Just diagonalize: a curvelet-based approach to seismic amplitude recovery

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Motivation

Migration generally does not correctly recover the amplitudes.

Least-squares migration is computationally unfeasible.

Amplitude recovery (e.g. AGC) lacks robustness w.r.t. noise.

Existing diagonal amplitude-recovery methods

- do not always correct for the order (1 - 2D) of the Hessian [see Symes '07]
- do not invert the scaling robustly

Moreover, these (scaling) methods assume that there

- are no conflicting dips (conormal) in the model
- is infinite aperture
- are infinitely-high frequencies
- etc.

Curvelets & seismology

Wish list

A transform that

- detects the reflectors without **prior** information on the **geologic** dips
- is **sparse**, i.e. the magnitude-sorted coefficients decay **fast**
- is relative **invariance** under the **demigration-migration**, i.e. sparse on migrated images

Curvelets

- were “born” from studying high-frequency solution operators for wave propagation*
- diagonalization of migration operators**

*See work by Stein, Smit, Donoho, Candes & Demanet

** Main motivation for Douma & de Hoop and Chauris

Nonlinear approximation

Migrated mobil data set

reconstructed data with $p=99$

Nonlinear approximation

Recovery from largest 3 %

reconstructed data with $p=3$

Nonlinear approximation

Difference

Nonlinear approximation rates

Imaged Mobil data

Reflectivity SEG AA'

Curvelets & wave propagation

Theoretical results that claim that curvelets near diagonalize migration operators [Demanet et. al, de Hoop]

Encouraging results for constant velocity media [Douma & de Hoop; Chauris]

Challenge: discrete curvelets move off the grid

- interpolation
- definition of curvelet molecules [Demanet et. al, de Hoop]

In not so smooth media curvelets spread significantly

Curvelet propagation

Curvelet propagation

Major challenge. Limit ourselves to migration amplitude recovery!

“Imaged” curvelet

Hessian/Normal operator

[Stolk 2002, ten Kroode 1997, de Hoop 2000, 2003]

Alternative to expensive least-squares migration.

In high-frequency limit Ψ is a PsDO

$$(\Psi f)(x) := (K^T K f)(x) = \int_{\mathbb{R}^d} e^{-ix \cdot \xi} a(x, \xi) \hat{f}(\xi) d\xi$$

- pseudolocal
- singularities are preserved

Corresponds to a spatially-varying dip filter after appropriate preconditioning ($\Rightarrow$ zero order).

Invariance under Hessian matrix

- curvelets remain invariant
- approximation improves for higher frequencies

Diagonal approximation of the Hessian

Existing scaling methods

Methods are based on a diagonal approximation of Ψ .

- Illumination-based normalization (Rickett '02)
- Amplitude preserved migration (Plessix & Mulder '04)
- Amplitude corrections (Guitton '04)
- Amplitude scaling (Symes '07)

We are interested in an 'Operator and image adaptive' scaling method which

- estimates the action of Ψ from a reference vector close to the actual image
- assumes a smooth symbol of Ψ in space and angle
- does not require the reflectors to be conormal $\Leftrightarrow$ allows for conflicting dips
- stably inverts the diagonal

Approximation

Theorem 1. *The following estimate for the error holds*

$$\|(\Psi(x, D) - C^T \mathbf{D}_\Psi C) \varphi_\mu\|_{L^2(\mathbb{R}^n)} \leq C'' 2^{-|\mu|/2},$$

where C'' is a constant depending on Ψ .

Allows for the decomposition

$$\begin{aligned} (\Psi \varphi_\mu)(x) &\simeq (C^T \mathbf{D}_\Psi C \varphi_\mu)(x) \\ &= (A A^T \varphi_\mu)(x) \end{aligned}$$

with $A := \sqrt{\mathbf{D}_\Psi} C$ and $A^T := C^T \sqrt{\mathbf{D}_\Psi}$.

Approximation

$$\begin{aligned} y(x) &= (\Psi m)(x) + e(x) \\ &\simeq (AA^T m)(x) + e(x) \\ &= Ax_0 + e, \end{aligned}$$

- Wavelet-vagulette like [Donoho, Candes]
- Amenable to nonlinear recovery

Estimation of the diagonal scaling

Diagonal estimation

(a)

(b)

Seismic amplitude recovery

Recovery

Final form

$$\mathbf{y} = \mathbf{A}\mathbf{x}_0 + \boldsymbol{\varepsilon}$$

with $\mathbf{x}_0 = \boldsymbol{\Gamma}\mathbf{C}\mathbf{m}$ and $\boldsymbol{\varepsilon} = \mathbf{A}\mathbf{e}$.

Solve

$$\mathbf{P} : \begin{cases} \min_{\mathbf{x}} J(\mathbf{x}) & \text{subject to } \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = (\mathbf{A}^H)^{\dagger} \tilde{\mathbf{x}} \end{cases}$$

with

$$J(\mathbf{x}) = \overbrace{\alpha \|\mathbf{x}\|_1}^{\text{sparsity}} + \underbrace{\beta \left\| \boldsymbol{\Lambda}^{1/2} \left(\mathbf{A}^H \right)^{\dagger} \mathbf{x} \right\|_p}_{\text{continuity}} .$$

Gradient of the reference vector

Application to the SEG AA' model

Example

SEGAA' data:

- “broad-band” half-integrated wavelet [5-60 Hz]
- 324 shots, 176 receivers, shot at 48 m
- 5 s of data

Modeling operator

- Reverse-time migration with optimal check pointing (Symes '07)
- 8000 time steps
- modeling 64, and migration 294 minutes on 68 CPU's

Scaling requires 1 extra migration-demigration

bandpass-filtered reflectivity

migrated image

reference vector

imaged reference vector

bandpass-filtered reflectivity

migrated image

reference vector

diagonal approximation

bandpass-filtered reflectivity

migrated image

reference vector

norm-one recovered

Migrated data

*Amplitude-corrected & denoised
migrated data*

Nonlinear data

Conclusions

Curvelet-domain scaling

- handles conflicting dips (conormality assumption)
- exploits invariance under the PsDO
- robust w.r.t. noise

Diagonal approximation

- exploits smoothness of the symbol
- uses “neighbor” structure of the curvelet transform

Results on the SEG AA' show

- recovery of amplitudes beneath the Salt
- successful recovery of clutter
- improvement of the continuity

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