Gap Winds in a Fjord. Part II: Hydraulic Analog

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ABSTRACT

A simple shallow-water model of gap wind in a channel that is based upon hydraulic theory is presented and compared with observations and output from a 3D mesoscale numerical model. The model is found to be successful in simulating gap winds. The speed and depth of gap wind flow is strongly controlled by topography. Horizontal or vertical channel contractions can act to force strong, shallow supercritical flow downwind and light, deep subcritical flow upwind. Force-balance analysis of the hydraulic model output confirms mesoscale model results and indicates that the prime force balance in gap wind is between external pressure gradient and friction for supercritical flow and between external pressure gradient and height pressure gradient for subcritical flow. This force balance changes near channel controls when the balance is between advection and height pressure gradient. Sensitivity analyses show positive sensitivity of gap wind speed to changes in discharge and external pressure gradient, negative sensitivity to changes in friction and boundary layer height at the channel exit, and mixed sensitivity of gap wind speed to changes in reduced gravity.

1. Introduction

Part I of this two-part paper introduced, provided a context, and discussed observations and 3D mesoscale numerical modeling of gap winds in Howe Sound. Since gap winds are a low-level flow of dense air, constrained horizontally by fjord walls and vertically by an inversion, they would seem to resemble the flow of water in a channel. Moreover, results from numerical modeling show similarities between gap wind flow and hydraulic flow (Part I): descending, supercritical flow in zones of strong wind; and sharply ascending flow where wind rapidly becomes light and subcritical. For these reasons, it seems that hydraulic channel flow theory is a potentially useful analog to gap wind flow (Jackson 1993).

This paper will extend classical hydraulic theory to include synoptic pressure gradient, friction, and channels of variable cross section. The model thus created will then be applied to gap wind in Howe Sound for the case discussed in Part I and compared with observations and output from a 3D mesoscale numerical model (RAMS). Finally, the model will be used to test the sensitivity of gap wind to initial and boundary conditions.

2. Hydraulic theory

Hydraulic theory has been used extensively for many years (often in a civil engineering context) to study the flow of water in open channels. The developments described here are based upon simplifications to shallow water theory and follow closely those which can be found in classical hydraulics textbooks, such as Henderson (1966). The exception here is that gravity g is replaced by reduced gravity g' , a different formulation for friction is used, and an externally imposed synoptic pressure gradient is included (something that is clearly negligible when considering the flow of water).

Previous atmospheric applications of hydraulics can be found in the strong katabatic wind theory of Ball (1956), which has been applied by Arakawa (1968) and Pettre (1982) to strong wind flow in valleys (which considered flow through rectangular channels assuming no friction or synoptic pressure gradient). Pettre (1982) studied the Mistral—a low-level flow similar to gap wind that occurs through the Rhône Valley in France. He applied hydraulic theory for a rectangular channel of constant elevation with no external pressure gradient or friction and used it to interpret aircraft observed winds along the valley. He found that observed strong flow occurred downwind of the major horizontal contraction in the valley, which was in agreement with hydraulic theory. A sharp decrease in wind speed near the end of the valley was attributed to a hydraulic jump.

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To derive the appropriate momentum equation, consider two-layer incompressible channel flow with the upper, less dense (ρ_2) fluid at rest, while the lower moving fluid of higher density (ρ_1) is relatively shallow (height h). Assume flow in the lower layer is parallel to the ground and constant in the cross channel and vertical directions, varying only in the down-channel direction. This is meant to be analogous to gap winds' lower, nearly neutral layer of strongest winds surmounted by a stable layer. Further, assume the hydrostatic approximation holds, and the time and length scales are sufficiently small that the earth's rotation can be ignored. Let the channel be aligned along the x direction with the configuration and variables shown in Fig. 1, and assume the flow is steady state. The steady-state assumption is supported by RAMS numerical modeling results (Part I) in which the local velocity tendency was small.

