Modular Spectrum Utilization for Next-Generation Fixed Transmission Networks

by

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Abstract

Internet application and mobile service demands have continuously driven the need for increased data rates throughout global communication networks. One bottleneck of global network capacity is the backhaul network, which is responsible for connecting intermediate links throughout the core network. The backhaul network is commonly established either via high capacity optical fibre lines or low cost fixed microwave wireless antennas. Due to their low cost, microwave links are currently used for over 50% of the world’s backhaul networks, as opposed to optical fibre connections. However, these links must follow strict regulation on the wireless bandwidths and pulse shapes they are allowed to occupy.

The goal of this thesis is to improve the spectral efficiency within a transmission channel’s regulated pulse shapes. The spectrum regions we intend to utilize are the sidebands outside the primary pulse, or spectrum “skirts”. While these skirts are commonly intended for adjacent channel usage, it has been shown that they typically go unused. We investigate two modular methods to take advantage of these unused sidebands, a superposition and multi-carrier approach. While the first method overlaps a primary and secondary pulse in the time domain, the other places the pulses adjacent to one another in the frequency domain.

The superposition method must deal with the interference caused by overlapping pulse shapes. Simulations show that the primary stream cancellation is essential to this system’s performance. The multi-carrier method requires custom pulse shapes that fit under the spectrum mask to match performance. Simulation results demonstrate that without phase noise impairments, the superposition
scheme yields the highest performance. However when transmitting with phase noise, the multi-carrier scheme is predicted to transmit the most data under practical channel conditions.
Lay Summary

Improvements to wireless microwave transmission technology have become necessary to meet the modern demands for data transmission throughout global communication networks. While microwave spectrum usage is highly regulated, one overlooked feature for potential data rate improvement is the spectrum “skirts”. These skirts can be understood as side regions within the frequency domain that are intended for adjacent communication links. However, it has been found that these adjacent links typically go unused. This thesis proposes several modular schemes in order to take advantage of these skirts, while ensuring that common spectrum regulations are not violated. The superposition scheme is the main focus of this thesis, but an additional multi-carrier approach is also evaluated. These schemes are finally validated and compared to show overall communication system improvements.
Preface

This dissertation is original, unpublished, independent work by the author, J. Naterer.
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<td>AIR</td>
<td>Achievable Information Rate</td>
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<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BCJR</td>
<td>Bahl, Cocke, Jelinek and Raviv</td>
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<tr>
<td>BICM</td>
<td>Bit-Interleaved Coded Modulation</td>
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<tr>
<td>CAGR</td>
<td>Compound Annual Growth Rate</td>
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<td>C-RAN</td>
<td>Centralized Radio Access Networks</td>
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<tr>
<td>DAC</td>
<td>Digital-to-Analog Converter</td>
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<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
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<td>DP</td>
<td>Dual Polarization</td>
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<td>DVB</td>
<td>Digital Video Broadcasting</td>
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<tr>
<td>ETSI</td>
<td>European Telecommunications Standards Institute</td>
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<tr>
<td>FBMC</td>
<td>Filter Bank Multi-Carrier</td>
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<tr>
<td>FEC</td>
<td>Forward Error Correction</td>
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<td>FTN</td>
<td>Faster-than-Nyquist</td>
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<td>GMI</td>
<td>Generalized Mutual Information</td>
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<td>Abbreviation</td>
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<td>--------------</td>
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<tr>
<td>IC</td>
<td>Interference Cancellation</td>
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<tr>
<td>IID</td>
<td>Independent and Identically Distributed</td>
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<tr>
<td>IoT</td>
<td>Internet of Things</td>
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<tr>
<td>IP</td>
<td>Internet Protocol</td>
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<tr>
<td>ISI</td>
<td>Inter-Symbol Interference</td>
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<tr>
<td>LDPC</td>
<td>Low-Density Parity Check</td>
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<tr>
<td>LLR</td>
<td>log likelihood ratios</td>
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<tr>
<td>MAP</td>
<td>Maximum A Posterior Probability</td>
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<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
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<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiple Access</td>
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<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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<tr>
<td>RAN</td>
<td>Radio Access Network</td>
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<tr>
<td>RC</td>
<td>Raised Cosine</td>
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<tr>
<td>RRC</td>
<td>Raised-Root Cosine</td>
<td></td>
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<tr>
<td>SIC</td>
<td>Successive Interference Cancellation</td>
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<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
<td></td>
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<td>XPI</td>
<td>Cross-Polarization Interference</td>
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Chapter 1

Introduction

1.1 Overview
The insistent rise of Internet application and mobile service demand continuously increases the data requirements for global communication networks. Furthermore, the fifth generation of wireless communications places greater emphasis on data requirements for use-cases such as ultra reliable low latency communications in stock trading, and massive machine type communications in consumer Internet of Things (IoT) devices. Referencing quantitative predictions, Cisco’s most recent Visual Networking Index Forecast predicts that global fixed broadband speeds will double and mobile cellular speeds will triple between 2018 and 2023 [1]. Global machine-to-machine connections are estimated to grow from 1.2 billion to 4.4 billion at a Compound Annual Growth Rate (CAGR) of 30% within the same time period, and Global Data Center Networking is expected to grow at a CAGR of 12%. These speeds along with other network demands are predicted to result in a CAGR of 26% for global Internet Protocol (IP) traffic.

Gradually over the years, the bottleneck of Radio Access Network (RAN) capacity has shifted from the radio interface to the backhaul network. This backhaul network is responsible for connecting intermediate links throughout the core network, or more specifically, connecting base stations with their associated mo-
bile switching nodes. One paradigm shift that has assisted the advancement of backhaul network capacity is the Centralized Radio Access Networks (C-RAN) architecture, which shifts many base station responsibilities to a centralized cloud location, connected via a fronthaul network. This change combats the densification of base stations in small-cell backhaul networks and improves efficiency by pooling baseband resources.

Backhaul links are commonly established either by high capacity optical fibre lines, or low cost fixed microwave wireless antennas. While optical fibre can achieve significantly higher data rates and larger capacities, it is typically associated with a much higher cost. However, a study in 2014 observes that optical fibre backhaul is not available nationwide in Europe and current microwave networks cannot sustain the traffic growth of LTE and LTE-A [2], signifying the importance of improving current microwave technologies.

Traditionally, these microwave links have used carrier frequencies between 6-42GHz, while millimetre wave research is pushing towards the V-Band (57 - 66 GHz) and the E-Band (71 - 76 & 81 - 86 GHz) to allow larger bandwidths and capacity. These higher frequencies along with additional technologies such as antenna polarization, link aggregation, and adaptive coding & modulation drive down operating costs for providers while increasing flexibility and efficiency.

1.2 Problem Statement

The goal of this project is to improve the spectral efficiency within a single link microwave transmission channel. The proposed changes require no major architectural changes and have the ability to increase capacity without leasing additional frequency bands. While the target application is towards microwave links, the techniques discussed are equally applicable to any communication channel with a defined spectrum limit. Throughout this work we use the limits defined by the European Telecommunications Standards Institute (ETSI) as our primary spectral constraints [3].

Spectrum limit standards are typically enforced to ensure that adjacent chan-
nels do not interfere with one another. However, it is observed in many microwave links that adjacent channels at high carrier frequencies often go unused throughout the year. This is due to the large number of channels still available for licensing or purchase at high frequency bands. Therefore, this thesis intends to utilize the spectrum “skirt” regions while still conforming to the spectrum limits defined by ETSI. We also assume that transmission is maintained at high Signal-to-Noise Ratio (SNR) values, as experienced by fixed microwave links for the majority of time each year.

We introduce two transmission architectures to take advantage of the side skirts, a superposition and multi-carrier approach. While the first method overlaps a strong primary and weaker secondary pulse, the second method places two weaker pulses adjacent to the strong primary pulse while remaining orthogonal in the frequency domain. The superposition method must deal with the interference caused by overlapping pulse shapes while the multi-carrier method requires custom pulse shapes to fit under the spectrum mask. Transmission impairments such as a dispersive channel and phase noise are also accounted for as experienced in practical microwave link settings. The final pulse shapes selected for comparison are illustrated in Fig. 1.1. It can be seen that the superposition scheme overlaps a weaker and wider pulse below the spectrum skirts, while the multi-carrier scheme adds a custom-shaped pulse that does not interfere with the primary pulse shape.

1.3 Thesis Contributions

Our main contributions throughout this work is the development of the superposition approach and the addition of dual-polarization to the multi-carrier approach. Typically, the transmission of data over a microwave link requires an extensive amount of experimental data to acquire reliable performance results. Therefore, we model and simulate a microwave system via analytic relations that have been verified in practical scenarios.

We implement a superposition microwave link that can reconstruct and subtract a primary stream in order to detect a secondary stream under ideal channel
Figure 1.1: Spectrum skirt pulse shapes for 25.6 MHz bandwidth RRC, superposition, and multi-carrier transmission schemes.

conditions. We notice that the reconstruction accuracy is essential to this system’s performance and must be as precise as possible in order to achieve maximum data rates. Next, we add phase noise and dispersive channel impairments under the assumption that the receiver has full knowledge of the channel conditions. Once adding phase noise estimators and channel equalizers to relax these constraints, we introduce additional techniques to reduce equalization overhead and improve phase noise estimates.

After tuning superposition transmission parameters to reach the scheme’s maximum potential, we compare it to the multi-carrier approach and notice that it is outperformed under practical channel conditions. We deduce that this is due to the interference caused by overlapping signals, and how the multi-carrier approach is more effective at utilizing the spectrum skirts with minimal interference. Once
having the ability to commit to one approach, we implement dual-polarization with linear equalizers. The linear equalizer’s dual convolution operation does require breaking each transmission packet into smaller chunks to fit into memory, but eventually this addition nearly doubles data rates. We believe that this multi-carrier approach is the most effective tool at utilizing spectrum skirts to improve spectral efficiency.

