Construction and calibration of an extension spring scale

Abstract

A spring scale was constructed by attaching several extension springs in series. The springs were acquired by disassembling a collection of staplers; however, these springs proved to be limited in their use, since they did not have a clearly defined elastic limit. The spring scale constructed has a *k* value of 7.05 ± 5.7x10^{-3} N/m. The measurement error was calculated to be equivalent to 0.34 grams. The scale is accurate to 1 gram and has a functional range from 60 grams to 450 grams.

Introduction

The goal of this project was to build an accurate, functioning spring scale. Springs have the useful property of stretching a distance proportional to the force applied to the spring. This property of springs only occurs within a certain range of force; I will refer to this range as the Hookian range. Hooke's law is a very well known equation:

Eq. 1 $$F = -kx.$$

Where the restoration force *F* of the spring is proportional to the length *x* the spring has been stretched or compressed (known as deflection) times the spring constant *k.* The spring constant *k* differs from spring to spring and has units of N/m. The negative sign is used to signify that the spring's restoration force acts opposite to deflection. In this article I refer to the force applied to the spring, which acts in the same direction as spring displacement, thus the negative term is omitted.

Hooke's law is quite simple, and has been modified to include variables for wire diameter, spring diameter, number of coils or turns, and the rigidity of the spring [1]. These more complex models are beyond the scope of this project, where Hooke's law suffices.

Springs behave according to Hooke's law if not stretched beyond their elastic limit [2]. The elastic limit is the amount of force that can be applied to a spring before it permanently deforms [2]. Quality springs made of steel or steel alloys have a definite, easily measured elastic limit [2]; they experience an insignificant amount of deformation when stretched below this limit. Copper and nickel based alloys do not behave this way [2]; they do not have a definite elastic limit. These springs deform more gradually than their steel counterparts [2]. Since there is no definite elastic limit, a different term known as proof stress is used [2]. Proof stress is the amount of force needed to cause the spring to permanently elongate by 0.2% [2], although other elongation ratios may be used.

Extension springs also display a property known as initial tension. Initial tension is the force that presses the coils of extension springs together when not under load [2]. The initial tension force must be overcome before the spring begins to stretch [2].

Methods

There were several requirements for the spring to be used in the scale's construction. I was biased towards a precise scale, thus a spring with a low *k* value was needed since it would provide the most deflection per unit of mass added. Spring length was also an important factor, since a longer spring would experience more deflection

before reaching its elastic limit. Long springs with a low *k* value were not easily accessible to me. The only springs suitable for this project were very expensive and would have required an inordinate amount of time and effort to acquire.

I purchased 4 staplers and used the springs inside the staplers. I determined the *k* value of each individual spring. The method for determining the *k* value of each spring was as follows: I attached the spring of interest to the end of a metrestick fastened vertically to a wall. The length of the spring was recorded to the nearest millimetre. Standard weights were then attached to the spring and the length of deflection was measured to the nearest millimetre. I increased the load weight in 10, 20, and 50 gram intervals until reaching 400 grams of weight. Due to the distance between the spring and metrestick, there was some parallax viewing error. I estimated the uncertainty to be no larger than 0.6 mm.

I found the linear least squares fit to the linear range of mass-extension data. When fitting my model I did not use the non-linear data points that occur at low-weight loads, since at these points the spring is not yet within a Hookian range. The slope of each data fit (in N/m) corresponds to the *k* value of the spring of interest. None of the springs used had a satisfactory *k* value. I calculated that three of the springs would need to be attached in series to attain the desired *k* equivalent. I calculated the *k* equivalent values using equation 5, which was derived as follows. First, I rewrote Hooke's law in terms of deflection:

Eq. 2 $$\frac{F}{k} = x.$$

The total length of deflection in a series of springs will be equal to the sum of the deflection of each individual spring. The force acting on each of these springs is assumed equal, since the tension is constant throughout the series of springs:

Eq. 3 $$x_{eq} = x_a + x_b + x_n = \frac{F_a}{k_a} + \frac{F_b}{k_b} + \frac{F_n}{k_n} = F(\frac{1}{k_a} + \frac{1}{k_b} + \frac{1}{k_n}).$$

So:

Eq. 4 $$k_{eq} = \frac{F}{x_{eq}}.$$

Eq. 5 $$k_{eq} = \left[\frac{1}{k_a} + \frac{1}{k_b} + \frac{1}{k_n}\right]^{-1}.$$

The initial tension of each spring was estimated by using the x-intercept of each regression line. The x-intercept corresponds to the amount of force that must be applied to a model spring before it begins to stretch. Although linear spring models were not accurate at predicting spring deflection under light loads, the initial tension estimates were quite accurate at predicting the force that needed to be applied before the springs behaved according to Hooke's law. I chose the springs to be connected in series by their initial tension: a smaller initial tension was preferred since they would likely have a larger

functional range. The springs were arranged in order of decreasing initial tension, the spring with the largest initial tension placed above, the spring with the lowest below. Using this arrangement, the mass of each spring played a slight role in reducing the initial tension of the spring above.

I performed an elastic limit test on the remaining spring. To determine the elastic limit of the spring, I began adding mass to it, beginning at 400 grams. I increased the mass in 20 gram intervals until the spring length could no longer be evaluated due to the spring's continuous deformation. There appeared to be an exponential relationship between mass added and deflection. I linearized the data on a semi-log plot, and found a linear fit to the model. The model could then be converted back into an exponential function that modeled the deflection.

Spring	*k* (N/m)	Estimated initial tension (N)
A	19.72 ± 0.053	8.76×10^{-2}
B	23.50 ± 0.048	2.856×10^{-1}
C	20.76 ± 0.047	3.532×10^{-1}
D	23.11 ± 0.065	5.080×10^{-1}

Figure 1. Spring data collected for each spring individually. Estimated initial tension estimated using regression models.

Results and Discussion

The experimental *k* value of the spring scale was very close to the expected *k.* I anticipated a spring *k* of 7.07 N/m, which was calculated using equation 5 and the data presented in figure 1. The actual *k* of the spring scale was 7.05 ± 5.7×10^{-3} N/m. This value was calculated as the others were, by

