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WHAT PP PARAMETERS NEED MEASURING FROM 200 TO 525 MeV?

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Introduction

It is well known that the pp elastic scattering amplitudes are uniquely determined up to about 600 MeV.[1] However, although the phase shift solution is unique, it is not as precise as one might wish, particularly in low partial waves (which contain the essential physics) and at the higher energies. We investigate the improvements possible in the phase shifts from fresh measurements in the range 200 to 520 MeV, readily accessible at TRIUMF, particularly of Wolfenstein parameters D, R, A, R′ and A′. We assume a reasonable goal for fresh measurements to be ±0.8% for $d\sigma/d\Omega$, ±0.01 for P and ±0.025 for other parameters, and seek a combination of measurements which reduces the errors on all observables to these levels. A subsequent report will deal with the less healthy np system.

Method

First we use all existing data to do single energy phase shift analyses at 200, 320, 380, 425 and 520 MeV. The data set is very close to that of Arndt *et al.*[1] However, we keep, with increased errors, a few data points which they reject. We also add the recent Maryland measurements of $d\sigma/d\Omega$[2] and Geneva data on P at small angles.[3] Our phase shift solutions and error matrices closely reproduce the published values of Arndt *et al.*

From these solutions we predict, using the full error matrix, values and errors for all the common experimental observables from 10 to 170 deg (Tables IV). These errors immediately indicate what most needs measuring, and at what level of accuracy. We choose convenient groups of new measurements and find their effects on the error matrix of the phases (Tables V), until the errors on all observables are below the limits specified in the Introduction.

The Choice of Variables

Phases up to and including $^3H_6$ are treated as variables; higher partial waves are set to OPE values. At 320 MeV we include inelasticity in $^3P_1$ only. At 380, 425 and 520 MeV we allow inelasticity in $^1D_2$ only. This is undoubtedly an over-simplification. However, we find that the elastic amplitudes are generally insensitive to small inelasticity (whose effect can be simulated by a small change in phase), and our conclusion is that the small inelasticity parameters are better determined by studying the inelastic channels.

We do not attempt an energy-dependent analysis, since our feeling is that at this level of accuracy the energy dependence must be flexible enough to accommodate an S-shaped excursion through the chosen energies (due to poles below threshold and inelasticity above 500 MeV). We omit very little data of consequence at intermediate energies: data close to our chosen energies E_i are reliably adjusted to E_i using the energy dependence of the phases.

We have investigated the effect of varying g^2, the π^0NN coupling constant. At 200 MeV the error on g^2 from existing data is ±3.4, and at 320 MeV it is ±3.8. Data at 425 and 520 MeV give no real determination of g^2. We conclude that g^2 is fixed essentially by the lower energy data (particularly around 140 MeV and near threshold), and that it would unfairly inflate the errors at the higher energies to treat g^2 as a variable. Accordingly, most results are presented with g^2 fixed at 14.4. We remark, as is well known, that g^2 is essentially determined by data at small angles. Values of D and A′ are particularly sensitive to it. Table I shows the accuracy with which g^2 is determined by various measurements at 200 MeV.

Table II shows the effect on χ^2 of fixing some or all of the H- and G-waves at OPE values. Existing data show little deviation from OPE in the H-waves, except perhaps in $^3H_4$ at 200 and 520 MeV. Exchanges of heavier mesons are to be determined from the lower partial waves, particularly P, D and F. We maintain that it should be possible subsequently to predict the tiny contributions from heavy boson exchange to G- and H-waves, and hence to reduce the errors on the low partial waves by a bootstrap operation. From an experimental viewpoint one wants to be sure that fresh experiments are more powerful than this type of theoretical constraint. As an example, we show in Table III the reduction in the errors in low partial waves at 320 MeV when G- and H-waves are fixed at OPE. (Small contributions from heavy mesons will not affect the errors significantly.) From this and Table V(b) we can judge which experiments are more powerful than theory, and the conclusion is that theory alone is not enough.

