Seismic imaging and processing with curvelets

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joint work with Deli Wang

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Combinations of ***parsimonious*** *signal representations with nonlinear* ***sparsity*** *promoting programs hold the* ***key*** *to the next-generation of seismic data processing algorithms ...*

Since they

- *allow for formulations that are* ***stable*** *w.r.t.*
  - *noise*
  - *incomplete data*
  - *moderate phase rotations and amplitude errors*

Finding a **sparse** representation for seismic data & images is complicated because of

- wavefronts & reflectors are multiscale & multi-directional
- the presence of caustics, faults and pinchouts

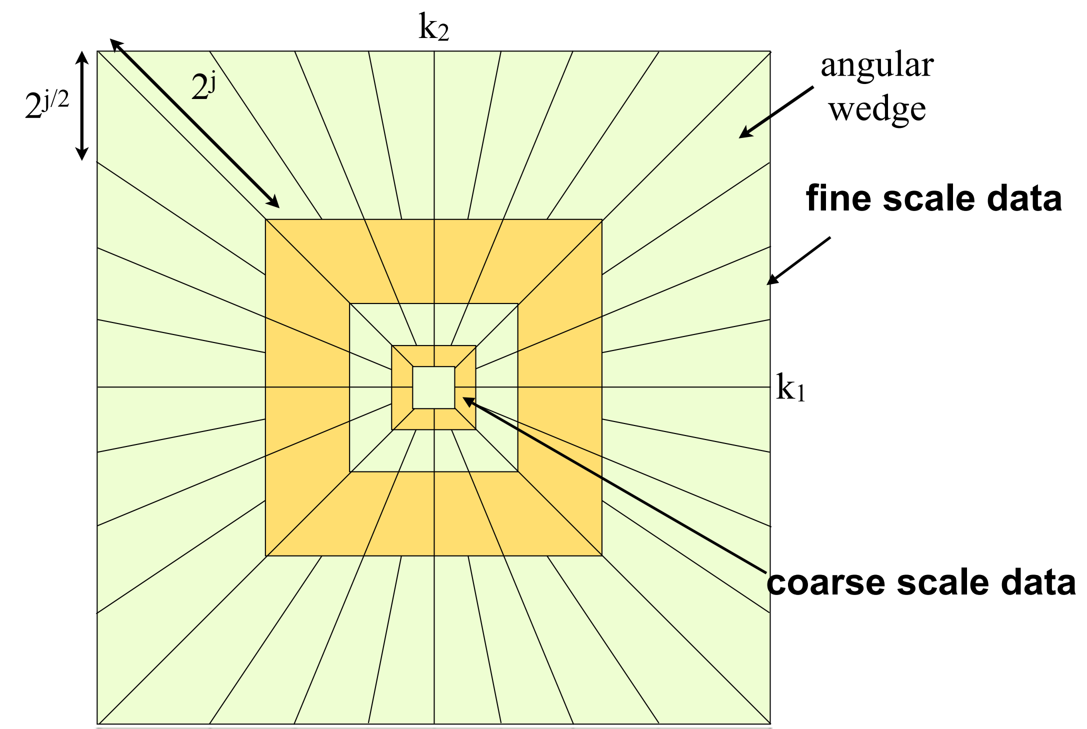
The curvelet transform

Representations for seismic data

Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

Properties curvelet transform:

- **multiscale:** tiling of the FK domain into dyadic coronae
- **multi-directional:** coronae sub-partitioned into angular wedges, # of angle doubles every other scale
- **anisotropic:** parabolic scaling principle
- **Rapid decay space**
- **Strictly localized in Fourier**
- **Frame with moderate redundancy (8 X in 2-D and 24 X in 3-D)**



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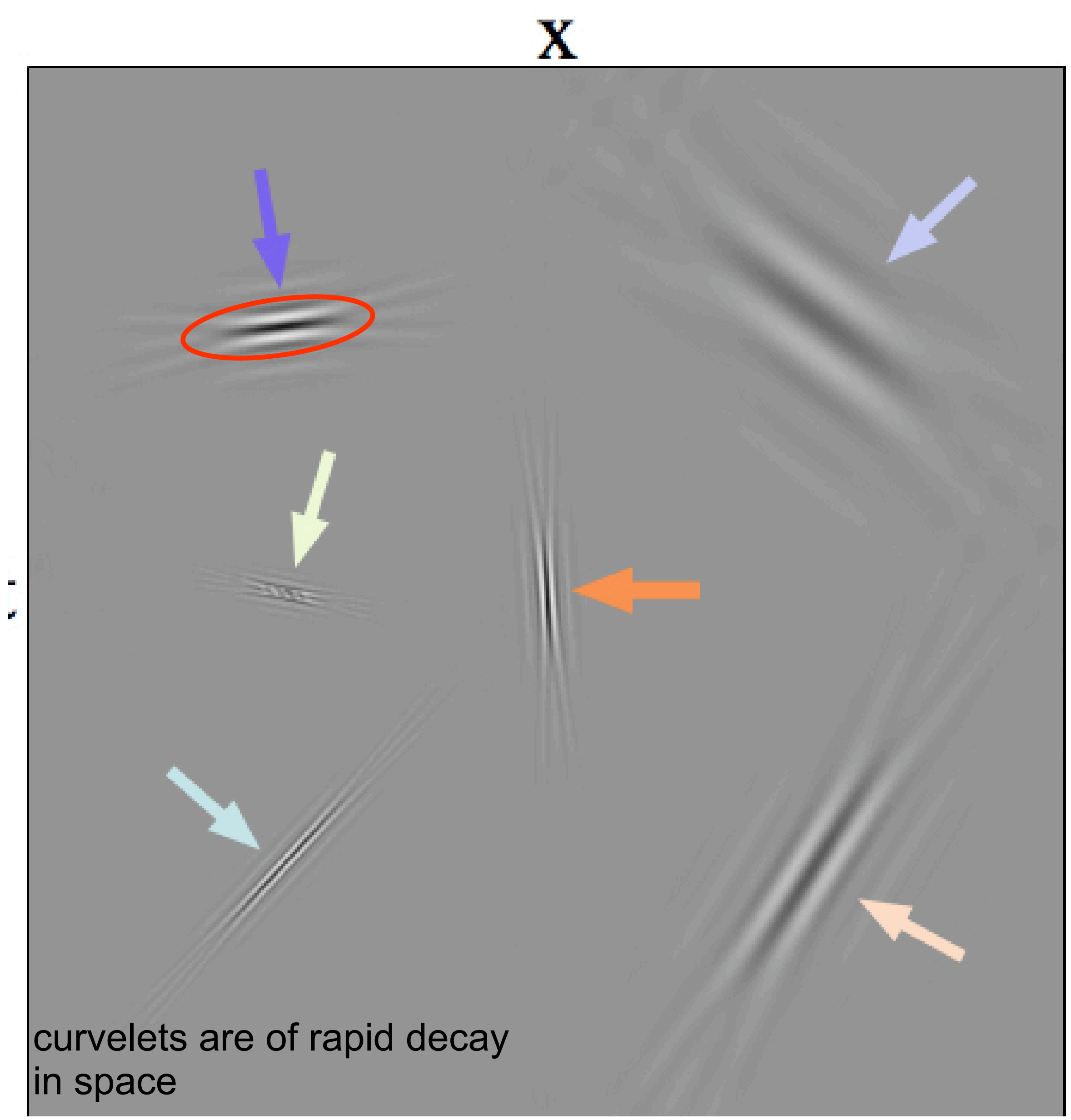
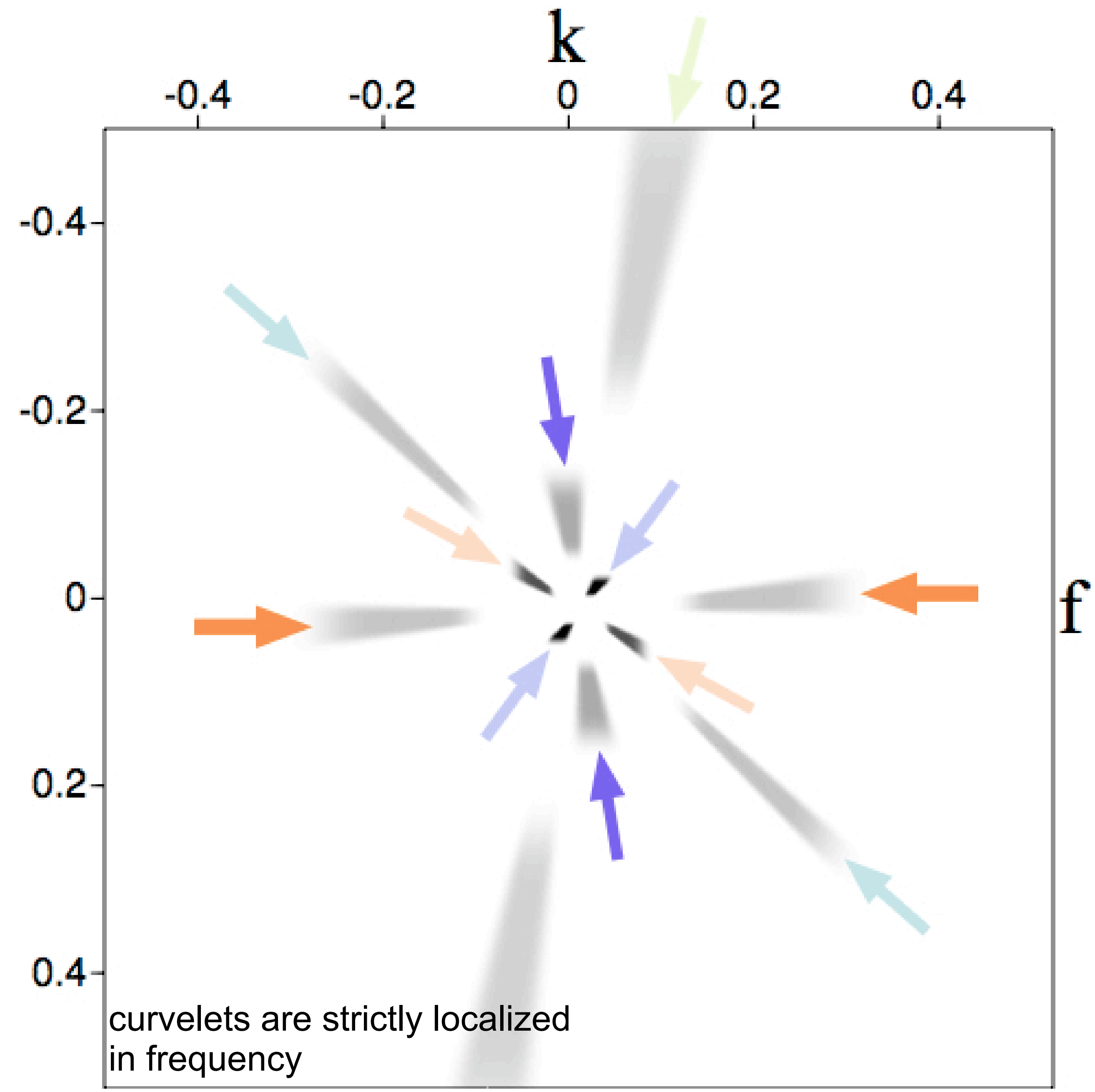
2-D curvelets



x-t



f-k

Oscillatory in one direction and smooth in the others!

Obey *parabolic* scaling relation $\text{length} \approx \text{width}^2$

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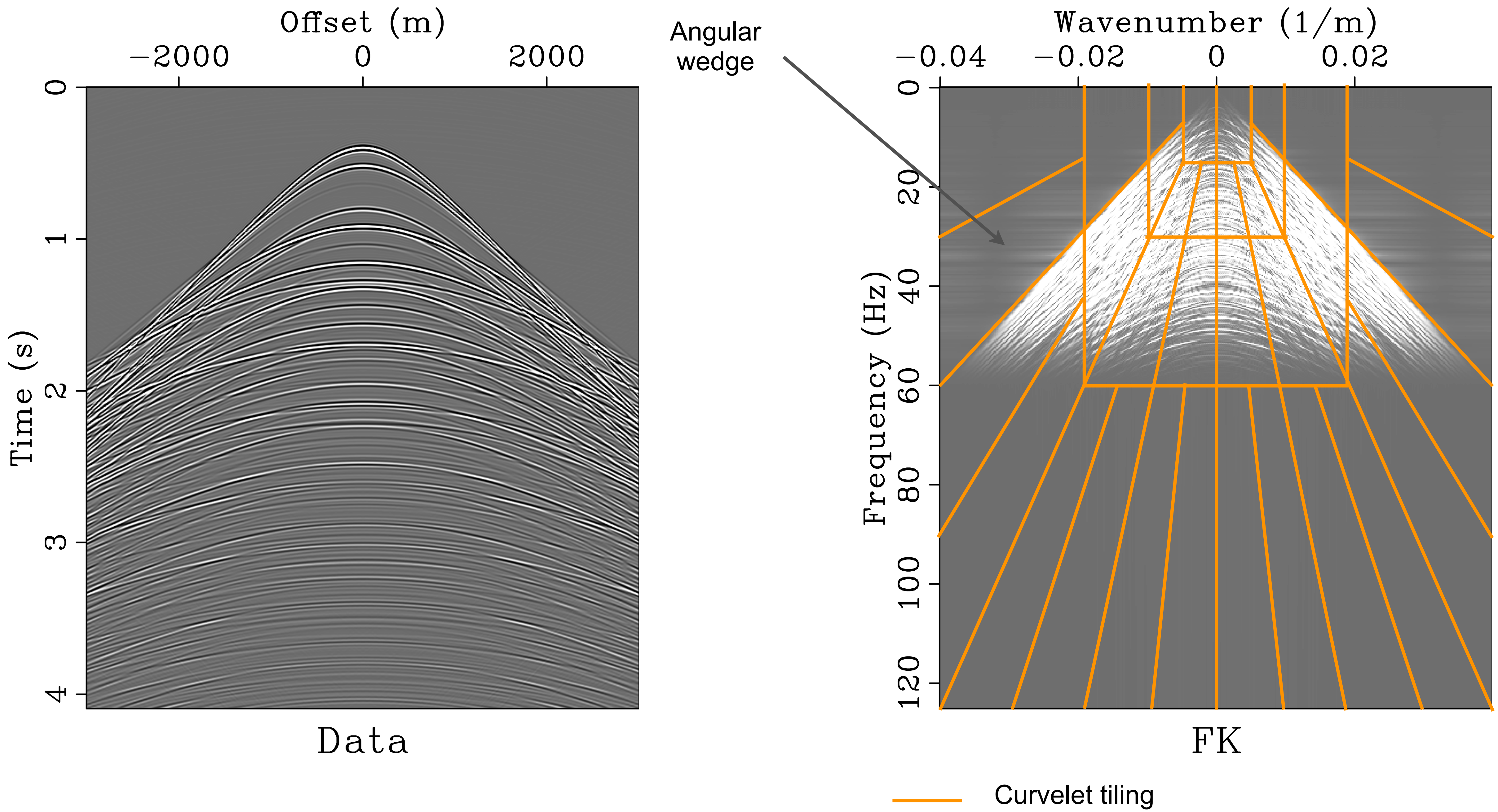
Curvelet tiling & seismic data



# of angles doubles every other scale doubling!

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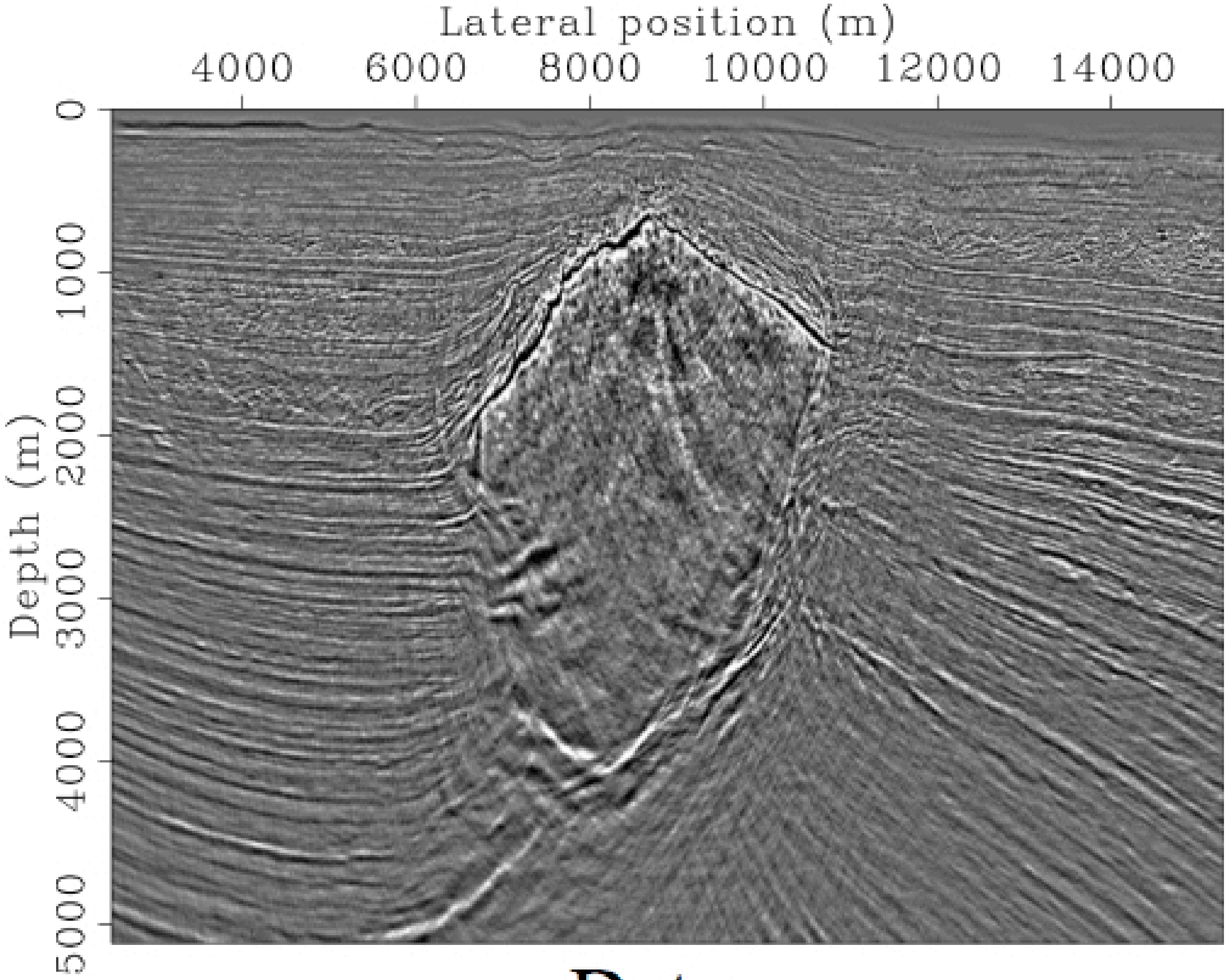
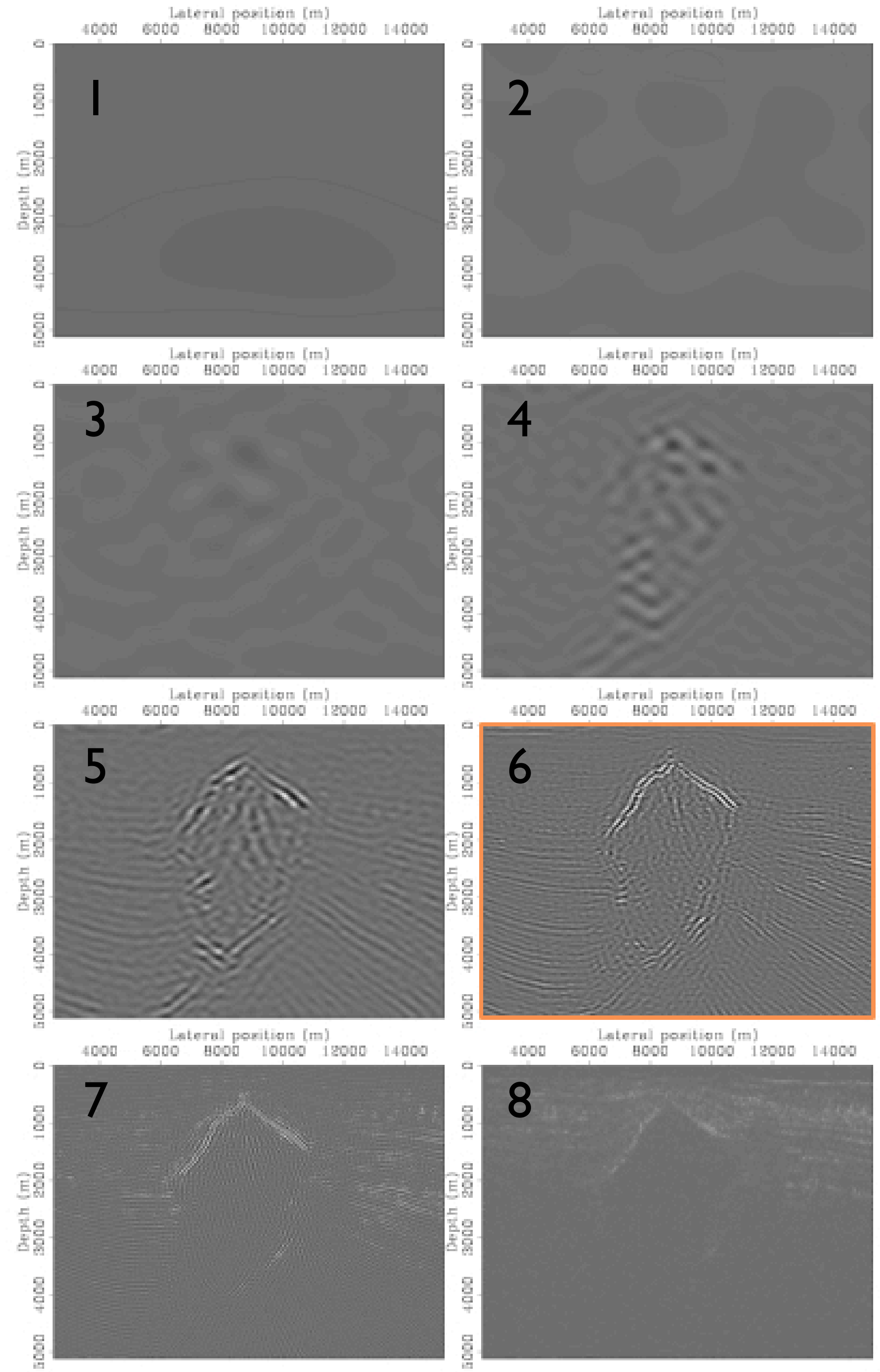
Real data frequency bands example



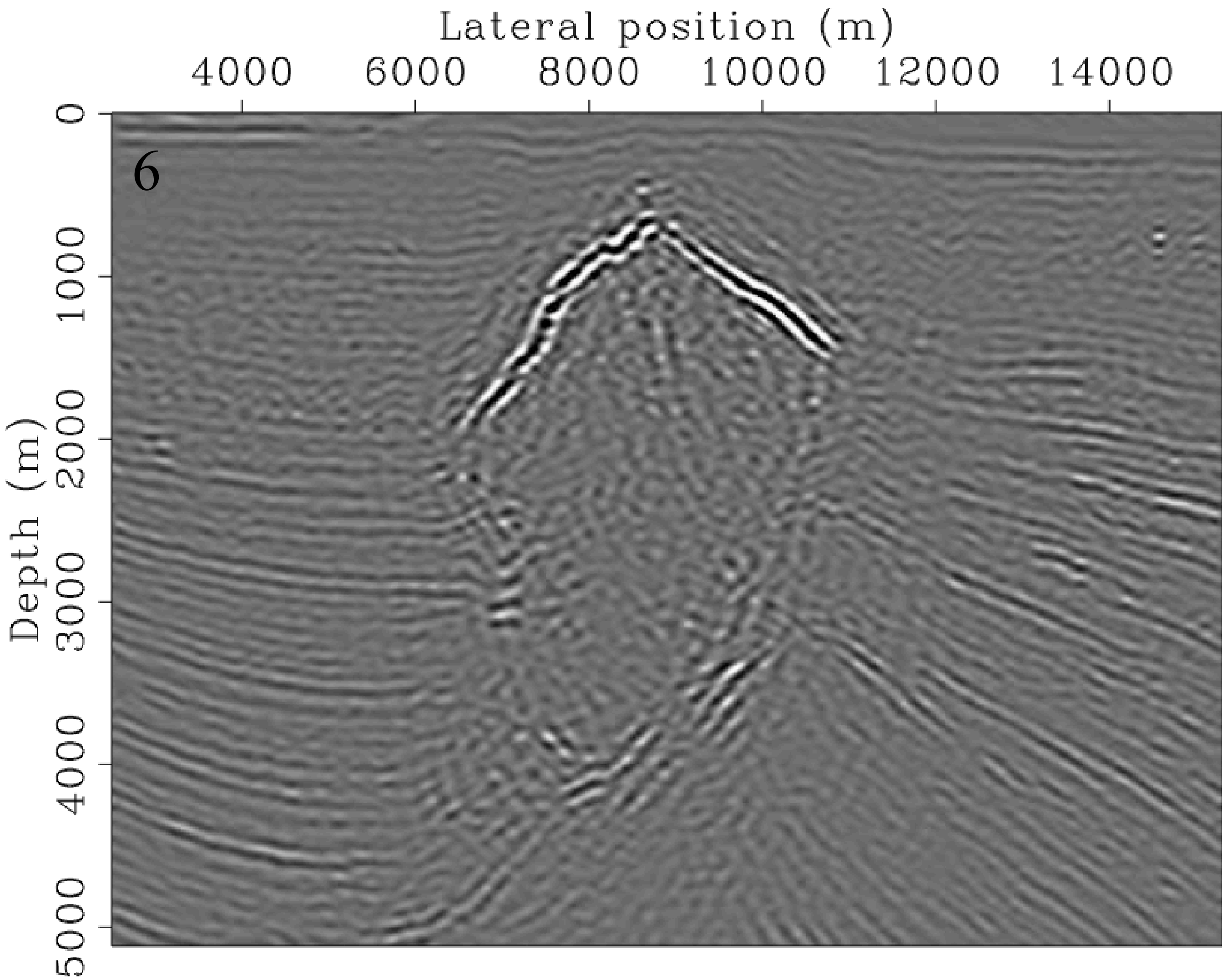
Data

Data is multiscale!



