F-Note Acoustic Efficiency of an Upright Piano over Seven Octaves

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Abstract

Pianos are machines. Their sound output is determined by acoustic efficiency which is a measure of acoustic energy produced per input mechanical work. The piano as an instrument is unique from other chordophones like violins and guitars as it uses hammers to cause string vibrations. Where work is done directly on the strings in a guitar, pianos utilize a two-step procedure in converting mechanical work into acoustic energy. First, mechanical energy is used to depress a piano key which lifts a hammer. Second, this hammer strikes the strings and acoustic energy is produced. Thus, pianos should be less efficient than strict chordophones. An index of acoustic efficiency as a function of frequency was generated using an F-note from each of the seven full octaves of a Yamaha upright piano (T118PE). Experimental results indicate an inversely proportional relationship between acoustic efficiency and key frequency.

Introduction

The piano is an interesting machine. It is a string instrument like violins and guitars, but it is also a percussion instrument due to the fact that hammers are what actually strikes the piano strings. The purpose of the piano is to convert mechanical work done by the pianist into acoustic energy or sound. Simply put, acoustic efficiency is the answer to the question, "How much sound is the pianist getting out of his/her work?" In a piano, mechanical energy put into the keys is transferred along a lever which lifts up the piano hammer that strikes the strings. In this process alone, two actions are happening; mechanical energy is being transferred from the key to the hammer, which strikes the strings, causing the strings and the soundboard to convert the mechanical energy into acoustic energy. Thus, the dual-action nature of a piano should be less efficient than simple chordophones like violins and guitars. I hypothesized that acoustic efficiency will vary with respect to frequency and that a trend will emerge. I predicted that acoustic efficiency and sound persistence will decrease with increasing key frequency based on qualitative observations made prior to the experiment. Base notes were perceived to acoustically persist longer and louder (woofer-like), whereas treble notes were never heard for too long or too loud.

Mechanical Energy Measurements

The first step in determining the acoustic efficiency of a piano is in devising a method to generate constant input work. This can be done with the aid of gravity, which provides a constant downward (vertical) force on a given mass. As the desired direction of motion in depressing a piano key is also in the vertical direction, the application of an external gravitational force (ie, separate mass) provides work on the piano keys (1).

$$\mathbf{W} = |\mathbf{F}\triangle\mathbf{d}| = |\triangle\mathbf{U}| = |\mathbf{mg}\triangle\mathbf{h}| \qquad \mathbf{(1)}$$

The actual mechanical energy measurement utilized a light string fixated to the key being tested by strong cellophane tape. The light string was used to hang a 0.684kg metronome. The mass was allowed to hang freely, which caused a complete depression of the key being tested (1.1cm below its resting height) (fig. 1). To generate a sound, the key was lifted to its rest height manually and let go, falling as a result of the constant force imparted by gravity on the metronome. This mechanism was used throughout the experiment to produce a constant mechanical work input.

Fig. 1: **The experimental setup for constant work generation. (Cat is the metronome)**

Acoustic Energy Measurements

Measuring acoustic energy presents many challenges; first, it is virtually impossible to directly measure the acoustic power output from an analog musical instrument like the piano. Only intensity measurements are possible. But due to the piano's large size, it cannot be approximated as a point source even in fairly large spaces. Second, sound waves tend to sustain in closed spaces in the form of reverberations and standing waves. These two phenomena tend to reinforce acoustic intensity at regions near the surfaces of the room/space as the weakened transverse sound waves reflect back before decaying to negligible intensities. As a result, acoustic intensity inside a closed space does not adhere to the inverse square law. Thus, it is usually not surprising to find an almost constant magnitude of acoustic intensity at all points in a room. Nonetheless, a satisfactory range of possible acoustic energy values were derived by assuming the surface area of the acoustic power spread to be that of the surface area of the enclosed space or room. This approximation worked on the assumption that since the true acoustic power output from the piano is a constant value, acoustic intensity measurements in a larger room with a larger surface area, as opposed to a smaller room with a smaller surface area, would be consistently lower at every point in space enclosed by the room. The critical assumption made was that for enclosed rooms that are not too large (ex. 15m x 15m x 4m) acoustic intensity levels decrease linearly with the increase in surface area. Thus, this assumption simplified the acoustic power spread by asserting that the acoustic power produced by the piano was spread out evenly over the surface area of the enclosing room itself.