The Choice of Measurements

Tables IV show the errors on all the common experimental observables from 10 to 170 deg. The situation deteriorates rapidly as the energy rises. Except at 520 MeV, $d\sigma/d\Omega$ and P are already predicted with an accuracy close to our goal. Tables V display the diagonal elements of the error matrix resulting from existing data W, plus various hypothetical addition. The angular range is arbitrarily broken up into three sections, 0-60 deg, 60-120 deg and 120-180 deg, which require rather different experimental configurations (dictated by the range of the recoil proton). We choose a set of measurements common to all energies and concentrate on Wolfenstein parameters rather than correlation parameters, since we believe they are easier to measure. We assume that one would measure P with an accuracy ±0.017 and Wolfenstein parameters with an accuracy ±0.025, *with a common normalization* with uncertainty ±0.025; the P measurements serve only to normalize the Wolfenstein parameters. We find that R is generally less useful than D, but the experimental configuration is so similar that we assume both would be measured in practice.

We summarize our general observations, and then discuss individual energies.

1) Measurements at one particular angle often greatly reduce the errors on other observables at the same angle. However, measurements at angles more than 15 deg apart are largely independent, i.e. a measurement at small angles does not usually reduce the errors near 90 deg. An exception is that accurate measurements from 0 to 60 deg generally reduce errors in the range 120 to 180 deg (i.e. transfer parameters) to a low level.

2) It is essential to have good (1%) $d\sigma/d\Omega$ data over the full angular range, in order to fix the normalization of the large amplitudes. The normalization of existing $d\sigma/d\Omega$ data is frequently unreliable, and the normalization is therefore fixed by the total cross-section. This is a rather unhappy situation, since no cross-check is available. We suspect that the increase in χ^2 going from energy-independent analysis to energy-dependent analysis as observed by Arndt *et al*. is largely due to normalization problems.

3) D is sensitive to tensor parameters $\overline{\varepsilon}_2$ and $\overline{\varepsilon}_4$ and g^2; it is a vital parameter over the full angular range 0 to 120 deg.

4) R is sensitive to singlet phases; it is less useful than D, and could be omitted from 60 to 125 deg at 425 MeV.

5) Measurements of D and R from 13 to 115 deg reduce errors on most observables to our goal and therefore largely exhaust the necessary measurements. Table IV(e) illustrates this at 425 MeV. However, some further measurements are necessary and vary with energy. R′ can be measured with a configuration very similar to R, and for this reason we have systematically included R′ (33 deg) in Tables V; its weight is not great, and one angle between 0 and 60 deg is enough, except perhaps at 520 MeV where measurements at larger angles would help. More useful, but much more difficult, is A′ (longitudinal spin before and after the scatter); it is highly desirable at all energies from 0 to 60 deg. A is a somewhat less satisfactory substitute (e.g. Table V(b)).

Off-Diagonal Elements of the Error Matrix

Generally these are small and follow the trend of the diagonal elements. However, a striking correlation exists at all energies between $^3P_1$ and $^3P_2$ (Table VI). This is because they are the largest phases and dominate $d\sigma/d\Omega$ and σ_T at all energies; their similar effect on $d\sigma/d\Omega$ (except in the Coulomb interference region) causes a strong correlation in the phase shift solution. The χ^2 surface in the two-dimensional sub-space of the $^3P_1$ and $^3P_2$ variables has contours which are narrow ellipses, and the eigenvalues λ of the error matrix, given by

$$\lambda = \frac{1}{2}\left(E_{11} + E_{22}\right) \pm \left(E_{12} + \frac{1}{4}\left(E_{11} - E_{22}\right)^2\right)^{\frac{1}{2}},$$

are of greater interest than the diagonal elements themselves; they are shown in Table VI. Any measurement which reduces the correlation helps substantially in reducing the larger eigenvalue. In this respect, the otherwise ineffective R′(33 deg) measurement is useful in eliminating a strong correlation between $^3P_1$, $^3P_2$ and $^3H_5$, hence its inclusion in the selection of measurements. (However, A′ would do the same job better.)

Conclusions

We discuss our requirements at individual energies.

200 MeV: The data are already of such accuracy that the only worth-while additions are i) D (13 deg) to fix g^2 with greater accuracy, ii) D and R close to 90 deg, and iii) A′ (13-53 deg).

