Use of a DSLR Camera and Integrating Sphere To Determine The Luminance of The Moon

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Submitted March 21, 2014
Abstract
A Nikon D80 DLSR CCD camera is used to measure the luminance of the Moon arriving at the Earth’s surface. The camera is calibrated using an Integrating Sphere, a Tenma 7250 illuminance metre, and incandescent light bulbs of 60W and 40W. The weighted average camera calibration constant $K_c$ is determined to be $3.56\pm0.03$. Photos of the moon are taken over the period of March 9-18, 2014, then analyzed using aperture photometry in Maxim DL to obtain average absolute pixel brightness of the lunar disk. Upon correcting for the effects of attenuation by atmospheric extinction, increasing lunar phase is correlated with decreasing luminance, with luminance ranging from $(1.2\pm0.1)\times10^3$cd/m$^2$ to $(5.3\pm0.7)\times10^3$cd/m$^2$ over phase angles of $16.86^\circ \leq \Phi \leq 71.24^\circ$. The variability of measured luminances is attributed to non-linearity of light detection response with respect to changing camera settings in JPEG format, imperfect diffuse light reflection within the Integrating Sphere, and variability in atmospheric conditions.

Introduction

The luminance of an object is a an intrinsic property of light sources describing the brightness of light reflecting off of a surface as perceived by the human eye, (Hiscocks 2008, Taylor 2000). It differs from a similar quantity, illuminance, which is a measure of the incident light upon a surface (Figure 1). Nevertheless, both of these quantities are used in the field of photometry, which by definition pertains to the measurement of light in the visible spectrum; this contrasts with radiometry, which concerns all wavelengths of the electromagnetic spectrum.

![Diagram from www.enablingenvironments.com.au.](image)

**Figure 1:** Visual depiction of the photometric quantities of luminous flux, luminous intensity, illuminance, and luminance$^1$.

Despite the fact that luminance may be measured precisely via a luminance metre, such instruments typically cost upwards of hundreds or thousands of dollars. As such, we explore a relatively inexpensive method of determining the luminance of the moon, using a CCD

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(charge-coupled device) found on a Nikon D80 Digital Single Lens Reflex (DSLR) camera. CCDs operate by allowing incident photons with sufficiently high energy to strike the detector surface, releasing electrons and resulting in the emission of an electron (Peterson 2001), in accordance with the photoelectric effect. They consequently convert photons into digital signals (pixels), effectively counting photons and recording them digitally.

Photometry can be classified into differential, relative and absolute photometry. This study seeks to determine the luminance of the moon via absolute photometry, in which a calibrated instrument is used to determine absolute, as opposed, to relative quantities. In contrast, differential photometry employs known 'calibration stars' in order to provide brightness standards with which target stars may be compared, while relative photometry relates available stars in a frame relative to each other (Romanishin 2006).

Despite the enhanced sensitivity and precision of CCDs and telescopes employed by professional astronomers, DLSR photometry has been shown to be a cost-efficient method of performing absolute photometric measurements (Hiscooks 2013). However, being an easy method for obtaining quantitative data, the light-detecting instrument—the digital camera, in this experiment—must be calibrated in order to calculate relative pixel intensity 'greyscale' values to absolute SI photometric units, such as luminance.

Methods and Background

A) Calibration and use of the Integrating Sphere

Digital SLR cameras have previously been used for the purpose of absolute photometry (Hiscooks 2011). The amount of light received by a digital camera (Hiscooks 2012) is given by

\[ N_d = \frac{K_c S t L_s}{f_s^2} \]  

(1)

where \( N_d \) is the mean pixel brightness in units in greyscale,
\( K_c \) is the experimentally determined camera calibration constant,
\( t \) is the exposure time (the length of time for which the camera shutter is open) in seconds,
\( S \) is the ISO light sensitivity in relative units,
\( f_s \) is the camera’s aperture width (also called F-number or focal ratio) in relative units, and
\( L_s \) is the luminance in candelas per square metre,

which we denote the ‘camera calibration equation.’ Theoretically, Equation (1) allows for the conversion of arbitrary pixel brightness values in greyscale units, to the luminance \( L_s \), defined as the luminous flux per unit area, in SI units of candelas per metre squared. If camera settings of exposure time, ISO light sensitivity and f-stop are known, one need merely calculate the mean pixel \( N_d \) and the
luminance in order to determine $K_c$, an arbitrary constant based upon the particular camera employed.

Previous research (Hiscocks 2013) has indicated that an Integrating Sphere apparatus, consisting of a hollow sphere with two circular holes—one for the light source and one for the photodetector—is an optimal luminance standard. This means that the emitted light from any point along its surface area is observed to be the same all around the sphere.

A further condition of the Integrating Sphere is that it must follow Lambert’s Cosine Law, which states that a Lambertian Reflector is one which diffuses light perfectly isotropically, such that the surface’s luminance is invariant with respect to an observer’s viewing angle (Taylor 2000). For a sphere obeying this law, the luminance can be directly calculated from the illuminance at an exit port in the sphere via the following relationship\(^2\)

\[
L = \frac{EP}{\pi}
\]

where $E$ is the illuminance (in lux) measured by the lux metre at the exit port and $P$ is the reflectance coefficient of the interior sphere surface.

