Dynamics of Structures

CIVL 507 Term Paper

A Short Note on Floor Vibration Management and Control

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October 17, 2015

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1. Abstract

Modern large span floors and light weight structural systems tend to vibrate under service loads. As a matter of fact, the effect of floor vibration might have negative impacts on daily routines of a structure ranging from a single family household to a rotating machinery in a chemical plant and even a multi-billion dollar hydro power plant. Anywhere alongside this broad spectrum, human comfort, performance of equipment and or structural serviceability may be compromised.

Investigating floor vibration has a unique set of properties for which it becomes a suitable topic to apply theory of structural dynamics. This particular system could potentially be simplified enough to fit within a classic dynamics problem. A quick glance at available publications on this topic indicates that numerous approaches are convention in various parts of the world. This provides us the opportunity to familiarize with a few and match them with the topics discussed within the context of CIVL507 course.

A literature review is first conducted regarding the vibration problems associated with the floors, and various techniques to minimize the impact of the vibration (e.g. rehabilitation or design state procedures) A background information will be prepared on various levels of human perception of vibration in terms of acceleration. A couple of methods will then be identified and handpicked to characterize the problem based on Single Degree of Freedom approach. Two case studies will also be provided in which the abovementioned methods are implemented (at least one method per case). The first case will be a conventional solution at the design level which result in a desired effective mass stiffness and or damping mechanism in order to remain within the desired comfort levels. The second case will simulate the response of a TMD compensated floor system in response to a harmonic excitation .

2. Background Information

Serviceability of a civil system may be viewed as a quality metrics or quality goal. Serviceability (rather than strength) is indeed one important design requirement for vibration-sensitive floor construction. Current architectural trends expect elimination of full height partitions, heavy filling cabinets, and large bookshelves and so on. These factors one way or another are part of the available dissipation mechanisms and their absence results in decrease of load as well as damping in general. According to Ebrahimpour and Sack (2005) a modern electronic office may typically have a bay length of 12 m with slab thickness between 100 mm and 130 mm while the corresponding figures for a traditional office are 7.5 m long bays and 140 mm to 190 mm thickness for slabs.

Middleton and Brownjohn (2010) argue that today instead of deflection, vibration serviceability has become a major concern for the following reasons. First, construction techniques have changed, allowing for longer span floors with much lighter construction. Second, occupancy of `normal' floors has changed. With moves towards open plan, paperless offices and increasing use of computer equipment, the mass of non-structural components supported has in general been reduced. In addition, huge technological advances have been made in medical, scientific and micro-manufacturing and as these disciplines move towards greater dimensional precision, the type of equipment used becomes extremely sensitive to vibration.

2.1. *Modeling Methodologies*

Floor vibration is managed by employing a variety of tools and techniques with various level of complexity. Early studies focused the research on the tuned mass dampers (TMDs). Allen and Pernica (1984) used TMDs consisting of wooden planks with weights on top for the reduction of annoying vibrations due to human walking. A vibration absorber is an oscillator of much smaller mass (m) than that of the floor structure (M), but with the same natural frequency. In some other cases (Mehdi Setareh (2002); M. Setareh, Ritchey, Baxter, and Murray (2006); M. Setareh, Ritchey, Murray, Koo, and Ahmadian (2007)) used Ground Hooked TMD, Pendulum TMD to control the floor vibrations due to walking and dancing in auditorium floor.

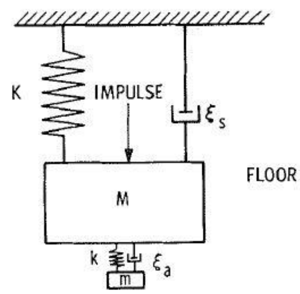

Figure 1. Idealization of floor structure with vibration absorber $\frac{K}{M} = \frac{k}{m} = (2\pi f)^2$

The concept of TMD is of particular interest in this report. In most cases, the resulting floor system is idealized in Figure 1. How the vibration absorber works is illustrated in Fig. 2, which shows the resulting natural vibration of the idealized floor system following a single footstep impulse.

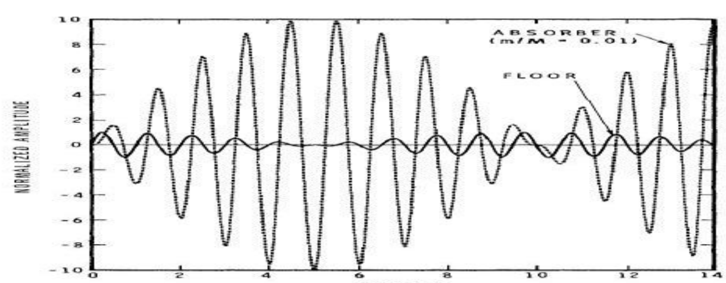

Figure 2.Transfer of vibration energy in an undamped tuned system (Allen, Rainer, & Pernica, 1985)

There are also more sophisticated methodologies to ensure serviceability of a given floor system. Namely Son, Kawachi, Matsuhisa, and Utsuno (2007) present a proof-of-concept using a momentum exchange impact damper. The impact damper consists of a spring and a mass that is contact with the floor. When a falling object collides with the floor, the floor interacts with the damper mass, and the momentum of the falling object is transferred to the damper. They claimed this configuration has the capacity to reduce the vibration up to 25%.

The work in aerospace on dynamic behavior and damping in composite Orthogrid and Isogrid systems are closely related to the passive control of floor vibrations. Although not specifically used for vibration control, advanced composites such as carbon fiber reinforced polymer (CFRP) strips and viscoelastic materials are used in civil engineering to strengthen existing structures (Ebrahimpour & Sack, 2005).

2.2. Active Control and Hybrid Approach Concepts

Optimal Control Theory is predominantly used in a wide variety of control mechanisms and there are plenty of reports, procedures and guidelines accordingly. Active Damping and Active Isolation are discussed in a great detail by Preumont and Seto (2008). The authors introduce active vibration control through the use of smart materials and structures, semi-active control devices and a variety of feedback options; they then discuss topics including methods and devices in civil structures, modal analysis, active control of high-rise buildings and bridge towers, active tendon control of cable structures, and active and semi-active isolation in mechanical structures.

