Nonlinear characteristics of the surface air temperature over Canada

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[1] Nonlinear characteristics of the Canadian surface air temperature (SAT) were investigated by applying a neural-network-based nonlinear principal component analysis (NLPCA) method to the SAT anomaly data for individual seasons. The SAT data were separated into three subsets: data for 1900–1949 and 1900–1995 over southern Canada (south of 60°N), called S0049 and S0095, respectively, and data for 1950–1995 over the entire country, called C5095. The NLPCA was computed for the three data sets separately. The leading NLPCA modes from C5095 and S0095 show similar results: the nonlinearity is strong in winter (December, January, and February, DJF) and fall (September, October, and November, SON), but is much weaker in spring (March, April and May, MAM) and summer (June, July, and August, JJA), manifesting the seasonal dependence of the nonlinearity in the Canadian SAT. No significant nonlinearity is detected from data set S0049, even for the winter and fall seasons, indicating interdecadal dependence of the nonlinearity. The leading NLPCA mode combines the effects of Pacific-North America (PNA) pattern and North Atlantic Oscillation (NAO) on the Canadian winter SAT. A possible reason for the existence of nonlinearity in the winter SAT only after 1950 is that the NAO manifested its strong negative phase from the 1950s to the early 1970s. INDEX TERMS: 3309 Meteorology and Atmospheric Dynamics: Climatology (1620); 3220 Mathematical Geophysics: Nonlinear dynamics; 4215 Oceanography: General: Climate and interannual variability (3309); KEYWORDS: surface air temperature, Canada, neural network, nonlinear principal component analysis (NCPCA), interdecadal

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1. Introduction

[2] North America is an area of interest for climate studies related to the El Niño-Southern Oscillation (ENSO) phenomenon [Horel and Wallace, 1981; Wallace and Gutzler, 1981; Trenberth et al., 1998]. The traditional view is that the climate variations associated with El Niño-Southern Oscillation (ENSO) are linear, with anomalies during the El Niño phase being the reverse of those during the La Niña phase [e.g., Ropelewski and Halpert, 1989; Bunkers et al., 1996]. However, recent evidence shows that the atmospheric responses to warm and cold events were not exactly opposite. Richman and Montroy [1996] examined the composite January temperature and precipitation patterns over the United States and parts of Canada associated with El Niño and La Niña events. Their results suggest that El Niño and La Niña have their own unique characteristics in terms of temperature and precipitation, so the responses are not linear. Asymmetric spatial patterns of the Canadian surface air temperature (SAT) and precipitation associated with the

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Southern Oscillation (SO) were detected by Shabbar and Khandekar [1996] and Shabbar et al. [1997]. Further evidences of nonlinear response of North America climate to ENSO were provided by Hoerling et al. [1997], who suggested that the midlatitude atmospheric response to the different phases of the SO is inherently nonlinear, due to differences in the locations of the intense tropical Pacific SST-induced deep convection between El Niño and La Niña events. From the phase shift in the midlatitude geopotential height anomalies during the opposite phases of the SO, they concluded that midlatitude temperature and precipitation patterns should also have nonlinear relations with the SO. The robustness of nonlinear climate response to ENSO's extreme phases was then confirmed by four GCMs [Hoerling et al., 2001]. A nonlinear identification of the atmospheric response to ENSO was also addressed by Hannachi [2001] using state-of-the-art general circulation models.

[3] Describing the nonlinear behavior in the midlatitude climate and its nonlinear relations to ENSO is a great challenge. Standard multivariate statistical techniques such as principal component analysis (PCA, also known as EOF analysis) and canonical correlation analysis (CCA) are linear methods. Composite analysis does not assume linearity, but

	C5095				S0095				S0049				
	DJF	MAM	JJA	SON	DJF	MAM	JJA	SON	DJF	MAM	JJA	SON	
1	44.6	46.1	40.6	40.1	58.2	54.9	48.3	55.4	61.4	55.3	47.9	60.6	
2	24.7	17.5	15.3	29.4	21.3	21.3	16.5	24.9	19.0	20.9	15.7	20.9	
3	11.0	14.2	14.3	10.6	7.5	8.2	11.2	5.8	6.1	7.5	11.1	5.7	
4	7.3	6.1	6.5	5.7	4.1	3.8	6.0	3.3	4.2	3.5	7.1	2.9	
Σ_1^4	87.6	83.9	76.7	85.8	91.1	88.2	82.0	90.0	90.7	87.2	81.8	90.1	

Table 1. Variance Explained by the Four Leading EOF (PCA) Modes of the Seasonal SAT Anomalies^a

^a Values are in percent. Given are anomalies for 1950–1995 over the entire country (C5095) and for 1900–1995 and 1900–1949 over southern Canada (south of 60° N, S0095 and S0049, respectively). The bottom line shows the sum of the four modes.

