Determining the mathematical relationship between the thermal energy applied to a squash ball and the consequential gain in hang time of the ball

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Abstract
During the game of squash, the transfer of thermal energy to the squash ball is integral to increase the bounce height of the ball. This investigation involves the transfer of thermal energy to squash balls of the same brand and specification to observe the effect on hang time. Preliminary work involved determining the specific heat capacity of the squash balls. On completing the investigation, the results showed a linear relationship between the thermal energy applied and the hang time.

Introduction
Dunlop, the most popular manufacturer of squash balls, produces various types of squash balls, with the slowest ball being the double yellow dot ball. This ranking of speed given to each level of ball is related to the bounce height of the ball when it is first used without being heated, with the slower squash balls having a lower natural bounce height. The single yellow dot squash ball will be investigated as it has a low bounce height at room temperature, and so can have a wider range of thermal energies applied to it. The different levels of balls have varying elastic properties due to the different combination of polymers used to form them [1]. To produce the elastic properties desired in the squash balls, elastomers are most likely used, as they produce a material that has "the ability to be stretched to at least twice their original length and to retract very rapidly to approximately their original length" [1]. This elastic property would produce a large hang time for the squash ball. As it is commonly known that a squash ball bounces higher as it is heated up, the point of this investigation is to mathematically quantify this increase, so as this specific game play property of hang time of the yellow dot squash ball can be mathematically described. Throughout the investigation there are two terms that must be clearly defined: 'Hang time is how long the ball is in the air. Rebound height is how high the ball bounces'[2]. The calculation of hang time is shown as Equation (1).

$$t = 2\sqrt{\frac{2x}{g}}$$ Eq. (1) [2]

where
t = hang time
x = rebound height
g = acceleration of gravity

The equation used to determine the amount of thermal energy applied to a material is:

$\Delta E = mc\Delta\theta$ [3] Eq.(2).

Where E is the energy supplied or removed from the material, m is the mass of the sample; c is the specific heat capacity of the material and $\Delta\theta$ is the change in temperature in ^{o}C.

Method

Produce Calibration Curve for Thermistor

Prepare a beaker of water which is at 5°C. Submerge the thermistor in the water, and take a reading of the resistance. Remove the thermistor from the water and increase the temperature of the water by 5°C. Submerge the thermistor in the water and again take a reading of the resistance. Continue up to 70°C and plot a graph (refer to Fig.1) of Resistance vs. Temperature using the data.

Determine Submersion Time to heat Squash Balls

First acclimatise a 'sacrificial' squash ball to room temperature, and then pierce a small hole in it, and insert the thermistor inside the squash ball, trying to retain as much of the air as possible inside the squash ball. Bring the temperature of the ball down to 5°C from room temperature. Repeat three times to get an average value. Once the ball is at 5°C, then determine how long it takes to raise the temperature of the ball to 10°C. Repeat three times. Determine how long it takes to raise the temperature of the squash ball to 15°C and repeat three times. Do the same to raise the temperature to 20°C and attain an average value. Then determine the time required to raise the temperature from 65°C to 70°C and repeat three times. If the average times required to raise the temperature by 5°C agree to within one minute, assume this value as a minimum average for the rest of the investigation. The time required to reduce the temperature to 5°C will probably be slightly longer, as the equipment available does not include anything which causes rapid cooling, and so the cooling process will have its own average value for time. If they do not agree to within one minute, repeat the procedure until they do so.

Determine the Specific Heat Capacity of the Squash Ball

Weigh the squash ball to be tested and record the mass. Using weighing by differences, determine the mass of a known volume of water in a beaker. Record the temperature of this water, and keep the thermometer in this beaker of water. Heat the squash ball by 10°C using the previously determined submersion time, then submerge the heated squash ball in the beaker of water of known temperature. Measure the temperature rise of the water by observing the change on the thermometer, until the squash ball and water come into equilibrium, and there are no more fluctuations in the reading of temperature, which should occur at the pre-determined submersion time for the squash ball to come into equilibrium with the water.