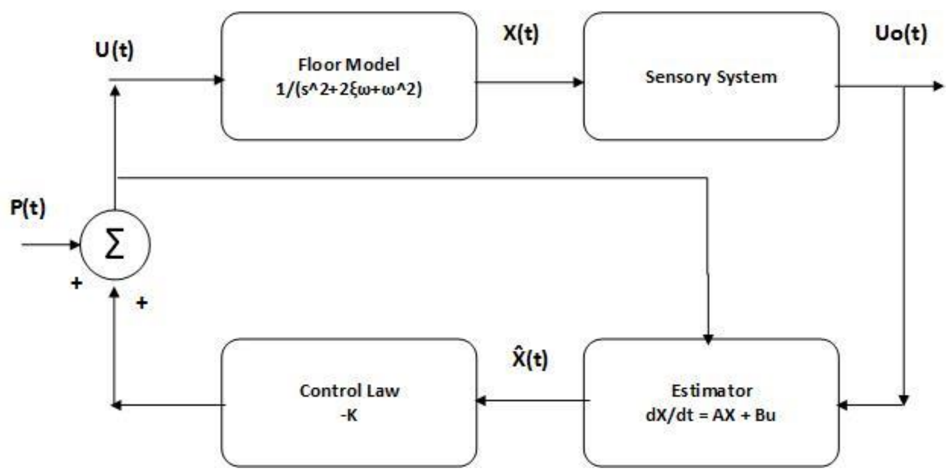

Figure 3. Active Control Loop Block Diagram

Critical infra-structures such as power plants or bridges may deteriorate over time, that is, deviation of structural characteristics from the original design values. Subsequently structural systems would respond to excitations relatively differently than what it is expected. However replacement of machinery or structural system could become cumbersome and this should make a case justifiable for active control approach. A full state control law using *State Space* (Figure 3) method may be designed to manipulate damping mechanism of a given floor system modeled based on SDOF concept.

2.3. Scope of Work

Subsequent sections of this report attempt to frame the problem at hand by addressing the following challenges:

- Analytical model of the excitation
- Analytical model of the floor system
- Characteristics of the response spectra
- Characteristics of the design/redesign objectives

Vibration perceptibility is another aspect of floor vibration problem. According to Naeim (1991), several factors influence the level of perception and the degree of sensitivity of people to vibrations. His classification includes but not limited to: position of the human body, excitation source characteristics, exposure time, floor system characteristics, level of expectancy, and type of activity engaged in. Prior to investigation of analytical modeling techniques, it is constructive to study the perception criteria of vibration in the context of user-structure interaction where user may corresponds to human or machinery. In both cases, vibration management plan is designed to ensure flawless system performance.

3. The Definition of Comfort Levels

Human response to floor motion is a very complex phenomenon may be described by the magnitude of the motion, the environment surrounding the sensor, and the human sensor. For example, people dining beside a dance floor, lifting weights beside an aerobics gym, or standing in a shopping mall, will accept about 1.5 percent g (i.e. %1.5g). In contrast, people in offices or residences do not like "distinctly perceptible" vibration (i.e. peak acceleration of about 0.5 percent of the acceleration of gravity, g (Murray, Allen, & Ungar, 1997; Naeim, 1991)).

The perception of vibration also depends on frequency content of the floor response. For instance, the above limits are for vibration frequencies between 4 Hz and 8 Hz. Outside this frequency range, people accept higher vibration accelerations as shown in Error! Reference source not found.. It is necessary to stimate the acceleration response during the design of the structure to assess the whether or not the level of vibration will be acceptable for the occupants.

Acceleration limits as recommended by the International Standards Organization (International Standard ISO 2631-2, 1989), adjusted for intended occupancy. According to ISO Standard the acceleration limits are in terms of RMS values as a multiple of the baseline line curve shown in Figure 4a. The multipliers for the proposed criterion, which is expressed in terms of peak acceleration, are 10 for offices, 30 for shopping malls and indoor footbridges, and 100 for outdoor footbridges. For design purposes, the limits can be assumed to range between 0.8 and 1.5 times the recommended values depending on the duration of vibration and the frequency of vibration events.

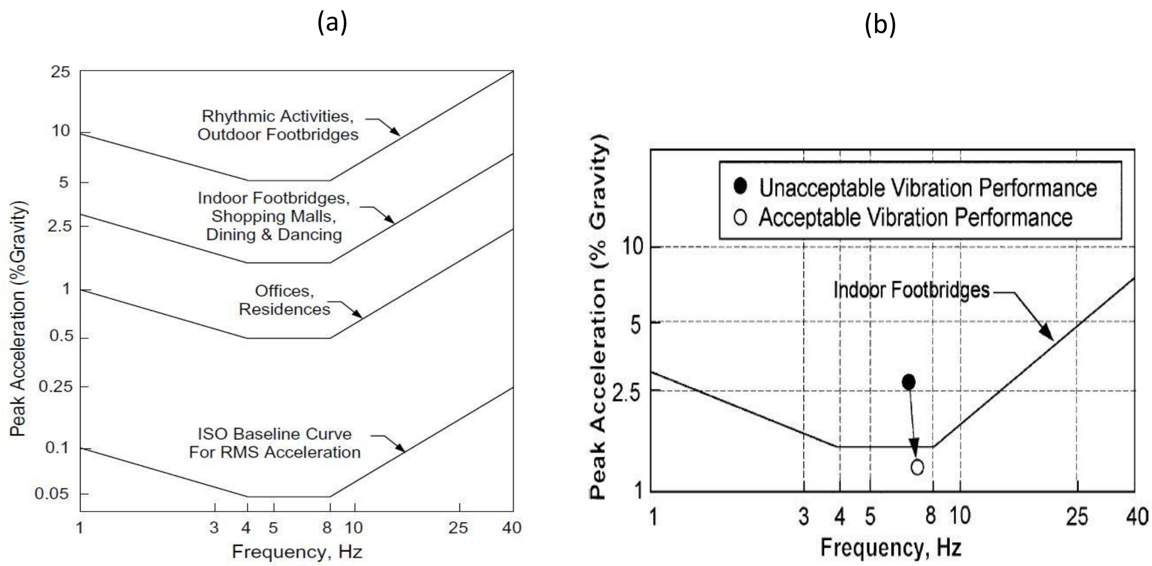

Figure 4. a) Recommended peak acceleration for human comfort for vibrations due to human activities (Allen and Murray, 1993; ISO 2631-2: 1989). b) AISC recommended vibration criterion for the footbridge.

It is worth noting that human perception criteria might not be suitable to characterize a machinery-structure interaction. As a matter of fact, Middleton and Brownjohn (2010) reports that during the 1970s, prompted by lack of guidance by machine manufacturers, Bolt Beranek & Newman Inc. (BBN) attempted to create a generic set of criteria specific to sensitive machinery. These criteria are known as the VC curves and are currently widely used.

3.1. Standard and Guidelines

3.1.1. American Institute of Steel Construction

The modeling technique outlined by the American Institute of Steel Construction (AISC) Design Guide 11 has been extensively used in North America for several years. For human comfort, the AISC method assumes the floor behavior is governed by the resonant response of the fundamental floor mode. A steady-state floor response is calculated assuming the walker and vibration-sensitive receptor are both located at the position of the maximum modal displacement (center of the bay) to produce a worst case response. The natural frequency of the floor is estimated from the maximum static deflection of the bay under consideration due to the acting dead and live loads. Such a simplistic method cannot easily predict the natural frequency when the floor layout is irregular, such as would occur around shafts and/or in non-rectangular buildings.

