ESSAYS ON THE OPERATIONS AND MANAGEMENT OF TRANSPORTATION SYSTEMS

by

Wenyi Xia

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The following individuals certify that they have read, and recommend to the Faculty of Graduate and Postdoctoral Studies for acceptance, the dissertation entitled:

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Abstract

This dissertation addresses three important issues in transportation and logistics, and provides insights for efficiently operating and managing transportation and logistics systems.

The first essay (in Chapter 2) investigates the effects of airlines and high-speed rail (HSR) cooperation in a region with multiple airports. This essay is motivated by the rapid development of HSR, which has greatly promoted regional connectivity and air-rail intermodal transport. It shows under what conditions the air-rail cooperation is feasible, and how the air-rail cooperation can help relieve airport congestion. Specifically, when the HSR operator cares more about social welfare, or when the airlines are collusive, the airlines and HSR operator are more likely to cooperate. The air-rail cooperation helps to relieve airport congestion by diverting traffic to uncongested airports in the multiple-airport region.

The second essay (in Chapter 3) investigates how seaports can better adapt to climate change by determining the optimal timing and scale of investments in protective measures and throughput capacity. This study is motivated by the alarming rate of global warming and the threats imposed to vulnerable transportation infrastructure. This essay takes into account the uncertainty about the probability and magnitude of climate-change-related disasters, the irreversibility of infrastructure investments, and information accumulation. This essay highlights the importance of considering the two types of investment (protection and capacity) together, because the seaport can adapt in different ways as future climate gets worse, such as investing more in protection, holding off capacity investment, or adjusting its fee.

The third essay (in Chapter 4) proposes a new structural estimation approach which does not require individual choice data to estimate passengers’ mode choice of airport ground transportation. This approach is applied to Incheon International Airport. The estimation results suggest that passengers who fly Korean airlines or have experienced longer waiting times are less likely to take public transport. I formulate a nonlinear program that incorporates passengers’ choice behavior with the objective of maximizing train ridership. I show that the current train schedule is not
efficiently matched with the intraday demand, and could be substantially improved to achieve higher ridership without adding trains.
Lay Summary

This dissertation investigates three emerging and challenging topics in transportation and logistics sector. The first topic is about airlines and high-speed rail cooperation in a region with multiple airports. It shows how the air-rail cooperation can help relieve airport congestion by re-allocating traffic, and under what conditions the air-rail cooperation is feasible. The second topic is about how seaports can better adapt to climate change by determining the optimal timing and scale of investments in protective measures and throughput capacity, given the uncertainties associated with the likelihood and severity of climate-change-related disasters. The third topic proposes an estimation method that does not require individual choice data to identify factors affecting passengers’ mode choice of airport ground transportation, and proposes a schedule optimization method that incorporates passengers’ choice behavior. Overall, the dissertation provides insights for more efficiently operating and managing transportation and logistics systems, and opens avenues for future research.
Preface

A version of chapter 2 has been published. Xia, W., Jiang, C., Wang, K., Zhang, A., 2019. Air-rail revenue sharing in a multi-airport system: Effects on traffic and social welfare. Transportation Research Part B: Methodological, 121, 304-319. The research idea was identified together by Changmin Jiang, Kun Wang, and myself. Kun Wang and I developed the initial analytical framework. I was responsible for the theoretical analysis, numerical analysis, model extensions, and writing and revising the manuscript for publication. Professor Anming Zhang provided advice and made edits on the manuscript.

Chapter 3 was initiated by Professor Lindsey and myself. My contributions include developing and solving the analytical model, implementing numerical analysis, and writing the manuscript. The supervisory committee provided advice. Professor Lindsey made edits on the manuscript.

I was the primary investigator for Chapters 4. I was responsible for identifying the estimation strategy, conducting empirical estimation, solving the nonlinear optimization program, and writing the manuscript. Jong Hae Choi provided the data. Kun Wang was involved in the early stage of the estimation development. The supervisory committee provided advice.
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<td>$\omega$</td>
<td>The weight on the HSR’s own profit in its objective function</td>
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<td>$\tau$</td>
<td>The fee that the port charges for per unit cargo shipped</td>
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<td>$K$</td>
<td>The port’s capacity investment</td>
</tr>
<tr>
<td>$I$</td>
<td>The port’s protection investment</td>
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<td>$q$</td>
<td>Total demand</td>
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<td>$x$</td>
<td>The perceived probability of a disaster</td>
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<td>$\rho(q)$</td>
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<td>$g(K,q;\delta)$</td>
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<tr>
<td>$f(K,I;\theta,m)$</td>
<td>The disaster damage cost per unit cargo incurred by the shippers</td>
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<td>$F(K,I;\eta,M)$</td>
<td>The disaster damage cost per unit capacity incurred by the port</td>
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<tr>
<td>$D(K,I;\eta,M)$</td>
<td>The total disaster damage incurred by the port</td>
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Two-period model:

Parameter | Definition
---|---
\(x_1\) | The probability of a disaster in period 1
\(x_{2H}\) | The probability of a disaster in period 2 in the pessimistic state
\(x_{2L}\) | The probability of a disaster in period 2 in the optimistic state

**Chapter 4**

Parameter | Definition
---|---
i | Index for arriving flight
\(m\) | Index for airport ground transportation mode (bus, train or outside option)
j | Index for the scheduled time of the airport ground transportation mode
d | Index for the final destination (Daejeon, Daegu, or Busan)
\(U_{imd}\) | Indirect utility of an air passenger arriving on flight \(i\) choosing transport mode \(m\) at the scheduled time \(j\) to destination \(d\)
\(V_m\) | Constant utility of choosing transport mode \(m\)
\(TT_{mjd}\) | In-vehicle travel time by mode \(m\) at scheduled time \(j\) to destination \(d\)
\(WT_{imd}\) | Waiting time of an air passenger arriving on flight \(i\) choosing transport mode \(m\) at the scheduled time \(j\) to destination \(d\)
\(P_{md}\) | Ticket price by transport mode \(m\) to destination \(d\) (all trains or buses have the same price regardless of their departure times)
\(X_i\) | Characteristics of arriving flight \(i\), including flight type, flight distance, region of the origin airport, and whether the flight is operated by a Korean airline
\(\epsilon_{imd}\) | Unobserved idiosyncratic utility shock
\(\Theta\) | Parameters \((V_m, \theta_1, \theta_2, \theta_3, \theta_4)\) to estimate
\(S_{imd}\) | Probability of a passenger arriving on flight \(i\) choosing transport mode \(m\) at scheduled time \(j\) to destination \(d\)
\(\hat{Q}_{mjd}\) | Estimated ridership of transport mode \(m\) scheduled at time \(j\) to destination \(d\)
\(A_i\) | The number of passengers on flight \(i\)
\(r_d\) | The share of passengers on each flight whose final destination is \(d\)
$I(m, j, d)$ The set of arriving flights whose passengers can take transport mode $m$ scheduled at time $j$ to destination $d$

$Q_{mjd}$ The actual ridership of transport mode $m$ to destination $d$ scheduled at time $j$

$n$ The total number of buses and trains to all destinations in the study period

$a = 1, 2$ The two terminals at ICN

$t$ The time of the day $(0 \leq t \leq 24)$

$\lambda_a(t)$ The arrival rate of airport passengers

$k$ Index for the trains ($k = 1, ..., K$) according to the order of their departure times

$k(t)$ Index of the first train that departs after $t$

$y_k$ The departure time of the $k^{th}$ train

$B_d(t)$ The departure time of the next bus to destination $d$ that departs after $t$

$S_{d}^{trn}$ The probability of passengers arriving at time $t$ that would take train to destination $d$
Acknowledgements

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To my parents, Li Ye and Xia Xiaoming, for their unconditional love and support through the ups and downs in my life.
Chapter 1: Introduction

This dissertation is a collection of three independent essays that investigate three major research topics in transportation and logistics: (i) air and rail intermodality in a multi-airport region, (ii) seaport capacity investment and adaptation to climate change, and (iii) choice modeling and schedule optimization for passenger transport. The three topics in this dissertation are motivated by the current emerging and challenging issues in the transportation and logistics sector. This dissertation aims to investigate the abovementioned three topics through theoretical modeling and empirical approaches, with the hope to identify optimal policies and strategies. Since each essay investigates an independent topic, I will briefly summarize the motivation, methodology, findings, and contribution of each essay in this chapter, and leave a more comprehensive discussion of the research topic to each chapter.

The first essay in Chapter 2 is motivated by the rapid development of high-speed rail (HSR) in the past decade, which has greatly promoted regional connectivity. Although HSR and air transport have long been considered as competitors, the complementarity between the two modes has been gradually enhanced. One source of such complementarity is that HSR facilitates the formation of multi-airport regions by linking airports that are previously independent of each other. As a result, the complementarity between the two transportation modes may serve as a way to efficiently re-allocate traffic within the newly generated multi-airport region and relieve hub airport congestion. The complementarity between the two modes in a multi-airport region can be further enhanced by certain types of cooperation. A particular scheme of cooperation has been observed in both Asia and Europe: airlines or airports cooperate with the HSR operator to offer intermodal passengers discounts. This cooperation can be achieved when the operators of the two modes share the revenue from the multi-modal program. Chapter 2 thus develops an analytical framework to investigate the effects of air-rail revenue sharing in a multi-airport region, with each operator retaining its own objective function. We investigate an air-rail revenue sharing mechanism under various scenarios and show that when the HSR operator is social welfare oriented, or when the airlines are collusive in the multi-airport region, the revenue sharing agreement is more likely to be reached. Furthermore, airport congestion has an important implication for air-rail revenue sharing in the
sense that such cooperation is welfare-enhancing by efficiently diverting passengers from congested airports to uncongested ones. Finally, the Pareto efficient range and Pareto improvement range of revenue sharing amount are identified, and the implication for negotiations between the two transport operators are discussed. This chapter contributes to the existing literature on air-rail intermodality by investigating a specific air-rail revenue sharing mechanism and by modeling the strategic behavior of HSR in a multi-airport region, which have not been well captured and explored.

The second essay in Chapter 3 is motivated by the alarming rate of climate change and the threats imposed to vulnerable transportation infrastructure. Maritime transport is responsible for over 80 per cent of global trade by volume, and more than 70 per cent of trade by value (UNCTAD, 2017). Seaports thus provide crucial linkages in global supply-chains and serve a vital function within the local, regional, and global economy. However, due to their location along shorelines, seaports are particularly vulnerable to coastal natural disasters and extreme weather events associated with sea level rise and storm activities. Thus, adaptation to reduce the impacts of climate change is inevitable. In the meantime, maritime transport has been growing, despite rising climate-related threats. Many ports around the world are undergoing or plan to undergo expansions in response to the growth in demand and expanded vessel sizes. As a result, seaports worldwide have to reconcile two types of investments: investment in throughput capacity (henceforth “capacity investment”) and investment in adaptation to climate change (henceforth “protection investment”). Capacity investment is essential to accommodate rising traffic volumes, reduce congestion, and maintain long-term competitiveness, while protection investment improves the seaports’ resilience to climate change. Determining when and how much to invest is challenging, due to the irreversibility of the two investments (in the sense that they cannot be recovered once made), and the uncertainty associated with climate change. With this as background, this chapter investigates the optimal timing and scale of protection and capacity investments of seaports, given uncertainty about the probability and magnitude of climate-change-related disasters. We show that a higher disaster probability or magnitude reduces equilibrium capacity and traffic volume, but may increase or decrease protection. In terms of protection investment, the port prefers to wait if the probability of a disaster can change a lot. It also prefers to wait if the disaster probability is currently low in order
to avoid overinvestment if the climate remains relatively benign. However, unless the present-value cost of protection investment declines in the future, the port will not hold back on current protection investment and plan to invest more in all future states. As far as capacity, the port prefers to postpone capacity investment if the probability of a disaster can fall, but prefers to invest in advance if the climate is likely to get worse. The results are similar for private and public ports. This chapter contributes to the existing literature on climate change adaptation and investment under uncertainty by considering the interdependence of capacity and protection investments for seaports that are congestion-prone in a dynamic model with information accumulation.

The third essay in Chapter 4 aims to identify factors affecting passenger choice of airport ground transportation modes using aggregate data. Incheon International Airport (ICN), known as the gateway of Korea, is connected to several major Korean cities by high-speed trains and intercity buses. The data I received from ICN records the number of passengers on each train and bus, the scheduled and actual arrival times for each flight, and the number of passengers on each flight. A limitation of the data is that it does not identify individual passenger’s choice. The challenge is that, without individual-level data, classical methods used to estimate consumer choice, such as the multinomial logit model, cannot be applied. Therefore, I develop a structural estimation method adaptable to this data. My method estimates the ridership of each train and bus based on a structural discrete choice model. This approach essentially converts unobserved individual choice probability to the observable ridership. The model coefficients are computed by the nonlinear least-squares estimation which minimizes the distance between estimated and observed ridership. The estimation results suggest that passengers who fly Korean airlines or have experienced longer waiting time are less likely to take public transportation, while passengers who fly low-cost airlines are more likely to take public transportation. In addition, waiting time has a much stronger negative impact than in-vehicle travel time on passengers’ choice of public transportation. This approach allows counterfactual analysis to estimate what ridership of airport ground transport services would be if the schedules were adjusted. In the current practice, the train schedules are not efficiently matched with either the intraday demand or passengers’ choice. Thus, I formulate a nonlinear program with the estimated parameters to determine the optimal train schedule with the objective of maximizing the train ridership. I demonstrate that the proposed schedule solved from
the nonlinear program could increase the train ridership by 31.3% without adding extra trains. By counterfactual analysis, I also estimate the monetary gains or losses of increasing or decreasing train frequency in consumer surplus and the train operator’s revenue. The estimates can provide insights for the train operator to determine the optimal train frequency. This chapter contributes to the existing literature on mode choice modeling and scheduling by proposing and estimating a structural discrete choice model with aggregate-level data (while the existing literature mostly uses individual-level data), and by proposing a nonlinear optimization program that takes into account passenger choice behavior in scheduling. The estimation results also deliver important information for understanding airport passengers’ choice behaviors and allow airport managers to optimize public transport schedules to achieve higher efficiency.
Chapter 2: Air-rail revenue sharing in a multi-airport system

2.1 Introduction

The rapid development of high-speed rail (HSR) in the past decade has greatly promoted regional connectivity by linking airports that are previously independent of each other. Although the rapid expansion of HSR networks is considered largely as a threat to air transport, a less recognized fact is that complementarity between the two modes has been gradually enhanced. One source of such complementarity is that HSR facilitates the formation of a multi-airport system (MAS), thus efficiently re-allocating traffic within the newly generated MAS and relieving airport congestion. For instance, the TEN-T (The Trans-European Transport Networks) Priority Project 2, completed in 2012, involved a rail link connecting Brussels Airport with the existing Brussels-Antwerp-Amsterdam and Paris-Brussels-Köln-Amsterdam-London HSR networks. The completion of the link helps to enhance co-modality, improve accessibility, and reduce congestion.

The complementarity between air and HSR in an MAS can be further enhanced by certain types of cooperation. A particular scheme of air-HSR cooperation has been observed amid the creation of MAS: airlines or airports cooperate with the HSR to offer air-HSR multi-modal passengers free HSR tickets, with an objective of attracting traffic from nearby airports. This cooperation can be achieved when the HSR shares part of the revenue with the airlines. For example, since 2013, Spring Airlines, the first and largest low-cost carrier (LCC) in China, has been providing Beijing

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1 Empirical studies have found that competition from HSR has exerted a downward pressure on airfares, flight frequencies, and air traffic (e.g., Albalate and Bel, 2012; Givoni and Dobruszkes, 2013; Dobruszkes et al., 2014; Albalate et al., 2015; Wan et al., 2016; Chen, 2017; Wang et al., 2018). Recent studies have also identified complementarity between air transport and HSR (e.g., Brida et al., 2017; Qin, 2018; Zhang et al., 2018; Liu et al., 2018).


3 This type of air-HSR cooperation shows certain features that are distinct from the more fully integrated multimodal services, such as integrated ticketing, baggage handling, schedule coordination, and delay warranties. In our case, airlines/airports and the HSR do not need to integrate their service or do joint marketing. This is a low level of air-HSR cooperation, but is easier to achieve without much investment needed (e.g., integrate the ticket reservation systems, etc.). See Jiang et al. (2017) and Li et al. (2018) for the discussion on the levels of air-HSR partnerships.
passengers who choose Shijiazhuang Airport (SJW) as their origin airport free HSR tickets between Beijing and Shijiazhuang. Beijing and Shijiazhuang are 280 km (kilometers) apart, with an HSR ride of about 1 hour and 20 minutes. Spring Airlines is not able to obtain regulatory approvals to enter Beijing Capital Airport (PEK), due to the restrictive regulations that strongly favor the state-owned Air China that uses the airport as its hub (Wang et al., 2017), whereas SJW is an airport with sufficient capacity, welcoming Spring Airlines to enter. Thus, the HSR linkage between Beijing and Shijiazhuang creates an MAS, allowing Spring Airlines to compete for air traffic from Beijing. A second example occurred in 2014 when Tianjin, a city 130 km away from Beijing, launched a similar program. Tianjin Airport (TSN) fully reimburses passengers their HSR tickets between Beijing and Tianjin if they fly from Tianjin. An HSR trip between Beijing and Tianjin takes only 40 minutes. In contrast to the program provided by Spring Airlines with the purpose to compete, the program launched by TSN aims to coordinate with PEK, a congested gateway with substantial air travel demand, so as to better allocate traffic (the two airports are owned by the same parent company). A similar air-HSR cooperation scheme has also been observed in Europe. Air France/KLM offers a free HSR ride (around 1 hour 30 minutes) from Brussels-Midi/Zuid railway station to Amsterdam Schiphol Airport (AMS). Free HSR tickets are also provided by Air France/KLM between Antwerp Central Station and AMS, with an HSR ride of 54 minutes, and between Brussels-Midi/Zuid railway station and Paris-Charles de Gaulle Airport (CDG), with an HSR ride of about 1 hour and 35 minutes. Passengers in Belgium who

4 There are several reasons why Spring Airlines did not choose TSN (which is closer to Beijing than SJW) to launch its air-HSR program. First, SJW and Shijiazhuang HSR station are located closer to each other compared with the case of Tianjin (5 vs. 25 minutes’ transportation time), thus facilitating easier air-HSR connection. Second, Shijiazhuang Municipal Government signed a “strategic cooperation and development” contract with Spring Airlines in 2011 and offered the airline favorable conditions to operate at the airport. The number of passengers served by Spring Airlines at SJW increased from less than 200,000 in 2011 to more than 2,000,000 in 2016.

5 There are 84 high-speed trains per day running from Beijing to Shijiazhuang (one-way), and 154 high-speed trains per day running from Beijing to Tianjin (one-way), featuring high HSR service frequencies, which may explain that schedule coordination or delay warranties of such air-HSR cooperation may not be necessary in China. According to the data from TSN, in 2014, more than 368,000 passengers utilized the HSR ticket reimbursement service at the airport.

6 There are multiple ways through which a free HSR ride can be provided to air-HSR multi-modal passengers. A passenger may first buy the HSR ticket, and then present it at the airport for a refund, as in the Tianjin case. Passengers may also book an air-HSR service through the airline, and pick up the free HSR ticket at the HSR station or the airport by showing the flight ticket and valid proof of identity, as in the case of Spring Airlines and Air France/KLM.
used to travel to/from Brussels Airport (BRU) can now choose to travel to/from AMS or CDG with the air-HSR service, intensifying the competition between AMS/CDG and BRU. In Taiwan, EVA Air partnered with Taiwan HSR Corporation (THSRC) to offer passengers (flying to Europe, Australia, New Zealand, or North America) from southern Taiwan a free HSR ride to Taoyuan, although the network structure does not feature an MAS as Taoyuan International Airport is the dominant international gateway in Taiwan.

Motivated by the commonly observed air-HSR cooperation, this chapter studies the effects of air-HSR revenue sharing in an MAS when the airlines offer transfer passengers free HSR tickets and the HSR shares revenues with the airlines. For a more comprehensive analysis, two scenarios are considered in order to investigate the conditions under which this revenue sharing mechanism can take place, and their corresponding market implications. The MAS could be either subject to airline competition so that the air-HSR multi-modal program is used by one airline to compete with another, or controlled by collusive airlines so that the air-HSR program is used to allocate air traffic between the two airports more efficiently. Both cases are observed in practice. As discussed above, for the competing case, we observe that Spring Airlines in China employs the air-HSR multimodal program to compete with major Chinese airlines at PEK, and Air France/KLM adopts the same strategy to compete with airlines at BRU. For the collusive case, we observe that TSN and PEK belong to the same parent company, and are both the hubs of Air China. The air-HSR program at TSN is thus intended to better allocate traffic between the two airports, rather than enhancing the competition. Therefore, in reality, air-HSR cooperation can facilitate either inter-airport competition or inter-airport coordination. Moreover, more and more MAS are formed because of the HSR linkage. For example, recently, Thailand approved a US$7.2 billion HSR project to link its three main airports: Don Mueang (DMK), Suvarnabhumi (BKK) and U-Tapao (UTP), with the

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7 In practice, the air-HSR revenue sharing program can be launched between airlines and HSR, or between airport and HSR, or among airlines, airports, and HSR. However, involving airport in such revenue sharing mechanism may generate further complications. One is the vertical relation between airport and airlines. That is, the pricing is determined in a vertical structure with the upstream airport charging the airline for using the infrastructure, while the downstream airline charging end consumers. The other is the ownership structure of airports, as the airport can be public (social welfare maximizing) or private (profit maximizing), while the airline industry is mostly deregulated or corporatized, thus featuring profit maximization. Therefore, for the sake of tractability, in this chapter, we focus on the revenue sharing between airlines and HSR, but we will keep on our research project along this line.
latter two being the hubs for Thai Airways. As BKK and DMK are both operating beyond capacity, UTP can become an alternative with the HSR linkage, serving the purpose of diverting traffic from Bangkok’s two congested airports, which may correspond to the case of collusive airlines in an MAS. The HSR between Guangzhou and Hong Kong that opened in September 2018 offers an opposite example. The HSR linkage may intensify competition between Hong Kong Airport (HKG) and Guangzhou Baiyun Airport (CAN), which serve respectively as the hubs for Cathay Pacific and China Southern Airlines, corresponding to the case of competing airlines in an MAS. Although any air-HSR multimodal program has not yet been in place for the above two examples, our analysis may shed light on the policy implications if such program were to take place, especially nowadays connecting airports with HSR networks has become a popular practice. In addition, we also modelled airport congestion which allows the possibility of diverting passengers from a more congested airport to a less congested one, since the airport that is engaged in the air-HSR multimodal program in general is less congested than the one not involved in such program. Last, we allow the HSR to have various types of ownership, by taking into account possibilities between profit maximization and social welfare maximization in its objective function. In terms of the revenue sharing amount for each free HSR ticket provided, we identify the Pareto efficient range within which the airline and the HSR can negotiate, with the equilibrium depending on the two parties’ bargaining power. We find that when the HSR is oriented towards social welfare maximization, or when the airlines are collusive in the MAS, a revenue sharing agreement is likely to be reached. When such a revenue sharing mechanism is in place, the transfer traffic is stimulated while the direct traffic is reduced. Furthermore, total traffic and consumer surplus in the network will always increase. With collusive airlines in the MAS, welfare strictly increases with revenue sharing, but with competing airlines, welfare may increase or decrease. Moreover, airport congestion also has an important implication on this revenue sharing mechanism in the sense that such cooperation is welfare-enhancing, owing to an efficient diversion of passengers from congested airports to less congested ones. Last, we show that economies of traffic density in airline industry enhance the possibility of revenue sharing.