320 MeV: Measurements of D and R from 13 to 115 deg make a valuable reduction in the present errors. Either A′ or A is desirable from 13 to 53 deg, A′ being the better.

425 MeV: The greatest need is for D and R from 0 to 60 deg. However, to reduce the errors on all observables to the goal given in the Introduction one would like measurements of D from 65 to 115 deg, A′ from 0 to 60 deg, and $d\sigma/d\Omega$ across the full angular range.

520 MeV: The present χ^2 minimum is very ill defined, and one really needs extensive new measurements. One gets a vast reduction in the errors from measurements of D and R from 0 to 120 deg. New 1% measurements of $d\sigma/d\Omega$ are also important, and C_{NN} from 30 to 90 deg is highly desirable. For the sake of completeness, Table V(d) lists other measurements which would finally reduce the errors on all observables to the arbitrary level specified in the Introduction; however, it will be better to make fresh judgements after the first round of new measurements.

380 MeV: Present data do not give a well-defined χ^2 minimum. However, much of the data at this energy is of high quality, and the addition of D and R from 0 to 115 deg gives an excellent minimum. Again A′ in the range of 0 to 60 deg is desirable. It seems presently that sizable errors in the predictions for C and $A_{\alpha\beta}$ parameters at small angles can only be eliminated by measuring one of these parameters from 0 to 60 deg, the one with the greatest sensitivity being A_{ZZ}. However, 380 MeV is unique in this respect, and it may be an artifact which disappears once data define a true χ^2 minimum.

Acknowledgements

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Table I. The error on g^2 from measurements at D(13 deg) and A′(11.5 deg) at 200 MeV as a function of the accuracy of the measurements.

Data	δg^2
Present data	±3.40
δD(13 deg) = ±0.025	±1.67
δD(13 deg) = ±0.017	±1.47
δD(13 deg) = ±0.010	±1.32
δD(13 deg) = ±0.005	±1.26
δA′(11.5 deg) = ±0.025	±1.77
δA′(11.5 deg) = ±0.017	±1.68
δA′(11.5 deg) = ±0.010	±1.62

Table II. Values of χ^2 with various G- and H-waves fixed at OPE values.

Variables	g^2	Energy (MeV)			
		200	320	425	520
All G- and H-waves free	free	50.6	117.5	-	-
All G- and H-waves free	14.4	50.9	117.9	137.6	128.4
$^3H_6$ fixed at OPE	14.4	52.9	122.2	137.6	132.5
$^3H_6$ and $^3H_5$ fixed at OPE	14.4	54.4	124.6	140.2	134.3
All H-waves fixed at OPE	14.4	61.7	125.2	141.5	162.3, unstable
G- and H-waves fixed at OPE	14.4	71.8	125.4	160.1	161.7

Table III. The effect on the diagonal elements of the error matrix (degrees^2) of fixing H-waves at OPE values at 320 MeV. (g^2 is fixed at 14.4)

Variables	$^3P_0$	$^1S_0$	$^3P_1$	$^3P_2$	$\overline{\varepsilon}_2$	$^3F_2$	$^1D_2$	$^3F_3$	$^3F_4$	$\overline{\varepsilon}_4$	$^3H_4$	$^1G_4$	$^3H_5$	$^3H_6$
H-waves free	2.48	1.57	0.79	0.28	0.15	0.29	0.18	0.36	0.060	0.11	0.14	0.07	0.28	0.036
H-waves at OPE	1.87	0.98	0.53	0.19	0.12	0.25	0.13	0.33	0.040	0.054	-	0.039	-	-
G- & H-waves at OPE	1.82	0.89	0.42	0.15	0.11	0.23	0.13	0.33	0.039	0.051	-	-	-	-

Table IV(a). Predicted errors on common observables at 200 MeV from the phase shift analysis of existing data. (g^2 fixed at 14.4; phases varied up to $^3H_6$)

