A new paradigm for proppant schedule design

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Abstract

This study introduces a novel methodology for the design of the proppant pumping schedule for a hydraulic fracture, in which the final proppant distribution along the crack is prescribed. The method is based on the assumption that the particles have relatively small impact on the fracture propagation, unless they reach the tip region. This makes it possible to relate the proppant velocity to the clear fluid velocity inside the fracture, which is calculated assuming no proppant. Having the history of the clear fluid velocity distribution, the prospective proppant motion can be computed. Then, volume balance is used to relate the final concentration at some point inside the fracture to the corresponding input concentration at a specific time instant, which helps to avoid solving an inverse problem. One exceptional feature of the approach lies in the fact that it is applicable to multiple fracture geometries and can be implemented using various hydraulic fracturing simulators. To verify the technique, two fracture geometries are considered: Khristianovich-Zheltov-Geertsma-De Klerk (KGD) and pseudo-3D (P3D). It is shown that the developed approach is capable of properly estimating the pumping schedule for both geometries. In particular, the proppant placement along the fracture at the end of the pumping period, calculated according to the adopted proppant transport model, shows close agreement with the design distribution. The comparison with Nolte’s scheduling scheme shows that the latter is not always accurate, and cannot capture the essential differences between the schedules for the fracture geometries considered.

1 Introduction

Hydraulic fracturing (HF) is a process in which a viscous fluid injected into a fracture drives crack propagation. One of the most common applications is the stimulation of oil and gas wells [1], in which HF is used to break the rock to produce high conductivity channels. To avoid the fracture closing after pumping has stopped and the fluid has leaked off, various proppant additives are pumped at some stage. Despite the fact that many studies have been devoted to proppant transport modelling and investigating the effects of settling [2, 3, 4], only a few consider the design of a proppant schedule [5, 6, 7]. The appropriate proppant schedule is as important as the correct prediction of the fracture footprint, since it directly affects the proppant distribution inside the fracture, and thus influences the conductivity and the production rate.

One of the most common approaches in generating the pump schedule is the schedule due to Notle [6]. This is a very convenient option, as it provides an analytical formula for the schedule for a given efficiency, total pumping time, and a desired (uniform) concentration inside the fracture at the end of the job. The approach is based on the conservation of volume, and the estimation of the total volume of fluid that is leaked during the HF treatment to estimate the pad length. A power law-type schedule is then suggested and the exponent is calculated based on the proppant volume balance. Not being tied to any fracture geometry, this scheduling approach is considered applicable to multiple fracture geometries, such as PKN or radial. The universality, seeming reasonable accuracy, and the ease of use are, possibly, the main reasons why this scheduling is commonly used, see e.g. [1, 8]. There is an alternative method, developed in [7], which suggests using an iterative scheme together with an appropriate proppant transport model to solve an inverse
The procedure for the method, then Sections 3 and 4 illustrate the implementation respectively for the KGD and P3D fracture geometries, and, finally, Section 5 provides comparison between different schedules (including Nolte’s schedule) and discusses the applicability and possible extensions of the approach.

2 Idea behind estimation of pumping schedule

There are several things that have to be prescribed before attempting to obtain the pumping schedule: i) the geometry of the hydraulic fracture (HF), ii) the size of the fracture at the end of pumping, and iii) the desired proppant distribution inside the fracture at the end of the job. Here it is assumed that the properties of the rock, the fluid and the proppant, as well as the pumping rate are all known and fixed. The first item, i.e. the geometry, can be interpreted as the type of the HF model that is used for the design, such as KGD, radial, PKN, P3D, or a fully planar HF solver. The second item, i.e. the size of the fracture, is just the length (or radius) of the HF, that is determined during the design process. The last item, which concerns the desired proppant pattern inside the HF, could be a separate topic for research, since it is unclear what the optimal distribution of the proppant is that would give the best production rate, see e.g. [11] where the residual fracture opening supported by proppant is analyzed or [12] where the effect of the proppant distribution on the conductivity is studied. Since the investigation of the optimal proppant pattern is beyond the scope of this study, it is assumed that the desired distribution of the proppant concentration is given, and hence the schedule should be sufficiently adaptable to be able to accommodate any designed pattern. The main objective of this study is the calculation of the pumping schedule for given properties of the materials, HF geometry, the design length of the HF, and the desired proppant distribution at the end of the fracturing job.

