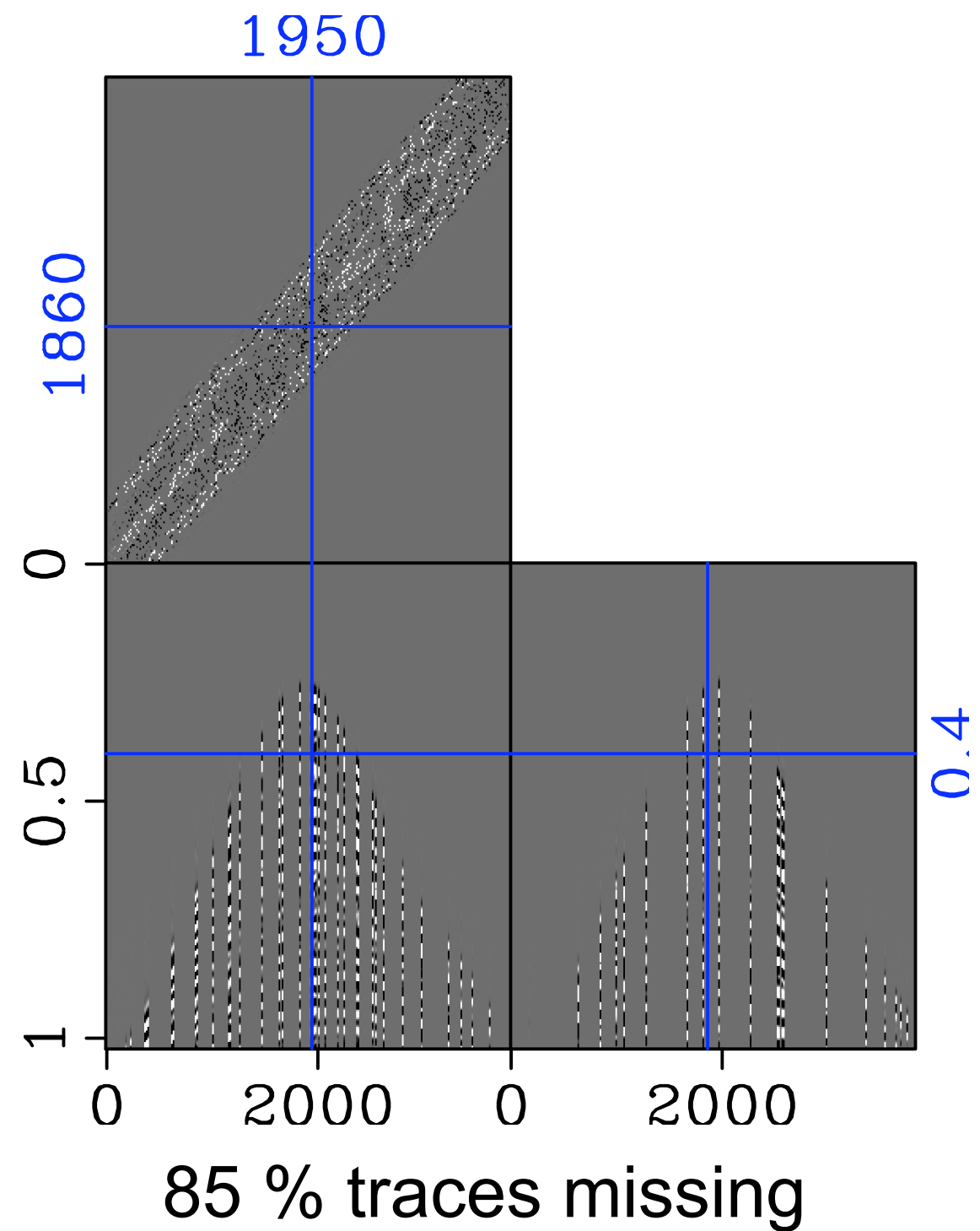
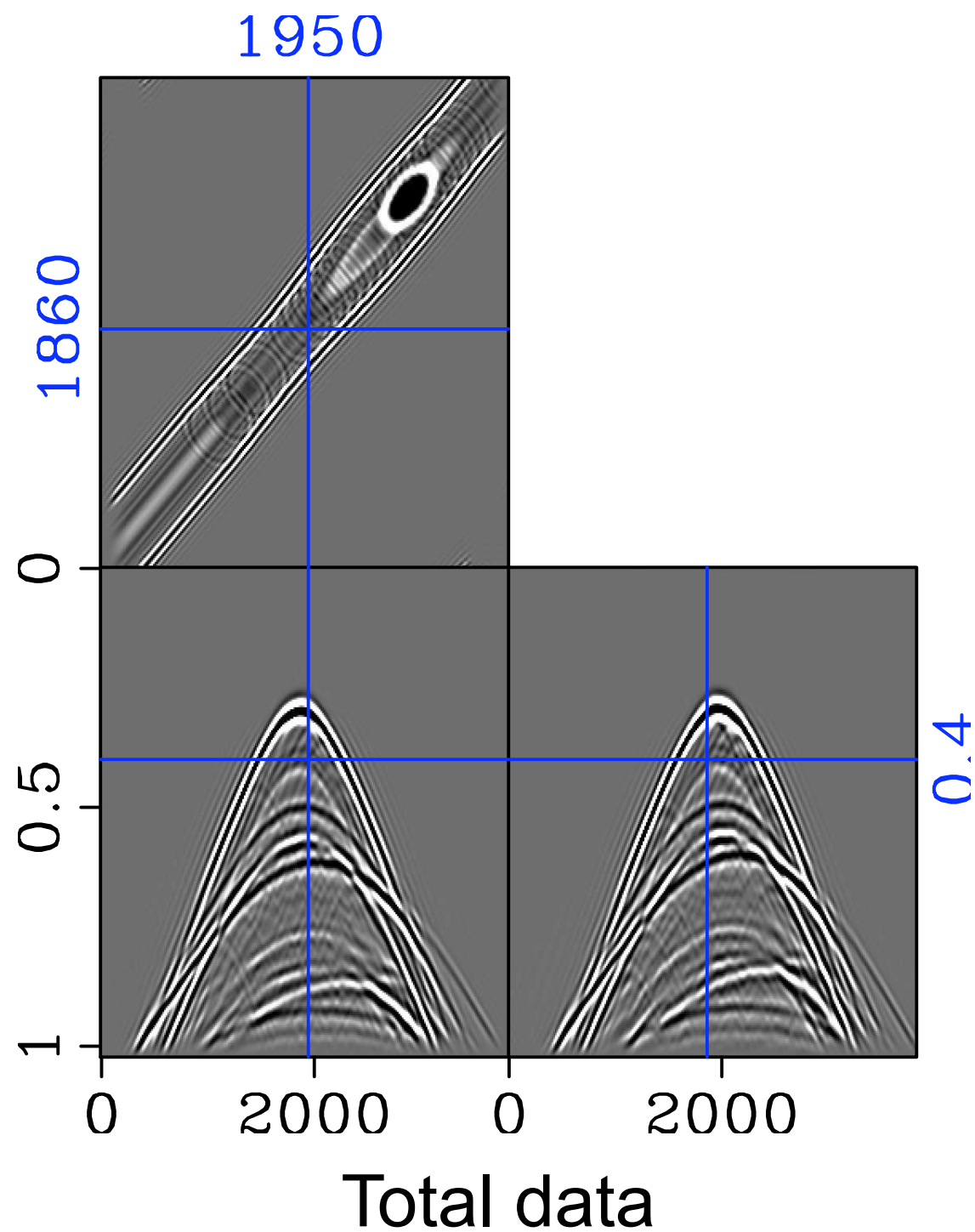


# Surface-related multiple prediction from incomplete data

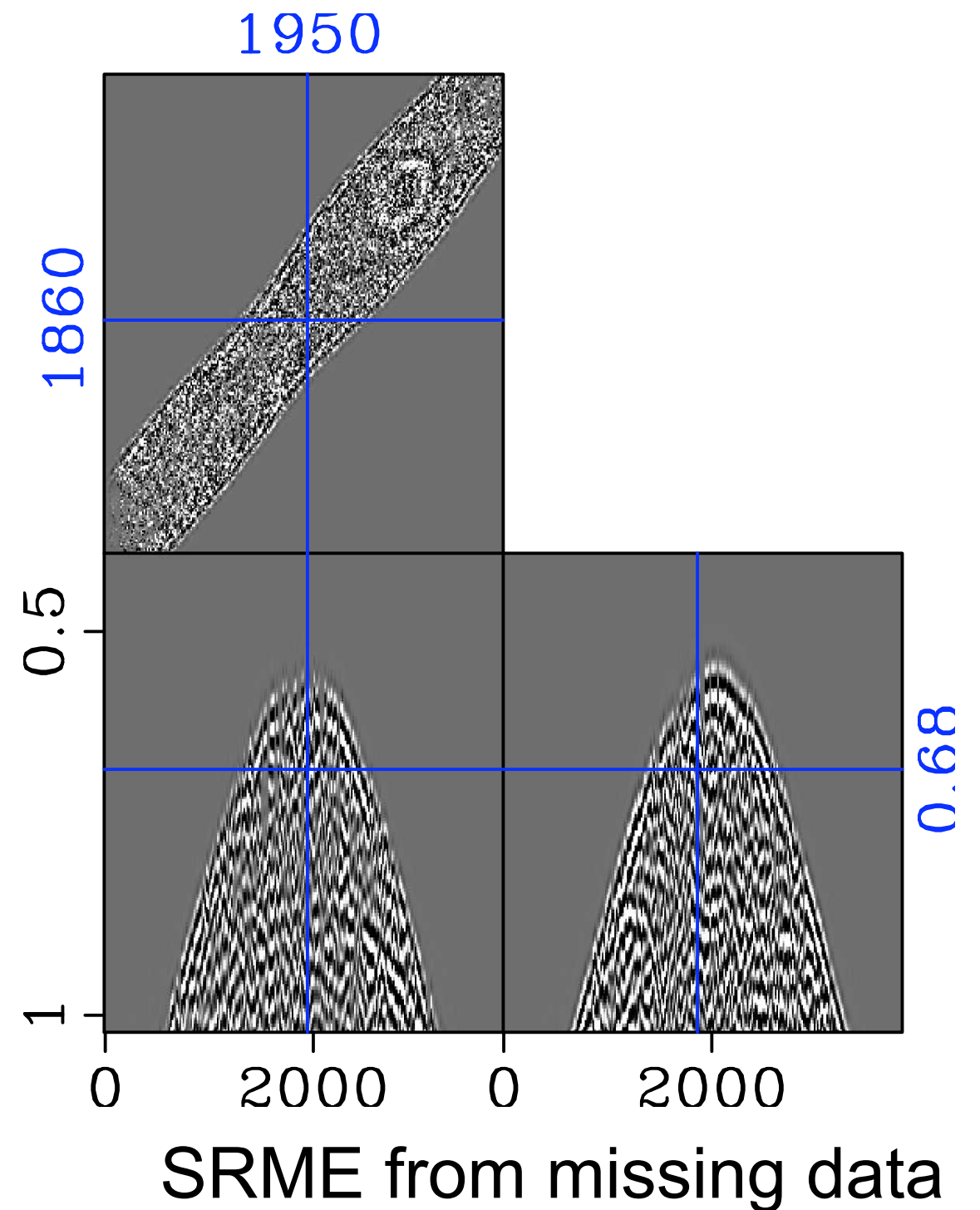
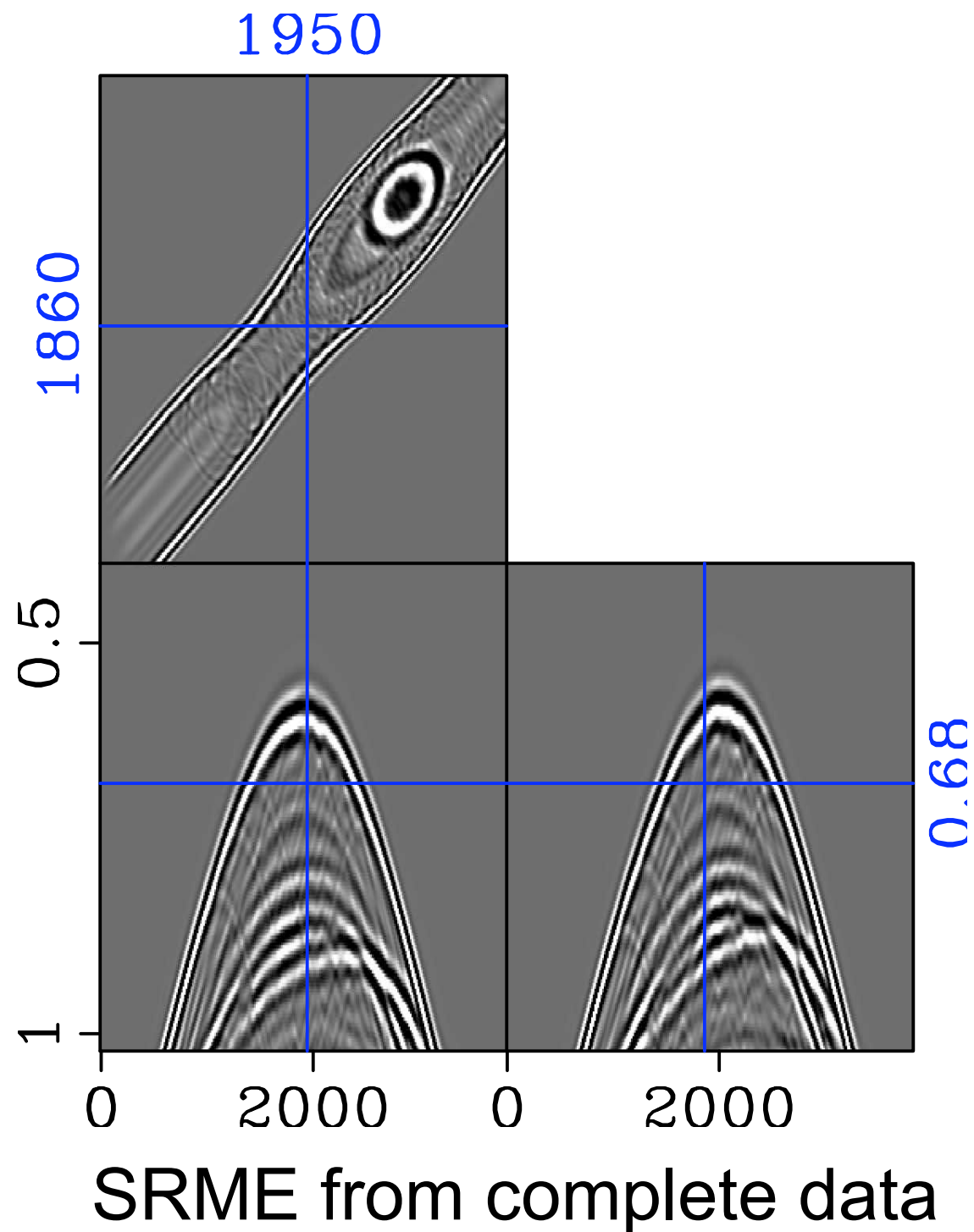
Felix J. Herrmann

joint work with Deli Wang and Gilles  
Hennenfent.

