Recent developments in curvelet-based seismic processing

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Combinations of ***parsimonious*** *signal representations with* ***nonlinear*** *sparsity promoting programs hold the* ***key*** *to the next-generation of seismic data processing algorithms ...*

Since they

- *allow for a formulation that is* ***stable*** *w.r.t. noise & incomplete data*
- *do not require* ***prior*** *information on the* ***velocity*** *or* ***locations*** *&* ***dips*** *of the events*

Seismic data and images are complicated because

- wavefronts & reflectors are multiscale & multi-directional
- the presence of caustics, faults and pinchouts

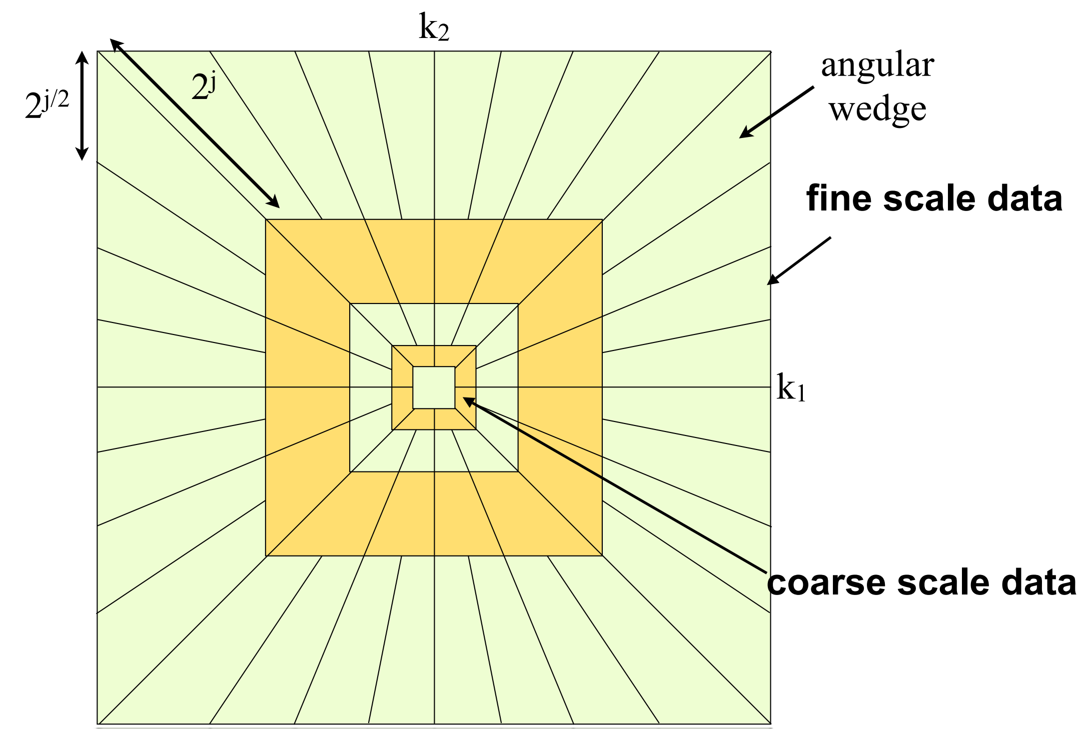
Curvelets

Representations for seismic data

Transform	Underlying assumption
FK	plane waves
linear/parabolic Radon transform	linear/parabolic events
wavelet transform	point-like events (1D singularities)
curvelet transform	**curve-like events (2D singularities)**

Properties curvelet transform:

- **multiscale:** tiling of the FK domain into dyadic coronae
- **multi-directional:** coronae sub-partitioned into angular wedges, # of angle doubles every other scale
- **anisotropic:** parabolic scaling principle
- **Rapid decay space**
- **Strictly localized in Fourier**
- **Frame with moderate redundancy**



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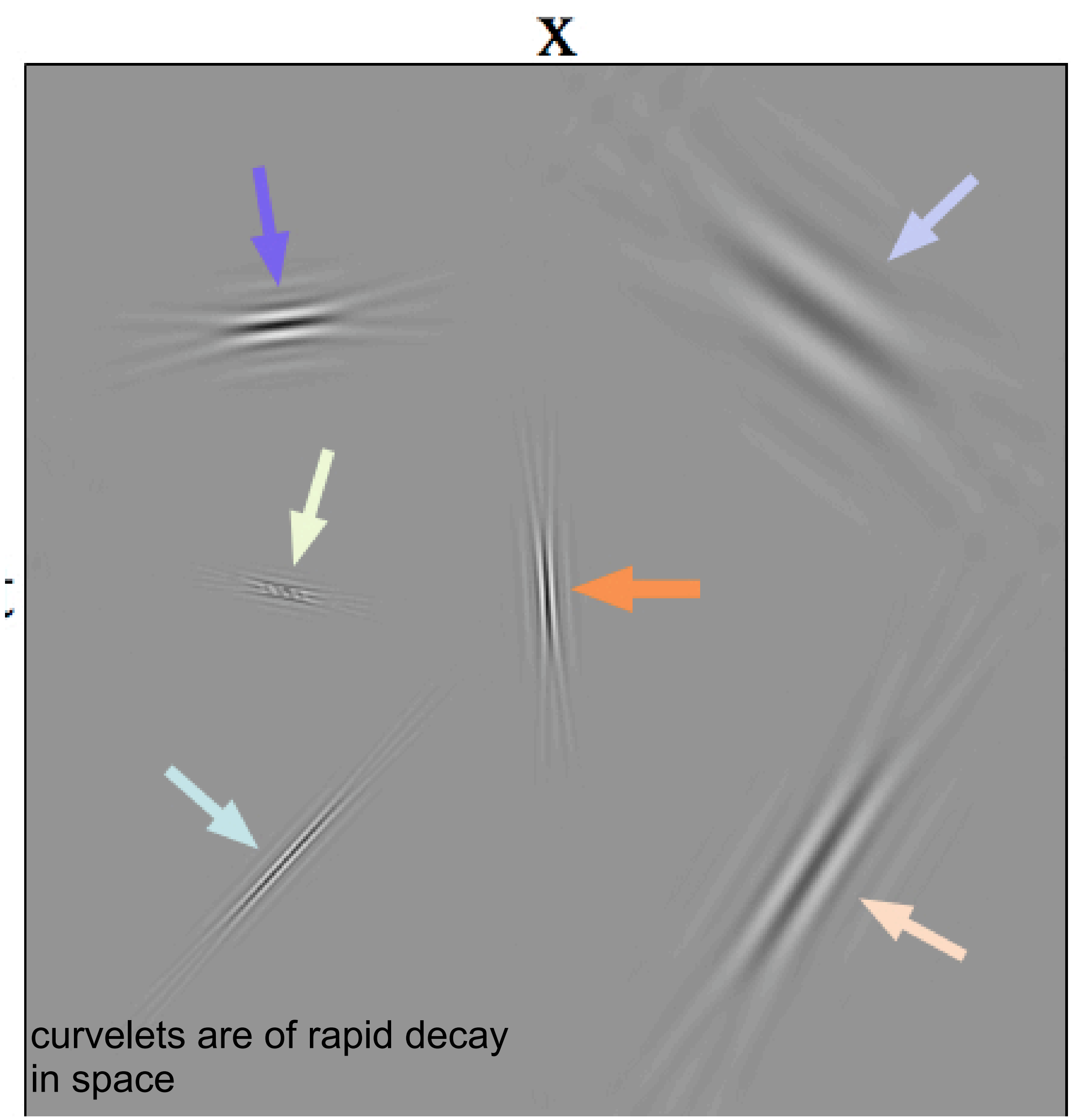
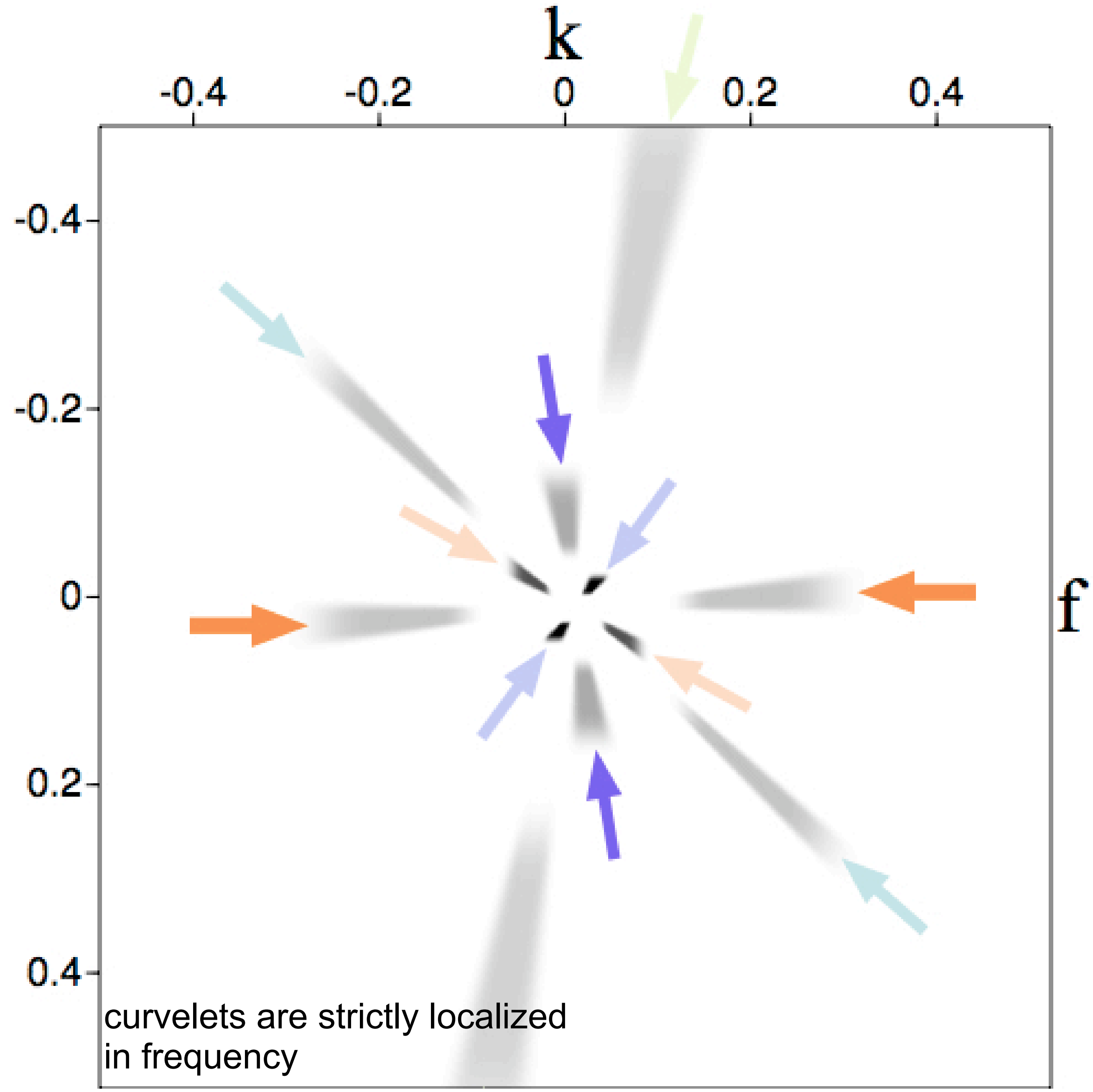
2-D curvelets



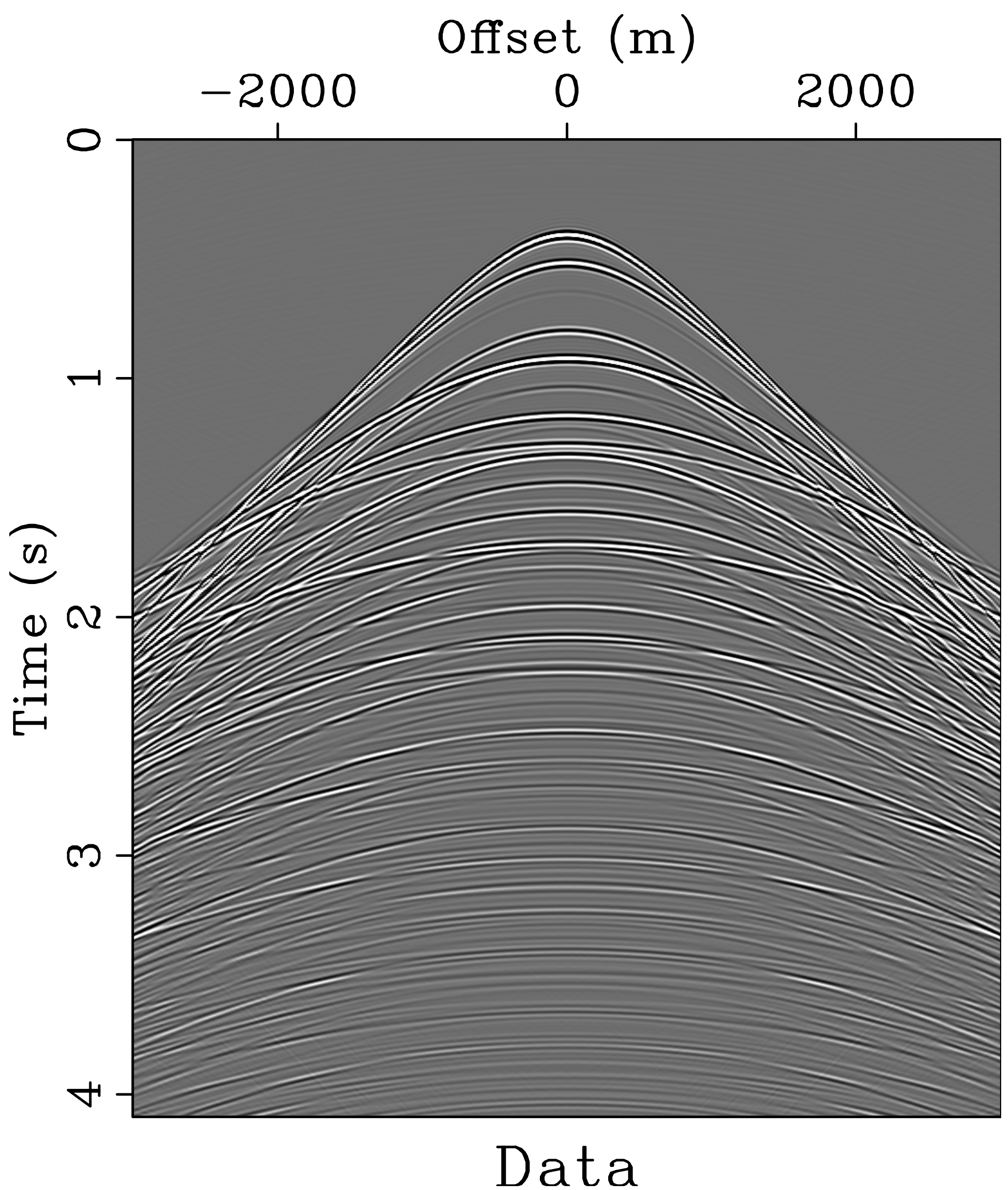
x-t



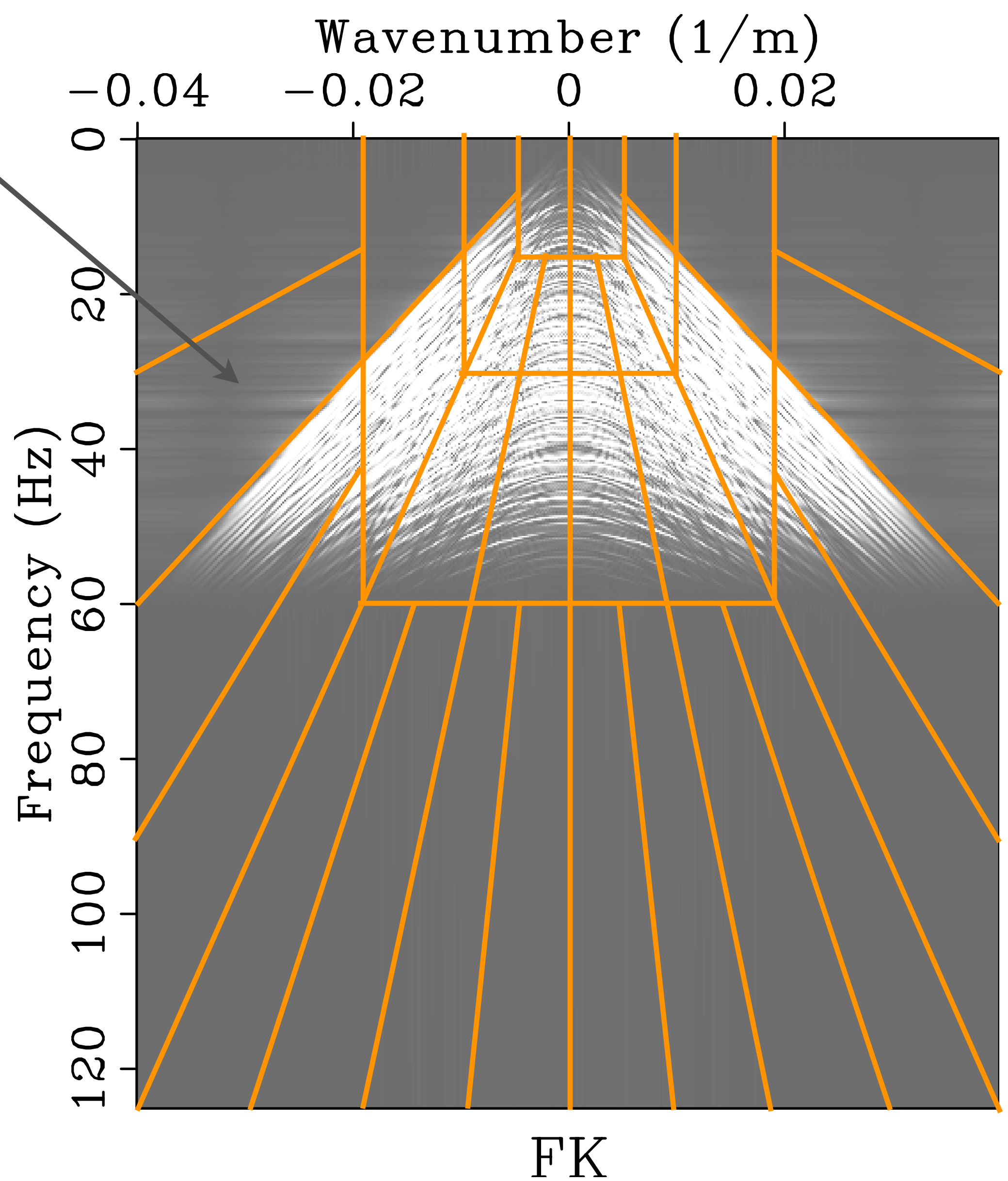
f-k

Oscillatory in one direction and smooth in the others!