The contributions of this study are multi-folds. In terms of methodology, it contributes to three streams of literature. First, for the air-HSR cooperation literature, this chapter serves as the first
study to investigate a specific air-HSR revenue sharing mechanism, complementing existing studies that relay almost exclusively on the assumption of air-HSR joint profit maximization (e.g., Socorro and Viecens, 2013; Jiang and Zhang, 2014; Takebayashi, 2016; Xia and Zhang, 2016, 2017; Jiang et al., 2017). Besides, most of these studies focus on a hub-and-spoke network, while the present study focuses on a network of an MAS. Second, for the MAS literature, existing studies on MAS focus largely on passengers’ airport choice (e.g., Furuichi and Koppelman, 1994; Windle and Dresner, 1995; Basar and Bhat, 2004; Hess and Polak, 2005), airport-airline choice (e.g., Pels et al., 2000, 2001; Ishii et al., 2009), access mode-airport choice (Pels et al., 2003), and flight schedule coordination (Takebayashi, 2012), but HSR’s strategic behavior in an MAS is not well captured and explored. The present chapter is one of the few studies to consider HSR as an active decision-maker in an MAS. Third, a key insight of our analysis is that air-HSR revenue sharing is essentially a type of third-degree price discrimination, because the HSR is able to identify those air-HSR transfer passengers from the local HSR passengers. This provides HSR an additional instrument to maximize its objective (i.e., social welfare or own profit). This insight is similar to the one pointed out by Bilotkach (2005) and Czerny (2009) in the literature on airline alliances. Our analysis is in a similar spirit, but the revenue sharing cooperation involves two different transport modes (HSR and air) in the context of an MAS.

8 Studies that incorporate HSR in an MAS include Terpstra and Lijesen (2015), Takebayashi (2015, 2018), and Takebayashi and Onishi (2018). Terpstra and Lijesen (2015) empirically show that larger airports with a strong market position can benefit more from HSR connection. Takebayashi (2015) finds that the degree of airport-HSR connectivity can affect the market shares of the two airports. Takebayashi (2018) studies the efficiency of an MAS with HSR connecting the airports. Takebayashi and Onishi (2018) discuss the policies for the reliever airport to regain traffic when the main airport is dysfunctional due to a catastrophe, with the two airports linked by HSR. However, these studies treat HSR linkage as an exogenous factor for reducing travel time/cost between airports; they have not taken into account the active decision-making of the HSR.

9 Czerny (2009) examines the price effects of a complementary airline alliance on both the transfer (“connecting”) and the local passengers. He finds that “code-sharing” between airlines – perhaps the most popular form of airline alliances – can serve as a means for price-discriminating local passengers from connecting passengers. This point has also been made previously by Bilotkach (2005) in a different code-sharing model, where the number of “stops” (resulted from code-sharing) serves as a measure of product differentiation and a price-discrimination method.

10 Our study is also related to recent studies on airport-airline revenue sharing and its impacts on airlines, airport, and social welfare (e.g., Fu and Zhang, 2010; Zhang et al., 2010; D’Alfonso and Nastasi, 2014). These studies focus on sharing revenue from “concession services” (i.e., non-aircraft related operations in terminals and on airport land) that are largely complementary to the demand for aviation services. This study only considers aviation demand and abstracts away concession services.
This study also offers several policy implications under various scenarios. First, when the HSR is more profit-oriented, the air-HSR revenue sharing agreement is less likely to form. In this case, government intervention, such as a subsidy to the airlines or the HSR, could be encouraged to facilitate the air-HSR cooperation which helps to relieve airport congestion and enhance social welfare. Second, when the revenue sharing amount is negotiated between the airline and HSR, the resultant equilibrium can depend on the bargaining power of the two parties. However, such a negotiated equilibrium may not lead to the greatest welfare. In such cases, a third part or government arbitrator could be deployed to help the airlines and the HSR reach an equilibrium that is closest to the social optimum.

The rest of the chapter is organized as follows. Section 2.2 sets up the basic model. The cases of competing airlines and collusive airlines are discussed in Section 2.2.1 and Section 2.2.2, respectively. Section 2.3 investigates the conditions for the revenue sharing mechanism to be achieved, and explores the effects of airport congestion and HSR ownership structure on the equilibrium. Section 2.4 extends the analysis to identify the Pareto improvement range and Pareto efficient range of the revenue sharing amount, and to account for endogenous HSR price and economies of traffic density in airline industry. Section 2.5 summarizes the study.

### 2.2 Basic model

Consider the simplest network structure with which our main questions on air-HSR revenue sharing in an MAS can possibly be addressed. The structure is depicted in Figure 2.1, where the region under consideration has two airports: airport 1 is a congestible airport, whilst airport 2 is an airport with lower utilization of capacity (i.e., traffic volume divided by the airport designed capacity), and thus has no congestion and more incentive to attract traffic.\(^\text{11}\) Destination 3

\(^{11}\) For instance, in the Kansai region of Japan, there are two major airports, namely Itami Airport (smaller capacity) and Kansai Airport (larger capacity). Due to its convenient location within the Osaka city, Itami Airport is preferred by Kansai passengers, thus being very congested. While Kansai airport is less congested and has incentive to attract traffic from Itami through express rail (with about an 1.5-hour ride). Both airports are the hubs of All Nippon Airways and Japan Airlines.
aggregates the common (outside of the region) destinations served by airlines at airports 1 and 2. An HSR line links airports 1 and 2 that are located close to each other, rendering air service commercially unviable between them.\textsuperscript{12}

![Diagram of network structure of an MAS with HSR link]

Figure 2.1 The network structure of an MAS with HSR link

Passengers in the origin-destination (OD) market between city 1 and city 3 can either fly directly, or fly via airport 2 with the 1-2 leg served by HSR and the 2-3 leg served by air.\textsuperscript{13} Without HSR linkage, however, passengers in market 1-3 can only fly directly. A simple analysis can show that with HSR linkage, the total traffic volume in the OD market 1-3 increases and social welfare also increases no matter air-HSR cooperation is in place or not. Thus, the HSR linkage is pro-competitive by offering an alternative to travel in market 1-3, and further creates a regional MAS. In the following analysis, we focus on the OD market 1-3 and abstract away the OD market 2-3.\textsuperscript{14} As a result, the HSR linkage may have the potential to help airport 2 compete with airport 1, or to better re-allocate the air traffic in the MAS.

\textsuperscript{12} We consider that Destination 3 is far away so that the HSR linkage between cities 1 and 3 is either less competitive than airlines or infeasible (e.g., the overseas market). In this sense, we abstracted away the head-on competition between HSR and airlines in the OD market 1-3. This also helps us focus on the main economic insights for the air-rail revenue sharing mechanism in the MAS.

\textsuperscript{13} It is possible that traffic from airport 2 is routed through airport 1 with the linkage of HSR. In those cases, airport 1 is usually not subject to serious congestion while economies of traffic density are strong. To simplify the analysis, we abstract away this possibility.

\textsuperscript{14} We note that some of our conclusions do not alter when including the OD market 2-3, but some of the results become dependent on the market size of market 2-3. Incorporating market 2-3 into the analysis results in much more tedious derivations, however.
As shown in Figure 2.1, we refer to the airline operating on route 1-3 as “airline 1”, and the airline operating on route 2-3 as “airline 2”. To carry out a comprehensive analysis as mentioned earlier, we consider that airline 1 and airline 2 can either be independent (referred to as “competing airlines”) or belong to one single decision-maker (referred to as “collusive airlines”). As indicated earlier, we consider air-HSR cooperation through a setting in which the airline offers the transfer passengers free HSR tickets, and the HSR shares some of the revenue with the airline. With competing airlines, air-HSR cooperation is assumed to be reached between the HSR and airline 2. When the two airlines compete, airline 2 could be aggressive to attract passengers away from airport 1 by offering free HSR tickets. With collusive airlines, it is however irrelevant which airline offers free HSR tickets as the two maximize their joint profit.

For the benchmark case where no cooperation between HSR and airline is reached, we introduce a variant of quadratic net utility function as proposed by Singh and Vives (1984). For a representative passenger in the OD market 1-3, the utility function is specified as follows: \[ U = \alpha_1(q_1 + q_T) - \frac{1}{2}(q_1^2 + q_T^2 + 2\beta q_1 q_T) - q_1 P_1 - q_T(P_2 + P_{HSR} + t) - \lambda q_1^2, \] (2.1)

where \( q_1 \) is the direct air traffic in market 1-3, \( q_T \) is the transfer air traffic in market 1-3 via airport 2, \( \alpha_1 \) indicates the potential market size of the OD market 1-3, \( \beta \) indicates the substitutability between the two alternatives with \( 0 < \beta < 1 \), which captures the horizontal differentiation between direct travel from airport 1 and transfer travel via airport 2, \( P_1 \) is the airfare of direct flight on route 1-3, \( P_2 \) is airfare of direct flight on route 2-3, and \( P_{HSR} \) is the HSR price on route 1-3. Travel in the OD market 1-3 via airport 2 also incurs an extra time cost \( t \), which include the extra HSR travel time, transfer time between HSR station and airport, as well as the travel time difference between routes 2-3 and 1-3. We assume that the flying time is the same between routes 2-3 and 1-3 (net of congestion), so \( t \) can be interpreted as the HSR transfer time cost. Time cost \( t \) captures the vertical differentiation between the direct flight and transfer via airport 2. Since airport

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15 This approach has been widely used in transportation literature (e.g., Fu et al., 2006; Flores-Fillol and Moner-Colonques, 2007; Oum and Fu, 2007; Socorro and Viecens, 2013; Jiang and Zhang, 2014; D’Alfonso et al., 2015, 2016).
can be congested, passengers may also incur a quadratic congestion cost of $\lambda q_1^2$. The parameter $\lambda$ measures the passenger’s sensitivity and disutility to airport congestion. We abstract away the congestion cost incurred by airline 1 for model tractability. However, relaxing this assumption does not change the main results, as will be carefully discussed in Section 2.3.

With the representative passenger maximizing a net utility $U$ with respect to $q_1$ and $q_T$, the demand functions without air-HSR cooperation are given in the below equations:

$$q_1 = \frac{\alpha_1 - P_1 + \beta(P_2 + P_{HSR} + t - \alpha_1)}{1 - \beta^2 + 2\lambda}$$  \hspace{1cm} (2.2)

$$q_T = \frac{\beta(P_1 - \alpha_1) + (2\lambda + 1)(\alpha_1 - P_2 - P_{HSR} - t)}{1 - \beta^2 + 2\lambda}$$  \hspace{1cm} (2.3)

As can be observed from Eq. (2.2) and Eq. (2.3), a high HSR price or transfer time cost discourages transfer travel demand while encourages direct travel demand. A large market size enhances both direct and transfer travel demand, but congestion at airport 1 decreases direct travel demand. As will be discussed in detail, the revenue sharing takes the form that for each free HSR ticket the airline provides, the HSR will return a fixed amount to the airline. When the local HSR traffic between cities 1 and 2 is abstract away, the HSR price and the return amount is co-linear in the HSR’s objective function in the sense that only the difference between the two matters. Treating both the HSR price and the return amount as decision variables would result in multiple equilibria. Thus, we treat the HSR price as fixed and the return amount as the decision variable in the following analysis. The exogenous HSR price can also be justified for two reasons. First, HSR price in many countries does not vary as much as airfares, especially when the HSR is publicly owned or controlled by the government (e.g., China, Japan, South Korea, Taiwan).\footnote{For example, in Japan, there is a JPY200 (US$1.8) discount during off-peak season and a JPY200 (US$1.8) surcharge during peak season for the Tokaido line, but other than that the price is fixed. In South Korea, the HSR prices from Seoul to Daegu, Daejeon and Busan are fixed. In China, the HSR ticket price is imposed with a baseline price of 0.45 RMB/km with a maximum of only 5% fluctuation (Wang et al., 2017). In Taiwan, HSR offers only a limited quantity of early bird discount, but when the early bird tickets are sold out, regular and fixed fares apply.} Second, the air-HSR transfer passengers might only account for a small portion compared to the local HSR traffic, thus imposing a negligible impact on $P_{HSR}$. In Section 2.4.2, we take into account the local
HSR traffic and endogenize HSR price by numerical analysis. We show that the main results still hold.

When the airline provides free HSR ticket for passengers transferring via airport 2, the net utility function becomes Eq. (2.4), as transfer passengers do not need to pay for the HSR ride.

\[
U = \alpha_1 (q_1 + q_T) - \frac{1}{2} (q_1^2 + q_T^2 + 2\beta q_1 q_T) - q_1 P_1 - q_T (P_2 + t) - \lambda q_1^2
\]  
(2.4)

The demand functions with air-HSR cooperation can be derived by applying \( P_{HSR} = 0 \) in Eq. (2.2) and Eq. (2.3). We consider a three-stage game for the air-HSR cooperation. At the first stage, the airline decides whether to initiate cooperation with the HSR in which case it will provide free HSR ticket for transfer passengers. At the second stage, the HSR decides whether to cooperate, and if so, how much revenue it would like to share with the airline. Specifically, the HSR can return an amount \( \delta \) to the airline for each free HSR ticket, thus sharing \( \delta q_T \) of its total revenue \( P_{HSR} q_T \) with the airline. In this way, the HSR essentially charges each transfer passenger \( P_{HSR} - \delta \), lower than the full HSR price \( P_{HSR} \) paid by local HSR passengers.\(^{17}\) It is thus equal to a third-degree price discrimination because the HSR is able to identify the market segment (local vs. transfer passengers) and charges different prices.\(^{18}\) It is apparent that the HSR can be better off under this third-degree price discrimination, because the benchmark case with no revenue sharing is essentially a special case with the \( \delta \) value set to be zero.\(^{19}\) At the third stage, given the revenue sharing amount \( \delta \), the two airlines decide their airfares simultaneously. In Section 2.4.1, we extend

\(^{17}\) Although the specific contract terms between the HSR and airlines/airports of this air-HSR cooperation program are not publicly available, we propose a possible revenue sharing mechanism, which could be a practical air-HSR cooperation approach. This is meaningful especially when existing studies on air-HSR cooperation almost exclusively impose joint profit maximization.

\(^{18}\) Avenali et al. (2018) examine two air-HSR partnerships without a full air-HSR integration: (i) an agreement under which the airline buys train seats and sells the intermodal product (service), and (ii) an agreement where a joint venture is formed to be in charge of the intermodal product. The authors find that both agreements increase traffic in the network, and the joint-venture agreement benefits air passengers independent of hub congestion and mode substitution. However, Avenali et al. (2018) consider one hub airport with the HSR feeding traffic to spoke destinations. The present study, instead, focuses on an MAS with efficient re-allocation of traffic among airports made possible by the HSR-airport connection.

\(^{19}\) It should be noted that if HSR does not share any revenue, the airline incurs the full cost of the free HSR tickets provided to transfer passengers. It can be shown that the airline will pass 100% of the HSR ticket price to airfare, thus leading to the same outcome as the benchmark case of no revenue sharing.
our discussion to allow $\delta$ to be negotiated between airlines and the HSR, and we show that the main findings are consistent. In the next sections, we analyze the market equilibria with competing airlines and with collusive airlines.

### 2.2.1 Equilibrium with competing airlines

We first derive the equilibrium for the benchmark case where no free HSR ticket is provided. Consider that airline 1 and airline 2 compete in price to maximize their own profits: $\pi_1 = (P_1 - c_1)q_1$ and $\pi_2 = (P_2 - c_2)q_T$, where $q_1$ and $q_T$ are derived in Eq. (2.2) and Eq. (2.3).\(^{20}\) We normalize airlines’ marginal costs to zero ($c_1 = c_2 = 0$) for model tractability. This assumption will be relaxed in Section 2.4.2, where we conduct numerically analysis to account for economies of traffic density in airline industry. The equilibrium prices and quantities with competing airlines in the benchmark case are provided in Appendix A.1.

In the case of revenue sharing, we solve the model by backward induction. At the third stage, the two airlines maximize their own profits, $\pi_1 = P_1q_1$ and $\pi_2 = (P_2 - P_{HSR} + \delta)q_T$, with respect to price $P_1$ and $P_2$. The optimal $P_1$ and $P_2$ can be solved as a function of $\delta$ in the below equations:

$$P_1 = \frac{(2\lambda + 1)(\alpha_1(2 - \beta) + \beta(P_{HSR} + t - \delta)) - \alpha_1\beta^2}{4 - \beta^2 + 8\lambda},$$

$$P_2 = \frac{\beta^2(t - \alpha_1) + 2(2\lambda + 1)(P_{HSR} - t + \alpha_1 - \delta) - \alpha_1\beta}{4 - \beta^2 + 8\lambda}.$$  

(2.5)  
(2.6)

The best response functions show that a larger $\delta$ reduces both $P_1$ and $P_2$, indicating that revenue sharing intensifies competition between airline 1 and 2. At the second stage, the HSR maximizes a weighted sum of profit and social welfare $\omega \pi_{HSR} + (1 - \omega)SW$ with respect to $\delta$, where $\omega \in [0,1]$ is the weight on the HSR’s own profit.\(^{21}\) At the first stage, airline 2 decides whether it would

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\(^{20}\) The formulation of Bertrand competition follows the same approach of, among others, Bilotkach (2005) and Adler and Smilowitz (2007).

\(^{21}\) The similar approach (i.e., HSR is assumed to maximize a weighted sum of social welfare and profit) has been used in Basso and Zhang (2010), Yang and Zhang (2012), and D’Alfonso et al. (2015). These studies recognize a budget constrained or partly corporatized public owned transport entity (airport, and railway) bearing both social welfare and own profit in its objective.
like to initiate the collaboration with the HSR given the amount $\delta$ that the HSR would offer at the second stage. The equilibrium results are collated in Appendix A.1.

### 2.2.2 Equilibrium with collusive airlines

When airlines 1 and 2 belong to the same company, they maximize joint profit, i.e., $\max_{P_1,P_2} P_1 q_1 + P_2 q_T$, where the demand functions of $q_1$ and $q_T$ are given in Eq. (2.2) and Eq. (2.3). We thus can solve for the equilibrium in the benchmark case, which is relegated in Appendix A.1.

In the revenue sharing case, airline 2 provides free HSR ticket to transfer passengers and the two airlines maximize their joint profit, $P_1 q_1 + (P_2 - P_{HSR} + \delta)q_T$, with a revenue $\delta q_T$ shared from the HSR. The best response functions given $\delta$ are derived in the below equations:

\[
P_1 = \frac{\alpha_1}{2} \tag{2.7}
\]

\[
P_2 = \frac{1}{2} (P_{HSR} - t + \alpha_1 - \delta) \tag{2.8}
\]

As shown in Eq. (2.7) and Eq. (2.8), collusive airfare will be charged on route 1-3 regardless of the revenue sharing amount from the HSR. This is different from the case of competing airlines, in which the revenue sharing amount will lower airfares on both route 1-3 and route 1-2 due to competition effect. At the second stage, the HSR maximizes a weighted sum of its own profit and the social welfare by choosing $\delta$, given the airlines’ best responses.\(^{22}\) At the first stage, the two airlines decide whether to implement the air-HSR cooperation, given the revenue sharing amount from the HSR. The equilibrium outcome is provided in Appendix A.1.\(^{23}\)

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\(^{22}\) In our model, the differences between the two transport modes (air vs. HSR) are characterized in the following aspects: (i) Airlines are maximizing profits while the HSR is maximizing a weighted sum of profit and social welfare, which corresponds to the fact that the airline industry is mostly privatized or corporatized while the HSR sector is more likely to be publicly owned. (ii) Congestion is considered for the airport but not for HSR, because compared with airport runway capacity, rail track capacity is much less constrained, especially for HSR operations. (iii) The two transport modes serve different types of routes (air serves long-haul while HSR serves short-haul). However, some features are not considered in the model. For example, trains can carry much more seats than airlines owing to the limited size of the aircraft. We assume away this capacity difference, but will consider this feature in our future work.

\(^{23}\) Second-order conditions are all satisfied.
2.3 Analysis

In this section, we investigate the conditions for the revenue sharing agreement to be reached, under both competing airlines and collusive airlines in the MAS. We also explore the effects of airport congestion at airport 1 and the ownership structure of the HSR.

2.3.1 Conditions for revenue sharing agreement

The revenue sharing agreement can be achieved only when both the HSR and the airline are better off from such cooperation. It can be shown that the HSR is always better off when it can determine the revenue sharing amount \( \delta \), because the HSR can actively engage in a third-degree price discrimination, having an additional instrument to optimize its objective. The airline will benefit from such agreement (i.e., its profit will increase) only when \( \delta \) is positive. With some tedious algebra, the condition for the endogenous variable \( \delta \) to be positive can be broken down into the relations among exogenous variables \( P_{HSR}, t, \omega \).