The measurements were conducted on an upright Yamaha piano, model T118PE in the recreation wing of Vancouver Coastal Health's George Pearson Centre. An enclosed space, the room had an approximately square floor and ceiling (15m x 15m) with rectangular walls (15m x 4m). The piano was placed in the middle of the room to obtain an even intraspatial acoustic power spread. Actual measurements were done at 6m from the piano, directly in the line of sight of the piano player. Instantaneous acoustic intensity values displayed on a decibel meter were recorded using a video camera. F notes from each of the 7 complete octaves were tested, namely F1 to F7. Each F-note tested consisted of 4 to 5 strokes of the piano key itself, repeated as data replicates to increase precision. By analyzing the footages, the refresh rate of the decibel meter was determined to be $1/15^{th}$ of a second. In particular, the acoustic intensity produced by the F5 note was recorded at 3m, 6m, and 8m from the piano (in the line of sight of the piano player) to determine the model of acoustic intensity decay in the room.

In order to obtain values of acoustic energy, acoustic intensity values were plotted against time. According to the physical definition of intensity, it is equal to power per surface area, thus no acoustics -specific equations were necessary in calculating acoustic energy output. The actual steps taken

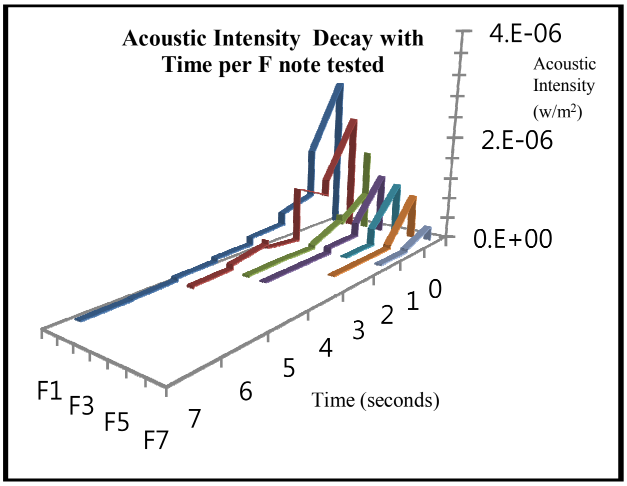

Fig. X: For acoustic intensity decay visualization, (no numerical significance). No single model fits the whole curve for a given frequency.

involved first decomposing the intensity definition into intensity equaling energy per area and time, thus, enabling the integral of acoustic intensity with respect to time to account for the acoustic energy per time component of equation (2). Since the projected surface area (of the room) was constant throughout data acquisition, it was left outside of the integral expression. Unfortunately, no single model was able to account for all of the acoustic intensity values in a single disturbance as initial spikes showing sudden increases in acoustic intensity values following a key stroke prevented the use of decay models (fig. x). Thus, trapezoidal rule was chosen to approximate the integral due to its finite-interval, numerical approach to calculating areas under curves generated by connecting the discrete acoustic intensity values with respect to time (2).

Acoustic Energy Output **(2)**

$$= A\int_{\beta}^{\alpha} I \cdot dt \approx A \cdot \sum_{i=1}^{n} \left(\frac{I_{(i+1)} + I_{(i)}}{2}\right) \cdot \left(t_{(i+1)} - t_{(i)}\right)$$

As 4 to 5 plots of acoustic intensity vs. time graphs were generated per F-note tested, integral values were averaged to represent that specific F-note. Limits of integration were not particularly affective of the actual magnitude of the approximation as disturbances were usually about 15~20dB above the rest sound levels corresponding to acoustic intensity values greater by a factor of 30 to 100. Thus, the main bulge of the acoustic intensity vs. time plot accounted for virtually the entire trapezoidal sum. Nonetheless, a strict scheme for defining limits of integration was used to systemize the procedure. Each sum began with the data point just before a key stroke caused a significant spike in acoustic intensity to account for the area under the increasing acoustic intensity plot. The sum ended when the acoustic intensity values first reached that of the average rest acoustic intensity levels.