Curvelet tiling & seismic data



Angular wedge



Curvelet tiling

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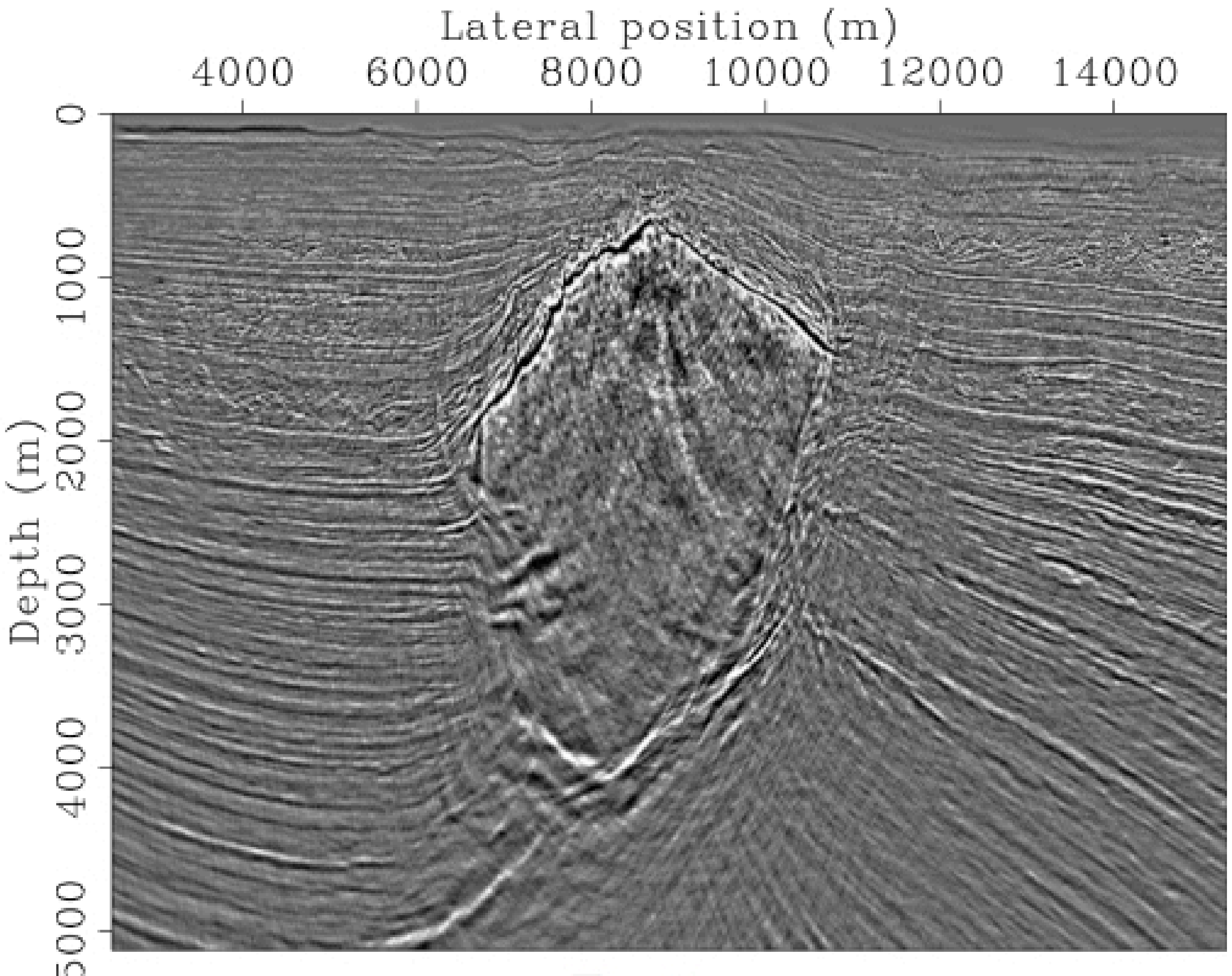
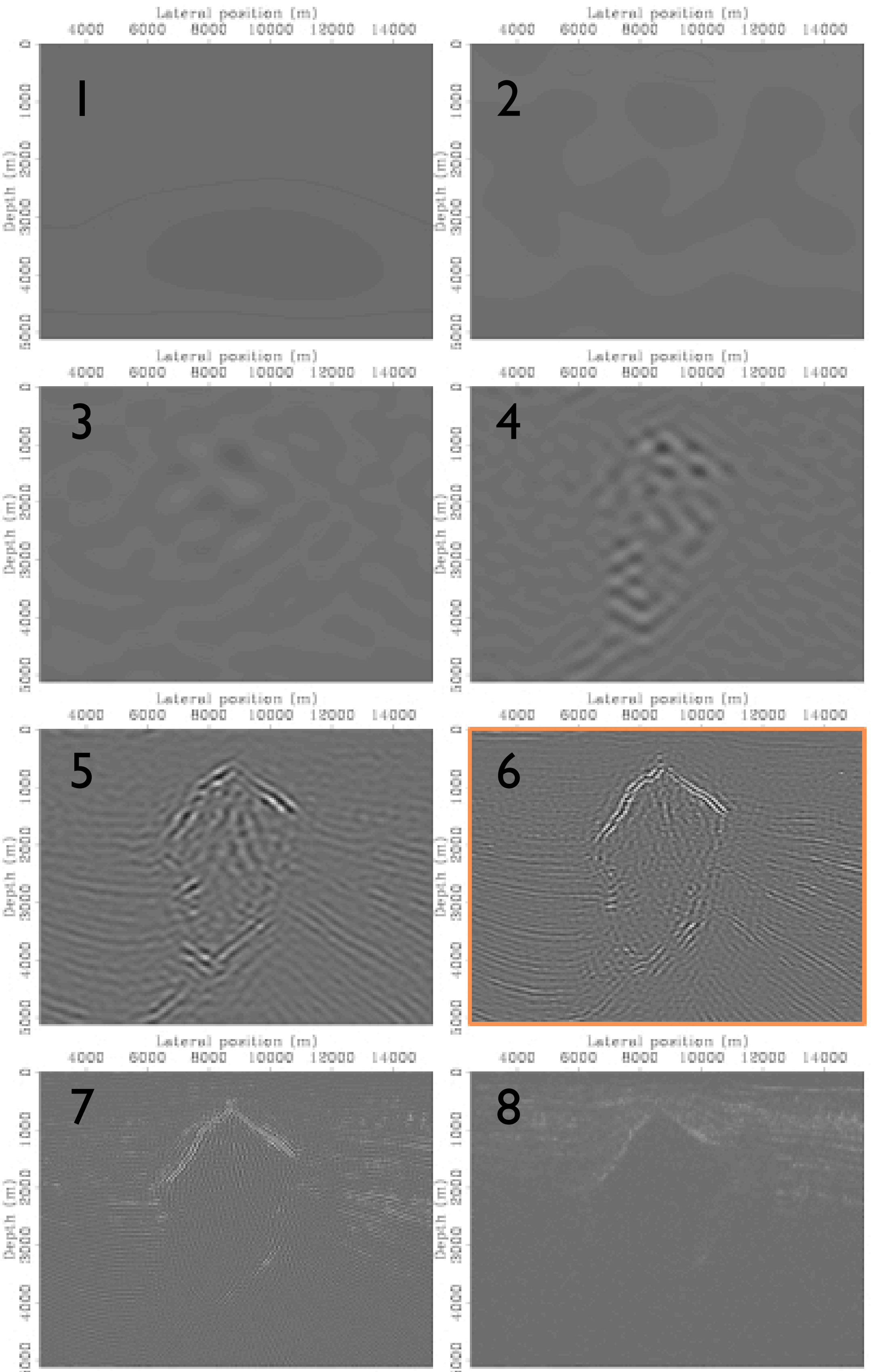
Real data frequency bands example



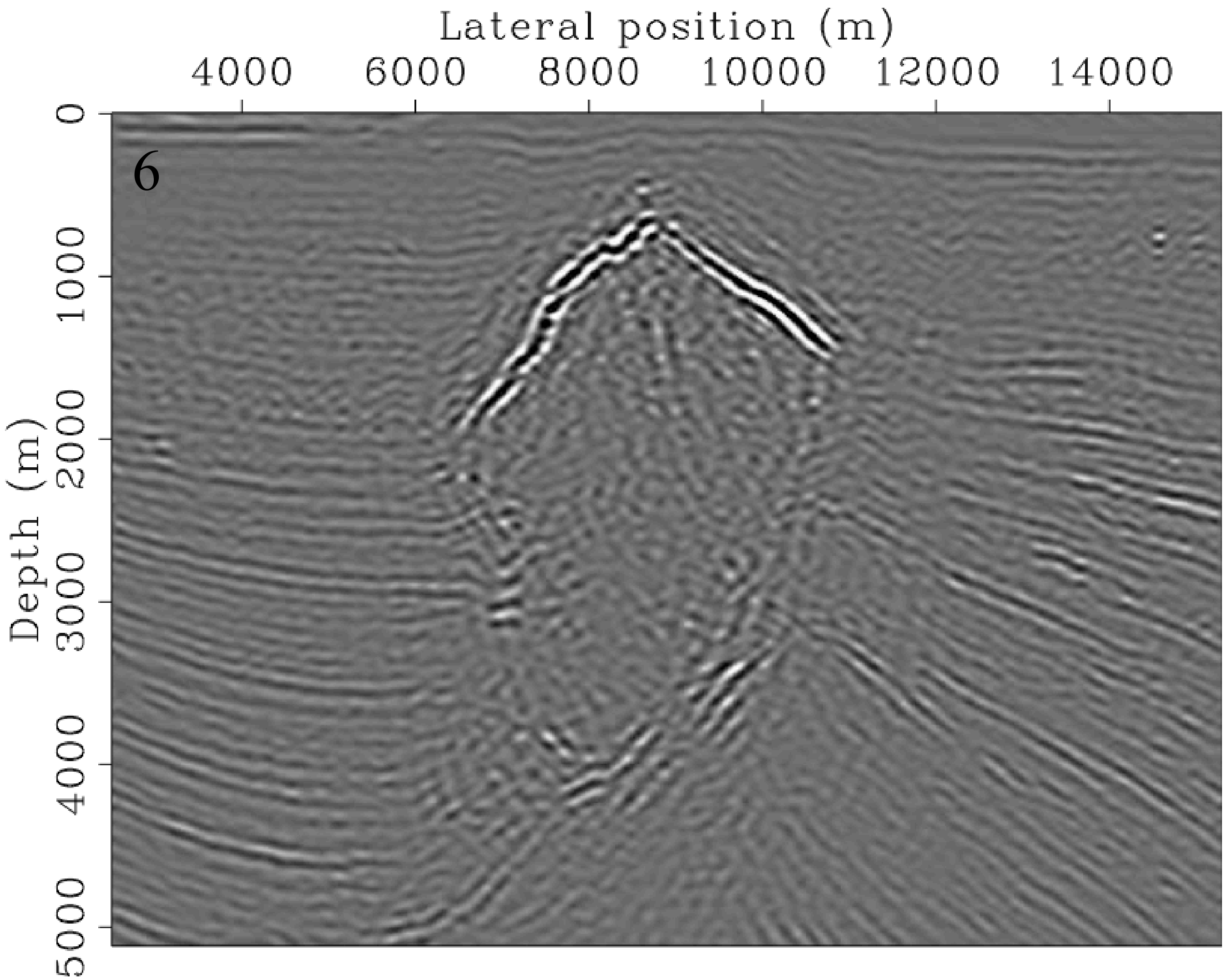
Data

Data is multiscale!



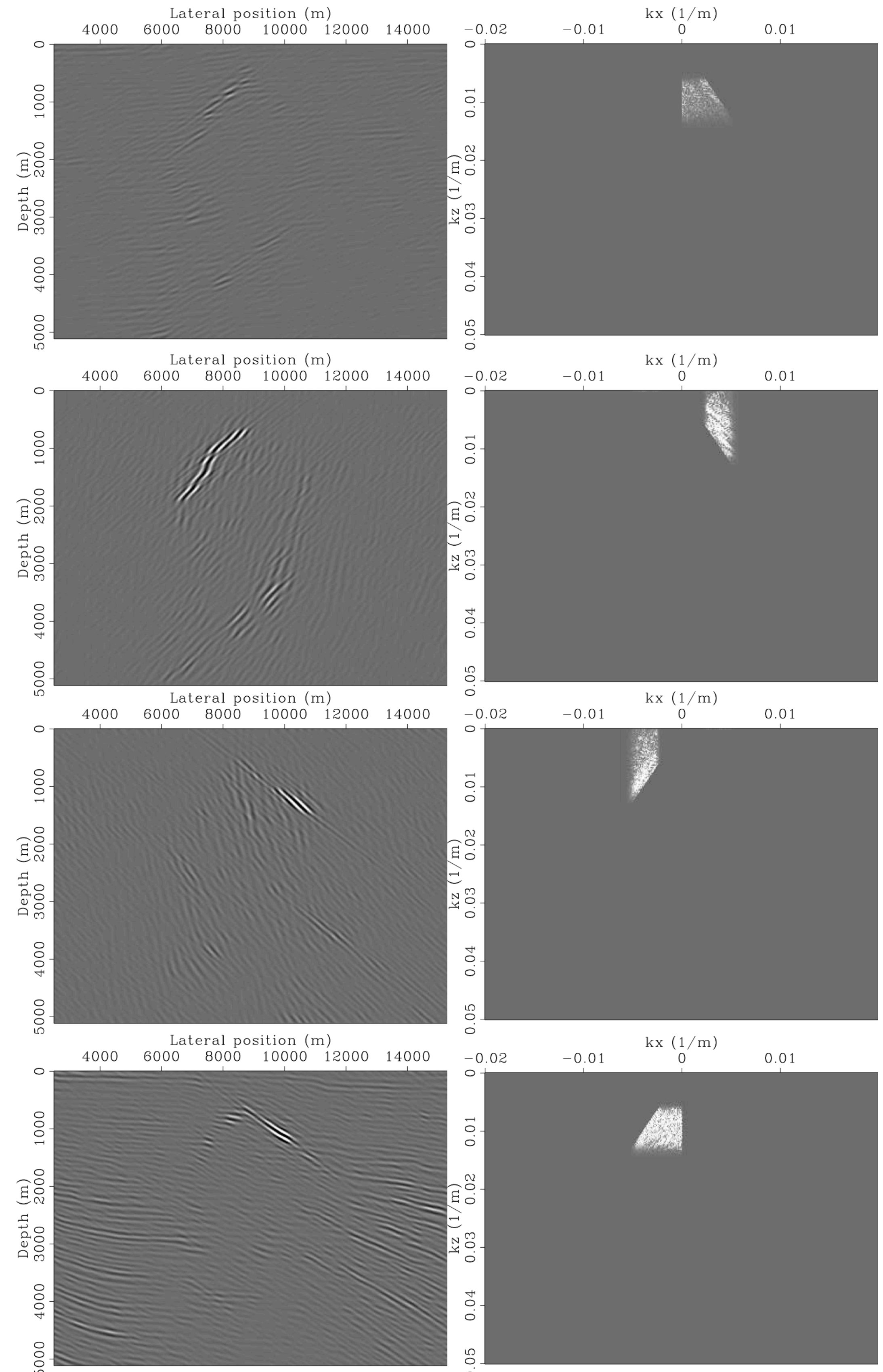
Decomposition in frequency bands

Single frequency band angular wedges



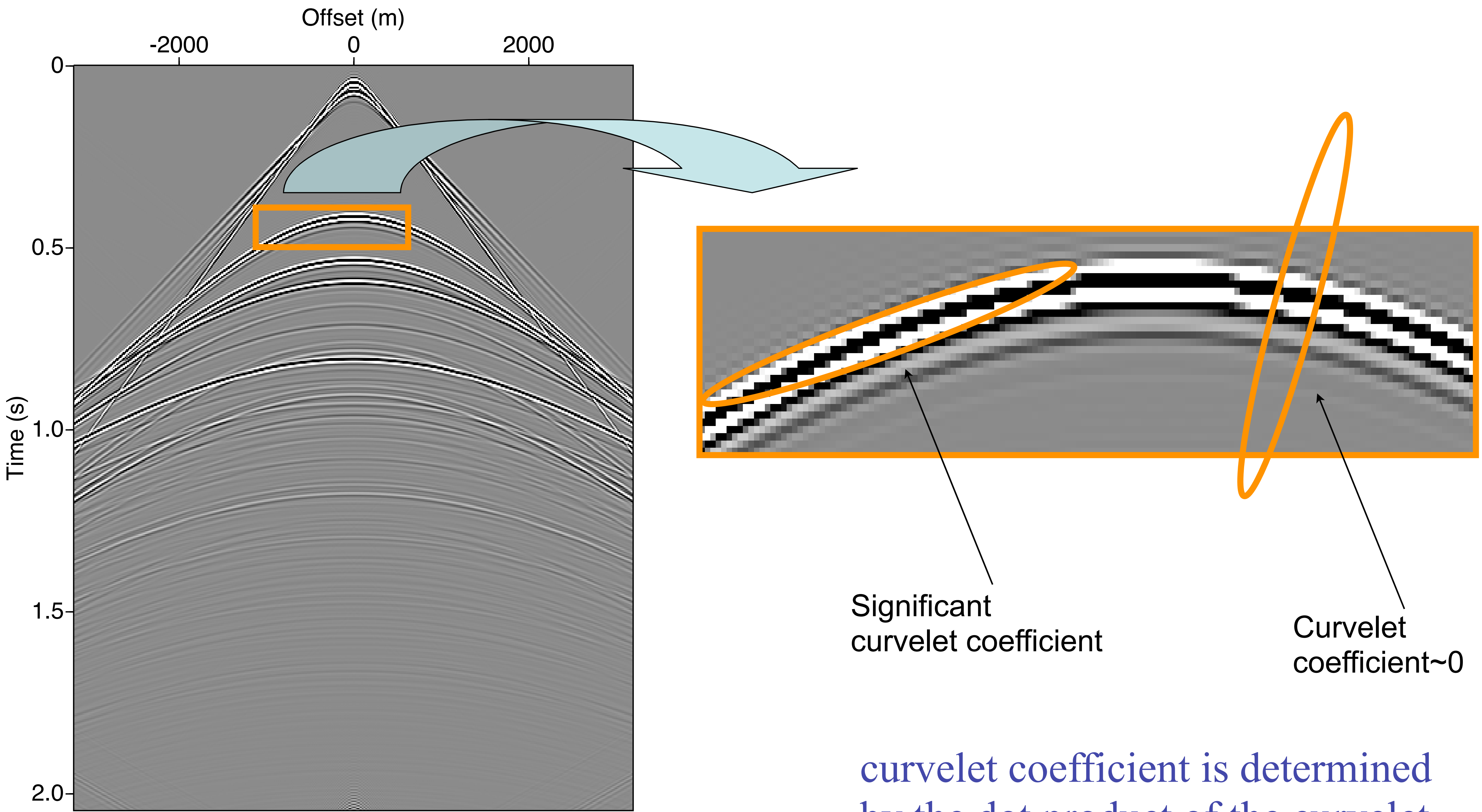
6^{th} scale image

Data is multidirectional!



