Drop Encapsulation

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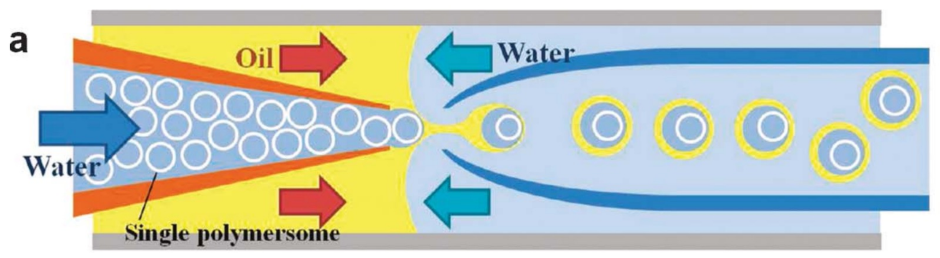
Background and Motivation

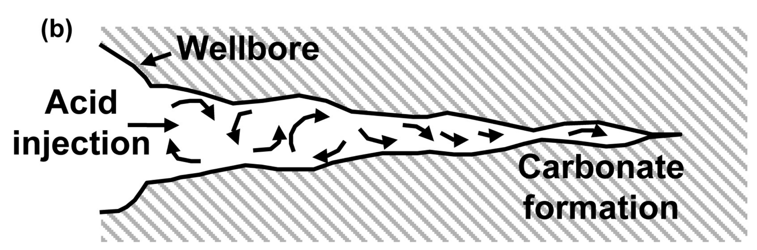
In Microscale, surface tension is a dominate force, allowing to encapsulate drops in another fluid which has numerous application in microfluidics devices.



Micro drop Encapsulation [1]

Is it feasible to encapsulate a macro size drop?

This opens up possibilities for application in large scale industries (e.g. **petroleum**, **food**, **personal care** and **drug industries**)



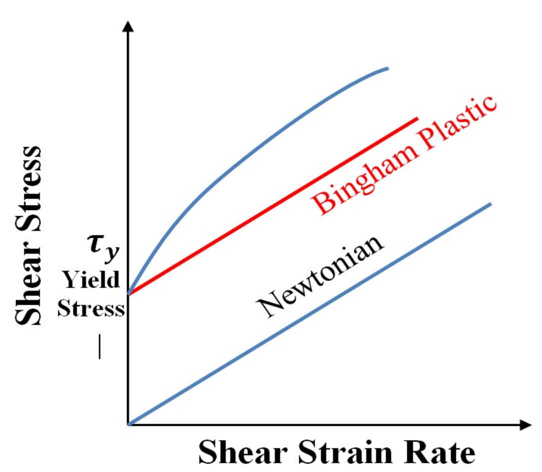
Acid Fracturing, a potential application of macro drop encapsulation

It is observed that Stable multi-layer flow can be achieved using a yield stress fluid [2].

Hypothesis: **To encapsulate a Macroscale drop in a yield stress fluid (visco-plastic fluid).**

Visco-plastic fluids (fluids with yield stress)

Examples of visco-plastic fluids

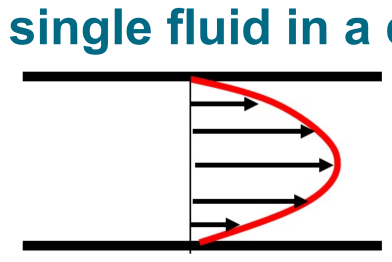


$$\tau_{ij} = \left(\mu + \frac{B}{\dot{\gamma}}\right)\dot{\gamma}_{ij} \iff \tau > B$$

