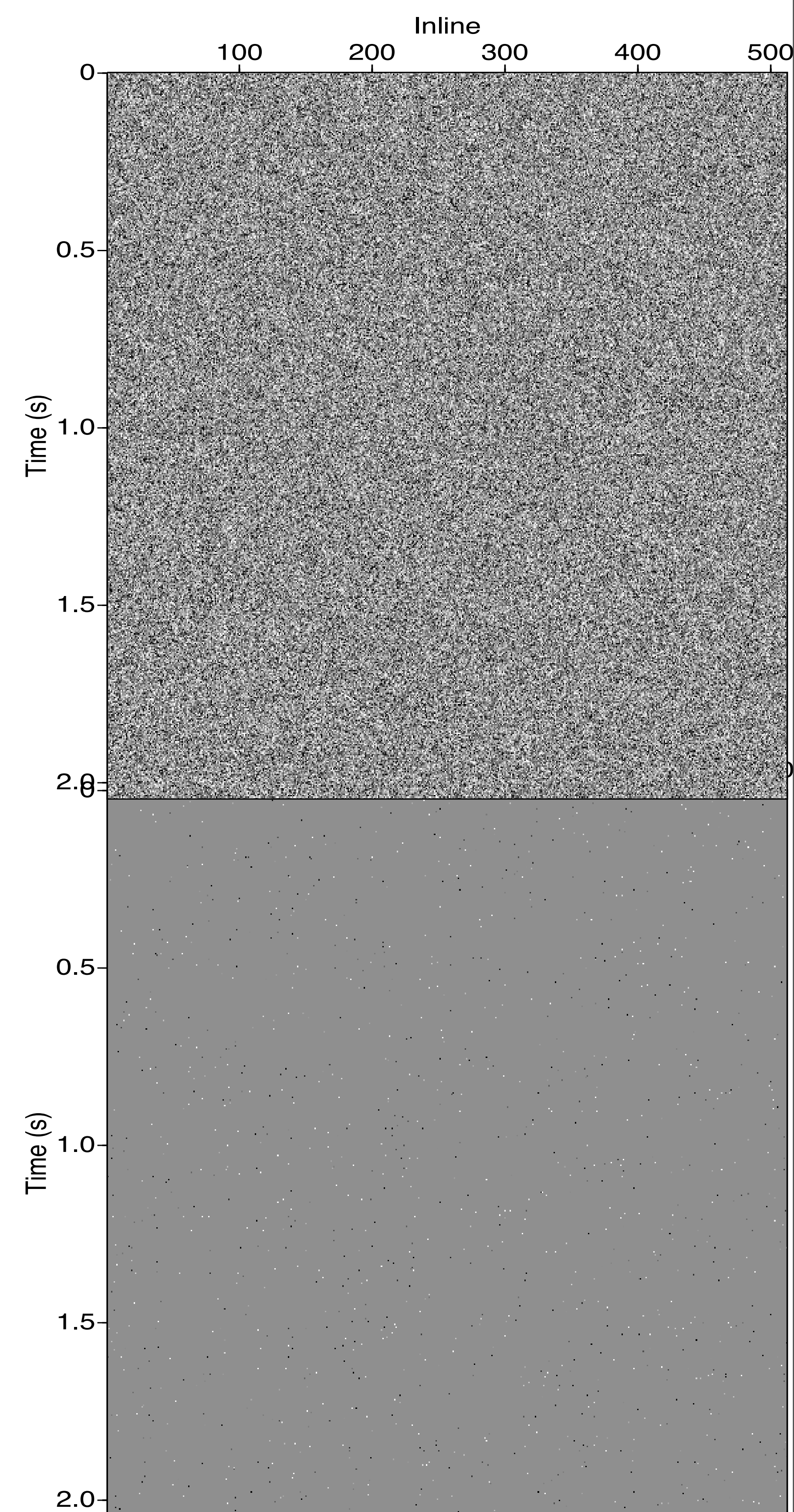
Non-linear:

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon$$

- **model Cauchy (sparse)**

Problem:

- **data does not contain point scatterers**
- **not sparse**



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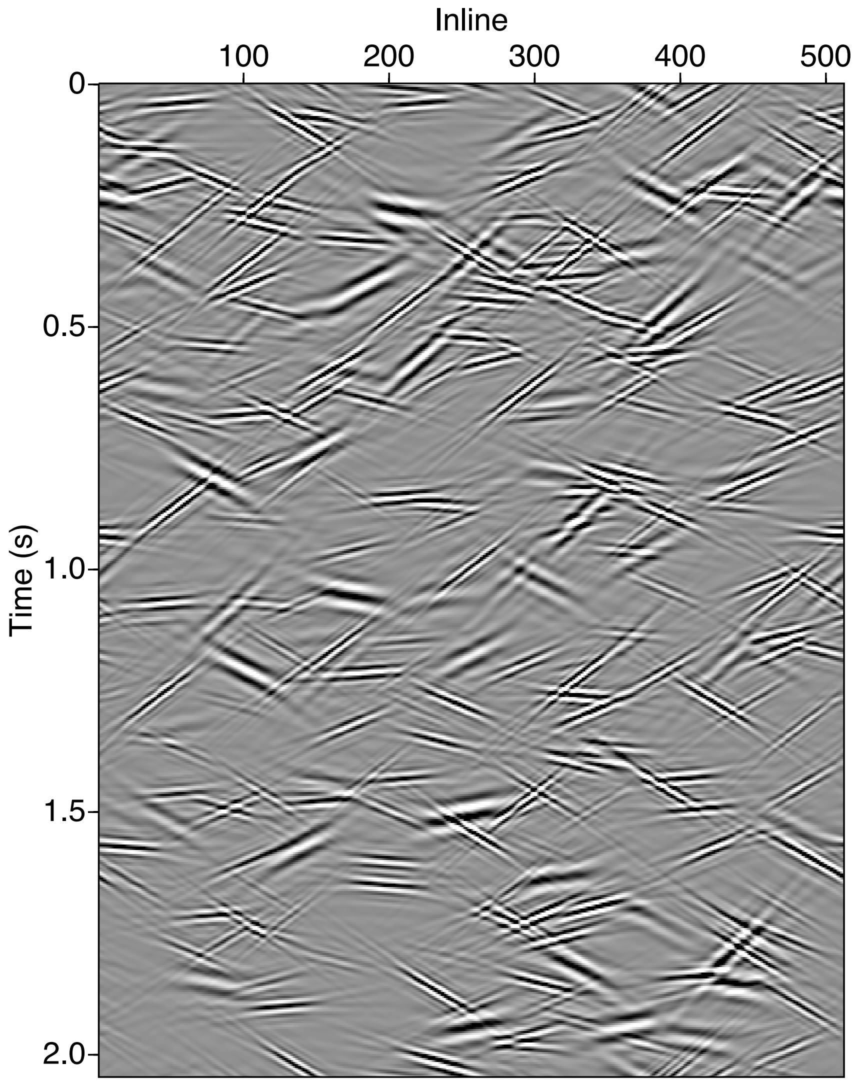
Our contribution

Model as superposition of little plane waves.

Compound ***modeling*** operator with curvelet ***synthesis***:

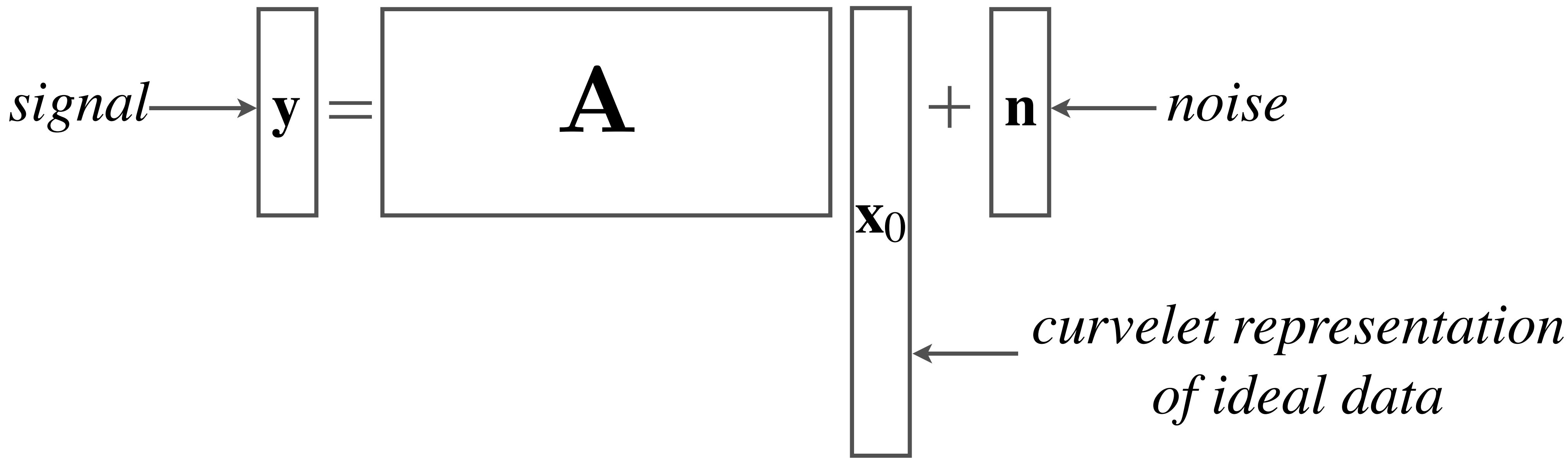
$$
\begin{aligned}
\mathbf{K} &\mapsto \mathbf{K}\mathbf{C}^T\\
\mathbf{m}_0 &\mapsto \mathbf{x}_0\\
\tilde{\mathbf{m}} &= \mathbf{C}^T\tilde{\mathbf{x}}
\end{aligned}
$$



Exploit ***parsimoniousness*** of curvelets on seismic data & images ...

Sparsity-promoting program

Problems boil down to solving for $\mathbf{x}_0$



with

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = \mathbf{C}^T \tilde{\mathbf{x}} \end{cases}$$

- exploit sparsity in the curvelet domain as a prior
- find the sparsest set of curvelet coefficients that match the data, i.e., $\mathbf{y} \approx \mathbf{K}\mathbf{C}^T\tilde{\mathbf{x}}$
- invert an underdetermined system

Seismic wavefield reconstruction with CRSI

Sparsity-promoting inversion*

Reformulation of the problem

signal → $\mathbf{y} = \mathbf{R}\mathbf{C}^H \mathbf{x}_0 + \mathbf{n}$ ← *noise*

$\mathbf{x}_0$ ← *curvelet representation of ideal data*

Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI)

- look for the **sparsest/most compressible**, **physical** solution

KEY POINT OF THE RECOVERY

$$\mathbf{P}_\epsilon : \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \overbrace{\|\mathbf{W}\mathbf{x}\|_1}^{\text{sparsity constraint}} \quad \text{s.t.} \quad \overbrace{\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2}^{\text{data misfit}} \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{C}^T \tilde{\mathbf{x}} \end{cases}$$

* inspired by Stable Signal Recovery (SSR) theory by E. Candès, J. Romberg, T. Tao, Compressed sensing by D. Donoho & Fourier Reconstruction with Sparse Inversion (FRSI) by P. Zwartjes