The constant $P$ term has been appended in order to account for cases when the sphere is not a perfectly diffusively reflector; conversely, when this is the case, we have the special case $P=1$. For the experimental Integrating Sphere, the interior surface was coated with Cloverdale Hi-Hide Eggshell Interior Paint (serial number 0350W), for which the manufacturer specifies a reflectance of $84.5\pm1.0\%$. For this project, the 34cm diameter hollow sphere was constructed by covering an inflatable beach ball with several layers of paper mache. Then, after several days of drying, the sphere was cut in half and its inner surface was coated with paint. The two hemispheres were reassembled using tape, with two holes cut out: a larger 6.1cm diameter hole for the light source (further elaborated upon below) and a smaller 2.6 cm hole for the two light-measuring devices: a Nikon D80 DSLR camera (with a AF-S Nikkor 18-70 mm 1:3.5-4.5G ED lens and Tiffen 67mm HAZE-1 UV filter), and a Tenma 72-7250 illuminance (lux) metre.

The sphere ports were situated at an angle of around 90 degrees from one another, such that light from the bulb would not be directly incident upon the lux metre (Figure 2). To further alleviate this potentially undesirable effect, a rectangular barrier known as a baffle (Ducharme et. al. 1997), was placed on the inside surface of the sphere, in-between the light bulb and lux metre. Composed of several layers of white printer paper, the baffle reflects light around the sphere— which would otherwise be directly incident upon the sphere exit port—in an attempt to ensure that light arriving at the exit port was largely non-directional.

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\(^2\) See Hiscocks 2012, section 5.2 for a detailed derivation of the luminance-illuminance relationship pertaining to a light source entering an Integrating Sphere.
**Figure 2:** Diagrammatic representation of an Integrating Sphere (diameter=33cm). The light source enters the sphere through the entrance aperture and then reflects uniformly off of the diffusely reflecting coating before reaching the light-detecting lux metre. A baffle is installed halfway in between the entrance and exit ports to reduce direct transmission of light from the light bulb to detector.

Calibration began by placing a 60W Sylvana incandescent bulb barely inside the large sphere port. This was done by allowing the bulb to run for several minutes so as to reduce potential variation in light output due to temperature fluctuations in the tungsten filament. The camera was then centred at the exit sphere port, adjusting image magnification such that this port nearly, though not entirely filled the camera’s field of view (Figure 3). Camera settings were adjusted so that the pixel value histogram obtained a maximum value of approximately 80%, so reduce the possibility of light oversaturation in the image. A lux metre was then placed at the same position and an illuminance measurement was taken. The same process was then repeated using a 40W incandescent bulb of the same model. All measurements were taken in a completely dark room so in order to reduce interference from external lighting in the calibration. At all times (as well as for moon observations), a UV filter was mounted on the lens to prevent UV light from entering and being recorded by the CCD. This is essential since the luminosity function essential to photometry covers only the visible EM spectrum.
Figure 3. Setup of the Integrating Sphere apparatus used for camera calibration. Light from the light bulb enters the sphere at a hole in the sphere’s backside (denoted the entrance port, though not visible), and exits at the exit port, where the camera and then the lux metre detect light.

B) Lunar Observation Method

We selected clear nights to observe the moon, mounting the camera on a tripod. Despite the proximity of the observing location to the urban area of Vancouver, British Columbia and its associated light pollution, an area of the UBC campus close to tall trees and relatively secluded from apartment lighting or roadside lamps was selected.

Camera settings were recorded and manipulated in “Manual” mode in order to prevent over-saturation, after which 10-20 pictures were taken using JPEG (8-bit) format. Settings of the camera were chosen by first looking at the saturation of the photo and then adjusting the exposure time $t$, ISO light sensitivity $S$, and aperture width $f_s$ such that the maximum RGB values remained below approximately 80% of saturation. Specifically, a low ISO setting was chosen in most data sets in order to avoid grain and quality deterioration in the photos - this was especially important in light of the fact that photos were taken in the dark. Camera zoom was set to maximum in order to increase the absolute number of ‘bright’ pixels. To reduce uncertainty, no pictures were taken with visible clouds in the direct path of light. Furthermore, considering that changes in atmospheric conditions (such as cloud cover and position) could affect luminance measurements, pictures were taken in quick succession over a maximum time interval of two minutes, minimizing the time interval and thus reducing potential error\(^3\). Finally, the camera’s focal length was set to infinity to allow for the most effective image focus.