For areas with sensitive equipment, the AISC considers three walking speeds: 50, 75, and 100 steps per minute. An empirical force coefficient is estimated as a function of the walking pace, and the weight of the walker. Simple beam theory is employed to calculate the floor deflection, and the natural frequency. The floor response velocity is then calculated as a function of the force coefficient, floor natural frequency, and floor deflection.

3.1.2. Steel Construction Institute and Concrete Centre

The methodologies proposed by The Steel Construction Institute (SCI P354) and The Concrete Centre (CCIP- 016) for calculating footfall-induced floor vibrations are based on employing a Finite Element (FE) model to predict the mode shapes and natural frequencies of the floor. The scope of this report is limited and FE application may fit into a broader statement of work.

3.2. Vibration Indicator

Plenty of research papers study the property of their vibration of interest in terms of acceleration (Allen & Pernica, 1984; Murray et al., 1997; Nguyen, Saidi, Gad, Wilson, & Haritos, 2012; Mehdi Setareh, 2002; M. Setareh et al., 2006; M. Setareh et al., 2007). In 2009, (Brownjohn & Middleton) study the response of a high frequency floor system which is predominantly relevant to the category of machine-structure interaction. The authors discuss that many of the processes in vibration sensitive areas are photographic in nature (i.e. using photosensitive sensors). Such processes can tolerate limited blurring, which is defined as the distance travelled during the exposure, i.e. velocity. The velocity criteria appear constant within a class of machine with respect to frequency. Also, using the frequency domain a conversion can be made between displacement, velocity and acceleration, with velocity only one integration or differentiation step from the other metrics. In contrary displacement is mainly observed for seismic analysis of long period structures and may be less relevant to floor vibration problem.

4. Various Sources of Floor Vibration

The primary source of vibration in most facilities is human activity. As people walk, the impact from each footfall induces floor motions that may easily transmit to nearby spaces. Quantifying vibration from walking, whether through measurement of existing spaces or numerical predictions for guiding the design of a new facility, is a complex task. This task is complicated in part by the availability of a number of vibration measurement and prediction methodologies, each associated with both similar and unique assumptions.

Murray et al. (1997) explain that most floor vibration problems involve repeated forces caused by machinery or by human activities such as dancing, aerobics or walking, although walking is a little more complicated than the others because the forces change location with each step. In some cases, the applied force is sinusoidal or nearly so. In general, a repeated force can be represented by a combination of sinusoidal forces whose frequencies, f, are multiples or harmonics of the basic frequency of the force repetition, e.g. step frequency, f_p for human activities.

The key reason for studying human–structure dynamic interaction and pedestrian synchronization is because when walking on more or less perceptibly moving structures are increasingly giving serious cause for concern in vibration serviceability design.

There are two types of model in literature: time-domain models and frequency-domain models. For analytical and simulation purposes, some researchers prefer to employ a more inclusive form of excitation as the input to a floor system (Ebrahimpour & Sack, 2005; Middleton & Brownjohn, 2010; Mehdi Setareh, 2002; M. Setareh et al., 2007) which is represented in frequency domain:

$$P = P_0 e^{j\omega t}$$

Equation 1

And the steady state response should also take the form of:

$$X = X_0 e^{j\omega t}$$

Equation 2

Where ω is the frequency of excitation and X_0 is a complex value. This form is material in this report and will be discussed in subsequent sections.

4.1. *Vibrations due human related activities*

Racic, Pavic, and Brownjohn (2009) have provided a comprehensive study in modeling human related activities. They claim , under the assumption that an individual generates identical and therefore perfectly repeatable footfalls with the period T, the vertical walking force $F_P(t)$ can be represented in time domain as a sum of Fourier harmonic components, that is:

$$F_p(t) = F_0 \{1 + \sum_{i=1}^{N} \lambda_i \,.\sin(2\pi f t - \phi_i)\}$$

Equation 3

Where P is the subject's static weight (N), i the order number of the harmonic, N the total number of contributing harmonics, λ_i the Fourier coefficient of the i^{th} harmonic generally known as dynamic loading factor (DLF), fp the activity rate (Hz) and ϕi the phase shift of the i^{th} harmonic (rad). According to Murray et al. (1997) this model is valid for low frequency floor systems.

As a general rule, the magnitude of the dynamic coefficient decreases with increasing harmonic, for instance, the dynamic coefficients associated with the first four harmonics of walking are 0.5, 0.2, 0.1 and 0.05, respectively. In theory, if any frequency associated with the sinusoidal forces matches the natural frequency of a vibration mode, then resonance will occur, causing severe vibration amplification (Chopra, 2012) .

Recommended values for α_i are given in **Error! Reference source not found.** under the assumption that nly one harmonic component of **Error! Reference source not found.** is used since all other harmonic vibrations are small in comparison to the harmonic associated with resonance (Murray et al., 1997).

Harmonic i	Person Walking		Aerobics Class		Group Dancing	
	f, Hz	α_i	f, Hz	α_i	f, Hz	α_i
1	1.6–2.2	0.5	2-2.75	1.5	1.5-3	0.5
2	3.2–4.4	0.2	4-5.5	0.6	—	—
3	4.8–6.6	0.1	6-8.25	0.1	—	—
4	6.4–8.8	0.05	—	—	—	—
*dynamic coefficient = peak sinusoidal force/weight of person(s).						

Table 1. Common Forcing Frequencies (f) and Dynamic Coefficient (a.k.a. DLF or Dynamic Loading Factor) (α_i)

Middleton and Brownjohn (2010) have provided four important conclusions from the various investigations into walking forces are:

1. The general shape of a footfall force is similar, and is independent of the individual who is walking, his/her weight, pace rate and stride length.
2. The maximum force increases if the weight, stride length or pace rate of the person increases.
3. A normalized force can be obtained by normalizing to weight, pace rate and stride length. For higher harmonics, pace rate and stride length have little influence so only normalization with weight is necessary.
4. The average pacing rate of a person is 2 Hz with an average velocity of slightly over 1 m/s, but this also depends on sex, race, build and situation.

During walking, human produces dynamic loading with three components: vertical, horizontal-lateral and horizontal-longitudinal. Typical shapes of walking forces are shown in the following

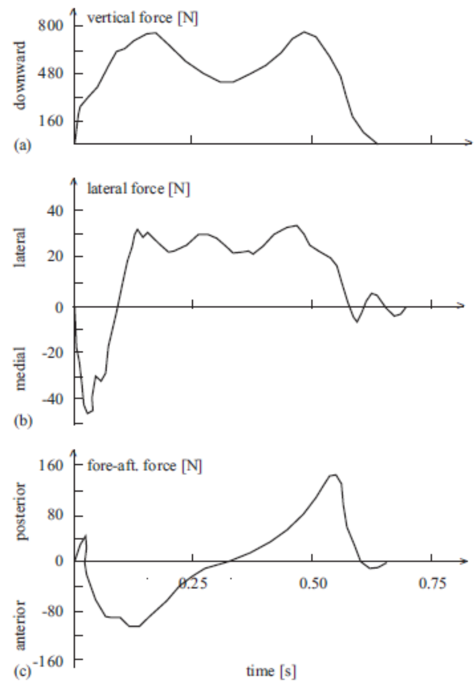

Figure 5. Typical shapes of walking force in (a) vertical, (b) lateral and (c) longitudinal direction (Racic et al., 2009)

Racic et al. (2009) also pointed out that increasing walking velocity led to increasing step length and peak force magnitude i.e. dynamic effect of the forces was changing with time (Figure 5). A continuous walking or running force can be obtained artificially by combining individual forces and accounting for overlaps. Shape of running forces differs from those of walking in having only one peak (Figure 6).

