NOTES AND CORRESPONDENCE

Application of Two-Dimensional Terrain Height Spectra to Mesoscale Modeling

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ABSTRACT

A caution is offered with regard to the use of one-dimensional terrain spectra to indicate the grid resolution needed to resolve terrain forcing in mesoscale numerical modeling exercises. To illustrate this, two-dimensional terrain height spectra are presented for two contrasting terrains: a relatively direction free topography (a portion of southcentral British Columbia, Canada) and a highly ordered topography (a portion of the ridge and valley terrain in Pennsylvania). Isoamplitude plots of the two spectra show clearly the morphological differences between the two regions and indicate the degree of directionality of the ordered terrain.

An investigation of the wavenumber dependence of the terrain height spectra shows the spectral roll-off for the first case to be essentially independent of direction and to decay roughly as wavenumber to the −5/2 power over a wavenumber range of 0.04 to 8.33 km^{-1}. By contrast, the spectral roll-off in the second case is strongly dependent on direction with an exponent that may be either greater than or less than the convergence limit (−2.0 for the amplitude spectrum) indicated by Young and Pielke.

1. Introduction

Young and Pielke (1983) and Young *et al.* (1984) have drawn attention to the application of terrain height variance spectra to the numerical modeling of mesoscale atmospheric flows. The spectra they present are all wavelength–space representations of linear sections through the terrain of interest. In order to represent the spectral content of a segment of terrain, they average the spectra from a number of such sections. Pielke and Kennedy (1980) mention the possible application of two-dimensional spectral analysis to terrain within the context of atmospheric mesoscale modeling but do not present any analyses. The objective of this contribution is to expand on the advantages of the full two-dimensional spectral analyses over the more restricted one-dimensional form and to present the results from analyses of two terrain types.

2. Two-dimensional versus one-dimensional terrain height spectra

The major advantage of two-dimensional terrain height spectra within the context of atmospheric forcing is that they explicitly retain any directionality in the topography. Pielke and Kennedy (1980) and Rayner (1972) demonstrate how directionality of terrain is represented in the spectra. Rayner's (1972) isopleth plots of spectral amplitude in wavenumber space are particularly useful since perfectly circular isopleths centered on the (0, 0) point should represent a totally directionless topography. Any systematic deviation from circularity would indicate directional bias.

In the case of directionless topography, one-dimensional spectral analyses are sufficient to represent the topography, and any numerical modeling grid design or observational network design may be performed on the basis of the shape of the one-dimensional spectra averaged over a number of linear sections.

If the two-dimensional spectrum shows significant directional bias (in the sense of the spectral shape being dependent on direction), that bias may be examined by comparing the spectral shape along various sections of the spectrum radiating from the (0, 0) wavenumber point. These sections would represent the spectrum of all sections through the topography in a direction given by the orientation of the section in the spectral space. In either case, the full two-dimensional spectrum is the most complete representation of the scales of terrain variability.

3. Data and methods

Two Digital Elevation Models (DEM) were chosen to demonstrate the two-dimensional spectra. An apparently random topography is represented by a DEM of St. Mary Lake, Canada (49°40′N, 116°15′W) (NTS 82 F/9 at 1:50,000) digitized at a grid spacing of 60 m over a domain of 21.54 × 30.18 km (360 × 504 points). It is believed that this DEM has very high reliability since it has been subjected to rigorous quality control tests within the departments of Computer Science and Forestry at The University of British Columbia where it is used for research and teaching purposes. A contrasting topography, having considerable directional bias, is represented by a DEM of Blair's Mills, Pennsylvania (77°41′N, 40°19′W) (USGS, 1:24,000 scale, Blair's Mills, Pennsylvania), digitized at a grid spacing

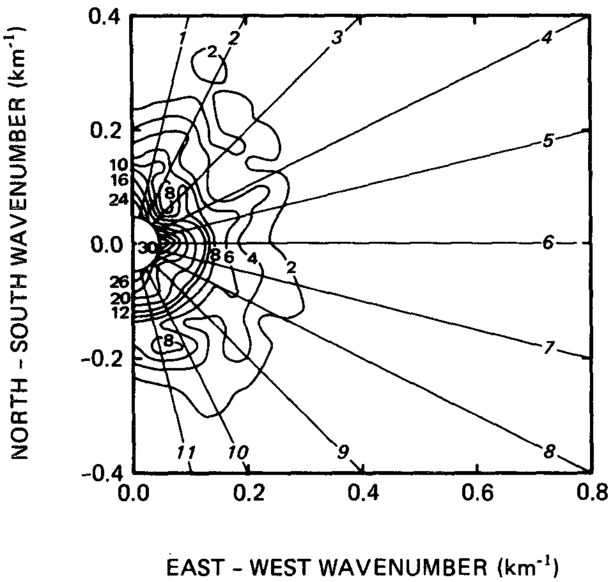

FIG. 1a. Isoamplitude plot of the terrain height spectrum for St. Mary Lake, British Columbia.