To design the pumping schedule, it is imperative to know where the proppant would go. This can be achieved by estimating the proppant velocity inside HF as a function of time and space. At the same time, assuming that the proppant concentrations are sufficiently small, the proppant velocity can be related to the fluid velocity, where the latter is calculated assuming no proppant. So, the tentative procedure for the pumping schedule design is the following: i) run an appropriate HF solver without proppant, record time histories of all velocity components, width and fracture footprint, and find the time required to achieve the desired fracture size/footprint, ii) find the location of the proppant, injected at time instant \( t_i \) by “tracking” its position with time by integrating the actual velocity field, and iii) use conservation of volume to relate the input concentration (at any given time \( t_i \)) to the concentration at the end of the pumping. It is important to note, that this procedure is applicable to any HF solver and can be implemented as a separate module. Also, the solution for the proppant location resembles the Lagrangian approach used in continuum mechanics, and has a clear advantage over the “classical” Eulerian approach (i.e. solving the advection equation for proppant transport) by making it possible to relate the final proppant distribution to the schedule concentration. Another important advantage of the proposed approach is that any desired concentration distribution can be achieved through the appropriate scheduling, which opens interesting research possibilities for the optimal proppant placement inside the HF.

To illustrate the methodology, two fracture geometries are examined: KGD and P3D. The schedules for
both are verified numerically by using a proppant transport model described in [13].

3 Pumping schedule for KGD fractures

To address the proppant scheduling problem in a more straightforward way, first the one-dimensional KGD fracture geometry is analyzed. Given the properties of the rock and the fracture design length \( l_e \), an appropriate HF simulator (without the proppant) can be used to deduce the total injection time \( t_e \), as well as to record the histories of the fracture footprint \( l(t) \), width \( w(x,t) \), and average fluid velocity \( V_f(x,t) \). Due to the nonuniform distribution of the proppant across the fracture width, the average proppant velocity is higher than that of the slurry. It follows from [13], in the limit of low concentrations, that the ratio between these two corresponding velocities is \( \beta = 1.2 \). This can be understood in the following way: the proppant tends to concentrate near the centre of a channel, so it flows with nearly the maximum velocity, which is always higher than the average velocity in the channel. By denoting the position of the particles at time instant \( t_i \), injected at \( t_i \), by \( x(t_i) \), the governing equation for the proppant front is

\[
\frac{\partial x}{\partial t} = \beta V_f(x,t), \quad x(t_i) = 0, \quad t_i \leq t \leq t_e,
\]

where the parameter \( t_i \) enters the problem through the initial condition, so that (1) is actually an ordinary differential equation. The solution of (1) allows us to find \( x(t_i) \), i.e. the current location of the proppant, which is injected at any time instant \( t_i \). Fig. 1 shows the schematics of the proppant front movement in the fracture. The schedule for the proppant injection concentration is shown in the top right picture, and the red area is proportional to the total amount of proppant pumped during the interval \( \Delta t \), which is \( \phi_0 Q_0 \Delta t \), where \( Q_0 \) is the slurry injection rate. This proppant is schematically shown at \( t = t_i \) in the top left picture. At this time instant, the particles leave the borehole and enter the fracture. The same proppant (half of it due to the symmetry), but now at time \( t > t_i \), and inside the fracture, is shown in the bottom picture. To calculate the “pad” time, \( t_p \), or the time up to which pure fluid is pumped and before the proppant injection (see Fig. 1), one may use the solution for \( x(t_i) \) and require

\[
x(t_e, t_p) = l_e,
\]

i.e. determine \( t_p \) by requiring that the particles reach the crack tip by the end of pumping. Note that for a more accurate prediction for bigger proppants, it is reasonable to replace \( l_e \) in (2) by a smaller length, \( \overline{l}_e \), where the latter is calculated based on