is restricted to the analysis of differences between specific phases of the SO or equatorial sea surface temperature (SST) indices. Are there general methods which can describe the nonlinearity of the midlatitude climate variability and its nonlinear relations to ENSO? Recently, neural networks (NN) [Hsieh and Tang, 1998] have been used for nonlinear PCA (NLPCA) [Kramer, 1991] and nonlinear CCA (NLCCA) [Hsieh, 2000]. The tropical Pacific SST and sea level pressure fields have recently been analyzed by the NLPCA [Monahan, 2001] and by NLCCA [Hsieh, 2001a], where SST was found to exhibit considerable nonlinearity, while the sea level pressure was found to be less nonlinear. Also, NLPCA was applied to the analysis of the winter Northern hemisphere atmospheric variability [Monahan et al., 2000]. Hsieh [2001b] pointed out that NLPCA unifies the PCA and rotated PCA approaches.

[4] In this paper, the NLPCA model of *Hsieh* [2001b] will be applied to study the SAT variability over Canada. The motivation of this paper is to examine the nonlinear characteristics of the Canadian SAT using NLPCA, before we build our prediction models using NLCCA to link the SAT and the tropical Pacific SST. The paper is organized as follows: The data are briefly introduced in section 2. The leading NLPCA modes for the Canadian SAT data for 1950–1995 over the entire country, and for 1900–1995 and 1900–1949 over southern Canada (south of 60° N) are presented in section 3. Some possible dynamics related to the NLPCA mode of Canadian winter SAT are discussed in section 4. A Summary with concluding remarks is given in section 5. Details of the NLPCA model are described in Appendix A.

2. Data

[5] The basic data we used in this study is the gridded monthly mean SATs interpolated from the station observations for the period of January 1900 to December 1995 [*Vincent and Gullet*, 1999]. With this data (up to 1998), *Zhang et al.* [2000] analyzed the trend of Canadian SAT in the 20th century. Following *Zhang et al.* [2000], we separated the data into three subsets: data for 1900–1949 and for 1900–1995 over southern Canada (south of 60°N), called S0049 and S0095, respectively, and data for 1950–1995 over whole Canada, called C5095. There are two reasons for doing so: (1) the limited data availability in northern Canada prior to 1950; (2) a check on the signifi-

cance of the NLPCA results. If the NLPCA results are reproducible from different data sets, then the nonlinearity can be regarded as robust rather than sampling dependent. The SAT data have an approximately linear trend [see *Zhang et al.*, 2000, Figure 3], so linear detrending was performed first. Monthly SAT anomalies were calculated by removing the climatological monthly mean based on the whole period of each data set. The anomaly data were then smoothed with a 3-month running mean. The SAT anomalies were separated into four seasons: SAT anomalies in December, January, and February (DJF) were used to form a data set for the winter season, March, April, and May (MAM) for the spring, June, July, and August (JJA) for the summer, and September, October, and November (SON) for the fall.

[6] The 500-mb geopotential height came from the National Centers for Environmental Prediction's (NCEP) reanalysis data sets for the period from January 1948 to December 1995 with a 2.5° grid over a global domain [Kalnay et al., 1996]. The monthly SST was from the reconstructed global historical SST data sets by Smith et al. [1996] for the period 1950–2000 with a resolution of 2° by 2° over global oceans. Anomalies for the 500-mb height and SST fields are calculated with respect to climatological monthly means for 1950–1995. Linear detrending and 3-month running mean were then performed on both data sets.

[7] The winter (December through March) index of the North Atlantic Oscillation (NAO), defined as the difference of normalized sea level pressure (SLP) between Lisbon, Portugal and Stykkisholmur/Reykjavik, Iceland since 1864 to 2001, was provided by Dr. Jim Hurrel of the National Center for Atmospheric Research (NCAR). The SLP anomalies at each station were normalized by dividing the seasonal mean pressure by the long-term (1864–1983) standard deviation. The NAO index used in this paper is an update of the time series published by *Hurrel* [1995].

3. The Leading NLPCA Mode of Canadian SAT 3.1. C5095

[8] Prior to NLPCA, traditional PCA (or EOF analysis) was performed on the SAT anomalies of each season. Variance contributions from the four leading modes are listed in Table 1 and the spatial patterns for the three leading modes are shown in Figure 1. We can see that spatial

Figure 1. (opposite) The first four empirical orthogonal functions (EOFs) of the seasonal surface air temperature (SAT) anomalies for 1950–1995 over the entire country (C5095). From top to bottom, the four rows represent the winter (a-c), spring (d-f), summer (g-i) and fall (j-l), respectively. Solid curves denote positive contours, dashed curves, negative contours, and thick curves, zero contours. The contour interval is 0.02. The EOFs are normalized to unit norm.