Specifically, with competing airlines, if the HSR is sufficiently social welfare maximizing (i.e., \( 0 \leq \omega < \omega^D \), air-HSR cooperation can be formed when the transfer time cost \( t \) is sufficiently small (\( t \leq \bar{t}^D \)). If \( t \) is not sufficiently small (\( t > \bar{t}^D \)), the HSR price \( P_{HSR} \) should be sufficiently high (\( P_{HSR} > \bar{P}_{HSR}^D \)) for achieving the cooperation.\(^{24}\) When the transfer cost \( t \) is sufficiently small, a welfare-oriented HSR is more likely to encourage transfer traffic than a profit-oriented HSR. With the air-HSR cooperation, transfer passengers pay less, which prompts airline 1 to reduce airfare for direct passengers. The social benefit of the intensified competition outweighs the social loss from the reduced profit of airline 1, incentivizing HSR to share revenue with airline 2. However, when \( t \) is large, the HSR will share revenue with airline 2 only if the HSR ticket price \( P_{HSR} \) is very expensive. In this case, the high HSR ticket price discourages transfer traffic to a socially sub-optimal level, such that a welfare-oriented HSR has incentive to stimulate transfer traffic through revenue sharing with airline 2. In contrast, when the HSR is more profit-oriented (i.e., \( \bar{\omega}^D \leq \omega \leq 1 \)), air-HSR cooperation can be formed if and only if \( P_{HSR} \) is high enough (i.e.,

\(^{24}\) The mathematical expressions for the boundary values (e.g., \( \bar{\omega}^D, \bar{t}^D, \bar{P}_{HSR}^D \)), and those in the following analysis, are all collated in Appendix A.1.
$P_{\text{HSR}} > \bar{P}_{\text{HSR}}$. When the HSR price is very high, the HSR is willing to cooperate with airline 2 because more profits can be generated from an increase of transfer traffic by reducing the price for transfer passengers through revenue sharing. However, when the HSR price is low, the HSR does not have an incentive to cooperate, because the existing transfer traffic is already considerably high, and the HSR does not have any incentive to further increase it by sharing revenue with the airline. It is also noticed that this condition does not depend on $t$, because when the HSR is sufficiently profit maximizing, it does not care much about the transfer time cost $t$ incurred by transfer passengers. As will be illustrated in Figure 2.2 in Section 2.3.2, the condition for a profit-oriented HSR (i.e., $\omega^D \leq \omega \leq 1$) to reach revenue sharing agreement is a subset of the condition for a welfare-oriented HSR (i.e., $0 \leq \omega < \omega^D$). Thus, when the HSR cares more about its own profit, the condition for revenue sharing becomes more restrictive.

With collusive airlines, we show that when the HSR is sufficiently social welfare maximizing (i.e., $0 \leq \omega < 1/3$), revenue sharing agreement can always be achieved, while when the HSR is sufficiently profit maximizing (i.e., $1/3 \leq \omega \leq 1$), revenue sharing agreement can be achieved only if the HSR price is sufficiently high ($P_{\text{HSR}} > \bar{P}_{\text{HSR}}$). Unlike the competing airlines, a welfare-oriented HSR is always willing to cooperate so as to reduce the airfares, because high airfares from collusive airlines dominate the effects of transfer time cost $t$ and HSR price $P_{\text{HSR}}$. When the HSR is profit-oriented, the intuition for reaching revenue sharing agreement is basically the same as discussed in the case of competing airlines. In addition, similar as the case of competing airlines, the air-HSR cooperation with collusive airlines is also more likely to be achieved when $\omega$ is small.

The analytical conditions can be linked to real-world cases. Specifically, the HSR in China is more welfare maximizing (the HSR price is low under government regulation), whereas the HSR in Europe may be regarded as more profit maximizing (the HSR price is relatively high compared with the prices of LCCs in Europe). In the case of Spring Airlines (more related to competing airlines and welfare maximizing HSR in our model), since the transfer time cost is low, the revenue sharing can be achieved. In the case of Tianjin Airport (more related to collusive airlines and welfare maximizing HSR in our model), the revenue sharing can be achieved, not affected by the
HSR price per se. In the case of Air France/KLM (more related to competing airlines and profit maximizing HSR in our model), the revenue sharing could also be feasible since the HSR service is expensive. To summarize the above conditions for air-HSR cooperation, we have Proposition 1.25

**Proposition 1:**
An air-HSR revenue sharing agreement is more likely to be reached with a welfare-oriented HSR in an MAS.

We next compare the conditions for air-HSR cooperation between competing airlines and collusive airlines. In the following analysis, we use superscript $R$ to denote the equilibrium for the case of revenue sharing and duopoly (i.e., competing airlines), and superscript $M$ for the case of revenue sharing and monopoly (i.e., collusive airlines). Similarly, superscript $B$ and $D$ denote the case of benchmark (i.e., no air-HSR cooperation) and duopoly (i.e., competing airlines) and the case of benchmark and monopoly (i.e., collusive airlines), respectively. We find that $\delta_{RM} > 0$ (i.e., achieving cooperation with collusive airlines) is necessary but not sufficient for $\delta_{RD} > 0$ (i.e., achieving cooperation with competing airlines), meaning that if air-HSR cooperation can be achieved when airlines are competing, it can also be achieved when airlines are collusive, but not vice-versa. In addition, we have $\delta_{RM} > \delta_{RD}$. Proposition 2 is thus reached as follows:

**Proposition 2:**
(i) An air-HSR revenue sharing agreement is more likely to be reached with collusive airlines than with competing airlines. (ii) The revenue sharing amount per passenger provided by the HSR is larger with collusive airlines than with competing airlines.

To see the intuition for Proposition 2, we note that *ceteris paribus* the collusive airlines charge higher airfares at both airports than the competing airlines do, leading to both smaller transfer and

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25 The proofs of the propositions are relegated to Appendix A.2.
direct traffic. Air-HSR cooperation can effectively reduce the airfares at both airports, thus stimulating overall traffic. Since transfer and overall traffic is smaller with collusive airlines, the HSR is thus more motivated to engage in revenue sharing program regardless of the weight on profit or social welfare. The same intuition applies when comparing the revenue sharing amount $\delta$ per transfer passenger. In order to effectively increase traffic, the HSR offers more revenue sharing amount to collusive airlines than to competing airlines, because of the higher airfares charged by the collusive airlines.

Table 2.1 Equilibrium comparison between benchmark and revenue sharing cases

<table>
<thead>
<tr>
<th>Competing airlines</th>
<th>Collusive airlines</th>
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</thead>
<tbody>
<tr>
<td>airline 1</td>
<td>airline 1</td>
</tr>
<tr>
<td>airline 2</td>
<td>airline 2</td>
</tr>
<tr>
<td><strong>Price</strong></td>
<td></td>
</tr>
<tr>
<td>$p_{1BD} &gt; p_{1RD}$</td>
<td>$p_{1BM} = p_{1RM}$</td>
</tr>
<tr>
<td>$p_{2BD} \geq p_{2RD}$</td>
<td>$p_{2BM} \geq p_{2RM}$</td>
</tr>
<tr>
<td>$p_{2BD} + p_{HSR} &gt; p_{2RD}$</td>
<td>$p_{2BM} + p_{HSR} &gt; p_{2RM}$</td>
</tr>
<tr>
<td><strong>Traffic volume</strong></td>
<td></td>
</tr>
<tr>
<td>$q_{1BD} &gt; q_{1RD}$</td>
<td>$q_{1BM} &gt; q_{1RM}$</td>
</tr>
<tr>
<td>$q_{2BD} &lt; q_{2RD}$</td>
<td>$q_{2BM} &lt; q_{2RM}$</td>
</tr>
<tr>
<td>$Q_{BD} &lt; Q_{RD}$</td>
<td>$Q_{BM} &lt; Q_{RM}$</td>
</tr>
<tr>
<td><strong>Profit</strong></td>
<td></td>
</tr>
<tr>
<td>$\pi_{1BD} &gt; \pi_{1RD}$</td>
<td>$\pi_{1BM} &gt; \pi_{1RM}$</td>
</tr>
<tr>
<td>$\pi_{2BD} &lt; \pi_{2RD}$</td>
<td>$\pi_{2BM} &lt; \pi_{2RM}$</td>
</tr>
<tr>
<td>$\pi_{HSR} \geq \pi_{HSR}$</td>
<td>$\pi_{HSR} \geq \pi_{HSR}$</td>
</tr>
<tr>
<td>$\pi_{1BD} + \pi_{2BD} \geq \pi_{1RD} + \pi_{2RD}$</td>
<td>$\pi_{1BM} + \pi_{2BM} &lt; \pi_{1RM} + \pi_{2RM}$</td>
</tr>
<tr>
<td>$\pi_{total} \geq \pi_{total}$</td>
<td>$\pi_{total} \geq \pi_{total}$</td>
</tr>
<tr>
<td><strong>Welfare</strong></td>
<td></td>
</tr>
<tr>
<td>$CS_{BD} &lt; CS_{RD}$, $SW_{BD} \geq SW_{RD}$</td>
<td>$CS_{BM} &lt; CS_{RM}$, $SW_{BM} &lt; SW_{RM}$</td>
</tr>
</tbody>
</table>

Table 2.1 compares equilibrium outcomes of benchmark and revenue sharing cases under both competing airlines and collusive airlines. As shown in Table 2.1, a direct passenger pays equal or lower price under air-HSR revenue sharing (i.e., $p_{1BM} = p_{1RM}, p_{1BD} > p_{1RD}$), and a transfer passenger pays strictly less under revenue sharing (i.e., $p_{2Bi} + p_{HSR} > p_{2Ri}$, where $i = D$ or $M$). This demonstrates a price reduction effect of such revenue sharing mechanism.\textsuperscript{26} The transfer

\textsuperscript{26} Note that despite the similarity (cooperation between two monopolies and revenue sharing between airlines and HSR both reduce price), our result is essentially different from the elimination of double marginalization. Double marginalization is an externality that results from separate pricing of two monopolies (one upstream, one downstream). This externality can be eliminated by joint pricing which reduces the two separate markups to one (Tirole, 1988). But
traffic is also stimulated under revenue sharing (i.e., \( q_T^{Bi} < q_T^{Hi} \)), but the direct traffic is reduced (i.e., \( q_1^{Bi} > q_1^{Ri} \)). Nevertheless, total traffic in the network always increases (i.e., \( Q^{Bi} < Q^{Ri} \)), as well as consumer surplus (i.e., \( CS^{Bi} < CS^{Ri} \)), indicating that consumers benefit from such cooperation. Airline 2 also benefits from revenue sharing (i.e., \( \pi_2^{Bi} < \pi_2^{Hi} \)), which is essentially the participation constraint for achieving such agreement under competing airlines, but airline 1 is worse-off (i.e., \( \pi_1^{Bi} > \pi_1^{Ri} \)). The HSR’s profit may increase or decrease with revenue sharing (i.e., \( \pi_{HSR}^{Bi} \gtrless \pi_{HSR}^{Hi} \)), but the value of the HSR’s objective function (i.e., \( \omega \pi_{HSR} + (1 - \omega)SW \)) increases, which is again the participation constraint for such agreement. With collusive airlines, social welfare strictly increases with revenue sharing (i.e., \( SW^{BM} < SW^{RM} \)), but with competing airlines, social welfare may increase or decrease (i.e., \( SW^{BD} \gtrless SW^{RD} \)). The ambiguity in social welfare comparison can be attributed to the fact that the profit loss of airline 1 from revenue sharing may dominate the overall benefits to passengers, to airline 2 and to the HSR, resulting in a net social welfare loss. This will not happen with collusive airlines, because the profit loss of airline 1 will not be large enough to override the benefits due to the participation constraint that guarantees the two airlines overall benefit from the revenue sharing program. The result that only airline 1 does not benefit from the revenue sharing program can be attributed to the setup that the airline 1 is assumed to incur no congestion cost (for model tractability). If diverting traffic away benefits airline 1 by reducing the congestion at airport 1, social welfare comparison under competing airlines may no longer be ambiguous, with the potential result that \( SW^{BD} < SW^{RD} \). In addition, relaxing this setup does not change Proposition 1 and 2. First, since now the revenue sharing program benefits airline 1 as well, a social welfare-oriented HSR will have more incentive to engage in the cooperation, verifying Proposition 1. Second, since the participation constraint under collusive airlines is easier to be met because both airlines now benefit from the cooperation, while the participation constraint under competing airlines remains the same, Proposition 2(i) can be verified. Last, since collusive airlines will accept a lower \( \delta \), the difference between \( \delta^{RM} \) and \( \delta^{RD} \) our result is different in the following aspects. First, airlines and HSR do not make decisions jointly. Instead, they make their own decisions in different stages. Second, since airlines and HSR have their own objective functions, the joint profit of airlines and HSR is not maximized, different from the double-marginalization setup. Third, our model does not have two separate markups in the benchmark case, because HSR price is exogenous. Last, passengers in the market 1-3 have alternative travel options, unlike the double-marginalization setup, where going through the two monopolies is the only option.
will be smaller than when congestion cost of airline 1 is assumed away, but $\delta^{RM} > \delta^{RD}$ will still hold, verifying Proposition 2(ii).

2.3.2 Comparative statics

2.3.2.1 Effects of the congestion cost

In this subsection, we examine the impact of the congestion cost $\lambda$ incurred by passengers at airport 1. We illustrate the conditions discussed in Section 2.3.1 in Figure 2.2 (competing airlines) and Figure 2.3 (collusive airlines), where the shaded area indicates the range of parameters making revenue sharing feasible.

Figure 2.2 Conditions for revenue sharing with competing airlines and increasing congestion
We next analyze how the boundary values \((\bar{\omega}^D, \bar{P}^D_{HSR}, \bar{P}^M_{HSR})\) change with \(\lambda\). With \(0 \leq \omega < \bar{\omega}^D\), we can show that \(\bar{P}^D\) increases and \(\bar{P}^D_{HSR}\) decreases with \(\lambda\). As illustrated in Figure 2.2(a), the shaded area expands with \(\lambda\) when \(0 \leq \omega < \bar{\omega}^D\), implying that the revenue sharing is more likely to be formed with a larger \(\lambda\). We can further separate the range \(\bar{\omega}^D \leq \omega \leq 1\) into \(\omega \leq \bar{\omega}^D\), where \(\bar{P}^D_{HSR}\) may increase or decrease with \(\lambda\), and \(\bar{\omega}^D \leq \omega \leq 1\), where \(\bar{P}^D_{HSR}\) unambiguously increases with \(\lambda\). Therefore, as illustrated in Figure 2.2(c), with \(\omega \leq 1\), the shaded area shrinks with \(\lambda\), implying that the condition for air-HSR cooperation becomes more restrictive. However, when \(\omega \leq \omega \leq \omega^D\), the effect of congestion, as illustrated in Figure 2.2(b), is ambiguous. With collusive airlines, we show that \(\bar{P}^M_{HSR}\) increases with \(\lambda\), implying that when \(1/3 \leq \omega \leq 1\), the higher the congestion cost, the less likely the revenue sharing can be formed (as shown in Figure 2.3(b)), whereas when \(0 \leq \omega < 1/3\), congestion cost does not have an effect on the revenue sharing (as shown in Figure 2.3(a)). The results are summarized in Proposition 3.

**Proposition 3:**

(i) With competing airlines, when the HSR is sufficiently social welfare maximizing (i.e., \(0 \leq \omega < \bar{\omega}^D\)), the higher the congestion cost, the more likely air-HSR revenue sharing will be reached; while when the HSR is sufficiently profit-maximizing (i.e., \(\omega^D \leq \omega \leq 1\)), the higher the congestion cost, the less likely air-HSR revenue sharing will be reached.
(ii) With collusive airlines, when the HSR is sufficiently social welfare maximizing (i.e., $0 \leq \omega < 1/3$), air-HSR revenue sharing can always be achieved regardless of the congestion cost; while when the HSR is sufficiently profit-maximizing (i.e., $1/3 \leq \omega \leq 1$), the higher the congestion cost, the less likely air-HSR revenue sharing will be reached.

We provide some intuitions as follows. When passengers are sensitive to airport congestion (hence a large $\lambda$), revenue sharing should be encouraged to divert the traffic from the congested airport to the uncongested one, which is also socially optimal. As shown in Proposition 3(i), with competing airlines, when the HSR is welfare-oriented, the higher the congestion cost, the more motivated the HSR is to share revenue with airlines 2 in order to effectively divert traffic to the uncongested airport. However, congestion does not have an effect with collusive airlines and welfare-oriented HSR, since revenue sharing can always be achieved. However, if the HSR is profit-oriented, the higher the congestion cost, the less motivated the HSR is to engage in the revenue sharing, because the increasing congestion at the congested airport endogenously encourages passengers to choose the uncongested airport by taking HSR for transfer. Thus, the HSR does not have an incentive to cooperate with airlines. Propositions 3 implies that increasing airport congestion does not always make revenue sharing more feasible. The objective of the HSR plays an essential role here. Therefore, in such a case, government intervention can be called for, because diverting traffic to the uncongested airport can be social welfare improving. This finding suggests the necessity to consider the active decision making of the HSR in forming such cooperation, which has been ignored in the existing literature.

In addition, we derive how the equilibrium changes with the congestion cost parameter $\lambda$ under collusive airlines.\(^{27}\) We find that, in the revenue sharing case, when $0 \leq \omega \leq 1/3$, $\delta^{RM}$ increases, but $P_2^{RM}$ decreases with $\lambda$; while when $1/3 < \omega \leq 1$, $\delta^{RM}$ decreases, but $P_2^{RM}$ increases with $\lambda$. In addition, regardless of $\omega$, $q_1^{RM}$ always decreases with $\lambda$, $q_T^{RM}$ always increases with $\lambda$, and $Q^{RM}$, $CS^{RM}$ and $SW^{RM}$ may increase or decrease with $\lambda$. In the benchmark case, $q_T^{BM}$, $\pi_2^{BM}$, $\pi_{HSR}^{BM}$ increase with $\lambda$; $q_1^{RM}$, $Q^{BM}$, $\pi_1^{BM}$, $CS^{BM}$ decrease with $\lambda$; $SW^{BM}$ may increase or decrease with $\lambda$.

\(^{27}\) There is no analytically clear-cut result regarding how the equilibrium changes with $\lambda$ under competing airlines.
These findings are intuitive. Specifically, the transfer traffic always increases, while the direct traffic always decreases, with the congestion cost. In the revenue sharing case, when the HSR is welfare-oriented, the revenue sharing per passenger that the HSR offers ($\delta^{RM}$) increases with the congestion cost, because more revenue sharing is an effective way to divert traffic to the uncongested airport, which is welfare-enhancing. However, when the HSR is profit-oriented, $\delta^{RM}$ decreases with the congestion cost, because the transfer traffic is already large enough, leaving the HSR with less incentive to share revenue with airlines.

2.3.2.2 Effects of the HSR ownership structure

In this subsection, we investigate the effects of $\omega$, the weight that the HSR puts on own profit. Table 2.2 summarizes how the equilibrium changes with $\omega$. We summarize the main findings in Proposition 4.

<table>
<thead>
<tr>
<th>Market structure</th>
<th>Cases</th>
<th>Changes with parameter $\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competing airlines</td>
<td>$t &gt; \hat{t}^D$</td>
<td>$\delta^{RD}, q^{RD}<em>{T}, \pi^{RD}</em>{2}$, $p^{RD}<em>{1}, q^{RD}</em>{1}, \pi^{RD}<em>{1}, p^{RD}</em>{2}, SW^{RD}$, $Q^{RD}, CS^{RD}, \pi^{RD}<em>{HSR}$, $\omega \pi^{RD}</em>{HSR} + (1 - \omega) SW^{RD}$</td>
</tr>
<tr>
<td></td>
<td>$t &lt; \hat{t}^D$</td>
<td>$\delta^{RD}, q^{RD}<em>{T}, \pi^{RD}</em>{2}, p^{RD}<em>{1}, q^{RD}</em>{1}, \pi^{RD}<em>{1}, p^{RD}</em>{2}, SW^{RD}$, $Q^{RD}, CS^{RD}$, $\omega \pi^{RD}_{HSR} + (1 - \omega) SW^{RD}$</td>
</tr>
<tr>
<td></td>
<td>$\omega^{RD}, q^{RD}<em>{T}, \pi^{RD}</em>{2}$, $p^{RD}<em>{1}, q^{RD}</em>{1}, \pi^{RD}<em>{1}, p^{RD}</em>{2}, SW^{RD}$</td>
<td>$\omega \pi^{RD}_{HSR} + (1 - \omega) SW^{RD}$</td>
</tr>
<tr>
<td>Collusive airlines</td>
<td>$-\omega^{RM}, q^{RM}<em>{T}, \pi^{RM}</em>{2}$, $p^{RM}<em>{1}, q^{RM}</em>{1}, \pi^{RM}<em>{1}, p^{RM}</em>{2}, SW^{RM}$, $Q^{RM}, CS^{RM}$, $\omega \pi^{RM}_{HSR} + (1 - \omega) SW^{RM}$</td>
<td>$\omega^{RM}, \omega \pi^{RM}<em>{HSR} + (1 - \omega) p^{RM}</em>{1}$</td>
</tr>
</tbody>
</table>
**Proposition 4:**

(i) With competing airlines, when the transfer time cost $t$ is sufficiently low (high), the more welfare-oriented the HSR is, the larger (smaller) the revenue sharing amount per passenger is provided to the airline;

(ii) With collusive airlines, the more welfare-oriented the HSR is, the larger the revenue sharing amount per passenger is provided to the airlines regardless of the transfer time cost.