\[
w(\overline{l}_e, t_e) = 2a,
\]
where $a$ is a particle radius. This ensures that there is no proppant in the places where it cannot physically fit. Given the desired or design proppant concentration distribution inside the fracture $\phi_d(x)$, the balance of mass can be used to obtain
\[
\frac{1}{2} Q_0 \phi(t_i) \Delta t = w(x(t_e, t_i), t_e) [x(t_e, t_i) - x(t_e, t_i + \Delta t)] \phi_d(x(t_e, t_i)),
\]
where the left side reflects the volume of the proppant injected between $t_i$ and $t_i + \Delta t$ (the 1/2 factor comes from the symmetry), while the right side calculates the same volume at the end of the fracturing job. By taking a limit of $\Delta t \rightarrow 0$ in (3), the result can be simplified to
\[
\phi(t_i) = -\frac{2w(x(t_e, t_i), t_e)}{Q_0} \frac{\partial x(t_e, t_i)}{\partial t_i} \phi_d(x(t_e, t_i)), \quad t_i > t_p,
\]
which allows us to calculate the proppant schedule for any desired concentration distribution along the fracture. Note that due to the 1D nature of the geometry, $Q_0$ is the injection rate per unit length, i.e. measured in m$^2$/s. Also note that any function $\phi_d(x)$ can be used in (4), i.e. any proppant distribution can be achieved without introducing extra complexities.

**Numerical examples.** From the numerical implementation point of view, the task of finding the pumping schedule according to (4) requires an appropriate numerical scheme for the solution of (1), numerical differentiation in (4), as well as an extensive use of interpolation. The interpolation is used in both (1), since $V^I(x, t)$ is computed for a discrete set of $x$ and $t$, and (4) for the evaluation of $w(x(t_e, t_i), t_e)$ and $\partial x(t_e, t_i)/\partial t_i$ (the $x$ values are first interpolated and then numerically differentiated). To achieve high accuracy and to preclude possible oscillations (that can be caused by spline interpolation), the built-in Matlab function “PCHIP” (Piecewise Cubic Hermite Interpolating Polynomial) is used for the interpolation. To deal with the numerical solution of (1), the 4th order Runge-Kutta method is used.

To verify the proposed approach, an example problem is considered. The set of chosen parameters is $E' = 25$ GPa for the plane strain modulus, $\mu = 0.1$ Pa·s for the shear viscosity of the fracturing fluid, $Q_0 = 10^{-3}$ m$^2$/s for the inlet flux, $C' = 5 \times 10^{-5}$ m/s$^{1/2}$ for the Carter’s leak-off coefficient, $K_{ic} = 1$ MPa·m$^{1/2}$ for the fracture toughness and $a = 0.2$ mm for the particle radius. The design length of the fracture is set to $l_e = 100$ m, while the target concentration is considered to be uniform and is equal to $\phi_d = 0.2 \times \phi_m$. Here $\phi_m = 0.585$ is the maximum volume concentration that can be achieved [14, 13]. Note that, assuming the proppant mass density of 2300 kg/m$^3$, this concentration can be translated to approximately 2.6 lbs/gal. The HF simulator for KGD fractures, described in [13], is used to calculate the duration of the HF treatment and to record the history of the average velocity without proppant, which is then used to calculate the schedule. Given the pumping schedule, the same simulator is used for the verification of the design.
Fig. 2 shows the schedule, calculated according to equation (4), and the concentration distribution along the fracture at the end of the simulation (black solid lines). In addition, these results are compared to Notle’s scheduling [6] (blue solid lines). Note that Notle’s model including the correction for the pad length is used, see the Appendix for the description of Notle’s pumping schedule. Despite using the correction, the prediction of Notle’s model notably underestimates the pad size, which causes premature tip screen-out leading to a fracture length under 80 m (as opposed to the 100 m designed length). At the same time, the current approach shows better performance, as it just slightly distorts the designed 100 m fracture length, and produces a nearly uniform concentration distribution along the crack. Note that the spikes in the concentration correspond to plug formation and reflect its size. For the current schedule, the plug just started to form and therefore does not significantly affect the fracture propagation. The “dip” at the proppant distribution is due to both, numerical diffusion and the effect of coupling between proppant transport and HF propagation, namely the change of the slurry viscosity with concentration. Note that the schedule for the current approach has a region in which the concentration exceeds $\phi_d$, while the resultant concentration inside the fracture is approximately equal to $\phi_d$. This is due to the fact that the ratio between particle and slurry velocities is $\beta = 1.2$, which effectively reduces the mixture concentration by $\beta$ when it reaches a steady flow (i.e., according to the proppant transport model assumptions, when it enters the fracture).