With these assumptions, the momentum equation is

$$\underbrace{u \frac{du}{dx}}_1 + \underbrace{g' \frac{dh}{dx}}_2 + \underbrace{g' \frac{de}{dx}}_3 + \underbrace{\frac{1}{\rho_1} \frac{dP}{dx}}_4 - \underbrace{\frac{1}{\rho_1} \frac{d\tau}{dz}}_5 = 0, \quad (1)$$

where u is the layer mean down-channel speed; h is the thickness of the lower layer; P is large scale pressure at the top of the layer; e is the channel elevation; τ is the eddy stress; $g' = g(\rho_1 - \rho_2)/\rho_1 = g(\theta_2 - \theta_1)/\theta_1$ is reduced gravity; ρ_1 and ρ_2 are air densities in the lower and upper layers; and θ_1 and θ_2 are potential temperatures in the lower and upper layers. In (1), term 1 is velocity advection; term 2 is a horizontal pressure gradient force due to a gradient in the thickness of the dense air; term 3 is a horizontal pressure gradient force due to changes in valley bottom elevation; term 4 is an external pressure gradient force imposed at the top of the system (alternately referred to as a "synoptic pressure gradient"); and term 5 is friction.

The equation of continuity in the outflowing air is

$$Q = uA = uh\bar{b} = uDb_T = \text{constant}, \quad (2)$$

where Q is total discharge that is conserved along the channel, A is channel cross-sectional area, $\bar{b}$ is mean width, $D = A/b_T$ is hydraulic depth, and b_T is the width of the channel at the top of the lower layer.

If one divides (1) by g' and uses continuity to replace u by Q/A , the result is

$$-\frac{Q^2}{g'A^3} \frac{dA}{dx} + \frac{dh}{dx} + \frac{de}{dx} + \frac{1}{g'\rho_1} \frac{dP}{dx} - \frac{1}{g'\rho_1} \frac{d\tau}{dz} = 0, \quad (3)$$

where, by the chain rule,

$$\frac{dA}{dx} = \frac{d(h\bar{b})}{dx} = \bar{b} \frac{dh}{dx} + h \frac{d\bar{b}}{dx}. \quad (4)$$

If one defines the channel slope, S_0 as positive "down hill", then $S_0 = -de/dx$. If the external pressure gra-

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FIG. 1. Definition sketch for airflow in a channel: ρ_1 and ρ_2 are density of the lower and upper layer; h is the height of the lower (gap wind) layer; e is the channel-bottom elevation above datum; $\bar{b}$ is the mean channel width; b_T is the width at the top of the lower layer; D is the hydraulic depth; and u is the layer mean gap wind speed in the down-channel (x) direction.

dient is defined as S_p and made positive for decreasing pressure along x , then $S_p = -(g'\rho_1)^{-1}dP/dx$. The effect of the external pressure gradient S_p is similar to that of channel slope S_0 so that increasing S_p is like "tilting" the channel.

The surface stress τ_s is set equal to $\rho_1 Cu^2$, where C is a stability corrected drag coefficient and u is the 10-m wind speed. According to Deardorff (1972), if h is the height of the planetary boundary layer (in this case the neutral layer of gap wind flow), and $\tau = 0$ at $z = h$, then the friction term can be approximated:

$$\frac{1}{\rho_1} \frac{d\tau}{dz} \approx \frac{-1}{\rho_1} \frac{\tau_s}{h} = \frac{-Cu^2}{h}, \quad (5)$$

which assumes the drag is exerted throughout the boundary layer. Under neutral and stable conditions, the stress does not decrease linearly with height, so C is multiplied by 2.8 (Deardorff 1972).

The momentum equation can now be written:

$$\left(1 - \frac{Q^2}{g'A^3} \bar{b}\right) \frac{dh}{dx} = S_p + S_0 + \frac{hQ^2}{g'A^3} \frac{d\bar{b}}{dx} - \frac{Cu^2}{g'h}. \quad (6)$$

This equation is similar to that of standard hydraulics treatments (for example, Henderson 1966), except that gravity has been replaced by reduced gravity, an external pressure gradient is included, and friction is parameterized as above. Despite its apparent simplicity, the equation can describe a variety of complex flow situations. It incorporates pressure gradients that are externally imposed due to changing channel elevation and changing boundary layer depth. It can incorporate friction and channels of varying cross section.

a. The energy equation

The momentum equation gives considerable qualitative channel-flow information and could also be used to quantitatively find the flow. The traditional approach in hydraulics, however, is to use an integrated form of the momentum equation that is a statement of conservation of energy and a form of the Bernoulli equation. The energy equation can be found by starting with the equation of momentum (1), dividing it by g' , substituting (5) for the friction term, and substituting S_p for the external pressure gradient as before, resulting in

$$\frac{u}{g'} \frac{du}{dx} + \frac{dh}{dx} + \frac{de}{dx} - S_p + \frac{Cu^2}{g'h} = 0. \quad (7)$$