Re-arranging Eq. (2):

$$c_{ball} = \frac{\Delta E_{ball}}{m_{ball}\,\Delta\theta_{ball}}$$

Using the assumption of $\Delta E_{ball} = -\Delta E_{water}$ assume the following:

$$c_{ball} = \frac{\Delta E_{water}}{m_{ball}\,\Delta\theta_{ball}}$$

Thus:

$$c_{ball} = \frac{m_{water}\,c_{water}\,\Delta\theta_{water}}{m_{ball}\,\Delta\theta_{ball}}$$

Repeat three times to obtain an average possible value for the SHC (specific heat capacity) of the squash ball.

Measuring Bounce Heights of Squash Balls

Set the three squash balls to be tested on the table so as they acclimatise to room temperature. Discard the sacrificial squash balls previously used. Set up the bounce height test area, by securing a metre rule to a level, pale background using masking tape. Use a spirit level to ensure the metre rule is secured perpendicular to the floor. Set up the camera securely and safely so as a good quality video can be taken of the squash ball bounce. Next, using boiling water and cold water prepare a solution of the desired temperature for the squash ball, and keep a thermometer in the solution to ensure the temperature is kept constant. Once the water is at the correct temperature, the squash balls must all be submerged in the water at the same time using small beakers or glasses. Once all three squash balls have been submerged beneath the water, the stop watch must immediately be started to time the determined minimum submersion time. After the submersion time is up, start the video camera recording, and remove the squash balls from the water bath, and take them to the test area to be dropped. Drop all three squash balls from the top of the metre rule, one after the other. It is important this is done quickly so as not too much heat energy is lost from the balls to the atmosphere of the room. Once all three squash balls have been tested, the video recording can be stopped and the squash balls returned to the water bath to be heated to the next temperature. Repeat this method for the whole temperature range from 5-70^oC. Ensure that the videos of the bounces are appropriately ordered, so as the analysis of the data will be much easier with a systematic ordering of the results. Using computer software, the videos of the bounces must be slowed down to smallest frame intervals, and the highest bounce height determined from one of the static frames.

Results and Discussion

Determining the Specific Heat Capacity of the Squash Balls:

Using the assumption that $\Delta E_{ball} = -\Delta E_{water}$, as stated in the introduction, the following calculation was carried out for the first trial in determining the SHC:

$$C_{ball} = \frac{0.04033\text{kg} \times 4.2\text{KJ/Kg}^o\text{C} \times 4.5^o\text{C}}{0.02352\text{kg} \times 20.5^o\text{C}} = 1.581 \text{ KJ/Kg}^o\text{C} +/- 0.719. \text{ Eq.(3)}$$

Another two trials were carried out, as shown in Table 1. The possible relative error for each trial was determined by adding the relative errors in quadrature, as shown for the example equation (3), with the values of 4.5^oC and 20.5^oC coming from the changes in temperature of the ball and water found in equation (3) above:

$$\text{Relative Error} = \sqrt{\left(\frac{0.000005\,kg}{0.04033\,kg}\right)^2 + \left(\frac{2℃}{4.5℃}\right)^2 + \left(\frac{0.000005\,kg}{0.02352\,kg}\right)^2 + \left(\frac{2℃}{20.5℃}\right)^2} = 0.455 \text{ (3sf). Eq. (4)}$$

To then determine the actual error in the SHC calculated, the relative error was simply multiplied by the SHC. These results are tabulated in Table 1. Once the actual errors had been determined, the standard deviation of the errors and the average of the errors were determined to draw comparisons between the two. As shown in table 1, the calculated standard deviation (0.001) is much lower than the propagated average error of 0.76. Because of this an estimation of +/-0.80 kJ/kg^{o}C has been made to accommodate all possible ranges of errors calculated. This gives a maximum SHC of 2.35 kJ/kg°C and a minimum of 0.75 kJ/kg°C.