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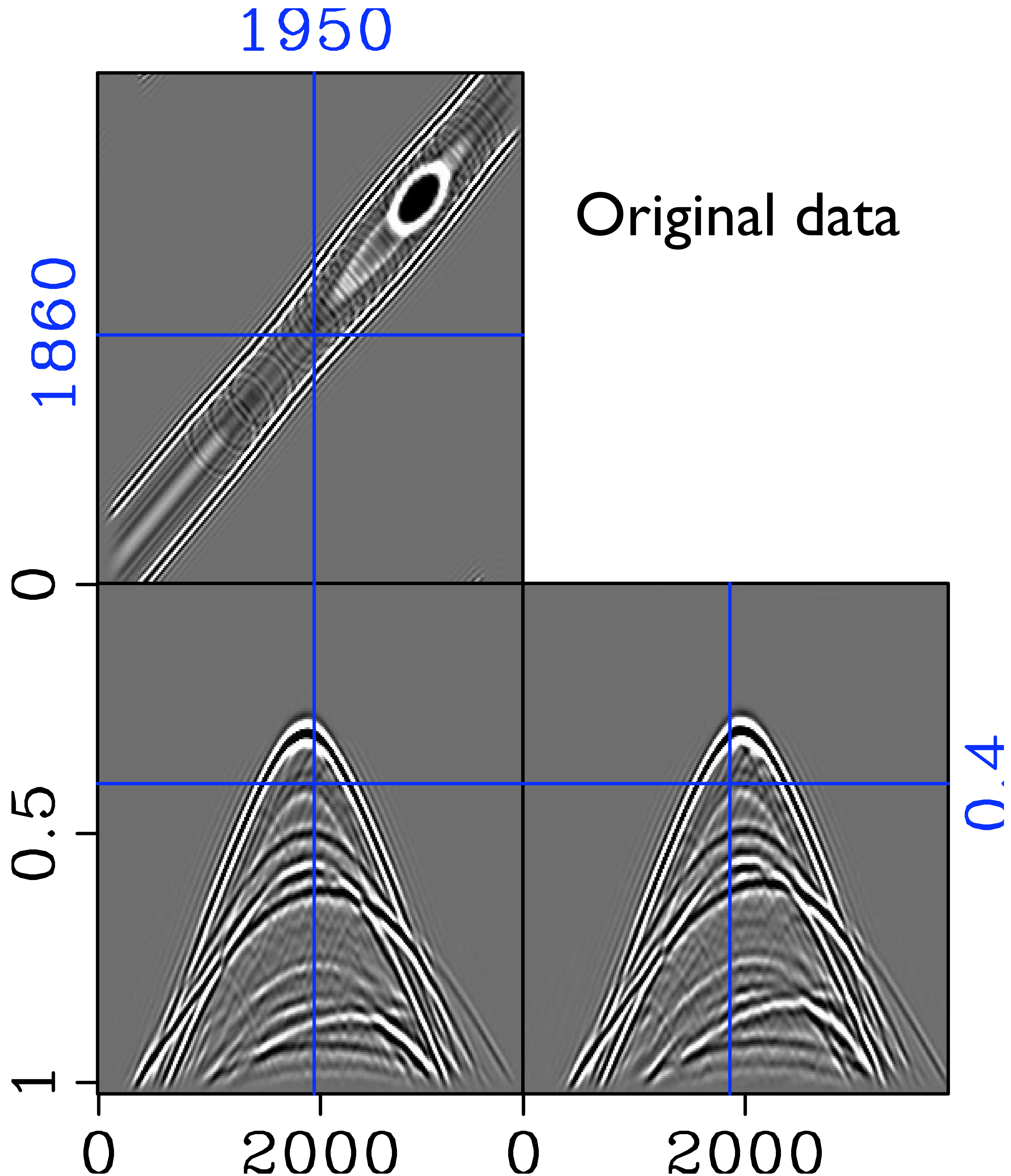
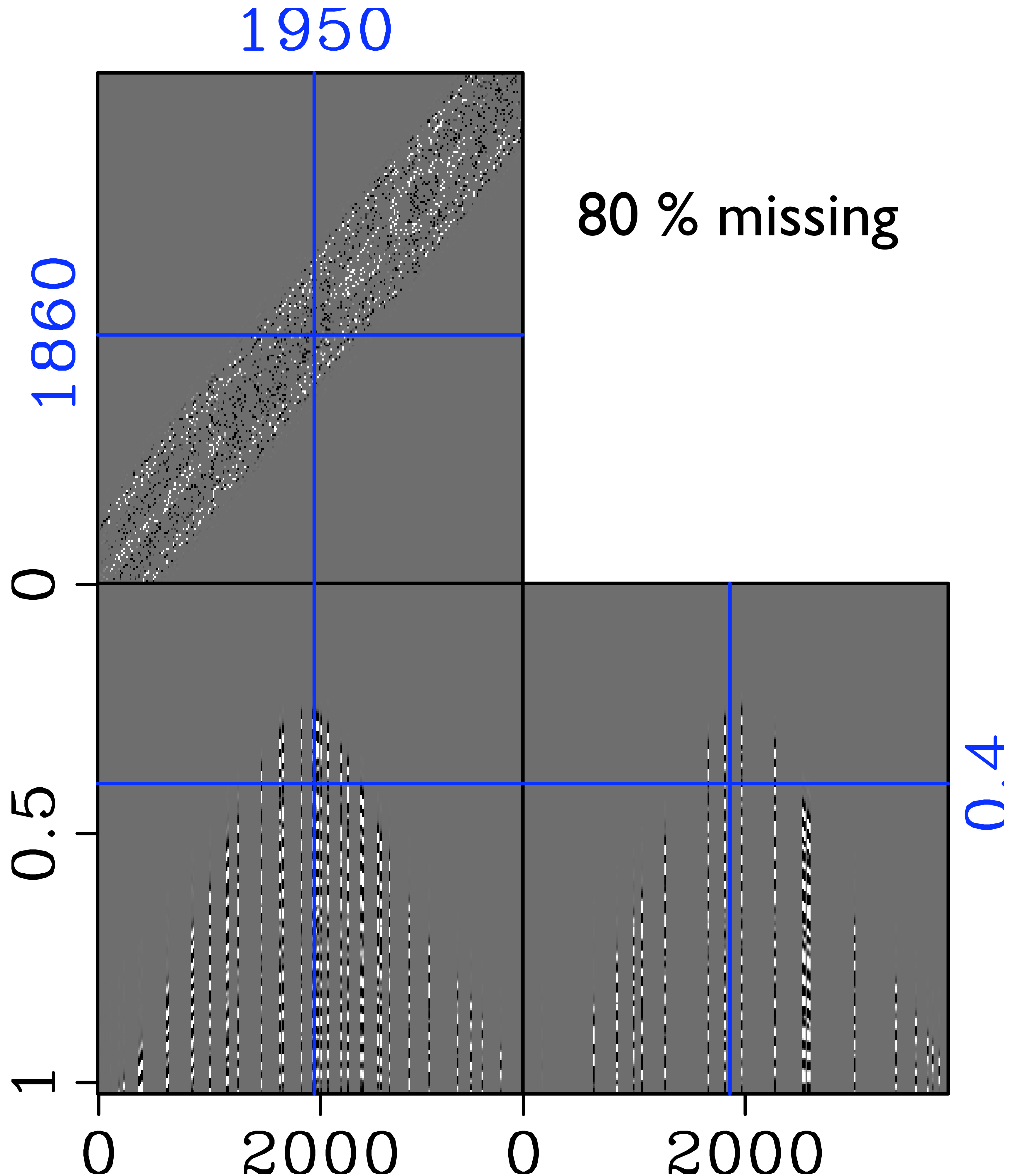
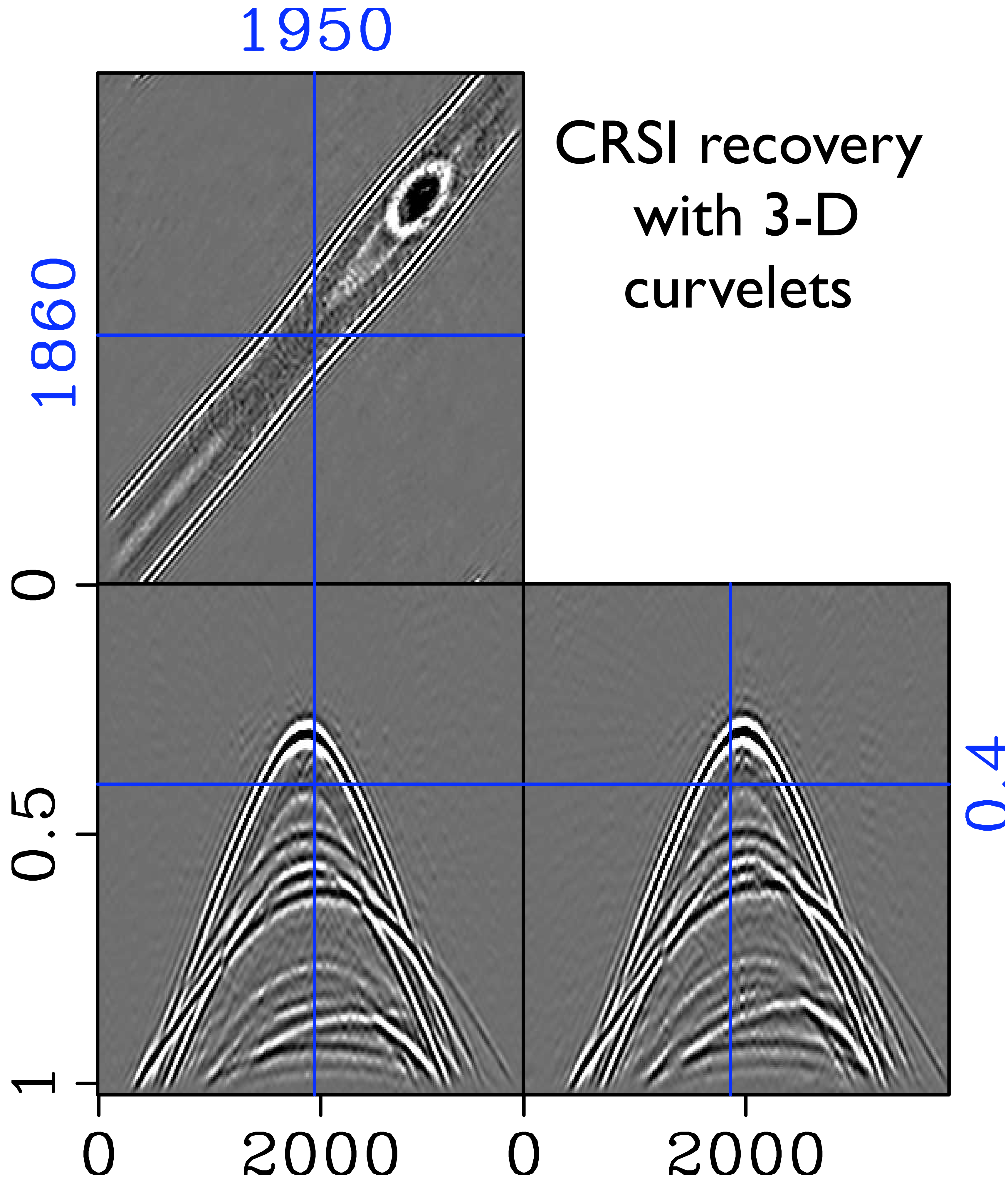

1950
1860
Original data
0
0.5
1
0.4
0
2000
0
2000


1950
1860
80 % missing
0
0.5
1
0.4
0
2000
0
2000



CRSI recovery
with 3-D
curvelets
1950
1860
0.4
0
0.5
1
0
2000
0
2000

Adaptive curvelet-domain matched filtering

Forward model

Linear model for amplitude mismatch:

$$(Bf)(x) = \int_{x\in\mathbb{R}^d} e^{jk\cdot x} b(x,k)\hat{f}(k)\mathrm{d}k$$

$$\begin{aligned} B &= \text{Pseudodifferential operator} \\ b(x,k) &= \text{the symbol} \end{aligned}$$

- spatially-varying dip filter
- zero-order Pseudo

After discretization

$$\mathbf{f} = \mathbf{B}\mathbf{g}$$

- linear operator
- **f** and **g** known
- matrix **B** is full and not known ....

Forward model

Diagonal approximation in the curvelet domain:

$$\begin{aligned} \mathbf{f} &= \mathbf{Bg} \\ &\approx \mathbf{C}^T \mathrm{diag}\{\mathbf{w}\}\mathbf{Cg} \end{aligned}$$

- curvelet domain scaling
- opens the way to an estimation of **w**

Examples:

	B	**f**	**g**
migration	$\mathbf{K}^T\mathbf{K}$	**migrated "image"**	**"reflectivity"**
multiple removal	**obliquity factor**	**total data**	**predicted multiples**

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Key idea

Problems with estimating **w**

- inversion of an *underdetermined* system
- *over* fitting
- *positivity* and reasonable *scaling* by **w**

Solution:

- use *smoothness* of the symbol
- formulate *nonlinear* estimation problem that minimizes

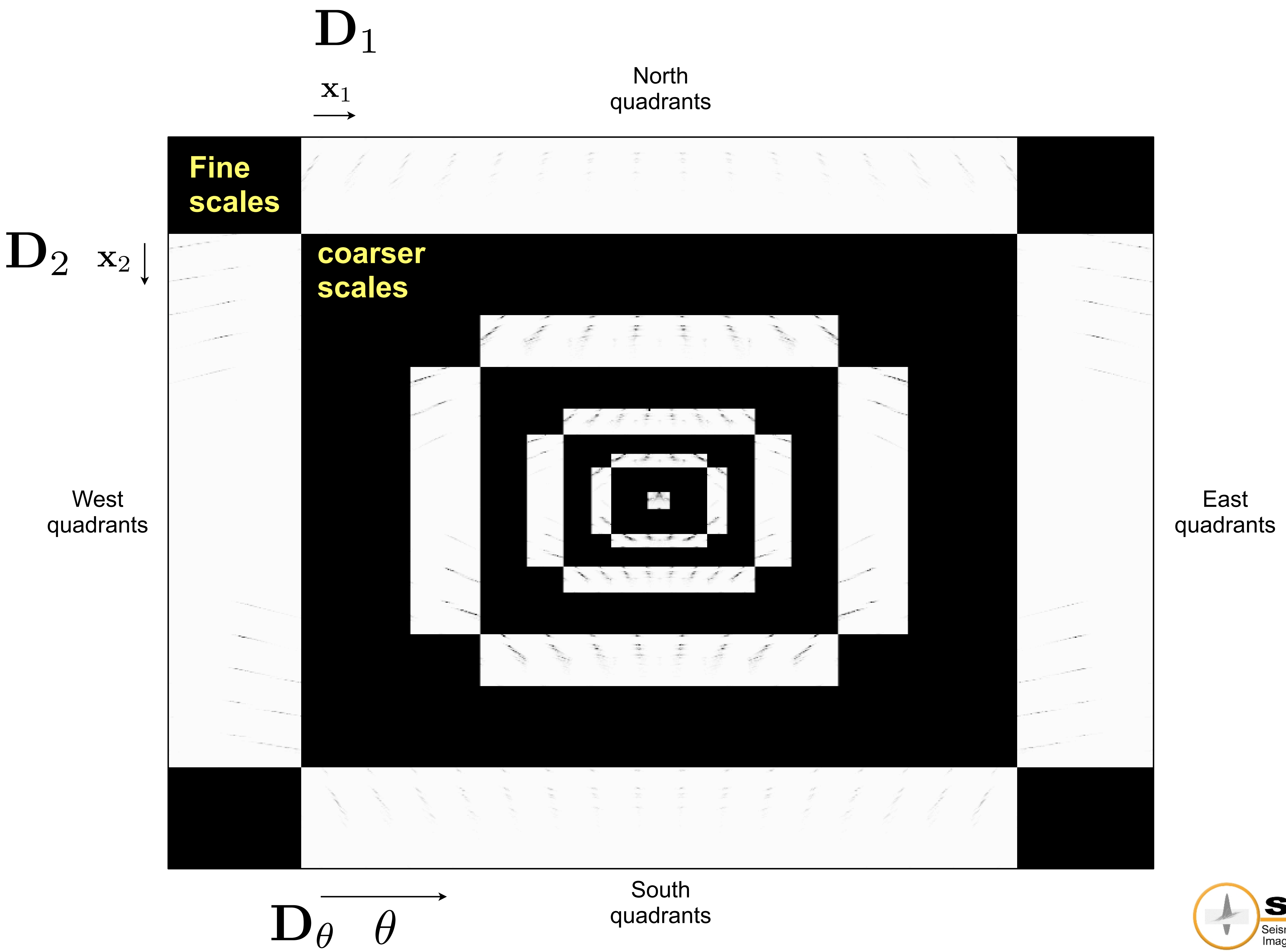
$$J_\gamma(\mathbf{z}) = \frac{1}{2}\|\mathbf{d} - \mathbf{F}_\gamma e^{\mathbf{z}}\|_2^2,$$

with

$$\mathrm{grad} J(\mathbf{z}) = \mathrm{diag}\{e^{\mathbf{z}}\}\left[\mathbf{F}^T\left(\mathbf{F}e^{\mathbf{z}} - \mathbf{d}\right)\right]$$

- solve with l-BFGS

Key idea



$\mathbf{D}_1$
$\mathbf{x}_1$
North quadrants
$\mathbf{D}_2$
$\mathbf{x}_2$
Fine scales
coarser scales
West quadrants
East quadrants
South quadrants
$\mathbf{D}_\theta$
θ

Key idea

Impose *smoothness* via following system of equations

$$\begin{aligned} \mathbf{f} &= \mathbf{C}^T \mathrm{diag}\{\mathbf{C}\mathbf{g}\}\mathbf{w} \\ \mathbf{0} &= \gamma \mathbf{L}\mathbf{w} \end{aligned}$$

with

$$\mathbf{L} = \begin{bmatrix} \mathbf{D}_1^T & \mathbf{D}_2^T & \mathbf{D}_\theta^T \end{bmatrix}^T$$

first-order differences in *space* and *angle* directions for each *scale*. Equivalent to

$$\tilde{\mathbf{w}} = \arg\min_{\mathbf{w}} \frac{1}{2}\|\mathbf{b} - \mathbf{P}[\mathbf{w}]\|_2^2 + \gamma^2\|\mathbf{L}\mathbf{w}\|_2^2$$

with

$$\mathbf{P} = \mathbf{C}^T \mathrm{diag}\{\mathbf{C}\mathbf{g}\}$$

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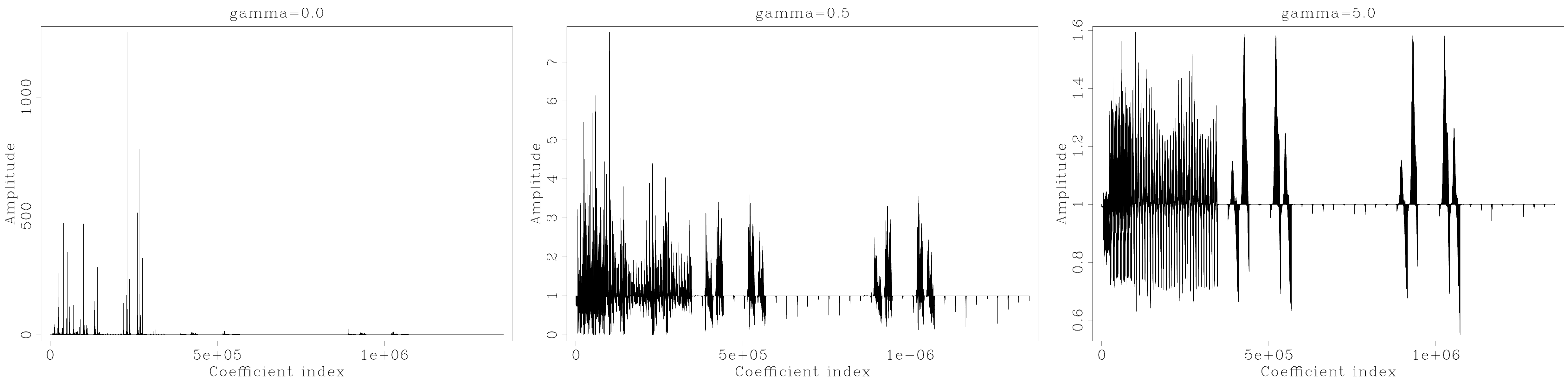
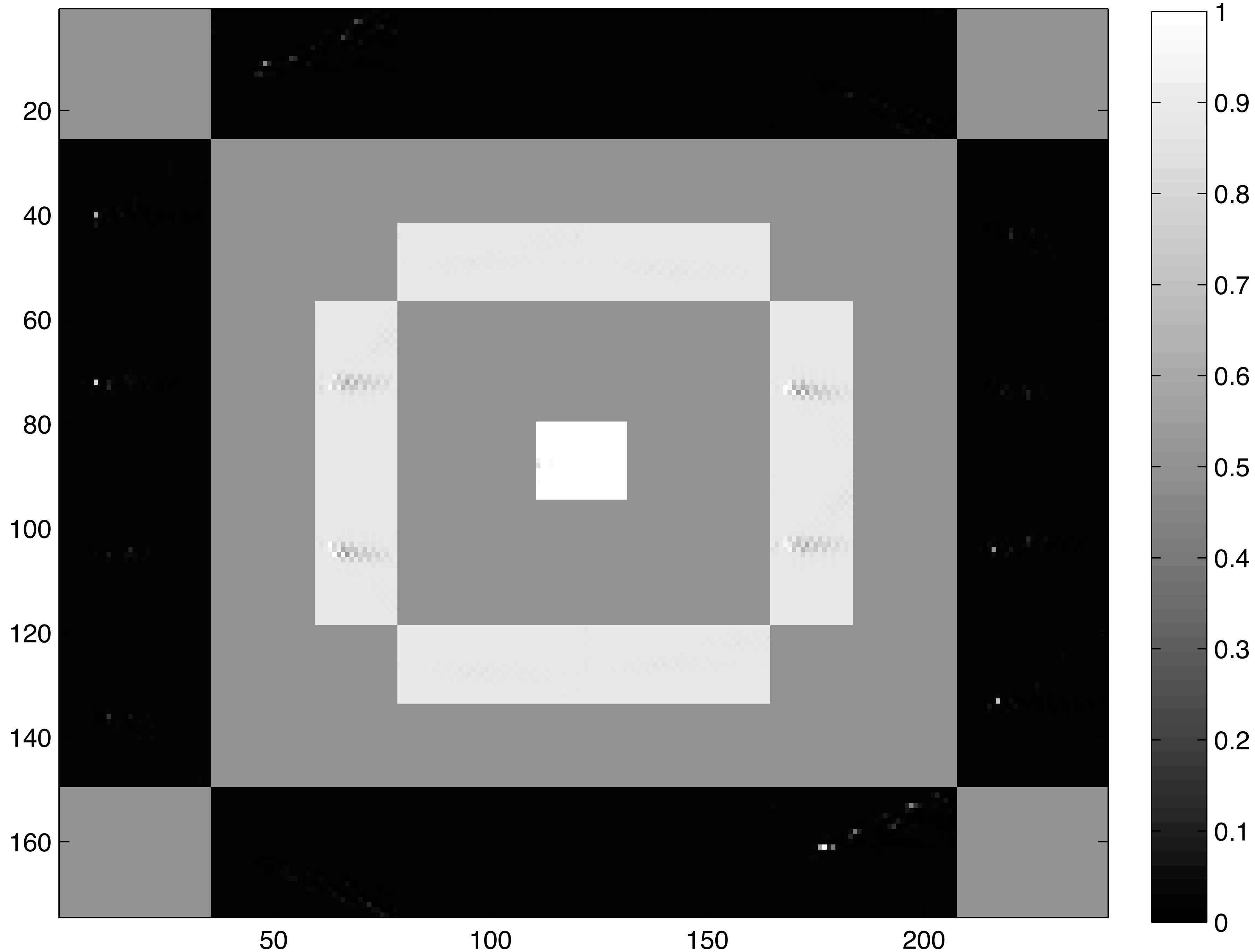
Smoothness penalty