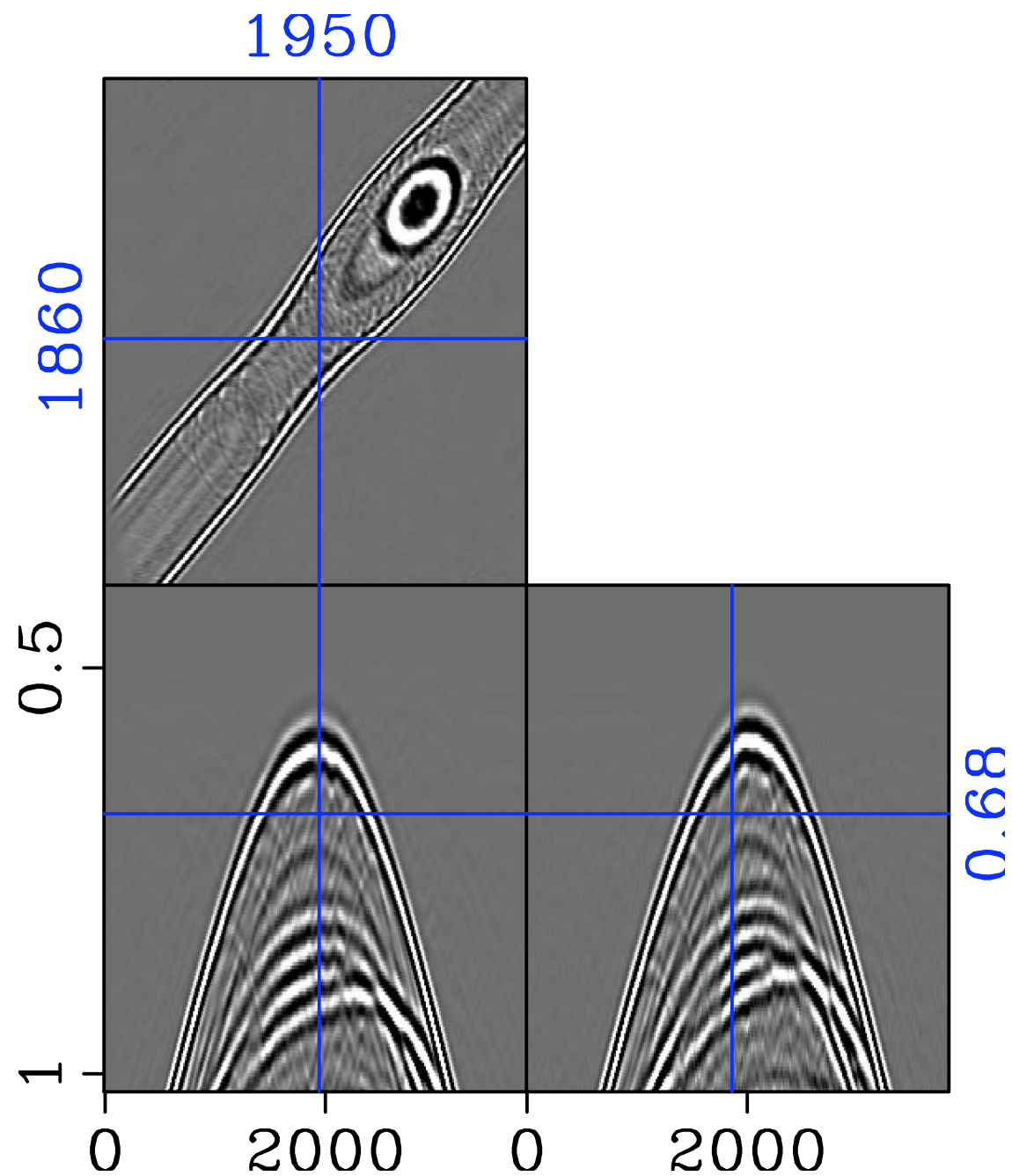
# The problem



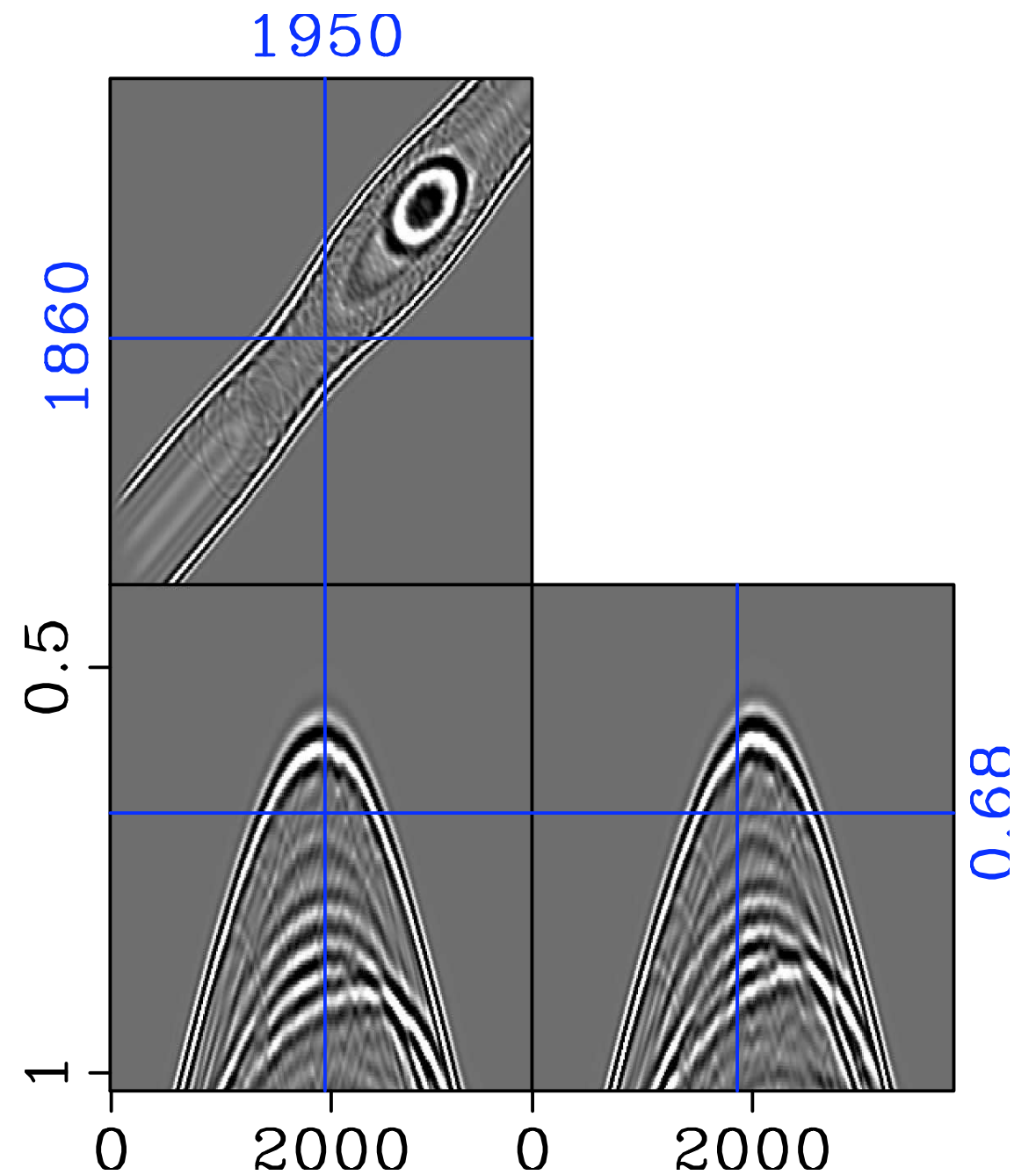
# The problem cont'd



# Our solution



SRME from recovered data



SRME from original data

# Motivation

Data-driven (SRME) multiple prediction requires fully sampled data.

The Focal transform (Berkhout & Verschuur '06) allows for

- mapping of multiples => primaries
- incorporation of *prior* information in the recovery

Present a curvelet-based scheme for sparsity-promoting

- recovery of the data
- prediction of primaries and surface-related multiples

# The curvelet transform

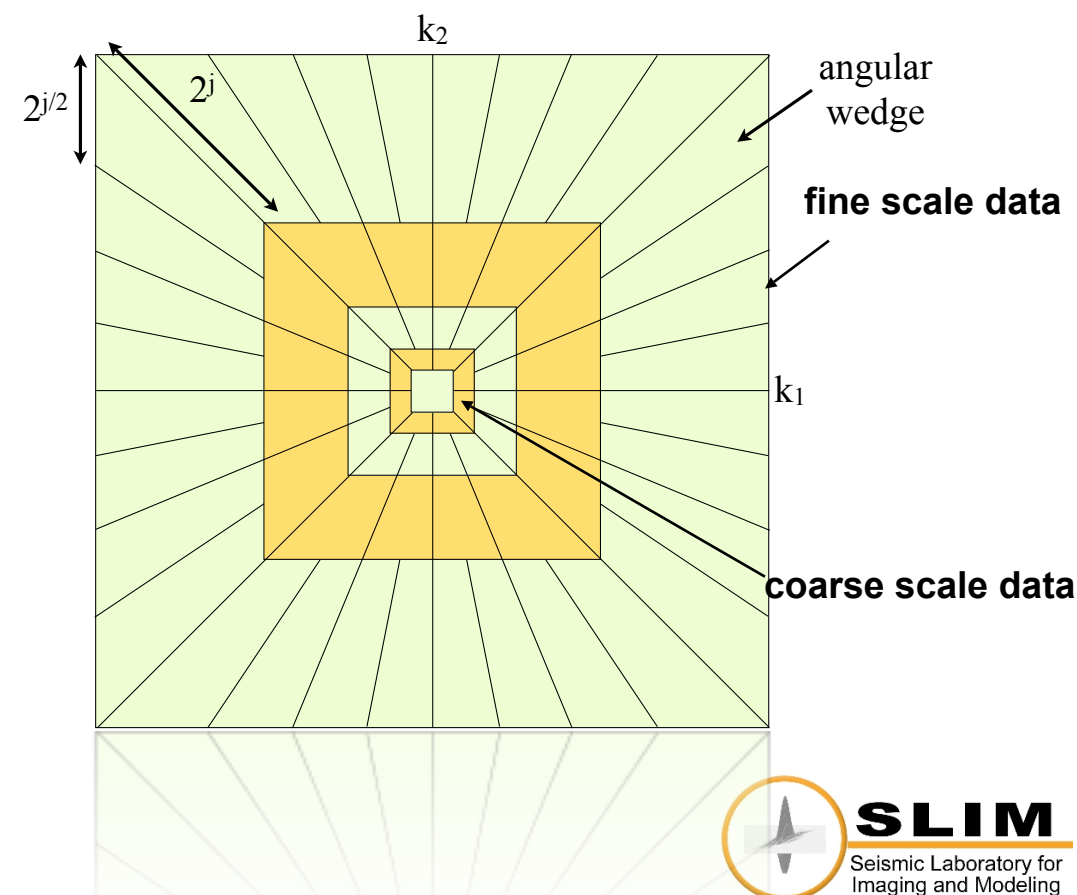


# Representations for seismic data

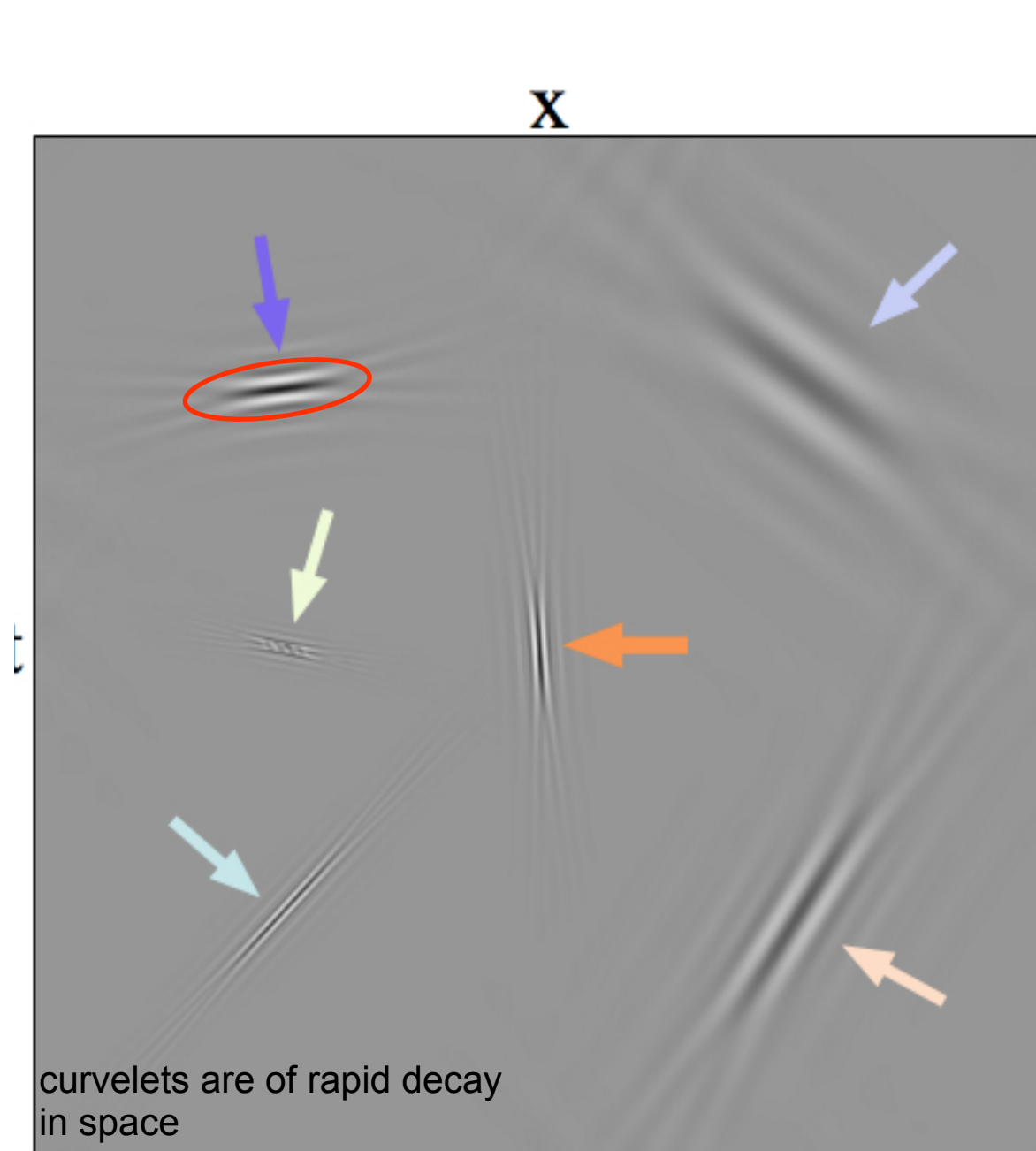
| Transform                        | Underlying assumption                       |
|----------------------------------|---------------------------------------------|
| FK                               | plane waves                                 |
| linear/parabolic Radon transform | linear/parabolic events                     |
| wavelet transform                | point-like events (1D singularities)        |
| <b>curvelet transform</b>        | <b>curve-like events (2D singularities)</b> |

## Properties curvelet transform:

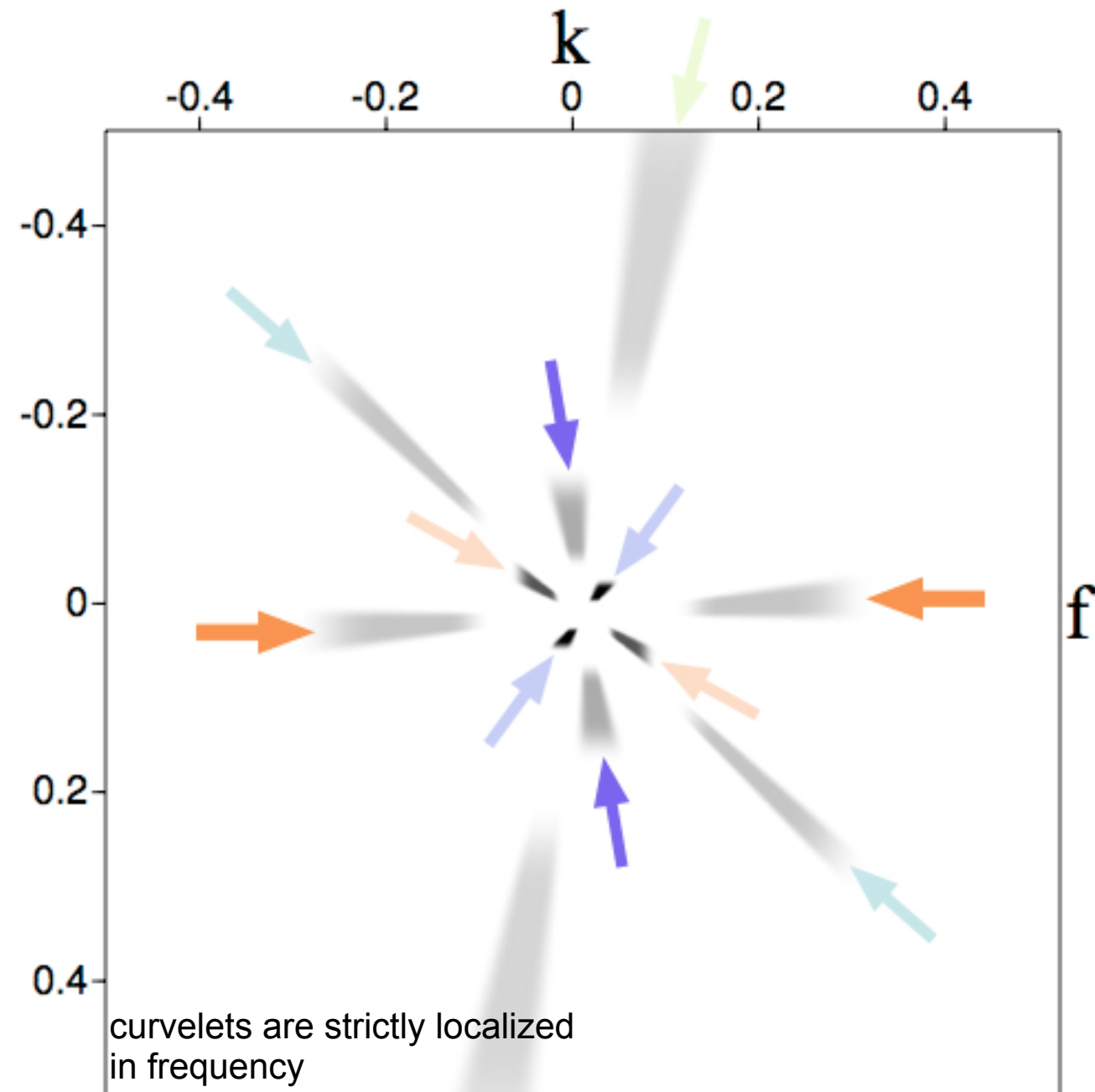
- **multiscale:** tiling of the FK domain into dyadic coronae
- **multi-directional:** coronae sub-partitioned into angular wedges, # of angle doubles every other scale
- **anisotropic:** parabolic scaling principle
- **Rapid decay space**
- **Strictly localized in Fourier**
- **Frame with moderate redundancy (8 X in 2-D and 24 X in 3-D)**



# 2-D curvelets



$x-t$



$f-k$

**Oscillatory in one direction and smooth in the others!**

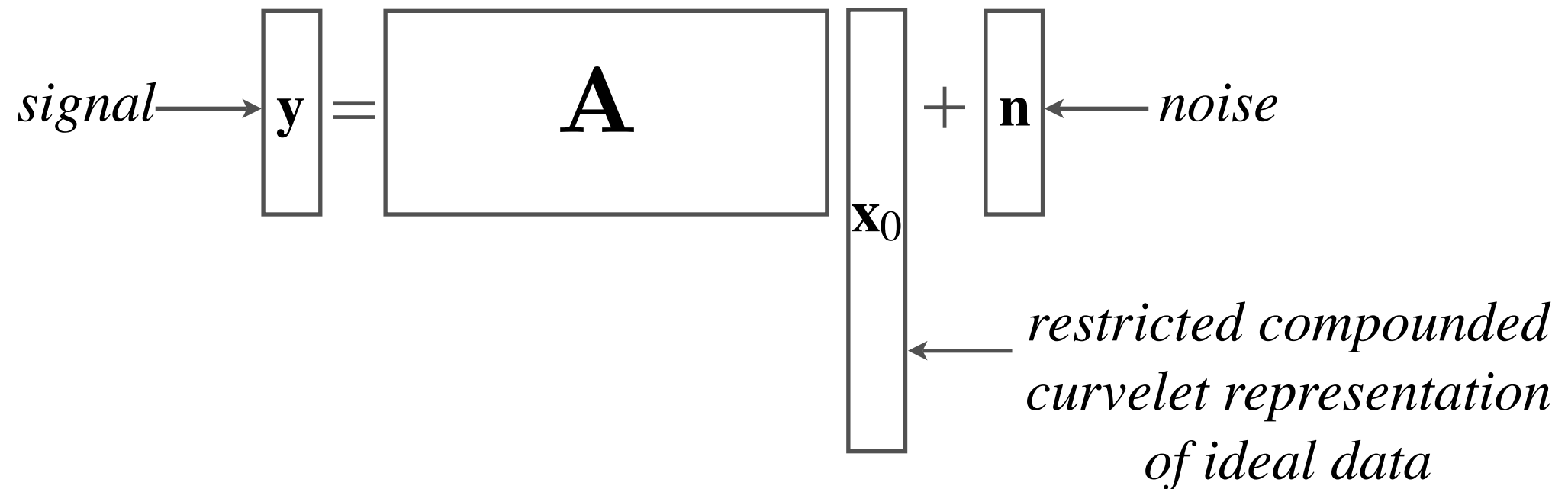
**Obey *parabolic* scaling relation  $\text{length} \approx \text{width}^2$**

# Curvelet sparsity promotion



# Sparsity-promoting program

Solve for  $\mathbf{x}_0$



with

$$\mathbf{P}_\epsilon : \begin{cases} \tilde{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{x}\|_1 & \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{m}} = \mathbf{C}^T \tilde{\mathbf{x}} \end{cases}$$

- exploit sparsity in the curvelet domain as a prior.
- find the sparsest set of curvelet coefficients that match the data.
- invert an underdetermined system.

# *Focused recovery with curvelets*

---

joint work with Deli Wang (visitor  
from Jilin university) and Gilles  
Hennenfent



# Focused recovery

***Non-data-adaptive*** Curvelet Reconstruction with Sparsity-promoting Inversion (CRSI) derives from ***sparsity*** of seismic data.