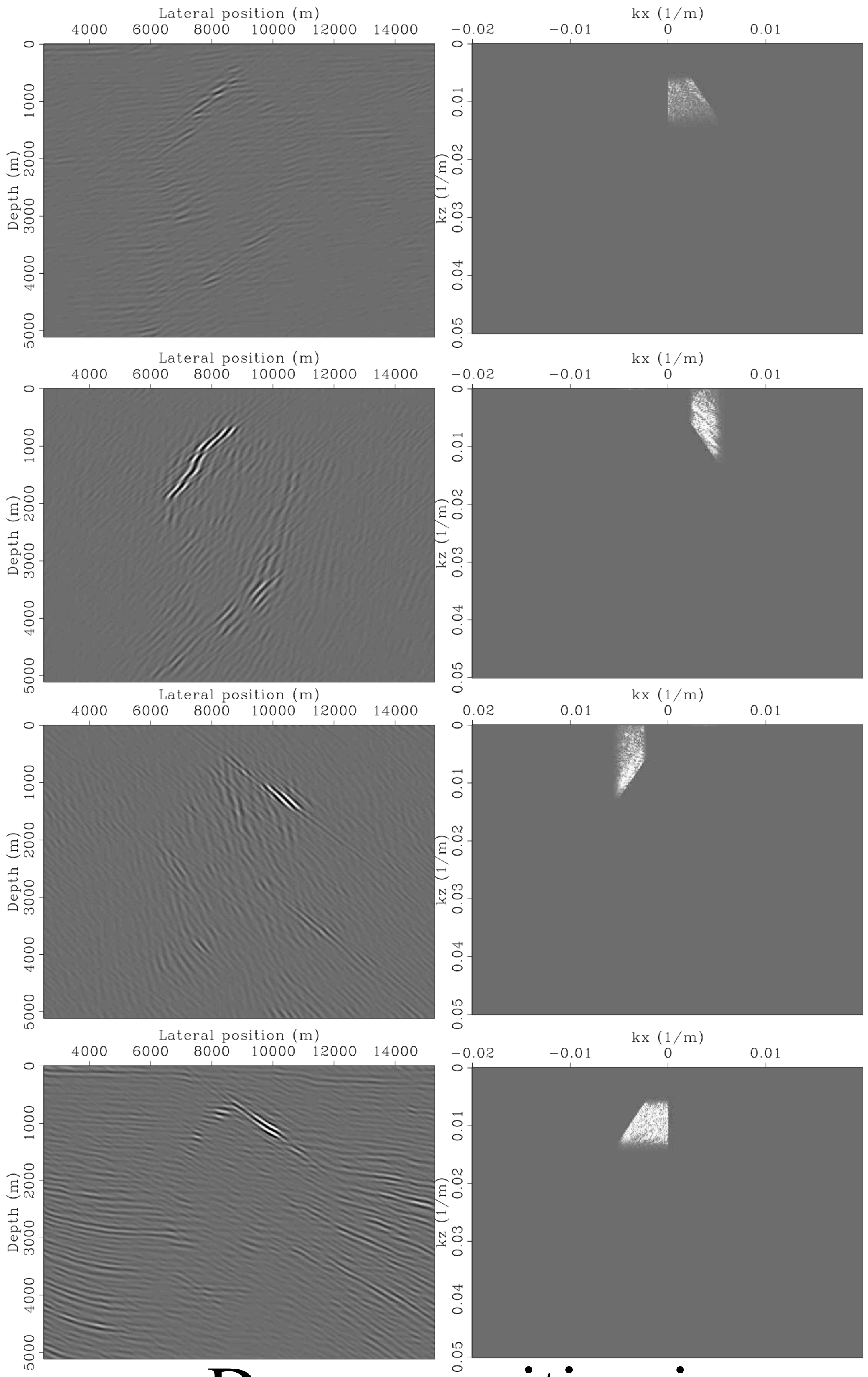
Decomposition in frequency bands

Single frequency band angular wedges



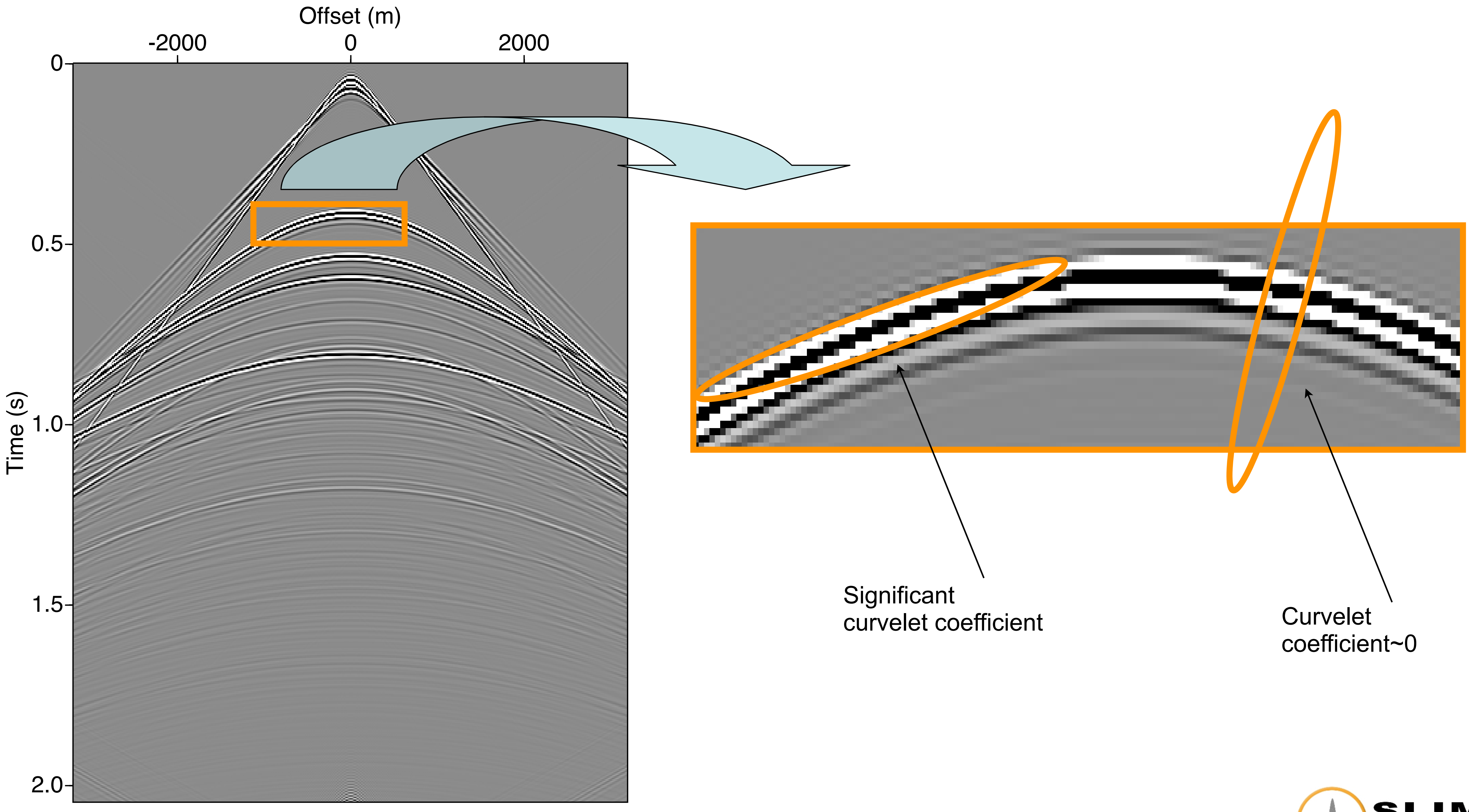
6th scale image

Data is multidirectional!



Decomposition in angular wedges

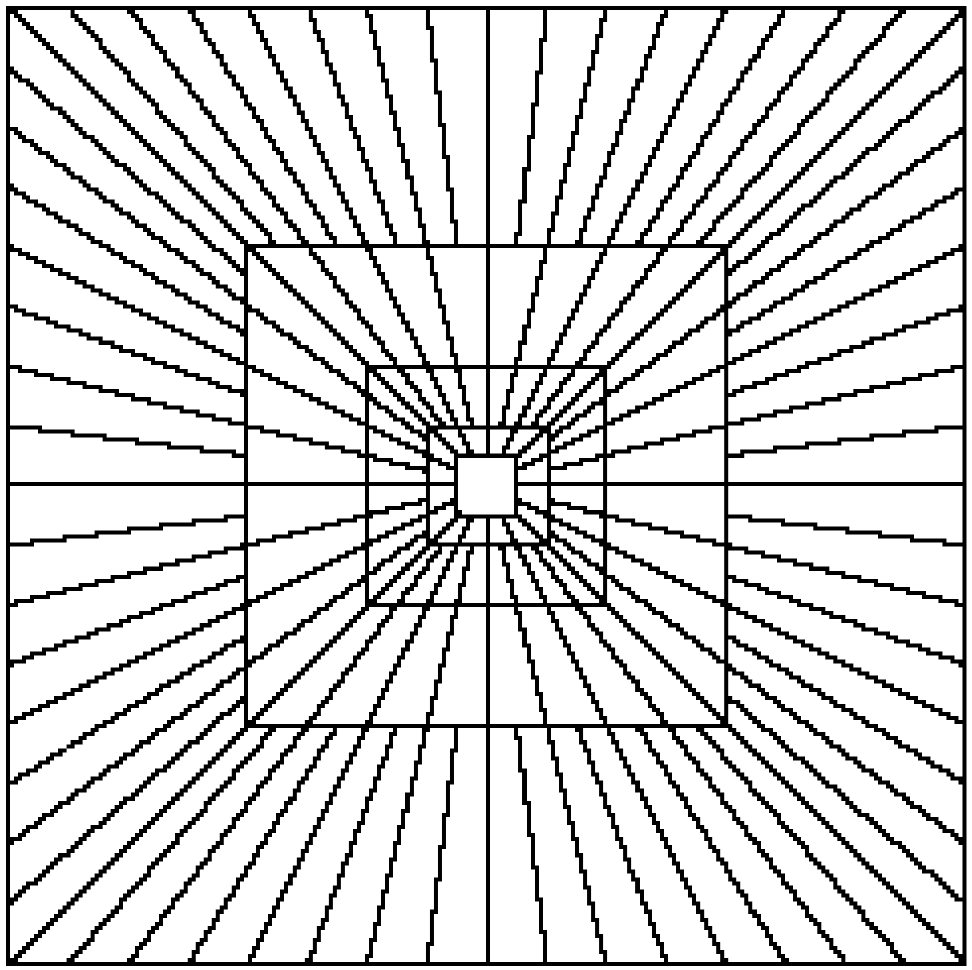
Wavefront detection

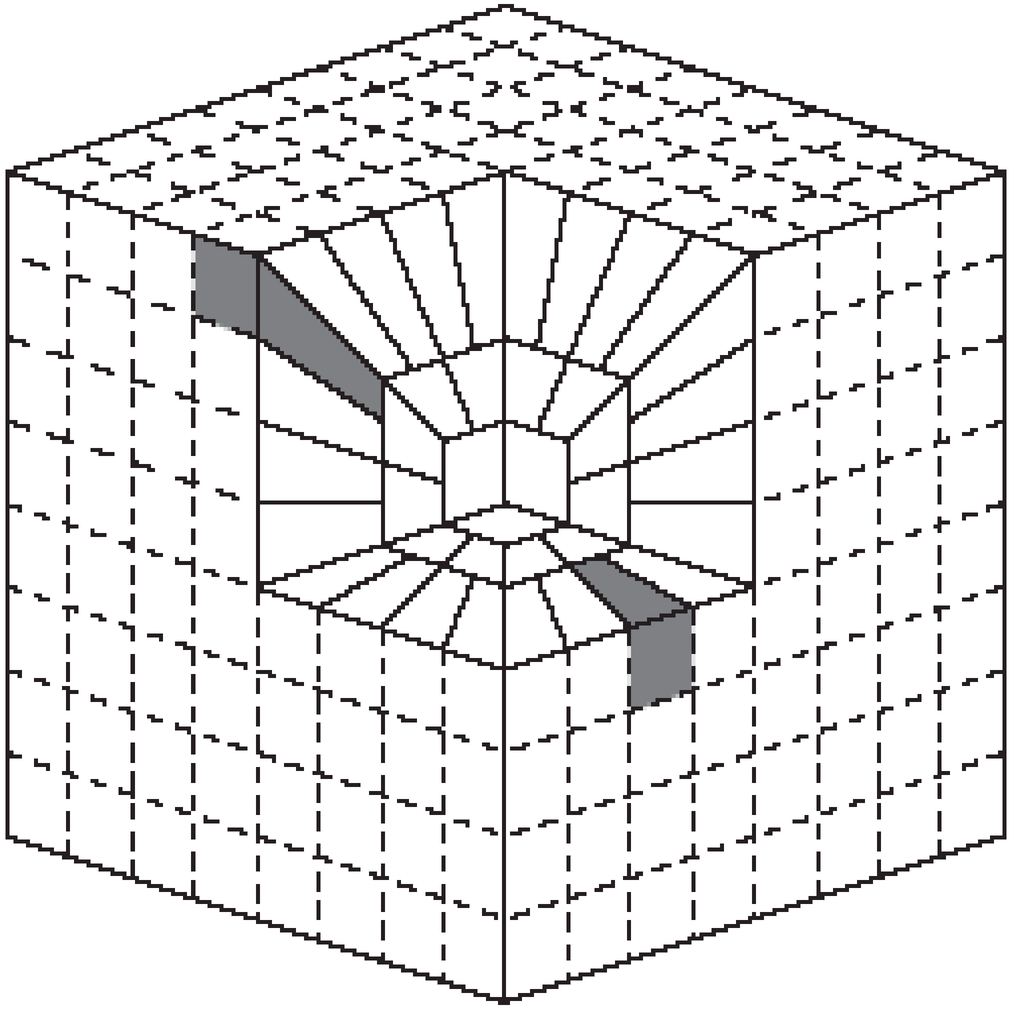


Extenstion to 3-D

Cartesian Fourier space

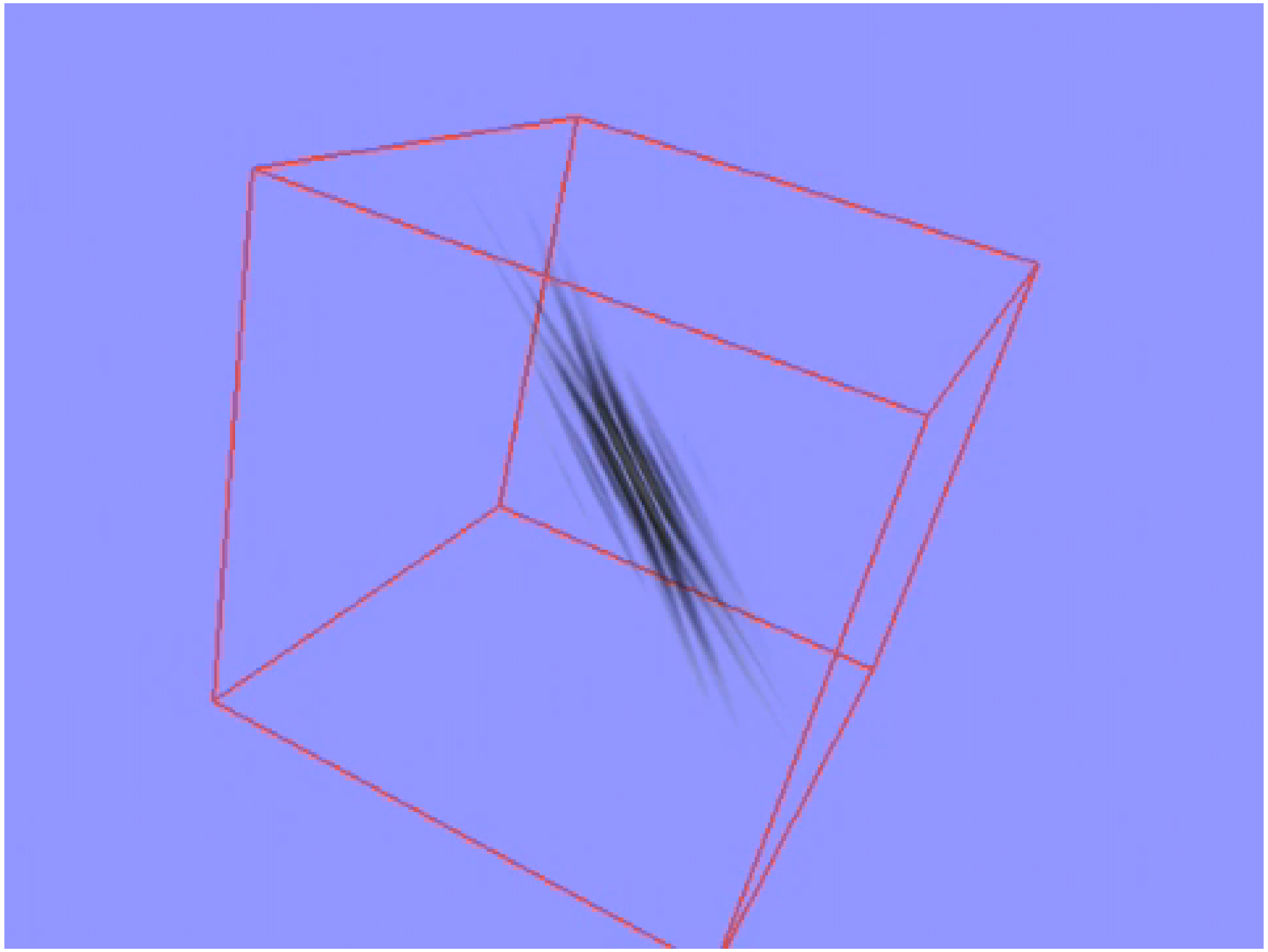
[courtesy Demanet '05, Ying '05]





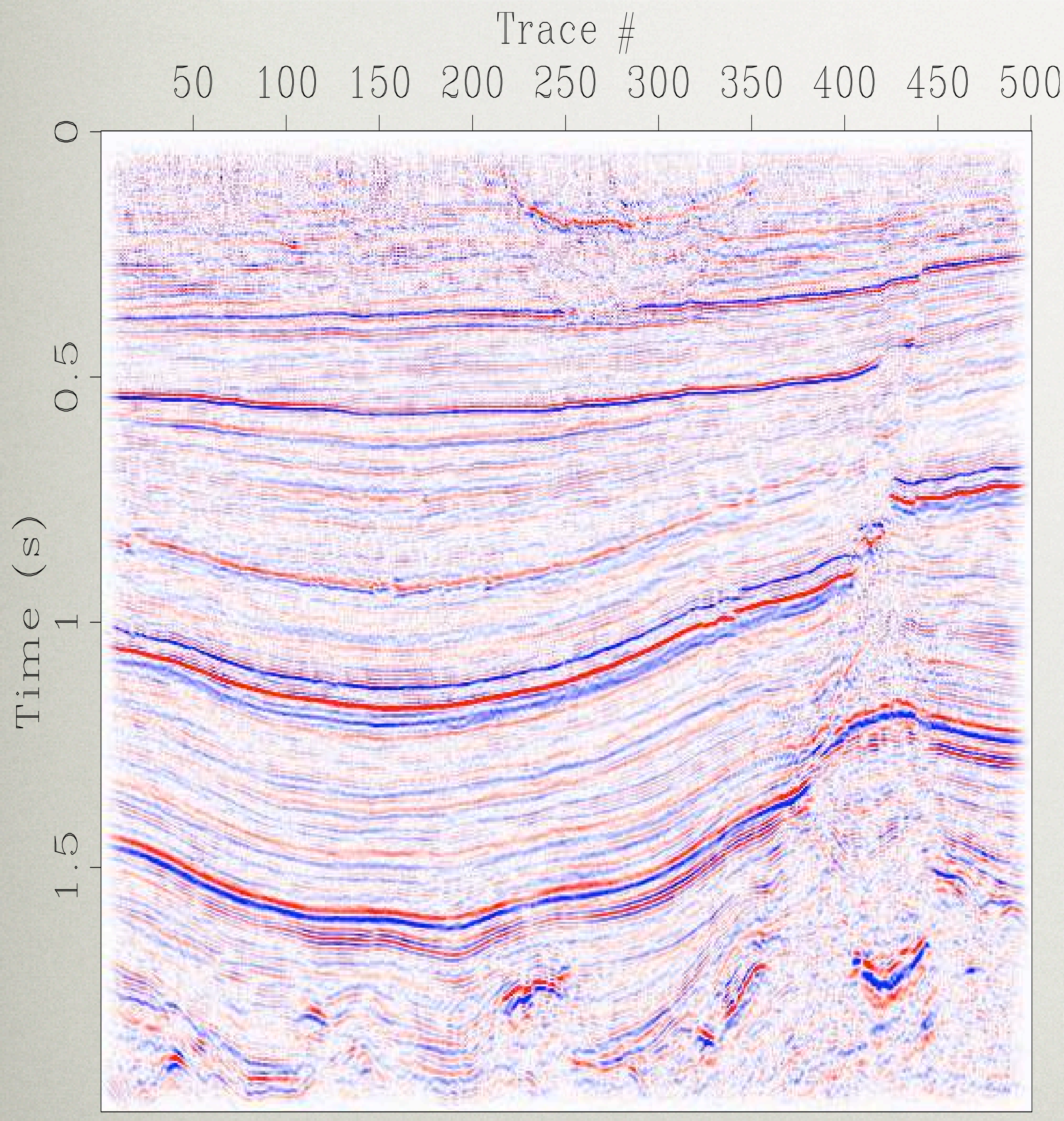
Curvelets live in a wedge in the 3 D Fourier plane...