With competing airlines, the amount of revenue sharing per passenger offered by the HSR ($\delta^{RD}$) is not monotone in $\omega$. When $t$ is large ($t > \hat{t}^D$), a welfare-oriented HSR (hence with a smaller $\omega$) intends to discourage the transfer traffic by reducing $\delta$, because the high additional transfer time cost makes air-HSR revenue sharing less attractive from a social welfare perspective. In contrast, when $t$ is sufficiently large, a profit-oriented HSR (hence with a larger $\omega$) intends to encourage the transfer traffic by increasing $\delta$, because the transfer time cost is less of a concern for the HSR while the impeded transfer traffic harms its profit. Other variables change with $\delta$ accordingly. For instance, with an increasing $\delta$, airfares at the two airports ($P_{1}^{RD}$ and $P_{2}^{RD}$) decrease, because of the intensified competition. As a result, the transfer traffic $q_{T}^{RD}$ increases, while the direct traffic $q_{1}^{RD}$ decreases, but total traffic still goes up. Passengers are overall better off because they pay strictly less, which also outweighs the transfer time cost. Intuitively, airline 2 benefits from more revenue sharing amount, but airline 1 is worse off. However, when $t$ is sufficiently small, the opposite will happen. Specifically, a welfare-oriented HSR intends to encourage transfer traffic by offering more revenue sharing amount to airline 2, whereas a profit-oriented HSR tends to discourage transfer traffic by offering smaller $\delta$.

With collusive airlines, we find that $\delta^{RM}$ decreases with $\omega$ regardless of transfer time cost, which indicates that a more welfare-oriented HSR always provides a larger revenue sharing amount to the collusive airlines in order to encourage transfer traffic. With an increasing $\delta$, other variables change accordingly as in the case of competing airlines, except that $P_{1}^{RM}$ is subject to no effect because the collusive airfare is always charged. No matter airlines are competing or collusive, the HSR’s profit increases with $\omega$, while social welfare decreases with $\omega$, which is intuitive. However,
the HSR’s objective function decreases with $\omega$, implying that the effect of $\omega$ on social welfare dominates that on the HSR’s profit.

2.4 Model extensions

In this section, we extend our discussion to (1) identify the Pareto efficient range and Pareto improvement range of revenue sharing amount, and (2) account for endogenous HSR price and airline economies of traffic density by numerical analysis.

2.4.1 Pareto improvement and efficient ranges of revenue sharing amount

By comparing the equilibrium between the cases where the HSR and the airlines decide $\delta$, we can obtain the Pareto improvement range of $\delta$ where increasing $\delta$ benefits both airlines and HSR, and the Pareto efficient range of $\delta$ where the revenue sharing scheme should be reached through negotiation, with the realized $\delta$ depending on the bargaining power of the HSR and the airlines. It is possible to use a Nash bargaining model to quantify the impact of bargaining power on the equilibrium $\delta$, but, unfortunately, closed-form analytical results cannot be obtained. Nevertheless, we are still able to qualitatively infer the impact of bargaining power on the equilibrium $\delta$. The detailed equilibrium analysis is relegated to the Appendix A.3. The main intuitions are discussed here.

We show than the objective function of HSR is concave in $\delta$, while the objective function of the airlines is convex in $\delta$. We plot the objective functions of both the HSR and the airlines as a function of $\delta$ in Figure 2.4. For easier reference in following discussion, we use “HSR-proposing” to represent the case where the HSR decides $\delta$, and “air-proposing” to represent the case where the airlines decide $\delta$. In Figure 2.4, $\delta^{HSR}$ and $\delta^{Air}$ denote the equilibrium revenue sharing amount in the HSR-proposing case and air-proposing case, respectively, and $\delta^{Indiff}_{HSR}$ denotes the $\delta$ that makes the HSR indifferent between revenue sharing and no revenue sharing. Since the result in

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28 In Appendix A.4, we briefly discuss how to use a Nash bargaining model to quantify the effect of bargaining power.

29 Since the transfer traffic is 0 for $\delta = \bar{\delta}$, the profit of the airlines is 0 for $\delta \leq \bar{\delta}$, as shown in Figure 2.4.
this subsection holds under both competing airlines and collusive airlines, we omit the superscript for the two cases in Figure 2.4.

\[ \delta^H = 0 \text{ is equivalent to the benchmark case (no cooperation)} \]

\[ \delta = 0 \text{ is equivalent to the benchmark case (no cooperation)} \]

**Figure 2.4 Objective functions of HSR and airlines in the revenue sharing amount**

In the HSR-proposing case, the airlines will accept the \( \delta^H \) proposed by the HSR, because the airlines are strictly better off than the case where \( \delta = 0 \) (i.e., the benchmark case with no revenue sharing). In the case of air-proposing, since the airlines’ profit is monotonically increasing in \( \delta \), the airlines will propose a \( \delta \) as large as possible, as long as the participation constraint of the HSR is satisfied (\( \delta^{Air} = \delta_{indif}^{HSR} \)). As illustrated in Figure 2.4, the coalition, which is defined as the set for Pareto improvement (i.e., both players can be better-off than the endowment or benchmark), is \([0, \delta^H]\), and the core, which is defined as the set for Pareto efficiency (i.e., neither player can
be better-off without making the other worse-off), is \([\delta^{HSR}, \delta^{Air}]\). Thus, the equilibrium \(\delta\) subject to negotiation falls in the range \([\delta^{HSR}, \delta^{Air}]\), with the realized value depending on the bargaining power of the HSR and the airlines. When the HSR (airline) has more bargaining power, the equilibrium \(\delta\) is closer to \(\delta^{HSR}\) (\(\delta^{Air}\)). Note that the analytical results in Section 2.3 still hold in the air-proposing case.

### Table 2.3 Comparison between HSR proposing \(\delta\) and airline proposing \(\delta\)

<table>
<thead>
<tr>
<th></th>
<th>Competing airlines</th>
<th>Collusive airlines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>(p_{1}^{RD-H} &gt; p_{1}^{RD-A})</td>
<td>(p_{1}^{RM-H} = p_{1}^{RM-A})</td>
</tr>
<tr>
<td></td>
<td>(p_{2}^{RD-H} &gt; p_{2}^{RD-A})</td>
<td>(p_{2}^{RM-H} &gt; p_{2}^{RM-A})</td>
</tr>
<tr>
<td>Traffic volume</td>
<td>(q_{1}^{RD-H} &gt; q_{1}^{RD-A})</td>
<td>(q_{1}^{RM-H} &gt; q_{1}^{RM-A})</td>
</tr>
<tr>
<td></td>
<td>(q_{T}^{RD-H} &lt; q_{T}^{RD-A})</td>
<td>(q_{T}^{RM-H} &lt; q_{T}^{RM-A})</td>
</tr>
<tr>
<td></td>
<td>(Q^{RD-H} &lt; Q^{RD-A})</td>
<td>(Q^{RM-H} &lt; Q^{RM-A})</td>
</tr>
<tr>
<td>Profit</td>
<td>(\pi_{1}^{RD-H} &gt; \pi_{1}^{RD-A})</td>
<td>(\pi_{1}^{RM-H} &gt; \pi_{1}^{RM-A})</td>
</tr>
<tr>
<td></td>
<td>(\pi_{2}^{RD-H} &lt; \pi_{2}^{RD-A})</td>
<td>(\pi_{2}^{RM-H} &lt; \pi_{2}^{RM-A})</td>
</tr>
<tr>
<td></td>
<td>(\pi_{HSR}^{RD-H} \leq \pi_{HSR}^{RD-A})</td>
<td>(\pi_{HSR}^{RM-H} &gt; \pi_{HSR}^{RM-A})</td>
</tr>
<tr>
<td>Consumer surplus</td>
<td>(CS^{RD-H} &lt; CS^{RD-A})</td>
<td>(CS^{RM-H} &lt; CS^{RM-A})</td>
</tr>
<tr>
<td>Social welfare</td>
<td>(SW^{RD-H} \leq SW^{RD-A})</td>
<td>(SW^{RM-H} \leq SW^{RM-A})</td>
</tr>
</tbody>
</table>

Note: The superscript “-H” and “-A” indicates the equilibrium in the case of HSR-proposing and air-proposing, respectively.

We compare the equilibrium between HSR-proposing and air-proposing in Table 2.3. Airfares are lower with air-proposing than with HSR-proposing because of the larger revenue sharing amount gained by the airlines. Transfer traffic and total traffic are both stimulated with air-proposing, but direct traffic decreases. Passengers and airline 2 both benefit from the air-proposing, but airline 1 is worse-off. Air-proposing reduces the HSR’s profit under collusive airlines, but the effect is ambiguous under competing airlines. The social welfare comparison is also ambiguous, with the result depending on the HSR’s weight on its profit \(\omega\), HSR price \(P_{HSR}\), and transfer time cost \(t\). If the HSR is a pure welfare-maximizer, HSR-proposing case leads to higher social welfare than the
air-proposing case, which is intuitive. However, if the HSR puts some positive weight on its own profit, there is no clear-cut analytical result on social welfare comparison.

### 2.4.2 Endogenous HSR price and economies of traffic density

In this subsection, we conduct numerical analysis to examine the effects of endogenous HSR price and economies of traffic density in the airline industry. To consider endogenous HSR price, we have to reformulate the model by taking into account the local HSR market between cities 1 and 2 in Figure 2.1. This is because when the HSR can adjust its price with air-HSR cooperation for transfer traffic, its price for the local passengers is changed correspondingly, affecting the HSR’s total profit. Not taking into account the local market 1-2 would result in multiple equilibria in the revenue sharing case if both the HSR price and $\delta$ are endogenous. The parameter calibration and detailed analysis are relegated to Appendix A.5 and A.6. Here we discuss the main findings.

We first investigate how the local HSR market size $\alpha_{HSR}$ will affect the conditions for revenue sharing. We show that when the HSR tends to be profit-maximizing (i.e., $\omega$ is large) and when the size of the HSR local market $\alpha_{HSR}$ is small, revenue sharing cannot be achieved. This is because the small $\alpha_{HSR}$ limits the HSR to charge high $P_{HSR}$. As revenue sharing would further reduce the price paid by transfer passengers, a profit-maximizing HSR does not have an incentive to engage in revenue sharing. We next investigate how $\alpha_{HSR}$ affect the revenue sharing amount $\delta$, and show that $\delta$ is higher when $\alpha_{HSR}$ is larger, because a larger $\alpha_{HSR}$ contributes to a higher $P_{HSR}$, which enables the HSR to provide a larger $\delta$. Last, we examine the effect of economies of traffic density in airline industry. We find that the stronger the economies of traffic density, the less welfare-oriented the HSR has to be to achieve revenue sharing. The intuition is that when economies of traffic density is strong, airlines have incentive to boost air traffic in the network due to the decreasing marginal cost. As the revenue sharing program increase both transfer air traffic and overall air traffic, airlines are willing to accept a smaller $\delta$ to reach cooperation with the HSR than when economies of traffic density is weak or not present, thus imposing less constraint on the

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30 See Basso and Jara-Díaz (2006) for a detailed discussion of the differences among economies of density, economies of scale, and economies of scope.
welfare-maximizing nature of the HSR. Therefore, economies of traffic density enhance the possibility for revenue sharing. We also show numerically in the Appendix A.5 that the analytical results in Section 2.3 do not change when taking into account endogenous HSR price and economies of traffic density in airline industry.

2.5 Concluding remarks

This chapter analyzes an air-HSR cooperation mechanism connecting an MAS, where the airline provides transfer passengers free HSR tickets and the HSR shares revenue with the airline for each HSR ticket. In contrast to the commonly assumed joint profit maximization in air-HSR cooperation studies, our proposed mechanism has low transaction cost and does not require full air-HSR integration. Such revenue sharing also provides the HSR with an additional instrument to practice third-degree price discrimination between local HSR passengers and air-HSR transfer passengers. The conditions for the revenue sharing formation, given either collusive or competing airlines, are explored. We find that the air-HSR revenue sharing can be achieved as long as the HSR is willing to share a positive amount of revenue with the airline. Whether this condition can be met depends on the HSR price, transfer time cost and the HSR’s weight on social welfare. Specifically, we find that when the HSR is more profit-oriented, regardless of competing airlines or collusive airlines, air-HSR cooperation can be achieved only when the HSR price is too expensive, which impedes the transfer traffic. However, with competing airlines, when the HSR is more welfare-oriented, air-HSR revenue sharing can be achieved either when the transfer time cost is sufficiently low, in which case transfer should be encouraged from a social welfare optimal perspective, or when both the transfer time cost and the HSR price are sufficiently high, in which case the transfer traffic is discouraged to a socially sub-optimal level and thus should again be stimulated. With collusive airlines, when the HSR is more welfare-oriented, air-HSR revenue sharing can always be achieved because the high airfares dominate the effects of the transfer time cost and the HSR price. Since airfares at both airports are lower with competing airlines, air-HSR cooperation, ceteris paribus, is more likely to be achieved with collusive airlines as a way to reduce airfares and stimulate transfer traffic. Furthermore, we find that a welfare-oriented HSR has more incentive to share revenue with the airline than a profit-oriented HSR.
We also find that the airport congestion has important implications on air-HSR revenue sharing. Specifically, such cooperation is conducive to social welfare by more effectively diverting passengers from congested airports to less congested ones. However, when the HSR is more profit-oriented, an increasing congestion at one airport makes revenue sharing less likely to occur, because the fact that passengers endogenously choose the uncongested airport gives a profit-oriented HSR no incentive to further expand transfer traffic. Government interventions may thus be needed to help achieve cooperation that can be welfare enhancing. Last, we extend our study to find the Pareto efficient range of revenue sharing amount for the HSR and the airlines to negotiate. It is found that the amount of revenue sharing per passenger is larger in the case of air-proposing than in the case of HSR-proposing. Although passengers benefit more from air-proposing, the social welfare comparison is ambiguous. We also find by numerical analysis that strong economies of traffic density in airline industry enhance the possibility for revenue sharing. Last, by conducting a case study that uses real-world data to calibrate the model parameters, we show that taking into account endogenous HSR price and the local HSR market does not qualitatively change the results of the study.

However, some limitations exist, mainly due to our model simplifications for tractability. First, in order to focus on the function of air-HSR inter-modality in an MAS, we ignore the potential competition between the airlines and the HSR. It is well expected that the implication of such inter-modality on competition and hence social welfare will be very different and a lot more intricate. Second, we abstract away the vertical structure between airlines and airports to focus on intermodal cooperation between airlines and the HSR. However, airport-airline vertical relation could in fact have implications on the formation of revenue sharing with the HSR. For example, it is conjectured that a revenue sharing agreement initiated by an airport might be somewhat different from that by an airline. The inter-airlines competition within one airport or across two airports in the MAS can also be an important avenue for future investigations. In addition, we assume the airlines to be profit maximizing, because airlines are mainly private entities. However, airports can have various ownership types, probably having partial or full government ownership. We did not explore this aspect on airport ownership because we have already modelled the diverse ownership of the HSR,
and discussion on airport ownership can significantly complicate the model derivation. The future study can consider the airport to maximize a weighted aggregate of profit and social welfare.
Chapter 3: Port adaptation to climate change and capacity investments under uncertainty

3.1 Introduction

Climate change is manifest as global warming, changing precipitation patterns, rising sea levels, and more frequent extreme weather events. The evidence of ongoing climate change is irrefutable, and much (and perhaps most) of it is attributable to human activity (IPCC, 2014a). Records show that the mean sea-level rise has increased from 1.7 millimetres per year (mm/y) between 1901 and 2010 to 3.2 mm/y between 1993 and 2010 (IPCC, 2013). Sea levels are expected to continue to rise throughout the 21st century if global warming continues at its current rate. Climate change has significant economic, social, and environmental impacts (See, for example, Cho et al., 2001; Gasper, et al., 2011; Carleton and Hsiang, 2016; Tol, 2018). To reduce the negative impacts of climate change, there are two main policy responses – mitigation and adaptation. As the transportation sector is a significant and growing contributor to greenhouse gas emissions (GHG), there is a huge literature on how the transportation sector can minimize its emissions. Furthermore, there are various actions to reduce carbon emissions in the transportation sector, such as increasing the use of renewable fuels; setting emission regulations; designing more fuel-efficient aircrafts, ships, and road vehicles; promoting electric vehicles and greener transportation modes (e.g., walking or biking); and better planning of routes and logistics. However, the literature on how transportation infrastructure can better adapt to climate change still lags behind the literature on abatement, and adaptation itself has also been slow to gain momentum. One reason is that clear

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31 IPCC (2014a) confirms that human influence on the climate system is clear and growing, with impacts observed across all continents and oceans. Many of the observed changes since the 1950s are unprecedented over decades to millennia. The IPCC is now 95 percent certain that humans are the main cause of current global warming.

32 Mitigation is a human intervention to reduce the sources or enhance the sinks of greenhouse gases (IPCC, 2014b), whilst adaptation is the process of adjustment to actual or expected climate and its effects (IPCC, 2014c). Mitigation, by addressing the root causes, is necessary to reduce the rate and magnitude of climate change, while adaptation is essential to reduce vulnerability to the harmful effects of climate change that cannot be avoided.

33 For instance, the transportation sector accounted for the largest portion (29%) of total U.S. GHG emissions in 2017 (EPA, 2019). By comparison, the contributions of other sectors were: electricity (28%), industry (22%), agriculture (9%), commercial (6%), and residential (5%).
performance measures for adaptation are not as tangible as measures for mitigation such as emissions standards. Another reason is that the great uncertainties associated with climate changes make investment decisions difficult.

This paper investigates the optimal timing and scale of adaptation (i.e., protection) and capacity investments of seaports, given uncertainty about the probability and magnitude of climate-related disasters. Maritime transport is responsible for over 80 per cent of global trade by volume, and more than 70 per cent of trade by value (UNCTAD, 2017). Seaports thus provide crucial linkages in global supply-chains and serve a vital function within the local, regional, and global economy. However, due to their location along shorelines, seaports are particularly vulnerable to coastal natural disasters and extreme weather events associated with sea level rise and storm activities (Chang, 2000; Chang and Nojima 2001; Becker et al., 2013; Nursey-Bray et al., 2013; Scott et al., 2013). The past decade has witnessed increasingly frequent coastal natural disasters and climate-related disruptions around the world, causing substantial social and economic costs. For instance, Hurricane Katrina in 2005 severely damaged the Port of Gulfport, Mississippi. Hurricane Sandy in 2012 disrupted maritime operations and damaged facilities at the Port of New York and New Jersey. The entire port was closed for a week, and total damages to its facilities were estimated at USD 2.0 billion (AON Benfield Corporation, 2013; Sturgis et al., 2014). The ports of North Carolina experienced an 11-day closure due to Hurricane Florence in September 2018, and later that month operations at the Port of Hong Kong were suspended for several days because of Typhoon Mangkhut.

Despite rising climate-related threats, maritime transport has been growing. According to UNCTAD (United Nations Conference on Trade and Development) estimates, the total volume of world maritime trade reached 11 billion tons in 2018: an all-time high. In addition, an estimated 793.26 million TEUs (twenty-foot equivalent units) in 2018 were handled in container ports worldwide: an increase of 35.3 million TEUs over 2017 (UNCTAD, 2019). UNCTAD projects
that international maritime trade will continue rising at a compound annual growth rate of 3.4 per cent over the 2019–2024 period.\textsuperscript{34}

Many ports around the world are undergoing or plan to undergo expansions in response to the growth in demand and expanded vessel sizes. The Port of Prince Rupert, a fast growing port for transpacific trade in North America, has agreed to an expansion plan that will increase its current capacity from 1.35 million TEUs to 1.8 million TEUs by 2022. Due to the expanded Panama Canal, most ports serving the US container market are in varying stages of planning for expansions in order to handle larger ships and increased volume of container traffic (Fan et al., 2012; Wang and Pagano, 2015).\textsuperscript{35} For instance, in February 2020, the Georgia Ports Authority announced plans to double capacity at the Port of Savannah to 9 million TEUs by 2030.\textsuperscript{36} South Korea plans to invest more than US$35Bn to expand cargo-handling capacity at 12 ports to 48.73 million TEUs by 2040 from 27.17 million TEUs in 2017. The Vietnamese and Thai governments are also considering expanding their ports, as many companies are constructing new factories in the two countries (Chen and Liu, 2016).

As a result, ports worldwide have to reconcile two types of investments: investment in throughput capacity (henceforth “capacity investment”) and investment in adaptation to climate change (henceforth “protection investment”). Capacity investment is essential to accommodate rising traffic volumes, reduce congestion, and maintain long-term competitiveness (Luo et al., 2012), while protection investment improves ports’ resilience to climate change. However, ports currently appear to be focused only on capacity investment by developing or upgrading their facilities to keep up with increasing shipping traffic. Becker et al. (2012) found that only 3.2% of the port

\textsuperscript{34} Despite trade tensions and growth in protectionism, the global outlook on maritime trade volumes tends to be positive. See Table 1.13 of UNCTAD (2019) for a summary of international maritime trade volume forecasts for 2017–2026 by different research institutions and agencies.

\textsuperscript{35} Wang and Pagano (2015) examine the effects of Panama Canal expansion on nine container ports in the US.

\textsuperscript{36} Port of Savannah, a major seaport in Southeast U.S., handled 8.5 percent of U.S. containerized cargo volume in 2017. See the press release of their major capacity expansion plans: \url{https://gaports.com/press-releases/gpa-unveils-major-expansions/}.
authorities they surveyed planned to build protective structures, and 22% had no plans to develop them within the next 10 years.

Shippers prefer to operate at ports with sufficient capacity to avoid lengthy delays. Shippers and port authorities alike also prefer port facilities that are resilient to extreme weather and other shocks. However, capacity and protection are very costly, and determining optimal levels of investments in them is a difficult problem because overinvestment and underinvestment are both dangers. Both types of investment are irreversible in the sense that they cannot be recovered once made. Consequently, there is an option value to waiting until the case for investment is clear. However, investments take time. Postponing capacity investments can result in congestion and lost business. Similarly, postponing protection investments may prove disastrous if a disaster happens because it will be too late to act. Due to the uncertainties associated with the likelihood and severity of climate-change-related disasters at the regional and local level, planning and financing protection investment are especially challenging in terms of the scale of the required investments, and the need to time them appropriately. In addition, port capacity and protection investments are interdependent in the sense that a larger port may need more protection measures.

