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MAGNETIC FIELD TOLERANCES FOR A SIX-SECTOR 500 MeV H^- CYCLOTRON

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TRI-67-2 ERRATUM

The right hand sides of Eq. 5 and 6 should be multiplied by γ^{-2} to take account of the changed energy for an equilibrium orbit at a given radius when the field deviates from isochronism. This leads to the cancellation of the factor $(1 + \mu')$ in Eq. 9, 22, 25 and 27. The trim coil spacing is thus relaxed by a factor γ, but since γ does not exceed 1.53 and because an upper limit is imposed on the spacing the total number of coils required is only reduced by 3 from 35 to 32, or 54 to 51.

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ABSTRACT

This report discusses the tolerances which beam dynamical requirements place on the accuracy to which the cyclotron magnet must be constructed and its magnetic field measured. The tolerance most significantly affecting manufacturing methods is probably that demanded of the spiral shape of the sectors to keep vertical focusing within acceptable limits. At large radii the precision required reaches ±0.033" and will necessitate a shimming programme subsequent to manufacture. The associated ±7 G tolerance on the field flutter should be easier to achieve.

Isochronism sufficient to give 36% microscopic duty factor can be provided by a radial field gradient correct to ±2 G/ft together with 35 circular trim coils. Separated turn acceleration would require ±1 G/ft with 54 trim coils and very closely controlled dee voltage, radio frequency and magnet excitation.

To avoid the poor energy resolution resulting from large radial betatron oscillations, the first harmonic field amplitude must not exceed 0.2 G; this demands 72 harmonic trim coils, ±0.14° accuracy in placing the sectors, and uniformity in their reluctance to 0.5%. Finally, to keep electric stripping of the H^- ions within ±10% limits, the hill magnetic field must meet ±0.4% limits over the outer 20 in.

C O N T E N T S

1. INTRODUCTION

The intensity and shape of the magnetic field of a sector-focused cyclotron are determined by the need to keep the H^- ion beam isochronous and focused within certain limits as it is accelerated. It is the choice of these limits which decides the accuracy to which the specified magnetic field must be achieved. In practical terms this determines

(i) the dimensional tolerances and material specifications for the magnet;

(ii) the extent to which shimming may be profitably pursued;

(iii) the precision required in magnetic field measurements.

Our method follows that of Richardson.[1,2] The tolerances quoted are to be regarded as standard deviations of a normal distribution about the specified value (i.e. there is a 32% chance the actual value will lie outside the tolerance limits). They are also combined as standard deviations. At first sight this may seem to run counter to our usual notions that engineering construction tolerances have rectangular probability distributions. However, most quantities depend on a number of individual measurements, and as their errors are combined a normal distribution is soon approached.

The design specifications and tolerances recommended are summarized at the end of this report.

2. ISOCHRONISM

An ion of charge q, rest mass m_o and velocity v orbiting in a cyclotron will have a mean angular frequency

$$\omega = \frac{v}{r} = \frac{q\overline{B}}{m} = \frac{q\overline{B}}{\gamma m_o} \qquad (1)$$

where $\overline{B}$ is the time average of the axial magnetic field it encounters, and its mass m is proportional to the relativistic factor

$$\gamma = \frac{m}{m_o} = 1 + \frac{T}{m_o c^2} = \frac{1}{\sqrt{1 - \beta^2}} \qquad (2)$$

T being the kinetic energy of the ion and $\beta \equiv v/c$. In a sector-focused cyclotron the radius of an ion varies with azimuth because of the orbit scalloping; the "average radius" r = orbit length/2π. We also note from (1) that the momentum

$$p = mv = \beta\gamma m_o c = \sqrt{\gamma^2 - 1}\, m_o c = q\overline{B}r. \qquad (3)$$

For isochronous acceleration of the ions by an applied radio frequency of $\omega_d = \nu\omega'$ (where $\omega' \simeq \omega$ and ν is the harmonic order) we must require that ω be nearly constant, and therefore that $\overline{B}$ vary with radius as

$$\overline{B} = \frac{m\omega}{q} = \frac{\gamma m_o \omega}{q} \equiv \gamma B_c = \frac{B_c}{\sqrt{1 - (r/r_c)^2}} \qquad (4)$$

where this equation defines "central field" B_c. The "cyclotron radius" r_c is defined by $r_c \equiv c/\omega$, so that $r = \beta r_c$. Note that $B_c r_c = m_o c/q = 1233.5$ kG-in for all H^- cyclotrons.

In this section we will be concerned with evaluating the precision with which (4) must be satisfied, taking account of the allowable phase excursions of the ions, and the help of the circular trim coils in making small corrections to $\overline{B}$.

2.1 Phase Acceptance and Duty Factor

If the orbit frequency of the ion is not precisely a subharmonic $\frac{1}{\nu}$ of the applied voltage, there will be a variation in the phase α. (α is defined with respect to the RF wave such that $\alpha = 0°$ for maximum acceleration.) The change in phase per ion turn is given by

$$\frac{\delta\alpha}{\delta n} = 2\pi\nu \frac{\omega - \omega'}{\omega} = 2\pi\nu \frac{\delta B}{\overline{B}} \tag{5}$$

where δB is the difference between the actual and the isochronous fields. Then, if V_d is the peak dee voltage,

$$\cos\alpha \frac{\delta\alpha}{\delta T} = \frac{2\pi\nu\delta B}{4qV_d\overline{B}} . \tag{6}$$

To find the phase change over a given radial interval we note from (3) that, without invoking any isochronous assumptions, we may write

$$T + m_o c^2 = \gamma m_o c^2 = m_o c^2 \sqrt{1 + (r\overline{B}q/m_o c)^2} \tag{7}$$

so that

$$\frac{dT}{dr^2} = \frac{q^2\overline{B}}{2m_o\gamma} (\overline{B} + r\frac{d\overline{B}}{dr}) . \tag{8}$$

Thus the change in phase over a radial interval from r_1 to r_2 may be written, with some shuffling, and using (3) again,

$$\sin\alpha_2 - \sin\alpha_1 = 2\pi\nu \frac{m_o c^2}{4qV_d} \int_{r_1}^{r_2} (1 + \mu') \frac{\delta B}{B_c} \beta^2 \frac{dr}{r} \tag{9}$$

where $\mu' \equiv (r/\overline{B}) (d\overline{B}/dr)$. For an isochronous field where $\overline{B}$ satisfies (4),

$$\mu' = \gamma^2 - 1. \tag{10}$$

Examining relation (9) we see that the <u>spread</u> in $\sin\alpha$ remains invariant with radius (provided $|\alpha| \leq \frac{1}{2}\pi$) because all the ions have the same shape for the plot of their history of $\sin\alpha$ vs. radius (Fig. 1), the only difference being a displacement along the sin α axis corresponding to a difference in the initial value sin α_1. But in order to have continuous, single-pass acceleration we must always have $\frac{1}{2}\pi > \alpha > -\frac{1}{2}\pi$ so that the energy gain per turn

$$\delta T/\delta n = 4\, qV_d \cos\alpha \tag{11}$$

is positive. The ions that wander beyond these limits experience deceleration and are lost for our purposes.

Now picture the situation in which axial injection is used to start ions out at all possible phases with respect to the RF. If strict isochronism obtained, all ions would have sin α = constant independent of radius, and all would be accelerated to the final radius (with varying number of turns required for each phase). In practice, however, the plots of sin α vs. radius for different particles will wander in parallel paths (see Fig. 1) and those which exceed the above limits will be wiped out. Let us say that the maximum variation in sin α for a single particle during the acceleration process is sin α_u - sin α_ℓ. Then the total spread in sin α at any radius for the particles which will survive the acceleration process is given by

$$\begin{aligned} S &\equiv \sin\alpha_f - \sin\alpha_b \\ &= 2 - (\sin\alpha_u - \sin\alpha_\ell) = \text{constant} \end{aligned} \tag{12}$$

where α_f is the most forward phase and α_b is the most backward phase to be found in all the particles at that radius. However, the duty factor D at a particular radius is defined by

$$D \equiv (\alpha_f - \alpha_b)/2\pi \tag{13}$$

in terms of the spread in α, rather than the spread in sin α, and hence varies with radius. Thus for a given value of S the duty factor D will be a maximum when α_f or α_b = 90° and a minimum when $|\alpha_f| = |\alpha_b|$. For example, if S = 1.809 with α_f (or α_b) = ±90° and α_b (or α_f) = ±54° then D = 40%, but if the spread is symmetric so that $|\alpha_f| = |\alpha_b|$ = 64.8° then D = 36%.

In general

$$\cos\,(\alpha_f - \alpha_b)_{max} = \cos\, 2\pi D_{max} = 1 - S \qquad (14)$$

$$\cos\,(\alpha_f - \alpha_b)_{sym} = \cos\, 2\pi D_{sym} = 1 - \tfrac{1}{2}S^2. \qquad (15)$$

The variation of D_{max} and D_{sym} with S are shown in Fig. 2. Unfortunately, there are reasons (below) connected with acceleration time for taking the symmetric case as most representative of a practical situation.

2.2 Dependence of Beam Loss on Phase and Duty Factor

Because of induced radioactivity and other considerations, it is desirable to confine the injected beam to those phase angles which continue to satisfy the requirement $\frac{1}{2}\pi \geqslant \alpha \geqslant -\frac{1}{2}\pi$ throughout the acceleration process. This factor gives us a limit on the number of trim coils and the tolerance on the magnetic field. In addition there is another factor which plays a role in TRIUMF, and that is the lifetime of the H^- ions for gas and electric stripping.

Let us pursue our investigation under the assumption that the excitation of the magnet and the gas pressure are held constant. Then the loss in beam per unit energy interval due to both kinds of stripping will depend on particle phase in the following way:

$$\frac{dN}{dT} \propto N\ (\text{path length}) \propto N\ (\text{number of turns}) \propto \frac{N}{\delta T} \propto \frac{N}{\cos\,\alpha} \qquad (16)$$

As shown above, the field will be designed to minimize the variation of phase during acceleration, so for any particular particle we will assume $\alpha \simeq$ constant.

Let f = fraction of beam remaining after acceleration at phase α,

f_o = fraction of beam remaining after acceleration at phase $\alpha = 0°$.

Then integrating the above relation from injection to final energy we have

$$\ell n \frac{N}{N_i} \equiv \ell n\, f = \frac{\ell n\, f_o}{\cos\,\alpha} \qquad (17)$$

$$\text{so } f = (f_o)^{\sec\,\alpha}\,. \qquad (18)$$

Fig. 3 shows a plot of the fraction of beam remaining, f, as a function of the phase angle α for three different values of f_o. If the phase angles spread from $- D\pi$ to $+ D\pi$ (symmetric case) then

$$\bar{f} = \frac{\int_o^{\frac{1}{2}D\pi} f\, d\alpha}{\int_o^{\frac{1}{2}D\pi} d\alpha} = \frac{2}{\pi D} \int_o^{\frac{1}{2}D\pi} f_o^{\sec\,\alpha} d\alpha \qquad (19)$$

is the net average value of the beam remaining with a duty factor D. The results are shown in Fig. 4.