1.4 Thesis Outline

This thesis is organized as follows. In Chapter 2 we introduce the system model of a general point to point microwave communication link and channel impairments such as dispersive fading and phase noise. In Chapter 3 we discuss the superposition transmission model and several novel improvements to improve channel estimation and phase noise compensation. In Chapter 4 we discuss the multi-carrier transmission model and introduce dual-polarization to significantly improve the maximum achievable data rate. Chapter 5 evaluates the results of various numerical simulations in order to compare the two modular spectrum utilization schemes. Finally, Chapter 6 draws conclusions from the results and suggests potential future work regarding the outcomes of this thesis.
Chapter 2

System Model

In order to effectively discuss spectrum utilization methods, this chapter outlines the overall microwave link transmission model used throughout the thesis. We begin by introducing a baseband transmission model with QAM modulation and general pulse shape filters to fit within our spectral mask. We then introduce channel coding in order to improve system performance. Next, we discuss a dispersive Rummler channel impairment, and how linear Minimum Mean Square Error (MMSE) equalization is required to reduce the effect of inter-symbol-interference. Finally, phase noise is added to our overall channel model and handled via a discretized sequence estimator.

2.1 Overview

The purpose of a digital communication system is to transmit information over an analog channel between a transmitter and a receiver [4]. This information signal has the characteristics of a random process in order to convey information most efficiently. While these signals are always transmitted over an analog channel, we typically analyze the effects of impairments in the digital domain.

The most common transmission model is the Additive White Gaussian Noise (AWGN) channel. It contains an additive noise whose power distribution is uni-
form across the frequency band of the communication system. The noise samples follow a Gaussian normal distribution with an average value of zero. The continuous-time representation of this channel is expressed as

\[ r(t) = s(t) + n(t), \]  

where \( s(t) \) is the initial signal transmitted at power \( P_s \), and \( n(t) \) denotes the AWGN with a constant power spectral density across all frequencies.

The process of converting data between a digital and analog domain is called modulation and demodulation respectively. The modulation schemes are associated with alphabets, where a specific number of digital bits represents a specific symbol in the alphabet’s finite signal space. In order to simplify probabilistic relations, each symbol in the modulation alphabet is considered equally likely to be transmitted and received.

We use filters to transmit the data over a finite frequency range, allowing us to transmit multiple signals simultaneously. Using a finite frequency range allows us to transmit multiple signals simultaneously. This frequency range is also referred to as the bandwidth and is equivalent to the symbol rate of the transmission system. While transmitting each symbol over a finite bandwidth causes the signal to theoretically occupy an infinite space in time, a slight tolerance in the form of spectrum skirts are commonly allowed which cause minimal interference to the adjacent symbols sent. In order to receive the data, the signals are filtered to consider the frequency band of interest, and then sampled at constant time intervals.

The most common transmitter and receiver pulse shape is the Raised-Root Cosine (RRC) pulse, which is easy to implement with electronics and fits well under the required spectral mask constraints. Since the two pulse shapes are identical, they are referred to as matched filters, an optimal solution to receiving data over an AWGN channel. A signal that passes through both of these filters encounters an Raised Cosine (RC) pulse. This RC pulse follows the Nyquist Inter-Symbol Interference (ISI) criterion which ensures that at each sampling instance of the received signal, the time shifted adjacent signals do not interfere.
The block diagram to illustrate the signal chain is displayed below in Fig. 2.1. At the transmitter end, the binary data is first passed to a channel encoder to improve error performance, then mapped to a signal constellation in preparation for digital to analog conversion. The newly formed symbols then pass through a digital RRC pulse shaping filter at an oversampled rate, then are converted into an analog signal by the Digital-to-Analog Converter (DAC). Finally, the analog signal is upconverted to the carrier frequency and transmitted over the intended channel. At the receiver end, the process unfolds mostly in reverse order. However, instead of the pulse shaping filter, an RRC matched filter is employed to maintain the ISI criterion. Additionally, linear equalization and phase noise tracking is employed to handle the impairments that are discussed in more detail in the following sections.

2.2 QAM Modulation

Quadrature Amplitude Modulation (QAM) is the most common modulation pattern choice for mapping binary data to a finite symbol space. This modulation format simultaneously transmits two carriers known as the in-phase and quadrature components respectively. These two parallel streams are each associated with an amplitude and can be visualized together as real and imaginary components in a constellation diagram. When mapping the binary data to symbols, a Gray code
is commonly used to ensure that neighbouring constellations only differ by one bit. Various symbol mappings are illustrated in Fig. 2.2 as a square or symmetric cross shape depending on the bit order.

The in-phase and quadrature baseband components can be expressed as

\[ x_I(t) = \sum_n a_i[n] p(t - nT) \]
\[ x_Q(t) = \sum_n a_q[n] p(t - nT), \]

where \( T \) denotes a sampling time interval, and \( p(t) \) denotes our pulse shape which satisfies the Nyquist ISI criterion. The terms \( a_i \) and \( a_q \) represent the discrete-time amplitudes of the in-phase and quadrature components.

At the transmitter side, the overall QAM passband signal is expressed as

\[ s(t) = x_I(t) \cos(2\pi f_c t) - x_Q(t) \sin(2\pi f_c t), \]

where \( f_c \) denotes the carrier frequency, and \( \cos(2\pi f_c t) \) and \( \sin(2\pi f_c t) \) denote the orthogonal carriers which allow simultaneous transmission of two data streams.

At the receiver end, the overall QAM baseband signal is expressed as

\[ x_I(t) = \text{LPF}[\cos(2\pi f_c t) s(t)] \]
\[ x_Q(t) = \text{LPF}[\sin(2\pi f_c t)s(t)], \] 

where LPF represents our low-pass matched filter associated with \( p(t) \).

An example of our simulation matching analytical results is illustrated in Fig. 2.3 with the probability of symbol error \([4]\), where \( m \) denotes the set of \( M = 2^m \) possible symbol mappings. The x-axis sweep parameter is the SNR, representing the ratio between the transition power and discrete-time AWGN variance.

### 2.3 Achievable Information Rate

In addition to uncoded SER curves, another metric which takes into account the channel capacity can be used to accurately compare operation parameters. The
channel capacity is difficult to measure, but is effectively expressed by the maximum net rate at which information can be reliably transmitted, or an Achievable Information Rate (AIR). The Generalized Mutual Information (GMI) estimates this AIR for bit-wise decoders by using the statistical distributions, or log-likelihoods, of each received sample [5,6]. It has been shown that the GMI is a more reliable measure than the uncoded SER when applying channel coding to a communication system [7]. While the GMI is measured in bits per channel use, it can give an estimate of the overall data rate in bits/s by using the transmission system’s baud rate.

In order to calculate the GMI, we estimate the symbol-wise mutual information between each transmitted and received symbol. The derivation assumes a memoryless channel with a uniform discrete input distribution, such as a QAM constellation. The average mutual information between the transmitted and received symbols is expressed as

$$I(X;Y) = \mathbb{E}_{XY} \left[ \log_2 \frac{p_{Y|X}(Y|X)}{p_Y(Y)} \right],$$

(2.7)

where $X$ is the random variable denoting the input element belonging to its respective QAM constellation with cardinality $M = 2^m$, while $Y$ is the random variable denoting the channel output element. The probability $p_{Y|X}$ denotes the conditional memoryless channel transition distribution while the probability $p_Y$ denotes the channel output density.

By utilizing the weak law of large numbers, the following expression converges to the actual mutual information via Monte-Carlo simulations:

$$\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \log_2 \frac{p_{Y|X}(y_i|x_i)}{p_Y(y_i)} \to I(X;Y).$$

(2.8)

While the transition probabilities can take on any distribution, we use an AWGN channel where $p_{Y|X}(y|x_i) \sim \mathcal{N}(x_i, \sigma^2_N)$. The channel is assumed to be memoryless and the input variables are generated from a uniform distribution.
We also assume a bit-wise channel where dependencies between bits within the same symbol are neglected, known as Bit-Interleaved Coded Modulation (BICM). The GMI is then estimated according to [8, Eq. (6)] as

$$R_{\text{GMI}} = m - \frac{1}{N} \sum_{k=1}^{m} \sum_{i=1}^{N} \log_2 \left( 1 + \exp \left( (-1)^{b_{k,i}} \text{LLR}_{k,i} \right) \right). \quad (2.9)$$

The log likelihood ratios (LLR) are given by

$$\text{LLR}_{k,i} = \log_2 \frac{\sum_{x \in \mathcal{X}_1^k} \exp \left( -\frac{|y_i - x|^2}{\sigma_N^2} \right)}{\sum_{x \in \mathcal{X}_0^k} \exp \left( -\frac{|y_i - x|^2}{\sigma_N^2} \right)}, \quad (2.10)$$

where $\mathcal{X}_1^k$ is the set of constellation points where the $k$th bit is 1, and $\mathcal{X}_0^k$ is the set of constellation points where the $k$th bit is 0.

and can be calculated via the max-log approximation

$$\text{LLR}_{k,i} = \log_2 \frac{\max_{x \in \mathcal{X}_1^k} \exp \left( -\frac{|y_i - x|^2}{\sigma_N^2} \right)}{\max_{x \in \mathcal{X}_0^k} \exp \left( -\frac{|y_i - x|^2}{\sigma_N^2} \right)}. \quad (2.11)$$

Returning to the simulations in Fig. 2.3, the AIR for BICM over an AWGN channel are illustrated in Fig. 2.4. As expected, the AIR reaches increasingly higher maximums at higher QAM orders and SNR values. We label our y-axis in units of bits / RRC channel use, where the denominator corresponds to the current transmission system in use. This will be used as a benchmark to compare other schemes through the remainder of this thesis.

### 2.4 Spectrum Use

We briefly review the use of frequency bands relevant to backhaul networks [9]. Below 3 GHz frequencies have low channel attenuation to travel long distances, but have limited bandwidths available. Applications typically include utilities,
Figure 2.4: AIR for an AWGN channel with $2^{10}$-QAM, $2^{12}$-QAM, $2^{14}$-QAM, and $2^{15}$-QAM constellations.

public safety, and communication to remote locations. The systems which use spectrum from 3 GHz to 10 GHz can accomplish 50km hops, and are used for long-haul connections in the backhaul network. Due to the reasonable bandwidths available with long distance capabilities, they continue to play a key roll in fixed wireless systems. The main bands currently in use today fit between 10 GHz to 57 GHz whose systems are capable of 20km distances. In North America, the bands between 11 GHz - 23 GHz are primarily used for microwave backhaul.