Decomposition in angular wedges

Wavefront detection



curvelet coefficient is determined by the dot product of the curvelet function with the data

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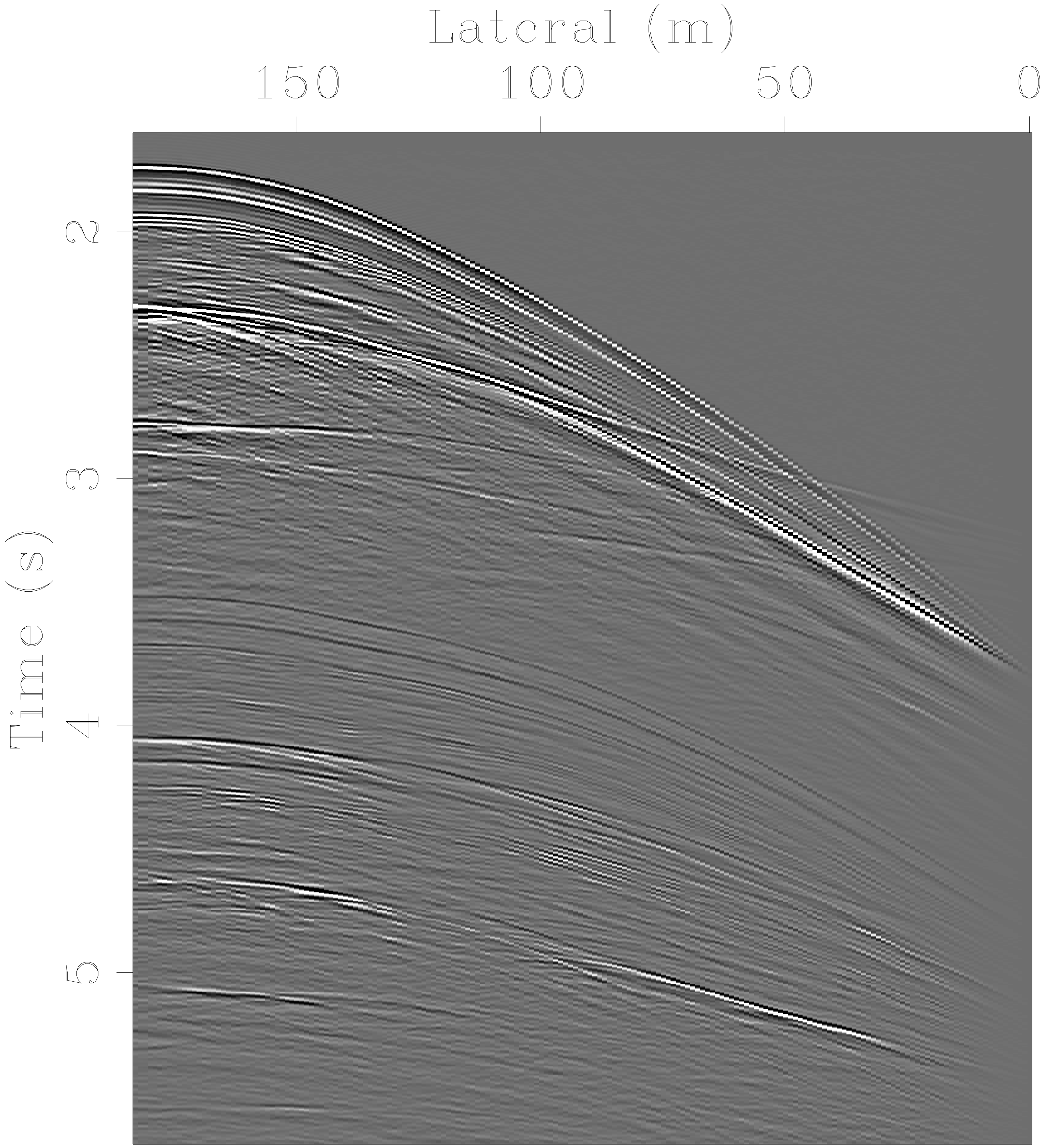
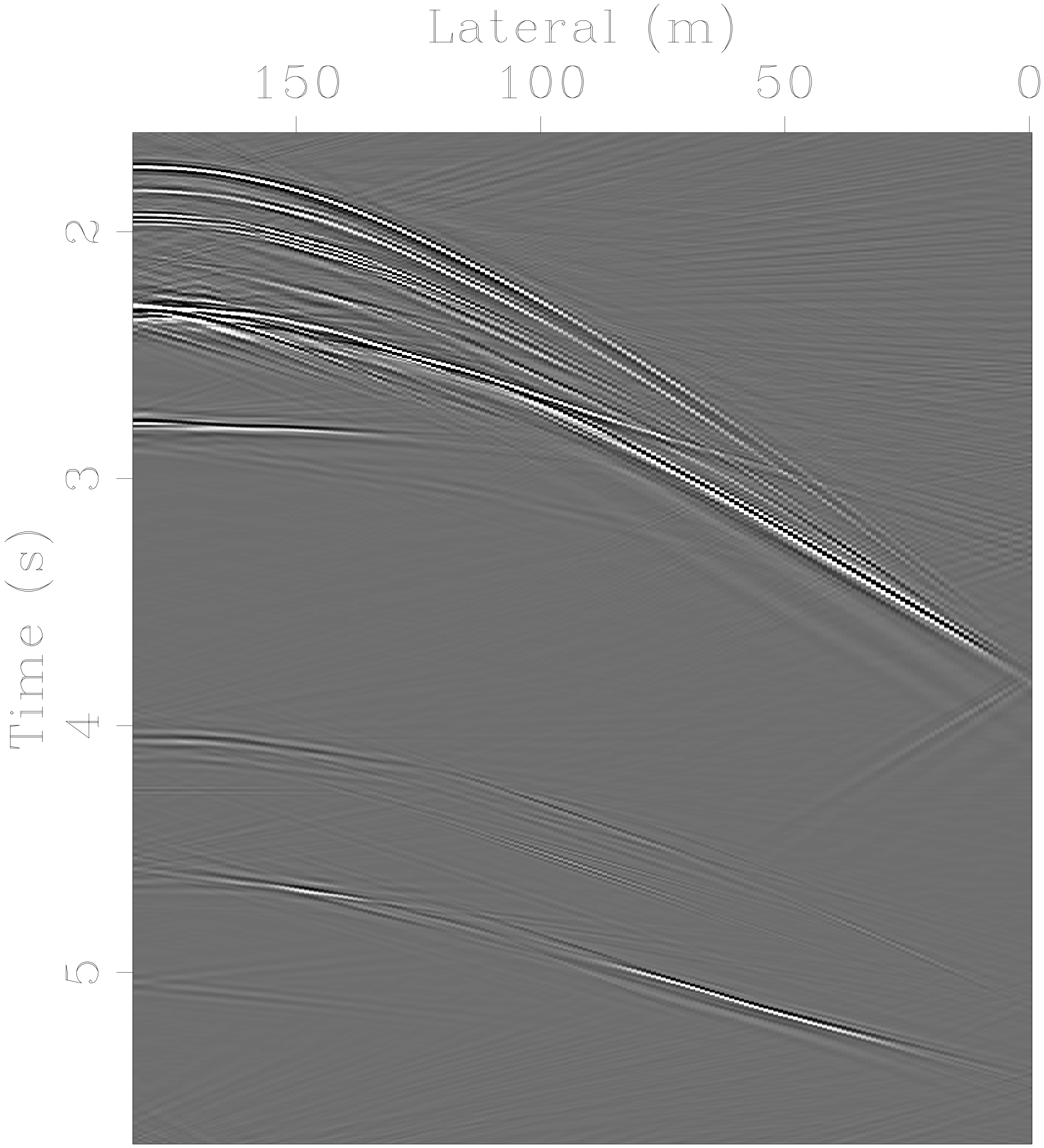
Nonlinear approximation