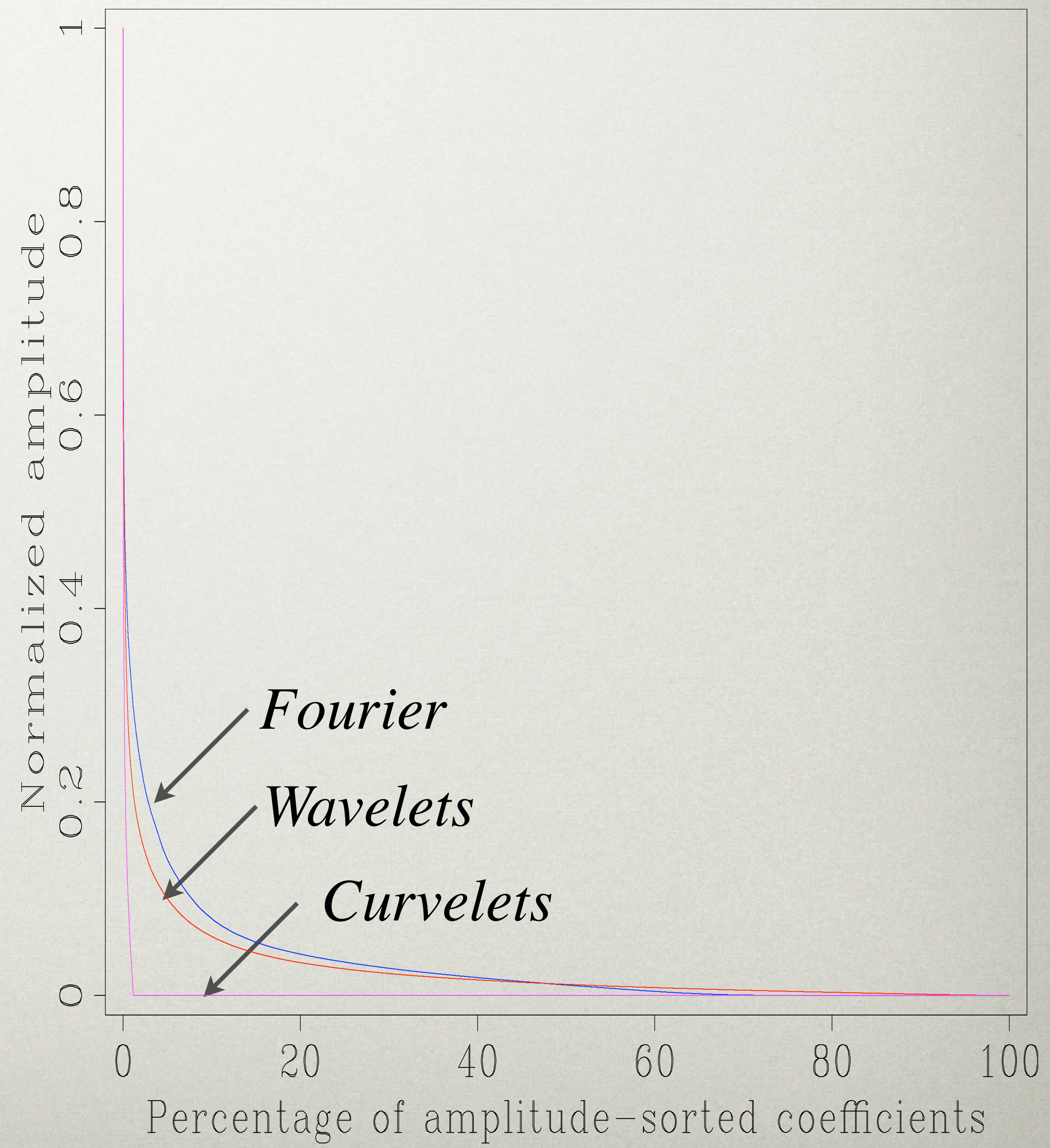
3-D curvelets



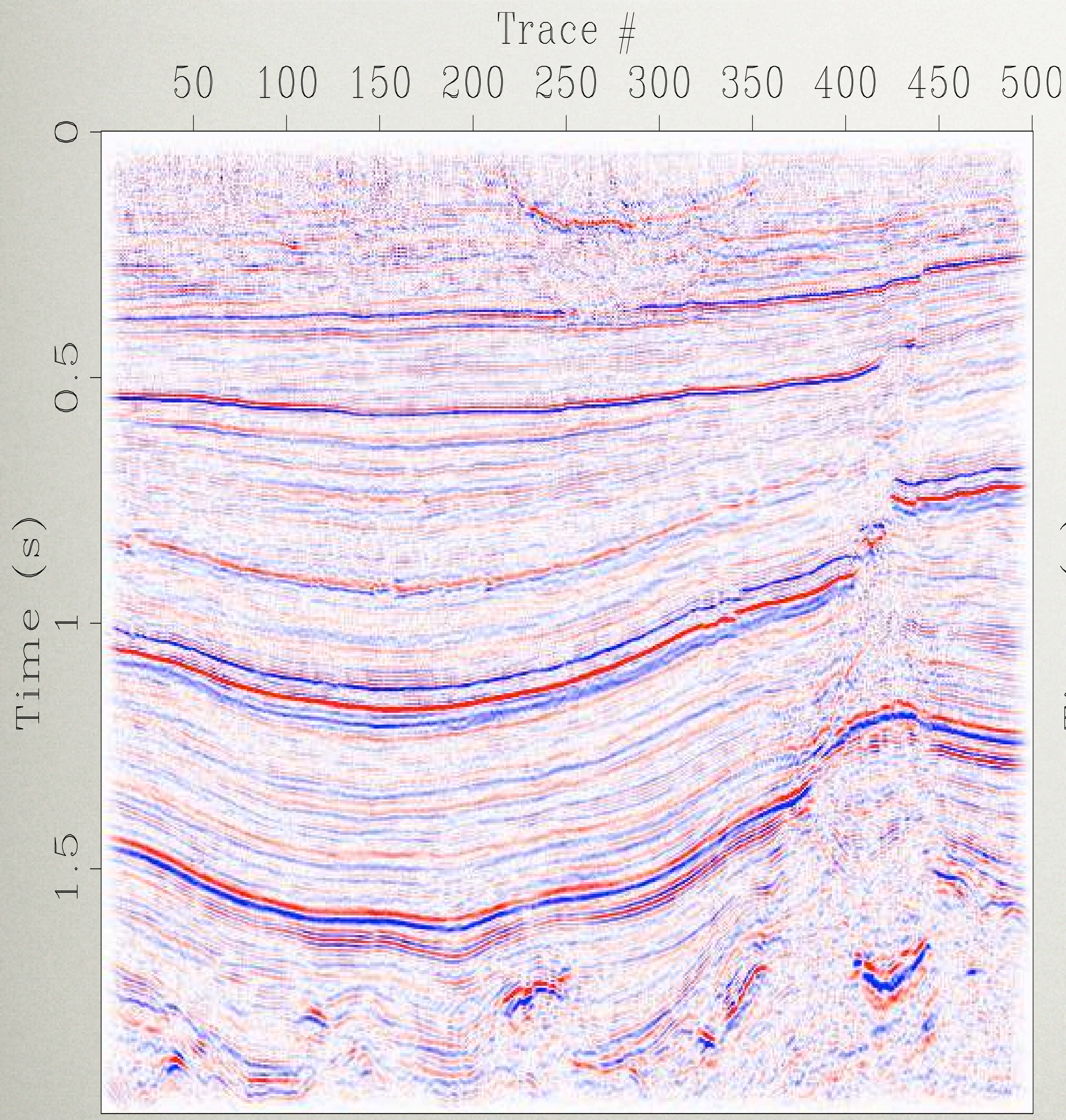
Curvelets are oscillatory in one direction and smooth in the others.

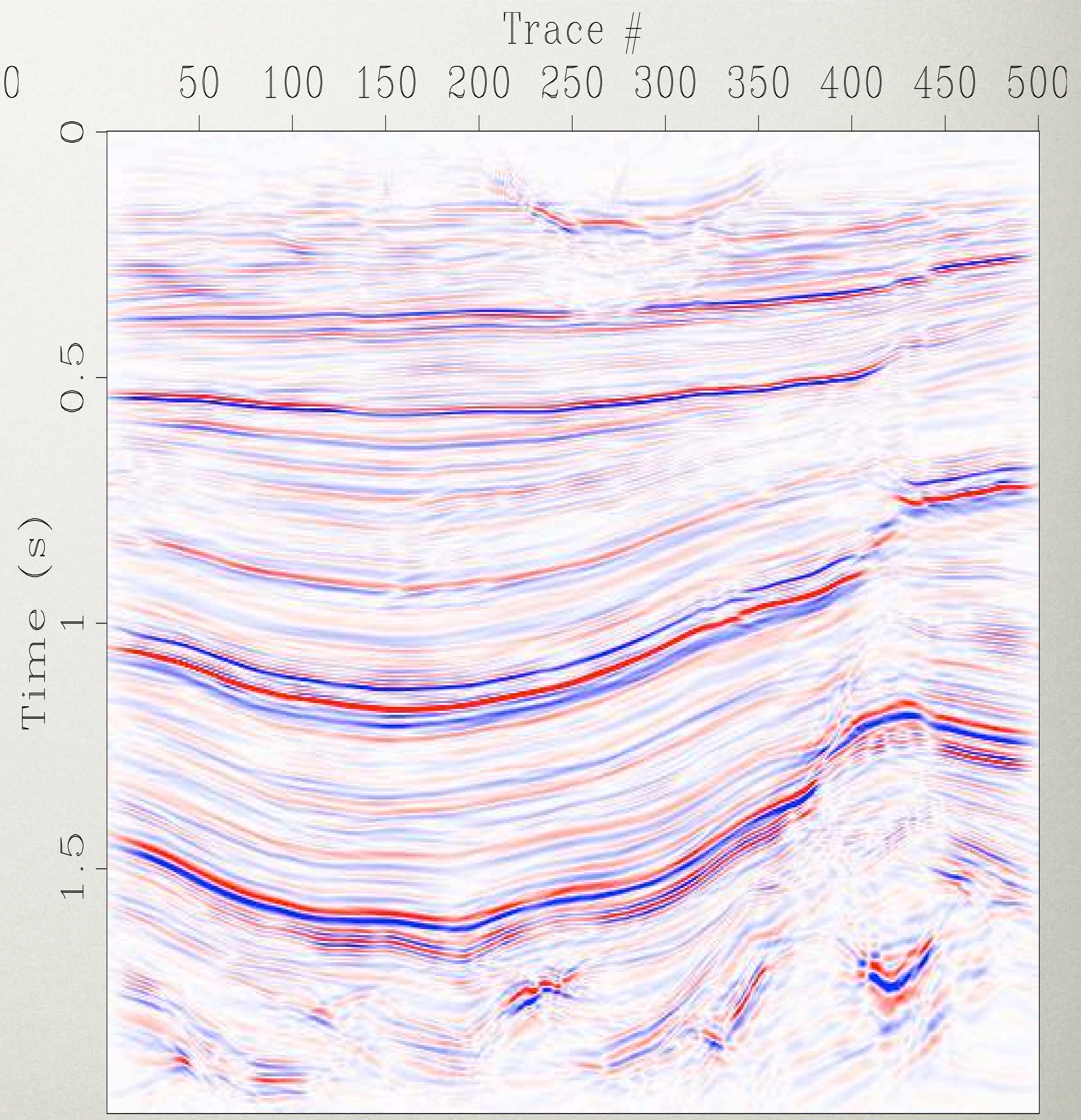
Coefficients Amplitude Decay In Transform Domains





Partial Reconstruction Curvelets (1% largest coefficients)





SNR = 6.0 dB

Curvelet sparsity promotion

Forward model

Linear model for the measurements of a function $\mathbf{m_0}$:

$$\mathbf{y} = \mathbf{K}\mathbf{m}_0 + \mathbf{n}$$

with

$$\begin{aligned}
\mathbf{y} &= \text{data} \\
\mathbf{K} &= \text{the modeling matrix} \\
\mathbf{m}_0 &= \text{the model vector} \\
\mathbf{n} &= \text{noise}
\end{aligned}$$

- inversion of **K** either ill-posed or underdetermined.
- seek a **prior** on **m.**

Key idea

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon$$

↑ *sparsity enhancement* ↑ *data misfit*

When a traveler reaches a fork in the road, the l1 -norm tells him to take either one way or the other, but the l2 -norm instructs him to head off into the bushes.

John F. Claerbout and Francis Muir, 1973

New field "compressive sampling": D. Donoho, E. Candes et. al., M. Elad etc.

Preceded by others in geophysics: M. Sacchi & T. Ulrych and co-workers etc.

Linear quadratic (lsqr):

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_2 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon$$

- **model Gaussian**

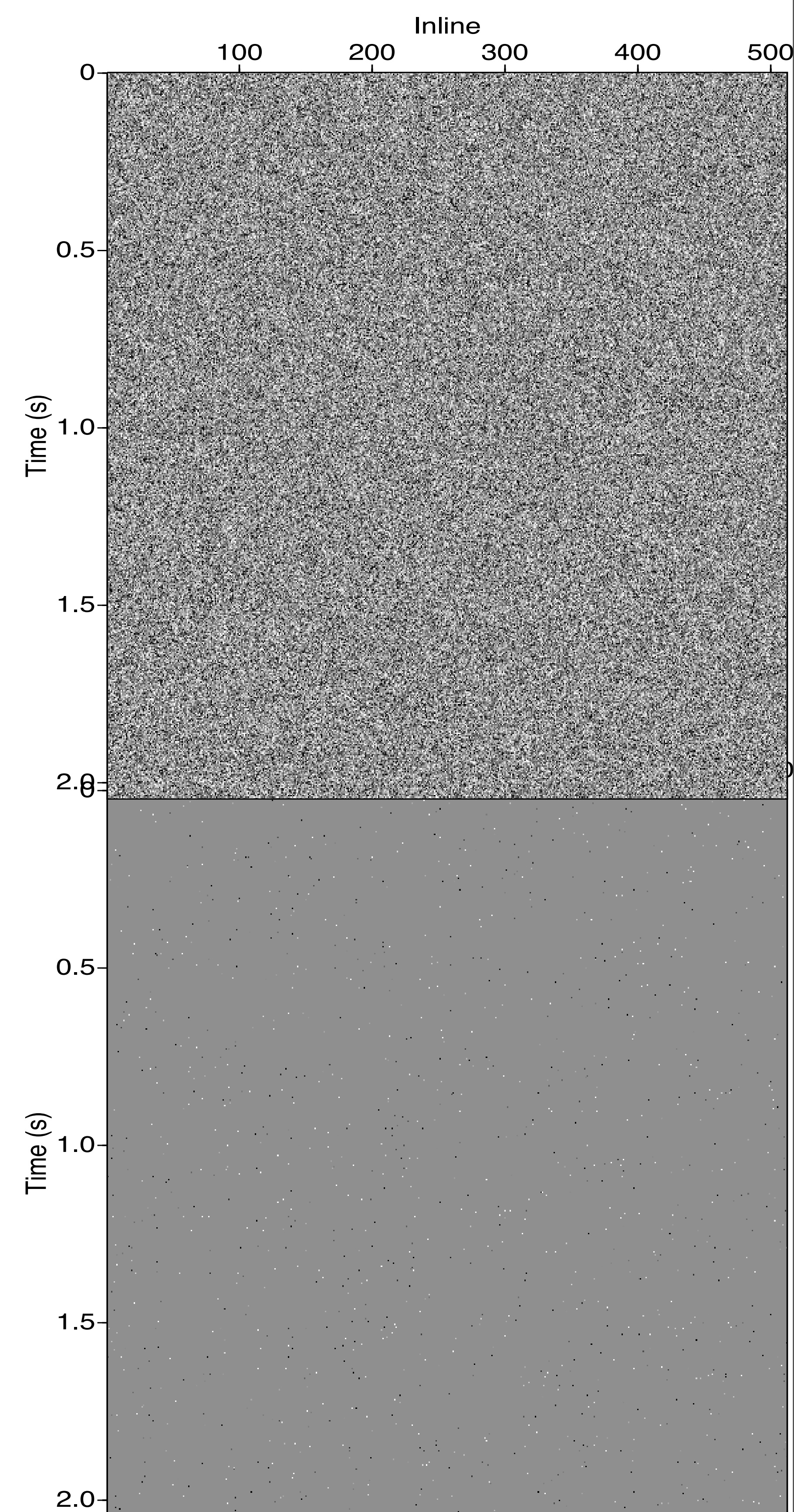
Non-linear :

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon$$

- **model Cauchy (sparse)**

Problem:

- **data does not contain point scatterers**
- **not sparse**

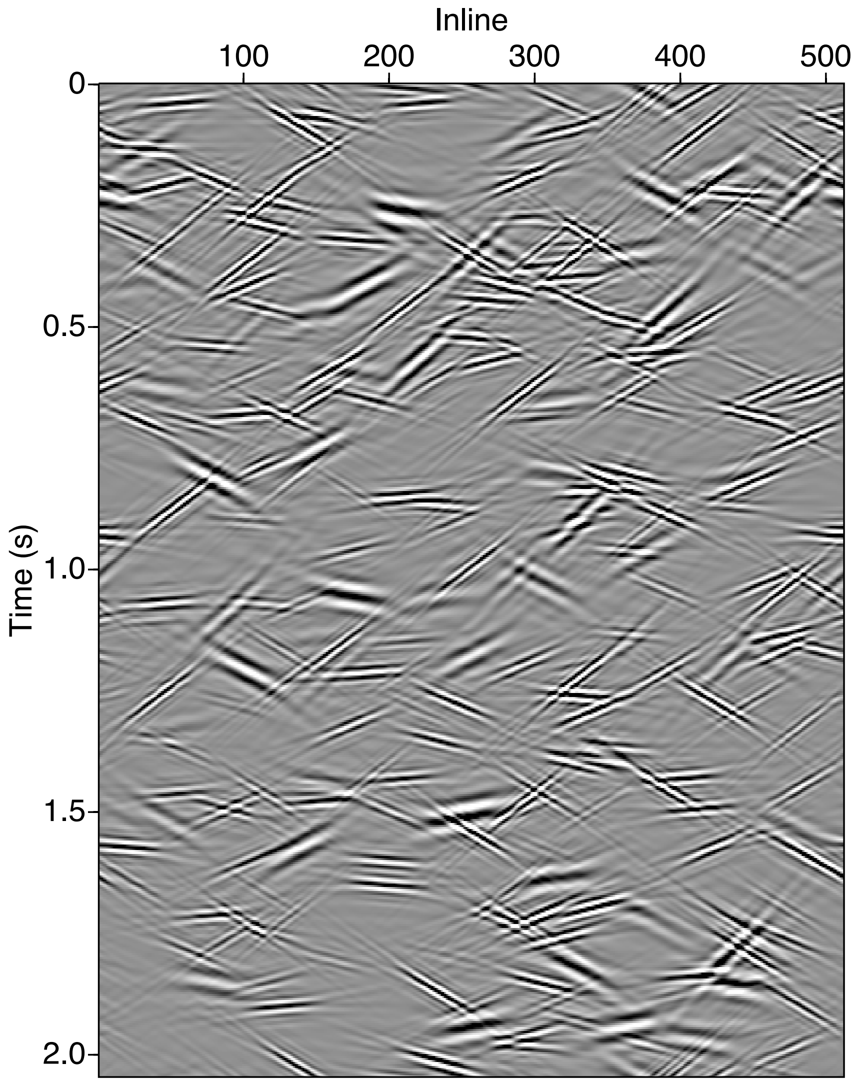


Our contribution

Model as superposition of little plane waves.

Compound ***modeling*** operator with curvelet ***synthesis***:

$$
\begin{aligned}
\mathbf{K} &\mapsto \mathbf{K}\mathbf{C}^T \\
\mathbf{m}_0 &\mapsto \mathbf{x}_0 \\
\tilde{\mathbf{m}} &= \mathbf{C}^T\tilde{\mathbf{x}}
\end{aligned}
$$



Exploit ***parsimoniousness*** of curvelets on seismic data & images ...

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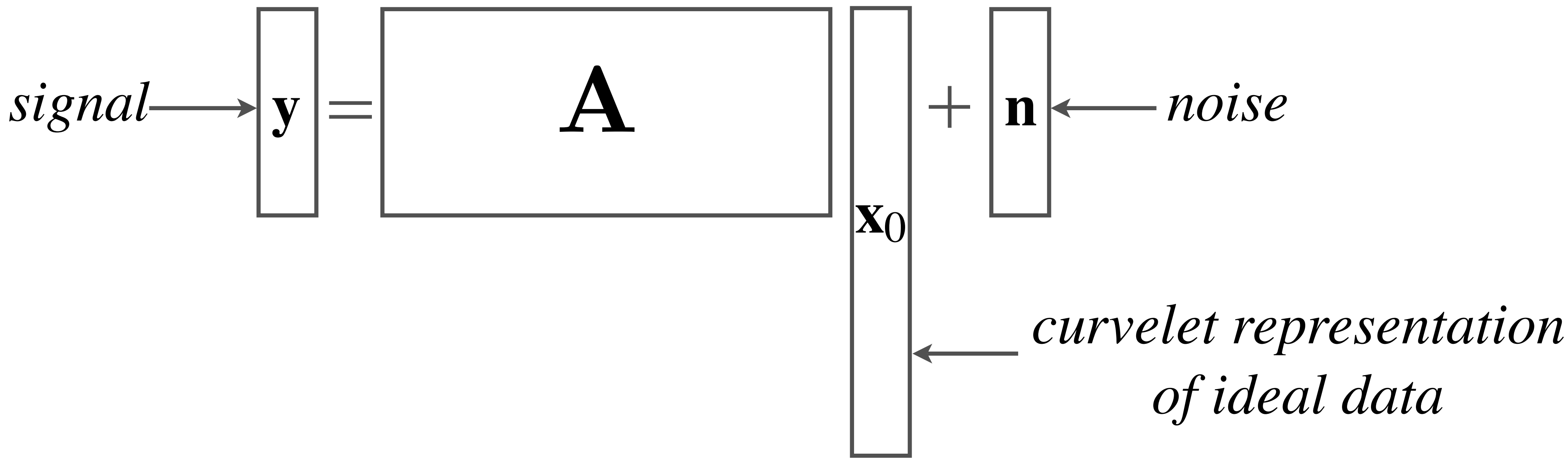
Sparsity-promoting program

Problems boils down to solving for $\mathbf{x}_0$



with

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = \mathbf{C}^T \tilde{\mathbf{x}} \end{cases}$$

- exploit sparsity in the curvelet domain as a prior
- find the sparsest set of curvelet coefficients that match the data, i.e., $\mathbf{y} \approx \mathbf{K}\mathbf{C}^T\tilde{\mathbf{x}}$
- invert an underdetermined system

Solver

Initialize:

$i = 0$; $\mathbf{x}^0 = \mathbf{0}$;

Choose: L, $\|\mathbf{A}^T\mathbf{y}\|_\infty > \lambda_1 > \lambda_2 > \cdots$

while $\|\mathbf{y} - \mathbf{A}\mathbf{x}^i\|_2 > \epsilon$ **do**

for $l = 1$ to L **do**

$$\mathbf{x}^{i+1} = T^s_{\lambda_i}\left(\mathbf{x}^i + \mathbf{A}^T\left(\mathbf{y} - \mathbf{A}\mathbf{x}^i\right)\right)$$

end for

$i = i + 1$;

end while

$\widetilde{\mathbf{f}} = \mathbf{C}^T\mathbf{x}^i$.