The 57 GHz - 66 GHz V-Band shows promising results, but experience strong atmospheric absorption between 59 - 64 GHz. While these frequencies could be used for private small cell links, the outer frequencies can be utilized for their large available bandwidths. Unfortunately, these bands have seen scattered regulations between the US and Europe, and will likely not see significant use in the
upcoming push for 5G backhaul usage [10]. Finally, the 71 - 76 and 81 - 86 GHz E-Band is also capable for high-capacity transmission, as they do not encounter the atmospheric effects at 59-64 GHz frequencies. While this band is deemed as incapable of supporting 5G use, it is a strong candidate for a global backhaul band.

In terms of spectral usage throughout the thesis, the standard parameters we use for our benchmark RC transmission system are as follows:

- Symbol Rate: \( R_{sym} = 25.6 \times 10^6 \) symbols/s
- Symbol Duration: \( T_{sym} = 1/R_{sym} = 39.06 \) ns
- Rolloff Factor: \( \beta_{RC} = 0.15 \)
- Bandwidth: \( B_{RC} = (1 + \beta_{RC})R_{sym} = 29.44 \) MHz
- Oversampling Factor: \( SPS_{RC} = 2 \)
- Sampling Interval: \( T_{sam} = T_{sym}/SPS_{RC} = 19.53 \) ns
- Sampling Rate: \( R_{sam} = 1/T_{sam} = 51.2 \times 10^6 \) samples/s
- Frequency Band: 17-30 GHz [3]

2.5 Channel Coding

In order to improve error-rate performance, Forward Error Correction (FEC) may be implemented to reduce the error rate to near zero. While algebraic block and convolutional codes offer simplicity and low latency, they cannot achieve the performance of other modern graph based coding schemes. Low-Density Parity Check (LDPC), Polar, and Turbo codes have been thoroughly compared according to throughput per chip area, low complexity, latency, and suitability at short and long block lengths [11]. Based on previous comparison studies, LDPC codes
$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

**Figure 2.5:** (7,4) Hamming code parity check matrix with Tanner graph.

have traditionally been selected for many high throughput and low latency scenarios while Polar codes have been chosen for ultra reliable control information.

LDPC codes can be implemented or viewed as block codes or convolutional codes. However, these codes can conveniently be visualized with a tool called the Tanner Graph. Fig. 2.5 illustrates an example of a Tanner Graph with seven variable nodes and three check nodes. The circular variable nodes correspond to the number of codeword symbols (column count) while the square check nodes correspond to the parity check equations (row count). While the code displayed is not an LDPC code, they generally have a relatively small number of graph edges since the parity check matrix is considered low-density or sparse.

By representing the parity check matrix as a graph-like problem, modern graphical model message passing algorithms can be utilized. While simple bit-flipping algorithms exist based on a tree representation of the Tanner Graph, the most notable algorithm used for decoding LDPC codes is belief propagation, or the sum-product algorithm [12]. Instead of performing hard-decisions iteratively until a convergence is met, soft probability values are used to speed up the process. The sum-product algorithm efficiently calculates the marginals

$$f_i(x_i) = \sum_{x_j} f(x_1, x_2, \ldots, x_n),$$

(2.12)

for each bit using the full factor graph.
It makes use of how the global function \( f(x_1, x_2, \ldots, x_n) \) is a factor of some local functions depending on a subset of variables. For example, if the global function can be expressed as the product

\[
\begin{align*}
f(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) &= g_1(x_1)g_2(x_2)g_3(x_1, x_2, x_3, x_4)g_4(x_4, x_5, x_6) \\
g_5(x_5)g_6(x_6, x_7, x_8)g_7(x_7),
\end{align*}
\]

then

\[
\begin{align*}
f_4(x_4) &= \left( \sum_{x_1, x_2, x_3} g_1(x_1)g_2(x_2)g_3(x_1, x_2, x_3, x_4) \right) \left( \sum_{x_5, x_6} g_4(x_4, x_5, x_6)g_5(x_5)g_6(x_6, x_7, x_8)g_7(x_7) \right)
\end{align*}
\]

can be used to yield less computation relative to the general case.

The process of decoding begins by estimating an initial node or symbol, and then sending messages out in opposite directions across the graph edges to continue estimations until convergence. Note that the algorithm is expected to converge in a finite number of steps if the graph is cycle-free, while graphs with cycles cannot guarantee convergence.

While Polar codes are a more recent invention that show promise, we selected LDPC codes to model high throughput transmission. LDPC codes are currently used in Digital Video Broadcasting, 10 Gigabit Ethernet, and WiFi MIMO standards.

The BER curves for transmission with various QAM constellation sizes over an AWGN channel are illustrated in Fig. 2.6. The code rate applied is a 4/5 code provided by MATLAB’s Digital Video Broadcasting (DVB) standard library. This library provides a limited number of available LDPC parity check matrices at a block length of 64800 bits, in order to simulate LDPC correction. This block length is a constraint that must be considered when generating data streams, and padding is often added to ensure that the block sizes are divisible by the QAM
order and pilot spacing that is discussed below. Also, there are only ten available code rates to choose from, which limit simulation precision in desired BER dropoff location.

The dropoff locations are estimated via the AIR curves in Fig. 2.4. By tracing a horizontal line at the AIR value of the QAM order multiplied by the code rate, the intersection of the simulated curve is the SNR value predicted for dropoff. For example, by drawing a horizontal line at 8 bits and matching it with the \( m = 10 \) curve, the SNR value of 25.48dB is predicted for a dropoff. For these particular simulations, the actual dropoff occurs between 1.0-1.5dB after the ideal dropoff.
2.6 Dispersive Fading

Aside from the AWGN channel, dispersive fading is another impairment that must be handled when operating at microwave frequencies on a line-of-sight path. Dispersive fading can be understood as an inter-symbol interference, where one symbol interferes with other symbols at different time instances. It is also associated with multi-path fading, where a radio signal reaches the receiver at different time instances due to taking various propagation paths. While this impairment takes place in the passband frequency range, the baseband continuous-time representation of a dispersive fading channel is expressed as

\[ r(t) = \int_{-\infty}^{\infty} s(\tau)c(t-\tau)d\tau + n(t), \]  

(2.14)

where \( c(t) \) is the cascade of our lowpass transmit and receive RRC filters, and some dispersive channel \( h(t) \).

Our system uses a Rummler two-path model [13] that is typically utilized when operating at carrier frequencies below 11 GHz [14,15]. While our spectrum mask assumes using higher microwave carrier frequencies between 17-30 GHz [3], we follow the channel models that have been utilized by similar microwave system designs [16].

The complex baseband frequency response is modelled as

\[ H(f) = a \left[ 1 - be^{j2\pi(f-f_0)\tau_0} \right], \]

(2.15)

where the coefficients \( a \) and \( ab \) represent the first and second transmission paths respectively, while the time constant \( \tau_0 \), typically fixed at 6.3ns, represents an expected delay between the two paths. The frequency \( f_0 \) corresponds to the offset with respect to the carrier frequency where the attenuation notch is located. This notch can be modelled by the depth and shape via the \( a \) and \( b \) parameters. The small time delay was chosen [13] to allow the \( a \) and \( b \) values to effectively determine the shape of the frequency response. The specific value of 6.3ns was chosen.
to align with a factor multiple of 1.1 MHz, the frequency spacing at which the channel attenuation was originally measured during experiments.

The transmission path coefficients $a$ and $b$ are generated via the random variables

$$A = -20\log_{10}(a)$$
$$B = -20\log_{10}(1-b),$$

where $B$ denotes the notch depth and $A$ denotes the overall attenuation.

While the notch depth can be set as a constant, it is typically modelled as a random variable according to an exponential distribution with mean $\mu_B = 3.8$ dB. The overall attenuation follows a normal distribution $A \sim \mathcal{N}(A_0, 5)$, but also depends on the notch depth as

$$A_0 = 24.6 \frac{B^4 + 500}{B^4 + 800}. \quad (2.17)$$

In order to generate the offset frequency, a relative phase $\phi = 2\pi f_0 \tau_0$ of the second multipath is generated according to the piecewise function

$$P_\Phi(\phi) = \begin{cases} 
1/216 & |\phi| \leq \pi/2 \\
1/1080 & \pi/2 < |\phi| < \pi.
\end{cases} \quad (2.18)$$

While a time varying Rummler channel is possible to model, it is enough to assume a particular notch depth and offset frequency and model many simulations based on a select number of Rummler channels. In our simulations, we typically look at three channels, each with a decreasing level of performance corresponding to increasing notch depth and notch frequency depending on the method used.

### 2.7 Equalization

In order to handle dispersive fading caused by multiple transmission paths, an equalizer must be utilized to handle the time-shift echoes of the desired signal. While many linear equalizers utilize adaptive filter taps in practice, we assume
that the training phase is complete, and a stable channel prediction is achieved. Many equalizers today are implemented digitally and must take into account the in-phase and quadrature components of the complex input. Equalizers may also take samples than the signal rate to better handle frequency distortion near the bandwidth edges. These fractionally-spaced equalizers are known to outperform many synchronous ones with the same number of taps [16].

We represent the continuous-time transmitted and received signals as

\[
s(t) = \sum_k I[k] p(t - kT)
\]

\[
r(t) = \sum_k I[k] c(t - kT) + n(t),
\]

where

\[
c(t) = \int_{-\infty}^{\infty} p(\tau) h(t - \tau) d\tau,
\]

denotes the cascade of the band-limited RC pulse shape \(p(t)\) and the channel response \(h(t)\). \(I[k]\) represents discrete-time information sequence of QAM symbols and \(n(t)\) represents the continuous-time additive white Gaussian noise.

After receiving the data sequence, we apply a finite-length equalizer filter to receive symbol estimates expressed as

\[
\hat{I}[k] = \sum_j f[j] r[k - j],
\]

where \(f[k]\) denotes the equalizer filter taps.

