Bottom-trapped Subinertial Motions over Midocean Ridges in a Stratified Rotating Fluid

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ABSTRACT

Linear analytical solutions for bottom-trapped subinertial oscillatory flow over simple ridge topographies in a stratified (two-layer) rotating fluid are presented. Results are compared to moored current meter observations of bottom-intensified motions over the Endeavour Segment of Juan de Fuca Ridge in the northeast Pacific. The solutions reproduce many of the observed features including preferential amplification of the clockwise rotary component of velocity over the ridge and increased velocity amplification with proximity to the ridge crest. For a given internal deformation radius, the degree of current amplification increases with increased bottom slope, ridge height, and oscillation frequency. Amplification decreases with increased width of the ridge relative to the deformation radius.

1. Introduction

The global midocean ridge system is one of the most extensive topographic features on the surface of the earth. Sections of the ridge system are found in all the world's oceans for a total length in excess of 50 000 km and are centers of volcanic flux that add an average of 20 km³ of new crust to the seafloor each year (Ridge Steering Committee 1988). Midocean ridges are also regions of enhanced oceanographic activity arising through hydrothermal venting and the interaction of bottom currents with the rapidly sloping topography. With vertical reliefs of hundreds to thousands of meters, ridge segments provide a focus for the trapping and amplification of near-bottom motions.

The present analysis was motivated by the observation of bottom-intensified subinertial flow in moored current records collected from the Endeavour Segment of Juan de Fuca Ridge in the northeast Pacific Ocean (Thomson et al. 1990). Endeavour Ridge is a narrow (10-km wide) north-south trending feature with an overall extent of 300 km and a vertical relief of 200 to 300 m in typical water depths of 2000 to 2500 m. Marked near-bottom amplification of currents over the ridge occurs within the diurnal tidal band and the low-frequency "weather" band (periods of 3–10 days). In contrast, there is no near-bottom amplification of ridge currents in the prominent semidiurnal tidal band.

Similar findings for the weather band are reported by Cannon and Thomson (1989) and Cannon and Pashinski (1987, 1990) for currents observed along more southern segments of Juan de Fuca Ridge. In all cases, amplification for the subinertial motions occurs within the lower portion of the water column and is almost entirely confined to the clockwise rotary component of the current. Current amplification is greatest over the crest of the ridge and diminishes rapidly toward deeper water on either side of the ridge (G. Cannon, personal communication). Amplification within the weather band is greatest in winter and appears to be strongly linked to the intensity of surface winds.

The effect of abrupt or sloping topography on deep-sea subinertial motions in a rotating fluid has been considered by Rhines (1969), Thompson and Luyten (1976), Johnson (1985), and Gill et al. (1986). Brink (1989) and Chapman (1989) have investigated trapped motions around isolated symmetrical seamounts while Willmott and Grimshaw (1991) and Allen (1988) have considered subinertial motions over rectangular topography such as escarpments. In related papers, Killworth (1989a,b) has described the scattering of a coastal Kelvin wave impinging on a deep-sea ridge. A portion of the incident wave energy is found to travel outward along the equatorial side of the ridge as a bottom-trapped Rossby wave. The free-wave solution with no alongridge variation is a steady flow.

In this paper, we develop a simple analytic model to account for the near-bottom amplification of subinertial currents observed over the central portion of Endeavour Ridge. Model results are derived for a uni-

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formly rotating two-layer flow with a deep upper layer in which motions are assumed to be independent of the alongridge direction. Although the model is highly idealized, the results are considered representative of the mesoscale subinertial motions observed near the midsection of the 300-km long ridge section. The time-dependent flow over the ridge is examined for a hierarchy of simple ridge geometries (top-hat, triangular, and hyperbolic sech-squared function). Detailed solutions are given for triangular ridges since these are considered closely representative of oceanic ridges. In order to compare the model results with the current meter observations, we look for bottom-trapped motions forced by spatially uniform rectilinear barotropic currents. Particular emphasis is on motions within the tidally forced diurnal band and meteorologically forced weather band since it is in these bands that observed bottom amplification is most pronounced. For a given frequency and rate of rotation, amplification of the subinertial motions over the ridge is found to be governed by the strength of the barotropic forcing term, the slope of the ridge and height of the ridge relative to the depth of the lower layer.

Organization of the paper is as follows. In section 2, we derive the equations of motion for a two-layer rotating fluid with one-dimensional bottom topography. The method of solution for a hierarchy of ridges (top-hat, triangle, and hyperbolic sech-squared function) is presented in section 3 for a 1½-layer fluid in which the upper layer is assumed to be infinitely deep. In section 4 we present a detailed analysis of motions over a triangular ridge for different values of ridge width, ridge height, and barotropic forcing frequency. In section 5, results for the triangular ridge model are compared with results from current meter observations at Endeavour Ridge. A summary and conclusions are provided in section 6.

2. Governing equations

a. Model derivation

The governing equations for linear, inviscid, Bousinesq, and hydrostatic shallow-water flow in the upper layer of a stratified (two-layer) rotating ocean are

$$\frac{\partial U}{\partial t} - fV = -g \frac{\partial \xi}{\partial x} + \frac{\tau^x}{\rho H}, \quad (2.1a)$$

$$\frac{\partial V}{\partial t} + fU = -g \frac{\partial \xi}{\partial y} + \frac{\tau^y}{\rho H}, \quad (2.1b)$$

$$H \frac{\partial U}{\partial x} + H \frac{\partial V}{\partial y} + \frac{\partial \xi}{\partial t} - \frac{\partial \eta}{\partial t} = 0, \quad (2.1c)$$

where U , V are the horizontal velocity components in the x , y directions, respectively, t is the time, H is the depth of the upper layer, ξ is the elevation of the surface, η is the elevation of the interface, f ($\approx 10^{-4} \text{ s}^{-1}$) is the Coriolis parameter, g is the acceleration due to gravity,

and τ^x , τ^y are the surface wind-stress components. The corresponding equations for the lower layer (to order $g'/g \approx 10^{-3} - 10^{-4}$) are

$$\frac{\partial u}{\partial t} - fv = -g' \frac{\partial \eta}{\partial x} - g \frac{\partial \xi}{\partial x}, \quad (2.2a)$$

$$\frac{\partial v}{\partial t} + fu = -g' \frac{\partial \eta}{\partial y} - g \frac{\partial \xi}{\partial y}, \quad (2.2b)$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = 0 \quad (2.2c)$$

in which u , v are the velocity components and h is the depth of the lower layer. The reduced density is $g' = g\Delta\rho/\rho$, where $\Delta\rho$ is the difference in density between the two layers and ρ is a typical density.