With this as background, we investigate how a private or public port’s choice of fee and capacity affects its choice of protection, and how its choice of fee and protection affects its choice of capacity. We also explore when a port prefers to invest in capacity and protection early on, and when it prefers to hold off and wait for better information. We begin with a one-period model to investigate the capacity, protection, and pricing decisions of the port under uncertainty. We first analyze the “partial equilibrium” in which the port optimizes only one decision variable at a time while holding the other two fixed. We then consider the “full equilibrium” in which the three decision variables are optimized simultaneously. We show that for both public and private ports, the port charge and capacity are submodular, while capacity and protection are supermodular, and the charge and protection are also supermodular. In partial equilibrium, a higher disaster probability or magnitude induces the port to reduce the charge and capacity, but increase protection. In full equilibrium, the disaster probability and other parameters affect each decision variable both directly and indirectly via the other decision variables. The direct effect and indirect effect may go
in opposite directions. Using specific disaster functions, we show that a higher disaster probability or magnitude reduces equilibrium capacity and traffic volume, but has an ambiguous effect on protection. We also show that the comparative statics results in partial equilibrium and full equilibrium can be different, which highlights the importance of considering the two types of investment together – especially for ports that are expanding capacity to accommodate growing demand.

We then extend the static model to a two-period model in order to investigate the optimal timing of investments. On the one hand, because of uncertainty about the speed of climate change and the irreversibility of infrastructure investment, it can be advantageous to wait for better information before investing. On the other hand, if protection investments are not made in a timely manner, port activities can be disrupted, infrastructure can be destroyed, and traffic can be lost. According to Global Risks Report (2019), in the case of flooding events total spending on recovery is almost nine times higher than on prevention measures taken earlier on. Thus, there is a trade-off between making large and early investments for protection against near-term disasters, and postponing investment to reduce the present-value costs and avoid wasting resources in the event that climate change does not proceed as quickly as feared. We incorporate information accumulation in the model by assuming that the port has better knowledge of potential disasters in the second period based on the realized state in the first period. We find that unless the present-value cost of protection declines, the port will invest in protection in period 1, and invest further in period 2 only if the risk of disaster in period 2 is high. We also show that the port is more likely to add capacity in a low-risk state and to add protection in a high-risk state. In addition, we show numerically that the port prefers to postpone capacity investment if the probability of a disaster can fall, but to invest in advance if the climate is likely to get worse. In terms of protection investment, the port prefers to wait if the probability of a disaster can change a lot. It also prefers to wait if the disaster probability is currently low in order to avoid overinvestment if the climate remains relatively benign. All these results hold for both private and public ports.

The study offers managerial insights for ports and their stakeholders to develop appropriate capacity and adaptation management strategies to climate change. Although the focus is on
seaports, the general lessons also apply to protection investments for other types of vulnerable infrastructure such as airports, railway lines, coastal roads, and buildings and transportation facilities built on permafrost that is in danger of melting.

The rest of the chapter is organized as follows. Section 3.2 reviews the literature. Section 3.3 sets up the static model, and analyzes the pricing, capacity, and protection investment decisions of the port both independently and simultaneously, considering separately public and private ports. Section 3.4 describes the two-period model, and explores it both analytically and numerically. Section 3.5 concludes.

3.2 Literature review

This research is related to three streams of literature: (1) irreversible investment under uncertainty, (2) optimal capacity and pricing decisions of congestible facilities, and (3) adaptation strategies to climate change.

In their seminal work on investment under uncertainty, Dixit and Pindyck (1994) highlight the option value of waiting for better information when investment is irreversible. Most of the subsequent literature has assumed that uncertainty is manifest as an exogenous parameter that evolves over time according to geometric Brownian motion. Optimal investment timing is determined by a threshold of the stochastic process (either output price or demand) above which the investment is undertaken (Dixit, 1989). Many studies have followed this approach to analyze optimal timing and scale of irreversible investments. Among others, Dixit (1995) incorporates scale economies in the classic models of irreversible investment. Bar-Ilan and Strange (1996) take into account time to build in the investment decisions. Décamps et al. (2006) consider the choice among two alternative investment projects of different scales under output price uncertainty. These papers focus on the investment decisions of an individual firm or a single project. Extensions have been made to introduce competition. Balduressson (1998) studies an oligopoly where firms facing stochastic demand use capacity as a strategic variable. Novy-Marx (2007) incorporates firm heterogeneity in firms’ investment timing decisions. Huisman and Kort (2015) considers
investment timing and scale as entry deterrence/accommodation strategies in a duopoly setting with uncertain demand. Pricing is not the core issue in these studies because firms are assumed to be price takers in their input and output markets.\(^{37}\) In contrast, pricing can be important for ports which typically have market power, and can alleviate congestion by raising their fees.

Pricing and capacity investment decisions for congestible facilities have been widely studied in various contexts. For example, De Borger and Van Dender (2006) study the strategic interactions between two congestible facilities that make sequential decisions on capacities and prices. Zhang and Zhang (2006) consider congestion pricing and capacity choice of an airport with a focus on the vertical airline-airport market structure. Basso and Zhang (2007) incorporate downstream market structure when investigating rivalry between congestible facilities. De Borger et al. (2007) study pricing and investment decisions on a congested transport corridor where the elements of the corridor are controlled by different governments. Balliauw et al. (2019) consider investment timing of two ports engaging in Cournot competition under demand uncertainty in a leader-follower game. Except for Balliauw et al. (2019), these studies employ one-period models, and none considers adaptation investment to climate change or information accumulation under uncertainty.

Studies on port adaptation strategies are now emerging. Xiao et al. (2015) examine the optimal timing of protection investment over a two-period horizon. They assume that the probability of disasters is known more precisely in period 2 than period 1, and investigate whether protection investment should be made in period 1 or period 2. Their model features a cost-minimizing landlord port and does not consider port charges. Wang and Zhang (2018) incorporate duopoly port competition in protection investment, and examine the impacts of competition on the scale of a port’s investment in a one-period model. Randrianarisoa and Zhang (2019) extend Wang and Zhang (2018) to a two-period model. Like Xiao et al. (2015), they assume that ports can only invest in protection in either period 1 or period 2, not both. Asadabadi and Miller-Hooks (2018)  

\(^{37}\) Studies in the real options literature have considered endogenous output prices, but their focus is on non-storable/perishable commodities (e.g., Balduresson 1998, 1999; Aguerrevere, 2003). Our study considers the pricing of a congestible facility.
use a single-period model to investigate port protection investment in a maritime network characterized by co-opetition. Chan et al. (2016) use Monte Carlo simulation to show that the dynamic, learning-based investment strategy which updates estimates of storm surges is especially advantageous when the underlying storm generation process is either unknown, or growing in intensity. McDaniels et al. (2008) propose a framework to identify factors that can improve the resilience of infrastructure systems to extreme events and to characterize alternatives regarding mitigation and adaptation strategies.

No study has yet considered both protection and capacity investments, and the interactions between them. We do so here. First, we consider a one-period model, and then a two-period model in which a port can undertake each type of investment in period 1, period 2, or both periods.

3.3 The one-period model

In this section, we first characterize demand for port services. We next analyze the port’s decisions on capacity and protection investment, as well as the port charge. With regard to development in port governance, there is a global trend towards greater private-sector participation in both port service operations and port infrastructure financing (UNCTAD, 2016). We thus first analyze the decisions of a private port that maximizes profit, and then conduct the same analysis for a public port that maximizes social welfare (thus the “first-best” outcome). We compare the decisions of a private port and a public port to shed light on whether regulations in port activities in pricing and investments are needed.

38 According to Rodrigue (2020), there are five main port management models based upon the respective responsibilities of the public and private sectors. They include the public-service port, the tool port, the landlord port, the corporatized port and the private service port. The port authority of public service ports performs the whole range of port-related services. The tool port differs from the public-service port only by the private handling of its cargo operations. Landlord ports represent the most common management model in which infrastructure is leased to private operating companies with the port authority retaining ownership of the land. The port authority of corporatized ports essentially behaves as a private enterprise, but the ownership remains public. The port authority of private service ports is entirely privatized with almost all the port functions under private control.
3.3.1 Demand for port services

Our model features a congestible seaport (hereafter “port”) facing climate-change-related disasters, and atomistic users of the port. The generalized cost for users is the sum of three components: (1) port charge, (2) congestion cost at the port facilities, and (3) damage cost to the vessel or cargo if a maritime disaster happens. Let $\tau$ denote the fee that the port charges for per unit cargo shipped. The congestion cost that shippers incur at the port depends on the demand for the use of port services and the port’s capacity. We assume the congestion cost takes the following function form:

$$g(K, q; \delta) = \delta \frac{q}{K},$$

(3.1)

where $q$ is total demand, $K$ is the port’s capacity investment or “capacity” for short, and $\delta, \delta > 0$, is an exogenous parameter that depends on time cost of delay. Eq. (3.1) is homogeneous of degree zero in usage and capacity. This is a standard assumption in the literature on congestible facilities (e.g., Arnott and Kraus, 1993; de Palma and Lindsey, 1998; Lindsey, 2009). The same congestion function specification has been used in, for example, De Borger and Van Dender (2006), Basso and Zhang (2007), Xiao et al. (2007) and De Borger et al. (2008).

If a disaster happens at the port, shippers also incur a damage cost per unit of cargo of $f(K, I; \theta, m)$, where $I$ is the port’s protection investment, $\theta$ is a measure of effectiveness of the protection, and $m$ is a measure of the severity of a disaster. Note that we use a semicolon (;) to separate exogenous parameters from endogenously-determined variables. Shippers decide how much to use the port based on the expected generalized price

$$p(q) = \tau + g(K, q; \delta) + x f(K, I; \theta, m),$$

(3.2)

where $x$ is the perceived probability of a disaster. The port and shippers have a common prior about $x$ based on scientific research, as well as historical data on sea level rise, weather, and extreme events.

The disaster damage function is assumed to have the following properties: 39

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39 The assumptions in Eqns. (3.3) – (3.4) apply if $I$ and $K$ are both strictly positive. Some of the assumptions do not apply with $I = 0$ or $K = 0$. These corner solutions are ruled out as uninteresting.
According to Eq. (3.3), the disaster cost borne by shippers increases with capacity, decreases with protection, and is convex in both capacity and protection. This formalizes the logic that a larger port has more infrastructure that could be damaged with knock-on effects on shippers. Protection reduces disaster damage for shippers, but is subject to decreasing returns. The inequality $\frac{\partial^2 f}{\partial l^2} > 0$ implies that the marginal benefit from protection for shippers is greater for a larger port. The effects of parameters $m$ and $\theta$ on function $f$ are recorded in Eq. (3.4). Since $m$ is a measure of damage severity, it follows that $\frac{\partial f}{\partial m} > 0$. Inequality $\frac{\partial^2 f}{\partial l^2} > 0$ in Eq. (3.4) indicates that the disaster damage rises more quickly with capacity when damage is severe. Inequality $\frac{\partial^2 f}{\partial m^2} < 0$ implies that the marginal benefit from protection is larger for major disasters. For protection effectiveness, $\theta$, the condition $\frac{\partial f}{\partial \theta} < 0$ is intuitive. Inequality $\frac{\partial^2 f}{\partial K \partial \theta} < 0$ indicates that the disaster damage rises less quickly with capacity when the protection is more effective. Since the existing literature that explores both capacity and protection is scarce, there is no well-defined damage function for port/shippers incorporating both capacity and protection. The existing literature that only explores protection investment (e.g., Xiao et al., 2015; Wang and Zhang, 2018; Randrianarisoa and Zhang, 2019) impose a linear disaster damage function in protection investment. Therefore, we adopt a general disaster damage function that has plausible first, second and cross derivatives.

A final assumption is

$$\frac{\partial g}{\partial K} + x \frac{\partial f}{\partial K} < 0.$$  

(3.5)

---

40 This assumption plausibly holds locally in the neighborhood of the optimal solution. However, it is also possible that protection has locally increasing returns. For example, a two-metre-high seawall might be useless for protection against high water, whereas a three-metre-high wall might provide adequate protection except in extreme disasters.
This assures that the sum of congestion costs and expected damage costs decreases with capacity, so that fears of climate change do not dominate the port’s investment decisions.

Turning to demand, we assume that aggregate shippers’ demand for port service is given by the linear inverse demand function \( \rho(q) = a - bq \). Demand to use the port is thus price elastic, although competition between ports is not explicitly modeled. Setting \( \rho(q) \) equal to \( p(q) \) in Eq. (3.2) yields a reduced-form demand function \( q(\tau, K, I; x, \theta, \delta, m, a, b) \), which has the following properties (see Appendix B.1 for details):

\[
\frac{\partial q}{\partial \tau} < 0, \quad \frac{\partial q}{\partial K} > 0, \quad \frac{\partial q}{\partial I} > 0,
\frac{\partial q}{\partial x} < 0, \quad \frac{\partial q}{\partial \theta} > 0, \quad \frac{\partial q}{\partial \delta} < 0, \quad \frac{\partial q}{\partial m} < 0, \quad \frac{\partial q}{\partial a} > 0, \quad \frac{\partial q}{\partial b} < 0.
\] (3.6)

A higher port charge reduces demand, but more capacity and protection raise demand. A higher disaster probability \( x \), a higher congestion cost \( \delta \), or a higher damage cost \( m \) reduces demand, while demand increases with more effective protection for shippers, \( \theta \). Demand can be increased by increasing parameter \( a \) which shifts the demand curve upwards, or by reducing parameter \( b \) which causes the demand curve to flatten and rotate outwards. For brevity, we will refer to changes to parameter \( a \) as “additive demand shocks”, and changes to \( b \) as “multiplicative demand shocks”.

3.3.2 Pricing and investment of a private port

Now consider a private port which is assumed to choose \( \tau, K, \) and \( I \) to maximize its profit.\(^{41}\) The port’s expected profit is:

\(^{41}\) In this paper, we assume the pricing and investment decisions are made by one port entity. According to Transport Canada, the Canada Port Authority is responsible for setting the port fee and making commercial and investment decisions (see https://www.tc.gc.ca/en/services/marine/ports-harbours/list-canada-port-authorities.html). However, terminal operators can be influential in a port’s investment decisions. For instance, Wang and Zhang (2018), Randrianarisoa and Zhang (2019), and Wang et al. (2020) take into account the governance structure within a port where the port authority charges the downstream terminal operator and the terminal operator charges the downstream shippers. In Wang and Zhang (2018), the port adaptation investment is made by both the port authority and the terminal operator. In Randrianarisoa and Zhang (2019) and Wang et al. (2020), the port adaptation investment decision is only made by the port authority. Wang et al. (2020) consider multiple downstream terminal operators. We will consider the role of terminal operators for future study.
\[ \Pi = (\tau - c)q - xD(K, I; \eta, M) - c_kK - c_lI, \]  

(3.7)

where \( c \) is the unit cost of handling cargo, \( c_k \) is the unit cost of capacity, and \( c_l \) is the unit cost of protection. Unit costs are constant so that economies of scale or scope are absent. Function \( D(K, I; \eta, M) \) is the total disaster damage incurred by the port. It will be written as \( D(K, I; \eta, M) = F(K, I; \eta, M)K \), where \( F(K, I; \eta, M) \) is the disaster damage cost per unit capacity incurred by the port. Parameter \( \eta \) is a measure of the protection effectiveness for the port, and \( M \) is a measure of the severity of a disaster suffered by the port which includes the costs of repairs, any unrepaired damages, and lost business. We assume the same properties in Eqns. (3.3) – (3.4) hold for \( F(K, I; \eta, M) \):

\[
\begin{align*}
\frac{\partial F}{\partial K} > 0, & \quad \frac{\partial^2 F}{\partial K^2} > 0, & \quad \frac{\partial F}{\partial I} < 0, & \quad \frac{\partial^2 F}{\partial I^2} > 0, & \quad \frac{\partial^2 F}{\partial K \partial I} < 0, \\
\frac{\partial F}{\partial M} > 0, & \quad \frac{\partial^2 F}{\partial K \partial M} > 0, & \quad \frac{\partial^2 F}{\partial I \partial M} < 0, & \quad \frac{\partial F}{\partial \eta} < 0, & \quad \frac{\partial^2 F}{\partial K \partial \eta} < 0.
\end{align*}
\]  

(3.8)

In the following subsections, we first analyze the partial equilibrium in which the port optimizes only one decision variable at a time while holding the other two fixed. We then consider the full equilibrium in which the three decision variables are optimized simultaneously.

3.3.2.1 Partial-equilibrium analysis

**Pricing**

The first-order condition (FOC) with respect to the port charge \( \tau \) is:

\[ \Pi_\tau = q + (\tau - c) \frac{\partial q}{\partial \tau} = 0, \]  

(3.10)

where the subscript denotes a partial derivative. The second-order condition (SOC) is:

\[ \Pi_{\tau \tau} = 2 \frac{\partial q}{\partial \tau} + (\tau - c) \frac{\partial^2 q}{\partial \tau^2} = 2 \frac{\partial q}{\partial \tau} < 0, \]  

(3.11)

which is satisfied because, with linear demand, \( \frac{\partial^2 q}{\partial \tau^2} = 0 \). From the equilibrium condition \( \rho(q) = p(q), \frac{\partial q}{\partial \tau} = \frac{1}{\frac{\partial p}{\partial q} \frac{\partial q}{\partial \tau}} < 0 \). Substituting this into Eq. (3.10) yields
\[ \tau^* = c + q \frac{\partial g}{\partial q} - q \frac{\partial \rho}{\partial q}. \] (3.12)

The optimal port charge is the sum of the marginal cost of service, the standard Pigouvian congestion charge, and a markup. In Appendix B.1 we show that

\[ \frac{\partial \tau^*}{\partial K} < 0, \quad \frac{\partial \tau^*}{\partial I} > 0. \] (3.13)

Thus, the optimal port charge is a decreasing function of capacity, and an increasing function of protection. The port capacity has two opposing effects on the marginal profit of port charge. Increasing \( K \) raises demand, but a higher \( K \) also strengthens the demand-reducing effect of port charge (\( \frac{\partial^2 q}{\partial \tau \partial K} < 0 \)). The net effect is negative at the optimal port charge: \( \frac{\partial \tau^*}{\partial \tau} < 0 \). Since \( \frac{\partial^2 q}{\partial \tau \partial I} = 0 \), port protection only has one effect on the marginal profit of port charge: a higher \( I \) raises demand. Thus, a higher \( I \) induces a higher port charge.

### Capacity investment

The FOC for capacity is:

\[ \Pi_k = (\tau - c) \frac{\partial q}{\partial K} - x \frac{\partial D}{\partial K} - c_k = 0. \] (3.14)

The SOC is:

\[ \Pi_{kk} = (\tau - c) \frac{\partial^2 q}{\partial K^2} - x \frac{\partial^2 D}{\partial K^2} < 0, \] (3.15)

which is satisfied (see Appendix B.1). Now consider how the other two decision variables affect optimal capacity. For the charge:

\[ \frac{\partial K^*}{\partial \tau} = -\frac{\Pi_{k\tau}}{\Pi_{kk}}, \]

where \( \Pi_{k\tau} = \frac{\partial q}{\partial K} + (\tau - c) \frac{\partial^2 q}{\partial \tau \partial K} \). The port price has two opposing effects on the marginal profit of capacity. Increasing \( \tau \) raises the marginal profit from larger capacity, but a higher \( \tau \) also weakens
the demand-increasing effect of capacity \( \frac{\partial^2 q}{\partial q \partial K} < 0 \). The net effect is ambiguous in general. However, if the toll is set at its optimal level, \( \tau^* \), the overall effect is negative. Hence,

\[
\frac{\partial K^*}{\partial \tau} \leq 0, \quad \frac{\partial K^*}{\partial \tau} \bigg|_{\tau^*} < 0.
\]

(3.16)

Appendix B.1 also establishes that

\[
\frac{\partial K^*}{\partial I} > 0.
\]

(3.17)

Thus, consistent with intuition optimal capacity is an increasing function of protection.

**Protection investment**

Optimal port protection is determined by the FOC:

\[
\Pi_I = (\tau - c) \frac{\partial q}{\partial I} - x \frac{\partial D}{\partial I} - c_I = 0.
\]

(3.18)

The marginal benefit from protection consists of extra profits generated by more demand as a result of protection, and damage avoided by more protection. At the optimum, this balances the extra protection cost. The SOC holds (see Appendix B.1):

\[
\Pi_{II} = (\tau - c) \frac{\partial^2 q}{\partial I^2} - x \frac{\partial^2 D}{\partial I^2} < 0.
\]

(3.19)

As shown in Appendix B.1,

\[
\frac{\partial I^*}{\partial \tau} > 0, \quad \frac{\partial I^*}{\partial K} > 0.
\]

(3.20)

Thus, protection is an increasing function of both the charge and capacity.

We depict the relationships between the port’s charge, capacity, and protection in Figure 3.1. Consider first the charge and capacity. Since the two variables are submodular \( \frac{\partial \tau^*}{\partial K} < 0, \frac{\partial K^*}{\partial \tau} < 0 \),
an exogenous increase in the charge induces the port to reduce capacity, and an exogenous increase in capacity induces the port to reduce the charge. There is a causal loop relationship between the charge and capacity shown by the upper counterclockwise loop. The causal loop is positive because it includes two negative linkages. Capacity and protection are supermodular $\left( \frac{\partial K^*}{\partial I} > 0, \frac{\partial I^*}{\partial K} > 0 \right)$. An exogenous increase in capacity induces the port to increase protection, and an exogenous increase in protection induces the port to increase capacity. Hence, there is another positive causal loop relationship between capacity and protection shown by the lower counterclockwise loop. The charge and protection are also supermodular $\left( \frac{\partial \tau^*}{\partial I} > 0, \frac{\partial I^*}{\partial \tau} > 0 \right)$, and there is a third positive causal loop, shown on the right.

![Causal loop diagram for port charge, capacity, and protection](image)

**Figure 3.1 Causal loop diagram for port charge, capacity, and protection**

We can also derive the effects of exogenous variables on optimal pricing, capacity, protection, and output in partial-equilibrium analysis (see Appendix B.1 for details). The results are summarized in Table 3.1.

