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THE PHASE SPACE ACCEPTANCE OF A HELICAL QUADRUPOLE CHANNEL OF FINITE LENGTH

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ABSTRACT

The acceptance region in the four-dimensional phase space of a helical quadrupole channel has been investigated numerically. The two transverse directions are strongly coupled and the projection on the (x, x') plane of the four-dimensional acceptance region is not independent of (y, y'). If such a channel is used as a pion or a muon channel, it has to be matched to quadrupoles and bending magnets so that its effective acceptance would become smaller than the acceptance region in the four-dimensional phase space.

In calculating the acceptance, a circular aperture has been assumed. Also, the axial magnetic field B_z is assumed to be small so that the transverse equations of motion are linear.

1. INTRODUCTION

A possibility of using a helical quadrupole as a muon channel has recently been discussed by R.M. Pearce[1]. By numerically integrating the equations of motion, he showed that, for a point source on the axis, the helical channel is stronger in the focusing action than an alternating-gradient channel. Since the construction of such a helical quadrupole seems to be technically feasible and the cost could be considerably less than that of an ordinary AG channel[2], it is interesting to investigate its acceptance in detail and to compare with the performance of a conventional AG muon channel[3].

The overall acceptance $A(x, x', y, y')$ of an infinitely long helical quadrupole in the four-dimensional phase space $(x, dx/dz, y, dy/dz)$ has already been calculated by Salardi et al.[4] for the case where the period λ of the pole tip rotation is much larger than the aperture R so that the axial component of the magnetic field, B_z, can be neglected. The acceptance $A(x, x', y, y')$ for a given aperture R is proportional to λ^{-2}. For a reasonable choice of $\lambda/R = 20$, the four-dimensional acceptance of a helical quadrupole for the optimum momentum is about 20% larger than the corresponding acceptance of AG channels[5]. One definite disadvantage of a helical channel is its low acceptance for momenta smaller than the optimum momentum, p(optimum). The lowest acceptable momentum is about 0.8p(optimum) compared to 0.6 to 0.7 p(optimum) for an AG channel. The normalized four-dimensional acceptance calculated by Salardi, $A(x, x', y, y')\lambda^2/R^4$ is shown in Fig. 1 as

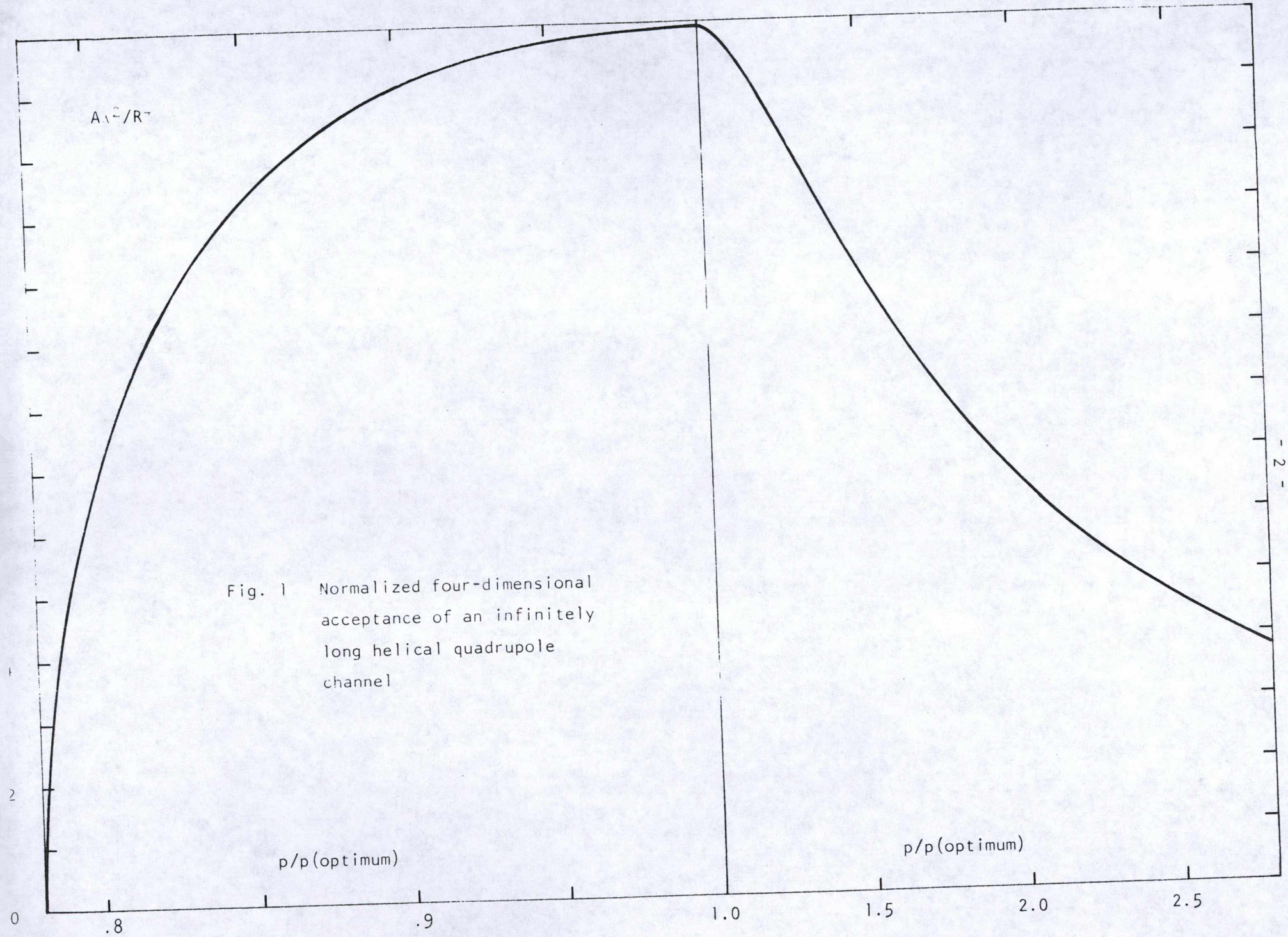

Fig. 1 Normalized four-dimensional acceptance of an infinitely long helical quadrupole channel

a function of p/p(optimum). For a muon channel, it is usually desirable to choose channel parameters such that the acceptance for the pion momentum, p_π, and for the muon momentum, p_μ, is the same. Fig. 2 indicates the decrease of this acceptance as the ratio p_π/p_μ deviates from unity.

The most important difference of a helical quadrupole from AG channels (and, at least in the first order in x, x', y and y', from bending magnets) is the strong coupling of two transverse directions, x and y. One cannot discuss the two-dimensional acceptance in (x, x') phase space independent of (y, y'). This situation is of course the same for solenoid channels[6] and for coaxial channels[7]. When these channels are used together with ordinary quadrupoles and bending magnets, as is usually the case, perfect matching of acceptances is impossible and the acceptance of a helical quadrupole (or of a solenoid) diminishes effectively.

In the following sections, computer programs for studying the acceptance of a helical channel of a <u>finite</u> length are described together with some results. In particular, as a reasonable measure of the effective two-dimensional acceptance, an acceptance area in (x, x') phase space common to all accepted values of y and y' can be found by these programs. Throughout the following discussion, the axial magnetic field B_z is neglected and notations are the same (unless specifically explained) as in RMP[1]. For the sake of convenience, notations used in the article by Salardi et al.[4] are listed in Table 1 together with corresponding notations used in RMP.

