Calibration and Tests of a Yaw Sphere–Thermometer System for Sensible Heat Flux Measurements

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ABSTRACT

A yaw sphere-thermometer assembly, to measure sensible heat flux density by the eddy correlation method, was built following the design of Tanner and Thurtell. Wind tunnel experiments indicate that the sphere constant should be 1.57, which is significantly less than the theoretical value of 2.25. The effects of tilt indicate that heat fluxes may be in error by 5% per degree of tilt in unstable conditions and up to 11% per degree in stable conditions. Field comparisons of the heat fluxes measured by the yaw sphere-thermometer system and a Bowen ratio apparatus produced satisfactory agreement.

1. Introduction

Knowledge of the vertical eddy fluxes of physical entities, such as sensible heat, water vapor and momentum, are basic to a rational understanding of the atmospheric boundary layer. The accurate determination of these fluxes in the field is not easily accomplished, and remains at the core of micrometeorological research. In the last decade much of this research has centered around the application of the eddy correlation method, since it possesses the advantage of directly measuring eddy fluxes.

Measurement of the sensible heat flux H via the eddy correlation technique requires very fast-response sensing of the air temperature and vertical wind velocity. The stringent requirements of the wind sensor are hardest to meet. One of the most promising approaches is the pressure-sphere anemometer (e.g., Wesely *et al.*, 1970, 1972). Recently, Tanner and Thurtell (1970) showed the promise of a relatively simple yaw sphere-thermometer (YST) system for the measurement of H incorporating this pressure-sphere principle. Our instrument closely followed their design. When directed into the wind the yaw sphere generates a pressure between two ports that is proportional to the product of the horizontal and vertical winds. The analog pressure signal from an electrical pressure transducer is filtered and used to drive a resistance-thermometer bridge. The output (proportional to the product of the mean horizontal wind speed $\bar{u}$ and the instantaneous heat flux) is integrated, and divided by $\bar{u}$ from a sensitive cup anemometer to give H.

This paper deals with three important aspects of the YST system. First, the need for direct calibration of the yaw sphere is investigated on the basis of wind tunnel experiments. Second, the effects of tilting the yaw sphere are analyzed. Finally, the results of a field comparison between the YST system and a Bowen ratio apparatus are presented.

2. Review of yaw sphere-thermometer theory

In real fluid flow, the pressure distribution at points on a sphere can be written

$$P=P_s+(\rho/2)V^2(1-b\sin^2\psi),\ \psi<60°, \qquad (1)$$

where P_s is the static pressure, ρ the air density, V the air speed, and ψ is the angle between $\mathbf{V}$ and the radius vector of the point. The sphere constant b is a function of the Reynolds number Re[$=Vd/\nu$, where d is the diameter of the sphere and ν the kinematic viscosity of the fluid], but is relatively constant for $2000<\mathrm{Re}<200{,}000$. In ideal, irrotational fluid flow, the theoretical value of b is 9/4.

On directing the sphere azimuthally into the wind, as shown in Fig. 1, the components of the wind vector with respect to the x,z plane (formed by the yaw sphere ports and its bisector) are

$$u=|\mathbf{V}|\cos\alpha,\ v=0,\ \ w=|\mathbf{V}|\sin\alpha,$$

where α is the angle between the wind vector and the bisector of the ports. The pressure difference between the ports of the yaw sphere is then given by

$$\Delta P=\rho b(\sin\theta)uw, \qquad (2)$$

where θ is the included angle between the ports.

The pressure difference (ΔP) is converted to an analog signal and passed through a high-pass filter

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which is subsequently used to drive a resistance-thermometer bridge. Amplification and integration of this bridge output, expressed in Reynolds' notation, gives

$$\bar{E}_0 = \rho b(\sin\theta)GBM(\bar{u}\overline{w'T'} + \bar{w}\overline{u'T'} + \overline{u'w'T'}), \quad (3)$$

where G is the amplifier gain, B the bridge constant, and M the pressure transducer constant.