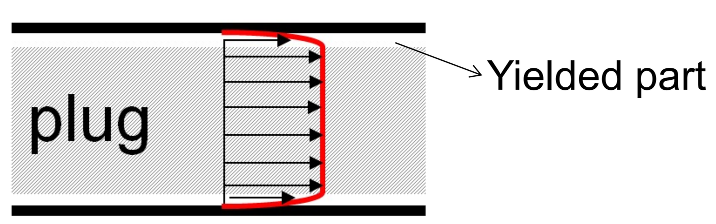
$$\dot{\gamma} = 0 \iff \tau < B$$

$$B = \frac{\hat{\tau}_y \hat{D}}{\hat{\mu}\hat{U}}$$

Flow of a single fluid in a duct



Newtonian Fluid

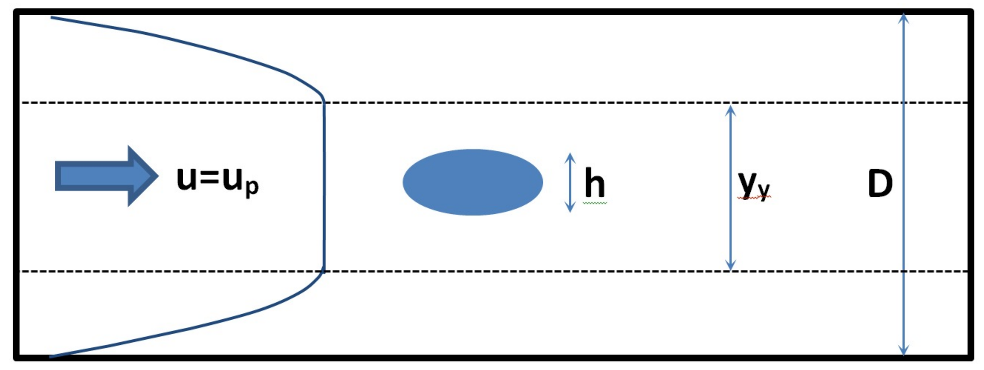


Bingham Fluid

Question 1: What is the largest size of encapsulated drop?

Question 2: Where is the position of true plug?

Schematic sketch of the problem



Theoretical approach (perturbation method)

Flow inside the drop

- Assumption: Low Reynolds number (**Stokes Flow** $\nabla^4\psi = 0$)
- Boundary Conditions: Continuity of velocity, normal and tangential stresses at the interface
- General from of Solution
  - $\psi(r,\theta) = F(r)G(\theta)$
  - $F^{(4)} - 4F''' + 2(2-n^2)F'' + (3+4n^2)F' + n^2\,(n^2-4)F = 0$
  - $G(\theta) = \begin{cases}\sin n\theta \\ \cos n\theta\end{cases}$
- The solution to this biharmonic problem is $\psi = 0$

Flow outside the drop

- Assumption
  - Lubrication approximation
  - Slender drop
- Asymptotic analysis
  - $u = u_0 + \epsilon\, u_1 + \cdots$
  - $v = v_0 + \epsilon\, v_1 + \cdots$
  - $p = p_0 + \epsilon\, p_1 + \cdots$
- 0-order solution

$$u_0 = \frac{B}{2(\,y_y - h)}\left(\left(1-y_y\right)^2 - \left(y-y_y\right)^2\right)$$

$$u_{p0} = \frac{B}{2(\,y_y - h)}\left(1-y_y\right)^2$$

$$y_y^3 - 3hy_y^2 + y_y\left(6h - 3 - \frac{6}{B}\right) + 2 + 6\frac{h}{B} - 3h = 0$$

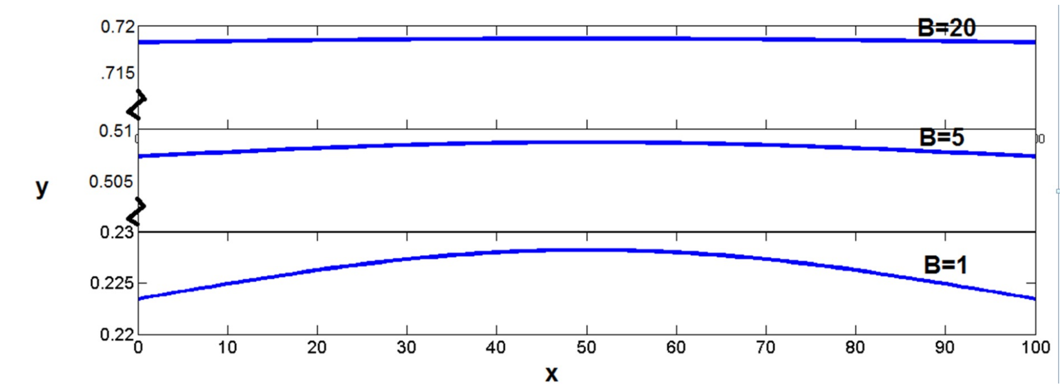
$\boldsymbol{u_{p0} = u_{p0}(x)}$ $\Longrightarrow$ **Lubrication Paradox !**