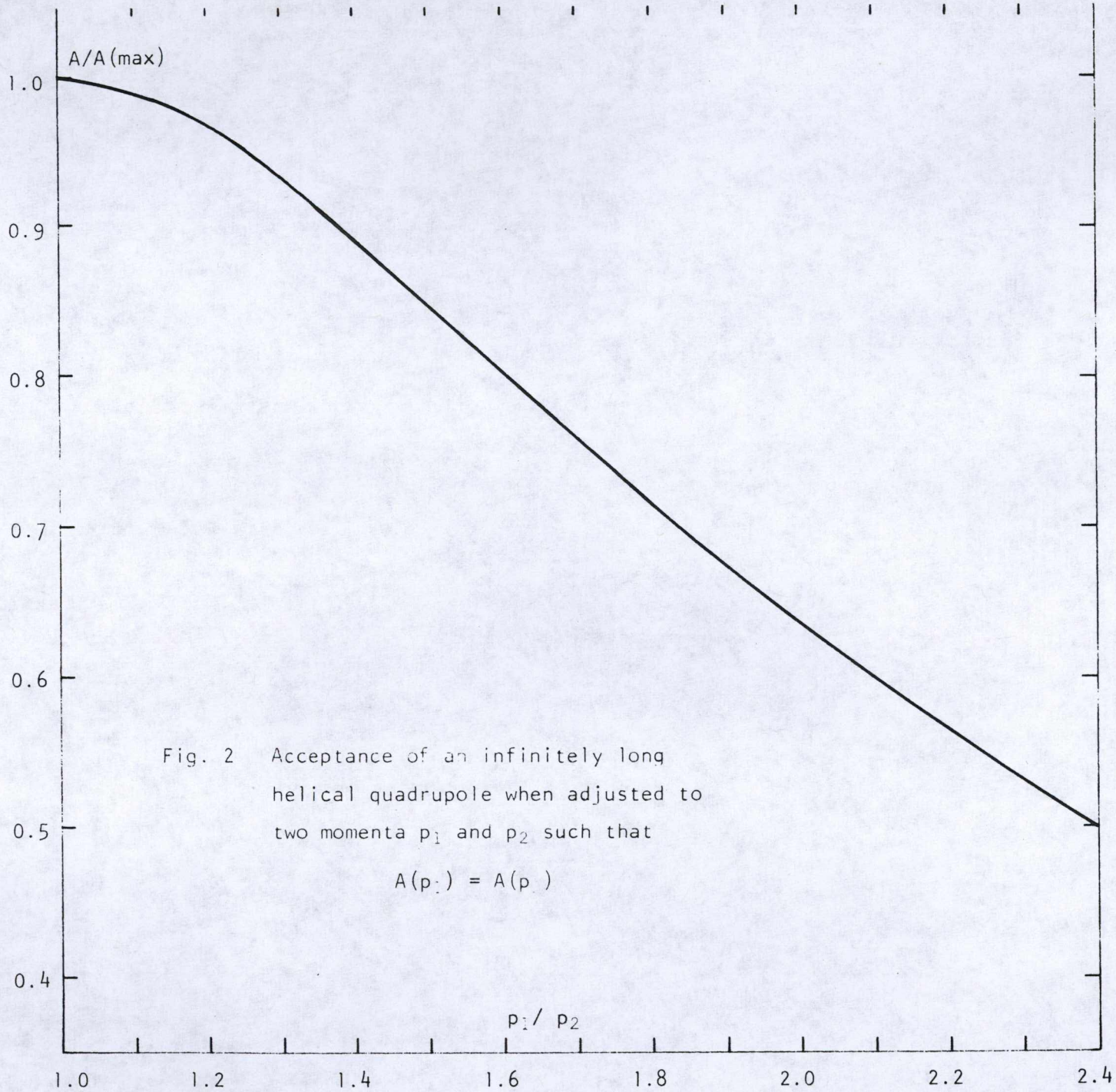

Fig. 2 Acceptance of an infinitely long helical quadrupole when adjusted to two momenta p_1 and p_2 such that

$$A(p_1) = A(p_2)$$

Table 1

List of Symbols Used in RMP[1] and in Salardi et al.[4]

RMP (also in this report)	Salardi et al
λ	L
k^2	K
$2\pi/\lambda$	ω
a	$(1 - a^2)/[4(1 + a^2)]$
X, Y	$\underline{x}, \underline{y}$
X', Y'	$(L/4\pi)\underline{x}', (L/4\pi)\underline{y}'$
z	$(4\pi/L)z$
ψ	$\omega_1 z$
ϕ	$\omega_2 z$
$(2\pi/\lambda)\sqrt{1 - 4a}$	ω_1
$(2\pi/\lambda)\sqrt{1 + 4a}$	ω_2
$\sqrt{(1 - 4a)/(1 + 4a)}$	a

2. TRAJECTORIES AND THE FOUR-DIMENSIONAL ACCEPTANCE

Exact solutions for fixed transverse coordinates x, x', y, x' and for rotating coordinates X, X', Y, Y' due to L. Teng[8] are given in RMP. Since the aperture is assumed to be circular, both coordinate systems should give the same acceptance. The rotating system is chosen such that the Y axis is always in the focusing direction. It is thus obvious that the maximum possible excursion of a particle will be in the Y direction.

From the expressions given by R.M. Pearce[1], exact solutions for X and Y are

$$X(z) = k_1(-D \cos \psi(z) + C \sin \psi(z)) + (1/k_2)(-B \cos \phi(z) + A \sin \phi(z))$$

$$Y(z) = A \cos \phi(z) + B \sin \phi(z) + C \cos \psi(z) + D \sin \psi(z) \qquad (1)$$

where

$$k_1 = (1 - 4a)^{\frac{1}{2}} \quad , \quad k_2 = (1 + 4a)^{\frac{1}{2}} \quad ,$$

$$\psi(z) = k_1 z/2 \quad , \quad \phi(z) = k_2 z/2 \quad ,$$

$$A = y_o + x_o'/2a \quad , \quad B = k_2 y_o'/2a \quad ,$$

$$C = -x_o'/2a \quad , \quad D = -(x_o + y_o'/2a)/k_1 \ . \qquad (2)$$

Note here that $x_o \equiv X(z=0)$, etc. and z is the axial distance in units of $\lambda/4\pi$ so that x_o' and y_o' have the same dimension (length) as x_o and y_o. Stable motion exists only for $|a| < \frac{1}{4}$ and a is positive when Y is the focusing direction. By rewriting the expression (1) in the form

$$X(z) = [(A^2+B^2)^{\frac{1}{2}} k_2] \sin(\phi-\phi_o) + [k_1(C^2+D^2)^{\frac{1}{2}}] \sin(\psi-\psi_o)$$

$$Y(z) = (A^2+B^2)^{\frac{1}{2}} \cos(\phi-\phi_o) + (C^2+D^2)^{\frac{1}{2}} \cos(\psi-\psi_o) \tag{3}$$

with $\phi_o = \tan^{-1}(B/A)$ and $\psi_o = \tan^{-1}(D/C)$,

one sees immediately that $\max.|Y(z)| = \max.|X(z)|$ since $k_1 \leq 1$ and $k_2 \geq 1$. The four-dimensional acceptance at $z = 0$, when the aperture is R, is determined by the condition

$$(A^2 + B^2)^{\frac{1}{2}} + (C^2 + D^2)^{\frac{1}{2}} \leq R. \tag{4}$$

By evaluating the four-dimensional volume in x_o, x'_o, y_o, y'_o space bounded by the condition (4), Salardi et al.[4] obtained the analytic expression for the acceptance $A(x, x', y, y')$,

$$A(x, x', y, y') = (R^4/\lambda^2)(2\pi^4/3)(4a)^2(1-4a)^{\frac{1}{2}}/(1+4a)^{\frac{1}{2}}. \tag{5}$$

However, this is strictly true only for an infinitely long (that is, channel length $\gg \lambda/k_1$) channel. For a channel of a finite length, $\cos(\phi-\phi_o)$ and $\cos(\psi-\psi_o)$ may not take the limiting value (+1 or -1) at the same position within the channel so that the quantity on the left side of (4) could be larger than R. For example, with $\phi_o = 0$ and $\psi_o = \pi$, $\max.|Y(z)| = |A| + |C|$ at $z = 2n\pi/k_1$ (n=1, 2, ---) for (k_2/k_1) = even integers but $\max.|Y(z)| < |A| + |C|$ for any finite value of z when (k_2/k_1) = odd integers. Corresponding values of a are, for example, 0.15 and 0.2, respectively.