The nature of this output signal needs careful consideration. In Eq. (3) the moment $\overline{u'T'}$ characterizes the turbulent heat flow in the direction of the mean wind velocity. One would expect this quantity to be negative for unstable, and positive for stable stratification (Monin and Yaglom, 1971). Direct measurements show that on the average, the ratio $\overline{u'T'}/\overline{w'T'}$ grows with increasing stability [e.g., Zubkovskii and Tsvang, 1966; Wesely *et al.*, 1970; Sheppard (see Monin and Yaglom, 1971)] and indicate that $\overline{u'T'}$ is larger than $\overline{w'T'}$. However, since $\bar{w}$ is typically very small compared to $\bar{u}$, the term $\bar{w}\overline{u'T'}$ will be small compared to $\bar{u}\overline{w'T'}$, and will be assumed negligible. If we can assume the triple moment $\overline{u'w'T'}$ to be small when compared to the term $\bar{u}\overline{w'T'}$, then as shown by Tanner and Thurtell (1970) the expression for $\bar{E}_0$ reduces to

$$\bar{E}_0 \approx \rho b(\sin\theta)GBM\bar{u}\overline{w'T'}, \quad (4)$$

and since the sensible heat flux can be written as $H = \rho c_p \overline{w'T'}$, then

$$H \approx c_p[b(\sin\theta)GBM]^{-1}\bar{E}_0/\bar{u}. \quad (5)$$

Thus, the sensible heat flux can be determined from the yaw sphere-thermometer system output if $\bar{u}$ is measured nearby at the same height. Tanner and Thurtell (1970) suggested the use of a cup anemometer to find $\bar{u}$. Kondo *et al.* (1971) and Hyson (1972) suggest that cup anemometers may overestimate the mean wind speed by as much as 5%. Slight underestimation of H may therefore be expected if $\bar{u}$ is measured with a cup anemometer.

3. Determination of the sphere constant

The sphere constant was derived from data collected in a series of wind tunnel experiments with the yaw sphere. The sphere ports were aligned azimuthally into the direction of the mean flow thus allowing α (the angle of attack of the flow to the yaw sphere axis) to be measured directly. Eq. (2) can be rewritten

$$b = 2\Delta P/(\rho|\mathbf{V}|^2 \sin 2\alpha \sin\theta). \quad (6)$$

All the terms on the right-hand side of Eq. (6) being known ($\theta = 45°$ for our sphere), the sphere constant could be evaluated. A series of measurements were made at various wind speeds with $6000 < \mathrm{Re} < 20{,}000$ for a constant angle of attack α. Then the axis of the yaw sphere probe was tilted through a series of angles, $|\alpha| < 10°$. Inherent limitations of the wind tunnel did not allow larger angles of attack, nor to calibrate the sphere (5 cm diameter) for $\mathrm{Re} < 6000$. The sphere constant was obtained from measurements both with and without a grid in the flow.

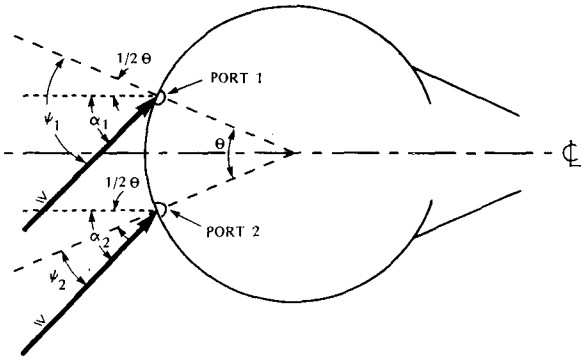

FIG. 1. Schematic representation of the angles ψ, α, θ, and the wind vector $\mathbf{V}$ on the yaw sphere.

In the wind tunnel, the angle of attack was measured as that between the horizontal and the apparent axis of the yaw sphere probe (i.e., the sphere and its supporting stem). Any inherent misalignment in the yaw sphere axis and supporting stem would manifest itself as a constant error in the measurement of α in the wind tunnel. Let this tilt error be δ. With the assumption that 2δ is small

$$\sin(2\alpha + 2\delta) \approx \sin 2\alpha + 2\delta \cos 2\alpha,$$

and Eq. (6) becomes

$$b = 2\Delta P/[\rho|\mathbf{V}|^2(\sin 2\alpha + 2\delta\cos 2\alpha)\sin\theta]. \quad (7)$$

The above expression will be written

$$\tan 2\alpha + 2\delta = (1/b)[2\Delta P/(\rho|\mathbf{V}|^2\cos 2\alpha \sin\theta)], \quad (8)$$

so that a plot of $\tan 2\alpha$ vs $2\Delta P/\rho|\mathbf{V}|^2\cos 2\alpha\sin\theta$ yields a slope of $1/b$, and intercept values of -2δ on the abscissa and $2b\delta$ on the ordinate. Fig. 2 represents such a graph for $V = 4$ and 6 m sec^{-1} in flow without a grid. The sphere constant obtained was 1.57 with a tilt error of $\sim 1°$. This value for b is significantly less than the theoretical value of 2.25. Data from the grid flow experiments yielded a b estimate within 6% of the 1.57 value. Little emphasis is placed on this variation since the ΔP trace on the chart recorder could not be resolved to better than $\pm 10\%$ for the grid turbulence experiment. In another series of wind tunnel measurements, the fast-response resistance thermometer (used with the yaw sphere to measure the sensible heat flux) was mounted at the side of the sphere in its usual position for field measurements. This configuration had no noticeable effect on the