Another way of evaluating these results is to assume that the amount of beam that can be extracted is limited by the amount of beam lost in the accelerator. Then if I and I_o are the allowable initial beam intensities for $D \neq 0$ and $D = 0$ respectively, we have

$$I\,(1 - \bar{f}) = I_o\,(1 - f_o) = \text{constant} \qquad (20)$$

where for TRIUMF this constant is the equivalent of 20 μA at 500 MeV. Then increasing the duty factor from 0 to D changes the extractable beam in the ratio

$$\frac{I\bar{f}}{I_o f_o} = \left(\frac{1 - f_o}{1 - \bar{f}}\right) \frac{\bar{f}}{f_o}\,. \qquad (21)$$

Fig. 5 shows the results for the fraction of the beam current that can be extracted with the duty factor D compared to that which could be extracted if all the beam were accelerated on the peak of the RF wave. From this figure we see that a duty factor of 40% will reduce the extractable beam by one third.

In practice, a more critical limitation on the microscopic duty factor is provided by the beam dynamics in the central region.

For instance, ions with phase $-55° < |\alpha| < +35°$ do not gain enough energy to clear the centre post on the first half-turn, and even if they did the centre point spread would become unacceptably large. A further factor is the strength and phase dependence of the vertical electric focusing over the first few turns, which make it difficult to match the magnetic focusing over a wide phase range. These considerations limit the attainable microscopic duty factor to about 25% in the central region.

2.3 Tolerance on the Radial Field Gradient and Design of the Trim Coils

When a circular trim coil is powered, the field inside the coil is raised or lowered uniformly, while outside the coil the shift is much smaller and in the opposite direction.[3] In the region of the coil itself there is therefore a change in the radial field gradient ($d\bar{B}/dr$). When a spiral ridge cyclotron is tuned up by the use of trim coils, the process involves adjusting the gradient $d\bar{B}/dr$ in the radial interval governed by the particular trim coil in such a way that the phase of the ions is constrained to move through the minimum possible change while being accelerated to final energy. Over the small radial interval $\delta r \ll r$ governed by one of the trim coils the integral in (9) giving the phase change may be approximated by taking average values of the quantities over that interval (these being indicated by brackets) yielding

$$\sin \alpha_2 - \sin \alpha_1 = 2\pi\nu \frac{m_o c^2}{4qV_d} \langle 1 + \mu' \rangle \langle \beta^2 \rangle \left\langle \frac{\delta B}{B_c} \right\rangle \frac{\delta r}{r}. \quad (22)$$

Deviations from isochronism with a wavelength $\lambda \gg 2\delta r$ or $\lambda \ll 2\delta r$ are relatively easily corrected. The most difficult situation occurs when $\lambda \simeq 2\delta r$, especially if the extrema coincide with the coil positions; if the tolerance to which the radial gradient has been corrected by shimming is denoted by $\pm\Delta\left(\frac{dB}{dr}\right)$ (tolerances are indicated by the symbol Δ throughout this report), then in the least favourable case the extreme deviation from the isochronous field is given by

$$\delta B \simeq \frac{\delta r}{2}\,\Delta\left(\frac{d\bar{B}}{dr}\right) \tag{23}$$

and to first order the result for the mean deviation is

$$<\delta B> = \frac{\delta r}{4}\,\Delta\left(\frac{d\bar{B}}{dr}\right) . \tag{24}$$

We insert this in the above relation and obtain

$$\left(\frac{\delta r}{r}\right)^2 = \frac{4qV_d}{m_o c^2} \quad \frac{4B_c}{r\Delta\left(\frac{d\bar{B}}{dr}\right)} \quad \frac{2 - S}{2\pi\nu\ <1+\mu'>\ <\beta^2>} \tag{25}$$

where, as above, we denote the allowable wandering of sin α by 2 - S, and S is related to the duty factor D as shown in Fig. 2.

From the discussion above, it seems reasonable to adopt the goal of D = 36% for TRIUMF for the symmetric conservative case, giving 2 - S = 0.191. For the tolerance on the magnetic field gradient we take $\Delta\left(\frac{d\bar{B}}{dr}\right)$ = ±2 G/ft = 0.17 G/in, which may be compared to the required isochronous values

$$\frac{d\bar{B}}{dr} = (\gamma^2 - 1)\frac{\bar{B}}{r} = \frac{\beta\gamma^3 B_c}{r_c} \tag{26}$$

of 33 G/ft = 2.74 G/in at 50 MeV (r = 129 in) and 245 G/ft = 20.4 G/in at 500 MeV (r = 311 in), given B_c = 3.00 kG, r_c = 411 in. Then taking $4qV_d$ = 0.4 MeV per turn and ν = 5 we obtain

$$\left(\frac{\delta r}{r}\right)^2 = \frac{0.18''}{r\ <1 + \mu'>\ <\beta^2>} = \frac{0.18''}{r<\beta^2\gamma^2>} \tag{27}$$

For example, at 500 MeV, $\beta^2\gamma^2$ = 1.35 and $\left(\frac{\delta r}{r}\right)^2$ = 4.4 x 10^{-4}. Thus $\delta r/r$ = 2.1 x 10^{-2} and the spacing between trim coils should be 6.5 in at this radius. Proceeding in this way one can work out the required positions of the trim coils to give a symmetric duty factor of 36% when faced with deviations from the isochronous gradient of ± 2 G/ft. The following table gives the radial positions of these coils in inches:

324	277	214	144	74	
318	269	204	134	64	
312	261	194	124	54	
306	252	184	114	44	
299	243	174	104	34	
292	234	164	94	24	
285	224	154	84	14	Total 35 coils

Additional coils at small and large radius are put in on the basis of experience with other cyclotrons. In particular, the trim coil spacing is not permitted to exceed 10 in, the expected width of the region over which a trim coil can affect $d\overline{B}/dr$.

The coils should be circular, not scalloped to follow ion orbits; however, small deviations to circumvent pumping ports or skyhooks would be acceptable, preferably being repeated every 60° or 120°.

The power provided to each trim coil must be sufficient to enable it to change the radial gradient by $\pm\Delta\left(\frac{d\overline{B}}{dr}\right) = \pm 2$ G/ft. The central 80% of the field rise $D\overline{B}$ across a trim coil occurs[3] in a radial interval which depends a little on the coil width, but which closely approaches half the pole gap g for relatively narrow coils such as will be used for TRIUMF. Here g = 20 in, so we require

$$D\overline{B} \simeq \pm \frac{100}{80} \frac{g}{2} \Delta \left(\frac{d\overline{B}}{dr}\right) = 2.1 \text{ G.} \tag{28}$$

Now for a trim coil near the maximum radius r_m, presumably

$$\frac{D\overline{B}(r_m)}{(NI)_{tc}} \simeq \frac{d\overline{B}(r_m)}{d(NI)_{mc}} \tag{29}$$

where $(NI)_{tc}$ is the excitation (in ampere-turns) required for the trim coil pair, and $(NI)_{mc}$ is that for the main coil pair. At smaller radii the trim coils are more efficient[3] so that, inserting numerical values from model measurements,[4] we have

$$(NI)_{tc} \lesssim D\bar{B}\,\frac{d(NI)_{mc}}{d\bar{B}(r_m)} \tag{30}$$

$$= \frac{2.1 \times 0.084 \times 720{,}000}{0.04 \times 4740} = 670 \text{ Amp-turns/pair}$$

Model measurements are recommended to check these conclusions for the TRIUMF magnet. If it were ever desired to accelerate ions to a significantly (say 5%) greater energy than 500 MeV, at the same r_m, considerably more excitation would be required to trim the field sufficiently. Meanwhile a somewhat higher excitation should be considered for safety and flexibility.

2.4 Seventh Harmonic Acceleration

The above design is based on the assumption of a factor of five between the resonator frequency and the ion frequency ($\nu = 5$). On the other hand, if one wants to increase this ratio to seven ($\nu = 7$), expression (25) above becomes

$$\left(\frac{\delta r}{r}\right)^2 = \frac{.13''}{r\ \langle\beta^2\gamma^2\rangle} \tag{31}$$

Under these circumstances one can either increase the number of trim coils from 35 to 39 and keep the tolerance on the magnetic field gradient at ±2 G/ft or one can keep the number of trim coils at 35 and refine the tolerance to ±1.4 G/ft. An alternative procedure would be to leave the number of trim coils at 35 and the field gradient tolerance at ±2 G/ft and accept the reduced duty factor which would result, namely 2 - S = 0.267 instead of 0.191 and the symmetric duty factor D = 33% instead of 36%.

2.5 Operation at Low Dee Voltages

The threshold voltage V_{th} is defined as the minimum peak dee voltage for which particles can be accelerated. As the dee voltage is lowered the phase wander will increase and the accelerable phase spread S decrease, since

$$2 - S = \sin\alpha_u - \sin\alpha_\ell \propto \nu/V_d. \tag{32}$$

Threshold occurs when S, and hence the duty factor, reaches 0, so that for a given harmonic number

$$2V_{th} = (2 - S)V_d. \tag{33}$$

For $\nu = 5$ and $S = 1.809$ for $V_d = 100$ kV, we find $V_{th} = 9.6$ kV. For $\nu = 7$ V_{th} will remain at 9.6 kV provided the field tolerance or the number of trim coils is adjusted to keep the same S; but if they are left unchanged so that S falls to 1.733, then V_{th} will rise to 13.4 kV.

Operation of the cyclotron at a dee voltage below the design value will result in an increased fraction of the beam being stripped and lost. This is because the number of turns required to get to full energy, and hence path length in the cyclotron, will be increased, by

(i) the reduced energy gain per turn at a given phase
(ii) the increased phase wander carrying ions to less favourable phases.

The fractional loss will approach 100% as V_d drops to V_{th}. On the other hand the accelerable duty factor falls as V_d is reduced, so that the beam current loss will rise at a slower rate, reach a maximum at some intermediate V_d and fall to zero at V_{th}. Unfortunately a quantitative estimate of the dependence of the stripping on V_d does not seem to be possible without knowing exactly how $\overline{B}$ deviates from isochronism and the consequent form of the phase wander. Nevertheless it is clearly advisable to operate at a reduced beam current for dee voltages significantly below 100 kV but significantly above V_{th}.

2.6 Separated Turn Acceleration

The separated turn acceleration of ions in a cyclotron is a concept observed at low radii or at low energies but it has never been achieved at or near the full output energy of a cyclotron. In this concept the injected blob or "fish" of ions, having a certain azimuthal spread $\delta\theta$ and a certain radial spread δr, maintains its physical integrity throughout the acceleration history and emerges as a pulse of ions in a certain time δt with a spread in energy δT.