Table 1.Tabulated Results of SHC of Squash Ball:

SHC of Squash Ball/ kJ/kg°C	Relative Error in Specific Heat Capacity	Actual Error in SHC	Standard Deviation	Average SHC/ kJ/kg°C	Average Error
1.58	0.46	0.72	0.001	1.55	0.76
1.51	0.51	0.77			
1.55	0.51	0.79			

The SHC of the squash ball was determined as a whole object instead of determining the SHC of the material and enclosed gas separately, as the SHC of the gas would be too small to measure with the available equipment. As Dunlop is very secretive about the mix of polymers used in the production of the squash balls, an elastomer, in particular, butyl rubber, has been assumed as the main constituent of the material of the squash ball, as it often used in the manufacture of sports equipment such as basketballs and squash balls. Cross referencing this calculated SHC with the documented value of 1.85 kJ/kg°C [3] for vulcanised butyl rubber, the result seems reasonable. The SHC of the vulcanised form of the butyl rubber has been used as cross-reference, as this is most likely the form of the butyl rubber used in the production of the squash balls, due to this increased cross-linking between the elastomer molecules.

Calibration Curve for thermistor:

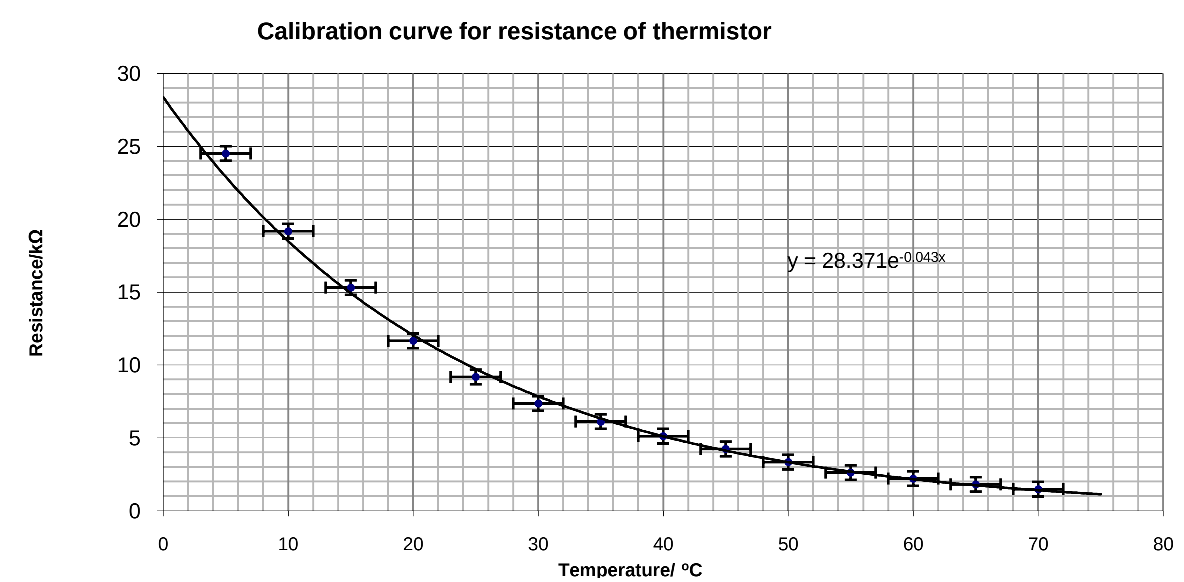

Fig.1 Plot used for conversion of thermistor readings of resistance into temperatures. Vertical Error bars were determined by observing the fluctuations in resistance readings, which was +/- 0.5kΩ. Horizontal error bars are +/- 2°C fluctuations in the temperature, due to inaccuracies in the thermometer, and difficulty of maintaining a volume of water at constant temperature using just ice and water.

The calibration curve shown in Fig.1 allowed for the conversion of resistance readings of the squash ball into temperature readings. Minor gridlines were included on this graph to allow for easier extrapolation of corresponding temperatures from resistance values. The x-axis was kept in the units of °C as Kelvins produced an inappropriate axis scale.

