Garage and Curbside Parking Competition with Search Congestion*

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Abstract

We consider a parking market with privately operated parking garages, publicly operated curbside parking, and drivers who differ in parking duration. In equilibrium, long-term and short-term parkers can allocate themselves to garages and curbside parking in various ways. The equilibrium is generally inefficient because garage operators exercise market power, and drivers have to search for curbside parking which creates a search cost externality. A city planner can achieve the social optimum without regulating garage prices if it can charge differentiated curbside parking fees. Many cities still have parking meters that can only levy uniform fees and it is very costly to replace them. However, numerical results show that the efficiency loss due to uniform pricing is modest in most cases.

JEL Classification: D62; L13; R41; R48

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1 Introduction

Downtown parking markets can be rather complex. Parking is often available both on and off the street. Parking garages provide bulk capacity at discrete locations, and can extend over multiple storeys above or below ground. The friction of space gives them market power. Curbside parking, on the other hand, is located more widely but it is frequently in short supply and difficult to find. According to some estimates, cruising for parking accounts for roughly thirty percent of traffic at certain times of day (Shoup, 2005, 2006). Time spent searching for parking increases the full price or generalized cost of curbside parking, and limits the degree to which curbside parking constrains garages’ market power.

Garage and curbside parking differ in how they are priced. Garages generally cater to drivers who park for different lengths of time, and they usually charge hourly parking fees that vary with parking duration. Curbside parking is typically priced at a uniform hourly rate in North American cities where conventional parking meters are used. However, nonlinear pricing is sometimes practiced in cities where labor is relatively cheap. For example, in Istanbul employees collect parking fees using hand terminal technology and the hourly fees vary by parking duration. Finally, administration arrangements vary. Curbside parking is publicly operated in most cities whereas garage parking can be publicly or privately operated and/or regulated. For example, some Dutch cities such as Maastricht and Almere regulate garage parking fees while garages in London and Boston are free to choose their prices.

In this paper we study downtown parking markets in which spatial competition between garage and curbside parking, nonlinear pricing, and curbside parking search congestion are simultaneously at play. To facilitate analysis, the model is kept simple by treating total parking demand as fixed, ignoring through traffic congestion, and considering only two types of individuals that differ in the amount of time they wish to park. Nevertheless, curbside parking search congestion creates an interdependence between parking submarkets and non-convexities in garages’ profits, and the derivation of market equilibria in this setting is new to the spatial competition literature. In such an environment, we attempt to answer some questions about downtown parking markets: How does competition between parking garages play out when curbside parking is available as a substitute? How can garage parking fee schedules be explained? How should curbside parking fees be set to control cruising congestion and parking garage market power? Is a uniform hourly fee optimal, or should hourly fees be varied with parking duration? Is regulation of garage parking necessary to achieve a social optimum, or can curbside parking fees do the job?

Several strands of literature cover part of the ground required to address these questions
(see Arnott, 2011 and Inci, 2015 for comprehensive literature reviews). Some studies consider curbside parking in isolation (Arnott and Inci, 2006, 2010). Others incorporate garage parking but omit heterogeneity with respect to parking durations (Arnott and Rowse, 2009; Arnott, Inci and Rowse, 2015). Parking duration is considered by Glazer and Niskanen (1992) and Calthrop and Proost (2006), and parking time limits by Arnott and Rowse (2013). Spatial competition between parking garages has been studied (Arnott and Rowse, 1999, 2009; Anderson and de Palma, 2004, 2007; Calthrop and Proost, 2006) but without including price discrimination. Yet other studies analyze parking and pricing of parking when traffic congestion occurs at a bottleneck (Arnott, de Palma and Lindsey, 1991; Zhang et al., 2005; Fosgerau and de Palma, 2013). Shoup (2005, 2006), Arnott and Inci (2006, 2010), and Arnott, Inci and Rowse (2015) emphasize the importance of curbside parking search congestion externalities in downtown districts, and van Ommeren, Wentink, and Dekkers (2011) and van Ommeren, Wentink, and Rietveld (2012) estimate the externalities empirically. Perhaps the most thorough analysis of parking market competition to date is by Arnott (2006) who analyzes spatial competition between parking garages in his core model, which he later enriches to include curbside parking. However, he assumes that drivers are identical and he does not consider price discrimination.

Understanding the effects of driver heterogeneity and price discrimination on downtown parking markets is important. It highlights the market power of parking garages that should be considered in designing optimal parking policy. For example, a parking garage may increase the parking fee it charges to short-term parkers, some of who will then park on the curb and increase curbside parking search congestion. This in turn increases demand from long-term parkers for parking in the garage, which allows the garage to charge a higher fee to them. Such effects cannot be studied in models with identical drivers and no price discrimination. Another important insight is that if parking garages price discriminate according to parking duration, it is welfare-enhancing to differentiate curbside parking fees as well. Indeed, in our model the social optimum can be achieved without any regulation of garage parking fees by differentiating curbside parking fees.

Spatial competition and price discrimination have been extensively studied in the industrial organization literature (see Gabszewicz and Thisse (1986), Varian (1989), and Stole (2007) for literature reviews). Spatial competition models such as Salop’s (1979) allow for the possibility that some potential customers choose not to buy a product or service from any

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1 Although parking meters in many North American cities can only charge uniform fees, the practice of not charging for parking in the evening introduces an element of second-degree price discrimination. Time limits on parking are common in other countries. However, varying curbside parking rates in hourly increments according to duration of stay is very rare.
firm, but select an outside option instead. These models can be adapted to the downtown parking market by treating parking garages as firms offering services that differ by location, and curbside parking as an outside option that is ubiquitous. The standard models typically assume that utility from the outside option is exogenous. However, in the parking market expected utility from curbside parking decreases with the number of individuals who use it because of search congestion. Our setting is unique in incorporating such an endogenous outside option into a Salop-type model.

A few empirical studies of competition in parking markets have recently appeared. Kobus, Gutierrez-i-Puigarnau, Rietveld and van Ommeren (2012) examine the effects of parking fees on drivers’ choices between curbside and garage parking. Froeb, Tschantz, and Crooke (2003), Choné and Linnemer (2012), and De Nijs (2012) focus on the effects of mergers in the parking industry. Lin and Wang (2014) examine the relationship between competition and price discrimination. Several general lessons emerge from these studies which inspired the general structure of our model. First, hourly garage parking fees generally decline steeply with parking duration. Put another way, total payment or outlay is an increasing but steeply curved concave function of parking duration. Second, the degree of curvature in the outlay curve declines with increased competition. Third, the (short-run) marginal supply cost of parking is close to zero for garages. Fourth, drivers are reluctant to walk more than a few blocks from a parking garage to their destination.

The paper is organized as follows. Section 2 describes the model. Section 3 characterizes various possible equilibrium allocations of driver types between garage and curbside parking. Section 4 derives the socially optimal allocation of driver types between garage and curbside parking space, and shows how the allocation can be decentralized using differentiated hourly curbside parking fees. Section 5 uses a numerical example to illustrate how the welfare gains from implementing optimal differentiated fees depend on such parameters as the distance between parking garages, parking search costs, and walking time costs. Section 5 also assesses the relative efficiency of setting optimal uniform curbside fees. Section 6 discusses extensions, and Section 7 concludes. An (online) appendix provides derivations of the various equilibria, and technical and computational details.

2 The Model

Consider a fixed set of individuals (henceforth drivers) who travel to a downtown area by car. Drivers differ in their destinations and lengths of stay. A long-term parker (denoted by
L) requires parking for $l_L$ hours while a short-term parker (denoted by $S$) requires parking for $l_S$ hours, where $l_S < l_L$. A type $i$ driver, $i = L, S$, receives a benefit of $B_i$ from a trip. The $B_i$s are large enough that all potential trips are made, and total parking demand is therefore price inelastic.

Each driver has a given trip destination. Destinations are uniformly distributed around a circle with densities $d_L$ for long-term parkers and $d_S$ for short-term parkers. Demand for parking by type $i$ drivers in hours per unit distance is $h_i \equiv d_i l_i$. Parking is available at parking garages and on the curb. Curbside parking is operated publicly and distributed continuously around the circle. Parking garages have fixed locations a distance $D$ apart. Each garage is operated by a separate private firm. Garage parking space is lumpy because of scale economies in garage capacity (Arnott, 2006).

Curbside parking in many cities is priced at a constant fee per hour. However, to allow for price discrimination and the use of curbside parking fees to enhance market efficiency, it is assumed that curbside parking fees can differ, with type $i$ drivers paying an hourly fee of $p_{ci}$, $i = L, S$. Short-term parkers therefore pay $p_{cS} l_S$ to park for $l_S$ hours, and long-term parkers pay $p_{cL} l_L$ to park for $l_L$ hours. Depending on how parking fees are levied and enforced, incentive compatibility constraints may apply. If $p_{cS} < p_{cL}$, a long-term parker might be able to save money by interrupting his visit, returning to his car, and either moving it to another parking spot or feeding the meter. But doing so would be inconvenient, and we rule it out. If, alternatively, $p_{cS} > p_{cL}$, it is possible that $p_{cS} l_S > p_{cL} l_L$. A short-term parker could then stay at the destination an extra $l_L - l_S$ hours and save $p_{cS} l_S - p_{cL} l_L$ on the parking bill. The driver might also be able to pay the long-term charge and then leave after $l_S$ hours. To admit this possibility we will entertain the incentive compatibility constraint

$$p_{cS} l_S \leq p_{cL} l_L.$$  

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2One interpretation is that long-term parkers are commuters and short-term parkers are making business trips. Another is that long-term parkers are multipurpose or comparison shoppers who shop for an extended period of time, whereas short-term parkers are one-stop shoppers who need to park for a shorter time.

3Parking is sometimes also available at surface lots. Surface lots are typically built as transitory uses of land after buildings are torn down, and therefore offer only temporary additional space to park. Surface lots are similar to garages in that they do not contribute appreciably to search congestion. In our model setting, lots can be treated as equivalent to garages. There are also parking spaces supplied by businesses to employees (van Ommeren and Wentink, 2012) and customers (Hasker and Inci, 2014), which we ignore. Their presence lowers garages’ local market power.

4We abstract from safety issues in different parking forms. In some countries, a lot of crime is committed in parking garages. According to the Bureau of Justice Statistics, more than 1 in 10 property crimes occurred in parking lots or garages between 2004 and 2008 in the US. In Istanbul, cars parked on the curb have a higher risk of burglary or damage due to accidents than cars parked in garages.
This constraint is not imposed in the analysis of Sections 3 and 4, but it is addressed in the numerical analysis of Section 5. To concentrate on the behavior of garage operators, curbside parking fees $p_{cL}$ and $p_{cS}$ are treated as exogenous until Section 4.