(a) EOF1



(b) EOF2



(c) EOF3



(d) EOF1



(e) EOF2



(f) EOF3



(g) EOF1



(h) EOF2



(i) EOF3







(l) EOF3

(j) EOF1



Figure 2. A schematic diagram of the NN model for calculating nonlinear PCA (NLPCA). There are three "hidden" layers of variables or "neurons" (denoted by circles) sandwiched between the input layer \mathbf{x} on the left and the output layer \mathbf{x} on the right. Next to the input layer is the encoding layer (with *m* hidden neurons), followed by the "bottleneck" layer (with a single neuron u), which is then followed by the decoding layer. A nonlinear function maps from the higher dimension input space to the lower dimension bottleneck space, followed by an inverse transform mapping from the bottleneck space back to the original space represented by the outputs, which are to be as close to the inputs as possible by minimizing the cost function $J = \langle ||\mathbf{x} - \mathbf{x}'||^2 \rangle$. Data compression is achieved by the bottleneck, with the bottleneck neuron giving u, the nonlinear principal component. The actual NLPCA used in this paper has four inputs (and four outputs), has m between 2 and 3, and adds a weight penalty term in the cost function to alleviate overfitting (see Appendix A).

patterns of individual seasons are very similar to each other, except that mode 2 and mode 3 of the spring data are interchanged. The first mode is uniform in sign (except the west coast for the summer) by indicating either warm or cold conditions over the whole domain, while the second mode displays a southwest-northeast contrast. For each season, the first mode can always explain 40–50% of the total variance, and the second mode about 20%. The four leading modes explain altogether about 80% of the total variance. The four leading principal components (PCs) (i.e., EOF time coefficients) are thus used as the inputs to the NLPCA model (Figure 2).

[9] The first NLPCA modes of SAT anomalies for four seasons are shown in Figure 3. For convenience, 3-dimensional figures are used, where we can see not only the NLPCA curve in the PC₁-PC₂-PC₃ 3-D space but also its projections on PC1-PC2, PC1-PC3 and PC2-PC3 planes. In winter (Figure 3a), we can find a notable curve in the PC_1 -PC₂ plane, which indicates considerable nonlinearity, relative to the PCA (straight line). 53.7% of the variance of the original data is explained by the NLPCA mode 1, versus 44.6% by the PCA mode 1. The MSE (mean square error) for the NLPCA mode is much smaller than that for the PCA mode with a ratio of 0.865. Here MSE is the mean square of the distance between a data point and its projection onto the NLPCA (or PCA) mode, i.e., the unexplained variance. The MSE ratio can be considered an indicator of the nonlinearity: As a ratio of 1 or close to 1 means the NLPCA is essentially linear, while a smaller MSE ratio means stronger nonlinearity.

[10] Figures 3b and 3c show relatively weak nonlinearity in the spring and summer data, with MSE ratios of 0.965 and 0.977, respectively, and with slightly higher variance contributions (Table 2) relative to their PCA mode 1 counterparts (Table 1). The nonlinearity is then enhanced during the fall, as manifested in the increased curvature (Figure 3d). The NLPCA mode 1 for the fall data accounts for 51.8% of the total variance, versus 40.1% by the PCA mode 1. The MSE ratio is 0.833, indicating a little higher nonlinearity than during winter.

[11] Unlike a PCA mode, which produces a fixed spatial pattern (the EOF), the NLPCA mode does not give a single characteristic spatial pattern. For a specific value of the NLPC u (see Appendix A), we can use the NN to map u in the bottleneck neuron onto \mathbf{x}' in the output layer (Figure 2) using equations (5) and (6) (in Appendix A). Note that u can be regarded as a simple curvilinear coordinate system on the NLPCA curve, and therefore each value of *u* corresponds to a pattern within the first four PCs. Figure 4 shows the SAT anomaly patterns corresponding to minimum u and maximum u. We can see that the spatial patterns on opposite extremes of u are now asymmetric, i.e., no longer mirror images. The asymmetry is enhanced as the nonlinearity increases. The spatial patterns associated with minimum u and maximum u are basically symmetric for the summer (Figures 4e and 4f), and slightly more asymmetric for the spring, with the warm center shifted southward relative to the cooling center in Figures 4c and 4d. The spatial asymmetry gets enhanced in the fall and winter. For the fall, the minimum and maximum u patterns have respectively negative and positive anomalies basically covering the whole domain. But the cold pattern is centered over the northeast (Figure 4g), and the warm pattern is centered toward the west (Figure 4h). The spatial patterns for opposite values of u are also quite asymmetric during the winter. At maximum u, there are positive anomalies over all of Canada (Figure 4b), similar to the EOF_1 (Figure 1a), with the warm center over western Canada. At minimum u, there are negative anomalies to the southwest and positive anomalies to the northeast, similar to the EOF_2 (Figure 1b). It is not surprising because the NLPCA mode 1 combines PC1 and PC2 (Figure 3a).

[12] It is worth noting that the winter SAT anomaly pattern associated with maximum u (Figure 4b) is similar to the composite results during El Niño years [*Hoerling et al.*, 1997, 2001], while the SAT anomaly pattern associated with minimum u (Figure 4a) is different from their composite results during La Niña years. Therefore, the NLPCA mode reveals the possible effects from more than ENSO on the Canadian SAT. This will be further discussed later in this paper.

