Comment on "Diffraction of Solitary Kelvin Waves at Cape Mendocino"

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1. Introduction

Dorman (1985) has recently presented evidence that the propagating "bump" in the marine layer along the central California coast is a form of internal solitary Kelvin wave that is trapped by the coastal topography. In his discussion Dorman notes that the progression of the leading edge of this wave, as indicated by the edge of the overcast stratus cloud on satellite images of the coast, apparently ceases at the sharp convex bend formed by Cape Mendocino. Since the linear theory of Kelvin wave diffraction (Miles, 1972) indicates that the wave should in fact propagate around the bend, Dorman speculates that an increased mean northerly wind exists north of the Cape which is strong enough to prevent further wave progression.

According to Dorman (see his Fig. 8), wind speeds just north of the Cape decrease from 15 m s^{-1} twelve hours or so before the arrival of the leading edge of the wave to 0–5 m s^{-1} as the wave passes overhead. Over the following day, the winds increased to 5–10 m s^{-1}. This mean wind is to be compared with the theoretical phase speed of the wave of 14 m s^{-1}. This value for the phase speed has been computed from linear theory using data for the height of the marine layer and the temperature difference between the marine and upper layers at Gualala station which is located about 150 km to the south of the Cape (Gualala being the nearest station for which data exists). Although the wave speed is observed to decrease slightly as the wave propagates north, it is unlikely that it will have been reduced sufficiently below 14 m s^{-1} for the wind to act as an effective block. Instead, as is shown in the next section, the theory of Miles (1977) for the diffraction of solitary waves by an irregular coastline offers a more plausible explanation for the observed inability of the wave to propagate around Cape Mendocino.

2. Application of Miles' theory

Miles (1977) has shown that a nonlinear solitary wave cannot turn through an angle at a convex bend in the boundary of greater than

$$\theta_c = (3\alpha)^{1/2}, \tag{1}$$

where α is the ratio of the wave amplitude to the depth of the fluid layer, without undergoing separation or other loss of identity. Although the theory has been derived for a nonlinear surface solitary wave whose displacement profile $\eta(x)$ is given by the well known sech^2x solution to the Korteweg-deVries (KdV) equation, it can readily be extended to the internal Kelvin wave case. Indeed, Smith (1972) and Grimshaw (1977, 1985) have shown that nonlinear (i.e., solitary) Kelvin waves satisfy a form of the KdV equation and therefore have the sech^2x profile. Dorman has in fact assumed this profile for his solitary Kelvin wave model [see his Eq. (7) but note the typographic omission of the power 2] so the nonlinear theory of Miles (1977) is directly applicable. Thus, if the angle θ formed by the coastal topography at Cape Mendocino is greater than θ_c then the theory would seem to offer a convenient explanation for the observation that the progression of the solitary wave ceases there. In addition, the interaction of the flow separated by the Cape with the mean northerly wind offshore could provide a mechanism for the formation of the cyclonic eddy that is observed to occur just at its southern edge.

To determine the magnitude of θ_c estimates of the wave parameter α and hence the fluid layer depth and wave amplitude are required. The marine layer depth has been observed at Gualala to be 480 m. Unfortunately, no data for the wave amplitude exist but an indirect measure can be obtained from Eq. (2) by using the observed pressure difference (see Fig. 13 in Dorman's paper) between the top and bottom of the fluid layer formed by the propagating wave:

$$\frac{p}{p_0} = \exp\left(\frac{gH}{287.04T}\right), \tag{2}$$

where p_0 is the pressure at the top of the layer, p that at the layer base, g is the acceleration due to gravity, H is the thickness of the layer (i.e. the solitary wave amplitude) and T is the temperature. From Dorman, $p/p_0 = 1.009$, $T = 298$ K which gives a solitary wave amplitude H of 77.13 m and hence a value for α of

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0.161. Since the first order formula (Christie et al., 1978) for the phase speed c of an internal solitary wave depends on α, and an observed estimate for c exists, a check on the above calculation is straightforward:

$$c = [g'h(1 + \alpha)]^{1/2}. \quad (3)$$

In (3) g' is the reduced gravity of the two layer atmosphere and h is the depth of the lower layer. Dorman estimates c to lie in the range 11–16 m s^{-1} (i.e., the observed effective speed of propagation of the wave of 6 m s^{-1} plus the opposing mean wind of 5–10 m s^{-1}). From Dorman's data for Gualala, $g' = 0.408$ m s^{-2} and $h = 480$ m yielding an estimated range for α of 0–0.307 [negative values of α corresponding to c less than $(g'h)^{1/2}$ are excluded]. The estimate of $\alpha = 0.161$ is therefore consistent. Substituting this value into (1) shows that the critical angle for separation of this solitary wave at a convex bend is 39.8°. Since the bend of the 1000 m contour at Cape Mendocino is of the order of 50° (note that Dorman's Fig. 1 for the 609 m contour is misleading as it does not extend far enough up the coast to give an accurate estimate of the bend in the topography), it is clear that according to the nonlinear theory the solitary wave should indeed not propagate around the Cape.

Nonlinear behavior may also partially explain another of the questions raised by Dorman, namely that as the solitary wave propagates north it is observed to decrease in amplitude. Dorman postulates that this amplitude decay is due either to energy radiation or to turbulent drag. The latter mechanism seems more likely in light of the results from some rotating tank experiments performed by Maxworthy (1983). In these experiments, it is observed that the amplitude of a propagating internal solitary wave decays exponentially in time as a result of the drag exerted by inertial waves generated in the homogeneous fluid above and below the solitary wave.

3. Discussion

It has been illustrated that the nonlinear theory of solitary wave diffraction of Miles (1977) is consistent with the observation that the disturbance in the marine layer is unable to propagate around the convex bend formed by the coastal mountains at Cape Mendocino. Invoking the nonlinear theory does not depend on the magnitude of the mean flow and is therefore to be preferred over Dorman's speculation that it is an increase in the mean northerly wind north of the Cape that prevents the propagation of the wave there. In addition, the nonlinear separation of the solitary wave from the topography provides a source mechanism for the formation of the observed eddy at the Cape. Finally, we note that by identifying the propagating disturbance in the Californian marine layer as a solitary Kelvin wave, Dorman is in fact requiring his model to satisfy the nonlinear theory. Since the application of this theory has adequately explained the observations, the speculation of an increased mean wind is no longer necessary.

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