Back substitution of the derived linear solutions into the full nonlinear equations verifies that the nonlinear terms are small compared to the retained terms in the equations of motion. Further justification for neglect of the nonlinear terms is provided by the fact that the Rossby number, Ro ($\equiv U/fW$), is much less than unity for flow speeds $U \approx 0.1 \text{ m s}^{-1}$ over ridges of width $W \approx 10 \text{ km}$. Our neglect of bottom friction is less easy to justify. For values of the vertical eddy viscosity of 10^{-3} to $10^{-2} \text{ m}^2 \text{ s}^{-1}$, near the upper range of accepted estimates, the effect of frictional terms could be important. For midrange values of viscosity, however, the Ekman pumping term (which dominates the frictional stress) is about two orders of magnitude smaller than the topographic forcing term (udh/dx) at diurnal frequencies. The effect of frictional terms is discussed more completely in section 6.

We consider an infinitely long ridge that straddles the y axis with its center at $x = 0$. Since the flow is independent of y , we assume oscillating solutions of the form

$$Z = (Z_o(x) + Z_*(x))e^{i\omega t}, \quad (2.3)$$

where ω is the forcing frequency and Z is each of U , V , u , and v . The “*” variables are the response of the system to the barotropic flow over the topography while the “o” variables denote the background barotropic flow. The expressions for ξ and η are of the form (2.3), but ξ_o and η_o must be functions of both x and y (the variation with respect to y will be on the large, barotropic length scale) to allow a uniform background flow. The background flow is derived from the relations,

$$i\omega U_o - fV_o = -g \frac{\partial \xi_o}{\partial x} + \frac{\tau^x}{\rho H}, \quad (2.4a)$$

$$i\omega V_o - fU_o = -g \frac{\partial \xi_o}{\partial y} + \frac{\tau^y}{\rho H}, \quad (2.4b)$$

$$i\omega \xi_o - i\omega \eta_o + H \frac{dU_o}{dx} = 0, \quad (2.4c)$$

$$i\omega u_o - fv_o = -g \frac{\partial \xi_o}{\partial x} - g' \frac{\partial \eta_o}{\partial x}, \quad (2.4d)$$

$$i\omega v_o - fu_o = -g \frac{\partial \xi_o}{\partial y} - g' \frac{\partial \eta_o}{\partial y}, \quad (2.4e)$$

$$i\omega \eta_o + h_o \frac{du_o}{dx} = 0, \quad (2.4f)$$

where h_o is the depth in the region of uniform topography on either side of the ridge. For the baroclinic variables, the equations become

$$i\omega U_* - fV_* = -g \frac{d\xi_*}{dx}, \quad (2.5a)$$

$$i\omega V_* + fU_* = 0, \quad (2.5b)$$

$$i\omega \xi_* - i\omega \eta_* + H \frac{dU_*}{dx} = 0, \quad (2.5c)$$

$$i\omega u_* - fv_* = -g \frac{d\xi_*}{dx} - g' \frac{d\eta_*}{dx}, \quad (2.5d)$$

$$i\omega v_* + fu_* = 0, \quad (2.5e)$$

$$i\omega \eta_* + h \frac{du_*}{dx} + u_* \frac{dh}{dx} = -\frac{d}{dx} (h_\delta u_o) \quad (2.5f)$$

in which $h_\delta = h - h_o$. The right-hand side of (2.5f) is the usual forcing term for the standing wave response generated by barotropic flow over topography (cf. Rhines 1969; LeBlond and Mysak 1979).

Equations (2.5a)–(2.5f) can be combined to derive a single general equation governing the baroclinic response to a barotropic flow across a ridge in a two-layer fluid. The derivation of this equation is given in appendix A. For Endeavour Ridge, a reasonable approximation to the stratification is an upper layer about four times deeper than the near-uniform lower layer. Thus, to facilitate solutions to this problem, we will approximate the two-layer structure by a finite lower layer and an infinite upper layer. Mathematically, we let $U_* \rightarrow 0$ as $H \rightarrow \infty$ so that HU_* remains finite.

Combining (2.5a)–(2.5f) and restricting attention to barotropic flows with no variation in the x direction¹ yields a second-order equation for u_*

$$g'h \frac{d^2 u_*}{dx^2} + 2g' \frac{dh}{dx} \frac{du_*}{dx} - Fu_* = -g' \frac{d^2 h}{dx^2} (u_o + u_*), \quad (2.6)$$

where $F = f^2 - \omega^2$.

Once the values of u_* are found, the total velocities and interface height are given by

$$u = (u_o + u_*)e^{i\omega t}, \quad (2.7a)$$

$$v = \left(v_o - \frac{f}{i\omega} u_* \right) e^{i\omega t}, \quad (2.7b)$$

$$\eta = -\frac{1}{i\omega} \left(h \frac{du_*}{dx} + \frac{dh}{dx} (u_o + u_*) \right) e^{i\omega t}. \quad (2.7c)$$

b. Translation of analytic results to variables measured in the field

The effect of the ridge on the model flow structure can be illustrated in a straightforward way using the Cartesian representation in (2.7). Specifically, the value of u_* in (2.7a) provides a direct measure of the fractional change in the flow associated with barotropic motions over the ridge; however, to demonstrate preferential amplification of the clockwise rotational component of the current—an important “signature” of topographic influence on the subinertial motions—we need to express the current velocity in terms of its rotary components (Gonella 1972; Mooers 1973). In addition to showing rotational differences, the rotary ellipse format is useful to examine the spatial uniformity of the observed flow in a given frequency band. A brief description of the rotary formulation is presented below; further details can be found in appendix B.

Using complex notation, the Cartesian velocity components u , v can be written in rotary ellipse form (e.g., Thomson and Middleton 1985):

$$u + iv = \exp \left(i \frac{(\epsilon^+ + \epsilon^-)}{2} \right) \times \left[(A^+ + A^-) \cos \left(\frac{(\epsilon^+ + \epsilon^-)}{2} + \omega t \right) + i(A^+ - A^-) \sin \left(\frac{(\epsilon^+ + \epsilon^-)}{2} + \omega t \right) \right], \quad (2.8)$$

in which the counterclockwise (clockwise) rotating vector has magnitude A^+ (A^-) and is ϵ^+ (ϵ^-) radians counterclockwise from the x axis at time $t = 0$. The ellipse is oriented at an angle

$$\theta = \frac{(\epsilon^+ + \epsilon^-)}{2} \quad (2.9)$$

counterclockwise from the x axis. The current rotates counterclockwise if $A^+ > A^-$ and clockwise if $A^- > A^+$. In the ellipse format (2.8), $A^+ + A^-$ is the length of the semimajor axis and $|A^+ - A^-|$ is the length of the semiminor axis. In the case of the Endeavour Ridge, the major axis is oriented alongridge when $\theta \approx 70^\circ$ and crossridge when $\theta \approx 160^\circ$. The rotary coefficient r gives the degree and sense of rotation of the current vectors and is defined (Gonella 1972) as

$$r = \frac{S^+(\omega) - S^-(\omega)}{S^+(\omega) + S^-(\omega)}, \quad (2.10)$$

where S^- and S^+ are, respectively, the clockwise and

¹ Strictly speaking, the solution to (2.4) requires variation with respect to x , but it is on the large barotropic length scale.

counterclockwise rotary spectra of the current at frequency ω . Estimates of the clockwise and counterclockwise rotary current components A^- and A^+ can be found from the relation $(A^-, A^+) = ([S^- \cdot BW]^{1/2}, [S^+ \cdot BW]^{1/2})$, where BW is the bandwidth of the spectral estimate. According to (2.10), $r = 0$ if the current is rectilinear (flat ellipse), and $r = +1$ (-1) if the current ellipse is circular and counterclockwise (clockwise) rotary.