Although the four-dimensional region defined by (4) is generally of a complicated shape, its two-dimensional projection can be expressed in a simple analytic form for some special cases. Some of these are discussed in the Appendix.

3. ACCEPTANCE OF FINITE CHANNELS AND THE "EFFECTIVE" ACCEPTANCE

When the length of the channel is finite, the condition (4) is too restrictive in defining the acceptable phase space region. Since exact solutions for X and Y are given as a linear function of x_o, x_o', y_o, y_o', it is convenient to use polygons to represent acceptable areas in (x, x') or (y, y') phase space[9].

Unless the channel is very short (channel length $\lesssim \lambda/k_1$), the largest excursion within the channel would always occur in Y direction so that the condition for the acceptable region is

$$R \geqq Y(z) \geqq -R \text{ for all values of } z. \tag{6}$$

For a given pair of (y_o, y_o'), the condition (6) defines two parallel lines in (x_o, x_o') space. The true acceptance in (x_o, x_o') space is the envelope of these parallel lines evaluated at all values of z within the channel. For practical purposes, however, it is sufficient to take discrete values of z and regard the resulting polygon as a good approximation of the true acceptance. From Eq. (1), parallel lines evaluated at z are

$$u(z)\, x_o + v(z)\, x_o' + w(z,\, y_o,\, y_o') = R \text{ and } -R \tag{7}$$

where $\quad u(z) = -\sin\psi(z)/k_1$

$$v(z) = (\cos\phi(z) - \cos\psi(z))/(2a)$$

$$w = y_o \cos\phi(z) + y_o' \,(k_2 \sin\phi(z) - \sin\psi(z)/k_1)/(2a).$$

The procedure is of course quite similar if one wants to find the acceptance in $(y_o,\, y_o')$ space for a given pair of $(x_o,\, x_o')$.

The condition (7) is invariant under the transformation

$$(x_o,\, x_o',\, y_o,\, y_o') \rightarrow (-x_o,\, -x_o',\, -y_o,\, -y_o')$$

so that the acceptance has to be studied only for $y_o > 0$ or $x_o > 0$.

Furthermore, unless the channel is very short (channel length $\lesssim \lambda/k_1$), the transformation

$$(x_o,\, x_o',\, y_o,\, y_o') \rightarrow (-x_o,\, x_o',\, y_o,\, -y_o')$$

or $\quad (x_o,\, x_o',\, y_o,\, y_o') \rightarrow (x_o,\, -x_o',\, -y_o,\, y_o')$

would keep the acceptance unchanged. This can be seen from Eqs. (2) and (3) where these transformations merely change the sign of ϕ_o and ψ_o so that two oscillations, $\cos(\phi - \phi_o)$ and $\cos(\psi - \psi_o)$, are shifted in phase by at most 2π. Thus, the area to be studied in the four-dimensional phase space is $x_o,\, x_o',\, y_o,\, y_o' \geqq 0$.

When the channel is to be matched to quadrupoles or bending magnets, only that portion of the (x_o, x_o') polygons which are common to all values of (y_o, y_o') could be utilized effectively. In general, it is very difficult to shape a beam such that its distribution in (x_o, x_o') space depends in a specific manner on (y_o, y_o'). As a measure of this effective acceptance, one can consider the overlapping portion of a polygon with the standard polygon (for $y_o = y_o' = 0$). This is illustrated in Fig. 3 where shaded areas are considered to be unacceptable even though they satisfy the condition (6).

4. COMPUTER PROGRAMS: HELIX 1 AND HELIX 2

4.1 HELIX 1 (see pages 21-26)

For given values of the channel length and the parameter a, this program calculates acceptance polygons and their areas in (y_o, y_o') space for mesh points in (x_o, x_o') space. It can also be used for obtaining polygons in (x_o, x_o') space for mesh points in (y_o, y_o') space.

Input parameters:

card 1 (2I5, 3F10.)

IXY = 1 for polygons in (y_o, y_o') space,
2 for polygons in (x_o, x_o') space.

NZ($\leq$ 500) = total number of points along z direction where two parallel lines (7) are to be evaluated.

DZ = interval of equally spaced points along z.

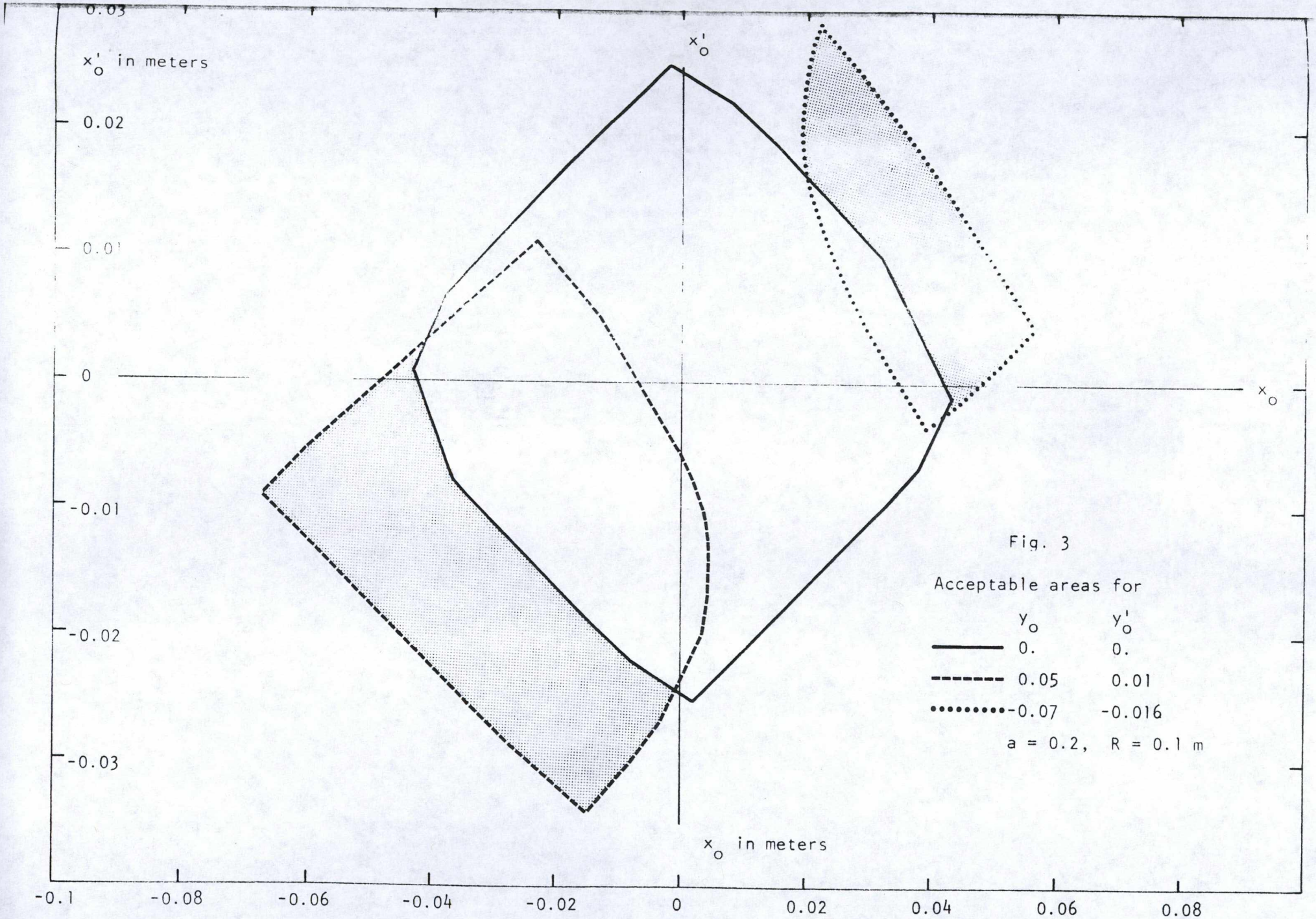

Fig. 3

Acceptable areas for

Parallel lines are calculated at

$$z = DZ,\ 2(DZ),\ 3(DZ),\ 3(DZ),\ \ldots\ldots,\ (NZ)(DZ)$$

and the channel length (in units of $\lambda/r\pi$) is (NZ)(DZ).