To highlight the versatility of the proposed scheduling procedure, Fig. 3 shows the pumping schedule and resultant proppant distribution for regular ($\phi_d = 0.2\times\phi_m$) and small ($\phi_d = 0.02\times\phi_m$) concentrations for a “zebra” configuration. This configuration suggests that at the end of the pumping, the proppant should be concentrated in three equal stripes, placed equidistantly inside the crack. This configuration is inspired by Schlumberger’s HiWAY proppant placement, for which the particles are injected in pulses, that produce bridges at the end of pumping. This increases the permeability, as hydrocarbons can flow between the bridges. In practice HiWAY technology suggests numerous pulses, while only three are considered here since a large number of pulses would require a much finer mesh, which is computationally demanding. As can be seen from the results, the calculated schedule indeed leads to the desired proppant placement within adequate accuracy. As was the case for Fig. 2, the discrepancy for the regular concentration comes mainly from numerical diffusion and the coupling between HF propagation and proppant transport. Since the coupling is minimal for small concentrations, the blue line (which corresponds to the small concentration solution) can be used to estimate the discrepancy caused by the numerical scheme only. Hence, the difference between the solutions for regular and small concentrations indicates the effect of coupling. Even though the effect of coupling leads to visible differences, the overall accuracy of the approach is still satisfactory. It is important to note that higher concentrations can lead to bigger differences, which is the limitation of the proposed design approach. At the same time, it is remarkable that the higher viscosity of the slurry (due to the presence of proppant) does not alter the final fracture length appreciably. This is because the biggest pressure gradients are near the fracture tip, where the width is minimal, while the rest of the fracture is
subject to smaller pressure gradients. When the proppant is introduced, the higher viscosity of the slurry perturbs mostly the small pressure gradients, which are away from the tip, and thus does not significantly affect the pressure distribution and fracture propagation. When the proppant eventually reaches the tip region, it starts to disturb the fracture, but it is already too late since the fracturing job is over once the proppant reaches the crack tip. In other words, even though the particles change the viscosity of a slurry, the time interval during which the proppant can affect the fracture behaviour is small, which makes the consequences of the coupling effect relatively insignificant.

Gravitational settling. One assumption that is implicitly stated is that the fracture is symmetric, which in particular implies that there is no gravitational settling. As shown in [13], gravitational settling may break the symmetry as the proppant sinks towards one side of the fracture and causes early tip screen-out. It is possible to account for gravitational settling in the schedule, but in this case the design should be performed for the downward fracture (i.e. finish pumping when the proppant reaches the downward moving crack tip). In the opposite case, the downward fracture could be screened, which can affect the other portion of the fracture and cause inapplicability of the scheduling approach. When gravitational settling is present, equation (1) should be replaced with

$$\frac{\partial x}{\partial t} = \beta V_f(x,t) + V_s, \quad x(t_i, t_i) = 0, \quad t_i \leq t \leq t_e,$$

where $V_s$ is the settling velocity, which can be estimated for small particle concentration as

$$V_s = \frac{2a^2}{9\mu f} (\rho_p - \rho_f) g,$$

where $a$ is a particle radius (it is assumed that all particles are spherical and equal in size), $\rho_p - \rho_f$ is the difference between particle and fluid mass densities, $\mu_f$ is the fluid viscosity, while $g$ is the gravitational acceleration. As with (1), equation (5) can be solved numerically and the solution can be used to calculate the schedule according to (4).

4 Pumping schedule for P3D fractures

The design of a pumping schedule for the P3D geometry [15] is conceptually similar to that for the KGD model, but there are several notable differences. One of them comes from the fact that a line source is used in the P3D model, as opposed to a point source. At the same time, another difference is related to the two-dimensional nature of the proppant flow, and the presence of the vertical velocity component. For the purpose of schedule calculation, it is assumed that the gravitational settling is negligible, and hence the vertical component of the velocity is zero along $x$ axis ($z = 0$), see Fig. 4. In this case, the proppant that is pumped in the vicinity of $z = 0$ remains close to the $x$ axis during the fracture growth. This fact allows us to “track” the proppant along the $x$ axis and not to consider its vertical migration. The schematics of the P3D fracture together with the “proppant tracking box” are shown in the Fig. 4. The function $x(t, t_i)$ has the identical meaning as for the KGD fracture, it calculates the position of the proppant at time $t$, that was injected at time $t_i$. As will be shown later, the height of the box, $\Delta z(t)$, varies with time due to the presence of the vertical velocity component. Since the particles can easily be traced along the $x$ coordinate, it is natural to establish the target concentration along the $x$ axis as well. As for the KGD fracture, the simulations without proppant are first performed, and for given problem parameters and design length $l_e$, the pumping time $t_e$ is calculated, and the histories of the appropriate quantities are recorded. Knowing the history of the $x$ component of the average fluid velocity at $z = 0$, $V_x^f$, the motion of the particles can be described by solving