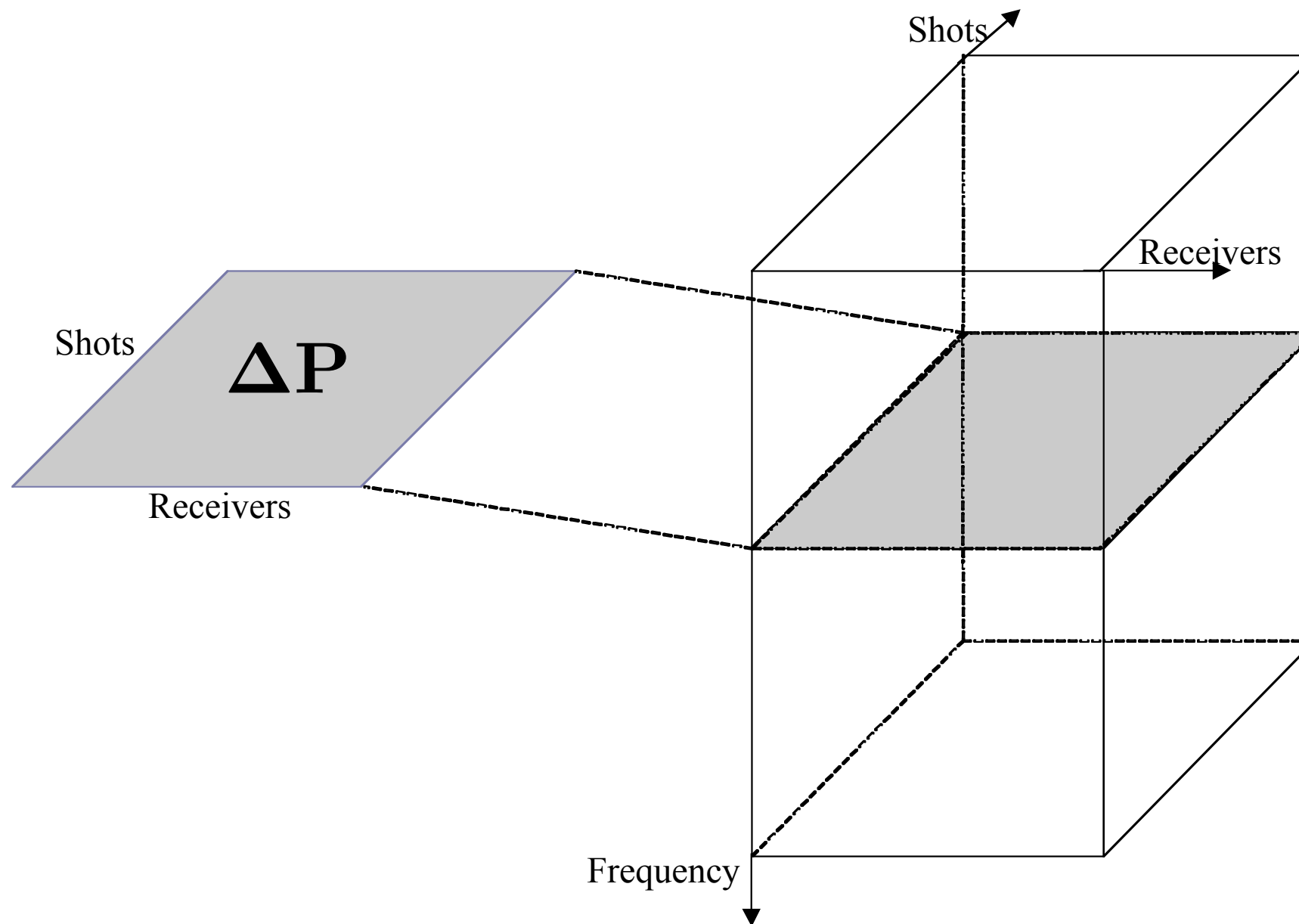
Berkhout and Verschuur's ***data-adaptive*** Focal transform derives from ***focusing*** of seismic data by the major primaries.

Both approaches entail the ***inversion*** of a linear operator.

Combination of the two yields

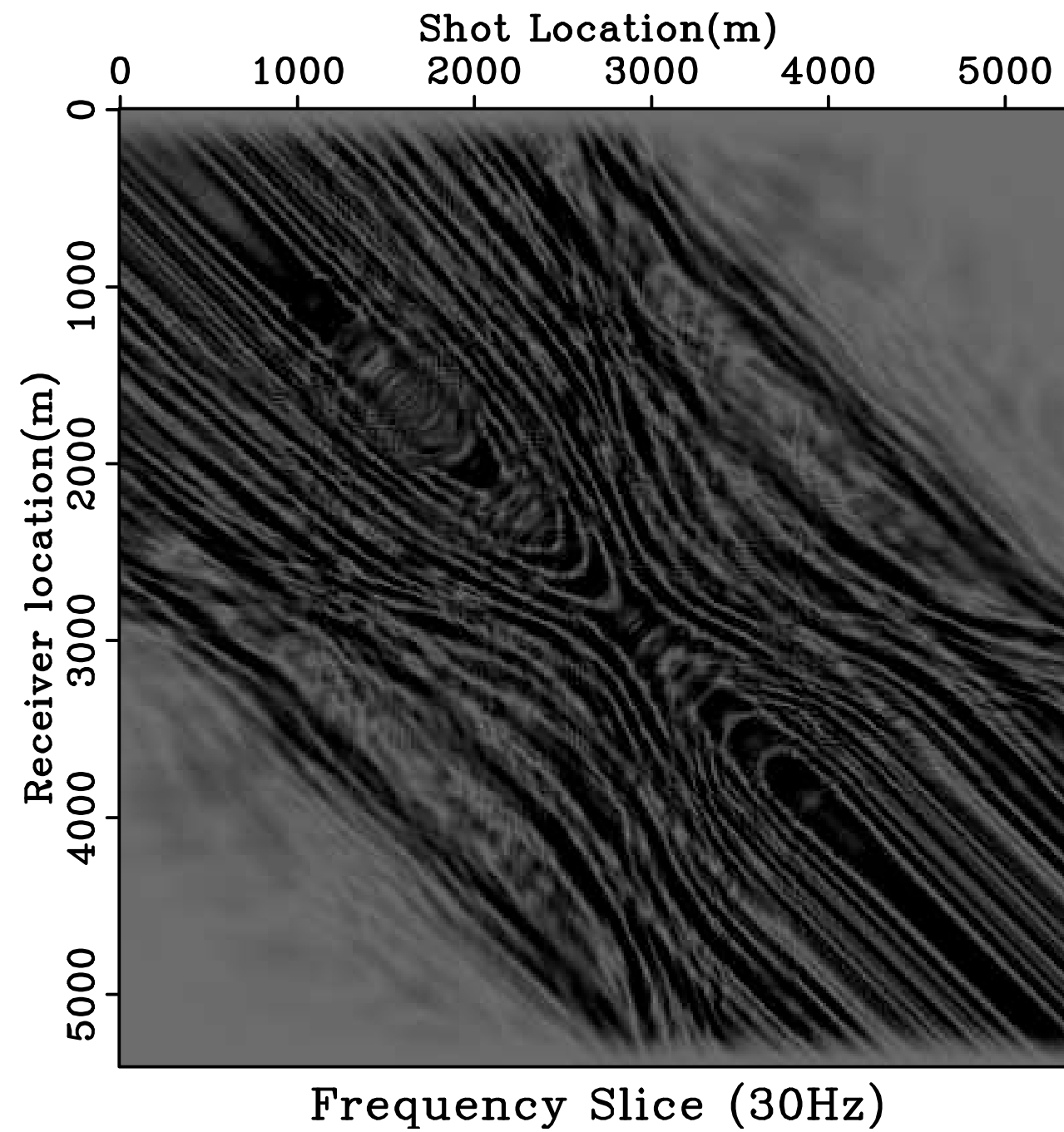
- improved focusing => more sparsity
- curvelet sparsity => better focusing

# Primary operator



Frequency slice from data matrix with dominant primaries.

# Primary operator



# Primary operator

**Primaries to first-order multiples:**

$$\Delta \mathbf{p} \mapsto \mathbf{m}^1 = (\Delta \mathbf{P} \mathcal{A} *_{t,x} \Delta \mathbf{p})$$

**First-order multiples into primaries:**

$$\mathbf{m}^1 \mapsto \Delta \mathbf{p} \approx (\Delta \mathbf{P} \mathcal{A} \otimes_{t,x} \Delta \mathbf{p})$$

**with the acquisition matrix**

$$\mathcal{A} = \left( \mathcal{S}^\dagger \mathbf{R} \mathcal{D}^\dagger \right)$$

**“inverting” for source and receiver wavelet wavelets geometry and surface reflectivity.**

# Curvelet-based Focal transform

Solve with 3-D curvelet transform

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{x}\|_1 & \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{S}^T \tilde{\mathbf{x}} \end{cases}$$

with

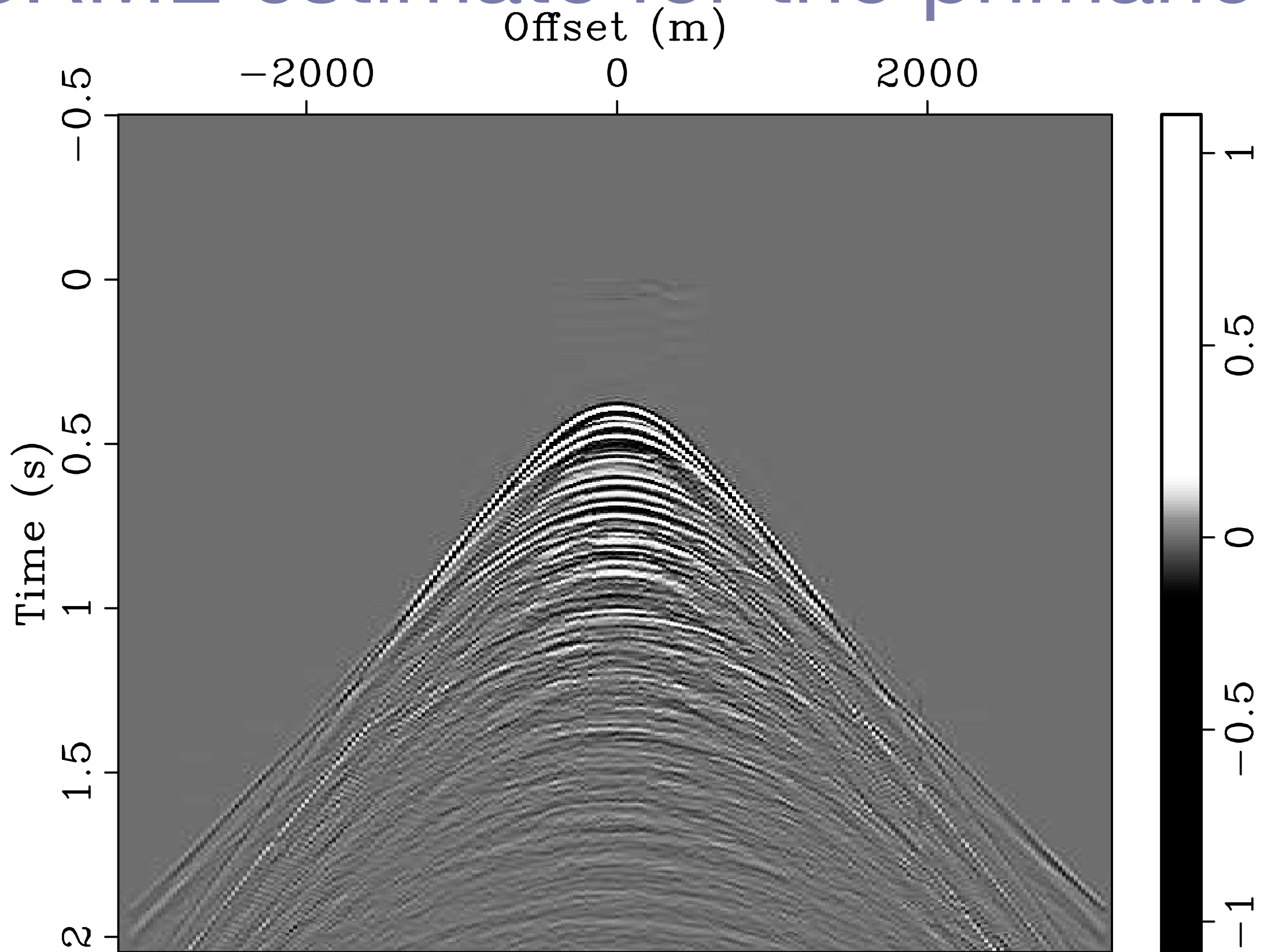
$$\mathbf{A} := \Delta \mathbf{P} \mathbf{C}^T \quad \text{and} \quad \Delta \mathbf{P} := \mathbf{F}^H \text{block diag}\{\Delta \mathbf{p}\} \mathbf{F}$$

$$\mathbf{S} := \mathbf{C}$$

$$\mathbf{y} = \mathbf{P}(:,)$$

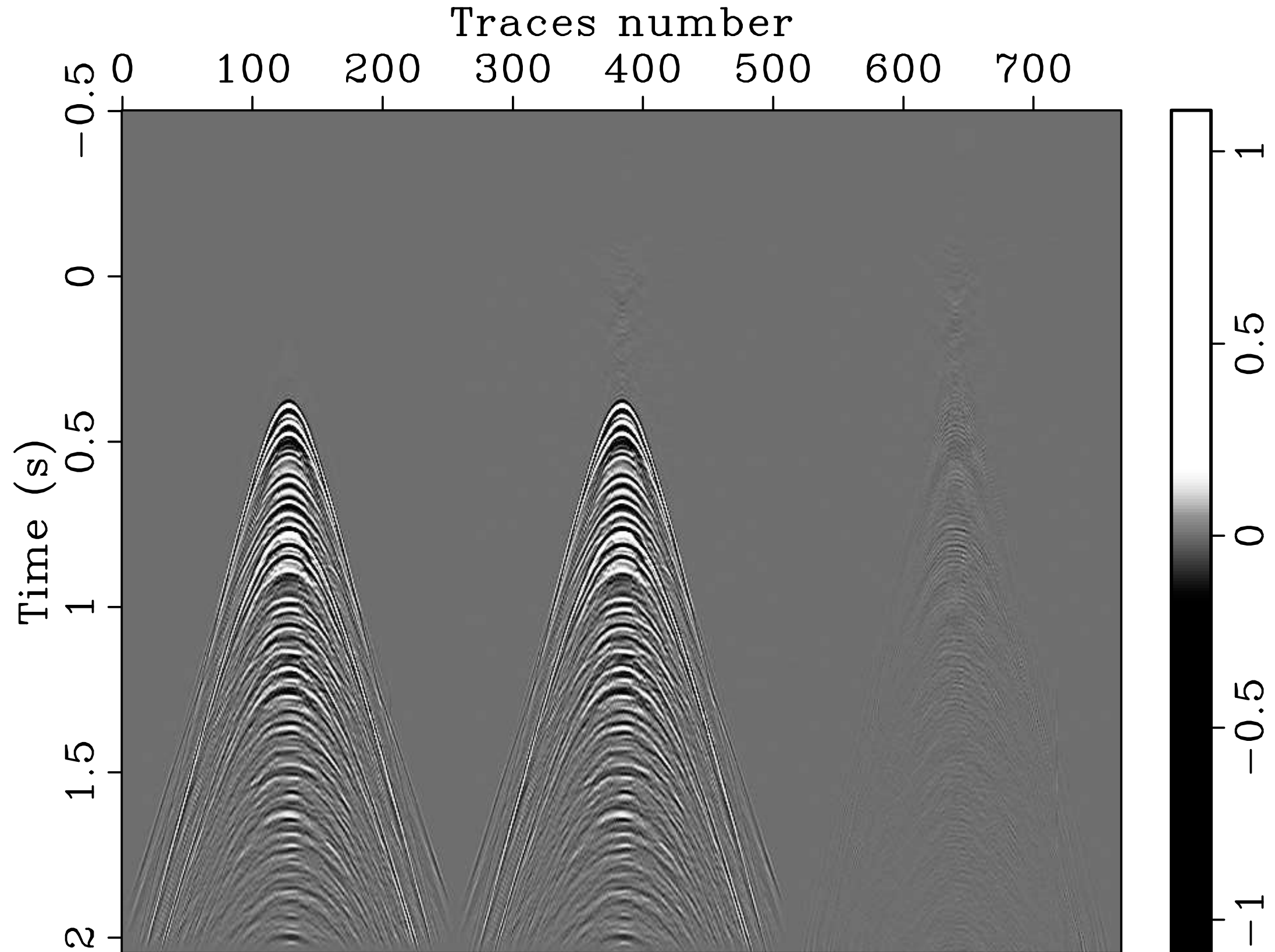
$$\mathbf{P} = \text{total data.}$$

# SRME estimate for the primaries



SRME primaries

# Difference



# Recovery with focussing

Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{x}\|_1 & \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{S}^T \tilde{\mathbf{x}} \end{cases}$$

