Relationship between viscosity and sugar concentration in aqueous sugar solution using the Stokes' Law and Newton's First Law of Motion

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Abstract

In this experiment, the relationship between viscosity and sugar concentration in aqueous sugar solution was determined by calculating viscosity with Stokes' Law and Newton's First Law of Motion. In 9 different concentrations of sugar solution, the viscosities of the 9 different solutions were measured by acquiring the average velocity of the marble to reach certain height within the solution. From the experiment, the result shows that the viscosity and concentration of sugar solution has two possible relationships: linear (η=1.19 · concentration+0.94) or exponential relationship ($\eta = 0.95e^{\text{concentration}}-0.006$).

1. Introduction

Viscosity is a measurement of a frictional force (resistance) of a fluid. Fluids resist such an applied force (or another object's motion through the fluids) with each layer's motion with different velocities. Kinematic viscosity is a measure of a fluid's resistance under the influence of the force of gravity. Kinematic viscosity is usually measured by a capillary viscometer, in which by observing the fluid's velocity to reach the bottom of the long tube. Throughout many experiments, it was shown that the more viscous the fluid is the slower it travels; the less viscous the fluid is, the faster it travels [1]. The SI unit of kinematic viscosity is $cm^2 \cdot s^{-1}$, usually measured in Stoke (St). This can be converted into a dynamic viscosity unit, Pascal second (Pa· s or N $m^{-2} \cdot s$). In this experiment, the viscosity of the fluid will be measured by the velocity of a marble travelling within the fluid instead of using the capillary viscometer.

Thus, I need to apply Newton's First Law of Motion, which states that if the vector sum of all forces acting on an object is zero, then the acceleration of the object is zero, and its velocity is constant. Since a marble accelerates first due to gravity and slows down after, there is non-zero force acting on the marble. Also, the force of viscosity acts as a friction force in this case. Using Stokes' Law, $V_{iscosity} = 6\pi\eta r v$, we can expand it to Thus, the net force, the gravity force, is the vector sum of all possible applied forces on it, which is expressed as followed.

Force of gravity = Force of Viscosity + Force of Buoyancy (**1**)

$$m g = 6 \pi \eta r v + \frac{4}{3} \pi r^3 \rho g \quad (2)$$

$$\eta = \frac{(m g - 4/3 \pi r3 \rho g)}{6 \pi r v} \quad (3)$$

$$\eta = \frac{2g\Delta\rho r^2}{9 v} \quad (4)$$

This hypothetical formula indicates that we can calculate the viscosity by substituting in variables (m= mass of the marble; g= gravity = $9.81m^2/s$; r= radius of the marble; ρ = density of the liquid; v = velocity of the marble travelling).In a literature, it is also hypothesized that the sugar concentration increases viscosity exponentially [1]. Therefore, in this experiment, the mathematical relationship of the concentrations of the sugar solutions and the calculated viscosity will be interpreted by graphing the absolute viscosity against concentration.

The study of viscosity of pure liquids or aqueous solutions is essential in many fields and engineering [2]. Specifically, the relationship between sugar solution concentration and its viscosity has already been used for sugar concentrations in nectars in flowers [1]. Since the viscosity of nectars affects other organisms which helps in pollination, the study of sugar concentration and viscosity can be interpreted and used in a biologically [1]. This can be used in biological system, such as the nectar sugar concentration. Also, the relationship between sugar concentration and its viscosity can be also utilized in food processing [2].

2. Method

2.1 Data taking techniques

I proposed Eq. (2) can be used for calculating viscosity of fluid in this case of experiment, but doing so requires data on measurement of density of aqueous solution, radius and mass of the marble used, and the time the marble took to travel certain distance within the liquid. First of all, in order to calculate the viscosity of the sugar solution by the marble's velocity, it is necessary to find the finite value of the variables. Thus, the radius and the mass of the marble were measured, in order to calculate the density of the ball. For each trial, certain cup(s) of sugar was put in a pot with a boiled water to dissolve the sugar. The solution was diluted to 2 liter by adding more water. This step was necessary because there should have been enough volume of sugar solution in the tube to fill the certain height of the marble travelling. Referring to Figure 1, in order to carry out the experiment, the sugar solution was transferred to a long tube, which was weighed by difference and recorded to calculate the density of the liquid. After measuring the height (h in Figure 1) of the liquid the marble travelled, the time the marble took to reach the bottom of the tube was recorded ($t_f - t_i$ in Figure 1). The marble was dropped three times to acquire the average time it travelled. For each trial, new sugar solution with different concentration, the velocity of the marble, and the density of the liquid were calculated each time.