The time required to find a curbside parking space is assumed to be proportional to the total number of hours of curbside parking that is occupied between adjacent garages, $T$. If all drivers of both types park on the curb, $T = (h_L + h_S)D$. A type $i$ driver incurs a search cost of $k_iT$, where $k_i > 0$ is the type-specific unit search cost.\(^5\) The generalized cost of curbside parking for a type $i$ driver is therefore $p_{ci}l_i + k_iT$, and the net benefit from a trip is

$$B_i = p_{ci}l_i - k_iT, \quad i = L, S.$$  \(2\)

Parking garages charge an hourly fee of $p_{gL}$ for parking $l_L$ hours, and an hourly fee of $p_{gS}$ for parking $l_S$ hours. The time required to locate a garage and park a vehicle there is assumed to be negligible. However, garage customers have to walk from the garage to their destination and back. A type $i$ driver incurs a round-trip walking time cost of $w_ix$, where $x$ is the distance between the parking garage and the destination, and $w_i$ is the type-specific walking time cost per unit distance in both directions combined. Similar to the case with

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\(^5\)One interpretation is that drivers require time to find a vacant spot, but can secure one close enough to their destination that walking time between the parking space and the destination is negligible. An alternative interpretation is that on-street parking is available not only on the circle, but also on intersecting roads and/or on roads that run parallel to the circle one or more blocks away. As nearby spots fill up, drivers have to travel further from the circle in order to find parking. Time is spent driving to a vacant spot, walking to the circle, and later walking back to the vehicle. Since occupancy of space is proportional to length of stay, in both interpretations the time cost depends on the total number of hours that curbside parking is used, $T$; i.e. the number of vehicles that use curbside parking multiplied by average parking duration. The functional dependence of the search cost function on $T$ depends on a number of factors (Brooke et al., 2014): where curbside parking is located relative to destinations on the circle, use of curbside parking for other purposes (e.g., residential parking, employee parking, retail deliveries), parking turnover rate, prevalence of illegal parking and rule enforcement, driver familiarity with parking supply and preferences (e.g., for walking distance, security and shade), and drivers’ choice of search strategies. One assumption is that search cost is inversely proportional to the fraction of parking spots that are vacant (Axhausen et al., 1994; Anderson and de Palma, 2004). If the number of parking spots is fixed, search cost increases to infinity as the occupancy rate of parking approaches 1. However, in the model here destinations and curbside parking space are both dispersed over space. As more drivers seek curbside parking, the range of locations over which they search expands. If curbside parking is equally prevalent on and off the circle, drivers might search within a diamond-shaped area centred about their destination. Since the number of spaces available within the area would rise with the square of its linear dimension, expected search costs might increase with the square root of the number of searchers. However, this would not be the case if parking availability decreases with distance from the circle. A further complication is that search costs tend to be higher for individuals with destinations midway between garages since parking search will be most intense there. The linear functional form adopted in equation (2) is an intermediate case between concave and convex functional dependencies that has the advantage of analytical tractability.
curbside parking, an incentive compatibility constraint may apply:

\[ p_{ss} l_S \leq p_{gL} l_L. \]  (3)

As discussed later, it is also possible for the short-term garage parking fee to exceed the long-term fee (i.e., \( p_{ss} > p_{gL} \)).

A type \( i \) driver who parks at a garage a distance \( x \) from his destination incurs a generalized cost of \( p_{gi} l_i + w_i x \) and gains a net benefit of

\[ B_i - p_{gi} l_i - w_i x, \quad i = L, S. \]  (4)

This specification embodies the assumption that a driver has a fixed parking duration which is equal to the sum of walking time to/from the destination and the visit duration. Visit duration therefore decreases with distance from a parking garage. This assumption simplifies the analysis. It also precludes the possibility for garage operators to price discriminate between drivers on the basis of their walking distance.

A parking garage incurs a cost of \( \alpha \) for each hour that a car is parked.\(^6\) Thus, it earns a profit of \( (p_{gi} - \alpha) l_i \) from a type \( i \) driver. To assure that parking garages can earn positive profits in equilibrium, it is assumed that the generalized cost of curbside parking when all drivers of both types park on the curb exceeds the supply cost of garage parking during their visit:

\[ p_{ci} l_i + k_i (h_L + h_S) D > \alpha l_i, \quad i = L, S. \]  (5)

Following common practice in the literature on spatial competition, attention is focused on symmetric equilibria in which all garages employ the same parking fee schedules. Consider one garage called the “home garage.” Given the simple, deterministic nature of demand, competition on the circle is localized and the home garage only competes directly either with curbside parking or with the nearest garages on either side.\(^7\) In any candidate symmetric

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\(^6\)This cost includes any costs related to transactions, security, vehicle damage, and wear and tear on the garage. Since the cost of curbside parking is set to zero in the model, it is assumed that garage parking is more costly to provide than curbside parking. Garage parking usually entails construction above or below ground, while curbside parking requires only allocation of existing road space. We neglect the opportunity cost of using space for curbside parking rather than moving traffic. Arnott, Inci, and Rowse (2015) account for this trade-off.

\(^7\)If the home garage sets parking fees much lower than its neighbors’ fees, it would gain the business not only of all customers located between it and its nearest neighbors but also some customers located on the far side of the neighbors. The literature has sometimes adopted a ‘no-mill-price-undercutting’ assumption to preclude hyper-competitive pricing and customer ‘leapfrogging’ of this sort. The potential profitability of undercutting is muted in the model here because utility from the outside alternative—curbside parking—
equilibrium, the nearest neighbors adopt the same parking fee schedules as the home garage. The home garage therefore attracts type \(i\) drivers from a distance \(x_i\) on either side. It earns a profit of \(2(p_{gi} - \alpha)h_ix_i\) from type \(i\) drivers, and a total profit of

\[
\pi (p_{gL}, p_{gS}) = 2(p_{gL} - \alpha)h_Lx_L + 2(p_{gS} - \alpha)h_Sx_S.
\]

The home garage chooses \(p_{gL}\) and \(p_{gS}\) to maximize this profit expression.

3 Market Equilibrium

In any symmetric equilibrium, the home garage competes for each type of driver either with curbside parking or with its nearest garage neighbors. There are nine possible market configurations in all. Two are illustrated in Figure 1 by showing a section of the circle between the home garage and one of its neighbors. In panel (a), garage parking fees are so low that no one parks on the curb. The market boundary for type \(i\) drivers between the home garage and its neighbor is located at a distance \(x_i\) from the home garage and a distance \(x_0^i = D\) from the neighbor. A type \(i\) driver with a destination on the market boundary is indifferent between parking at the home garage and parking at the neighbor, while preferring both to parking on the curb. We call this driver the “marginal type \(i\) driver”. The boundaries can differ for long-term and short-term parkers although in a symmetric equilibrium both boundaries are located mid-way between the garages (i.e., \(x_L = x_S = D/2\)). Because no drivers park on the curb, \(T = 0\) in this configuration. We call this market configuration regime \(Lg + Sg\), meaning that both driver types use only parking garages.

Another configuration in which some drivers park on the curb is shown in panel (b) of Figure 1. Type \(i\) drivers located within a distance \(x_i\) of the home garage park there, those located within a distance \(x'_i\) of the neighbor park there, and those located in the central region with a span of \(D - x_i - x'_i\) park on the curb. Thus, the marginal type \(i\) driver who is located a distance \(x_i\) from the home garage is indifferent between parking at the home garage and on the curb. Similarly, the marginal type \(i\) driver located distance \(x'_i\) from the neighbor is indifferent between parking at the neighbor and curbside parking. Hence, the total number of hours spent parking on the curb is

\[
T = h_L (D - x_L - x'_L) + h_S (D - x_S - x'_S).
\]
Figure 1: Alternative market configurations

We call this market configuration regime $Int$, meaning that each driver type uses both garage and curbside parking, and the market boundaries are in the interior of the market segment between the two garages (i.e., $0 < x_L < D/2$ and $0 < x_S < D/2$).

The full set of nine market configurations is depicted in Figure 2. The location of the home garage’s market boundary for long-term parkers is plotted on the horizontal axis, and the location of the boundary for short-term parkers is plotted on the vertical axis. In regime $Lg + Sg$, shown in panel (a) of Figure 1, both driver types park exclusively at garages. In regime $Int$, shown in panel (b) of Figure 1, both types split between garages and the curb. In regime $Lg$, long-term parkers use only garages while short-term parkers split between garages and curbside parking (i.e., $x_L = D/2$ and $0 < x_S < D/2$). Regime $Sg$ is defined analogously. In regime $Lc$, long-term parkers use only curbside parking while short-term parkers split between garages and the curb (i.e., $x_L = 0$ and $0 < x_S < D/2$). Regime $Sc$ is defined analogously. In regime $Lc + Sc$, both types of drivers park only on the curb (i.e., $x_L = x_S = 0$). In regime $Lg + Sc$, long-term parkers use only garages while short-term parkers use only the curb (i.e., $x_L = D/2$ and $x_S = 0$). Regime $Lc + Sg$ is defined analogously. Equation (7) is modified accordingly for each regime. Which one of the regimes prevails in equilibrium depends on parameter values including the curbside parking fees, $p_{cL}$.
Since $p_{cL}$ and $p_{cS}$ are treated as given in this section, the equilibrium will be called the "current equilibrium". Regime $Int$ is examined in the next subsection in more detail in order to provide a better sense of the model’s properties and the nature of equilibrium. All other regimes are examined in the (online) appendix.

### 3.1 Equilibrium for regime $Int$

In regime $Int$, shown in panel (b) of Figure 1, some drivers of each type park at garages and the rest park on the curb. The home garage chooses $p_{gL}$ and $p_{gS}$ to maximize its profit given in (6). Several conditions must hold for this regime to be an equilibrium. First, the marginal driver of each type must be indifferent between parking at the home garage and parking on the curb:

$$B_i - p_{gi}l_i - w_i x_i = B_i - p_{ci}l_i - k_i T, \quad i = L, S.$$  \hspace{1cm} (8)

Second, the net benefit to a marginal type $i$ driver of parking at the home garage must be strictly greater than the net benefit of parking at the neighbor:

$$B_i - p_{gi}l_i - w_i x_i > B_i - p'_{gi}l_i - w_i (D - x_i), \quad i = L, S. \hspace{1cm} (9)$$
This guarantees that some drivers of each type park on the curb. Third, if practically relevant, the incentive compatibility constraint in (3) must hold.

The market boundaries, \(x_L, x_S, x'_L, x'_S\), and total time spent parking on the curb, \(T\), can be solved using the two conditions in (8), the two analogous conditions for the neighboring garage, and equation (7). There are five linear equations in five unknowns. The solution for \(x_L\) is

\[
x_L = \frac{w_L w_S k_L (h_L + h_S) D + w_S k_L h_L l_L (p_gL - p_gS)}{w_L (w_L w_S + 2k_L h_L w_S + 2k_S h_S w_L)} + \frac{(w_S + 2k_S h_S) l_L (p_cL - p_gL) - k_L h_S l_S (2p_cS - p_gS - p'_gS)}{w_L w_S + 2w_S k_L h_L + 2w_L k_S h_S}.
\]

The formula for \(x_S\) is obtained from (10) by interchanging the \(L\) and \(S\) subscripts. As shown in online Appendix A.1, the equilibrium is stable in the sense that a perturbation in the number of drivers of either type who park on the curb induces adjustments that return the system to equilibrium asymptotically.

A notable feature of (10) is that the home garage’s market boundary depends on the parking fees set by the neighbor, \(p'_gL\) and \(p'_gS\), even though the garages do not compete directly with each other. The garage markets are interdependent because the fees charged by one garage affect the number of drivers who choose to park on the curb, the intensity of curbside parking search congestion, and hence the demand for parking at the neighboring garages.

The first-order conditions for an interior profit maximum for the home garage are

\[
\begin{align*}
\frac{\partial \pi}{\partial p_{gL}} &= 2h_L \left( x_L + (p_gL - \alpha) \frac{\partial x_L}{\partial p_{gL}} \right) + 2h_S (p_gS - \alpha) \frac{\partial x_S}{\partial p_{gL}} = 0, \\
\frac{\partial \pi}{\partial p_{gS}} &= 2h_S \left( x_S + (p_gS - \alpha) \frac{\partial x_S}{\partial p_{gS}} \right) + 2h_L (p_gL - \alpha) \frac{\partial x_L}{\partial p_{gS}} = 0.
\end{align*}
\]

These equations yield closed-form but very long expressions for \(p_{gL}\) and \(p_{gS}\), as well as long expressions for \(x_L\) and \(x_S\) when the formulas for \(p_{gL}\) and \(p_{gS}\) are substituted into (10) and its counterpart for \(x_S\).