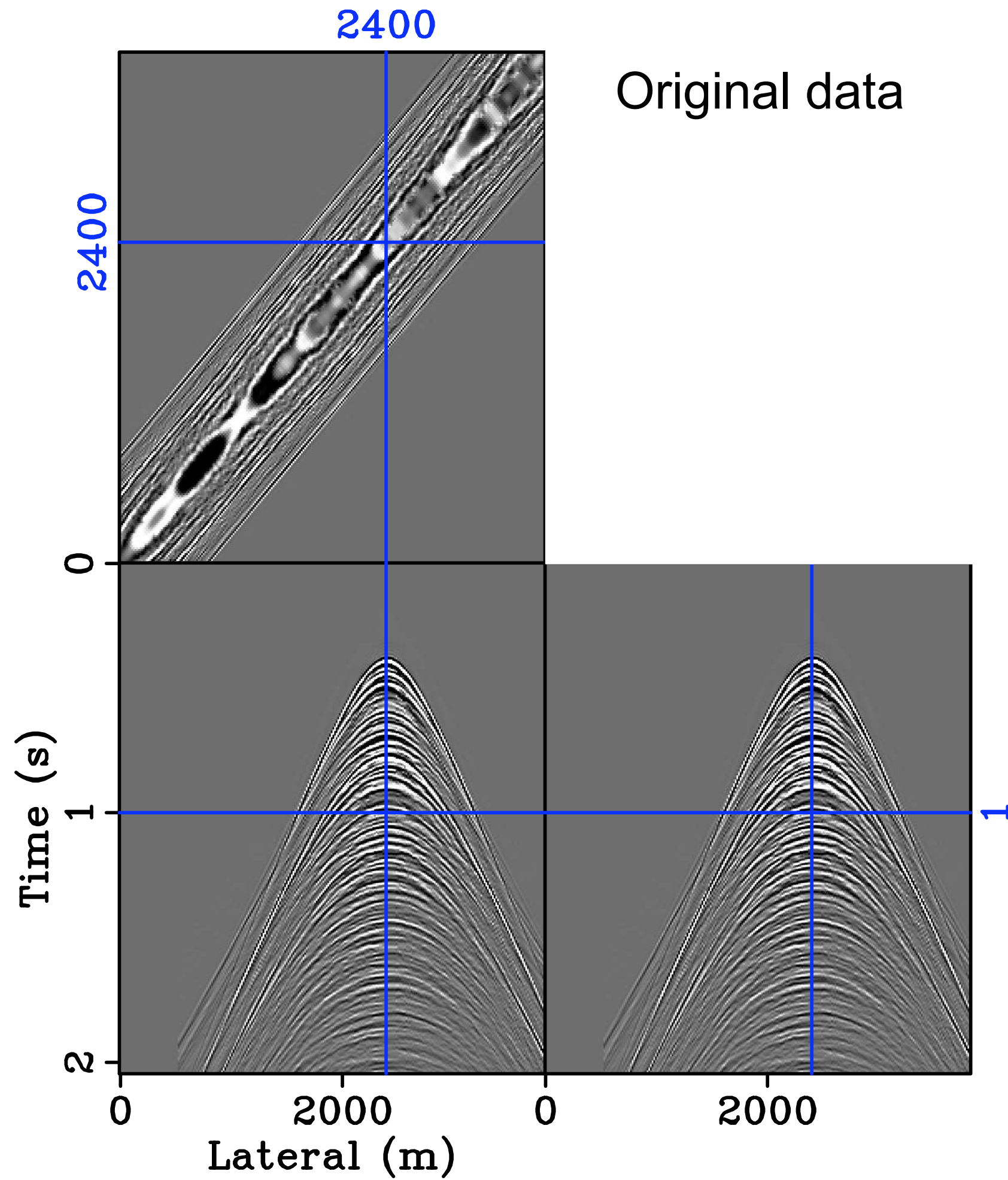
with

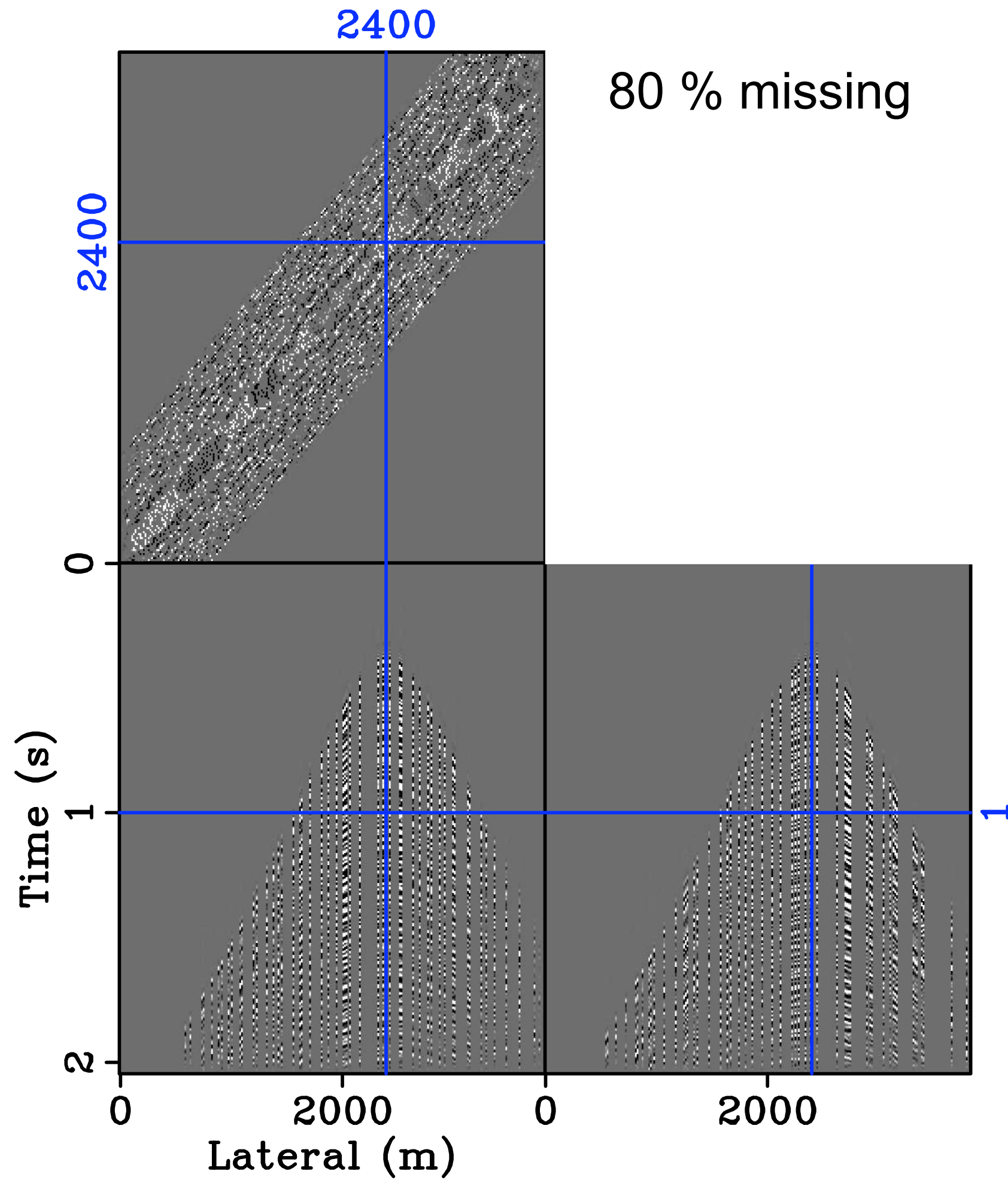
$$\mathbf{A} := \mathbf{R}\Delta\mathbf{P}\mathbf{C}^T$$

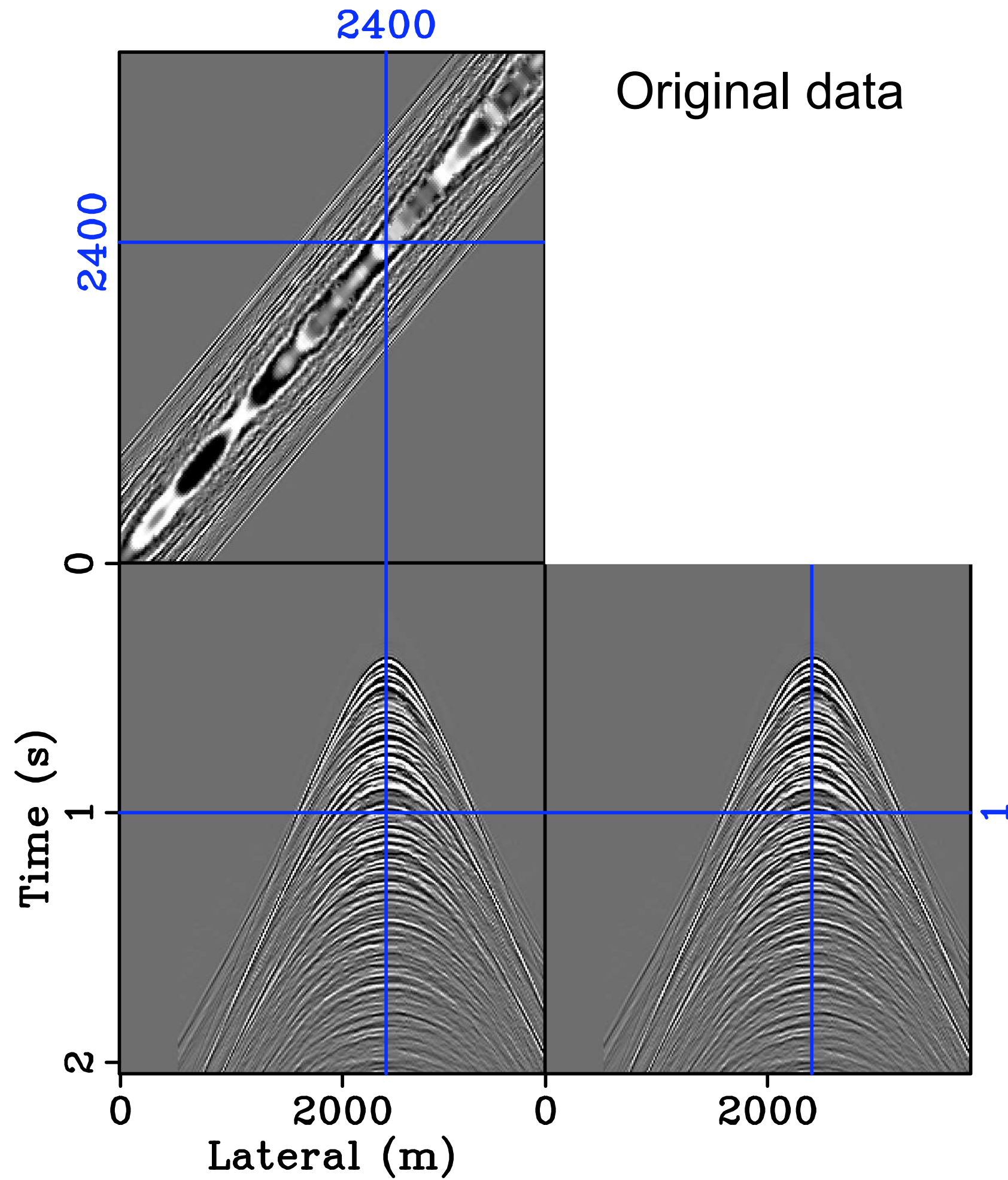
$$\mathbf{S}^T := \Delta\mathbf{P}\mathbf{C}^T$$

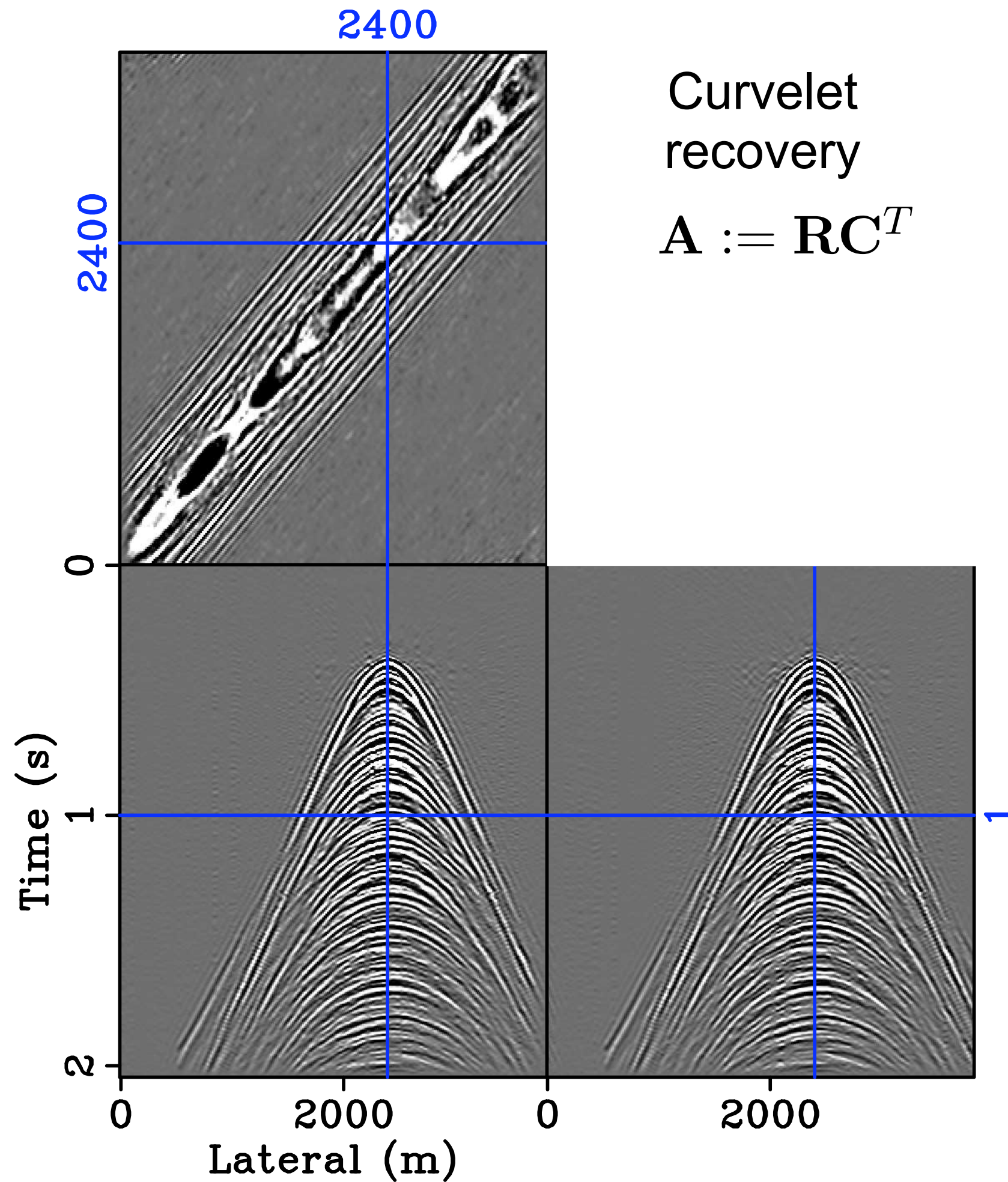
$$\mathbf{y} = \mathbf{R}\mathbf{P}(:)$$

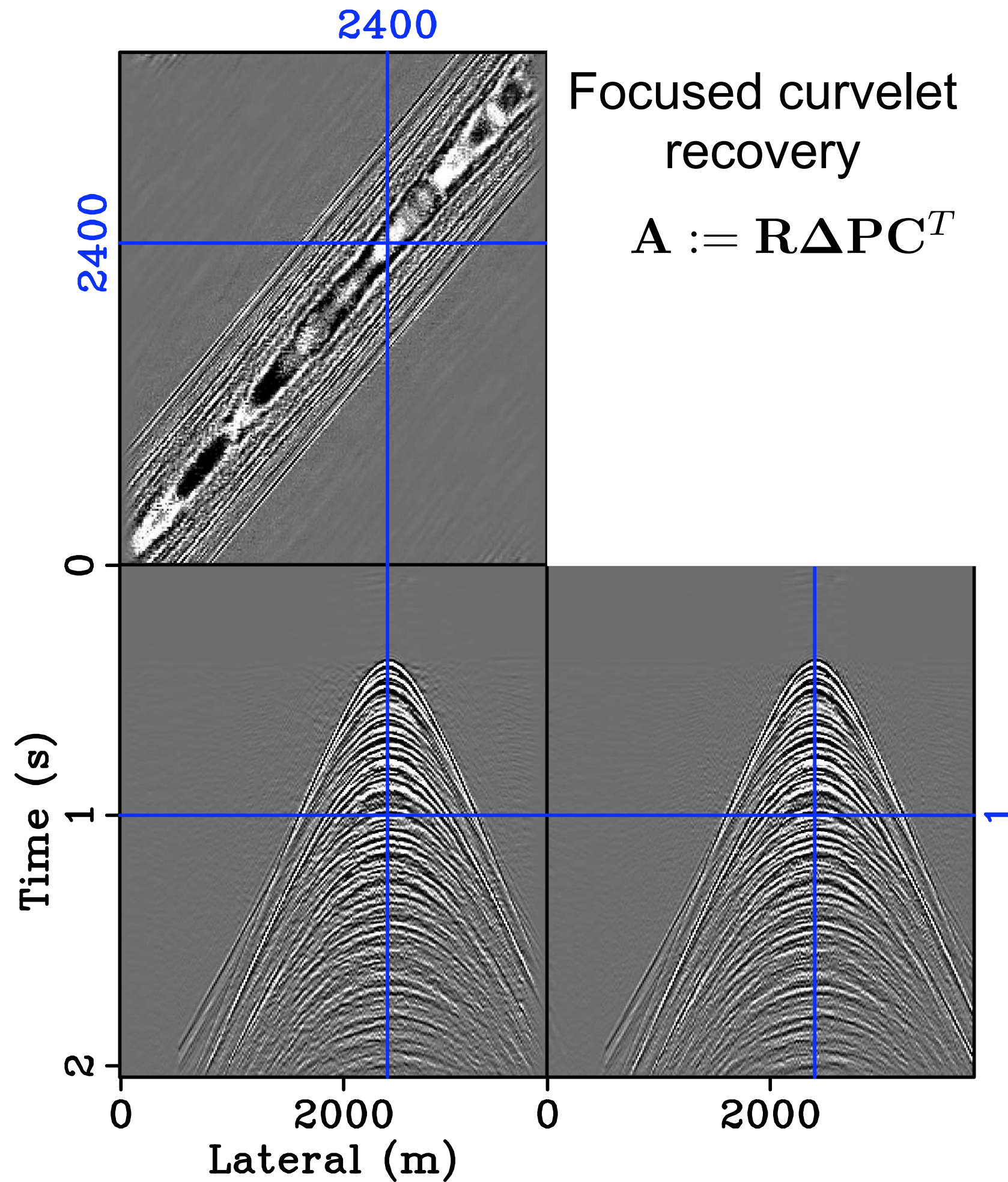
$$\mathbf{R} = \text{picking operator.}$$