As a result of the acceleration process the ions receive an energy $T = \sum_{n=1}^{N} 4qV_D \cos \alpha_n$ where n is a running index indicating the number of ion revolutions up to a total number N. Suppose we enquire about the possibility of limiting the energy spread to $\delta T = \pm 50$ keV. This can be achieved by limiting the variation of the phase α to $\pm\Delta\alpha$ and the dee voltage V_D to $\pm\Delta V_D$. In order to minimize the spread in energy δT for a given variation in phase $\pm\Delta\alpha$ we must have the average phase of the ions centred at $\alpha = 0°$. In this case we can use $\cos \alpha = 1 - \frac{\alpha^2}{2} + \ldots$ and consider that α has a normal distribution centred at $\alpha = 0°$. We see that T will not have a normal distribution but will have a skew distribution with its maximum near

$$T = T_{max}(1 - \tfrac{1}{2}(\Delta\alpha)^2). \tag{34}$$

Thus we take

$$\frac{2\delta T}{T} \simeq \frac{(\Delta\alpha)^2}{2} = \frac{2 \times 50}{500 \times 10^3} = 2 \times 10^{-4} \tag{35}$$

and so $\Delta\alpha = \pm 2 \times 10^{-2}$ rad $\simeq \pm 1°$. If we substitute this as $\sin \alpha_2 - \sin \alpha_1 = 4 \times 10^{-2}$ in the previous section, we see that this requirement decreases the tolerance on the isochronous condition by a factor of 4.8 from the previous value of 0.191.

This tightening of tolerances could be met by requiring a tolerance of ±1 G/ft on the radial gradient instead of the previous ±2 G/ft and by increasing the number of circular trim coils in the ratio $\sqrt{4.8/2} = \sqrt{2.4} = 1.55$. That is, we would require 54 trim coils for $\nu = 5$ or 60 trim coils for $\nu = 7$ (where ν is the harmonic number).

Separated turn acceleration also exerts stringent requirements on the radio-frequency accelerating voltage. It is clear from the above relation that the dee voltage must be regulated (including ripple) to be constant to one part in 10^4. It may be that the best way of accomplishing this regulation would be to base it on the radial position of the beam.

Since $\frac{500\ \text{MeV}}{400\ \text{keV}} = 1250$ turns are involved in the acceleration, the magnetic field and radio frequency must be held constant to one part in $360 \times 1250\ \nu = 2.25 \times 10^6$.

The injection time will be confined to $\pm 1°$ out of the whole RF cycle. This means a pulse length of $\frac{2}{360} \times \frac{1}{25} \times 10^{-6} \simeq 0.2$ n sec and a duty factor D = 0.6%.

A more detailed analysis of the interplay between the tolerances on the shape of the magnetic field on the one hand and the tolerances on the frequency of the RF and magnet regulation on the other hand for separated turn acceleration will be presented as an addendum to this report. These considerations do not materially affect the tolerances given here.

3. VERTICAL FOCUSING

The vertical focusing needed to keep ions near the median plane of an isochronous cyclotron is provided by the "edge focusing" which occurs when ions cross a boundary between high and low magnetic field regions at an angle to the normal. The focusing requirement thus leads naturally to the characteristic spiral sector shape of the magnet poles - a form which also enables the average field $\overline{B}$ to be varied radially to satisfy isochronism. In this section we shall be investigating the tolerances allowable on this pole shape.

The focusing power of the sectors depends on two more or less independent factors. The first is the azimuthal "flutter" in the magnetic field, defined by

$$F^2 \equiv \frac{<(B - \overline{B})^2>}{\overline{B}^2} = \frac{<B^2> - \overline{B}^2}{\overline{B}^2} \tag{36}$$

and determined mainly by the height and azimuthal width of the pole pieces; however, these also determine $\overline{B}$, which must be maintained isochronous, so the range of flutter available is not unlimited. The second factor is the magnitude of the spiral angle $\tan \varepsilon = r d\theta/dr$.

Assuming that the restoring forces are linear on average, the strength of the vertical focusing can be conveniently described by ν_z, the number of vertical betatron oscillations per revolution. According to the flat field, hard edge, approximation, which is sufficiently accurate for our purposes, ν_z in a sector-focused cyclotron is given by

$$\nu_z^2 = -\mu' + F^2 (1 + 2 \tan^2\varepsilon) . \tag{37}$$

As explained in Sec. 2.1 above, the logarithmic field gradient

$$\mu' \equiv \frac{r}{\overline{B}} \frac{d\overline{B}}{dr} = \gamma^2 - 1 \tag{10}$$

is necessarily positive in order to satisfy isochronism, and therefore necessarily defocusing. It is counteracted by the focusing term $F^2(1 + 2\tan^2\varepsilon)$ provided by the flutter and spiral angle as described above. The angle ε here is the mean spiral angle of a sector, given in terms of the spiral angles of its focusing and defocusing edges ε_f, ε_d by (cf. Fig. 6)

$$\tan\varepsilon_f = \tan\varepsilon - \tan\mu_o \tag{38}$$

$$\tan\varepsilon_d = \tan\varepsilon + \tan\mu_o \tag{39}$$

Here μ_o is the angle of flare required to increase the angular sector width η_o sufficiently to maintain $\overline{B}$ isochronous: in our approximation of flat hill and valley fields B_H and B_V, and where there are N sectors, we have

$$\tan\mu_o \equiv \frac{r}{2}\,\frac{d\eta_o}{dr} = \frac{\pi}{N}\,(\gamma^2 - 1)\;\frac{\overline{B}}{B_H - B_V}\,. \tag{40}$$

Since the TRIUMF cyclotron magnet will always be used at substantially the same excitation, it is more economic to provide the field variations described by F and $\tan\varepsilon$ by means of shaping the steel parts of the magnet rather than by auxiliary electric currents in coils. This situation lies in contradistinction to that applying in the case of multi-particle cyclotrons such as ORIC, the Berkeley 88 inch, etc. It follows that the tolerances on F and $\tan\varepsilon$ will be reflected in tolerances on the steel parts. No corrections to these quantities will be made with coils.

In order to calculate the tolerances on F and $\tan\varepsilon$ we see from (37) that we must first decide how much freedom can be allowed ν_z. This will depend on three factors - the range of ν_z accessible, the need to avoid undesirable values, and the variability permissible.

3.1 The Range of ν_z Accessible

From (10) we see that μ' increases more and more rapidly with energy so that the focusing term $F^2\ (1 + 2\tan^2\varepsilon)$ in (37) must be increased commensurately to keep $\nu_z^2 > 0$. This becomes most difficult at maximum radius, especially since the flutter available there is limited by the joint requirements of low maximum field B_H to avoid excessive electric stripping of H^- ions, but high average field $\overline{B}$ to keep the cyclotron magnet to a reasonable size. This economic consideration in fact dictates that the focusing will be weak, say $0 < \nu_z < 1$.

The preliminary cyclotron design (271'' maximum orbit radius) thus required only a minimal ν_z^2 of 0.05 ($\nu_z = 0.23$ or one complete oscillation every four turns). At 500 MeV, $\gamma^2 - 1 = 1.35$ so that a focusing term $F^2(1 + 2\tan^2\varepsilon)$ of 1.40 was required, to be provided by $F^2 = .076$ and $\tan\varepsilon = 2.96$ ($\varepsilon_f = 62.5°$, $\varepsilon_d = 76.0°$). In practice, when model measurements were made,[5] neither the flutter nor the spiral angle required could be obtained near maximum radius, although at smaller radii they were generally a little more than adequate, giving $\nu_z \simeq 0.3$. Since it did not seem possible to increase the flutter at a 271'' maximum radius, and since increasing the spiral angle was expected to be relatively ineffective (it eventually brings the sectors closer together and begins to reduce F^2 faster than it increases $(1 + 2\tan^2\varepsilon)$), the maximum radius was increased to 302''. Model measurements showed that this increase was indeed an improvement - the flutter F^2 rose to 0.1 at maximum radius giving positive focusing there, while at smaller radii $\nu_z \simeq 0.4$. The further increase in radius to the present 311'' design, made to compensate for the much shorter than expected lifetimes recently found[6] for H^- ions in the electric field range appropriate to the TRIUMF cyclotron, should ease the focusing problem still more.

3.2 Undesirable Values of ν_z

There is no importance in achieving any precise value of ν_z. There are, however, special values which must be avoided - namely resonances between the vertical oscillations and

(a) the structure of the magnetic field

(b) the generally much-larger-amplitude radial oscillations.

The nearest serious resonance is $\nu_z = \frac{1}{2}$ which we take as an absolute upper limit. The effects of other resonances will need to be further investigated. Fortunately, experience with sector-focused cyclotrons so far has shown that their large energy gain per turn enables many resonances to be traversed without discomfort.

As well as being kept away from resonance values, ν_z^2 must not be allowed to drop abruptly below zero for anything of the order of of $<|2\pi\nu_z|>^{-1}$ turns, or serious defocusing will result. The adiabatic approach of ν_z^2 to zero is discussed below.

3.3 Variability Permissible in ν_z

Changes in the strength of the vertical focusing will be accompanied by changes in the maximum amplitude z_m and maximum angle of divergence ζ_m of the vertical betatron oscillations. Suppose

$$z = z_m \sin \nu_z \omega t \tag{41}$$

and

$$\zeta = \zeta_m \cos \nu_z \omega t \tag{42}$$

Then ζ_m and z_m are related by

$$\zeta_m = \nu_z \left(\frac{\omega}{v}\right) z_m \simeq \left(\frac{\nu_z}{r}\right) z_m. \tag{43}$$

Also, in the absence of coupling with the radial oscillations the beam emittance E_z in vertical phase space will be conserved:

$$E_z = \pi m v \zeta_m z_m = \pi m_o \omega \gamma z_m^2 \nu_z = \text{constant}. \tag{44}$$

Assuming that changes in γ and ν_z take place adiabatically (i.e. slowly compared to the period of an oscillation), we can use this relation to predict the associated variations in amplitude and divergence

$$z_m \propto \frac{1}{\sqrt{\gamma\nu_z}} \qquad (45)$$

$$\zeta_m \propto \frac{1}{r}\sqrt{\frac{\nu_z}{\gamma}} . \qquad (46)$$

Neglecting the γ factor, which only contributes 24% damping all the way from 0 - 500 MeV, we see that $z_m \propto 1/\sqrt{\nu_z}$ so that a fall in ν_z by a factor four will produce a two-fold increase in amplitude. In comparison, the effect of a change in ν_z on the angular divergence will generally be masked by the collimation produced by acceleration to higher velocities.

Clearly we must not allow ν_z to vary by too large a factor, and especially it must not be allowed to come too close to zero. Furthermore, it is undesirable for vertical focusing to be stronger at injection, or weaker at extraction, than in the main body of the machine. While magnetic focusing is naturally weak in the central region, phase dependent electric focusing effects will have to be carefully considered. At the periphery too, care will be needed to ensure that ν_z does not fall off too rapidly, causing beam blow-up there.

