# A8-3 Model Rocket Impulse Measurement 

Magnus Haw<br>Science One 2008-09

May 10, 2009


#### Abstract

An experiment was performed to measure the impulse of the A8-3 Estes model rocket engine. The measurements were obtained by mounting an engine vertically onto a frame and measuring the changing weight of the assembly during the firing period for six trials. Thrust was calculated by the difference in measured weight from the rest weight while accounting for the mass loss rate. The impulse of each of the six thrust curves was averaged to find the final impulse measurement: $2.36 \pm .16 \mathrm{~N}$-s. Estimates of the average thrust and exhaust velocity were also calculated from the impulse measurement.


## 1 Introduction

The purpose of this experiment is to measure the impulse of an A8-3 rocket engine. The impulse for a rocket engine, $I_{r}$, is defined as:

$$
\begin{equation*}
I_{r}=\int_{0}^{t_{f}} T(t) d t=\Delta p \tag{1}
\end{equation*}
$$

where $T(t)$ is the thrust of the rocket, $t_{f}$ is the thrust duration, and $\Delta p$ is the change in momentum. The impulse is important because it gives a normalized value for the launch capacity of the rocket and is used to classify rockets into different ranks (A, B, C class etc. with A class being the least powerful). Since the change in momentum cannot be measured directly, this experiment found the impulse by measuring the thrust of the engine as a function of time.

The thrust is the force (generated by mass outflux from the engine) that the engine exerts on the rocket. The equation for thrust is derived in Equations 2-5. Given $\mathrm{m} \& \mathrm{v}$ are the rocket mass and velocity and m' \& v' are the exhaust mass and exhaust velocity:

$$
\begin{align*}
\frac{d p}{d t} & =\frac{d}{d t}(m v)+\frac{d}{d t}\left(m^{\prime} v^{\prime}\right)=0  \tag{2}\\
0 & =m \frac{d v}{d t}+v \frac{d m}{d t}+m^{\prime} \frac{d v^{\prime}}{d t}+v^{\prime} \frac{d m^{\prime}}{d t} \tag{3}
\end{align*}
$$

Since the exhaust experiences no acceleration after being vented, $\frac{d v^{\prime}}{d t}=0$ and $\frac{d m^{\prime}}{d t}=-\frac{d m}{d t}$ because the total mass of the rocket/exhaust system is conserved:

$$
\begin{align*}
m \frac{d v}{d t} & =\frac{d m}{d t}\left(v^{\prime}-v\right)  \tag{4}\\
T(t) & =-\frac{d m}{d t} u \tag{5}
\end{align*}
$$

where u is the velocity of the exhaust relative to the rocket [3]. This experiment seeks to measure thrust by measuring the changing weight during the thrusting period. To find $\mathrm{T}(\mathrm{t})$, the rest weight is subtracted from the measured weight (Equation 6).

$$
\begin{equation*}
T\left(t_{i}\right)=W\left(t_{i}\right)-M\left(t_{i}\right) * g . \tag{6}
\end{equation*}
$$

The mass loss rate must also be modeled because it changes the rest mass of the rocket; however, since the burn time is so short ( 0.8 s ) and the mass loss rate is quite small $(4 \mathrm{~g} / \mathrm{s})$, this experiment uses a linear approximation for $\mathrm{M}(\mathrm{t})$ (Equation 7).

$$
\begin{equation*}
M(t)=M_{0}-\frac{\Delta M_{\max }}{t_{f}} * t \tag{7}
\end{equation*}
$$

After finding $T\left(t_{i}\right)$, the general expression for the impulse $I_{r}$ at a sample rate $\frac{1}{\Delta t}$ is:

$$
I_{r} \approx \sum_{0}^{t_{f}}\left[W\left(t_{i}\right)-M\left(t_{i}\right) * g\right] \Delta t
$$



Figure 1: Vertical and Horizontal Frames: These photos show the vertical and horizontal frame structures, respectively. The image on the left shows the vertical frame (20x20x30 cm ) and the pre-ignition setup (Trial 6). The photo on the right shows the horizontal frame (10x20x20cm) during ignition (Trial 5).

## 2 Methods

This experiment measured the thrust of the A8-3 model rocket engine by securing rockets to a frame and measuring the weight of the rocket-frame system during ignition. The support frame was constructed out of half-inch pvc pipe with angle braces mounted on the top to secure the engine (see Figure 1).

For each trial, the engine would be mounted onto the frame with screws via the angle braces and adjusted until the nozzle was pointed vertically upward. Then the system would be placed on the digital scale and the rocket would be fired. A digital camera was set up to record the scale display values.

This procedure was repeated for seven vertical trials and two horizontal trials. The horizontal trials used a different frame configuration (see Figure 1) with the rocket set up horizontally and were designed to measure the mass loss rate of the rocket. However, due to the induced torques on the frame, this setup was unable to measure the mass loss rate. The first vertical trial also failed because the camera was incorrectly set up.

Additionally, the frame rate of the digital camera was tested against WWV time (standard time) to determine whether the factory value ( 15 hz ) was accurate. This was done by using the camera to record WWV time over 15 minutes and comparing the two elapsed times.