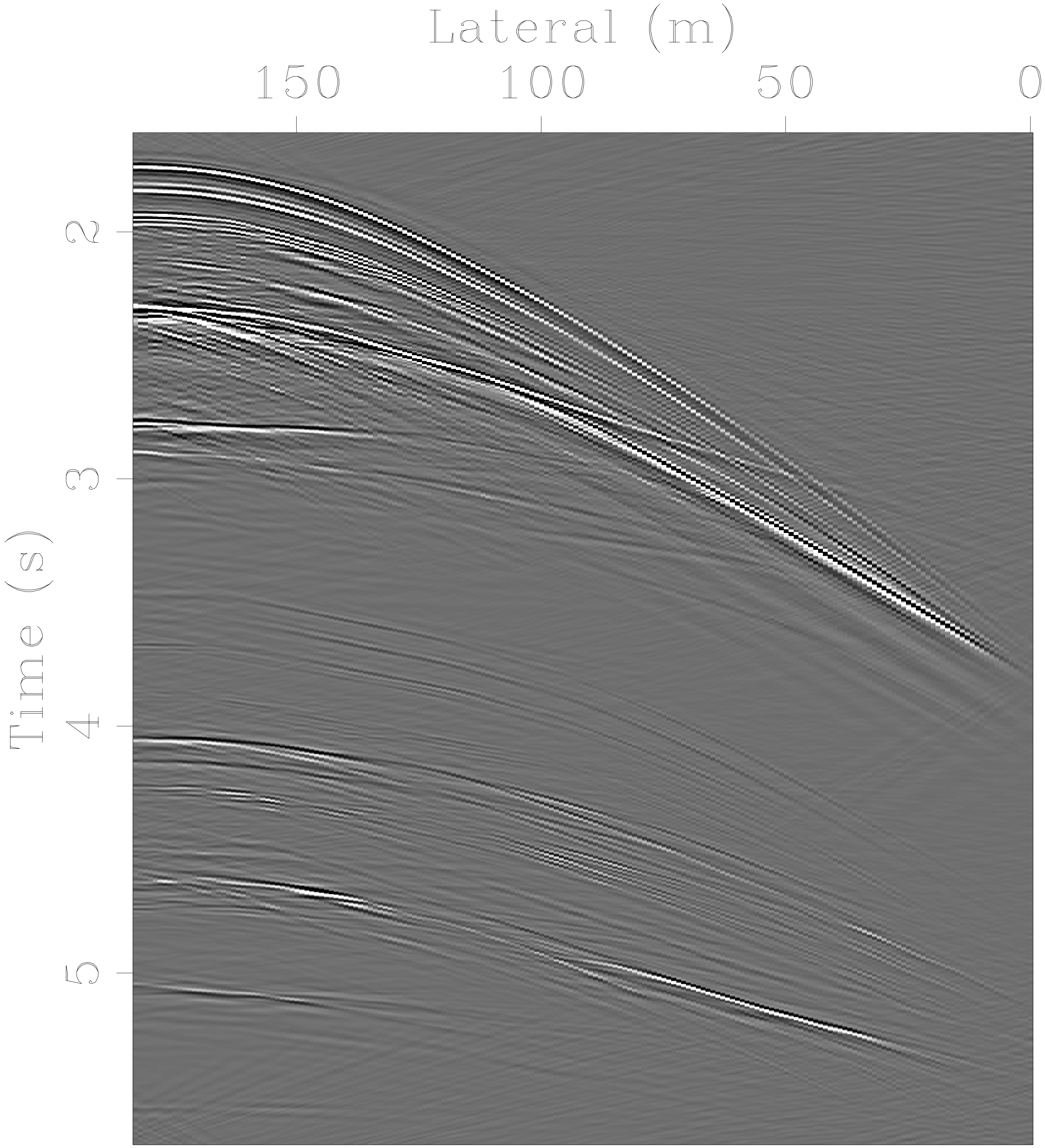
reconstructed data with p=99

Nonlinear approximation



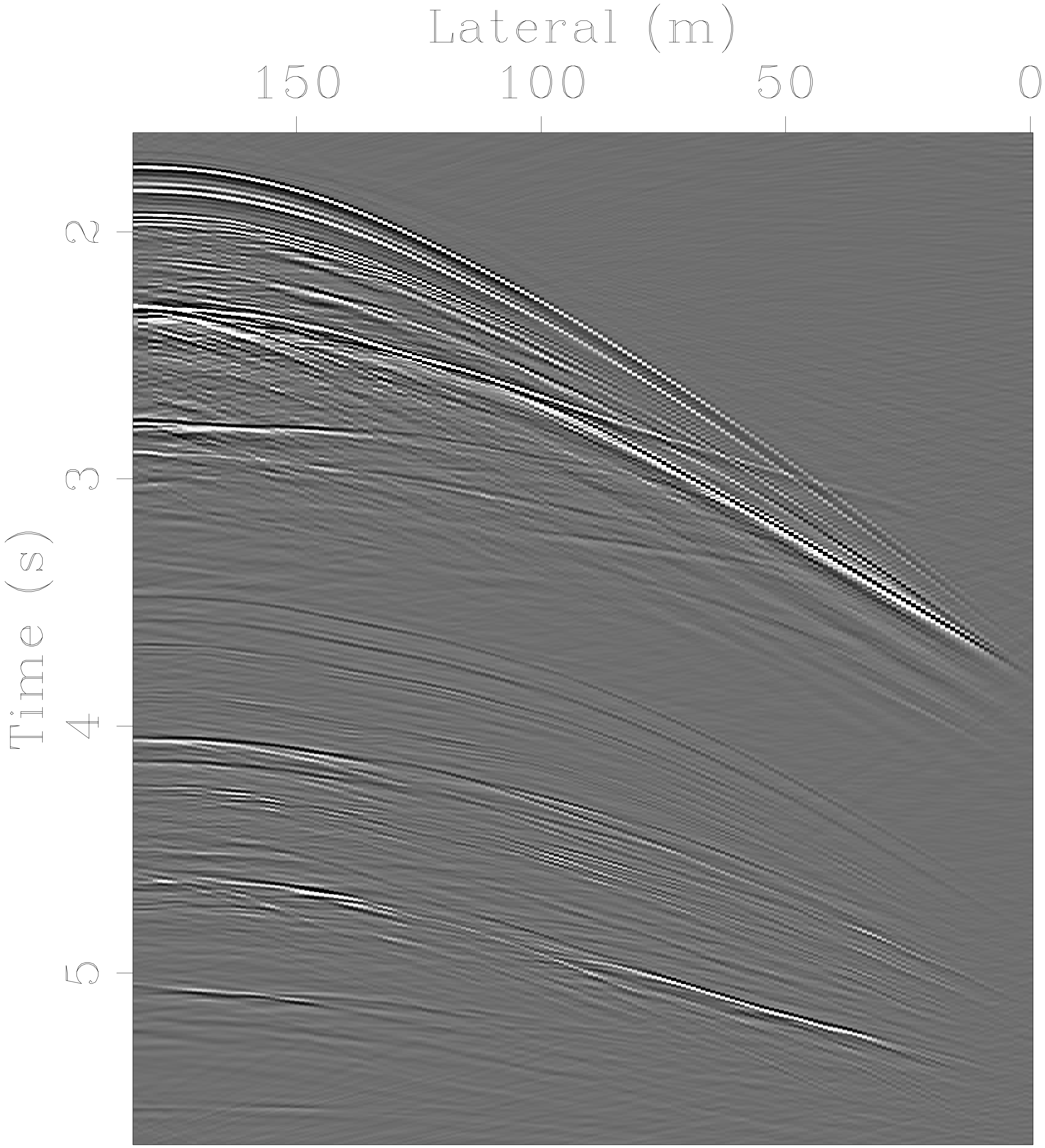
reconstructed data with p=1

Nonlinear approximation



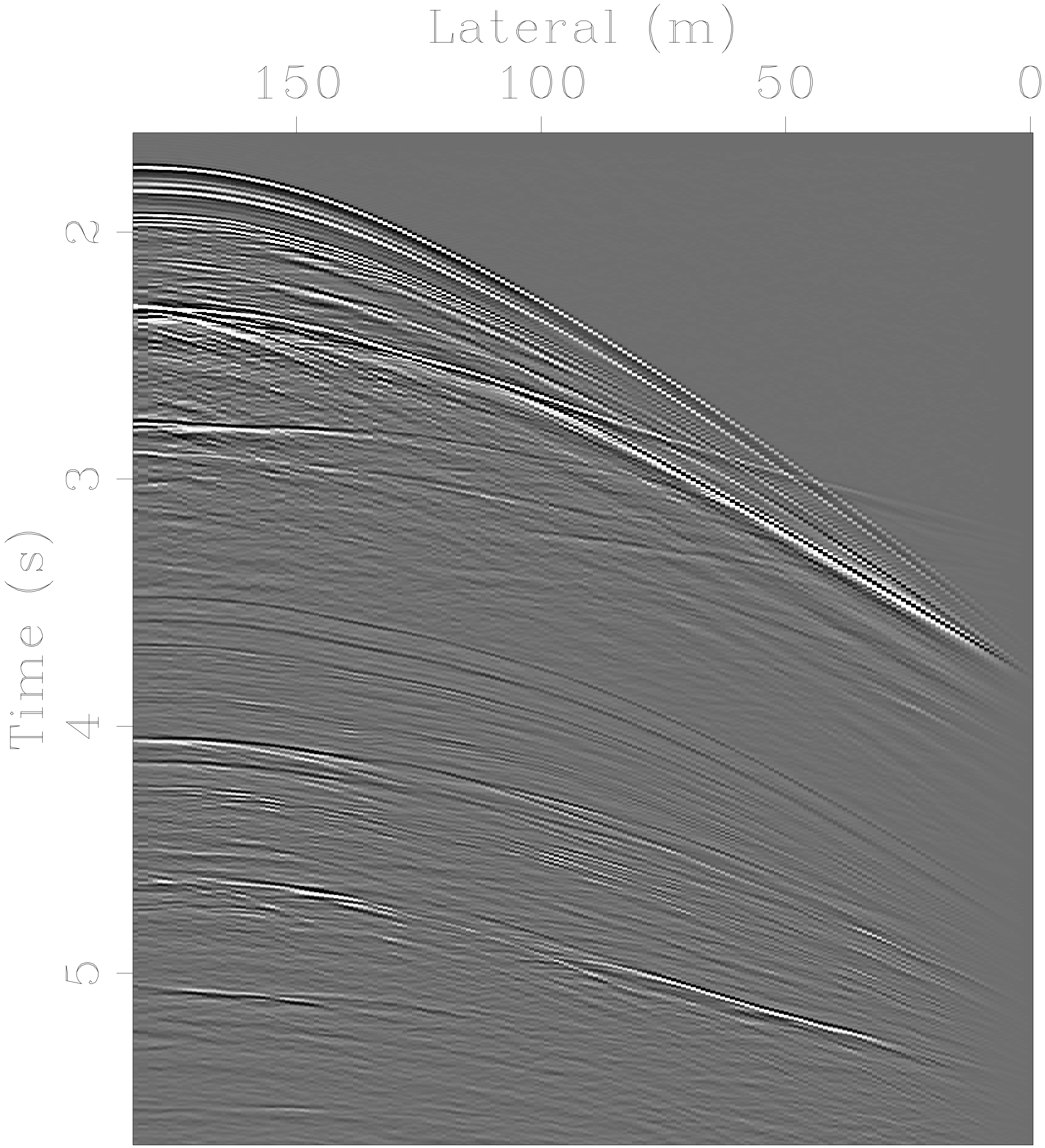
reconstructed data with p=5

Nonlinear approximation



reconstructed data with p=10

Nonlinear approximation



reconstructed data with p=99

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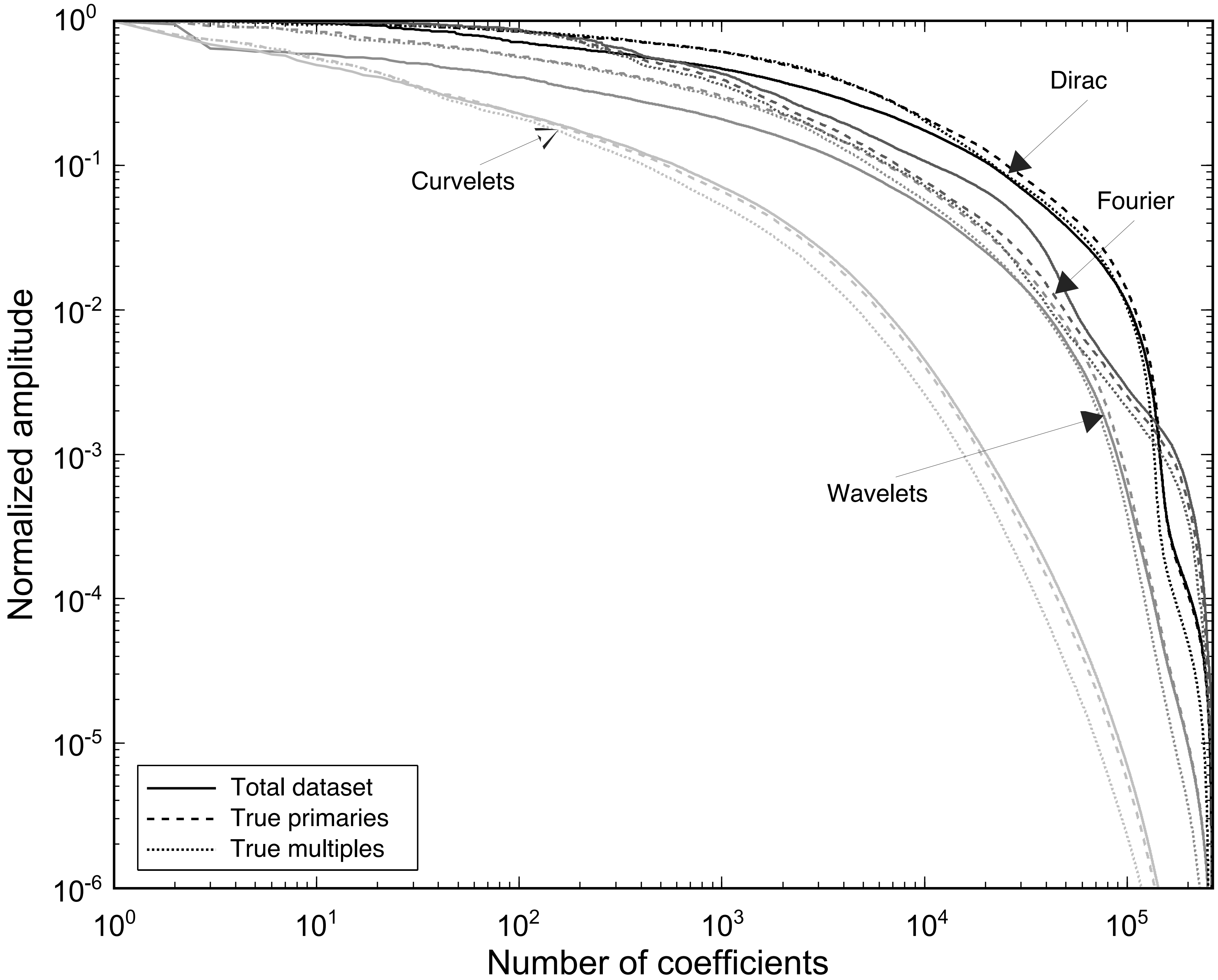
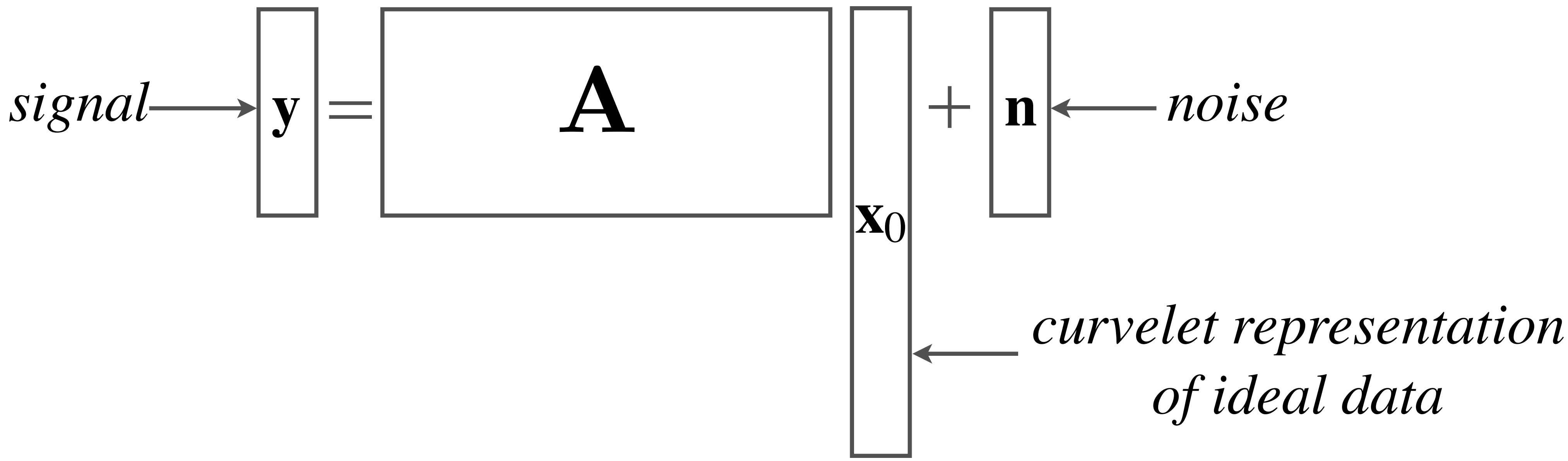
Nonlinear approximation rates



Sparsity promoting inversion

Key idea



$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon$$

sparsity enhancement *data misfit*

When a traveler reaches a fork in the road, the l1 -norm tells him to take either one way or the other, but the l2 -norm instructs him to head off into the bushes.

John F. Claerbout and Francis Muir, 1973

New field “compressive sampling”: D. Donoho, E. Candes et. al., M. Elad etc.

Preceded by others in geophysics: M. Sacchi & T. Ulrych and co-workers etc.

Applications

Sparsity promotion can be used to

- recovery from incomplete data: "Curvelet reconstruction with sparsity promoting inversion: successes & challenges and "Irregular sampling: from aliasing to noise"
- migration amplitude recovery: "Just diagonalize: a curvelet-based approach to seismic amplitude recovery
- ground-roll removal: "Curvelet applications in surface wave removal"
- multiple prediction: "Surface related multiple prediction from incomplete data"
- seismic processing: "Seismic imaging and processing with curvelets"

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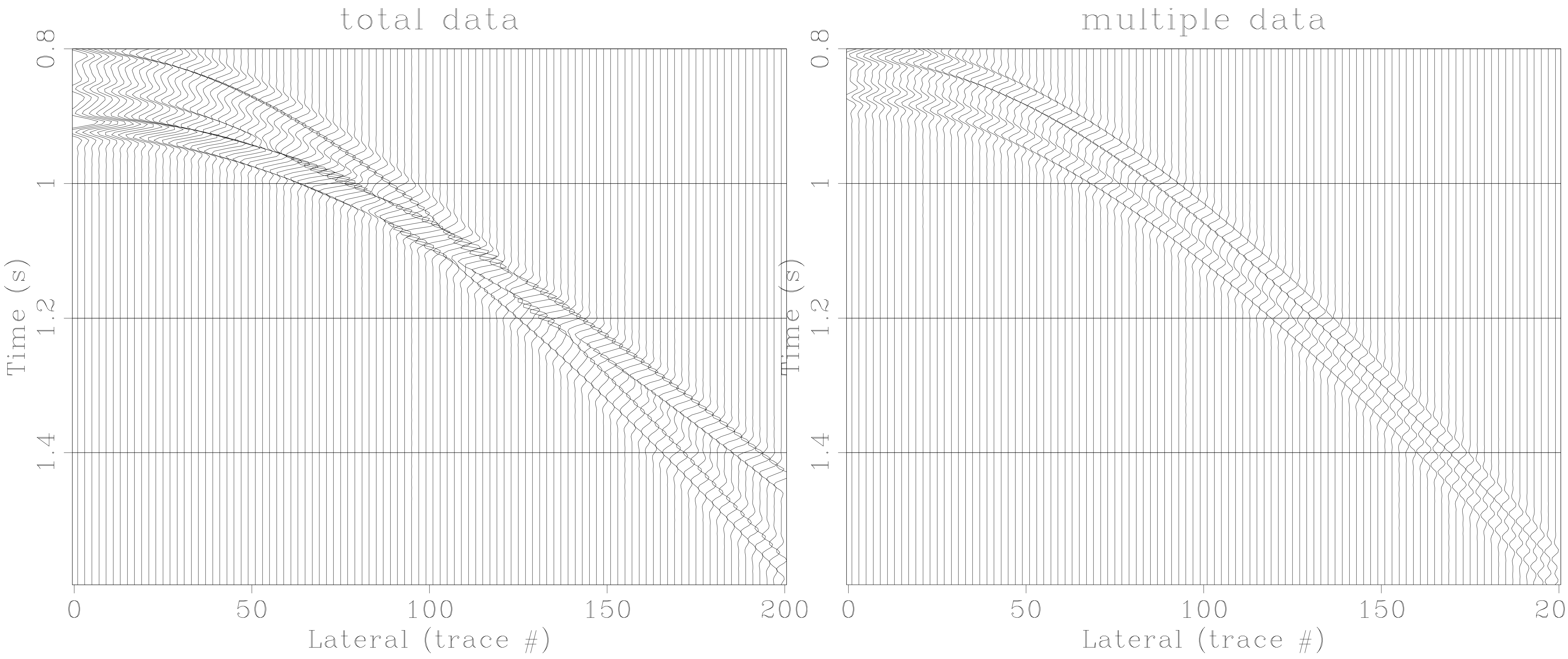
Primary-multiple separation

Joint work with Eric Verschuur, Deli Wang, Rayan Saab and Ozgur Yilmaz

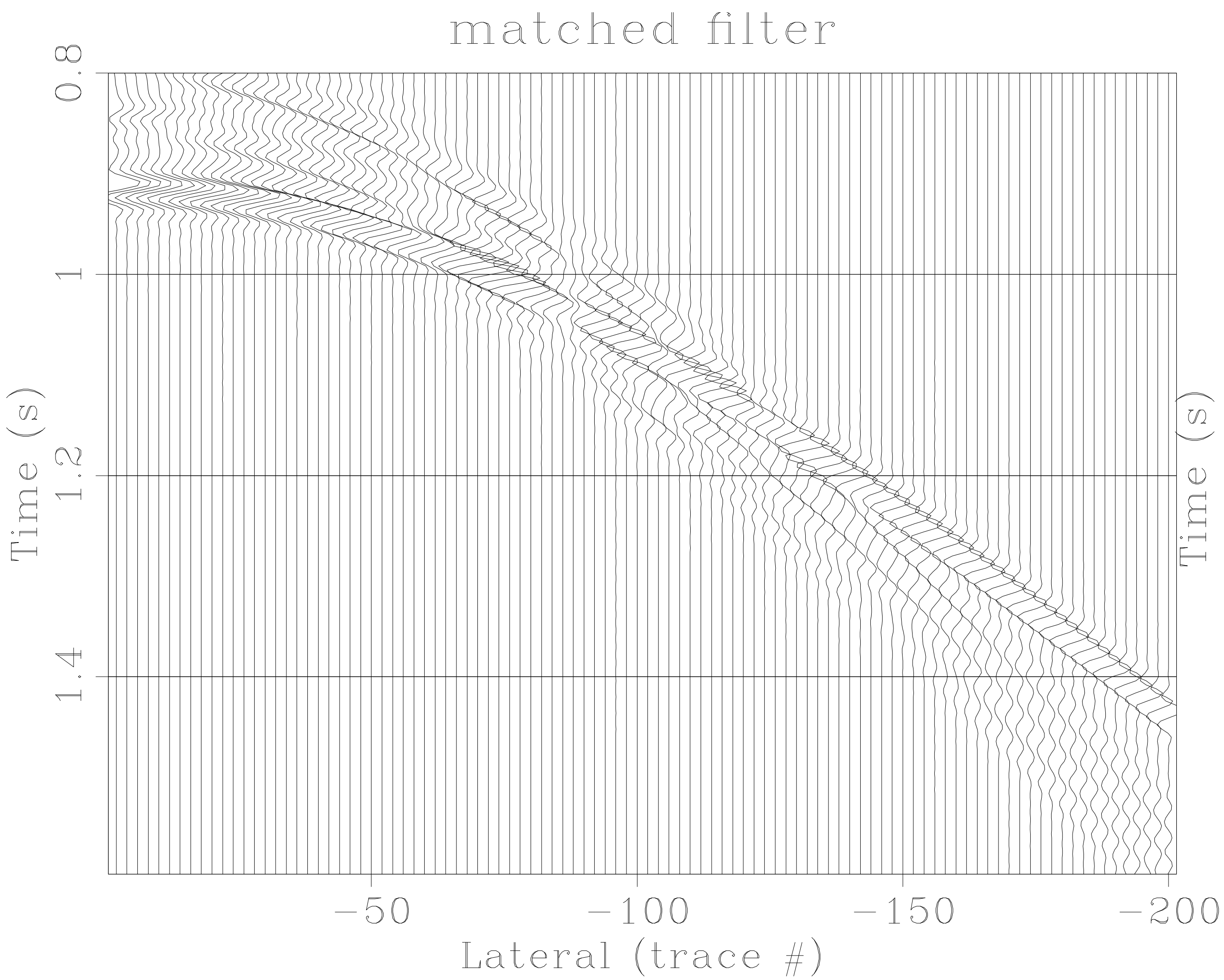



Move-out error



Multiple prediction with erroneous move out.