$$\frac{\partial x}{\partial t} = \beta V_x^f(x,t), \quad x(t_i, t_i) = 0, \quad t_i \leq t \leq t_e,$$

which is identical to (1). Also, the “pad” time, $t_p$, is calculated in a similar fashion as

$$x(t_e, t_p) = l_e,$$
where the final length of the fracture $l_e$ can be replaced by $\bar{l}_e$ for bigger proppants. Here $\bar{l}_e$ is determined from

$$w(\bar{l}_e, t_e) = 2a,$$

where $w(x, t)$ is the fracture width along $x$ axis for $z = 0$. With reference to Fig. 4, the volume of the proppant at the injection point and at the end of the fracturing can be equated to find

$$\phi(t_i) V^f_x(0, t_i) w(0, t_i) \Delta z(t_i) \Delta t = \left[ x(t_c, t_i) - x(t_c, t_i + \Delta t) \right] w(x(t_c, t_i), t_c) \Delta z(t_c) \phi_d(x(t_c, t_i)),$$  \hspace{1cm} (7)

where $\phi_d(x)$ is the design concentration distribution along the $x$ axis, $V^f_x(0, t_i)$ is the average fluid velocity at the inlet (and $z = 0$), while $w(0, t_i)$ is the corresponding width of the fracture at that point. By noting that

$$\frac{\Delta z(t_c)}{\Delta z(t_i)} = \beta \int_{t_i}^{t_c} \frac{\Delta V^f_x(x(t, t_i), t)}{\Delta z(t_i)} \, dt,$$  \hspace{1cm} (8)

where $\Delta V^f_x$ is the difference between the vertical components of the fluid velocity at the top and bottom of the “proppant tracking box”, equation (7) can be simplified to

$$\phi(t_i) = - \frac{w(x(t_c, t_i), t_c)}{w(0, t_i) V^f_x(0, t_i)} \frac{\partial x(t, t_i)}{\partial t_i} \left( 1 + \beta \int_{t_i}^{t_c} \frac{\partial V^f_x(x(t, t_i), t)}{\partial z} \, dt \right) \phi_d(x(t_c, t_i)).$$  \hspace{1cm} (9)

Note that $\partial V^f_x/\partial z$ is evaluated at $z = 0$ and its history has to be precomputed in addition to the history of the horizontal velocity component $V^f_x$. It is important to understand that $\beta$ should not enter on the left side in (7), since the ratio between the proppant and slurry fluxes is $\phi(t_i)$, while the slurry flux is proportional to $V^f_x(0, t_i)$. At the same time, since the proppant’s vertical velocity (as opposed to the fluid’s) is responsible for the vertical “box” growth, $\beta$ appears in (8). As with the expression for the KGD fracture (4), the relation (9) can be used to design a proppant schedule for any target concentration profile along the $x$ axis $\phi_d(x)$, which adds versatility to the approach.

**Numerical examples.** To illustrate these developments for P3D fractures and to check the validity of (9), several numerical examples are considered. The parameters used for the calculations are $E' = 25$ GPa for the plane strain modulus, $\mu = 0.1$ Pa·s for the shear viscosity of the fracturing fluid, $Q_0 = 10^{-2}$ m$^3$/s for the total inlet flux, $H = 25$ m for the reservoir layer, $\Delta \sigma = 2.5$ MPa for the magnitude of the stress barriers $C' = 5 \times 10^{-5}$ m/s$^{1/2}$ for the Carter’s leak-off coefficient, $K_{1c} = 1$ MPa·m$^{1/2}$ for the fracture toughness, $a = 0.2$ mm for the particle radius, $g = 9.8$ m/s$^2$ for the gravitational acceleration and $\rho^p - \rho^f = 1300$ kg/m$^3$. 

