Similarity Equations for Wind and Temperature Profiles in the Radix Layer, at the Bottom of the Convective Boundary Layer

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ABSTRACT

In the middle of the convective boundary layer, also known as the mixed layer, is a relatively thick region where wind speed and potential temperature are nearly uniform with height. Below this uniform layer (UL), wind speed decreases to zero at the ground, and potential temperature increases to the surface skin value. This whole region below the UL is called the radix layer (RxL), and is of order hundreds of meters thick. Within the bottom of the RxL lies the classical surface layer (order of tens of meters thick) that obeys traditional Monin-Obukhov similarity theory.

The RxL depth is shown to depend on friction velocity, Deardorff velocity, and boundary layer depth. The wind RxL is usually thicker than the temperature RxL. Using RxL depth, UL wind speed, and UL potential temperature as length, velocity, and temperature scales, respectively, one can form dimensionless heights, velocities, and temperatures. When observations obtained within the RxL are plotted in this dimensionless framework, the data collapse into similarity curves. This data collapse is tightly packed for data collected over single-location homogeneous surfaces, and shows more scatter for data collected along 72-km flight tracks over heterogeneous surfaces. Empirical profile equations are proposed to describe this RxL similarity. When these profile equations are combined with the flux equations from convective transport theory, the results are new flux–profile equations for a deep region within the bottom of the convective boundary layer.

These RxL profile similarity equations are calibrated using data from four sites with different roughnesses: Minnesota, BLX96-Lamont, BLX96-Meeker, and BLX96-Winfield. The empirical parameters are found to be invariant from site to site, except for the profile shape parameter for wind speed. This parameter is found to depend on standard deviation of terrain elevation, rather than on the aerodynamic roughness length. The resulting profile equations could be useful for calculating wind loading on bridges, wind turbine power estimation, air pollutant transport, or other applications where wind speeds or temperatures are needed over the bottom hundreds of meters of the convective boundary layer.

1. Introduction

The term mixed layer (ML) is used in this paper to represent the whole convective boundary layer that is nonlocally statically unstable (Stull 1991), and which is undergoing vigorous convective overturning associated with coherent thermals rising from the warm underlying surface. Within this ML, winds are zero near the ground (Fig. 1a) and smoothly increase with height until finally becoming tangent to a vertically uniform, subgeostrophic, wind speed layer in the mid-ML (Santoso and Stull 1998a). Across the top of the ML, the winds increase to their nearly geostrophic magnitudes above. Analogous profiles exist for potential temperature (Fig. 1b).

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Above the UL is the entrainment zone, a transition layer between the UL and the nearly geostrophic free atmosphere above. Within the entrainment zone are subadiabatic temperature profiles, overshooting thermals, intermittent turbulence, and wind shear (Deardorff et al. 1980). Both free convection and entrainment scales are important here (Sorbjan 1999). Because of the intermittency, the average top ($z_i$) of the ML is near the middle of the entrainment zone.

At the very bottom of the ML is the surface layer (SL), the nearly constant flux region where Monin-Obukhov (MO) similarity theory applies (Businger et al. 1971; Dyer 1974). In this layer the wind profile is nearly logarithmic with height ($z$) and is dominated by mechanically generated small-eddy turbulence within the wall shear flow (Stull 1997a). The dominant SL scales required for similarity of mean profiles (rather than for similarity of mean gradients) are aerodynamic roughness length ($z_0$), friction velocity ($u^*$), and Obukhov length ($L$):

$$L = -\frac{u^*_0}{\frac{g}{T_v} \frac{\partial \theta^*}{\partial z}} = -\frac{1}{k} \left( \frac{u^*_0}{w^*_0} \right)^3 z,$$

where $k \approx 0.4$ is the von Kármán constant.

There is a region or gap between the top of the SL and the bottom of the UL where SL similarity theory was not designed to work, and indeed where it has been shown (Santoso and Stull 1998a) to give rather poor results. In this region, one might expect that both SL and ML scales should be important.

b. The radix layer

To better explain the portion of the boundary layer below the UL, Santoso and Stull (1998a) analyzed data from the 1973 Minnesota field experiment (Izumi and Caughey 1976), and identified a radix layer (RxL) that obeys a similarity scaling different than MO. The word “radix” means “origin” or “root” in Latin, because the roots of convective thermals are in this layer. The new RxL scaling was found to apply to the whole region between the surface and the bottom of the UL, and thus includes the traditional SL as a subdomain. Typical depths of the RxL are on the order hundreds of meters for wind profiles and tens of meters for temperature profiles.

Based on the definitions above there is superposition of layers; namely, the ML contains the RxL as a subdomain, and the RxL contains the SL as a subdomain (Fig. 1). At the top of the RxL, wind speed ($M$) and potential temperature ($\theta$) become tangent to the UL, allowing one to define the top of the RxL as the lowest altitude where $\partial M/\partial z = 0$ or $\partial \theta/\partial z = 0$. Within the RxL, one finds that SL scales decrease in importance with increasing altitude, while ML scales increase.

Using data collected in the Minnesota field experiment, Santoso and Stull (1998a) proposed new similarity equations for wind and potential temperature profiles within and above the RxL. When the dimensionless ratio of wind speed or potential temperature divided by their UL values were plotted against dimensionless ratio of height divided by RxL depth, the data points collapsed quite tightly around the proposed similarity curves. There was also evidence that some of the parameters in these similarity equations were universal.

While these results showed promise for universal sim-
ilarity profiles in the RxL, there were two drawbacks. First, the success of the theory depended on knowledge of the RxL depth, but this depth was difficult to pinpoint from the observations because of the gradual blending of the wind and potential temperature profiles into the UL. Second, the mechanisms that control the RxL depth had not yet been identified in that first paper (Santoso and Stull 1998a).

c. Goals

The purpose of this work is to improve the data-analysis methodology, thereby allowing development of well-defined similarity profiles for the RxL. We utilize Buckingham Pi dimensional analysis to identify the relevant dimensionless groups of variables, and to help find the parameters that control RxL depth. We also employ convective transport theory (CTT; Stull 1994) to relate surface fluxes to ML scales, and combine these with RxL profiles to create new flux-profile relationships.

As motivation for our work, we review in section 2 basic premises and limitations underlying SL similarity theory. Then in section 3 we develop parameterizations for the RxL depths and improve RxL similarity theory for wind and potential temperature profiles. In section 4 we again use data from the Minnesota field experiment to recalibrate our new parameterizations and similarity equations. In section 5, data from Boundary Layer Experiment 1996 (BLX96) is used to determine whether RxL profiles depend on surface conditions such as roughness. In section 6 we compare our results against independent data collected during the Koorin field campaign. A flux-profile relationship is proposed in section 7, followed by summary and recommendations in section 8. Tables of field experiment data and error analyses are included in the appendixes.

2. Basic premises for SL similarity theory

Monin-Obukhov similarity theory has been the favored tool in deriving SL profile equations. The theory is built on the following premises. First, turbulent vertical fluxes are assumed approximately uniform with height in the SL. This simplifies the theory by allowing flux variations to be neglected. It also constrains its applicability to the bottom 10% of the ML, assuming $\bar{u}/\bar{w}$ variations to be neglected. It also constrains its height in the SL. This simplifies the theory by allowing the small eddies to be as small as 1% of $z$, during free convection (Dyer 1967).

Second, turbulence is assumed to be generated predominantly by mechanical shear flow near the ground. Such turbulence is idealized as consisting of small eddies, causing downgradient local transport. These characteristics lead to the widely known “law-of-the-wall” for statically neutral flow, which has a solid theoretical basis and strong support from laboratory experiments. For slightly nonneutral conditions, small eddies can still dominate at heights less than $|L|$, which is often used to define the top of the SL within which MO theory is valid.

A third premise, although rarely acknowledged, is that feedback exists between the mean flow and turbulence within the SL. Namely, 1) turbulence transports momentum, 2) momentum-flux divergence alters the mean-wind profile, and 3) shear in the mean-wind profile generates the small-eddy turbulence that feeds back to (1). Thus, this feedback loop is closed for shear-driven SLs: namely, at heights less than $|L|$. However, there is no reason why MO similarity theory should work for the strongly convective ML at heights above $|L|$, because small-size eddies are less important there. Also, the feedback loop is broken for that situation (Stull 1997a). Namely, 1) turbulence still transports momentum and 2) momentum-flux divergence modifies the mean wind profile. However, the mean wind profile does not generate large-eddy turbulence. Instead, 3) at heights above $|L|$, turbulence is driven by convective instabilities caused by the warm underlying surface. In the limit of strong surface heat flux and very weak mean winds (i.e., $as u/w$ becomes very small), the Obukhov length becomes very small (see appendixes A and B), resulting in an extremely shallow SL. For the BLX96 field experiment described later in this paper, we find that SL similarity theory is valid only for the bottom 0.2%–2% of the ML, in agreement with Dyer’s (1967) findings.