Our goal is to minimize the error of symbol detection denoted as

\[
e[k] = I[k] - \hat{I}[k],
\]

by utilizing the MMSE criterion expressed as an expectation

\[
E|I[k] - \hat{I}[k]|^2.
\]
We then assume that the covariance between the error and received signal sequence $r[k]$ is zero. This orthogonality principle is expressed as

$$E[\varepsilon[k]r^*[k-l]] = 0 \quad -\infty < l < \infty.$$  \hfill (2.25)

By substituting the symbol estimate (2.22) into (2.25) and incorporating the expectation over $l$ into the summation, we simplify the expression as

$$E \left[ \left( I[k] - \sum_j f[j]r[k-j] \right) r^*[k-l] \right] = 0 \quad (2.26a)$$

$$\sum_j f[j]E(r[k-j]r^*[k-l]) = E(I[k]r^*[k-l]), \quad -\infty < l < \infty. \quad (2.26b)$$

We simplify the left term as

$$E(r[k-j]r^*[k-l]) = \sum_{n=0}^{L} c^*[n]c[n+l-j] + \sigma_N^2 \delta[l-j], \quad (2.27)$$

by observing that it is the autocorrelation of the channel response $c[k]$ with an isolated noise term at power level $\sigma_N^2$.

We simplify the right term as

$$E(I[k]r^*[k-l]) = c^*[-l], \quad (2.28)$$

by observing that it is the channel response reversed.

In order to practically implement this linear equalizer, the convolution operation is modelled with a matrix multiplication by rearranging the channel vector into a Toeplitz Matrix. This matrix follows the property where each diagonal descending from left to right is constant. The newly formed convolution operation is expressed as

$$r[k] = \sum_{l=0}^{L} c[k-l]I[l] \quad (2.29a)$$

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By utilizing this matrix rule, equations (2.26b), (2.27), and (2.28) are used to solve for the equalizer coefficients

\[
\mathbf{r} = \begin{bmatrix}
c[0] & 0 & \ldots & 0 \\
c[1] & c[0] & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
c[L-1] & c[L-2] & \ldots & c[0]
\end{bmatrix} \begin{bmatrix}
I[0] \\
I[1] \\
\vdots \\
I[k]
\end{bmatrix}.
\]

(2.29b)

By utilizing this matrix rule, equations (2.26b), (2.27), and (2.28) are used to solve for the equalizer coefficients

\[
(C^H C + \sigma_N^2 I)\mathbf{f} = \mathbf{c}^* 
\]

(2.30a)

\[
\mathbf{f} = (C^H C + \sigma_N^2 I)^{-1}\mathbf{c}^*,
\]

(2.30b)

where \(\mathbf{f}\) denotes the vector of equalizer filter coefficients, and \(C\) denotes the Toeplitz representation of the channel response. While this equalization method requires knowledge of the noise power at the receiver, we assume that this value is known throughout our model.

Fig. 2.7 and Fig. 2.8 illustrate the results of a \(2^{12}\) QAM constellation under various Rummler channel instances once equalization is applied. While all AIRs reach the same final value, an increase in notch depth shifts the performance requirements by several dB of SNR. The effects of a Rummler channel are exaggerated at higher QAM orders, and the 3.5dB notch channel instance is typically used for future simulations.

### 2.8 Phase Noise

Another significant impairment we consider is phase jitter of the transmitter and receiver voltage controlled oscillators. Regarding a QAM constellation map, this effect results in random rotations of each symbol around the mapping. This impairment is especially damaging at the high QAM orders we simulate.

While this impairment takes place in the passband frequency range, the baseband continuous-time representation of a phase noise impaired channel is ex-
Figure 2.7: BERs for a $2^{12}$-QAM AWGN channel under various Rummler channel instances.

Figure 2.8: AIR for a $2^{12}$-QAM AWGN channel under various Rummler channel instances.
pressed as

\[ r(t) = s(t)e^{j\phi(t)} + n(t), \quad (2.31) \]

where \( n(t) \) denotes the continuous-time representation of the AWGN, and \( \phi(t) \) is the continuous-time phase noise.

For our simulation purposes, we model the phase noise channel as a discrete-time impairment expressed as

\[ r[k] = s[k]e^{j\phi[k]} + n[k], \quad (2.32) \]

where \( n[k] \) denotes the discrete-time AWGN with variance \( \sigma_N^2 \), and \( \phi[k] \) denotes a discrete-time representation of the phase noise.

A Wiener process is used to model phase noise variation, which implies an accumulation of phase noise steps. This process is expressed as

\[ \phi[k] = \phi[k-1] + \Delta[k], \quad \Delta[k] \sim \mathcal{N}(0, \sigma_\phi^2) \quad (2.33) \]

\[ p(\phi[k] | \phi[k-1]) = \frac{1}{\sqrt{2\pi\sigma_\phi^2}} e^{-\frac{|\phi[k] - \phi[k-1]|^2}{2\sigma_\phi^2}}, \quad (2.34) \]

i.e., each step is a zero-mean Independent and Identically Distributed (IID) Gaussian variable with variance \( \sigma_\phi^2 \). The transition probabilities \( p(\phi[k] | \phi[k-1]) \) are modelled by a normal distribution.

We make use of the Bahl, Cocke, Jelinek and Raviv (BCJR) algorithm \[17,18\] to perform Maximum A Posterior Probability (MAP) pilot-assisted phase noise sequence estimation at high-order QAM transmission. For this purpose, we discretize the phase noise to form a trellis. Following \[17\], we define a phase noise resolution that is fine enough for high accuracies, and a maximum span region. Pilot symbols are transmitted to limit the maximum phase span region of the trellis at constant intervals so the phase noise does not expand indefinitely.

The aforementioned BCJR algorithm \[18\] is a general algorithm not specific to phase noise, that is used to optimally estimate a posteriori probabilities of a
hidden finite Markov process. It generates forward and backward probabilities expressed as

$$\alpha_k(\phi) = \sum_{\phi'} \alpha_{k-1}(\phi') \cdot \gamma_k(\phi', \phi) \quad \forall \phi', \phi \in \Phi$$

(2.35a)

$$\beta_k(\phi) = \sum_{\phi''} \beta_{k+1}(\phi'') \cdot \gamma_k(\phi, \phi'') \quad \forall \phi, \phi'' \in \Phi,$$

(2.35b)

where the phase states $\phi', \phi, \phi''$ represent the previous, present, and future phase state values respectively. The $\gamma$ term must also be calculated, which represents the probability of the received symbol given two adjacent phase states. This probability is expressed as

$$\gamma_k(\phi', \phi) = p(\phi'|\phi) \cdot \frac{1}{\sqrt{\pi\sigma_\xi^2}} \cdot e^{-\frac{|r[k]|e^{-j\phi - \hat{s}[k]}|^2}{\sigma_\xi^2}} \quad \forall \phi', \phi \in \Phi$$

(2.36a)

$$\gamma_{k+1}(\phi, \phi'') = p(\phi|\phi'') \cdot \frac{1}{\sqrt{\pi\sigma_\xi^2}} \cdot e^{-\frac{|r[k]|e^{-j\phi'' - \tilde{s}[k]}|^2}{\sigma_\xi^2}} \quad \forall \phi, \phi'' \in \Phi,$$

(2.36b)

where $\hat{s}[k]$ denotes the symbol we have tentatively decided upon. These values must be calculated for every possible symbol within the QAM mapping.

As a final step, the estimator selects the phase noise state value that maximizes the a posteriori probability $\hat{\phi}[k] = \arg\max_{\phi} \alpha_k(\phi) \cdot \beta_k(\phi)$.

In order to simulate the phase noise, a random step model must be selected to determine the variance of the Markov process. In order to account for the
bandwidth, phase noise measurements are typically measured in units of dBc/Hz, representing the noise power $K_0$ relative to the carrier frequency contained in a 1 Hz bandwidth centred at the offset frequency $f_{\text{offset}}$ [19]. The denominator can be modified in order to apply phase noise at sample rate instead of symbol rate, expressed as

$$\sigma_\phi^2 = \frac{K_0 4 \pi^2 f_{\text{offset}}}{f_{\text{sym}}}. \quad (2.38)$$

Without a Rummler channel, the uncoded BER and AIR for $2^{12}$-QAM AWGN transmission under varying phase noise levels are illustrated in Fig. 2.9 and Fig. 2.10 respectively. We use a signal bandwidth of 25.6 MHz and a phase noise offset frequency of 100 kHz. While a noise floor appears at an uncoded BER order of $10^{-3}$, we can obtain practically zero error rates with strong enough FEC coding. The error floor is dependent on many factors, but QAM order seems to be the most significant due to the state space of the BCJR algorithm.
Figure 2.9: BERs for a $2^{12}$-QAM AWGN channel at various phase noise levels with $p = 50$ pilot spacing.

Figure 2.10: AIRs for a $2^{12}$-QAM AWGN channel at various phase noise levels with $p = 50$ pilot spacing.
Chapter 3

Superposition Transmission for Using Spectrum Skirts

In this chapter, we introduce the superposition transmission scheme for efficient spectrum utilization through microwave links. We begin by describing the system’s baseband transmission model and illustrating the general additive transmission and sequential receive processes. We then outline a novel channel estimation method which makes use of the narrowband channel knowledge in order to extrapolate the wideband channel to reduce equalization overhead. Finally, we outline a phase noise re-estimation method which uses FEC-aided symbol estimates in order to improve signal reconstruction accuracy.

3.1 System Model

The primary spectrum utilization method we study is an aggressive approach, which overlaps two carrier signals at the transmitter. Our superposition transmission scheme makes use of Interference Cancellation (IC) [20] at the receiver in order to sequentially detect a strong primary and weak secondary signal. While some interference cancellation techniques operate in parallel over multiple iterations to improve signal detection.
We apply Successive Interference Cancellation (SIC), which decodes signals successively. Once one signal is successfully detected, it is subtracted from the full combination of signals before the next signal is detected. While the first signal must deal with the interference from the second signal, the second signal enjoys the benefit of having the first signal’s interference already removed, albeit an approximate removal. SIC is suitable for our superposition transmission due to the large power disparity of the superposed signals.