2.2 Description of the experiment setup

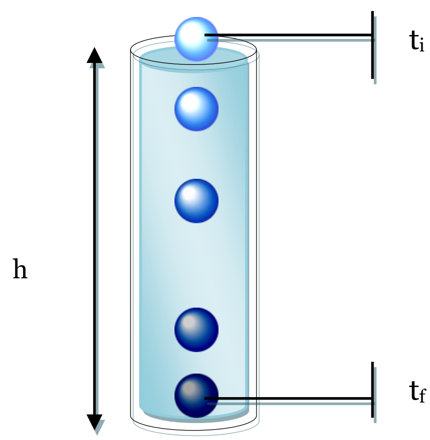


Figure 1 Description of the experiment setup: the marble was dropped right above where the liquid is (the top of the tube). The time the marble took to reach from the top of the tube to the bottom of the tube ($\Delta t = t_f - t_i$) was measured in each trial. The concentration of the solution differed in each trial (0M, 0.0400M, 0.0800M, 0.120M, 0.160M, 0.200M, 0.240M, 0.280M, and 0.320M). Also the height (h) the marble travelled was measured with a ruler, as well.

3. Results & Discussion

3.1 Measurements

Table 1 These following data were taken by using several different measuring devices, such as, cooking scale, ruler, and a timer. The uncertainties of the mass, length, and time were estimated by the half of the least digit the devices can produce. The densities of the aqueous sugar solutions and marble were calculated by mass divided by volume and the velocities of the marble travelling were calculated by distance divided by the time interval. All the data in the table were used to calculate the viscosity of a solution using the Eq. (4)

Aqueous Sugar Solution	Marble	Velocity measurement

	Mass (kg)	Volume (L)	Density (kg/m3)	Mass (g)	Radius (mm)	Density (kg/m3)	Average time (s)	Uncertainty (s)	Distance (m)	Velocity (m/s)
1	2.000	2.000	1000	5.000	6.600	4152	0.649	0.500	0.215	0.331
2	2.287	2.000	1144	5.000	6.600	4152	0.686	0.500	0.215	0.313
3	2.350	2.000	1175	5.000	6.600	4152	0.694	0.500	0.215	0.310
4	2.468	2.000	1234	5.000	6.600	4152	0.716	0.500	0.215	0.300
5	2.509	2.000	1255	5.000	6.600	4152	0.754	0.500	0.215	0.271
6	2.629	2.000	1315	5.000	6.600	4152	0.920	0.500	0.215	0.246
7	2.707	2.000	1354	5.000	6.600	4152	0.940	0.500	0.215	0.227
8	2.840	2.000	1420	5.000	6.600	4152	1.244	0.500	0.215	0.173
9	2.923	2.000	1462	5.000	6.600	4152	1.635	0.500	0.215	0.131

3.2 Uncertainty calculation

Table 2 These data were calculated by using the uncertainty measurements of measurement limitations. The partial derivative in Mechanics Lab Manual of error propagation method [2] was used, which is described more thoroughly below.

	Uncertainty of velocity (m/s)	**Uncertainty of density (kg/m3)**	**Uncertainty of radius (m)**	**Uncertainty of viscosity (Pa s)**
1	0.231	0.250	0.0000500	0.712
2	0.464	0.286	0.0000500	0.675
3	0.217	0.294	0.0000500	0.673
4	0.156	0.309	0.0000500	0.663
5	0.128	0.314	0.0000500	0.660
6	0.127	0.329	0.0000500	0.646
7	0.122	0.339	0.0000500	0.638
8	0.094	0.355	0.0000500	0.631
9	0.085	0.366	0.0000500	0.617

Since the variables are the values of experimental measurements, there are uncertainties due to measurement limitations which are propagate within the function, Eq. (4) [2]. The uncertainty propagations were done using partial derivatives in many physics lab and the same partial derivatives method was used to calculate uncertainty of viscosity using the partial derivative of Eq. (4), as following [2]:

$$d\eta = \left| \frac{2g\Delta\rho r^2}{9\,v} \cdot dp \right| + \left| \frac{4g\Delta\rho r}{9\,v} \cdot dr \right| + \left| \frac{2g\Delta\rho r}{9\,v^2} \cdot dv \right| \quad (5)$$