SA = parameter a (between 0 and 0.25).

R = aperture radius in meters (= 0.1).

The value of R is not really needed since x_o, x_o', y_o, y_o' are all proportional to R, due to the linear approximation for the equations of motion. The choice of 0.1 is purely for convenience.

card 2 (2I5, 2F10.)

NYNET ($\leq$ 50) = total number of mesh points in x_o direction (IXY = 1) or y_o direction (IXY = 2) including both end points.

NYPNET ($\leq$50) = total number of mesh points in x_o' direction (IXY = 1) or y_o' direction (IXY = 2) including both end points.

YMAX = largest value of x_o(IXY = 1) or y_o(IXY = 2) of the mesh; usually this is equal to R.

YPMAX = largest value of x_o'(IXY = 1) or y_o'(IXY = 2) of the mesh; with R = 0.1, YPMAX $\cong$ 0.03 - 0.05.

Total number of points in the mesh is (NYNET)(NYPNET) with

x_o = 0, YMAX/(NYNET-1), 2(YMAX)/(NYNET-1), ... , YMAX;

x_o' = 0, YPMAX/(NYPNET-1), 2(YPMAX)/(NYPNET-1), . . , YPMAX

for IXY = 1.

Output parameters:

For each mesh point (x_o, x_o') or (y_o, y_o'), coordinates (y_o, y_o') or (x_o, x_o') of the acceptance polygon are given as well as its area. The maximum number of vertices is internally limited to 25 in the program.

$$IXY = 1 \qquad X = y_o, \qquad XP = y'_o$$
$$\qquad\qquad Y = x_o, \qquad YP = x'_o$$
$$IXY = 2 \qquad X = x_o, \qquad XP = x'_o$$
$$\qquad\qquad Y = y_o, \qquad YP = y'_o$$

Note that the axial distance is in units of $\lambda/4\pi$ so that

$$(dx_o\ /ds) = (4\pi/\lambda)\ x'_o$$
$$(dy_o\ /ds) = (4\pi/\lambda)\ y'_o$$

where $s \equiv (\lambda/4\pi)z$ is the real distance (in meters, for example).

4.2 <u>HELIX 2</u> (see pages 27-35)

This is identical to HELIX 1 but polygons at mesh points are overlapping portions only. The effective four-dimensional phase space acceptance is then numerically integrated. If the aperture radius R is given in meters, the four-dimensional acceptance in (x_o, x'_o, y_o, y'_o) space is in $(\text{meter})^4$. In order to get the acceptance in $(x_o, dx_o\ /ds, y_o, dy_o\ /ds)$ space, one must multiply this by $(4\pi/\lambda)^2$. Input cards are identical to those for HELIX 1 except that NYNET must be an odd number.

Both programs could be used for a number of different parameters by just adding two input cards for each case. There should be a blank card at the end of all input data cards.

5. RESULTS

So far, two programs have been used for the following choice of parameters:

R = 0.1, a = 0.05, 0.1, 0.15, 0.2, 0.215, 0.23,

NZ = 314, DZ = 0.1, IXY = 2,

YMAX = 0.1, YPMAX = 0.03 or 0.04 depending on the choice of a,

NYNET = 11, NYPNET = 11 or 13.

The channel is "long" for a = 0.05 - 0.2 but "short" for a = 0.23. Results for a = 0.215 and 0.23 may not be as accurate as for smaller values of a. Total computer time with Yale IBM 7094(DCS) was about 80 minutes.

All results are available in four IBM printouts. Normalized four-dimensional acceptances are shown in Fig. 4 as a function of a; the solid line is for an infinitely long channel, the dashed line is for a channel of length 31.4 $\lambda/4\pi$ and the dotted line is the effective acceptance of the same finite channel. Although the finite channel gives a larger acceptance (dashed line) compared to an infinitely long channel (solid line), its effective acceptance (dotted line) could be as small as one-half of that.

It is interesting to compare the effective acceptance of a helical channel with the acceptance of a conventional AG channel. With a reasonable choice of the AG channel parameters,

S/L = 0.05 S = distance between two adjacent quadrupoles

L = length of each quadrupole

R/L = ¼ R = radius of quadrupole aperture,

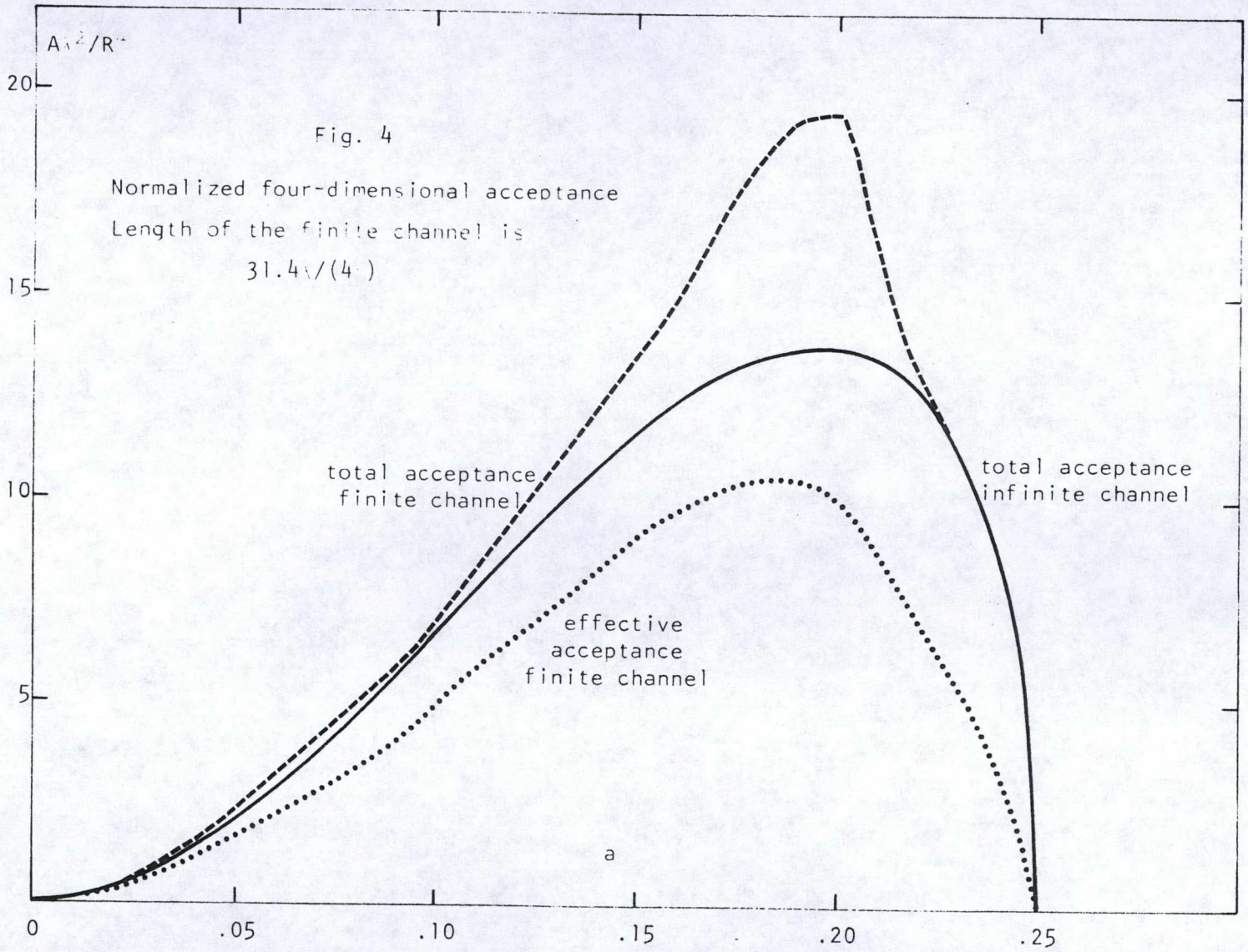

Fig. 4

Normalized four-dimensional acceptance

Length of the finite channel is

31.4λ/(4)