3.2. S0095

[13] Similarly, traditional PCA was performed on the SAT anomalies of S0095 for each season to compress the data and the first three EOFs are shown in Figure 5. The EOF1 and EOF2 are roughly consistent with those of C5095, while the EOF3 patterns are different from those shown in Figure 1. The variance contributions for the four leading modes are listed in Table 1. The first four PCs are used as the inputs to the NLPCA.



Figure 3. The first NLPCA mode for the seasonal SAT anomalies of C5095 plotted as (overlapping) squares in the PC_1 - PC_2 - PC_3 3-D space. The linear (PCA) mode is shown as a dashed line. The NLPCA mode and the PCA mode are also projected onto the PC_1 - PC_2 plane, the PC_1 - PC_3 plane, and the PC_2 - PC_3 plane, where the project NLPCA is indicated by (overlapping) circles, and the PCA by thin solid lines, and the projected data points by scattered dots. Panels (a), (b), (c) and (d) correspond to the winter, spring, summer and fall, respectively.

[14] The first NLPCA modes of SAT for four seasons are shown in Figure 6, where we can see similar results as extracted from C5095 although the NLPC curves are not exactly the same. The strongest nonlinearity occurs in winter with a MSE ratio of 0.879, followed by the fall with a MSE ratio of 0.908. The NLPCA modes explain 63.4% and 60.8% of the total variance of the winter and fall SAT anomalies, respectively, versus 58.2% and 55.4% by the PCA modes. Figures 6b and 6c show rather weak nonlinearity in the spring and summer data (see also Table 2). [15] In the spatial patterns (Figure 7), for the spring (Figures 7c and 7d) and summer (Figures 7e and 7f), the SAT anomalies on opposite extremes of the NLPC u are basically similar except for a sign change, resembling their EOF1 patterns(Figures 5d and 5g). More asymmetry can be seen in the winter (Figures 7a and 7b) and fall (Figures 7g and 7h), and the spatial patterns are in reasonably good agreement with those shown in Figures 4a and 4b and Figures 4g and 4h, respectively. Although C5095 and S0095 are partially overlapping (not completely independent), the

Table 2. Explained Variance by the NLPCA Mode 1 (Ev) and the Ratio (R) Between the MSE of the NLPCA Mode 1 and That of the PCA Mode 1^a

	C5095				S0095				S0049			
	DJF	MAM	JJA	SON	DJF	MAM	JJA	SON	DJF	MAM	JJA	SON
Ev	53.7	48.4	41.2	51.8	63.4	55.2	48.8	60.8	62.7	56.3	48.3	61.9
R	0.865	0.965	0.977	0.833	0.879	0.983	0.986	0.908	0.951	0.960	0.973	0.958

^a Values are in percent.



Figure 4. The SAT anomaly patterns (in $^{\circ}$ C) of the NLPCA mode 1 extracted from C5095, as the NLPC (i.e., *u*) of the first NLPCA mode takes its minimum and maximum values. Contour interval is 1 $^{\circ}$ C. The four rows from top to bottom display the winter (a,b), spring (c,d), summer (e,f) and fall (g,h) patterns, respectively.







(b) EOF2



(c) EOF3



(d) EOF1



(e) EOF2



(f) EOF3



(g) EOF1



(h) EOF2



(i) EOF3



Figure 5. Similar to Figure 2, but for S0095. For better visualization, the values have been multiplied by 100 and drawn with contour interval of 2.



Figure 6. Similar to Figure 3, but for S0095.

similarity of the NLPCA results from two data sets suggests the significance of the seasonal dependence of the nonlinearity of the Canadian SAT, at least for the data after the 1950s.

3.3. S0049

[16] S0049, which is completely independent of C5095, is considered in this section. Spatial patterns of the first three PCA modes, shown in Figure 8, are well consistent with those extracted from S0095 (Figure 5, except for the opposite sign in EOF2 for the winter, spring and summer). Detailed information for the PCA modes are listed in Table 1. The first four PCs serve as the inputs to the NLPCA.

[17] The first NLPCA modes of SAT anomalies for four seasons are shown in Figure 9. If disregarding the few outlier points (e.g., in Figure 9d), no notable nonlinearity can be seen in any of the four seasons. Table 2 indicated that the MSE ratios are all above 0.95 and the explained variance percentage differences between nonlinear and linear modes are also small (not over 2.0%), implying rather weak nonlinearity. The spatial anomaly patterns associated with minimum u and maximum u are generally symmetric for all four seasons (figures not shown), confirming the weak nonlinearity again.

[18] Since S0095 exhibits considerable nonlinearity in winter and fall, while S0049 does not show apparent non-linearity for all four seasons, nonlinearity for the winter and fall exists mainly in the data after 1950s, suggesting interdecadal dependence in the nonlinearity of the Canadian SAT.