![Figure 4: Schematics of the P3D fracture with the proppant “tracking” region.](image)
for the difference between particle and fluid mass densities, see Fig. 4 and [15, 13] for more details. Note that
the gravitational settling is formally included in the simulations, but, since a relatively small particle size is
considered, there is almost no distortion in the symmetry due to settling. As noted in [13], the dimensionless
parameter that determines the settling extent is
\[ G_s = \frac{16\Delta \rho a^2 g Q_o E^3 (t_e - t_p)}{3\Delta \sigma^4 H^4}, \]
where \( t_e \) is the total pumping time, while \( t_p \) is the time at which the proppant is first injected, see the top right
picture in Fig. 1. The parameter \( G_s \) reflects the ratio between proppant travel time and the settling time,
so when \( G_s \gg 1 \), then settling occurs before the end of the pumping, while if \( G_s \ll 1 \), then, practically, the
gravity does not alter the particle distribution. For the set of parameters under consideration, \( G_s = 0.035 \),
which indeed suppresses the effect of settling. As with the KGD fracture geometry, the design length of the
fracture is set to \( l_c = 100 \) m, while the target concentration is considered to be uniform and is equal to
\( \phi_d = 0.2 \times \phi_m \), where \( \phi_m = 0.585 \). The HF simulator for P3D fractures, described in [13], is used to calculate
the duration of the HF treatment and to record the history of the average \( x \) component of the velocity and
the derivative of the vertical velocity component (assuming no proppant), which are then used to calculate
the schedule using (9). Numerical techniques, that were used for dealing with the KGD fracture scheduling
in Section 3, are utilized for the numerical solution of (6) and for the interpolation, which are both nessecary
for the evaluation of (9). Given the pumping schedule, the same HF simulator for the P3D fracture, this
time with proppant, is used for verification purposes.

To evaluate the accuracy of the proposed scheduling, the top left picture in Fig. 5 compares the schedule
that is calculated according to (9) with that suggested by Nolte [6]. The differences are similar to those found
for the KGD model, see Fig. 2, namely, Nolte’s approach underestimates both the time of the first proppant
injection, \( t_p \), and the maximum concentration near the end of pumping. The consequences are similar as well,
i.e. Nolte’s schedule leads to premature tip screen-out (which in turn leads to a shorter fracture length) and
smaller concentration near the inlet, see Fig. 5. With regard to the accuracy of the current approach, there
is also a “dip” near the fracture tip and a small plug starts to form thereafter. Despite the fact that P3D
and KGD consider different types of fractures, the reasons for the “dip” are similar, namely, the coupling
between proppant transport and HF propagation and numerical diffusion. Note that the term with the
vertical velocity derivative in (9) plays an important role, and its absence can lead to observable inaccuracy
of the final concentration, while the \( t_p \) stays unaffected. Of course, the degree of influence depends on the
problem parameters, and in particular on the fracture growth in the vertical direction and the specificity
of the line source implementation in the HF simulator (i.e. the variation of source intensity versus \( x \) at
\( x = 0 \)). In addition, the developed proppant plug on the bottom right picture in Fig. 5 has a strange shape,
namely, the particles are concentrated near the top and the bottom of the fracture, leaving the central part
underpropped. Unfortunately, this is due to an inaccuracy in the P3D model [15, 13], in which a uniform
pressure along every vertical cross-section is assumed, which in turn leads to unrestricted motion of the
shurry in the vertical direction. The fact that the shurry is transported to the tip region mainly through
the central part of the fracture, and the leak-off occurs uniformly along the height, together lead to strong
off-central vertical velocities in the tip region, which bring the proppant away from the centre to the sides
of the fracture. This feature is more pronounced for the smaller particle size as considered in this paper,
while bigger particles form a plug some distance away from the crack tip, and are influenced to a much lesser
extent, see [13]. Note that even though Fig. 5 indicates smaller sensitivity of the proppant placement to
the schedule type, than Fig. 2, one should always keep in mind that those are the examples for one set of
parameters, and some variations are possible for different problem parameters.