together with the assumption that two channels have the same period λ of the pole rotation,

$$\lambda = 2X \text{ (magnetic period of the AG channel)}$$
$$= 6\ L, \tag{8}$$

one gets the relation

$$R/\lambda = 1/24.$$

This value is consistent with the assumption that the axial magnetic field B_z of the helical quadrupole is negligible. The maximum four-dimensional acceptance for each channel is *

helical channel	$10.4\ R^4/\lambda^2$
AG channel	$16.2\ R^4/\lambda^2$.

The required field gradient of the helical channel is about 70% of what is needed for AG channel quadrupoles. On the other hand, only 70% of the total length will be occupied by quadrupoles in the AG channel. When the channel is to be used as a muon channel, it is desirable to have the same acceptance for pions and muons. For example, assume that $p_\mu/p_\pi = 1.33$ (collection of forward-decaying muons). The acceptance one can get is then

helical channel	$9.3\ R^4/\lambda^2$
AG channel	$15.3\ R^4/\lambda^2$.

---

* The AG channel is assumed to be infinitely long here. This is a good approximation when the channel has more than 8 magnets (four magnetic periods) and it is operated near the optimum momentum.

It is of course possible to increase the effective acceptance of the helical channel by taking a smaller value for λ than given by (8). The ratio R/λ is then increased and the effect of the axial magnetic field may not be entirely ignored. Although the axial field seems to increase the focussing action of a helical channel,[1] the coupling of two directions x and y becomes more complicated (equations of motion contain x' and y') and it is by no means obvious that this would cause an increase in the effective acceptance. Another disadvantage of the helical channel is its inferior performance for $p < p$ (optimum). The acceptance is zero at $p = 0.75$ p (optimum) whereas the AG channel still maintains 57% of the maximum acceptance at this momentum. The low momentum cut-off of the AG channel is p (cut-off) = 0.64 p (optimum).

The author is grateful to members of the TRIUMF Project at the University of Victoria for stimulating discussions.

REFERENCES

1. R.M. Pearce, submitted to Nuclear Inst. and Methods, 1969. This will be referred to as RMP.

2. Roland Cobb, private communication.

3. For example, see "Stopped Muon Channel", Los Alamos Design Status Report. V.W. Hughes, S. Ohnuma, K. Tanabe, H. Vogel (1969).

4. G. Salardi, E. Zanazzi, and F. Uccelli, Nuclear Inst. and Methods, 59 (1968), 152.

5. See, for example, Internal Report Y-12, Yale University, October 1964, p. IV-182.

6. R. Helm, SLAC-4 ("Red" Stanford Report), August 1962.

7. S. van der Meer, CERN 62-16, April 1962; E. Regenstreif, CERN 64-41, September 1964.

8. Lee Teng, ANAL-55, February 1959.

9. A. Citron, J. Fronteau and J. Hornsby, CERN 63-30, August 1963; J. Fronteau and J. Hornsby, CERN 62-36, November 1962.

APPENDIX

The acceptable region in the four-dimensional space given by the condition (4) takes a complicated shape in general. However, its two-dimensional projection for some special cases can be represented by simple analytic expressions.

1. Parallel Beams $(x_o' = y_o' = 0)$

$$|y_o| + |x_o|/k_1 \leq R \qquad \text{Fig. 5a}$$

2. Point Source $(x_o = y_o = 0)$

$$u = x_o'/(2a) \ , \ v = y_o'/(2a)$$

$$(u^2 + k_2^2 v^2)^{\frac{1}{2}} + (u^2 + v^2/k_1^2)^{\frac{1}{2}} \leq R \qquad \text{Fig. 5b}$$

Since $k_1 k_2 = (1 - 16a^2)^{\frac{1}{2}} \leq 1$, this area lies between two ellipses

$$u^2 + k_2^2 v^2 \leq (R/2)^2$$

$$u^2 + v^2/k_1^2 \leq (R/2)^2 \qquad \text{broken lines in Fig. 5b}$$

3. $y_o' = 0$ $\quad u = x_o/k_1 \ , \quad v = x_o'/(2a)$

$$v \leq (R-y_o)/2 - \tfrac{1}{2}u^2/(R-y_o) \quad \text{for} \quad v \geq -y_o \ ,$$

$$v \geq \tfrac{1}{2}u^2/(R+y_o) - (R+y_o)/2 \quad \text{for} \quad v \leq -y_o \ . \qquad \text{Fig. 5c}$$

Note that, for $y_o = R$, the acceptable area in (x_o, x_o') space reduces to a line