\(^3\) Another reason to minimize the time interval is the changing of the moon’s position within the sky during the night. In particular, its zenith angle is always changing, which affects the amount of atmosphere that light from the moon must pass through on its way to the camera at the Earth’s surface.
Results and Data Analysis

Data analysis shall be treated in two parts: Integrating Sphere calibration (A) and analysis of lunar observations with subsequent determination of lunar luminance (B). During both processes, multiple exposures were stacked into one ‘master’ photo, so as to simplify data analysis as well as increase signal-to-noise ratio. In the process of stacking, small changes were observed in the positions of objects with respect to the frame of the photo; to compensate, a computational aligning technique is used before stacking. While this technique was usually successful, auto alignment sometimes produced bad results, in which case manual alignment was implemented.

A) Calibration using the Integrating Sphere

The image processing software Maxim DL (Diffraction Limited) was used to analyze pixel brightness values, in order to obtain an average for each picture. For each picture, a rectangular area of approximate dimensions 1250x1250 (~1.5*10^6 pixels) was selected inside the frame to obtain a mean pixel value for stacked image set (Figure 4). Due to the diffraction of incoming light into the lens and consequent impossibility of focusing at the sphere exit port, the area of analysis was selected to be the middle of the hole, since the outer part of the hole was not resolved. Standard deviation of the pixel values, determined by the software, was assumed to be the uncertainty. This uncertainty was then propagated with the uncertainty in lux metre measurements to determine the uncertainty in $K_e$. The average $K_e$ was then obtained via weighted averaging of all measurements.

Figure 4: Photograph taken through the exit port of the integrating sphere, converted from RGB colour to black and white using Maxim DL astronomical image processing software. Mean pixel brightness (‘Average’) and standard deviation (‘Std Dev’) of the highlighted area is then calculated using the software, with the area being manually selected as a square within the confines of the circle.
Various combinations of camera settings were tested in order to determine $K_c$ value by the following procedure. Specifically, 5 photos were taken were taken for a given combination of settings, after which the process was repeated by varying one setting twice while keeping the others fixed (Table 1). A weighted average for $K_c$ was calculated to be $3.56 \pm 0.03$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
Trial & Power of & ISO & Aperture & Exposure & Average Pixel & Standard & Illuminance & Luminance & Camera \\
Number & Incandescent & (5) & width (f) & time (t [s]) & Value & Deviation of & (E [lux]) & (Ls [cd/m²]) & Calibration \\
Sample & Light Bulb & & & & (N_d [greyscale]) & Average Pixel & & & Constant \\
Size & (P [W]) & & & & & Value & & & (K) \\
\hline
1 & 60 & 100 & 3.5 & 0.005 & 165.24 & 2.23 & 3710±50 & 1,080±10 & 3.76±0.07 \\
2a & 60 & 320 & 5.6 & 0.005 & 159.90 & 2.78 & 3480±80 & 1,010±20 & 3.10±0.09 \\
2b & 60 & 320 & 5.6 & 0.004 & 142.65 & 3.02 & 3480±80 & 1,010±20 & 3.46±0.11 \\
2c & 60 & 320 & 5.6 & 0.003 & 126.85 & 2.89 & 3480±80 & 1,010±20 & 3.94±0.13 \\
3a & 60 & 250 & 5.6 & 0.005 & 141.45 & 3.24 & 3480±80 & 1,010±20 & 3.51±0.11 \\
3b & 60 & 200 & 5.6 & 0.005 & 128.17 & 3.13 & 3480±80 & 1,010±20 & 3.98±0.13 \\
3c & 60 & 160 & 5.6 & 0.005 & 110.000 & 3.08 & 3480±80 & 1,010±20 & 3.39±0.10 \\
4a & 60 & 200 & 4.5 & 0.005 & 169.17 & 3.17 & 3480±80 & 1,010±20 & 3.38±0.11 \\
4b & 60 & 200 & 5 & 0.005 & 136.50 & 2.93 & 3480±80 & 1,010±20 & 4.19±0.15 \\
4c & 60 & 200 & 6.3 & 0.005 & 106.66 & 3.02 & 3480±80 & 606±17 & 3.30±0.11 \\
5 & 40 & 100 & 3.5 & 0.008 & 156.50 & 5.31 & 2110±50 & 612±15 & 3.91±0.16 \\
6a & 40 & 200 & 4.5 & 0.008 & 158.32 & 2.23 & 2090±60 & 606±17 & 3.30±0.11 \\
6b & 40 & 200 & 5 & 0.008 & 169.17 & 2.45 & 2090±60 & 606±17 & 3.49±0.12 \\
6c & 40 & 200 & 5.6 & 0.008 & 136.61 & 2.56 & 2090±60 & 606±17 & 3.92±0.14 \\
7a & 40 & 400 & 6.3 & 0.008 & 150.41 & 2.61 & 2090±60 & 606±17 & 3.08±0.10 \\
7b & 40 & 320 & 6.3 & 0.008 & 134.54 & 2.90 & 2090±60 & 606±17 & 3.44±0.12 \\
7c & 40 & 250 & 6.3 & 0.008 & 117.75 & 2.74 & 2090±60 & 606±17 & 3.85±0.14 \\
8a & 40 & 250 & 4.5 & 0.006 & 121.20 & 2.56 & 2090±60 & 606±17 & 5.09±0.11 \\
8b & 40 & 250 & 4.5 & 0.004 & 134.54 & 2.47 & 2090±60 & 606±17 & 4.59±0.16 \\
8c & 40 & 250 & 4.5 & 0.003 & 107.10 & 2.67 & 2090±60 & 606±17 & 4.58±0.17 \\
\hline
\end{tabular}
\caption{Calibration of Digital Camera via Integrating Sphere}
\end{table}