increasing smoothness



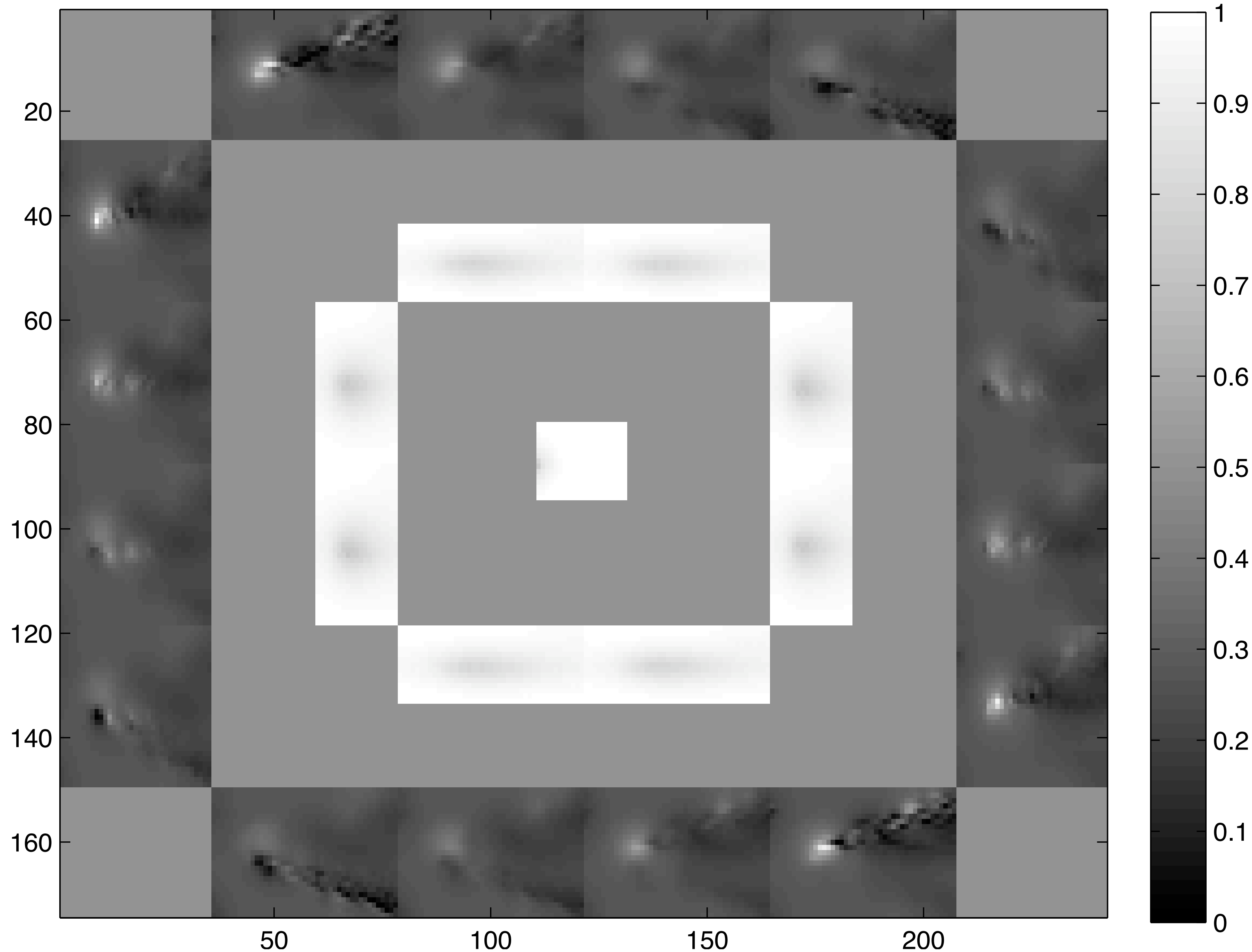
- reduces overfitting
- scaling is positive and reasonable

Smoothness penalty



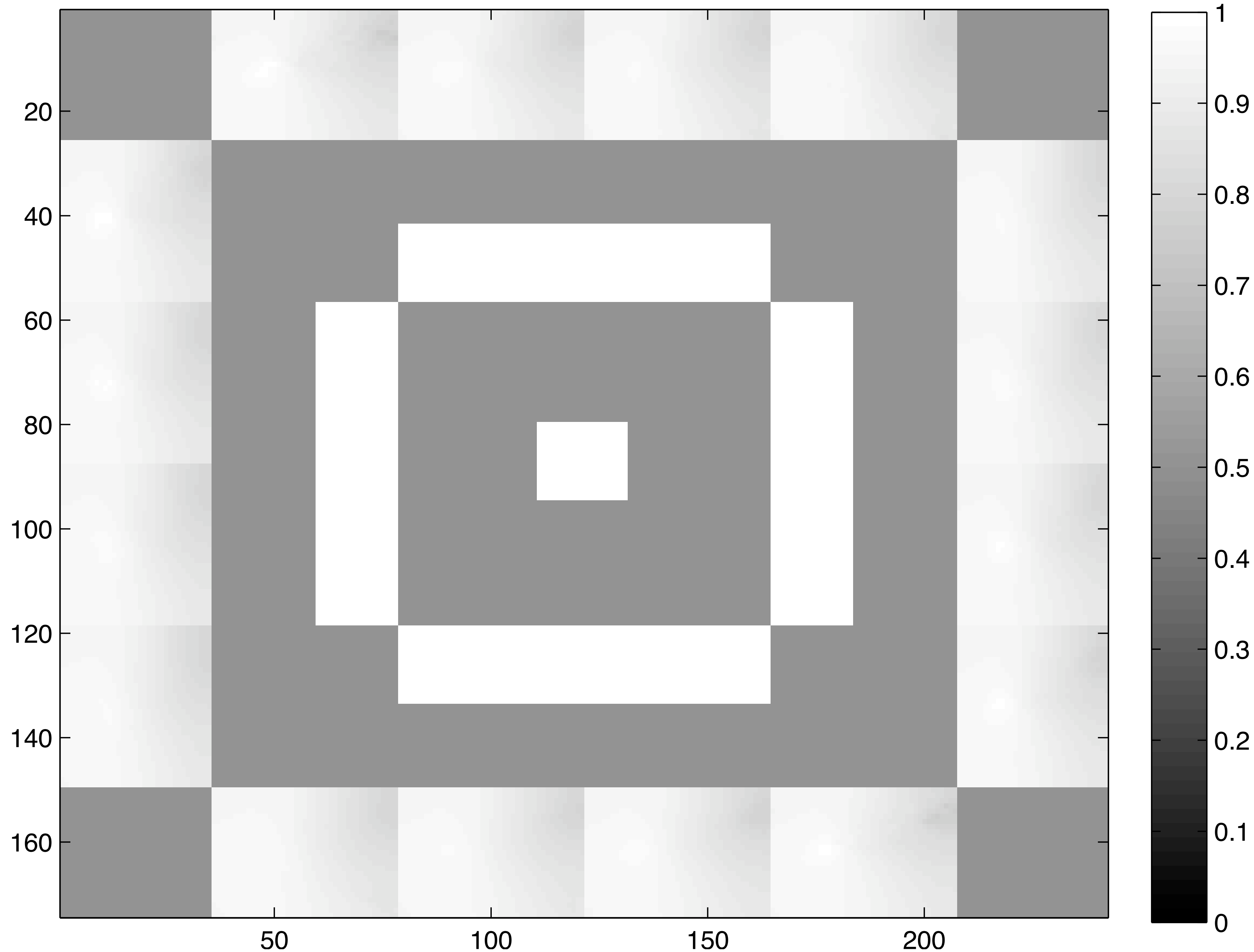
$\gamma = 0$

Smoothness penalty



$\gamma = 1/2$

Smoothness penalty



$\gamma = 5$

Seismic amplitude recovery

Matching procedure

Compute *reference* vector <=> defines **g**

- migrate data
- apply spherical-divergence correction

Create "data" <=> defines **f**

- demigrate
- migrate

Estimate *scaling* by inversion procedure

Define *scaled* curvelet transform

Recover migration amplitudes by *sparsity* promotion.

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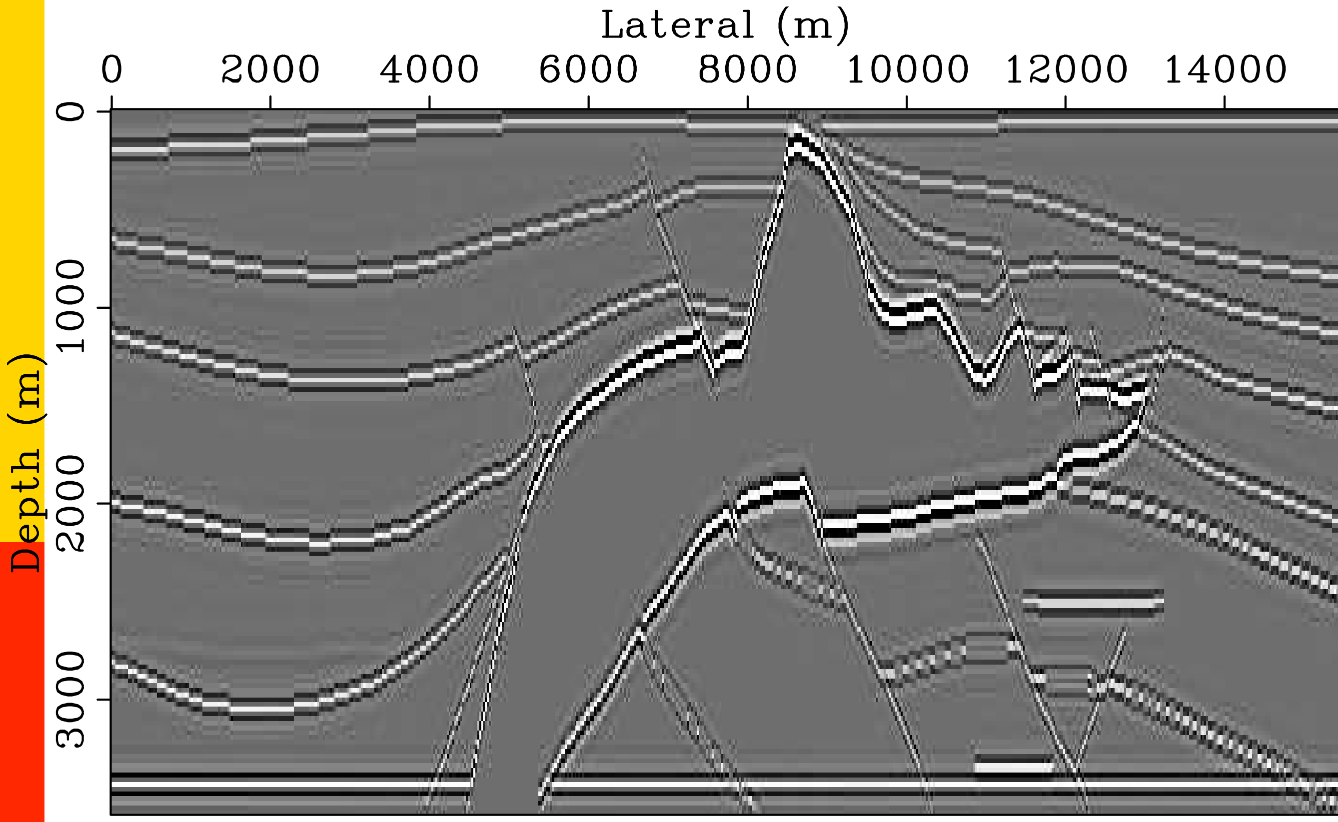
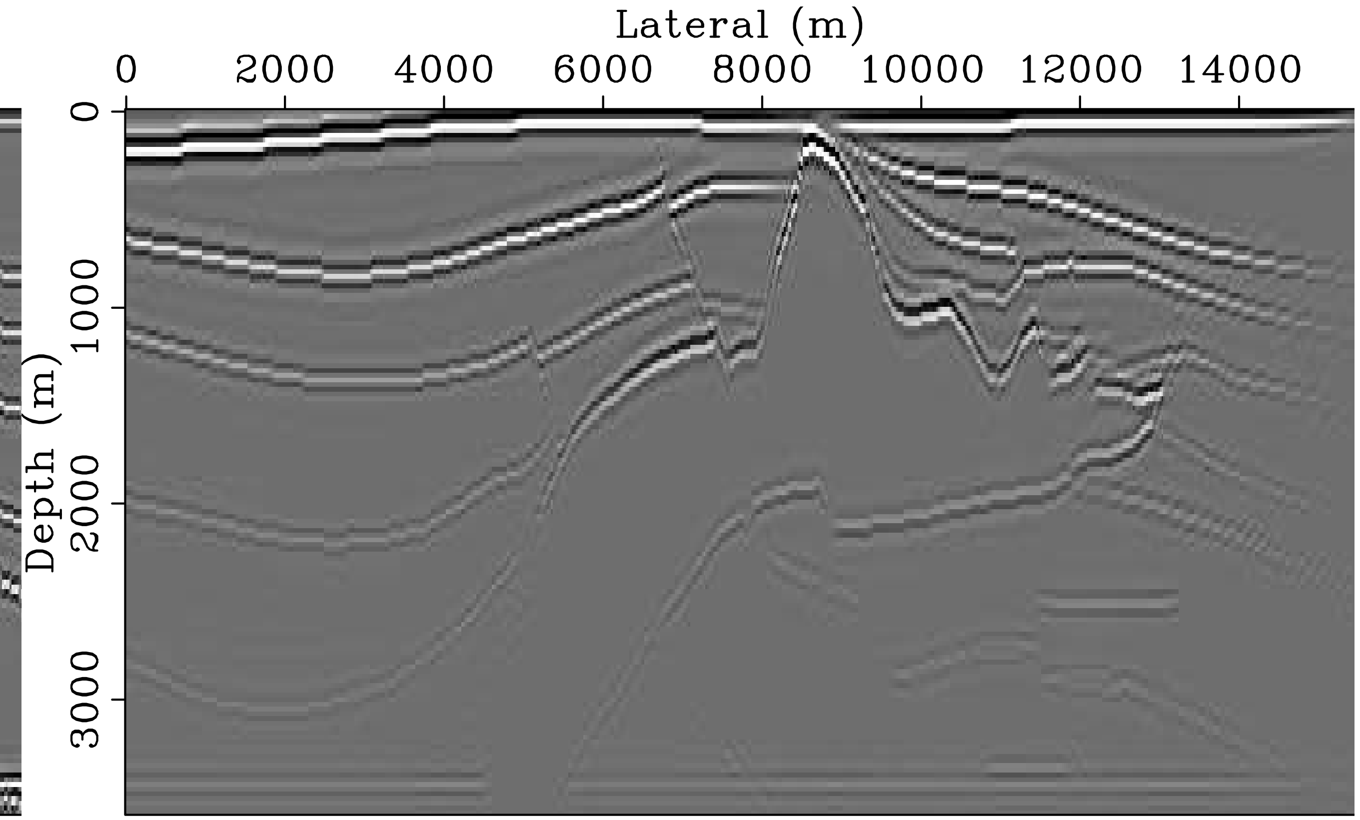
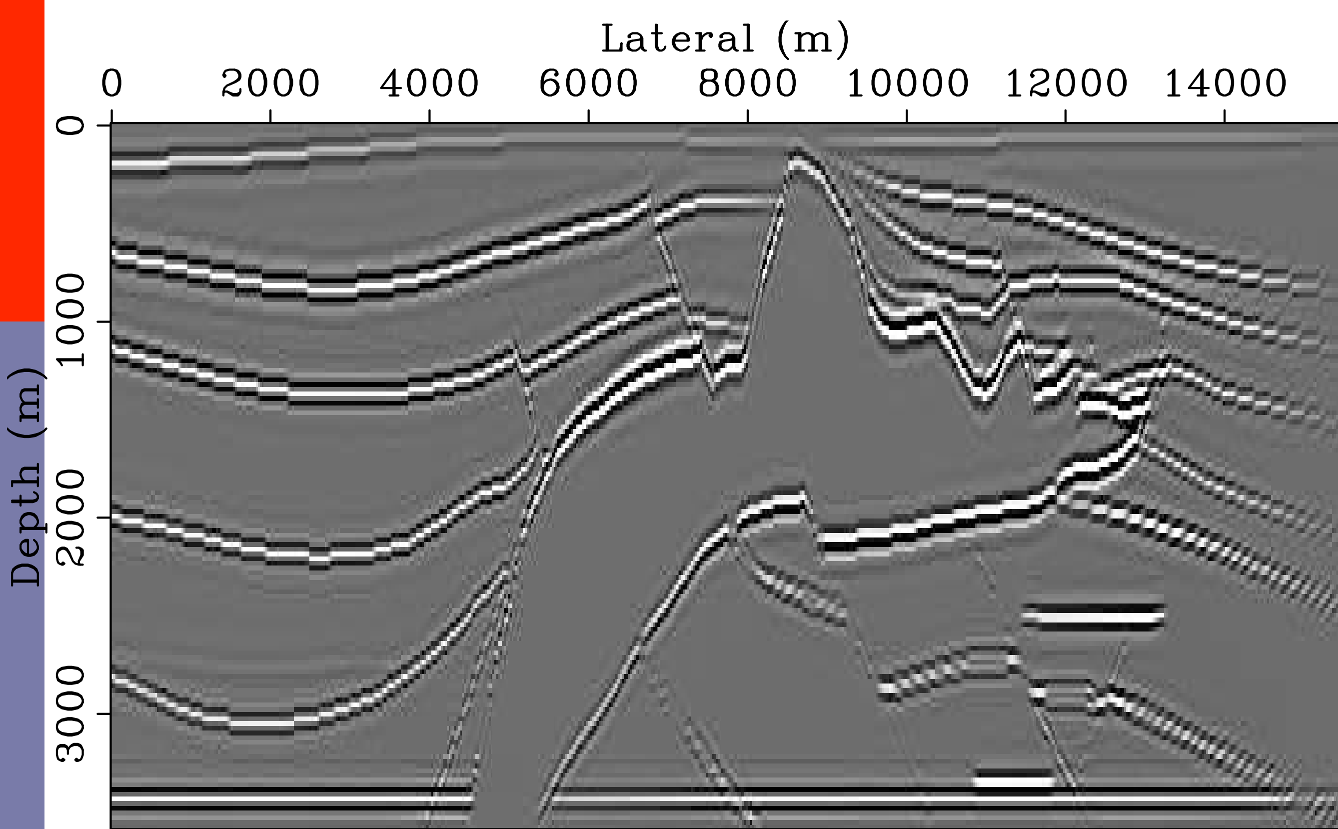