3.4 Tolerances on the Flutter and Spiral Angle

In light of these considerations, especially the improved flutter obtained by increasing the magnet radius, we are in a position to re-examine the design specification for ν_z. In particular, it might be advantageous to demand a higher value than the conservative preliminary one of 0.05 for ν_z^2, in order to keep away from the defocusing region without imposing very restrictive tolerances on ν_z^2. The value and tolerance we suggest are

$$\nu_z^2 = 0.125 \pm 0.05 \qquad \nu_z = 0.35 \begin{array}{l} + .07 \\ - .08 \end{array} \qquad (47)$$

From the model experiments we know this to be in an accessible range of values; moreover, it places ν_z^2 squarely between the two chief undesirable values 0 and $(0.5)^2$, with 2.5 tolerances of leeway on either side. In so far as the tolerances can be regarded as standard deviations, there is a 1.2% probability that one or other of the limits will be reached. If ν_z^2 fell by two tolerances from 0.125 to 0.025, the resulting beam amplitude increase would only be by a factor 1.7.

The point of making the tolerances symmetric in ν_z rather than ν_z^2 is, of course, so that we may set symmetric tolerances on the magnet parameters in the equation

$$\nu_z^2 = -\mu' + F^2(1 + 2 \tan^2\varepsilon). \tag{37}$$

Now the tolerance $\Delta\mu'$ is already settled by the assignment $\Delta(d\overline{B}/dr) = \pm 2$ G/ft made in Sec. 2.3, for at a given radius r the percentage error permitted on $\overline{B}$ is negligible compared to that on $d\overline{B}/dr$, and therefore $\Delta\mu' \simeq (r/\overline{B})\ \Delta(d\overline{B}/dr)$. Thus at 500 MeV we assign values and tolerances to (37) as follows

$$0.125 \pm 0.05 = -1.35 \pm 0.01 + 1.475 \pm 0.05.$$

(Throughout this section we shall illustrate the general procedure by quoting numerical values for the 500 MeV radius, where the tolerances are tightest. Values for smaller radii are listed in Section 6.2. A "Mark V" design is assumed.)

The percentage tolerance available on the focusing term $F^2\ (1 + 2 \tan^2\varepsilon)$ amounts to $\pm 3.3\%$ at 500 MeV. To the spiral term, where the situation is most critical, we assign a percentage tolerance 0.955 as large:

$$\frac{\Delta(1 + 2 \tan^2\varepsilon)}{1 + 2 \tan^2\varepsilon} = 0.955\ \frac{\Delta\left[F^2\ (1 + 2 \tan^2\varepsilon)\right]}{F^2\ (1 + 2 \tan^2\varepsilon)} \tag{48}$$

i.e. 3.2% at 500 MeV. The percentage tolerance available for the flutter is then 0.30 as large:

$$\frac{\Delta F^2}{F^2} = 0.30 \frac{\Delta\left[F^2\ (1 + 2\tan^2\varepsilon)\right]}{F^2\ (1 + 2\tan^2\varepsilon)} \tag{49}$$

i.e. 1.0% at 500 MeV. As mentioned above, model magnet measurements yield values of 0.10 for the flutter factor $F^2 = <(B-\overline{B})^2>/\overline{B}^2$ at maximum radius so that with a tolerance $\Delta F^2/F^2 = \pm 1.0\%$ then $\Delta F/F = \pm 0.5\%$ and $\Delta <(B-\overline{B})^2>^{\frac{1}{2}} = \pm 7.4$ G. This tolerance appears to be readily achievable and would be checked by a suitable averaging of 720 field measurements at 0.5° intervals around a circular path; fewer measurements would be required at small radii.

The above assignment for the flutter leaves for the spiral angle $(1 + 2\tan^2\varepsilon) = 14.75$ or $\tan\varepsilon = 2.63$, and for its tolerance (48) gives

$$\Delta\tan\varepsilon = \frac{\Delta(1 + 2\tan^2\varepsilon)}{4\tan\varepsilon} \tag{50}$$

$$= 0.955 \times 0.033 \times \frac{(1 + 2\tan^2\varepsilon)}{4\tan\varepsilon} = 0.045.$$

Recalling that $d(\tan\varepsilon) = \sec^2\varepsilon\, d\varepsilon$ we write

$$\Delta\varepsilon = \frac{\Delta\tan\varepsilon}{1 + \tan^2\varepsilon} = \pm 5.6 \text{ mrad} = \pm 0.32°.$$

From a construction point of view we are interested in the tolerances on the focusing and defocusing edges of the hill. From (38) and (39) above, given $\tan\mu_o = 1.27$ at 500 MeV, we find $\tan\varepsilon_f = 1.36$ ($\varepsilon_f = 53.6°$) and $\tan\varepsilon_d = 3.90$ ($\varepsilon_d = 75.6°$). Also we see that

$$2\tan\varepsilon = \tan\varepsilon_f + \tan\varepsilon_d. \tag{51}$$

In order to see how to divide the available tolerance between ε_f and ε_d, we introduce, in view of (51), a weighting angle λ such that

$$\Delta\tan\varepsilon_d = \Delta\varepsilon_d/\cos^2\varepsilon_d = 2\cos\lambda.\Delta\tan\varepsilon \tag{52}$$

$$\Delta\tan\varepsilon_f = \Delta\varepsilon_f/\cos^2\varepsilon_f = 2\sin\lambda.\Delta\tan\varepsilon.$$

Now in the outer regions of the magnet the tolerances turn out to be small enough that shimming techniques will have to be used to achieve them. If we assume that the work involved in shimming is inversely proportional to the angular tolerance aimed at, then the choice of λ which minimizes the sum of the work required on the two edges is given by

$$\tan^3\lambda = \cos^2\varepsilon_d/\cos^2\varepsilon_f. \tag{53}$$

Then at the 500 MeV radius, $\tan\lambda = 0.53$ and $\Delta\varepsilon_d = \pm 4.7$ mrad, $\Delta\varepsilon_f = \pm 17$ mrad.

Now consider the specification of the spiral in polar co-ordinates, r and θ. Over short distances, the accuracy on r can be made as precise as one pleases while over large distances the tolerance is quite relaxed. Thus we can write for the azimuthal tolerance (see Fig. 6)

$$\Delta y_i = \frac{x}{\sqrt{2}}\,\Delta\tan\varepsilon_i = \frac{h}{\sqrt{2}}\cos\varepsilon_i\,\Delta\tan\varepsilon_i = \frac{h}{\sqrt{2}}\,\frac{\Delta\varepsilon_i}{\cos\varepsilon_i} \quad [i=f,d] \tag{54}$$

where we suppose that the measurements from which the spiral angle is determined are made at regular intervals h along the sector edges. The factor $\sqrt{2}$ appears because the tolerance available has to be divided between the two endpoints of each interval. Putting h = 10'', equal to half the magnet gap, then

$$\Delta y_d = 0.707 \times 10 \times 0.0047/0.25 = 0.13''$$

and

$$\Delta y_f = 0.707 \times 10 \times 0.017/0.60 = 0.20''$$

give the required tolerances on the azimuthal positions for the focusing and defocusing edges of the hill.

In the manufacture of the spiral sectors and the checking thereof, it may be more desirable to lay a straight edge, wire, or line of sight across the chord of the spiral. The length of h can again be determined as well as necessary to make $\Delta h \to 0$.

To get the tolerance on the shape of the spiral hill in terms of a distance a perpendicular to the tangent to the spiral, one can make use of the relation

$$\Delta a_i = \Delta y_i \cos \varepsilon_i = \frac{h}{\sqrt{2}} \cos^2 \varepsilon_i \, \Delta \tan \varepsilon_i = \frac{h}{\sqrt{2}} \Delta \varepsilon_i . \quad [i = f, d] \tag{55}$$

From above $\Delta a_d = \pm 0.033''$

and $\Delta a_f = \pm 0.12''$.

Fortunately, tolerances as tight as these are only required close to the 500 MeV radius r_m. The table in Sec. 6.2 shows that Δa_d and Δa_f rise to ±0.10'' and ±0.15'', respectively, at $r = 0.90\ r_m$ and to ±0.20'' and ±0.35'' at $r = 0.80\ r_m$. (It should be borne in mind that until the spiral sector shape has been finalized, these values can be regarded as preliminary only. The spiral angles of the edges, and hence their tolerances, are particularly sensitive to small changes in the design.)

If the smaller tolerances quoted above were to be transferred directly to tolerances on the shape of the hill pole pieces, it appears that we would end up with a very expensive magnet indeed. Fortunately, a logical and ordered programme of shimming the contours of the pole pieces can provide us with the desired tolerances. For example, if the manufacturing tolerance on the pole piece contours is $\pm\frac{1}{4}$ inch, it would appear reasonable to shave all the iron contours by $\frac{1}{2}$ inch and plan on adding shims of the necessary thickness (along the direction a) to achieve the desired magnetic field contours. One point which should be answered by model magnet studies is the required vertical extent of these shims. Would, for instance, two inches be enough?

4. RADIAL MOTION - TOLERANCES ASSOCIATED WITH THE FIRST HARMONIC

Expressed in a Fourier expansion of the magnetic field as a function of azimuth, the TRIUMF magnet will be designed to have large amplitudes in the sixth and eighteenth harmonics. This expansion may be written:

$$B - \overline{B} = \sum_{k=1}^{\infty} b_k \sin (k\theta - \phi_k) \tag{56}$$

and we adopt the usage that k = 1 is called the first harmonic. This first harmonic component of the field is probably the most difficult magnetic quantity to measure. Also it is difficult to make quantitative predictions of the precise effects of a certain amplitude of first harmonic on beam quality, energy spread, etc. The most reliable method is to follow a complete beam path on the computer (with acceleration) from injection to extraction after introducing various amounts of first harmonic at various radii. This would have to be done for very many beam paths and would require a large amount of computer time. In the interim, we follow the procedure of Reference 2, particularly Sections 3.5 and 4.3.

The equation for radial motion with a first harmonic forcing term may be written

$$\frac{d^2x}{d\theta^2} + \nu_r^2 x = \frac{b_1 r}{\overline{B}} \cos \theta \tag{57}$$

where x is the radial displacement from the equilibrium orbit. The resulting motion is thus the "interminable beat" of a forced but undamped oscillator; close to the resonance $\nu_r = 1$ the slowly varying amplitude of these oscillations is given by

$$A_x = \frac{b_1 r}{\overline{B}(\nu_r - 1)} \sin \left[(\nu_r - 1)\pi n\right] \tag{58}$$

where n is the number of turns made subsequent to starting from an equilibrium orbit. This must be compared with the amplitude of the oscillations inherent in the radial emittance E_r of the injected beam (cf. (44) and (45) above):

$$x_m = \sqrt{\frac{E_r}{\pi m_o \omega \gamma \nu_r}} . \tag{59}$$

To keep A_x the same fraction of x_m as in conventional cyclotrons (for the same emittance, same energy and hence supposedly the same ν_r), we must therefore have

$$\frac{A_x}{x_m} \propto \frac{b_1 r\sqrt{\omega}}{B_c} \propto \frac{b_1}{B_c^{3/2}} = \text{constant.} \tag{60}$$

If we take the Berkeley 88" cyclotron as our model (B_c = 16 kG and b_1 < 2G), then for TRIUMF the tolerance on the first harmonic field amplitude must be $(16/3)^{3/2}$ = 12 times more severe, or b_1 < 0.2 G. We propose that this criterion be satisfied by means of coils and that the magnetic field measurement program only be required to detect the presence of amplitudes of 1 G in the first harmonic.