While MO similarity theory identifies the dimensionless groups that are relevant in the SL (e.g., $z/\nu, z/\zeta, L$), no form of dimensional analysis is able to give the relationships between these groups. Such relationships must be estimated from field experiments. For convective conditions, empirical SL relationships have been suggested by Swinbank (1968), Zilitinkevich and Chalikov (1968), Businger et al. (1971), Dyer (1974), Dyer and Bradley (1982), Foken and Skeib (1983), Högström (1988), Kader and Perpeckin (1989), Kader and Yaglom (1990), Frenzen and Vogel (1992), Brutsaert (1999), and others.

All the premises listed earlier in this subsection imply that the MO similarity theory is strongly dependent on surface characteristics, but is virtually independent of factors higher in the ML. For this reason one cannot expect the empirical SL equations to merge smoothly into the UL, because no information about the UL is considered in those equations. As pointed out by Panofsky (1978), convective-matching-layer and free-convective-layer formulations (Priestley 1955; Kaimal et al. 1976) fail near the bottom of the UL, where the shear and potential-temperature gradient approach zero.

The limited altitude range of applicability of traditional MO SL similarity was illustrated by Santos and Stull (1998a), who tested the Businger et al. (1971)–Dyer (1974) similarity equations against Minnesota field experiment data. In the SL, all the data collapsed to a
single curve when plotted as dimensionless groups based on SL similarity. However, at higher altitudes the data scattered away from a universal curve. Namely, MO similarity theory works well in the SL, but is less successful higher in the RxL and in the UL. It is obviously desirable to find a new similarity theory that can be applied over a greater depth. One approach is to include $z_{\text{UL}}/L$ into MO SL profile theories, such as described by Khanna and Brassier (1997, 1998) and Johansson et al. (2001). An alternative approach, taken here, is to refine a RxL similarity theory that is not based on MO SL similarity.

3. RxL theory

a. Review of previous work

1) RxL profile equations

The first step in any similarity analysis is to hypothesize which variables are relevant to the physics. Because thermal structures exist within the whole ML including the SL, we can infer that similarity theory for RxL should depend on both SL and ML parameters. For wind and potential temperature profiles, Santos and Stull (1998a) hypothesized that RxL depths ($z_{\text{RM}}$ and $z_{\text{R,0}}$, for wind and potential temperature, respectively) are the relevant height scales, and that the winds and potential temperature in the UL ($\overline{M}_{\text{UL}}$ and $\overline{\theta}_{\text{UL}}$) are the relevant velocity and temperature scales. Additional constraints are that the partial derivatives of the mean variables with respect to height are zero at the top of (and above) the RxL, and mean variables in the UL are constant with height.

Based on Minnesota data, the following empirical relations (Santoso and Stull 1998a) were suggested to describe the vertical profiles of mean wind speed $\overline{M}$ and potential temperature $\overline{\theta}$ within and above the RxL.

\[
\overline{M}(z) = \begin{cases} 
\overline{M}_{\text{UL}} \left( \frac{z}{z_{\text{RM}}} \right)^{A_1} \exp \left[ A_2 \left( 1 - \frac{z}{z_{\text{RM}}} \right) \right] & \text{for } z \leq z_{\text{RM}} \\
\overline{M}_{\text{UL}} & \text{for } z > z_{\text{RM}} 
\end{cases}
\]

\[
\overline{\theta}(z) - \overline{\theta}_{\text{UL}} = \begin{cases} 
\left( \overline{\theta}_0 - \overline{\theta}_{\text{UL}} \right) \left( 1 - \left( \frac{z}{z_{\text{R,0}}} \right)^{A_1} \exp \left[ A_2 \left( 1 - \frac{z}{z_{\text{R,0}}} \right) \right] \right) & \text{for } z \leq z_{\text{R,0}} \\
0 & \text{for } z > z_{\text{R,0}} 
\end{cases}
\]

where $\overline{\theta}_0$ is potential temperature near the surface; overbars represent a horizontal-average ergodic approximation to the ensemble average; and $A_1$ and $A_2$ are empirical constants.

Both sets of equations above satisfy the desired zero gradient at the top of (and above) the RxL (Arya 1999; Santos and Stull 1999a). Both sets also show that the mean profiles and the vertical gradients are continuous and smoothly merge at the top of RxL. Namely, the RxL profiles are tangent to the UL at finite height as observed, rather than asymptotically approaching the UL at infinite height, or rather than crossing the UL at an arbitrary matching height (Panofsky 1978).

A nonlinear regression (Bevington 1969; Press et al. 1992) was used to determine best-fit parameters $\overline{M}_{\text{UL}}$, $z_{\text{RM}}$, and $A_1$ for wind; and $\overline{\theta}_0$, $\overline{\theta}_{\text{UL}}$, $z_{\text{R,0}}$, and $A_2$ for potential temperature, for each dataset from the Minnesota experiment. An iterative process was used to minimize the sum of squared deviations between the regression equation (RxL and UL taken together) and the data. Among best-fit parameters, $\overline{M}_{\text{UL}}$, $z_{\text{R,0}}$ and $\overline{\theta}_{\text{UL}}$ were found to be the most statistically robust, having least root-mean-squared (rms) errors. The RxL depths, $z_{\text{RM}}$ and $z_{\text{R,0}}$ calculated individually using this method (with zero vertical gradient as the desired constraint) were very sensitive to even small errors in measured data. With such high uncertainties, the attempted parameterizations of the RxL depths as a function of SL and ML variables were less successful.

2) Convective transport theory (CTT)

Thermal diameters are sufficiently large (order of the mixed layer depth of 1–2 km) that there is a very large central core in each thermal that is protected from small-eddy lateral entrainment (Crum et al. 1987). Within this core, air parcels from near the surface are moved to the middle and the top of the convective boundary layer (BL) with virtually no dilution (Stull and Eloranta 1985). Because the buoyant thermals are anisotropic with more energy in the vertical, they can efficiently transport heat, momentum, and moisture vertically away from the surface.

Stull (1994, 1997b) utilized these characteristics of thermals to show that during free-convective conditions the surface vertical turbulent flux $\overline{w^* \Psi'}$, of any mean variable $\Psi$ is proportional to $w^*$ times $\Delta \overline{\Psi}$, the mag-
nitude of the difference of $\psi$ between the surface skin and the UL:

$$w'\psi' = C_\psi w_\psi \Delta \bar{\psi},$$  \hfill (4)$$
where $C_\psi$ is an empirical mixed layer transport coefficient. In the notation of this paper $\Delta \bar{\psi} = \bar{\psi}_{\text{skin}} - \bar{\psi}_{\text{UL}}$, and $\Delta M = \bar{M}_{\text{skin}} - \bar{M}_{\text{UL}} = -\bar{M}_{\text{UL}}$, because $\bar{M}_{\text{skin}} = 0$ by definition. The CTT concept was found applicable for many different sites (Kustas et al. 1996; Greisberger and Stull 1999), particularly when one accounts for the variation of $C_\psi$ (Santoso and Stull 1998b).

b. New RxL depth equations

To overcome the sensitivity problem of RxL depths, we now reverse our steps. First we parameterize the RxL depths as functions of both ML and SL scales, because of the superposition of large buoyantly driven and small shear-driven eddies in the RxL. The large eddies scale to $z_\text{R}$ and $w_\text{s}$ (Deardorff 1970, 1972; Deardorff et al. 1980; Wyngaard et al. 1971; Kaimal et al. 1976; Sorbjan 1986; Stull 1988). The small eddies scale to the RxL depth ($z_{\text{RM}}$ and $z_{\text{RS}}$) and $u_\text{s}$. The small-eddy length scale of RxL depth was chosen because at the bottom of the ML the shear needed to generate shear-driven eddies exists only within the RxL. That is, mean-shear-driven eddies exist everywhere within the RxL, they dominate only in the SL, and they do not exist in the UL.

Using Buckingham Pi analysis (Stull 1988) with this set of scales $(z_\text{R}, z_\text{i}, u_\text{R}, w_\text{R})$ yields RxL dimensionless groups of $z_\text{RM}, z_\text{RS}$, and $u_\text{s}/w_\text{s}$, where we use $z_\text{R}$ as a generic RxL depth. The relationship between these groups is not given by Buckingham Pi analysis; it must be found empirically. The relationship that was found to work best for wind is

$$z_{\text{RM}} = E_\text{M} \left( \frac{u_\text{R}}{w_\text{R}} \right)^{B_\text{M}} z_\text{i},$$  \hfill (5a)$$
where $B_\text{M}$ and $E_\text{M}$ are empirical constants, and the subscript $M$ denotes wind speed. For potential temperature (subscript $\theta$) the corresponding relationship is

$$z_{\text{R}\theta} = E_\text{M} \left( \frac{u_\text{R}}{w_\text{R}} \right)^{B_\text{M}} z_\text{i},$$  \hfill (5b)$$
where $B_\theta$ and $E_\theta$ are empirical constants that are not necessarily the same as those for momentum. Relationships (5) agree with the suggestions of Plate (1997) that the altitude of the base of the UL should be proportional to $u_\text{s}/w_\text{s}$. The adequacy of (5) will be tested with Minnesota data in section 4.