B) Lunar Observation and Luminance Determination

Camera photographs were taken on 6 days over the period from March 9 - March 18, 2014 (Table 2), with supplementary information such as lunar zenith and phase angles obtained associated with each observing date obtained from the “Sun and Moon Azimuth Table” (a database provided by the United States Naval Observatory).
To initiate image processing, sets of lunar images were ‘stacked’ (combined into one representative image) using Maxim DL software, effectively averaging the pixel brightnesses $N_d$ across each data set (Table 3). Then, the method of aperture photometry was employed. First, a circular area was drawn around the outline of the lit region, representing light from the moon (Figure 5). In cases where the lit region was hemi-spherical, a small area within the region was selected. In both cases, the mean pixel brightness was determined by the software, in addition to its associated standard deviation. Then, using the same tool, a large circle (called the annulus) was drawn outside the moon, measuring the average brightness of the background surrounding the moon. This background value was subtracted from the mean moon pixel brightness to obtain a new background-corrected mean brightness.

Table 3. Determination of Mean Lunar Pixel Brightness via Aperture Photometry

<table>
<thead>
<tr>
<th>Observation Date</th>
<th>Number of Photos stacked</th>
<th>Time [PST (24h)]</th>
<th>ISO</th>
<th>Aperture Width ($F_s$ [f-stop number])</th>
<th>Exposure Time ($t$ [s])</th>
<th>Mean Pixel Brightness of Moon [greyscale]</th>
<th>Mean Pixel Brightness of Background [greyscale]</th>
<th>Mean Moon Pixel Brightness after Bkgd. Subtraction [greyscale]</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/09/2014</td>
<td>15</td>
<td>23:14</td>
<td>800</td>
<td>7.2</td>
<td>0.00625</td>
<td>200.74 ± 19.32</td>
<td>2.18 ± 0.48</td>
<td>198.56 ± 19.33</td>
</tr>
<tr>
<td>03/10/2014</td>
<td>21</td>
<td>21:48</td>
<td>400</td>
<td>4.5</td>
<td>0.002</td>
<td>181.66 ± 15.65</td>
<td>1.21 ± 0.41</td>
<td>180.45 ± 15.60</td>
</tr>
<tr>
<td>03/11/2014</td>
<td>15</td>
<td>21:40</td>
<td>320</td>
<td>5.6</td>
<td>0.0025</td>
<td>143.71 ± 19.64</td>
<td>1.09 ± 0.28</td>
<td>142.62 ± 19.64</td>
</tr>
<tr>
<td>03/13/2014</td>
<td>15</td>
<td>01:53</td>
<td>320</td>
<td>5.6</td>
<td>0.002</td>
<td>129.28 ± 26.37</td>
<td>1.60 ± 0.18</td>
<td>127.68 ± 26.37</td>
</tr>
<tr>
<td>03/14/2014</td>
<td>14</td>
<td>20:40</td>
<td>320</td>
<td>5.6</td>
<td>0.002</td>
<td>153.17 ± 18.80</td>
<td>2.23 ± 0.40</td>
<td>150.94 ± 18.80</td>
</tr>
<tr>
<td>03/18/2014</td>
<td>13</td>
<td>00:06</td>
<td>200</td>
<td>5.0</td>
<td>0.0025</td>
<td>148.57 ± 16.49</td>
<td>1.39 ± 0.12</td>
<td>147.18 ± 16.49</td>
</tr>
</tbody>
</table>

4 The United States Naval Observatory (USNO). *Sun or Moon Altitude/Azimuth Table*. April 22, 2013.
5 The zenith angle is complementary to the altitude.
7 Ibid.
8 Whereas $\Phi > 0$ is indicative the moon is in approaching full moon (waxing), $\Phi < 0$ indicates leaving full moon (waning).
9 The method of aperture photometry employed was derived from that used by Mighell (1999).
Figure 5. Determination of the Moon’s luminance with aperture photometry. The average value of the background (the night sky) is taken to be that of the outer ring, which is then subtracted from the mean value of the innermost circle.

C) Determination of Lunar Luminance with Correction for Atmospheric Extinction

Having determined the calculated pixel value $N_d$ and the average camera calibration constant $K_c$, luminance values of the moon arriving at the Earth’s surface were calculated in Table 4 for each observation date, using the camera calibration equation: effectively the inverse of the process used to calibrate the camera.