Angle (c.m. deg)	$\delta\sigma/\sigma$	δP	δD	δR	δA	$\delta R'$	$\delta A'$	δC_{KP}	δC_{NN}	δC_{KK}	δC_{PP}	δA_{XX}	δA_{ZZ}	δA_{ZX}
10	0.011	0.003	0.021	0.017	0.008	0.004	0.047	0.010	0.020	0.010	0.046	0.010	0.047	0.006
20	0.005	0.004	0.021	0.013	0.012	0.009	0.019	0.014	0.036	0.018	0.034	0.014	0.038	0.012
30	0.005	0.003	0.012	0.010	0.008	0.010	0.036	0.010	0.018	0.017	0.044	0.016	0.045	0.010
40	0.005	0.003	0.011	0.009	0.010	0.011	0.036	0.005	0.008	0.023	0.043	0.017	0.038	0.019
50	0.006	0.004	0.013	0.011	0.012	0.011	0.022	0.004	0.005	0.022	0.029	0.013	0.020	0.020
60	0.006	0.005	0.013	0.012	0.017	0.012	0.019	0.005	0.004	0.025	0.026	0.011	0.011	0.024
70	0.005	0.005	0.021	0.019	0.021	0.018	0.023	0.010	0.007	0.030	0.030	0.009	0.010	0.030
80	0.006	0.003	0.032	0.027	0.015	0.025	0.015	0.015	0.012	0.023	0.021	0.012	0.017	0.022
90	0.008	0.000	0.038	0.031	0.008	0.029	0.008	0.017	0.014	0.007	0.007	0.014	0.022	0.000
100			0.035	0.029	0.021	0.028	0.021							
110			0.024	0.023	0.025	0.021	0.026							
120			0.014	0.016	0.018	0.012	0.020							
130			0.013	0.018	0.013	0.010	0.020							
140			0.013	0.024	0.011	0.007	0.027							
150			0.020	0.021	0.011	0.016	0.025							
160			0.021	0.012	0.015	0.017	0.014							
170			0.028	0.005	0.010	0.023	0.005							

Table IV(b). As IV(a) at 320 MeV.

Angle (c.m. deg)	$\delta\sigma/\sigma$	δP	δD	δR	δA	$\delta R'$	$\delta A'$	δC_{KP}	δC_{NN}	δC_{KK}	δC_{PP}	δA_{XX}	δA_{ZZ}	δA_{ZX}
10	0.024	0.009	0.061	0.038	0.017	0.014	0.079	0.022	0.037	0.024	0.070	0.024	0.073	0.018
20	0.011	0.011	0.050	0.026	0.029	0.026	0.033	0.027	0.053	0.028	0.041	0.022	0.044	0.027
30	0.009	0.010	0.035	0.037	0.043	0.032	0.062	0.018	0.033	0.035	0.028	0.034	0.026	0.020
40	0.008	0.009	0.041	0.034	0.050	0.035	0.067	0.034	0.031	0.033	0.017	0.041	0.017	0.028
50	0.008	0.007	0.042	0.031	0.041	0.029	0.051	0.044	0.026	0.019	0.018	0.037	0.040	0.028
60	0.009	0.006	0.035	0.023	0.042	0.029	0.046	0.034	0.023	0.016	0.027	0.025	0.042	0.022
70	0.007	0.007	0.035	0.017	0.046	0.028	0.048	0.030	0.019	0.020	0.028	0.021	0.037	0.025
80	0.007	0.005	0.055	0.016	0.028	0.020	0.033	0.042	0.022	0.020	0.021	0.035	0.048	0.020
90	0.008	0.000	0.078	0.017	0.029	0.016	0.031	0.048	0.027	0.014	0.014	0.044	0.056	0.000
100			0.083	0.023	0.057	0.016	0.057							
110			0.066	0.035	0.064	0.019	0.066							
120			0.050	0.044	0.060	0.029	0.061							
130			0.053	0.057	0.062	0.042	0.067							
140			0.048	0.061	0.066	0.045	0.072							
150			0.042	0.046	0.060	0.032	0.056							
160			0.055	0.022	0.055	0.023	0.028							
170			0.064	0.012	0.043	0.044	0.009							

Table IV(c). As IV(a) at 425 MeV.