These parameterizations of the RxL depths have a functional form similar to that of the Obukhov length (2), except that the exponents might be found to differ. We will show in section 4 that there is a simple relationship between RxL depth, Obukhov length, and ML depth.

c. Revised RxL profile equations

Based on the arguments above we propose new empirical equations for wind and potential temperature profiles in the RxL ($z \leq z_\text{R}$) and UL ($z > z_\text{R}$):

$$\frac{M(z)}{M_{\text{UL}}} = F(z)$$  \hfill (6a)$$
$$\frac{\bar{\psi}(z) - \bar{\psi}_{\text{UL}}}{\bar{\psi}_{\text{skin}} - \bar{\psi}_{\text{UL}}} = 1 - F(z),$$  \hfill (6b)$$
where $F(z)$ is a function of the form

$$F(z) = \frac{\left( \frac{z}{z_\text{R}} \right)^A \exp[A \left( 1 - \left( \frac{z}{z_\text{R}} \right)^2 \right)]}{1} \quad \text{for } z \leq z_\text{R},$$
$$\frac{z}{z_\text{R}} \quad \text{for } z > z_\text{R},$$  \hfill (7)$$
and $A$ and $D$ are empirical profile-curvature parameters. Based on CTT, we use $\bar{\psi}_{\text{skin}}$ in (6) instead of $\bar{\psi}_{\theta}$ from (3).

We use two shape parameters $(A, D)$ in (7) instead of only one $(A)$ in (3) because we found the profile shape to depend on both meteorological and surface characteristics. The formulation of (7) is designed to allow separation of these two effects, with $D$ depending on surface characteristics and $A$ on meteorological characteristics. The net result is that the exponential portion of (7) contributes more than it did in (3), relative to the “power law” portions of those equations.

Substituting (5) into (7) yields a new functional equation for the wind and potential temperature profiles

$$F(z) = \frac{(\xi_{\text{R}}^A \exp[A(1 - \xi_{\theta}^2)])}{1} \quad \text{in the RxL}$$
$$\frac{1}{\xi_{\theta}} \quad \text{in the UL}$$  \hfill (8)$$
where the dimensionless height is

$$\xi_{\text{R}} = \frac{1}{E_\text{M} \bar{M}_{\text{UL}}} \left( \frac{w_\text{R}}{u_\text{R}} \right)^B \left( \frac{z}{z_\text{R}} \right),$$
$$\xi_{\theta} = \frac{1}{E_\theta \bar{M}_{\text{UL}}} \left( \frac{w_\text{R}}{u_\text{R}} \right)^B \left( \frac{z}{z_\text{R}} \right)\left( \frac{z}{z_\text{R}} \right)$$

and where $(z_\text{R}, A, B, D, E, \xi_{\text{R}}) = (z_{\text{RM}}, A_M, B_M, D_M, E_M, \xi_{\text{RM}})$ for momentum, and $(z_\theta, A, B, D, E, \xi_{\text{R}}) = (z_{\text{RM}}, A_M, B_M, D_M, E_M, \xi_{\text{RM}})$ for potential temperature. As will be shown in section 6, all heights including the RxL depth should be measured from a datum defined as surface elevation plus displacement distance. That is, over dense forest canopies and other regions of significant displacement distance $z_\text{R}$ where the effective aerodynamic surface is above the physical surface, replace all heights $z$ in Eqs. (5)–(9) with $z_\text{R} - z_\text{R}$, where $z_\text{R}$ is height above the physical surface.

These revised profile equations [(6), (8), (9)] identically satisfy the zero-gradient condition at the top of the RxL for any values of the empirical parameters. The resulting mean profiles and vertical gradients in the RxL are continuous, and smoothly merge into the overlying UL. Also, the new profile equation is not an explicit
function of RxL depth, thereby eliminating a problem area of the previous parameterization. As will be shown later, an iterative-graphical procedure can be used to simultaneously solve for those B, E, and \( z_b \) values that allow the field-experiment data to collapse about a common profile curve with minimum scatter.

The equations for the RxL depth (5) and the dimensionless similarity equations for mean profiles [(6), (8), (9)] are the basis for the RxL theory as used in the remainder of this paper.

d. Comparison with traditional power laws

The traditional surface-layer power-law (SLPL) wind profile is \( \left( M/M_{10} \right) = \left( z/z_{10} \right)^p \), where \( M_{10} \) is the wind speed at reference height \( z_{10} \) (\( z_{10} = 10 \text{ m} \) for standard surface observations), and \( p \) is the power (Pasquill and Smith 1983; Panofsky and Dutton 1984; Arya 1988). The empirical parameters are \( M_{10} \), \( z_{10} \), and \( p \). This profile equation and its variations have been used extensively in fluid mechanics as well as in air pollution meteorology and wind engineering. Based on observed wind profiles, \( p \) is usually found to be a function of static stability and surface roughness. Such SLPLs have been found to represent the observed wind profiles over a greater depth than the Monin–Obukhov SL similarity profiles (Arya 1999). However, these SLPLs have not had the strong theoretical underpinning associated with the MO similarity theory.

There are four main differences between traditional SLPL and the new RxL profiles. First, the RxL profile includes the product of an exponential function times the power function of height. The exponential contribution is required to make the profile become tangent to the UL at the top of the RxL, as is observed (Santoso and Stull 1999a). Second, the RxL reference height is at the base of the UL (in the mid ML), rather than at 10 m in the SL. Thus, the RxL approach incorporates ML forcings into the parameterization. Third, RxL includes surface skin effects, because surface forcings are another important contributor to profile processes. Fourth, a limitation of RxL is that it is designed only for the statically unstable boundary layer, while traditional power laws have been designed for a wide range of static stabilities. Finally, both approaches include the effect of aerodynamic roughness, within the parameter \( p \) for SLPLs, or within the parameter \( C_b \) for RxL profiles (Santoso and Stull 1999a).

4. Calibration of empirical parameters

The parameters in any similarity theory must be determined empirically. As discussed in detail in the first RxL paper (Santoso and Stull 1998a), calibration of RxL parameters requires statistically robust wind and potential temperature data that are consistent and contiguous over a large range of heights from near the surface through the interior of the ML. One cannot use data that have profile gaps and mismatches, such as are typical of field experiments where instantaneous rawinsonde observations in the middle of the ML are combined with time-averaged observations from instrumented towers in the SL.

We use data from three field sites that satisfy the required statistical robustness. Data from one of the sites, Minnesota, is used in this section to find (i.e., calibrate) the RxL empirical parameters of Eqs. (5), (6), (8), and (9). In later sections, we determine the terrain roughness dependence of the parameters using BLX96 field data, and finally we compare the parameters with independent data from the Koorin field campaign. Although not used in the present study, valid data for testing RxL theory could also come from very tall (>300 m) towers, such as the Boulder Atmospheric Observatory in the USA, and the Cabauw tower in the Netherlands.

a. The Minnesota site

The Minnesota site was located at 48°34′N, and 95°51′W with elevation of 255 m above mean sea level (MSL). It was a flat, recently harvested, and plowed farm “square-mile section” (1.609 km on each side) with no aerodynamically significant vegetation close by. A uniform fetch of 10 km existed to the north, which was the predominant wind direction. Details about the Minnesota field experiment, including a description of the site, instruments, and experimental procedures, can be found in Izumi and Caughey (1976) and Kaimal et al. (1976). This ideal site is virtually horizontally homogeneous.

Profile robustness was achieved in the Minnesota field experiment by locating sensors at fixed heights on both a 32-m tower and along the tether of a tethered balloon and then averaging over 75-min at each height. The resulting profiles were statistically consistent, smooth, and contiguous. There were 11 runs that were obtained during the Minnesota experiments. Table A1 in appendix A lists key dates, times, and the scales from the Minnesota experiment that are important for the RxL analysis.

There were no measurements of surface skin temperatures during the Minnesota experiment. However, we can estimate them indirectly using the improved CTT equation for surface heat flux \( \bar{w} \bar{\theta}^* \), as proposed by Santoso and Stull (1998b):

\[
\bar{w} \bar{\theta}^* = \bar{w} \bar{\theta}^*_0 + C_{wH} \Delta \bar{\theta}_s
\]

where the heat-flux intercept parameter is \( \bar{w} \bar{\theta}^*_0 = 0.022 \text{ km s}^{-1} \) and the ML transport coefficient for heat is \( C_{wH} = 0.0039 \). One can rearrange (10) to solve for \( \Delta \bar{\theta} = \bar{\theta}_{\text{skin}} - \bar{\theta}_{\text{UL}} \), which can be used directly in the denominator of (6b).
Wind speed and potential temperature profiles that plagued the original RxL paper. A similar approach using \( \frac{(\bar{\theta}(z) - \bar{\theta}_{UL})(\bar{\theta}_{air} - \bar{\theta}_{UL})}{\zeta_{u}} \) versus \( \zeta_{u} \) allows one to find \( B \) and \( E \) for temperature.

The end result is shown by the data points in Fig. 2 for all 11 runs of the Minnesota experiment (ignore the solid line for now). The figure also includes plots in semi-log form to show more detail near the surface. The resulting best-fit parameter values (Table 1) are \( E_{u} = 1/2(\pm0.061) \) and \( B_{M} = 3/4(\pm0.007) \) for wind, and \( E_{u} = 1/7(\pm0.036) \) and \( B_{\theta} = 3/4(\pm0.005) \) for potential temperature, with \( \pm \) error given as standard deviations as discussed in appendix C. Because \( B \) for momentum and temperature are identical and equal to 3/4, we will use the symbol \( B = 3/4 \) without subscript, and will test later whether this value for \( B \) can be considered “universal.”