$$x_o = 0 \ , \quad -2aR \leq x_o' \leq 0 \ .$$

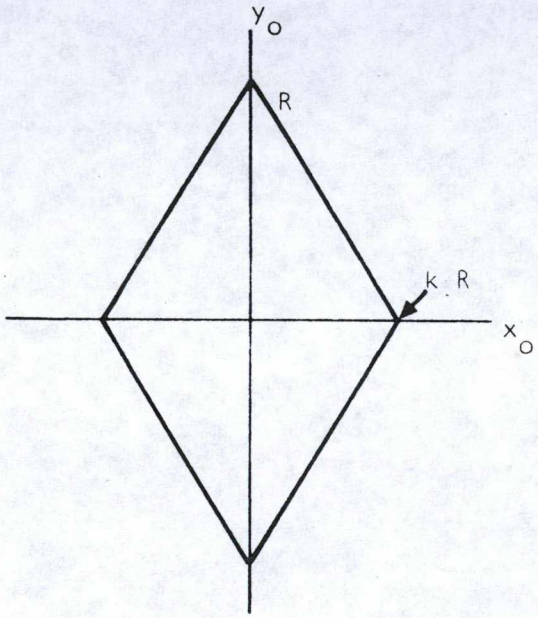

Fig. 5a

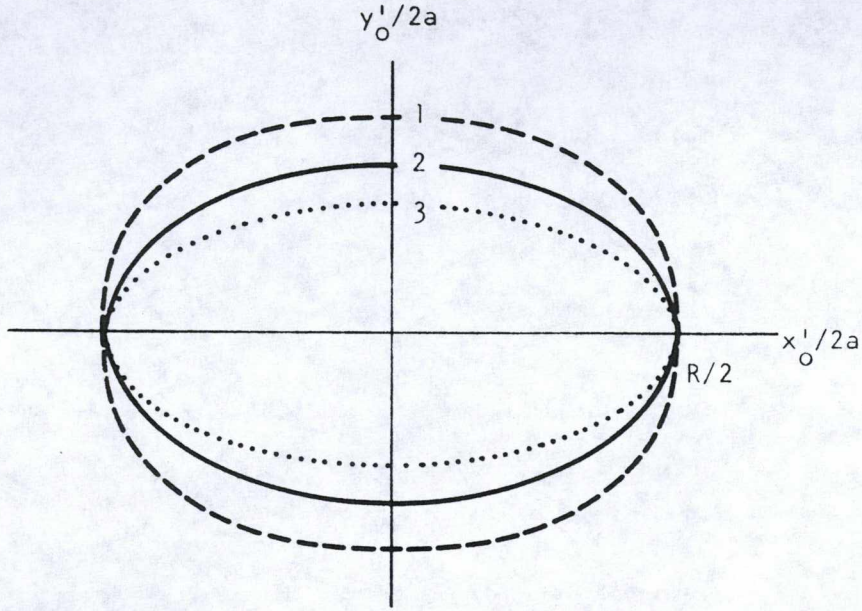

Fig. 5b

(1) $R/(2k_2)$

(2) $k_1 R/(1+k_1 k_2)$

(3) $k_1 R/2$

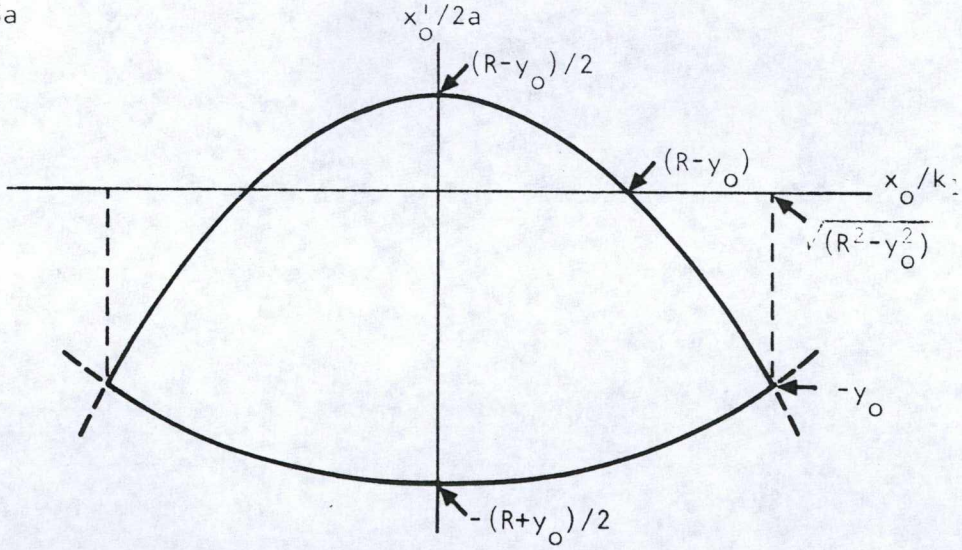

Fig. 5c

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      DIMENSION C(500),D(500),G1(500),G2(500),Y(50),YP(50),AY(4),WXI(4),
     1WXPI(4),WX(100),WXP(100)
 1000 FORMAT(2I5,4F10.6)
 1001 FORMAT(1H1,5X,3HA =F10.5,5X,3HR =F10.5///)
 1002 FORMAT(//10X,10H**********,3X,3HY =F10.5,10X,4HYP =F10.5,3X,
     110H**********/)
 1003 FORMAT(5X,10HACCEPTANCE,E20.5)
 1004 FORMAT(5HO X  /(5E20.5))
 1005 FORMAT(5HO XP /(5E20.5))
 1006 FORMAT(/5X,14HMAX. DISTANCE=F10.3)
 1007 FORMAT(2I5,4F10.5)
 1008 FORMAT(5X,15H(NO ACCEPTANCE)/)
      NXI=4
  500 READ(5,1000)IXY,NZ,DZ,SA,R
      IF(IXY.EQ.0)STOP
      WRITE(6,1001)SA,R
      WXI(1)=R
      WXI(2)=R
      WXI(3)=-R
      WXI(4)=-R
      WXPI(1)=0.5
      WXPI(2)=-0.5
      WXPI(3)=-0.5
      WXPI(4)=0.5
      A1=SQRT(1.-4.*SA)
      A2=SQRT(1.+4.*SA)
       DO 10 K=1,NZ
      Z=FLOAT(K)*DZ
  300 PS=A1*Z/2.
      PH=A2*Z/2.
      CPS=COS(PS)
      SPS=SIN(PS)
      CPH=COS(PH)
      SPH=SIN(PH)
      C(K)=-SPS/A1
      D(K)=(CPH-CPS)/2./SA
      G1(K)=CPH
      G2(K)=(A2*SPH-SPS/A1)/2./SA
   10  CONTINUE
      READ(5,1007)NYNET,NYPNET,YMAX,YPMAX
      DO 20 K=1,NYNET
   20 Y(K)=FLOAT(K-1)*YMAX/FLOAT(NYNET-1)
      DO 70 K=1,NYPNET
   70 YP(K)=FLOAT(K-1)*YPMAX/FLOAT(NYPNET-1)
      DO 30 I=1,NYNET
      YNOW=Y(I)
      DO 40 J=1,NYPNET
      YPNOW=YP(J)
      WRITE(6,1002)YNOW,YPNOW
      NX=NXI
      DO 51 L=1,NXI
      WX(L)=WXI(L)
      WXP(L)=WXPI(L)
   51 CONTINUE
      DO 50 K=1,NZ
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```
      AY(4)=R
      IF(IXY.EQ.1)GO TO 95
      AY(1)=C(K)
      AY(2)=D(K)
      AY(3)=YNOW*G1(K)+YPNOW*G2(K)
      GO TO 96
   95 AY(1)=G1(K)
      AY(2)=G2(K)
      AY(3)=YNOW*C(K)+YPNOW*D(K)
   96 CALL OUTPO(NX,WX,WXP,AY)
      IF(NX.GT.25) CALL REDUCE(NX,WX,WXP,25)
      CALL AREA(NX,WX,WXP,ACCEPT)
      IF(ACCEPT.LE.0.0) GO TO 90
   50 CONTINUE
   60 WRITE(6,1003)ACCEPT
      WRITE(6,1004)(WX(K),K=1,NX)
      WRITE(6,1005)(WXP(K),K=1,NX)
      GO TO 40
   90 WRITE(6,1008)
   40 CONTINUE
   30 CONTINUE
      GO TO 500
      END
```

```
      SUBROUTINE CUTPL(M,X,Y,A)
      DIMENSION X(100),Y(100),A(4),V(100),W(100)
      ZXIF(P)=-(A2*C+(A3-P)*DX)/DET
      YYIF(P)=(A1*C-(A3-P)*DY)/DET
      A1=A(1)
      A2=A(2)
      A3=A(3)
      N=0
      X(M+1)=X(1)
      Y(M+1)=Y(1)
      MM=M+1
      DO 50 I=2,MM
      DX=X(I)-X(I-1)
      DY=Y(I)-Y(I-1)
      C=X(I)*Y(I-1)-Y(I)*X(I-1)
      ELI=A(1)*X(I)+A(2)*Y(I)+A(3)
      ELIP=A(1)*X(I-1)+A(2)*Y(I-1)+A(3)
      DET=ELI-ELIP
      IF(A(4))5,55,5
    5 JJ=0
      IF(ELI-A(4))10,10,12
   10 JJ=1
      IF(ELI+A(4))12,11,11
   11 JJ=2
   12 KK=0
      IF(ELIP-A(4))15,15,20
   15 KK=1
      IF(ELIP+A(4))20,16,16
   16 KK=2
   20 K=3*JJ+KK+1
      GO TO (50,23,24,25,50,26,30,31,49),K
   23 N=N+1
      V(N)=ZXIF(-A(4))
      W(N)=YYIF(-A(4))
   24 N=N+1
      V(N)=ZXIF(A(4))
      W(N)=YYIF(A(4))
      GO TO 50
   25 N=N+1
      V(N)=ZXIF(A(4))
      W(N)=YYIF(A(4))
   26 N=N+1
      V(N)=ZXIF(-A(4))
      W(N)=YYIF(-A(4))
      GO TO 50
   30 N=N+1
      V(N)=ZXIF(A(4))
      W(N)=YYIF(A(4))
      GO TO 49
   31 N=N+1
      V(N)=ZXIF(-A(4))
      W(N)=YYIF(-A(4))
      GO TO 49