Angle (c.m. deg)	$\delta\sigma/\sigma$	δP	δD	δR	δA	$\delta R'$	$\delta A'$	δC_{KP}	δC_{NN}	δC_{KK}	δC_{PP}	δA_{XX}	δA_{ZZ}	δA_{ZX}
10	0.024	0.011	0.072	0.031	0.030	0.021	0.117	0.029	0.057	0.029	0.106	0.030	0.110	0.021
20	0.014	0.014	0.087	0.022	0.050	0.042	0.061	0.037	0.071	0.023	0.057	0.025	0.062	0.033
30	0.017	0.012	0.066	0.028	0.057	0.049	0.056	0.028	0.040	0.038	0.046	0.036	0.039	0.034
40	0.015	0.010	0.059	0.026	0.057	0.043	0.060	0.030	0.017	0.037	0.037	0.027	0.017	0.042
50	0.013	0.008	0.048	0.021	0.038	0.027	0.048	0.034	0.016	0.021	0.022	0.023	0.023	0.033
60	0.014	0.006	0.021	0.012	0.014	0.011	0.019	0.021	0.015	0.014	0.013	0.017	0.023	0.015
70	0.013	0.005	0.018	0.013	0.011	0.013	0.017	0.022	0.011	0.013	0.012	0.017	0.025	0.014
80	0.015	0.004	0.032	0.019	0.016	0.018	0.027	0.050	0.018	0.011	0.012	0.047	0.053	0.011
90	0.019	0.000	0.036	0.019	0.025	0.018	0.028	0.062	0.023	0.011	0.011	0.062	0.066	0.000
100			0.033	0.017	0.031	0.014	0.027							
110			0.022	0.013	0.023	0.012	0.020							
120			0.026	0.016	0.019	0.015	0.018							
130			0.049	0.027	0.041	0.022	0.040							
140			0.057	0.033	0.057	0.033	0.053							
150			0.055	0.028	0.056	0.035	0.042							
160			0.047	0.017	0.052	0.021	0.020							
170			0.048	0.010	0.052	0.035	0.009							

Table IV(d). Predicted errors on common observables at 520 MeV from the phase shift analysis of existing data. (g^2 fixed at 14.4; phases varied up to $^3H_6$) These errors almost certainly are an underestimate, since χ^2 = 128 for 72 deg of freedom in the phase shift fit.

Angle (c.m. deg)	$\delta\sigma/\sigma$	δP	δD	δR	δA	$\delta R'$	$\delta A'$	δC_{KP}	δC_{NN}	δC_{KK}	δC_{PP}	δA_{XX}	δA_{ZZ}	δA_{ZX}
10	0.101	0.019	0.068	0.121	0.077	0.073	0.165	0.143	0.193	0.267	0.209	0.290	0.200	0.123
20	0.052	0.031	0.069	0.066	0.097	0.105	0.046	0.179	0.066	0.078	0.226	0.117	0.182	0.189
30	0.036	0.027	0.098	0.060	0.084	0.091	0.037	0.131	0.094	0.083	0.141	0.070	0.125	0.142
40	0.015	0.013	0.103	0.094	0.103	0.095	0.046	0.107	0.098	0.100	0.089	0.085	0.125	0.098
50	0.012	0.009	0.116	0.096	0.099	0.101	0.057	0.118	0.127	0.104	0.091	0.082	0.163	0.090
60	0.011	0.006	0.117	0.088	0.073	0.083	0.050	0.121	0.124	0.072	0.095	0.083	0.184	0.055
70	0.011	0.004	0.106	0.087	0.078	0.078	0.070	0.140	0.111	0.046	0.091	0.117	0.192	0.027
80	0.011	0.003	0.092	0.100	0.086	0.079	0.096	0.196	0.130	0.039	0.100	0.162	0.258	0.017
90	0.019	0.000	0.069	0.096	0.097	0.085	0.109	0.231	0.149	0.074	0.074	0.185	0.301	0.000
100			0.058	0.071	0.104	0.082	0.099							
110			0.055	0.049	0.101	0.061	0.083							
120			0.048	0.073	0.104	0.043	0.100							
130			0.052	0.109	0.094	0.035	0.122							
140			0.084	0.117	0.093	0.050	0.120							
150			0.109	0.163	0.123	0.053	0.141							
160			0.095	0.219	0.192	0.111	0.172							
170			0.089	0.152	0.174	0.197	0.128							

Table IV(e). Predicted errors on common observables at 425 MeV, after the addition of PDR(13,33,53 deg) and R′(33 deg) to existing data. Otherwise, as Table IV(c).