- Higher order corrections to determine the position of true plug
- ϵ-order solution
  - $u_1(x,y) = \frac{\eta}{(y_y-1)^2}\left(3y^2 - y\left(2y_y+4\right) + 2y_y + 1\right)$
  - $\eta = \frac{u_p(x;h=0) - u_p\left(x,y_y;h=h(x)\right)}{\delta}$
  - $y_T = y_y + O(\epsilon)$ (y_T is the position of true plug)
  - $y_T = y_y + 2\left(\frac{\left(1-y_{y;h=0}\right)^2}{y_{y;h=0}} - \frac{\left(1-y_y\right)^2}{y_y}\right)\left(\frac{y_y-h}{y_y-1}\right)$

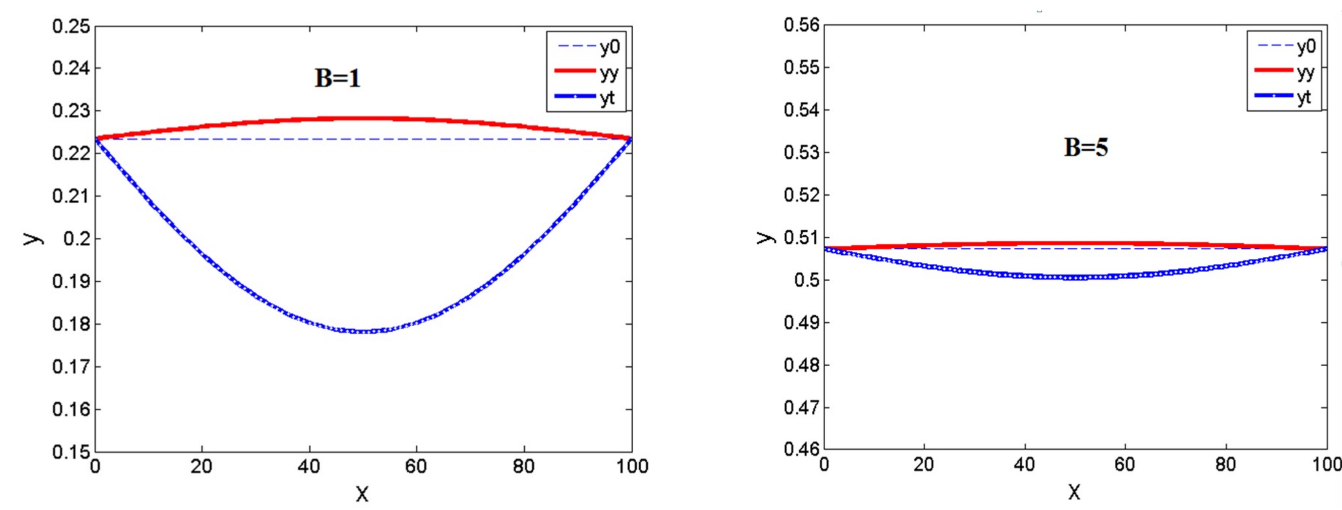
Position of true plug is determined.

Results

- Position of yielded and true plug interface



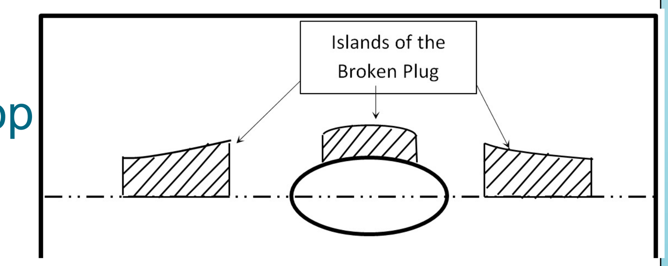
Yielded plug interface (0-order approximation)



Composite asymptotic solution compared to 0-order approximation

Future Work

- **Theory**
  - Islands of a broken plug
  - Solve for a Macroscale drop
- **Computation:**
  - Rheolef Finite Element Package
  - Adaptive Mesh
  - Augmented Lagrangian method handle the exact viso-plastic model



A broken plug

Preliminary computation results for drop encapsulation [3]

References

[1] Duncanson et al. “Microfluidic synthesis of advanced microparticles for encapsulation and controlled release” Lab Chip, 2012, 12(12):2135-45.

[2] Hormozi et al. “Extending the visco-plastic lubrication concept to near net shape products and encapsulation”, Annual transaction of the Nordic rheology society, Vol 20, 2012

[3] S. Hormozi & I. A. Frigaard (2012) Nonlinear stability of a visco-plastically lubricated viscoelastic fluid flow. Journal of Non-Newtonian Fluid Mechanics. 160-170, 61-73.