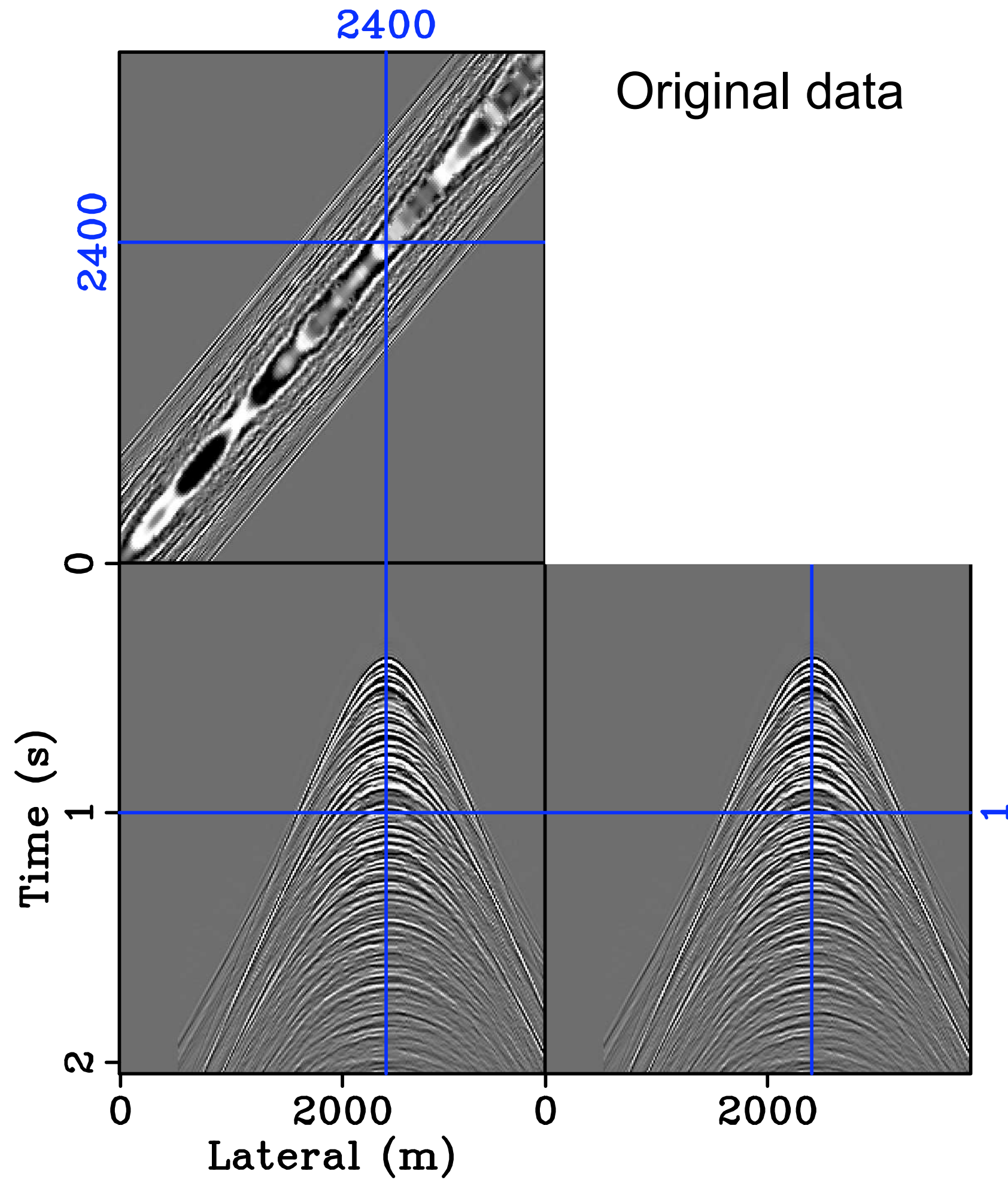


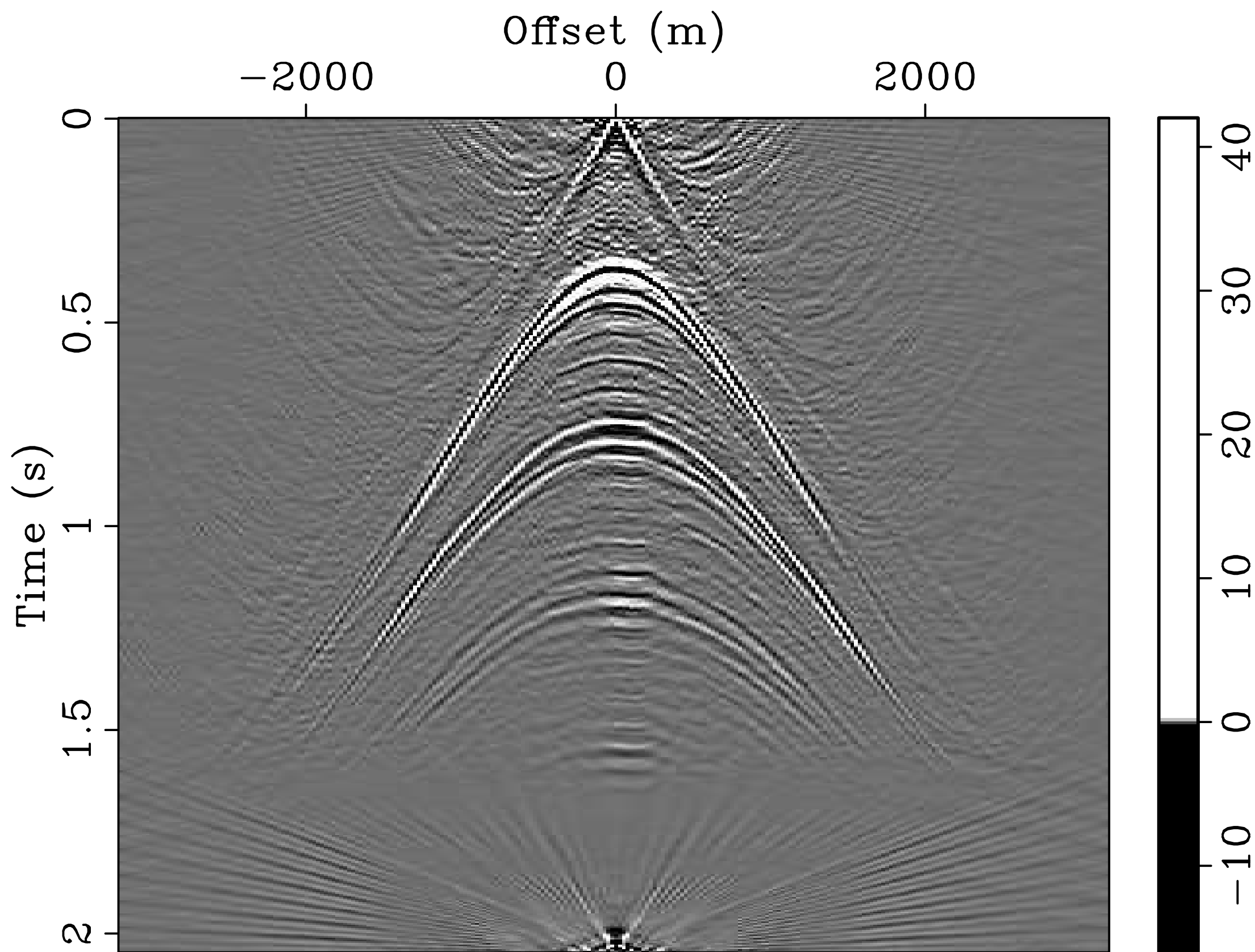












Recovered Data in Focal Domain

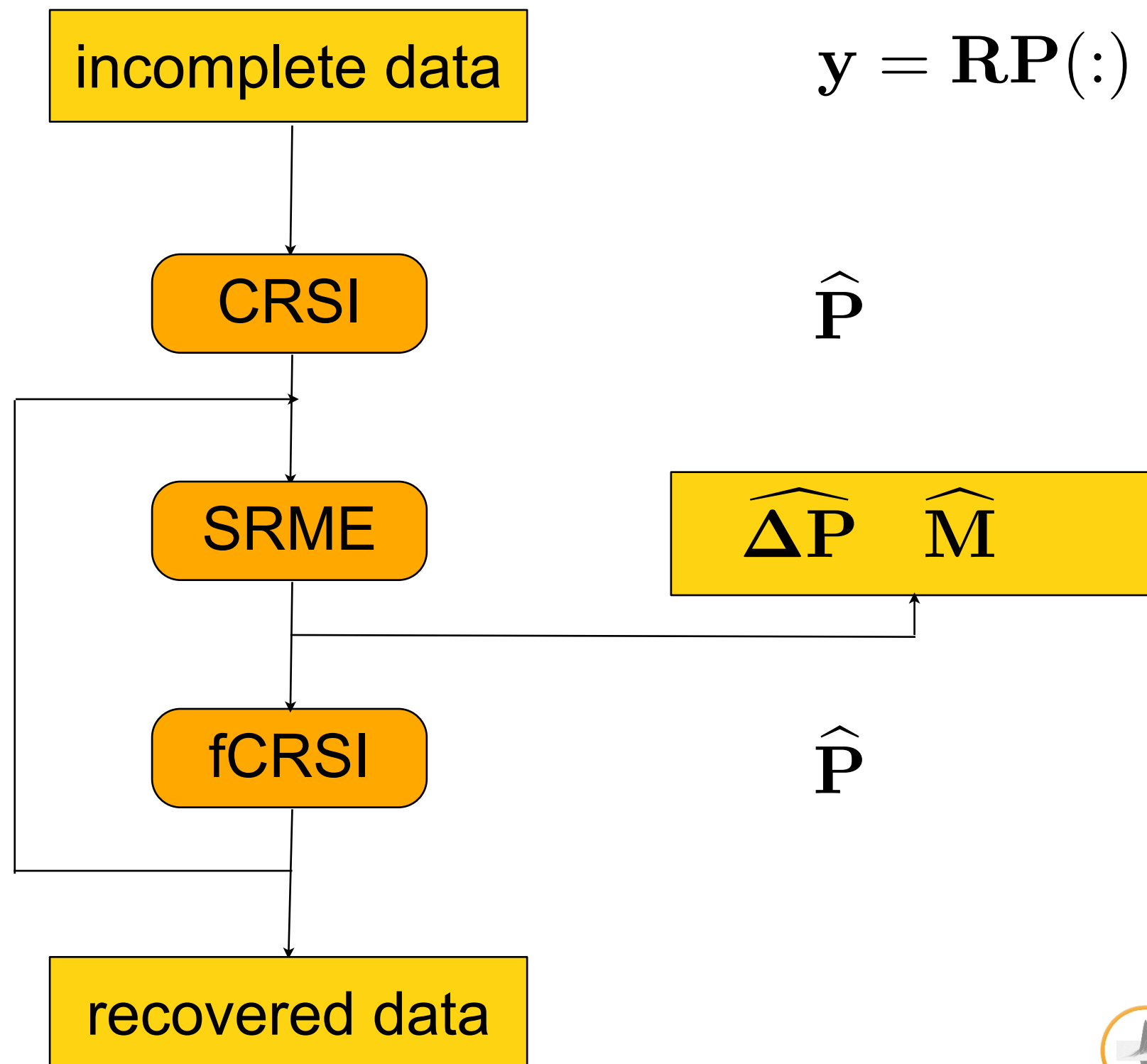
# Nonlinear primary-multiple prediction

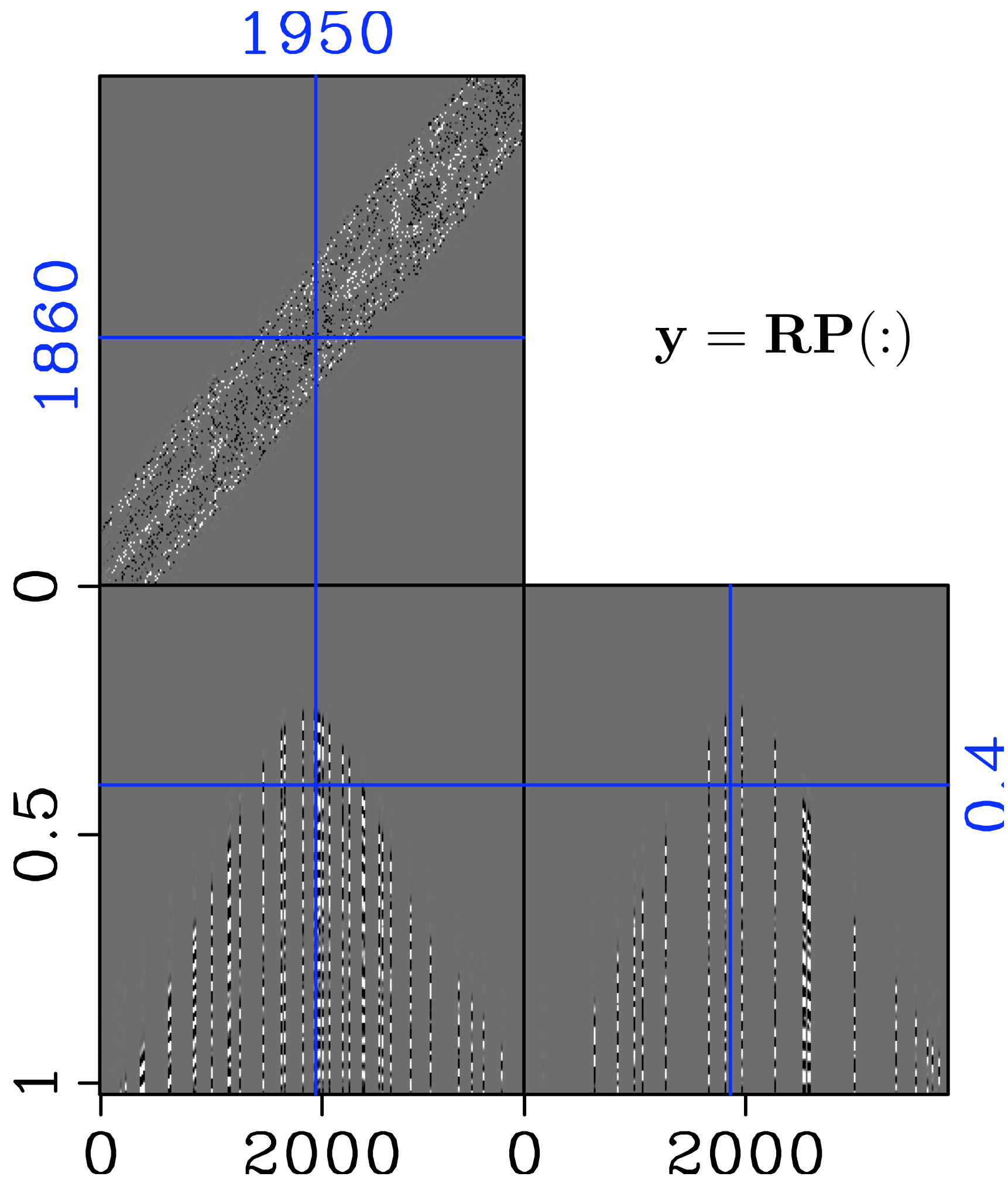
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joint work with Deli Wang (visitor from Jilin university) and Eric Verschuur