Angle (c.m. deg)	$\delta\sigma/\sigma$	δP	δD	δR	δA	$\delta R'$	$\delta A'$	δC_{KP}	δC_{NN}	δC_{KK}	δC_{PP}	δA_{XX}	δA_{ZZ}	δA_{ZX}
10	0.020	0.008	0.020	0.014	0.014	0.013	0.043	0.011	0.042	0.023	0.055	0.023	0.056	0.008
20	0.013	0.009	0.021	0.015	0.018	0.017	0.022	0.014	0.033	0.018	0.041	0.016	0.042	0.014
30	0.015	0.008	0.019	0.015	0.024	0.019	0.035	0.012	0.018	0.020	0.031	0.019	0.026	0.017
40	0.014	0.008	0.021	0.014	0.031	0.021	0.039	0.015	0.013	0.019	0.022	0.016	0.013	0.021
50	0.013	0.006	0.019	0.013	0.025	0.017	0.028	0.015	0.011	0.012	0.015	0.013	0.014	0.015
60	0.013	0.005	0.012	0.009	0.011	0.009	0.012	0.010	0.010	0.008	0.011	0.008	0.014	0.007
70	0.011	0.005	0.015	0.010	0.010	0.010	0.012	0.017	0.010	0.009	0.010	0.013	0.021	0.009
80	0.015	0.004	0.020	0.014	0.011	0.013	0.015	0.032	0.017	0.010	0.010	0.030	0.037	0.007
90	0.019	0.000	0.023	0.014	0.016	0.013	0.017	0.039	0.021	0.011	0.011	0.037	0.044	0.000
100			0.025	0.012	0.022	0.011	0.020							
110			0.020	0.011	0.018	0.009	0.016							
120			0.017	0.012	0.016	0.010	0.015							
130			0.024	0.019	0.030	0.013	0.029							
140			0.023	0.025	0.036	0.015	0.036							
150			0.023	0.023	0.030	0.017	0.028							
160			0.029	0.014	0.034	0.015	0.015							
170			0.040	0.008	0.041	0.023	0.007							

Table V(a). Diagonal elements of the error matrix (degrees^2) at 200 MeV from existing data, W, and various additions to it. In the additions $\delta P = \pm 0.017$ and other errors = ± 0.025. g_0^2 is fixed at 14.4.

Data	$^3P_0$	$^1S_0$	$^3P_1$	$^3P_2$	$\overline{\varepsilon}_2$	$^3F_2$	$^1D_2$	$^3F_3$	$^3F_4$	$\overline{\varepsilon}_4$	$^3H_4$	$^1G_4$	$^3H_5$	$^3H_6$
W	0.264	0.272	0.042	0.028	0.016	0.059	0.078	0.031	0.027	0.007	0.050	0.010	0.035	0.022
W + PDR (13,33,53 deg)	0.245	0.249	0.035	0.025	0.014	0.056	0.064	0.027	0.025	0.005	0.038	0.009	0.026	0.018
Ditto + R′(33 deg)	0.244	0.249	0.034	0.024	0.014	0.056	0.061	0.026	0.025	0.005	0.037	0.009	0.026	0.018
Ditto + PDR (65-115 deg, 10 deg steps)	0.087	0.088	0.025	0.020	0.008	0.032	0.018	0.023	0.019	0.003	0.028	0.004	0.020	0.014
Ditto + A′(13,33,53 deg)	0.072	0.066	0.022	0.018	0.006	0.029	0.011	0.015	0.018	0.002	0.015	0.004	0.011	0.008
W + PDR(65-115 deg, 10 deg steps)	0.096	0.110	0.029	0.022	0.009	0.036	0.024	0.027	0.021	0.004	0.035	0.005	0.025	0.017
Ditto + A′(13,33,53 deg)	0.077	0.070	0.023	0.019	0.006	0.030	0.012	0.016	0.019	0.003	0.017	0.004	0.011	0.009

Table V(b). As V(a) at 320 MeV.