3. Analytical models

We now seek solutions to the topographically forced motions for simple models of the ridge geometry. The "top hat" ridge, triangular ridge, and sech-type ridge provide a hierarchy of topographic sophistication that helps illustrate the method of solution as well as reveal differences in the effects of ridge structure on deep oceanic motions. For convenience, we drop the subscript "*" Unless stated otherwise, velocity components without subscripts denote perturbation velocity components.

a. Top-hat ridge

The top hat is the most straightforward geometrical model for a midocean ridge. For this model, we consider a flat-topped step of width W and height, Δh ($|x| \leq W/2$), centered along the y axis at $x = 0$ in water depth h (Fig. 1). Symmetry in the flow requires that the cross-ridge gradient of the velocity perturbation, du/dx , vanish at $x = 0$. The appropriate solution to (2.6) is then

$$u = C \cosh(x/R'_\omega), \quad |x| < W/2, \quad (3.1)$$

where $R'_\omega = [g'(h - \Delta h)(f^2 - \omega^2)^{-1}]^{1/2}$ is the deformation radius appropriate to oscillating flow over the ridge. The required solutions on either side of the ridge follow from the need for finite perturbations far from the ridge, so that

$$u = A \exp(-|x|/R_\omega), \quad |x| > W/2 \quad (3.2)$$

where $R_\omega = [g'h(f^2 - \omega^2)^{-1}]^{1/2}$ is the deformation radius for oscillating flow over the region of uniform depth, away from the ridge. There are two matching conditions at $x = \pm W/2$: First, the interfacial elevation must be continuous so that the velocity remains finite, and second, the total cross-ridge flux (perturbation plus barotropic component) must be continuous. The latter condition implies

$$(u_o + u|_{-W/2+\delta})(h - \Delta h) = (u_o + u|_{-W/2-\delta})h,$$

or

$$u|_{-W/2+\delta} = \frac{u_o \Delta h + hu|_{-W/2-\delta}}{h - \Delta h}, \quad (3.3)$$

where the distance δ is arbitrarily small. The interface height is given by (2.5f),

FIG. 1. Cross-ridge variation for a top-hat ridge. (a) The normalized perturbation velocity u/u_o , (b) the interface depth η , and (c) the rotary coefficient r for the total current, $u + u_o$. Here, $g' = 1.0 \times 10^{-4} \text{ m s}^{-1}$ and motions are forced at frequency $\omega/f = 0.6$. The ridge has a width $W/R = 2.5$ and a height $\Delta h/h = 0.5$.

$$\eta = -\text{sgn}(x) \frac{Ah}{R_\omega \omega} \exp(-|x|/R_\omega), \quad |x| > W/2, \quad (3.4a)$$

$$\eta = \frac{C(h - \Delta h)}{R'_\omega \omega} \sinh(x/R'_\omega), \quad |x| < W/2, \quad (3.4b)$$

where sgn is the sign function, equal to 1 if $x > 0$ and -1 if $x < 0$. The amplitude of the cross-ridge (x) component of the perturbation velocity at the ridge boundary is given by

$$u(\pm W/2) = A = -u_o \frac{\Delta h}{h} \times \left[e^{W/2R_\omega} / \left(\frac{R'_\omega}{R_\omega} \coth(W/2R'_\omega) + 1 \right) \right], \quad (3.5)$$

and the perturbation velocity at the center of the ridge by

$$u(0) = C = u_o \frac{\Delta h}{h - \Delta h} \left[1 / \left(\cosh(W/2R'_\omega) + \frac{R_\omega}{R'_\omega} \sinh(W/2R'_\omega) \right) \right]. \quad (3.6)$$

Examples of the cross-section of the perturbation velocity and interface elevation are presented in Fig. 1. As for all models, the maximum cross-ridge velocity at the crest of the ridge is in phase with the background barotropic current. By (2.7b), the alongridge velocity v has the same form as u but is -90° out of phase and f/ω stronger. We further note that the interfacial height η has odd symmetry while the current has even symmetry. Currents are clockwise rotary over the ridge and counterclockwise rotary over the offridge region.

b. Triangular ridge

In contrast to the top-hat ridge, the triangular ridge has finite uniform slopes but a vanishingly narrow crest. Although this particular model is discussed more thoroughly in the next section, selected results are presented for direct comparison with the top-hat and sech-type ridges. We consider a linear ridge with triangular cross section and slope, $s = \Delta h/(W/2)$, lying along the y axis centered at $x = 0$ (Fig. 2). The forcing term on the right-hand side of (2.6) is nonzero only where the gradient of the bottom slope is nonzero; that is, at $x = 0$, and $\pm W/2$ where it takes the form of a Dirac delta function.

Over the flat oceanic regions on either side of the ridge, the perturbation flow has the form (3.2),

$$u = B \exp(-|x|/R_\omega), \quad |x| > W/2 \quad (3.7)$$

where B is determined by matching solutions at the borders of the ridge. Unlike the case for the top-hat ridge, complete solutions of (2.6) for the triangular ridge must be found using numerical techniques. The following "shooting" method was used. For a trial solution B , the equation was integrated numerically from $x = -W/2$ to $x = +W/2$. The parameter B was then adjusted until a consistent solution was found; that is, until the solutions for both velocity and the velocity derivative over the ridge matched the solution, (3.7), at $x = +W/2$. As the problem has even symmetry, the actual integration was performed only from $x = -W/2$ to $x = 0$. To find the boundary condition relating u and $du/dx = 0$ we integrate (2.6) over an infinitesimal region, $[-\delta, \delta]$, centered at $x = 0$. The only contributions to the integral come from the d^2/dx^2 terms because the perturbation velocity must be continuous. Thus,

$$(h - \Delta h) \left(\frac{du}{dx} \Big|_{\delta} - \frac{du}{dx} \Big|_{-\delta} \right) = -(u_o - u) \left(\frac{dh}{dx} \Big|_{\delta} - \frac{dh}{dx} \Big|_{-\delta} \right).$$

FIG. 2. As for Fig. 1 but for a triangular ridge (see insert).