Next, knowing constants \( E \) and \( B \), the RxL depths for wind and temperature can be calculated using (5) for each Minnesota run. Once the RxL depth is known, then those data points above the RxL depth can be averaged for momentum and temperature, with \( \pm \) error given as standard deviations as discussed in appendix C. Because \( B \) for momentum and temperature are identical and equal to 3/4, we will use the symbol \( B = 3/4 \) without subscript, and will test later whether this value for \( B \) can be considered “universal.”

To get \( A \) and \( D \), a nonlinear regression is applied to minimize the sum of squared errors between the regression equations (6), (8), and (9) and the data, with the RxL and UL taken together when calculating squared errors. In Fig. 2, this corresponds to finding the solid line that best fits the cloud of data points. This procedure is done separately for wind and temperature. Using all 11 Minnesota runs, one finds that \( A_{M} = 1/4(\pm0.017) \) and \( D_{M} = 1/2(\pm0.06) \) for the wind profile, and \( A_{\theta} = 1/2(\pm0.01) \) and \( D_{\theta} = 1/5(\pm0.038) \) for the potential temperature profile. The lines plotted in Fig. 2 show the resulting best-fit profile equations [(6), (8), (9)].

In Fig. 2 the potential temperature data points are packed around the best-fit curves quite tightly. The wind speed data points also pack tightly around the best-fit curves.

![Fig. 2](image.png)

**Fig. 2.** (a, b) Wind speed \( M \) and (c, d) potential temperature \( \theta \) profiles for the Minnesota field experiment, plotted in dimensionless form using radix layer similarity theory. (a) and (c) Linear axes to focus on the UL and top of the RxL; (b) and (d) Semilog axes to focus on the middle and bottom of the RxL.

### b. Methodology—Separation of parameters

Parameters \( A \) and \( D \) appear only in the profile shape equation (8), while parameters \( B \) and \( E \) appear in the dimensionless height \( \zeta_{u} \) definition (9). This separation of parameters allows one to empirically solve for the best-fit values of \( B \) and \( E \) first, independent of the as-yet-unknown values of \( A \) and \( D \). Such an approach overcomes the sensitivity problem in calculating the RxL depths that plagued the original RxL paper.

To get \( B \) and \( E \) for momentum, first estimate the values of wind speed in the UL. Then, plot the observed data in the form of dimensionless wind \( \frac{\bar{M}(z)}{\bar{M}_{UL}} \) against dimensionless height \( \zeta_{u} \). The result will be a cloud of points clustered along a vertical profile. An iterative approach is then used to vary parameters \( B \) and \( E \) until the cloud of points exhibits the tightest packing. Such an approach overfits the data points above the RxL depth can be averaged for momentum and temperature, with \( \pm \) error given as standard deviations as discussed in appendix C. Because \( B \) for momentum and temperature are identical and equal to 3/4, we will use the symbol \( B = 3/4 \) without subscript, and will test later whether this value for \( B \) can be considered “universal.”

Next, knowing constants \( E \) and \( B \), the RxL depths for wind and temperature can be calculated using (5) for each Minnesota run. Once the RxL depth is known, then those data points above the RxL depth can be averaged for momentum and temperature, with \( \pm \) error given as standard deviations as discussed in appendix C. Because \( B \) for momentum and temperature are identical and equal to 3/4, we will use the symbol \( B = 3/4 \) without subscript, and will test later whether this value for \( B \) can be considered “universal.”

To get \( A \) and \( D \), a nonlinear regression is applied to minimize the sum of squared errors between the regression equations (6), (8), and (9) and the data, with the RxL and UL taken together when calculating squared errors. In Fig. 2, this corresponds to finding the solid line that best fits the cloud of data points. This procedure is done separately for wind and temperature. Using all 11 Minnesota runs, one finds that \( A_{M} = 1/4(\pm0.017) \) and \( D_{M} = 1/2(\pm0.06) \) for the wind profile, and \( A_{\theta} = 1/2(\pm0.01) \) and \( D_{\theta} = 1/5(\pm0.038) \) for the potential temperature profile. The lines plotted in Fig. 2 show the resulting best-fit profile equations [(6), (8), (9)].

In Fig. 2 the potential temperature data points are packed around the best-fit curves quite tightly. The wind speed data points also pack tightly around the best-fit curves.

### Table 1. Best-fit radix-layer parameter values for the homogeneous terrain of Minnesota and Koorin, with the \( \pm \) errors given as standard deviations. For the heterogeneous terrain of BLX96, the \( \pm \) errors are roughly 5 to 10 times larger for wind parameters, and 2 times larger for temperature parameters (see error values given in section 5c). The standard deviation of terrain elevation is \( \sigma_{z} \).

<table>
<thead>
<tr>
<th>Radix parameter (dimensionless)</th>
<th>A</th>
<th>B</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential temperature</td>
<td>0.50 (\pm0.01)</td>
<td>0.75 (\pm0.005)</td>
<td>0.20 (\pm0.038)</td>
<td>0.143 (\pm0.036)</td>
</tr>
<tr>
<td>Wind speed</td>
<td>0.25 (\pm0.017)</td>
<td>0.75 (\pm0.007)</td>
<td>( D_{M} = a + b \cdot \sigma_{z} )</td>
<td>0.50 (\pm0.061)</td>
</tr>
</tbody>
</table>

For \( a = 0.35 (\pm0.10) \) and \( b = 0.018 (\pm0.004) \) m\(^{-1}\).
curves, except near the bend in the line where the best-fit curve is slightly to the right of the data in the top half of the RxL. Namely, the RxL similarity winds are too fast by 3% for heights of $0.5 < \zeta_M < 1$. This range of heights is roughly 3–50 times the value of $|L|$, a region where MO similarity theory gives a much poorer fit to the data, as demonstrated by Santosos and Stull (1998a). For the Koorin field experiment data shown later, the best-fit curve is slightly off in the other direction (4% too slow) in the top half of the RxL. Further improvements to the curve fit in this region might be possible with different profile equations (8), or better estimates of displacement distance (see section 6).

The statistical spreads of the data points from their similarity curves, measured by the standard deviations of nondimensional wind and potential temperature, are $\sigma_{UL} = 0.028$ (dimensionless), and $\sigma_{UL} = 0.013$ (dimensionless), respectively. These statistics were used for error propagation calculations in appendix C to estimate the error bounds of the similarity parameters $A, B, D,$ and $E$ that were presented in Table 1.

The Minnesota field experiment data do not allow us to test whether the RxL profiles vary with surface roughness length because all the observations were made at one location. Surface-roughness effects will be examined in section 5 using data from the BLX96 experiment, which was conducted over three sites with different land surface characteristics.

c. Relationship between RxL depth, ML depth, and Obukhov length

Because the Obukhov length and RxL depth are both boundary layer length scales, it is instructive to compare them. Combining the definition for Obukhov length (2) with the definitions for RxL depth (5), and using the best-fit parameters found above, one finds the following relationship between RxL depth and Obukhov length for the statically unstable boundary layer:

$$z_R = E\left[k(\zeta^2)\right]^{1/4},$$

(11a)

where $(z_0, E)$ is $(z_{BL}, E_{BL})$ for wind speed, and $(z_{BL}, E)$ for potential temperature. The equation above shows that RxL depth is more strongly related to $z_\theta$ than to $L$. The equation can also be rewritten in nondimensional form $z_R/z_i$ to show the relationship with the convective boundary layer stability parameter $-z_\theta/L$:

$$\frac{z_R}{z_i} = E \cdot k^{1/4} \left(\frac{-z_\theta}{L}\right)^{-1/4}.$$  

(11b)

The Obukhov length is more strongly dependent on the ratio of $u_*/w_*$ than the RxL depth. Similar to Obukhov depth, the RxL depths are indirectly related to surface roughness length $z_\theta$ via the friction velocity $u_*$. Table A2 in appendix A includes the RxL depth, the Obukhov length $L$, and $0.1z_i$ for comparison of the various length scales. Plots of Eq. (11b) for each experimental run are given in Fig. A1 in appendix A, to illustrate the robustness of this relationship.

5. Dependence on terrain roughness

To determine whether surface conditions such as aerodynamic roughness $z_\theta$ or topography influence the RxL profiles and depths, data from Boundary-Layer Experiment 1996 (BLX96) is analyzed next. Components of the BLX96 experiment were designed specifically to examine this issue.

a. Site characteristics

During BLX96 the University of Wyoming King Air aircraft instrumented with boundary layer/turbulence sensors was flown over three different tracks in Oklahoma and Kansas having different land use and surface roughness. The three flight tracks were named after nearby villages: Lamont and Meeker (in Oklahoma) and Winfield (in Kansas). Relatively flat topography, large areas of uniform land use, and frequent fair weather motivated the selection of the field site. A total of 12 good flights were made between 15 July and 13 August 1996.