4. Dynamics Related to Canadian SAT NLPCA Mode

4.1. Winter 500-mb Height

[19] As the strong nonlinearity tends to occur in winter, we will examine the winter season more closely. Regressions of a winter SAT EOF PC onto the global 500-mb height anomalies (1950–1995) reveal the spatial patterns of the 500-mb heights covarying with the PC, which was standardized first. The first winter SAT PC regressed onto the simultaneous 500-mb height anomalies revealed a Pacific-North American (PNA) pattern, where there is generally positive height anomalies over Canada, and negative height anomalies over the midlatitude North Pacific and the southeastern United States when the PC is positive (Figure 10a), corresponding to warming over whole Canada (Figure 4b). The second SAT PC regressed onto the winter



Figure 7. Similar to Figure 4, but for S0095.



(a) EOF1



(b) EOF2



(c) EOF3



(d) EOF1



(e) EOF2



(f) EOF3



(g) EOF1





(h) EOF2



(i) EOF3



Figure 8. Similar to Figure 5, but for S0049.



Figure 9. Similar to Figure 3, but for S0049.

500-mb height anomalies gave an NAO pattern, with a large negative anomaly centered over southwestern Greenland and northeastern Canada, and a large positive anomaly centered over western Europe and a positive anomaly over the west coast of Canada when the PC is positive (Figure 10b), corresponding to warm western Canada and cool eastern Canada (Figure 4a with signs reversed). Since major nonlinearity occurs between PC1-PC2 (Figures 3a and 6a), the leading NLPCA mode thus combines the effects of PNA and NAO on the Canadian winter SAT, suggesting that the effects of PNA and NAO on Canada may not be totally independent. It appears that the SAT PC₂ tends to take on negative values when PC1 takes on significant negative values (Figure 3a), i.e., negative NAO effects, a warm eastern Canada-cool western Canada (Figure 1b with signs reversed), tends to concur with negative PNA effects (cool Canada) (Figure 1a with signs reversed).

4.2. The Global SST

[20] Similar regressions of a winter SAT PC onto the global SST anomalies were performed. The regressions of the first winter SAT PC onto SST anomalies yielded a spatial pattern resembling the Pacific Decadal Oscillation

(PDO) [*Trenberth and Hurrell*, 1994]; that is, when the PC is positive (corresponding to warm Canada, Figure 1a), there is cool SST over the central-western midlatitude North Pacific and warm waters off the west coast of North America (Figure 10c), which is consistent with the ocean's response to the PNA pattern in the atmosphere. The third SAT PC regressed on the SST anomalies manifested a pattern somewhat similar to ENSO, i.e., warm SST in the eastern-central equatorial Pacific (Figure 10d) when the third PC is positive, corresponding to warm southern Canada and cool northern Canada (Figure 1c).

4.3. Nonlinearity and the NAO

[21] We take a closer look at the NLPCA mode for S0095 and S0049 in Figure 11 (only projections on the PC_1-PC_2 plane are shown). S0095 and its NLPCA mode are shown by the symbols of plus and overlapping circles, while S0049 and its NLPCA mode, by scattered dots and squares, respectively. For comparison, the signs of PC_2 , also the PC_2 at the output layer of NLPCA model of S0049 are reversed since the EOF2 of S0049 (Figure 8b) is opposite in sign to the EOF2 of S0095 (Figure 5b). We can see the symbols of "+" and "." at the same time (1900–1949) are



Figure 10. The regression coefficients between the winter first PC (of the Canadian SAT anomalies of C5095) and the global simultaneous (a) 500-mb height anomalies, and (c) SST anomalies; and the regression coefficients (b) between the winter second PC and the 500-mb height anomalies, and (d) between the winter third PC and the SST anomalies. The regression coefficient shown is the slope, with the PC treated as the independent variable in the regression. Contour interval is 10 in panel (a) and (b), and 0.1 in panel (c) and (d), respectively. The SAT PCs were standardized before performing regression.

close but not exactly overlapping, as the EOFs for two data sets are similar but not identical (Figures 5a-5c and Figures 8a-8c). Regarding the NLPCA modes, if we ignore the data points of S0095 in the big circle (Figure 11), the two NLPC curves could be very similar, i.e., it is these points that brings the nonlinearity to S0095. These data points are to be found in the winters of 1916, 1933, 1937, 1951, 1955, 1956, 1965,1966, 1968, 1969, 1977 and 1982. These years have negative NAO index values (Figure 12), except for 1933, 1937 and 1982, and most of them are within the period of 1951–1977. Actually, after a 7-year running mean, the NAO index (thick line in Figure 12) is basically positive from 1900 to 1930, normal in the 1930s and 1940s and negative from 1952 to the early 1970s and then turns to be positive after the mid 1970s, reaching its positive extreme in the early 1990s. It is possible that the existence of the nonlinearity in the Canadian winter SAT only after the 1950s is due to the strong negative phase of the NAO occurring from 1950s to the early 1970s. The negative NAO pattern (Figure 10b with signs reversed) and the resulting SAT patterns shown in Figures 4a and 7a are more likely to occur during the negative phase of the NAO.