The continuous-time representation of our superposition transmission model is expressed as

\[
s(t) = \sum_k I_1[k]p_1(t - kT_1) + \sum_k I_2[k]p_2(t - kT_2),
\]

(3.1)

where \(I_1[k]\) and \(I_2[k]\) are the QAM symbols of the primary and secondary stream scaled by power \(P_1\) and \(P_2\) respectively. The pulse shapes \(p_1(t)\) and \(p_2(t)\) represent the narrowband and wideband pulses used to shape our primary and secondary pulses, while \(T_1\) and \(T_2\) denote each stream’s symbol interval. While we assume \(T_1 = 2T_2\) throughout our simulations to ease the sampling process, this is not required.

The continuous-time representation of the signal we receive is expressed as

\[
r(t) = e^{j\phi_t(t)} \int_{-\infty}^{\infty} s(\tau)e^{j\phi_r(\tau)}h(t - \tau)d\tau + n(t)
\]

(3.2)

where \(h(t)\) denotes our multipath Rummiller channel and \(n(t)\) denotes the zero-mean additive Gaussian noise. The signals \(\phi_t(t)\) and \(\phi_r(t)\) represent the transmitter and receiver phase noise processes. After receiving \(r(t)\), the primary and secondary streams are separated with a narrowband and wideband receive filter respectively.

Regarding pulse shapes, the superposition scheme allows the second stream to incorporate a larger bandwidth to effectively utilize the spectrum skirts. The combined pulse shapes with bandwidths 25.6 MHz and 51.2 MHz are displayed in Fig. 3.1, relative to a standard benchmark RC scheme.
The difference of the filter gain at frequency 0 is chosen as 40dB to allow usage of a channel bandwidth of exactly double the primary stream. Using this bandwidth allows for straightforward pulse synchronization. A gap of roughly 2dB from the spectral mask is used to allow for fluctuations of the transmitter electronics.

The transmitter architecture in Fig. 3.2 is similar to the RC scheme in Chapter 2, with a signal chain added to the primary stream before analog up-conversion and antenna transmission. The QAM orders of each stream may vary based on channel and phase noise conditions.

The receiver architecture in Fig. 3.3 is more complex, as it requires removal of the primary stream to detect the second stream. Once the primary stream has been decoded, it is re-encoded and sent over an estimation of how the channel appears. After re-applying channel impairments, it is subtracted from the received signal at a larger bandwidth, and the secondary stream can effectively be decoded. The main bottleneck of this receiver architecture is the effective reconstruction of the primary stream. While an accurate secondary stream equalization and phase noise

![Graph showing Amplitude Frequency Response](image)

**Figure 3.1:** Pulse shape for 51.2 MHz superposition transmission scheme.
compensation is important at lower signal power, the residual ISI and phase noise of an incorrectly reconstructed primary stream is amplified when detecting the secondary stream due to the large power difference.

### 3.2 Channel Estimation

One challenge related to the equalization of the spectrum skirts is the necessity of wideband channel knowledge. While the primary stream can send pilot symbols in order to estimate the narrowband Rummel channel, the secondary stream is at a much weaker SNR and requires knowledge of the wideband Rummel channel response for equalization and detection. Adaptive equalization or training is possible at the lower SNR, but another approach is to use the information from the narrowband channel response to predict a larger portion of the channel bandwidth.
Referring to Fig. 3.3, knowledge of a larger channel bandwidth is required within the primary stream reconstruction pipeline where the Rummler channel is re-applied, and in the secondary stream linear equalizer. The straightforward solution to both of these problems is to send a set of training symbols over the full bandwidth, while at a lower power to conform to the pulse shape requirements. The more novel approach is to use the information from the narrowband channel response to predict a larger portion of the channel bandwidth.

Before discussing the novel channel estimation approach, we introduce another more simple method to obtain the Rummler channel response required for the primary reconstruction stream. Referring to Fig. 3.3, the reconstruction stream only requires the cascade of the Rummler channel and a single narrowband \( \frac{1}{T_1} \) RRC filter. This is still a narrowband channel response, with a slightly less steep rolloff due to only one RRC filter being required. We obtain this single rolloff channel response by removing one RRC from our estimated narrowband channel in the frequency domain.

We begin by modelling the discrete-time representation of our overall channel as

\[
c[k] = \sum_l h[l]p'[k - l],
\]

(3.3)

where \( h[l] \) denotes our discrete-time dispersive Rummler channel and \( p'[k] \) denotes the discrete-time RC pulse shape where \( p[k] \) is a single RRC pulse. We can also model our overall channel as

\[
c[k] = \sum_l c'[l]p[k - l],
\]

(3.4)

where \( c'[k] \) denotes the cascade of our Rummler channel and a single RRC pulse.

We then can remove a single RRC rolloff by modelling the overall channel in the frequency domain as

\[
C[m] = C'[m] \cdot P[m]
\]

(3.5a)
\[ C'[m] = \begin{cases} \frac{C[m]}{P[m]} & P[m] \geq \xi \\ 0 & \text{otherwise} \end{cases} \tag{3.5b} \]

where \((C[m], C'[m], P[m])\) represent the Discrete Fourier Transform (DFT) of their respective time-domain representations. The threshold \(\xi\) is used to consider the region where we can reliably remove the RRC response. Typically \(\xi = 0.1\) is used to ignore the frequency response region that is effectively zero. The DFT is calculated via the Fast Fourier Transform (FFT) operation. We cannot remove both RRC filters via this method because of how they zero the channel response, an operation that is practically irreversible by a single division operation.

The primary issue with this method is the inaccuracies introduced by each DFT operation, and the numerical imprecision of being unable to remove the RRC rolloff effect at frequencies below the specified cutoff value.

The other method for determining the Rummler channel response is to predict the full bandwidth, required for the secondary stream equalizer and usable by the primary reconstruction stream. We utilize a number of assumptions that have been found similar to other research ventures in sampling and reconstruction. Many signals typically are limited by a finite number of degrees of freedom, specifically in terms of a limited number of samples at particular units of time. These signals that exhibit a limited number of degrees of freedom are considered to have a finite rate of innovation and can be accurately reconstructed in the presence of noise [21]. The algorithm proposed to solve finite rate of innovation problems only requires a non-aliased input stream, and knowledge of the pulse shape associated with the overall signal.

Our Rummler channel model fits the description of having a finite rate of innovation as it is limited to two primary taps separated by a constant time delta. We consider this finite stream of pulses as

\[ c(t) = \sum_{l=1}^{L} c_l p'(t - t_l), \tag{3.6} \]
where \( p'(t) \) is our known RC pulse shape, and \( \{t_l, c_l\}_{l=1}^L \) correspond to the unknown time delays and amplitudes. We assume that the number of pulses are known \( L = 2 \) and the time delays are restricted to the finite sampling time interval \( t \in [0, T) \).

We now consider the signal \( c(t) \) to be periodic with a sufficiently large interval \( T \) so that its continuous-time Fourier series expansion is expressed as

\[
c(t) = \sum_{k=-\infty}^{\infty} C[k] e^{-j \frac{2\pi}{T} kt},
\]

(3.7)

where \( C[k] \) are the corresponding Fourier series coefficients. It can then be shown that the coefficients \( C[k] \) satisfy

\[
C[k] = \frac{1}{T} P'(\frac{2\pi}{T} k) \sum_{l=1}^{L} c_l e^{-j \frac{2\pi}{T} k t_l},
\]

(3.8)

where \( P'(f) \) represents the continuous-time Fourier transform of \( p'(t) \). By removing the RC coefficients, the Rummler channel response can be expressed as

\[
X[k] = \frac{1}{T} \sum_{l=1}^{L} c_l e^{-j \frac{2\pi}{T} k t_l} = \frac{1}{T} \sum_{l=1}^{L} c_l z_l^k,
\]

(3.9)

where \( z_l = e^{-j \frac{2\pi}{T} t_l} \) represent the \( L = 2 \) complex exponentials we hope to find.

We then utilize the subspace method to obtain these exponentials, outlined in the following algorithm. This algorithm is explained in more detail in [21].
Algorithm: Subspace method for Rummler channel estimation

1: Construct a $M \times N$ Hankel Matrix


where $M + N = K$.

2: Compute the singular-value decomposition $X = USV^H$.

3: Find the largest $L = 2$ largest singular values and the corresponding left and right singular vectors $U_s$ and $V_s$.

4: Compute estimates of $\hat{z}_l$ from (3.9) as the eigenvalues of the matrix $Z = U_s^+ \cdot \bar{U}_s$ where the overbar and underbar symbols represent omitting the first and last row of each matrix respectively.

In order to practically obtain $X[k]$, we calculate $N_{\text{dft}} = 4K$ coefficients of the discrete Fourier transform (DFT) instead of using the continuous-time Fourier transform (3.9). We utilize only $K$ of the available coefficients as we assume that our known narrowband channel estimate $C[k]$ is four-times oversampled.

Given $\hat{z}_l$ from the subspace method, we estimate the Rummler channel time delays as

$$\hat{t}_l = N_{\text{dft}} \frac{T \angle \hat{z}_l}{2\pi}. \quad (3.11)$$

To obtain the channel coefficients, we transform (3.9) into a Vandermonde matrix and solve the system of equations as

$$P = \begin{bmatrix} 1 & \hat{z}_0 & \hat{z}_0^2 & \cdots & \hat{z}_0^{K-1} \\ 1 & \hat{z}_1 & \hat{z}_1^2 & \cdots & \hat{z}_1^{K-1} \end{bmatrix} \quad (3.12)$$

$$\hat{c}_l = P^+ \cdot X[k]. \quad (3.13)$$

One modification we make to the original subspace algorithm is iterating over a number of $M$ values to generate various unique $X$ matrices (3.10). We then
select a final set of $\{\hat{t}_l, \hat{c}_l\}_{l=1}^L$ coefficients that minimize the mean squared error between the known narrowband channel $c[k]$, and a reconstruction of the narrowband channel $\hat{c}[k]$ using such coefficients.