## 3 Results and Discussion

### 3.1 Thrust as a function of time

The data extracted from the six usable vertical trials $(2,3,6,7,8, \& 9)$ was reduced according to Equation 6 to calculate the thrust over time (Figure 2).

The general shape of the thrust curve is a smoothed peak skewed right. The maximum thrust recorded was 5.92 N at a time of 0.4 s and the thrust duration, $\mathrm{t}_{f}$, is 1.00 s . These curves differ from previously measured A8-3 thrust curves in that they do not have a region of constant thrust after the initial peak [1]. Additionally, the peak thrust is significantly less $(6 \mathrm{~N}$ versus 10 N$)$ and occurs at a different time ( 0.4 s versus 0.2 s ). Furthermore, the thrust duration $\left(\mathrm{t}_{f}\right)$ is longer than previous measurements ( 1.0 s versus 0.7 s ) [1]. This difference may be caused by the response time of the scale's digital display and the sampling rate.

Several distinct sources of error affect these measurements: scale error, non-vertical component error, timing error and extraction error. The extraction error is the main source of error and is the uncertainty associated with reading the values on the scale display. The display changed so quickly that the previous values would not have fully faded by the next video frame. This problem created multiple number overlaps for the ones digit making it difficult or impossible to identify the actual value. Since the ones digit was highly uncertain the extraction error added an uncertainty of $\pm 9$ grams. The timing error analysis comparing the camera time and WWV found that the camera takes 14.98991 frames $/ \mathrm{s}$ compared to the product value of 15.0 frames $/ \mathrm{s}$; over the thrust period this difference is negligible $(<.1 \%$ error). The other errors: scale error ( $<1 \%$ error) and the non-vertical component relative error $(<0.1 \%)$ were also negligible in comparison to the extraction error. Summing these errors in quadrature gives a total relative error of $3.3 \%$ for each data point (Figure 2).


Figure 2: Thrust Measurement: All curves except Trial 9 follow the same trend. The error bars represent $3.3 \%$ error. The uncertainty in timing is insignificant ( $<10^{-4} \mathrm{~s} /$ frame ) and is not shown.

### 3.2 Impulse Measurement

The impulse of each trial was calculated using finite difference approximation (midpoint rule). The individual impulses and the $95 \%$ confidence interval are shown in the following table.

Impulse Data

|  | Impulse (N-s) |
| :---: | :---: |
| Trial 2 | 2.29 |
| Trial 3 | 2.48 |
| Trial 6 | 2.33 |
| Trial 7 | 2.61 |
| Trial 8 | 2.46 |
| Trial 9 | 2.01 |
| Average Impulse | $\mathbf{2 . 3 6}$ |
| 95\% error bars | $\pm 0.16$ |

The confidence interval for this measurement encompasses both the factory value (2.5 $\mathrm{N}-\mathrm{s}$ ) and the value from T.Dooling ( $2.32 \mathrm{~N}-\mathrm{s}$ ) [1,2]. This demonstrates that both the mean impulse and the error are not unreasonable measurements because they agree with previous values. Given this agreement and the small relative error (7\%), it can be concluded that the method used in this experiment is adequate for measuring the impulse of a model rocket engine.

### 3.3 Average Thrust and Exhaust Velocity

The average thrust of the engine is simply the impulse divided by the thrust duration: $T_{\text {avg }}=\frac{2.36 \mathrm{Ns}}{1.00 \mathrm{~s}}=2.36 \mathrm{~N}$. Additionally, an order of magnitude approximation for the exhaust velocity can be obtained from this experiment by combining Equations 1 and 4. Assuming that the relative exhaust velocity, $u$, is constant and using the average propellant mass, $\Delta m=3.6 \mathrm{~g}$ :

$$
\begin{align*}
-u * \int_{0}^{t_{f}} \frac{d m}{d t} d t & =I_{r}  \tag{8}\\
|u| & =\frac{I}{\Delta m} \approx 650 \mathrm{~m} / \mathrm{s} \tag{9}
\end{align*}
$$

The fact that the exhaust velocity of a hobby rocket is almost twice the speed of sound gives one an appreciation of the physics involved in a rocket engine.

## 4 Conclusion

This measurement $(2.36 \pm .16 \mathrm{~N}-\mathrm{s})$ is a good estimate of the impulse of an A8-3 Estes rocket. If higher accuracy is needed, the most efficient way to reduce experimental error is to incorporate a data logger into the scale. This would increase the sample rate and remove the extraction error. This improvement might allow a conclusion to be drawn on which impulse value, Estes (2.5 N-s) or T. Dooling (2.32 N-s), is more accurate [1,2]. However, since the engines have a significant population variation in impulse ( $\pm .25 \mathrm{~N}-\mathrm{s}$ ), as measured by Estes, this could prove an unnecessary expenditure; additionally, measurement of higher powered engines would require significantly better equipment [2].

## 5 References

[1] Dooling, T.A. An Eight-Parameter Function for Simulating Model Rocket Engine Thrust Curves. Phys. Teach. 45, 280- 283 (2007).
[2] Estes Industries. Model Rocket Engine and Igniter Instructions. (2009).
[3] Nelson, R.A., Wilson, M.E. Mathematical Analysis of a Model Rocket Trajectory Part I: The powered phase. Phys. Teach. 14, 150- 161 (1976).