The comparative statics properties of equilibrium garage parking fees and market boundaries are mostly ambiguous. Table 1 lists some properties for \(p_{gL}\) and \(x_L\) for the case where curbside parking fees are undifferentiated (i.e., \(p_cL = p_cS = p_c\)).

\[
\text{The comparative statics properties for } p_{gS} \text{ and } x_S \text{ are mirror images of those for } p_{gL} \text{ and } x_L \text{ except for the condition } l_L \geq l_S, \text{ which is unchanged.}
\]
Table 1: Comparative statics properties of interior equilibrium

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<th>$l_L$</th>
<th>$l_S$</th>
<th>$w_L$</th>
<th>$w_S$</th>
<th>$p_c$</th>
<th>$d_L, d_S$, $k_L, k_S$</th>
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<tr>
<td>$p_{gL}$</td>
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<td>+ if $l_L = l_S$ and $k_L = k_S$</td>
<td>+ if $k_L = k_S$</td>
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<td>Same sign</td>
<td>Opposite sign</td>
<td>- if $l_L = l_S$</td>
<td>+ if $l_L = l_S$</td>
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It is possible in regime $Int$ for the hourly garage parking fee to be higher for long-term parkers than short-term parkers.\(^9\) Conversely, it is also possible for the long-term fee to be so much lower that incentive compatibility constraint (3) is violated. For example, this occurs if $l_L$ and $l_S$ are similar, and if short-term parkers have higher walking and search costs than long-term parkers. Section 5 includes a numerical example in which this happens.

One, degenerate, case in which (11) and (12) yield simple solutions occurs if search costs for curbside parking are zero ($i.e., k_L = k_S = 0$). In this case, garage parking fees and market boundaries work out to

$$p_{gi} = \frac{\alpha + p_{ci}}{2}, \quad i = L, S \tag{13}$$
$$x_i = \frac{(p_{ci} - \alpha) l_i}{2w_i}, \quad i = L, S. \tag{14}$$

\(^9\)For instance, assume for the moment that $p_{cL} = p_{cS}$ and $k_L/k_S = l_L/l_S$. One can easily show that $p_{gL} = p_{gS}$ in this case. Then, within a neighborhood of this point there are parameter combinations for which $p_{gL} > p_{gS}$.
If $k_L = k_S = 0$, curbside parking serves as an outside good with *exogenous* utility as in Salop’s (1979) model. Garages can then attract a type $i$ driver without losing money only if $p_{ci} > \alpha$. They set their fees for each driver type half way between the cost of providing garage parking, $\alpha$, and the fee for curbside parking, $p_{ci}$, as per equation (13). The garage market for each type in (14) varies proportionally with the type’s parking duration, $l_i$, and inversely with its walking cost to and from garages, $w_i$. The simplicity of formulas (13) and (14) shows that search costs are essential to make the model interesting and useful for policy analysis.

$$l w p c d k$$

| $p_g$ $- + + + +$ | $x$ Same sign as $p_c - \alpha$ $- + +/+ - +/+ -$ |

Table 2: Comparative statics properties of interior equilibrium with homogeneous drivers

A second case with a simple solution is the case in which drivers are identical so that there is effectively only one driver type. Garages then set only one parking fee, $p_g$, and there is only one market boundary, $x$, between garage parking and curbside parking. The home garage’s problem then is to choose a single parking fee, $p_g$, to maximize profit, $2(p_g - \alpha)dlx$, subject to the constraints $p_g l + wx = p_c l + kT$, $p_g l + wx' = p_c l + kT$, and $T = h(D - x - x')$. There is no incentive compatibility constraint with just one type. The symmetric equilibrium solution is given by

$$p_g = \frac{p_c w + wdkD + \alpha (w + kh)}{2w + kh}, \quad (15)$$
$$x = \frac{kh (w + kh) D + (p_c - \alpha) l (w + kh)}{(2w + kh) (w + 2kh)}. \quad (16)$$

The comparative statics properties of this simplified model, listed in Table 2, follow by differentiating (15) and (16). The effects of market density, $d$, and search time costs, $k$, are now definitive. If the density of parkers increases, and the curbside parking fee exceeds the supply cost of garage parking, the garage parking fee increases and the garage market expands. This happens because the garage market operates with constant returns to scale whereas, due to search costs, curbside parking suffers from diminishing returns. An increase in the size of the market therefore favors garage parking. If search costs for curbside parking increase, and the curbside parking fee is less than the supply cost of garage parking, the garage parking fee increases and the garage market expands. However, if curbside parking is very costly, garages have substantial market power and they can increase their fees so much
that the garage market area contracts.

Derivations of equilibria for the remaining regimes are relegated to (online) Appendices A.2-A.9. These derivations are novel in dealing with endogeneous outside options in Salop-type spatial competition models.

4 Social Optimum

Since total parking demand is fixed, the social optimum corresponds to a total cost minimum. Total costs are determined by how many drivers of each type park in garages, and how many park on the curb. All drivers who park at a garage should patronize the nearest one, and with garages spaced at equal intervals the social optimum can be derived by minimizing total costs within a distance $D/2$ on either side of the home garage.

Total costs ($TC$) are the sum of garage supply costs ($GC$), walking time costs ($WC$), and curbside parking search costs ($SC$). The component costs in a symmetric equilibrium are tallied as follows. The number of type $i$ drivers on either side of the home garage who park there is $2d_i x_i$. Type $i$ drivers park for $l_i$ hours, and it costs $\alpha$ dollars to provide an hour of parking. Therefore, garage operating costs are

$$GC = 2\alpha (h_L x_L + h_S x_S). \quad (17)$$

A type $i$ driver bound for a destination $x$ units away from a garage incurs a walking cost for the round-trip of $w_i x$. Total walking time for type $i$ drivers is $2d_i \int_0^{x_i} x dx$. Total walking time costs for all drivers are therefore

$$WC = 2d_L w_L \int_0^{x_L} x dx + 2d_S w_S \int_0^{x_S} x dx = d_L w_L x_L^2 + d_S w_S x_S^2. \quad (18)$$

There are $d_i(D-2x_i)$ type $i$ drivers who each park for $l_i$ hours on the curb and incur a search cost of $k_i T$, with $T$ given by (7). Total curbside parking search costs are therefore

$$SC = [k_L d_L (D-2x_L) + k_S d_S (D-2x_S)] [h_L (D-2x_L) + h_S (D-2x_S)]. \quad (19)$$

The social optimum is derived by choosing $x_L$ and $x_S$ to minimize total costs, $TC = GC + WC + SC$. In principle, the optimum can fall into any one of the nine regimes described in Section 3 for the current equilibrium. However, assumption (5) rules out regime
$Lc + Sc$ in which garage parking is not used. The model assumptions also rule out regime $Lg + Sg$ in which curbside parking is not used. The reasoning is as follows. First, the supply of curbside parking is fixed so that providing curbside parking has no opportunity cost. Second, administration costs of curbside parking and fee collection are assumed to be zero. Third, drivers do not contribute to through traffic congestion while they are searching for parking. Fourth, the marginal external cost of search approaches zero as hours of curbside parking used, $T$, approach zero. Finally, garage parking involves a supply cost per vehicle-hour of $\alpha > 0$, and drivers incur walking time costs when they use garages. Therefore, the marginal social cost of using a small amount of curbside parking is lower than the marginal social cost of using a small amount of garage parking so that some curbside parking should always be used.\(^\text{10}\)

It is straightforward to derive the social optimum for each of the seven remaining regimes. Formulas for the optimal market boundaries are complicated and opaque, and are not presented here (formulas for regime $Int$ are given in online Appendix A.10). In general the social optimum differs from the market equilibrium described in Section 3. In some cases the equilibrium regime differs as demonstrated in the numerical example of Section 5. In other cases the regime is the same, but garage fees differ. The market equilibrium is generally inefficient because it is distorted in two ways. First, parking garages exploit their market power by setting parking fees above marginal costs and discriminating between short- and long-term parkers. Second, parking garages do not internalize the congestion externality associated with search for curbside parking.

In the balance of this section we offer some general insights about the social optimum and how it can be decentralized. A numerical example is developed in the next section.

The social optimum can be achieved in several ways. The most direct approach is for the city to regulate garage parking fees or take over control of garages altogether. The city can implement first-best pricing of both garage and curbside parking. Denote this approach with a superscript $o$. Garage parking is priced at marginal supply cost so that $p_{gL}^o = p_{gS}^o = \alpha$. Curbside parking is priced to internalize the marginal external cost of search. Because this externality is proportional to parking duration, hourly curbside parking fees will be equal, (i.e., $p_{cL}^o = p_{cS}^o$). We will refer to the common value, $p_c^o$, as the “first-best uniform curbside parking fee”. Given total curbside parking search costs in (19), the marginal external hourly cost imposed by a type $i$ driver is $[k_L d_L (D - 2x_L) + k_S d_S (D - 2x_S)]i$, and the first-best

\(^{10}\)This differs from some other parking models such as that in Arnott, Inci, and Rowse (2015) where allocating space to curbside parking reduces road capacity, and cruising for parking contributes to congestion for through traffic.
uniform curbside fee is therefore \( p_c^* = k_Ld_L(D - 2x_L) + k_Sd_S(D - 2x_S) \).

Since total parking demand for each driver type is fixed, first-best pricing is not the only way to support the social optimum. Indeed, any set of garage and curbside parking fees that maintains the appropriate differential between garage and curbside parking fees for each driver type does the job. For example, in the symmetric equilibrium for regime \( \text{Int} \) the market boundary for long-term parkers given in (10) is a function of \( p_{cL} - p_{gL} \) and \( p_{cS} - p_{gS} \), but it does not depend on the levels of the fees. If the city can regulate garage fees, it can fix curbside fees, \( p_{cL} \) and \( p_{cS} \), at any level (subject to nonnegativity constraints) and then set \( p_{gL} \) and \( p_{gS} \), correspondingly. If parking garage operators can set their rates freely, the city can still attain the optimum by setting curbside fees appropriately. The hourly curbside parking fees for the two driver types will typically differ because their garage parking demand elasticities differ, and garages therefore impose different markups on their rates. We call the curbside parking fees in this case “first-best differentiated curbside parking fees” and denote them with a caret (i.e., \( \hat{p}_{cL} \) and \( \hat{p}_{cS} \)).

Differentiating curbside fees is difficult using conventional parking meter technology, and it may be opposed by interest groups or ruled out on public policy grounds. If so, hourly curbside fees are constrained to be uniform and the social optimum cannot, in general, be reached if garage parking is privately controlled. We denote this scheme by a superscript \( u \), and refer to the common fee, \( p_{c}^u \), as the “second-best uniform curbside parking fee”.

Regardless of which equilibrium regime prevails at the optimum, the first-best and second-best curbside parking fees defined above are positive for both driver types. This follows from two observations. First, curbside parking creates a negative search congestion externality. The first-best (Pigouvian) curbside parking fees are therefore positive. Second, when garage parking is privately controlled it is overpriced because the operators exercise market power. Because garage operators do not suffer from congestion themselves, they do not internalize congestion costs in the way that private toll road operators do (Verhoef, Nijkamp, and Rietveld, 1996). Indeed, garage operators benefit from the congestion induced by curbside parking search. Since curbside parking is a substitute for garage parking, second-best curbside parking fees exceed first-best fees.