Through the subspace method, we are able to obtain a wideband channel estimate from the known narrowband channel that is suitable for primary stream reconstruction, and secondary stream equalization. However, we observe that an accurate channel estimate is essential for the primary stream reconstruction. When performing secondary stream detection, small errors in the primary reconstruction stream are amplified due to the large signal power difference. These effects and overall improvements are illustrated in Chapter 5.

### 3.3 Phase Noise Estimation

Another unique challenge that faces the superposition transmission scheme is the necessity of precise phase noise estimation, especially for the reconstruction of the primary stream in order to estimate the secondary stream. The phase noise estimate is used in three locations of the receiver architecture displayed in Fig. 3.3. While the initial primary stream phase noise compensator cannot be easily improved, the phase noise re-application in the reconstruction phase is a much greater limit to system performance than the final phase noise compensator for the secondary stream.

Once the primary stream has been effectively detected and decoded, there is potential for a second phase noise estimate. As long we using FEC such as an LDPC code, we could assume every symbol is a pilot in terms of a BCJR-like algorithm, as long as the code has brought the BER to zero. This suggests that if this pilot-assumption method is utilized, the primary stream would have to be a low enough QAM order such that we could assume the BER would drop to zero.

Similar to Sec. 2.8, the discrete-time baseband representation of our impaired channel is expressed as

$$r[k] = s[k]e^{j\phi[k]} + n[k],$$  \hspace{1cm} (3.14)
where \( n[k] \) denotes the discrete-time AWGN with variance \( \sigma_N^2 \), and \( \phi[k] \) denotes a discrete-time representation of the phase noise. A Wiener process models the phase noise with zero-mean IID Gaussian variable steps with variance \( \sigma_\phi^2 \). The main change relative to Sec. 2.8 is that we now know the originally transmitted symbol \( s[k] \).

In order to estimate the phase noise, we use an alternative to the discretized phase BCJR algorithm [22]. This method assumes the phase noise process is a complex circularly symmetric Gauss-Markov process. This assumption results in forward and backward recursions in the form of a Kalman smoother. Similar to the BCJR, we estimate the phase noise by utilizing the distribution \( P(s[k]) \) of the transmitted symbol.

The sum-product algorithm we present from [22] is expressed as

\[
P(s[k]) \propto \int_0^{2\pi} p_f(\phi[k]) p_b(\phi[k]) f_k(s[k], \phi[k]) d\phi[k]
\]  

(3.15a)

\[
f_k(s[k], \phi[k]) = \exp \left( \frac{1}{\sigma_N^2} \Re[r[k]s[k]^* e^{-j\phi[k]}] - \frac{|s[k]|^2}{2\sigma_N^2} \right)
\]

(3.15b)

\[\approx \exp \left( -\frac{1}{2\sigma_N^2} |r[k] - s[k]e^{j\phi[k]}|^2 \right),\]

where \( p_f(\phi[k]) \) and \( p_b(\phi[k]) \) correspond to the forward and backward probabilities of the phase noise, and \( f_k(s[k], \phi[k]) \) corresponds to current stage’s probability. Note that the proportionality relationships used are allowed, since the algorithm allows scaling its messages by positive factors.

In order to simplify the desired distribution \( P(s[k]) \), the forward messages are expressed as

\[
p_f(\phi[k]) \approx \int_0^{2\pi} p(r[k] | \phi[k]) p_f(\phi[k - 1]) \delta(\phi[k] - \phi[k - 1]) d\phi[k - 1]
\]

(3.16a)
\[ p_f(\phi[k]) \simeq \int_0^{2\pi} \exp \left\{ \frac{r[k-1]s[k-1]^* e^{-j\phi[k-1]}}{\sigma_N^2} \right\} p_f(\phi[k-1]) \delta(\phi[k] - \phi[k-1]) d\phi[k-1] \]  

(3.16b)

\[ p_f(\phi[k]) \propto \exp \{ \text{Re}[a_{f,k} e^{-j\phi[k]}] \}, \]  

(3.16c)

where we assume that the posterior distribution \( p(r[k] \mid \phi[k]) \) follows a Gaussian PDF of the previously decoded symbol, and that channel phase is slowly varying for \( \sigma_\phi \to 0 \) such that a simple delta \( \delta(\phi[k] - \phi[k-1]) \) is incorporated into the final relation. The backward messages are simplified similarly.

By using (3.15a) and (3.16c), the desired posterior distribution \( P(s[k]) \) takes the form of a Tikhonov distribution expressed as

\[ P(s[k]) \propto \exp \left( -\frac{|s[k]|^2}{2\sigma_N^2} \right) I_0 \left( \left| a_{f,k} + a_{b,k} + \frac{r[k]s^*[k]}{\sigma_N^2} \right| \right) \]  

(3.17a)

\[ a_{f,k} = \gamma(\sigma_\phi^2, a_{f,k-1} + \frac{r[k-1]s^*[k-1]}{\sigma_N^2}) \]  

(3.17b)

\[ a_{b,k} = \gamma(\sigma_\phi^2, a_{b,k+1} + \frac{r[k+1]s^*[k+1]}{\sigma_N^2}) \]  

(3.17c)

\[ \gamma(x_1, x_2) = \frac{x_2}{1 + x_1 |x_2|}, \]  

(3.17d)

where \( I_0 \) represents the modified Bessel function. Fortunately, the desired posterior distribution is maximized at the location of the complex parameter of the modified Bessel function. This property is expressed as

\[ \hat{\phi}[k] = \arg \max_\phi P(s[k]) = \angle \left( a_{f,k} + a_{b,k} + \frac{r[k]s^*[k]}{\sigma_N^2} \right). \]  

(3.18)

Therefore, with one forward and backward recursion of all received symbols in a packet, the phase noise estimate can be improved. Similar to the channel re-estimation method, the phase noise estimate is essential for primary stream reconstruction. The overall improvements are illustrated in Chapter 5.
Chapter 4

Multi-Carrier Transmission for Using Spectrum Skirts

In this chapter, we introduce the multi-carrier transmission scheme for efficient spectrum utilization through microwave links. This multi-carrier system was primarily developed by a colleague in our lab group. The chapter begins by describing the system’s baseband transmission model and illustrating the general transmission and receive processes. We then describe the use of custom sideband pulse shapes to improve overall data rates. Our primary contributions involve the final section, with the addition of dual-polarization transmission.

4.1 System Model

The multi-carrier spectrum utilization method transmits three pulses simultaneously while remaining orthogonal in the frequency domain. This scheme is a method similar to the frequency multiplexing technique known as Filter Bank Multi-Carrier (FBMC) transmission. This technology sends multiple signals shifted to adjacent frequency offsets, and relies on well designed transmitter and receiver filters to neglect interference. It is commonly compared with Orthogonal Frequency Division Multiple Access (OFDM) but does not require a cyclic prefix or
suffer from high peak power.

The continuous-time representation of our multi-carrier transmission model is expressed as

\[ s(t) = \sum_k I_1[k]p_1(t - kT) + \sum_k I_2[k]p_2(t - kT) + \sum_k I_3[k]p_3(t - kT), \]

(4.1)

where \( I_1[k], I_2[k], \) and \( I_3[k] \) are the QAM symbols of the primary and secondary streams scaled by power \( P_1 \) and \( P_2 \) respectively. The pulse shapes \( p_1(t), p_2(t), \) and \( p_3(t) \) represent the pulse shapes of the single primary and two secondary streams transmitted at symbol interval \( T \). The frequency offsets for the secondary streams are incorporated into each pulse shape.

Regarding pulse shapes, Fig. 4.1 illustrates our primary pulse and one of the secondary pulses that occupy the spectrum skirt. For this pulse shape design, we set the power of the secondary RC pulses to -45dB to ensure that a 30 MHz offset frequency fits within the spectral mask. Since these pulses are orthogonal in the frequency domain, we have the option of using custom pulse shapes, which is discussed in a later section. While adding additional pulse shapes below the -50dB spectrum mask is possible, it is not feasible for reliable transmission for the SNR ranges we intend on using.

The transmitter architecture in Fig. 4.2 highly resembles the other transmitter schemes with the addition of two secondary streams before analog up conversion. However, the main difference between the superposition scheme is that the secondary pulses require a frequency offset, which does not require much more additional hardware complexity.

The receiver architecture in Fig. 4.3 is more simple than the superposition scheme as each pulse can be detected in parallel due to the minimal cross interference. However, one optimization that can be made as discussed in the previous section is using the channel and phase noise estimation of the primary stream to improve achievable rates in the secondary streams.
Figure 4.1: Pulse shape for 51.2 MHz multi-carrier transmission scheme.

Figure 4.2: Transmitter architecture for multi-carrier scheme.

4.2 Pulse Shape Design

As mentioned earlier, the secondary pulses do not necessarily have to conform to the typical RRC shape seen in the other schemes. Instead, our colleagues design a custom pulse to fill the spectrum mask. The problem may be formulated as an optimization problem to match the spectrum mask as closely as possible. This translates to minimizing the weighted sum squared error between the desired shape and the actual filter bank waveform. Note that these pulses are asymmetric and result in complex valued filter taps. Additionally, the filter coefficients are
Figure 4.3: Receiver architecture for multi-carrier scheme with linear equalization, phase noise tracking, and FEC coding.

made to be conjugate-symmetric in order to limit the phase response as linear. The translation of these properties to a frequency response is expressed as

\[ P(f) = e^{-j2\pi N f} \left( \tilde{v}_{re}(f)^T \tilde{p}_{re} + \tilde{v}_{im}(f)^T \tilde{p}_{im} \right) \quad (4.2a) \]

\[ \tilde{v}_{re}(f) = [1, 2\cos(2\pi f), 2\cos(4\pi f), \ldots, 2\cos(2\pi N f)]^T \quad (4.2b) \]

\[ \tilde{v}_{im}(f) = [1, 2\sin(2\pi f), 2\sin(4\pi f), \ldots, 2\sin(2\pi N f)]^T, \quad (4.2c) \]

where \( \tilde{p}_{re} \) and \( \tilde{p}_{im} \) denote the discrete-time filter coefficients and \( N \) denotes the number of filter coefficients we hope to use.