Move-out error



Curvelet-based result obtained by single soft threshold given by the predicted multiples

$$\tilde{\mathbf{s}}_1 = \mathbf{C}^T T_{\lambda|\mathbf{C}\breve{\mathbf{s}}_2|}(\mathbf{C}\mathbf{s})$$

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Approach

Bayesian formulation of the primary-multiple separation problem

- promotes ***sparsity*** on estimated primaries & multiples
- minimizes ***misfit*** between total data and sum of estimated ***primaries*** and ***multiples***
- exploits decorrelation in the curvelet domain
- **new:** minimizes ***misfit*** between estimated and (SRME) predicted ***multiples***

Separation formulated in terms of a sparsity promoting program robust under

- moderate timing and phase errors
- noise

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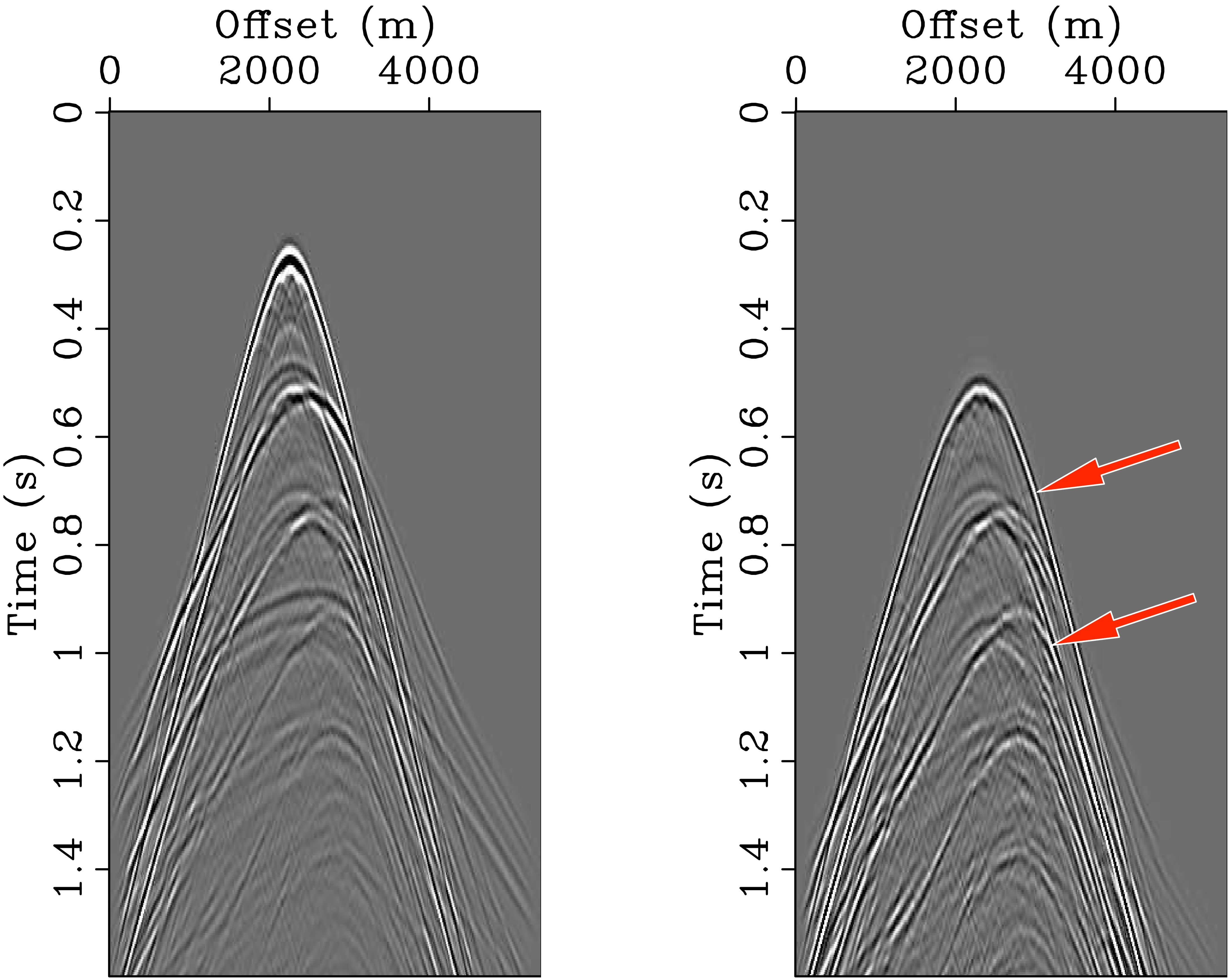
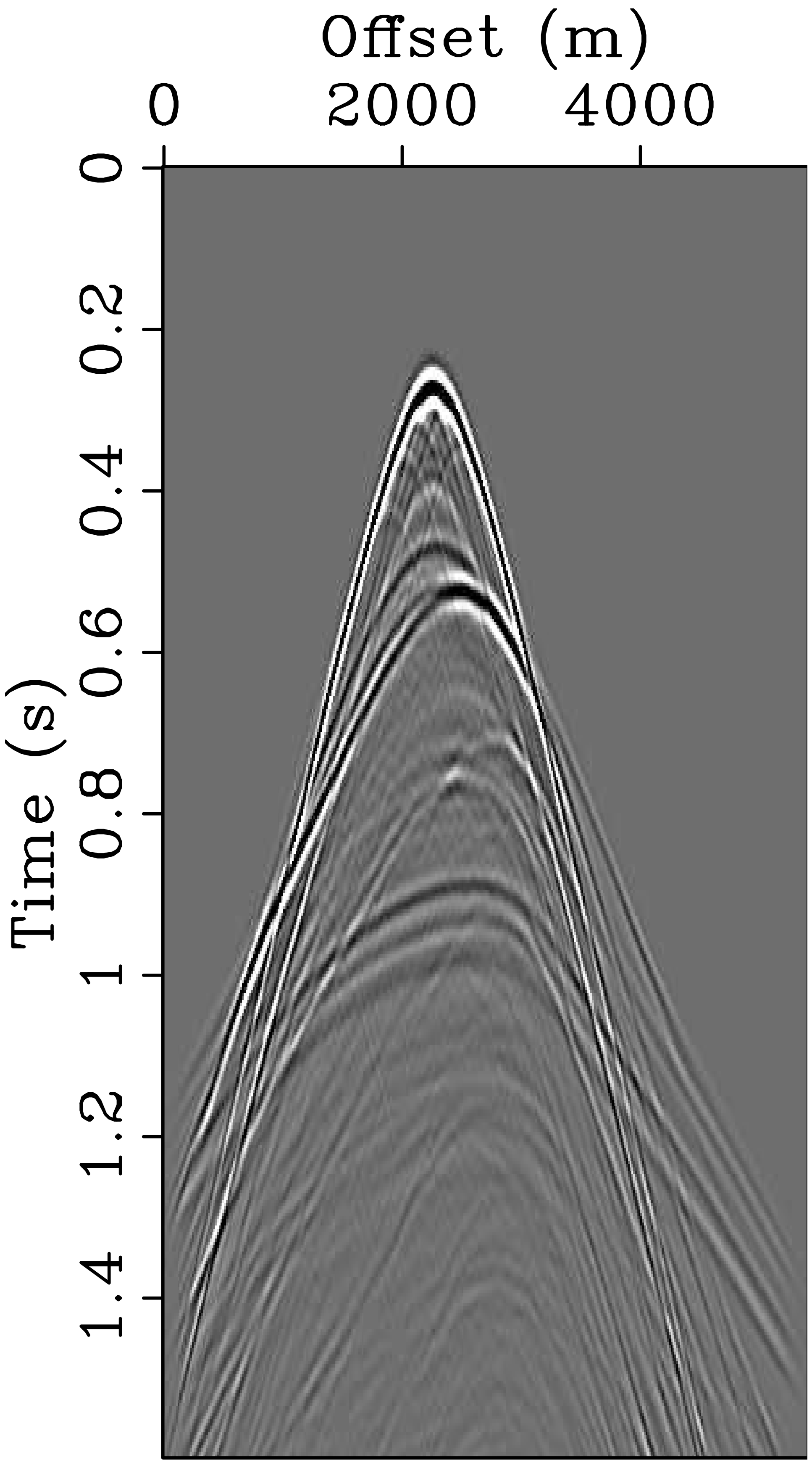
Synthetic example



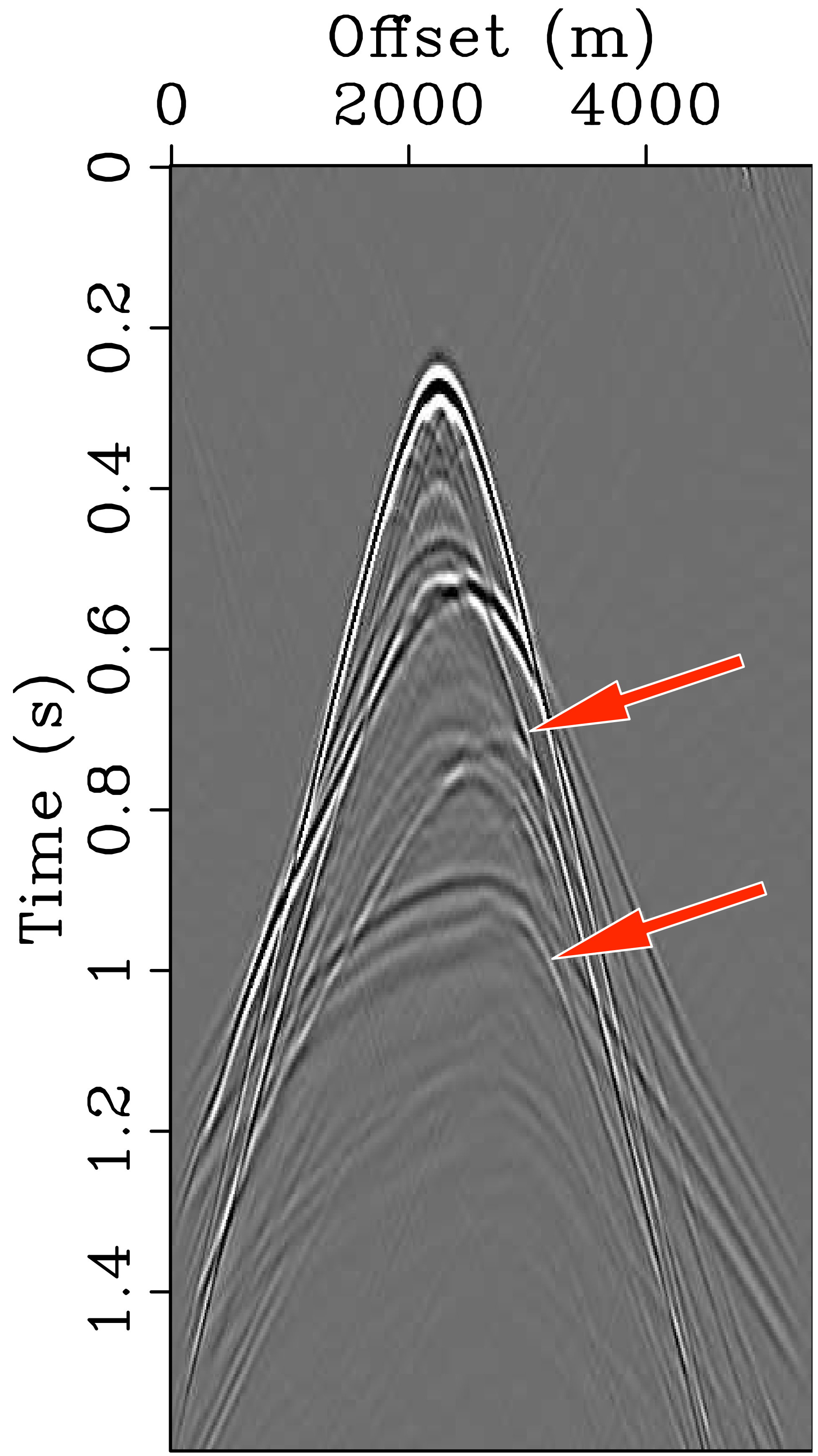
total data

SRME predicted multiples

Synthetic example



SRME predicted primaries



curvelet-thresholded

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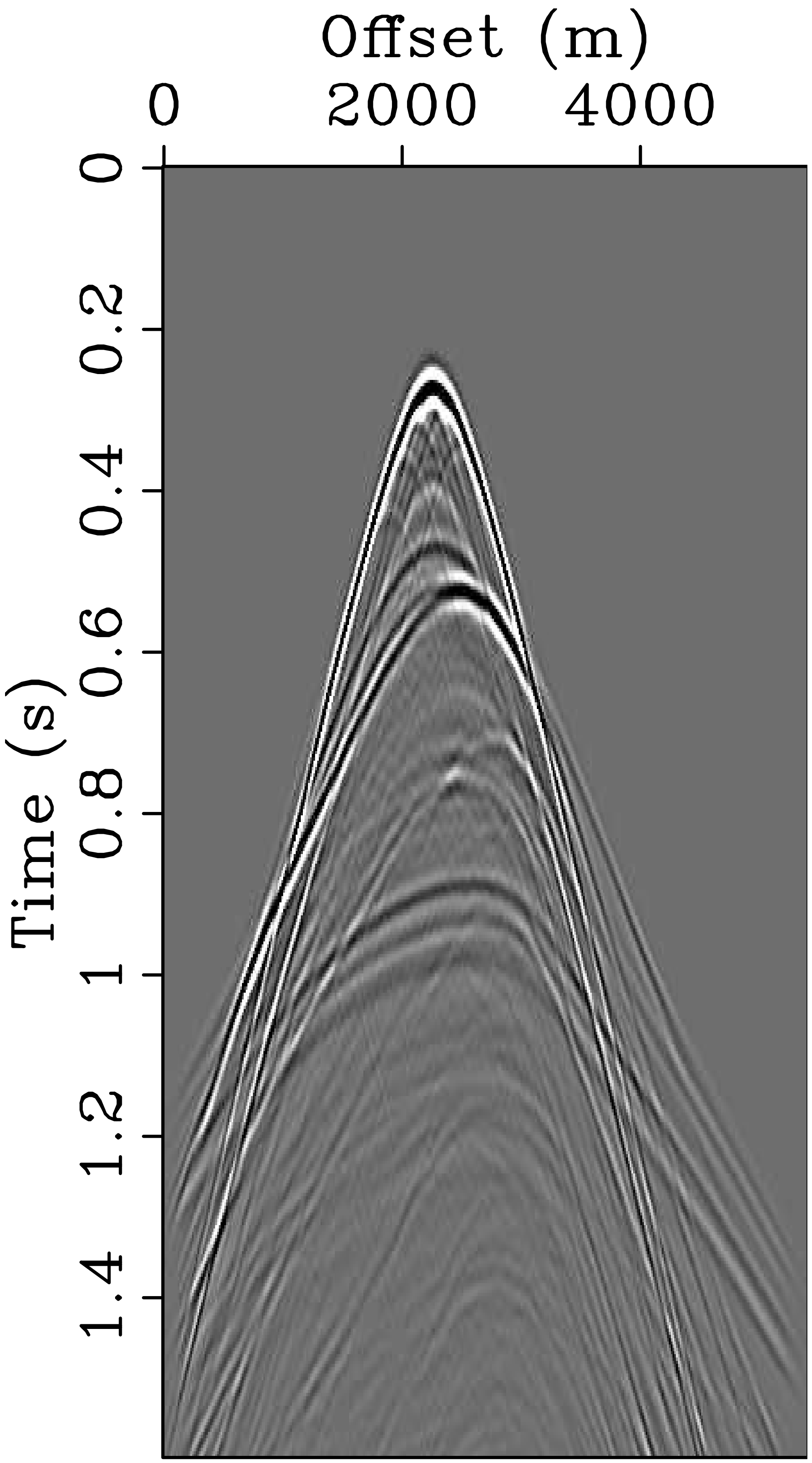
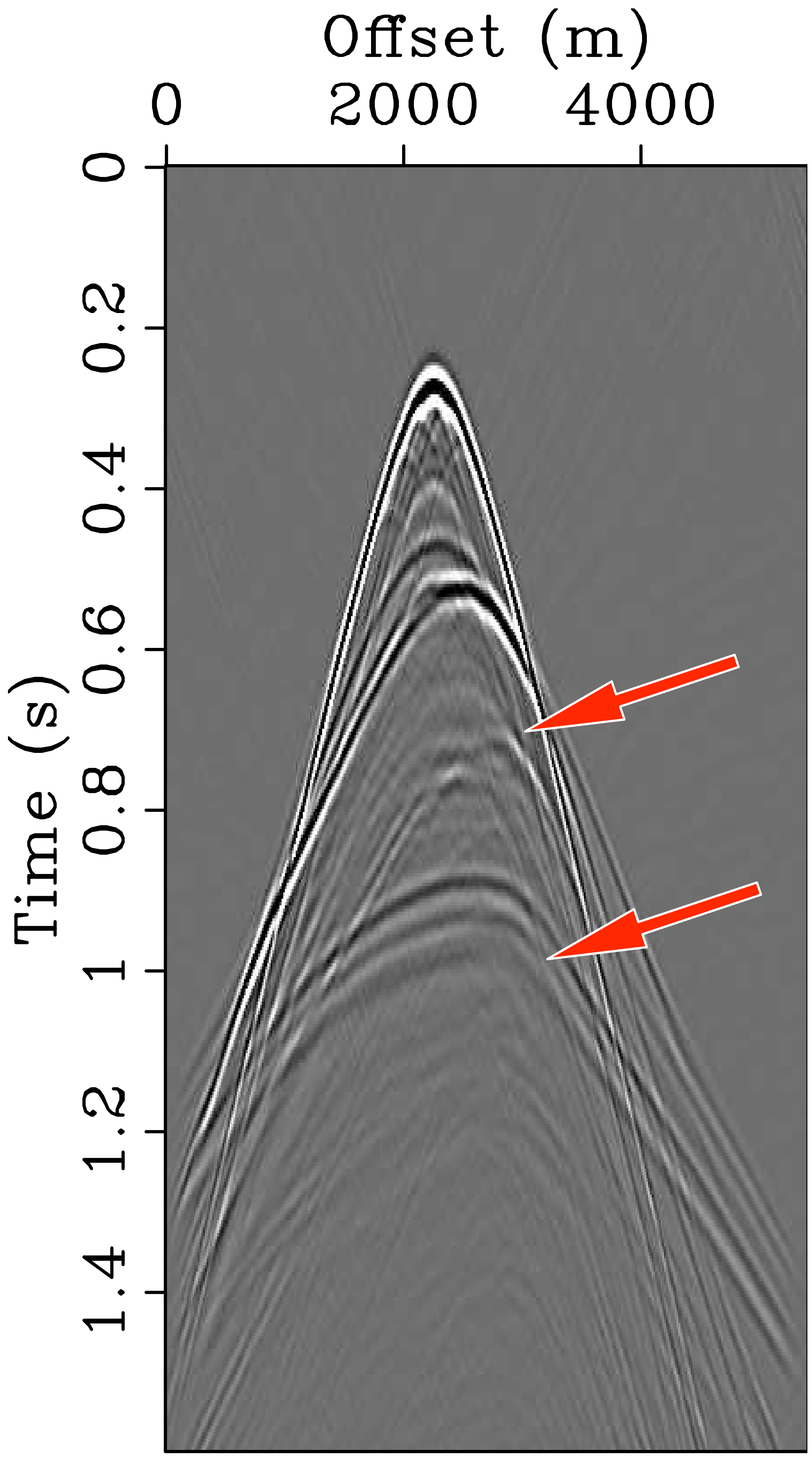
Synthetic example



SRME predicted primaries



estimated

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Curvelet-based recovery

joint work with Gilles Hennenfent

Sparsity-promoting inversion*

Reformulation of the problem

$$\overbrace{\mathbf{y}}^{\textit{signal}} = \mathbf{R}\mathbf{C}^H \underbrace{\mathbf{x}_0}_{\textit{curvelet representation of ideal data}} + \overbrace{\mathbf{n}}^{\textit{noise}}$$