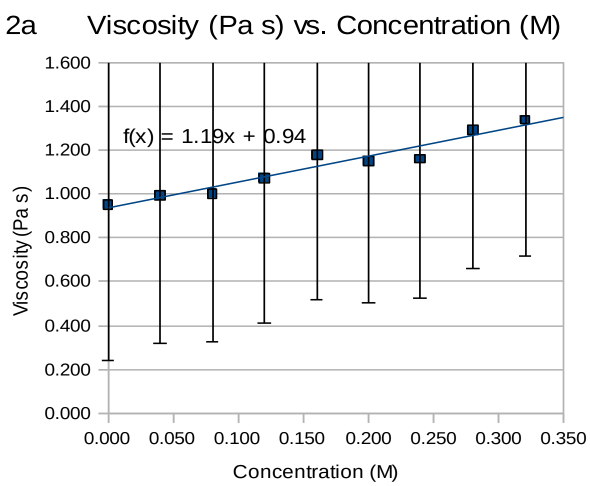
When measuring the uncertainties, it was recognized that measuring such a short time interval (less than 1 second) produces very high uncertainties in viscosity uncertainties. Also, the devices such as, a cooking scale or a measuring cup, were not precise enough for accurate measurements of the variables. Along with this, the water of the sugar solution may have been evaporated during the experiment, which could possibly change the values of density, as well. Along with this, legitimate assumption of calculating viscosity using Stokes' Law and Newton's First Law of Motion might have made few more uncertainties of the data.

3.3 Data in Graph

Table 3 The following data were used to create the graph below, Figure 2a and 2b. The data table was created by the viscosity calculation with the data in Table 1. Also, the uncertainties of the viscosity was acquired by using the partial derivatives method in Eq. (5) and the data in the Table2.

Concentration (M)	**Viscosity (Pa s)**	**Viscosity uncertainty (Pa s)**
0.000	0.950	0.712
0.040	0.993	0.675
0.080	0.999	0.673
0.120	1.072	0.663
0.160	1.176	0.660

0.200	1.152	0.646
0.240	1.160	0.638
0.280	1.291	0.631
0.320	1.337	0.617



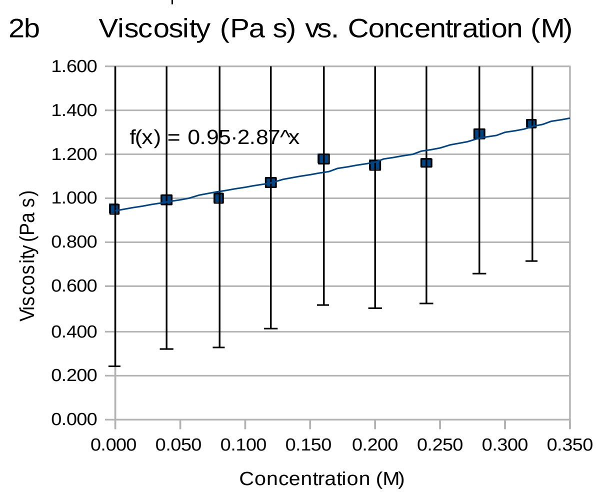


Figure 2 These graph were plotted using the data in Table 3, where the values of the viscosity were calculated using Eq. (4) with respect to the change in concentration. The uncertainties of the viscosity are computed by using the partial derivative of the Eq. (4). **Figure 2a.**This shows that the relationship between viscosity and concentration of sugar can be described as both linear (η =1.19*conc. +0.94) and **Figure 2b**. exponential (η = 0.95$e^{concentration}$-0.006)

Table 4 The following data are the used to linearize Figure 2. The model was created in order to acquire a directly proportional relationship between the viscosity and concentration of the sugar solution. The model shows that viscosity = 0.95 exp (conc.) – 0.006. The fact that the slope is 0.95 indicates that the initial viscosity is 0.944 Pa s.