This equation is now integrated along x :

$$\Delta \left(\frac{u^2}{2g'} + h + e \right) = \overline{\left(S_p - \frac{Cu^2}{g'h} \right)} \Delta x, \quad (8)$$

where the left-hand side is the change in total energy along x (the change in "head"), and the right-hand side contains the factors causing the total energy to change (friction and external pressure gradient). The overbar denotes an average over the horizontal interval Δx . The first two terms on the left-hand side of (8) define the flow specific energy:

$$E = \frac{u^2}{2g'} + h. \quad (9)$$

That (9) is actually a cubic in h can be easily seen in the case of a rectangular channel, where $Q = uhb$ with b the width. If u^2 is replaced by Q^2/h^2b^2 , then (9) becomes $h^3 - Eh^2 + Q^2/2g'b^2 = 0$. This equation has generally three roots, one of which is negative and can be ignored. The two positive roots represent different values of h at which the flow can have the same specific energy. This can be seen in Fig. 2, which schematically depicts h versus E for a particular discharge and width. The smaller h value (h_{sup}) is for fast, shallow, supercritical flow, while the larger (h_{sub}) is for slow, deep, subcritical flow. Flows through valleys of varying cross section are represented by a family of such curves. A similar analysis could be used for nonrectangular channels, except that in general the roots must be found iteratively. At the minimum in E , there is only one

FIG. 2. Schematic illustration of h versus specific energy for a particular flow condition (after Henderson 1966): h_c , h_{sub} , h_{sup} are the critical, subcritical, and supercritical heights; E_c is the critical specific energy.

positive root, at the critical height h_c , where the flow is said to be critical. As the critical height occurs at a minimum in specific energy, it can be found by making use of the substitution $u = Q/A$, taking the derivative of E with respect to h in (9), and setting it to zero (after Henderson 1966):

$$-\frac{Q^2}{g'A^3} \frac{dA}{dh} + 1 = 0 \quad (10)$$

$$\frac{u^2}{g'D} = 1, \quad (11)$$

where $dA/dh = b_T$ is the channel-top width and the hydraulic depth $D = A/b_T$ is as before. This leads to a definition of the Froude number:

$$F = \left(\frac{Q^2 b_T}{g'A^3} \right)^{1/2} = \frac{u}{(g'D)^{1/2}}. \quad (12)$$

The Froude number determines the flow regime: if the Froude number is less than 1, flow is subcritical; if it is greater than 1, flow is supercritical; and if it is equal to 1, flow is critical. Since the Froude number is the ratio of wind speed to the speed of long gravity waves, supercritical flow means that the fluid travels faster than gravity waves on the fluid interface. Therefore, in supercritical flow, fluid disturbances can only propagate downstream—no "information" can propagate upstream, so that the flow is controlled by upstream conditions. In subcritical flow, which is controlled by downstream conditions, information can propagate both upstream and downstream.

At each location along the fjord, it is possible to find the critical depth h_c : the depth at which the flow at that location would be critical ($F = 1$). This is found by

setting the Froude number in (12) to 1. For channels of rectangular cross section, this is $D_c = h_c = Q^2/g'A^2$ (after Henderson 1966). For irregular channels, this will in general have to be found iteratively. The critical speed can be easily found once h_c is known: $u_c = Q/A$, since A is a known function of h . If the flow depth is below the critical depth, then the speed will be greater than the critical speed, the flow will be supercritical with a Froude number greater than 1, and vice versa for subcritical flow. It is the occurrence of supercritical flow that creates the strongest winds in a channel. It is difficult to apply (6) where the flow transits between sub- and supercritical, since several of the terms become zero, making dh/dx large or infinite. An additional complication is that (6) does not incorporate the loss of energy due to friction that occurs in a turbulent hydraulic jump. This will be treated later by considering conservation of momentum.

In channels of constant slope and cross section, it is possible to define a flow state called *uniform flow* (Henderson 1966), in which state the fluid thickness is constant ($dh/dx = 0$) and the external pressure gradient and channel slope terms are exactly balanced by friction, since with constant cross section the contraction term (the third term on the right) of (6) is zero:

$$S_P + S_0 = C \frac{u^2}{g'h}. \quad (13)$$

In long channels of constant slope, roughness, and cross section, the flow would asymptotically tend toward uniform flow. It can be seen that if the channel is horizontal ($S_0 = 0$), the condition of uniform flow reduces to

$$u = \left(\frac{S_P g' h}{C} \right)^{1/2}. \quad (14)$$

A *control* in hydraulic theory is a channel feature (such as a contraction or change in slope) that fixes the flow in its locality. Flow passes smoothly from subcritical upstream of a control, to critical at a control, to supercritical downstream of a control. The transition from supercritical to subcritical, however, is not smooth, occurring as a *hydraulic jump*. The hydraulic jump location is determined by a downstream control and friction in the channel (Henderson 1966). Whether or not a feature will act as a control depends on both the boundary conditions (external pressure gradient, terrain slope, amount of horizontal contraction, and friction) and the flow itself (discharge which depends on speed and depth). A fixed boundary layer height at the channel terminus, like a reservoir, acts as a control on upstream flow.