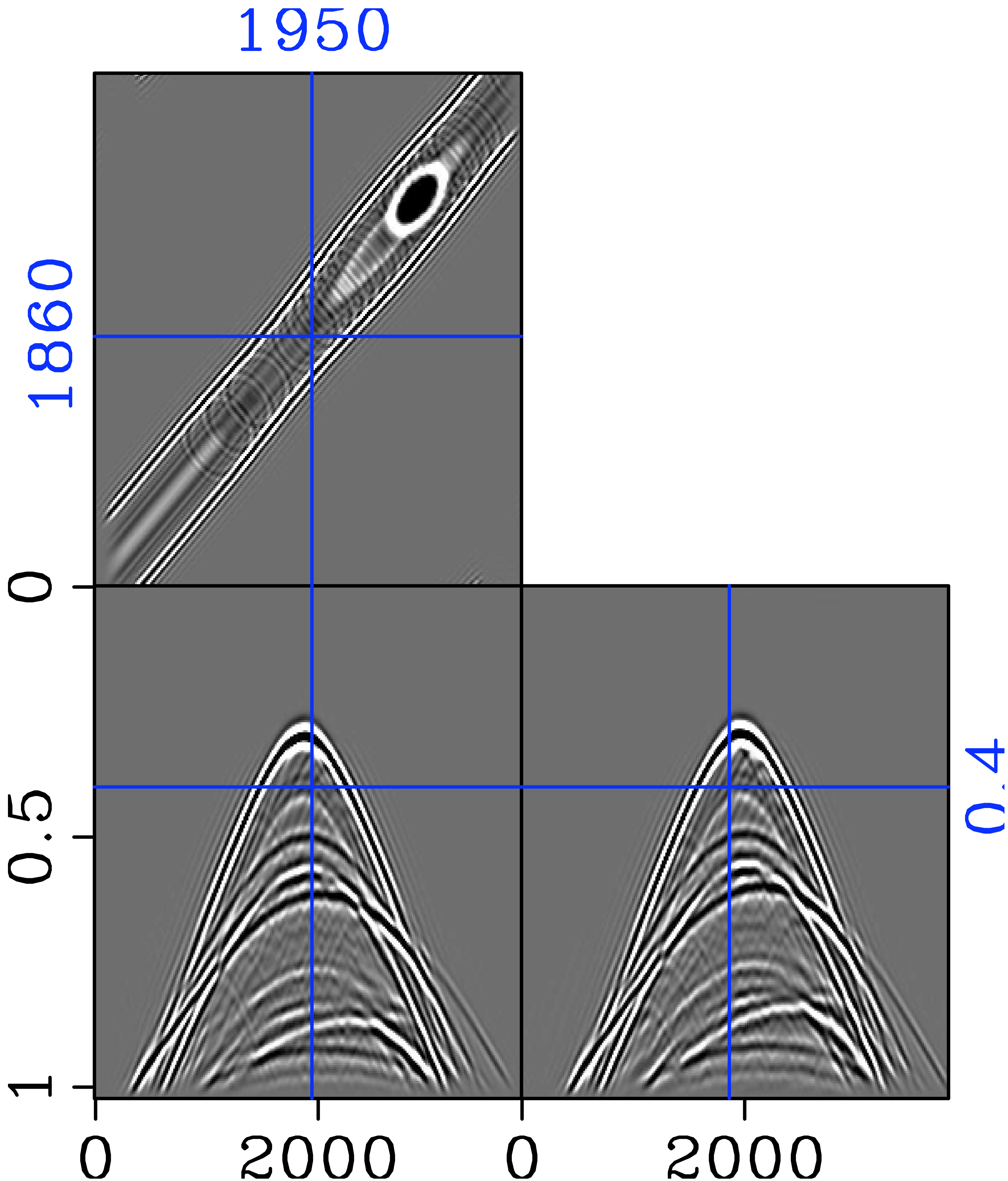
Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI)

- look for the **sparsest/most compressible**, **physical** solution

KEY POINT OF THE RECOVERY

$$\mathbf{P}_\epsilon : \begin{cases} \tilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \overbrace{\|\mathbf{W}\mathbf{x}\|_1}^{\text{sparsity constraint}} \quad \text{s.t.} \quad \overbrace{\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2}^{\text{data misfit}} \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{C}^T \tilde{\mathbf{x}} \end{cases}$$

* inspired by Stable Signal Recovery (SSR) theory by E. Candès, J. Romberg, T. Tao, Compressed sensing by D. Donoho & Fourier Reconstruction with Sparse Inversion (FRSI) by P. Zwartjes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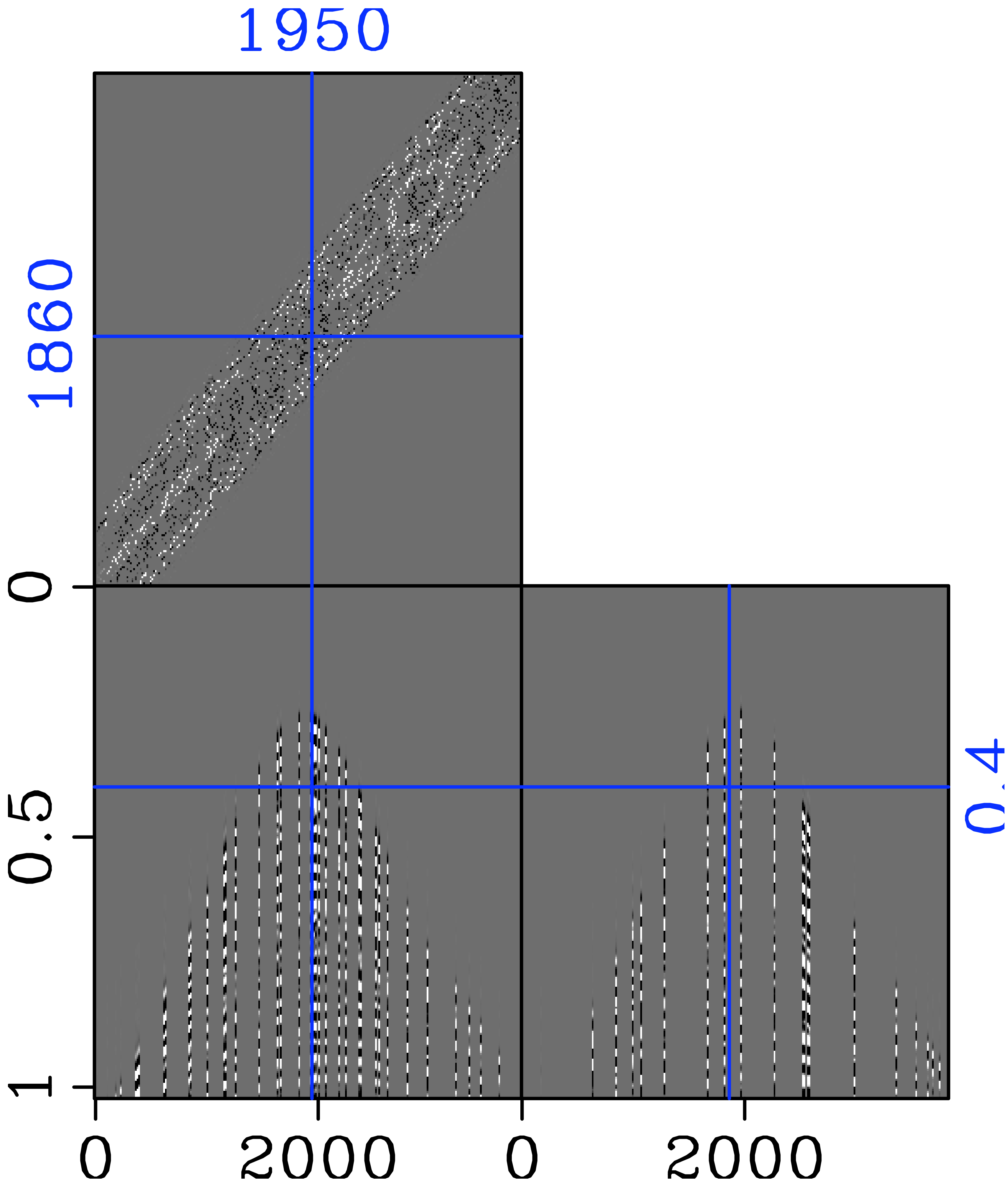

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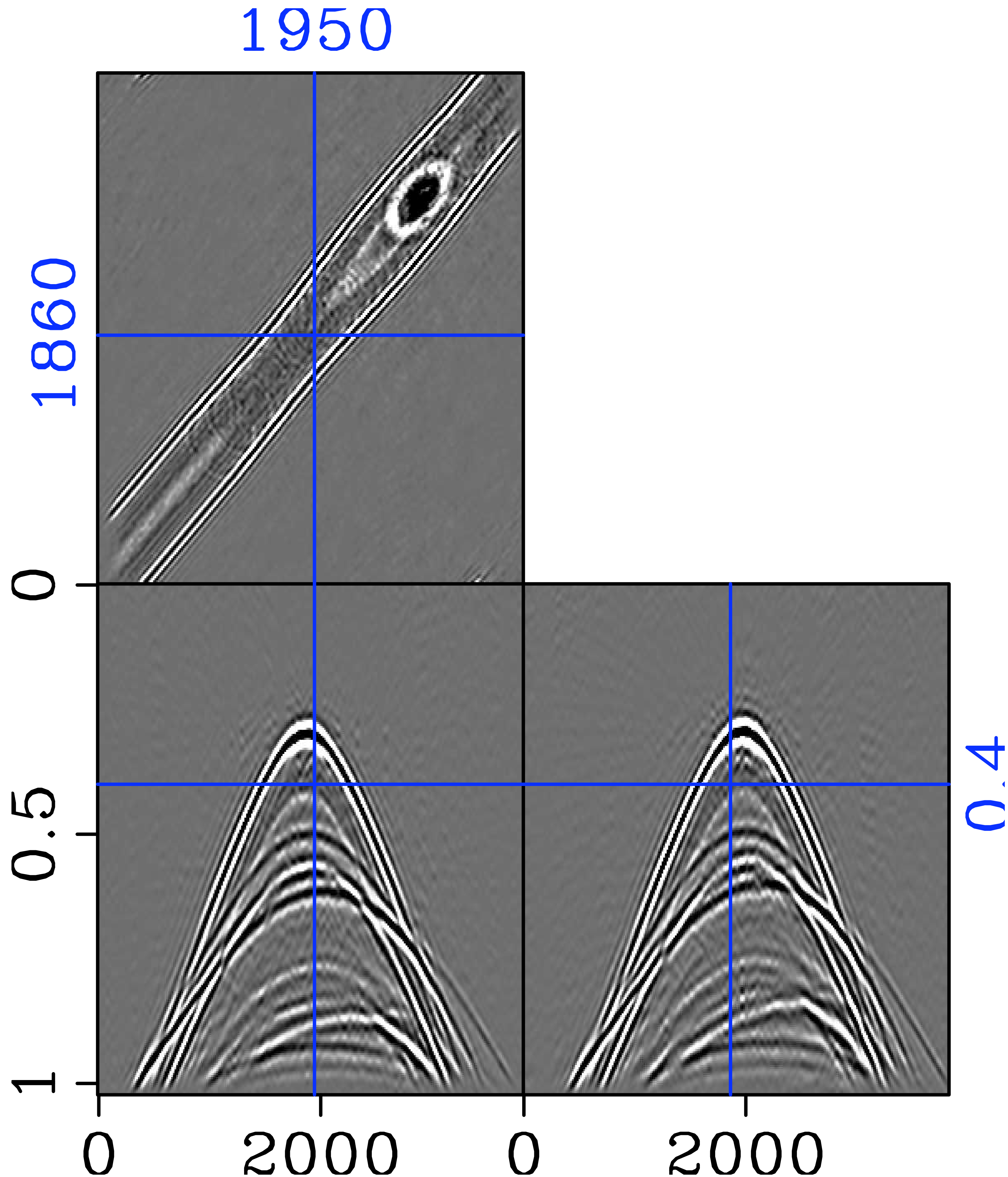

1950
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0.4
0
2000
0
2000

Focused recovery with curvelets

joint work with Deli Wang (visitor from Jilin university) and Gilles Hennenfent

Motivation

Can the recovery be extended to “migration-like” operators?

How can we incorporate ***prior*** information on the wavefield, e.g. information on major primaries from SRME?

How can we compress extrapolation operator?

Compound primary operator with inverse curvelet transform.

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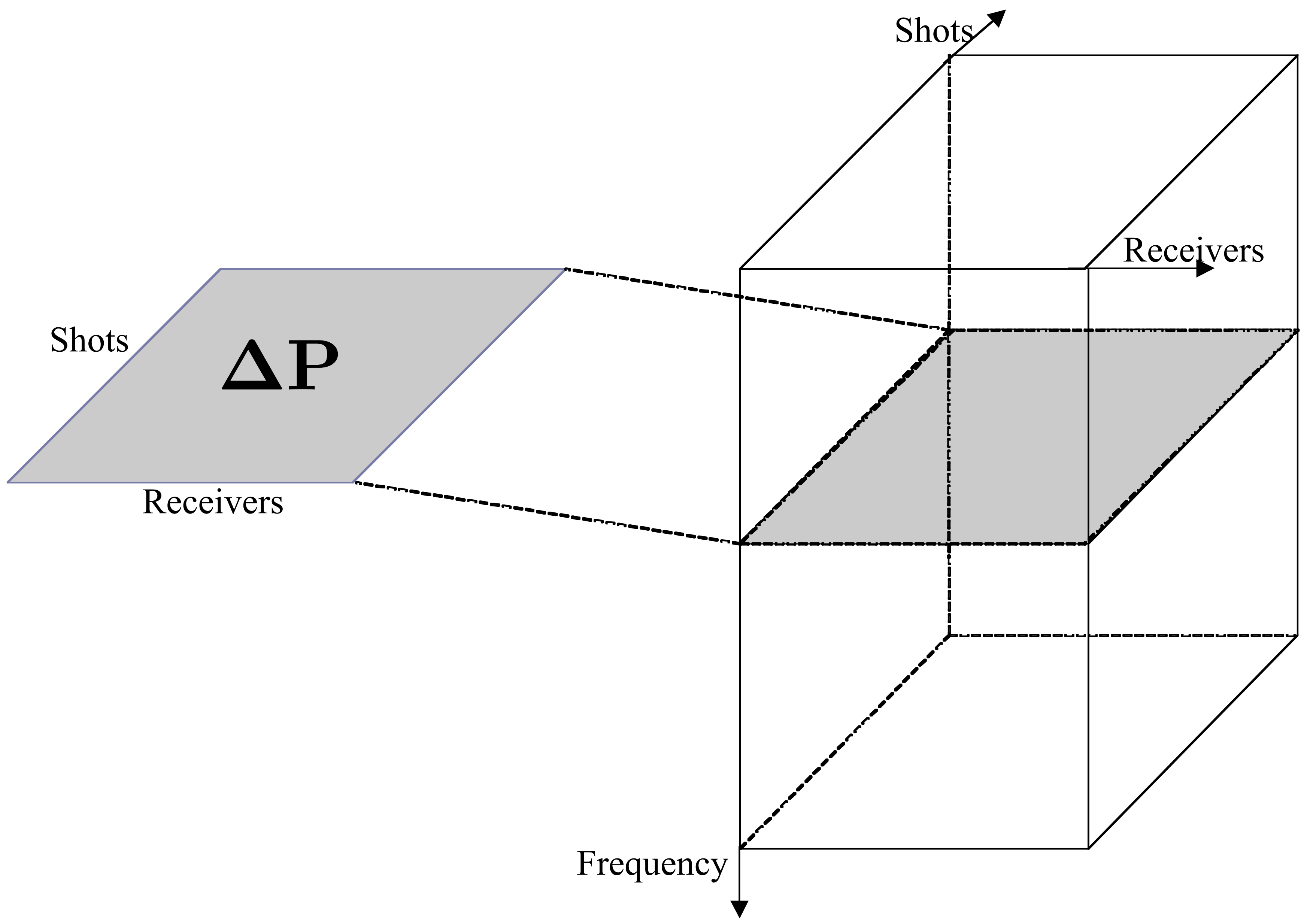
Primary operator

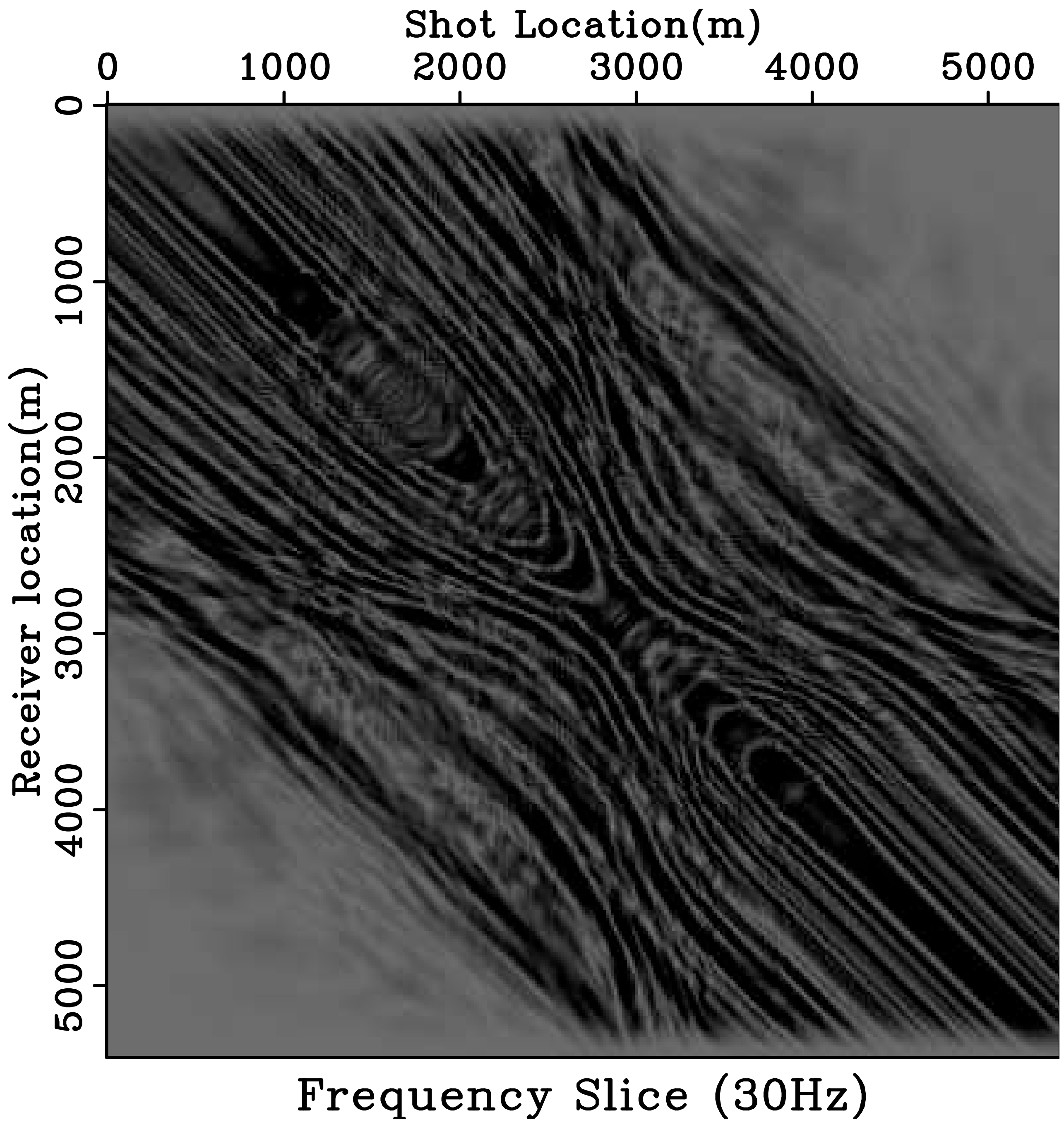
[Berkhout & Verschuur '96]



Frequency slice from data cube

Primary operator

[Berkhout & Verschuur '96]



Maps primaries into first-order multiples. So its inverse focuses ....