[22] To check that the downward curve in S0095 (Figure 11) is not due to the outliers from a single year, we removed the data from winter 1969 (which contributed the two points with the most negative PC_2 values) and recomputed the NLPCA solution, which again yielded the downward curve found in Figure 11.

5. Concluding Remarks

[23] NLPCA was performed on three subsets of the Canadian seasonal SATs: 1900–1949 and 1900–1995 over southern Canada (south of 60°N), and 1950–1995 over the entire country, with the three data sets named S0049, S0095, and C5095, respectively. The SAT anomalies were found to be generally linear during 1900–1949 for all seasons in S0049. However, after 1950, the SAT anomalies showed considerable nonlinearity in winter (DJF) and fall (SON) (but much weaker nonlinearity in spring and summer), indicating interdecadal and seasonal dependence in the nonlinearity of the Canadian SAT. During winter, the leading NLPCA model reveals asymmetric SAT anomaly patterns: At one extreme of the NLPCA mode, there are



Figure 11. Projections of S0095 and its leading NLPCA mode onto the PC_1 - PC_2 plane, shown by the symbol of plus (+) and overlapping circles. Similarly, projections of S0049 and its leading NLPCA mode, by scattered dots and overlapping squares, respectively. The big circle includes the data points which possibly contribute to nonlinearity of SAT anomalies after 1950. The straight line represents the PCA mode 1.

negative NAO effects with a warm eastern Canada-cool western Canada pattern (Figure 4a), and at the other extreme, positive PNA effects with warming over all of Canada (Figure 4b). The negative NAO effects also tend to concur with negative PNA effects, suggesting that the PNA

and NAO effects on Canadian SAT may not be independent, and the NLPCA has been successful in combining the effects of the two.

[24] The absence of notable nonlinearity in the SAT anomalies during 1900–1949 was puzzling. Since non-



Figure 12. Time series of the winter (December through March) index of the North Atlantic Oscillation (NAO), defined as the difference of normalized sea level pressure (SLP) between Lisbon and Stykkisholmur (Lisbon minus Stykkisholmur) since 1864 to 2001, shown with the hollow bars. A 7-year running mean of the NAO index is displayed by the thick line.

linearity was found in the data during 1900–1995 (S0095), this implied nonlinearity of considerable strength in the data after 1950. In fact, the NLPCA mode 1 results from data set S0095 agreed well with those from data set C5095, confirming the robustness of the nonlinearity found in the SAT anomalies in the later period of 1950–1995. Still, there is a concern that the nonlinearity detected in the 1950-1995 period was due to sampling, as nonlinearity was not detected in the 1900-1949 period. Indeed, with short climate records, this possibility cannot be ruled out. However, in the last section, our test with the outlier winter1969 data removed from the NLPCA calculation at least eliminated the possibility that the detected nonlinearity arose from a single outlier year. Furthermore, as negative NAO effects contributed to the nonlinearity in the winter SAT, and strong negative NAO occurred during the 1950s to the early 1970s but not during the 1900–1949 period, this could be a reason for the absence of significant nonlinearity in the winter SAT data before 1950.

[25] The temporal relationship between the NAO and ENSO was investigated by *Huang et al.* [1998] using a multi-resolution cross-spectral technique. Their results show significant coherence between NAO and Niño3 SST in about 70% of the warm events from 1900 to 1995; that is, the strong El Niño events concur with the positive phase of the NAO. During relatively weak Niño3 SST anomalies, they found a teleconnection pattern which shows a strong negative phase of the NAO and a pattern resembling a weak eastward-shifted negative PNA pattern. Interestingly, their results generally corroborate the findings in this paper.

Appendix A: The NLPCA Method

[26] A variable **x**, which consists of *l* spatial stations and *n* observations in time, can be expressed in the form $\mathbf{x}(t) = [x_1, \ldots, x_l]$, where x_i $(i = 1, 2, \ldots, l)$, is a time series of length *n*. PCA is to find a scalar variable *u* and an associated vector **a**, with

$$u(t) = \mathbf{a} \cdot \mathbf{x}(t), \tag{A1}$$

so that

$$\langle \| \mathbf{x}(t) - \mathbf{a}u(t) \|^2 \rangle$$
 is minimized, (A2)

where $\langle \cdots \rangle$ denotes a sample or time mean. Here *u*, called the first principal component (PC), is a time series resulting from a linear combination of the original variables x_i , while **a**, the first eigenvector of the data covariance matrix (alsocalled an empirical orthogonal function, EOF), often describes a spatial pattern. From the residual, $\mathbf{x} - \mathbf{a}u$, the second PCA mode can similarly be extracted, and so on for the higher modes. In practice, the common algorithms for PCA extract all modes simultaneously by calculating the eigenvalues and eigenvectors of the data covariance matrix.