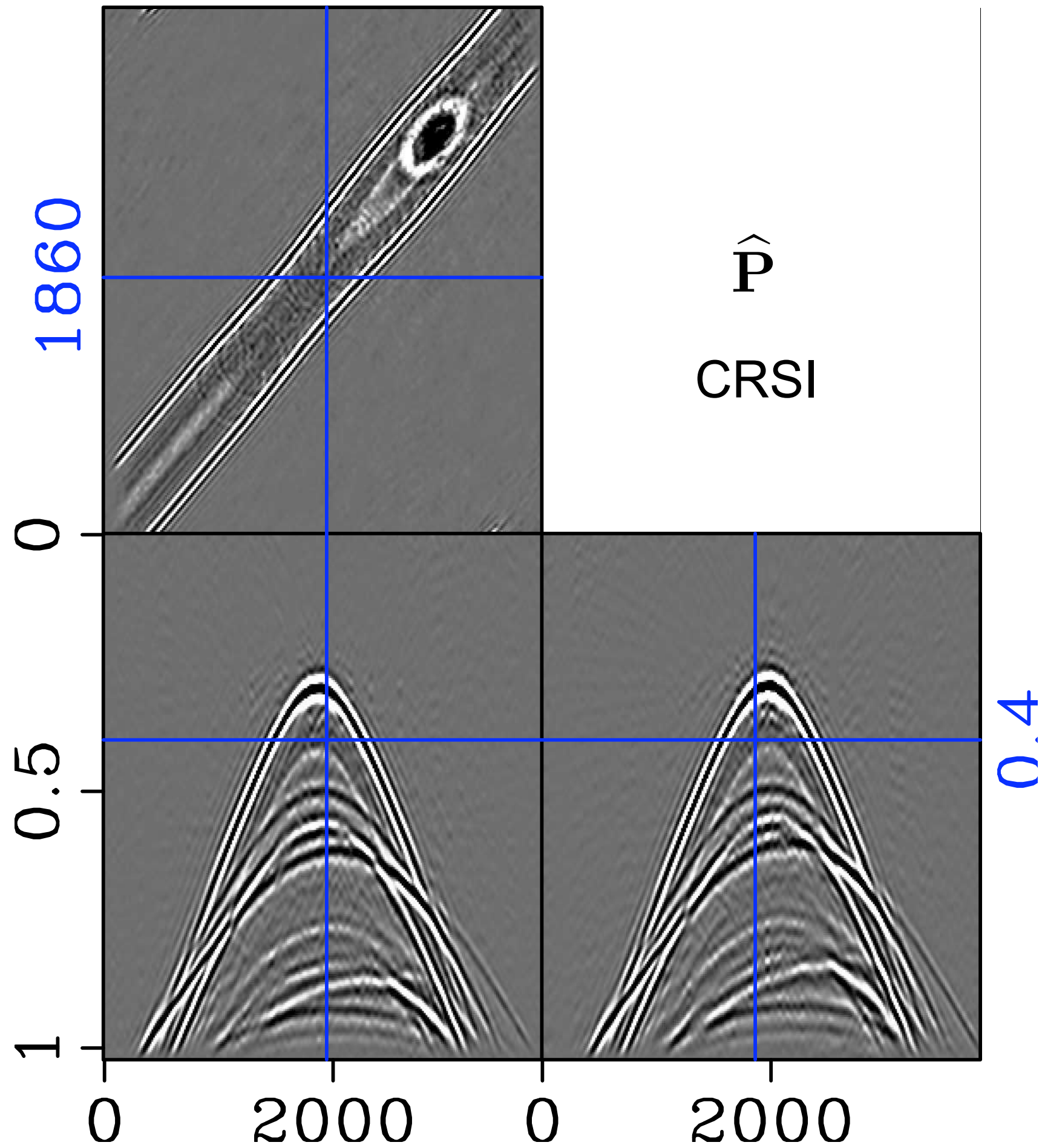


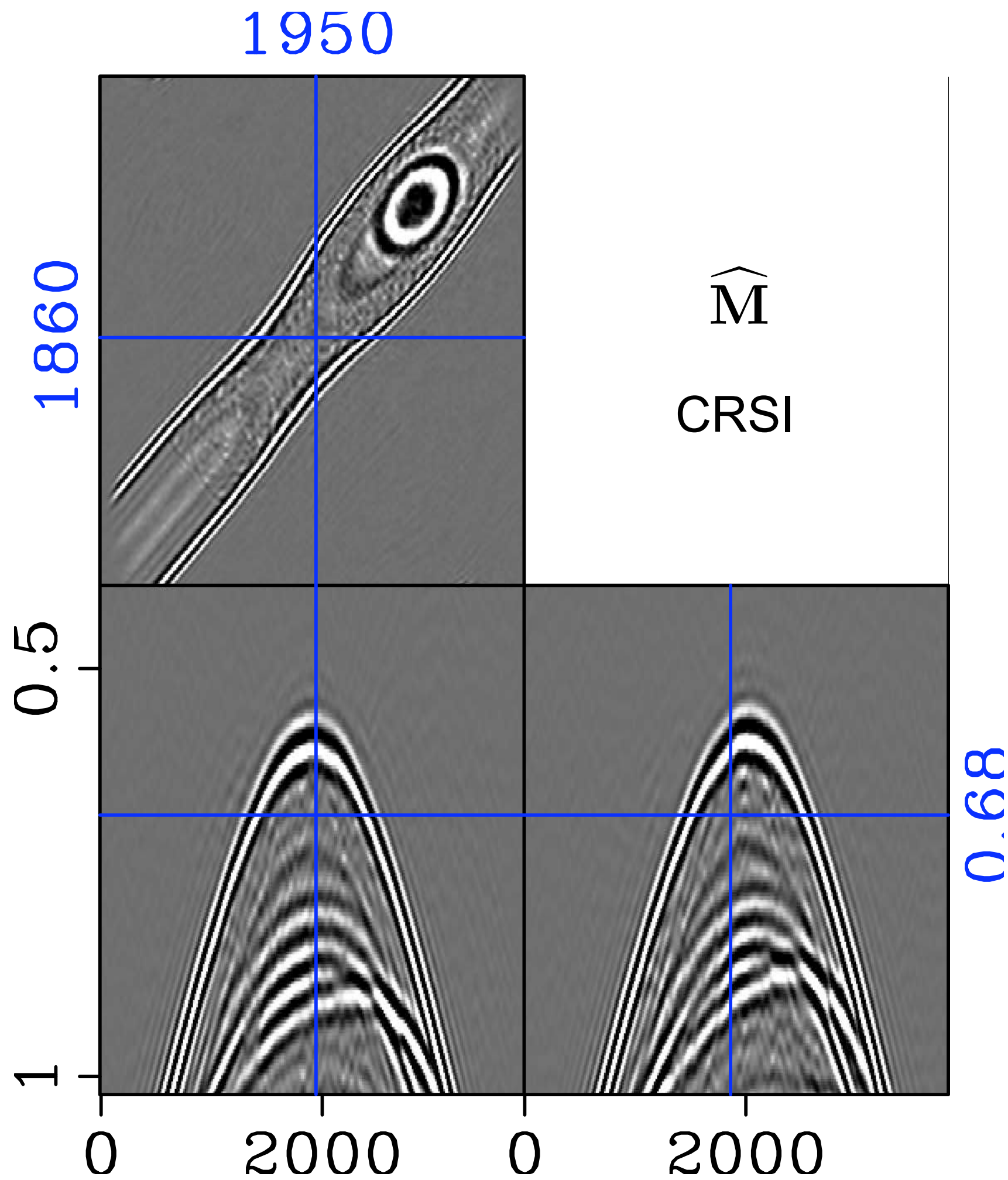
# Multiple prediction with fCRSI

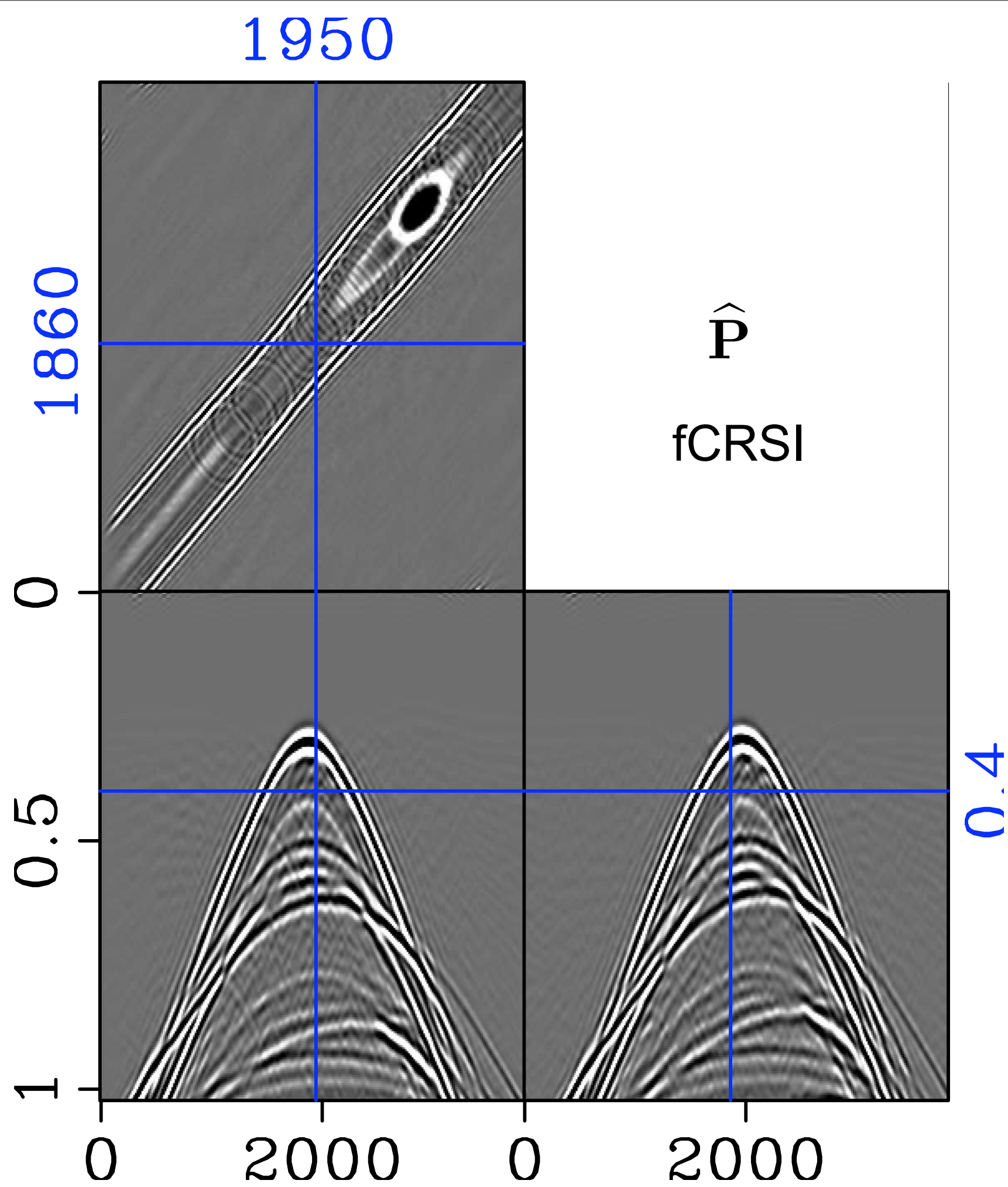


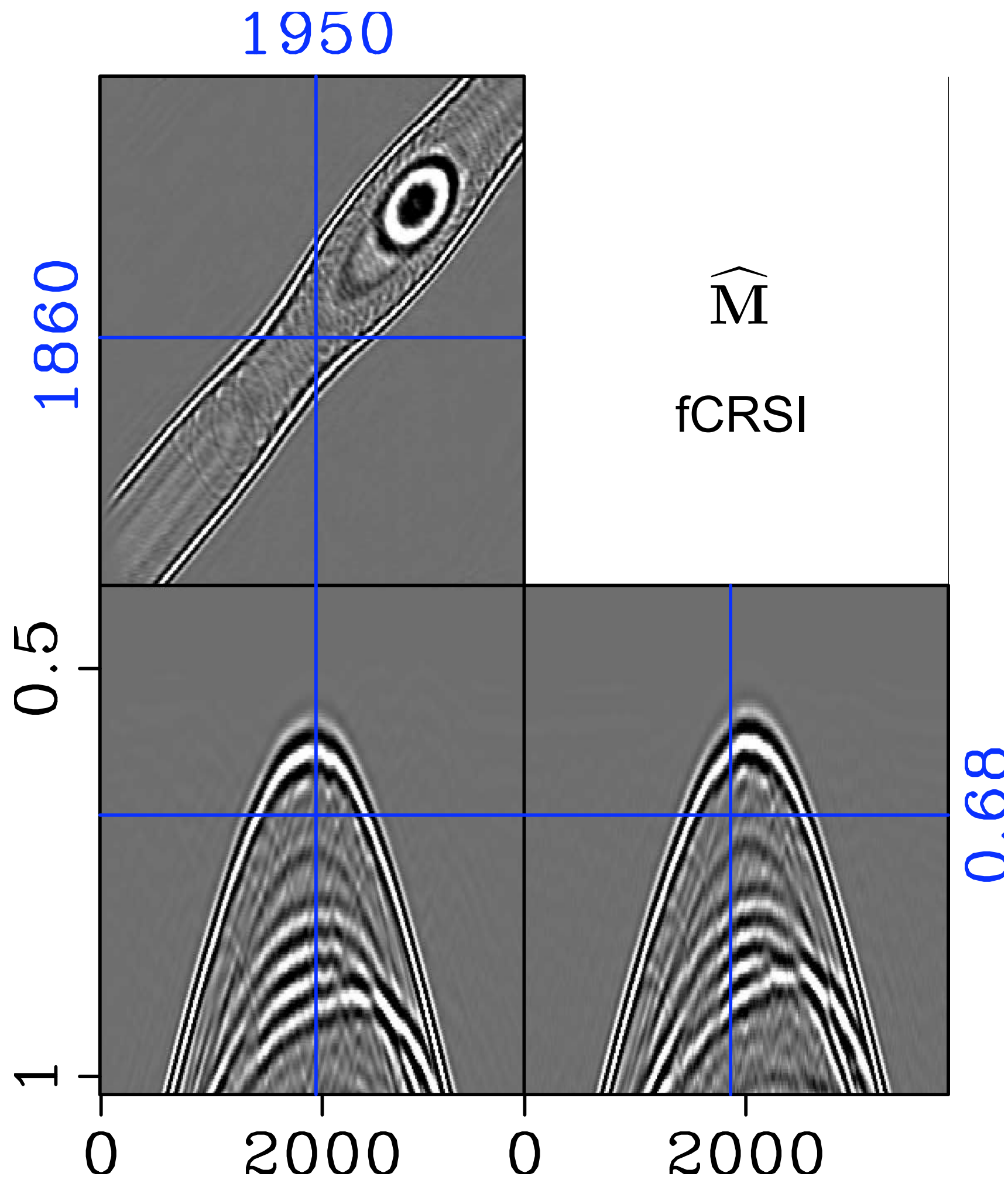


1950

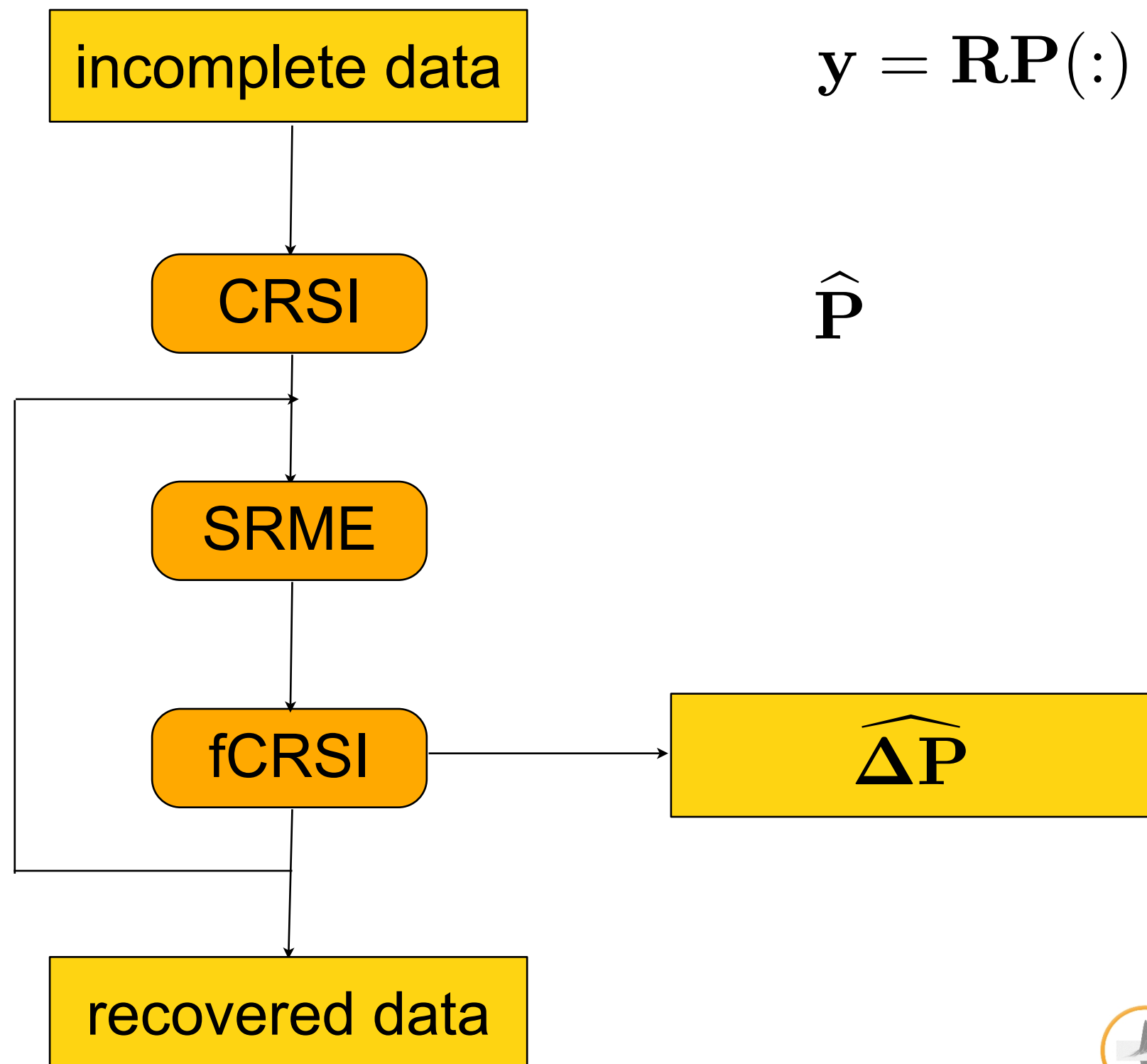








# Primary prediction with fCRSI



# Curvelet-based Focal transform

Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{x}\|_1 & \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{S}^T \tilde{\mathbf{x}} \end{cases}$$