## 5 A Numerical Example

The numerical example is designed to establish a sense of which of the various candidate equilibrium regimes prevails under alternative parameter assumptions. It also compares the
first-best and second-best pricing schemes described in Section 4. Online Appendix A.11 summarizes the numerical solution procedure that was used to derive equilibria.

5.1 Parameterization

Base-case parameter values for the example are listed in Table 3. Some of the values are taken from Arnott (2006), Arnott and Inci (2006, 2010), and Arnott and Rowse (2009). Suitable values for the $d_i$, $l_i$, $w_i$, and $k_i$ parameters depend on such factors as trip purpose and income, and possibly also trip frequency and familiarity with parking availability. There is a large empirical literature on values of driving time and walking time, and how they vary with individual socioeconomic characteristics. There are very few estimates of the value of time spent cruising for parking, and such estimates cannot be translated into values for the $k_i$ parameters because search time is measured in terms of aggregate vehicle-hours of parking rather than minutes. Another complication is that suitable parameter values depend on how long-term and short-term parkers are interpreted. To be agnostic, we have chosen for the base case identical parameter values for the two driver types except for parking duration.

<table>
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<tr>
<th>$p_{cL} = p_{cS}$</th>
<th>$D$</th>
<th>$\alpha$</th>
<th>$l_L$</th>
<th>$l_S$</th>
<th>$w_L = w_S$</th>
<th>$k_L = k_S$</th>
<th>$d_L = d_S$</th>
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<td>2 hr</td>
<td>1 hr</td>
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<td>$0.16/vehicle-hr$</td>
<td>100/mile</td>
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</table>

Table 3: Base-case parameter values

The curbside parking fee for both types is $1.00 per hour; the distance between two consecutive parking garages is 0.125 miles; the cost of providing a parking garage space is $2.50 for each hour that a car is parked; parking duration is 2 hours for long-term parkers and 1 hour for short-term parkers; walking time cost for both types is $16 per mile; unit parking search cost for both types is $0.16 per vehicle-hour; and the density of trip destinations for each type is 100 per mile. With these parameter values, the cost of walking for the round-trip between a garage and the mid-point between parking garages is $2.00 for each type. For short-term parkers, the generalized cost of curbside parking ranges from $p_{cS} = 1.00$ if no one uses curbside parking to $p_{cS} + k_S T = p_{cS} + k_S (h_L + h_S) D = 8.00$ if all drivers of both types park on the curb. The corresponding minimum and maximum values for long-term parkers are $2.00 and $8.00, respectively.

11From a meta-analysis of UK values of travel time Abrantes and Wardman (2011) report that the value of time spent searching for parking is 38% higher than the value of time spent on normal driving.
5.2 Results

Results for various parameter combinations are reported in Tables 4-6. As shown in Table 4, for the base case (case 1), the current equilibrium, denoted by a *, falls in regime \textit{Int}. Hourly garage parking fees are \( p_{gL}^* = 2.81 \) and \( p_{gS}^* = 3.74 \). Only 9 percent of long-term parkers use garages, but 97 percent of short-term parkers do so. Total costs are about $85 per garage. This value should be interpreted as costs incurred every 2 hours: the parking duration of long-term parkers.

The social optimum falls into regime \textit{Sg} (see Table 5). All short-term parkers use garage parking, and a larger fraction (33 percent) of long-term parkers use garages than in the current equilibrium. This pattern reflects the underusage of garages in the current equilibrium, described earlier, that results from a combination of underpricing of curbside parking and exercise of garage market power. Total costs in the social optimum are about $81 which is roughly 5 percent lower than in the current equilibrium.

The social optimum can be attained by pricing garage parking at marginal social cost (i.e., \( p_{gL}^* = p_{gS}^* = 2.50 \)), and charging curbside users the marginal search congestion externality cost. For the base-case parameter values the optimal curbside fee is \( p_{c}^* = 1.33/\text{hr} \) which is not much higher than the $1.00/\text{hr} assumed for the current equilibrium. Consider the long-term parker who is indifferent between parking on the curb and at the home garage. To park at the home garage he would pay a total parking fee of \( p_{gL}^* l_L = 2.5 \times 2 = 5 \) and incur a walking time cost of \( w_L l_L = 16 \times (0.125 \times 0.5 \times 0.33) = 0.33 \) for a total cost of $5.33. If he parked on the curb instead, he would pay a parking fee of \( p_{c}^* l_L = 1.33 \times 2 = 2.66 \) and incur a search time cost of \( k_L T = k_L d_L l_L (D - 2x_H) = 0.16 \times 100 \times 2 \times 0.083 = 2.66 \) for the same total cost of $5.33. All long-term parkers with destinations closer to the home garage than this individual strictly prefer parking at the home garage. All long-term parkers with destinations further away from the home garage than this individual strictly prefer parking on the curb. Meanwhile, all short-term parkers are strictly better off parking at a garage. The driver bound for a destination midway between garages pays a parking fee of \( p_{gS}^* l_S = 2.5 \times 1 = 2.50 \) and incurs a walking cost of \( w_S l_S = 16 \times (0.125 \times 0.5) = 1 \) for a total cost of $3.50. Other short-term parkers incur a lower cost. Any short-term parker who parked on the curb would pay a parking fee of \( p_{c}^* l_S = 1.33 \times 1 = 1.33 \), and incur a search time cost of \( k_S T = 2.66 \) for a total cost of $3.99: higher than the cost of garage parking.

As explained in Section 4, even if garages are operated privately and free of regulation, the social optimum can be decentralized by setting first-best differentiated curbside parking fees. For Case 1, \( \hat{p}_{cL} \) is $1.61, and \( \hat{p}_{cS} \) is at least $2.83 to deter short-term parkers from parking.
on the curb. Starting from the current equilibrium, garages raise the rate for short-term parkers from $p^*_{gs} = $3.74 to $p_\text{gs} = $4.50. However, they lower the rate for long-term parkers slightly from $p^*_{gL} = $2.81 to $p_\text{gL} = $2.78 because short-term parkers no longer park on the curb which makes curbside parking more attractive to long-term parkers. This illustrates how curbside parking congestion creates an interdependence between the parking markets for long-term and short-term parkers.

The second-best uniform curbside parking fee is $p^u = $1.85 (see Table 6) which is intermediate between the first-best differentiated fees. The uniform fee fails to support the social optimum. Its relative efficiency can be measured by the ratio

$$\text{eff} = \frac{TC^* - TC^u}{TC^* - TC^o} \in [0, 1],$$

where $TC^*$ denotes total costs in the current equilibrium, $TC^o$ total costs in the first-best optimum, and $TC^u$ total costs with the uniform curbside parking fee. For the base-case parameter values, $\text{eff} = 0.77$ so that the second-best uniform fee achieves more than three-quarters of the efficiency gains from first-best differentiated fees.

Results for eight alternative parameter combinations are reported in Tables 4-6, with parameter variations identified in bold type in Table 4.

Case 2: The hourly curbside parking fee for each driver type is raised from $1.00 to $4.00. The current equilibrium shifts to regime $L_g$. Because long-term parkers park for twice as long as short-term parkers, they are hit twice as hard by the higher curbside parking fees and all of them end up parking at garages. By contrast, short-term parkers shift toward curbside parking because of reduced search congestion. The curbside parking fee is higher than the garage parking fees in this case, which is the practice in the Netherlands and Belgium. Similar to the base case, congestion in curbside parking creates an interdependence between the parking markets. The social optimum is unchanged, of course, by the change in current curbside parking fees. However, the welfare gain from optimal curbside parking fees more than quadruples compared to the base case, and the relative efficiency of the second-best uniform curbside parking fee rises to 94 percent.

Case 3: Curbside parking is made free. The current equilibrium shifts to regime $L_c$, and both driver types use less garage parking than in the base case. The welfare gain from optimal curbside parking fees is similar to Case 2, and the relative efficiency of the second-best uniform curbside parking fee rises to 93 percent.

Case 4: Long-term parkers are assumed to be commuters who park for 9 hours. The
current equilibrium shifts to regime $S_g$ with all short-term parkers using garages. Long-term parkers patronize mainly curbside parking although the proportion is less than in the base case equilibrium. Total parking costs increase to nearly three times their base-case level. The increase is due to the higher resource costs of accommodating long-term parkers at garages for 9 hours as well as the greater contribution of long-term parkers to curbside congestion due to their longer stays. The social optimum is similar to the current equilibrium, and the efficiency gains from optimal curbside parking are modest. Moreover, the optimum can be supported by charging any non-negative hourly fee for short-term curbside parking so that the fee can be set at the optimal level for long-term parking of $1.35$ and the second-best uniform curbside parking fee achieves 100 percent efficiency.

Case 5: Distance between garages is doubled. The current equilibrium remains in regime $Int$, and the social optimum remains in regime $S_g$. But total costs increase greatly, and optimal curbside parking fees are appreciably higher than in previous cases. Garage profits are also much higher. The second-best uniform curbside parking fee of $3.34$ achieves about 90 percent of the welfare gains from the first-best differentiated curbside fees. A problematic feature of the differentiated fees is that short-term parkers pay a total of $p_{cS}l_S = 5.39$ for parking which slightly exceeds the payment by long-term parkers of $p_{cL}l_L = 5.14$. Incentive compatibility constraint (1) is therefore violated. If the incentive compatibility constraint is practically relevant, it can be accommodated by imposing it on the optimization problem while still treating $p_{cL}$ and $p_{cS}$ as independent control variables. Doing so would affect the solution very little in this instance because the constraint is only violated by a small margin.

Case 6: Curbside parking search costs are doubled. The current equilibrium shifts to regime $S_g$. It does not differ markedly from the social optimum so that the benefits from adjusting curbside parking fees are modest.

Case 7: Curbside parking search costs are reduced by half. Both the current equilibrium and the social optimum shift to regime $L_c$. The social optimum is supported by rather modest first-best differentiated curbside fees although incentive compatibility constraint (1) is again violated.

Case 8: Walking time costs are doubled. The market power of garages increases and they increase their hourly fees slightly. The combined effect of more onerous walking and higher garage parking fees induces short-term parkers to reduce their use of garage parking.

Case 9: Walking time costs are halved. All short-term parkers now use garage parking in both the current equilibrium and the social optimum, and the optimum can be supported by charging for short-term parking any fee over $1.41/hr$. Since this minimum is below the
optimal fee of $1.45/hr for long-term parking, the second-best uniform curbside parking fee is $1.45/hr and supports the social optimum.

Further outcomes can be realized by changing several parameters at once. For example, it is possible to create an equilibrium configuration in which long-term and short-term parkers are fully segregated between curbside and garage parking, and to create a social optimum in which all long-term parkers park in garages.

Overall, the numerical example illustrates a range of ways in which drivers are allocated to parking space in the various pricing schemes. Consistent with usual practice, in most cases the hourly fee for long-term garage parking is lower than the fee for short-term parking but the total outlay is higher for long-term parking. In general, the current equilibrium features too little garage parking because curbside parking is underpriced. (Case 2 is an exception.) The welfare gain from efficient curbside parking varies strongly with parameter values. It is largest in Case 5, where the distance between garages is doubled, because costs are tallied over a large market area and because the greater friction of space endows garages with more market power.