However, the fact that zenith angle was not held constant means that a correction must be applied, as the zenith angle dictates the amount of atmosphere that light must traverse to reach the Earth’s surface. Previous research has lead to the development of various physical models for the attenuation of light in the atmosphere (Courter 2003, Green 1992), termed ‘atmospheric extinction’. In accordance with the following model conceived by Ackermann and Goesele (2013), it is assumed that light from the Moon travels in a straight line towards Earth until reaching sea level, during which it traverses a column of air in the atmosphere termed an ‘airmass’ (denoted as $A$). During its interactions with the atmosphere, the light is scattered primarily by two modes: Rayleigh scattering and aerosol scattering, for which average values of $R_{ray} = 0.1451$ and $R_{aer} = 0.12$ are assumed (Ackermann and Geosele 2013).
Accordingly, the lunar luminance upon accounting for atmospheric extinction $L_m$ is given by the following set of equations:

$$
L_m = L_m(\text{observed})F
$$

$$
F = 2.512(R_{Ray} + R_{aer})A
$$

$$
F = 2.512^{0.1451 + 0.12}A
$$

$$
F = 1.2766A
$$

$$
A = \frac{1}{\cos(z) + 0.25e^{-1.1\cos(z)}}
$$

where $F$ is a factor by which the experimentally observed luminance must be multiplied in order to correct for atmospheric extinction,

$R_{Ray}$ is the Rayleigh scattering coefficient,

$R_{aer}$ is the aerosol scattering coefficient,

$A$ is given by the Rozenberg equation (Rozenberg 1966), and

$z$ is the zenith angle in degrees, defined as the angle between a line normal to the Earth’s surface at the observing location and another line extending toward the moon.

It should be noted that $A$ and thus $F$ will be at a minimum value of one (representing one airmass) when the phase angle $\Phi$ is equal to zero, corresponding to the full moon. This is because light from the moon traverses the least amount of atmosphere at this point, and is therefore scattered to the least extent. Conversely, the amount of atmosphere traversed increases as zenith angle increases (towards the horizon), resulting in a larger correction factor $F$ and a commensurately larger corrected lunar luminance $L_m$ (Table 4). It follows from Equation (3) that the moon luminance observed $L_m(\text{observed})$ which has been attenuated by the atmosphere, will be less than that which would be observed in the absence of the atmosphere, $L_m$.

The altitude (angle of elevation) was obtained by inputting position (latitude and longitude) and time coordinates (time zone with respect to GMT) into a reference database (United States Naval Observatory) for each observation date. Then, using the fact that zenith angle is complementary to altitude, lunar luminance accounting for atmospheric extinction was calculated (Table 4, Figure 6). Values of $L_m$ are decrease as the phase angle increases, or as the Moon regresses from full Moon and approaches New Moon (Figure 6).
Table 4: Summary of Moon Data & Luminance Calculations with Atmospheric Extinction

<table>
<thead>
<tr>
<th>Moon Observation Dates [m/d/y]</th>
<th>Time [PST (24h)]</th>
<th>Lunar Phase Angle (°)</th>
<th>Percentage of Moon Disk Illuminated (%)</th>
<th>Zenith Angle (°)</th>
<th>Moon Luminance at Earth's Surface ($L_{m(observed)}$) [cd/m²]</th>
<th>Moon Luminance Accounting for Atmospheric Extinction ($L_{m}$ [cd/m²])</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/09/2014</td>
<td>23:14</td>
<td>71.24</td>
<td>66.08</td>
<td>53.7 ± 0.5</td>
<td>(0.58 ± 0.06) x 10^3</td>
<td>(1.2 ± 0.1) x 10^3</td>
</tr>
<tr>
<td>03/10/2014</td>
<td>21:48</td>
<td>60.79</td>
<td>74.40</td>
<td>37.4 ± 0.3</td>
<td>(1.3 ± 0.1) x 10^3</td>
<td>(2.1 ± 0.2) x 10^3</td>
</tr>
<tr>
<td>03/11/2014</td>
<td>21:40</td>
<td>49.92</td>
<td>82.19</td>
<td>35.8 ± 0.1</td>
<td>(1.6 ± 0.2) x 10^3</td>
<td>(2.5 ± 0.3) x 10^3</td>
</tr>
<tr>
<td>03/13/2014</td>
<td>01:53</td>
<td>37.58</td>
<td>89.62</td>
<td>62.4 ± 0.5</td>
<td>(1.8 ± 0.4) x 10^3</td>
<td>(4.8 ± 0.8) x 10^3</td>
</tr>
<tr>
<td>03/14/2014</td>
<td>20:40</td>
<td>16.86</td>
<td>97.85</td>
<td>49.2 ± 0.3</td>
<td>(2.1 ± 0.3) x 10^3</td>
<td>(4.1 ± 0.6) x 10^3</td>
</tr>
<tr>
<td>03/18/2014</td>
<td>00:06</td>
<td>19.40</td>
<td>97.16</td>
<td>60.3 ± 0.2</td>
<td>(2.1 ± 0.2) x 10^3</td>
<td>(5.3 ± 0.7) x 10^3</td>
</tr>
</tbody>
</table>

