CORRESPONDENCE

Comments on "The Determination of View-Factors in Urban Canyons"

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Johnson and Watson (1984) present a general expression for the analytic evaluation of sky view-factors at all positions on the floor of an urban canyon of finite length. Their techniques are of great utility in evaluating the surface radiant energy budget of urban surfaces and will aid in work directed towards an understanding of the urban heat island under conditions dominated by surface exchanges of longwave radiation. While not rare, such conditions are a special case in which many other mechanisms influencing cooling rates are absent or negligible (Oke, 1981, 1982).

One such mechanism arises from the possibility of the usually relatively polluted urban air being radiatively active in the longwave band. The cooling rates of air volumes within the urban canopy layer would in this case be partly determined by the divergence (either positive or negative) of longwave radiative fluxes. While Atwater (1971) has investigated this effect for the urban boundary layer, an extension into the urban canopy layer has yet to be made. Essential to such an extension is knowledge of the wall view-factors of the surface facets defining infinitesimal volumes within the urban canyon.

FIG. 1. Model for radiative exchange between a surface element and a wall of finite length. The angles β , δ_1 and δ_2 are in vertical planes and are all positive while the angles γ_1 and γ_2 are in the horizontal plane with γ_2 positive and γ_1 negative.

An analysis similar to that of Johnson and Watson (1984) may be employed to yield the appropriate view-factors. Their Eq. (19) represents the view-factor of a finite wall and an infinitesimal surface element when the surface element is in a plane perpendicular to the wall and, using their notation, takes the form:

$$\psi_{w,1} = \frac{1}{2\pi} \{ (\gamma_2 - \gamma_1) + \cos\beta [\tan^{-1}(\cos\beta \tan\gamma_1) - \tan^{-1}(\cos\beta \tan\gamma_2)] \}. \quad (1)$$

The other limiting case that must be considered in the present context is the one in which the surface element is in a plane parallel to the wall. In this case, geometric considerations lead to a view-factor of:

FIG. 2. Model for radiative exchange between a volume at position $P(y, z)$ within an urban canyon similar to that studied by Nunez and Oke (1976). The canyon dimensions are: $H = 7.91$ m, $h = 5.59$ m, $w = 7.54$ m, and a total length of 79 m.

TABLE 1. Wall view-factors for the six wall segments in Fig. 2 from horizontal and vertical surface elements.

Wall segment number	Horizontal surface element		Vertical surface element	
1	$\psi_{w,1}(1) = \frac{1}{2} \left(1 - \frac{y}{[(h-z)^2 + y^2]^{1/2}} \right)$	(-)	$\psi_{w,2}(1) = \frac{1}{2} \left(\frac{(h-z)}{[(h-z)^2 + y^2]^{1/2}} \right)$	(+)
2	$\psi_{w,1}(2) = \frac{1}{2} \left(1 - \frac{(w-y)}{[(h-z)^2 + (w-y)^2]^{1/2}} \right)$	(-)	$\psi_{w,2}(2) = \frac{1}{2} \left(\frac{(h-z)}{[(h-z)^2 + (w-y)^2]^{1/2}} \right)$	(-)
3	$\psi_{w,1}(3) = \frac{1}{2} \left(1 - \frac{y}{(z^2 + y^2)^{1/2}} \right)$	(+)	$\psi_{w,2}(3) = \frac{1}{2} \left(\frac{z^2}{(h^2 + z^2)^{1/2}} \right)$	(+)
4	$\psi_{w,1}(4) = \frac{1}{2} \left(1 - \frac{(w-y)}{[z^2 + (w-y)^2]^{1/2}} \right)$	(+)	$\psi_{w,2}(4) = \frac{1}{2} \left(\frac{z^2}{[z^2 + (w-y)^2]^{1/2}} \right)$	(-)
5	$\psi_{w,2}(5) = \frac{1}{2} \left(\frac{y}{(z^2 + y^2)^{1/2}} \right)$	(+)	$\psi_{w,1}(5) = \frac{1}{2} \left(1 - \frac{z}{(z^2 + y^2)^{1/2}} \right)$	(+)
6	$\psi_{w,2}(6) = \frac{1}{2} \left(\frac{(w-y)}{[z^2 + (w-y)^2]^{1/2}} \right)$	(+)	$\psi_{w,1}(6) = \frac{1}{2} \left(1 - \frac{z}{[z^2 + (w-y)^2]^{1/2}} \right)$	(-)

$$\psi_{w,2} = \frac{1}{2\pi} [\sin\gamma_2 \tan^{-1}(\tan\delta_2 \sin\gamma_2) + \sin\beta \tan^{-1}(\cot\delta_2 \sin\beta) + \sin\gamma_1 \tan^{-1}(\tan\delta_1 \sin\gamma_1) + \sin\beta \tan^{-1}(\cot\delta_1 \sin\beta)], \quad (2)$$

where the angles β , γ_1 , γ_2 and δ_1 , δ_2 are as defined in Fig. 1.

Equations (1) and (2) may now be used to determine the wall (or sky) view-factors of surface elements anywhere within the volume of the urban canyon,

and in either of the two orientations given. These surface elements may then be taken as the faces of volume elements in a rectangular array, and used in finite element modeling of radiative transfer processes. This procedure may be simply illustrated by applying the equations to the urban canyon investigated by Nunez and Oke (1976). For simplicity, they restricted themselves to measurements in a cross-sectional plane at the midcanyon point (the plane is at right angles to the canyon axis). Our calculations are applied to elements within that plane and would be appropriate

FIG. 3. The sky view-factor of a sky-facing surface element within the urban canyon of Fig. 2.

FIG. 4. The sky view-factor of a surface element facing the west wall of the urban canyon of Fig. 2.

as the starting point for a two-dimensional radiative transfer model.

It is of interest that in this case, the exact view-factors are within 0.1% of the approximate values obtained by assuming the canyon to be infinitely long. Because of this, we use the approximate forms of Eqs. (1) and (2), viz:

$$\psi_{w,1} \approx \frac{1}{2} \left[1 - \frac{y}{(h^2 + y^2)^{1/2}} \right], \quad (3)$$

$$\psi_{w,2} \approx \frac{1}{2} \frac{h}{(h^2 + y^2)^{1/2}}, \quad (4)$$

where we have dropped Johnson and Watson's (1984) angular notation in favor of a linear one. This change facilitates the use of Cartesian coordinates in the plane of interest with the origin of the coordinate system defined as shown in Fig. 2. Figure 2 defines the coordinate system used in this calculation and indicates how the canyon walls must be divided in order to use Eqs. (3) and (4) to determine the wall view-factors as functions of y and z for the two surface element orientations. Table 1 gives the wall view-factors for each of the six wall segments for horizontal and vertical surface elements. It must be recognized that any "volume" in this two-dimensional scheme is represented by upper and lower as well as left and right surface elements. The bracketed signs in Table 1 indicate this by giving the sign of the

radiative component for which the given view-factor is appropriate.

As an illustration of these view-factors applied to the canyon of Nunez and Oke (1976), Fig. 3 gives the sky view-factor for a horizontal surface element, which is

$$\psi = [1 - \psi_{w,1}(1) - \psi_{w,1}(2)], \quad (5)$$

and Fig. 4 gives the wall view-factor for a vertical surface element facing the west wall which is:

$$\psi = \psi_{w,2}(1) + \psi_{w,2}(3) + \psi_{w,1}(5).$$

The techniques described by Johnson and Watson (1984) and in this communication provide the means for determining view-factors within urban canyons which are essential prerequisites for an understanding of the energetic basis of urban heat islands.

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