A second-best uniform hourly curbside parking fee supports the social optimum in some cases where the allocation of driver types is partly segregated between garages and the curb. In other cases, it falls short of the social optimum but still performs relatively well. Finally, the first-best uniform curbside fees are always lower than the first-best differentiated and second-best uniform curbside parking fees. This is because garage pricing in the first-best case is priced at marginal social cost rather than priced at a markup by private operators. The first-best uniform curbside fees vary relatively little over the cases compared to the first-best differentiated and second-best uniform curbside parking fees because curbside congestion varies less from case to case than does garage market power.
Parameter value

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<th>$l_S$ (hr)</th>
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<th>$w_S$ ($/hr$)</th>
<th>$k_L$ ($/hr$)</th>
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<th>Regime</th>
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Notes:
1. * denotes current equilibrium.
2. Parameter values held constant are $\alpha = 2.5$, $d_L = 100$, and $d_S = 100$.
3. Parameter variations are identified in bold type.
4. Minimum values are underlined.

Table 4: Results for numerical example: current equilibrium
Social optimum

MSC pricing of curbside and garage parking

Differentiated curbside fees and private garage operation

<table>
<thead>
<tr>
<th>Case</th>
<th>Regime</th>
<th>( x_L^o ) (frac. of ( D ))</th>
<th>( x_S^o )</th>
<th>( TC^o ) ($)</th>
<th>( \text{Gain} ) ($)</th>
<th>( p_c^o ) ($/hr)</th>
<th>( p_{cL}^o ) ($/hr)</th>
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<th>( \hat{p}_c ) ($/hr)</th>
<th>( \hat{p}_{cL} ) ($/hr)</th>
<th>( \hat{p}_{cS} ) ($/hr)</th>
<th>( \hat{p}_L ) ($/hr)</th>
<th>( \hat{p}_S ) ($/hr)</th>
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<td>2.66</td>
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Notes:
1. \(^o\) denotes first-best optimum with uniform curbside parking fees and marginal social cost pricing at garages.
2. \(^\circ\) denotes first-best optimum with differentiated curbside parking fees.
3. \( MSC \) denotes marginal social cost.
4. Parameter values held constant are \( \alpha = 2.5, d_L = 100, \) and \( d_S = 100. \)
5. Minimum values are underlined. Maximum values are overlined.

Table 5: Results for numerical example: first best
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<th>Case</th>
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<th>Uniform curbside fee</th>
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<tr>
<td>9</td>
<td>Sg</td>
<td>1.45</td>
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</table>

Notes:
1. " denotes second-best optimum with uniform curbside fees.
2. Parameter values held constant are $\alpha = 2.5$, $d_L = 100$, and $d_S = 100$.

Table 6: Results for numerical example: second best
6 Extensions

The analysis in this paper could be extended in various directions. One obvious priority is to extend it from two driver types to multiple types or a continuum of types. Doing so would not only be more realistic, but also permit a more precise analysis of price discrimination according to parking duration as well as a comparison with garage pricing in practice. A typical garage in Lin and Wang’s (2014) dataset posts five fees, and most garages set between three and eight fees. Working with a continuum of types would be easier than with discrete types insofar as there would be fewer equilibrium regimes to deal with. However, it would be necessary to adopt a joint frequency distribution of types in three dimensions (i.e., parking duration, walking time cost, and search time cost) for which empirical data are lacking. It would still be necessary to contend with possible non-concave garage profit functions. Furthermore, basic insights would probably be more difficult to obtain from a continuum model.

A second extension would be to allow parking duration to depend on the cost of parking rather than being fixed. Third, alternative functional forms for curbside parking search costs could be entertained. Fourth, garage capacity constraints could be introduced. Parking capacity is an effective policy tool that can be used by public authorities. Many cities and towns regulate parking capacity by imposing minimum and maximum parking requirements. Unfortunately, adding capacity constraints to a model can introduce kinks in profit functions and create problems with the existence of pure-strategy equilibria. Froeb, Tschantz, and Crooke (2003) and Arnott (2006) discuss these problems in the context of parking markets. These difficulties can be finessed by assuming Cournot competition, but this approach is unsatisfactory for studying driver heterogeneity and price discrimination.

Alternative market structures are yet another extension that we will briefly consider here. Monopoly control of garage parking is one possibility. The polar opposite of perfect competition is implausible because of entry barriers. Parking garages have scale economies and substantial construction costs, and it can take many years to recoup investments. Opportunities to enter are limited by zoning regulations and shortage of space. Furthermore, even if entry were free, garages would still price above marginal costs in order to recoup their entry costs.

More plausible than either polar case is an intermediate market structure of oligopolistic competition between a mix of public and private multi-garage chains. Such markets are

\[12\text{ For example, a build-operate-transfer contract for parking garages in Istanbul was offered with an operating period of 30 years, yet it attracted no applicants.}\]
prevalent in many cities. A general spatial analysis would be challenging, but some insights can be derived from the circle model. If all chains are privately owned one property of equilibrium is immediate: if no chain owns garages adjacent to each other on the circle, the equilibrium is the same as if all garages were separately owned — as is the case in the model here. This is so because every garage is bounded by rival garages on either side. Rivals set their parking fees simultaneously and independently, curbside rates are (still) exogenous, and curbside parking search zones do not extend beyond the area between adjacent garages. A private chain therefore cannot boost profits at one of its garages by adjusting parking rates at its other garages.

The analysis is more complicated in a mixed oligopoly with one publicly operated chain that has a goal of maximizing social surplus in the whole market. The public operator can try to reduce the distortions due to garage market power and search congestion externalities by adjusting fees at public garages, but it cannot attain the social optimum because fees at private garages will still be nonoptimal. In this sense, public control over just part of the garage parking market is less effective at achieving market efficiency than simply pricing curbside parking efficiently.

More definitive results can be derived in specific settings. One such case is a configuration with public and private garages alternating around the circle, and curbside fees still set uniformly and exogenously. As shown in online Appendix A.12, when there is only one driver type and equilibrium falls in regime $Int$, the maximum efficiency of mixed oligopoly can be established analytically. We can define the relative efficiency index of mixed oligopoly, $eff^m$, by replacing $TC^u$ with $TC^m$ in equation (20), where $TC^m$ denotes total costs in the mixed oligopoly equilibrium. Index $eff^m$ turns out to be independent of parameters $\alpha$, $p_c$, and $D$, and it depends on the other parameters only via the ratio $v \equiv w/(kd)$.

With $v = 0$, $eff^m = 0$. $eff^m$ rises with $v$ to a maximum value of 0.586 at $v = 1.49$, and then slowly declines. With the base-case parameter values of the numerical example, $eff^m = 0.45$ if all drivers are of type $L$, and $eff^m = 0.54$ if all drivers are of type $S$.

Mixed oligopoly is therefore most efficient when the market distortion due to private garage market power (proportional to $w$) is comparable in magnitude to the distortion due to search cost externalities (proportional to $kd$). Nevertheless, although the public operator controls half the parking garages in the market, at best it can achieve less than 60 percent of the efficiency gained by letting private garages price as they please and optimizing curbside parking efficiently.

13 Amsterdam, Hong Kong, Istanbul, and Paris are some major cities with a mix of publicly and privately operated garage parking facilities. De Nijs (2012) and Choné and Linnemer (2012) provide detailed descriptions of the parking market structure in Paris.

14 Braid (1986) derives this result for the case of retail chain stores.
parking fees. The relative efficiency of mixed oligopoly would be even lower if, perhaps due to x-inefficiency, public garages are operated less inefficiently than private garages.

7 Conclusions

In this paper, we have developed a simple spatial parking model. Individuals drive downtown to visit destinations that are uniformly distributed around a circle. There are two types of drivers who differ in how long they need to park. Privately-operated parking garages compete for business with each other and with publicly-operated curbside parking space. Garage parking is free of traffic congestion, but curbside parking is scarce and drivers incur search costs that increase with the total number of vehicle-hours of curbside parking that is used.

The model has several notable properties. First, unlike in standard models of spatial competition the utility derived from curbside parking (the outside good) is endogenous because it decreases with the number of drivers who use it. This creates an interdependence between parking garages even when they do not compete for customers with each other directly. If one garage changes its parking fees, it affects the number of drivers who park on the curb, and the change in parking search congestion affects demand at neighboring garages. An increase in fees benefits rivals (and a reduction hurts them) in the same qualitative way as when garages compete directly across a common border. Second, curbside parking congestion creates an interdependence between the demand for garage parking by the two driver types. For example, if a garage raises its fee for short-term parking, some short-term parkers switch to curbside parking. This increases search congestion for long-term parkers and induces some of them to switch from curbside parking to the garage. Demand for garage parking from the two market segments is therefore interdependent, and partly substitutable, even though garages are not constrained by capacity in the number of vehicles they can accommodate.

The parking market in the model is distorted in two ways. First, search for curbside parking creates a negative congestion externality among drivers looking for curbside parking. Second, parking garages have market power due to their discrete locations and the friction of space, and they exploit it by setting parking fees above marginal costs. They also price discriminate by setting different hourly fees for short- and long-term parking. Under plausible assumptions, the parking outlay is increasing and concave with parking duration, but exceptions are possible.
The degree of market failure varies with the distance between parking garages, parking search costs, and (crucially) the “current” fees charged for curbside parking. In the base model, the public authority lacks regulatory power over garage prices, but it can set curbside parking fees to alleviate search congestion. Moreover, since total parking demand is inelastic it can achieve the social optimum by charging differentiated curbside parking fees. In some cases a uniform hourly curbside parking fee can also achieve full efficiency. If garage parking rates can be regulated, the social optimum can also be reached regardless of curbside fees by adjusting garage fees to maintain the appropriate differential between garage and curbside rates. At least in the mixed oligopoly extension of the model we consider, operating some of the parking garages publicly without also optimizing curbside fees yields only modest efficiency gains. Moreover this simple extension shows that, in the case in which there are privately owned parking garage chains, the equilibrium is the same as if all parking garages were separately owned if no chain owns garages adjacent to each other.

References


Online Appendix to

GARAGE AND CURBSIDE PARKING COMPETITION WITH
SEARCH CONGESTION

(not intended for publication)

EREN INCI AND ROBIN LINDSEY
A Appendix (not intended for publication)

A.1 Stability of equilibrium in regime Int

Following standard procedures, the stability of an interior equilibrium for given garage and curbside parking fees can be established by constructing, for each driver type, excess demand functions for parking at the home garage and parking at the neighbor. Using (7), equations (8)-(9) can be written as excess demand functions:

\[ E_L = p_c l_L + k_l (h_l (D - x_l - x'_l) + h_s (D - x_s - x'_s)) - p_y l_L - w_L x_L \]  
\[ E_S = p_c s_L + k_s (h_l (D - x_l - x'_l) + h_s (D - x_s - x'_s)) - p_y s_L - w_S x_S \] 
\[ E'_L = p_c l_L + k_l (h_l (D - x_l - x'_l) + h_s (D - x_s - x'_s)) - p'_y l_L - w_L x'_L \] 
\[ E'_S = p_c s_L + k_s (h_l (D - x_l - x'_l) + h_s (D - x_s - x'_s)) - p'_y s_L - w_S x'_S. \]  