Concentration (M)	Viscosity (Pa s)	m*exp(concentration)+b	(Model-Actual)²	Slope
0.000	0.950	0.943	0.00005	0.95
0.040	0.993	0.986	0.00005	**y-intercept**
0.080	0.999	1.031	0.00103	-0.006
0.120	1.072	1.078	0.00004	**Sum of (Model-Actual)²**
0.160	1.176	1.127	0.00240	0.0082536
0.200	1.152	1.178	0.00068	**Average difference of Model and Actual values**
0.240	1.160	1.230	0.00496	0.0009171
0.280	1.291	1.286	0.00002	
0.320	1.337	1.344	0.00005	

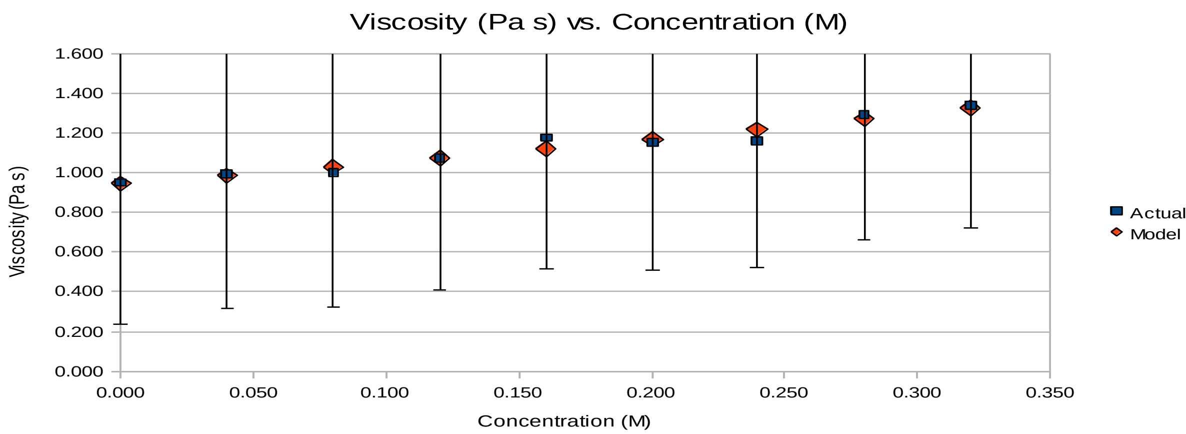


Figure 3 The comparison between the actual data and model equation values is displayed in the following graph. The model (∴ viscosity= $0.95e^{\text{concentration}}$-0.006) produces similar values to the actual measured viscosity. All of the points from the model fit within the error bars and physically interact with most of the points.

3.4 Data analysis

When interpreting the data in Table 3, the calculated viscosity of water (η =950) agreed with the other experiment, which states that the pure water viscosity in room temperature (20°C to 25°C) is approximately from 1.002 to 0.8904 [4]. Thus, it can be assumed that the method of calculating the viscosity of fluid and the result was reasonably accurate [4].

Then, when the graph was plotted in order to see the trend of viscosity change with the concentration variation, the graph showed both the linear and exponential relationship as in Figure 2a and 2b. However, in the study of nectar, it is hypothesized that viscosity increases exponentially with a constant increase in sugar concentration [1]. Thus, to interpret the data more clearly, the exponential value of measured sugar concentration is calculated and plotted with the concentration of the sugar and displayed in Figure 3. By linearization of the graph, it is seen that the slope of 0.95 and the y-intercept of -0.006, the graph resulted with the average difference of the calculated and the model value of 0.0009171. This small value of average difference between the calculated and model value show that the model can be an "optimal" fit to describe the measured value of viscosity of sugar solution. The modelling of the function shows that the mathematical relationship between viscosity and sugar concentration of the sugar aqueous solution is approximately exponential ($\eta = 0.95e^{\text{concentration}}-0.006$) or possibly linear ($\eta=1.19 \cdot \text{concentration}+0.94$). In other words, the viscosity can be increased exponentially with a constant increase in sugar concentration in the aqueous sugar solution as hypothesized in other literature.

4. Conclusion

Throughout the experiment, the result shows two possible relationships between the viscosity and the sugar concentration: linear and exponential relationship. Thus, either sugar concentration of the aqueous sugar solution or the viscosity of the sugar solution in room temperature can be determined with one of the other two values. However, further study or experiment on the model of sugar concentration and its viscosity should be done with more precise device in order to confirm or determine if the relationship of the two variables is actually linear($\eta=1.19 \cdot \text{concentration}+0.94$) or exponential ($\eta = 0.95e^{\text{concentration}}-0.006$).

5. References

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