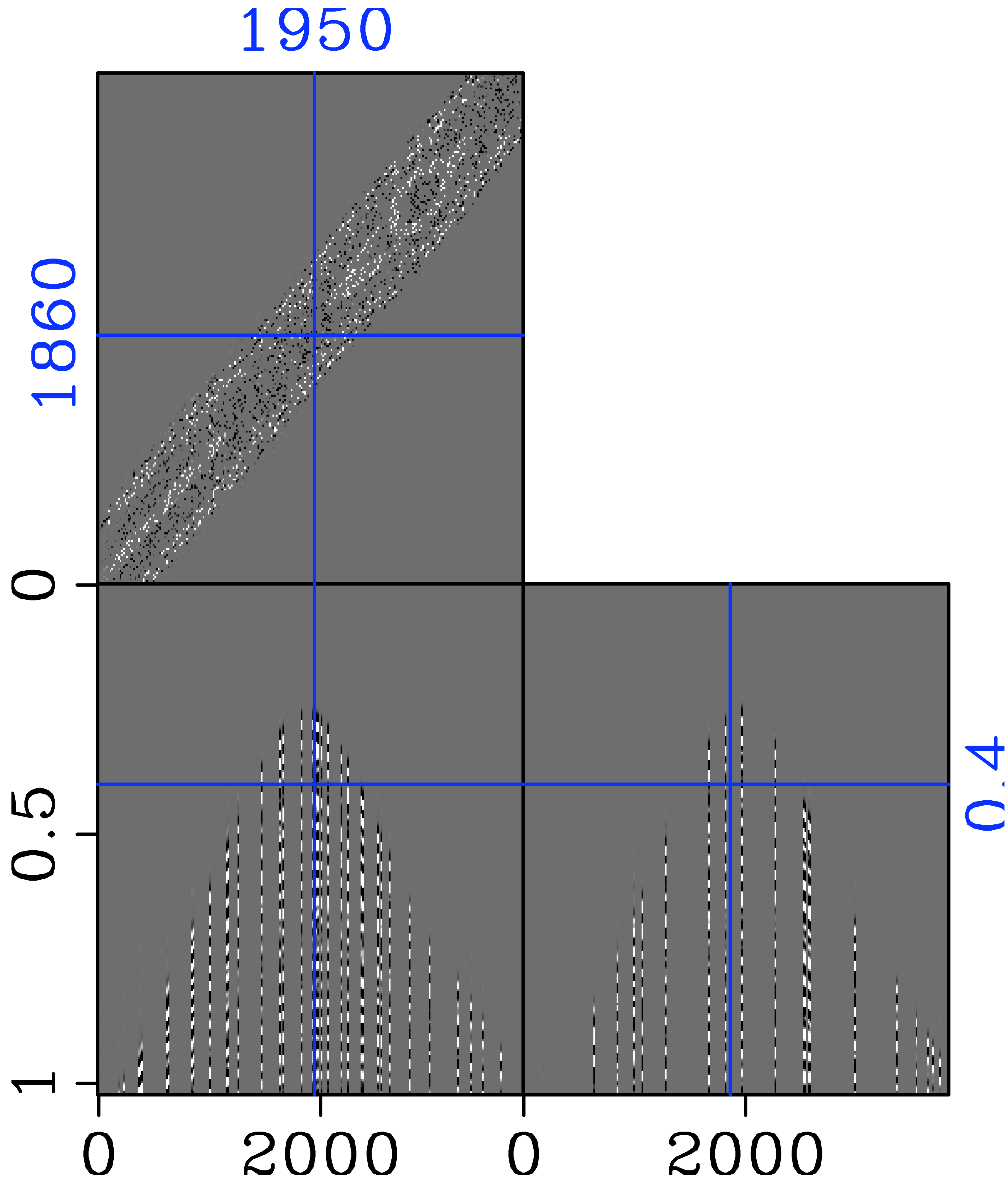
Recovery with focussing

Solve

$$\mathbf{P}_\epsilon : \qquad \begin{cases} \widetilde{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \widetilde{\mathbf{f}} = \mathbf{S}^T \widetilde{\mathbf{x}} \end{cases}$$

with

$$\begin{aligned} \mathbf{A} &:= \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \text{and } \boldsymbol{\Delta}\mathbf{P} := \mathbf{F}^H \text{block diag}\{\Delta p\}\mathbf{F} \\ \mathbf{S}^T &:= \boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T \\ \mathbf{y} &= \mathbf{R}\mathbf{P}(:) \\ \mathbf{R} &= \text{picking operator.} \end{aligned}$$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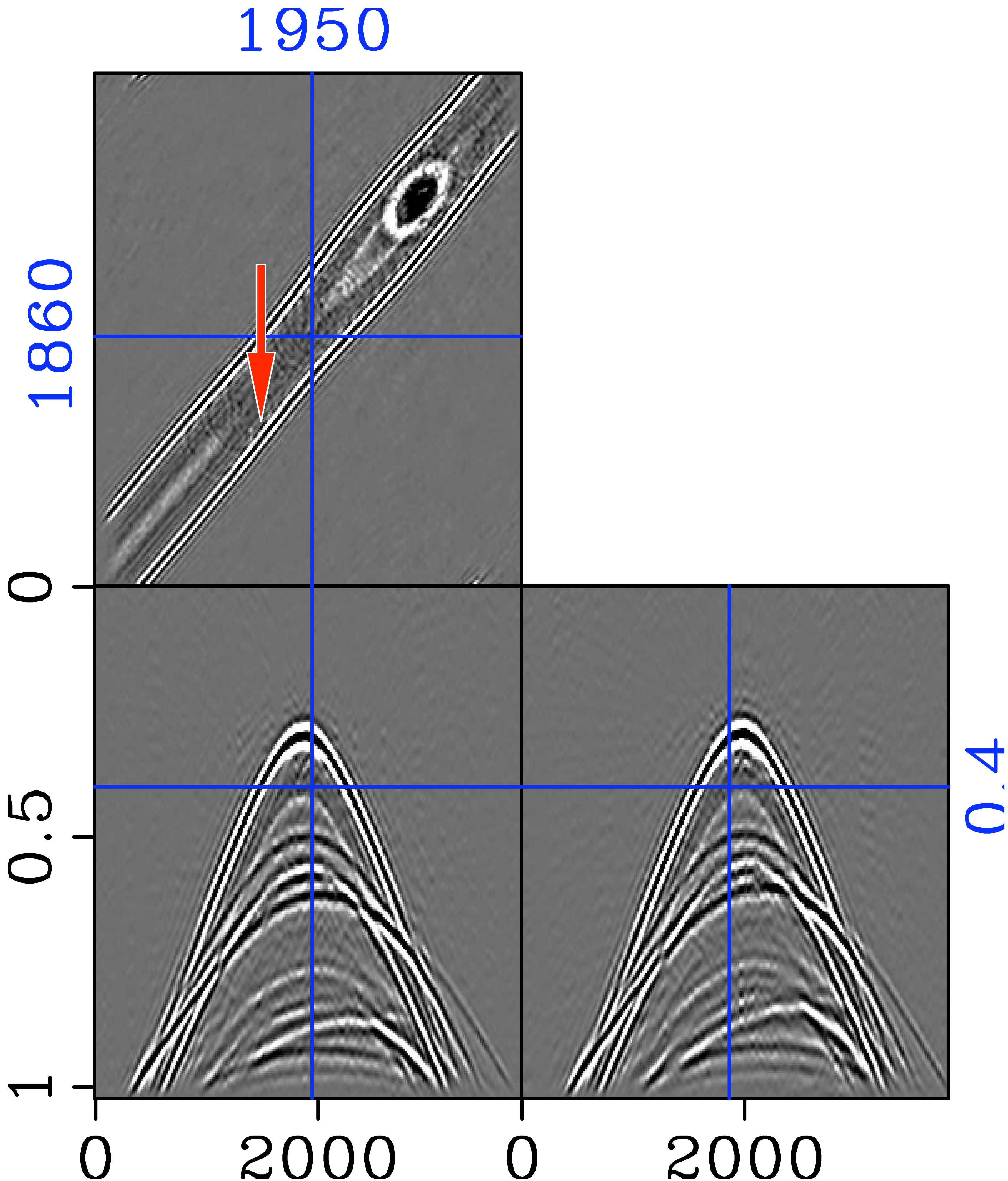

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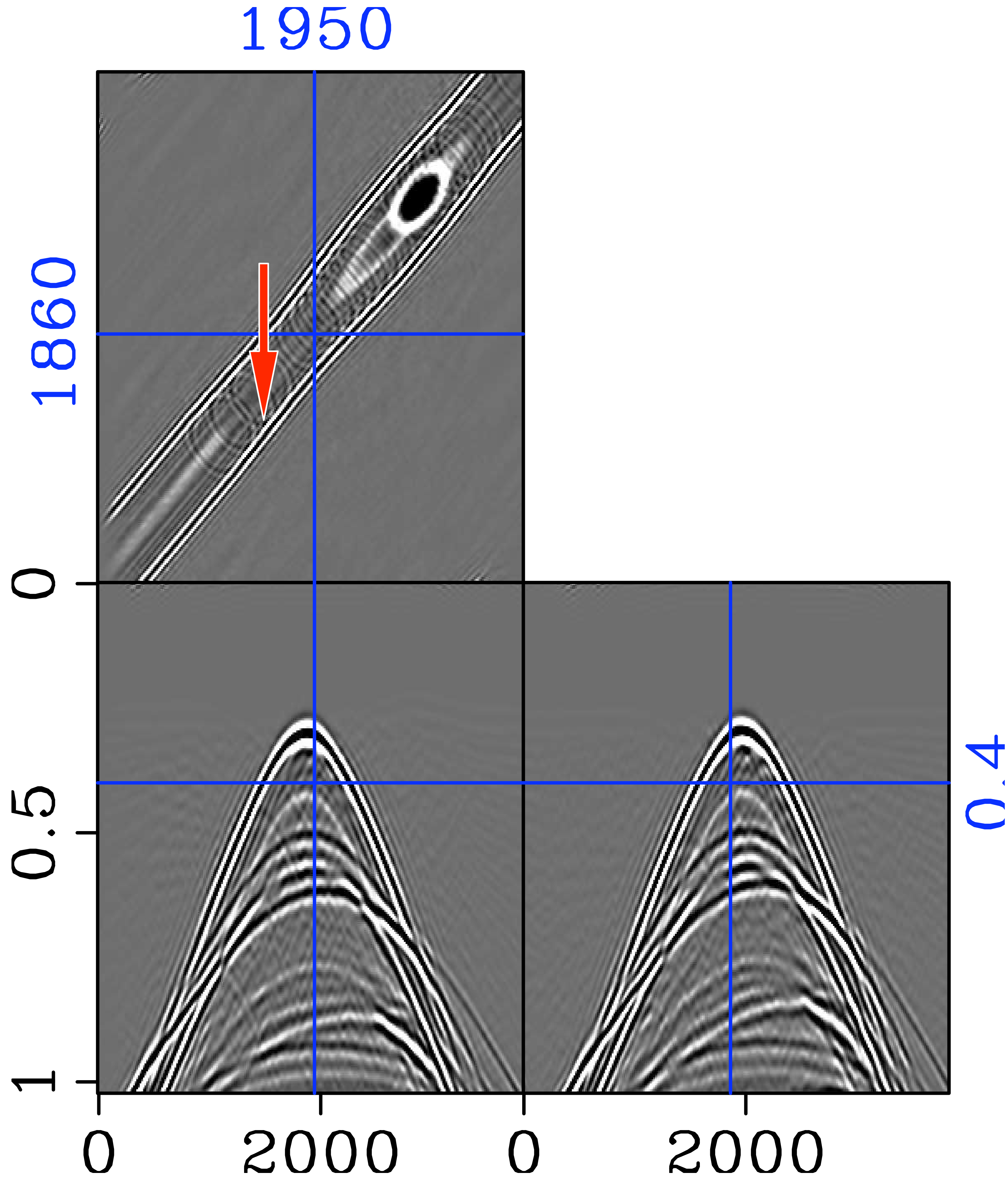