Experimental Results & Discussion

$$\varepsilon = \frac{\text{Acoustic Energy Out}}{\text{Mechanical Energy In}} \qquad (3)$$

To obtain acoustic efficiency, output acoustic energy was divided by input mechanical energy (eqn. 3). Acoustic efficiency was calculated per F-note from F1 to F7 (fig. 3). As predicted, the magnitude of acoustic efficiency decreased with increasing frequency. When the acoustic efficiency was plotted against frequency, a trend appeared modeled by the relationship shown on figure 2 (y_1). From a physical perspective, a correlation between frequency and acoustic efficiency would likely be a quantized one in that the constant power of equation y_1 would be either -1 or -0.5. Since -0.65 is closer to -0.5, an inverse square root relationship is suggested between frequency and acoustic efficiency (eqn. 4).

$$\textbf{Acoustic efficiency, } \varepsilon \propto \frac{1}{\sqrt{\text{frequency}}} \qquad (4)$$

With a maximum acoustic efficiency of about 3% to a minimum of about 0.2%, ε-values from fig. 3 appear to be fully valid when compared to classical guitar acoustic efficiency values determined under more precise conditions. The acoustic efficiency of a classical guitar was reported to be in the range of 11~17% from the Moschioni article[1]. Since guitars are single-action, in that sound is produced through direct, manual excitation of the strings without a secondary agent (in pianos, hammers), a lower acoustic efficiency is expected for pianos. Thus, 11% would be the absolute upper-bound for acoustic efficiency in pianos. Since no lower bound (other than zero) could be implied from the experimental data, it would be reasonable to assume that the acoustic efficiency values obtained are within logical bounds.

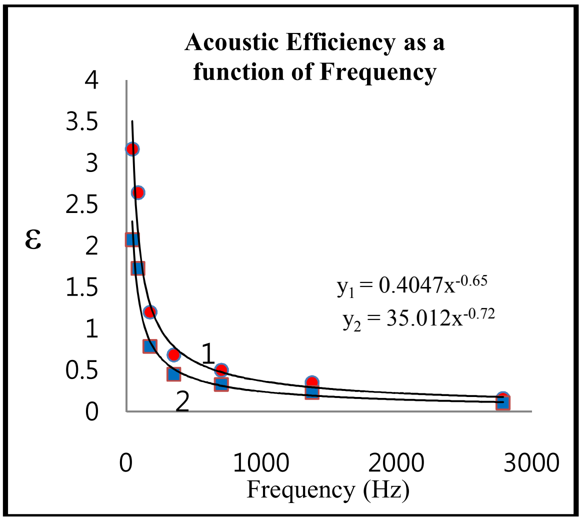

Fig. 2: Acoustic efficiency calculated with respect to 1) surface area of the room; 2) inverse square model ($4\pi r^2$, r=6m). The region between the two fits is where the true acoustic efficiency value is likely to be found in. Error bars were avoided as the largest uncertainty estimate was around a factor of 3 considering figure 4.

Key Frequency	Acoustic Efficiency, ε (%)	Complete Acoustic Intensity Decay Time (s)	Time Constant of Initial Acoustic Intensity Decay prior to beating (s)
F1, 44Hz	3.16 ~ 2.08	7.08±0.3	1.12±0.3
F2, 87Hz	2.64 ~ 1.73	4.23±0.3	1.06±0.3
F3, 176Hz	1.20 ~ 0.79	3.36±0.3	0.64±0.3
F4, 352Hz	0.69 ~ 0.45	2.76±0.3	0.53±0.3
F5, 703Hz	0.50 ~ 0.33	1.59±0.3	0.24±0.3
F6, 1374Hz	0.35 ~ 0.23	0.49±0.3	0.19±0.3
F7, 2787Hz	0.16 ~ 0.10	0.45±0.3	0.30±0.3

Fig. 3: Each F-key characterized; uncertainty values for column 3 and 4 are total estimates only. Individual standard deviation values were much smaller.

The internal microphone of a net book was originally planned for use in recording acoustic intensity values, but technical problems arose in that the perceived intensities in the computer were extremely small (12dB at rest). Nonetheless, the frequency at which the netbook program took data points allowed for the visualization of each acoustic intensity decay curve. As the curve shapes were repeated for a given note under replicates, it became clear that the netbook uncertainty was in the magnitudes of the acoustic intensity values only. Thus, observing that the decay time for acoustic intensity decreases with increasing frequency (ie, sound dissipates faster), netbook data was used to generate column 3 and 4 in fig. 3. Surprisingly, both plots were proportional to the inverse square root of frequency just as in the acoustic efficiency vs. frequency plot.