   55 JJ=0
```

```
      IF(ELI)57,56,56
   56 JJ=1
   57 KK=0
      IF(ELIP)59,58,58
   58 KK=1
   59 K=2*JJ+KK+1
      GO TO (50,24,30,49),K
   49 N=N+1
      V(N)=X(I)
      W(N)=Y(I)
   50 CONTINUE
      M=N
      IF(M)75,75,60
   60 XMIN=1.E10
      YMIN=1.E10
      XMAX=-1.E10
      YMAX=-1.E10
      DO 68I=1,N
      IF(V(I)-XMIN)61,62,62
   61 XMIN=V(I)
   62 IF(V(I)-XMAX)64,64,63
   63 XMAX=V(I)
   64 IF(W(I)-YMIN)65,66,66
   65 YMIN=W(I)
   66 IF(W(I)-YMAX)68,68,67
   67 YMAX=W(I)
   68 CONTINUE
      SIZE1=XMAX-XMIN
      SIZE2=YMAX-YMIN
      SIZ=SIZE1+SIZE2
      IF(SIZ.EQ.0.0) N=1
      M=1
      X(1)=V(1)
      Y(1)=W(1)
      IF(N.LE.1) GO TO 75
      DO 72I=2,N
      IF(SIZE1.LE.0.0.AND.SIZE2.GT.0.0) GO TO 80
      IF(ABS(V(I)-X(M))/SIZE1-.00005)81,70,70
   81 IF(SIZE2.LE.0.0) GO TO 72
   80 IF(ABS(W(I)-Y(M))/SIZE2-.00005)72,70,70
   70 M=M+1
      X(M)=V(I)
      Y(M)=W(I)
   72 CONTINUE
   75 RETURN
      END
```