Figure 6: Lunar luminances accounting for atmospheric extinction, as a function of phase angle. Red circles represent luminance measurements without atmospheric correction ($L_{m(observed)}$) while black circles represent luminance measurements with atmospheric correction ($L_{m}$). Decreasing phase angle indicates waxing of the Moon - corresponding to an increase in percentage of surface illuminated - with full moon attained at $\Phi = 3.53^\circ$ for this lunar cycle.
To convert these luminances to the apparent magnitudes commonly used to quantify celestial bodies—a logarithmic scale—the following formulae are used, given by Karboulonis (2014) and Hiscocks (2011), respectively:

\[ m = S_b - 2.514 \log_{10} \left[ \pi (0.5A_d)^2 \right] \]

(4)

\[ S_b = - \log_{10} \left( \frac{10.8 \times 10^3 L_m}{0.4} \right) \]

where \( m \) is the apparent magnitude

\( S_b \) is the surface brightness of the moon in magnitudes per square arcsecond

\( A_d \) is the angular diameter in arcseconds

Combining equations:

\[ m = - \log_{10} \left( \frac{10.8 \times 10^3 L_m}{0.4} \right) - 2.514 \log_{10} \left[ \pi (0.5A_d)^2 \right] \]

(5)

The angular diameter of the moon is referenced from the Horizons Web Interface (NASA Jet Propulsion Laboratory), with the uncertainty estimated by examining the variation in \( A_d \) over the course of several minutes (Table 5). These results are summarized a phase curve (Figure 7), comparing experimental apparent magnitudes with those obtained from a the Horizons Database (NASA Jet Propulsion Laboratory). It should be noted that the leftmost point at \( \Phi = 16.86^\circ \) was in fact taken after Full moon (waning); phase angle was taken to be symmetrical at equal angles away from full moon.

<table>
<thead>
<tr>
<th>Moon Luminance Accounting for Atmospheric Extinction ((L_m \text{ [cd/m}^2\text{]}))</th>
<th>Angular Diameter of Moon ((A_d \text{ [arcseconds]})</th>
<th>Apparent Magnitude ((m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1.2 \pm 0.1) \times 10^3)</td>
<td>(1,790.7 \pm 0.2)</td>
<td>(-11.25 \pm 0.11)</td>
</tr>
<tr>
<td>((2.1 \pm 0.2) \times 10^3)</td>
<td>(1,791.3 \pm 0.2)</td>
<td>(-11.79 \pm 0.09)</td>
</tr>
<tr>
<td>((2.5 \pm 0.3) \times 10^3)</td>
<td>(1,791.3 \pm 0.2)</td>
<td>(-11.99 \pm 0.15)</td>
</tr>
<tr>
<td>((4.8 \pm 0.8) \times 10^3)</td>
<td>(1,786.9 \pm 0.2)</td>
<td>(-12.72 \pm 0.23)</td>
</tr>
<tr>
<td>((4.1 \pm 0.6) \times 10^3)</td>
<td>(1,810.5 \pm 0.2)</td>
<td>(-12.55 \pm 0.14)</td>
</tr>
<tr>
<td>((5.3 \pm 0.7) \times 10^3)</td>
<td>(1,851.9 \pm 0.2)</td>
<td>(-12.90 \pm 0.12)</td>
</tr>
</tbody>
</table>
Figure 7. Lunar phase curve depicting apparent magnitude as a function of phase angle. Black circles represent values of apparent magnitude, converted from luminance. Green triangles representing historical data from the Horizons Web Interface (NASA Jet Propulsion Laboratory) are provided for comparison, with inputs of (49.2500° N, 23.1000° W), altitude 80m, and GMT -8.

Discussion

Collectively, experimentally obtained luminance values both with and without atmospheric correction, in addition to their associated magnitude conversions, exhibit a negative correlation with respect to increasing phase angles (Figures 6, 7), where more negative magnitudes are defined to be greater, corresponding to a brighter moon. Qualitatively, an intuitive explanation for this tendency is that the moon receives, being closer to the sun, receives greater solar illumination at lower phase angles, which is subsequently reflected to observers on Earth.

In Figure 7, comparison with historical lunar magnitudes from the Horizons Web-Interface Database (NASA Jet Propulsion Laboratory) suggests that the experimentally determined luminance values are more negative than should be expected. Specifically, experimental magnitudes are more negative by an average of 0.68 magnitudes, which is to say that this project’s calculations overestimate the brightness of the moon. Moreover, the degree and consistency of this offset implies the presence of systematic error, which will be the predominant focus of discussion in ensuing sections.
If one is to use the Horizons data as a reference with which assess the accuracy of experimental values, it should be noted that the value of the measurement at $\Phi=38^\circ$ appears anomalously large. This may be explained by the fact the moon’s zenith angle of $62.4^\circ$ at this time is quite large (being a maximum within the dataset); as such, the atmospheric extinction equations employed (Equation 3) may not adequately account for extinction at high zenith angles.