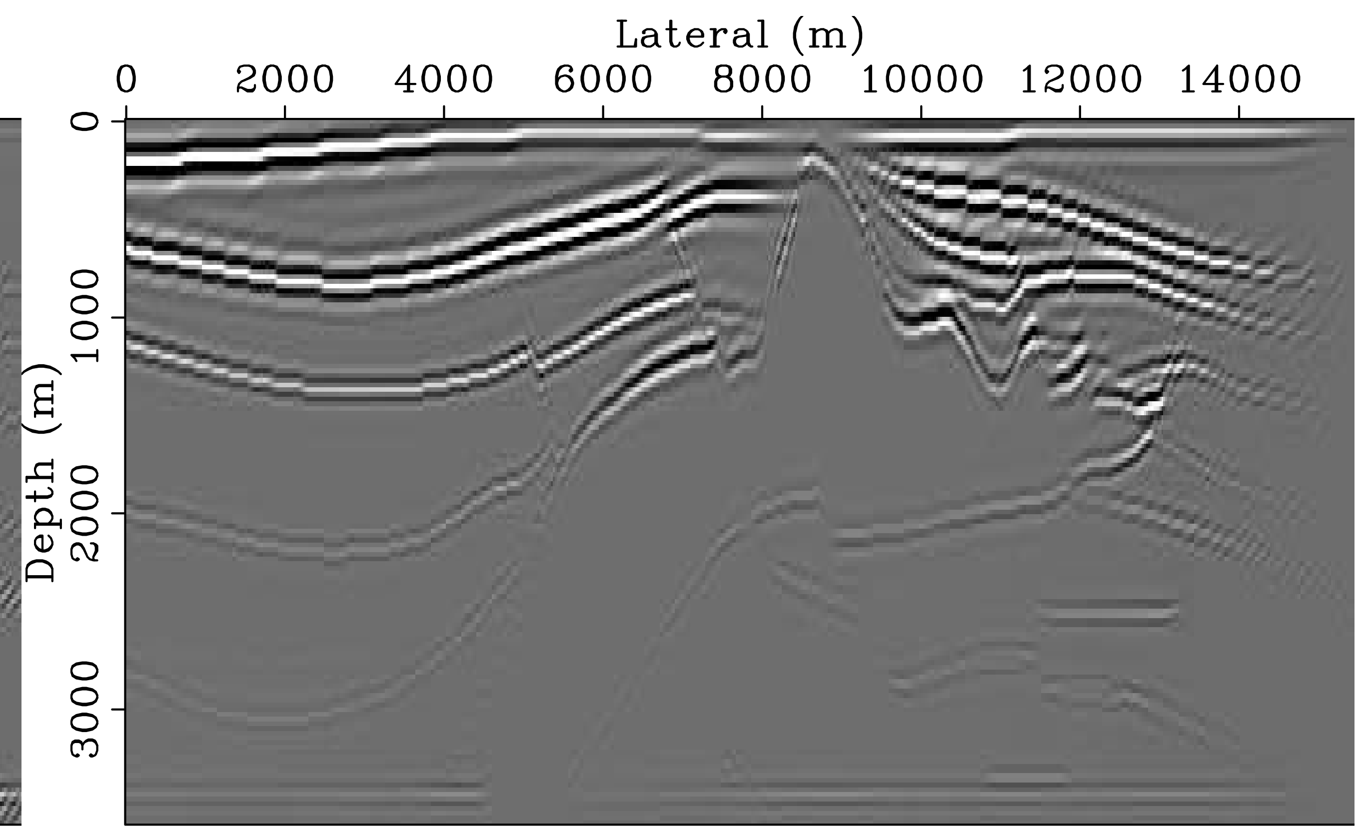
bandpass−filtered reflectivity



migrated image



reference vector



imaged reference vector

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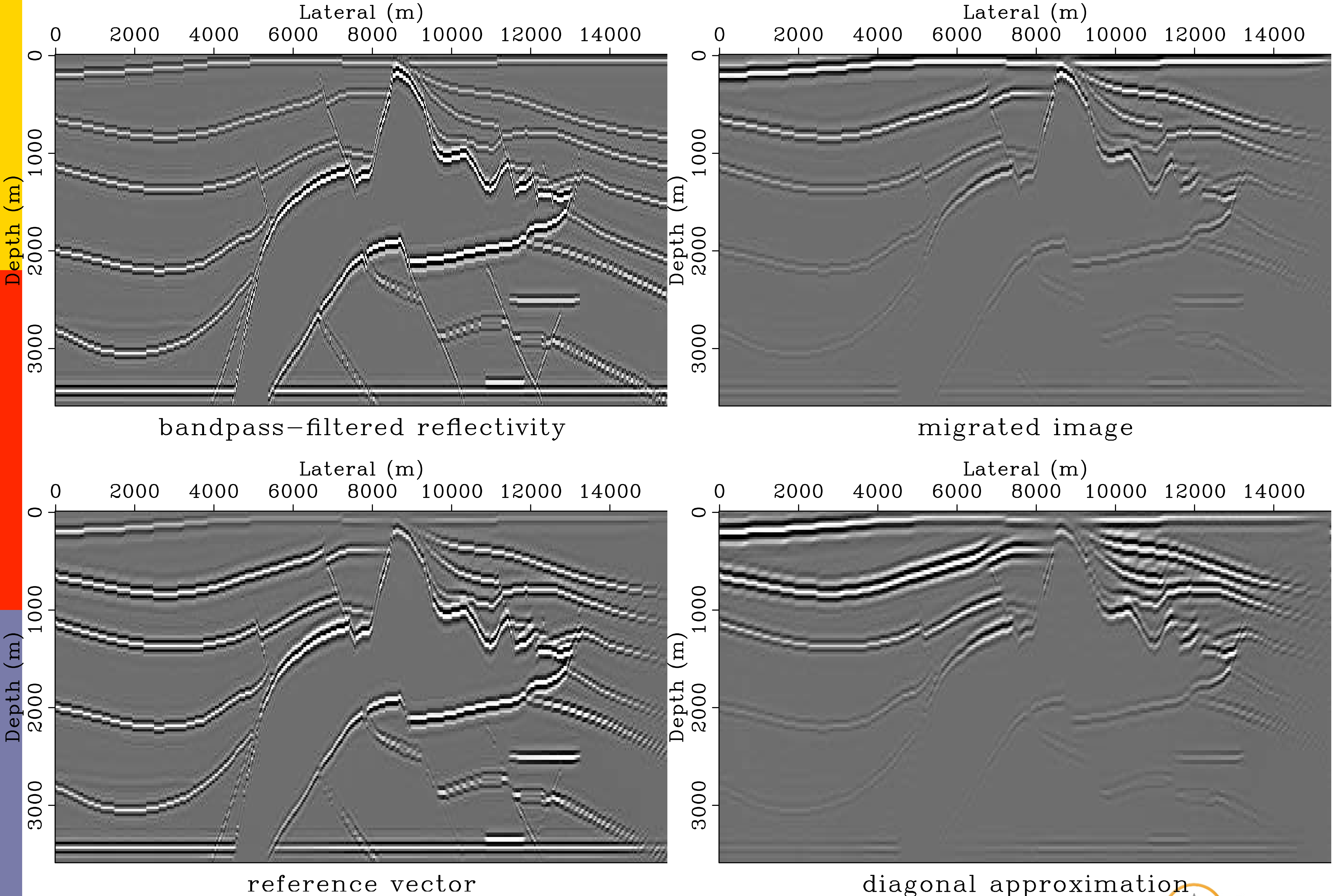
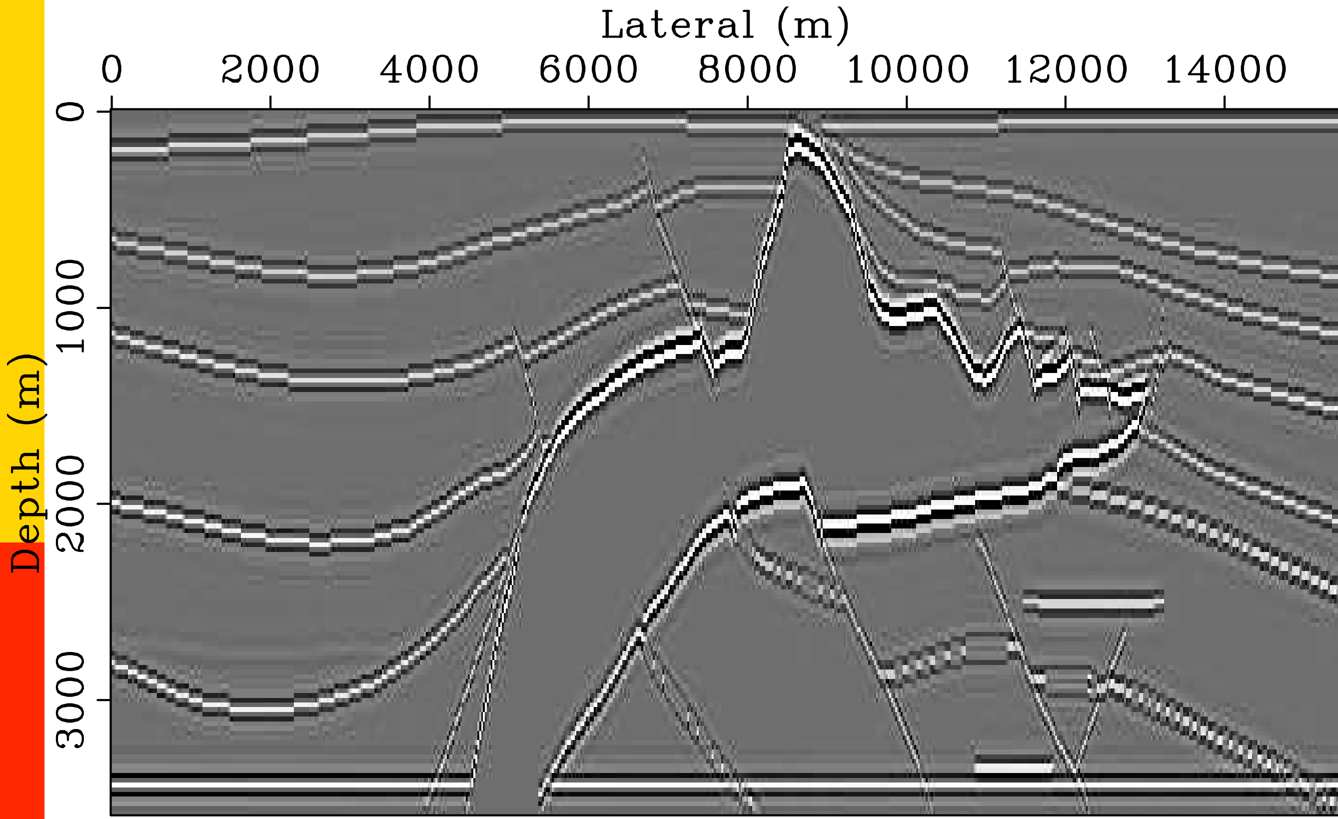
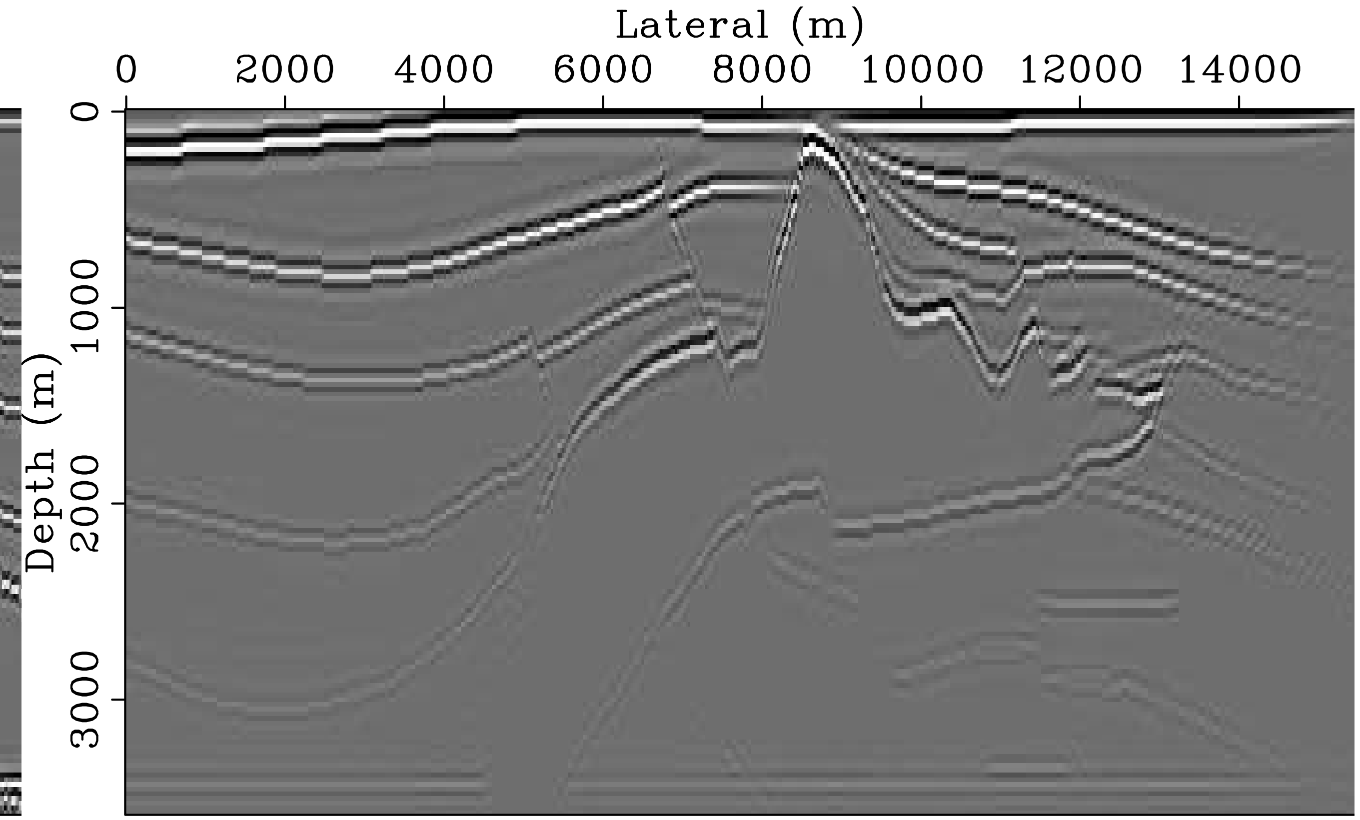