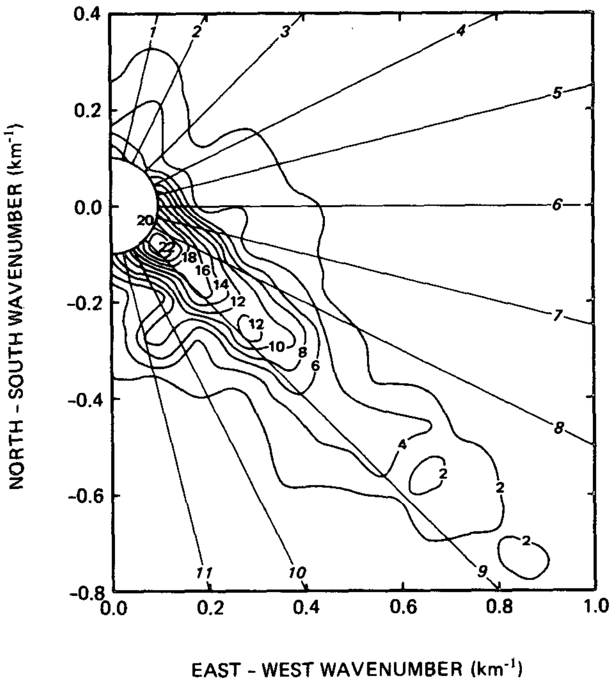

FIG. 1b. Isoamplitude plot of the terrain height spectrum for Blair's Mills, Pennsylvania.

of 30 m over a domain of 13.80 × 11.04 km (463 × 369 points). Full details of this DEM, which is one of the standard USGS 7.5 minute DEMs, are available in Elassal and Caruso (1983). Apart from appropriate adjustments to accommodate the differences in grid spacing and overall size, the two DEMs were subjected to the same analytical procedure.

The data arrays were first averaged, and the average elevation subtracted from each point in order to remove the large amplitude spike at zero wavenumber which represents the average terrain elevation. The resulting arrays were then multiplied by a circular filter having a value of unity over most of the domain and a cosine taper to zero at the fringes (the taper width being roughly 9.5% of the radius). Justice (1981) recommends this procedure in order to reduce the variance introduced into the spectral estimates by the edge discontinuity. The circular arrays were then transformed using a standard two-dimensional discrete Fourier transform software package.

Since the spectral estimates are derived at discrete wavenumbers, profiles in wavenumber space are most efficiently extracted at angles having rational ratio tangents [i.e., arctan(1/1), arctan(1/2), arctan(1/4), etc.]. These profiles were then fitted (using the least-squares method) to functional relations of the form $S = ak^b$ where S is the terrain height spectrum in units of meter per kilometer squared,[1] k the wavenumber in units of per kilometer, b is dimensionless, and a has units consistent with dimensional homogeneity (and is hence dependent on b).

4. Results

a. *Isoamplitude plots*

The results of these analyses are shown in Figs. 1a, b which are isopleths of S (normalized by the total terrain height) in wavenumber space. While the max-

TABLE 1. Parameters of the least-squares best fit relation of the form $S = ak^b$ for eleven sections (given on Fig. 1b) through the terrain height spectrum for St. Mary Lake; r is the correlation coefficient.

Section	a	b	r
1	0.74	−2.57	−0.97
2	0.72	−2.51	−0.97
3	0.59	−2.33	−0.96
4	0.86	−2.59	−0.98
5	0.72	−2.61	−0.96
6	0.76	−2.48	−0.97
7	0.84	−2.77	−0.97
8	0.70	−2.55	−0.97
9	0.61	−2.24	−0.96
10	0.73	−2.51	−0.97
11	0.67	−2.56	−0.96
Mean	0.72 ± 0.08	−2.5 ± 0.1	

[1] In this communication, the computed spectra are two-dimensional amplitude spectra of terrain height. The magnitude of the spectrum at any position in wavenumber space is thus the amplitude of the harmonic component represented by the wavenumbers (north–south and east–west components). Previous communications on this topic (Young and Pielke, 1983; Young *et al.*, 1984) have presented variance spectra which are the square root of the amplitude spectra. In order to compare this work with the aforementioned, the fitted exponent (b) must be halved.

imum wavenumbers resolvable by these analyses are 8.3 and 16.6 km^{-1} for St. Mary Lake and Blair's Mills, respectively, the wavenumber domains in Figs. 1a, b are considerably narrower since neither of these terrains contains significant structure at such high wavenumbers. In addition to being truncated at large wavenumbers, the spectral regions around zero wavenumber have been excluded for wavenumbers smaller than the smallest resolvable value. This lower limit is determined by the extent of the digitized domain employed in the analysis.