**Error Analysis**

*i) Calibration*

Upon examining Table 1, it is clear that the calculated values of $K_c$ for each permutation of camera settings are variable. This implies that the camera settings used were not linear since $K_c$ was not constant, as should have been the case to justify the use of Equation (1). For example, for measurements 7a to 7b to 7c in which ISO decreases from 400 to 320 to 250 while exposure time and aperture width remain constant, $K_c$ increases from 3.08 to 3.44 to 3.85. Furthermore, average pixel brightness values $N_d$ decrease more slowly from 7a-7c: while $S$ (ISO) decreases by a factor of $400/250 = 1.6$, $N_d$ only decreases by a factor of $150.41/117.75 \approx 1.3$. This implies that ISO values decreased at a less than linearly proportional rate. A similar deviation occurs for the exposure time $t$ from 2a-2c, as well as for the aperture width $f$, from 6a-6c\(^{10}\), indicating that all of the Nikon D80’s camera settings exhibit a non-linear response. It is inferred that the nature of this non-linearity is such that each change in setting changes the CCD’s photon gathering properties to a lesser degree than Equation (1) would dictate.

A more advanced DSLR camera with greater linearity in its settings would be of greatest benefit, due to the potential for more uniform $K_c$ value across the parameter space of camera settings. Otherwise, a wider variety of settings could be tested; for instance, one could test six permutations of ISO values while the other camera settings are held constant, rather than three permutations. A line of best fit could then be calculated. Then, the other camera settings (aperture width and exposure time) could each be systematically varied in this manner in order to construct a new camera calibration equation. This would allow for the precise quantification of observed non-linearities in each camera setting.

Regarding the Integrating Sphere, it fulfilled the role of being a uniform, diffuse Lambertian Reflector to an extent. While the percent difference in pixel brightness (grey value) between the left and right of camera images was typically less than 10\%, images were always brighter towards the left, in the direction of the entrance port (Figure 8). This can accounted for by the simple fact the light source was located in this direction; thus, due to imperfect screening by the baffle, the left side was always at least slightly brighter than the right: an effect often observed in the form of a gradual fading.

\(^{10}\) Though aperture width is in the denominator of Equation (1) unlike ISO and exposure time, the deviation in $K_c$ due to changing aperture width occurs in the same direction as for ISO and exposure time, resulting in the same conclusion about the non-linearity of aperture width.
toward the left-hand side of most images. In terms of the steep drop-off in gray value towards the left and right edges (Figure 8), the fact that the camera was not able to focus at these locations due to extreme proximity to the lens may have played a role. The picture was not resolved properly at these extremes and therefore would not produce accurate pixel values. As such, only the central portion of images were analyzed for average pixel brightness, in order to reduce this distorting effect.

![Graph showing distribution of pixel brightness values](Image)

**Figure 8:** Distribution of Pixel Brightness Values (‘Grey Values’) along a horizontal line (yellow) within the Integrating Sphere exit port, from left to right. Though relatively constant within the central regions, a slight increase in brightness is discernible toward the left of the photograph. From photo stack taken with settings of the 40W bulb, ISO 320, aperture width (f-stop) 6.3, and 0.008s exposure time.

According to the results of the calibration, a model that used to measure $K_c$ was not particularly consistent, as experimental $K_c$ values varied by up to 43%. There are two probable inferences: firstly, that the model does not accurately relate the given variables, and secondly, the camera having inaccurate projection of the settings selected. There is also a small uncertainty in between each measurement as the tripod could have moved or the luminance of a light bulb has slightly changed, but these uncertainties were under control and were reduced as much as possible. Therefore, a separate experiments should be carried out as these uncertainties produced the largest fraction of variability of obtained $K_c$ values. Using a different camera in calibration can possibly shed light on accuracy of the model used.
ii) Lunar Observation and Luminance Measurements

Regarding calculated lunar luminance values with atmospheric correction, relative uncertainties are in the range of 9-21%. Quantitatively, the two most substantial contributors to this uncertainty were the diverse topography of the lunar surface and atmospheric extinction. The first resulted in an intermixing of dark and bright patches of the lunar surface in photographs during image processing of lunar photographs (Figure 5), as demonstrated by large standard deviations in the range of 9-20% of the mean pixel brightness. The second, concerning atmospheric extinction, was due to the fact that the mathematical model presented by Ackerman and Goesele (2013) relies upon an averaged value of the extinction coefficient \(k\), for which it is difficult to determine a precise value. Seeing as the value of \(k\) varies significantly depending on local atmospheric conditions, a more precise accounting of atmospheric effects would require knowledge of the specific optical properties of the atmosphere directly overlying the observation site.

Another issue was systematic error attributed to sky light pollution at the observing site (a suburban university campus), which increased the average brightness of the sky and thus reduced the signal to noise ratio, or SNR (Romanishin 2006). However, the fact that moon photos were stacked would have reduced the SNR by a factor of 5-10 depending on the number of photos stacked. Furthermore, the use of stacking in our photos would have improved the SNR by a factor of the square root of \(N\), where \(N\) is the number of photos taken per observation set (Starizona). The stacking process would have also reduced the effect of ‘dark pixels’: pixels whose values were observed to be abnormally low, upon examination of histograms in a spreadsheet. This is particularly important because individual photos were observed to contain pixels of 0 brightness. Stacked photos had significantly more rounded curves in their associated distance-brightness graphs, with fewer such anomalous pixels.