b. The hydraulic jump

When flow transits from super- to subcritical in a hydraulic jump, there is considerable energy loss due

to turbulence. This energy loss is not represented on the right-hand side of (8), so the energy equation breaks down and cannot be used to find the flow. Instead, conservation of momentum through the jump can be used to find the jump location (Henderson 1966).

Consider the slab of air just large enough to contain a hydraulic jump depicted schematically in Fig. 3. Ignore the frictional drag of the channel, external pressure gradients, and channel slope, which are minor terms compared to the energy lost in the hydraulic jump. Then the change in momentum across the slab must equal the difference in hydrostatic forces acting on each face of the slab:

$$\Delta(Q\rho_1 u) = Q\rho_1 u_2 - Q\rho_1 u_1 \quad (15)$$

$$= F_{H1} - F_{H2}, \quad (16)$$

where F_H is the hydrostatic force acting on the face of each end of the slab. The hydrostatic thrust is equal to the mean pressure due to the weight of air times the cross-sectional area:

$$F_{H1} = [\rho_2 g(H - h_1) + \rho_1 g(h_1 - \bar{h}_1)]A_1 + \rho_2 g(H - \bar{h}_1)(A_2 - A_1) \quad (17)$$

$$F_{H2} = [\rho_2 g(H - h_2) + \rho_1 g(h_2 - \bar{h}_2)]A_2, \quad (18)$$

where subscripts 1, 2 refer to conditions on either side of the jump; $\bar{h}$ is the height of the centroid of cross-sectional area A ; $\bar{h}_j$ is the height of the centroid of the jump. Equating the change in momentum to the differ-

FIG. 3. Forces acting on a slab of air on either side of a hydraulic jump. Here F_{H1} and F_{H2} are the hydrostatic forces acting on either side of the jump; h_1 and h_2 are the gap wind height on either side of the jump; $\bar{h}_1$ and $\bar{h}_2$ are the cross-sectional centroid heights on either side of the jump; $\bar{h}_j$ is the centroid height of the jump; and u_1 and u_2 are the wind speeds on either side of the jump. The parameter H is an arbitrarily large height.

ence in forces, after some manipulation, results in the following:

$$\left[\frac{Q^2}{g'A_2} + A_2(h_2 - \bar{h}_2) \right] - \left[\frac{Q^2}{g'A_1} + A_1(h_1 - \bar{h}_1) \right] = 0. \quad (19)$$

For rectangular channels, this equation can be solved directly for the two heights on either side of the jump (h_1 and h_2), leading to the well-known hydraulic jump equation (Henderson 1966):

$$\frac{h_2}{h_1} = \frac{1}{2} [(1 + 8F_1^2)^{1/2} - 1], \quad (20)$$

where F_1 is the upstream Froude number. For nonrectangular channels, (19) must be solved iteratively to yield h_1 and h_2 .

3. The hydraulic model

With an equation for conservation of energy (8), which is applicable on either side of a transition from sub- to supercritical flow, and conservation of momentum (19), which can be used to find the conditions on either side of a transition from super- to subcritical flow, it is possible to construct a model describing the flow of a relatively dense fluid (in this case cold air) underlying less dense fluid (in this case warmer air) in a channel.

The energy equation (8) is used to find the flow in the following manner. The horizontal domain along the channel is divided into a number of evenly spaced horizontal grid points. Given the flow information at one grid point (subscript a), the information at a neighboring grid point (subscript b) is found by iteratively solving (8) with varying h_b until the change in "head" [left-hand side of (8)] equals the factors causing the head to change [right-hand side of (8)]. The new a flow variables are set to the past b variables, and this is repeated to find flow information for the next horizontal grid location. In this manner, the energy equation (8) is solved over finite steps, in an upstream direction (usually) for subcritical flow and in a downstream direction (always) for supercritical flow. This is called the *step method* in classical hydraulics. The equation is always solved in reaches between control points, with a complete solution for the entire fjord being comprised of the superposition of solutions over individual reaches. For subcritical flow, the influence of downstream control points can extend upstream through other potential control points as far as necessary. Likewise, supercritical flow resulting from an upstream control point can extend downstream through other potential control points if necessary.