---

42 A loop with an even number of negative links is a positive feedback loop, and a loop with an odd number of negative links is a negative feedback loop.
<table>
<thead>
<tr>
<th>Climate and damage parameters</th>
<th>Charge ($\tau$)</th>
<th>Capacity ($K$)</th>
<th>Protection ($I$)</th>
<th>Output ($q$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$ (disaster probability)</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$m$ (disaster intensity to shippers)</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$M$ (disaster intensity to port)</td>
<td>0</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>$\theta$ (protection effectiveness to shippers)</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>+</td>
</tr>
<tr>
<td>$\eta$ (protection effectiveness to port)</td>
<td>0</td>
<td>+</td>
<td>?</td>
<td>0</td>
</tr>
<tr>
<td>Demand parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a$ (additive demand shocks)</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>$b$ (multiplicative demand shocks)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cost parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$ (congestion cost to shippers)</td>
<td>0</td>
<td>?</td>
<td>-</td>
<td>?</td>
</tr>
<tr>
<td>$c$ (port operating cost)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$c_k$ (cost of capacity investment)</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$c_l$ (cost of protection investment)</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

The disaster probability ($x$) and magnitude ($m, M$) have similar effects on the port’s decisions, except that $M$ does not affect the optimal port charge because $M$ has no effect on demand. A higher probability/magnitude reduces the charge and capacity, but increases protection. A port whose protection measures for shippers/port are more effective (higher $\theta$, $\eta$) can charge a higher fee, and is inclined to have larger capacity, but protection effectiveness can have a non-monotone effect on protection in the sense that a higher protection effectiveness may not necessarily induce more protection. In terms of demand parameters, positive additive demand shocks (higher $a$) boost the charge and capacity, and positive multiplicative demand shocks (lower $b$) boost capacity and protection. Since $\Pi = \frac{\partial q}{\partial b} + (\tau - c) \frac{\partial^2 q}{\partial \tau \partial b}$ where $\frac{\partial q}{\partial b} < 0$ and $\frac{\partial^2 q}{\partial \tau \partial b} > 0$, the multiplicative demand shocks $b$ has two opposing effects on the marginal profit of port charge: an increase in $b$ reduces...
demand and congestion, but weakens the demand-reducing effect of port charge. We show in Appendix B.1 that at the profit-maximizing solution the two opposing effects cancel out ($\Pi_{rb} | \tau^* = 0$), which gives $\frac{\partial \tau^*}{\partial b} = 0$. This result is due to the linear inverse demand function and linear congestion function in $q$. In terms of cost parameters, an increase in unit operating cost ($c$) boosts the charge but decreases capacity and protection. The congestion cost $\delta$ has an ambiguous effect on capacity because of two countervailing effects: when $\delta$ is high, the congestion-relieving effect of capacity is strong, but demand is low. By contrast, protection decreases with $\delta$ because a larger $\delta$ results in smaller demand, which requires less protection. The effects of investment costs, ($c_k$, $c_I$), are obvious. The exogenous parameters affect the partial-equilibrium output in two ways: (1) through the direct effect on output, and (2) through the indirect effect from the decision variable. Parameters $M$, $\eta$, $c$, $c_k$, and $c_I$ do not directly affect demand, so their effects on partial-equilibrium output follow the indirect effect. When $\tau$ is the decision variable, the direct and indirect effects on partial-equilibrium output have opposite signs for parameters $x$, $m$, $\theta$, $a$, $b$, and $\delta$. For instance, a higher disaster probability $x$ directly reduces demand, but also lowers the optimal port charge, which positively affects demand. Nevertheless, it can be shown that the direct effect dominates the indirect effect at the optimal port charge. When $K$ is the decision variable, the direct and indirect effects on partial-equilibrium output exhibit the same signs for parameters $x$, $m$, $\theta$, $a$, and $b$. For instance, a higher disaster probability $x$ directly reduces demand, and also reduces optimal capacity which further reduces demand. When $I$ is the decision variable, the direct and indirect effects on partial-equilibrium output exhibit different signs for parameters $x$ and $m$, but it is not clear which one dominates. For instance, a higher disaster probability $x$ directly reduces demand, but increases optimal port protection which boosts demand. It is unclear which effect dominates.

### 3.3.2.2 Full-equilibrium analysis

In this section, we investigate the effects of the exogenous parameters in a full equilibrium in which the three decision variables are optimized simultaneously. We use two approaches, one with a general disaster function, and the other with a specific function.
**General disaster function**

We first consider the effect on the port’s charge of an increase in the probability of a disaster, \( x \). As shown in Table 3.1, a higher probability reduces the charge because demand falls. This effect is shown in Figure 3.2 by the red link extending from the disaster probability node to the charge node. If capacity and protection remain fixed, only this direct effect is at play. Hence, the charge unambiguously falls. In the longer run, the port may be able to adjust its capacity, its protection, or possibly both. If it can adjust capacity but not protection, the negative link from the disaster probability node to the capacity node comes into play, and so do the two negative links between capacity and charge. The increase in \( x \) acts directly to reduce capacity. However, the reduction in the charge encourages the port to increase capacity. The prospective change in capacity, either up or down, further induces a second-round change in the charge in the opposite direction. A priori, it is unclear whether in the full equilibrium the charge increases or decreases.

![Figure 3.2 Full-equilibrium effects of a change in disaster probability](image)

If, instead, the port can adjust protection, but not capacity, the positive link from the disaster probability node to the protection node comes into play, and so do the two positive links between protection and the charge at the far right of Figure 3.2. The increase in \( x \) acts directly to increase protection, which encourages the port to increase the charge. The positive indirect effect on the
charge acts in the opposite direction to the direct negative effect. The prospective change in the port charge, either up or down, induces a second-round change in protection in the same direction, and so on. A priori, it is again unclear whether the equilibrium charge increases or decreases. If the port can adjust both capacity and protection, all the links in Figure 3.2 are active. The increase in disaster probability has a direct negative effect on the charge, and two indirect positive effects via changes in capacity and protection. The net effect is, again, a priori unclear.

The effects of an increase in disaster probability on capacity and protection can be analyzed similarly. Once more, the signs of the effects are uncertain. Changes in the other parameters can also be examined using variants of Figure 3.2. The results are summarized in Table B.1 in Appendix B.1. In term of protection investment, all but two of the comparative statics effects have definite signs as shown in Table 3.1. By contrast, in Table B.1, seven of them are uncertain when full equilibrium is considered. In term of capacity investment, only one of the comparative statics effects has ambiguous sign in Table 3.1, but seven of them are uncertain in Table B.1. This highlights the importance of considering the two types of investment together – especially for ports that are expanding capacity to accommodate growing demand.

**Specific disaster function**

There are many ambiguities in the full-equilibrium analysis using general functional forms. To obtain further results, we assume functions $f$ and $F$ take the form:

$$f(K, I; \theta, m) = me^{-\theta I K}, \quad F(K, I; \eta, M) = Me^{-\eta I K}$$  \hfill (3.21)

These functional forms do not satisfy the properties in Eqs. (3.3) – (3.4) and (3.8) – (3.9) globally, but in the numerical analysis in Section 3.4.4 we find that they are satisfied over a wide range of parameter values. The ratio $I/K$ represents protection investment per unit of capacity. Given Eq. (3.21), damage costs decrease with per-unit protection, but the benefits are subject to decreasing returns to scale. Let $\xi$ denote an exogenous variable of interest. The comparative statics effects of $\xi$ on the equilibrium values of $\tau^*$, $K^*$, and $I^*$ are given by:
\[
\begin{pmatrix}
\frac{d\tau^*}{d\xi} \\
\frac{dK^*}{d\xi} \\
\frac{dl^*}{d\xi}
\end{pmatrix}
= 
\begin{pmatrix}
\Pi_{\tau\tau} & \Pi_{\tau k} & \Pi_{\tau I} \\
\Pi_{k\tau} & \Pi_{kk} & \Pi_{kl} \\
\Pi_{l\tau} & \Pi_{l k} & \Pi_{l I}
\end{pmatrix}^{-1}
\begin{pmatrix}
\Pi_{\tau\xi} \\
\Pi_{k\xi} \\
\Pi_{l\xi}
\end{pmatrix}
\]  
(3.22)

The full effect of \(\xi\) on equilibrium traffic volume is:

\[
\frac{dq^*}{d\xi} = \frac{\partial q}{\partial \tau} \frac{d\tau^*}{d\xi} + \frac{\partial q}{\partial K} \frac{dK^*}{d\xi} + \frac{\partial q}{\partial I} \frac{dl^*}{d\xi}.
\]  
(3.23)

The results are summarized in Table 3.2.

### Table 3.2 Comparative statics for a private port in full equilibrium: Specific disaster functions

<table>
<thead>
<tr>
<th>Climate and damage parameters</th>
<th>Charge ((\tau))</th>
<th>Capacity ((K))</th>
<th>Protection ((I))</th>
<th>Output ((q))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x) (disaster probability)</td>
<td>+ if (\theta \geq \eta)</td>
<td>-</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>(m) (disaster intensity to shippers)</td>
<td>?</td>
<td>-</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>(M) (disaster intensity to port)</td>
<td>+</td>
<td>-</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>(\theta) (Protection effectiveness to shippers)</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>+</td>
</tr>
<tr>
<td>(\eta) (Protection effectiveness to port)</td>
<td>-</td>
<td>+</td>
<td>?</td>
<td>+</td>
</tr>
</tbody>
</table>

**Demand parameters**

| \(a\) (additive demand shocks) | + | + | + | + |
| \(b\) (multiplicative demand shocks) | 0 | - | - | - |

**Cost parameters**

| \(\delta\) (congestion cost to shippers) | - | ? | ? | - |
| \(c\) (port unit operating cost) | + | - | - | - |
| \(c_k\) (cost of capacity investment) | + | - | - | - |
| \(c_l\) (cost of protection investment) | - | - | - | - |

Note: (1) Shaded cells identify effects of definite sign that are ambiguous with general functions. (2) Wolfram Mathematica was used to derive the comparative statics, assuming that the assumptions in Eq. (3.3), (3.4), (3.5), (3.8), and (3.9) hold. In some cases, additional assumptions are required. For instance, \(\frac{\delta f^2}{\partial k \partial t} = \frac{\theta(K-\theta t)}{k^3} f\). To satisfy the
assumption $\frac{\partial f^2}{\partial k T} < 0$, it is necessary to impose the condition $K < \theta T$. This qualification applies to all of the analysis using specific disaster functions.

Table 3.2 shows that the disaster probability ($x$) and disaster damage ($m$ or $M$) decrease equilibrium capacity. Consistent with intuition, greater risks from climate change discourage a port from expanding capacity. However, the effect of risks on protection is unclear. Greater risks call directly for more protection, but smaller capacity requires less protection. Greater protection effectiveness ($\theta$ or $\eta$) increases capacity because the port is better protected, but it may increase or decrease the amount of investment in protection. Greater protection effectiveness also boosts the port charge despite the larger capacity. Additive demand shocks ($a$) increase the port charge and capacity, while multiplicative demand shocks ($b$) have no effect on the charge. A higher congestion cost reduces the charge while having ambiguous effects on capacity and protection. All the parameters have unambiguous effects on output. Greater climate risks (i.e., higher $x$, $m$, or $M$) reduce output; protection effectiveness ($\theta$ or $\eta$) increases output; additive demand shocks ($a$) increase output, multiplicative demand shocks ($b$) reduce output; and all cost parameters reduce output.

### 3.3.3 Pricing and investment of a public port

In this section, we consider the decisions of a public port that aims to maximize social welfare (as opposed to profit in the above analysis). Specifically, the public port maximizes the following social welfare function:

$$W = \int_0^q \rho(\zeta) d(\zeta) - \rho(q)q + \Pi,$$  \hspace{1cm} (3.24)

where $\rho(q) = a - bq$ is the inverse demand function and the port profit $\Pi$ is defined in Eq. (3.7). The FOCs for the three decision variables are given by, respectively:

$$W_\tau = q + \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial q}{\partial \tau} = 0,$$  \hspace{1cm} (3.25)

$$W_k = \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial q}{\partial K} - x \frac{\partial D}{\partial K} - c_k = 0,$$  \hspace{1cm} (3.26)
\[ W_i = (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial q}{\partial I} - x \frac{\partial D}{\partial I} - c_i = 0, \tag{3.27} \]

where the subscript denotes a partial derivative.

For Eq. (3.25), it can be shown that the optimal port charge for a welfare-maximizing port is the sum of the marginal cost of service and the standard Pigouvian congestion charge:

\[ \tau^w = c + q \frac{\partial g}{\partial q}, \tag{3.28} \]

where the superscript \( w \) indicates welfare maximizing. Unlike the profit-maximizing charge for private port in Eq. (3.12), the welfare-maximizing charge excludes a markup \( -\frac{\partial \rho}{\partial q} q \).

We show in Appendix B.2 that the SOC's given by Eqns. (3.29) – (3.31) are satisfied:

\[ W_{\tau \tau} = \frac{\partial q}{\partial \tau} \left( 2 - \frac{\partial q}{\partial \tau} \frac{\partial \rho}{\partial q} \right) < 0, \tag{3.29} \]
\[ W_{kk} = (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial K^2} - x \frac{\partial^2 D}{\partial K^2} - \frac{\partial \rho}{\partial q} \left( \frac{\partial q}{\partial K} \right)^2 < 0, \tag{3.30} \]
\[ W_{II} = (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial I^2} - x \frac{\partial^2 D}{\partial I^2} - \frac{\partial \rho}{\partial q} \left( \frac{\partial q}{\partial I} \right)^2 < 0. \tag{3.31} \]

### 3.3.3.1 Partial-equilibrium analysis

Using the same procedures as in Section 3.3.2.1, in Appendix B.2 we derive the following results for a public port:

\[ \frac{\partial \tau^w}{\partial K} < 0, \quad \frac{\partial \tau^w}{\partial I} > 0, \]
\[ \frac{\partial K^w}{\partial \tau} < 0, \quad \frac{\partial K^w}{\partial I} > 0, \]
\[ \frac{\partial I^w}{\partial \tau} > 0, \quad \frac{\partial I^w}{\partial K} > 0. \]

We summarize the above results in Proposition 1.
**Proposition 1:** For both private and public ports: (1) the port charge and protection are supermodular: a higher port charge induces more protection, and more protection induces a higher port charge; (2) capacity and protection are supermodular: higher capacity warrants more protection, and more protection encourages higher capacity; (3) the port charge and capacity are submodular: more capacity induces a lower charge, and a higher charge suppresses capacity.

We can also derive how in partial equilibrium the optimal values of \( \tau, K, \) or \( I \) vary with parameters given general disaster functions. The results are summarized in Table 3.3.

<table>
<thead>
<tr>
<th></th>
<th>Charge ((\tau))</th>
<th>Capacity ((K))</th>
<th>Protection ((I))</th>
<th>Output ((q))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Climate and damage parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x ) (disaster probability)</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( m ) (disaster intensity to shippers)</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( M ) (disaster intensity to port)</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>( \theta ) (Protection effectiveness to shippers)</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>+</td>
</tr>
<tr>
<td>( \eta ) (Protection effectiveness to port)</td>
<td>0</td>
<td>+</td>
<td>?</td>
<td>0</td>
</tr>
<tr>
<td><strong>Demand parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a ) (additive demand shocks)</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( b ) (multiplicative demand shocks)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Cost parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta ) (congestion cost to shippers)</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>( c ) (port operating cost)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( c_k ) (cost of capacity investment)</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_l ) (cost of protection investment)</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: (1) Shaded cells identify results that differ from a private port. (2) Specific disaster functions are required to evaluate \( \frac{\partial w}{\partial x} \) and \( \frac{\partial w}{\partial m} \), but all other derivatives are evaluated with the general disaster functions (see Appendix B.2).
The comparative statics in partial equilibrium are mostly the same as for a private port. Only three differences in the comparison between public and private ports in partial equilibrium are found: (1) \( \frac{\partial \tau^*}{\partial b} = 0 \), but \( \frac{\partial \tau^w}{\partial b} < 0 \); (2) \( \frac{\partial \tau^*}{\partial \delta} = 0 \), but \( \frac{\partial \tau^w}{\partial \delta} < 0 \); (3) \( \frac{\partial \tau^*}{\partial a} = 0 \), but \( \frac{\partial \tau^w}{\partial a} > 0 \). There are two influences of parameter \( b \) on the private charge. An increase in \( b \): (i) reduces demand and thus congestion, but (ii) increases the markup component. The effects on the congestion component and markup component cancel out. For a public port, only influence (i) is relevant. Hence, the public charge decreases with the value of parameter \( b \). An increase in \( \delta \): (i) reduces demand but increases congestion cost, which gives a net positive effect on the congestion component, and (ii) decreases the markup component. The effects on the congestion component and markup component cancel out. For a public port, only influence (i) is relevant. Hence, the public charge increases with the value of parameter \( \delta \). Since \( \frac{\partial q}{\partial i} = 0 \), the parameter \( a \) does not affect the demand-enhancing effect of \( I \). Therefore, from Eq. (3.18), \( \frac{\partial \Pi}{\partial a} = 0 \), which gives \( \frac{\partial I^*}{\partial a} = 0 \) for a private port. For a public port, an increase in \( a \) raises demand, and thus increases the markup component in Eq. (3.28). As a result, an increase in \( a \) increase the marginal welfare benefit of \( I \) (\( \frac{\partial W}{\partial a} > 0 \)), which gives \( \frac{\partial \tau^w}{\partial a} > 0 \).

### 3.3.3.2 Full-equilibrium analysis

For a public port, the comparative statics effects of exogenous parameter \( \xi \) on the equilibrium values of \( \tau^w, K^w, \) and \( l^w \) are given by:

\[
\begin{pmatrix}
\frac{d \tau^w}{d \xi} \\
\frac{d K^w}{d \xi} \\
\frac{d l^w}{d \xi}
\end{pmatrix} =
\begin{pmatrix}
W_{\tau \tau} & W_{\tau k} & W_{\tau l} \\
W_{k \tau} & W_{kk} & W_{kl} \\
W_{l \tau} & W_{lk} & W_{ll}
\end{pmatrix}
^{-1}
\begin{pmatrix}
W_{\tau \xi} \\
W_{k \xi} \\
W_{l \xi}
\end{pmatrix}
\]  

(3.32)

The full effect of \( \xi \) on equilibrium traffic volume is:

\[
\frac{dq^w}{d \xi} = \frac{\partial q}{\partial \xi} + \frac{\partial q}{\partial \tau} \frac{d \tau^w}{d \xi} + \frac{\partial q}{\partial K} \frac{d K^w}{d \xi} + \frac{\partial q}{\partial l} \frac{d l^w}{d \xi}.
\]  

(3.33)
Table 3.4 Comparative statics for a public port in full equilibrium: Specific disaster functions

<table>
<thead>
<tr>
<th>Climate and damage parameters</th>
<th>Charge ($\tau$)</th>
<th>Capacity ($K$)</th>
<th>Protection ($I$)</th>
<th>Output ($q$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$ (disaster probability)</td>
<td>+</td>
<td>-</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>$m$ (disaster intensity to shippers)</td>
<td>+</td>
<td>-</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>$M$ (disaster intensity to port)</td>
<td>+</td>
<td>-</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>$\theta$ (Protection effectiveness to shippers)</td>
<td>-</td>
<td>+</td>
<td>?</td>
<td>+</td>
</tr>
<tr>
<td>$\eta$ (Protection effectiveness to port)</td>
<td>-</td>
<td>+</td>
<td>?</td>
<td>+</td>
</tr>
</tbody>
</table>

Demand parameters

<table>
<thead>
<tr>
<th>Demand parameters</th>
<th>Charge ($\tau$)</th>
<th>Capacity ($K$)</th>
<th>Protection ($I$)</th>
<th>Output ($q$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$ (additive demand shocks)</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$b$ (multiplicative demand shocks)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Cost parameters

<table>
<thead>
<tr>
<th>Cost parameters</th>
<th>Charge ($\tau$)</th>
<th>Capacity ($K$)</th>
<th>Protection ($I$)</th>
<th>Output ($q$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$ (congestion cost to shippers)</td>
<td>+</td>
<td>?</td>
<td>?</td>
<td>-</td>
</tr>
<tr>
<td>$c$ (port operating cost)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$c_k$ (cost of capacity investment)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$c_I$ (cost of protection investment)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: Shaded cells identify results that differ from a private port.

The comparative statics properties for a public port are shown in Table 3.4. For capacity and protection, they are the same as for a private port. They differ for the port charge because the public port cares about shippers’ welfare, whereas the private port does not.

3.3.4 Comparison between private and public ports

Having analyzed the behaviors of private and public ports, we now compare them. Comparing Table 3.2 and Table 3.4, we summarize the results in Proposition 2.

**Proposition 2:** For both public and private ports: (1). A higher disaster probability/magnitude reduces capacity and traffic volume, and has an ambiguous effect on protection. (2). More effective
protection induces a larger capacity and traffic volume, but may increase or decrease protection. (3). Positive demand shocks increase equilibrium capacity, protection, and traffic volume. (4). Higher unit costs of operation and investments induce lower capacity, protection, and traffic volume.

Comparative static results in partial-equilibrium for private and public ports can be compared analytically to show which factors have a stronger effect on one versus the other. The results are summarized in Table 3.5.

In term of optimal port charge, we show that capacity reduces optimal port charge, but the negative effect is stronger for a public port. Protection increases optimal port charge, but the positive effect is stronger for a private port. Disaster probability ($x$) or magnitude ($m$) negatively affects port pricing, but the negative effect is stronger for a private port. Protection effectiveness ($\theta$) positively affects port pricing, but the positive effect is also stronger for a private port. The port-specific parameters ($M, \eta, c_k,$ and $c_I$) do not affect demand, and hence do not affect pricing.