We note that the maximum correction provided by the first harmonic coils in the design of the TRIUMF proposal (as in the UCLA proposal[7]) is ±2.5 G. In view of the many types of constructional defects which can contribute to a first harmonic, and in view of our desire to assign a tolerance of 1 G to the contribution of each of these types of defects, we recommend that the correction amplitude achievable from the first harmonic coils be increased to ±10 G. We also recommend that the number of independent radial regions for first harmonic correction be increased from four to twelve.

4.1 Azimuthal Positioning of the Magnet Sectors

Azimuthal misplacement of one of the hill sectors and yokes can produce a first harmonic. Let us assume a simplified model and calculate the tolerance $\Delta\theta$ on the azimuthal placement of the hill (see Fig. 7). We assume flat-topped hills and valleys of equal azimuthal width and $B_H - B_V = 2\delta B$. Then if we place the $\theta = 0$ origin correctly, we can ignore ϕ_1, and the expression for the amplitude of the first harmonic becomes

$$b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi}(B - \overline{B}) \sin\theta \, d\theta. \tag{61}$$

When there is no error, $b_1 = 0$ because $(B - \overline{B})$ is an even function. But when one of the hills is displaced, the integrals over the six hills and six valleys no longer cancel out exactly, and we are left with

$$\pi b_1 = -\int_{-45}^{-15+\Delta\theta} \delta B \sin\theta \, d\theta + \int_{-15+\Delta\theta}^{15+\Delta\theta} \delta B \sin\theta \, d\theta - \int_{15+\Delta\theta}^{45} \delta B \sin\theta \, d\theta$$

$$= 4\delta B \sin 15° \sin\Delta\theta \simeq 4\delta B \, \Delta\theta \sin 15°. \tag{62}$$

Thus

$$\frac{b_1}{\delta B} = \frac{4}{\pi} \Delta\theta \sin 15° \simeq 0.33 \, \Delta\theta.$$

Now if $b_1/\delta B$ is to be kept to 1/1200, we must have

$$\Delta\theta \simeq \frac{1}{400} \simeq \pm 0.14°$$

as the tolerance on the placement of the azimuthal position of one of the hills.

At the extraction radius of 311" the above tolerance $\Delta\theta$ corresponds to a linear positioning tolerance along the orbit of ±0.78". At smaller radii the linear tolerance would, of course, be smaller.

4.2 Magnetic Uniformity of the Sectors

An important aspect of the first harmonic tolerance is its influence on the required uniformity of construction and chemical composition of the various hills. Some appreciation of the requirements may be obtained by making an analysis in terms of an "effective fractional change in total reluctance" $\Delta Z_t/Z_t$. The concept of reluctance refers to the picture of the magnetic circuit where

$$\iint B da = \text{flux} = \frac{\text{magneto-motive force}}{\text{total reluctance}} = \frac{0.4\ \pi NI}{Z_t}. \tag{63}$$

$Z_t = \Sigma Z$ is the sum of all the reluctances in the magnetic circuit, combined in the same way as resistances in an electric circuit.

Now suppose one of the six hills, though otherwise identical, has a reluctance differing from the other five by ΔZ_t. Then the flux density would be

$$B = \frac{B_H}{1 + \Delta Z_t/Z_t} \simeq B_H\ (1 - \Delta Z_t/Z_t). \tag{64}$$

For the purpose of the Fourier analysis we place this hill at $\theta = 90°$. Then

$$\pi b_1 = \int_{75}^{105} \delta B\ (1 - \Delta Z_t/Z_t) \sin\theta\, d\theta + \int_{-105}^{-75} \delta B \sin\theta\ d\theta \tag{65}$$

where we have written down the parts of the integral which do not match out to zero. Thus we obtain

$$\pi b_1 = -2\delta B (\Delta Z_t/Z_t) \cos 75° \tag{66}$$

and

$$\left|\frac{\Delta Z_t}{Z_t}\right| = \frac{\pi b_1}{\delta B}\ \frac{1}{0.518} \simeq \frac{1}{200}$$

where we have again assumed a limit of $b_1 = 1$ G and $\delta B = 1200$ G.

The discrepancy in the reluctance of one of the hills may arise from a number of possible causes. For example, the chemical composition of the steel in one of the hills may differ from the average to the extent that its effective permeability differs from the average by $\Delta\mu$.

We can make an approximate estimate of the tolerance on the effective permeability by making use of the efficiency ε of the magnet (see Section 3.2 of Reference 2). In simple terms, the efficiency $\varepsilon \equiv \langle B_g\rangle/\langle B_g\rangle_i$ where $\langle B_g\rangle$ is the actual mean field in the magnet gap and $\langle B_g\rangle_i$ is the mean field one would obtain if the reluctance of the iron were negligible and the reluctance of the fringing field were infinite. These latter assumptions are so far from being true in the TRIUMF magnet that $\varepsilon \simeq 35\%$.

In the magnetic circuit picture we let

Φ_f = fringing flux
Φ_g = flux across hill gap
Z_f = reluctance corresponding to fringing flux
Z_g = reluctance across gap
Z = total reluctance of rest of magnet.

We can simplify the analysis according to the relation

$$\Phi_g = \frac{0.4\ \pi NI}{Z + \left(\frac{Z_f Z_g}{Z_f + Z_g}\right)} - \Phi_f \tag{67}$$

so if we assume $\Phi_f \simeq \frac{1}{2}\Phi_g$ then $Z \simeq (4/3)Z_g$ for $\varepsilon \simeq 1/3$ and

$$\frac{\Delta\Phi_g}{\Phi_g} \simeq \frac{2}{3}\frac{\Delta Z}{Z}\ .$$

Thus we see that in this rough estimate we get the tolerance on the effective permeability to be given by

$$\frac{\Delta\mu}{\mu} \simeq \frac{\Delta Z}{Z} \simeq \frac{1}{150}.$$

It is clear from this analysis that the structure of the magnet must embody some concept of averaging the steel from different melts over the various hills so that on the average the above condition is satisfied as well as possible.

The possibility of having a small separately energized coil around the yoke of each of the six sectors should be investigated. These coils could trim out asymmetries such as those being discussed here.

In addition to variations due to chemical composition there may be variations from sector to sector due to accidental air gaps or variations in necessary air gaps in the yoke. In the same spirit of our rough approximation above we see that the tolerance $\frac{\Delta Z}{Z} \simeq \frac{1}{150}$ corresponds to a tolerance on an air gap in the yoke of $\frac{1}{150} \times \frac{2}{3} \times 20 = 0.08''$. This tolerance refers to an air gap transverse to the flux path in the magnet.

4.3 Effects of Subsidiary Structures

Care should be taken to consider the effects of subsidiary structures such as pumps, support beams, jacks, etc. on the first harmonic of the field. Some of these effects must be studied on the model magnet. If these effects exceed the tolerance given above, plans must be made either to shim them out or to convert them to field irregularities which have a six-fold symmetry.

5. ELECTRIC STRIPPING AND TOLERANCE ON THE HILL FIELD

Since the binding energy of the second electron in an H^- ion is 0.755 eV, it is relatively easy to remove. There are two processes which may occur during acceleration in the cyclotron which will result in the removal of this electron and the consequent loss of the ion from the beam. These are:
(a) scattering by residual gas in the cyclotron vacuum chamber;
(b) electric dissociation by motion through the magnetic field.
We shall only be concerned with the second of these here.

An H^- ion travelling through a stationary magnetic field B with velocity $\underline{v} = \underline{\beta}c$ experiences in its rest frame an electric field

$$\underline{E}(\mathrm{MV/cm}) = 0.3\ \gamma\ \underline{\beta} \times \underline{B}(\mathrm{kG}). \tag{68}$$

If the value of E becomes large enough, the potential barrier retaining the extra electron will become sufficiently distorted that quantum mechanical penetration of the barrier occurs, allowing the electron to escape and the resulting neutral atom to fly off at a tangent to the ion orbit. The amount of beam loss which can be tolerated from this cause sets an upper limit to the magnetic fields that may be utilized in an H^- cyclotron.

The lifetime τ of H^- ions in the electric field range of interest to TRIUMF ($E \simeq 2$ MV/cm) has recently been measured by Olsen et al.[6] As seen from Fig. 8, their results are well fitted by a curve of the form suggested by Hiskes[8]

$$\tau = \frac{A}{E} \exp\left(\frac{C}{E}\right) \tag{69}$$

but with new values for the coefficients:

$$A = 4.8 \times 10^{-14}\ \mathrm{sec.MV/cm}$$

$$C = 43.6\ \mathrm{MV/cm}.$$

The lifetime varies very rapidly with electric field (roughly an order of magnitude shortening of τ for a 10% increase in E) so that all the significant stripping in the cyclotron takes place within a narrow band of electric fields close to the maximum value reached; i.e. only on the magnetic hills and near maximum energy. Over such a narrow region (69) may be adequately approximated by the simple power law

$$\tau = \tau_o (E_o/E)^k \tag{70}$$

where the value of k is chosen to equalize the slopes $d\tau/dE$ of (69) and (70) at the point (E_o, T_o). This requires

$$k = 1 + \frac{C}{E} \tag{71}$$

or $k \simeq 23$ for $E \simeq 2$ MV/cm.

In the laboratory frame of reference the rate of loss of ions is given by

$$\lambda \equiv \frac{1}{N}\frac{dN}{dt} = -\frac{\alpha}{\gamma\tau} \tag{72}$$

where the γ factor takes account of time dilation and $\alpha \simeq \eta_o/2\pi$ is the fraction of an orbit where the magnetic field has the flat hilltop value B_H. The total loss must be obtained by integrating numerically from injection to extraction for the field shape of the particular design. In the case of the TRIUMF cyclotron the requirement that the total loss by electric stripping shall not exceed 16 μA (12%) determines a maximum value for B_H of 5.76 kG.

In view of (72), the tolerances on λ, α and E at a given energy are related by

$$\left(\frac{\Delta\lambda}{\lambda}\right)^2 = \left(\frac{\Delta\alpha}{\alpha}\right)^2 + \left(\frac{\Delta\tau}{\tau}\right)^2 = \left(\frac{\Delta\alpha}{\alpha}\right)^2 + \left(k\frac{\Delta E}{E}\right)^2. \tag{73}$$

We shall require the beam loss λ to be within 10% of its design value. The tolerance on the hill width α already implicitly set by the isochronous and vertical focusing requirements on the field shape, discussed in Secs. 2 and 3 above, will be much smaller than 10%, so that we may write

$$\frac{\Delta B_H}{B_H} \simeq \frac{1}{k}\frac{\Delta\lambda}{\lambda} \qquad (74)$$

$$\simeq \pm 0.4\%$$

or $\Delta B_H = \pm 23$ G where $B_H = 5.76$ kG. Since k is pretty well constant over the region in which electric stripping is significant ($1.8 < E < 2.0$ MV/cm), the same ±0.4% tolerance on B_H throughout that region (i.e. from 400 to 500 MeV, or 293" to 311") will keep the total loss within ±10% of that planned.