The Lamont flight track was predominantly over crop land. Vegetation coverage consisted of 60%–80% wheat fields, 40%–20% pasture, and a small number of trees less than 10 m tall. Roughly 40% of the cultivated fields were recently plowed at the time of the experiment, leaving reddish-brown bare soil. Terrain under the track was quite flat, but gently rising to the west, with elevations ranging from 320 to 425 m MSL. An estimate of average aerodynamic roughness based on averaged direct calculations using Monin-Obukhov profile equations and tables of roughness classification (Smedman-Högström and Högström 1978; Stull 1988; Wieringa 1980, 1986) was $z_\theta = 0.1$ m.

The Meeker track was over a region mainly covered by forest. Vegetation coverage consisted of 40%–50% pasture, 40%–60% wooded areas with trees less than 10-m tall, and 10%–30% cropland. The track had some small rolling hills that ranged from 40 to 60 m in height. The remaining terrain was relatively flat, with elevations from east to west ranging from 250 to 280 m MSL. The average aerodynamic roughness length was $z_\theta = 1.4$ m.

The Winfield track was primarily over pasture land. Vegetation coverage consisted of 30%–60% pasture, 10%–50% forested areas, with trees less than 10 m tall and more wooded at west end, and the remaining area was cultivated. Terrain was rising to the west, with elevations ranging from 250 to 400 m MSL. Small hills near the center of the track ranged from 70 to 100 m in height. The average aerodynamic roughness length was $z_\theta = 0.9$ m.

A more detailed description of the goals, sites, instruments, and experimental procedures of BLX96 can
be found in Stull et al. (1997) and Berg et al. (1997). Tables of BLX96 surface fluxes, boundary layer depth, and scaling parameters are given by Santoso and Stull (1998b).

b. Field experiment procedure and instrumentation

To investigate the RxL, a vertical zigzag flight pattern was flown over a 70-km horizontal distance to provide vertical profiles of horizontal mean wind, temperature, and humidity between altitudes of about 10 and 700 m above ground level (AGL). This spans the SL, the RxL, and the bottom of the UL. This zigzag flight pattern was selected as being the most likely to give statistically robust data in a heterogeneous, nonstationary boundary layer, based on earlier trials made by flying a virtual aircraft through synthetic boundary-layer turbulence (Santoso and Stull 1999b).

To get vertical profiles, the data are sorted by altitude into nonoverlapping bins of 2-m vertical depth. The average value within each bin is assigned to a height at the bin center. The resulting wind and temperature data show some scatter, but the large number of data points helps improve the statistical robustness of the sample. As an example, Fig. 3 shows plots of mean wind and potential temperature profiles for a single zigzag flight labeled as leg SS, obtained at the Meeker site on 2 August 1996. Each plotted data point at bin center is based on an average of at least 50 observations.

The lowest safe altitude that was reached during the flight was dependent on terrain conditions and obstructions near the surface. For the Lamont track, which was relatively flat with bare soil, most vertical zigzag flights reached altitudes lower than 10 m AGL. For the Meeker and Winfield tracks, which had more complicated terrain conditions and obstructions, the lowest altitudes were a bit higher than 10 m AGL, which was just above canopy heights.

A downward-looking Heiman pyranometer on the aircraft was used to get surface skin temperatures ($T_{\text{skin}}$) averaged along the flight track during zigzag legs. An adjustment of 0.1 K per 100 m between aircraft and surface was added to the measured $T_{\text{skin}}$ value to compensate for partial absorption of surface radiation by moisture in the atmosphere, and contamination by atmospheric reemission at a lower air temperature (Perry and Moran 1994; Greischar and Stull 1999). Because the other potential temperatures were defined with respect to a reference pressure of 100 kPa, $T_{\text{skin}}$ was also converted to a potential temperature ($\theta_{\text{skin}}$) for the same reference pressure.

To find the ML depth $z_*$, higher ascent/descent soundings were flown at the beginning, middle, and end of each of the 4-h flights. These were interpolated to the midtimes of the zigzag flights to account for the ML nonstationarity. Heat flux, buoyancy flux, and Deardorff velocity were calculated iteratively using (1), (10), and the following buoyancy flux equation $w^9\theta_{*}^9 = w^9\theta_s^9(1 + 0.61r) + 0.61\theta^9\bar{w}\bar{r}^9$, where $r$ is mixing ratio and $w^9\bar{r}^9$ is surface kinematic moisture flux.

Momentum flux magnitude (equal to the square of the friction velocity $u_*$) was estimated using CTT:

$$u_*^2 = C_{**}w_9M_{UL},$$

where the ML transport coefficients for momentum flux $C_{**}$ at Lamont, Winfield, and Meeker are 0.019, 0.028, and 0.040, respectively, as reported by Santoso and Stull (1998b) for BLX96. These coefficients were found to be dependent on surface roughness length. Table B1 in appendix B lists flight tracks, dates, midtimes, and the boundary layer scales for all the vertical zigzag legs.

c. RxL parameters for BLX96

Using information listed in Table B1 in appendix B, the depths of the RxL for wind speed and potential temperature are estimated using (5) for each leg. Because these RxL depths define the location of the bottom of the UL, the mean wind speed and potential temperature in the UL are determined next. Finally, these are all used to give dimensionless profiles of mean wind speed and potential temperature: $\overline{M(z)M_{UL}}$ versus $\zeta_{LM}$ and $(\overline{\theta(z) - \theta_{UL}})(\overline{\theta_{\text{skin}} - \theta_{UL}})$ versus $\zeta_{L\theta}$. Wind data from the Meeker 28 July leg AA were excluded from the Meeker plot, because wind was so slow (about 1.5 m s$^{-1}$) that $M_{UL}$ was of the same order as $w_*$, resulting in excessive sampling noise. Namely, the mean flow is effectively calm when $M_{UL} < w_*$. The dimensionless profiles for the remaining 18 cases are plotted in Figs. 4–6 for the Lamont, Meeker, and Winfield sites. The height parameters ($z_*, z_{RM}$, and $z_{RO}$) in these plots have been corrected by the displacement height $z_d$ due to pasture, cropland, and forest. As recommended by Garratt (1978), the displacement height...
$z_d$ is taken as 64% of the estimated roughness-element height average. The displacement heights for individual vegetation types (trees, pasture, etc.) were weighted by the relative coverage of those types under each flight path footprint to give flight-leg averaged values. The resulting displacement heights for Lamont, Meeker, and Winfield are 0.3, 2.7, and 1.8 m, respectively.

Next, the parameters $B$ and $E$ are calculated to determine if they are significantly different from those for Minnesota. As shown in Table 1 and appendix C, error bounds on the $B$ and $E$ parameters for the heterogeneous surfaces under the 72-km-long flight tracks of BLX96 are 5 to 10 times greater than for the homogeneous terrain of Minnesota. Within the scatter of these BLX96 datasets, we are unable to discern a significant difference from the Minnesota values. We hypothesize that $B$ and $E$ might be universal constants rather than parameters.

Table B2 in appendix B lists calculated RxL depths for wind and potential temperature for each leg of BLX96. Again, we include the traditional length scales of Obukhov length $L$ and $0.1z_i$ for comparison to the calculated RxL depths.

Because parameters $B$ and $E$ for the three BLX96 sites do not vary significantly from those of Minnesota, we wondered if parameters $A$ and $D$ are also constant. The RxL wind profile equations using the Minnesota parameter values of $A$ and $D$ are plotted in Figs. 4–6 as the dashed lines, along with the BLX96 data points. The solid lines in these figures represent the RxL profile using parameters that are the best fit for the BLX96 data using direct nonlinear regression. For potential temperature, the best-fit RxL parameters are $A_u = 0.50 \pm 0.01$ for Minnesota, $D_p = 0.199 \pm 0.051$, $0.198 \pm 0.060$, and $0.203 \pm 0.035$ for BLX96, compared to $D_p = 0.20 \pm 0.038$ for Minnesota. All of these BLX96 parameters yield profile curves that are virtually identical to those using the Minnesota parameters; hence
the dashed line is hidden behind the solid line for the potential temperature figures. Our tentative conclusion based on these two datasets is that $A_u$ and $D_u$ are constant. The scatter of the data points about the best-fit wind and temperature lines for BLX96 have standard deviations of $0.095, 0.095,$ and $0.147$ (dimensionless) and $0.011$ (dimensionless) for the Lamont, Meeker, and Winfield sites.

For wind, the Minnesota parameter values do not provide the best fit to the data for all sites, as exhibited by the difference between dashed and solid lines. Further analysis revealed that these site-to-site profile differences of wind could not be explained by parameter $A_u$. We found $A_M = 0.254 \pm 0.062$, $0.248 \pm 0.062$, and $0.245 \pm 0.017$ for the Lamont, Meeker, and Winfield BLX96 sites, compared to $A_M = 0.25 \pm 0.017$ for Minnesota. Namely, the best-fit $A_M$ value for BLX96 was negligibly different from the best-fit Minnesota value given the error bounds for each experiment, and will be considered a constant here.