Once the frequency responses are defined, the filter design optimization problem can be formulated as the minimization of a weighted sum squared error as

\[ \min_{\tilde{p}_{re}, \tilde{p}_{im}} \int_{-F_s/2}^{F_s/2} \left| D(f) - \left( \tilde{v}_{re}(f)^T \tilde{p}_{re} + \tilde{v}_{im}(f)^T \tilde{p}_{im} \right) \right|^2 df \quad (4.3) \]

subject to \( \left| \tilde{v}_{re}(f)^T \tilde{p}_{re} + \tilde{v}_{im}(f)^T \tilde{p}_{im} \right| \leq D(f), \forall f \in [-F_s/2, F_s/2], \)

where \( D(f) \) denotes the amplitude response of the desired spectrum mask and \( F_s \) denotes the sampling frequency of the desired spectrum mask. The optimized pulse shape selected for simulations is illustrated in Fig. 4.4. Relative to the center
frequency, the custom pulse spans from +5dB to -5dB in its active region, and drops down to -50dB when exiting the bandwidth intended for usage.

4.3 Dual-Polarization

We again emphasize that we did not complete the preceding work, as it was not the main focus of the thesis. The main focus of this thesis was the superposition scheme, and how it compares to another modular spectrum utilization scheme. However, we contribute to the multi-carrier scheme by implementing an additional technique known as Dual Polarization (DP), to significantly improve performance. While the superposition architecture has the option of supporting this method as well, the implementation is more straightforward with the separate non-overlapping pulse shapes within the multi-carrier scheme.

Many practical state-of-the-art microwave systems utilize a horizontal and vertical polarization transmitted at the same carrier frequency in order to double the data rate. However, these designs suffer from cross-talk between the two polar-
izations known as Cross-Polarization Interference (XPI). In order to combat this impairment, a synchronous approach may be taken after receiving time and frequency synchronized samples from both streams. By coordinating a XPI cancellation filter to remove interference simultaneously, the cross-talk can be effectively cancelled [23].

The continuous-time baseband model for DP transmission is expressed as

\[ s_i(t) = \sum_k I_i[k] p(t - kT), \]  

(4.4)

where \( I_i[k] \) for \( i \in \{1, 2\} \) denote the QAM symbols for the \( i \)th stream, while \( p(t) \) represents the transmission RRC pulse shape.

The continuous-time baseband model for the received signal is expressed as

\[ r_i(t) = e^{j\phi_{ti}(t)} \sum_{j=1}^{2} \int_{-\infty}^{\infty} s_j(\tau) e^{j\phi_{tj}(\tau)} h_{ij}(t - \tau) d\tau + n_i(t), \]  

(4.5)

where \( h_{ij}(t) \) denotes the effective cross-talk channel response, modelling the combined effects of multipath ISI and XPI. Similar to previous models, the noise \( n_i \) is AWGN, while \( \phi_{ti} \) and \( \phi_{ri} \) represent the transmitter and receiver phase noise processes.

This channel operation is also expressed in the block diagram of Fig. 4.5. Through this diagram, it can be seen that each stream has an effect on the adjacent stream through the channel filter, and that the phase noise is applied both at the transmitter and receiver. Previously, we assumed that the phase noise was combined due to only having one dispersive channel. However, with the addition of cross-talk, the phase noise must be properly applied at the transmitter and receiver. Nevertheless, the transmitter and receiver phase noise is treated as a single process by the phase noise cancellation block.

We utilize the BCJR algorithm to compensate for phase noise, but alternative methods exist. One such option is a MMSE-based strategy that utilizes a Master/Slave phase synchronized phase lock loop in order to jointly track the phase.
noise on each branch [24]. This method has the benefit of requiring no statistical knowledge of the phase noise process. Another method is an adaptive decision-feedback structure with linear precoding that effectively cancels interference and tracks the carrier phase [23]. This method has the benefit of increased Faster-than-Nyquist (FTN) rates to improve spectral efficiency. However, we move forward with the original BCJR algorithm due to its reliability, and to stay consistent within our project.

The addition of the dual channel also requires an increase of equalizer complexity. Instead of one time dispersive channel and one input stream, the equalizer must handle four channels and two input streams simultaneously. While many of the discussed joint phase noise and equalization techniques implement an adaptive non-linear scheme, a linear approach is still possible. In a practical setting, the linear approach would be broken down into a training phase to learn the channel, and operation phase for data transmission. However, in order to implement the joint dual-polarization / equalization block, slight adjustments must be made to our dispersive channel assumptions.

Firstly, while performing each equalization operation separately is possible, performing the full operation concurrently allows the information of one stream to help decode the opposite stream. While many assumptions made by MMSE linear equalization still stand, the matrices must be formulated differently to allow
a dual convolution. Secondly, In order to allow the input of both streams and all
four channels in a single convolution operation, the data matrix interleaves the
first stream data and the second stream data. The channel matrix is then grouped
in $2 \times 2$ squares and simulate a Toeplitz Matrix convolution operation. Neglecting
the impact of phase noise, the resulting matrices are expressed as

$$ r = Cs, \quad s = \begin{pmatrix} s_1[0] \\ s_2[0] \\ s_1[1] \\ s_2[1] \\ \vdots \\ s_1[k-1] \\ s_2[k-1] \end{pmatrix} $$

(4.6)

where $c_{i,j}$ correspond to the cascade of the transmitter and receiver filters, the
Rummler channel, and the cross-talk effects. Once the matrices are formed in this
pattern, the equalizer coefficients may be found by using the previous equalization
techniques discussed in Chapter 2.

Dual polarization is possible to implement with the superposition scheme, but
cross-channel talk would be difficult to reproduce for the reconstruction stream
and would severely dampen performance. Additionally, the multi-carrier sec-
ondary pulses are orthogonal in the frequency domain to the primary pulse, caus-
ing minimal interference for the dual XPI and ISI filter. Instead, we keep dual-
polarization isolated to the multi-carrier method and illustrate results in the follow- ing chapter.
Chapter 5

Numerical Results and Discussion

Throughout this chapter, we present the numerical results for the spectrum utilization methods outlined throughout this thesis. We begin by presenting the effects of each channel impairment on the superposition scheme, and then add the channel extrapolation and phase noise re-estimation methods. We then compare the superposition scheme with the multi-carrier scheme, and then conclude with the analysis of multi-carrier dual-polarization transmission.

Throughout this chapter, we primarily use the AIR to predict the performance of each system configuration. The AIR performance can be verified through simulation by applying a strong FEC code and ensuring that the BER drops off within a predicted SNR range. For example, for a system using a $2^{10}$-QAM constellation, when the simulated AIR reaches 5 bits per channel use, a 1/2 coding scheme could be applied to obtain a BER dropoff within a reasonable SNR range.

We assume that the primary stream of our superposition model has been perfectly detected when calculating the AIR for the secondary stream. This allows for the secondary stream to be detected with a primary stream reconstruction that only relies on the channel and phase noise estimates for strong performance. We can also validate this assumption by ensuring that the BER drops off within a reasonable range against the predicted SNR location.

We typically use a constellation size of $[2^{12}, 2^4]$-QAM for the primary and
secondary streams of the superposition scheme, and $[2^{14}, 2^4]$-QAM for the multicarrier scheme. We also consider a baseline “RC” scheme throughout many simulations, which utilizes a single RC pulse without making use of the spectrum skirts. This baseline RC method dictates the units of all of our AIR curves, bits per RRC channel use, meaning how many bits could be sent relative to a transmission scheme with a single RC pulse. Whenever phase noise is present, we use the BCJR algorithm with pilots sent every 50 symbols to compensate such phase noise.

5.1 Superposition Transmission with BCJR Phase Estimation

We first consider the superposition transmission scheme with BCJR phase estimation. We assume that we have full wideband channel knowledge after performing some pilot training phase. This section considers the transmission architecture before introducing improvements in Sec. 5.2 to illustrate a generalized overview of channel impairments.

We begin by comparing the pre-improvement superposition scheme against various Rummler channels. Fig. 5.1 illustrates the results for two Rummler channel instances with notch depths of 3.5dB and 7dB. A $2^{14}$-QAM RC scheme is also displayed to demonstrate that the superposition scheme is impacted by a severe notch depth by a similar amount. While the gaps between each simulation are slightly larger in the superposition case, the general shapes of the AIR curves remain the same, and performance is only slightly impacted.

The next set of results consider different levels of phase noise at the transmitter and receiver. Fig. 5.2 illustrates the AIR over an AWGN channel with levels of phase noise at -90dBc/Hz and -85dBc/Hz. This phase noise is compensated with the BCJR algorithm without the Tikhonov phase re-estimation method. The superposition results are impacted by increased phase noise much more than the $2^{14}$-QAM RC benchmarks. This is due to the necessity for accurate phase noise estimates during primary stream reconstruction. However, the superposi-
Figure 5.1: AIR for multiple Rummler channel instances with $2^{14}$-QAM RC transmission and $[2^{12}, 2^4]$-QAM superposition transmission.

...tion scheme still outperforms the RC benchmark in each phase noise instance.

5.2 Superposition Transmission with Improvements

We now introduce channel extrapolation via the subspace method and Tikhonov phase re-estimation outlined in Chapter 3 to improve data rate performance. We still use BCJR phase estimation for primary stream phase noise compensation, and also assume that the narrowband channel is known for channel extrapolation.

We first begin with analyzing the channel extrapolation methods by observing secondary stream AIR performance in Fig. 5.3. This simulation has no phase noise effects and uses the QAM orders of $2^{12}$ and $2^4$ for the primary and secondary stream respectively. Two relatively severe Rummler channels with notch depths of 7dB and 13dB are compared each with the RRC removal method, finite rate-of-innovation subspace method, and ideal channel knowledge. In each case tested,
Figure 5.2: AIR for various phase noise levels with $2^{14}$-QAM RC transmission and $[2^{12}, 2^{4}]$-QAM superposition transmission.

the subspace method outperforms the RRC removal by some margin. While the subspace approach does not reach perfect channel knowledge results, it is robust against moderately notched Rummler channel responses.