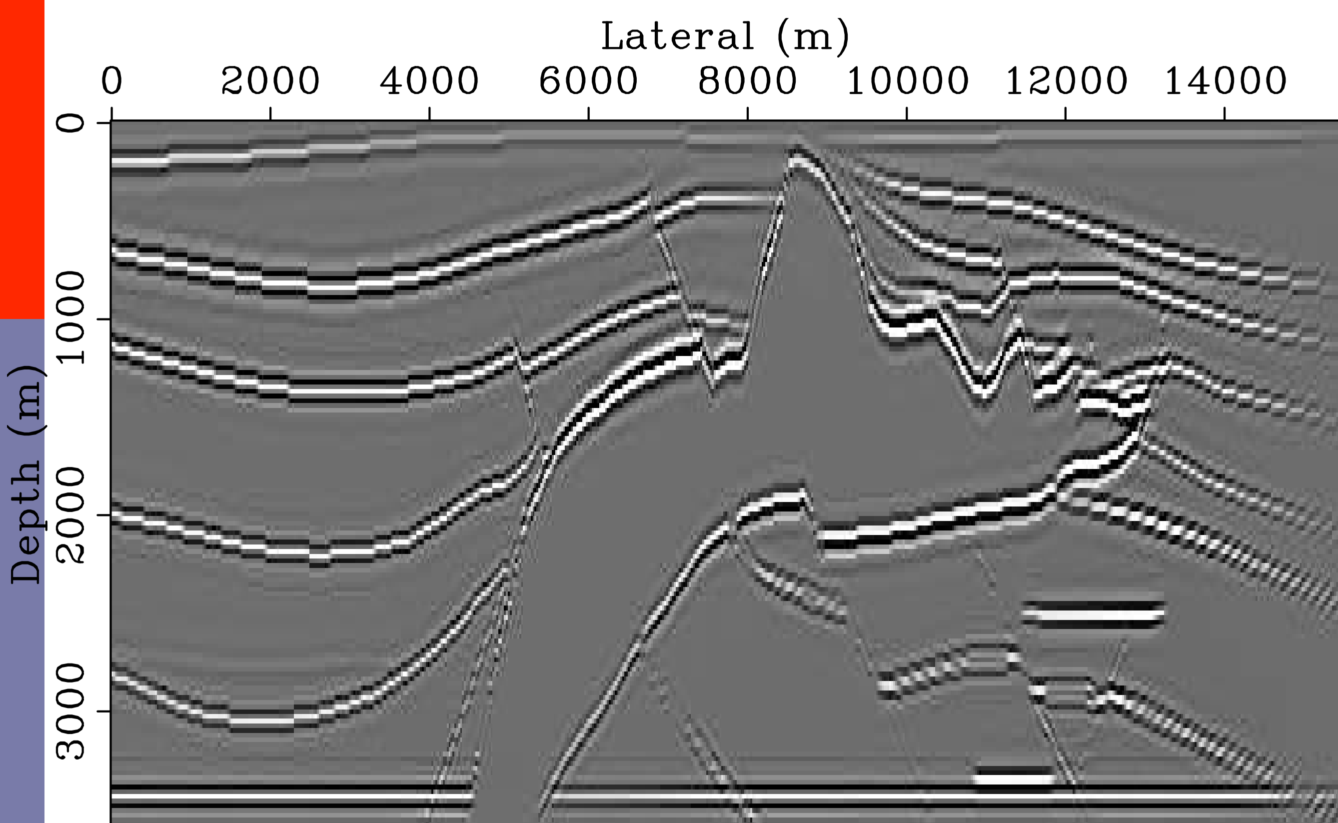
Lateral (m)
0 2000 4000 6000 8000 10000 12000 14000
Depth (m)
0 1000 2000 3000
bandpass−filtered reflectivity
migrated image
reference vector
diagonal approximation



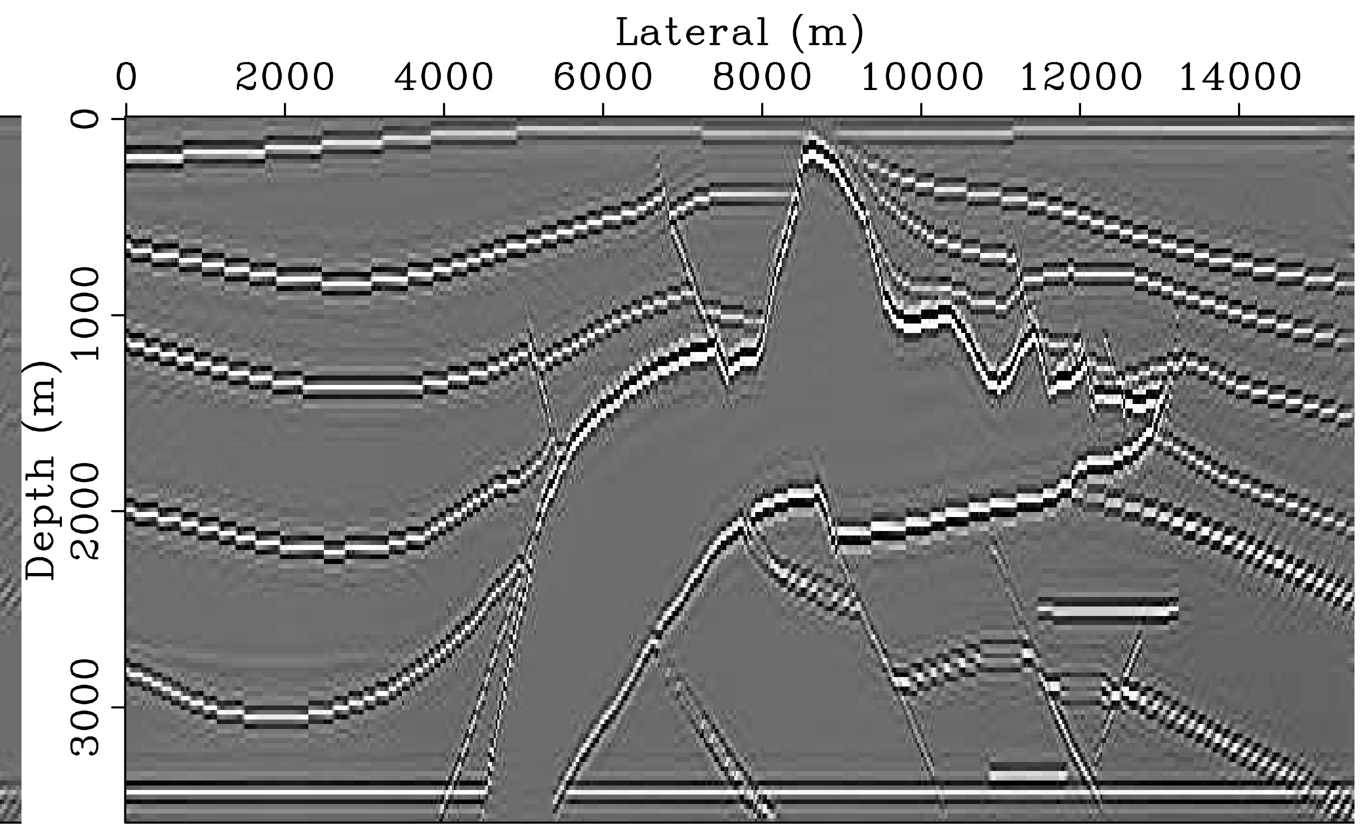
bandpass−filtered reflectivity



migrated image



reference vector



norm−one recovered

Primary-multiple separation

Matching procedure

Predict multiples <=> defines **g**

- apply conventional Fourier matched filtering

Consider total data as “*true*” multiples <=> defines **f**

- do not know *true* multiples
- use *total* data instead
- *minimize* energy mismatch

Estimate *scaling* by an *inversion* procedure.

Define scaled curvelet-domain threshold.

Separate primaries & multiples by *sparsity* promotion.

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Problem formulation

Signal model for total data

$$\mathbf{s} = \mathbf{s}_1 + \mathbf{s}_2$$

Multiple prediction by e.g. SRME may contain amplitude errors, i.e.,

$$\begin{aligned} \mathbf{s}_2 &= \mathbf{B}\breve{\mathbf{s}}_2 \\ \mathbf{s}_2 &\approx \mathbf{C}^T \mathrm{diag}\{\mathbf{w}\}\mathbf{C}\breve{\mathbf{s}}_2 \end{aligned}$$

Solve

$$J_\gamma(\mathbf{z}) = \frac{1}{2}\|\mathbf{s} - \mathbf{F}_\gamma e^{\mathbf{z}}\|_2^2,$$

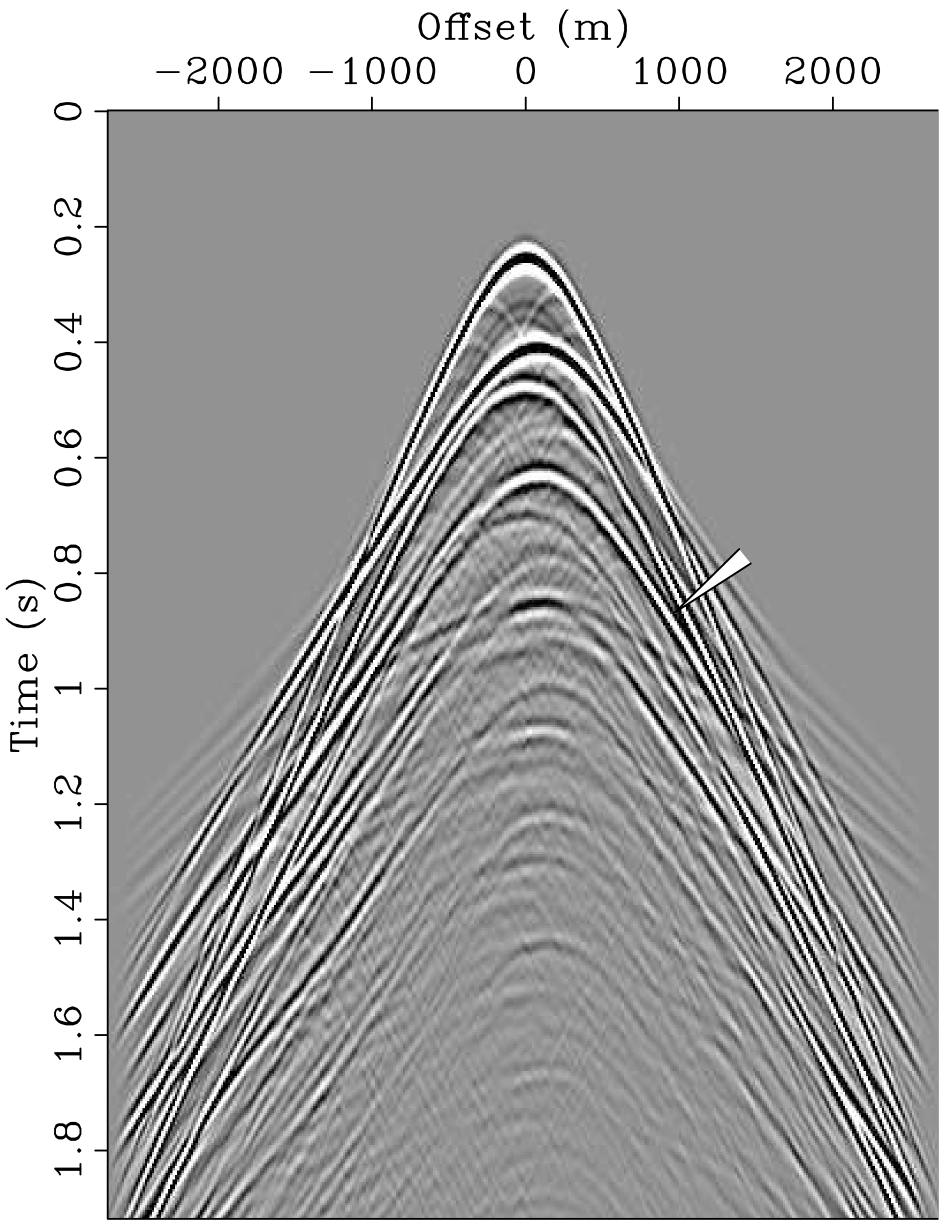
with **s** the total data. Use **z** to correct the predicted multiples, i.e.,

$$\breve{\mathbf{s}}_2 \mapsto \mathbf{C}^T \mathrm{diag}\{\tilde{\mathbf{w}}\}\mathbf{C}\breve{\mathbf{s}}_2 \text{ with } \tilde{\mathbf{w}} = e^{\tilde{\mathbf{z}}}$$