with

$$\mathbf{A} := \Delta \mathbf{P} \mathbf{C}^T$$

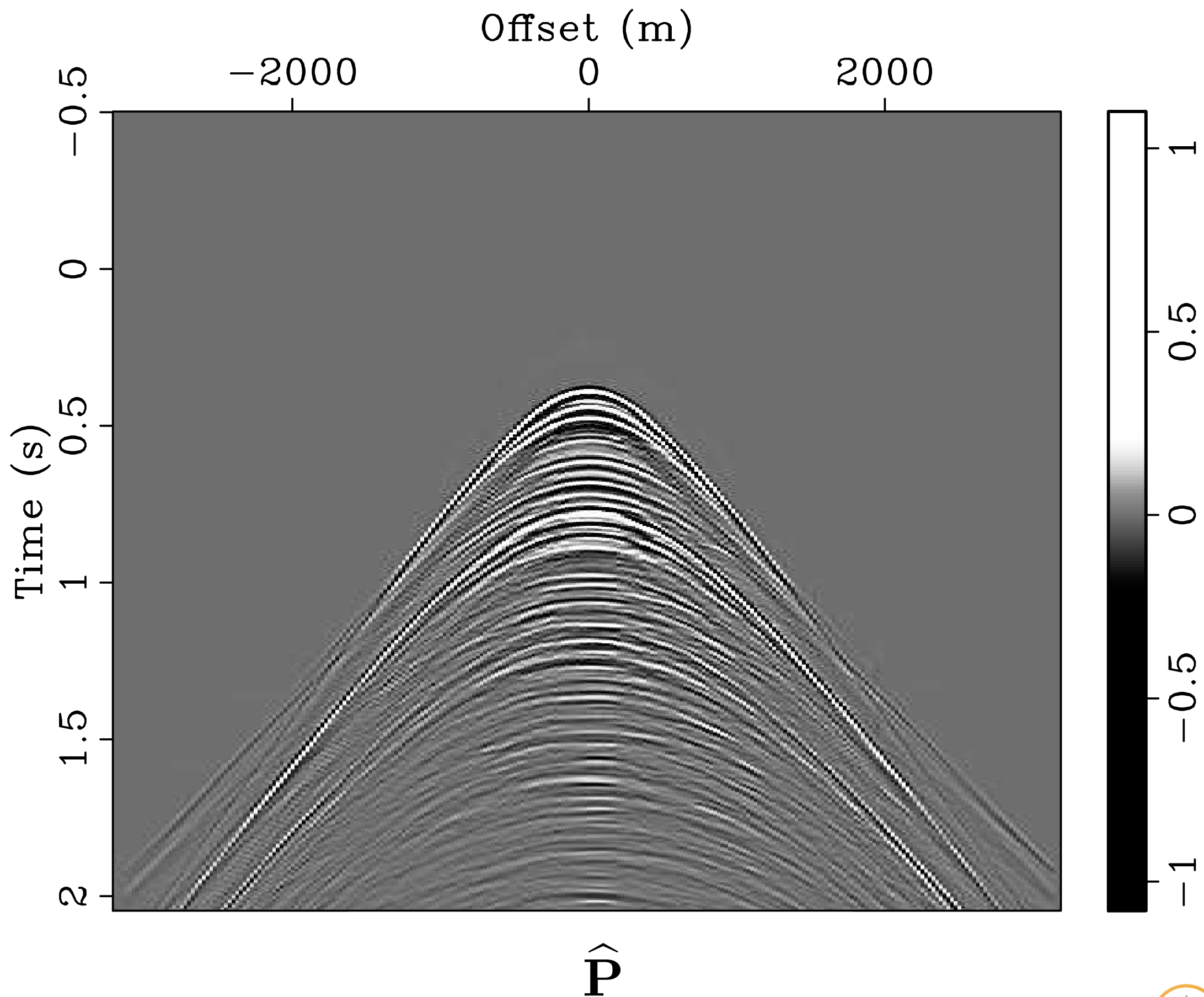
$$\mathbf{S} := \mathbf{C}$$

$$\mathbf{y} = \mathbf{P}(:,)$$

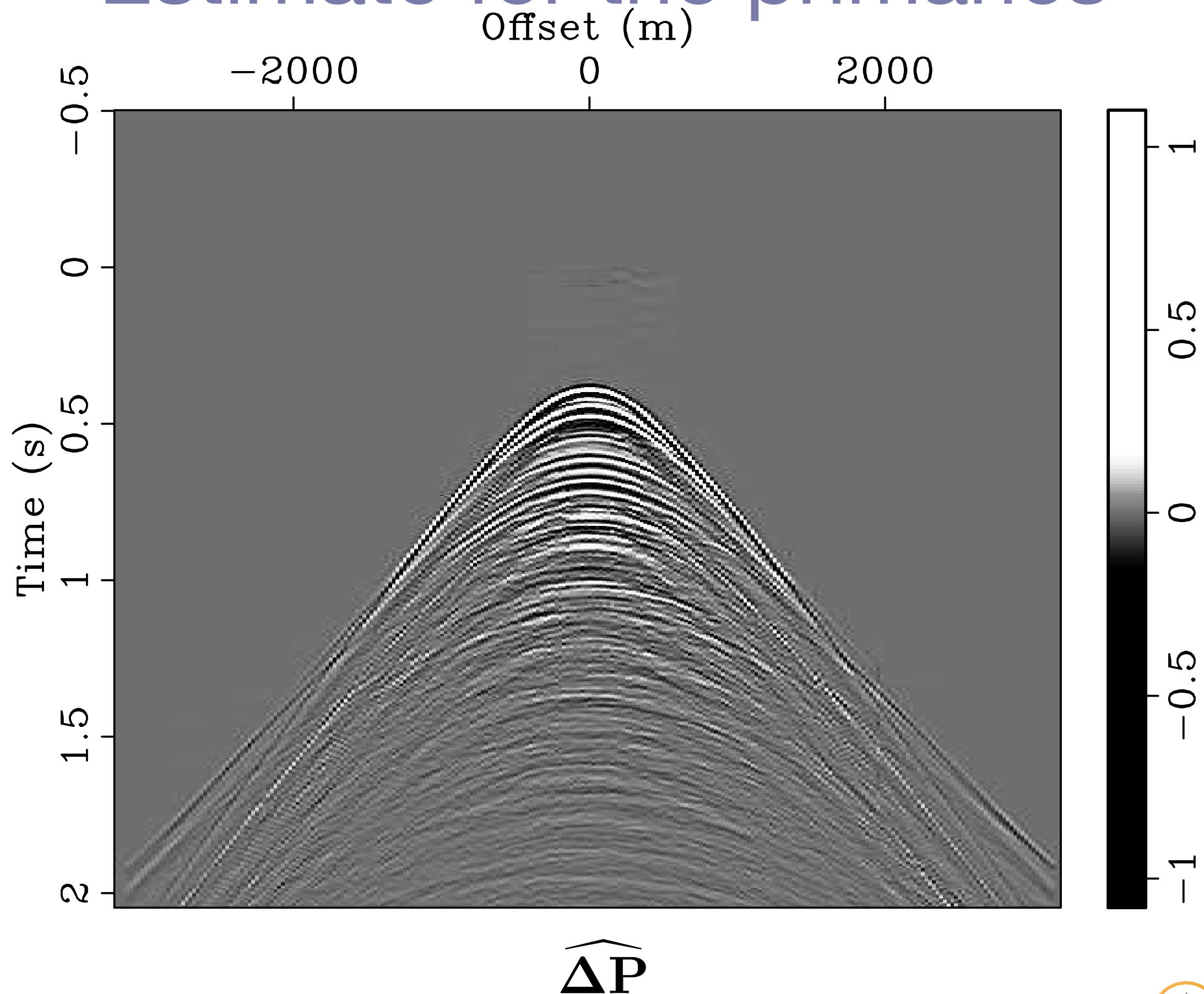
$$\mathbf{P} = \text{total data}$$

$$\tilde{\mathbf{f}} = \text{focused data.}$$

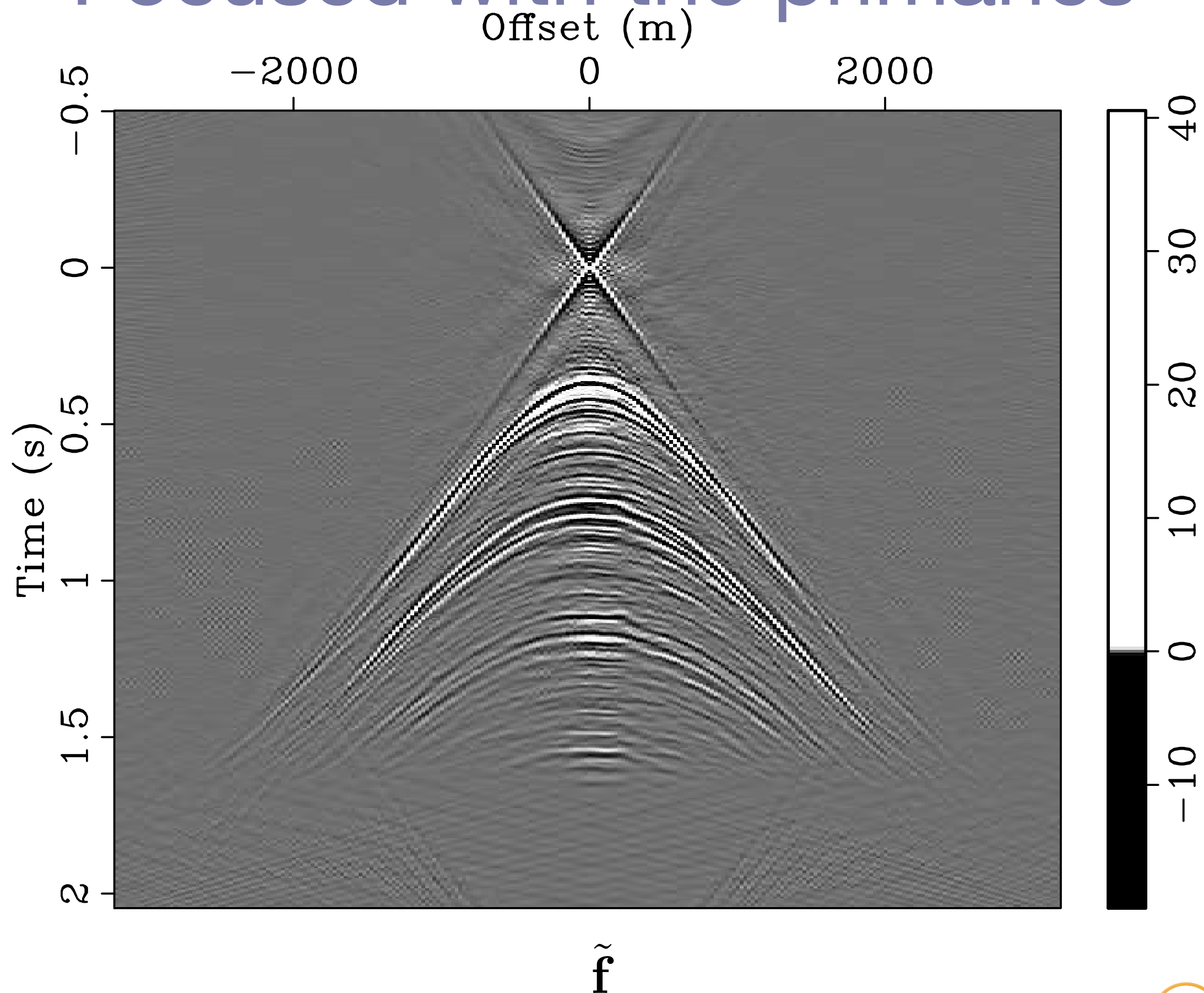
# Total data



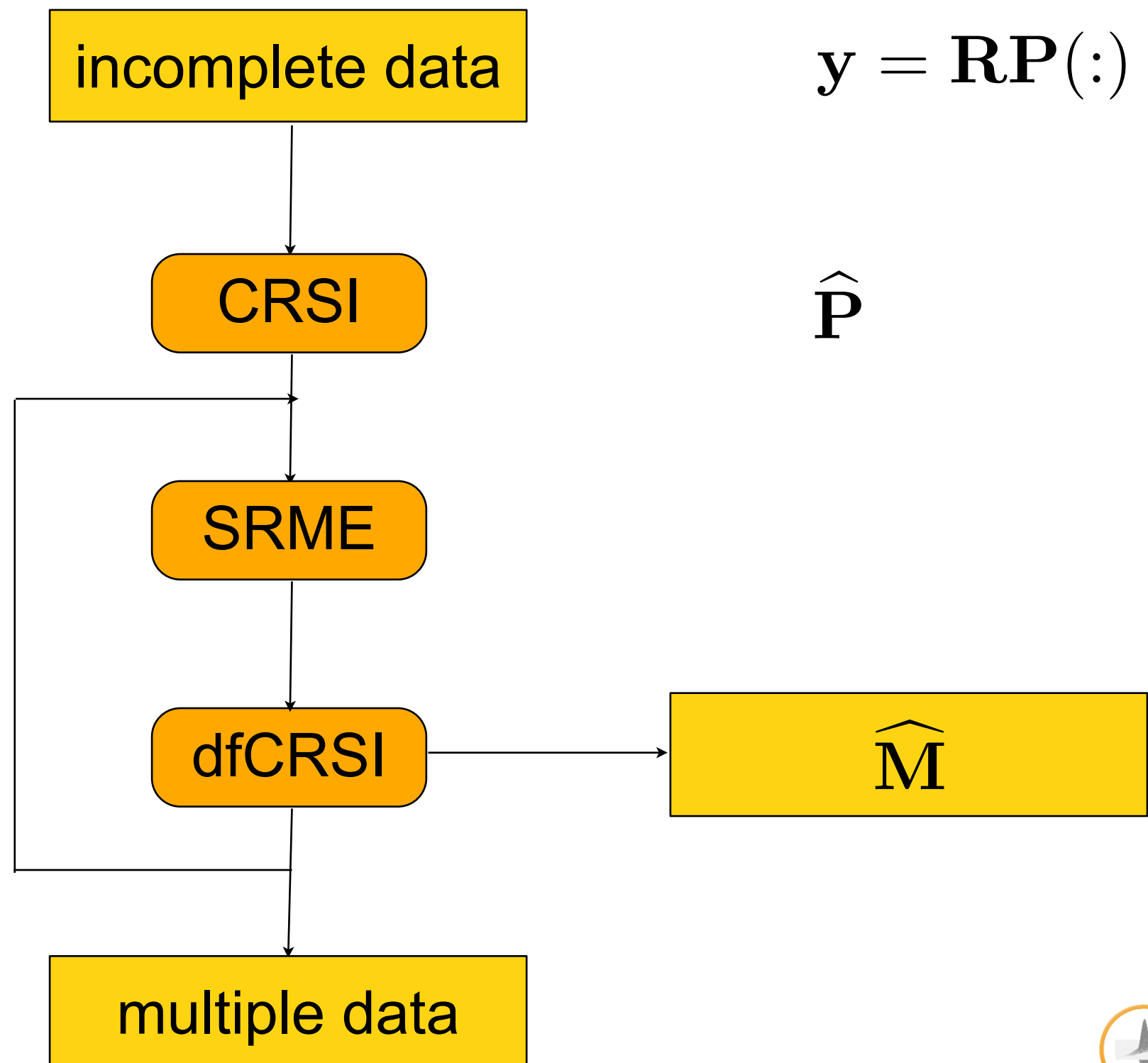
# Estimate for the primaries



# Focused with the primaries



# Multiple prediction with dfCRSI



# Curvelet-based deFocal transform

Solve

$$\mathbf{P}_\epsilon : \quad \begin{cases} \tilde{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{x}\|_1 & \text{s.t.} \quad \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2 \leq \epsilon \\ \tilde{\mathbf{f}} = \mathbf{S}^T \tilde{\mathbf{x}} \end{cases}$$

with

$$\mathbf{A} := \Delta \mathbf{P}^T \mathbf{C}^T \text{ and } \Delta \mathbf{P} := \mathbf{F}^H \text{block diag}\{\text{conj}(\Delta \mathbf{p})\} \mathbf{F}$$