To show the capabilities of the proposed scheduling paradigm, Fig. 6 shows the pumping schedule and
the results of the simulations for regular (\( \phi_d = 0.2 \times \phi_m \)) and small (\( \phi_d = 0.02 \times \phi_m \)) concentrations for P3D
fractures with “zebra” distributions of proppant. As for the KGD geometry, “zebra” configuration is defined
by requiring a specified proppant placement in three equal equidistant stripes along the \( x \) axis by the end
of the simulation. Due to the planar nature of the P3D model, the stripes form an interesting shape, which
reflects the velocity pattern inside the fracture. The comparison between the solutions for small and regular
concentrations allows us to estimate the effect of coupling between proppant transport and HF propagation.
This coupling is more pronounced than for the KGD geometry (see Fig. 3) and leads to some notable visual
Figure 5: Top left: comparison between the current and Nolte’s (efficiency $\eta = 0.39$) pumping schedules for P3D fracture. Top right: comparison between the normalized proppant concentration along the $x$ axis for the current and Nolte’s schedules with the design concentration distribution. Bottom left: footprint of the P3D fracture for the current schedule with the colour indicating the normalized proppant concentration $\phi/\phi_d$, so that the colour associated with 1 corresponds to the desired concentration specified in the design. Bottom right: footprint of the P3D fracture for Nolte’s schedule with the colour indicating the normalized proppant concentration $\phi/\phi_d$. 
Figure 6: Top left: pumping schedule for “zebra” proppant distributions for the P3D fracture. Top right: comparison between normalized proppant concentration along the $x$ axis for regular and small concentrations with the design concentration distribution. Bottom left: footprint of the P3D fracture for regular concentration with the colour indicating the normalized proppant concentration $\phi/\phi_d$. Bottom right: footprint of the P3D fracture for a small proppant concentration with the colour indicating the normalized proppant concentration $\phi/\phi_d$. 
distortions of the proppant pattern, see the bottom pictures in Fig. 6. Also note that both bottom pictures are not perfectly symmetric, which is due to the presence of small gravitational settling.

5 Comments

Comparison between different schedules. While Figs. 2 and 5 show the comparison between Nolte’s schedule and the predictions based on the equations (4) and (9), it is instructive to make a comparison for a broader range of efficiencies \( \eta \). To accomplish this goal, Fig. 7 shows the comparison between different schedules for \( \eta = 0.9, 0.5, \) and \( 0.1 \). Nolte’s schedules are compared to the corresponding KGD and P3D schedules, as well as to the schedule for a KGD fracture, which is affected by the presence of symmetric stress barriers. The stress barriers are placed 60 m from the inlet and have a magnitude \( \Delta \sigma = 2 \) MPa. To achieve the desired value of the efficiency for both KGD and P3D models, the Carter’s leak-off coefficient is adjusted. The comparison shows significant variability among different models. As mentioned previously, Nolte’s model underestimates \( t_p \) - the time instant at which the proppant is introduced, and the maximum concentration near the end of pumping. But what was not clear so far, is the big difference between the schedules for the KGD and P3D geometries, which clearly indicates that there is no universal schedule that can work for all fracture geometries. Even with the same fracture type (KGD), the introduction of the stress barriers affects the schedule to some extent. This also supports the fact that a universal schedule can not be generated. One peculiar feature that can be seen from Fig. 7 is the hierarchy between the schedules, namely Nolte’s schedule suggests the earliest proppant injection, followed by the corresponding KGD and, finally, P3D schedules. The difference between Nolte’s and the KGD schedules can be related to the ratio between particle and slurry average velocities, \( \beta = 1.2 \), which allows the proppant to reach the crack tip faster. At the same time, since the P3D model has another dimension, it effectively introduces another factor, which is the ratio between the peak proppant velocity and the average proppant velocity with respect to the vertical, i.e. the \( z \) direction. This can be seen on the bottom pictures in Fig. 6. The proppant at \( z=0 \) is already at the tip, but, on average, the boundary of the corresponding first “zone” of the proppant is some distance away form the tip. Note that the ratio between the peak proppant velocity and average slurry velocity is important, since the first “moves” proppant forward, while the second is responsible for the fracture growth. This ratio is higher for P3D than for KGD fractures, which allows proppant to reach the crack tip notably faster for a P3D geometry. While the differences between the schedules are prominent for high efficiencies, they become less pronounced for smaller efficiencies, see Fig. 7. Regarding the implementation, Nolte’s schedule is the easiest to deal with since analytical formulas are used, at the same time, once executed, the current approach makes it possible to calculate a reasonably accurate schedule on a computer in mere seconds for KGD and tens of seconds for P3D fractures.
Assumptions and limitations. Despite the fact that the scheduling has been verified numerically, it is essential to understand all the assumptions behind the model. First, it is assumed that the presence of proppant does not disturb fracture propagation. This is a critical ingredient for developing a pumping schedule, but, at the same time, it limits the applicability to relatively small concentrations, for which the viscosity of the slurry is not significantly perturbed by particles. However, for the design proppant concentration $\phi_d = 0.2 \times \phi_m$, which is used for the verification, the change in the apparent viscosity is approximately 20%, see the proppant transport model in [13]. Despite this notable change in the viscosity, Figs. 2 and 5 show good agreement between the designed proppant placement and the one that is calculated using the generated schedule. As discussed before, this is due to the fact that the particles spend little time at the near-tip region, and since this part of the fracture is primarily responsible for the propagation, the higher viscosity of the slurry does not alter the fracture footprint much. However, due to the nonlinear variation of the slurry viscosity versus particle concentration, higher design concentrations could introduce bigger discrepancies, so it is essential to verify the schedule via numerical simulation. In addition to the reduced accuracy for higher concentrations, the proposed schedule cannot be used for tip screen-out applications, since the proppant plug near the crack tip significantly changes the fracture behaviour relative to the corresponding fracture with pure fluid.