Drivers are assumed to adjust their parking location choices when out of equilibrium following the equations of motion

\[ \frac{dx_L}{dt} = \lambda_L E_L \]  
\[ \frac{dx_S}{dt} = \lambda_S E_S \] 
\[ \frac{dx'_L}{dt} = \lambda_L E'_L \] 
\[ \frac{dx'_S}{dt} = \lambda_S E'_S. \]  

where the speeds of adjustment for long and short-term parkers, \( \lambda_L \) and \( \lambda_S \) respectively, are both positive and can differ. Let \( x^*_L, x^*_S, x^*_L, \) and \( x^*_S \) denote the equilibrium values of \( x_L, x_S, x'_L, \) and \( x'_S, \) and define new variables \( y_L \equiv x_L - x^*_L, y_S \equiv x_S - x^*_S, y'_L \equiv x'_L - x^*_L, \) and \( y'_S \equiv x'_S - x^*_S. \) Given equations (A.1)-(A.8), the \( y \) variables follow the equations of motion

\[
\begin{bmatrix}
\frac{dy_L}{dt} \\
\frac{dy_S}{dt} \\
\frac{dy'_L}{dt} \\
\frac{dy'_S}{dt}
\end{bmatrix} =
\begin{bmatrix}
-\lambda_L (k_L h_L + w_L) & -\lambda_L k_L h_S & -\lambda_L k_L h_l & -\lambda_L k_L h_s \\
-\lambda_S k_S h_L & -\lambda_S (k_S h_S + w_S) & -\lambda_S k_S h_l & -\lambda_S k_S h_s \\
-\lambda_L k_L h_L & -\lambda_L k_L h_S & -\lambda_L (k_L h_L + w_L) & -\lambda_L k_L h_s \\
-\lambda_S k_S h_L & -\lambda_S k_S h_S & -\lambda_S k_S h_L & -\lambda_S (k_S h_S + w_S)
\end{bmatrix}
\begin{bmatrix}
y_L \\
y_S \\
y'_L \\
y'_S
\end{bmatrix}
\]  

Finally, assume that the \( y \) variables evolve out of equilibrium according to the equations

\[ y_L = Z_L e^{\mu t}, \quad y_S = Z_S e^{\mu t}, \quad y'_L = Z'_L e^{\mu t} \quad \text{and} \quad y'_S = Z'_S e^{\mu t}, \]

where the \( Z \) variables are constants.
The characteristic equation for the system (A.9) is

\[
\begin{vmatrix}
-\lambda_L (k_L h_L + w_L) & -\lambda_L k_L h_S & -\lambda_L k_L h_L & -\lambda_L k_L h_S \\
-\mu & -\lambda_S (k_S h_S + w_S) & -\lambda_S k_S h_L & -\lambda_S k_S h_S \\
-\lambda_S k_S h_L & -\mu & -\lambda_L (k_L h_L + w_L) & -\lambda_L k_L h_S \\
-\lambda_S k_S h_L & -\lambda_S k_S h_S & -\lambda_S k_S h_L & -\lambda_S (k_S h_S + w_S)
\end{vmatrix} = 0,
\]

(A.10)

which reduces to

\[
(\lambda_L w_L + \mu) (\lambda_S w_S + \mu) (\mu^2 + B\mu + C) = 0,
\]

(A.11)

where \( B = \lambda_S w_S + \lambda_L w_L + 2\lambda_L k_L h_L + 2\lambda_S k_S h_S \), and \( C = \lambda_L \lambda_S w_L w_S + 2\lambda_L \lambda_S (w_S k_L h_L + w_L k_S h_S) \). The first two terms of the product in (A.11) yield roots \( \mu_1 = -\lambda_L w_L \) and \( \mu_2 = -\lambda_S w_S \) which are both negative. It is straightforward to show that the third, quadratic, term in (A.11) has a positive discriminant. The two roots are therefore both real and take the form

\[
\mu_3 = \frac{-B - \sqrt{B^2 - 4C}}{2}, \quad \mu_4 = \frac{-B + \sqrt{B^2 - 4C}}{2}.
\]

(A.12)

Since \( B \) and \( C \) are both positive, \( \mu_3 \) and \( \mu_4 \) are both negative. All four roots of the characteristic equation are therefore negative, which proves that the system is stable.

A.2 Equilibrium for regime \( Lg \)

In regime \( Lg \), all long-term parkers park at garages while short-term parkers split between garages and the curb. The home garage maximizes its profit, given by (6), subject to several constraints. First, the marginal long-term parker must be indifferent between parking at the home garage and parking at the neighbor:

\[
B_L - p_{gL} l_L - w_L x_L = B_L - p'_{gL} l_L - w_L (D - x_L).
\]

(A.13)

Second, the marginal short-term parker must be indifferent between parking at the home garage and curbside parking:

\[
B_S - p_{gS} l_S - w_S x_S = B_S - p_{cS} l_S - k_S T.
\]

(A.14)
This condition is the same as condition (8) for regime \( Int \), except now \( T = h_S(D - x_S - x_L') \) because only short-term parkers use curbside parking. Third, the marginal long-term parker must weakly prefer parking at the home garage to curbside parking:

\[
B_L - p_{gL}l_L - w_L x_L \geq B_L - p_{eL}l_L - k_L T. \tag{A.15}
\]

Fourth, the marginal short-term parker must strictly prefer parking at the home garage to parking at the neighbor:

\[
B_S - p_{gL}l_S - w_S x_S > B_S - p_{gL}l_S - w_S (D - x_S). \tag{A.16}
\]

This condition assures that some short-term parkers prefer curbside parking to garage parking. Finally, if practically relevant, the incentive compatibility constraint given in (3) must hold. The equilibrium solution depends on whether condition (A.15) binds or not. The two cases are examined in turn.

### A.2.1 Long-term parkers strictly prefer garage parking to curbside parking

In this case, constraint (A.15) does not bind. The market boundary for long-term parkers is obtained from (A.13):

\[
x_L = \frac{D}{2} + \frac{p'_{gL} - p_{gL}}{2w_L} l_L. \tag{A.17}
\]

The market boundary for short-term parkers is determined by solving (A.14) and its counterpart for the neighboring garage. The solution for \( x_S \) is

\[
x_S = \frac{k_S h_S D + l_S p_{eS}}{w_S + 2k_S h_S} + \frac{k_S h_S p'_{gL} - (w_S + k_S h_S) p_{gL}}{w_S (w_S + 2k_S h_S)} l_S. \tag{A.18}
\]

Substituting (A.17) and (A.18) into (6), and differentiating with respect to \( p_{gL} \) and \( p_{gL} \), yields the home garage’s best response functions for garage parking fees:

\[
p_{gL} = \frac{\alpha}{2} + \frac{w_L D}{2l_L} + \frac{p'_{gL}}{2} \tag{A.19}
\]

\[
p_{gL} = \frac{\alpha}{2} + \frac{k_S d_s w_S D + w_S p_{eS} + k_S h_S p'_{gL}}{2 (w_S + k_S h_S)}. \tag{A.20}
\]
In a symmetric equilibrium, \( p'_{gL} = p_{gL} \) and \( p'_{gS} = p_{gS} \). Then, equations (A.19) and (A.20) yield the symmetric-equilibrium parking fees:

\[
p_{gL} = \alpha + \frac{w_LD}{l_L}, \tag{A.21}
\]
\[
p_{gS} = \frac{(w_S + k_S h_S) \alpha + k_S d_S w_S D + w_S p_{cS}}{2 w_S + k_S h_S}. \tag{A.22}
\]

By applying symmetry in (A.17) and substituting (A.22) into (A.18), we get the market boundaries:

\[
x_L = \frac{D}{2}, \tag{A.23}
\]
\[
x_S = \frac{w_S + k_S h_S}{(w_S + 2k_S h_S) (2w_S + k_S h_S)} \left( k_S h_S D + l_S (p_{cS} - \alpha) \right). \tag{A.24}
\]

Unlike regime \( \text{Int} \), in regime \( Lg \) the formulas for garage parking fees and market boundaries for one driver type do not depend on parameters specific to the other type because the two types no longer interact. Equation (A.21) can be rewritten as \( (p_{gL} - \alpha) l_L = w_L D \) which shows that the profit earned from a long-term parker is independent of parking duration, \( l_L \).\(^{15}\) The profit increases with the market power of a garage, which varies proportionally with the distance between garages, \( D \), and the walking time cost for long-term parkers, \( w_L \). The equilibrium garage parking fee for short-term parkers in (A.22) is a more complicated function. It increases, but less than proportionally, with the cost of providing service, \( \alpha \), and the curbside parking fee, \( p_{cS} \).\(^{16}\) The fee \( p_{gS} \) also increases with the distance between garages because more drivers then park on the curb, which increases search costs and makes curbside parking less attractive as a substitute for garage parking. Finally, \( p_{gS} \) decreases with parking duration, \( l_S \), but unlike for long-term parkers, the profit earned from short-term parkers increases with parking duration.

Similar to regime \( \text{Int} \), it is possible for the garage parking fee to be higher for long-term parkers than short-term parkers (i.e., \( p_{gL} > p_{gS} \)), and conversely also possible for the total outlay on garage parking to be lower for long-term parkers than short-term parkers (i.e., \( p_{gL} l_L < p_{gS} l_S \)). That is, the incentive compatibility constraint can be active. Section 5 provides an example with \( p_{gL} l_L < p_{gS} l_S \).

\(^{15}\)This result is attributable to the linearity of the model.
\(^{16}\)If \( \alpha \) and \( p_{cS} \) both increase by some amount \( \Delta \), then \( p_{gS} \) also increases by \( \Delta \) because total parking demand is price inelastic.
Two further derivatives of interest are

\[
\frac{\partial p_{gs}}{\partial w_s} = p_{cs} + k_s d_s D - \alpha, \quad (A.25)
\]
\[
\frac{\partial p_{gs}}{\partial k_s} = \alpha + \frac{2 w_s D}{l_s} - p_{cs} > 0, \quad (A.26)
\]

where \(^s\) means “identical in sign.” Assumption (5) does not assure that the right-hand side of (A.25) is positive, but it is likely to be satisfied unless the supply cost of garage parking is high. If the right-hand side is positive, the garage parking fee for short-term parkers rises as their walking time cost increases. Using (A.24) and the condition \(x_s < D/2\), it is easy to show that the right-hand side of (A.26) is positive. Therefore, \(p_{gs}\) rises with the search cost for curbside parking, as expected.

A.2.2 Long-term parkers indifferent between garage and curbside parking

If long-term parkers are indifferent between garage and curbside parking, constraint (A.15) binds and multiple symmetric equilibria for \(p_{gL}\) and \(p_{gS}\) can exist. This complication arises because the garage’s profit function is kinked. To see this, consider the candidate symmetric equilibrium shown in Figure 3. The full cost of parking at the home garage, line \(AB\), and the full cost of parking at the neighbor, \(CD\), intersect at point \(E\) where \(x_L = x'_L = D/2\). The cost of curbside parking, \(FG\), intersects the other two curves at point \(E\) as well. If the home garage lowers \(p_{gL}\), line \(AB\) shifts down and intersects line \(CD\) at point \(E_1\). All long-term parkers now strictly prefer parking at one of the garages to parking on the curb. If the home garage instead raises \(p_{gL}\), line \(AB\) shifts up. The market boundary with the neighbor is broken, and some long-term parkers now park on the curb. This raises the search cost of curbside parking, and line \(FG\) shifts upwards. A new market boundary between the home garage and curbside parking forms at point \(E_2\), and the equilibrium changes from regime \(Lg\) to regime \(Int\).