The disaster magnitude ($m$ and $M$) increases protection, but $M$ (disaster magnitude to port) has a stronger effect on private port, while $m$ (disaster magnitude to shippers) has a stronger effect on public port. The congestion cost ($\delta$) reduces optimal protection due to the reduction in demand, but this negative effect is stronger for a public port, because demand is higher for a public port, and has more scope to decline. The operating cost ($c$) and protection investment cost ($c_I$) both reduce optimal protection, but the effect is stronger for a private port.
### Table 3.5 Comparison between public and private ports in partial equilibrium\(^{43}\)

<table>
<thead>
<tr>
<th>Other decision variables</th>
<th>Charge ((\tau))</th>
<th>Protection ((I))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K) (capacity investment)</td>
<td>(0 &gt; \frac{\partial \tau^*}{\partial K} &gt; \frac{\partial \tau^w}{\partial K})</td>
<td>(\frac{\partial I^*}{\partial K} \leq \frac{\partial I^w}{\partial K})</td>
</tr>
<tr>
<td>(I/\tau) (protection investment/port charge)</td>
<td>(\frac{\partial \tau^*}{\partial I} &gt; \frac{\partial \tau^w}{\partial I} &gt; 0)</td>
<td>(\frac{\partial I^*}{\partial \tau} &gt; \frac{\partial I^w}{\partial \tau} &gt; 0)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Climate and damage parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x) (disaster probability)</td>
</tr>
<tr>
<td>(m) (disaster intensity to shippers)</td>
</tr>
<tr>
<td>(M) (disaster intensity to port)</td>
</tr>
<tr>
<td>(\theta) (Protection effectiveness to shippers)</td>
</tr>
<tr>
<td>(\eta) (Protection effectiveness to port)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (additive demand shocks)</td>
</tr>
<tr>
<td>(b) (multiplicative demand shocks)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cost parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta) (congestion cost to shippers)</td>
</tr>
<tr>
<td>(c) (port operating cost)</td>
</tr>
<tr>
<td>(c_k) (cost of capacity investment)</td>
</tr>
<tr>
<td>(c_i) (cost of protection investment)</td>
</tr>
</tbody>
</table>

Note: The rankings for \(\tau\) are derived using the general disaster functions (see Appendix B.3 for details). The rankings for \(I\) are derived using the specific disaster functions.

\(^{43}\) Comparisons for capacity \(K\) are intractable. By the implicit function theorem, \(\frac{\partial K^*}{\partial \xi} - \frac{\partial K^w}{\partial \xi} = -\frac{\pi_{kk}^\xi}{\pi_{kk}}(-\frac{W_{kk}^\xi}{W_{kk}}),\) where \(\xi\) is any parameter. Due to the complicated expressions for \(\pi_{kk}\) in Eq. (3.15) and \(W_{kk}\) in Eq. (3.30), the analytics are intractable even with the specific disaster functions.
Next, we compare the equilibrium $\tau$, $K$, and $I$. From the optimal private charge in Eq. (3.12), we have

$$W_\tau|_{\tau^*} = q + (\tau - c) \frac{\partial q}{\partial \tau} \bigg|_{\tau^*} - q \frac{\partial \rho}{\partial q} \frac{\partial q}{\partial \tau} \bigg|_{\tau^*} < 0,$$

which implies $\tau^w(K, I) < \tau^*(K, I)$. Thus, given the same $K$ and $I$, a private port charges higher than a public port.

Similarly, we can show that

$$W_K|_{K^*} = (\tau - c) \frac{\partial q}{\partial K} - x \frac{\partial D}{\partial K} - c_k \bigg|_{K^*} - q \frac{\partial \rho}{\partial q} \frac{\partial q}{\partial K} \bigg|_{K^*} > 0,$$

which implies $K^w(\tau, I) > K^*(\tau, I)$, and

$$W_I|_{I^*} = (\tau - c) \frac{\partial q}{\partial I} - x \frac{\partial D}{\partial I} - c_I \bigg|_{I^*} - q \frac{\partial \rho}{\partial q} \frac{\partial q}{\partial I} \bigg|_{I^*} > 0,$$

which implies $I^w(\tau, K) > I^*(\tau, K)$. Thus, given $\tau$ and $I$, a public port invests in more capacity than a private port. Similarly, given $\tau$ and $K$, a public port invests in more protection than a private port.

Although there are no closed-form solutions, we can still compare the full equilibrium for public and private ports, and derive the following proposition (see Appendix B.2 for proof).

**Proposition 3:** (1) Equilibrium capacity is larger for a public port than a private port; (2) Given the same, fixed port charge, equilibrium protection is larger for a public port than a private port; (3) Given the same, fixed protection, the equilibrium user charge is lower for a public port than a private port.

### 3.4 The Two-Period Model

In this section, we extend the static model to a two-period model in order to incorporate learning and investigate optimal investment timing. At the beginning of period 1, the port and shippers have
a common prior about the probability of a disaster, \( x_1 \). At the end of period 1, the state of period 1 is realized. Depending on whether a disaster has occurred, the results of climate research, and other sources of learning, port and shippers update their prior probability of a disaster in period 2. Two states are assumed to be possible. In the pessimistic state the probability is updated to \( x_{2H} \), and in the optimistic state the probability is updated to \( x_{2L} \) with \( x_{2L} < x_{2H} \). Superscripts \( H \) and \( L \) can be read as high risk and low risk, respectively. For brevity, the pessimistic state will sometimes be called period “2H”, and the optimistic state period “2L”.

The rest of the specification for the two-period model is similar to the one-period setting. In period 1, the expected price for shippers is

\[
\pi_1 = (\tau_1 - c_1)q_1 - x_1 D_1(K_1, I_1; \eta_1, M_1) - c_k K_1 - c_{l1} l_1,
\]

where \( D_1(K_1, I_1; \eta_1, M_1) = F_1(K_1, I_1; \eta_1, M_1) \) \( K_1 \) is the total damage cost incurred by the port. The port chooses \( K_1 \), \( I_1 \), and \( \tau_1 \) at the beginning of period 1.

At the beginning of period 2, the port decides whether to add more capacity and/or protection. Its expected profit in period 2 is

\[
\pi_{2s} = (\tau_{2s} - c_{2s})q_{2s} - x_{2s} D_{2s}(K_{2s}, I_{2s}; \eta_2, M_2) - c_k(K_{2s} - K_1) - c_{l2}(l_{2s} - l_1),
\]
where $D_{2s}(K_{2s}, I_{2s}; \eta_2, M_2)$, $c_2$, $c_{k2}$, and $c_{l2}$ are defined as for period 1. Since investments are irreversible, $K_{2s} \geq K_1$ and $I_{2s} \geq I_1$, so that neither capacity nor protection can be reduced in period 2.\(^{44}\) The port’s optimization problem can be formulated as follows:

$$
\max_{\{\tau_1, K_{2s}, I_{2s}, \tau_2H, K_{2H}, I_{2H}, K_{2L}, I_{2L}\}} \Pi = \pi_1 + \beta (x_1 \pi_{2H} + (1 - x_1) \pi_{2L}),
$$

(3.36)

subject to $K_{2H} \geq K_1, I_{2H} \geq I_1, K_{2L} \geq K_1, I_{2L} \geq I_1$,

where $\beta$ is the discount factor. Set up the Lagrangian:

$$
\mathcal{L} = \pi_1 + \beta (x_1 \pi_{2H} + (1 - x_1) \pi_{2L}) - u_1 (K_1 - K_{2H}) - u_2 (I_1 - I_{2H}) - u_3 (K_1 - K_{2L}) - u_4 (I_1 - I_{2L}),
$$

(3.37)

where $u_i, i = 1, 2, 3, 4$, are the nonnegative Lagrangian multipliers. The Karush–Kuhn–Tucker (KKT) conditions are:

$$
\frac{\partial \mathcal{L}}{\partial K_1} = \frac{\partial \pi_1}{\partial K_1} + \beta c_{k2} - u_1 - u_3 = 0.
$$

(3.38)

$$
\frac{\partial \mathcal{L}}{\partial K_{2H}} = \beta x_1 \frac{\partial \pi_{2H}}{\partial K_{2H}} + u_1 = 0.
$$

(3.39)

$$
\frac{\partial \mathcal{L}}{\partial K_{2L}} = \beta (1 - x_1) \frac{\partial \pi_{2L}}{\partial K_{2L}} + u_3 = 0.
$$

(3.40)

$$
\frac{\partial \mathcal{L}}{\partial I_1} = \frac{\partial \pi_1}{\partial I_1} + \beta c_{l2} - u_2 - u_4 = 0.
$$

(3.41)

$$
\frac{\partial \mathcal{L}}{\partial I_{2H}} = \beta x_1 \frac{\partial \pi_{2H}}{\partial I_{2H}} + u_2 = 0.
$$

(3.42)

$$
\frac{\partial \mathcal{L}}{\partial I_{2L}} = \beta (1 - x_1) \frac{\partial \pi_{2L}}{\partial I_{2L}} + u_4 = 0.
$$

(3.43)

$$
\frac{\partial \mathcal{L}}{\partial \tau_1} = \frac{\partial \pi_1}{\partial \tau_1} = 0, \quad \frac{\partial \mathcal{L}}{\partial \tau_2H} = \beta x_1 \frac{\partial \pi_{2H}}{\partial \tau_{2H}} = 0, \quad \frac{\partial \mathcal{L}}{\partial \tau_{2L}} = \beta (1 - x_1) \frac{\partial \pi_{2L}}{\partial \tau_{2L}} = 0.
$$

(3.44)

\(^{44}\)This specification implicitly assumes the following. First, period 2 inherits the capacity and protection decisions made in period 1. Second, any damage gets fully repaired at the end of period 1. However, it is possible that new technology could be installed following a disaster so that capacity and/or protection are more productive or effective in period 2 than period 1 (which could be captured by assuming $\theta_2 > \theta_1$ and $\eta_2 > \eta_1$). Alternatively, damage incurred in period 1 might not be fully repaired. The port might even be abandoned. We ignore these possibilities.
3.4.2 Public port

In this subsection, we specify the objective function and KKT conditions for a public port. A public port’s objective function in period 1 is the same as in Section 3.2:

\[ w_1 = \pi_1 + \int_0^{q_1} \rho_1(\zeta)d(\zeta) - \rho_1(q_1)q_1. \] (3.45)

At the beginning of period 2, the state of period 1 is realized, and the port decides whether to add more capacity and/or protection. Its expected profit in period 2 is

\[ w_{2s} = \pi_{2s} + \int_0^{q_{2s}} \rho_{2s}(\zeta)d(\zeta) - \rho_{2s}(q_{2s})q_{2s}. \] (3.46)

A public port’s optimization problem can be formulated as follows:

\[
\max_{\{r_1, k_1, l_1, r_{2H}, k_{2H}, l_{2H}, r_{2L}, k_{2L}, l_{2L}\}} W = w_1 + \beta(x_1w_{2H} + (1 - x_1)w_{2L}),
\]

\[ \text{s.t. } K_{2H} \geq K_1, l_{2H} \geq l_1, K_{2L} \geq K_1, l_{2L} \geq l_1, \] (3.47)

where \( \beta \) is the discount factor. Set up the Lagrangian:

\[ L = w_1 + \beta(x_1w_{2H} + (1 - x_1)w_{2L}) - u_1(K_1 - K_{2H}) - u_2(l_1 - l_{2H}) - u_3(K_1 - K_{2L}) - u_4(l_1 - l_{2L}), \] (3.48)

where \( u_i, i = 1, 2, 3, 4, \) are the nonnegative Lagrangian multipliers. The KKT conditions are:

\[
\frac{\partial L}{\partial K_1} = \frac{\partial w_1}{\partial K_1} + \beta c_{k2} - u_1 - u_3 = 0. \] (3.49)

\[
\frac{\partial L}{\partial K_{2H}} = \beta x_1 \frac{\partial w_{2H}}{\partial K_{2H}} + u_1 = 0. \] (3.50)

\[
\frac{\partial L}{\partial K_{2L}} = \beta (1 - x_1) \frac{\partial w_{2L}}{\partial K_{2L}} + u_3 = 0. \] (3.51)

\[
\frac{\partial L}{\partial l_1} = \frac{\partial w_1}{\partial l_1} + \beta c_{l2} - u_2 - u_4 = 0. \] (3.52)

\[
\frac{\partial L}{\partial l_{2H}} = \beta x_1 \frac{\partial w_{2H}}{\partial l_{2H}} + u_2 = 0. \] (3.53)

\[
\frac{\partial L}{\partial l_{2L}} = \beta (1 - x_1) \frac{\partial w_{2L}}{\partial l_{2L}} + u_4 = 0. \] (3.54)
\[
\frac{\partial L}{\partial \tau_1} = \frac{\partial w_1}{\partial \tau_1} = 0, \quad \frac{\partial L}{\partial \tau_{2H}} = \beta_1 \frac{\partial w_{2H}}{\partial \tau_{2H}} = 0, \quad \frac{\partial L}{\partial \tau_{2L}} = \beta (1 - x_1) \frac{\partial w_{2L}}{\partial \tau_{2L}} = 0. \quad (3.55)
\]

### 3.4.3 Analytical results

For both public port and private port, there are 16 possible cases as to whether the port will add capacity or protection in period 2. The cases are listed in Table 3.6. For instance, the case \(I_{2H} > I_1, I_{2L} > I_1, K_{2H} > K_1, K_{2L} > K_1\) means that the port will add both capacity and protection in period 2 no matter the state in period 2 turns out to be “H” or “L”.

#### Table 3.6 Binding constraint cases for two-period model

<table>
<thead>
<tr>
<th>Case</th>
<th>(u_2 = 0, u_4 = 0)</th>
<th>(u_2 = 0, u_4 &gt; 0)</th>
<th>(u_2 &gt; 0, u_4 = 0)</th>
<th>(u_2 &gt; 0, u_4 &gt; 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_1 = 0, u_3 = 0)</td>
<td>(I_{2H} &gt; I_1) and (I_{2L} &gt; I_1)</td>
<td>(I_{2H} = I_1) and (I_{2L} &gt; I_1)</td>
<td>(I_{2H} = I_1) and (I_{2L} = I_1)</td>
<td>(I_{2H} = I_1) and (I_{2L} = I_1)</td>
</tr>
<tr>
<td>(K_{2H} &gt; K_1) and (K_{2L} &gt; K_1)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>(u_1 &gt; 0, u_3 = 0)</td>
<td>(K_{2H} = K_1) and (K_{2L} &gt; K_1)</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>(K_{2H} &gt; K_1) and (K_{2L} &gt; K_1)</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>(u_1 = 0, u_3 &gt; 0)</td>
<td>(K_{2H} &gt; K_1) and (K_{2L} = K_1)</td>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>(u_1 &gt; 0, u_3 &gt; 0)</td>
<td>(K_{2H} = K_1) and (K_{2L} = K_1)</td>
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</table>

Note: The results in Table 3.6 are derived for general disaster functions.
- Cases 1, 5, 9, 13: Precluded if \(\beta c_{2H} - c_{1H} \geq 0\).
- Cases 3, 4: Precluded if all parameters except the disaster probability are the same in periods 1 and 2.
- Cases 11, 12, 15: Precluded if demand in period 2 is sufficiently high.

Of the 16 cases, 9 can be conditionally ruled out.

**Cases 1, 5, 9, 13:** If the present-value cost of protection in period 2 is equal to or higher than in period 1 \((\beta c_{2H} - c_{1H} \geq 0)\), adding protection in both periods 2H and 2L is ruled out. The port will
invest enough in period 1 that it will choose not to invest more in at least one of the two states of period 2. Thus, protection investment in at least one of periods 2H and 2L will be binding (i.e., \( I_{2H} > I_{2L} = I_1 \), or \( I_{2L} > I_{2H} = I_1 \), or \( I_{2H} = I_{2L} = I_1 \)). Among these three possibilities, investing in more protection in period 2L but not in period 2H (\( I_{2L} > I_{2H} = I_1 \)) is possible only if the port invests in more capacity in period 2L (\( K_{2L} > K_{2H} = K_1 \)). Therefore, investing in more protection in period 2H but not in period 2L is the most probable outcome. In contrast, if \( \beta c_{i2} - c_{i1} < 0 \), the port might hold back on protection in period 1 and invest more in period 2 regardless of the state.

Cases 3, 4: If all parameter values except for the disaster probability are the same in both periods, cases 3 and 4 can be excluded. Thus, if the port does not add more protection in period 2H, it will not add capacity in both periods 2H and 2L. This is intuitive because when no protection is added in a high-risk state, adding more capacity exposes the port to more potential damage, which is certainly not optimal.

Cases 11, 12, 15: If demand in period 2 is sufficiently high, it is never optimal to add capacity in period 2H but not period 2L, while adding protection in period 2L but not period 2H (case 11). In addition, for a given level of protection (\( I_{2L} = I_{2H} = I_1 \)), adding capacity in period 2H but not period 2L (case 12) is never optimal. Similarly, for a given level of capacity (\( K_{2L} = K_{2H} = K_1 \)), adding protection in period 2L but not period 2H (case 15) is never optimal.

The above results hold for both public and private ports. The proof for private ports is relegated to Appendix B.3, and the proof for public ports to Appendix B.4. The results are summarized in the following proposition:

Proposition 4: For both public port and private ports: (1) if the present-value cost of protection in period 2 is at least as high as in period 1 (\( \beta c_{i2} - c_{i1} \geq 0 \)), the port will not invest in protection in both states of period 2; (2) if demand in period 2 is sufficiently high, the port will not add capacity only in the high-risk state while adding protection only in the low-risk state; (3) if parameter values
other than the disaster probability are the same in both periods, the port will not add capacity in both states of period 2 if it does not add more protection in the high-risk state.

3.4.4 Numerical analysis

Further analytical results on the timing of investment in the two-period model are elusive. We thus proceed in this section numerically. We are especially interested in determining when the port prefers to make all its investments in period 1, and when it prefers to wait until period 2 and then invest in more capacity and/or protection depending on what state is realized.

Table 3.7 lists the baseline parameter values. Rather than representing any specific real-world case, the values are chosen to produce reasonable demand elasticities with respect to the port fee, port capacity, and protection. More important, they reveal some interesting intertemporal dependencies between the decision variables. The prior probability of a disaster in period 1 is set to $x_1 = 0.2$. In period 2L, the probability remains the same as in period 1 at $x_{2L} = 0.2$, whereas in period 2H the probability increases to $x_{2H} = 0.5$. Note that because of climate change, the ex ante expected probability of a disaster in period 2 can be higher than in period 1. Other parameter values are set to the same values in the two periods. Alternative parameter assumptions are explored in the sensitivity analysis.

<table>
<thead>
<tr>
<th>Table 3.7 Baseline parameter values</th>
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<tbody>
<tr>
<td>Demand intercept $a_1 = a_2 = 5$</td>
</tr>
<tr>
<td>Demand slope $b_1 = b_2 = 1$</td>
</tr>
<tr>
<td>Disaster cost to port $M_1 = M_2 = 1$</td>
</tr>
<tr>
<td>Unit handling cost $c_1 = c_2 = 0.1$</td>
</tr>
<tr>
<td>Unit protection cost $c_{t1} = c_{t2} = 0.2$</td>
</tr>
<tr>
<td>Unit capacity cost $c_{k1} = c_{k2} = 0.3$</td>
</tr>
<tr>
<td>Discount factor $\beta = 0.5$</td>
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</table>
Table 3.8 Baseline equilibrium for private and public port

<table>
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<tr>
<th>Table 3.8 Baseline equilibrium for private and public port</th>
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<tbody>
<tr>
<td><strong>Private Port</strong></td>
</tr>
<tr>
<td><strong>Period 1</strong></td>
</tr>
<tr>
<td>Port fee</td>
</tr>
<tr>
<td>Capacity</td>
</tr>
<tr>
<td>Protection</td>
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<tr>
<td>Traffic volume</td>
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<tr>
<td>Demand elasticities:</td>
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<tr>
<td>w.r.t. ( \tau )</td>
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<tr>
<td>w.r.t. ( K )</td>
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<tr>
<td>w.r.t. ( I )</td>
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<tr>
<td><strong>Public Port</strong></td>
</tr>
<tr>
<td><strong>Period 1</strong></td>
</tr>
<tr>
<td>Port fee</td>
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<tr>
<td>Capacity</td>
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<td>w.r.t. ( K )</td>
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<tr>
<td>w.r.t. ( I )</td>
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</tbody>
</table>

Table 3.8 presents the equilibrium solutions and demand elasticities with respect to (w.r.t.) the port fee, capacity, and protection for both private and public port. Several points are worth noting. First, regardless of the realized state in period 1, the port (either private or public) never invests in additional capacity in period 2. Second, in period 2L the port (either private or public) does not invest in additional protection either. However, it does invest in period 2H because the probability of a disaster rises significantly. Third, demand in period 2 is correspondingly more responsive to protection when the probability of disaster is high, and more responsive to capacity when the probability of disaster is low. Fourth, a public port charges a lower price and invests more in both
capacity and protection than a private port. Consequently, equilibrium output is higher for a public port than a private port.

In the remainder of this section, we conduct sensitivity analysis for some of the parameter values. For two reasons we limit attention in the text to private ports. First, we showed in Section 3.3 that, except for the port charge, the full-equilibrium comparative statics results are the same for private and public ports. Second, the two-period model results derived in the previous subsection hold for both private and public ports. We relegate the numerical analysis of a public port to Appendix B.4.

![Figure 3.3 Varying the probability of disaster $x_{2H}$, with $x_1 = x_{2L} = 0.2$ (private port)](image)

Figure 3.3 depicts the effects of varying the probability of disaster in period 2H, $x_{2H}$. The solution in period 2L is the same as in period 1 because all parameter values are the same. In period 2H, the port reduces the fee although traffic volume still falls. As $x_{2H}$ rises, the port invests in less capacity because it is more liable to be damaged as the future climate gets worse. Moreover, the port never invests in additional capacity in period 2 in either state. Consequently, traffic volume decreases with $x_{2H}$ in both periods and both states. If $x_{2H} \leq 0.3$, the probability of disaster is not much higher than in period 1. As $x_{2H}$ increases within this range, the port invests in more protection in period 1 because the protection is more valuable in both periods, but it does not invest in further protection in period 2. By contrast, if $x_{2H} > 0.3$, the difference in disaster probabilities between periods 2L and 2H is large enough that information about what happens becomes valuable. This gives the port an option value of waiting for better information in making irreversible investment decisions under uncertainty. The port refrains from building too much protection in period 1, and waits until the state is revealed. It invests in more protection in state 2H, but not in...
state 2L. Counterintuitively, the amount of protection installed in period 1 actually decreases (albeit slowly) with $x_{2H}$ even though the future climate is getting worse in a probabilistic sense. This happens because the amount of capacity requiring protection is declining.

In summary, the port can adapt in different ways as the future climate gets worse. If the climate is not too bad in period 2H (i.e., $x_{2H} \leq 0.3$), the port does not invest in more protection but merely reduces its fee. If, contrarily, the climate is bad (i.e., $x_{2H} > 0.3$), the port does build more protection, and by enough that it hardly adjusts its fee at all. Thus, once the probability of disaster exceeds a threshold value, the port alters how it responds. The shift in strategy is driven by the irreversibility of the investments.