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Applications

Problems in seismic processing can be cast in to $\mathbf{P}_\epsilon$

- stable under noise
- stable under missing data

Obtain a formulation that

- explicitly exploits compression by curvelets
- is stable w.r.t. noise
- exploits the "invariance" of curvelets under imaging

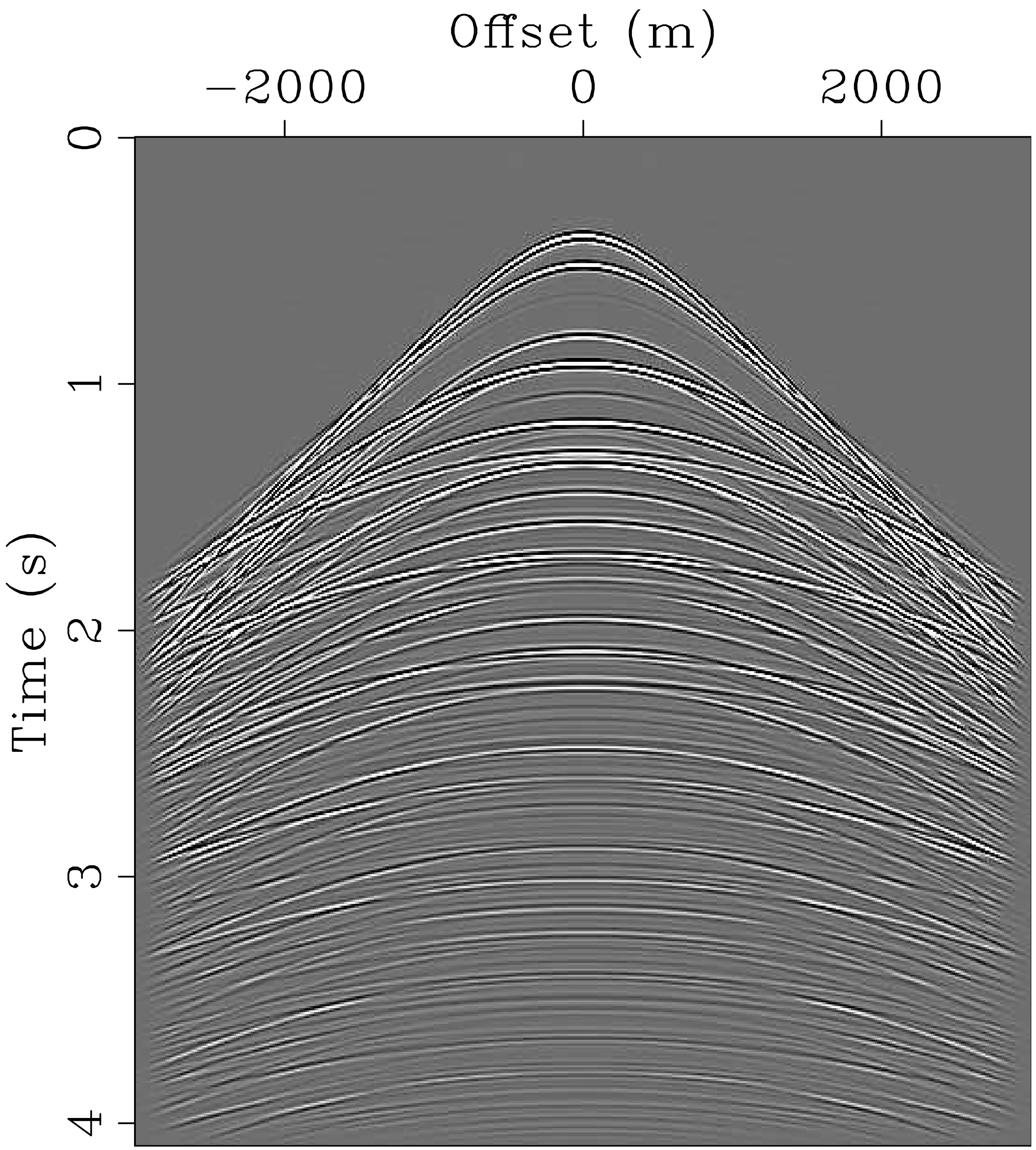
Applications include

- seismic data regularization
- primary-multiple separation
- seismic amplitude recovery

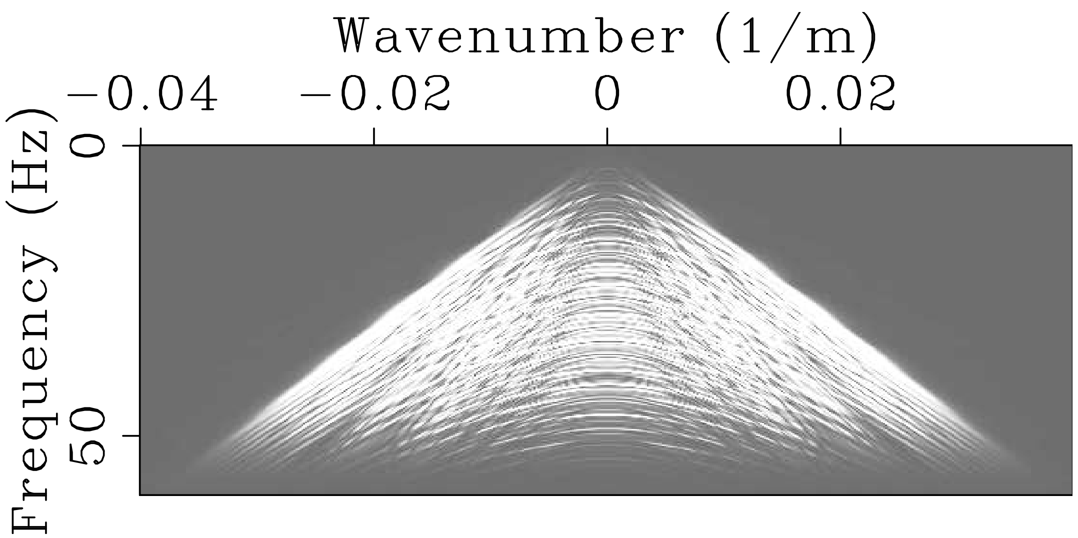
Seismic data regularization

joint work with Gilles Hennenfent

Motivation

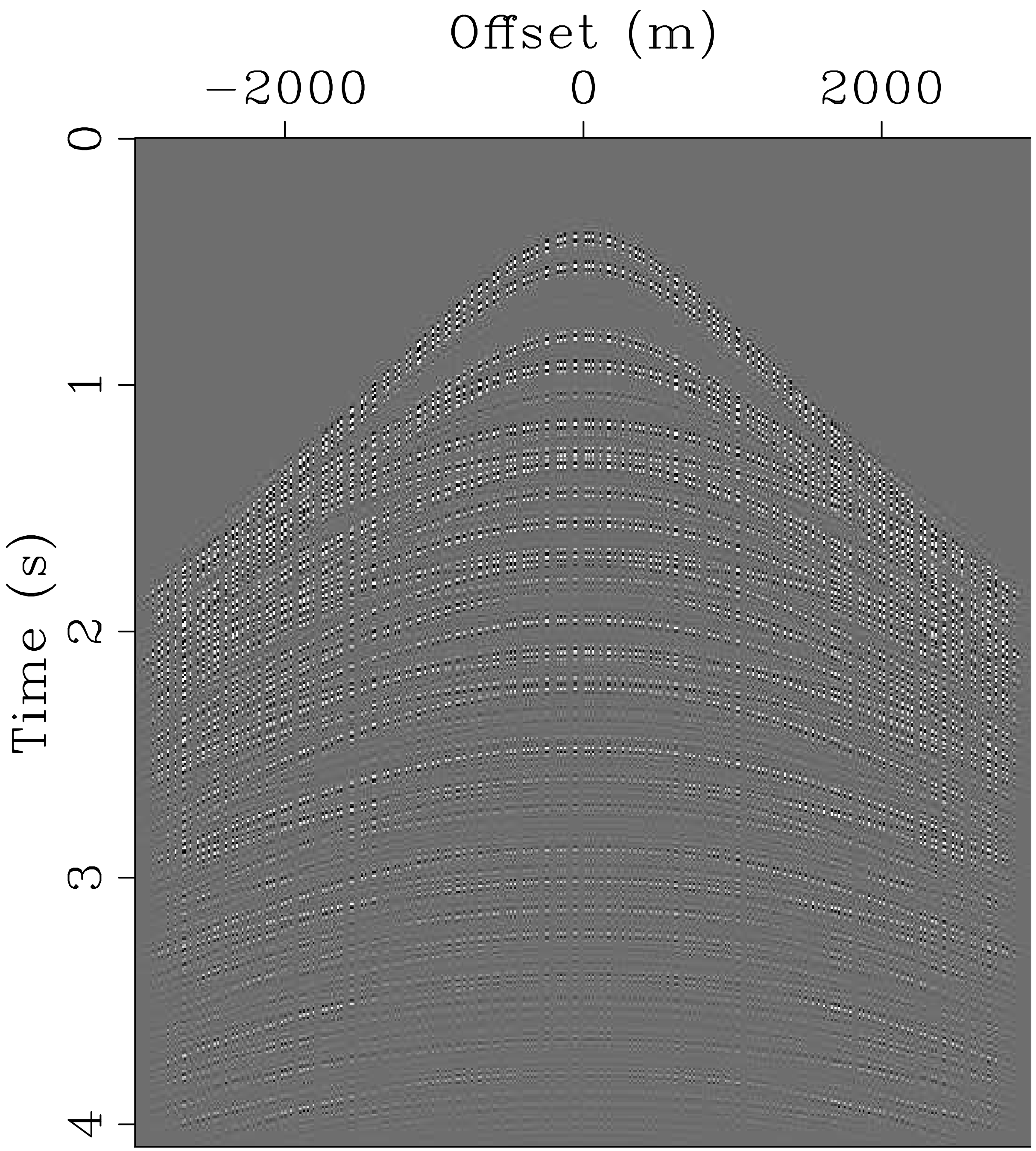


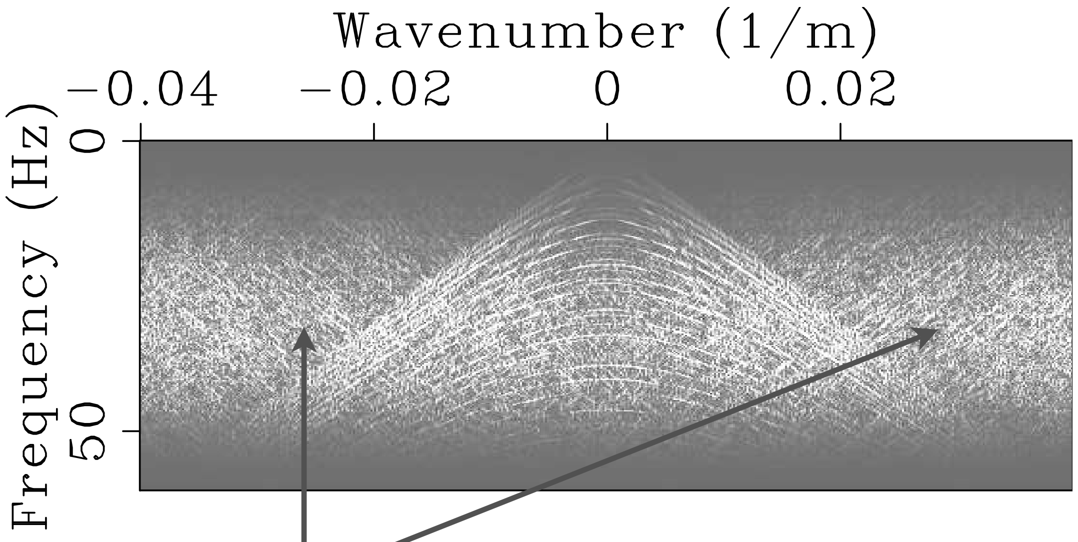
Offset (m)
−2000
0
2000
Time (s)
0
1
2
3
4



Wavenumber (1/m)
−0.04
−0.02
0
0.02
Frequency (Hz)
0
50

Irregular sub-sampling

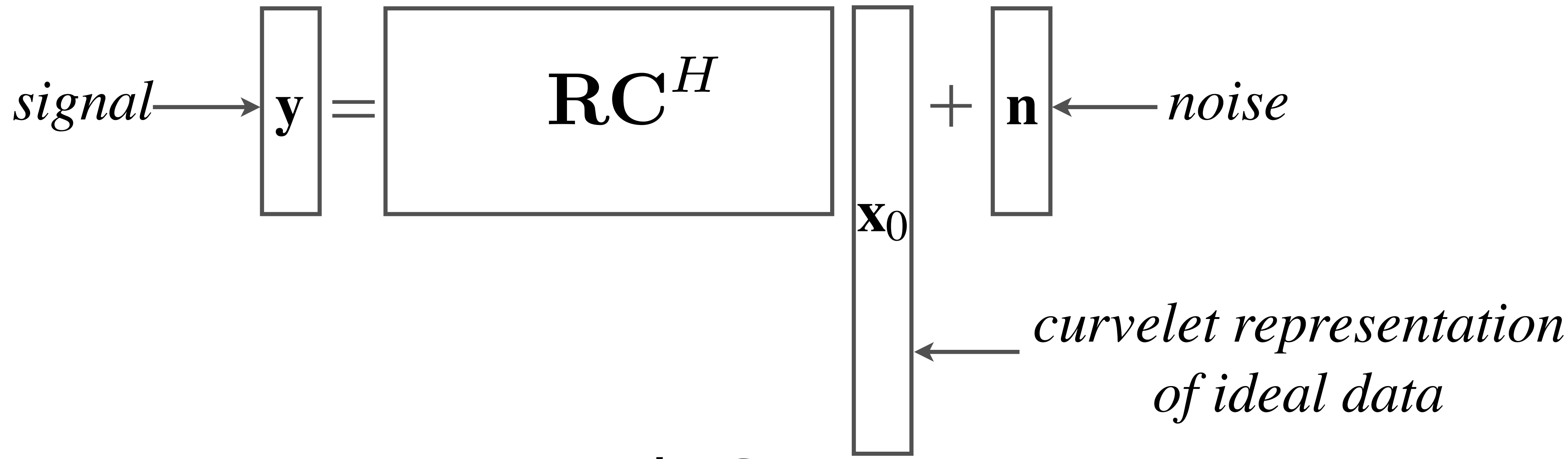




Noisy because of irregular sampling ...

Sparsity-promoting inversion*

Reformulation of the problem



Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI)

- look for the **sparsest/most compressible**, **physical** solution

KEY POINT OF THE RECOVERY

$$
\mathbf{P}_\epsilon : \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \overbrace{\|\mathbf{W}\mathbf{x}\|_1}^{\text{sparsity constraint}} \quad \text{s.t.} \quad \overbrace{\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2}^{\text{data misfit}} \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{C}^T \tilde{\mathbf{x}} \end{cases}
$$

* inspired by Stable Signal Recovery (SSR) theory by E. Candès, J. Romberg, T. Tao, Compressed sensing by D. Donoho & Fourier Reconstruction with Sparse Inversion (FRSI) by P. Zwartjes

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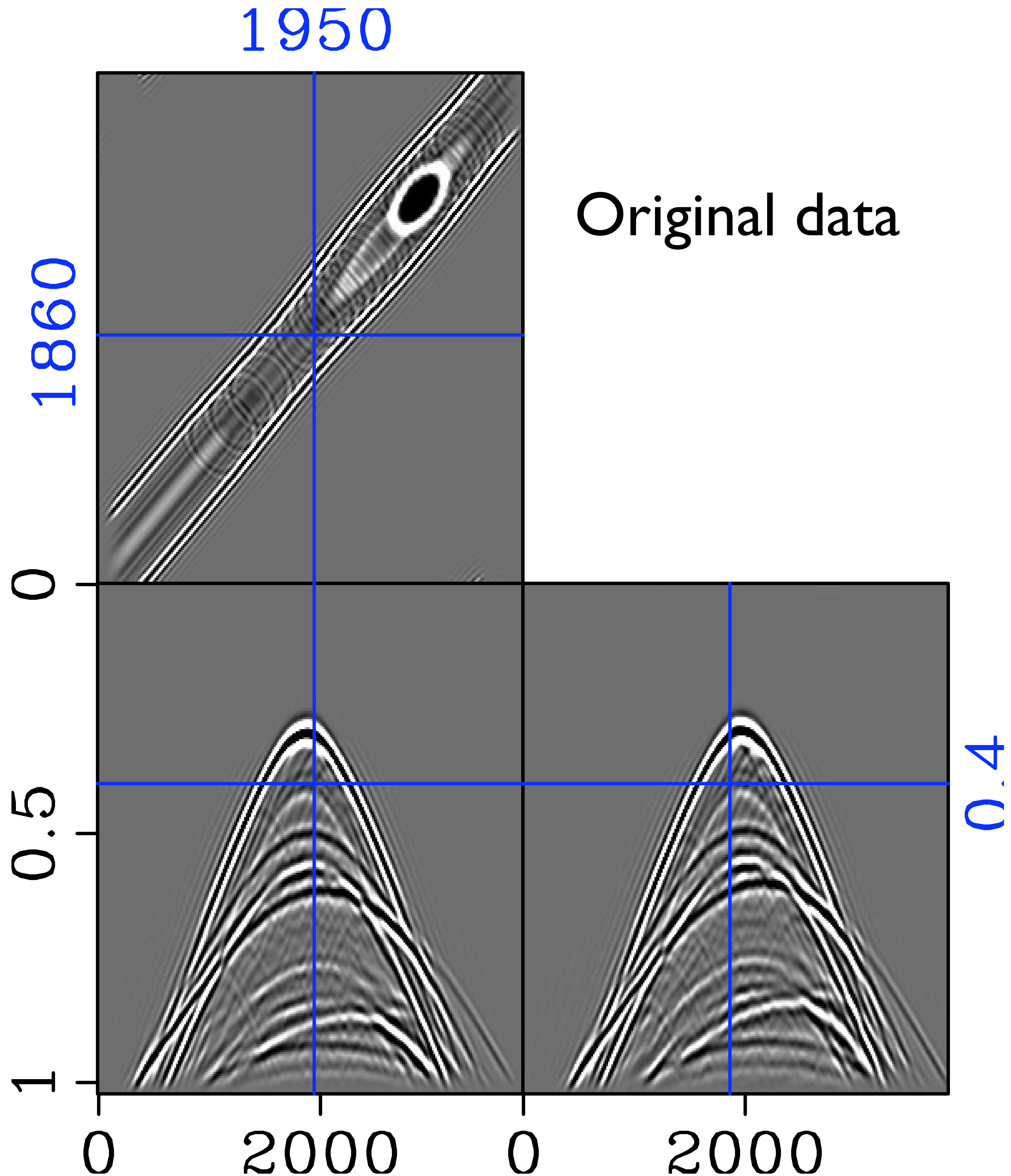
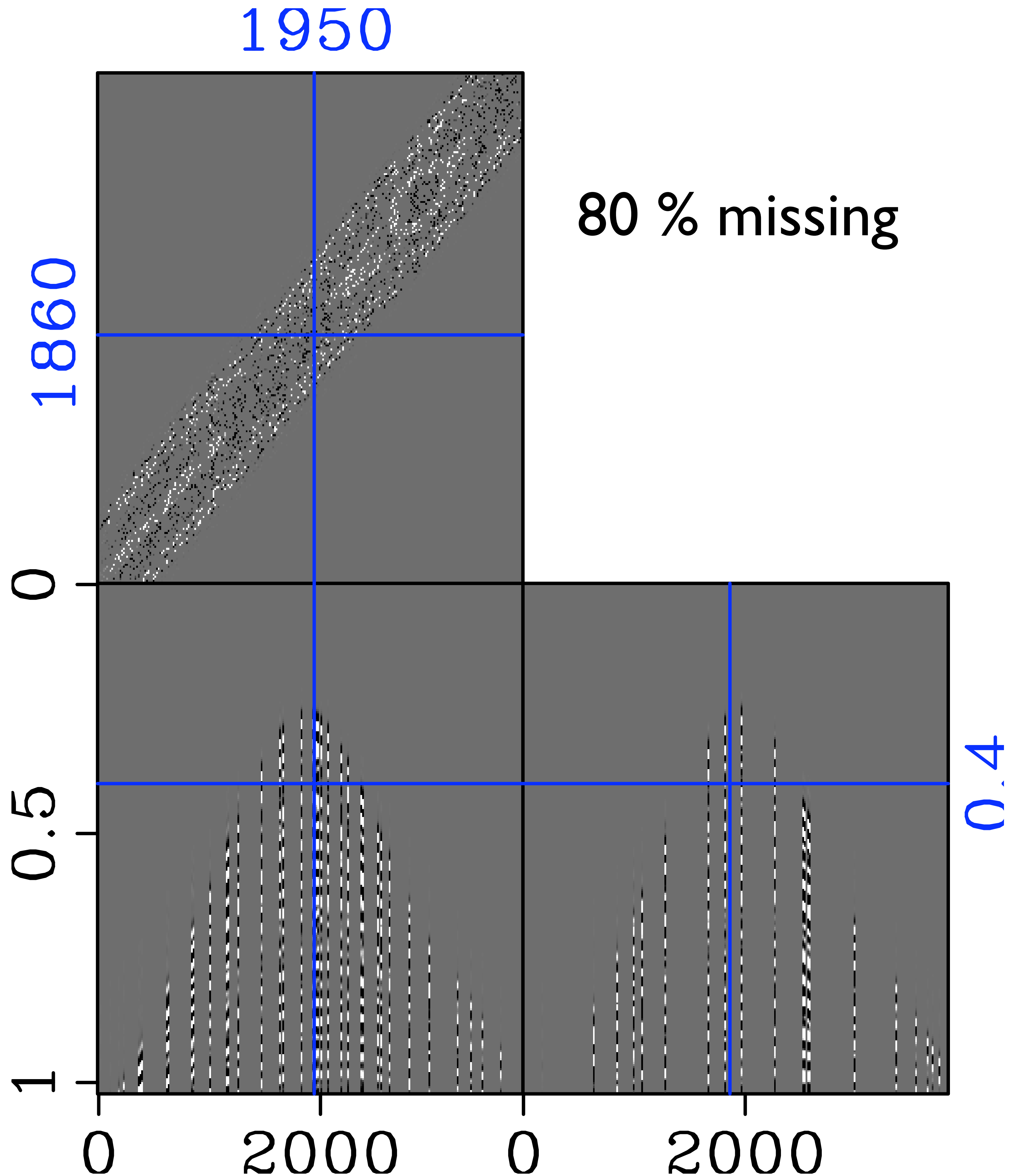
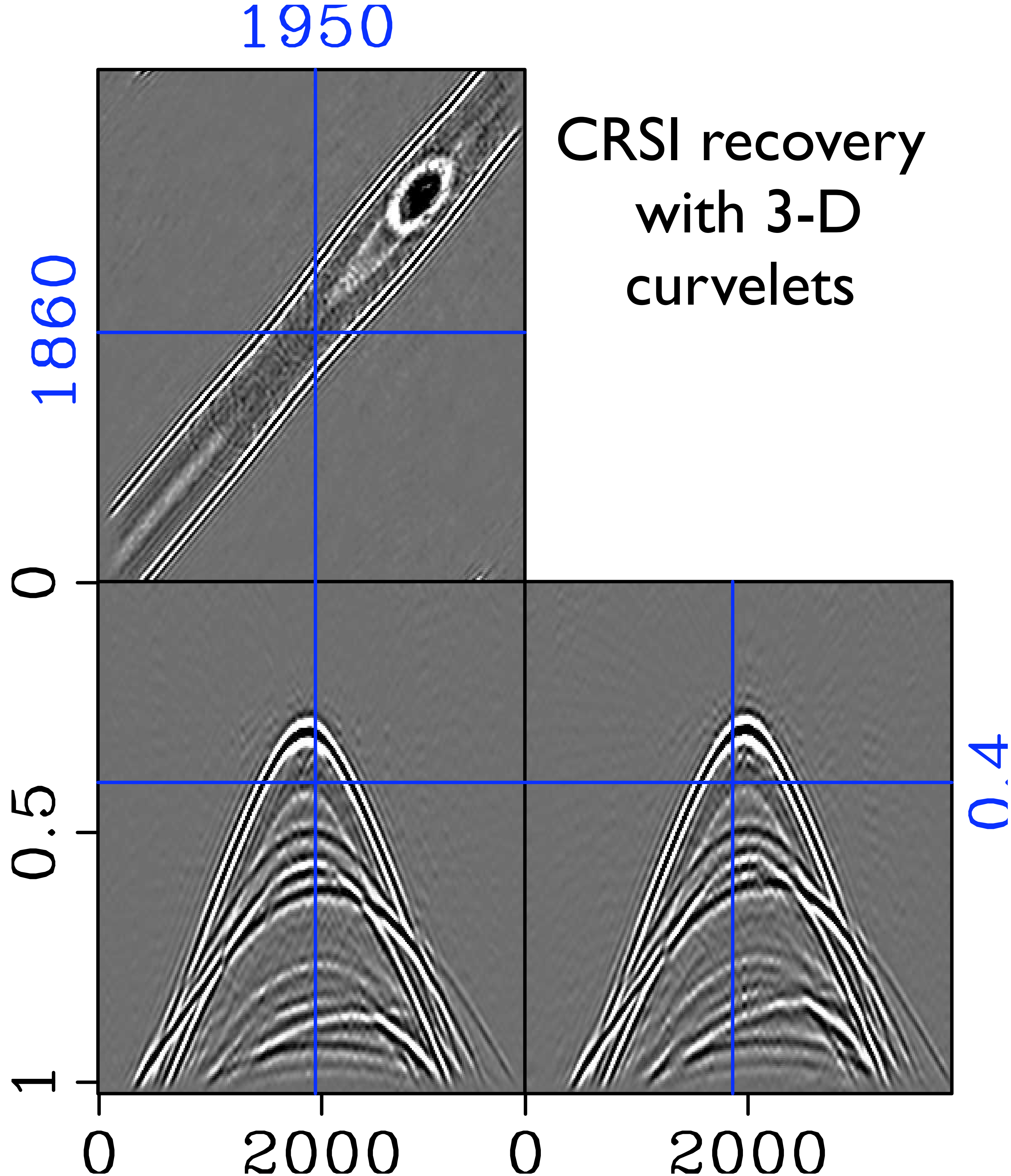