The hydraulic jump equation (19) is used to determine the location of a hydraulic jump in reaches where

there is supercritical flow from an upstream control point and subcritical flow from a downstream control point. This is again done iteratively by moving upstream from the downstream control point and finding the horizontal location where (19) is satisfied.

The computer code written to solve these equations is called *hydmod*. Hydmod includes important parameters such as: varying channel elevation and cross section (see Fig. 4 for cross section locations used); externally imposed pressure gradients; friction; and variations in the height of gap wind. There remain, however, a number of assumptions implicit in hydmod, and in its extension to gap winds. The hydraulic model (even as applied to water) is only valid where the flow is steady in time, gradually varying in space, 1D, and on slopes that are not too steep. Thus, transient phenomena such as waves are not incorporated. Flows that are rapidly varied in space, such as transitions between sub- and supercritical flow, are constructed by the superposition of gradually varying sections. It is assumed

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FIG. 4. Topography of Howe Sound region. The hydmod channel is defined by the thin down-channel lines. The heavy line through the middle of the channel is the horizontal coordinate. The numbered lines crossing the channel are locations of cross sections 2.5 km apart where terrain data are input. The stations indicated by symbols are wind observation points.

that energy losses due to channel sinuosity are negligible. The extension of hydraulics to gap wind adds other assumptions. The gap wind should bear some resemblance to a dense fluid flowing in a channel—a clearly marked lower, more dense layer surmounted by a definite inversion and a passive layer above. The hydraulic model does not incorporate entrainment of air from above or any other interactions with overlying air (other than an externally imposed pressure gradient), such as transport of momentum and friction. This could be a problem, especially in cases where lower-level air is not clearly of different density than the air above.

The hydmod wind output is not directly comparable to observed winds. Wind speed in hydmod is a layer mean, which is assumed to be constant across the entire channel. In fact, as observations and RAMS simulations have shown, wind speed is not constant across the channel where it widens significantly, with certain regions having greatly enhanced flow. The significant flow seems to be constrained by topography to the main channel along the eastern side of Howe Sound, with more stagnant flow elsewhere. Tests using actual terrain cross section data (Fig. 5) show simulated gap winds too shallow where the channel widens. This allows downstream conditions at the channel exit to easily induce slow subcritical flow by a “backwater” effect in the region near the channel exit. To circumvent this problem, a modified channel was created by connecting the main ridge lines of Anvil, Gambier, and Bowen Islands with a vertical “wall.” The wall not only links the significant topographic ridges, but also separates the region of stagnant flow in western Howe Sound from the gap flow, as suggested by observations. The location of this artificial wall, as well as the position and orientation of cross sections, are shown in Fig. 4. Most fjords do not widen as much as Howe Sound, so this type of modification should not generally be required.

The input data required for hydmod are

- Q , the volume discharge of outflowing air (from u and h at some place in the channel, in this case Squamish Town: u_s, h_s);
- a tabulation of cross-sectional area A by height and horizontal location along the channel;
- e , the elevation of the channel floor at various horizontal locations along the channel;
- θ_1, θ_2 , the potential temperature of the lower and upper air layers;
- dP/dx , the externally imposed horizontal pressure gradient along the channel;
- C , drag coefficient values at horizontal locations along the channel; and
- h_f , the height of gap wind at the end of the channel.

Uniform height and speed are used as the initial conditions at the start of the channel. The cross-sectional area above 2000 m, or above the highest terrain in each cross section, is assumed to be rectangular. None of the

FIG. 5. Gap wind flow for “most likely” input parameters for unmodified channel. Flow is from right (starting at Squamish Airport) to left (ending at Lookout Point). (a) Height of gap wind; (b) gap wind speed; (c) Froude number; and (d) force-balance components due to friction, advection, channel elevation changes, gap wind height changes, and external pressure gradient.

simulations made had flow depths as great as this. One pragmatic problem is that not all input data required for hydmod initialization are directly available from routine data. The most difficult parameters to specify are the gap wind height at some place in the channel and the channel end (h_s and h_f).