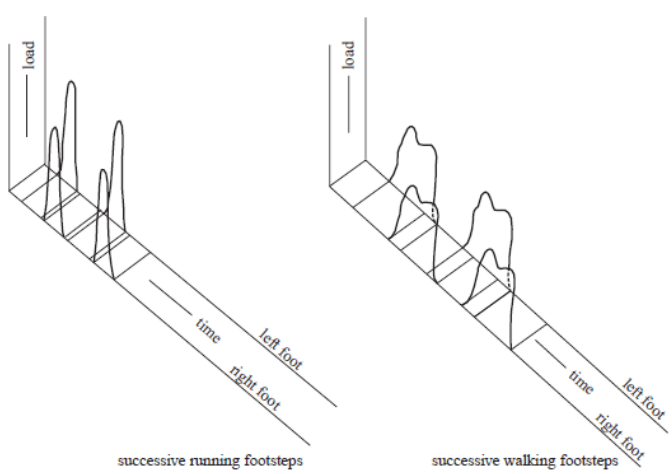

Figure 6. Typical pattern of running and walking forces

A single person walking has been studies for many years and the vertical component has been regarded as the most important of the three forces since it has the highest magnitude. However this conclusion underestimates the effect of lateral force induced by walking. Generally the frequency of the vertical direction force is in the range of 1.4-2.4 Hz and in the range of 0.7-1.2 Hz for lateral direction (half of which is for the vertical direction). For very flexible slender structures, the lateral mode vibration frequency may coincide to the range of frequency of the lateral load and cause resonant (or near resonant) condition.

4.2. Effect of Lateral Loading

Bachmann, Ammann, International Association for, and Structural (1987) reported[1] that the first and third harmonics (at 1 and 3 Hz, respectively) were dominant in the medial–lateral direction, whereas the first and the second harmonic (at 2 and 4 Hz, respectively) were the most significant in the case of longitudinal force. Interestingly, in the lateral case they found that third harmonic for some pedestrians might exceed the fundamental in amplitude.

5. Problem Statement

The question of how to efficiently manage unwanted floor vibration is central. There are plenty of modeling techniques in the area of floor vibration management. The vibration absorber which is a mechanical device often employed to decrease or eliminate unpleasant vibration. For brevity, the analysis of a single degree of freedom mass-damper is assumed to be accessible from structural dynamic textbooks. The basic principle of a vibration absorber is described in this section.

5.1. Modeling Approach

In its simplest form, a vibration absorber consists of one spring and a mass. Such an absorber system is attached to a SDF system, as shown in Figure 1. The equations of motion for the main system m_1, c_1, k_1 and the absorber m_2, c_2, k_2 are:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{u_1} \\ \ddot{u_2} \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{u_1} \\ \dot{u_2} \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} \ddot{u_1} \\ \ddot{u_2} \end{Bmatrix} = \begin{Bmatrix} p(t) \\ 0 \end{Bmatrix}$$

Equation 4

In Equation 4, $m, c \ and \ k$ correspond to mass, damping ratio and the stiffness of the element. Quite a few academic publication (Murray et al., 1997; Ritchey, Setareh, Wicks, & Murray, 2004) have provided various sets of hand calculation methods to characterize the structural properties of a given floor system. This report makes use of these methods where applicable however the verification of these techniques may lie outside the scope of this report. For completeness, the following notations are defined:

[1] This issue was noted in manuscript written by Racic et al. (2009)

$$\omega_i = \sqrt{\frac{k_i}{m_i}} \ ; i = 1,2$$

Equation 5

Where ω_i is the natural angular frequency of the independent system of mass-damper. In addition

$$\zeta_i = \frac{c_i}{2m_i\omega_i}; i = 1,2$$

Where $\zeta_i$is the damping factor of an independent mass-damper system.

Back to the model, it becomes evident that Equation 1 is the representation of the system by means of two coupled differential equations. There are a handful of approaches to characterize the solution of this system and frequency domain analysis is selected in order to shed light on various types of behavior to harmonic excitations[2].

A few observation immediately becomes available, should we neglect the damping on the main mass; that is, $\zeta_1 \approx 0$ where ζ_1 is the damping ratio of the floor system[3]. Transforming the system of equation to frequency domain and solving for $U_1(s)$ and $U_2(s)$ result the following expressions:

$$\begin{cases} [m_1 s^2 + c_2 s + (k_1 + k_2)].U_1(s) - (c_2 s + k_2).U_2(s) = P(s) \\ -(c_2 s + k_2).U_1(s) + [m_2 s^2 + c_2 s + k_2].U_2(s) = 0 \end{cases}$$

Equation 6

Let us go ahead and express $U_2(s)$:

$$\frac{U_1(s)}{P(s)} = \frac{1}{\det(.)}[m_2 s^2 + c_2 s + k_2]$$

Equation 7

$$\frac{U_2(s)}{P(s)} = \frac{-1}{\det(.)}[c_2 s + k_2]$$

Equation 8

[2] The complex form of Fourier series expansion is a reasonable approximation of any arbitrary real periodic function and it has been shown that human walking may indeed be modeled as such.

[3] $\zeta_1 = \frac{c_1}{2m_1\omega_1}$

Where det(.) refers to the characteristic equation of the system. U_1 is floor displacement and subsequently the floor acceleration is $s^2U(s)$ assuming zero initial conditions.

$$\frac{s^2U_1(s)}{P(s)} = \frac{1 \times s^2}{\det(.)}[m_2s^2 + c_2s + k_2]$$

Equation 9

It is now evident that floor acceleration has four zeros. Two of them are at the origin of the s-plane and may be considered trivial. The other two are complex conjugates:

$$s_{1,2} = \zeta_2\omega_2 \pm j\omega_2.(1 - (\zeta_2)^2)^{\left(\frac{1}{2}\right)}; j = \sqrt{-1}$$

Setting $= j\omega$, it becomes apparent that the amplitude of the floor acceleration[4] is zero at $\omega = \omega_2$. This fact suggests that the floor vibration at a particular frequency (i.e. ω_1 often natural frequency of the floor system[5]) may be canceled if the TMD's natural frequency is tuned properly, that is:

$$\omega_1 = \omega_2$$

Equation 10

Consequently, the analysis is advanced by finding the complete solution of the system of coupled differential equations. Setting $s = j\omega$ in Equation 6 and using the generalized excitation introduced in Equation 1 result in capturing the steady state frequency response as:

$$u_1(t) = \overline{U_1}e^{j\omega t} \quad ; \quad u_2(t) = \overline{U_2}e^{j\omega t}$$