1950
1860
Original data
0
0.5
1
0.4
0
2000
0
2000


1950
1860
80 % missing
0
0.5
1
0.4
0
2000
0
2000


1950
1860
CRSI recovery
with 3-D
curvelets
0
0.5
1
0.4
0
2000
0
2000

Primary multiple separation

Joint work with Eric Verschuur, Deli Wang, Rayan Saab and Ozgur Yilmaz




Motivation

Primary-multiple separation step is crucial

- moderate prediction errors
- 3-D complexity & noise

Inadequate separation leads to

- remnant multiple energy
- deterioration primary energy

Introduce a transform-based technique

- stable
- insensitive to moderate shifts & phase rotations

Exploit sparsity and parameterization transformed domain

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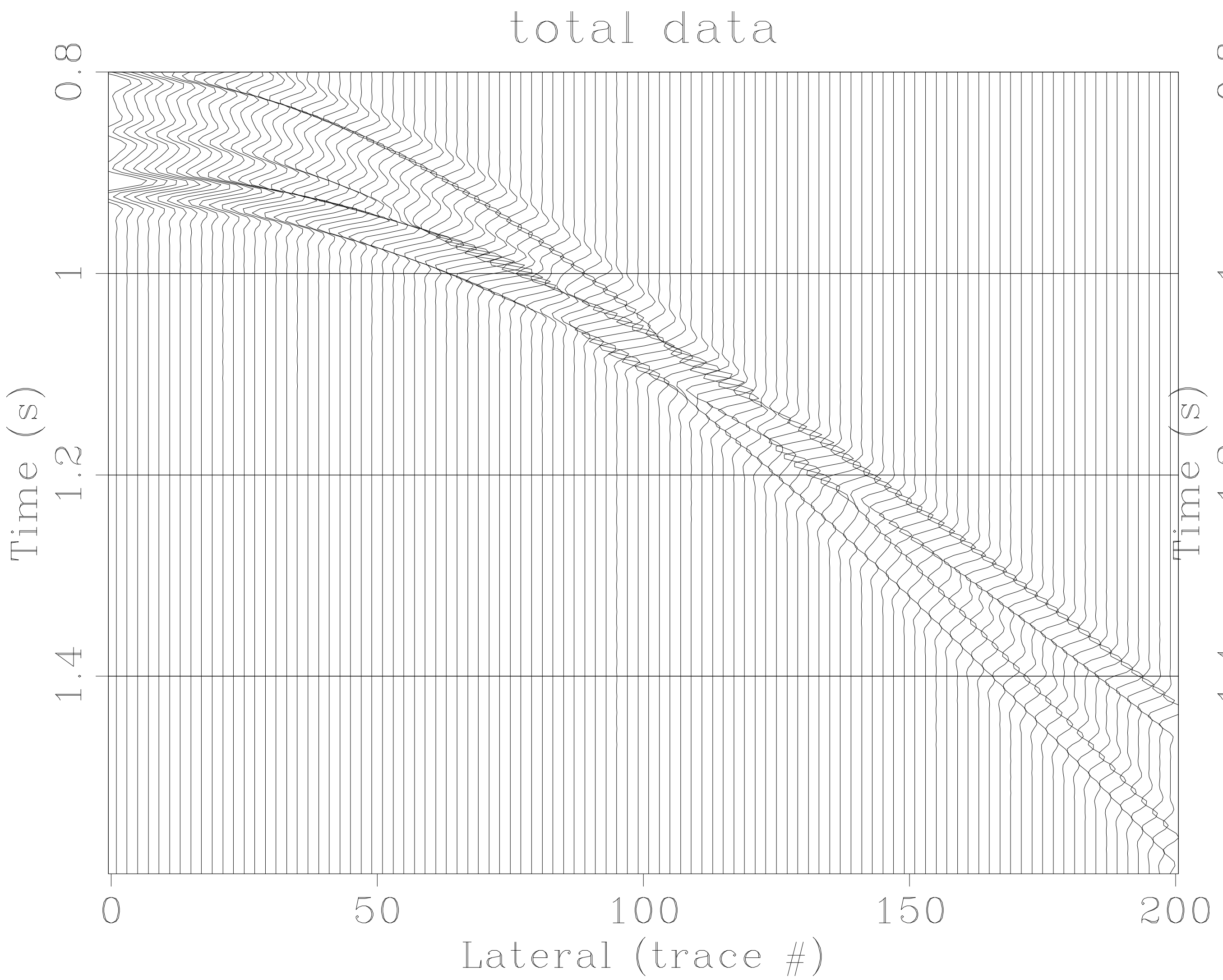
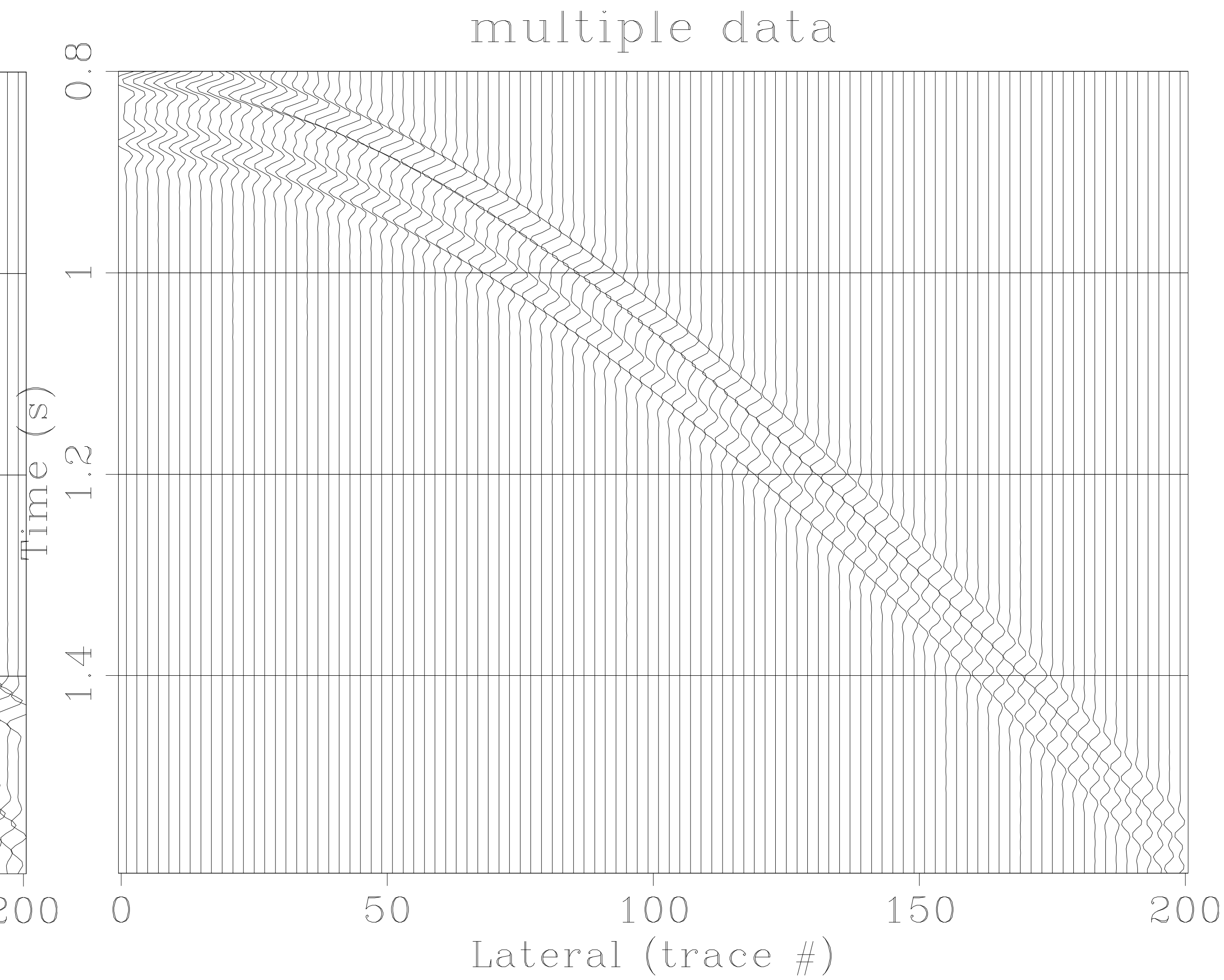
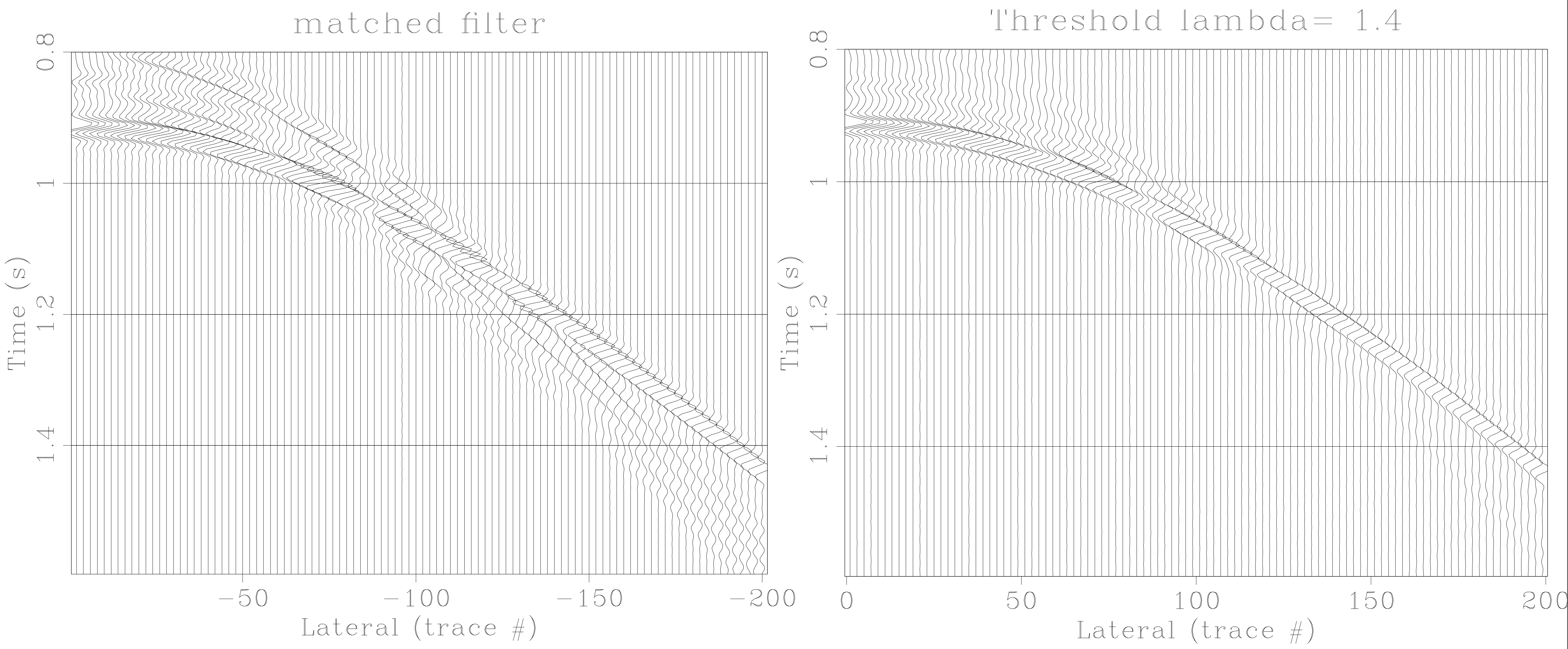
Move-out error





Move-out error



The problem

Sparse signal model:

$$\mathbf{y} = \mathbf{A}\mathbf{x}_0 + \mathbf{n},$$

with

$$\mathbf{A} = [\mathbf{A}_1 \quad \mathbf{A}_2] \quad \text{and} \quad \mathbf{x}_0 = [\mathbf{x}_{01} \quad \mathbf{x}_{02}]^T$$

- augmented synthesis and sparsity vectors
- index 1 <-> primary
- index 2 <-> multiple

The solution

The weighted norm-one optimization problem:

$$\mathbf{P_w}: \begin{cases} \min_{\mathbf{x}} \|\mathbf{x}\|_{\mathbf{w},1} \quad \text{subject to} \quad \|\mathbf{y}-\mathbf{A}\mathbf{x}\|_2 \leq \varepsilon \\ \hat{\mathbf{s}}_1 = \mathbf{A}_1\hat{\mathbf{x}}_1 \quad \text{and} \quad \hat{\mathbf{s}}_2 = \mathbf{A}_2\hat{\mathbf{x}}_2 \\ \text{given:} \quad \breve{\mathbf{s}}_2 \quad \text{and} \quad \mathbf{w}(\mathbf{y},\breve{\mathbf{s}}_2) \end{cases}$$

with

$$\begin{aligned} \mathbf{w} &:= \left[\mathbf{w}_1,\ \mathbf{w}_2\right]^T \\ \mathbf{A} &:= \left[\mathbf{C}^T,\ \mathbf{C}^T\right] \\ \breve{\mathbf{s}}_2 &:= \text{predicted multiples} \\ \breve{\mathbf{s}}_1 &:= \mathbf{S} - \breve{\mathbf{S}}_2 \end{aligned}$$

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Solution cont'd

The weights

$$\begin{cases} \mathbf{w}_1 := \max\left(\sigma \cdot \sqrt{2\log N},\ C_1|\breve{\mathbf{u}}_1|\right) \\ \mathbf{w}_2 := \max\left(\sigma \cdot \sqrt{2\log N},\ C_2|\breve{\mathbf{u}}_2|\right) \end{cases}$$

with

$$\begin{aligned} \breve{\mathbf{u}}_1 &\approx \mathbf{C}\breve{\mathbf{s}}_1 \\ \breve{\mathbf{u}}_2 &\approx \mathbf{C}\breve{\mathbf{s}}_2 \end{aligned}$$

- during minimization signal components are driven apart
- curvelet compression helps
- separates on the basis of position, scale and direction

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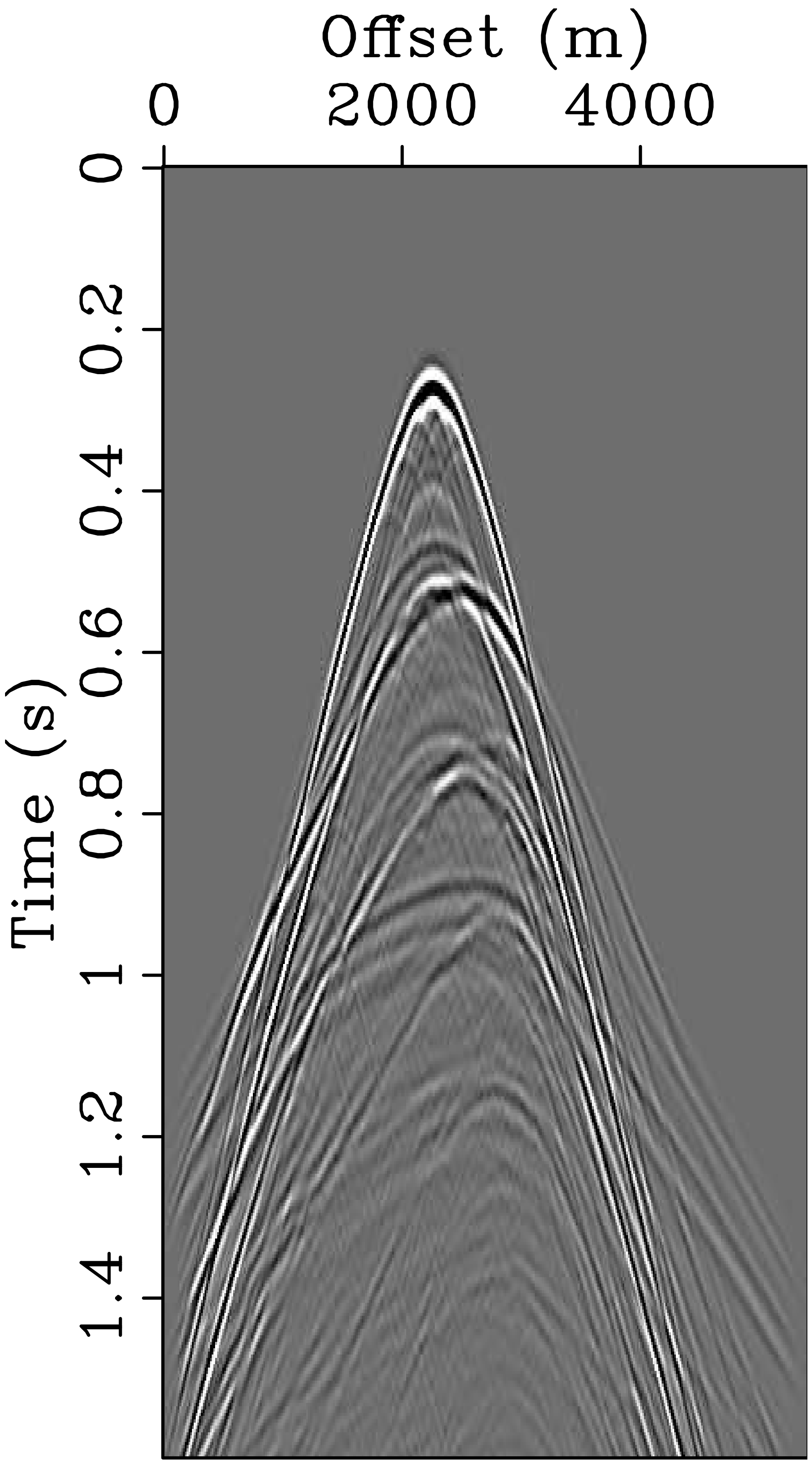
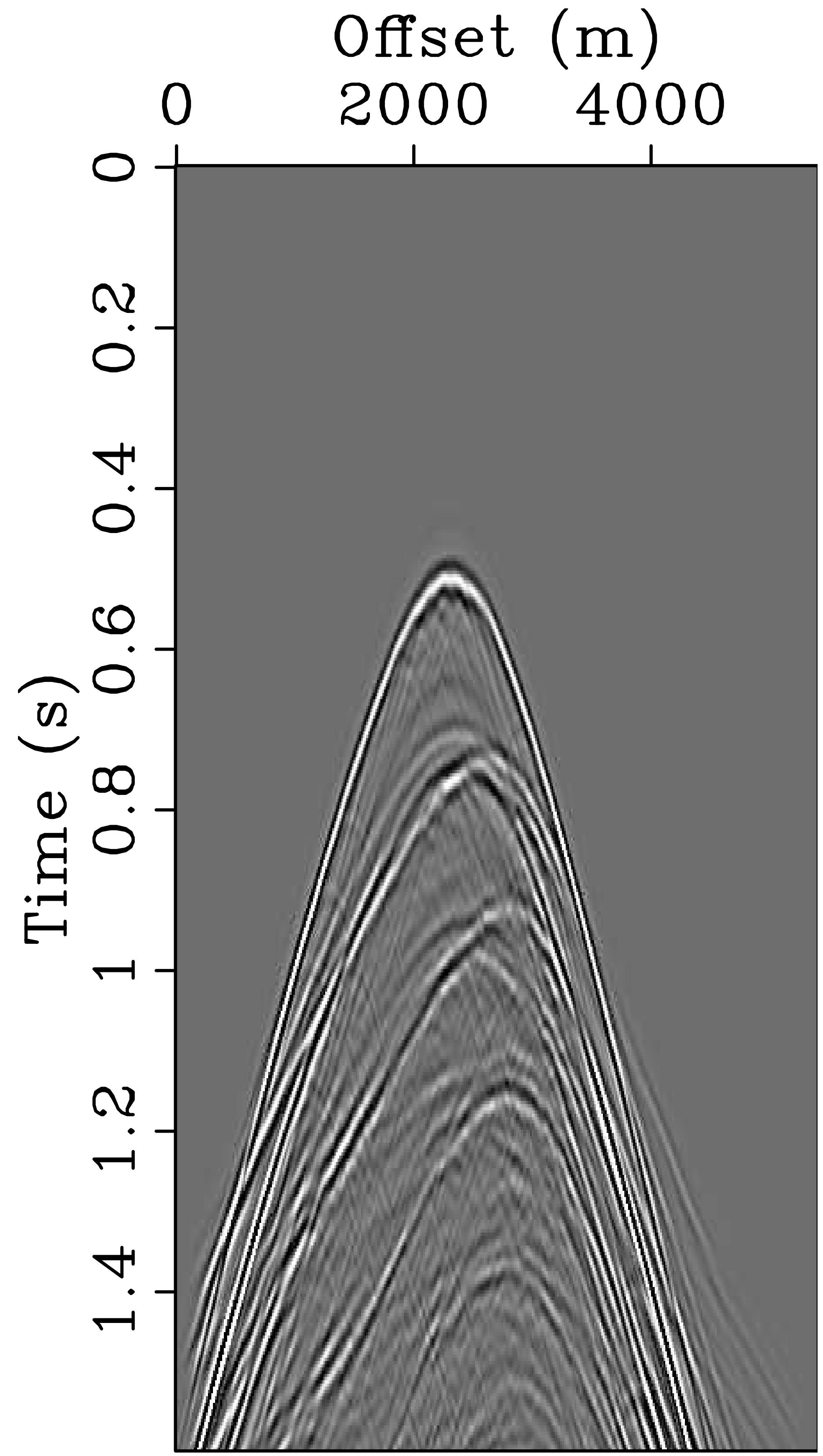
Synthetic example