```
      SUBROUTINE REDUCE(N,X,Y,MAXPT)
      DIMENSION X(100),Y(100)
      IF(N.LE.MAXPT) RETURN
   45 DIST=SQRT((X(N)-X(1))**2+(Y(N)-Y(1))**2)
      IK=N
      M=N-1
      DO 20 I=1,M
      DISTC=SQRT((X(I+1)-X(I))**2+(Y(I+1)-Y(I))**2)
      IF(DISTC.GE.DIST) GO TO 20
      DIST=DISTC
      IK=I
   20 CONTINUE
      IF(IK.LT.N)    GO 25
      X(1)=(X(1)+X(N))/2.
      Y(1)=(Y(1)+Y(N))/2.
      GO TO 35
   25 X(IK)=(X(IK)+X(IK+1))/2.
      Y(IK)=(Y(IK)+Y(IK+1))/2.
      IF(IK.EQ.M) GO TO 35
      LL=IK+1
      DO 30 J=LL,M
      X(J)=X(J+1)
   30 Y(J)=Y(J+1)
   35 N=N-1
      IF(N.GT.MAXPT) GO TO 45
      RETURN
      END
```

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```
      SUBROUTINE AREA (N,X,Y,A)
      DIMENSION X(100),Y(100)
      Z=0
      DO 20 I=2,N
      Z=Z+ABS((X(I)-X(I-1))*(Y(I-1)-Y(I))-(Y(I)-Y(I-1))*
     -        (X(I-1)-X(I)))/2.
   20 CONTINUE
      A=Z
      RETURN
      END
```

```
      COMMON C(500),D(500),G1(500),G2(500),WX(100),WXP(100),AY(4),
     1R,NX,YNOW,YPNOW,IXY
      DIMENSION Y(50),YP(50),WXI(4),WXPI(4),ACCEPT(500),WXF(100),
     1WXPF(100)
      DIMENSION APIS(51),APINT(50,51),AIY(50)
 1000 FORMAT(2I5,4F10.6)
 1001 FORMAT(1H1,5X,3HA =F10.5,5X,3HR =F10.5///)
 1002 FORMAT(//10X,10H**********,3X,3HY =F10.5,13X,4HYP =F10.5,3X,
     110H**********/)
 1003 FORMAT(5X,10HACCEPTANCE/(5E20.5))
 1004 FORMAT(5H0 X  /(5E20.5))
 1005 FORMAT(5H0 XP /(5E20.5))
 1006 FORMAT(/5X,14HMAX. DISTANCE=F10.3)
 1007 FORMAT(2I5,4F10.5)
 1008 FORMAT(5X,15H(NO ACCEPTANCE)//)
      NXI=4
  501 READ(5,1000)IXY,NZ,DZ,SA,R
      IF(IXY.EQ.0)STOP
      WRITE(6,1001)SA,R
      WXI(1)=R
      WXI(2)=R
      WXI(3)=-R
      WXI(4)=-R
      WXPI(1)=0.1
      WXPI(2)=-0.1
      WXPI(3)=-0.1
      WXPI(4)=0.1
      A1=SQRT(1.-4.*SA)
      A2=SQRT(1.+4.*SA)
       DO 10 K=1,NZ
      Z=FLOAT(K)*DZ
  300 PS=A1*Z/2.
      PH=A2*Z/2.
      CPS=COS(PS)
      SPS=SIN(PS)
      CPH=COS(PH)
      SPH=SIN(PH)
      C(K)=-SPS/A1
      D(K)=(CPH-CPS)/Z./SA
      G1(K)=CPH
      G2(K)=(A2*SPH-SPS/A1)/2./SA
   10  CONTINUE
      READ(5,1007)NYNET,NYPNET,YMAX,YPMAX
      DO 20 K=1,NYNET
   20 Y(K)=FLOAT(K-1)*YMAX/FLOAT(NYNET-1)
      DO 70 K=1,NYPNET
   70 YP(K)=FLOAT(K-1)*YPMAX/FLOAT(NYPNET-1)
      YNOW=0.
      YPNOW=0.
      NX=NXI
      DO 500 L=1,NXI
      WX(L)=WXI(L)
  500 WXP(L)=WXPI(L)
      DO 600 K=1,NZ
      CALL POLY(K)
```

```
      CALL AREA(NX,WX,WXP,ACC)
  600 CONTINUE
  601 NXF=NX
      DO 602 L=1,NXF
      WXF(L)=WX(L)
  602 WXPF(L)=WXP(L)
      APTS(1)=ACC
      WRITE(6,2000)ACC
 2000 FORMAT(//5X,20HCENTRAL ACCEPTANCE =E12.5)
      WRITE(6,1004)(WXF(L),L=1,NXF)
      WRITE(6,1005)(WXPF(L),L=1,NXF)
      DO 30 I=1,NYNET
      YNOW=Y(I)
  403 DO 40 J=1,NYPNET
      YPNOW=YP(J)
      WRITE(6,1002)YNOW,YPNOW
      IF(YNOW.EQ.0..AND.YPNOW.EQ.0.)GO TO 40
      NX=NXI
      ARAP=1000.
      DO 51 L=1,NXI
      WX(L)=WXI(L)
      WXP(L)=WXPI(L)
   51 CONTINUE
      DO 50 K=1,NZ
      CALL POLY(K)
      CALL AREA(NX,WX,WXP,ACCEPT(K))
      IF(ACCEPT(K).LE.0.)GO TO 90
   50 CONTINUE
   60 CALL INTPO(NXF,WXF,WXPF,NX,WX,WXP)
      IF(NX.GT.25)CALL REDUCE(NX,WX,WXP,25)
      CALL AREA(NX,WX,WXP,APT)
      IF(APT.LE.0.)GO TO 90
      APTS(J)=APT
      JNOW=J
      WRITE(6,2001)APT
 2001 FORMAT(5X,12HACCEPTANCE =E12.5)
      WRITE(6,1004)(WX(K),K=1,NX)
      WRITE(6,1005)(WXP(K),K=1,NX)
   40 CONTINUE
   90 WRITE(6,1006)
      JLST=JNOW+1
      APTS(JLST)=0.
      JCK=JNOW-2*(JNOW/2)
      IF(JCK.EQ.1)JLST=JNOW
      DO 450 JJ=1,JLST
  450 APINT(I,JJ)=APTS(JJ)
      JTOT=JLST-2
      AIY(I)=0.
      DO 453 KKJ=1,JTOT,2
  453 AIY(I)=AIY(I)+APINT(I,KKJ)+4.*APINT(I,KKJ+1)+APINT(I,KKJ+2)
      AIY(I)=AIY(I)*ABS(YP(J)-YP(I))/3.
   30 CONTINUE
      AIYYP=0.
      ITOT=NYNET-2
      DO 454 I=1,ITOT,2
  454 AIYYP=AIYYP+AIY(I)+4.*AIY(I+1)+AIY(I+2)
```

```
      AIYYP=AIYYP+ABS( [illegible] Y([illegible]))/3.
      WRITE([illegible],1111)AIY[illegible]
 1111 FORMAT(////10X,[illegible] AVERAGE [illegible] ,E15.[illegible])
      AIYYP=SQRT(AIYYP[illegible]
      WRITE([illegible],1112)AIY[illegible]
 1112 FORMAT(//10X,E15.[illegible] ,[illegible]**2)
      GO TO 501
      END
```

```
      SUBROUTINE CUTP(M,X,Y,A)
      DIMENSION X(100),Y(100),A(4),V(100),W(100)
      ZXIF(P)=-(A2*C+(A3-P)*DX)/DET
      YYIF(P)=(A1*C-(A3-P)*DY)/DET
      A1=A(1)
      A2=A(2)
      A3=A(3)
      N=0
      X(M+1)=X(1)
      Y(M+1)=Y(1)
      MM=M+1
      DO 50 I=2,MM
      DX=X(I)-X(I-1)
      DY=Y(I)-Y(I-1)
      C=X(I)*Y(I-1)-Y(I)*X(I-1)
      ELI=A(1)*X(I)+A(2)*Y(I)+A(3)
      ELIP=A(1)*X(I-1)+A(2)*Y(I-1)+A(3)
      DET=ELI-ELIP
      IF(A(4))5,55,5
    5 JJ=0
      IF(ELI-A(4))10,10,12
   10 JJ=1
      IF(ELI+A(4))12,11,11
   11 JJ=2
   12 KK=0
      IF(ELIP-A(4))15,15,20
   15 KK=1
      IF(ELIP+A(4))20,16,16
   16 KK=2
   20 K=3*JJ+KK+1
      GO TO (50,23,24,25,50,26,30,31,49),K
   23 N=N+1
      V(N)=ZXIF(-A(4))
      W(N)=YYIF(-A(4))
   24 N=N+1
      V(N)=ZXIF(A(4))
      W(N)=YYIF(A(4))
      GO TO 50
   25 N=N+1
      V(N)=ZXIF(A(4))
      W(N)=YYIF(A(4))
   26 N=N+1
      V(N)=ZXIF(-A(4))
      W(N)=YYIF(-A(4))
      GO TO 50
   30 N=N+1
      V(N)=ZXIF(A(4))
      W(N)=YYIF(A(4))
      GO TO 49
   31 N=N+1
      V(N)=ZXIF(-A(4))
      W(N)=YYIF(-A(4))
      GO TO 49

   55 JJ=0
```

```
      IF(LL.)57,[illegible]
   56 JJ=1
   57 KK=0
      IF(ELIP)59,[illegible]
   58 KK=1
   59 K=2*JJ+KK+1
      GO TO (50,24,30,49),[illegible]
   49 N=N+1
      V(N)=X(I)
      W(N)=Y(I)
   50 CONTINUE
      M=N
      IF(M)75,75,60
   60 XMIN=1.E10
      YMIN=1.E10
      XMAX=-1.E10
      YMAX=-1.E10
      DO 68I=1,N
      IF(V(I)-XMIN)61,62,62
   61 XMIN=V(I)
   62 IF(V(I)-XMAX)64,[illegible],63
   63 XMAX=V(I)
   64 IF(W(I)-YMIN)65,[illegible],66
   65 YMIN=W(I)
   66 IF(W(I)-YMAX)68,[illegible],67
   67 YMAX=W(I)
   68 CONTINUE
      SIZE1=XMAX-XMIN
      SIZE2=YMAX-YMIN
      SIZ=SIZE1+SIZE2
      IF(SIZ.EQ.0.0) [illegible]=1
      M=1
      X(1)=V(1)
      Y(1)=W(1)
      IF(N.LE.1) GO TO 75
      DO 72I=2,N
      IF(SIZE1.LE.0.0.AND.SIZE2.GT.0.0) GO TO 80
      IF(ABS(V(I)-X(M))/SIZE1-.00005)81,70,70
   81 IF(SIZE2.LE.0.0) GO TO 72
   80 IF(ABS(W(I)-Y(M))/SIZE2-.00005)72,70,70
   70 M=M+1
      X(M)=V(I)
      Y(M)=W(I)
   72 CONTINUE
   75 RETURN
      END
```

```
      SUBROUTINE REDUCE(N,X,Y,MAXPT)
      DIMENSION X(100),Y(100)
      IF(N.LE.MAXPT) RETURN
   45 DIST=SQRT((X(N)-X(1))**2+(Y(N)-Y(1))**2)
      IK=N
      M=N-1
      DO 20 I=1,M
      DISTC=SQRT((X(I+1)-X(I))**2+(Y(I+1)-Y(I))**2)
      IF(DISTC.GE.DIST) GO TO 20
      DIST=DISTC
      IK=I
   20 CONTINUE
      IF(IK.LT.N) GO TO 25
      X(1)=(X(1)+X(N))/2.
      Y(1)=(Y(1)+Y(N))/2.
      GO TO 35
   25 X(IK)=(X(IK)+X(IK+1))/2.
      Y(IK)=(Y(IK)+Y(IK+1))/2.
      IF(IK.EQ.M) GO TO 35
      LL=IK+1
      DO 30 J=LL,M
      X(J)=X(J+1)
   30 Y(J)=Y(J+1)
   35 N=N-1
      IF(N.GT.MAXPT) GO TO 45
      RETURN
      END
```

```
      SUBROUTINE AREA (N,X,Y,A)
      DIMENSION X(100),Y(100)
      Z=0
      DO 20 I=2,N
      Z=Z+ABS((X(I)-X(I-1))*(Y(I-1)-Y(I))-(Y(I)-Y(I-1))*
     -        (X(I-1)-X(I)))/2.
   20 CONTINUE
      A=Z
      RETURN
      END
```

```
      SUBROUTINE POLY(K)
      COMMON C(500),D(500),G1(500),G2(500),WX(100),WXP(100),AY(4),
     1R,NX,YNOW,YPNOW,IXY
      AY(4)=R
      IF(IXY.EQ.1)GO TO 10
      AY(1)=C(K)
      AY(2)=D(K)
      AY(3)=YNOW*G1(K)+YPNOW*G2(K)
      GO TO 20
   10 AY(1)=G1(K)
      AY(2)=G2(K)
      AY(3)=YNOW*C(K)+YPNOW*D(K)
   20 CALL OUTPO(NX,WX,WXP,AY)
      IF(NX.GT.25) CALL REDUCE(NX,WX,WXP,25)
      RETURN
      END
```

```
      SUBROUTINE INTPC(N1,X,Y,N2,V,W)
      DIMENSION X(100),Y(100),V(100),W(100),A(4)
      J=N1+1
      X(J)=X(1)
      Y(J)=Y(1)
      DO 20 I=2,J
      A(1)=Y(I)-Y(I-1)
      A(2)=X(I-1)-X(I)
      A(3)=Y(I-1)*X(I)-X(I-1)*Y(I)
      A(4)=0.
      CALL OUTPC(N2,V,W,A)
      IF(N2)20,21,20
   20 CONTINUE
   21 RETURN
      END
```