By symmetry du/dx and dh/dx are equal and opposite on either side of $x = 0$, which gives

$$\frac{du}{dx} = -s \frac{u_o + u}{h - \Delta h}, \quad (3.8)$$

as the boundary condition at $x = 0$, where $s = \Delta h/(W/2)$.

Examples of the cross section of the perturbation velocity and interface elevation are presented in Fig. 2. Unlike the flow over the top-hat ridge, the perturbation velocity u is a continuous function of x and its maximum value occurs at the crest rather than the edge of the ridge. The maximum negative velocity perturbation (which occurs at the edge of the ridge) is considerably smaller for the triangular ridge, indicating that the flat faces of the top-hat ridge form a formidable block to the flow.

c. Sech-type ridge

The hyperbolic sech(x) function squared (Fig. 3) is commonly used for models of continuous, smoothly varying ridgelike topography (e.g., Killworth 1989b). The solution proceeds similarly to that for the triangular ridge. Far from the center of the ridge, the depth

is essentially uniform and the perturbation velocity is given by (3.2). Equation (2.6) is then integrated from large negative x to the center of the ridge, where from symmetry, the cross-ridge derivative of the total velocity must be zero.

Examples of the cross section of the perturbation velocity and interface elevation are presented in Fig. 3. In the case of the triangular ridge, the perturbation velocity but not its derivative are continuous across the ridge. For the "smooth" sech-type ridge, both the velocity and its derivative are continuous in the cross-ridge direction. For a given ridge height, the velocities over the crest of the sech-type ridge are smaller since the smooth changes in slope (d^2h/dx^2) that force the flow [see (2.6)] are spread over the width of the ridge; in the case of a triangular ridge, the slope changes are isolated at the crest and the boundaries of the ridge.

d. Ridge protruding into the upper layer

If the ridge protrudes into the upper layer, it acts as a complete block to the lower-layer flow. The effect on the upper-layer flow can be neglected since the layer is assumed to have infinite depth in the present model.

FIG. 3. As for Fig. 1 but for a $\text{sech}^2(x/W_e)$ -type ridge (see insert). The value W_e was chosen so that the height of the ridge has fallen to $\Delta h/4$ by $W/2$. Thus, $W_e = W/(2 \text{arccosh} 2)$.

FIG. 4. Velocity field over a triangular ridge through one-half of a forcing period. Bottom arrows denote boundaries and crest of the ridge at $|x| = W/2R$ and $x = 0$, respectively. The ridge has a width $W/R = 0.75$ and a height $\Delta h/h = 0.4$. The oscillation period is $\omega/f = 0.65$.

Moreover, the horizontal positions, $\pm x_c$, where the density interface meets the ridge can be assumed constant since we are using linearized dynamics. The integration then proceeds as in section 3b, with the boundary condition that the total acrossridge velocity must be zero at x_c .

For a top-hat ridge, where the sides of the ridge are vertical walls, the solution to (2.6) with the boundary condition that the total velocity at $x = W/2$ must be zero is

$$u = -u_0 \exp\left(-\frac{|x| - W/2}{R_w}\right), \quad (3.9)$$

where u is the perturbation velocity. The total cross-slope velocity (2.7a) increases exponentially from zero moving away from the boundaries of the ridge and the alongridge velocity (2.7b) exponentially decreases from fu_0/ω to zero. The numerical solutions for the triangular and sech-type ridge are similar.

4. Numerical results

Periodic solutions

The subinertial flow for different ridge widths and heights in a fluid of varying lower-layer depth and density has been investigated in detail using the relatively simple triangular ridge model described in section 3b. Of the three ridge shapes, the triangular ridge most closely approximates the shape of Endeavour Ridge.

As illustrated by Fig. 4, the periodic barotropic flow

excites a standing wave response with subsequent enhancement of velocities over the ridge. Relatively strong anticyclonic (clockwise rotary) motions are generated over the central portion of the ridge while weaker cyclonic motions are generated over the outer flank and offridge regions. The observed pattern is intuitive in the sense that fluid columns moving over the ridge must, on average, be compressed thereby generating anticyclonic vorticity while those off the ridge are stretched thereby generating cyclonic vorticity. Because the model ridge is infinitely long, the generated standing wave has an infinite wavelength and has the form of a degenerate wave or a spatially uniform oscillation.

Although we have considered the response as a trapped wave, one can view the problem in terms of currents and pressures changing in phase with the applied barotropic velocity. The barotropic flow is compressed and accelerated as it crosses the ridge; however, the presence of the ridge reduces the total flux on the upstream side of the ridge, causing the interface to rise, and increases the total flux on the downstream side of the ridge, causing the interface to drop. As the tide turns, the pressure produced by the interface dominates the flow and the flow is along the ridge in quasigeostrophic balance with the interface slope.

We can examine in detail the changes in flow response to the ridge by varying one of the topographic or oceanic parameters while holding all others constant. Values are presented for the maximum velocity perturbations found at the ridge crest ($x = 0$) and for flow at the ridge boundary ($x = W/2$). Results for variable ridge width W/R , ridge height $\Delta h/h$, and forcing fre-

FIG. 5. Variation in the perturbation current amplitude for a triangular ridge as a function of the ridge width W/R for various values of frequency, ω/f . (a) The ridge crest ($x = 0$); and (b) the ridge boundary ($|x| = W/2R$). The ridge height, $\Delta h/h = 0.6$.

FIG. 6. Variation in the perturbation current amplitude for a triangular ridge as a function of the relative ridge height, $\Delta h/h$, for various values of ω/f . (a) The ridge crest ($x = 0$); and (b) the ridge boundary ($|x| = W/2R$). The ridge width, $W/R = 2.5$.

quency ω/f are presented in Figs. 5 to 7, respectively. Since $R = (g'h)^{1/2}/f$, the effect of increased lower-layer density on the solution is proportional to a decrease in $(W/R)^{1/2}$ in Fig. 5.

As indicated by Fig. 5, the amplitude of the cross-ridge perturbation velocity at the ridge crest decreases as the width of the ridge increases. Increasing the width of the ridge decreases the slope, dh/dx , and thus decreases the integrated curvature; for $\delta x \rightarrow 0$,

$$\int_{W/2-\delta x}^{W/2+\delta x} \frac{d^2 h}{dx^2} dx = 0 - \frac{dh}{dx} \Big|_{W/2-\delta x}, \quad (4.1)$$

which is the forcing term in (2.6). Note that it is the change in slope, the curvature, that generates the perturbation velocities in (2.6), not the slope itself. The velocity at the boundary of the ridge becomes more negative as the ridge width increases for narrow ridges but increases with ridge width for wider ridges. For narrow ridges the decreased absolute value of the velocity at the ridge boundary for decreasing width is due to some cancellation of the velocity field generated by

FIG. 7. Variation in the perturbation current amplitude for a triangular ridge as a function of the forcing frequency, ω/f , for various values of the ridge height, $\Delta h/h$. (a) The ridge crest ($x = 0$); and (b) the ridge boundary ($|x| = W/2R$). The ridge width $W/R \approx 2.5$.

the integrated curvature at the peak and opposing integrated curvature at the ridge boundary when these regions are closer than a deformation radius. From the

numerical results, significant cancellation is found for ridges with $W/R_\omega < 2.4$. The right-hand side of this inequality decreases slowly with $\Delta h/h$.