The terrain data (C , e , and A at 100-m intervals of h from the ground to 2000 m above ground) are input along the valley at regular horizontal intervals (every 2500 m in this case). Since the input intervals are too coarse to solve the energy equation accurately, data are interpolated horizontally using cubic splines to the much higher-resolution (62.5 m) model grid spacing. At high-resolution horizontal locations, cubic splines are computed for cross-sectional area A as a function of height, giving A at any height for any horizontal location along the fjord. Once initial data have been input and interpolated to the model resolution, the model computes critical and uniform flow at all points; finds all potential control points; computes sub- and supercritical flow up and downstream from each potential control point assuming critical flow at the control point; solves the actual flow (which may or may not become critical at any control point); and, finally, outputs and plots the results.

4. Results

Hydmod output will be compared with actual data and RAMS output (Part I). To simplify comparison with RAMS output, plots will be made with horizontal orientation and scale corresponding to that of the RAMS simulation. The mouth of Howe Sound (beyond Lookout Point) is on the left of each plot, and the entrance (beyond Squamish Airport) is on the right. Flow is from right to left. The numbers along the top of each plot correspond to the cross section number in Fig. 4.

a. Comparison with observations

Hydmod simulations are compared with observations and RAMS model output at four times. In order to do this, it is necessary to choose values of h_s , h_f , u_s , dP/dx , g' , and the drag coefficient C , which match observations. The drag coefficient is estimated to be 0.02 over land and 0.01 over water [based on Garratt (1977) and high form and frictional drag due to coniferous forests and islands]. There was difficulty estimating the external pressure gradient at specific times because of conflicting pressure data. The external pressure gradient computed from the pressure difference between Squamish town and Lookout Point was systematically larger than that computed between the Squamish River station (refer to Part I, Fig. 2 for location) and Lookout Point. As a consequence, results using both estimates of external pressure gradient at each time are shown. Reduced gravity g' is difficult to estimate because the air, while showing distinct layers,

is nevertheless continuously stratified. It is set at values corresponding to a potential temperature difference of 3° or 4°C, which are estimated from vertical profiles, as are h_s and u_s , leaving only h_f unobserved and unestimated. There is no easy solution to this problem, so h_f has been simply set to 600 m. This height is somewhat lower than suggested by RAMS output, which is reasonable given that the RAMS simulation performed poorly in the south because of too large a boundary layer height there (Part I). Higher values of h_f were found to degrade the flow representation near the channel terminus.

Table 1 gives the parameters used as input to hydmod for the four times simulated. Figure 6 compares hydmod gap winds with observations and RAMS simulation values. The comparison is between modeled volume-averaged wind (hydmod) and observed 10-m AGL wind at a point. Since 10-m winds are influenced by surface friction, one would expect hydmod winds to be somewhat stronger. Hydmod shows some success at replicating the observed gap wind flow at these times. The surface wind observations for the most part lie between the high and low external pressure gradient gap wind curves. In particular, hydmod simulates both the lowest wind speeds near the start of the channel (which is expected since this is near where the initial conditions are applied) and the highest wind speeds in supercritical flow farther down channel. The hydmod simulation (compared to observations) is qualitatively inferior to the RAMS simulation in the northern half of the channel but superior to RAMS in the southern half where the RAMS simulated winds were too light.

b. Sensitivity tests

Hydmod can be used to test the sensitivity of the flow to various parameters. The external parameters that can be tested are external (synoptic) pressure gradient; variations in discharge; height of gap wind at the channel exit; and reduced gravity. In addition the effect of

TABLE 1. Values of parameters used in hydmod comparisons with observations. SQR is Squamish River station, SQT is Squamish town site, LOO is Lookout Point. Refer to Part I, Fig. 2 for locations. Times are LST.

Input parameter	30 Jan 2300	31 Jan 0500	31 Jan 1100	31 Jan 1700
SQT height h_s (m)	1000	900	900	800
SQT speed u_s (m s ⁻¹)	5	8	7	7
Discharge Q (10 ⁶ m ³ s ⁻¹)	31.41	43.33	37.92	32.29
Ending height h_f (m)	600	600	600	600
SQR to LOO pressure gradient dP/dx (Pa m ⁻¹)	0.002	0.0025	0.0029	0.003
SQT to LOO pressure gradient dP/dx (Pa m ⁻¹)	0.0086	0.0083	0.0083	0.0092
Lower potential temperature θ_1 (K)	267	267	267	267
Upper potential temperature θ_2 (K)	270	271	271	270
Drag coefficient C	0.02 land; 0.01 water

FIG. 6. Comparison of hydmod wind output for two pressure gradient possibilities with observations and RAMS output for (a) 2300 LST 30 January; (b) 0500 LST 31 January; (c) 1100 LST 31 January; and (d) 1700 LST 31 January. Triangles connected by solid lines are observed down-channel wind speeds. Light dotted line is RAMS output down-channel wind. Heavy dashed lines are hydmod output wind for two pressure gradient possibilities.

internal parameters can be tested: the frictional roughness of the fjord; and the configuration of the channel itself. In order to test the sensitivity of the flow to various parameters, most probable values and the likely range of each must be specified (Table 2).