One can easily show that the home garage’s market shifts more quickly when \(p_{gL}\) increases than when it decreases, which suggests that the profit function has a “downward” kink at the candidate equilibrium. To see this, suppose that the marginal long-term parker is indifferent between the home garage, the neighboring garage, and curbside parking. If the home garage reduces \(p_{gL}\), the marginal long-term parker now strictly prefers garage parking. Regime \(Lg\) applies and the market boundary for long-term parkers is defined by (A.17):
$x_L = (D/2) + ((p'_L - p_{gL}) l_L)/(2w_L)$. The slope of the home garage’s market demand is

$$\frac{\partial x_L}{\partial p_{gL}}|_{Lg} = -\frac{l_L}{2w_L}.$$  

If the home garage instead increases $p_{gL}$, regime $Int$ prevails and the market boundary for long-term parkers is given by (10). The slope of the home garage’s market demand is

$$\frac{\partial x_L}{\partial p_{gL}}|_{Int} = -\frac{l_L}{2w_L} \frac{2h_l k_L w_S + 2l_L (w_S + 2h_S k_S) l_L}{2w_L w_S + 2h_l k_L w_S + 2h_S k_S w_L} < -\frac{l_L}{2w_L}.$$  

(A.27)

Since $|\frac{\partial x_L}{\partial p_{gL}}|_{Lg}| < |\frac{\partial x_L}{\partial p_{gL}}|_{Int}|$, the market demand curve for long-term parkers is kinked. This suggests that the profit function has a “downward” kink at the candidate equilibrium. However, because the cost of curbside parking rises if $p_{gL}$ is increased, some short-term parkers shift from curbside parking to the home garage, which increases the home garage’s profits from short-term parkers. The direction of the kink in the profit function is thus ambiguous.

The profit function is also kinked for changes in $p_{gS}$. If $p_{gS}$ rises, more short-term parkers
park on the curb, and curbside parking becomes less attractive for long-term parkers. Line $FG$ rises and the equilibrium for long-term parkers is unaffected. If $p_{gS}$ drops instead, fewer short-term parkers park on the curb, and curbside parking becomes more attractive for long-term parkers. Line $FG$ drops, and the home garage loses some long-term parkers. The home garage’s profit function therefore has a downward kink with respect to $p_{gS}$ at the candidate equilibrium. The kink occurs for similar reasons as in Salop’s (1979) model although utility from the outside good is exogenous in his model whereas utility from curbside parking is endogenous here.

The kinks explained above in the home garage’s profit function complicate the derivation of equilibria for regime $Lg$. Let $(p_{gL}^*, p_{gS}^*)$ be candidate symmetric equilibrium fees, $\pi^{Int}$ the home garage’s profit function in regime $Int$, and $\pi^{Lg}$ the home garage’s profit function in regime $Lg$. If condition (A.15) binds, several local conditions must be satisfied for a candidate symmetric equilibrium in regime $Lg$ to prevail. First, raising $p_{gL}$ slightly should not increase profit. Raising $p_{gL}$ slightly shifts the equilibrium from regime $Lg$ to regime $Int$. An increase is unprofitable if

$$\lim_{p_{gL} \to p_{gL}^*} \frac{\partial \pi^{Int}}{\partial p_{gL}} \leq 0.$$ 

This condition can be written as a lower bound on $p_{gL}^*$:

$$p_{gL}^* \geq p_{gL}^{Min}, \quad (A.28)$$

where $p_{gL}^{Min}$ is a complicated expression.

Second, lowering $p_{gL}$ slightly should not increase profit. A slight reduction leaves equilibrium in regime $Lg$. The requisite condition is therefore

$$\lim_{p_{gL} \to p_{gL}^*} - \frac{\partial \pi^{Lg}}{\partial p_{gL}} \geq 0.$$ 

This condition can be written as an upper bound on $p_{gL}^*$:

$$p_{gL}^* \leq p_{gL}^{Max} = \alpha + \frac{w_L D}{l_L}. \quad (A.29)$$

Third, raising $p_{gS}$ should not increase profits. Since a slight increase in $p_{gS}$ leaves equilibrium in regime $Lg$, the condition for deviation to be unprofitable is

$$\lim_{p_{gS} \to p_{gS}^*} - \frac{\partial \pi^{Lg}}{\partial p_{gS}} \leq 0,$$
which can be written as a lower bound on $p^*_g$:

$$p^*_g \geq p^*_{gS} = \frac{(w_s + k_s h_s) \alpha + k_s d_s w_s D + w_s p_{cS}}{2w_s + k_s h_s}. \quad (A.30)$$

Fourth, lowering $p_g$ should be unprofitable. Since lowering $p_g$ shifts equilibrium to regime $Int$, the relevant condition is

$$\lim_{p_g \rightarrow p^*_g} \frac{\partial \pi^{Int}}{\partial p_g} \geq 0.$$ 

This can be written as an upper bound on $p^*_g$:

$$p^*_g \leq p^*_{gS}, \quad (A.31)$$

where $p^*_{gS}$ is a complicated expression.

A final local condition for a symmetric equilibrium is that condition (A.15) in the text holds as an equality with $x_L = D/2$; this yields

$$p_g^* = \frac{2l_L (w_s + 2h_s k_s) p_{gL} - 2l_L (w_s + 2h_s k_s) p_{cL} + 4k_L h_s l_s p_{cS}}{4h_s l_s k_L} + \frac{(w_L (w_s + 2h_s k_s) - 2h_s w_s k_L) D}{4h_s l_s k_L}. \quad (A.32)$$
Figure 4 illustrates conditions (A.28)-(A.32) for a numerical example with parameter values $p_{cL} = p_{cS} = 2.5$, $D = 0.125$, $\alpha = 2.5$, $l_L = 2$, $l_S = 1$, $w_L = 8$, $w_S = 8$, $k_L = 0.2$, $k_S = 0.08$, $d_L = 100$, and $d_S = 100$. Inequality conditions (A.28)-(A.31) are all satisfied within the polygonal region of $(p_{gL}, p_{gS})$ with vertices at points $A$, $B$, $C$ and $D$. All points within this region are candidate equilibria. Condition (A.32) lies within the region between points $E$ and $F$. All points on the line segment $EF$ are therefore candidate symmetric equilibria. We arbitrarily choose point $E$ with the lowest garage parking fees as the equilibrium for regime $Lg$.

The necessary conditions for a profit maximum listed above are all local conditions. To confirm that regime $Lg$ is a global equilibrium it is also necessary to check that it yields a higher profit to the home garage than any other regime. Doing so entails consideration of non-local deviations by the home garage in garage parking fees that shift equilibrium to other regimes. This process is described in Appendix A.11 below. A systematic check is not practical analytically, but it is straightforward to do numerically as is done in Section 5.

A.3 Equilibrium for regime $Sg$

Regime $Sg$ is a mirror image of regime $Lg$. The solution can therefore be obtained from the solution to regime $Lg$ described in the text by interchanging $L$ and $S$ subscripts.

A.4 Equilibrium for regime $Lc$

In regime $Lc$, all long-term parkers park on the curb. Short-term parkers split between garages and the curb, and the marginal short-term parker is indifferent between parking at a garage and parking on the curb. Thus, the market boundary for short-term parkers is solved by using (8) with $T = h_L D + h_S (D - x_S - x_S')$ and the counterpart of (8) for the neighbor. In a symmetric equilibrium with $p_{gS}' = p_{gS}$, the solution is

$$x_S = \frac{k_S (h_S + h_L) D + l_S (p_{cS} - p_{gS})}{w_S + 2 k_S h_S}.$$  \hfill (A.33)

The home garage’s profit is given by (6) with $x_L = 0$: $\pi = 2 (p_{gS} - \alpha) h_S x_S$. The local profit-maximizing parking fee for short-term parkers works out to

$$p_{gS}^* = \frac{(w_S + k_S h_S) l_S \alpha + w_S l_S p_{cS} + w_S k_S (h_S + h_L) D}{2 w_S + k_S h_S} l_S.$$  \hfill (A.34)
Several conditions must be satisfied for this solution to be consistent with regime $Lc$ as well as with a global profit maximum. First, $p_{gL}$ must be high enough that no long-term parkers are drawn to park at a garage. The lower bound on $p_{gL}$ is an increasing function of $p_{gS}$ because a higher value of $p_{gS}$ induces more short-term parkers to use curbside parking, which makes curbside parking less attractive for long-term parkers, and increases their demand for garage parking. The constraint thus takes the form

$$p_{gL}^* \geq p_{gL}^{Min}(p_{gS}),$$

where $p_{gL}^{Min}(p_{gS})$ is a complicated, increasing, linear function of $p_{gS}$.

A second condition is that the home garage does not find it profitable to attract long-term parkers by setting $p_{gL}$ low enough. If it does attract any long-term parkers, the market moves into regime $Int$. Thus, the profit in regime $Int$ should be non-decreasing at $p_{gL}^{Min}(p_{gS})$: 

$$\frac{\partial \pi^{Int}}{\partial p_{gL}} \bigg|_{p_{gL}^{Min}(p_{gS})} \geq 0.$$ 

This condition can be written as an upper bound on $p_{gL}^*$:

$$p_{gL}^* \leq p_{gL}^{Max}(p_{gS}),$$

where $p_{gL}^{Max}(p_{gS})$ is another complicated, increasing, linear function of $p_{gS}$.

Further conditions must be imposed because the profit function is not globally concave. If $p_{gS}$ is increased sufficiently above the level given in (A.34), curbside parking may become so crowded that long-term parkers start to use garage parking. Let $p_{gS}|_{x_L=0}$ denote the threshold value of $p_{gS}$ if it exists. At this point, the profit function is kinked upward with respect to $p_{gS}$. A necessary condition for (A.34) to be a global profit maximum is therefore that the local maximum of regime $Lc$ be lower than the threshold parking fee at which the regime switches from regime $Lc$ to regime $Int$.

$$p_{gS}^* < p_{gS}|_{x_L=0},$$

Figure 5 depicts a case where inequality (A.37) is violated. In this example, the parking market switches from regime $Lc$ to regime $Int$ at point $B$. Thus, by default, the local maximum in regime $Lc$, represented by point $A$, must be below the local maximum in regime $Int$, represented by point $C$. This means that point $A$ cannot be the global maximum under
these conditions. If inequality (A.37) is satisfied, profit may still increase enough within the interior region that the global profit maximum lies within regime \(Int\) rather than regime \(Lc\). A fourth condition is therefore

\[
\pi^{Lc*} > \pi^{Int*},
\]

where \(*\) denotes the maximum of the respective regime.

In Figure 6, regime switching occurs (at point \(B\)) after the local maximum of regime \(Lc\) (point \(A\)); yet the local maximum of regime \(Int\), point \(C\), is higher. Thus, in this figure, condition (A.37) is satisfied but condition (A.38) is violated. Figures 7 and 8 show two further cases for which both conditions are satisfied.

Finally, multiple equilibria are a possibility. Figure 9 depicts an example with the same parameter values as for the base case of the numerical example in Section 5 except for the curbside parking fees: \(p_{cL} = p_{cS} = 0.7\), \(D = 0.125\), \(\alpha = 2.5\), \(l_L = 2\), \(l_L = 1\), \(w_L = 16\), \(w_S = 16\), \(k_L = 0.16\), \(k_S = 0.16\), \(d_L = 100\), and \(d_S = 100\). The locally optimal parking fee for short-term parkers in regime \(Lc\) is \(p_{gL}^* = 3.9\). Inequality conditions (A.35) and (A.36) are satisfied on line segment \(AB\). Conditions (A.37) and (A.38) are also satisfied since profits within regime \(Int\) are decreasing to the right of the line \(\partial \pi^{Int}/\partial p_{gS} = 0\). The choice of \(p_{gL}\) on segment \(AB\) is inconsequential because it does not affect parking choices for either type of driver.

The equilibrium conditions listed above are necessary conditions for an equilibrium in
Figure 6: Profit function in regime $Lc$, case (b)

Figure 7: Profit function in regime $Lc$, case (c)
Figure 8: Profit function in regime $Lc$, case (d)

Figure 9: Example with multiple equilibria in regime $Lc$
regime \( Lc \) to prevail. To confirm that regime \( Lc \) is a global equilibrium, it is also necessary to check that it yields a higher profit to the home garage than any other regime.