Data	$^3P_0$	$^1S_0$	$^3P_1$	$^3P_2$	$\overline{\varepsilon}_1$	$^3F_2$	$^1D_2$	$^3F_3$	$^3F_4$	$\overline{\varepsilon}_4$	$^3H_4$	$^1G_4$	$^3H_5$	$^3H_6$
W	2.47	1.62	0.81	0.29	0.15	0.29	0.19	0.36	0.061	0.11	0.14	0.073	0.28	0.035
W + PDR(13,33,53 deg)	1.86	0.52	0.54	0.18	0.11	0.25	0.10	0.33	0.036	0.024	0.058	0.034	0.18	0.024
Ditto + R′(33 deg)	1.74	0.49	0.44	0.13	0.092	0.25	0.091	0.31	0.032	0.024	0.057	0.033	0.14	0.021
Ditto + PDR(65-115 deg, 10 deg steps) = X	0.21	0.26	0.22	0.10	0.042	0.053	0.061	0.055	0.020	0.011	0.032	0.027	0.057	0.014
Ditto + A′(13,33,53 deg)	0.21	0.23	0.084	0.060	0.022	0.043	0.050	0.037	0.019	0.009	0.024	0.023	0.023	0.013
Ditto + A(13,33,53 deg)	0.20	0.022	0.065	0.047	0.021	0.040	0.048	0.033	0.018	0.009	0.024	0.022	0.022	0.013
X + A(13,33,53 deg)	0.21	0.27	0.11	0.058	0.029	0.044	0.060	0.040	0.019	0.011	0.030	0.024	0.036	0.014
W + PDR(65-115 deg, 10 deg steps)	0.50	0.61	0.42	0.21	0.074	0.11	0.13	0.067	0.029	0.029	0.045	0.043	0.089	0.021

Table V(c). As V(a) at 425 MeV.

Data	$^3P_0$	$^1S_0$	$^3P_1$	$^3P_2$	$\overline{\varepsilon}_2$	$^3F_2$	$^1D_2$	$^3F_3$	$^3F_4$	$\overline{\varepsilon}_4$	$^3H_4$	$^1G_4$	$^3H_5$	$^3H_6$
W	2.13	1.16	1.16	0.57	0.55	0.27	0.25	0.16	0.050	0.19	0.13	0.048	0.19	0.071
W + PDR(13,33,53 deg)	0.76	0.43	0.55	0.30	0.16	0.11	0.11	0.069	0.043	0.035	0.057	0.036	0.14	0.034
Ditto + R′(33 deg)	0.64	0.37	0.30	0.15	0.13	0.093	0.11	0.065	0.034	0.035	0.057	0.036	0.11	0.030
Ditto + PDR(65-115 deg, 10 deg steps)	0.40	0.28	0.25	0.13	0.090	0.075	0.096	0.057	0.031	0.021	0.045	0.032	0.069	0.027
Ditto + A′(13,33,53 deg)	0.36	0.27	0.19	0.11	0.064	0.061	0.081	0.046	0.028	0.016	0.036	0.032	0.030	0.024
Ditto + dσ/dΩ(10-90 deg, 5 deg steps)	0.29	0.23	0.11	0.075	0.056	0.033	0.056	0.023	0.011	0.014	0.029	0.013	0.023	0.014

Table V(d). As V(a) at 520 MeV.