For this reason, our investigation focused on the variation of $D_M$ with terrain characteristics. When holding $A_M$ constant, the nonlinear regression best-fit values of $D_M$ for BLX96 are $D_M = 0.62, 0.73,$ and $0.89 \pm 0.06$ for Lamont, Meeker, and Winfield, which are the solid lines plotted in Figs. 4–6. When these values of $D_M$ were plotted against aerodynamic roughness length $z_0$, no significant correlation was found (correlation coefficient $r = 0.24$). When plotted against the displacement height $z_d$, again no significant correlation was found. Based on this limited dataset, we conclude that neither aerodynamic roughness nor displacement height can explain the site-to-site variation of profile curvature (i.e., of parameter $D_M$). So there must be some other characteristic that causes $D_M$ to vary from site to site.

Next we investigated whether larger-scale topographic variability could explain the differences between the sites. Figure 7 shows the plots of surface elevations under the Lamont, Meeker, and Winfield flight tracks, which were flown approximately east–west. By eye, the Lamont surface topography (Fig. 7a) appears relatively smooth and flat and is related to the smallest $D_M$, while Winfield (Fig. 7c) has the roughest topography and is related to the largest $D_M$. These plots suggest that resolvable topographic characteristics might have caused the variation of $D_M$.

To better quantify such a relationship, we analyze the discrete variance (energy) spectrum of the surface topography for each site under the flight track. From horizontal low-level aircraft flights, measurements of aircraft pressure altitudes and radar altitudes AGL sampled at 1 Hz, are used to estimate surface elevations MSL under the aircraft. Taylor’s hypothesis is used with mean aircraft speed (order of 100 m s$^{-1}$) to convert the re-
sulting Fourier spectrum from frequency (after being demeaned and detrended) to wavenumber.

To allow better comparison of the discrete variance (energy) spectrum between sites, we truncate the time series from all sites to be the same length as the shortest time series (still corresponding to roughly 70-km horizontal flight distance) for each flight track. We flew many low-level flights over each site following virtually the same track based on GPS navigation. Our results show that the total spectral energy density (e.g., total variance) from flight to flight over the same site varies over only a small percentage (about 6%) of the total variance, supporting our above assumption. We found the standard deviation of topography elevations for the Lamont, Meeker, and Winfield tracks were $\sigma_z = 12.9$, 16.8, and 30.2 m, respectively. These are plotted as the (○) data points in Fig. 8.

Also in Fig. 8 is the corresponding data point (×) for the Minnesota field experiment. This experiment was over a single location rather than being along a flight track, and no terrain spectra were provided in the Minnesota data book. Therefore, we performed a spectral analysis of current digital elevation data (from the U.S. Geological Survey web site: 1-m contour accuracy, 30-s interval distance) for the Minnesota field site, assuming that topography has not changed significantly since 1973 (in contrast to aerodynamic roughness, which usually does change as vegetation and snow cover varies). We found the standard deviation of topography elevation $\sigma_z = 12.2$ m. Even though the crop characteristics and recent plowing of neighboring farm fields at Minnesota were similar to those at Lamont (and therefore might be expected to have nearly equal values of aerodynamic roughness length), the Minnesota site has slightly smoother resolvable surface topography. Thus, the Minnesota site has smaller standard deviation of elevation than any of the BLX96 sites, and indeed corresponds with the smaller best-fit value of $D_M$.

The variations of parameter $D_M$ were found to be correlated (with correlation coefficient $r = 0.92$) to the standard deviation topography elevation $\sigma_z$:

$$D_M = a + b\sigma_z,$$

where $a = 0.35 (±0.10)$ is dimensionless and $b = 0.018 (±0.004) m^{-1}$. This relation is shown in Fig. 8 as the straight line. The conclusion is that rougher terrain-elevation variations cause greater curvature in the wind speed profile, as indicated by large values of $D_M$.

Unlike the ML transport coefficient $C_{MT}$ for momentum fluxes that was found to be dependent on surface roughness length $z_0$ (Santoso and Stull 1998b), the curvature parameter $D_M$ for RXL wind profiles is dependent on standard deviation $\sigma_z$ of surface topography. These different dependencies are probably due to differences in their measurement heights. The momentum fluxes were measured close to the ground surface, therefore, local surface elements and obstacles that create the aerodynamic roughness have more influence on these near-surface ($z = tens$ of meters) measurements. For RXL wind profiles that span heights of order hundreds of meters, the local surface elements or obstacles are felt only by the near-surface part of the RXL wind profiles. The higher part of the wind profiles is not influenced by local surface elements, but are more likely influenced by much larger footprint area (e.g., surface topography). Thus, in hindsight it is not surprising we found that the RXL wind profiles are dependent on standard deviation of surface topography via the curvature parameter $D_M$.

An obvious question is why the RXL momentum profile apparently depends on terrain characteristics, while the temperature profile does not. One explanation is that the wind profile has surface skin values that are always zero, regardless of wind speed in the UL. Thus, the magnitude of wind shear and of surface stress must always reflect the frictional drag at the surface. Contrast this with potential temperature, where the surface skin temperature can increase as the UL layer warms during the day, in order to maintain the temperature difference necessary to drive surface heat fluxes sufficient to respond to solar forcings. A similar explanation also applies to CTT (Stull 1994), which was also found to have heat flux parameters independent of surface roughness, but momentum flux parameters that do depend on roughness (Santoso and Stull 1998b).

6. Comparison with Koorin data

From Fig. 8 in the previous section, data from both Minnesota and BLX96 were used to find a relationship between the standard deviation of resolvable terrain roughness and the parameter $D_M$. The other RXL profile parameters were suggested to be constants, based on those same field experiments. We will now compare these parameters in the RXL profile equations.
against independent data from the Koorin field experiment. The Koorin experiment was conducted in northern Australia at a site with small hills that ranged from 50 to 70 m in height [see Figs. 1.1 and 5.3 in Clarke and Brook (1979)]. Within 50-km radius from the observation site the surface topography was quite similar to Winfield in general. Vegetation coverage in that region consisted of a forest of well-spaced and uniformly distributed eucalyptus and acacia trees of height 5–10 m, with sparse grass beneath. The vegetation coverage over the Koorin site might be quite similar to that over Meeker. The roughness length at Koorin was $z_0 = 0.4$ m and the displacement distance was $z_d = 5.1$ m. Based on the resolvable terrain roughness (calculated from a 1:100 000 scale topographic map, with contour interval of 20 m), the standard deviation of terrain elevation is approximately $\sigma_z = 13.0$ m.

Measurements were taken at Koorin between 15 July and 13 August 1974, during the austral winter. Vertical fluxes of momentum, heat, and moisture were determined using the eddy correlation technique. Mean wind speeds were measured at heights of 11.55, 15.65, 22.15, 32.15, and 48.65 m on a tower, and temperature sensors were 0.63 m lower. All were above canopy-top heights. There were 118 experimental runs made during 30 days of convective conditions, giving a total of nearly 600 wind-profile data points. For temperature, there were only 23 experimental runs that can be used because of missing surface skin temperature measurements on the other days. In addition to the tower measurements, there were less-frequent radiosonde soundings to greater heights.

Observation periods for the mean winds, heat, moisture, and momentum fluxes were identical, but the averaging time for mean wind speed was an hour while for the fluxes it was half an hour. We will assume here that the measured fluxes are representative of 1-h fluxes. The long averaging time of the tower data in Koorin was fortuitous, because the resulting wind profiles have reduced sampling error. The ML depths $z_i$ were estimated from soundings, and were used in the calculation of the Deardorff velocity. RxL depths are then estimated using (5).

Unfortunately the radiosonde soundings cannot be used to give UL characteristics, because the soundings suffer from the lack of statistical robustness described earlier (the difficulty of merging instantaneous soundings with time-average SL and RxL winds). Instead, $\bar{M}_{ul}$ and $\bar{\theta}_{ul}$ must be inferred from RxL profiles. Equations (6), (8), and (9) can be used to solve for $\bar{M}_{ul}$ and $\bar{\theta}_{ul}$ values for each profile. Because the data are used to help determine one of the parameters ($\bar{M}_{ul}$ and $\bar{\theta}_{ul}$) in each profile equation, it means that we cannot independently test the quality of the proposed RxL profile equations and parameters. This is a very unfortunate limitation of the Koorin dataset. However, the available data do allow us to compare the shape or curvature of the profile equations [(6), (8), (9)] to the shape of the plotted data, because these shapes are not affected by the value of wind speed and potential temperature in the UL.

The RxL wind parameter values are taken as constants as previously discussed: $A_M = 1/4$, $B_M = 3/4$, and $E_M = 1/2$, except that $D_M = 0.59$ was found from Fig. 8 [or Eq. (13)] based on the resolvable terrain roughness of $\sigma_z = 13.0$ m at Koorin. The resulting RxL profile curves for wind are plotted in Figs. 9a and 9b, and fit the data quite well.