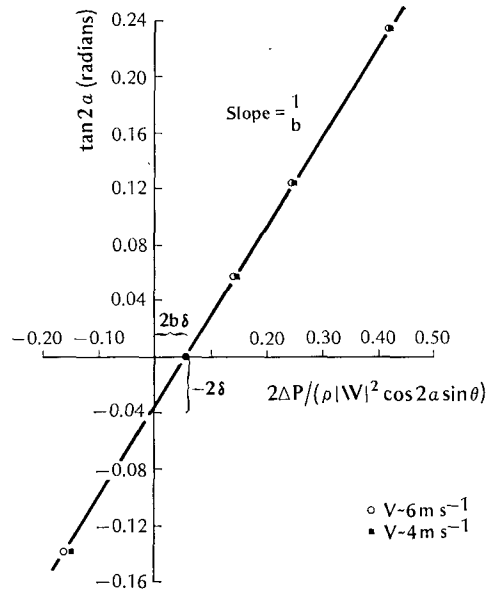

FIG. 2. Graphical determination of the sphere constant b and alignment error δ. The equation of the line is

$$\tan 2\alpha = -2\delta + (1/b)(2\Delta P/\rho|\mathbf{V}|^2 \cos 2\alpha \sin\theta).$$

The slope $(1/b)$ is 0.638, giving $b=1.57$; the y-intercept (-2δ) is -0.035, giving $\delta=0.0175$ rad $\approx 1°$; the x-intercept $(2b\delta)$ is 0.055, again giving $b\approx 1.57$.

pressure generated at the ports, or on the sphere constant.

A value of $b=1.79$ has recently been established for the original yaw sphere used by Tanner and Thurtell (Tanner, 1971, private communication). Values of $b<2.25$ have also been reported by Martinot-Lagarde *et al.* (1952), and Wesely *et al.* (1972) for other spheres with different port-hole configurations. The reason for these lower values has not been established; however, Thurtell (private communication, 1972) indicates that roughening the sphere surface gives a value closer to that predicted by theory.

While it was not possible to investigate the nature of b below Re=6000, we shall assume (after Martinot-Lagarde *et al.*, 1952) that it behaves approximately constant down to Re=2000. For our 5-cm yaw sphere this implies that the experimentally determined sphere constant should be used only when the mean wind speed is >60 cm sec^{-1}. In view of this, great caution must be used in interpreting results from the yaw sphere under very light wind conditions.

4. Effect of yaw sphere axis tilt

In Section 3, it was shown that any inherent misalignment in the yaw sphere axis could be determined from general considerations of the angle of attack of the flow incident on the sphere in a wind tunnel. Knowledge of this could then be applied in accurately aligning the axis of the sphere horizontally in field measurements. In practice, horizontal leveling of the yaw sphere axis may not always be assured. Consequently, some estimate of likely error in sensible heat flux measurements due to a tilt off-axis (due to construction mis-alignment or inaccurate leveling in the field) is given here.

Let us assume that the sphere is tilted off-axis so that the tilt angle δ is positive (i.e., upward; see Fig. 3). The components of the wind vector with respect to the x_T, z_T plane (formed by the ports and the tilt axis) are

$$u_T=(u\cos\delta - w\sin\delta),\ v_T=0,\ w_T=(u\sin\delta + w\cos\delta).$$

If we assume δ to be small, then $u_T\approx(u-\delta w)$ and $w_T\approx(u\delta+w)$. Substituting these approximations of u_T and w_T for the u and w components in Eq. (3) yields

$$\bar{E}_{0(\mathrm{TILT})}\approx\rho b(\sin\theta)GBM[(\bar{u}\overline{w'T'}+\bar{w}\overline{u'T'}+\overline{u'w'T'} + \delta(2\bar{u}\overline{u'T'}-2\bar{w}\overline{w'T'}+\overline{u'^2T'}-\overline{w'^2T'})]. \quad (9)$$