### A.5 Equilibrium for regime \( Sc \)

Regime \( Sc \) is a mirror image of regime \( Lc \), and the solution can be obtained from the solution to regime \( Lc \) by interchanging \( L \) and \( S \) subscripts.

### A.6 Equilibrium for regime \( Lc+Sg \)

In this regime, all long-term parkers park on the curb while all short-term parkers park at a garage. Thus, \( x_L = 0 \) and \( x_S = D/2 \) in a symmetric equilibrium. Total parking time on the curb is \( T = h_L D \). Several conditions must be satisfied for this regime to be an equilibrium. First, \( p_{gL} \) must be high enough that no long-term parkers are drawn to park at a garage, a condition that works out to

\[
p_{gL} \geq \frac{p_{cL} l_L + k_L h_L D}{l_L} \equiv p_{gL}^{Min}.
\]

Second, the curbside parking fee must be high enough that no short-term parkers are drawn to curbside parking, which defines an upper bound on the garage parking fee that can be charged to short-term parkers:

\[
p_{gS} \leq \frac{p_{cS} l_S + k_S h_L D - \frac{w_S D}{2}}{l_S} \equiv p_{gS}^{Max}.
\]

Third, it cannot be profitable to decrease \( p_{gL} \) and attract long-term parkers, which would cause a shift to regime \( Sg \):

\[
\frac{\partial \pi}{\partial p_{gL}} \bigg|_{p_{gL}^{Min}} \geq 0.
\]

Fourth, it cannot be profitable to increase \( p_{gS} \) and induce some short-term parkers to shift to the curb which would cause a shift to regime \( Lc \):

\[
\frac{\partial \pi}{\partial p_{gS}} \bigg|_{p_{gS}^{Max}} \leq 0.
\]

As is the case with other regimes, these four conditions are necessary conditions for a local maximum. To confirm that regime \( Lc + Sg \) is a global equilibrium, it is also necessary to
check that it yields a higher profit to the home garage than any other regime.

A.7 Equilibrium for regime \( Lg + Sc \)

This regime is a mirror image of regime \( Lc + Sg \). The solution can therefore be obtained from the solution to regime \( Lc + Sg \) by interchanging \( L \) and \( S \) subscripts.

A.8 Equilibrium for regime \( Lg + Sg \)

All drivers of both types in this regime use garage parking. Thus, \( x_L = D/2 \) and \( x_S = D/2 \) in a symmetric equilibrium. For this regime to be an equilibrium, the following conditions must be satisfied. First, \( p_{gL} \) must be low enough that no long-term parkers are drawn to park on the curb:

\[
p_{gL} \leq \frac{2p_{cL}l_L - w_LD}{2l_L} \equiv p_{gL}^{Max}.
\]

Second, a similar condition applies for short-term parkers:

\[
p_{gS} \leq \frac{2p_{cS}l_S - w_SD}{2l_S} \equiv p_{gS}^{Max}.
\]

Third, it cannot be profitable to increase \( p_{gL} \) above \( p_{gL}^{Max} \) which would cause a shift to regime \( Sg \):

\[
\frac{\partial \pi^{Sg}}{\partial p_{gL}} \bigg|_{p_{gL}^{Max}} \leq 0.
\]

Fourth, it cannot be profitable to increase \( p_{gS} \) above \( p_{gS}^{Max} \) which would cause a shift to regime \( Lg \):

\[
\frac{\partial \pi^{Lg}}{\partial p_{gS}} \bigg|_{p_{gS}^{Max}} \leq 0.
\]

These four conditions are necessary conditions for a local maximum. To confirm that regime \( Lg + Sg \) is an equilibrium regime it is also necessary to check numerically that it yields a higher profit to the home garage than any other regime.

A.9 Equilibrium for regime \( Lc + Sc \)

In this regime, all drivers of both types park on the curb. Thus, \( x_L = 0 \), \( x_S = 0 \), and \( T = (h_L + h_s)D \). Several conditions must be satisfied for this regime to be an equilibrium.
First and second, garage parking fees must be high enough that all drivers prefer to park on the curb. The requisite conditions are

\[ p_{gL} \geq \frac{p_{cL}l_L + k_L(h_L + h_S)D}{l_L} \equiv p_{gL}^{Min} \quad (A.47) \]

\[ p_{gS} \geq \frac{p_{cS}l_S + k_S(h_L + h_S)D}{l_S} \equiv p_{gS}^{Min} \quad (A.48) \]

Third, it cannot be profitable to decrease \( p_{gL} \) to attract long-term parkers and induce a shift to regime \( Sc \):

\[ \frac{\partial \pi^{Sc}}{\partial p_{gL}} \Bigg|_{p_{gL}^{Min}} \geq 0. \quad (A.49) \]

Fourth, it cannot be profitable to decrease \( p_{gS} \) to attract short-term parkers and induce a shift to regime \( Lc \):

\[ \frac{\partial \pi^{Lc}}{\partial p_{gS}} \Bigg|_{p_{gS}^{Min}} \geq 0. \quad (A.50) \]

Again, these four conditions are necessary conditions for a local maximum. To confirm that regime \( Lc + Sc \) is an equilibrium regime, it is also necessary to check numerically that the home garage cannot obtain positive profits in another regime.

**A.10 The social optimum: regime Int**

Because it is an interior solution, the social optimum in regime \( Int \) is characterized by first-order conditions \( \partial TC/\partial x_L = 0 \) and \( \partial TC/\partial x_S = 0 \) where \( TC \) is given by the the sum of (17), (18), and (19). The solution is

\[ x_L^{SO} \Big|_{Int} = \left( \frac{2h_S[d_L(k_l l_l - k_s l_L)^2 D - \alpha(k_L + k_S)l_l + \alpha k_s l_L]}{4d_L d_s (k_L l_L - k_s l_s)^2 - 4h_s k_s w_L - 4h_L k_L w_s - w_L w_s} \right) \quad (A.51) \]

\[ x_S^{SO} \Big|_{Int} = \left( \frac{2h_L[d_s(k_l l_L - k_s l_s)^2 D - \alpha(k_L + k_S)l_L + \alpha k_l l_s]}{4d_L d_s (k_L l_L - k_s l_s)^2 - 4h_s k_s w_L - 4h_L k_L w_s - w_L w_s} \right) \quad (A.52) \]

The second-order condition for a total cost minimum is \((4k_L h_L + w_L)(4k_S h_S + w_S) - 4d_L d_s (k_L l_L + k_s l_L)^2 > 0\).
A.11 Testing for a global equilibrium

As noted above, to establish that a candidate symmetric equilibrium for a given regime is a global equilibrium, it is necessary to check that the home garage cannot earn higher profits by deviating from the candidate and bringing about a change of regime. The following procedure was used for the numerical examples.

Let \((p^e_{gL}, p^e_{gS})\) denote candidate symmetric equilibrium parking fees and set the neighboring garage’s fees at these values. Then, allow the home garage to experiment with different combinations of \((p_{gL}, p_{gS})\) that shift equilibrium into other regimes. For example, suppose that the candidate equilibrium regime is \(Int\). Set \((p'_{gL}, p'_{gS}) = (p^{Int}_{gL}, p^{Int}_{gS})\). Then, consider deviations from \((p^{Int}_{gL}, p^{Int}_{gS})\) by the home garage. One possibility is that the home garage will reduce \(p_{gL}\) to the point where all long-term parkers park in garages. If short-term parkers continue to park both in garages and on the curb, the new regime becomes \(Lg\). Assume that regime \(Lg\) does prevail and derive profit-maximizing values of \((p_{gL}, p_{gS})\) for the home garage while holding \((p'_{gL}, p'_{gS})\) fixed at \((p^{Int}_{gL}, p^{Int}_{gS})\). Several consistency conditions have to be satisfied (the \(x_i\) must be non-negative, etc.). If the new regime passes this initial test, check whether the home garage’s profits are higher than in the candidate equilibrium. If they are higher, the candidate equilibrium fails. Otherwise, the candidate equilibrium passes this test and the next alternative regime is examined.

Because deviation by the home garage breaks symmetry, it is possible that the home garage and the neighbor will serve different markets. Consequently, several asymmetric regimes must be tested in addition to the symmetric regimes described in the text. The additional regimes are as follows:

- \(Lc(-)\): The home garage does not serve any long-term parkers, but the neighbor does serve some of them. Both home garage and neighbor serve some short-term parkers while the remaining short-term parkers park on the curb.
- \(-Lc\): Same as \(Lc(-)\) with the roles of home garage and neighbor reversed.
- \(Sc(-)\): The home garage does not serve any short-term parkers, but the neighbor does serve some of them. Both home garage and neighbor serve some long-term parkers while the remaining long-term parkers park on the curb.
- \(-Sc\): Same as \(Sc(-)\) with the roles of home garage and neighbor reversed.
- \(Lc(Lc)\): Neither the home garage nor the neighbor serves long-term parkers. This regime is qualitatively the same as regime \(Lc\) although it is not symmetric.
• $Sc(Sc)$: Neither the home garage nor the neighbor serves short-term parkers. This regime is qualitatively the same as regime $Sc$ although it is not symmetric.

• $Lg + Sc(−)$: The home garage and neighbor together serve all long-term parkers. The home garage does not serve any short-term parkers, but the neighbor does serve some of them.

• $Lc(−) + Sg$: The home garage does not serve any long-term parkers, but the neighbor does serve some. The home garage and neighbor together serve all short-term parkers.

• $Lc(Lc) + Sg$: Neither the home garage nor the neighbor serves long-term parkers. Together they serve all short-term parkers. This regime is qualitatively the same as regime $Lc + Sg$ although it is not symmetric.

• $Lg + Sc(Sc)$: The home garage and neighbor together serve all long-term parkers. Neither serves any short-term parkers. This regime is qualitatively the same as regime $Lg + Sc$ although it is not symmetric.

Depending on which candidate symmetric equilibrium is being tested, some of these additional regimes cannot be reached by profitable deviations by the home garage. But it is simpler to run through all the possibilities numerically than to determine analytically which are potentially profitable.

A.12 Mixed oligopoly with identical drivers

With a private oligopoly the equilibrium garage fee with one driver type in regime $Int$ is given in equation (15). For the mixed oligopoly the equilibrium fee for private garages works out to

$$p_g = \frac{5v^2 + 6p_v + 6\alpha v^2 + 8\alpha v + 2\alpha + (\alpha + p_v)v^3 + dkv(1 + v)(2 + v)D}{2(1 + 6v + 5v^2 + v^3)},$$

where $v \equiv w/(kdI)$. The second-best fee at public garages is

$$p_{g, pub} = \frac{(1 + 3v + 3v^2)\alpha + v(5 + 5v + v^2)p_v + (1 - v - v^2)dkvD}{1 + 6v + 5v^2 + v^3}.$$

Total costs in the private oligopoly and mixed oligopoly are given by long expressions.
The efficiency index defined in Section 6 works out to

\[ efm = \frac{v(v + 4)(10 + 52v + 63v^2 + 28v^3 + 4v^4)}{8(1 + 6v + 5v^2 + v^3)^2}. \]  \hspace{1cm} (A.53)

Equation (A.53) is plotted in Figure 10. Relative efficiency is zero at \( v = 0 \), increases steeply with \( v \) to a maximum value of \( efm = 0.586 \) at \( v = 1.49 \), and declines very slowly for larger values of \( v \).