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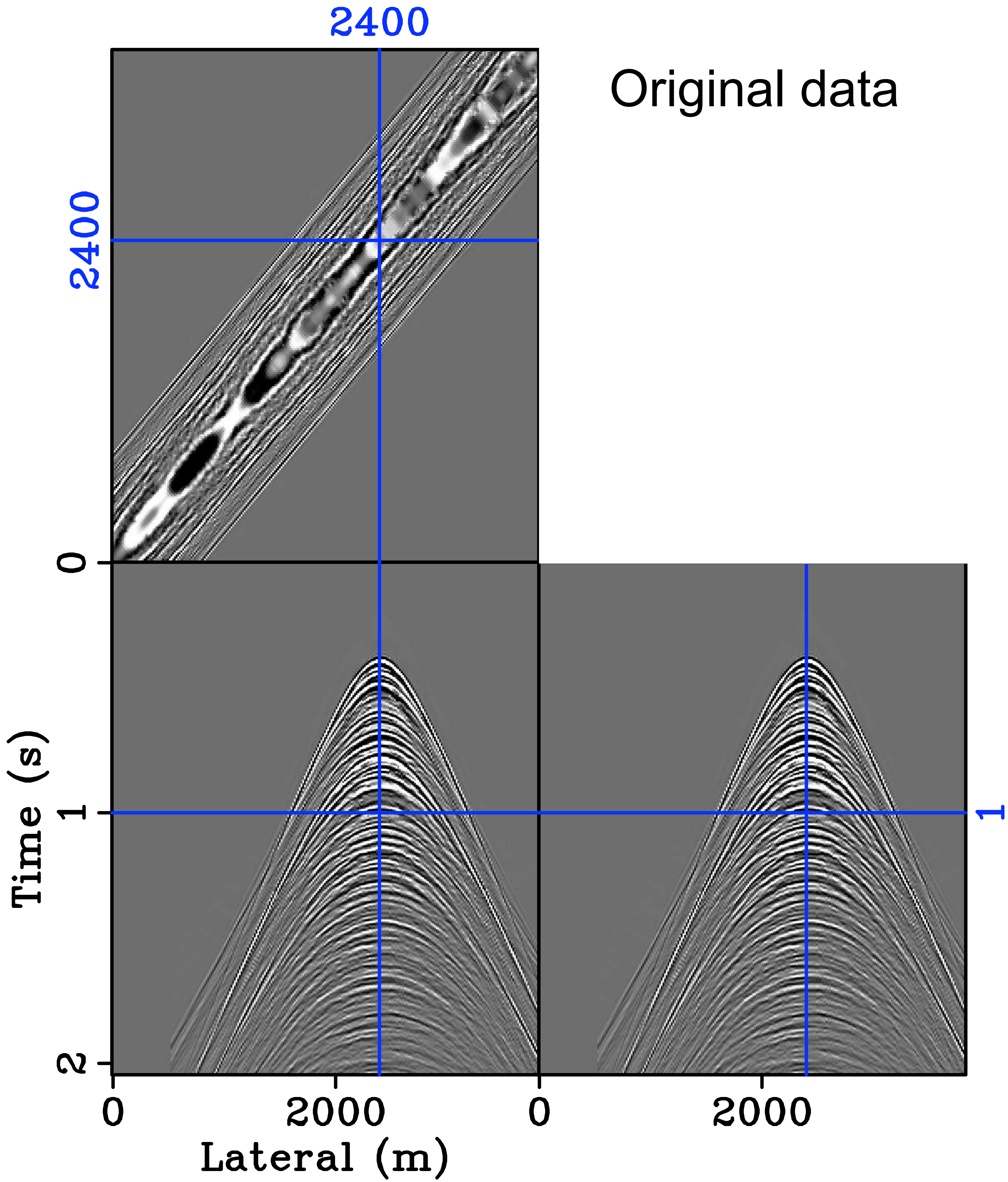
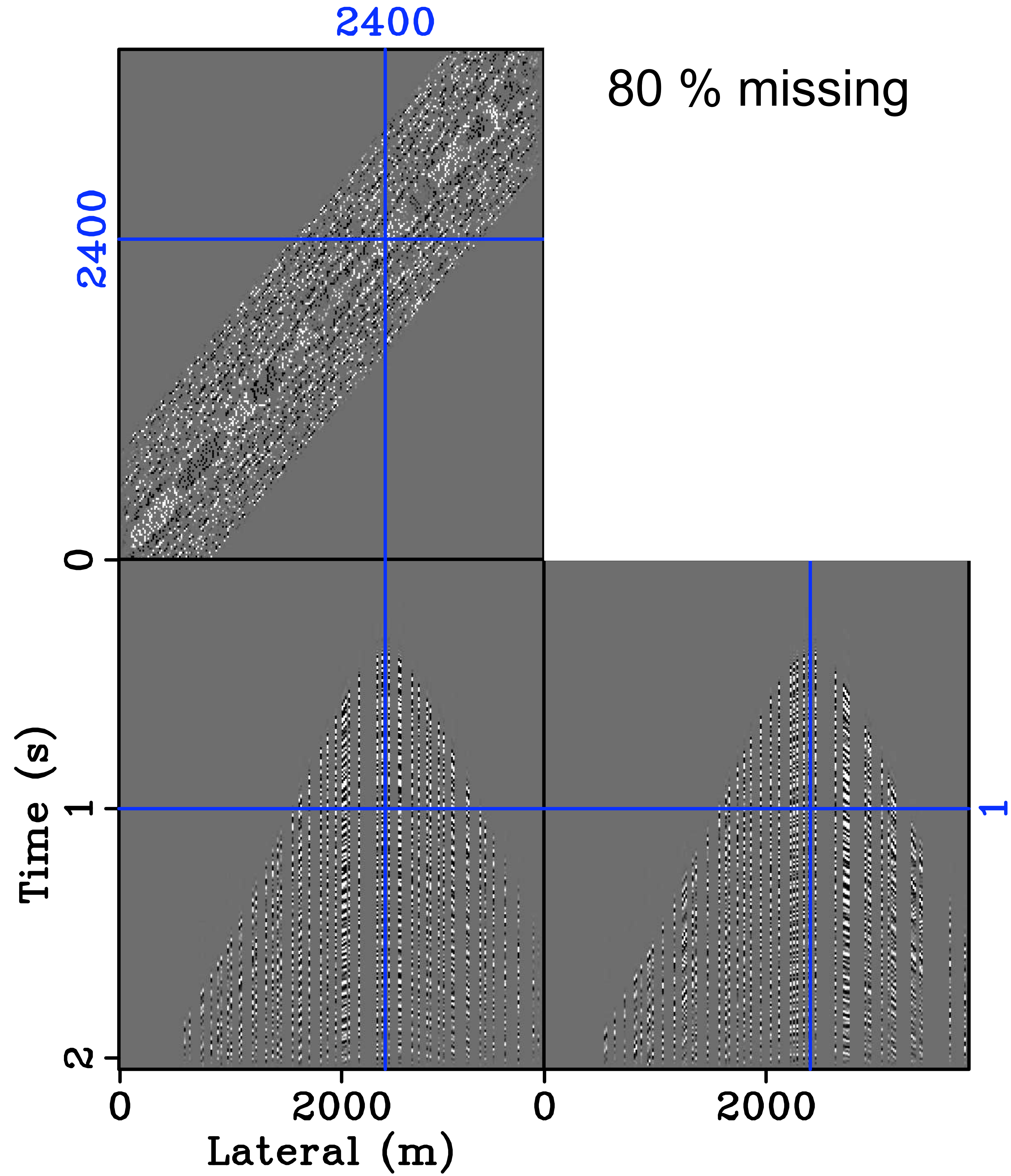
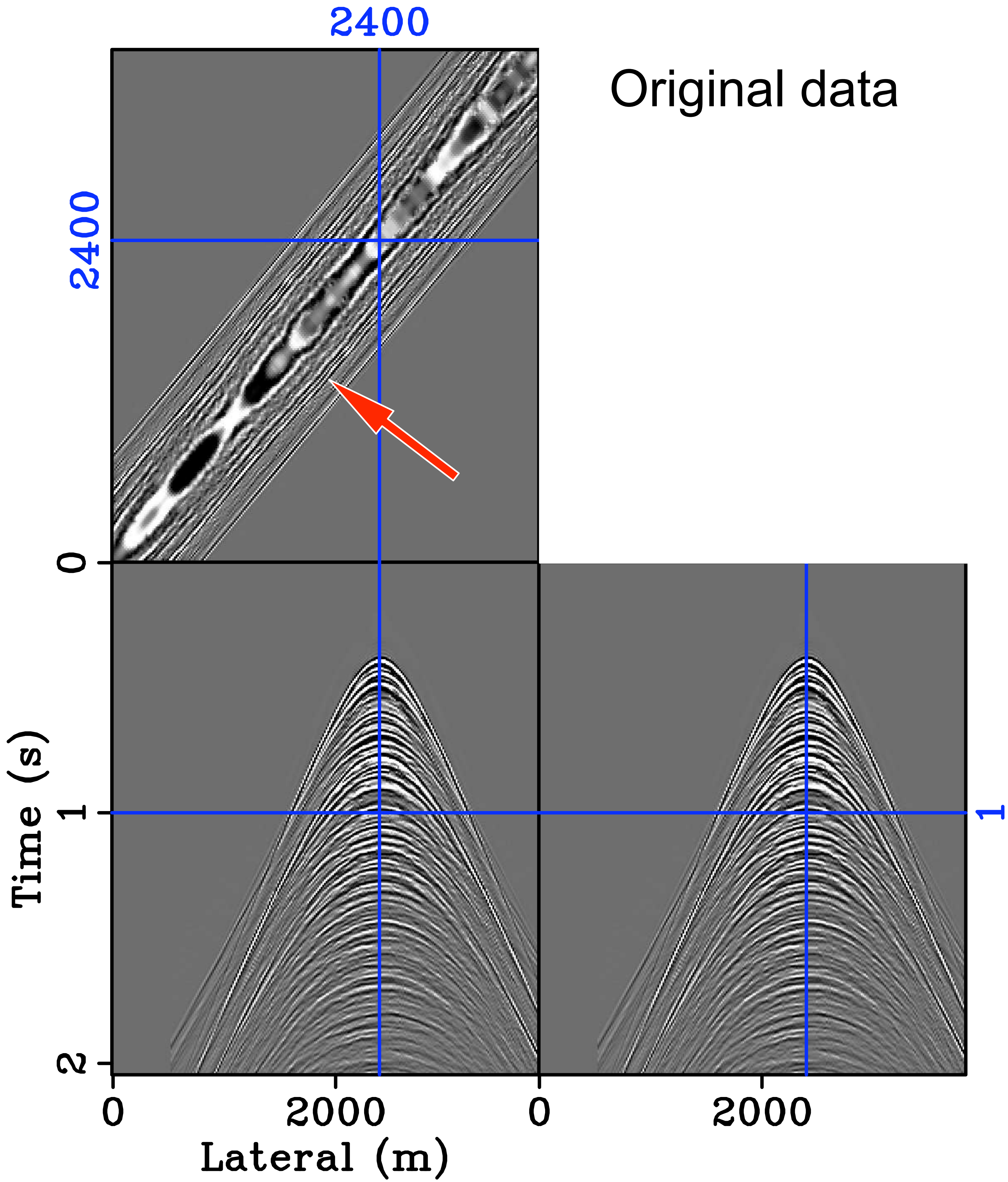
Original data





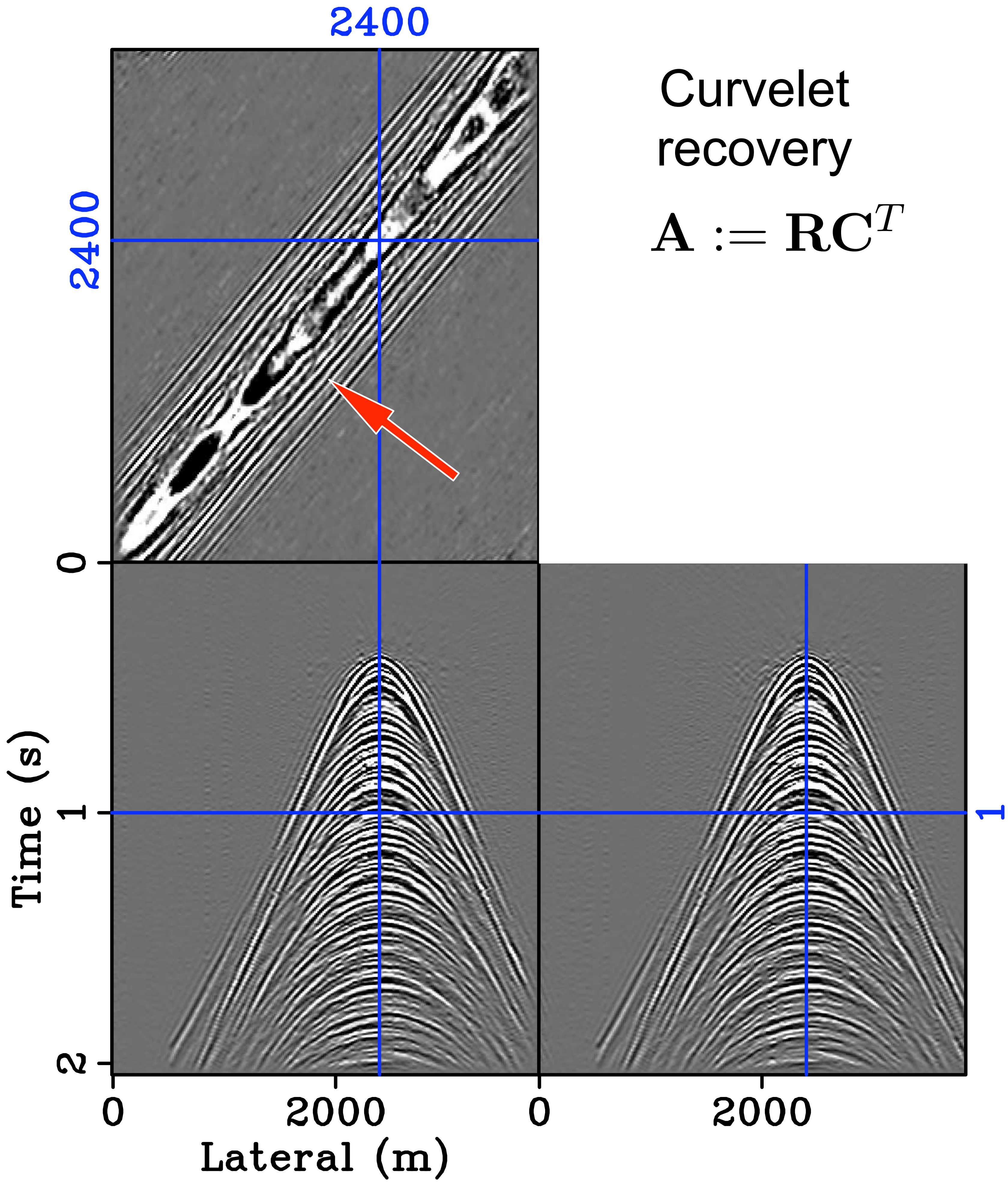
80 % missing

Original data



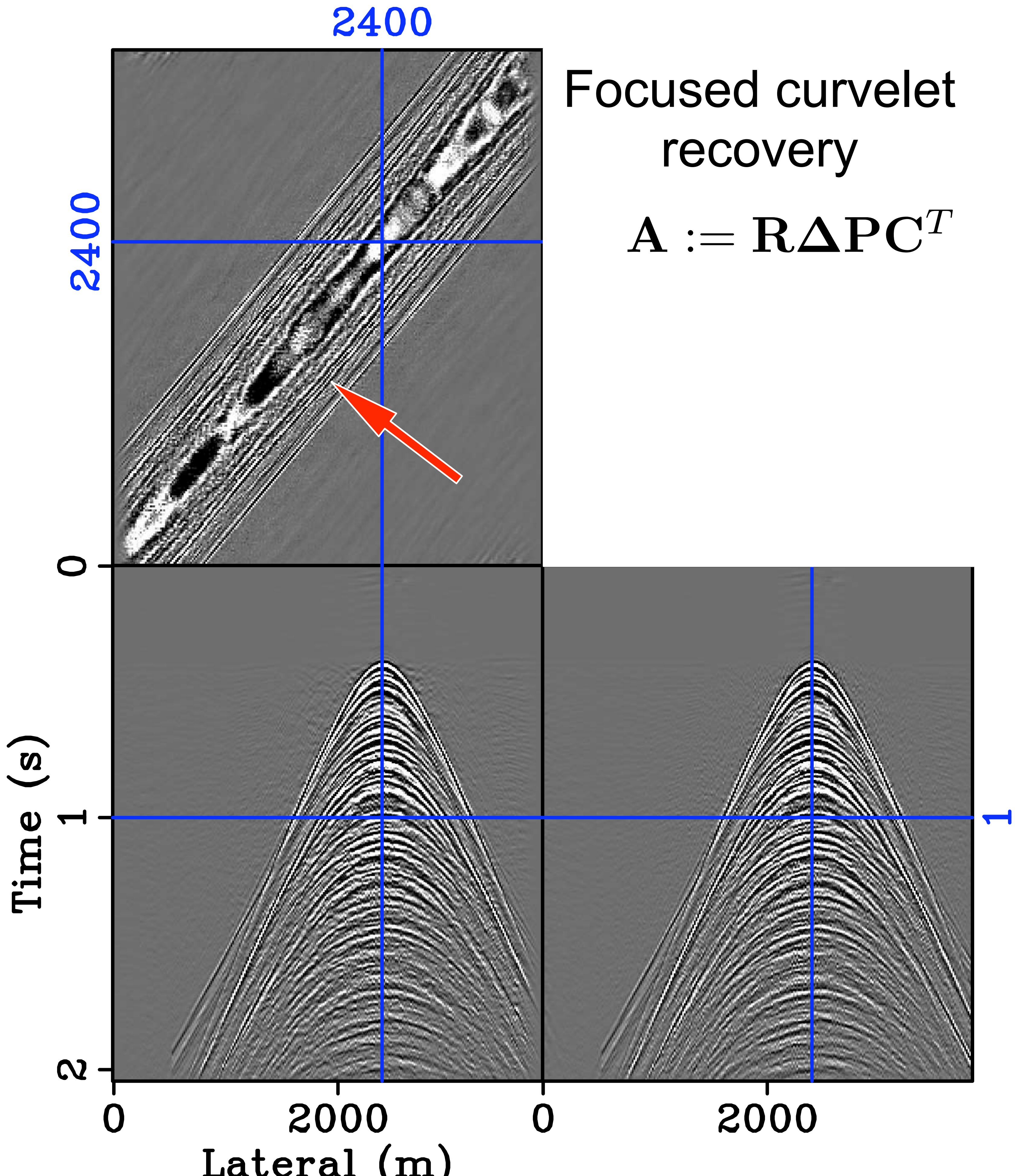
Curvelet recovery

$$\mathbf{A} := \mathbf{R}\mathbf{C}^T$$



Focused curvelet recovery

$$\mathbf{A} := \mathbf{R}\boldsymbol{\Delta}\mathbf{P}\mathbf{C}^T$$



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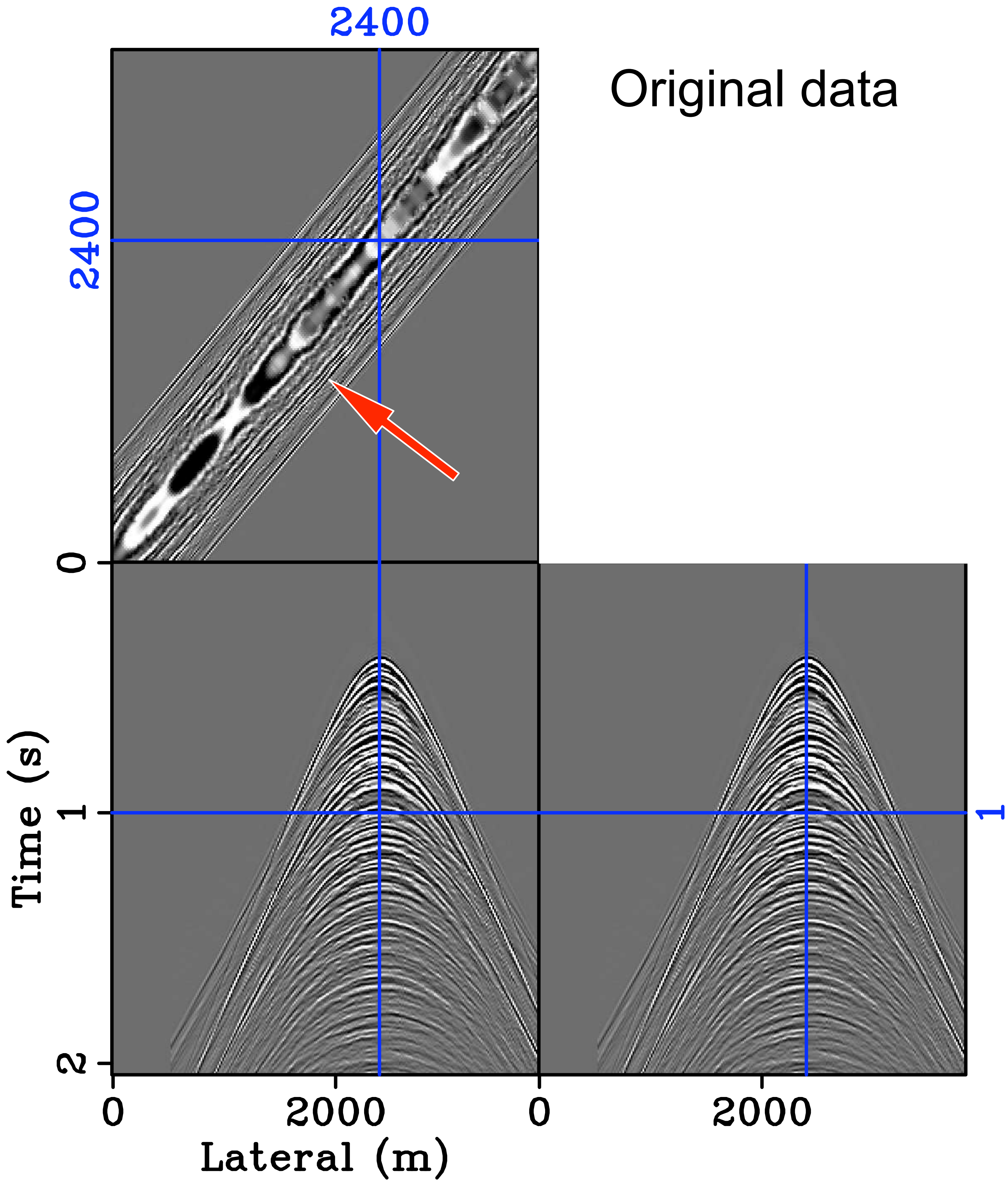
Original data



Conclusions

Curvelets represent a versatile transform that

- brings robustness w.r.t. moderate shifts and phase rotations to primary multiple separation
- allows for the nonlinear recovery for severely sub-Nyquist data
- leads to an improved recovery when compounded with "migration like" operators

Opens tentative perspectives towards a new sampling theory

- for seismic data
- that includes migration operators ...

Acknowledgments

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Eric Verschuur for providing us with the synthetic and real data examples.

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