Simulation results for the "most likely" parameter values are given in Fig. 7 and show flow becoming supercritical near the horizontal coordinate of -127 km between Squamish and Watts Point. The flow reverts to subcritical in a hydraulic jump near -131 km and becomes supercritical again at a control point near -134 km, which is the contraction south of Watts

Point, becoming subcritical again near -135 km where the channel widens near Defence Island. The flow becomes supercritical at a major control point near -142.5 km, which is the contraction between Anvil Island and Brunswick Point.

A force balance analysis of the hydraulic model output has been carried out. To facilitate comparison with RAMS output (Part I), the hydmod force balance has been made as compatible as possible with that of the RAMS simulation by multiplying (1) by -1 and plotting these values. The hydmod force balance (Fig. 7d), like the RAMS force balance (Part I, Fig. 20), indicates that zones of increasing winds are marked by large tendencies due to pressure gradients (primarily from horizontal gradients in gap wind height in the hydmod case), which are mostly balanced by advection. Friction becomes an important force in the areas of fast supercritical flow away from contractions, when the balance is mostly between friction and synoptic pressure gradient.

Sensitivity tests are carried out in the following way. All input parameters are held constant at the most likely value, except one that is varied through the probable range in ten increments. The results are summarized in plots of spatial maximum and mean gap wind versus the input parameter that was varied so that sensitivity of the wind to that parameter can be assessed. The results, shown in Fig. 8, show that increases in Q and dP/dx lead to increased gap wind speed (positive sensitivity). Increases in h_f and C lead to decreased gap wind speed (negative sensitivity). Increases in g' first increase and then decrease the maximum wind speed (i.e., there is an optimum g'), while the mean wind speed shows a slight but steady decrease.

The sensitivity of flow speed to Q (Fig. 8a) shows a large positive sensitivity. Three regions of flow can be noted from the maximum wind speed curves in Fig. 8a. The first zone, at low values of Q (i.e., low wind speed and depth u_s, h_s), has subcritical flow throughout the channel, with maximum speed at the channel contraction. The force balance is mainly between the external pressure gradient force and the height pressure gradient force. Advection becomes important near con-

TABLE 2. Values of parameters used in hydmod sensitivity tests.

Input parameter	Most likely value	Probable range
Discharge Q ($10^6 \text{ m}^3 \text{ s}^{-1}$)	23.06	3.22–58.34
Ending height h_f (m)	600	150–1500
External pressure gradient dP/dx (Pa m^{-1})	0.004	0.0–0.02
Lower potential temperature θ_1 (K)	267	267 (fixed)
Upper potential temperature θ_2 (K)	272	268–295
Drag coefficient C	0.02 land; 0.01 water	0.001–0.02

FIG. 7. Gap wind flow for "most likely" input parameters for "reduced" channel. Flow is from right (starting at Squamish Airport) to left (ending at Lookout Point). (a) Height of gap wind; (b) gap wind speed; (c) Froude number; and (d) force-balance components due to friction, advection, channel elevation changes, gap wind height changes, and external pressure gradient.

tractions, and gap wind height increases along the channel. The second flow regime in the sensitivity of wind to Q is when critical flow begins. This is indicated in

FIG. 8. Sensitivity of mean and maximum gap wind speed to changes in (a) discharge Q ; (b) external pressure gradient dP/dx ; (c) end of channel gap wind height h_f ; (d) drag coefficient C ; and (e) reduced gravity g' . Solid line is the spatial maximum wind speed; dashed line is the spatial average wind speed.

Fig. 8a by a relatively rapid increase in maximum wind speed. Figure 7, which depicts the flow for the "most likely" input parameters, shows this mixed-flow transition. The third sensitivity regime occurs with large Q , where the flow speed increases more slowly. Here the extent of supercritical flow is increasing, and the flow resembles Fig. 7, except the supercritical flow downstream of contractions is of greater extent.