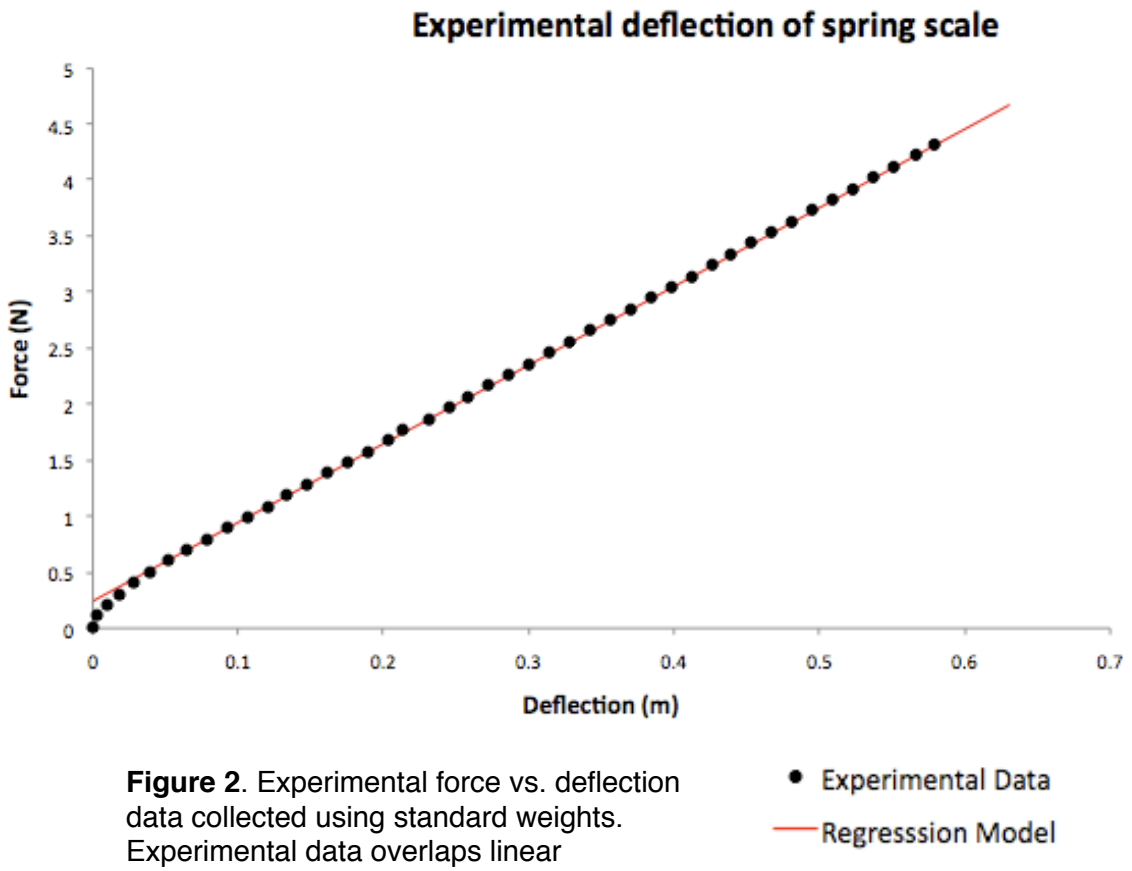

Figure 2. Experimental force vs. deflection data collected using standard weights. Experimental data overlaps linear regression model.

finding the linear least squares fit to the linear data presented in figure 2.

I determined the uncertainty in mass by using the data and regression line. In essence, the spring scale is a physical mathematical function, where mass is given as a function of deflection. I calculated the error in the scale by averaging the absolute differences (in grams) between the spring model and experimental mass. I calculated the error in the spring scale to be 0.34 grams. The average error in measurement, using the same method is 0.48 millimetres. This error is small enough to be accounted for by parallax viewing error.

I used the remaining spring to model the effect of adding excessive load weight to the spring, to the point where it surpasses its elastic limit. I plotted the deflection data on a semi-log plot and applied a linear fit to it, as shown in figure 5. I found the elongation in the tested spring to be as follows:

Eq. 6 $$E = 1.12 * 10^{-7} e^{1.531F}.$$

In the above equation, *E* is the elongation of the spring, namely, the difference between expected and actual deflection. *E* is measured in metres and *F* is the force applied to the spring in Newtons. I used this equation to construct the data tables shown in figures 3 and 4. Using this information, I estimated how much weight the scale can bear before elongation of the spring is noticeable. Although I would expect the proof stress values in all four springs to be similar (since they all came from the same source) the springs used in the scale may have different proof stress values than the test spring. Thus the maximum capacity is only an estimate, and may vary from spring to spring.

The proof stress value has a profound effect on the precision of the scale. If the proof stress value is proportional to over 0.5 grams of weight, my spring scale would no longer be accurate to 1 gram. I chose 0.2 grams equivalent to be my upper limit of elongation. This means that the proof stress would not cause the spring to elongate more than the equivalent of 0.2 grams of weight. I worked backwards through equation 6 to find the scale's maximum weight capacity. 0.2 grams equivalent of elongation (approximately 0.28 millimetres) occur when the scale is loaded with 449 grams of weight. This maximum capacity is quite low. The scale functions within a Hookian range from ~60 grams load to ~450 grams load. This is an effective range of 390 grams.

Elongation (% of total length)	Elongation (mm)	Proof stress (N)	Proof stress (g)
0.20%	0.24	5.01 ± 0.16	511 ± 16
0.078%	0.094	4.40 ± 0.16	449 ± 16

Figure 3. Proof stress and elongation data for elastic limit test spring.

Elongation (% of total length)	Elongation (mm)	Mass proportional to elongation (g)
0.20%	0.72	0.52
0.078%	0.28	0.20

Figure 4. Elongation and equivalent mass data for spring scale. Error due to mass-length conversion is negligible.