Since $\bar{w}$ is typically very small when compared to $\bar{u}$, the above expression reduces to

$$E_{0(\mathrm{TILT})}\approx\rho b(\sin\theta)GBM(\bar{u}\overline{w'T'}+2\delta\bar{u}\overline{u'T'}) \quad (10)$$

if we assume the triple moment terms to be negligible. Then

$$E_{0(\mathrm{TILT})}=\bar{E}_0+\bar{E}_{\delta 0},\quad \text{where } \bar{E}_{\delta 0}=\rho b(\sin\theta)GBM2\delta\bar{u}\overline{u'T'},$$

is the error caused by the tilt. This effectively produces an error in the heat flux

$$H_\delta\approx c_p[b(\sin\theta)GBM]^{-1}\bar{E}_{\delta 0}/\bar{u}=\rho c_p 2\delta\overline{u'T'}. \quad (11)$$

For a small positive tilt off-axis the YST system will therefore produce a heat flux measurement

$$H_{(\mathrm{TILT})}\approx H+H_\delta=\rho c_p(\overline{w'T'}+2\delta\overline{u'T'}). \quad (12)$$

Thus, for $\delta=1°$,

$$H_{1°}=\rho c_p[\overline{w'T'}+0.035\overline{u'T'}].$$

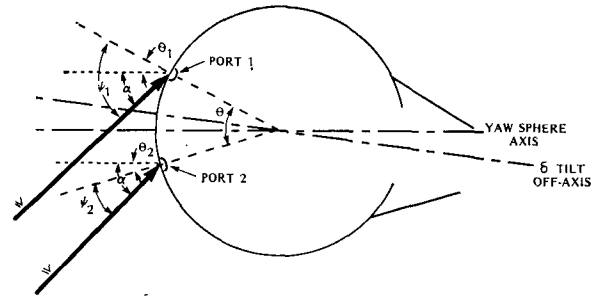

FIG. 3. The effect of tilt on the geometry of the sphere.

From previous studies concerning the dependence of $\overline{u'T'}/\overline{w'T'}$ on Richardson number, we note that the ratio increases from 0.14 in unstable to -3.2 in slightly stable conditions. Using these results, we can make some estimate of the error H_δ under different stability conditions. During the daytime with moderate instability, $-\overline{u'T'} \approx 1.4\ \overline{w'T'}$ so that $H_{1°}$ would then effectively include an error of $\sim 5\%$. On the other hand, under nighttime conditions with weak stability, $\overline{u'T'} \approx -3.2\ \overline{w'T'}$ in the limit. In this case $H_{1°}$ would include an error of $\sim 11\%$.

Although large tilt errors (of the order of 5°) should not occur in practice, we shall consider such an effect on our system to help delineate the error magnitude at small angles of tilt. For a 5° off-axis tilt of the sphere, the error in measuring the sensible heat flux would be $\sim 25\%$ in moderately unstable stratification. At night, with light winds and stable stratification, large tilts in the yaw sphere axis would lead to considerable errors in the measurements of the sensible heat flux. With conventional leveling devices, it should be possible to minimize this off-axis tilt in the sphere so that this source of error in the sensible heat flux measurements is within the range of the general heat flux spatial variability.

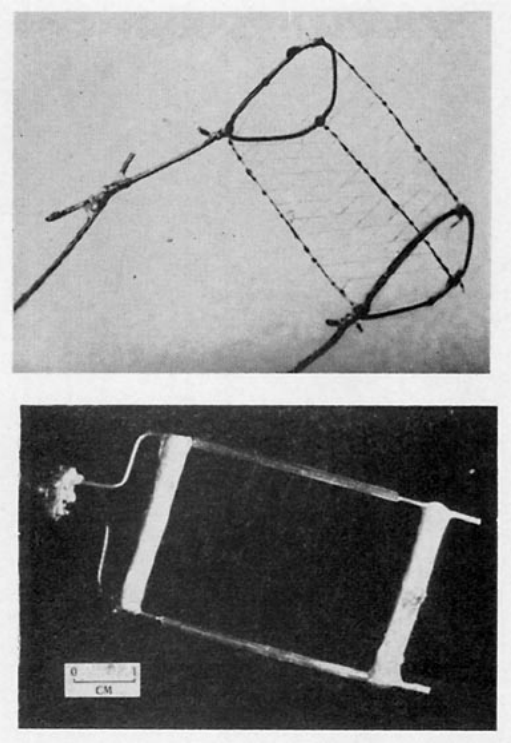

FIG. 4. Detail of the original fine-wire thermometer (top), and the modified ceramic-spacer design (bottom).