Discussion of Uncertainties

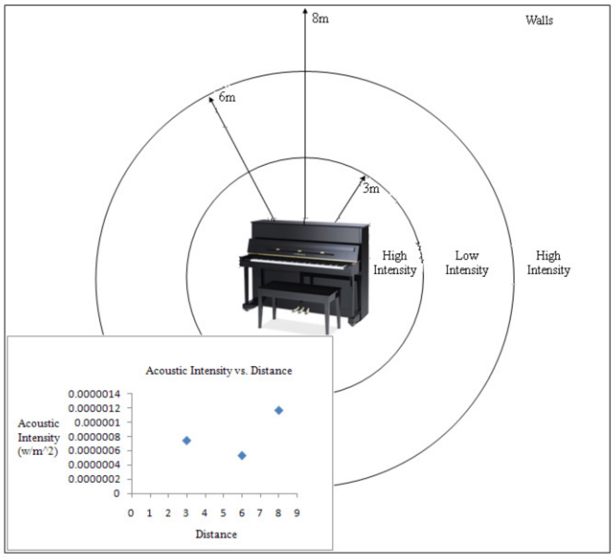

Fig. 4: "Donut in a room" attempt at explaining the scatterplot.

The greatest cause for uncertainty in this experiment originated from the disobedience of the inverse square law in indoor spaces. Furthermore, acoustic intensity measurements taken at three locations in the room (3m, 6m, and 8m from the piano in the player's line of sight), using the F5 note with a constant input of mechanical energy, showed that acoustic intensity values tended to be relatively large near the piano and near the walls. Lowest acoustic intensity was detected at the midpoint (6m) where all of the data for acoustic efficiency was taken. The room had a square base and ceiling with rectangular walls, thus acoustic intensity could have varied according to radial distance from the piano itself (fig. 4). Modeling of the obtained intensity variations according to radial distance considerations yielded a donut-shape localization of high/low acoustic intensities. As mentioned previously in the methods section, the reflective nature of the walls could have caused the incoming and reflected acoustic transverse waves to interfere constructively, thereby increasing the acoustic intensity in regions far from the acoustic power source (the piano) and close to the walls. A second point considered in attaching an overall uncertainty to the acoustic intensity values was the surface area consideration. Even though the $\int I dt$ = energy/surface area expression was evaluated to a fairly good precision due to the fact that the expression's precision depended on the calibration and the precision of the instruments used, the decision on the surface area was rather arbitrary and came with large uncertainties; the disobedience of the inverse square model failed to give a straightforward answer to the surface area problem. Thus, it appeared that the correct value for surface area lay between values given by the total surface area of the room and that of $4\pi(\text{radius})^2$ given by the inverse square law model. Any other uncertainties (standard deviation from each trials, etc) would be negligible after consideration of the surface area uncertainty. Another source of uncertainty was that the literature value for classical guitars gave a single acoustic efficiency value for varying frequencies. Thus, my data could have been affected by the delay in the decibel meter which might not have properly reported the maximum intensity values at higher frequencies which decayed faster.

Conclusion

Acoustic efficiency calculations indicated support for my hypothesis. Acoustic efficiency dropped from 3% at 44Hz to 0.2% at 2787Hz. A surprising finding was that all data reported in figure 3 columns 2,3,4 were very much proportional to the inverse square root of frequency. Overall, the results for acoustic efficiency was within the upper limit given by single-action guitars and was deemed to be reasonable as a lower bound could not be determined from the experimental data. The largest source of uncertainty came from the fact that acoustic intensity decay did not conform to the inverse square law. Thus, the constant surface area value used to calculate absolute acoustic energy output was somewhat arbitrary. However, the relative acoustic efficiency would remain unchanged since the surface area term is a constant and always positive.

Acknowledgements

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References

[1] Moschioni, G. and Saggin, B. A New method for Measurement of Acoustic Efficiency of Classical Guitars. *IEEE*. *Italy* **3**, 1953 – 1958 (2004).