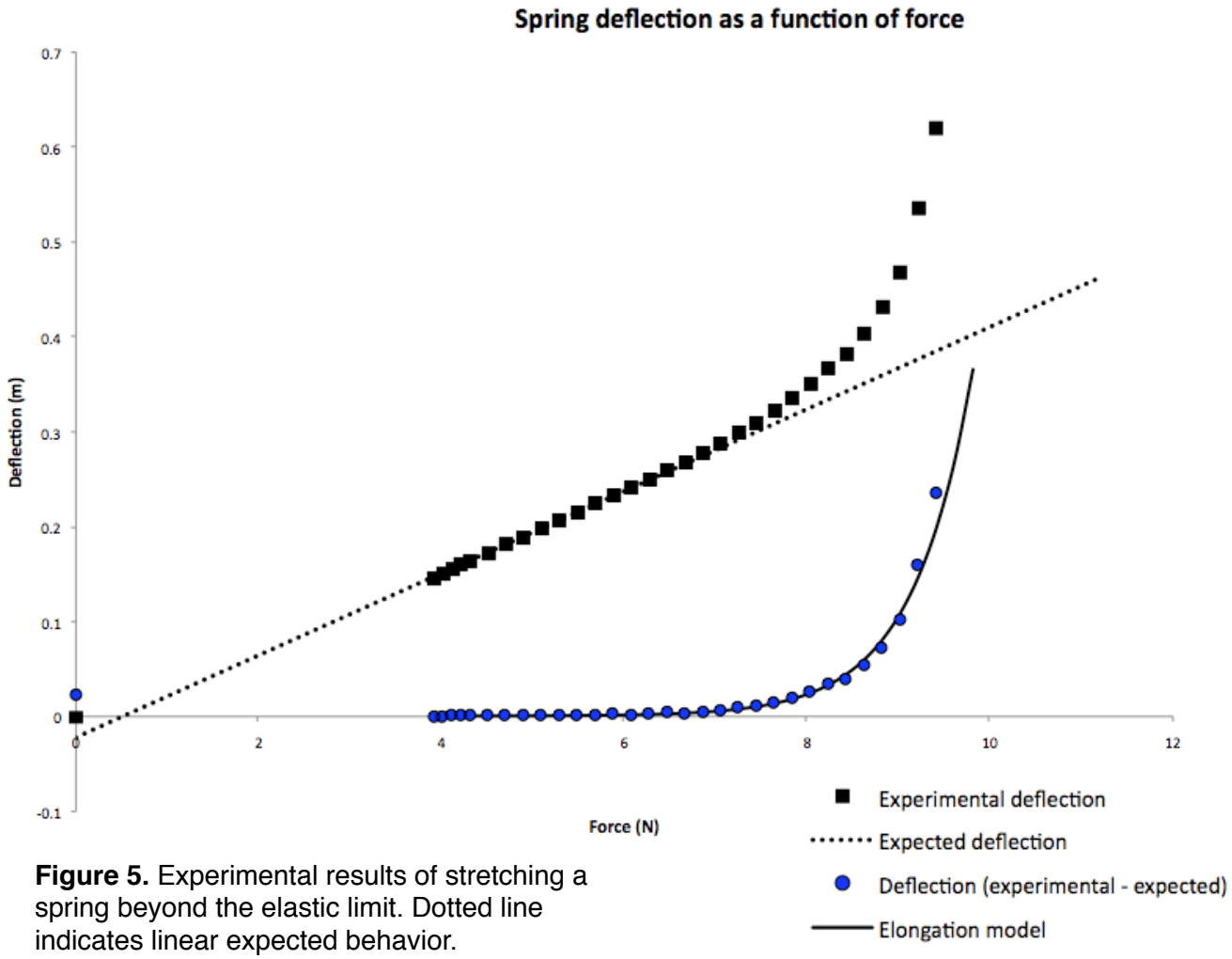

Figure 5. Experimental results of stretching a spring beyond the elastic limit. Dotted line indicates linear expected behavior.

Conclusion

My scale is functional and accurate within the described range. The range is quite limited by the spring's material. The scale's functional range could be greatly improved by using steel springs. As a way of measuring how useful my scale is compared to others, I assigned scores to different scales and balances depending on their range and precision. The range of the scale divided by the precision gives a dimensionless number, which I call the "scale score." My scale score was 390, a Kern top-loading balance received a score of 62,000 (620 g range, 0.01 g precision), and a Kern analytical balance a score of 3.2 million (320 g range, 0.1 mg precision). These scores indicate that my scale would be of little to no use in a laboratory setting.

References

1. Berry, W.R. Spring Design. London: Emmott & Company, 1961.

2. Carlson, Harold. Spring Designer's Handbook. New York: Marcel Dekker, 1978.