Determining the Submersion Time required for the squash balls:

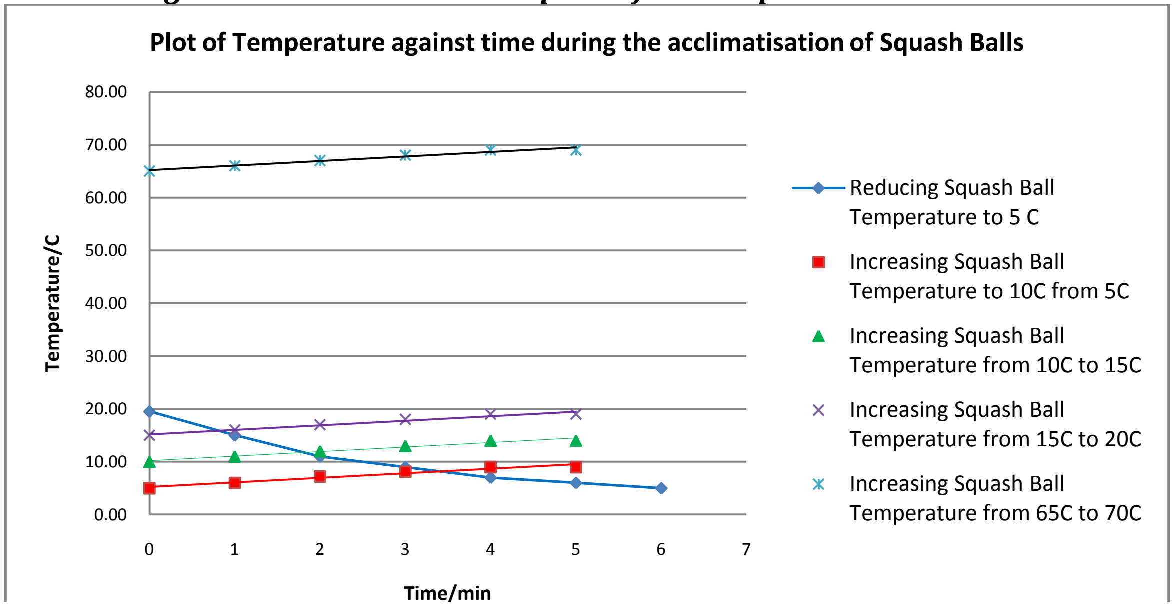

Fig.2 Average times to increase the temperature by 5°C were determined to be 5 minutes. To cool the squash ball down to 5°C from room temperature took 6 minutes on average, but the most important factor was allowing enough time so as the temperature no longer fluctuated, and so these average time values were used as minimum times rather than exact submersion times. The temperatures appeared to increase linearly initially, but then started to approach exponentially, just below the required temperature.

The first acclimatisation required is to bring the squash ball from room temperature down to 5°C. Three trial runs were carried out, producing an average time of 6 minutes, as shown in Fig.2. Trial runs were also carried out on raising the temperature by 5°C at the low and high end of the control temperatures used. These all averaged out to 5minutes each, also shown in Fig. 2. Having these averaged approximations, it could be assumed that it would take 6 minutes to cool the squash ball to 5°C from room temp, and that all of the subsequent 5°C increases after this point would take 5 minutes. This saved the squash ball from being cut open for each trial, which would drastically affect the structure of the ball.

Effects on Hang Time of Squash Ball:

As the thermal energy of the squash ball increased, so did the bounce height, and consequently the hang time of the squash ball. This is shown by the data in Table 2. The relationship between the thermal energy applied and the gain in hang time is represented by Fig 3. In which this linear relationship is clearly shown. The linear fit of this data produced the equation of:

Hang Time = (0.1085 x Thermal Energy of Squash Ball) + 0.234

Eq. (5)

This model to determine the hang time is only relevant to the yellow-dot squash ball type.