Next, we observe the impact of phase noise re-estimation over a standard AWGN channel with no dispersive channel effects and -90dBc/Hz phase noise. The results are illustrated with the secondary $2^{4}$-QAM AIRs in Fig. 5.4 with primary streams of $2^{12}$-QAM and $2^{14}$-QAM. These results compare the original BCJR estimation, the new Tikhonov re-estimation, and a genie re-estimation which is analogous to no phase noise.

We observe that regardless of QAM order, data rates can be improved with our new phase noise re-estimation method. Also, by observing the gaps between BCJR and Tikhonov phase estimation, we notice that an increased primary stream QAM order emphasizes the need for phase noise re-estimation. For clarification, we interpret that the two Tikhonov AIR curves align for $2^{12}$-QAM and $2^{14}$-QAM due to a saturation of the best possible estimate below genie-aided results.
Figure 5.3: Secondary stream AIR for various Rummiller channel reconstructions with $[2^{12}, 2^4]$-QAM superposition transmission.

Figure 5.4: Secondary stream AIR for various phase noise re-estimation methods with $[2^{12}, 2^4]$ and $[2^{14}, 2^4]$-QAM superposition transmission with phase noise at $-90$dBc/Hz.
5.3 Comparing Superposition and Multi-Carrier Transmission

Next, we introduce the multi-carrier (MC) transmission scheme to compare with the superposition (SP) scheme. In order to complete a fair comparison, we use the same number of equalizer taps for Rummler channel equalization and the same number of states in the BCJR phase estimation state machine. We consistently apply a Rummler channel with a 5dB notch along with phase noise at -90 dBc/Hz.

Fig. 5.5 shows the AIR comparison between the standard AWGN channel and impaired channel for the standard RC and two modular schemes. We selected the QAM order configurations which yielded the strongest AIR results under our 5dB notch Rummler channel instance and -90dBc/Hz phase noise level. The “DC” acronym represents a Rummler channel being present. While these QAM configurations performed best under the current impairments, different combinations may perform better at lower or higher SNR values. With this in mind, adaptive coding and modulation could be applied based on pre-determined SNR thresholds. For instance, the secondary streams could use $2^{3}$-QAM at low SNR values, and $2^{4}$-QAM at high SNR values.

Regardless of channel impairments, both modular schemes outperform the baseline RC scheme. Under the standard AWGN channel, the multi-carrier scheme performs the best at low SNRs while the superposition scheme reaches the highest maximum data rate at high SNRs. However, with the introduction of a dispersive channel and phase noise, the multi-carrier scheme outperforms the superposition scheme at all SNR values. This is due to the difficulty of reproducing channel impairments for the reconstructed primary stream for the superposition scheme.

We then predict the dropoffs at particular code rates using the AIR results. The BERs shown in Fig. 5.6 demonstrate how accurate dropoff estimations are. A LDPC coding scheme with rate 0.75 and codeword length 64800 bits is used for channel coding. The three code rates ($R_c$) are selected to attempt to fit the dropoffs near 45dB.

We first observe the non-steep dropoff of the RC scheme. The dropoffs of the
Figure 5.5: AIR for $2^{14}$-QAM RC, $[2^{12}, 2^4]$-QAM superposition, and $[2^{14}, 2^4]$-QAM multi-carrier transmission with phase noise at $-90\text{dBc/Hz}$ and a 5dB-notch Rummler channel.

Figure 5.6: BER for $2^{14}$-QAM RC, $[2^{12}, 2^4]$-QAM superposition, and $[2^{14}, 2^4]$-QAM multi-carrier transmission with phase noise at $-90\text{dBc/Hz}$ and a 5dB-notch Rummler channel.
superposition and multi-carrier scheme correspond to the secondary streams at low $2^4$-QAM orders. At these high SNR values, the primary streams already have practically zero error-rates due to their strong code rates. However, the RC scheme uses a much higher code rate for its $2^{14}$-QAM pulse, which results in the non-steep BER dropoff. We then observe how accurate the superposition dropoff estimates are relative to the multi-carrier scheme. This is explained by the stronger phase noise estimates that the superposition secondary stream has access to. While the multi-carrier scheme has a low QAM order, it also uses custom pulse shapes that are slightly more difficult to equalize.

Following these results, we wish to determine if the Rummler channel or phase noise is the primary bottleneck for the superposition scheme. The schemes shown in Fig. 5.7 use $2^{12}$-QAM for the primary streams of the superposition and multi-carrier schemes respectively, and 4 and 16-QAM for their secondary streams. Only the BER for the secondary streams are illustrated since the primary streams undergo perfect detection at the SNRs used. As seen in the BER curves, the superposition scheme performs the strongest under only dispersive channel conditions, but the introduction of phase noise significantly decreases performance.

5.4 Multi-Carrier Dual-Polarized Transmission

Finally, we implement dual-polarization alongside the multi-carrier architecture considering that it outperformed the superposition scheme under practical channel conditions. While dual-polarization introduces cross-channel interference, we may obtain a much higher data rate with sufficient cancellation methods.

To illustrate the impact of cross-channel interference on multi-carrier performance, we first show results for a benchmark RC scheme. Fig. 5.8 shows these results under various levels of cross-channel interference with a 3.5dB notched Rummler channel and phase noise at -90dBc/Hz.

The corresponding BERs are displayed in Fig. 5.9 under a 3/4 LDPC code rate. We notice that the cross talk curve takes much longer to dropoff relative to the RC benchmark. We explain this by referring to Fig. 5.8, where the -12dB
Figure 5.7: BER for $[2^{12}, 2^2/2^4]$-QAM superposition, and $[2^{12}, 2^2/2^4]$-QAM multi-carrier transmission with phase noise at $-90\text{dBc/Hz}$ and a 5dB-notch Rummler channel with a 0.75 rate LDPC code.

Figure 5.8: AIR for $2^{12}$-QAM RC dual-polarized transmission with phase noise at $-90\text{dBc/Hz}$ and a 3.5dB-notch Rummler channel.
cross-talk curve saturates at 10.5 bits. By using a 3/4 code rate at $2^{12}$-QAM, the predicted dropoff is located at 9 bits, which is close to the maximum data rate. By using a stronger code rate to obtain a predicted dropoff at a lower SNR, a steeper dropoff would be observed.

We finally introduce dual-polarization to the multi-carrier scheme. The multi-carrier scheme originally used $2^{14}$-QAM for its primary pulse, but must be reduced to $2^{12}$-QAM due to the cross-channel talk introduced by dual-polarization. Fig. 5.10 shows the results for our original multi-carrier and RC benchmark, along with a $2^{12}$-QAM dual-polarization RC transmission scheme and a $2^{12}$-QAM dual-polarization multi-carrier scheme with $2^4$-QAM sideband usage. We observe that the application of dual-polarization outperforms all previous results discussed within this thesis. While this transmission system requires a more complex ISI and cross-channel interference cancellation block, and the transmitter and receiver must be capable of receiving dual-polarized signals, the achievable data-rates can be significantly improved by implementing this technology.

Figure 5.9: BER for $2^{12}$-QAM RC dual-polarized transmission with phase noise at $-90$dBc/Hz and a 3.5dB-notch Rummler channel.
Figure 5.10: AIR for $2^{12}$-QAM RC and $[2^{12}, 2^4]$-QAM multi-carrier dual-polarized transmission with phase noise at $-90\text{dBc/Hz}$ and a 3.5dB-notch Rummier channel.
Chapter 6

Conclusions

In this thesis, we outlined two modular microwave transmission methods that utilize spectrum skirts to improve data-rate performance. We began by introducing a generalized microwave link transmission model that does not use the spectrum skirts, in order to establish a benchmark system. We also discussed system impairments such as multipath fading and phase noise, that would be present in practical microwave communication systems.

We then proposed a superposition transmission model that added a high-power narrowband and low-power wideband signal at the transmitter, and sequentially detected the signals at the receiver. We then introduced a novel channel estimation method which made use of the narrowband channel in order to extrapolate the wideband channel response to reduce equalization overhead. In order to detect the low-power secondary signal, the primary stream needed to be reconstructed for cancellation purposes. We improved this cancellation accuracy by utilizing FEC-aided symbol estimates to re-estimate the phase noise.

Next, we outlined a multi-carrier transmission model developed by a colleague that sent two low power secondary pulses orthogonal to the primary pulse in the frequency domain. These secondary streams were able to use custom pulse shapes due to their minimal interference from the high power primary stream. Our contributions included adding dual-polarization transmitter and receiver to improve
data-rates further. We finally compared these two modular methods via numerical results in our final section.

Both spectrum utilization methods allowed for easy modular implementation due to how their secondary streams could be simply enabled or disabled. While the superposition scheme offered more novel techniques such as Tikhonov phase re-estimation and channel extrapolation, the multi-carrier approach had the best performance under our selected channel impairments. Additionally, the ease of implementing dual-polarization for the primary stream of multi-carrier was another added benefit. Reconstructing a dual-polarized superposition primary stream would likely be too difficult and still under-perform the multi-carrier scheme. However, superposition did not require custom pulse shapes and outperformed the other scheme without phase noise, which would be useful in some applications.

Future work could include the pursuit of a single-carrier scheme with a single custom pulse shape. While this would require advanced equalization techniques to handle the ISI caused by the custom pulse shape, it could utilize more of the available bandwidth. Additionally, we made use of linear equalization to handle all dispersive channel impairments. An adaptive-equalization method could be implemented in order to test the effects of time varying dispersive channels, and the actual overhead required for a practical equalization architecture. Finally, one primary assumption made at the beginning of this thesis was to assume that the sidebands would go unused through the duration of a transmission. Each modular spectrum utilization method could be analyzed to demonstrate how they interfere with an adjacent pulse for the unlikely case that a simultaneous transmission were to occur. This could be accomplished with a small likelihood of adjacent channel transmission, or continuously transmitting an adjacent channel and measuring an overall performance degradation.
Bibliography


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