[27] The fundamental difference between NLPCA and PCA is that NLPCA allows a nonlinear continuous mapping from \mathbf{x} to u whereas PCA only allows a linear mapping. To perform NLPCA, the NN in Figure 2 contains three "hidden" layers of variables (or "neurons") between the input and output layers, called the encoding layer, bottleneck, and decoding layer, respectively.

[28] Following *Hsieh* [2001b], four transfer functions f_1 , f_2 , f_3 , f_4 are used to map from the input layer to the output layer ($\mathbf{x} \rightarrow \mathbf{h}^{(x)} \rightarrow u \rightarrow \mathbf{h}^{(u)} \rightarrow \mathbf{x}'$):

$$h_k^{(x)} = f_1((\mathbf{W}^{(x)}\mathbf{x} + \mathbf{b}^{(x)})_k), \tag{A3}$$

$$u = f_2(\mathbf{w}^{(x)} \cdot \mathbf{h}^{(x)} + \bar{b}^{(x)}), \qquad (A4)$$

$$h_k^{(u)} = f_3((\mathbf{w}^{(u)}u + \mathbf{b}^{(u)})_k),$$
 (A5)

$$x'_{i} = f_{4}((\mathbf{W}^{(u)}\mathbf{h}^{(u)} + \bar{\mathbf{b}}^{(u)})_{i}),$$
(A6)

where the capital bold font is reserved for matrices and the small bold font for vectors, **x** is the input column vector of length l, $\mathbf{h}^{(x)}$, a column vector of length m (m is the number of the hidden neurons in the encoding layer), $\mathbf{W}^{(x)}$ is an $m \times l$ weight matrix, $\mathbf{b}^{(x)}$, a column vector of length m containing the bias parameters, and $k \in [1, m]$. The bottleneck layer contains a single neuron, which represents the nonlinear principal component u. The decoding layer contain the same number of neurons m as the encoding layer, and the output layer is also a column vector of length l.

[29] The transfer functions f_1 and f_3 are generally nonlinear (here taken to be the hyperbolic tangent function), while f_2 and f_4 are taken to be the identity function. If the transfer functions f_1 and f_3 are also replaced by a linear function, NLPCA essentially reduces to PCA.

[30] The cost function $J = \langle ||\mathbf{x} - \mathbf{x}'||^2 \rangle$ is minimized by finding the optimal values of $\mathbf{W}^{(x)}$, $\mathbf{b}^{(x)}$, $\mathbf{w}^{(x)}$, $\bar{b}^{(x)}$, $\mathbf{w}^{(u)}$, $\mathbf{b}^{(u)}$, $\mathbf{W}^{(u)}$ and $\bar{\mathbf{b}}^{(u)}$. The MSE (mean square error) between the NN output \mathbf{x}' and the original data \mathbf{x} is thus minimized. [31] Generally, we can impose the constraint $\langle u \rangle = 0$, hence

$$\bar{\boldsymbol{b}}^{(x)} = -\langle \mathbf{w}^{(x)} \cdot \mathbf{h}^{(x)} \rangle. \tag{A7}$$

The total number of free (weight and bias) parameters to be determined is then 2lm + 4m + l. Furthermore, we adopt the normalization condition that $\langle u^2 \rangle = 1$. This condition is approximately satisfied by modifying the cost function to

$$J = \langle \| \mathbf{x} - \mathbf{x}' \|^2 \rangle + (\langle u^2 \rangle - 1)^2.$$
 (A8)

[32] The most serious problem with NLPCA is the presence of local minima in the cost function. As a result, optimizations started from different initial parameters often converge to different minima, rendering the method unstable. This problem can be effectively avoided by adding a weight penalty term into the cost function [*Hsieh*, 2001b].

$$J = \langle \| \mathbf{x} - \mathbf{x}' \|^2 \rangle + (\langle u^2 \rangle - 1)^2 + p \sum_{ki} (W_{ki}^{(x)})^2,$$
(A9)

where p is the weight penalty parameter. With p, the concavity of the cost function is increased, pushing the weights $\mathbf{W}^{(x)}$ to be smaller in magnitude, thereby yielding smoother and less nonlinear solutions than when p is small or zero. With a large enough p, the danger of overfitting is greatly reduced, hence the optimization can proceed until convergence to the global minimum.

[33] The nonlinear optimization was carried out by the MATLAB function "fminu," a quasi-Newton algorithm. Despite of the weight penalty, there is still no guarantee that the optimization algorithm reaches the global minimum. Hence an ensemble of 60 NNs with random initial weights and bias parameters was run. Also, 20% of the data was randomly selected as test data and withheld from the training of the NNs. Runs where the MSE was larger for the test data set than for the training data set were rejected to avoid overfitted solutions. Then the NN with the smallest MSE was selected as the solution. The NLPCA was run repeatedly with m = 2 and 3, and 30 values of p ranging from 0 to 0.18, then the solution with the smallest MSE was chosen as the desired solution.