The calculation of a pumping schedule, that is proposed in this paper, is always consistent with some hydraulic fracturing model, such as KGD or P3D model. For this reason, it is clear that the correctness of the HF model plays a crucial role in scheduling. This factor should not be underestimated, as, according to Fig. 7, there might be a significant difference among various fracture models. In addition, the scheduling is tailored to the specific proppant transport model [13], which, in particular, provides the value for $\beta$, see (1). The validity and the applicability range of this model are also essential for scheduling.

Possible extensions. One of the biggest advantages of the proposed scheduling procedure lies in its applicability to multiple hydraulic fracturing models. As an example, a HF simulator with a more accurate leak-off model can be used for the design. Another possibility is to use a HF simulator that accounts for turbulent motion near the wellbore. In the latter case, it might be necessary to introduce $\beta(x)$ in (1), as the proppant distribution along the width of the fracture for turbulent motion won’t resemble its laminar analog, and thus the proppant to slurry average velocity ratio could change.

Since the accuracy of the scheduling approach deteriorates for higher proppant concentrations, it might be useful to adjust the average velocity history iteratively. In other words, given an initial guess for a schedule, one may run the appropriate HF simulator with proppant and record the history of the average velocity. Then, this velocity history can be used to recalculate a schedule. This process can be repeated until the results converge. Note that the proppant velocity history can be recorded right away, which eliminates the use of $\beta$ in (1). The current design approach therefore represents the first step in such an iterative process. The iterative approach, although effective, is computationally demanding, and sacrifices the efficacy of the original non-iterative methodology.

6 Summary

This paper introduces a universal approach for designing a proppant schedule, which complements a given hydraulic fracture simulator. The main idea is based on the assumption that the proppant particles do not affect the fracture propagation until they reach the tip region. This makes it possible to precompute the history of the velocity distribution assuming no proppant, which is then used to evaluate the prospective proppant movement. Once the prospective movement is calculated, volume balance is used to relate the desired proppant concentration at a given point in space (concentration does not have to be uniform) to a corresponding input concentration at a certain time instant. In this way, the schedule is obtained without solving an inverse problem. The scheme is illustrated for two fracture geometries, namely, KGD and P3D. It is shown that Nolte’s schedule suggests earlier proppant injection, which leads to a premature tip screen-out, while the current approach produces more accurate results and does not alter the desired final fracture length. In addition, the effect of coupling between proppant transport and hydraulic fracture propagation is studied. As expected, smaller proppant concentrations lead to more accurate results, while the agreement
for higher concentrations is still adequate. The comparison between the schedules for different fracture geometries together with Notle’s schedule for different efficiencies shows that knowledge of the efficiency alone is not sufficient to predict the schedule. In particular, a noticeable difference between all of the models is observed. This demonstrates that there is no universal schedule that is applicable to multiple fracture geometries, while, on the other hand, the proposed technique is able to calculate the pumping schedule in an accurate and fast manner.

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**References**


Appendix: Nolte’s pumping schedule

This appendix summarizes Nolte’s pumping schedule that is used in this paper for comparison purposes. As follows from [6], the proppant concentration, which should lead to uniform distribution of proppant at the end of the fracturing job, can be written as

\[ \phi(t) = \phi_d \left( \frac{t - t_p}{t_e - t_p} \right)^{1 - \eta - f_d/\eta}, \quad t > t_p, \]

where \( \phi_d \) is the design concentration, \( t_e \) is the total injection time of the fluid and slurry, \( t_p = \left( (1 - \eta)^2 + f_d \right) t_e \) is the time at which proppant is introduced, \( \eta \) denotes the efficiency calculated as the ratio between the volume of the fracture and the total volume that is injected, and \( f_d = 0.05 \) is a correction factor. As discussed in [6], this correction factor makes it possible to match the data obtained through a numerical simulations. Note that it is implicitly assumed that \( \phi(t) = 0 \) for \( t < t_p \).