5. Thermometer and response

The fast-response thermometer was initially built following the design of Wesely *et al.* (1970). The thermometer element consisted of about 65 cm of platinum-coated tungsten wire with a diameter of 5.6 μm. This wire was welded to its stainless steel side support terminals with the aid of a Disa hot-wire anemometer welding assembly. The result is shown in Fig. 4.

Calculations by Wesely *et al.* (1970) indicate that for the resistance wire used in the thermometer, the time constant is ~ 1.5 msec in "still air," and ~ 0.6 msec in 10 m sec^{-1} winds. Up to a frequency of 20 Hz, reduction in amplitude should be $<2\%$ and phase shift $\sim 10°$. Solar heating of the fine resistance wire was shown to be negligible for eddy flux calculations.

Under field conditions the thermometer responded well, but its durability was not always satisfactory. As a result, a slightly modified design was developed. The thermometer element and support terminals remained the same. The triangular frame however was eliminated by winding the element directly around the side supports which were covered with an insulating layer of plastic tubing. The rigidity of the thermometer was maintained by inserting two thin ceramic spacers (Fig. 4). Ceramic was chosen because its thermal coefficient of expansion is similar to that of tungsten. Field tests of the two thermometer designs in two YST systems placed 2 m apart produced almost identical results under a variety of wind and cloud conditions. The new design has proved to be more durable.

Frequency response and phase shift of the complete YST system closely followed that reported by Tanner and Thurtell (1970). In the frequency domain the turbulent heat flux could be measured to an upper frequency limit of 8 Hz without significant attenuation.

6. Field test results

There is no standard against which to calibrate an instrument which measures sensible heat flux density. To gain confidence in the YST system, however, we compared the measurements against independent evaluation of H from a Bowen ratio apparatus.

The site chosen was an extensive flat grass surface at Ladner, British Columbia. The grass was approximately 90 cm tall, giving a roughness length of ~ 10 cm based on analysis of neutral wind profiles. The grass was dry, and the ground surface consisted of a dense old-grass litter layer. The instrument heights and mast locations ensured adequate height/fetch ratios for all wind directions except from the NNE–SSE sector. Winds were predominantly SW–NW during the test runs.

The YST system was mounted ~ 1.5 m above ground level, with a sensitive cup anemometer at the same height nearby (Fig. 5). The Bowen ratio apparatus was mounted at the same height, and 4 m from the YST system. Complete details of the Bowen

FIG. 5. The yaw sphere-thermometer system.

ratio system are given by Black and McNaughton (1971). Net radiation (Swissteco, Model S1) and soil heat flux density (flux plate) were also continuously recorded.

Heat fluxes were monitored continuously during daylight hours for a period of four days. The normal sample interval was set at 30 min for each system. Periodically it was necessary to check the electronic zero on the YST system, but the sample interval was never <20 min. This is in accord with the optimum sampling interval for H measurements set by Chou (1966).

The results of the comparison experiments are presented in Figs. 6 and 7. Clearly, with the possible exception of 24 August 1971 (Fig. 6a), the agreement between the independent sensible heat flux measurements (H_{YST} and H_β) is very promising. There are two possible explanations for the poor agreement on 24 August. First, wind direction was variable on this day, including flow from the east, where there was insufficient fetch. Second, net radiation was very variable in the afternoon contributing to a non-steady-state atmosphere. This makes it difficult to integrate the recorder trace and could lead to errors in Bowen ratio computations. In general, the energy partitioning

TABLE 1. Comparison of daytime cumulative sensible heat flux densities from the yaw sphere-thermometer (H_{YST}) and Bowen ratio (H_β) systems.

Date	H_{YST}/H_β	Duration (min)
24 August 1971	1.23	476
25 August 1971	0.99	458
26 August 1971	0.92	549
27 August 1971	0.98	582