The perturbation velocity at the ridge crest increases as the height of the ridge increases relative to the depth of the lower layer (Fig. 6). Since the motions are linear and inviscid, the velocity at the crest increases without bound as the peak of the ridge approaches the top of the lower layer. Once the peak penetrates into the upper layer, the maximum perturbation velocity occurs at the intersection of the ridge and the interface, where the perturbation velocity has equal magnitude but opposite direction to that of the barotropic background flow. At the boundaries of the ridge the velocity becomes monotonically more negative as the height of the ridge is increased, until the peak of the ridge penetrates the upper layer. At this point the velocity at the ridge boundary becomes positive and thereafter increases with increased ridge height.

As illustrated in Fig. 7, the perturbation velocity at the ridge crest increases as the frequency increases, whereas the velocity at the borders of the ridge initially becomes increasingly negative and then increases. The explanation for this is similar to that for the change in ridge width W/R . In particular, the modified deformation radius R_ω increases as the frequency increases so that the effective width of the slope, W/R_ω , decreases with frequency. For near-inertial motions, $\omega/f \rightarrow 1$, the perturbation velocity components increase without bound due to the singularity in $F = \omega^2 - f^2$ in (2.6). Although frictional and nonlinear effects would dampen the motions in this limit, we can expect strongly bottom-intensified motions in the near-inertial frequency band over the ridge (cf. Thomson et al. 1990).

In the low-frequency limit, $\omega/f \rightarrow 0$, the cross-ridge perturbation velocity u tends to a constant finite value

FIG. 8. Variation of the rotary coefficient r as a function of (a) ridge width, W/R ; and (b) ridge height, $\Delta h/h$, for different forcing frequencies, ω/f . (i) The ridge crest ($x = 0$); and (ii) the ridge boundary ($|x| = W/2R$).

of $O(u_0)$, whereas the alongridge component v increases without bound. This rapid increase in v is a direct result of the infinite length of the ridge. Because v is related to the time integral of that portion of the mass flux, $M = u_0 h - (u_0 + u)(h + \Delta h)$, that does not cross the ridge, v increases with time. In the more realistic case of a finite length ridge, the long-ridge mass flux would be able to spill around the ends of the ridge. The behavior at the low-frequency limit can also be explained in terms of resonance theory (cf. Chapman 1989) since the free-wave solution over a ridge having no alongridge variation is a steady flow (Killworth 1989a).

The effects of ridge dimensions and forcing frequency on the rotary coefficient are presented in Fig. 8. For low frequencies ($\omega/f < 0.25$), the region of near-circular clockwise rotary motions occurs over the flank

of the ridge but becomes increasingly more shifted to the crest of the ridge for higher frequencies. The decrease in $|r|$ for $\omega/f \rightarrow 0$ near the ridge crest indicates that current ellipses become flattened and more oriented parallel to the ridge axis for decreasing frequency. The change from clockwise to counterclockwise rotary motions takes place near $x/R = 0.75$ except for near-inertial motions ($\omega/f \rightarrow 1$) in which case the motions are clockwise rotary for all locations over the ridge. The perturbation velocity, alone, is circular clockwise rotary as expected for inertial motions.

5. Comparison with observations

Model results are readily compared with results from current meter observations collected at Endeavour Ridge. Detailed information on the locations, dura-

FIG. 9. Topography and mooring sites for the central region of Endeavour Ridge. Site A2a was occupied from September 1984 to August 1985. Sites A2, A3, R1, R2, T1, and T3 were occupied from September 1985 to June 1986 and site A2b from September 1986 to July 1987.

tions, and depths of current meter data collected in this location can be found in Thomson et al. (1989, 1990). Briefly, observations are from a three-year program of sediment trap and current meter moorings conducted by Oregon State University and the Institute of Ocean Sciences over the period September 1984–September 1987 (Fig. 9). From September 1984 to August 1985, a single current meter was moored 70 m above bottom at site A2a in the axial valley near the ridge crest. A total of five moorings extending up to 20 km from the central mooring site, A2, were deployed during the period September 1985–June 1986 and a single mooring, instrumented through the entire water column, was deployed at site A2b during the period September 1986–July 1987.

As illustrated by Fig. 10, near-bottom currents over Endeavour Ridge are dominated by motions in the tidal, inertial, and low-frequency bands. At the latitude (48°N) of the observations, the Coriolis frequency ($f = 0.0621$ cph) lies midway between the predominant semidiurnal frequency band and the slightly less energetic diurnal frequency band. Consistent with our model results, subinertial motions in the diurnal, near-inertial, and predominant low-frequency bands tend to be dominated by clockwise rotary currents. In con-

trast, motions in the superinertial semidiurnal band are more nearly rectilinear and show little well-defined rotary preference. The present analysis is concerned primarily with the observed near-bottom intensification within subinertial frequency bands. Specific attention is given to the comparatively energetic currents within the narrow diurnal tidal band and within the broader “weather” band centered at peak spectral periods of 3 to 8 days.

a. Diurnal tidal currents

Profiles of the rotary spectra, ellipse orientations (2.9), phase lags (B.4), and rotary coefficients (2.10) for the K_1 (0.0418 cph) tidal constituent observed near the crest of Endeavour Ridge are presented in Fig. 11. Similar plots are obtained for the less energetic O_1 (0.0387 cph) tidal constituent. The amplification of the diurnal currents is best illustrated by Fig. 11a, which shows ellipse characteristics for current meters at depths of 60, 486, 1486, 1686, 1936, and 2036 m in 2200 m of water at mooring A2b positioned near the ridge crest from September 1986 to July 1987 (Fig. 9). (The 1486-m depth current records are included in the analysis although there is some doubt about the reliability of the direction measurements obtained from the current meter.) Findings can be summarized as follows:

(i) Near-bottom amplification of the current is associated primarily with the clockwise rotary component of the current and is confined to within approximately 500 to 700 m of the seafloor. Comparison of the constituent amplitudes reveals a factor of 3 to 4 increase in the clockwise rotary component within the lower 500 m of the water column.

(ii) Current ellipse orientations are roughly uniform throughout the water column ($\theta \approx 110^\circ$ – 125°) but with a tendency to become oriented slightly more cross ridge near the bottom.