Data	$^3P_0$	$^1S_0$	$^3P_1$	$^3P_2$	$\overline{\varepsilon}_2$	$^3F_2$	$^1D_2$	$^3F_3$	$^3F_4$	$\overline{\varepsilon}_4$	$^3H_4$	$^1G_4$	$^3H_5$	$^3H_6$
W	19.2	35.8	8.77	1.53	9.09	2.12	23.6	3.55	0.54	2.00	1.10	4.49	1.36	0.40
W + PDR(13,33,53 deg)	6.33	5.50	2.88	0.62	1.32	0.42	4.60	0.47	0.37	0.56	0.35	1.21	0.31	0.17
Ditto + R′(33 deg)	5.87	5.33	1.94	0.52	1.32	0.40	3.10	0.45	0.16	0.45	0.25	0.99	0.31	0.16
Ditto + PDR(65-115 deg, 10 deg steps) = X	0.87	1.04	0.50	0.33	0.65	0.16	2.29	0.31	0.12	0.095	0.15	0.23	0.15	0.11
X + A′(13,33,53 deg)	0.71	0.98	0.39	0.19	0.51	0.12	1.56	0.24	0.086	0.089	0.097	0.20	0.10	0.072
X + dσ/dΩ(10-90 deg, 5 deg steps) = Y	0.62	0.78	0.28	0.17	0.47	0.072	1.31	0.21	0.035	0.075	0.059	0.17	0.066	0.028
Ditto + C_{NN}(30-90 deg, 10 deg steps) = Z	0.52	0.46	0.27	0.11	0.17	0.069	0.44	0.19	0.025	0.045	0.041	0.087	0.059	0.027
Y + A(65-115 deg, 10 deg steps)	0.58	0.75	0.25	0.10	0.22	0.053	0.58	0.16	0.028	0.058	0.045	0.14	0.051	0.024
Z + C_{KP}(90 deg)	0.51	0.46	0.26	0.10	0.13	0.063	0.39	0.19	0.025	0.040	0.041	0.087	0.058	0.027
Ditto + R(160 deg)	0.48	0.44	0.24	0.096	0.068	0.060	0.17	0.14	0.022	0.024	0.041	0.042	0.013	0.025

Table V(e). Diagonal elements of the error matrix (degrees^2) at 380 MeV from existing data, W, and various additions to it. In the additions $\delta P = \pm 0.017$ and other errors = ± 0.025. g_o^2 is fixed at 14.4. The minimum addition to existing data which gives a χ^2 minimum is shown in the first line of the table.

Data	$^3P_0$	$^1S_0$	$^3P_1$	$^3P_2$	$\overline{\varepsilon_2}$	$^3F_2$	$^1D_2$	$^3F_3$	$^3F_4$	$\overline{\varepsilon_4}$	$^3H_4$	$^1G_4$	$^3H_5$	$^3H_6$
W + PDR(13,33,53 deg)	1.89	1.28	0.93	0.47	0.22	0.48	0.085	0.48	0.14	0.068	0.22	0.043	0.28	0.17
Ditto + R′(33 deg)	1.88	0.75	0.79	0.38	0.22	0.40	0.063	0.47	0.11	0.057	0.22	0.039	0.19	0.14
Ditto + PDR(65-115 deg, 10 deg steps)	0.63	0.46	0.35	0.20	0.056	0.11	0.052	0.10	0.040	0.022	0.092	0.026	0.046	0.056
Ditto + A′(13,33,53 deg)	0.44	0.37	0.23	0.125	0.046	0.082	0.043	0.089	0.035	0.016	0.081	0.023	0.036	0.049
Ditto + A_{ZZ}(13,33,53 deg	0.29	0.32	0.084	0.061	0.035	0.076	0.033	0.049	0.034	0.013	0.044	0.015	0.024	0.027

Table VI. Diagonalization of the sub-error matrix in the $^3P_1$-$^3P_2$ space. E_{33} and E_{44} are the diagonal elements of the error matrix for $^3P_1$ and $^3P_2$, respectively, and E_{34} is the correlation term between them. Then λ_a and λ_b are the eigenvalues of the error matrix, ignoring correlation with other phases. Units are degrees^2.

	E_{33}	E_{44}	E_{34}	λ_a	λ_b
200	0.04214	0.02828	0.02784	0.0639	0.0064
320	0.84226	0.28903	0.40825	1.0588	0.0725
425	1.16435	0.56648	0.68271	1.6107	0.1201
520	8.77246	1.53218	1.08396	8.9313	1.3733
380	0.92729	0.46921	0.58576	1.3272	0.0693