Equation 11

It is worth noting that $\overline{U_1}$ and $\overline{U_2}$ are complex quantities therefore they can be represented in terms of phase and amplitude $(e.g. U_{1o}\ , U_{2o})$. The latter is focal in this report. Prior to presenting the response, it is useful to view this problem from a practical perspective. The reason for this underlies the main objective of this report which is finding feasible values for the TMD system. The displacement of the absorber at $\omega = \omega_2$ is :

$$U_{2o} = -\frac{P_o}{k_2}$$

Equation 12

[4] $abs(j\omega) = \omega$

[5] According to (Brownjohn & Middleton, 2008; Chopra, 2012; Murray et al., 1997; M. Setareh et al., 2006)

And the force acting on the absorber mass is:

$$k_2U_{2o} = \omega^2m_2U_{2o} = -P_o$$

Equation 13

According to Chopra (2012), this implies that the absorber system exerts a force equal and opposite to the exciting force. Thus, the size of the absorber stiffness and mass, k2 and m2, depends on the allowable value of U_{2o}. Obviously, a large absorber mass presents a practical problem. Therefore the ratio of the TMD mass to floor mass becomes a notable design parameter.

$$\mu = \frac{m_2}{m_1}$$

Equation 14

Further analysis which will be presented in this report demonstrates that the smaller the mass ratio μ, the narrower will be the operating frequency range of the absorber. From Equation 14, and Equation 5, it is inferred that:

$$\frac{k_2}{k_1} = \mu.\left(\frac{\omega_2}{\omega_1}\right)^2$$

Equation 15

In terms of facilitating the visualization of analysis, the frequency ratio f_r is introduced:

$$f_r = \frac{f_2}{f_1} = \frac{\omega_2}{\omega_1}$$

Equation 16

And finally the excitation frequency is normalized to ω_2 which will facilitate our interpretation, that is

$$\beta = \frac{\omega}{\omega_2} \qquad ; \qquad \beta f_r = \frac{\omega}{\omega_1}$$

Equation 17

A compact equation has been formed for a generalized excitation introduced in Equation 1 and the expression for floor system frequency response is obtained and presented in below:

$$\frac{\overline{U_1}}{(\frac{P_0}{k_1})} = \frac{(1-(\beta)^2 + j2\zeta(\beta))}{[1+\mu(f_r)^2 - (\beta f_r)^2 + j2\zeta\mu(\beta f_r)].[1-(\beta)^2 + j2\zeta(\beta)] - [\mu(f_r)^2 + j2\zeta\mu(f_r)(\beta f_r)]}$$

Equation 18

This is the frequency response of a TMD system (a.k.a Dynamic Amplification FactorR_d) to an arbitrary single tone input excitation and for the rest of this report, it will be used as a basis for our analysis. In order to provide graphical representation of response for various cases, Equation 18 has been programmed into Matlab.

5.2. Characteristics of the Response

It is apparent that R_d is in complex form. As it was discussed earlier, the quantity of interest is the absolute value of R_d. Ideally the TMD is tuned to natural frequency of the floor system and the mass ratio is large enough to provide an effective absorption of energy[6].

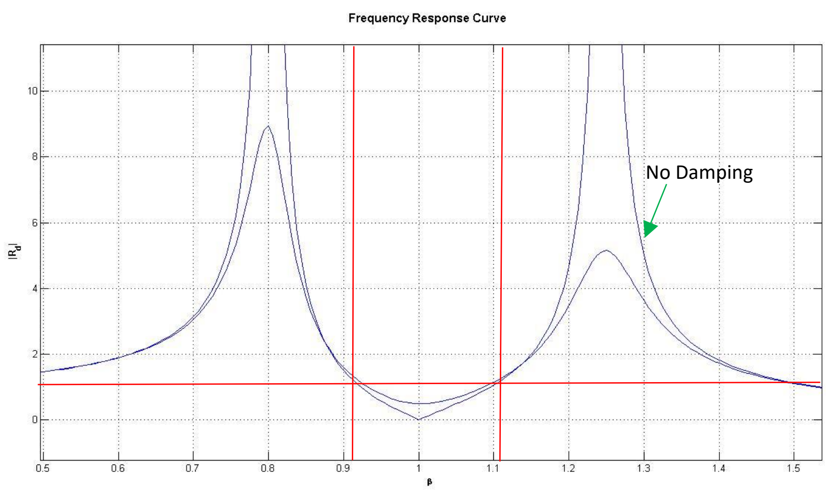

Figure 7. Amplitude response of the TMD for $\zeta = 0.05$ *and* $0, \mu = 0.2, f_r = 1$

The test case presented in Figure 7 is in agreement with the text book[7] and it confirms that the proposed model is valid. The solid red lines define the operating frequency range of the proposed model. The performance of the model may also be viewed in terms of mass ratio, frequency ratio as well as damping factor of the system.

[6] Large mass may be impractical as it was discussed earlier.

[7] Figure 12.2.1; Chopra, 3rd edition

It is of essence to see if the proposed model can also simulate the acceleration response factor. For this reason, the model is updated so that it corresponds to Equation 9 for the same test cases in Figure 7. This is shown in below.

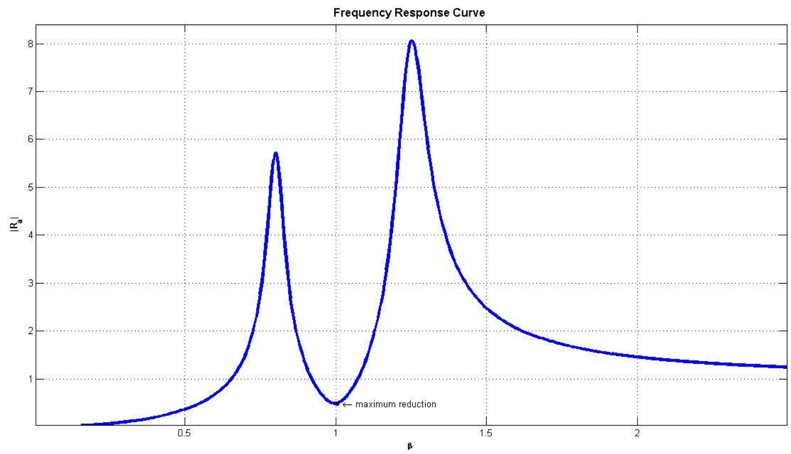

Figure 8. Amplification factor for floor acceleration,

Figure 8 shows slightly over 51% reduction in floor acceleration at the frequency of interest.