total data



SRME predicted multiples

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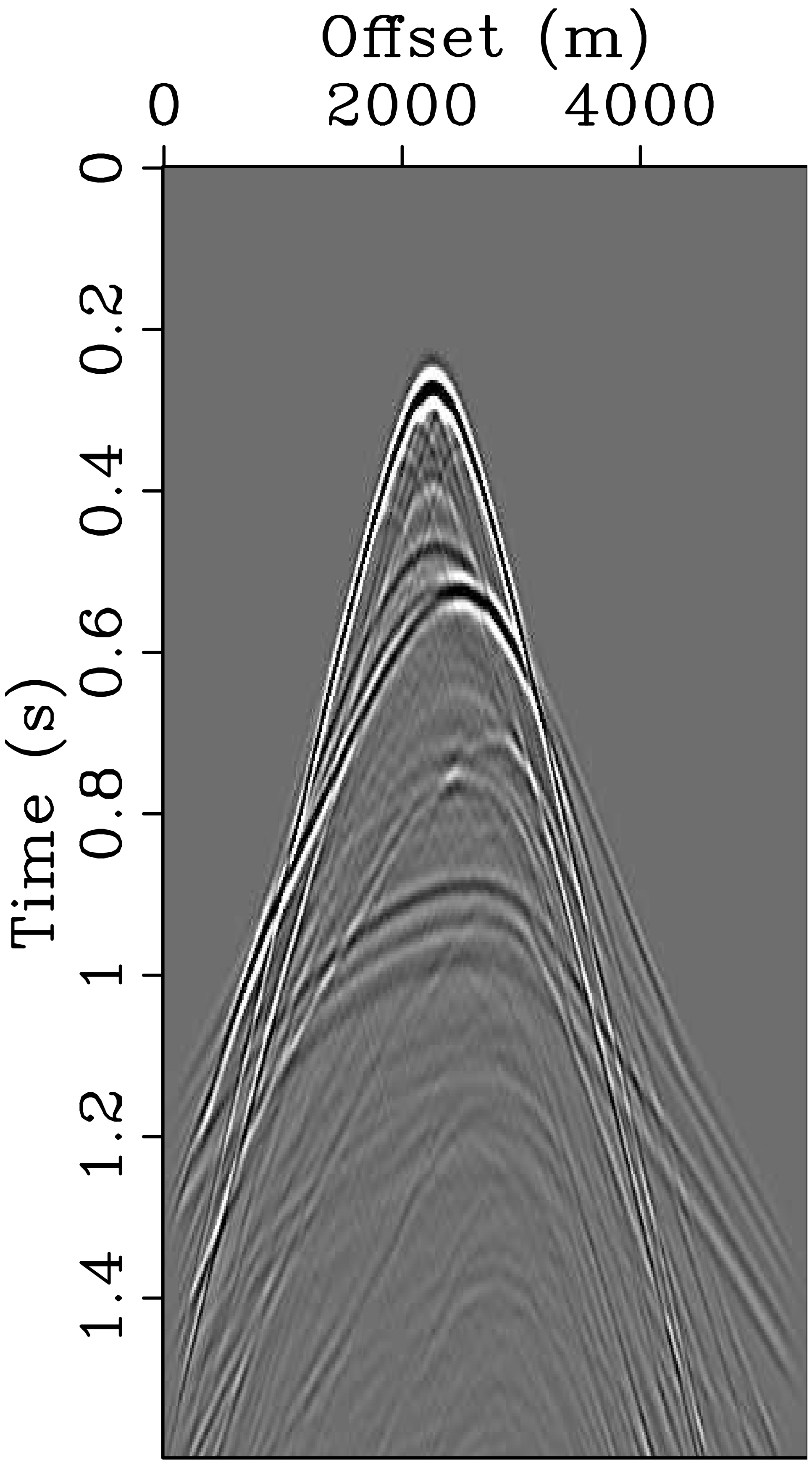
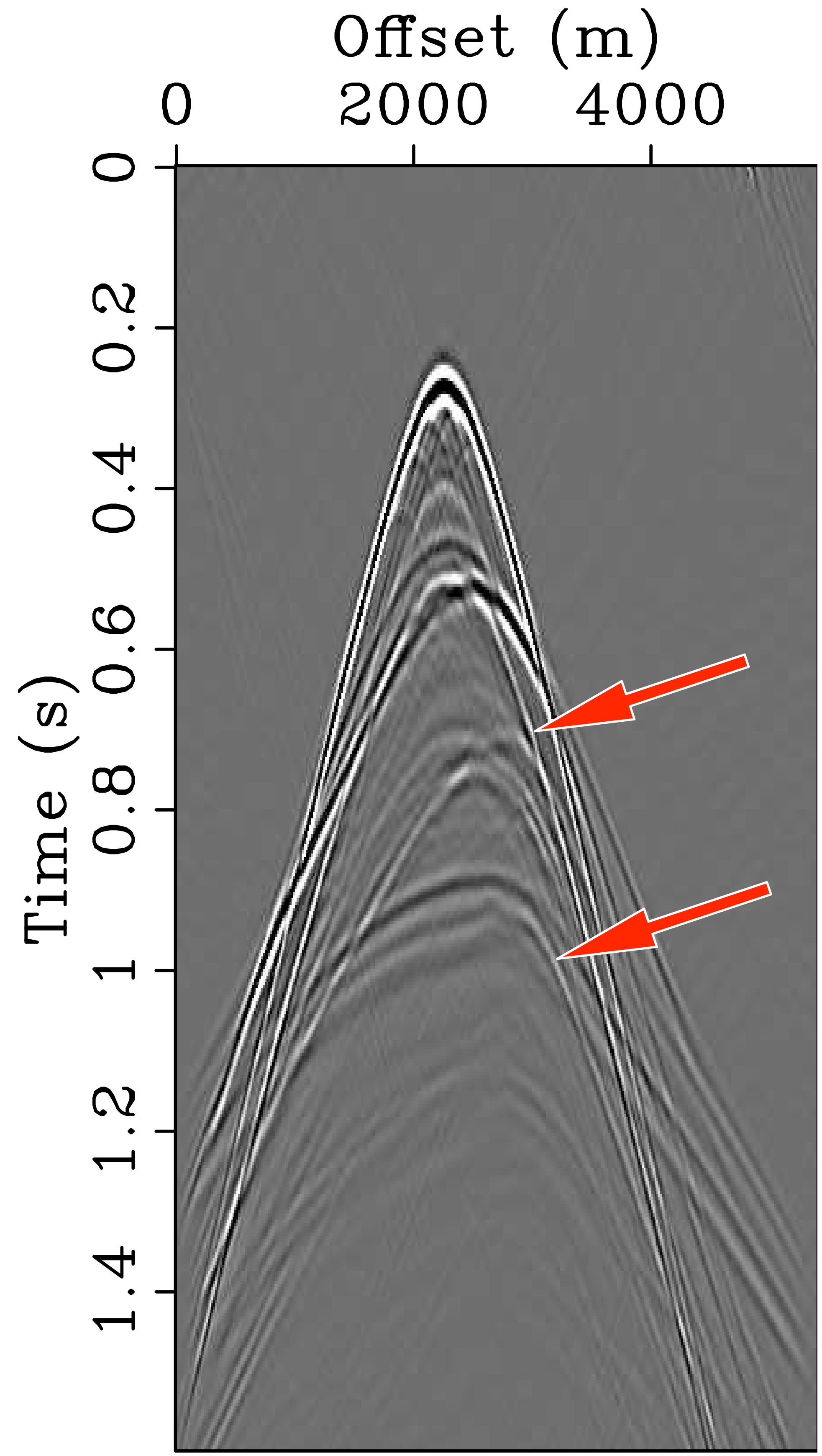
Synthetic example



SRME predicted primaries



curvelet-thresholded

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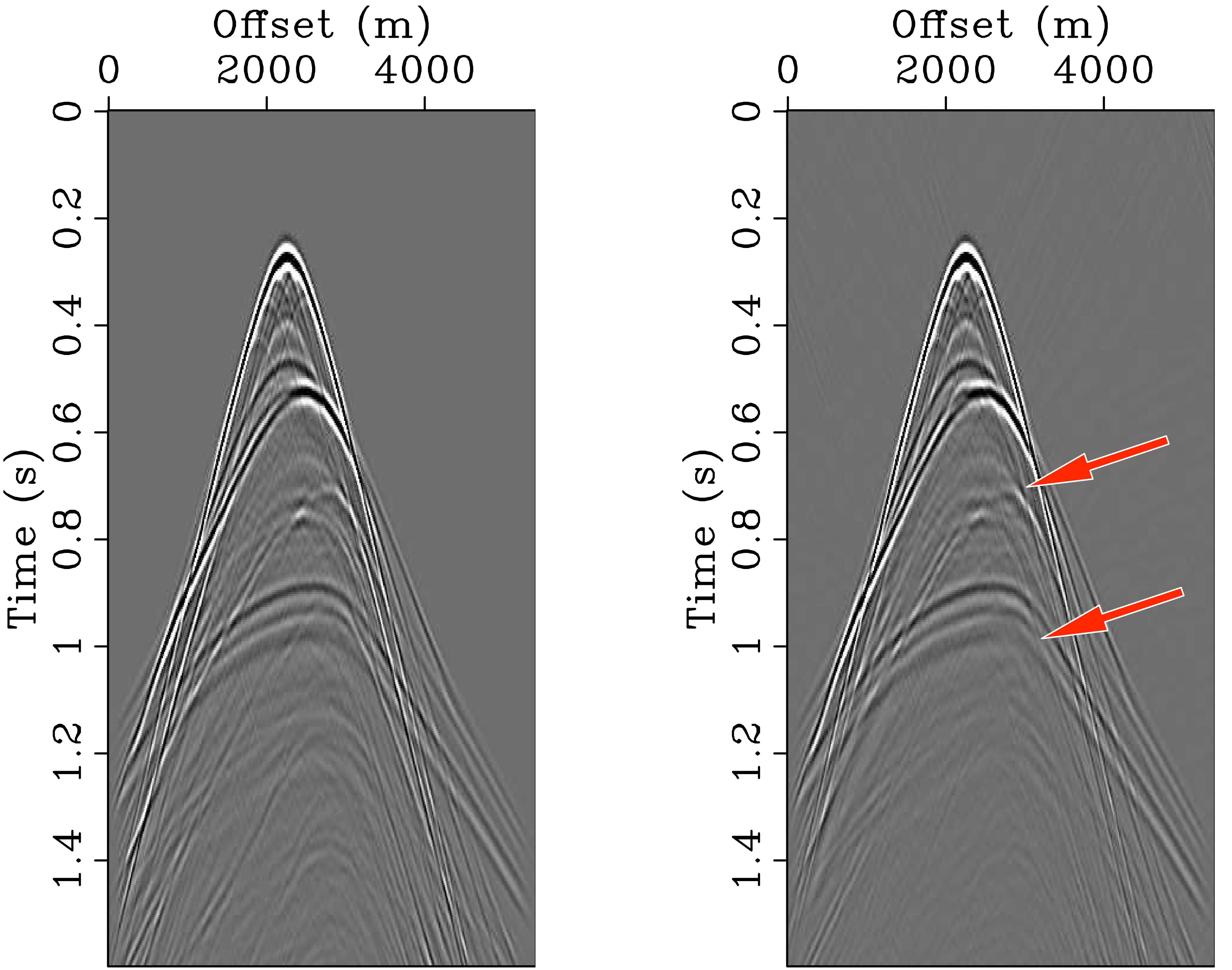
Synthetic example



SRME predicted primaries

estimated

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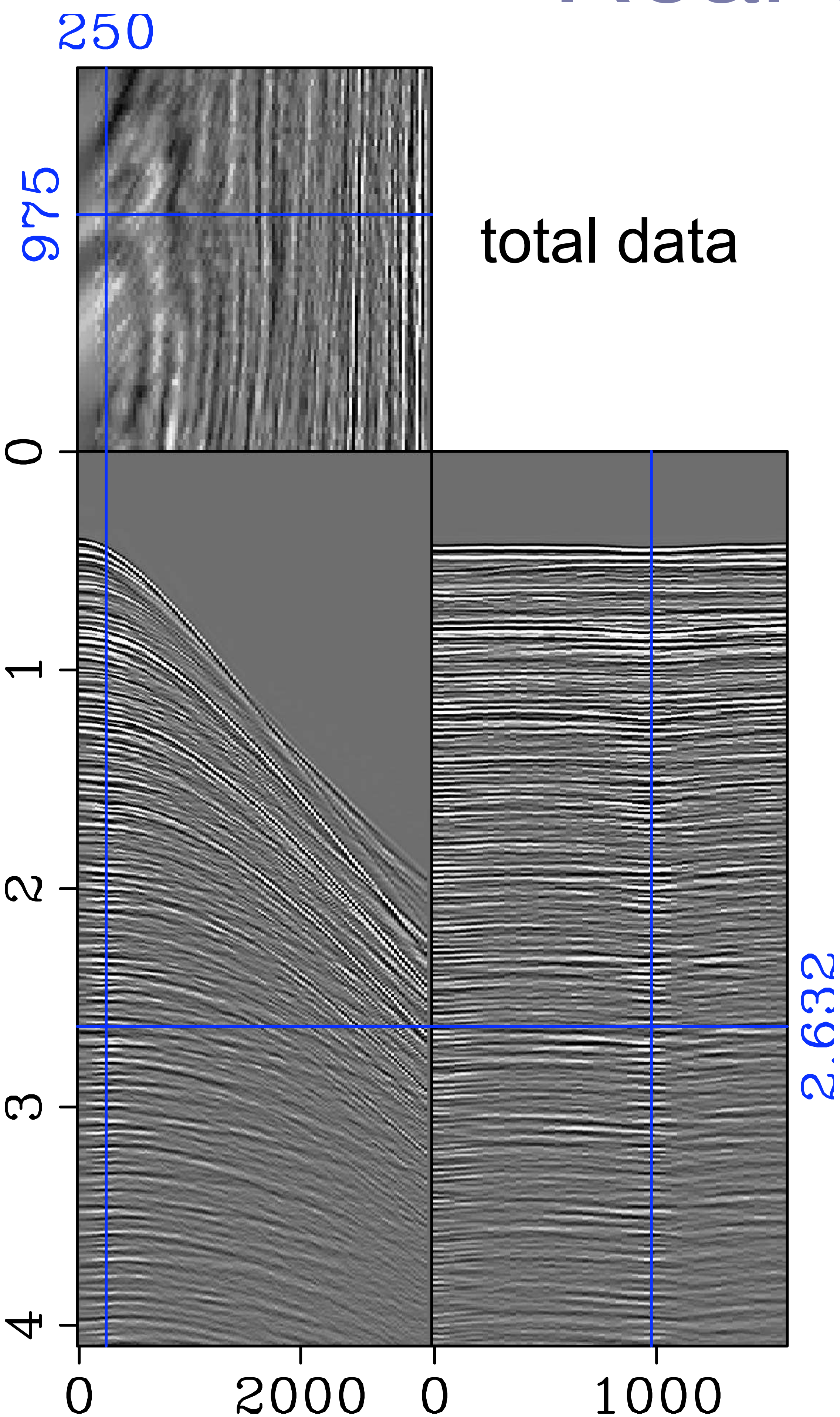
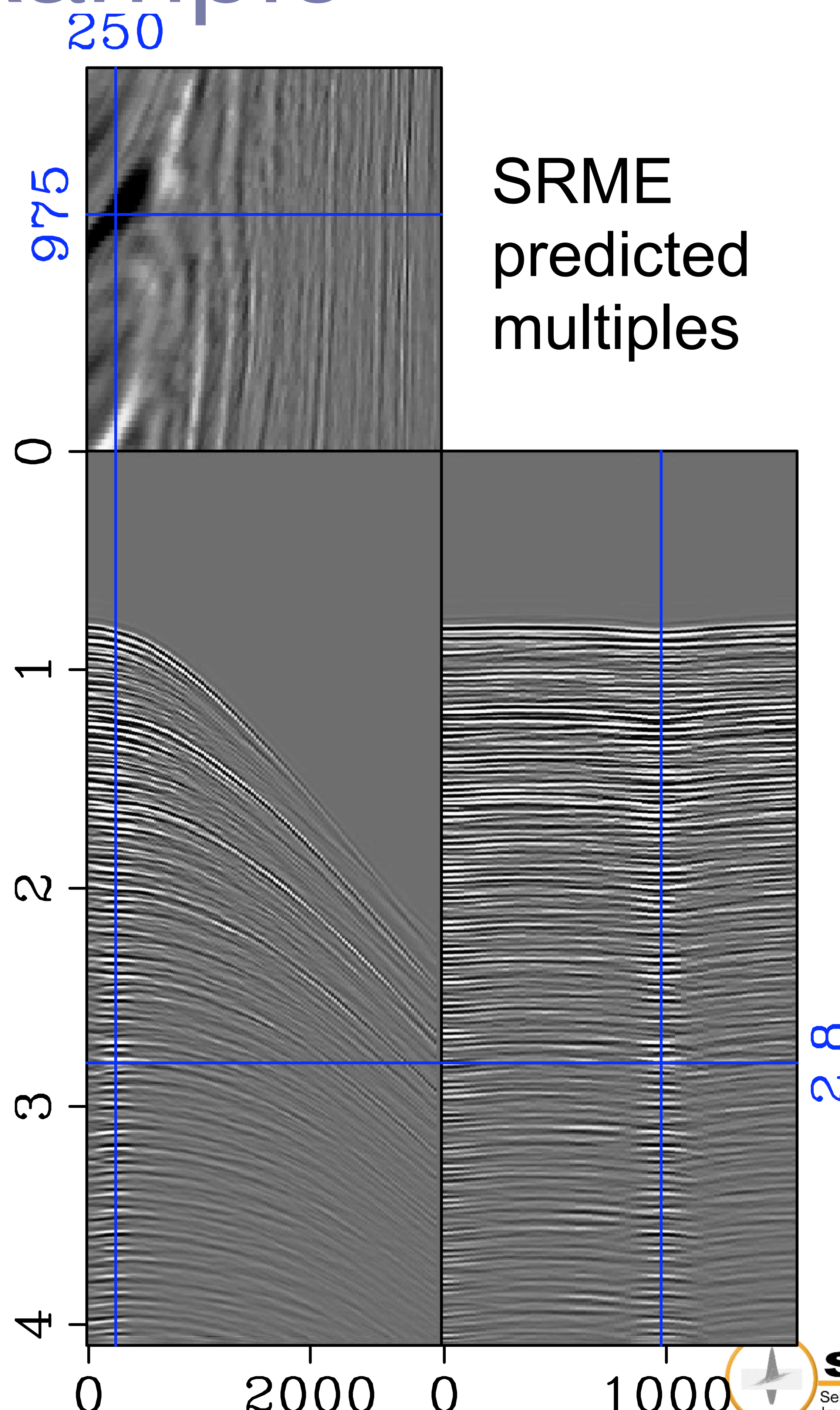
Real example



total data



SRME predicted multiples

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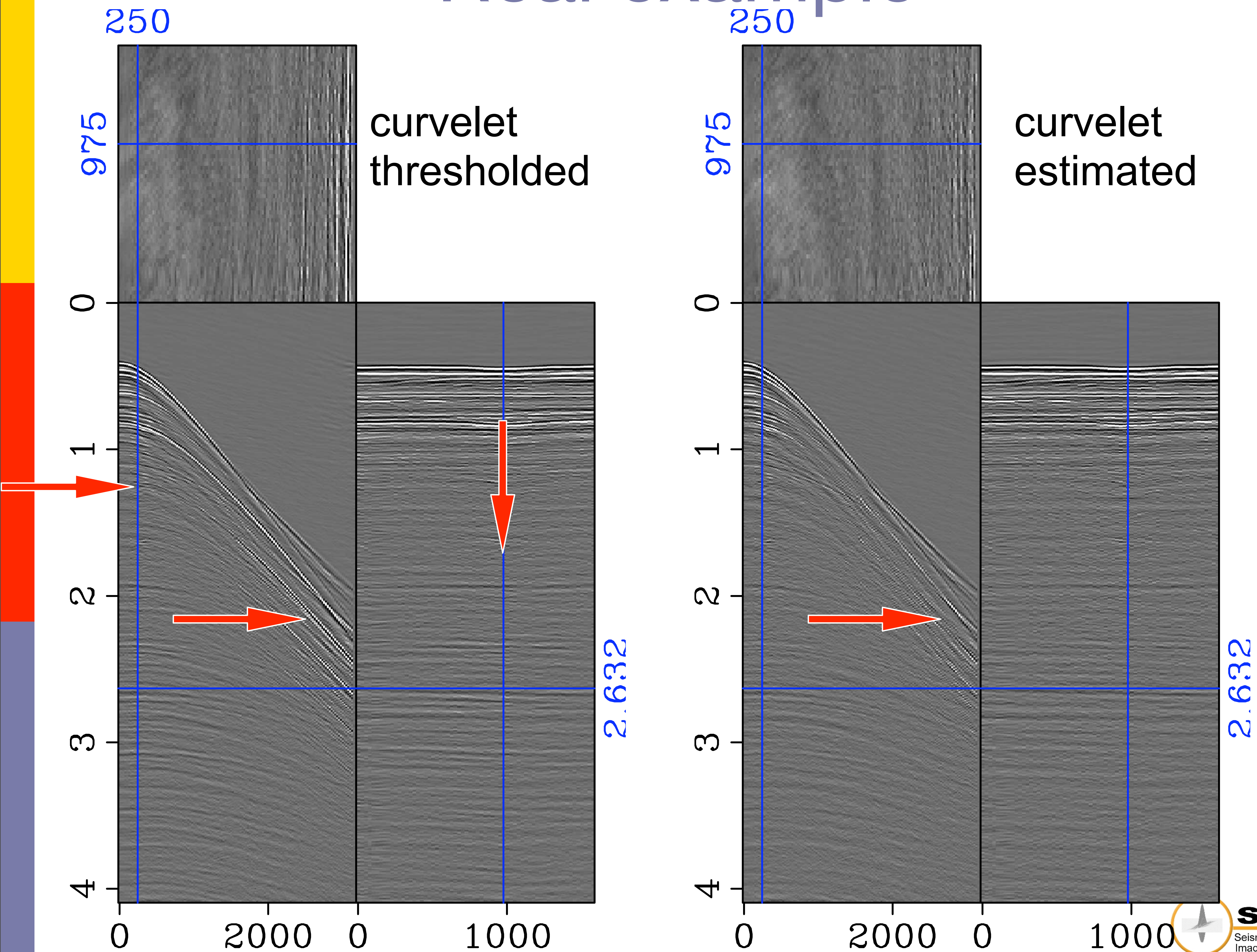
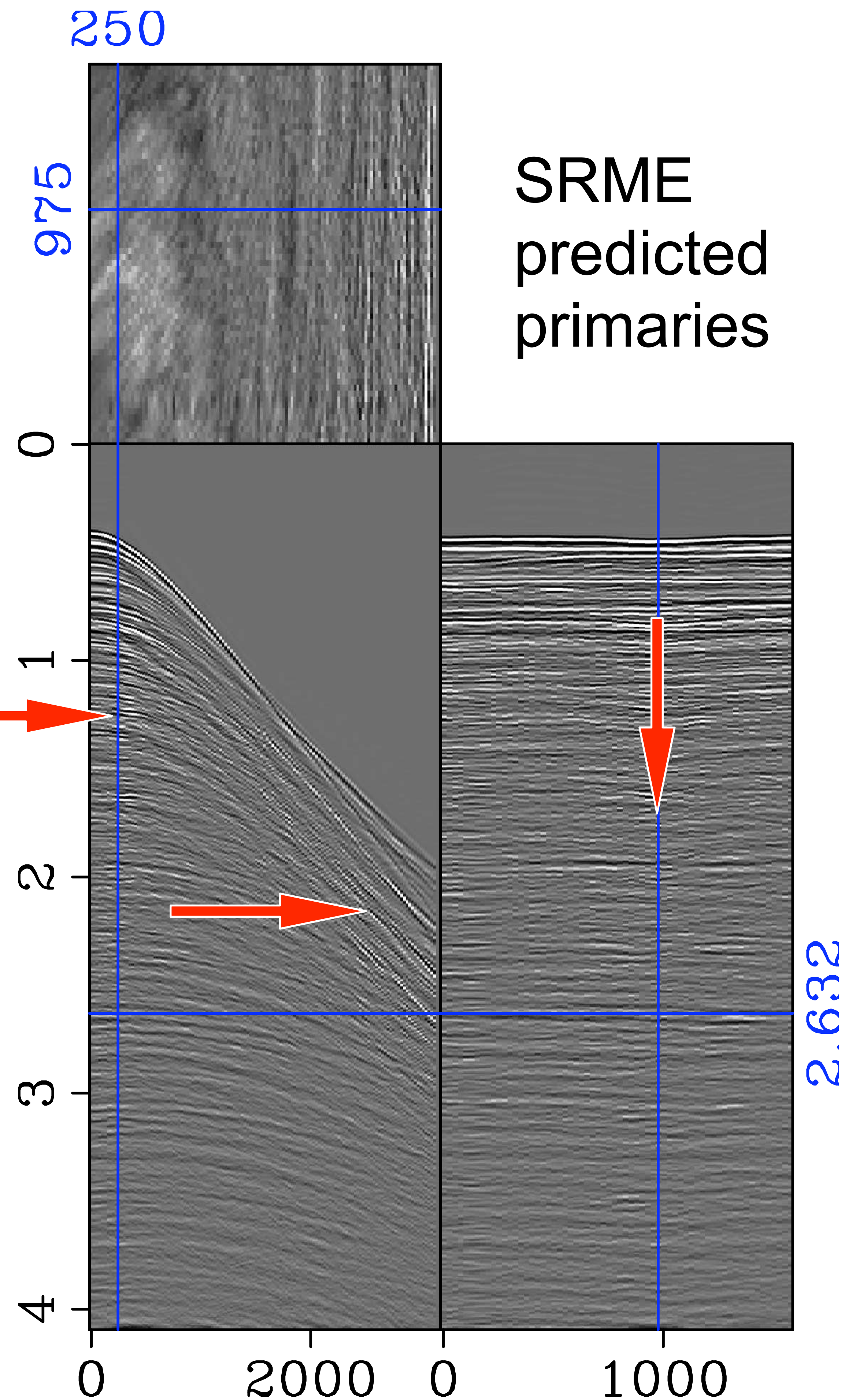
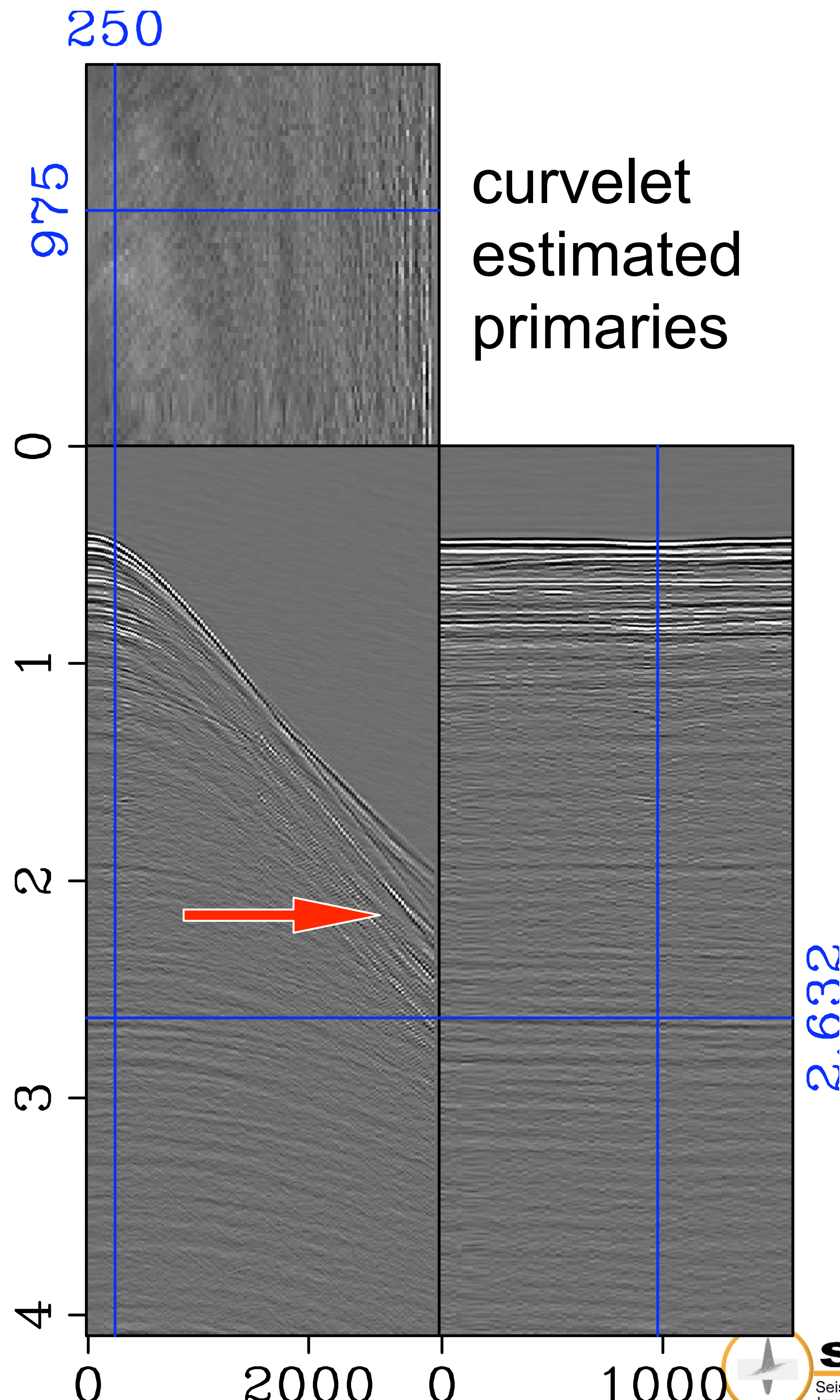
Real example