For potential temperature, it is not obvious what is the appropriate displacement height and what are representative skin temperatures. From photographs in the Koorin data book (Clarke and Brook 1979), it appears that the trees were sparsely leaved, and had large distances between neighboring trees, as would be expected in a semiarid region. Those photographs show most of the sunlight reaching the ground and the sparse grass, with relatively little intercepted by the trees. This
resulted in substantial differences between radiometric “skin” temperatures measured on tree leaves, grass, and bare soil (see plots and discussion by Stull 1994). In fact, leaf skin temperatures were cooler than the air temperature just above, in spite of the fact that there was vigorous convection associated with strong solar heating. Since this implies that most of the thermals were rising from the ground rather than from the sparse canopy top, it would be appropriate to set the temperature displacement distance to zero, and to use an area-average \( \theta_{\text{skin}} \) that “sees” the ground and grass, which is listed in the Koorin data book as the “black ball” method. The result of this approach is plotted in Figs. 9c and 9d, using the previously defined profile constants of \( A_\theta = 1/2, B_\theta = 3/4, D_\theta = 1/5, \) and \( E_\theta = 1/7 \).

Given the limitations of the comparison described above, at best we can conclude that the shape of the RxL wind and potential temperature profile equations agrees well with the shape of the data when the previously proposed “universal” constants are used. Also, the data collapse tightly into a single similarity curve after properly accounting for displacement distance.

7. Flux–profile relationships

The RxL profile equations are a function of the mean difference of wind speed or potential temperature between the surface skin and the UL. Similarly CTT (Stull 1994) gives surface fluxes as a function of the same wind or temperature difference. By combining both theories, one can relate fluxes to profiles, thereby giving new flux–profile relationships for the RxL.

Substituting CTT equations for momentum (12) and heat (10) fluxes into (6) yields flux–profile relationships for mean wind speed and potential temperature in the RxL:

\[
\overline{M}(z) = \frac{u^2_0}{C_{\theta \theta} w_\theta} F(\zeta, z_R, A, D) \tag{14a}
\]

\[
\overline{\theta}(z) - \overline{\theta}_{UL} = \left( w' \theta' - w' \theta'_{\text{UL}} \right) \frac{C_{\theta \theta} w_\theta}{\theta_{\text{UL}}} \times [1 - F(\zeta, z_R, A, D)], \tag{14b}
\]

where \( F \) has been defined by (8) and (9), and where the appropriate \( A \) or \( D \) parameters for wind or temperature must be used. Also, \( \zeta \) and \( z_R \) should be interpreted as distances above the displacement height, when the equations are applied to forest or urban canopy regions.

The flux–profile relationship for wind speed (14a) is dependent on a wide range of scales of terrain roughness. First, the ML transport coefficient for momentum flux \( C_{\theta \theta} \) depends on small-scale roughness elements that affect the aerodynamic roughness length \( z_0 \). Second, parameter \( D_\theta \) depends on resolvable-scale topographic variations that affect the standard deviation of terrain elevation \( \sigma_z \). Such dependence over the wide range of scales should be expected because the RxL profile equations were designed and calibrated as the average over a heterogeneous region (e.g., by using 72-km flight legs in BLX96), rather than being for one column over a single land use.

Equation (14) is easy to use to diagnose wind and temperature profiles given measurements of surface heat flux, momentum flux, and ML depth. However, it is more difficult to go in the other direction: to get the fluxes from the profiles. The reason is that the profile function \( F \) depends on the RxL depth \( z_R \), which itself is a function of the surface fluxes. For comparison, similar difficulties are encountered when trying to get the fluxes from the classical Businger-Dyer–profile relationships for the unstable surface layer, which are of the form

\[
\overline{M} = \frac{u^2_0}{k} \ln \left[ \frac{z - d}{\zeta_0} \right] - 2 \ln \left[ \frac{1}{2} + \frac{1}{2} \left( 1 - 15 \frac{(z - d - \zeta_0)}{L} \right)^{1/4} \right]
\]

\[
- \ln \left[ \frac{1}{2} + \frac{1}{2} \left( 1 - 15 \frac{(z - d - \zeta_0)}{L} \right)^{3/2} \right] + 2 \tan^{-1} \left[ \left( 1 - 15 \frac{(z - d - \zeta_0)}{L} \right)^{1/4} \right] - \frac{\pi}{2}, \tag{15}
\]

where the fluxes are hidden in each term containing Obukhov length \( L \). Nonetheless, there are many nonlinear regression packages available (Press et al. 1992) that can easily solve (14) for the fluxes, given observations of the mean wind and potential temperature profiles in the RxL. For situations where the fine vertical structure of the ML and SL are not well resolved, such as using satellite radiance estimates of ML temperature or global climate models with coarse vertical grid spacing, the solution of (14) for surface fluxes could be quite a valuable approach.

8. Summary and recommendations

The radix layer (RxL) is identified as the region between the surface and the base of the uniform layer (UL), in the bottom of the convective boundary layer. The classical surface layer is the bottom subdomain of the RxL. The depth of the RxL is given by (5), and depends both on mixed layer characteristics such as ML depth, and on surface characteristics such as heat and momentum fluxes. These new depth equations eliminate the uncertainty in RxL depth that was discussed in a previous paper (Santoso and Stull 1998a).

Similarity shapes of RxL wind and potential-temperature profiles are given by (6), (8), and (9). These equations are designed to give zero wind speed at the ground.
and to become tangent to the UL wind speed at the top of the RxL. These apply only to free convective conditions where thermals are the dominant turbulence process. They have been calibrated to work in mesoscale (order of 50 km) regions over heterogeneous surfaces. These equations are not applicable in the blending layer and the roughness sublayer, in the immediate vicinity of the individual roughness elements.

The RxL depth parameterization includes two parameters: B and E. The profile equations include two more parameters A and D. Based on the five field sites examined here (Minnesota, BLX96-Lamont, BLX96-Meeker, BLX96-Winfield, Koorin), all but one of the parameters are constant, suggesting that they might be universal. Only the D parameter for wind depends on standard deviation of surface terrain elevation $\sigma_s$, and surprisingly does not depend on traditional aerodynamic roughness length $z_0$. These parameter values are summarized in Table 1. For wind speed the terrain elevation parameters are $a = 0.35$ (dimensionless) and $b = 0.018 \text{ m}^{-1}$.

As shown in (11) there is a simple relationship between RxL depth and Obukhov length, although the RxL depth also includes a stronger dependence on ML depth. The RxL depth varies from day to day analogous to the variability of Obukhov length, depending on varying external forcings. The RxL depth for wind is in general greater than that for potential temperature. The parameterizations show that the ratio $z_{\text{rms}}/z_{R_0}$ is roughly 7:2.

Flux–profile equations for the convective RxL were suggested in (14), which extends over a greater depth than traditional surface-layer Monin-Obukhov flux–profile relationships. The dimensionless form of these flux–profile relationships is in terms of the mean profiles of wind and temperature, rather than of their gradients. As was discussed in a previous paper (Santoso and Stull 1998a), nondimensional gradient forms of SL flux–profile equations tend to hide or disguise errors and give false pictures of the accuracy of the relationships.

Finally, the data from Minnesota and Koorin show much less scatter than the data from BLX96. There are two reasons. One, the Minnesota and Koorin data are measured over single points on the earth’s surface having relatively homogeneous surroundings, compared to the BLX96 data, which was measured by aircraft over a 72-km mesoscale distance having surface heterogeneity. Second, the long time averages from multiple sensors at different heights during Minnesota and Koorin provided more robust statistics than the sequentially sampled heights by aircraft flying ascent–descent zigzag patterns during BLX96. For further tests of the equations presented in the paper, it would be very appropriate for other investigators to analyze meteorological data from very tall instrumented towers such as the Boulder Atmospheric Observatory or Cabauw, which would likely produce statistically robust data.

Potential applications for wind speed diagnoses using the RxL equations include air-pollutant transport, wind loading on bridges and other tall structures, and for calculating power output from wind turbines during convective conditions. The RxL potential temperature profile could be used to help determine chemical reaction rates of air pollutants within the RxL. Both profile equations could be used in global climate models to estimate vertically unresolved effects associated with relative coarse vertical grid spacing, but only during convective conditions.

Acknowledgments. This research was funded by the U.S. National Science Foundation (NSF) under Grant ATM-9411467. The University of Wyoming King Air aircraft is also sponsored by NSF. The Canadian Natural Science and Engineering Research Council and the Environment Canada Atmospheric Environment Service also provided significant grant support. The U.S. Department of Energy is gratefully acknowledged for their Grant DE-FG02-92ER61361, as well as for their data and field support at the Southern Great Plains Atmospheric Radiation Measurement program. Josh Hacker provided excellent daily forecasts that were used for flight planning and served as an airborne scientist, and Larry Berg also served as one of the airborne scientists.

APPENDIX A

Minnesota Measurements

Fig. A1. Lines show the theoretical relationship [Eq. (11b)] between radial-layer depth $z_r$ (made dimensionless with average mixed layer depth $z$) vs convective boundary layer stability parameter $z_r/z_L$, where $L$ is the surface-layer Obukhov length. Data points represent every run reported in appendixes A and B, with circles for wind, squares for temperature, open for Minnesota, and solid for BLX96. Solid line uses $E = 1/2$ for wind; dashed line uses $E = 1/7$ for temperature.
Table A1. Dates, times, and boundary layer scaling variables for the Minnesota field experiment. Friction velocity is $u_*$, surface kinematic heat flux is $\overline{w'\theta'}$, Deardorff velocity is $w_*$, mixed layer depth is $z_i$, Obukhov length is $L$. Central Daylight Time (CDT) = UTC − 5 h.