5.3. The Effect of Damping ζ

As a matter of fact, damping in this proposed model represents how the TMD mass interconnects with the floor system. To make this more tangible, let us assume a TMD with no damping. For a given mass ration, tuned at floor natural frequency, the overall system response is depicted in Figure 7. Physical interpretation of extremely high damping suggests that the TMD stiffness is dominated and can be ignored. This is identical to a situation that the floor mass is increased to prevent unwanted vibration (Figure 9) and a single degree of freedom adequately describes the behavior.

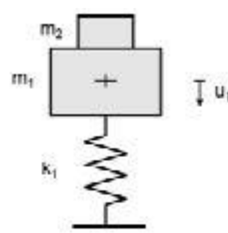

Figure 9. Representation of TMD with very large damping

In order to view the performance of a floor system as an SDOF, ζ in Equation 18 is set to a large value and mass ratio set to a small number which means the floor mass becomes a reasonable approximation of the total system mass.

If damping in the absorber is very large it resists displacement of the absorber mass and therefore prevents the transfer of vibration energy into the absorber, i.e., the absorber again becomes ineffective, as shown in Figure 10. To be fully effective the absorber should have an optimum damping ratio.

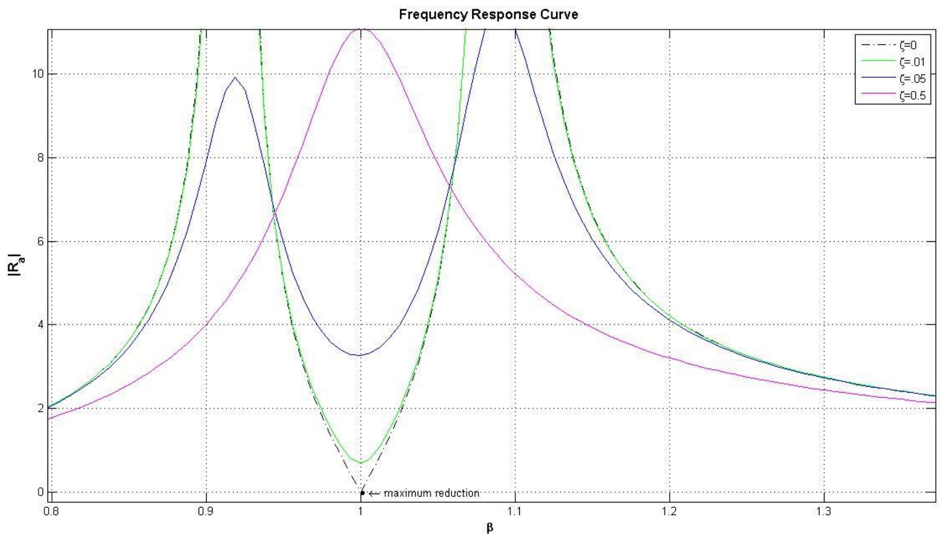

Figure 10. The variation of $|R_a|$ as ζ in increased

A series of numerical analysis were conducted to gain more insight about damping. It was noticed that this parameter introduces a trade-off or, in other words, an optimized value should exist.

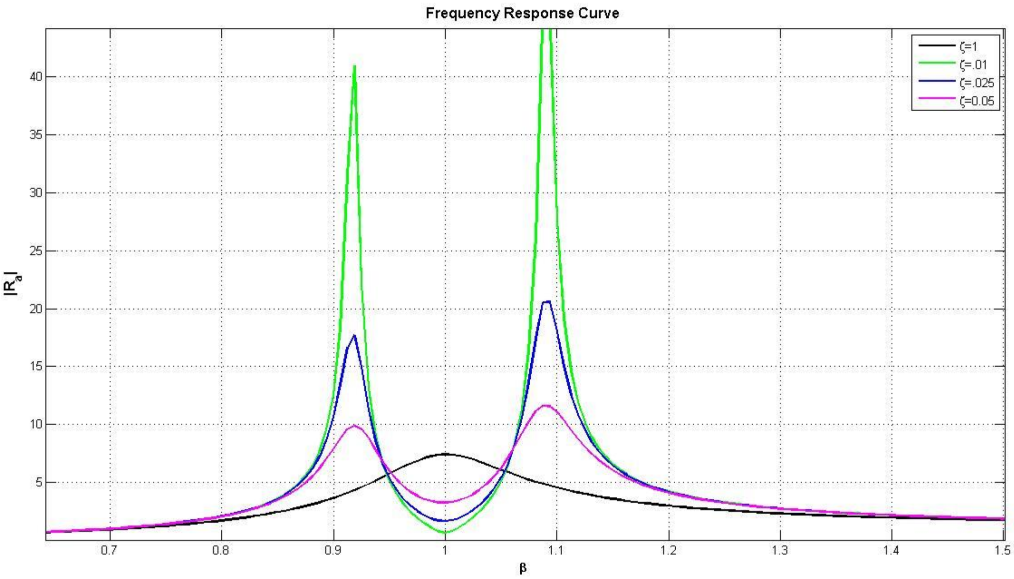

Figure 11. Amplification for $\zeta = 1, 0.01, 0.025, 0.05$ and $\mu = 0.03$ and $f_r = 1$

It is noted in Figure 11, once the damping is 1, no reduction is gained at $\beta = 1$. This observation is in agreement with what Allen and Pernica (1984) reports. When damping in the absorber is optimum, the effective damping ratio of the floor structure, ζ_1, is essentially determined by the number of cycles required to transfer vibration energy from the floor into the absorber ($n = \frac{1}{2}\sqrt{\frac{m_1}{m_2}}$). Based on this fact, he goes ahead and provides an estimate for the optimized value of damping:

$$\zeta = \zeta_2 \approx \frac{\ln 5}{\pi}\sqrt{\frac{m_2}{m_1}} \approx \frac{\sqrt{\mu}}{2}$$

Equation 19

Human-structure interaction is another factor which it turns out to impact the response characteristics. M. Setareh et al. (2006) have revealed that the introduction of human subjects in sitting and standing positions increased the floor natural frequency in addition to increasing the floor damping. A classic sensitivity analysis may be performed in order to simulate the effect of damping however it is important to consider the variation of all parameters simultaneously in order to achieve a realistic result.

5.4. The Effect of Frequency Ratio f_r

A successful TMD design must be effective as well as efficient. The effectiveness of TMD is, in a nut shell, the reduction factor which is the primary design objective. On the other hand, it also needs to be practical in terms of the value of f_r which is ideally unity. Therefore the question becomes to what degree the model is sensitive to an off-tuned TMD (i.e. $\omega_2 \neq \omega_1$).