There are other factors that should be taken into account but were assumed to be constant. Primarily, due to the fact that the albedo across the surface of the Moon varies depending upon the particular part of its surface being illuminated, the change of surface illuminance can cause a change in luminance. However, as the changes in distance are insignificant and the difference in surface observed is 9% at its maximum during the whole cycle, this effect was not considered significant. As another example, earthshine -an effect of light reflecting from the surface of the Earth and then being reflected at the surface of the Moon- is only noticeable during crescent phases of the Moon. The changes of distance from the Moon to Earth are 15 thousand kilometres at maximum in our measurements (3.7% of the apogee distance), while maximum difference possible is 42 thousand kilometres (The United States Naval Observatory). From the resulting change of distance, the libration effect also comes into effect, by which we can observe a slightly different lunar surface from time to time. The illuminance upon the surface of the Moon from the Sun is also assumed to be constant, since the Sun’s luminosity, temperature and distance is fairly constant with time. (The United States Naval Observatory).

There are multiple possibilities of error associated with CCD readout and the camera lens.
Hot and cold pixels being exceptionally rare in the CCD would not contribute to uncertainty in a large effect. Moreover, there is non-image thermal and electronic noise generated by the CCD, which could be remedied by taking a set of dark frames taken for each set of camera settings. Including flat field frames would help reduce internal reflections in the camera as well as accounting for the dust and various particles present on the surface of the lens (Starizona, Diffraction Limited). Though some dark and flat field frames were taken, it was decided that their effect would be negligible, with errors associated with them being at most 2% of the average pixel values.

**Future Directions**

In terms of improvements to future experiments, the increased resolution from utilizing RAW format would likely improve precision amongst $K_e$ values, and consequently precision in calculated lunar luminances. Of further benefit would be an integrating sphere of less imperfect spherical geometry, as well as greater reflectance. For instance, an alternate material and construction method could be chosen, such as that entailing the combination of two hemispherical metal mixing bowls. this would also have the ancillary benefit of potentially reducing the amount of light exiting the sphere through areas other than the exit port, such as through the narrow gaps present in the integrating sphere designed for this experiment. Furthermore, a sphere coating of flat, white paint containing barium sulfate could be utilized, due to its having a significantly higher reflectance of 94.9% (Nobel 2008) compared to 84.5% for the Cloverdale Hi-Hide Eggshell Interior Paint. Multiple applications of the paint would have also reduced the amount of light escaping. A combination of these measures would improve measurement of the mean pixel value at the sphere exit port, and consequently precision in the average calibration constant $K_e$.

To account for and improve accuracy in the midst of fluctuating atmospheric conditions, an alternate method to aperture photometry such as differential photometry could be utilized, in which target objects in the sky are compared with reference objects (typically stars) in order to ascertain their magnitudes (Romanishin 2006).

Due to prohibitive weather conditions that limited the number of observing nights, measurements of luminances at small phase angles could not be obtained. However, if such data were obtained in future experiments, a significant divergence from a linear trend would be expected; indicating that the lunar surface is non-Lambertian in nature. This would be due to the presence of aforementioned irregularities on the Moon’s surface in the form of craters. The result is that luminance increases to a disproportionately greater extent at low phase angles in what is known as the opposition effect (Buratti, Hillier, Wang 1996). Furthermore, due to the irregularities of the geometry of the surface, one could expect variation of albedo with respect to phase angle, and thus variation in the rate of change of luminance. Another contributor to this imperfection is the moon’s chemical structure, which allows incident sunlight to penetrate into several layers of subsurface and be released into various directions non-uniformly, at different wavelengths (Medkeff 2002).
Conclusion

Calculated lunar luminance values using an Integrating Sphere and a Nikon D80 DLSR camera suggested a negative correlation between the moon’s phase angle and luminance. Systematic errors were apparent, including factors such as imperfections in the uniformity of the integrating sphere and the extent to which it exhibited Lambertian reflectance, non-linear light response of the camera’s CCD with respect to the three camera settings, and atmospheric extinction that proved difficult to quantify precisely.

Considering the low cost of equipment used in this project, the fact that the determination of luminance and apparent magnitude resembles that obtained from a NASA database is encouraging. As such, calibration using the integrating sphere and aperture photometry should not be further explored as an inexpensive method for luminance determination. Greater availability of data, in terms of both camera setting permutations and lunar observation days, could substantially improve consistency of the camera calibration constant and the eponymous equation it dictates.

Acknowledgements

We thank our mentor Wayne Nagata for timely advice during the initial stages of our project, as well as James Charbonneau for giving us extremely valuable suggestions and ideas concerning our modest project when difficulties inevitably arose, in addition to lending us the Nikon digital camera.
References