The flow-speed sensitivity to dP/dx (external pressure gradient, Fig. 8b) is positive and similar to that for Q . The flow for maximum dP/dx of 0.02 is supercritical throughout the channel, except near the end, where the flow jumps to a subcritical state due to the end of channel control. The primary force balance is between external pressure gradient and friction. Advection and height pressure gradients are smaller but become important near the contraction.

The sensitivity of gap wind to h_f (Fig. 8c) is opposite to that for Q . Increasing values of h_f mean that subcritical flow controlled by the endpoint height is able to propagate farther upstream, resulting in decreased flow speeds in the channel.

The sensitivity of flow speed to increasing C (drag coefficient) is also negative with increasing drag coefficient resulting in decreasing wind speed (Fig. 8d), as would be expected. The sensitivity curve is quite flat for values of C larger than 0.003. This is because the frictional term in the force balance is typically of minor importance compared with changes in gap wind height, external pressure gradient, and advection. In the case of predominantly supercritical flow, the sensitivity would be much more negative (as it is for small C values), because the force balance is primarily between friction and external pressure gradient. Initially high flow speeds for low values of C (0.001) are due to propagation of supercritical flow through the contraction that jump to subcritical farther downstream in response to the upstream influence of h_f . Larger drag coefficients result in critical flow at the contraction, with subcritical flow extending upstream and supercritical flow downstream.

The sensitivity of gap wind flow to g' (reduced gravity) shown in Fig. 8e is mixed. The mean flow speed declines as g' increases, while the maximum speed increases and eventually begins to decline. The initial flow, for low values of g' , is nearly everywhere supercritical. Supercritical flow can exist for relatively small wind speeds with low g' because of the inverse relationship between g' and the Froude number. As for the case of supercritical flow with large dP/dx , a force analysis shows the main balance is between friction and external pressure gradient, and changes in height are only important near the contraction. A difference between the two situations (large dP/dx and small g') is that the force component magnitudes are much smaller for small g' , more closely resembling RAMS force balance component magnitudes. Because the force balance is between external pressure gradient and friction in

these situations, (15) would be an appropriate simple model to apply. This is expected, since under these conditions the pressure gradient resulting from variations in gap wind height, which is fundamental to hydraulic flow, will be small compared to other forces.

5. Discussion and summary

Open-channel hydraulic flow, based upon simplifications of the shallow-water equations, has been proposed as an analog to gap wind. This has been prompted by perceived similarities: gap winds are the flow of cold, dense low-level air horizontally constrained by a channel, which resembles the flow of water in a river, with the density difference across the inversion representing the water surface.

Hydmod is able to simulate the main features of gap wind and its variation along a channel with some accuracy. This implies the essence of gap wind is contained in the simply formulated physics of hydraulic theory. The application of hydraulic theory provides an understanding of spatial gap wind patterns, indicating where to expect strong gap wind. In situations where the flow is light and everywhere subcritical, the strongest wind can be found at the location of a horizontal contraction. When the flow is very strong and supercritical everywhere, the speed will be a minimum at contractions. The most striking result is when the flow becomes critical at a control point: gap wind height decreases and wind speed increases in a zone of fast supercritical flow that forms downstream. The fast supercritical flow transits suddenly to slow subcritical flow in a turbulent hydraulic jump. These zones of strong gap wind can be inferred both from observations of surface wind and RAMS vertical cross section output that depict descending isentropes in zones of accelerating wind and ascending isentropes in zones of decreasing wind.

For subcritical flow away from contractions, the primary force balance is between the pressure gradient due to gap wind height variations and the external pressure gradient. For supercritical flow away from contractions, the primary force balance is between friction and external pressure gradient, implying (15) is applicable in this situation. For flow near a contraction, the primary balance is between advection and pressure gradient due to height variations.

Sensitivity tests indicated the importance of various initial and boundary conditions to gap wind speed. Maximum and mean flow speed were positively sensitive to discharge and externally imposed pressure gradient. Maximum and mean wind speeds were negatively sensitive to gap wind height at the channel terminus and drag coefficient. The response of mean wind speed to increased reduced gravity was slightly negative. The response of maximum wind speed to increased reduced gravity was positive for small and moderately large values of g' , reaching an

optimum and becoming slightly negative at large values of g' . Thus, the effect of increased reduced gravity over most of the likely range was to increase gap wind variability (increase the wind maximum but decrease the mean).

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