or correct the thresholding

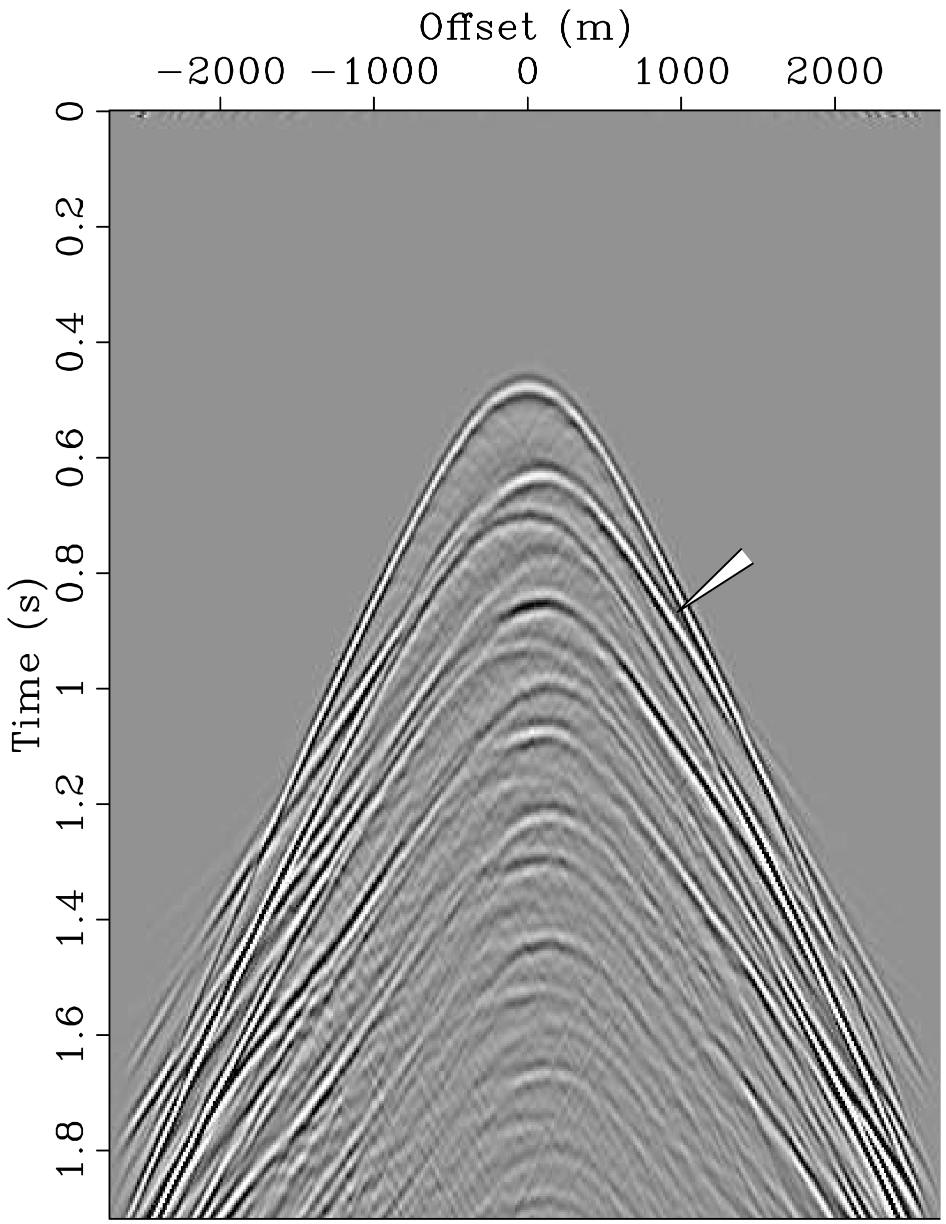
$$\mathbf{t} = \mathrm{diag}\{\tilde{\mathbf{w}}\}|\mathbf{C}\breve{\mathbf{s}}_2|$$

Synthetic example



Total data

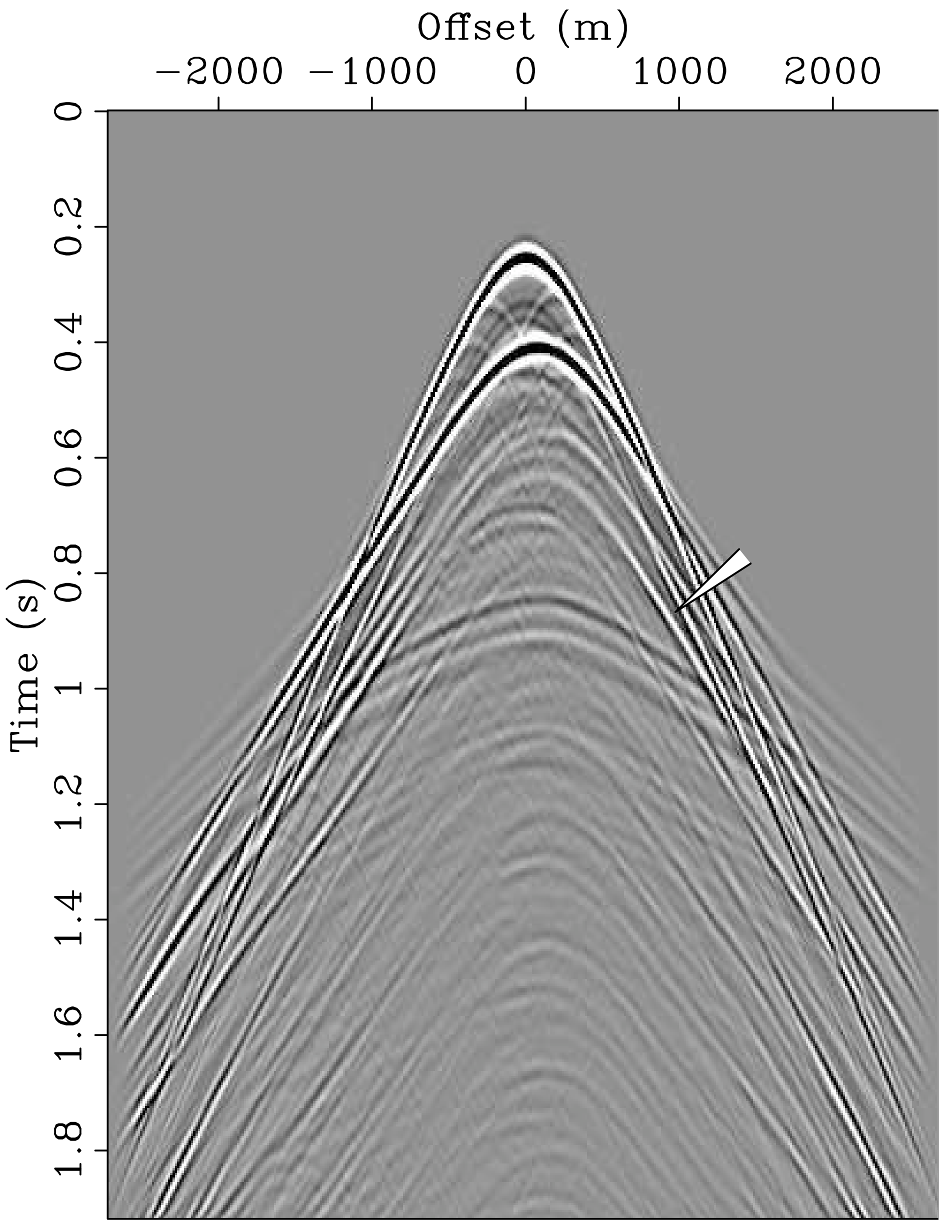
$\mathbf{s}$



SRME predicted multiples

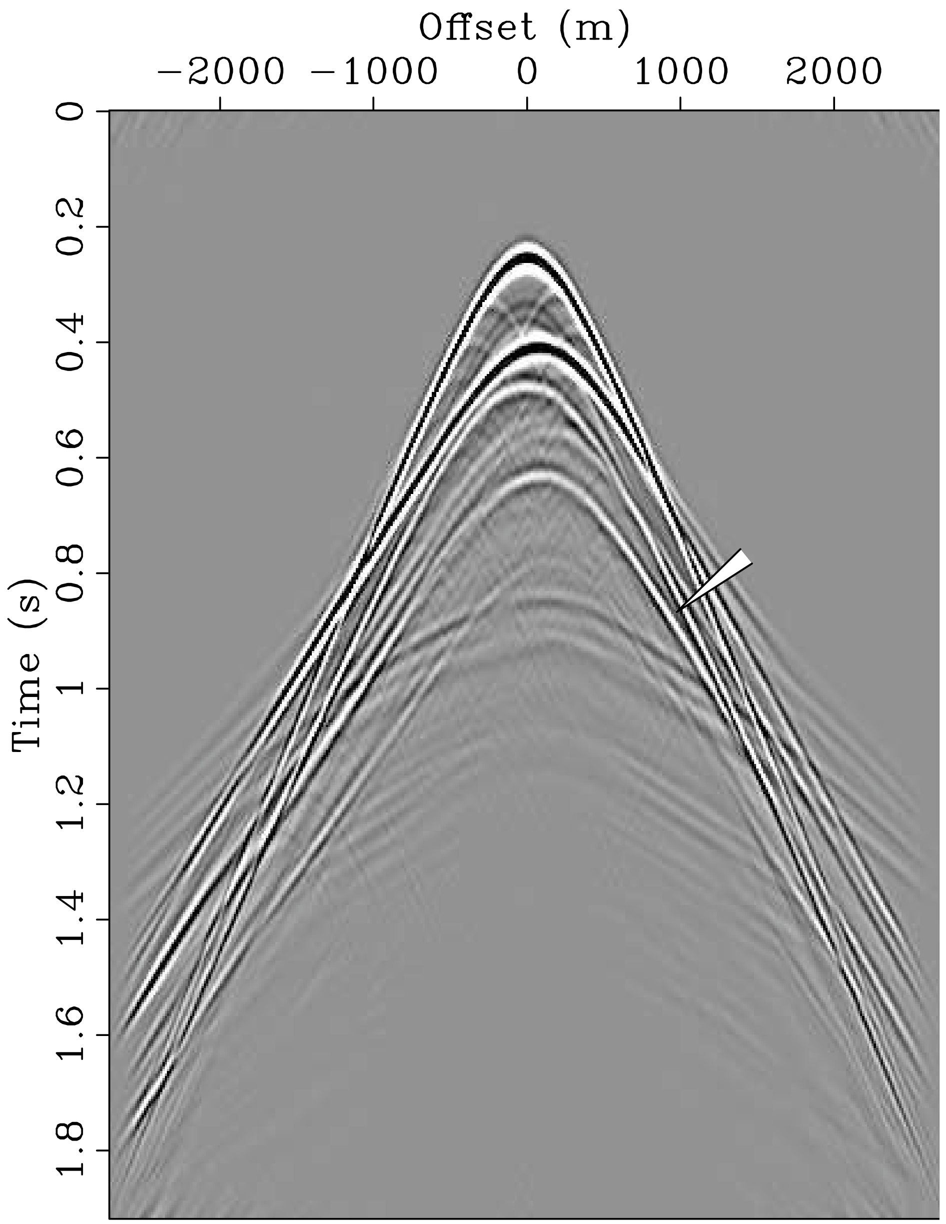
$\breve{\mathbf{s}}_2$

Synthetic example



SRME predicted primaries

$$\breve{\mathbf{s}}_1$$



Curvelet estimated primaries

$$\tilde{\mathbf{s}}_1 = \mathbf{C}^T T_{\mathbf{t}}(\mathbf{C}\mathbf{p})$$
$$\mathbf{t} = \mathbf{C}\breve{\mathbf{s}}_2$$

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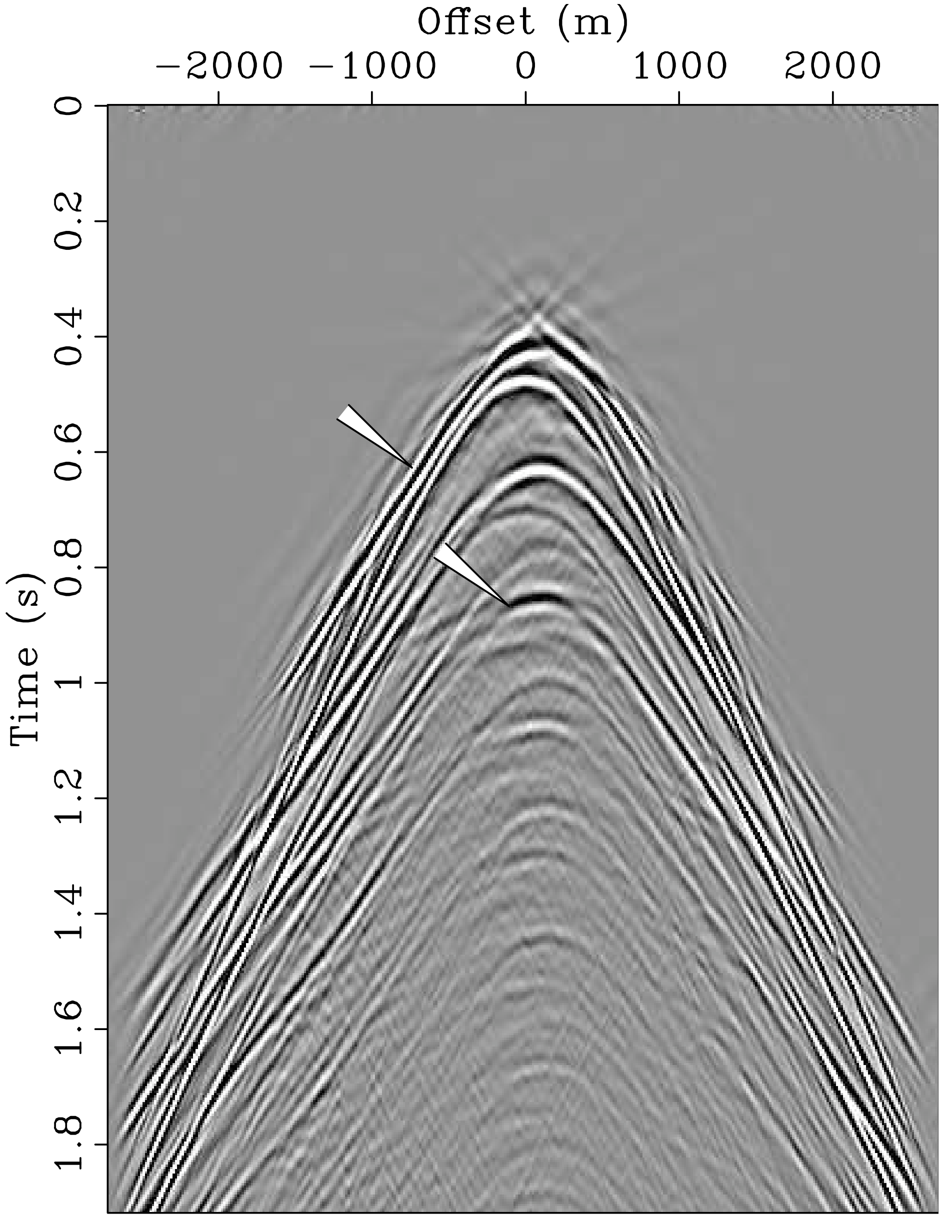
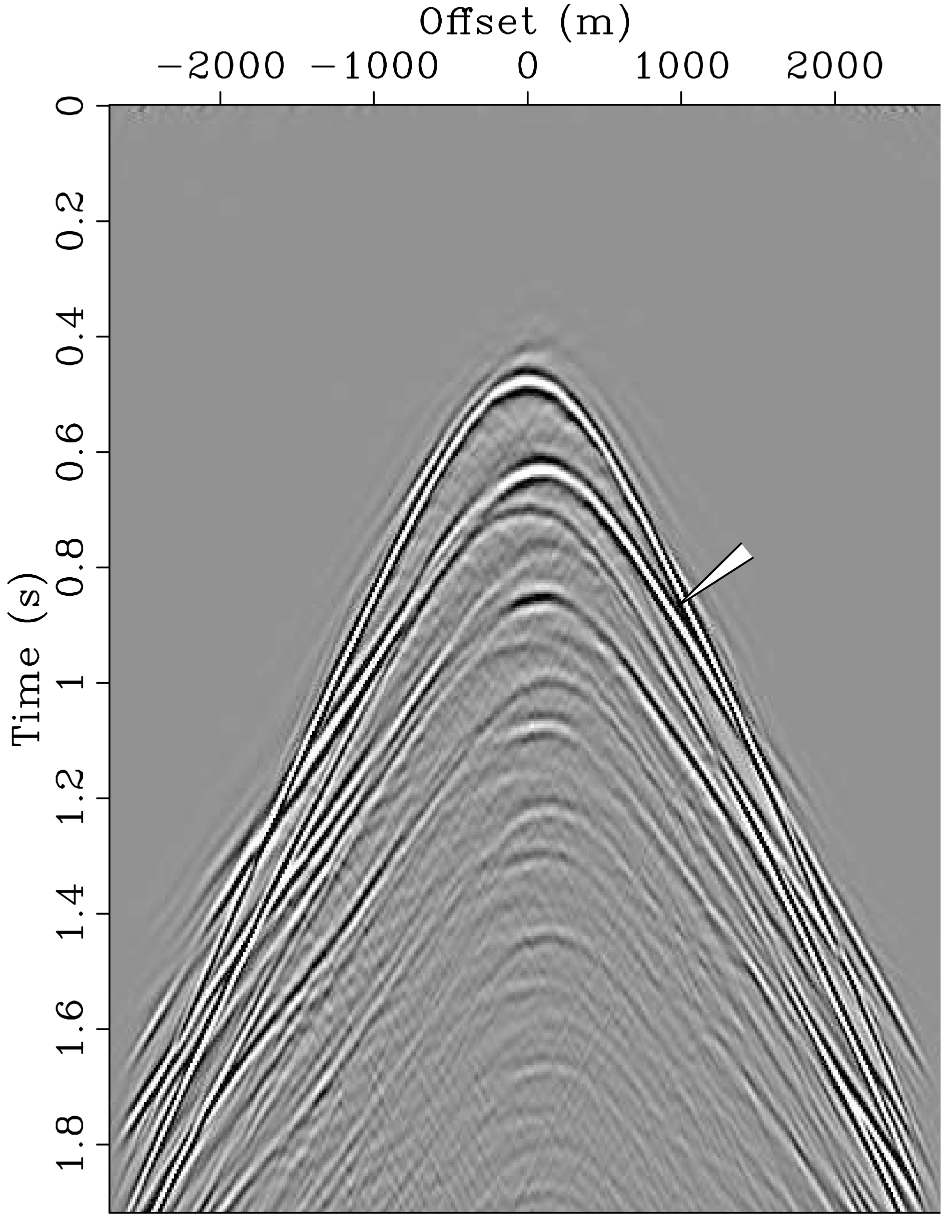
Synthetic example