<table>
<thead>
<tr>
<th>Run</th>
<th>Time (CDT)</th>
<th>Date (1973)</th>
<th>$u_*$ (m s$^{-1}$)</th>
<th>$w'\theta'$ (K m s$^{-1}$)</th>
<th>$w_*$ (m s$^{-1}$)</th>
<th>$z_i$ (m)</th>
<th>$-L$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A1</td>
<td>12:17–13:32</td>
<td>9/10</td>
<td>0.461</td>
<td>0.196</td>
<td>2.00</td>
<td>1250</td>
<td>38.23</td>
</tr>
<tr>
<td>2A2</td>
<td>13:32–14:47</td>
<td>9/10</td>
<td>0.454</td>
<td>0.209</td>
<td>2.23</td>
<td>1615</td>
<td>34.10</td>
</tr>
<tr>
<td>3A1</td>
<td>15:10–16:25</td>
<td>9/11</td>
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<td>0.186</td>
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<td>2310</td>
<td>20.96</td>
</tr>
<tr>
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<td>9/11</td>
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<td>2300</td>
<td>21.54</td>
</tr>
<tr>
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<td>9/15</td>
<td>0.194</td>
<td>0.069</td>
<td>1.35</td>
<td>1085</td>
<td>8.16</td>
</tr>
<tr>
<td>5A2</td>
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<td>0.241</td>
<td>0.210</td>
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<td>2095</td>
<td>5.08</td>
</tr>
<tr>
<td>5B1</td>
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<td>0.265</td>
<td>0.072</td>
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<td>1.89</td>
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<td>11.49</td>
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<td>0.099</td>
<td>1.58</td>
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Table B2. Estimates of radix-layer parameters for the Minnesota field experiment: $M_{sl}$ is wind speed in the uniform layer, $z_{est}$ is radix-layer depth for wind, $\theta_{est}$ is potential temperature in the uniform layer, $\Delta \theta$ is potential temperature difference between the surface skin and the uniform layer, $z_i$ is radix-layer depth for temperature. Also shown for comparison are the Obukhov length $L$ and 10% of the ML depth $z_i$, which are often used as depth scales for the classical surface layer.

<table>
<thead>
<tr>
<th>Run</th>
<th>$M_{sl}$ (m s$^{-1}$)</th>
<th>$z_{est}$ (m)</th>
<th>$\theta_{est}$ (K)</th>
<th>$\Delta \theta$ (K)</th>
<th>$z_i$ (m)</th>
<th>$-L$ (m)</th>
<th>0.1 $z_i$</th>
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</tr>
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<td>53.0</td>
<td>5.6</td>
<td>204</td>
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**APPENDIX B**

**BLX96 Measurements**

Table B1. The same as Table A1 but for the BLX96 field experiment.

<table>
<thead>
<tr>
<th>Track</th>
<th>Leg</th>
<th>Date (1996)</th>
<th>Midtime (CDT)</th>
<th>$\mu$ (m s$^{-1}$)</th>
<th>$w'\theta'$ (K m s$^{-1}$)</th>
<th>$w_*$ (m s$^{-1}$)</th>
<th>$z_i$ (m)</th>
<th>$-L$ (m)</th>
</tr>
</thead>
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<td>Lamont</td>
<td>AA</td>
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<td>13:51</td>
<td>0.309</td>
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<td>1010</td>
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</tr>
<tr>
<td></td>
<td>SS</td>
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<td>14:66</td>
<td>0.310</td>
<td>0.068</td>
<td>1.460</td>
<td>1194</td>
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</tr>
<tr>
<td></td>
<td>AA</td>
<td>7/27</td>
<td>12:78</td>
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<td>680</td>
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<td>14:54</td>
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<td>903</td>
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<td>1794</td>
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Table B2. The same as Table A2 but for the BLX96 field experiment. Site abbreviations are Lmt = Lamont, Mkr = Meeker, and Wfd = Winfield.

<table>
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<th>Date (1996)</th>
<th>$\bar{u}_{w}$ (m s(^{-1}))</th>
<th>$\bar{z}_{ms}$ (m)</th>
<th>Potential temperature</th>
<th>Wind</th>
<th>Other depths</th>
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<td>$\Delta \theta$ (K)</td>
<td>$\bar{z}_{ms}$ (m)</td>
<td>$L$ (m)</td>
<td>$0.1 \bar{z}_{i}$</td>
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APPENDIX C

Error Propagation Analysis

For this analysis, we assume that the greatest uncertainties (i.e., variances) are associated with nondimensional profile data [\(M(z)/\bar{u}_{UL}\) and (\(\bar{\theta}(z) - \bar{\theta}_{UL}\))/(\(\bar{\theta}_{skin} - \bar{\theta}_{UL}\))], and not in height (\(z\)), ML depth (\(z_{ms}\)), surface friction velocity (\(u_{w}\)), and Deardorff velocity (\(w_{d}\)). Following Bevington (1969), the square of the probable error \(\sigma_{x}^2\) in a parameter \(x\) depends on the uncertainties (variance) in nondimensional data \(\sigma_{x}^2\) by

\[
\sigma_{x}^2 = \sigma_{x}^2 \sum_{j=1}^{N} \left( \frac{\partial \theta}{\partial \psi_j} \right)^2,
\]

where \(\psi\) is the nondimensional wind or potential temperature and \(p\) is a parameter such as \(A, B, D,\) or \(E\). The partial derivative \(\partial \theta / \partial \psi\) is simply \((\partial \psi / \partial \theta)^{-1}\). Unfortunately, one cannot use this analytical method directly because the partial derivatives of the parameters \(A, B, D,\) and \(E\) with respect to the nondimensional profile variables contain a singularity (divide by zero) at the top of the RL and within the UL. These subdomains cannot be neglected, because the best-fit parameters \(A, B, D,\) and \(E\) were obtained by fitting the whole profile equation to data both below and above the top of the RL, specifically for the purpose of reducing sampling uncertainty.

To overcome this problem, synthetic data were created by introducing random perturbation errors into the nondimensional ideal profiles. Four experiments are conducted, where the random errors are set to yield standard deviations of \(\sigma_{\theta} = 0.01, 0.05, 0.10,\) and 0.15 (dimensionless). These error standard deviations span the range of observed scatter in the data points for the nondimensional Minnesota and BLX96 profiles, as reported in the body of this paper.

For each experiment, 10 different trials (i.e., different random perturbations from the ideal profiles) are used. For each trial of each experiment, nonlinear regression is used to find the best-fit parameters to the perturbed profiles. From the set of 10 values for each parameter for each experiment, the standard deviation of each parameter is computed as a measure of the parameter sensitivity to scatter in the data points. The results of sensitivity analyses for the parameters \(A, B, D,\) and \(E\) are shown in Table C1 and Fig. C1. It can be seen that the uncertainties in the parameters \(A, B, D,\) and \(E\) increase as the uncertainties in \(M(z)/\bar{u}_{UL}\) or \((\bar{\theta}(z) - \bar{\theta}_{UL}\))/(\(\bar{\theta}_{skin} - \bar{\theta}_{UL}\)) increase.

Finally, we found that parameters \(A\) and \(D\) vary oppositely in a special way. If \(A\) is fixed, then the uncertainty in \(D\) is relatively small, as given above. If \(D\) is fixed, then the uncertainty in \(A\) is relatively small, as given above. However, if both are allowed to vary, one finds a locus of sets of \(A\) and \(D\), which give profile

Table C1. Propagation of error (as a standard deviation \(\sigma\)) into the dimensionless profile parameters \(A, B, D,\) and \(E\) from errors in dimensionless wind or temperature observations \(\sigma_{\theta_{ms}}\), where subscript obs = \(M/\bar{u}_{UL}\) for wind and obs = \((\bar{\theta} - \bar{\theta}_{UL}\))/(\(\bar{\theta}_{skin} - \bar{\theta}_{UL}\)) for temperature.

<table>
<thead>
<tr>
<th>Observation error</th>
<th>(\sigma_{A})</th>
<th>(\sigma_{B})</th>
<th>(\sigma_{D})</th>
<th>(\sigma_{E})</th>
<th>(\sigma_{\theta_{ms}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.007</td>
<td>0.005</td>
<td>0.033</td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>0.029</td>
<td>0.009</td>
<td>0.105</td>
<td>0.098</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.065</td>
<td>0.020</td>
<td>0.164</td>
<td>0.152</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>0.072</td>
<td>0.028</td>
<td>0.253</td>
<td>0.236</td>
<td></td>
</tr>
</tbody>
</table>
curves with nearly identical goodness of fit. This suggests greater uncertainty in the value of both A and D taken together than would be inferred from the previous paragraph. Nonetheless, profile equations (6), (8), and (9) have a functional form that allows nonlinear regression packages to directly seek and quickly pinpoint a set of best-fit parameter values. These parameter values work well when the resulting similarity curve is plotted with the observation data. While beyond the scope of the present study, we recommend that other investigators be aware of a possible relationship between nonlinear regression packages and the resulting parameter values of A and B.

REFERENCES


temperature for computing regional scale sensible heat flux. 
*Bound.-Layer Meteor.*, 80, 205–221.