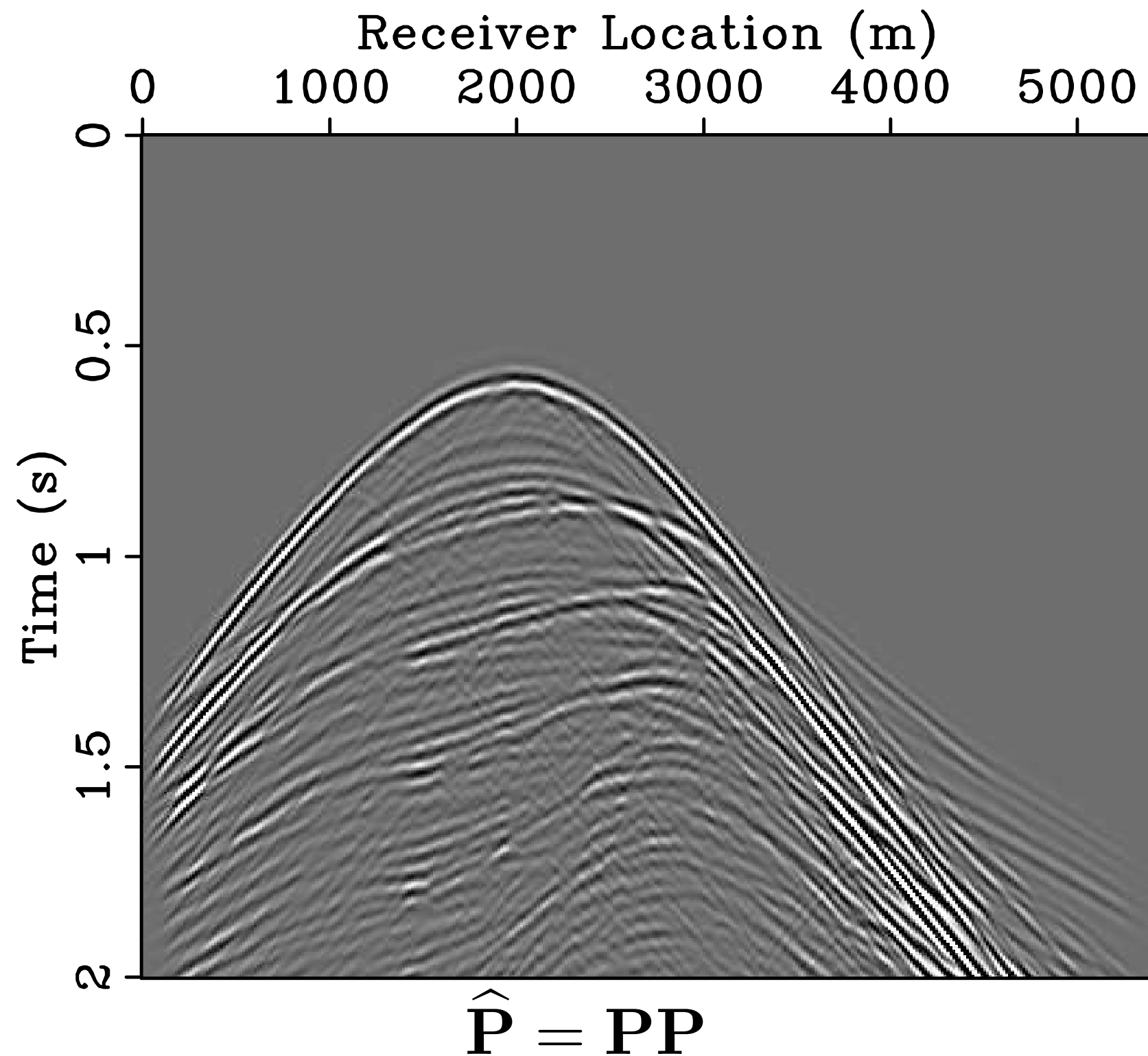
$$\mathbf{S} := \mathbf{C}$$

$$\mathbf{y} = \mathbf{P}(:,)$$

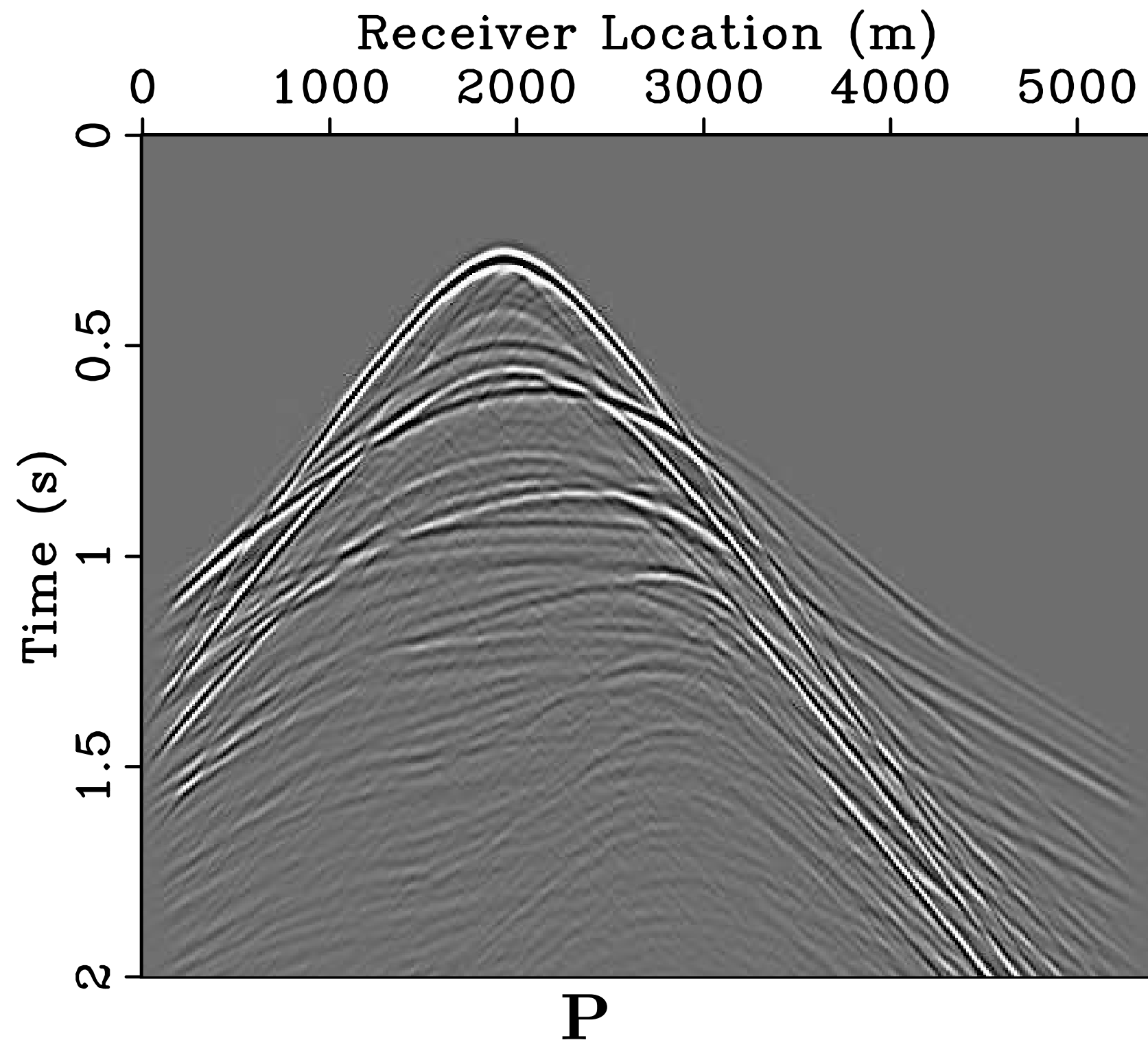
$$\mathbf{P} = \text{total data}$$

$$\tilde{\mathbf{f}} = \text{defocussed data.}$$

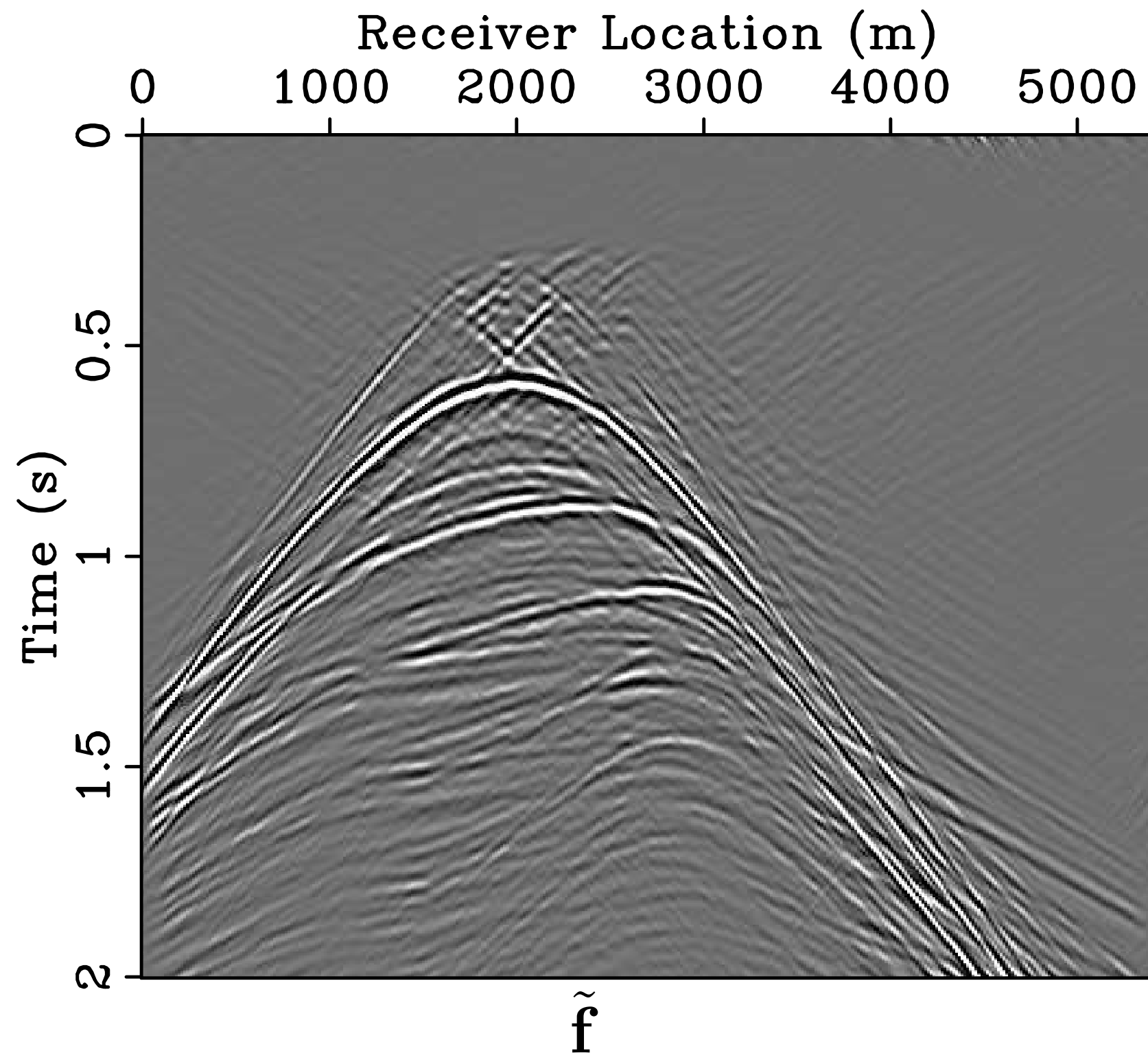
# SRME predicted multiples



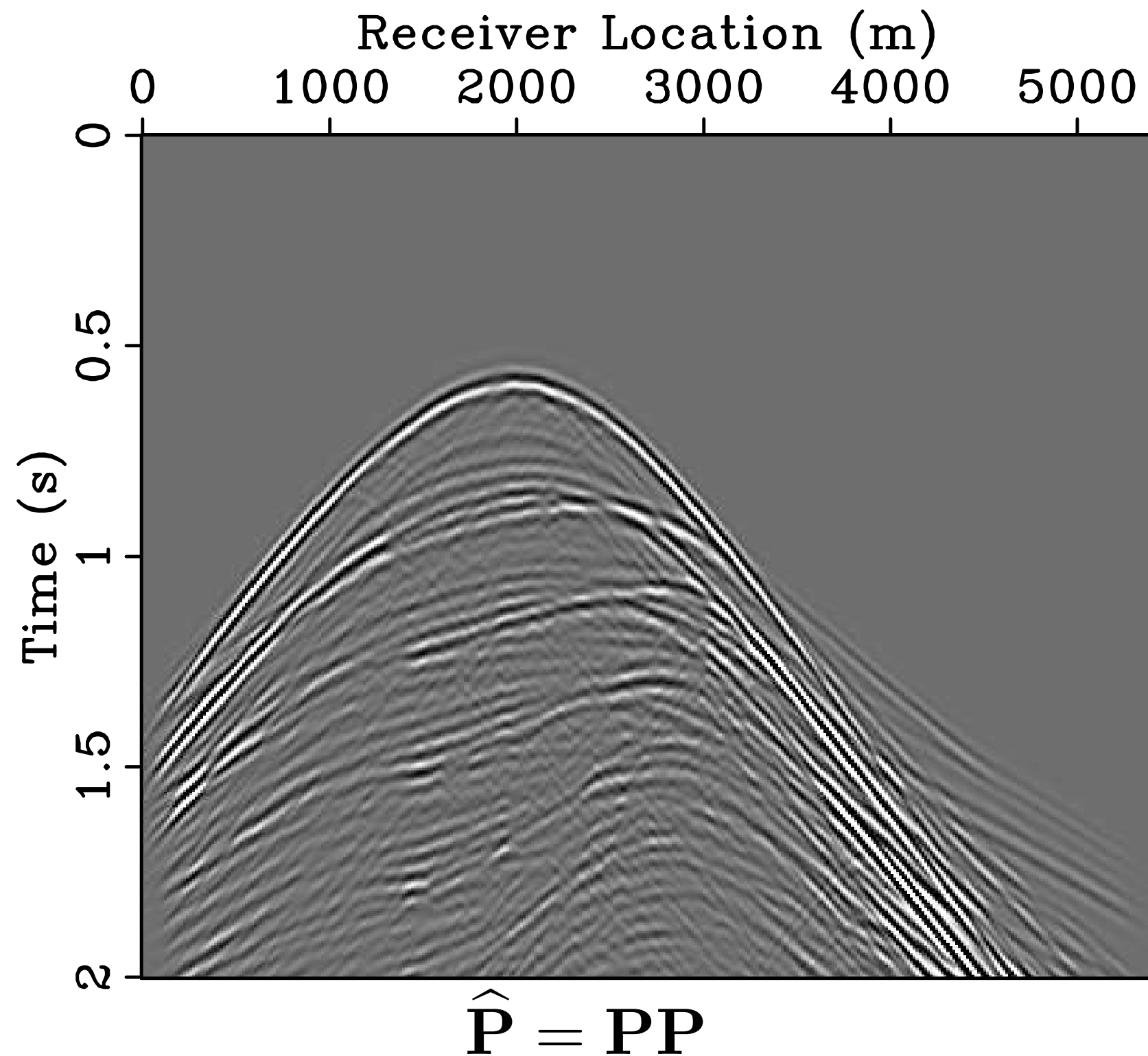
# Original data



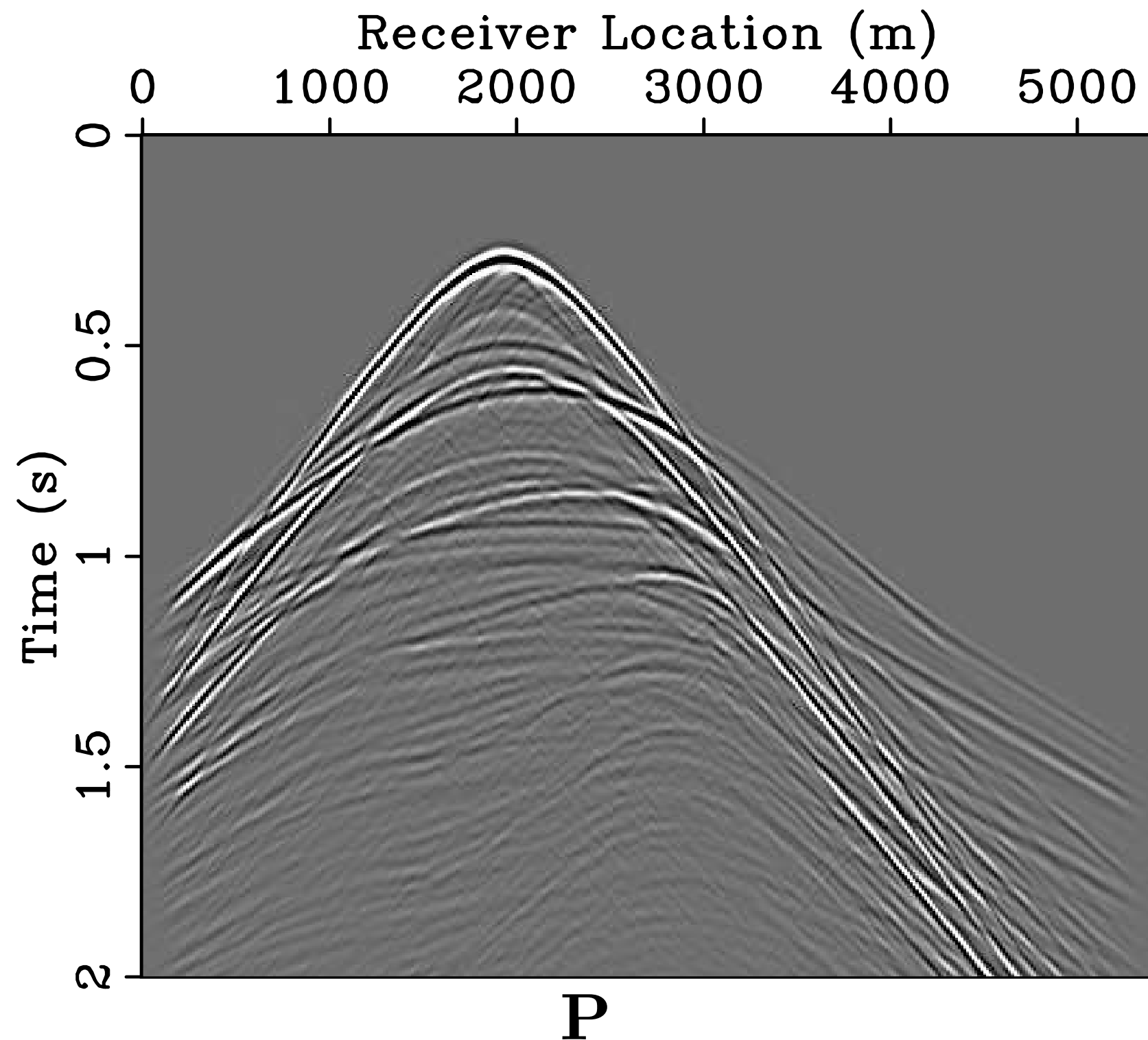
# Multiple estimate by dfCRSI



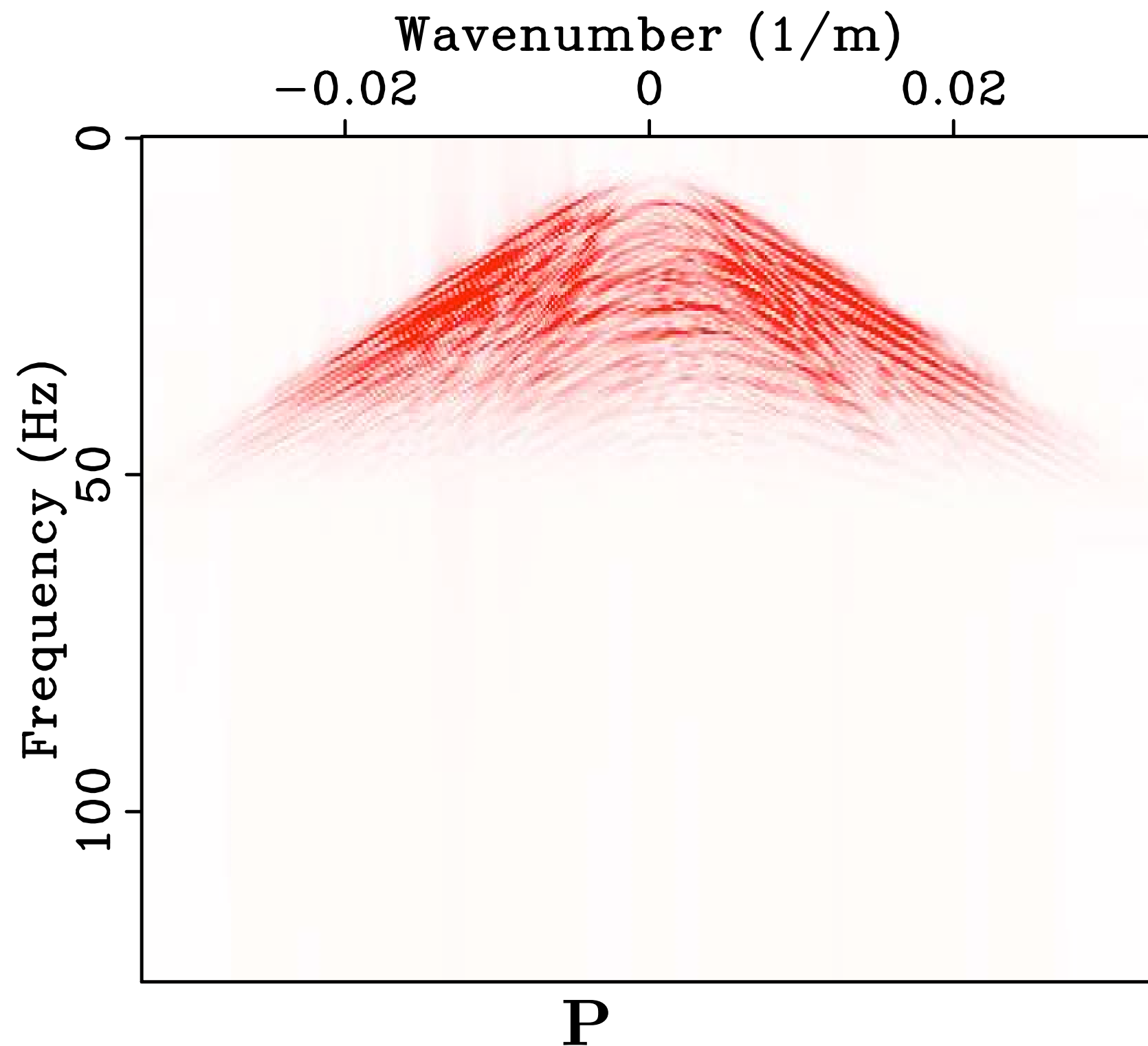
# SRME predicted multiples



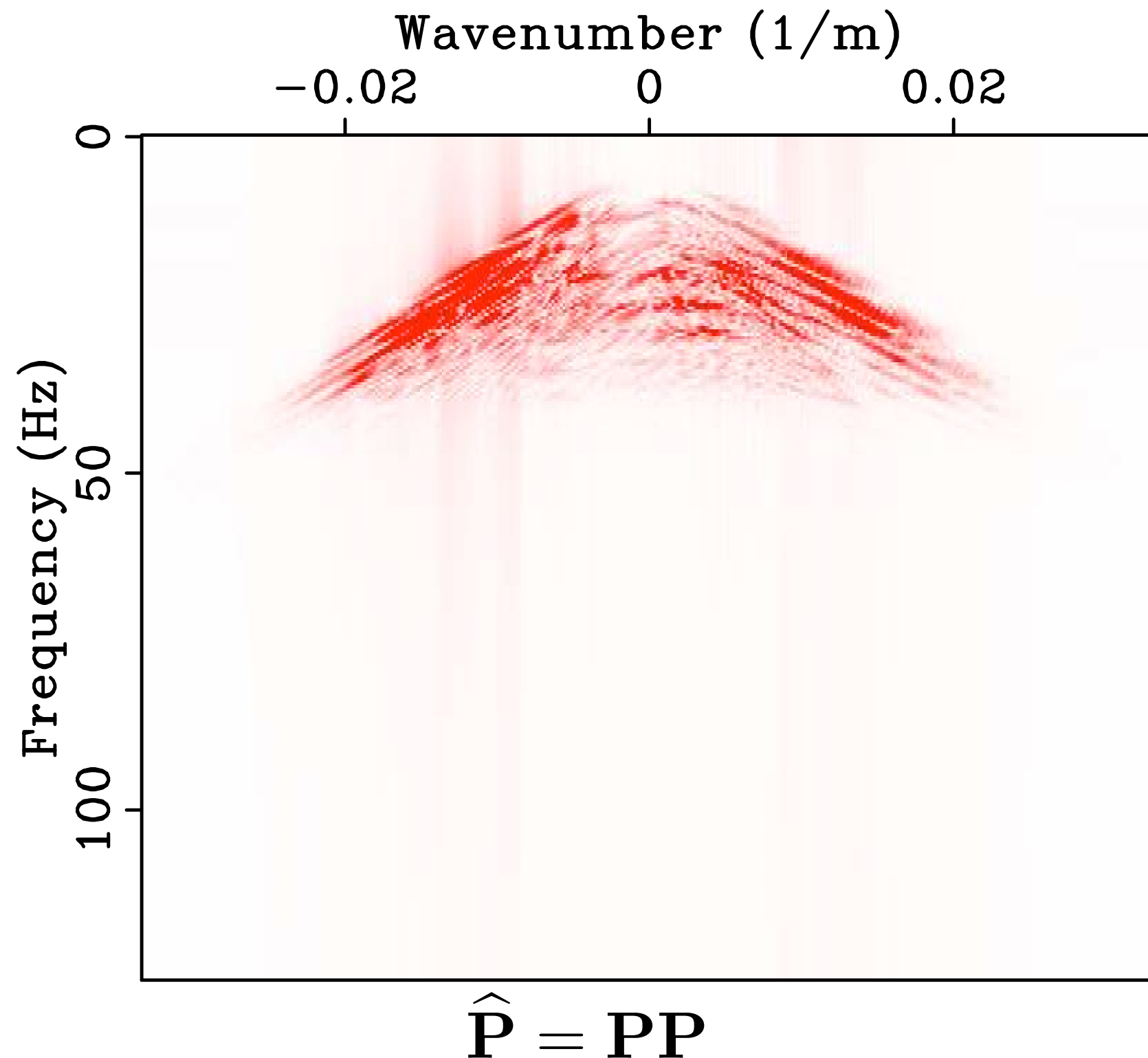
# Original data



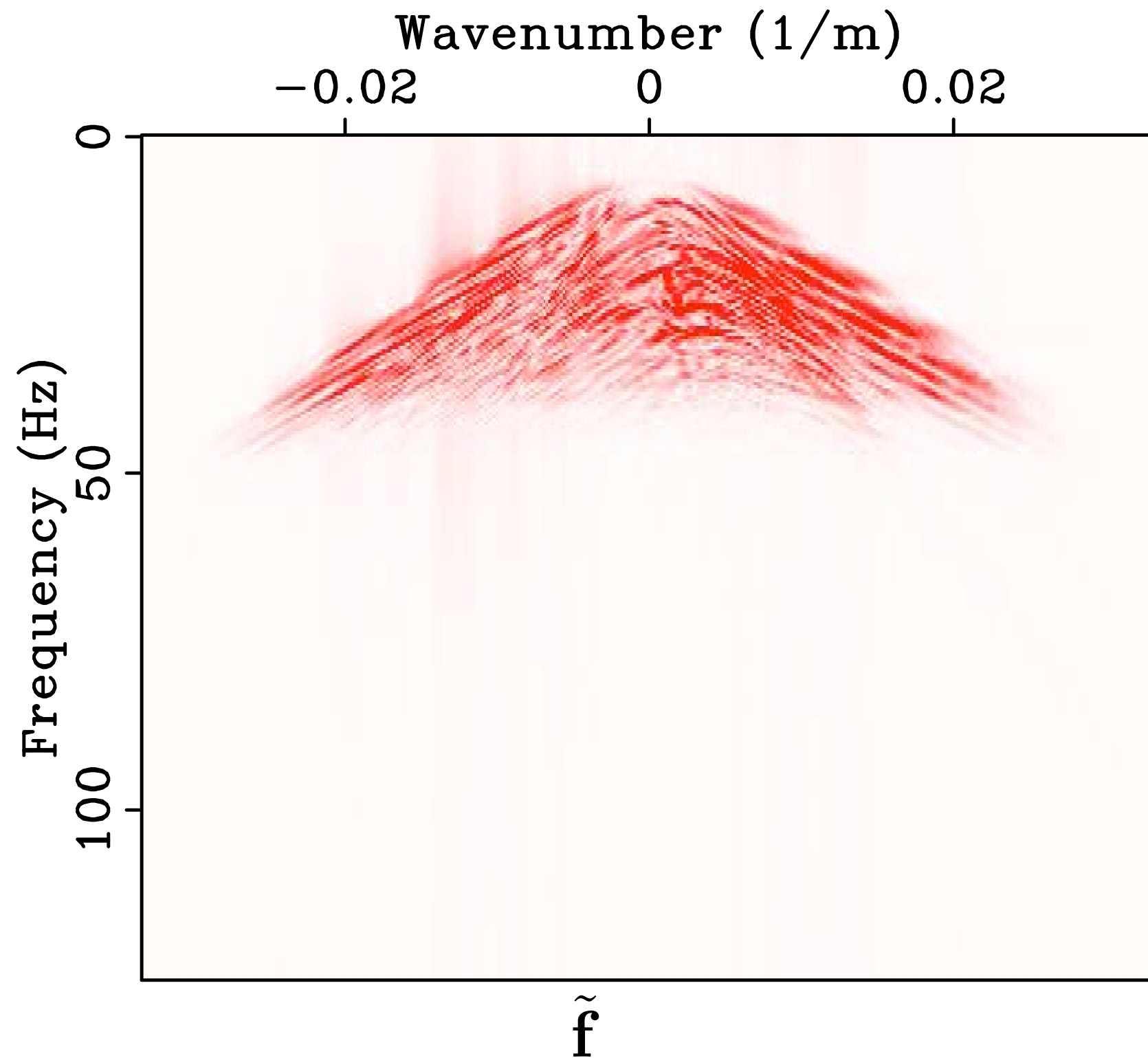
# Original data



# SRME predicted multiples



# Multiple estimate by dfCRSI



# Conclusions

## Focused CRSI

- improves the recovery and hence predicted multiples
- precursor of migration-based CRSI
- primary estimates have higher bandwidth (deconvolution of the source)

## deFocused CRSI

- improves the band width
- contains artifacts due to remnant multiple energy & X-terms

Curvelet-based approach improves the primary-multiple prediction.

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Dr. Verschuur for his synthetic data and the estimates for the primaries.

These results were created with Madagascar developed by Dr. Fomel.

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