Thermal Energy/KJ	Bounce Height/m	Hang time/s	Calculated Error of Hang time
0.18	0.07	0.24	0.33
0.36	0.09	0.27	0.28
0.55	0.11	0.30	0.27
0.73	0.12	0.31	0.26
0.91	0.14	0.34	0.26
1.09	0.16	0.36	0.26
1.28	0.18	0.38	0.26
1.46	0.19	0.39	0.26
1.64	0.21	0.41	0.26
1.82	0.22	0.42	0.26
2.01	0.24	0.44	0.26
2.19	0.28	0.48	0.26
2.37	0.30	0.49	0.26
2.55	0.31	0.50	0.26

Table 2.

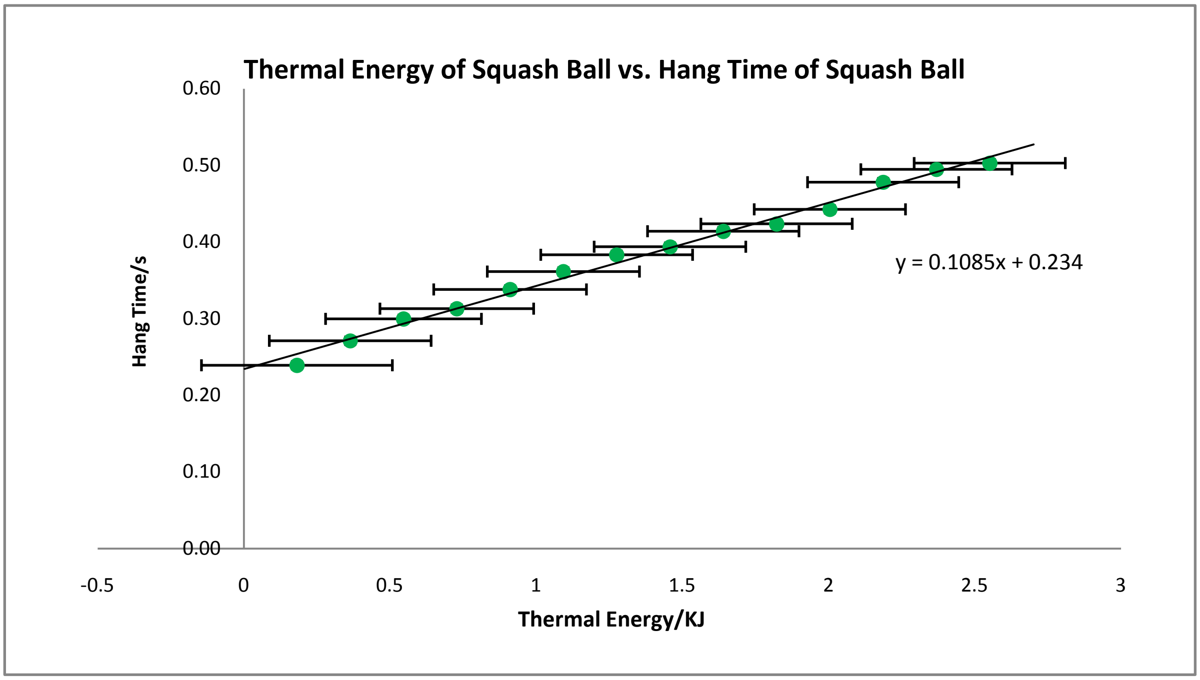

Fig. 3 Plot showing linear increase in hang time with increase in thermal energy of squash ball. Vertical error bars were determined by isolating the frame of the highest point of the bounce, and the difference between the peak height frame and the two adjacent frames were averaged to give the maximum possible change in height not captured by the video. The horizontal error bars were determined by propagating the error of the thermal energy, by adding all possible relative errors in quadrature.