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Seismic amplitude recovery

Joint work with Chris Stolk and Peyman Moghaddam

Motivation

Migration generally does not correctly recover the amplitudes.

Least-squares migration is computationally unfeasible.

Amplitude recovery (e.g. AGC) lacks robustness w.r.t. noise.

Existing diagonal amplitude-recovery methods

- do not always correct for the order (1 - 2D) of the Hessian [see Symes '07]
- do not invert the scaling robustly

Moreover, these (scaling) methods assume that there

- are no conflicting dips (conormal) in the model
- is infinite aperture
- are infinitely-high frequencies
- etc.

Existing scaling methods

Methods are based on a diagonal approximation of Ψ.

- Illumination-based normalization (Rickett '02)
- Amplitude preserved migration (Plessix & Mulder '04)
- Amplitude corrections (Guitton '04)
- Amplitude scaling (Symes '07)

We are interested in an 'Operator and image adaptive' scaling method which

- estimates the action of Ψ from a reference vector close to the actual image
- assumes a smooth symbol of Ψ in space and angle
- does not require the reflectors to be conormal <=> allows for conflicting dips
- stably inverts the diagonal

Our approach

"Forward" model:

$$
\begin{aligned}
\mathbf{y} &= \mathbf{K}^T\mathbf{K}\mathbf{m} + \varepsilon \\
&\approx \mathbf{A}\mathbf{x}_0 + \varepsilon
\end{aligned}
$$

with

$$
\begin{aligned}
\mathbf{y} &= \text{migrated data} \\
\mathbf{A} &:= \mathbf{C}^T\boldsymbol{\Gamma} \\
\mathbf{A}\mathbf{A}^T\mathbf{r} &\approx \mathbf{K}^T\mathbf{K}\mathbf{r} \\
\mathbf{K} &= \text{the demigration operator} \\
\boldsymbol{\epsilon} &= \text{migrated noise.}
\end{aligned}
$$

- diagonal approximation of the demigration-migration operator
- costs one demigration-migration to estimate the diagonal weighting

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Solution

Solve

$$\mathbf{P}: \quad \begin{cases} \min_{\mathbf{x}} J(\mathbf{x}) \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = (\mathbf{A}^{\mathbf{H}})^{\dagger}\tilde{\mathbf{x}} \end{cases}$$

with

$$J(\mathbf{x}) = \overbrace{\alpha\|\mathbf{x}\|_1}^{\text{sparsity}} + \beta \underbrace{\|\mathbf{\Lambda}^{1/2}\left(\mathbf{A}^{H}\right)^{\dagger}\mathbf{x}\|_p}_{\text{continuity}}.$$

Example

SEGAA’ data:

- “broad-band” half-integrated wavelet [5-60 Hz]
- 324 shots, 176 receivers, shot at 48 m
- 5 s of data

Modeling operator

- Reverse-time migration with optimal check pointing (Symes ‘07)
- 8000 time steps
- modeling 64, and migration 294 minutes on 68 CPU’s

Scaling requires 1 extra migration-demigration

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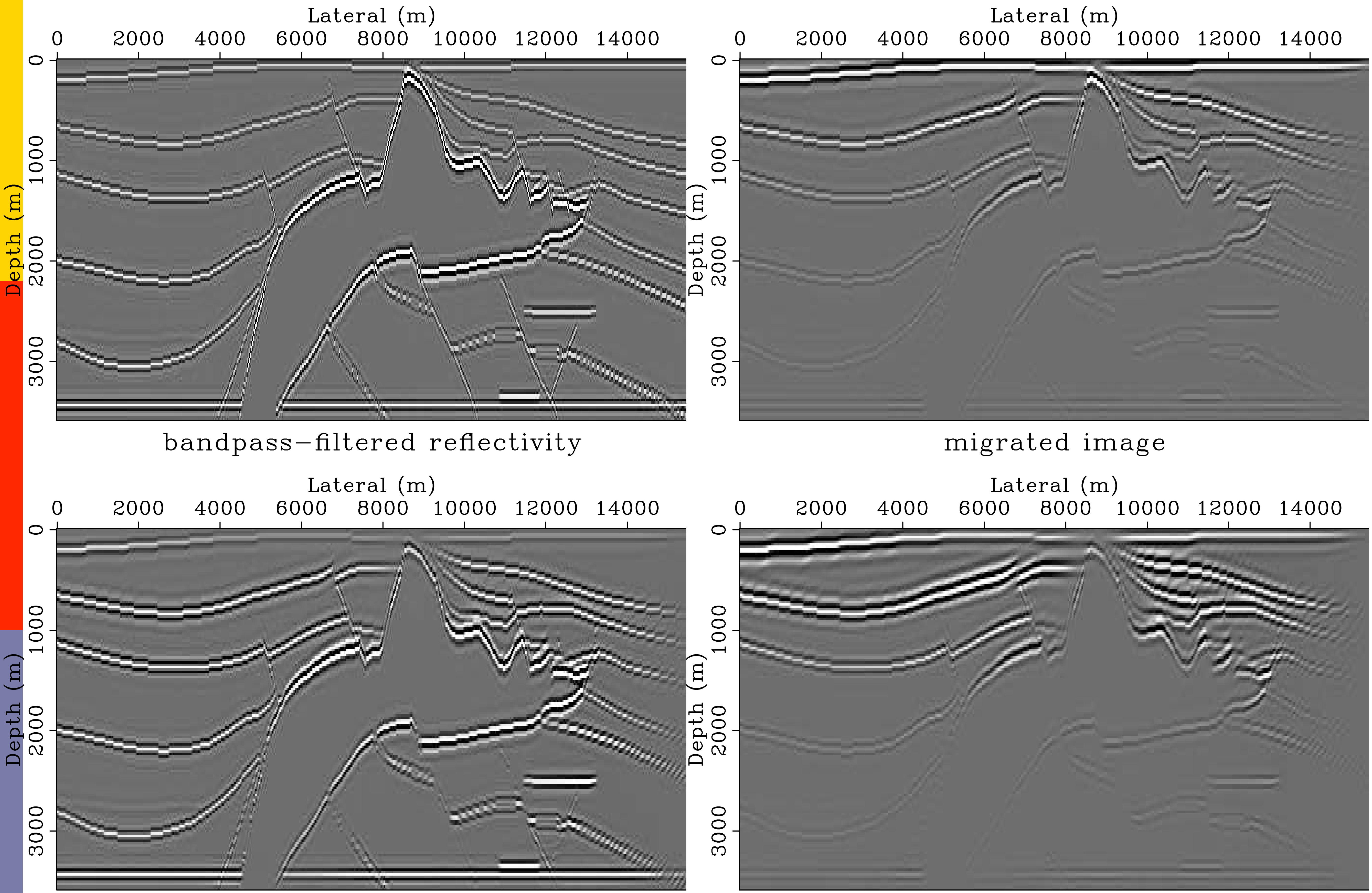
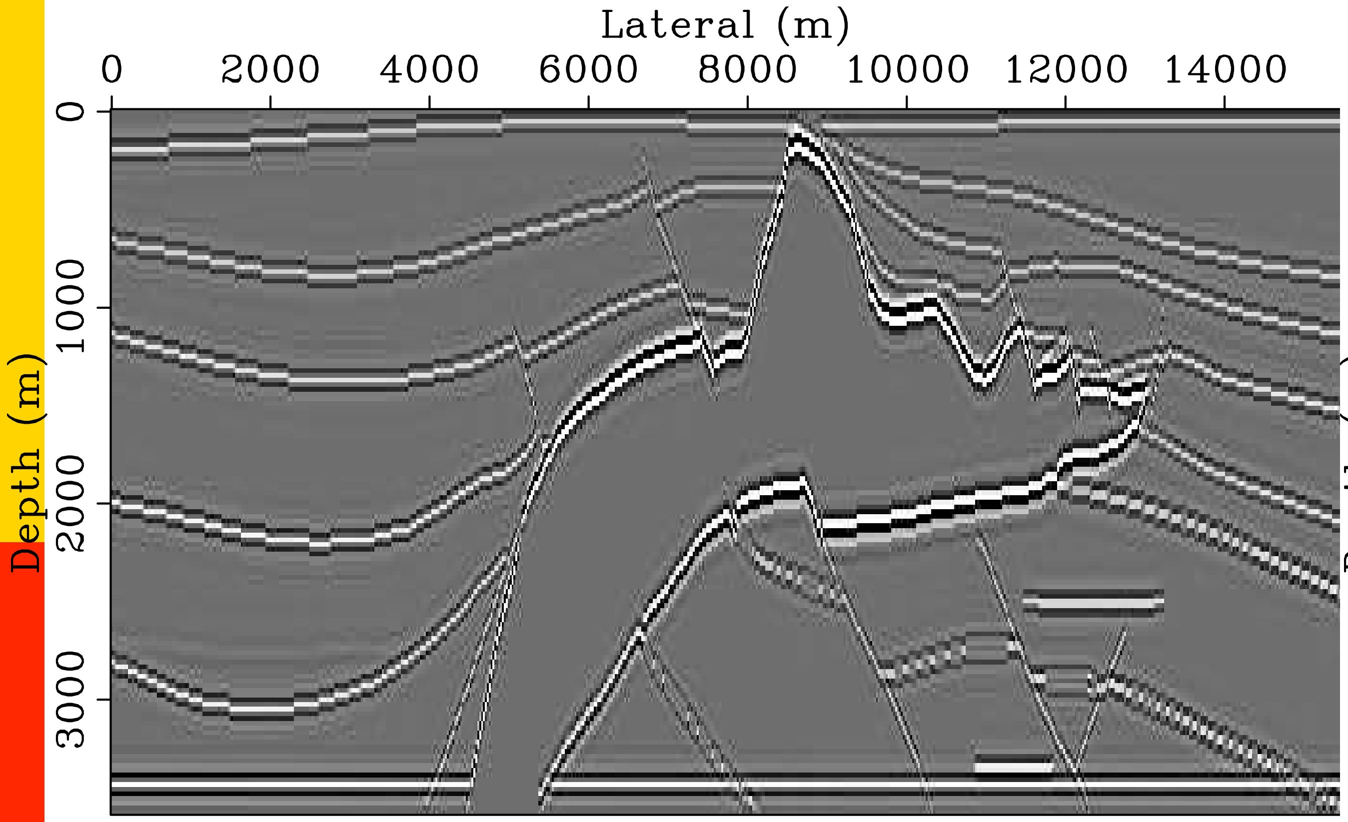