(iii) There is a small increase in phase of about 15° (1 h at diurnal periods) from the surface to the bottom. For the deepest records, the phase lag may increase to about 2 h relative to the near-surface values.

(iv) The rotary coefficient becomes increasingly more negative near the bottom due to the greater amplification of the clockwise component relative to the counterclockwise component.

The vertical structure of the diurnal currents measured at ridge sites A2, A3, R1, and R2 from September 1985 to June 1986 (Fig. 11b) is consistent with that obtained from site A2b (Fig. 11a). The exception is site A3 where the deeper records were obtained near the southern end of the axial valley at depths below the crest of the ridge. Motions here rotate counter to those at shallower depths and do not appear to be representative of motions above the ridge crest (cf. Thomson et al. 1989, 1990). There is no vertical amplification of the diurnal tidal motions at mooring site T3 located

FIG. 10. Rotary spectra at 1936-m depth at site A2b. Frequency resolution is 0.0005 cph (cycles per hour) and there are approximately six degrees of freedom per band. Spectral peaks at the inertial frequency f and the principal tidal frequencies are labeled. The counterclockwise rotary spectrum (S^-) is offset two units from the clockwise rotary spectrum (S^+).

FIG. 11. Profiles of rotary-current ellipses at the K_1 (diurnal) tidal frequency. (a) Mooring site A2b, September 1986–July 1987; (b) mooring sites for the period September 1985–June 1986. Panels show the rotary spectra S^+ and S^- , ellipse orientation (θ) measured counterclockwise from east, and the rotary coefficient r . The triangles in (a) are for the single current meter record at site A2a. Dual values in (b) are from current meters separated vertically by 1 m on the mooring.

in a comparatively flat offridge region 20 km to the south of the ridge.

The near-bottom amplification of the observed diurnal currents, the near uniformity of current ellipse alignment, and the increase in the ratio of the clockwise to counterclockwise flow are in qualitative agreement with the model predictions. The change in phase is presumably strongly affected by bottom friction, which is not accounted for in our model. To obtain a quantitative comparison of the observed diurnal currents with the two-layer model, we need estimates of the various model parameters based on the field data for Endeavour Ridge. The frequency ratio for the diurnal currents is given as $\omega/f \approx 0.65$ while the relative height of the ridge is assumed to be $\Delta h/h \approx 0.6$, correspond-

ing to a ridge of height 300 m in a lower layer of thickness 500 m. The latter depth is somewhat arbitrarily chosen as the depth range over which there is a noticeable increase in diurnal current amplitude with depth in Fig. 11. Since the current amplitude at the crest in the model is relatively insensitive to $\Delta h/H$ except near unity (Fig. 6), this choice seems reasonable. Based on observed density profiles from the vicinity of the ridge (cf. Fig. 13 of Thomson et al. 1990), the internal deformation radius

$$R = \frac{Nh}{f} \approx \frac{(g'h)^{1/2}}{f} \quad (5.1)$$

lies in the range $5 \leq R \leq 10$ km (N is the Brunt-

Väisälä frequency). Taking the ridge width as 10 km, the ratio W/R for Endeavour Ridge lies in the range $1 \leq W/R \leq 2$. The nearly rectilinear currents in the upper part of the water column (but below the surface layer) shown in Fig. 11a are oriented at an angle $\theta \approx 125^\circ$, which we can assume is the orientation of the barotropic flow (u_o, v_o) in the model. For this set of parameters, the model yields a clockwise rotary velocity amplification of 2.7 to 3.2 for the lower layer, where the smaller value corresponds to the wider slope. The orientation of the tidal ellipse, $\theta = 124.7^\circ$ to 127.4° , in the lower layer remains almost identical to that for the barotropic flow, and the phase lag G is larger than that of the barotropic current by 7.7° to 8.9° . Thus, we find that the model results are consistent with the observed maximum amplification of 3 to 4 in the lower layer and the small observed change in ellipse orientation as the bottom is approached. The rotary coefficient of -0.9 obtained from the model is slightly higher than the average value of $r \approx -0.8$ obtained for the current meter records below 1500-m depth in Fig. 11a; likewise, the 16° increase in phase G in the lower layer of the model is consistent with the 15° increase seen in the observations.

b. Semidiurnal tidal currents

Unlike the diurnal currents, rotary currents in the superinertial semidiurnal band attain maximum values in the upper layer of the ocean and remain roughly uniform with depth within the lower layer (Fig. 12). Furthermore, the orientation of the semidiurnal current ellipse undergoes a marked realignment along the direction of the ridge crest with increased proximity to the bottom, contrary to the near uniformity of align-

ment of the diurnal ellipses. The abrupt phase increase in the M_2 constituent of approximately 120° (4 h) between the upper layer and the lower layer is considerably greater than the maximum 30° change for the diurnal constituent and suggests the presence of baroclinic semidiurnal tides in the vicinity of the ridge. Similar results are obtained for the S_2 tidal constituent.

The pronounced difference in spatial structure between the diurnal and semidiurnal tidal currents is linked to dynamical differences between subinertial and superinertial motions. Since our model assumes $\omega/f < 1$, it cannot be expected to reproduce observed motions in the semidiurnal frequency band.

c. Weather-band currents

Low-frequency variability in the vicinity of Endeavour Ridge is characterized by oscillations in the 3–8-day weather band and in the marginally resolvable “red” end of the spectrum (periods greater than 40 days). Profiles of the rotary spectra, ellipse orientations (mean plus standard deviation), and the rotary coefficients for the weather band are presented in Fig. 13. Since peak spectral energy is not confined to a single band, the rotary spectra in these figures are based on the integrated spectral estimates over the range 0.005 to 0.015 cph (8.3 to 2.8 days). The ellipse orientation is derived from the 11 separate estimates within the frequency range while the rotary coefficient has been calculated directly from the integrated spectra.

Although much less pronounced than the diurnal motions, currents in the weather band undergo significant near-bottom amplification in the vicinity of the ridge. The flow also becomes more clockwise rotary and there is a tendency for ellipses to rotate slightly

FIG. 12. Same as (a) in Fig. 11 but for the M_2 (semidiurnal) tidal frequency. Vertical dashed line in panel 2 is the ridge orientation (70°T). Mooring sites other than A2b have poor vertical resolution and are not included.

more to the cross-ridge direction within close proximity to the bottom. Amplitude levels increase by roughly a factor of two within the lower kilometer of the water column.