It is predicted that a ‘faster’ squash ball, such as the blue-level ball, would have a much higher constant in the model, due the higher bounce height of a blue-level squash ball when at room temperature. The y-intercept of 0.234 in eq. (5) is the extrapolated value of the hang time for the yellow-dot squash ball if it were at 0^{o}C. Unfortunately, it was not possible to source any liquid nitrogen for the purpose of freezing the squash ball, and apparently, when high-school teachers have tried this before, the squash ball often ended up exploding, quite unexpectedly [4]. Although an ice and water mixture can reach a temperature of 0^{o}C, it is difficult to maintain this temperature for the mixture when at room temperature, and so freezing of the squash ball would be an interesting addition to any future investigations.

The measurements taken of bounce heights all had small relative errors, and although these vertical error bars are on the graph, they are so small that they are mainly covered by the data points. This suggests that the measurement of the bounce height did not produce a huge source of error in this investigation. The most significant error most likely came from trying to apply a specific thermal energy to the squash ball. One way in which this error may have occurred procedurally was using the assumption that the squash ball would come into equilibrium with the surrounding water within 5 minutes. It was best to use this assumption, however, than to drastically alter the structure of the squash ball by inserting a thermistor to determine the exact temperature. Another possible procedural error was during the transfer of the squash ball from the water to the metre rule, as heat may have been lost to the surrounding air. The most significant error was most definitely in the determination of the SHC of the ball, and so the energies of the ball will contain this systematic error. This is certainly one of the most important aspects which would need improvement in a future investigation, perhaps by determining the SHC of the polymer and enclosed gas separately.

Even with these numerous sources of error in the investigation, Chapman et al. [5] also found that the squash ball material displayed increased stiffness and therefore became bouncier also when heated. As stated by Lees [6]; “While different types of badminton shuttlecock and squash ball exist, there appears to be no recent literature investigating their characteristics.” Due to this it has been difficult to fully compare these findings with other published results, but taking into consideration the controlled possible errors, this linear fit to the data does seem reasonable. A possible explanation for the increased hang time of the squash ball with increasing thermal energy may be due to the heating of the air contained within the squash ball. As the air is heated, by fundamental thermodynamics it is known that the gas will expand. As the gas expands, the rubber of the squash ball will become stretched, and so this will lead to the ball becoming bouncier. Another possible explanation for the linear increase in hang time may have been due to energy changes in the molecules of the butyl-rubber. Butyl rubber is formed by the copolymerisation of 97% isobutylene and 3% isoprene [1]. In the structure of isobutylene (Fig.4), there is most likely steric hindrance of the methyl groups, which could also lead to repulsion forces between the adjacent hydrocarbon chains in the polymer producing a resilient material. As steric hindrance between adjacent groups can also restrict torsional bond angles, this could mean the flexibility of the structure is reduced, leading to a stiffer material and therefore a bouncier squash ball, with an increased hang time.

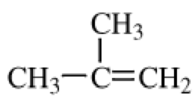

Fig. 4: Structure of isobutylene [1]

Conclusions

As a squash ball gains thermal energy, the bounce height of the squash ball increases linearly and consequently so does the hang time. This could be due to interactions within the molecule of the material, although the main factor is most likely due to the expansion of the enclosed gas. By determining the SHC of the squash ball to be 1.55kJ/kg^{o}C and comparing this to values of the SHC for butyl-rubber, we can conclude that the values are very close, and so the material of the squash ball is most likely a copolymer with includes butyl rubber. The mathematical relationship between the thermal energy and hang time for the single yellow dot ball is:

Hang Time = (0.1085 x Thermal Energy of Squash Ball) + 0.234. Eq. (5).

References

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[4] Malone, K., Tejwant, D. Beware the frozen squash ball. *Physics Teacher* **32** 351 (1994).

[5] Chapman, AE., Zuyderhoff, RN. Squash Ball Mechanics and implications for play. *Canadian journal of app. sport sci.* **11** 47-54 (1986).

[6] Lees, A. Science and the major racket sports review. *Journal of Sports Sci.***29** 707-732 (2003).