Let us consider two damping ratios for this analysis $\zeta = 0.01\ and\ \zeta = .08$; also consider the mass ratio is 0.03. The acceleration response for two cases for various off-tuned TMD:

$$f_r = [1\ .8\ .85\ .95\ 1.1]$$

The following figures depict the results:

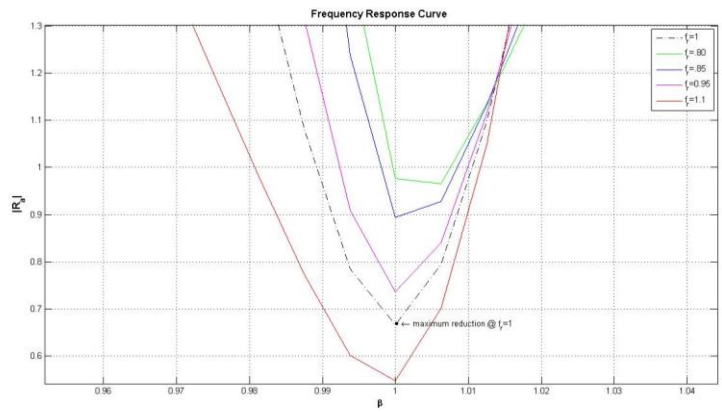

Figure 12. $\boldsymbol{\zeta = 0.01}$

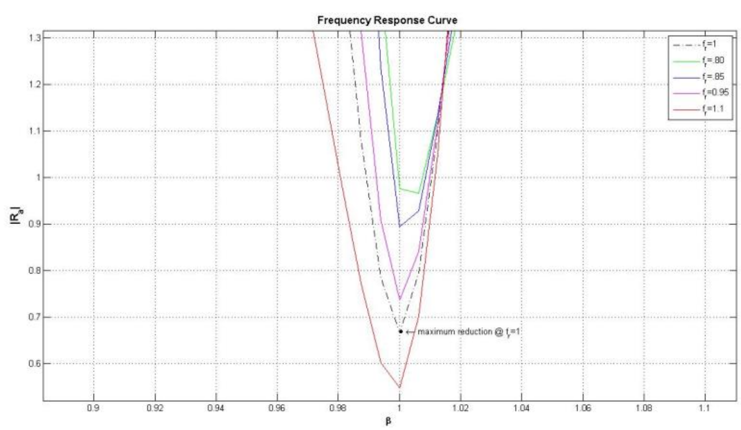

Figure 13. $\boldsymbol{\zeta = 0.08}$

It is observed that the TMD becomes less efficient as the frequency ratio is decreased. The value of f_r explains how precisely TMD is tuned to unwanted floor natural frequency which introduces two challenging task. One is the validity of estimation for floor natural frequency, that is, the structural property of the floor system and the other is the accurate implementation of TMD in terms of connections, component selection and last but not least the characteristics of input excitation.

5.5. The effect of mass ratio μ

The main source of off-tuning is variations in the floor mass. This problem is mainly due to the fact that the weight on the floor changes with variation in the live loading overtime. Only the off-tuning due to variations in the floor mass are present here. Studies of the effects of other sources of off-tuning on the performance of the TMD can be found in Ritchey et al. (2004). When the floor mass is reduced, the amount of off-tuning, i.e., the increase in the floor response, is less than when the floor mass is increased. This is due to the fact that a decrease in the floor mass results in an increase in the mass ratio, and therefore the TMD has more effect on attenuation of the floor vibration.

Mass ration also defines the practicality of the TMD. The more mass ratio means the heavier TMD and consequently this might become a practical downside while the performance is increased. A plenty of research have been conducted in order to identify an optimum mass ratio for a given floor system.

6. Case Studies

The closing section attempts to study the vibration of a given structural system in response to various loading. A number of tests were performed to evaluate the dynamic properties of the floor (Ritchey et al., 2004) and for the purpose this report, the structural properties of their test floor is employed (Table 2) for a geometrically simplified version of the structural system (Figure 15)

$\boldsymbol{m_1}$	$\boldsymbol{40\ lb\frac{s^2}{in}}$
$\boldsymbol{k_1}$	$\boldsymbol{27500\frac{lb}{in}}$
$\boldsymbol{m_2}$	$\boldsymbol{2\ lb-\frac{s^2}{in}}$
$\boldsymbol{f_r}$	$\boldsymbol{0.985}$
$\boldsymbol{\mu=\frac{m_2}{m_1}}$	$\boldsymbol{0.05}$
$\boldsymbol{f_1=\frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}}}$	**4.173078 Hz**

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m_2}}$$ **4.236627 Hz**

Table 2. Structural properties of the floor system and a given TMD characterized by mass and damping factor

The proposed model assumes that the floor system damping is negligible therefore the value of c_1 in Table 2 remains unused. Here is the expected floor response:

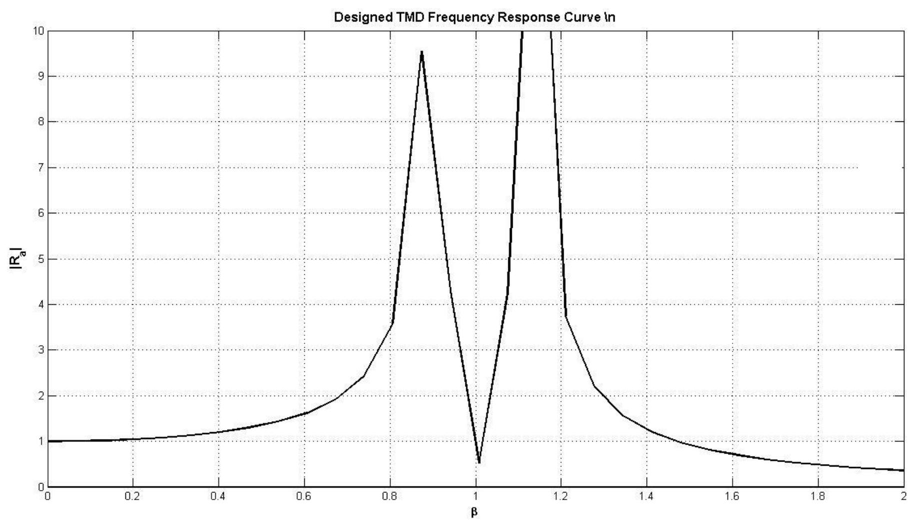

Figure 14. Floor frequency response according to design parameters in Table 2

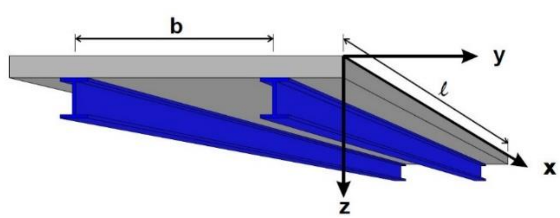

Figure 15. Graphical representation of floor vibration problem case #1 and #2

It is also worth mentioning that the time history of applied excitations are modeled at 200Hz sampling rate which is equivalent to the time interval of 4ms. As it was discussed under section 5.0, an arbitrary input excitation is expanded in terms of its Fourier series complex coefficients in order to represent the right side of Equation 12. In fact, the complex coefficients of Fourier series are related to Discrete Time

Fourier Transform of the signal. Therefore the The complex-valued coefficient Pj defines the amplitude and phase of the j^{th} harmonic. Appendix B contains the Matlab code for this endeavor.