Corrected multiples

$$\breve{\mathbf{s}}_2^{\text{corr.}} = \mathbf{C}^T \text{diag}\{\mathbf{w}\}\mathbf{C}\breve{\mathbf{s}}_2 \text{ for } \gamma = 0$$



Corrected multiples

$$\breve{\mathbf{s}}_2^{\text{corr.}} = \mathbf{C}^T \text{diag}\{\mathbf{w}\}\mathbf{C}\breve{\mathbf{s}}_2 \text{ for } \gamma = 0.5$$

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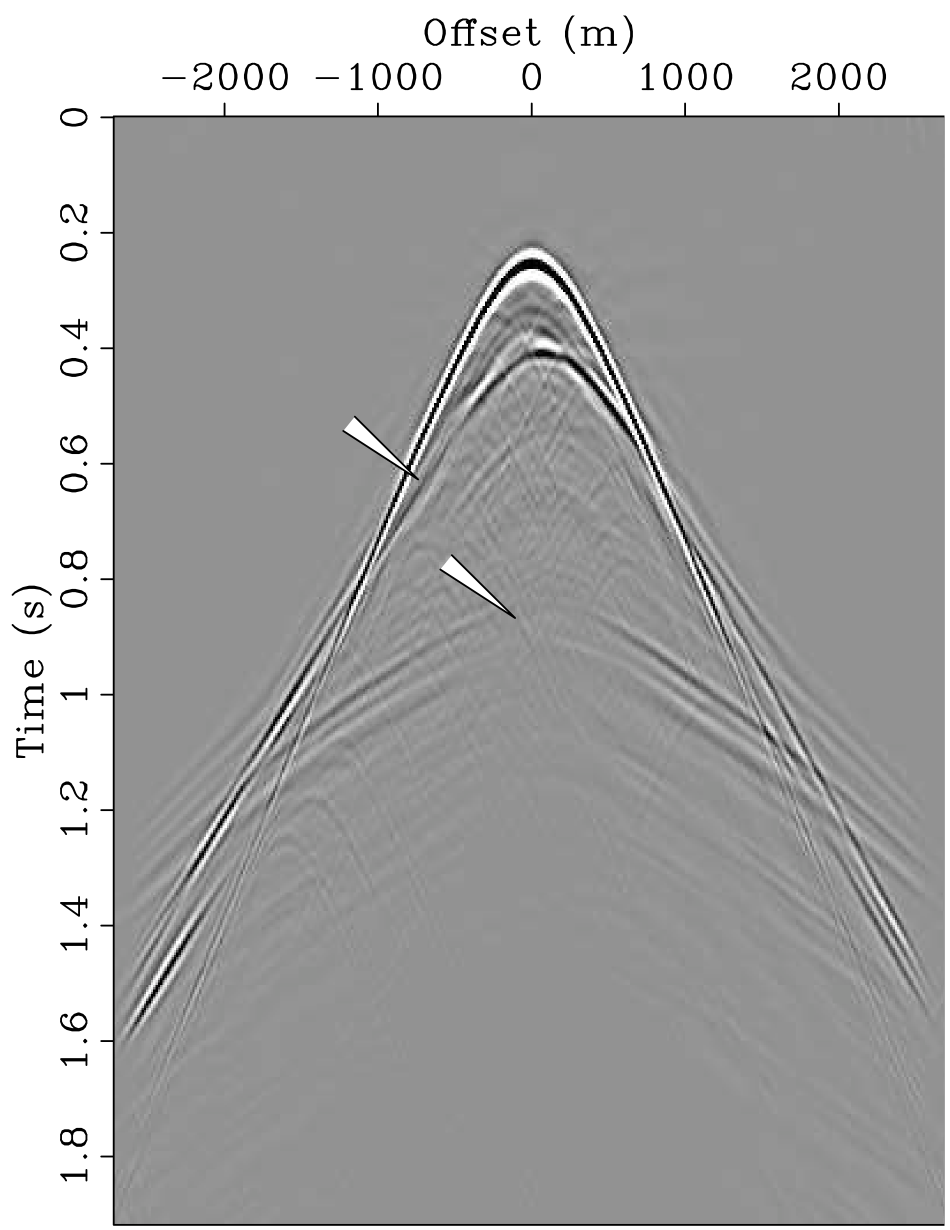
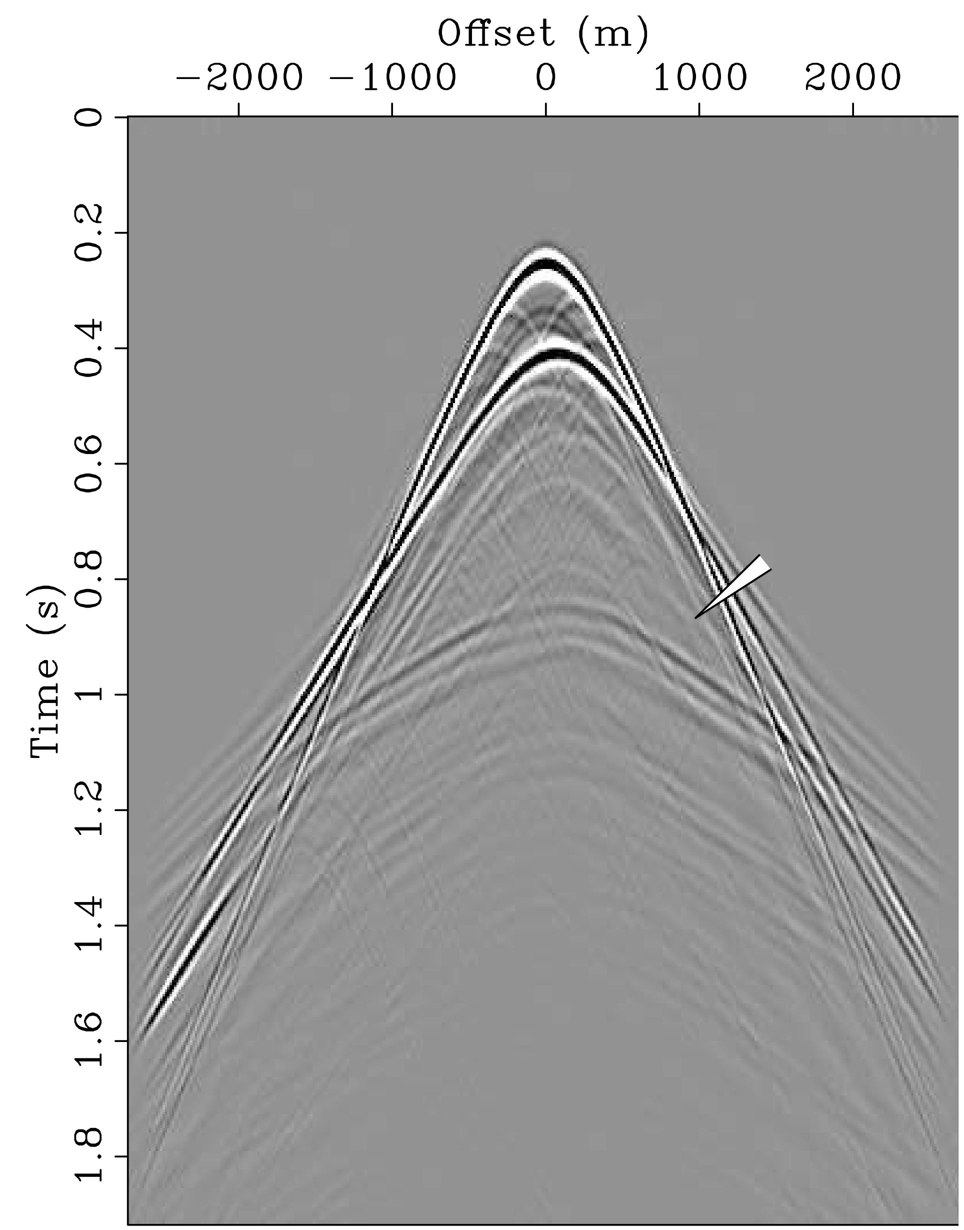
Synthetic example



Scaled thresholded primaries

$$\begin{aligned}\tilde{\mathbf{s}}_1 &= \mathbf{C}^T T_{\mathbf{t}}(\mathbf{C}\mathbf{p}) \\ \mathbf{t} &= \mathrm{diag}\{\mathbf{w}\}|\mathbf{C}\tilde{\mathbf{s}}_2|\end{aligned}$$



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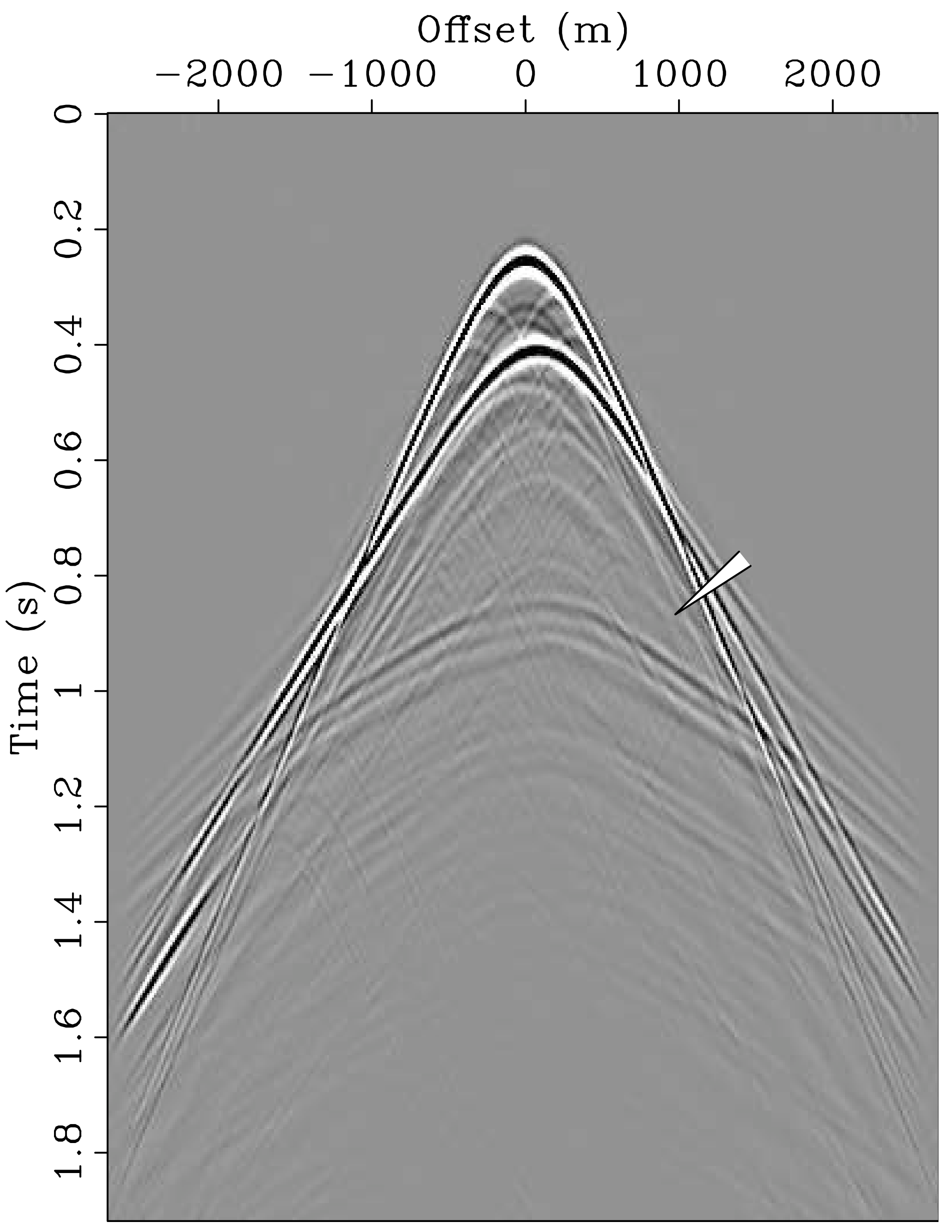
Synthetic example



Scaled thresholded primaries

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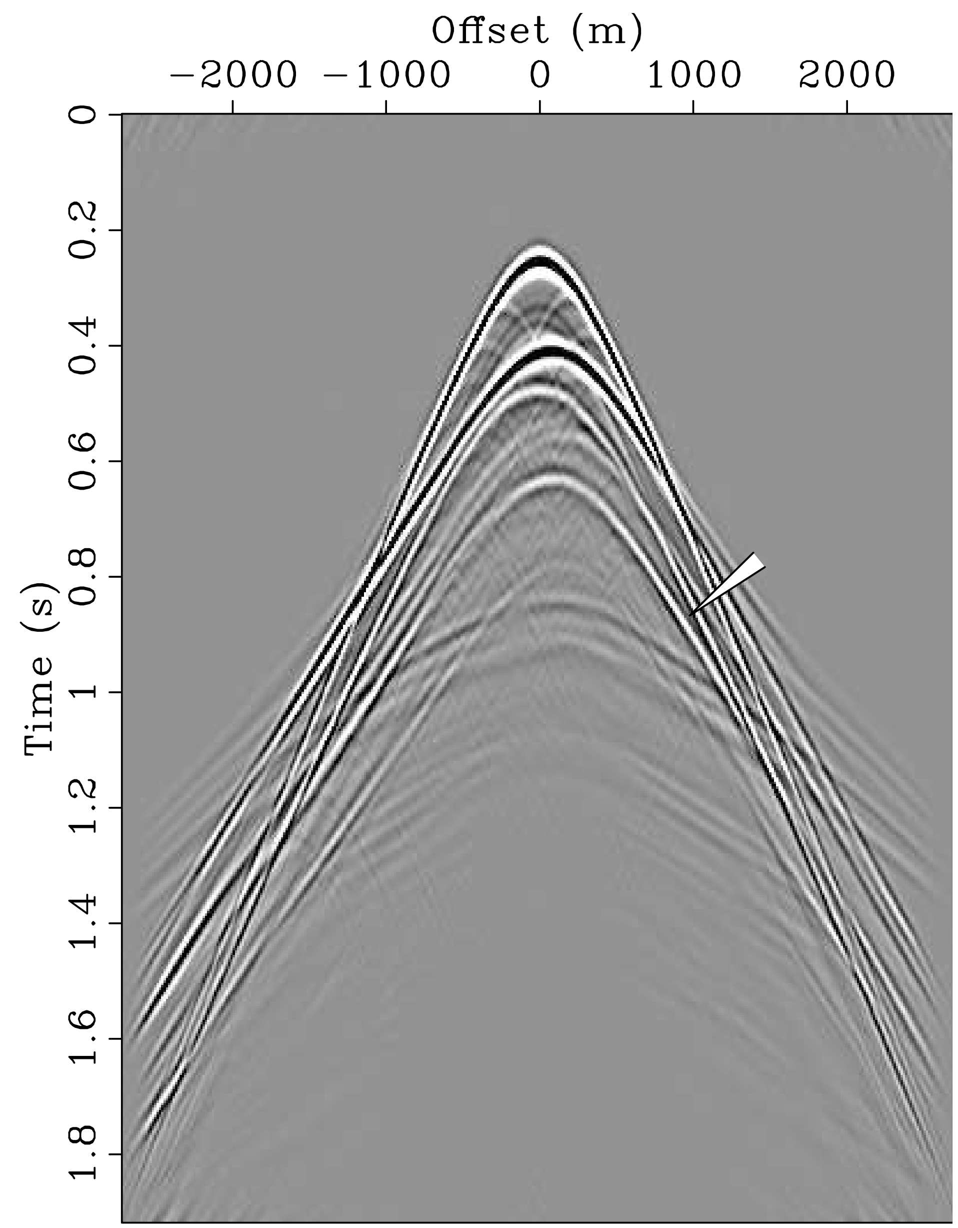