6. COMPILATION OF DESIGN PARAMETERS AND TOLERANCE SPECIFICATIONS

6.1 Isochronism and Separated Turn Acceleration

	Large Duty Factor Operation	Separated Turn Acceleration
Microscopic duty factor	36%	0.6%
Phase spread	±65°	±1°
Phase variation: $\sin \alpha_u - \sin \alpha_\ell$	0.191	0.04
Increase in stripping / Reduction in extractable beam (due to phase spread)	25%	0%
Threshold voltage	9.6 kV	
Tolerance required on $d\overline{B}/dr$ - before powering trim coils	±2 G/ft	±1 G/ft
Number of trim coils	35	54
Minimum trim coil spacing	6"	4"
Maximum trim coil spacing	10"	10"
Energy spread	±520 keV	±50 keV
Time variation of dee voltage	<1.5 in 10^3	<1 in 10^4
Time variation of (magnetic field, radio frequency)	<1 in 4×10^5	<1 in 2.25×10^6

6.2 Vertical Focusing

Number of vertical betatron oscillations per turn

$\nu_z^2 = 0.125 \pm 0.05$

$\nu_z = 0.35 \begin{smallmatrix} +0.07 \\ -0.08 \end{smallmatrix}$

Tolerances at approximately 30'' radial intervals

r (in.)	E (MeV)	$\Delta \langle (B-\overline{B})^2 \rangle^{\frac{1}{2}}$ (gauss)	$\Delta\varepsilon$ (mrad)	Δy_d (in.)	Δy_f (in.)	Δa_d (in.)	Δa_f (in.)
311.0	500	± 7	± 5.6	±0.13	±0.20	±0.033	±0.12
278.7	340	13	13	0.18	0.29	0.10	0.15
248.2	240	21	27	0.29	0.43	0.20	0.35
218.4	170	30	53	0.47	0.71	0.40	0.65
189.8	120	39	"	"	"	"	"
159.4	80	48	"	"	"	"	"
128.8	50	53	"	"	"	"	"
101.3	30	56	"	"	"	"	"

N.B. These values are based on the Mark V sector shape, scaled up from 302'' to 311'', and should be regarded as preliminary only.

6.3 First Harmonic

$\dfrac{\text{Amplitude of radial oscillations induced by first harmonic}}{\text{Amplitude of radial oscillations inherent in beam from ion source}}$ = Same as 88'' cyclotron

First harmonic field amplitude (ultimate)	<0.2 G
First harmonic field amplitude (by steel shimming)	<1 G
Number of harmonic coils	6 x 12
Field correction by harmonic coils	±10 G
Tolerance on azimuthal sector placement	±0.14°
Tolerance on uniformity of sector reluctance	±0.5%
Tolerance on uniformity of sector permeability	±0.7%
Tolerance on transverse air gaps in yoke	±0.080''

6.4 Electric Stripping

Permitted deviation in beam loss by electric stripping	± 10%
Tolerance on hill magnetic field B_H (293'' → 311'')	±0.4%

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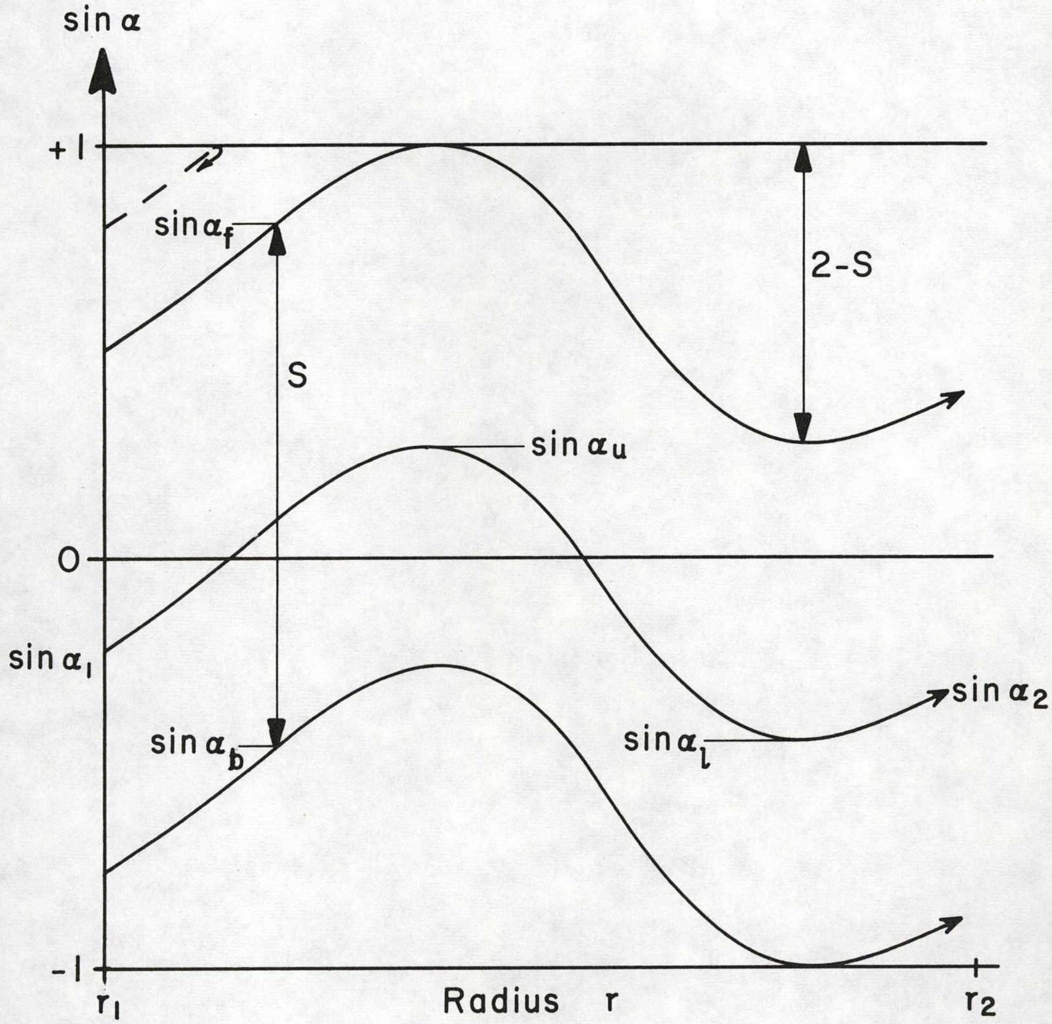

FIG. 1. Hypothetical phase history of ions between radii r_1 and r_2. The variation of sin α is independent of starting phase α_1 so that the accelerable ions lie within a fixed spread S in sin α.

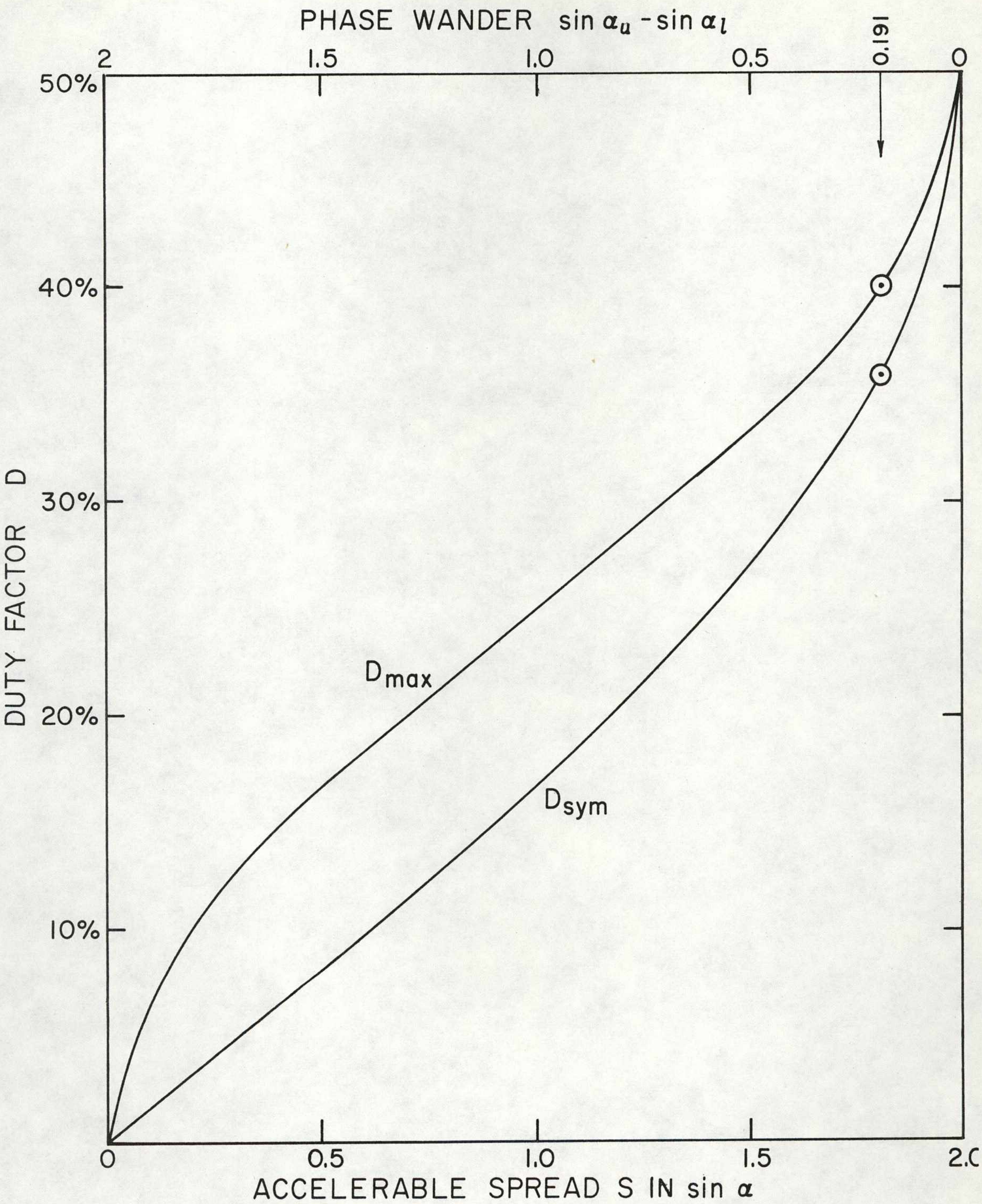

FIG. 2. Microscopic duty factor as a function of the accelerable spread S in sin α. For a given value of S the duty factor D may take any value between the upper limit D_{max} and the lower limit D_{sym}.