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References

- Bunkers, M. J., J. R. Miller, and A. T. DeGaetano, An examination of the El Niño or La Niña related precipitation and temperature anomalies across the northern plains, J. Clim., 9, 147–160, 1996.
- Hannachi, A., Toward a nonlinear identification of the atmospheric response to ENSO, J. Clim., 14, 2138–2149, 2001.
- Hoerling, M. P., A. Kumar, and M. Zhong, El Niño, La Niña and the nonlinearity of their teleconnections, J. Clim., 10, 1769–1786, 1997.
- Hoerling, M. P., A. Kumar, and T. Xu, Robustness of the nonlinear climate responses to ENSO's extreme phases, J. Clim., 14, 1277–1293, 2001.
- Horel, J. D., and J. M. Wallace, Planetary scale atmospheric phenomena associated with the Southern Oscillation, *Mon. Weather Rev.*, 109, 813– 929, 1981.
- Hsieh, W. W., Nonlinear canonical correlation analysis by neural networks, *Neural Networks*, 13, 1095–1105, 2000.
- Hsieh, W. W., Nonlinear canonical correlation analysis of the tropical Pacific climate variability using a neural network approach, *J. Clim.*, *14*, 2528–2539, 2001a.
- Hsieh, W. W., Nonlinear principal component analysis by neural networks, *Tellus, Ser. A*, 53, 599-615, 2001b.
- Hsieh, W. W., and B. Tang, Applying neural network models to prediction and data analysis in meteorology and oceanography, *Bull. Am. Meteorol.* Soc., 79, 1855–1870, 1998.
- Huang, J., K. Higuchi, and A. Shabbar, The relationship between the North Atlantic Oscillation and the El Niño-Southern Oscillation, *Geophys. Res. Lett.*, 25, 2707–2710, 1998.

Hurrel, J. W., Decadal trends in the North Atlantic Oscillation: Regional temperatures and precipitation networks, *Sciences*, 269, 676–679, 1995.

- Kalnay, E., et al., The NCEP/NCAR 40-year reanalysis project, Bull. Am. Meteorol. Soc., 77, 437–471, 1996.
- Kramer, M. A., Nonlinear principal component analysis using autoassociative neural networks, AIChE J., 37, 233–243, 1991.
- Monahan, A. H., Nonlinear principal component analysis, Tropical Indo-Pacific sea surface temperature and sea level pressure, *J. Clim.*, 14, 219– 233, 2001.
- Monahan, A. H., J. C. Fyfe, and G. M. Flato, A regime view of Northern Hemisphere atmospheric variability and change under global warming, *Geophys. Res. Lett.*, 27, 1139–1142, 2000.
- Richman, M. B., and D. L. Montroy, Nonlinearities in the signal between El Niño/La Niña events and North American precipitation and temperature, Preprints, 13th Conference on Probability and Statistics in the Atmospheric Sciences, pp. 90–97, Am. Meteorol. Soc., San Francisco, Calif., 1996.
- Ropelewski, C. F., and M. S. Halpert, Precipitation patterns associated with the high index phase of the Southern Oscillation, *J. Clim.*, 2, 268–284, 1989.
- Shabbar, A., and M. Khandekar, The impact of El Niño-Southern Oscillation on the temperature field over Canada, *Atmos. Ocean*, 34, 401–416, 1996.
- Shabbar, A., B. Bonsal, and M. Khandekar, Canadian precipitation patterns associated with Southern Oscillation, J. Clim., 10, 3016–3027, 1997.
- Smith, T. M., R. W. Reynolds, R. E. Livezey, and D. C. Stockes, Reconstruction of historical sea surface temperatures using empirical orthogonal functions, J. Clim., 9, 1403–1420, 1996.
- Trenberth, K. E., and J. W. Hurrell, Decadal atmosphere-ocean variations in the Pacific, *Clim. Dyn.*, 9, 303–319, 1994.
- Trenberth, K. E., G. W. Branstator, D. Karoly, A. Kumar, N.-C. Lau, and C. Ropelewski, Progress during TOGA in understanding and modelling global teleconnections associated with tropical sea surface temperatures, *J. Geophys. Res.*, 103, 14,291–14,324, 1998.
- Vincent, L. A., and D. Gullet, Canadian historical and homogeneous temperature datasets for climate change research, *Int. J. Clim.*, 19, 1375–1388, 1999.
- Wallace, J. M., and D. Gutzler, Teleconnection in the geopotential height field during the Northern Hemisphere winter, *Mon. Weather Rev.*, 109, 784–812, 1981.
- Zhang, X., L. A. Vincent, W. D. Hogg, and A. Niitsoo, Temperature and precipitation trends in Canada during the 20th century, *Atmos. Ocean*, 38, 395–429, 2000.

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