The significant near-bottom amplification of motions in the weather band is consistent with our analytical results; however, because the amplification is a weak function of frequency except as $\omega/f \rightarrow 1$ (Fig. 7), that the amplification in the weather band is considerably less than that in the diurnal band is probably related to two aspects of the wind forcing. The ellipse orientations in Fig. 13 suggest that the typical wind forcing at Endeavour Ridge is oriented almost parallel to the ridge axis so that the cross-ridge flow is quite weak; it is the cross-ridge component of the barotropic flow that excites the baroclinic motions. In addition, motions in the weather band are highly intermittent and have "random" phase from one wind event to another. Thus, for the same forcing magnitude, the time-integrated response of the wind-forced motions (as measured by the rotary spectra) will be less than for the constant tidal motions.

d. Low-frequency motions

Figure 14 shows the band-integrated current spectra for the two lowest frequency bands centered at 0.0005 and 0.0010 cph (periods 83 and 42 days). As might be expected, energy levels in the upper layer (depth < 500 m) are an order of magnitude greater than in the lower layer where motions are less directly affected by the wind. Currents within the lower layer undergo weak amplification several hundred meters above the ridge crest but do not indicate the consistent amplification with depth that characterizes the diurnal period motions. As with the weather-band motions discussed in the previous section, there is an increase in the

FIG. 14. Same as (a) in Fig. 11 but for the two lowest resolvable frequency bands centered at 0.0005–0.0010 cph. Spectra are integrated over the two bands and ellipse orientations give the mean plus standard deviation.

clockwise rotary component and a tendency for ellipses to align across ridge with proximity to the bottom.

6. Discussion and summary

Our analysis demonstrates that a subinertial, barotropic, oscillatory current with a nonzero cross-ridge component generates a standing, topographically trapped baroclinic wave over the ridge. Due to the infinitely long ridge approximation the wave has an infinite wavelength and appears as a spatially uniform oscillation. The wave is in phase with the barotropic flow—in the sense that the maximum cross-slope velocities occur at the same time—so that there is a net amplification of the currents over the ridge. Maximum reinforcement of the barotropic currents takes place over the peak of the ridge where the perturbation velocities are highest. The perturbation decreases over the flanks of the ridge, becoming negative (i.e., opposing the barotropic currents) near the boundaries of the ridge. The wave amplitude then decreases exponentially away from the ridge with an e -folding scale given by the modified Rossby radius, $(g'h/(f^2 - \omega^2))^{1/2}$, which for the diurnal tide in the vicinity of Endeavour Ridge is approximately 5–10 km.

The interface displacement is 90° out of phase with the cross-ridge perturbation velocity. Since the displacement is an odd function of x , the perturbation of the interface height passes through zero at the center of the ridge and reaches a maximum over the flanks of the ridge. The alongridge perturbation velocity v_* is -90° out of phase with the cross-ridge perturbation velocity u_* but has the same functional dependence along the x axis. The ratio of the velocity components is given by $|v_*|/|u_*| = f/\omega > 1$. Where the perturbation velocity reinforces the barotropic flow, currents

FIG. 13. Same as (a) in Fig. 11 but for the weather-band frequencies 0.005–0.010 cph. Spectra are integrated over the frequency band while ellipse orientations are the average plus standard deviation for the band. Ridge is oriented along 70° T.

are dominated by the clockwise rotary component of velocity (negative rotary coefficient). This occurs over the crest of the ridge where the generation of anticyclonic vorticity by compression of fluid columns leads to preferential amplification of the clockwise rotary motions. The rotary coefficient becomes positive over the boundaries of the ridge where the perturbation velocity is counter to the barotropic flow.

We have investigated the response for three distinct ridge models as functions of the forcing frequency, the ridge height, and ridge width, all suitably normalized. For all models, the amplitude of the velocity perturbation is proportional to the "curvature" of the ridge, measured by d^2h/dx^2 . Of the three topographic models considered, the perturbation response, for a given height of ridge, is strongest for the discontinuous top-hat ridge and weakest from the smooth sech-squared ridge. For ridges wider than the modified Rossby radius (see section 3a), increasing the height or (for continuous ridges) decreasing the width of the ridge increases the ridge slope and therefore the magnitude of the perturbation velocity. For ridges narrower than the modified Rossby radius, there is cancellation between the perturbations generated at regions of negative curvature and of positive curvature (the side boundaries and the crest, respectively, in the case of a triangular ridge). In the latter case, the perturbations over the flanks decrease with decreasing ridge width. The effect of changing the forcing frequency is similar to that of changing the width of the slope since the effective slope width is the width normalized by the modified Rossby radius. Increasing the frequency increases the modified Rossby radius, thereby decreasing the effective slope width and increasing the perturbation magnitude. If the effective slope width is of order unity, flow cancellation occurs and decreasing the frequency increases the perturbation magnitude over the flanks of the ridge.

The results of our two-layer model are found to reproduce several of the observed features of subinertial flow over Endeavour Ridge. In particular, we find that currents in the diurnal tidal and the wind-forced weather band over the ridge are amplified within the lower ≈ 500 m of the water column. For the diurnal tidal currents, the clockwise rotary component of the near-bottom flow exceeds that in the upper layer by a factor of 3 to 4. The weaker amplification observed in the weather band and low-frequency band is attributed to near along-ridge alignment of the current ellipses in these bands. Our model also accounts for the tendency of the current ellipses to become slightly more aligned in the cross-ridge direction with proximity to the bottom. As in the observations, the model gives only a small change in ellipse orientation with depth for the diurnal motions but a significant change in ellipse orientation for the weather and low-frequency bands.

The analysis in section 2 ignores nonlinear and bottom frictional effects. Neglect of nonlinear effects was justified in section 2a, where we showed that the Rossby

number is of order of 0.1, which is sufficiently small for nonlinear advection to be ignored in a first model. There are two effects of finite friction: a bottom stress and Ekman pumping. The first effect is the drag, which would appear as μu in the x -momentum equation (2.5d), and a similar term in the y -momentum equation (2.5e). To quantize this effect we wish to compare this term to the forcing term, udh/dx in (2.5f). We differentiate (2.5f) with respect to x and substitute into (2.5d) to eliminate η and produce one equation containing both the viscous drag term and the forcing term. The forcing is concentrated where the bottom changes slope. To average this effect we integrate the resultant equation over the width of the slope. Then the ratio of the viscous term to the forcing term reduces to

$$\frac{W^2 \mu \omega}{2h \Delta h g},$$

where W is the width of the slope, the natural horizontal length scale. Taking the limit $\omega \rightarrow f$ and the parameters for Endeavour Ridge (see section 5), the ratio is 0.0048 s/m μ and for even a large friction coefficient $\mu = 0.5$ cm s $^{-1}$ is small. Ekman pumping can be quantized by considering the Ekman spindown of vorticity compared to the topographic generation of vorticity. The vertical Ekman pumping is given by $\frac{1}{2}\delta_v \zeta$, where δ_v is the Ekman layer thickness $(2A_v/f)^{1/2}$, A_v is a vertical eddy viscosity, and ζ is the vorticity. The vertical velocity due to the topography is $u_o dh/dx$. The ratio of these two reduces to

$$\frac{1}{\sqrt{2}} \left(\frac{A_v}{f} \right)^{1/2} \frac{1}{\Delta h} \frac{f}{\omega},$$

which for $A_v = 10^{-4}$ m 2 s $^{-1}$ gives $0.007 f/\omega$, which is small for frequencies greater than $0.1 f$.