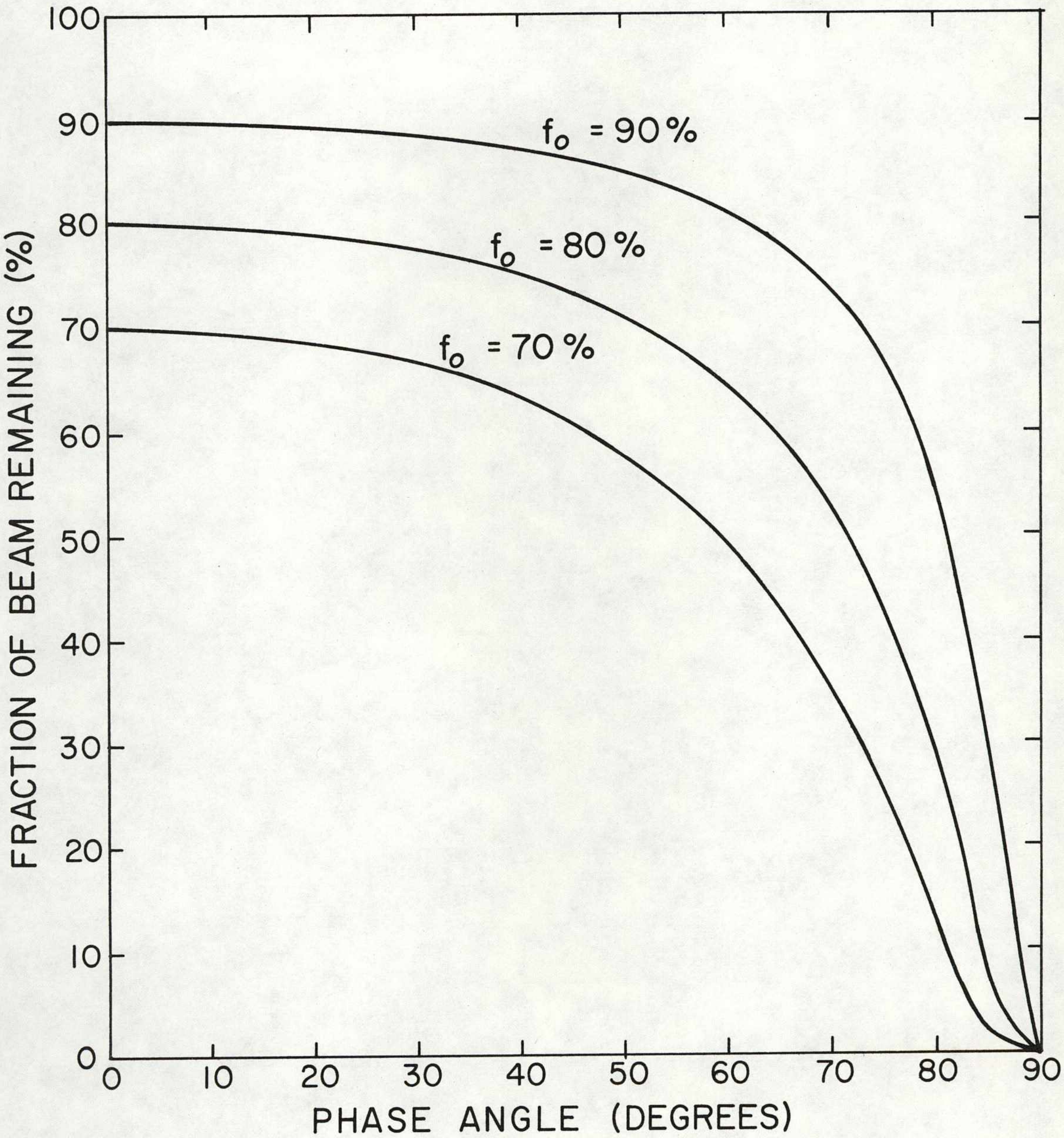

FIG. 3. Dependence of Beam Loss on RF Phase for 10%, 20% and 30% Loss at 0° Phase

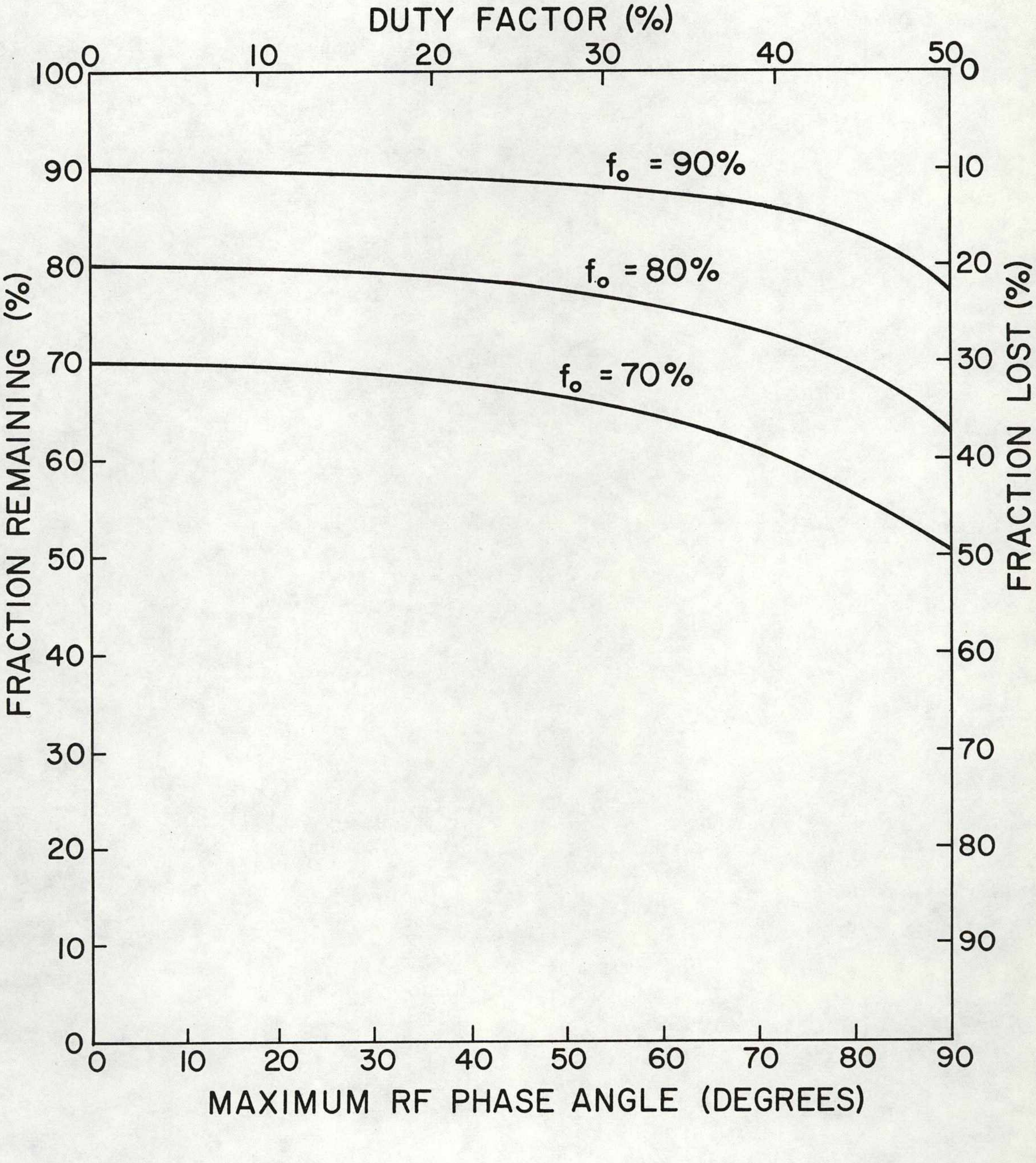

FIG. 4. Dependence of Beam Loss on Duty Factor for 10%, 20% and 30% Loss at 0° Phase

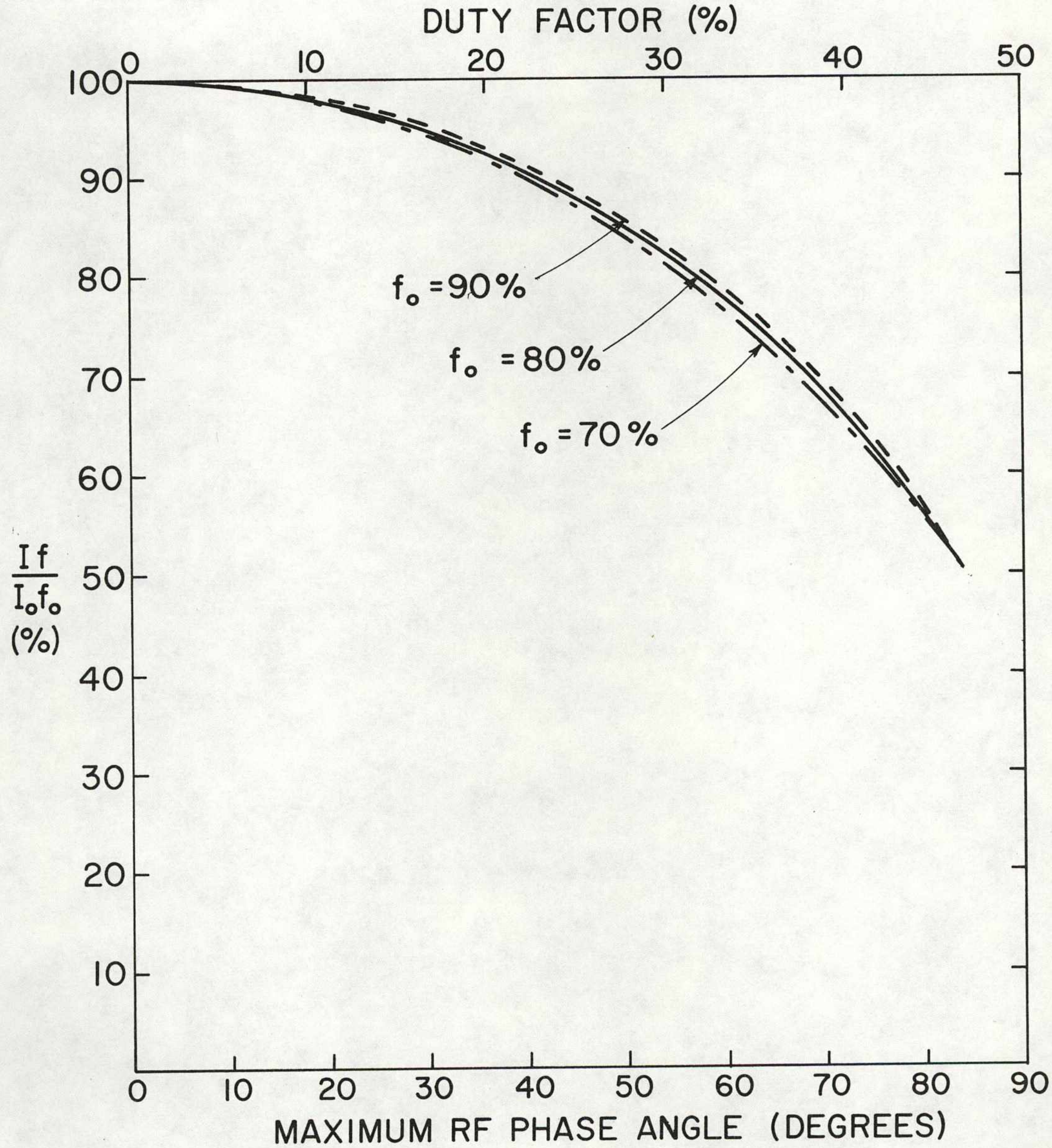

FIG. 5. Fraction of Beam Current Extractable vs. Microscopic Duty Factor, for Fixed Total Beam Current Loss