Curvelet estimated primaries

$$\begin{aligned}\tilde{\mathbf{s}}_1 &= \mathbf{C}^T T_{\mathbf{t}}(\mathbf{C}\mathbf{p})\\ \mathbf{t} &= \mathbf{C}\breve{\mathbf{s}}_2\end{aligned}$$

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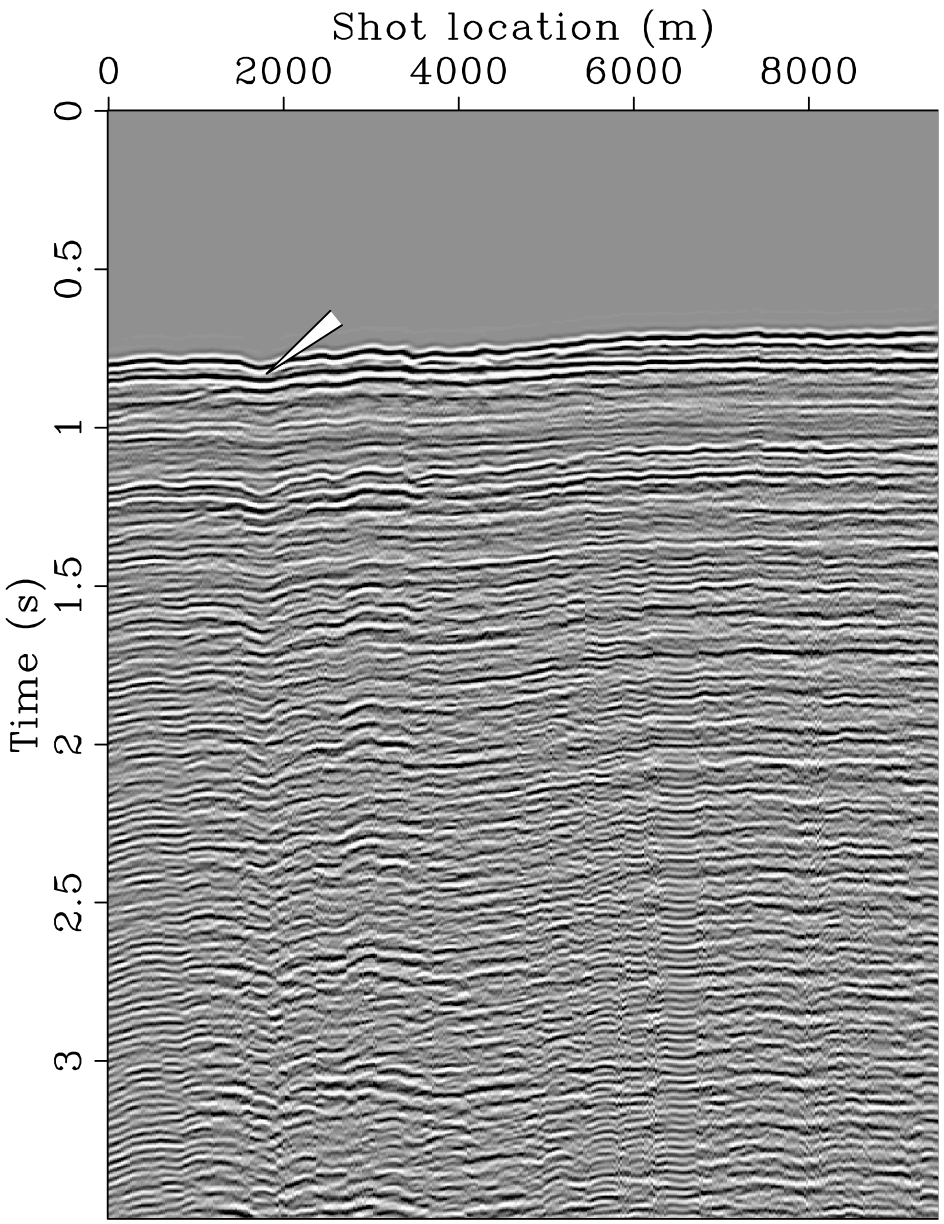
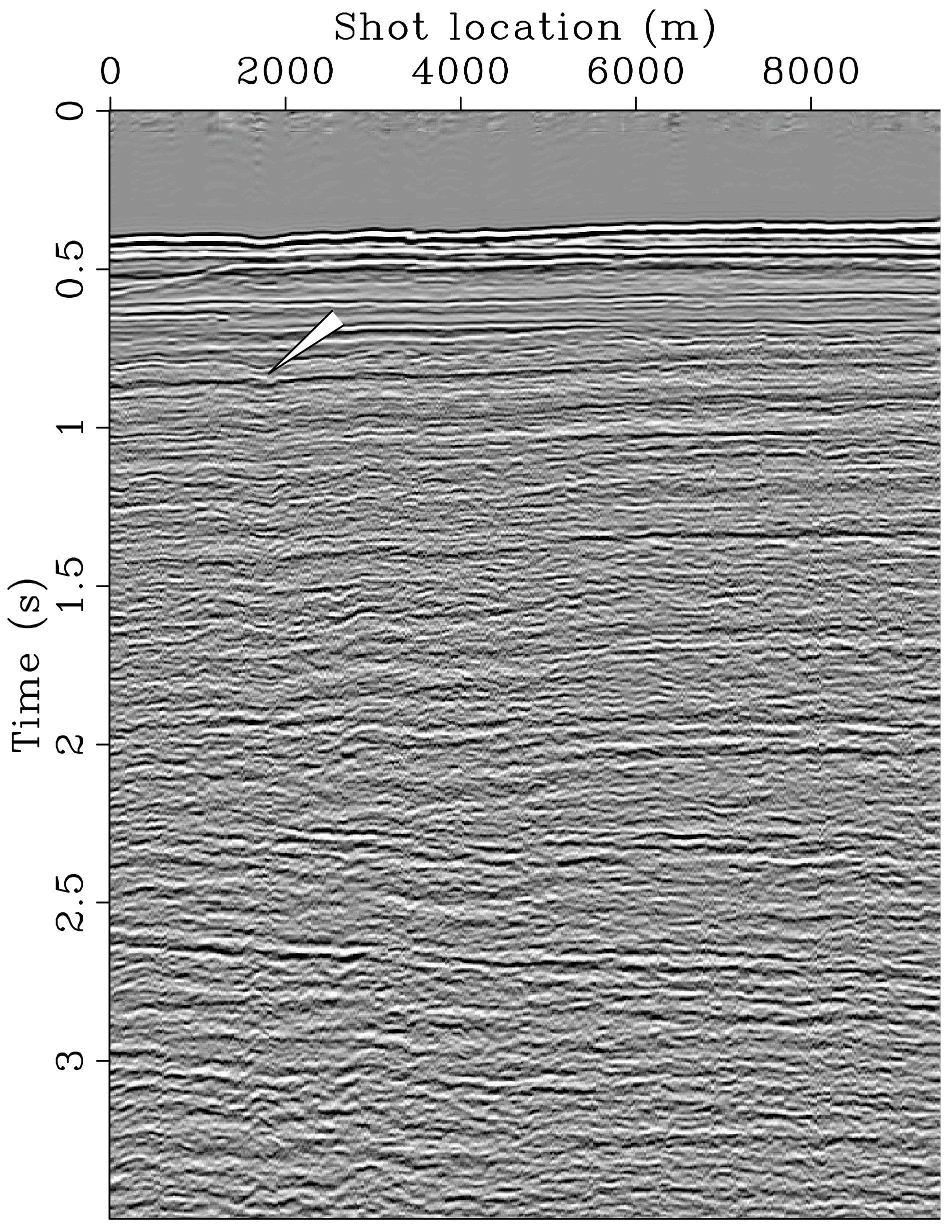
Real example



SRME predicted multiples

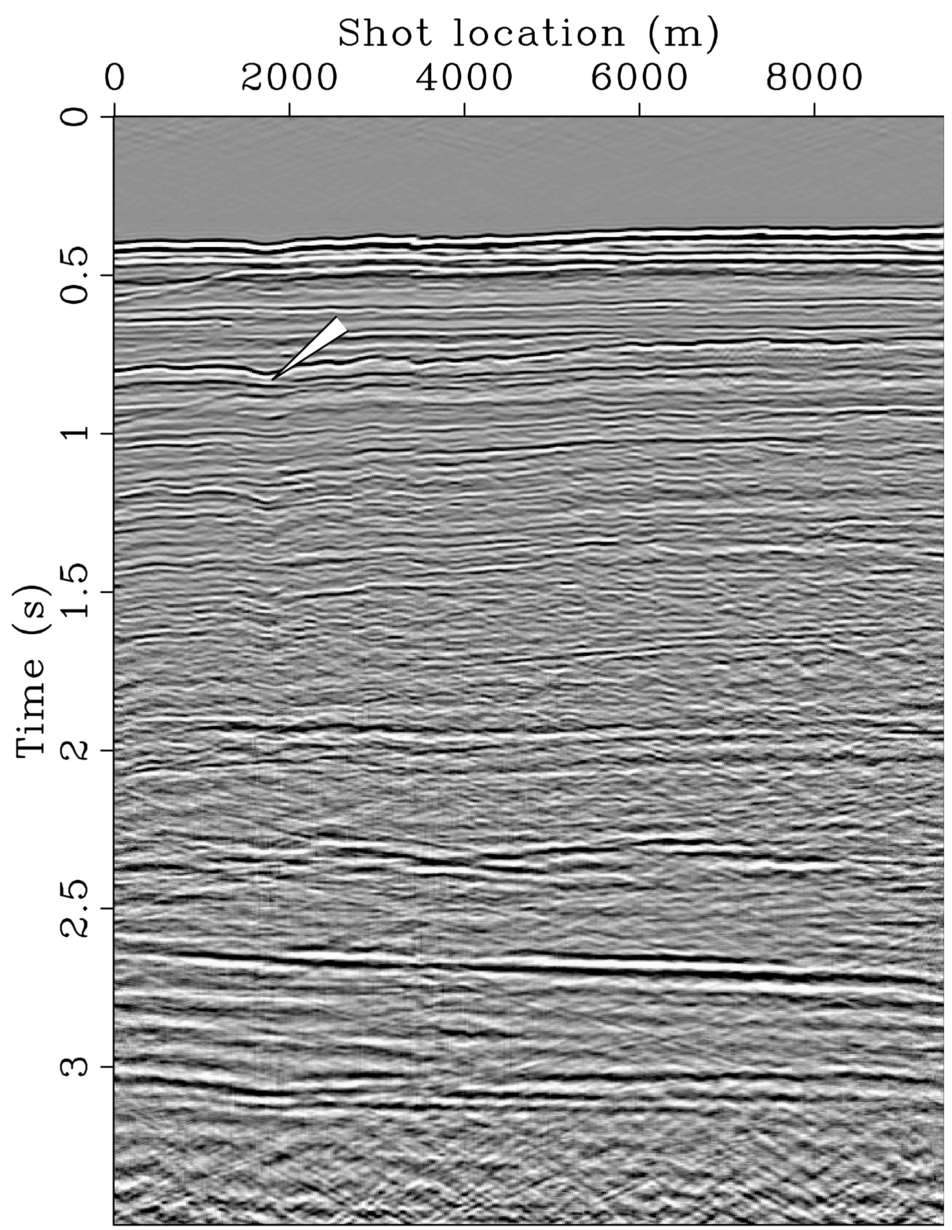
$\breve{\mathbf{s}}_2$



SRME predicted primaries

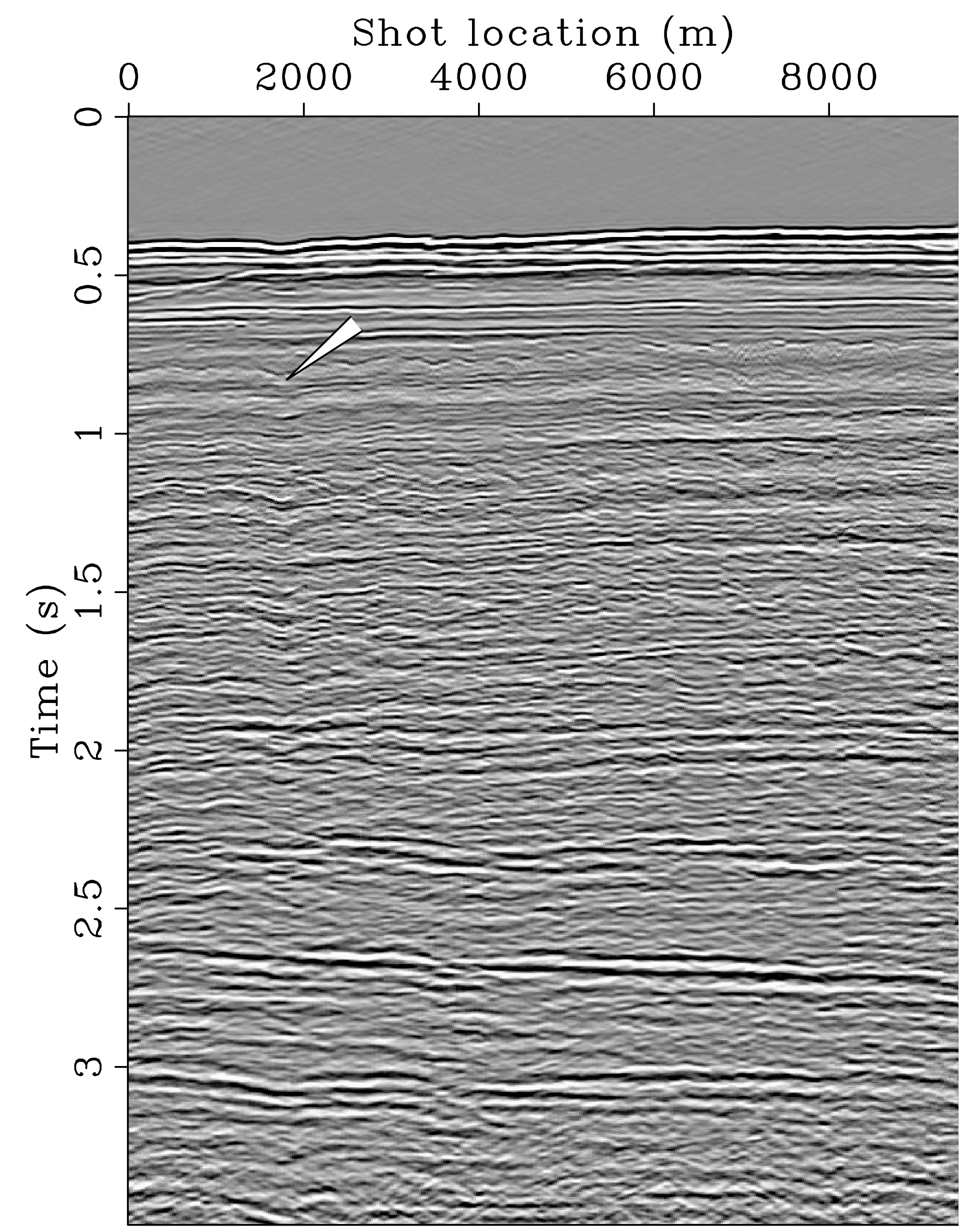
$\breve{\mathbf{s}}_1$

Real example



Thresholded primaries

$$\tilde{\mathbf{s}}_1 = \mathbf{C}^T T_{\mathbf{t}}(\mathbf{Cp})$$
$$\mathbf{t} = \mathbf{C}\breve{\mathbf{s}}_2$$



Scaled thresholded primaries

$$\tilde{\mathbf{s}}_1 = \mathbf{C}^T T_{\mathbf{t}}(\mathbf{Cp})$$
$$\mathbf{t} = \mathrm{diag}\{\mathbf{w}\}|\mathbf{C}\tilde{\mathbf{s}}_2|$$

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Conclusions

*Combining the parsimonious **curvelet** transform with **phase-space** structure allows us to*

- control diagonal estimation <=> over fitting
- handle data with conflicting dips
- stably recover & separate

Application

- improved migration-amplitude recovery
- improved primary-multiple separations

Future

- 3-D
- non-smooth symbols

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