bandpass−filtered reflectivity

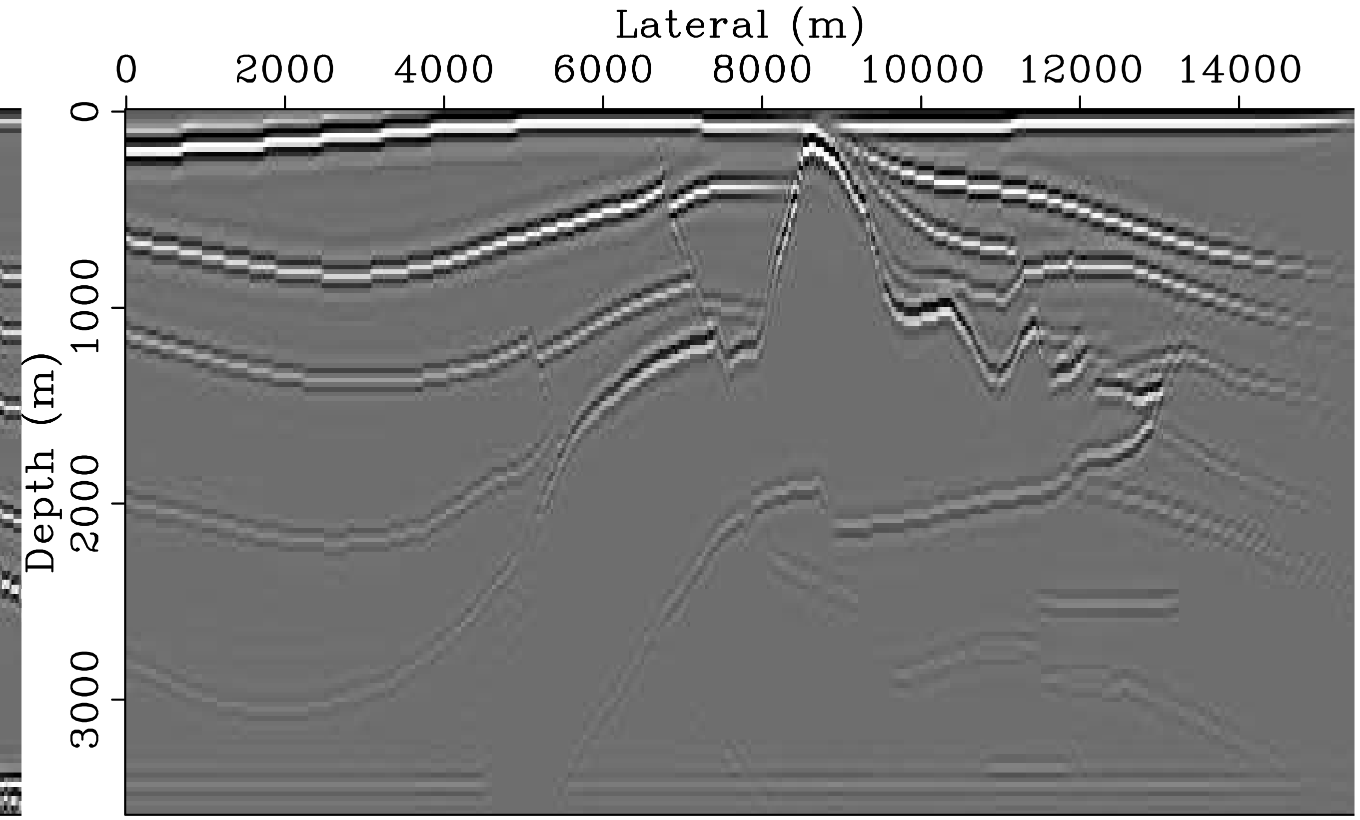
migrated image

reference vector

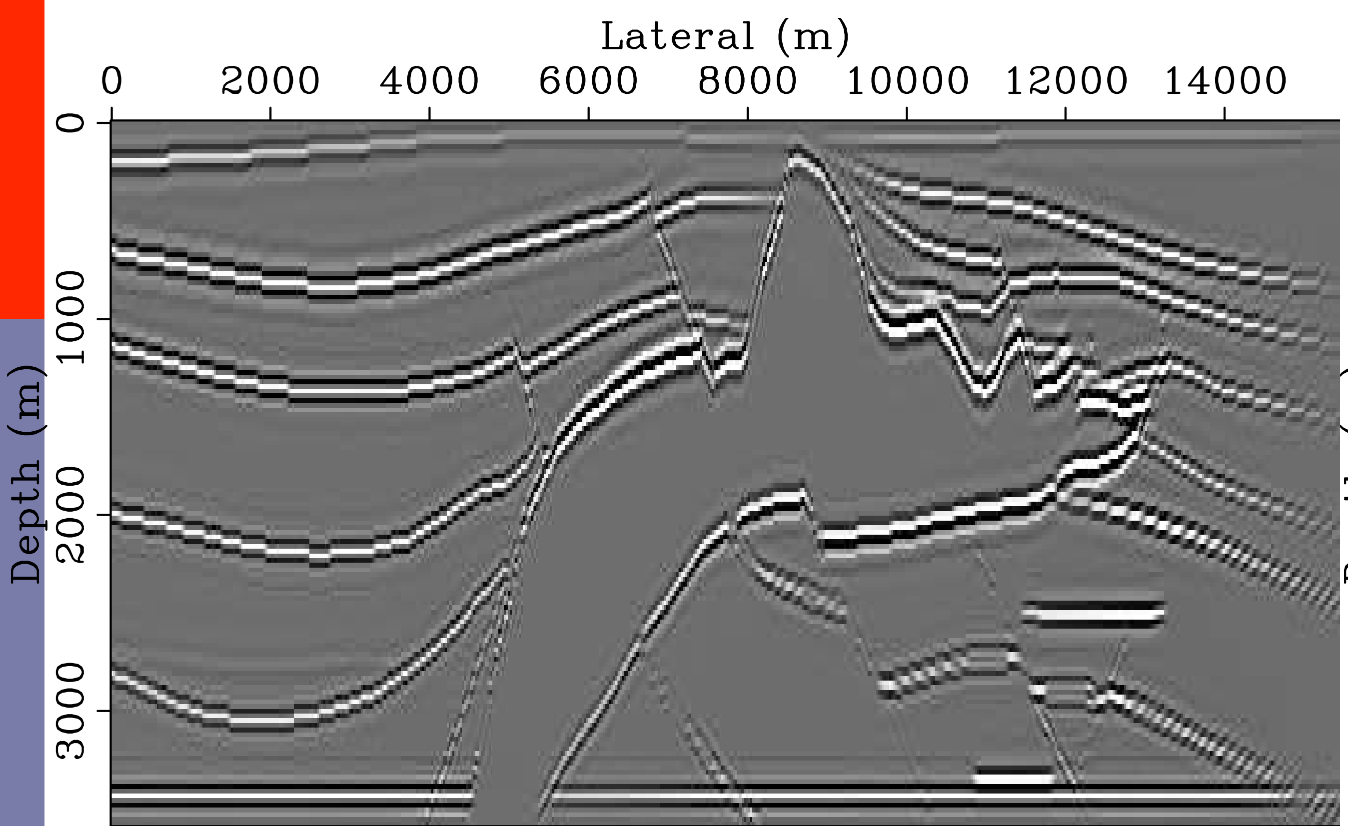
imaged reference vector



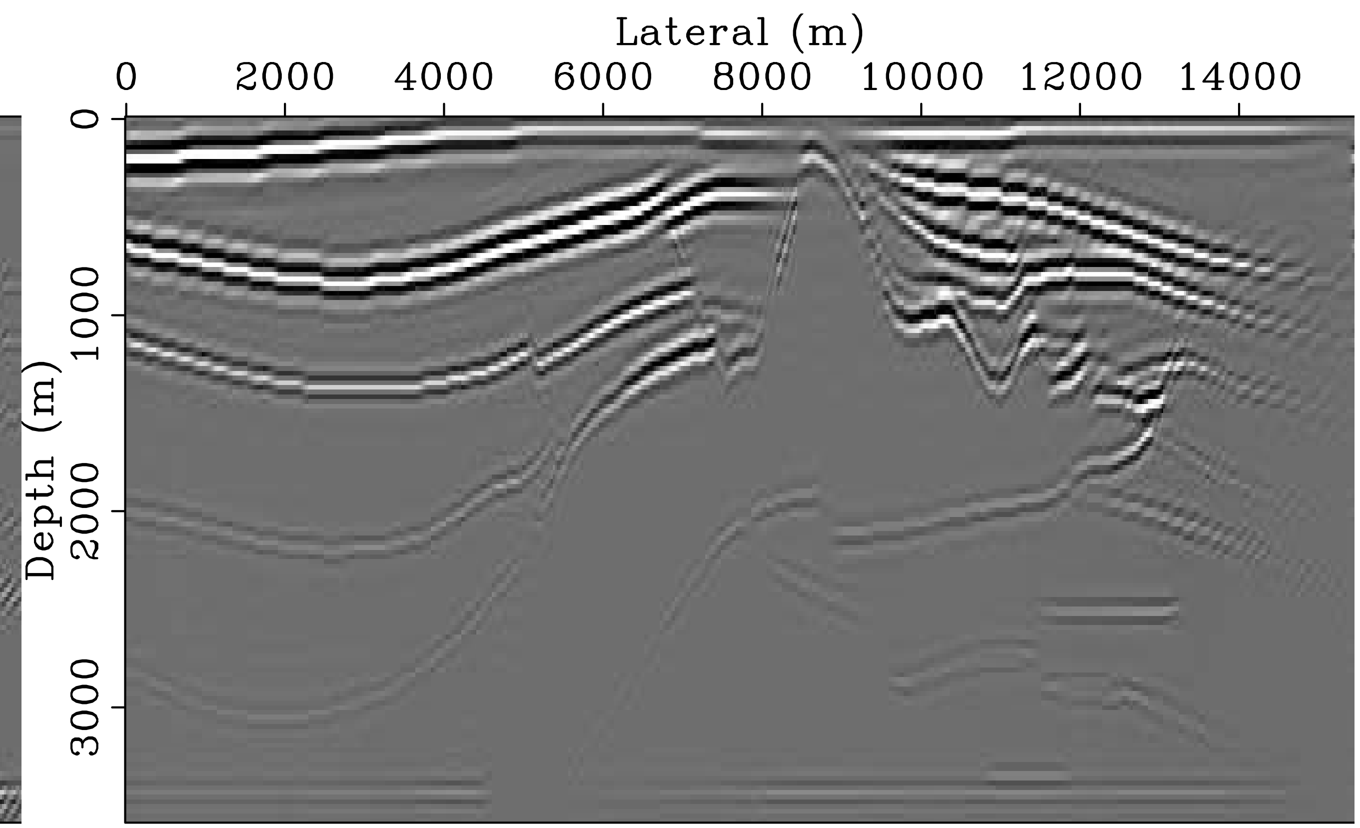
bandpass–filtered reflectivity



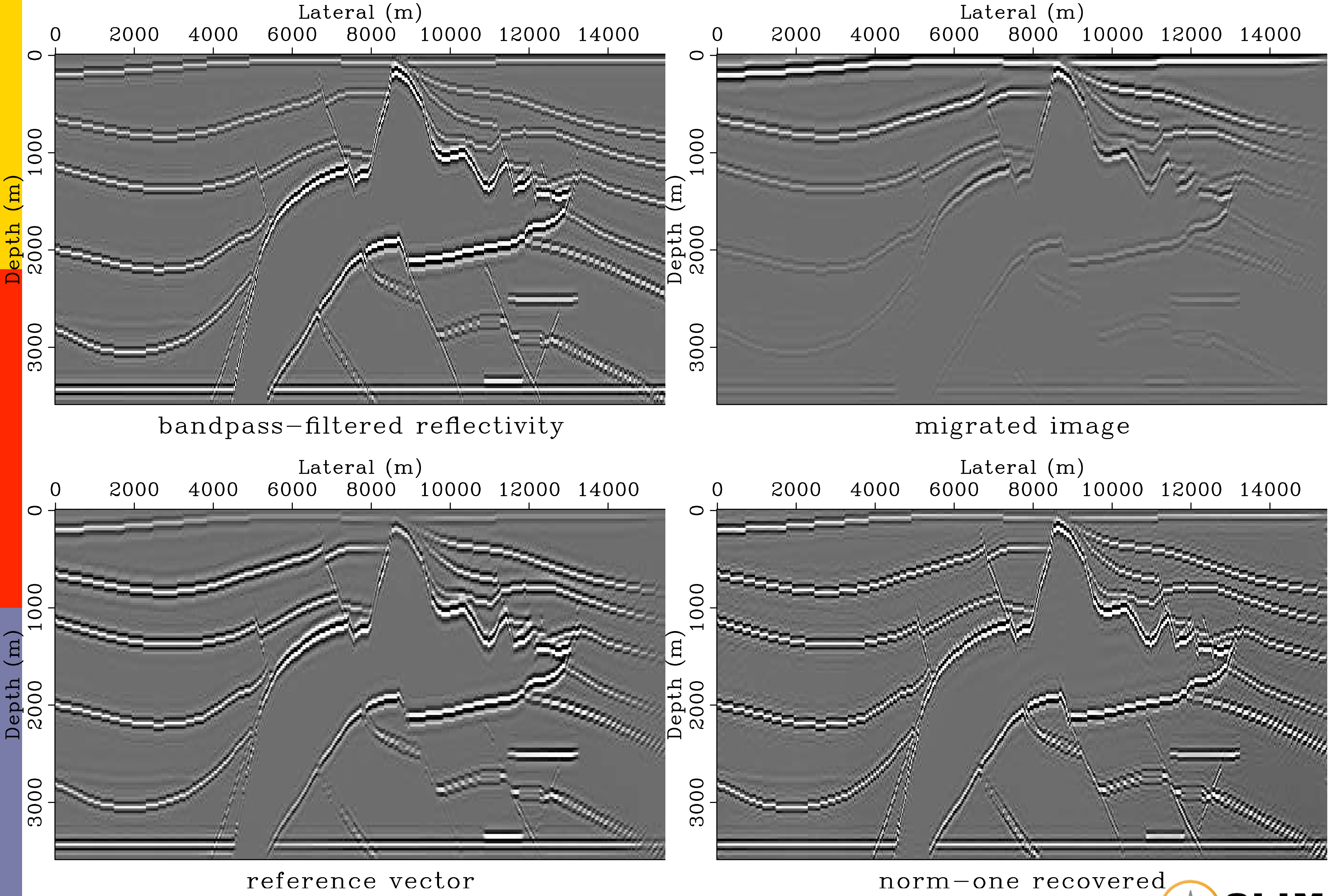
migrated image



reference vector



diagonal approximation



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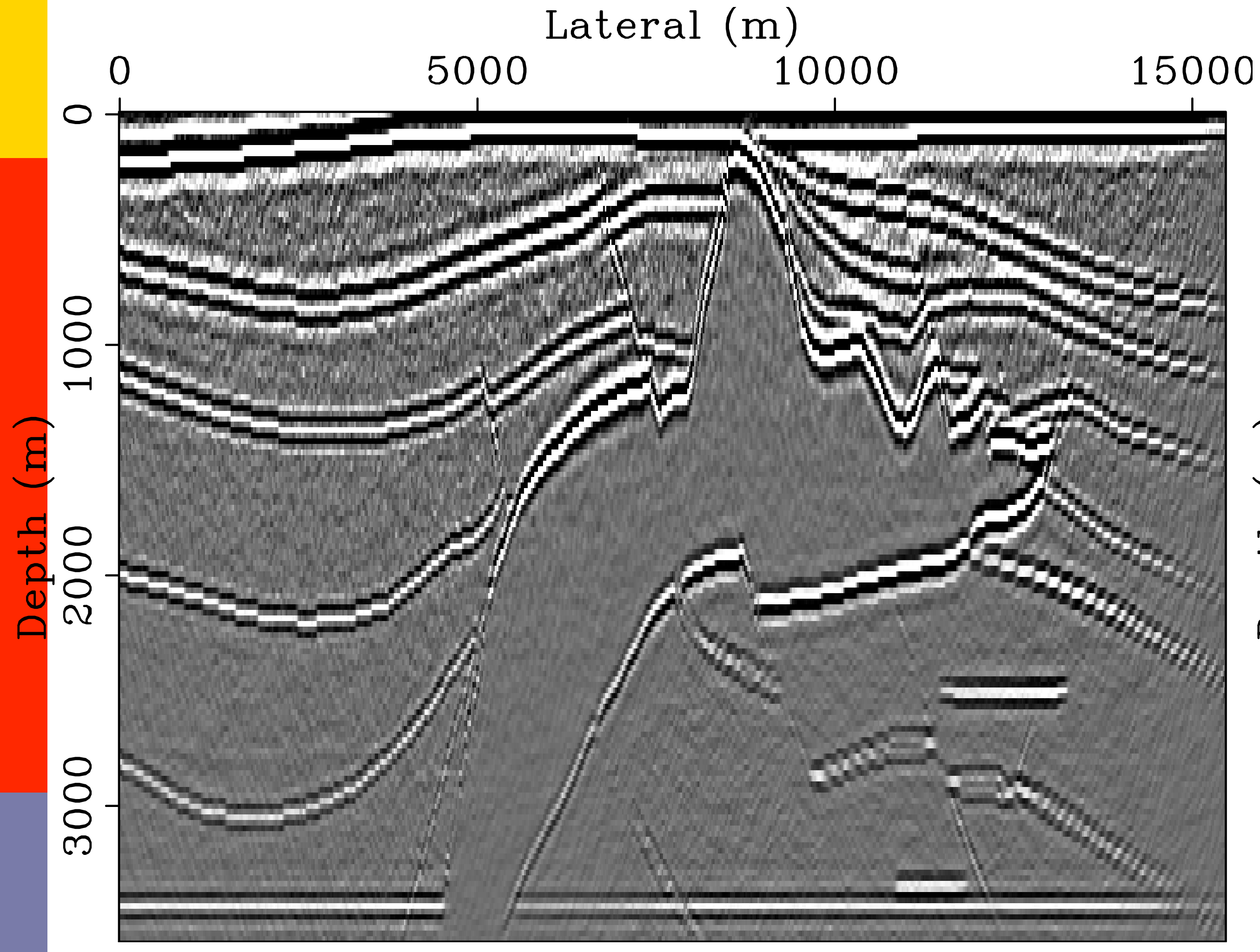
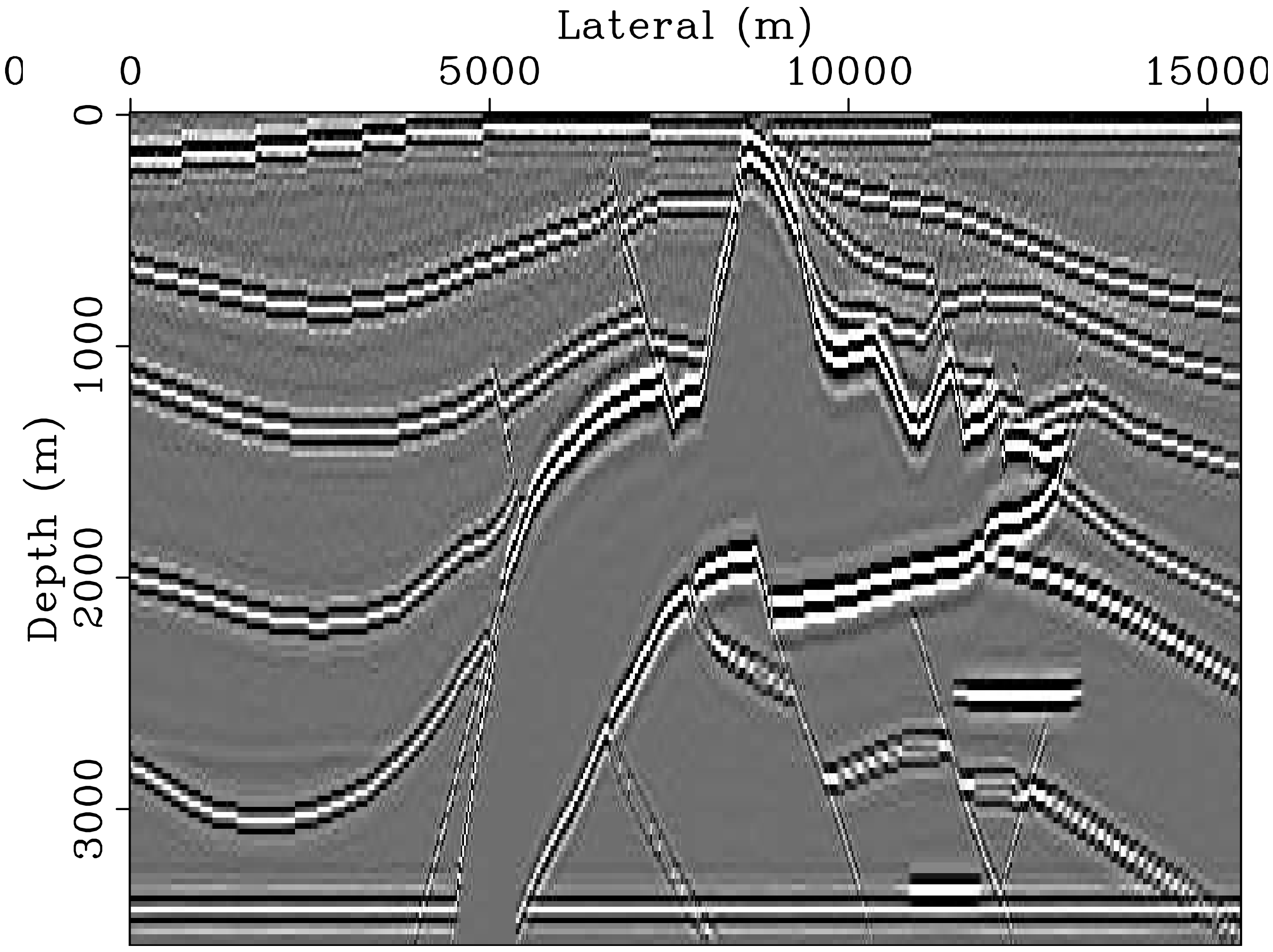
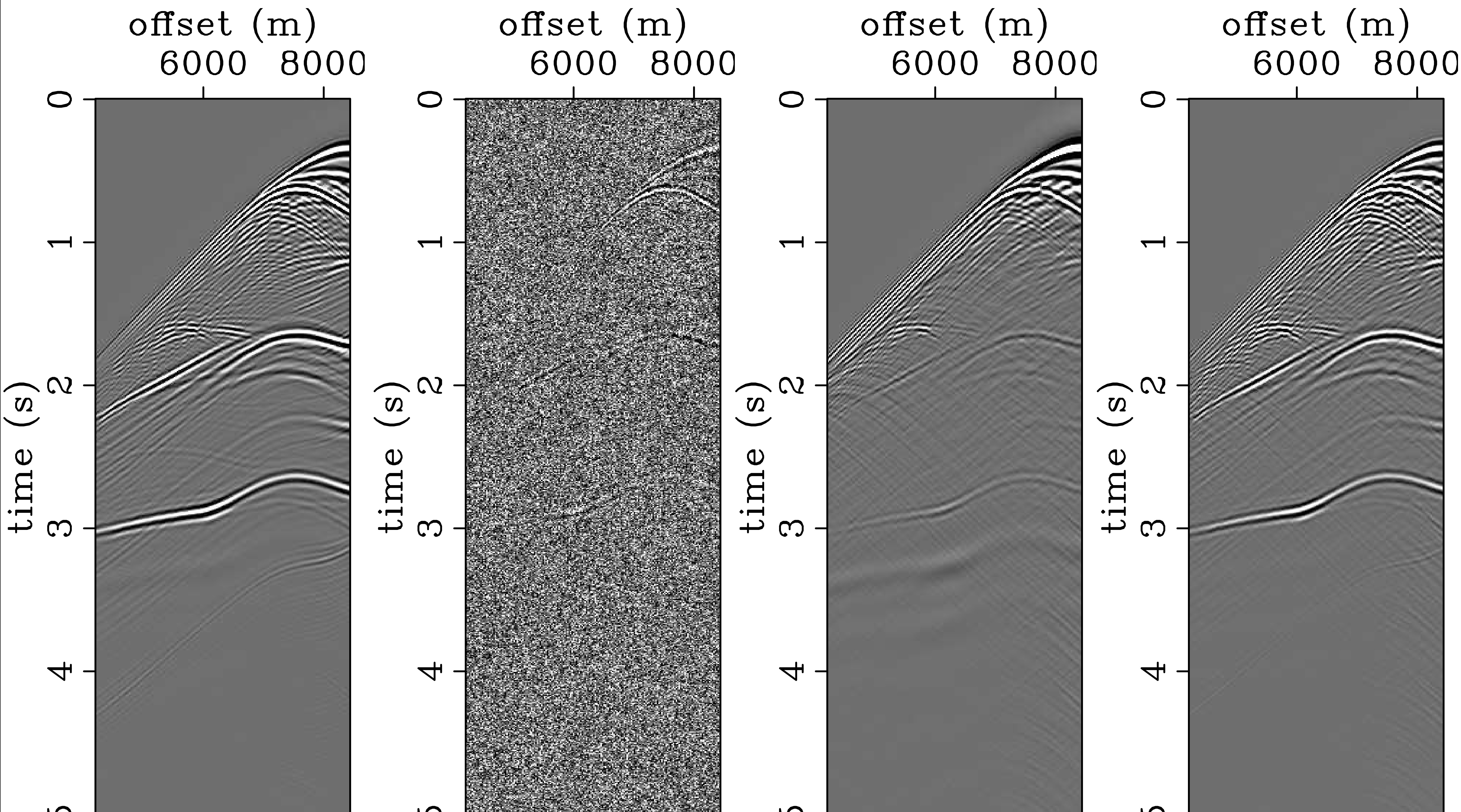


Migrated data

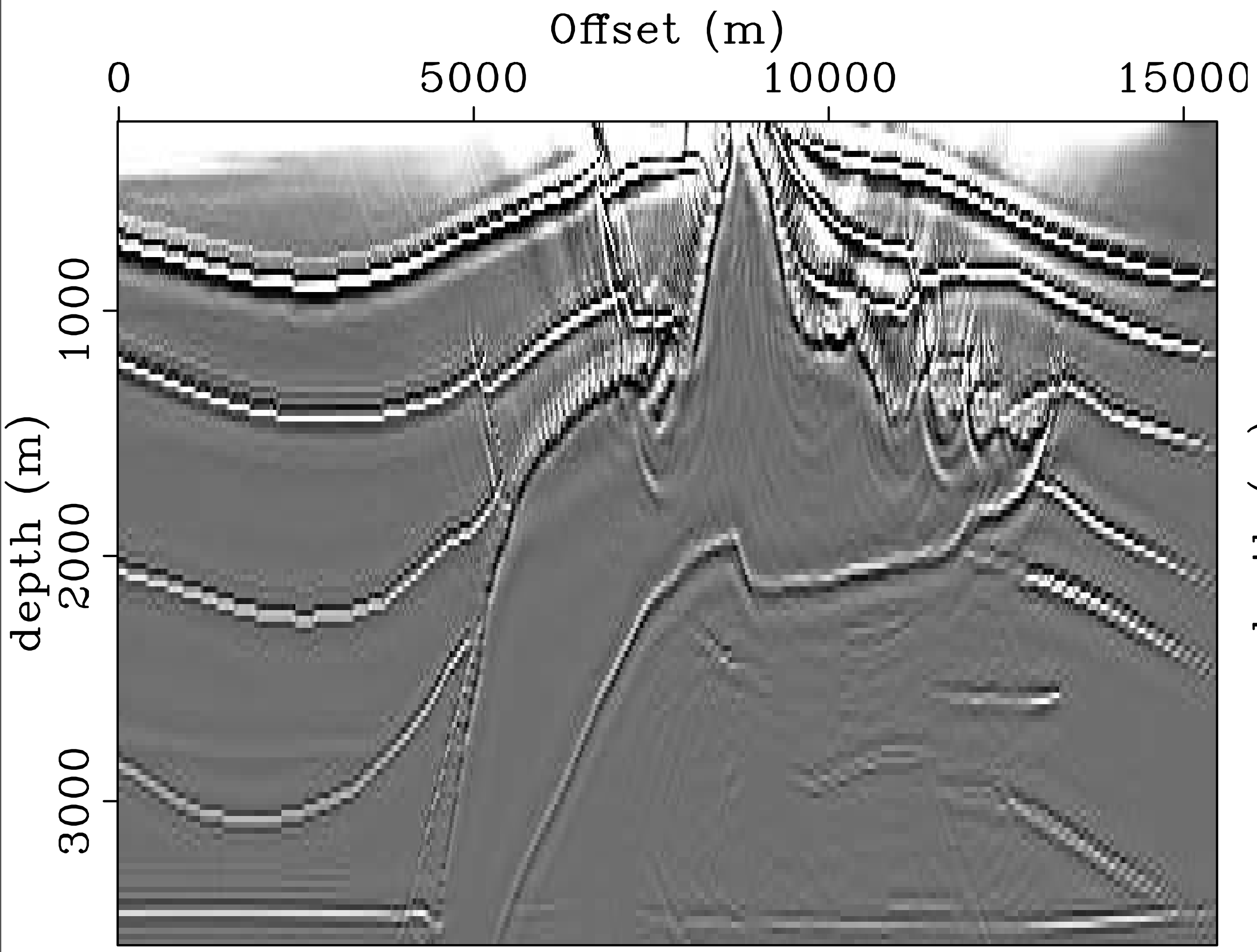


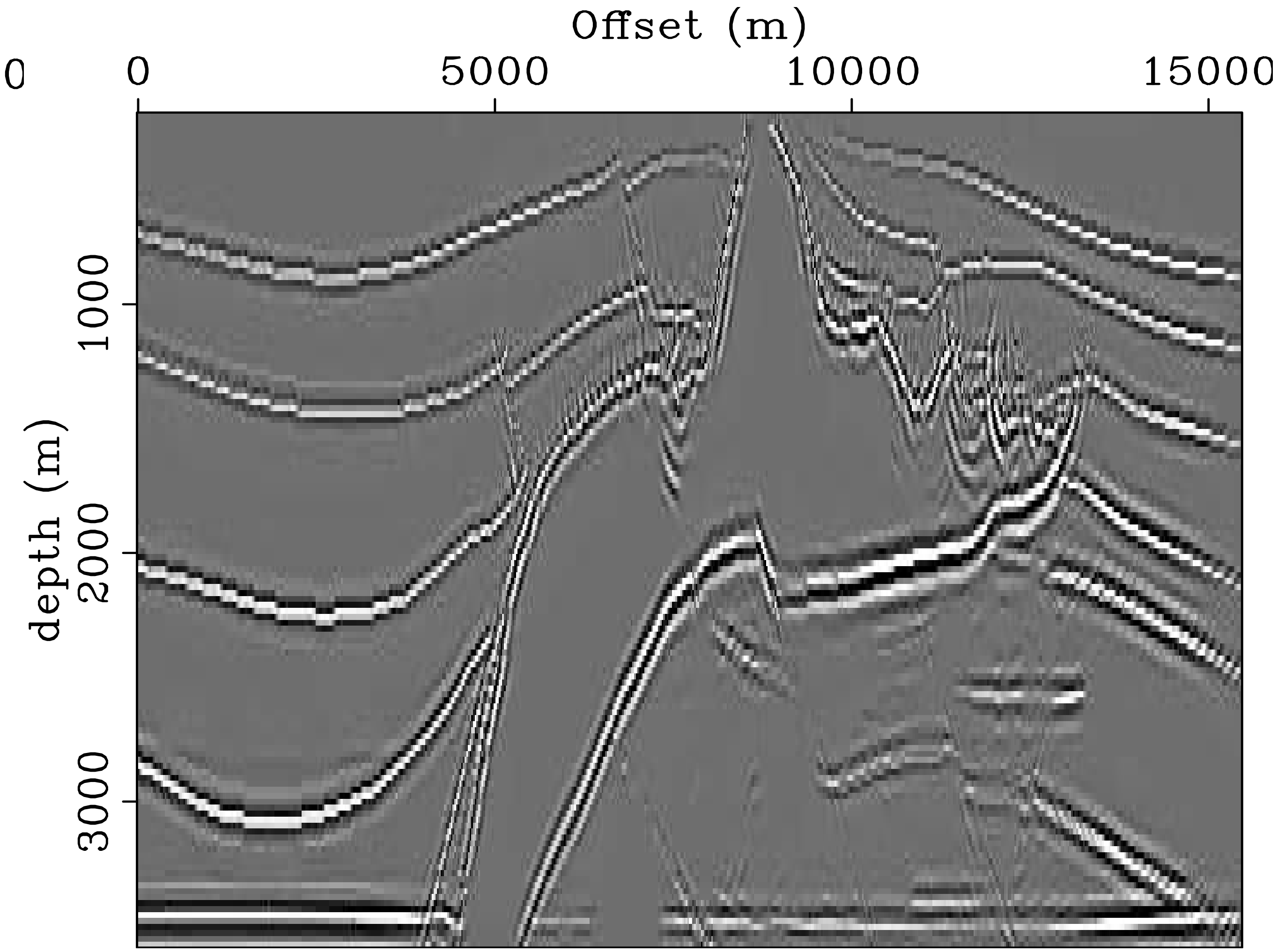
Amplitude-corrected & denoised migrated data



Noise-free data *Noisy data (3 dB)* *Data from migrated image* *Data from amplitude-corrected & denoised migrated image*

Nonlinear data





Conclusions

*The combination of the parsimonious **curvelet** transform with nonlinear **sparsity** & **continuity** promoting program allowed us to...*

- *recover seismic data from large percentages missing traces*
- *separate primaries & multiples*
- *recover migration amplitudes*

This success is due to the curvelet’s ability to

- detect wavefronts <=> multi-D geometry
- differentiate w.r.t. positions, angle(s) and scale
- diagonalize the demigration-migration operator

Because of their parsimoniousness on seismic data and images, curvelets open new perspectives on seismic processing ...

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Acknowledgments

The authors of CurveLab (Demanet, Ying, Candes, Donoho)

William Symes for the reverse-time migration code.

These results were created with Madagascar developed by Sergey Fomel.

This work was in part financially supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant (22R81254) and the Collaborative Research and Development Grant DNOISE (334810-05) of F.J.H. This research was carried out as part of the SINBAD project with support, secured through ITF (the Industry Technology Facilitator), from the following organizations: BG Group, BP, Chevron, ExxonMobil and Shell.