6.1. Case Study #1

First case study demonstrates the response of the floor-TMD to a pulse excitation with 500ms duration. This is a rough approximation of human heel drop who has a weight of 160 lb.

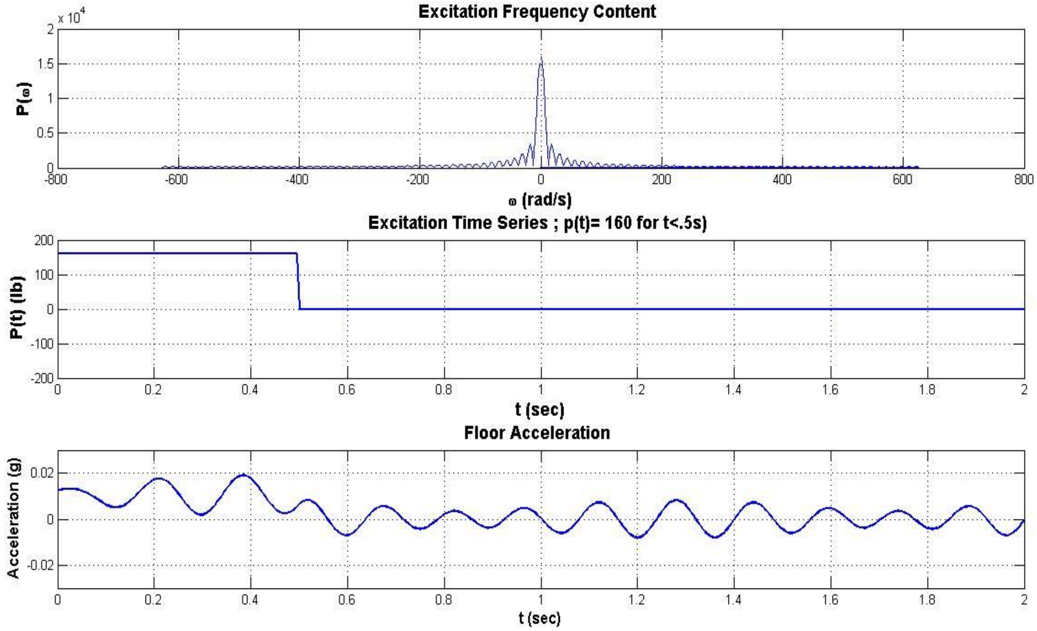

Figure 16. Simulation of human heel drop and corresponding floor response

6.2. Case Study #2

The following parameters are selected and the response of the structure to a harmonic excitation is evaluated:

$$p(t) = 160 \times \sin\ (10\pi t)\ lb$$

Equation 20

The following results indicate that the floor acceleration remains within 0.05g. This level of acceleration is relatively acceptable for aerobic activities and or outdoor footbridges.

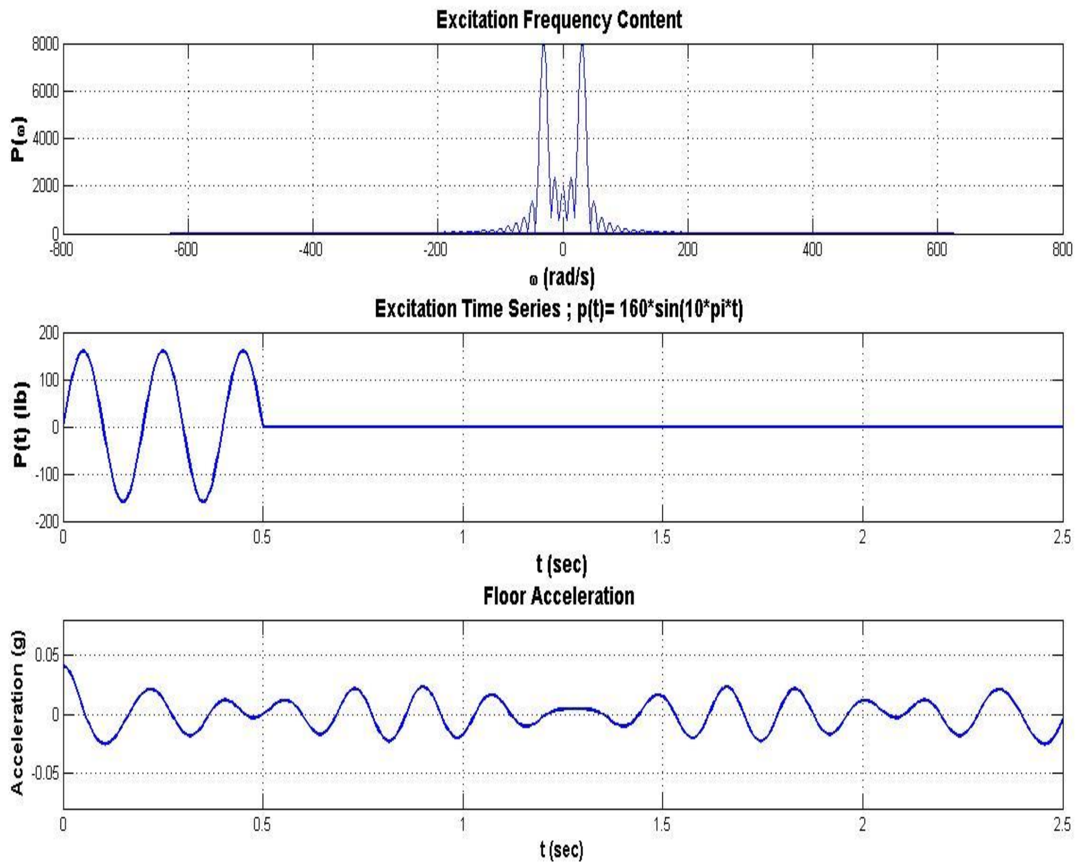

Figure 17. The response of the floor is acceptable for aerobic activities according to recommendation in Figure 4

7. Conclusion

Over 18 research have been reviewed in the area of floor vibration and it was found that TMD is an effective concept in order to reduce unwanted floor vibration. A simplified MDOF model was proposed and the model parameters were analytically obtained. It was shown that the model is valid based on available test cases. An two case studies were applied to the proposed model.

Despite the effectiveness of the model, it is now evident that more work may be required in order comprehensively understand and manage performance of a given floor system. It is of essence to consider the following opportunities for future works:

- To precisely estimate floor structural properties

- To investigate and implement more elaborated models such as Pendulum TMD (PTMD) (M. Setareh et al., 2006) , Skyhook TMD (STMD), Ground Hook TMD (GHTMD) , Semiactive TMD (SATMD) (M. Setareh et al., 2007).
- To apply more realistic methodologies such as Finite Element Modeling
- To apply optimization techniques such Genetic Algorithm in order to solve for an optimum set of mass ratio, damping and frequency ratio.

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