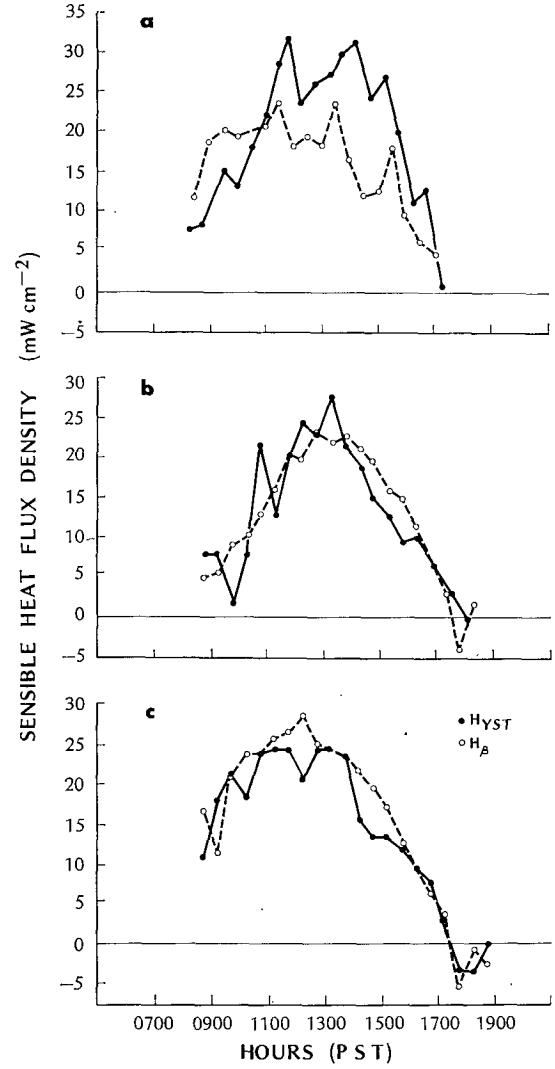

FIG. 6. Comparison of sensible heat flux densities from the yaw sphere-thermometer system (H_{YST}) and the Bowen ratio system (H_β) over grass at Ladner, B. C., for (a) 24 August, (b) 25 August and (c) 26 August, 1971.

resulted in $H \approx 0.5\ R_n$ (net radiation) at noon on most days.

Table 1 gives the cumulative sensible heat flux densities for each day for each method, expressed as a ratio. Sampling periods have been adjusted to conform to those of the YST system. Except for 24 August the difference between the two methods is <10%.

We may conclude there is good and consistent diurnal agreement between the YST and Bowen ratio approaches to evaluating H. Dyer and Hicks (1972) indicate that H may show a 10% spatial variability over uniform terrain, and this may account for a part of the differences between the H traces in Figs. 6 and 7. Other possible errors in the YST system include tilt error, overestimates of $\bar{u}$ by the cup anemometer,

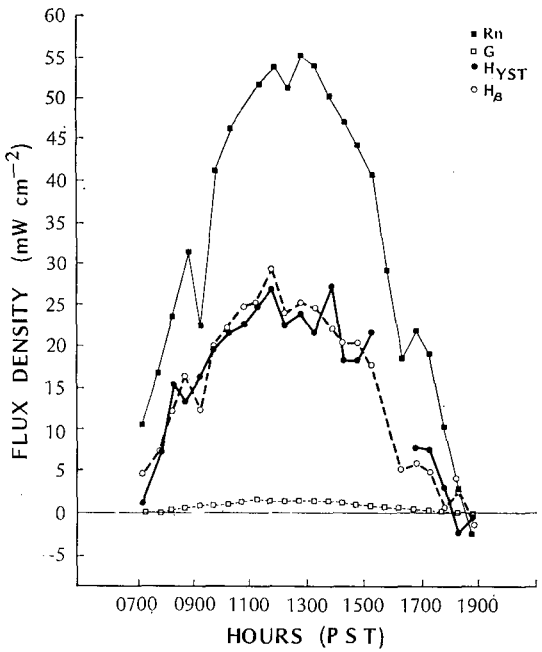

FIG. 7. Comparison of H_{YST} and H_β at Ladner, B. C., on 27 August 1971. Net radiation (R_n) and soil heat flux density (G) are included.

frequency response, and small zero drifts in the electronics. Similarly, the Bowen ratio method relies upon the assumption of constant similarity between the transfer coefficients for heat and water vapor.

7. Conclusion

A yaw sphere-thermometer system was constructed and successfully used to measure sensible heat fluxes. From wind tunnel experiments the sphere constant was determined to be 1.57. This value is significantly less than that predicted theoretically. It is important that the yaw sphere approach be modified to include this experimentally determined constant.

Analysis of the effects of tilting the yaw sphere axis indicates that an error of ~5% per degree of tilt is likely with moderately unstable conditions. This error may attain 11% per degree in very stable conditions.

Field comparisons of the YST system and a Bowen ratio apparatus yielded satisfactory agreement. A modified thermometer assembly was found necessary to provide durability in the field.

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