The pattern of the spectrum for St. Mary Lake (Fig. 1a) exhibits a large degree of symmetry about the (0, 0) point, and is representative of a topography with very little directionality. This is not surprising since the terrain in South Central British Columbia represents a glacially modified topography with very weak structural controls and a dendritic drainage system. By strong contrast, the spectrum for Blair's Mills (Fig. 1b) shows marked asymmetry, with variance contributions extending to much greater wavenumbers in a direction represented by negative (southward) north–south wavenumbers and positive (eastward) east–west wavenumbers. Such a pattern represents a strongly ordered terrain with a series of ridges running in a northeast to southwest direction (after Rayner, 1972). The terrain is in the ridge–valley country of Pennsylvania where the topography is associated with deeply eroded fold mountains and an associated trellis drainage system.

b. *Spectral roll-off*

Having established the asymmetry of the Blair's Mills spectrum and the relative symmetry of the St. Mary Lake spectrum, we consider it instructive to examine the spectral roll-off along various sections through the two spectra.

Figure 2a represents section 6 indicated on Fig. 1a. The points are tightly clustered around a line having a slope of −2.48 and intercepting the $k = 1$ km^{-1} axis at an ordinate of 0.76. The other sections through this spectrum have similar clustering, slopes and intercepts as given in Table 1. The homogeneity of the parameters a and b between sections is another reflection of the symmetry of the spectrum and hence the lack of directional bias in the terrain. Since the mean exponent b for this terrain is significantly less than −2.0, we may use the logic presented by Young and Pielke (1983) to

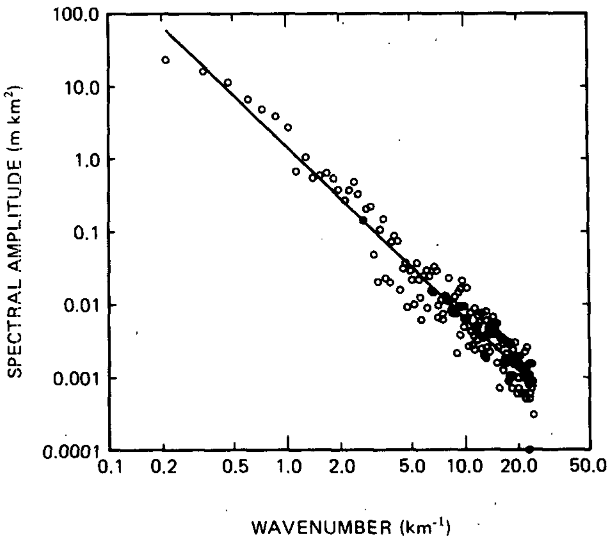

FIG. 2b. Terrain height spectrum along section 3 of Fig. 1b, representing the spectral roll-off in all northeast–southwest sections of the Blair's Mills terrain.

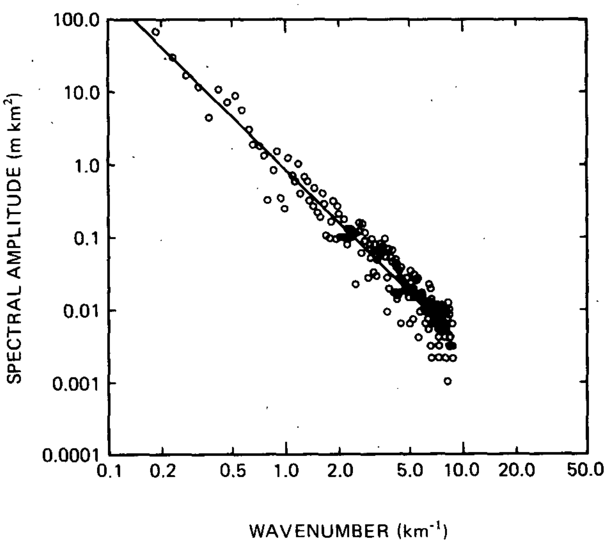

FIG. 2a. Terrain height spectrum along section 6 of Fig. 1a, representing the spectral roll-off in all east–west sections of the St. Mary Lake terrain.