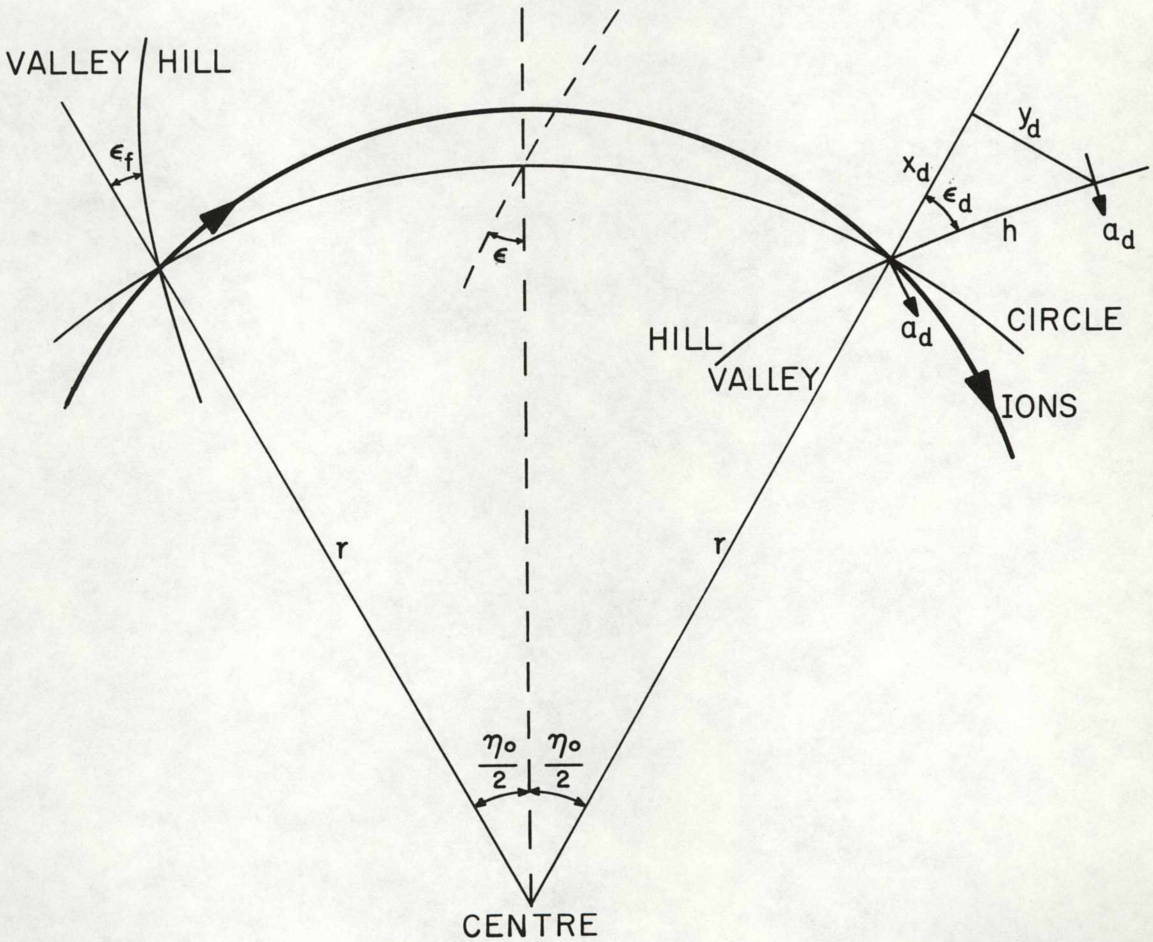

FIG. 6. Geometry of a Spiral Sector

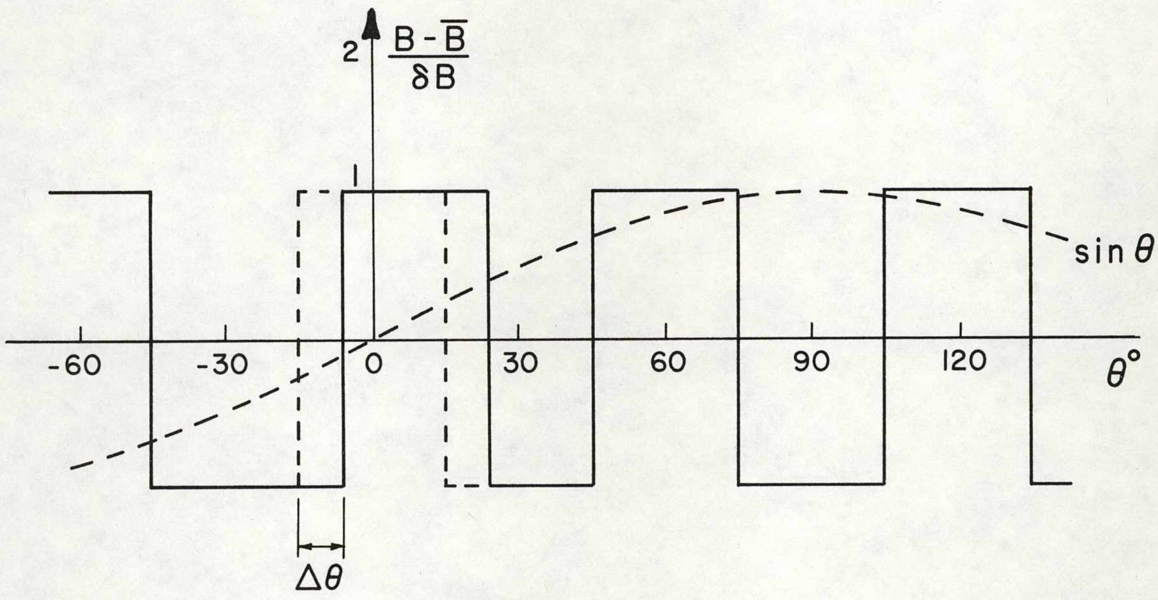

FIG. 7. Schematic field variation for azimuthal displacement of a hill. A simplified flat field hard edge model is used with equal hill and valley widths.

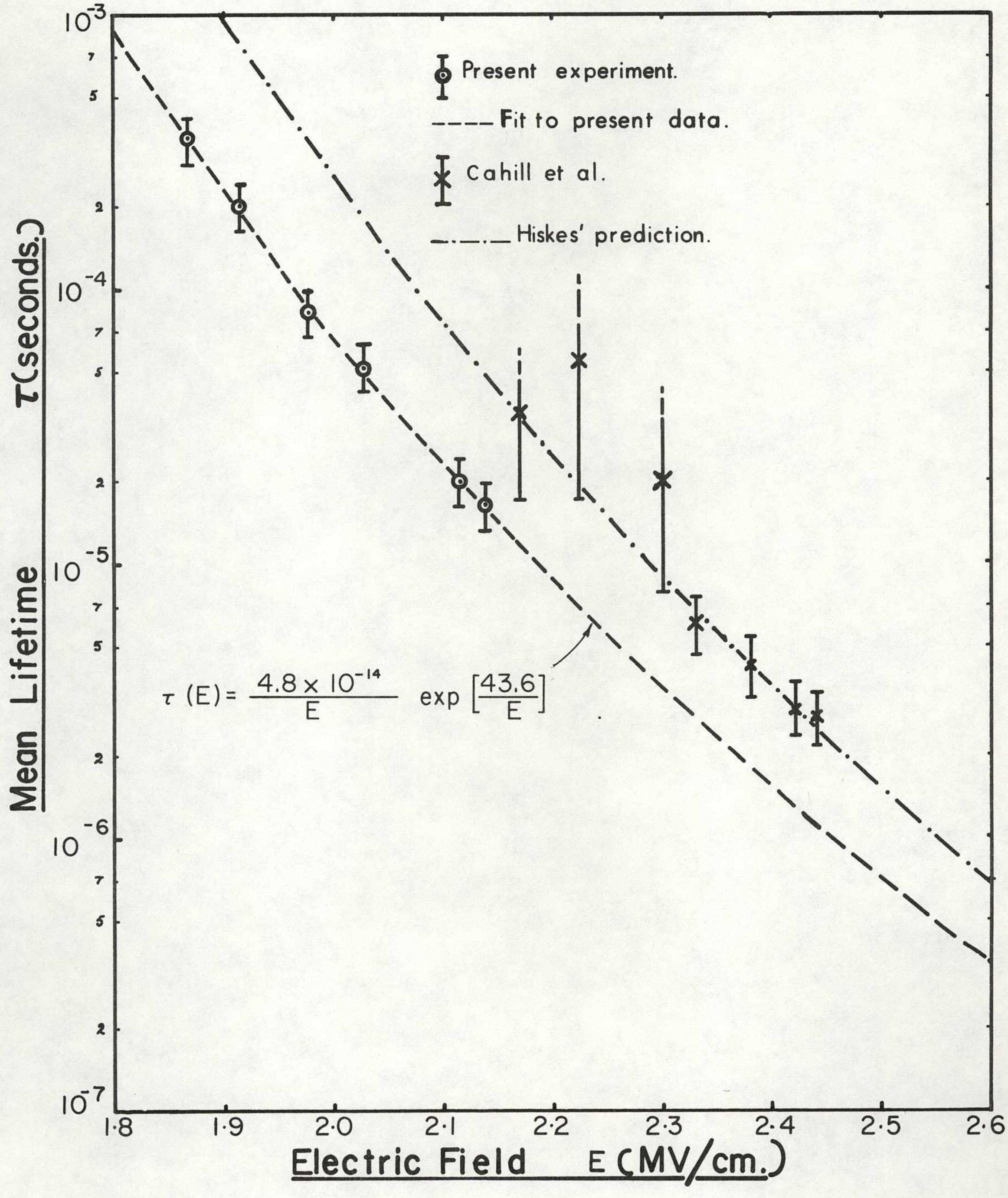

FIG. 8. Experimental H^- lifetimes as a function of electric field as determined by Olsen et al.[6] and by Cahill et al.[9] The prediction of Hiskes[8] and that from a fit to Olsen's data are also shown.