The results of our relatively simple 1½-layer model are encouraging; however, the extension to a continuously stratified model would allow a more thorough investigation of the vertical flow structure for comparison with observation. Also, the finite length of oceanic ridges must become important at low frequency to alleviate the unphysical behavior discussed in connection with the long-ridge component of flow (section 4a). Lastly, bottom friction is not included in our model but presumably has a significant effect on the amplitude and phase of the near-bottom flow.

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APPENDIX A

General Equation for Two Layers

If we go back to Eqs. (2.5a)–(2.5f) and do not make the approximation of an infinite upper layer, by eliminating η , ξ , V , and v we obtain two coupled governing equations for the cross-ridge perturbation velocity components,

$$FU_* = gH \frac{d^2 U_*}{dx^2} + gh \frac{d^2 u_*}{dx^2} + 2g \frac{dh}{dx} \frac{du_*}{dx} + g \frac{d^2}{dx^2} (h_\delta u_o) + gu_* \frac{d^2 h}{dx^2}, \quad (\text{A.1a})$$

$$Fu_* = gH \frac{d^2 U_*}{dx^2} + gh \frac{d^2 u_*}{dx^2} + 2g \frac{dh}{dx} \frac{du_*}{dx} + g \frac{d^2}{dx^2} (h_\delta u_o) + gu_* \frac{d^2 h}{dx^2} + g'h \frac{d^2 u_*}{dx^2} + 2g' \frac{dh}{dx} \frac{du_*}{dx} + g' \frac{d^2}{dx^2} (h_\delta u_o) + g'u_* \frac{d^2 h}{dx^2}. \quad (\text{A.1b})$$

If we make the standard approximation that $g' \ll g$, then substituting (A.1a) into (A.1b) gives the much simplified expression

$$Fu_* = FU_* - g'H \frac{d^2 U_*}{dx^2}, \quad (\text{A.2})$$

which is correct to order g'/g . If we then substitute back into (A.1a), we get

$$\frac{gg'hH}{F} \frac{d^4 U_*}{dx^4} + \frac{2gg'H}{F} \frac{dh}{dx} \frac{d^3 U_*}{dx^3} - g(H+h) \frac{d^2 U_*}{dx^2} - 2g \frac{dh}{dx} \frac{dU_*}{dx} + FU_* = \frac{d^2 h}{dx^2} \times \left(gU_* - \frac{gg'H}{F} \frac{d^2 U_*}{dx^2} \right) + g \frac{d^2}{dx^2} (h_\delta u_o), \quad (\text{A.3})$$

a fourth-order differential equation in a single variable U_* , which can then be solved numerically.

Equation (A.3) is the general equation governing the baroclinic response to an oscillatory barotropic flow across a ridge in a two-layer fluid. The 1½-layer case (2.6) can be derived as a special case of (A.3).

APPENDIX B

Useful Relations Between Model Variables and Measured Variables

The rectilinear barotropic current of maximum velocity M and at an angle α to the cross-ridge direction has rotary coefficients (marked “o”):

$$A_o^+ = A_o^- = \frac{1}{2} M, \quad (\text{B.1a})$$

$$\epsilon_o^+ = \epsilon_o^- = \alpha. \quad (\text{B.1b})$$

The total flow in the lower layer can be written

$$M \cos \alpha \cos \omega t + u_* \cos \alpha \cos \omega t + i(M \sin \alpha \cos \omega t - v_* \cos \alpha \sin \omega t), \quad (\text{B.2})$$

where u_* and v_* are the velocities that would be generated by $u_o = M$. Writing this equation in the form (2.8) gives coefficients

$$(A^+)^2 = \frac{1}{4} M^2 \sin^2 \alpha + \frac{1}{4} \cos^2 \alpha (M + u_* - v_*)^2, \quad (\text{B.3a})$$

$$(A^-)^2 = \frac{1}{4} M^2 \sin^2 \alpha + \frac{1}{4} \cos^2 \alpha (M + u_* + v_*)^2, \quad (\text{B.3b})$$

$$\tan \epsilon^+ = \frac{M \tan \alpha}{M + u_* - v_*}, \quad (\text{B.3c})$$

$$\tan \epsilon^- = \frac{M \tan \alpha}{M + u_* + v_*}. \quad (\text{B.3d})$$

The phase G is given by

$$\tan G = \frac{A^+ \sin \epsilon^+ + A^- \sin \epsilon^-}{A^+ \cos \epsilon^+ + A^- \cos \epsilon^-}, \quad (\text{B.4})$$

which can be expressed in terms of M , u_* , and v_* . The amplification of the clockwise rotary component is A^-/A_o^- , the increase in angle of orientation is $[(\epsilon^+ + \epsilon^-)/2] - \alpha$, and the change in the phase is $G - \alpha$.

The amplification, angle of orientation, and phase are not simple functions of u_* and α and are also functions of ω/f , as $v_* = fu_*/\omega$. A brief description of each of the four functions follows:

(i) The counterclockwise amplification A^+/A_o^+ is less than unity and decreases with increasing u_* for $u_*(f/\omega - 1) < M$ and increases monotonically for larger u_* . It is smallest for α small.

(ii) The clockwise amplification increases monotonically with u_* and is largest for α small.

(iii) For α positive, for small u_* , the angle of orientation is decreased in the lower layer relative to the upper layer (i.e., the ellipse becomes more aligned in the cross-ridge direction). For $u_*/M > w^2/(f^2 - w^2)$, for small α , however, the angle of orientation increases in the lower layer whereas for larger α it decreases. At $u_*(f/\omega - 1) = M$, A^+ reaches a minimum, and for $\alpha = 0$ the ellipse becomes a circle. Passing through this point, the major and minor axes invert and thus the angle of orientation changes by $\pi/2$. After this point there is still a dividing line between small angles for which the angle of orientation increases and large angles for which it decreases. The dividing line is given by,

$$\alpha = \begin{cases} \arctan \left[\left(\frac{f^2}{\omega^2} - 1 \right) \frac{u_*}{M} - 1 \right]^{1/2}, & u_* < 1 / \left(\frac{f}{\omega} - 1 \right) \\ \arctan \left[\left(\frac{f^2}{\omega^2} - 1 \right) \frac{u_*}{M} - 1 \right]^{1/2} - \frac{\pi}{2}, & u_* > 1 / \left(\frac{f}{\omega} - 1 \right). \end{cases} \quad (\text{B.5})$$

(iv) The phase G decreases for all positive α , increases for all negative α , and its absolute value increases as u_* or $|\alpha|$ increases.

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