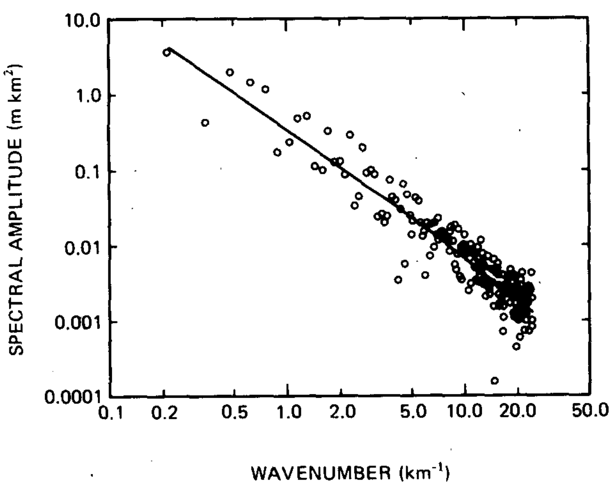

FIG. 2c. Terrain height spectrum along section 9 of Fig. 1b, representing the spectral roll-off in all southeast–northwest sections of the Blair's Mills terrain. There is a break in slope of the fitted lines, arbitrarily drawn at 5.5 km^{-1}.

design a mesoscale numerical modelling grid to study flow features induced by this terrain.

Figures 2b, c represent sections 3 and 9 on Fig. 1b and are examples of the complete set whose parameters are given in Table 2. The two lines on Fig. 2c are least-squares best fit lines to the low- and high-wavenumber portions of the spectral section, the distinction between low and high wavenumber being (arbitrarily) drawn at 5.5 km^{-1}. The exponent for these portions of the spectrum are -1.73 ± 0.02 and -1.94 ± 0.01, respectively. The 95% confidence limits on the two exponents indicate that the two sets of harmonics are from different populations. This is further evidence (in addition to that provided by Figs. 1a, b) that the differences between the two topographies are most evident in the low wavenumber spectra, but disappear at higher harmonics. The two distinct roll-off exponents seen in Fig. 2c are not evident in any of the other spectral sections from this terrain.

The greater variability of the parameters in Table 2 (when compared with those in Table 1) and generally smaller values of the correlation coefficients indicate the lack of homogeneity between sections and within sections. The variograms and associated fractal scales investigated by Mark and Aronson (1984) seem more capable of detecting the scale breaks in topographies such as this—they detect a scale break at 0.56 km^{-1} for the Blair's Mills quadrangle with a fractal scale of dimension 2.83 below this wavenumber and a fractal scale of dimension 2.48 above. Mark and Aronson (1984) conclude that the influences of structural control are apparent only at scales of 5 to 6 km and larger.

From the foregoing analysis, it is evident that a single spectral roll-off exponent cannot be used to represent the Blair's Mills terrain (or similar terrain types) because of the considerable directionality of topographic features. In addition, there is strong evidence that in certain directions, the roll-off exponent may be significantly larger than −2.0. In the face of this, it would be inadvisable to use a homogeneous mesoscale modelling grid to study atmospheric responses to forcing by this type of terrain.

TABLE 2. Parameters of the least-squares best fit relation of the form $S = ak^b$ for eleven sections (given on Fig. 1b) through the terrain height spectrum for Blair's Mills; r is the correlation coefficient.

Section	a	b	r
1	0.21	−2.0	−0.90
2	0.44	−2.1	−0.94
3	1.45	−2.5	−0.97
4	0.76	−2.4	−0.95
5	0.31	−2.2	−0.94
6	0.32	−1.6	−0.89
7	0.29	−2.1	−0.91
8	0.33	−2.0	−0.94
9	0.32	−1.9	−0.93
10	0.32	−2.0	−0.94
11	0.24	−1.9	−0.94
Mean	0.5 ± 0.3	−2.1 ± 0.3	

5. Conclusion

Two-dimensional terrain height spectra for two different terrain types have been presented. The two examples were chosen to show the differences between apparently isotropic (in the sense of being devoid of directional bias) and highly ordered terrain. Such differences are clearly illustrated by the two-dimensional terrain height spectra. The spectral roll-off for the random terrain is uniform in all directions with an exponent of less than −2.0, whereas the spectral roll-off for the ordered terrain is strongly directionally dependent. The ordered terrain has a strong harmonic component which results in the spectral roll-off being significantly larger than −2.0 along the direction of the major harmonics and not significantly larger than −2.0 for all other directions.

On the basis of these differences, it is recommended that caution be exercised in using one-dimensional terrain height spectra to design mesoscale meteorological modelling grids for ordered terrain.

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