Figure 3.4 Varying the probability of disaster $x_{2L}$, with $x_1 = 0.5$, $x_{2H} = 0.8$ (private port)

Figure 3.4 displays the effects of varying the probability of disaster in period 2L, $x_{2L}$. To allow for the possibility that the port’s assessment of the climate can change for the better, probabilities $x_1$ and $x_{2H}$ are set at relatively high values, with $x_1 = 0.5$ and $x_{2H} = 0.8$. If $x_{2L} \geq 0.4$, all three probabilities are reasonably similar. Thus, there is no option value of waiting for better information and the port makes all its capacity investment in period 1. In contrast, if $x_{2L} < 0.4$, the port does hold back on capacity to wait for the state to be revealed, and invests more in period 2L. Interestingly, capacity investment in period 1 is not monotone in $x_{2L}$. If $x_{2L}$ is small, a small increase in $x_{2L}$ induces the port to build more capacity because it plans to install sufficient additional protection to defend its larger facilities. However, if $x_{2L}$ is already large and increases further, the port reduces capacity because its protection is already high, and adding further protection is subject to diminishing returns. If $x_{2L} < 0.4$, the port holds back on protection because
information is valuable and there is an option value of waiting. Conversely, if $x_{2L} \geq 0.4$, the port makes all its protection in period 1 to cover both periods rather than waiting for more information. The port fee in period 1 and period $2H$ increases with $x_{2L}$ due to the substantial increase in protection. However, the fee decreases in period $2L$ because $x_{2L}$ negatively affects demand. Traffic volume in period $2L$ decreases with $x_{2L}$ as expected, but traffic volumes in periods 1 and $2H$ first increase and then decrease in parallel with capacity.

Figure 3.5 Varying the probability of disaster $x_1$, with $x_{2L} = x_1; x_{2H} = x_1 + 0.3$ (private port)

Figure 3.5 varies the three disaster probabilities together to examine the effect of a general deterioration in climate. As expected, the port decreases capacity and increases protection. The combined effect allows it to increase its fee, but traffic volume falls. All capacity is built in period 1. If the probabilities are relatively low (i.e., $x_1$ is below about 0.45) the port holds back on protection to avoid overinvesting, and adds more in period 2H.

Figure 3.6 Varying the demand intercept in period 2, $a_2$ (private port)

To examine the effect of more demand in period 2, we set the disaster probability to the baseline parameters. Figure 3.6 shows the results of increasing demand in period 2. The effects depend on
whether the increase is small, medium, or large. If it is small (i.e., \(a_2 \leq 6\)), the port still chooses to build capacity only in period 1. If the increase is medium (i.e., \(6 < a_2 \leq 7\)), the port holds back on capacity in period 1 and adds more in period 2L. Finally, if the increase is large (i.e., \(a_2 > 7\)), the port adds capacity in period 2 in both states. It holds back on capacity investment in period 1 to save on costs, and invests more in period 2 because the large increase in demand outweighs the risks. However, the port does not invest in further protection in period 2L until \(a_2\) is sufficiently large (\(a_2 > 11\)). Demand can also be increased by reducing the slope parameter, \(b_2\). Except for the fee, the effects are similar to increasing \(a_2\). See Figure B.3.1 in Appendix B.3.

The effects of varying the cost of capacity in period 2, \(c_{k2}\), are shown in Figure 3.7. If \(c_{k2}\) rises above its baseline value of 0.3, investment in period 2 becomes more expensive and this reinforces the port’s decision to make all its capacity investment in period 1. Only if the cost drops substantially (\(c_{k2} < 0.2\)) does the port hold back on capacity, and wait to see what happens. In period 2L, it takes advantage of the low cost to add more capacity. The port fee is highest in period 1. It drops slightly in period 2L due to the additional investment in capacity. The fee drops much further in period 2H. Traffic volume changes in the opposite direction to the fee.

Figure 3.7 Varying the cost of capacity in period 2, \(c_{k2}\) (private port)

Figure 3.8 shows the effects of varying the cost of protection in period 2, \(c_{I2}\). Similar to Figure 3.7, an increase in the cost above its baseline value of 0.2 reinforces the port’s decision to forego protection investment in period 2L. Only if the cost drops does the port hold back on protection in period 1. If the reduction in cost is modest (\(0.15 < c_{I2} < 0.3\)), it adds more protection only in
period 2H. If the reduction is large ($c_{I2} < 0.15$), it also adds protection in period 2L although by less than in 2H. The port also invests in more capacity in period 1 because it can add further protection in period 2 relatively cheaply. As $c_{I2}$ changes, the port’s fee rises or falls in tandem with its protection.

![Figure 3.8 Varying the cost of protection in period 2, $c_{I2}$ (private port)](image)

Figure 3.8 considers variations in the effectiveness of protection for shippers, $\theta_1$ and $\theta_2$. Since varying the two parameters independently yields similar results (see Figure B.3.4 in Appendix B.3), we hold $\theta_1 = \theta_2$ and refer to the common value as $\theta$. The amount of protection changes in a non-monotone way with $\theta$. It increases at first as protection becomes more effective, but eventually decreases as protection becomes so potent that less of it is needed. Consistent with intuition, port capacity, traffic volume, and the fee all increase monotonically with $\theta$. Varying the effectiveness of port protection, $\eta_1$ and $\eta_2$, has similar effects to varying $\theta_1$ and $\theta_2$ (see Figures B.3.5 and B.3.6 in Appendix B.3).

Figure 3.10 considers varying the congestion cost for shippers with $\delta_1 = \delta_2$, which we refer to as $\delta$. (See Figure B.3.7 in Appendix B.3 for varying $\delta_2$ independently, which provides similar insights.) When $\delta$ is small, providing more capacity is optimal when $\delta$ increases, as it relieves congestion. When $\delta$ is large, demand is substantially reduced. More capacity is no longer desirable as there is not much traffic that can benefit from it, so capacity is reduced. Protection changes in the same direction as capacity. Consistent with Table 3.2, both the charge and traffic volume
decrease with $\delta$. The results of sensitivity analysis with respect to other parameters, such as the disaster severity, $m$ and $M$, are reported in Appendix B.3.

![Figure 3.9 Varying the effectiveness of protection to shippers with $\theta_1 = \theta_2$ (private port)](image1)

![Figure 3.10 Varying the congestion cost to shippers with $\delta_1 = \delta_2$ (private port)](image2)

### 3.5 Concluding remarks

This study investigates the optimal timing and scale of port capacity and protection investments when there is uncertainty about the rate of climate change. We analyze how optimal capacity and protection interact with each other, and how the two types of investments affect the port fee. We show that the fee and capacity are submodular, while capacity and protection are supermodular, and the fee and protection are supermodular, too. In a partial equilibrium, a higher disaster probability or magnitude reduces the fee and capacity, but increases protection. Capacity investment induces the port to charge a lower price, and this effect is stronger for a public port than a private port. By contrast, protection investment induces the port to charge a higher price, and this effect is stronger for a private port. In a full equilibrium, using specific functional forms,
we show that a higher disaster probability or magnitude reduces capacity and traffic volume, while the effect on protection is ambiguous. More effective protection induces a larger capacity and traffic volume, but may increase or decrease protection. These results hold for both public and private ports. We show that comparative statics results for “partial equilibrium” and “full equilibrium” can differ in sign. This highlights the importance of considering the two types of investment together. When making protection investment decisions, it is important to take into account possible changes in productive capacity and port fees. These changes affect the port’s profits, and hence the value to the port of protecting its infrastructure. Similarly, when making capacity investment decisions, it is important to take into account possible changes in protection. Protection is an additional cost that should be included in a cost-benefit analysis of port expansion.

In the two-period model, we investigate the optimal timing of investments when information accumulation occurs between period 1 and 2. We find that if the present-value cost of protection investment does not decrease from period 1 to 2, the port will not invest in additional protection in period 2. The port is more likely to add capacity in a low-risk state, and to add protection in a high-risk state. In addition, the port prefers to postpone capacity investment if the probability of a disaster can fall, if demand increases substantially, or the cost of investment in capacity falls. Conversely, it prefers to invest in advance if the climate is likely to get worse.

In terms of protection, the port prefers to wait under several circumstances. It prefers to wait if the probability of a disaster can change a lot. Since changes in the disaster probability create an option value of waiting for better information, the port can adjust the amount of investment after the state of climate is revealed. It prefers to wait if the disaster probability is currently low in order to avoid overinvestment if the climate remains relatively benign. It also prefers to wait if protection becomes cheaper in period 2. Significantly, investments in capacity and protection can change in either the same or opposite directions. They move in tandem with the level of future demand. But if the probability of disaster rises, capacity falls while protection rises. If protection is already effective, a further improvement in its effectiveness induces the port to cut back on protection while expanding capacity. An increase in capacity tends to reduce the port fee because congestion
becomes less of a problem, but an increase in protection tends to increase the fee because shippers are willing to pay more to use a safer port.

The model could be extended to include a number of omitted considerations such as demand uncertainty, depreciation of capacity and protection infrastructure, and disaster-related damage to protection. Protection investments should also be made with due consideration for how long they will last, and whether technology will change. In extreme disasters, it may be cost-effective to abandon a port or relocate it to a less vulnerable site. The timing of the model could be refined to allow shippers to respond to events more quickly than ports. Economies or diseconomies of scale in protection investment could be introduced. Economies of scope in investment may also be relevant since it may be cost-effective to invest in capacity and protection at the same time to minimize the disruption to a port’s operations. Finally, competition among ports can also be introduced.
Chapter 4: Structural estimation of airport ground transport mode choice using aggregate data

4.1 Introduction

The airport ground transportation system is the key link connecting passengers and goods with the aviation system. With the increase in air travel demand, more ground trips are made to and from the airports for the purpose of air travel. As a result, a comprehensive multi-modal airport ground transportation system is critical for an airport’s long-term growth. In order to better operate and plan the ground transportation system at the airport, it is essential to understand the ground travel behavior of air passengers. Many studies have been done to identify how various factors, such as fare, travel time, schedule reliability, and service frequency, would influence air travelers’ mode choice of ground transportation modes. However, most of the existing studies rely on stated preference survey data. With the individual-level data, multinomial or nested logit models are used to estimate passengers’ mode choice. However, surveys could be costly to conduct as a large amount of individual-level data has to be collected. In addition, the respondents may not always be representative for the general population, which can lead to measurement errors and selection bias in the estimation.

In this study, I propose a new estimation approach which uses aggregate data to estimate air passengers’ mode choice of airport ground transportation. Unlike existing approaches, this method does not need any individual-level attributes and preferences. The estimation simply replies on the observed aggregate data, namely the ridership and schedule of public transport, the arrival time of each flight, and the number of passengers on each flight. Specifically, this method estimates the ridership of each airport ground transport service based on a discrete choice model that takes into account fare of the transportation modes, in-vehicle travel time, waiting time, and flight characteristics. The model coefficients can be computed by the nonlinear least-squares estimation that minimizes the distance between the estimated and observed ridership for each scheduled public transport service. Essentially, this approach converts unobservable choice probabilities into aggregate counts which can be compared to the observables.
To implement the proposed estimation approach, I utilize data from the Incheon International Airport (ICN), known as the gateway of Korea. ICN is connected to several major Korean cities, such as Daejeon, Daegu, and Busan, by high-speed rail. Passengers arriving at ICN can travel to those cities either by private or public transportation. The latter includes high-speed trains and intercity buses. With detailed flight arrival information and the schedules and ridership of the buses and trains, I investigate passenger choice behavior at ICN using the proposed estimation approach.

The estimation results suggest that passengers who fly Korean airlines or have experienced longer waiting times are less likely to take public transport, and passengers who fly low-cost airlines are more likely to take public transport. In addition, waiting time has a much stronger negative impact than in-vehicle travel time on passengers’ choice probability of public transport. The estimation results deliver important information for understanding airport passengers’ choice behaviors and allow airport managers to optimize public transport schedules to achieve higher efficiency. In current practice, the train schedules are not efficiently matched with the intraday demand. Thus, I conduct a counterfactual analysis to derive alternative train schedule by formulating a nonlinear program with estimated parameters to maximize the train ridership. This nonlinear program takes into account the effect of different train schedules on passengers’ choice, which is usually not captured in the existing literature. The new train schedule solved from the nonlinear program improves the total ridership by 31.3% over the existing train schedule at ICN.

This chapter makes four major contributions. First, I propose an identification strategy to calibrate a passenger choice model using aggregate ridership data, rather than individual choice data. Second, the estimation results reveal that waiting time have a significant negative effect on passengers’ choice of ground transportation modes. This finding provides managerial insights for the management of the airport ground transportation system. Third, using the estimation results, I formulate a nonlinear program to solve the train scheduling problem. This method complements the existing scheduling literature by modeling the effect of a schedule on customer demand. Last,

45 KTX (Korea Train eXpress) is South Korea’s high-speed rail system, operated by Korail, the national train operator.
several counterfactual analyses have been done to suggest alternative schedules for the train operator in order to maximize train ridership. The analysis is practically meaningful, as the train operator has been facing low ridership at ICN. The counterfactual analysis reveals that the actual train schedule departing from ICN is suboptimal, and may result in huge revenue loss for the train operator, as well as the surplus loss for air travelers. I show that the proposed schedule can be realized with the current fleet capacity. Therefore, it is promising to implement the proposed train schedule in practice.

The rest of the chapter is organized as follows. Section 4.2 reviews the literature. Section 4.3 introduces the study setting and the data. Section 4.4 proposes a structural estimation approach to derive the estimation of model coefficients and summarizes the estimation results. Section 4.5 performs a counterfactual analysis which formulates a nonlinear program to solve the optimal train schedules. Section 4.6 concludes the study and proposes future research questions.

4.2 Literature review

This study closely relates to two streams of literature. The first is the airport ground transport mode choice, and the second is the literature on waiting time and public transport scheduling. I review the relevant papers in the following sections.

4.2.1 Airport ground transport mode choice

Airport ground transport attributes and their effects on passengers’ airport access mode choice have been extensively studied in the past decades. Harvey (1986) is one of the earliest empirical studies to model air passengers’ access mode choice to the airport, using survey data from San Francisco Bay Area. Later on, many studies have followed up to empirically identify air travelers’ airport access mode choice, and the responsiveness to factors such as cost, travel time, schedule

46 Since the launch of high-speed train service in 2014 at ICN, the ridership has been a problem mainly due to the low train frequency compared to bus frequency. The deteriorating ridership has eventually forced Korail to suspend train operation between ICN and Daejeon/Daegu/Busan in March, 2018. Now, air passengers traveling to Daejeon/Daegu/Busan from ICN by train have to first take the train to Seoul and from Seoul transfer to another train to Daejeon/Daegu/Busan.
reliability, and service frequencies. For instance, Tsamboulas and Nikoleris (2008) investigate the public transport passengers’ willingness-to-pay for reducing access time to Athens International Airport. Keumi and Murakami (2012) consider the choice of access modes to hub airports in Japan and find that the passengers face a trade-off between long travel times at local airports and long waiting times at hub airports. Koster et al. (2011) analyze the trip timing decisions of travelers going to the airport and find that passengers’ departing time is strategically correlated with the expected airport congestion. Tam et al. (2008) assess the magnitude of safety margin, the additional time that passengers allocate for airport ground access trips, and find that business air passengers place a significantly higher value on both travel time and safety margin for their ground access to Hong Kong International Airport.

In addition, with the increasing presence of multiple departure airports in metropolitan areas, airport accessibility is also incorporated to study multi-airport competition and passengers’ airport/airline choice. For example, Windle and Dresner (1995) study the effect of access time on passengers’ airport choice in the Washington DC area and show that airport access time and flight frequencies are significant predictors of airport choice. Başar and Bhat (2004) study the passengers’ choice of airports in San Francisco Bay Area and quantify the effect of ground access cost and time on passengers’ choice. Pels et al. (2000) analytically investigate the effects of an improvement in airport accessibility on airport and airline competition in multi-airport regions. Pels et al. (2001) estimate passengers’ sequential choice of airport and airline in a multiple airport region and find that access time is of vital importance for the choice of an airport by air travelers.

Despite the extensive literature, several research gaps still exist. First, most of the above studies use individual-level data obtained from stated preference surveys or revealed preference surveys to study air travelers’ surface mode choice. Second, the above studies all focus on airport access mode choice, while little attention has been paid to the analysis of airport egress mode choice. Third, the passenger flows determined by flight schedules to and from the airport have not been featured in the existing studies on airport ground transport mode choice.
4.2.2 Waiting time and public transport scheduling

Passenger waiting time is an important criterion in evaluating the quality of public transportation, one of the most essential services in urban areas. Several studies have proposed methods for waiting time estimations in public transport models. For example, Amin-Naseri and Baradaran (2014) develop formulas to derive accurate estimation of the average passenger waiting time with dependent or independent headways and under uniform passenger arrival times. Ingvardson et al. (2018) estimate passenger waiting times by using a mixture arrival distribution consisting of a uniform and a beta distribution. Hsu (2010) develops a passenger transfer waiting time model for a connecting service at multi-modal stations, and shows that the operational coordination between the connection service and the feeder service is essential to improve the transfer waiting time.

Passengers usually perceive waiting time at public transport stops to be more onerous than in-vehicle time. Studies show that passengers are more sensitive to waiting time than they are to in-vehicle travel times (Eberlein et al., 1998; Larsen and Sunde, 2008). Jansson (1993) shows that the smaller the waiting time, the more inclined the passenger is to choose this service instead of other modes of transport. Since waiting time strongly influences the attractiveness and use of public transport, several studies have investigated the optimal scheduling of public transport. For instance, Vansteenwegen and Van Oudheusden (2007) develop a linear program to improve the timetables for a passenger railway network with the objective of minimizing passenger waiting cost. de Palma and Lindsey (2001) analyze the optimal timetable for a given number of public transport vehicles on a single transit line with the objective of minimizing riders’ total schedule delay when each rider’s preferred travel time is uniformly distributed.

There are some gaps remaining in the literature. First, most of the studies assume passenger arrival times to follow a certain distribution, while very few studies use the real passenger arrival data in waiting time estimation. Second, few studies have discussed the empirical evidence in quantifying the effect of waiting time on mode choice. Third, although various optimization programs have been proposed to optimize public transport timetables, few studies have considered the effect of transport timetable on passenger choice in formulating their optimization problem. Cordeau et al.
(1998) provide a review of optimization models for train routing and scheduling.

4.3 Study setting and data description

4.3.1 Study setting

ICN is one of the busiest airports in the world. In 2018, a total of 67.7 million passengers used ICN, including 33.9 million arrivals and 33.8 million departures. Air passengers can choose from a range of various transport modes to commute to ICN. In particular, ICN is co-located with the intercity rail station, which largely facilitates trains to travel to and from the airport. The Gyeongbu high-speed rail line is Korea’s first and the main high-speed rail line that connects Seoul and Busan (the second most-populous city after Seoul in Korea), with a maximum speed of 305 km/h. In 2014, Korail, the national railway operator in Korea, started to operate direct high-speed rail services between ICN and the major Korean cities on the Gyeongbu high-speed rail line, with the ICN-Seoul section running on conventional rail. Figure 4.1 illustrates the railway network in Korea.

This study aims to identify factors that may affect airport passengers’ choices of ground transport modes for intercity travels. I focus on air passengers’ egress mode choice from the airport to their destinations, instead of access mode choice to the airport, because it is difficult to obtain unbiased estimation of public transport schedules for the latter. Specifically, when studying the airport egress mode choice, one can use the flight arrival times plus the transfer times as the passengers’ arrival times at the public transport platform, which is exogenous; while for the airport access mode choice, it is difficult to determine when the passenger chooses to take the transport mode. In particular, passengers’ departure times may endogenously depend on the airport congestion level as passengers are strategic. For instance, Koster et al. (2011) find that for the airport access mode choice, passengers’ departing time is strategically correlated with the expected airport congestion. Arnott et al., (1988) analyze the departure time decisions of heterogeneous morning commuters who differ in: (1) travel time and schedule delay costs, (2) relative costs of early and late arrival, (3) desired arrival time. Thus, the departing time is endogenous for the airport access mode choice, which complicates the empirical estimation.
Seoul is excluded as an egress destination in this study because the ground transport between ICN and Seoul is characterized by various modes (e.g., subway, bus, train, taxi, private car, etc.) and high frequencies. It is also difficult to distinguish the mode choice between air passengers and
airport employees from aggregate data. Therefore, this study focuses on the mode choice from ICN to three largest cities (Daejeon, Daegu, and Busan) in the southern part of Korea. Passengers arriving at ICN can travel to these cities either by private or public transport. The latter includes high-speed trains and intercity buses.

4.3.2 Data description

Detailed daily information is collected for the arriving flights at ICN and for the airport egress modes (bus and train) to three destinations (Daegu, Daejeon, and Busan).

<table>
<thead>
<tr>
<th>Table 4.1 Characteristics of arriving flights at ICN per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of arriving flights</td>
</tr>
<tr>
<td>Departure region:</td>
</tr>
<tr>
<td>Asia</td>
</tr>
<tr>
<td>Northeast Asia</td>
</tr>
<tr>
<td>Southeast Asia</td>
</tr>
<tr>
<td>Central + South Asia</td>
</tr>
<tr>
<td>Europe</td>
</tr>
<tr>
<td>North and Central America</td>
</tr>
<tr>
<td>Southwest Pacific</td>
</tr>
<tr>
<td>Middle East</td>
</tr>
<tr>
<td>Eastern Africa</td>
</tr>
<tr>
<td>Flight type:</td>
</tr>
<tr>
<td>Low-cost carriers</td>
</tr>
</tbody>
</table>

For each arriving flight, the data records the airline, flight number, origin airport, scheduled and actual flight arrival time, arriving terminal, the number of passengers on board, and flying distance. For the two public transport modes (i.e., bus and train), data is available on the daily schedules from ICN to the three destinations, the in-vehicle travel time, ticket price, and the ridership on
The data covers 3623 flights that arrived during the study period, which is from July 15 to July 22, 2017, the peak season at ICN. Table 4.1 summarizes the characteristics of arriving flights on each day. It can be seen that Northeast Asia and Southeast Asia rank the first and second, respectively, in terms of the number of arriving flights, with their total share summing up to 80.7%. In addition, low-cost carriers (LCCs) account for about 30.1% arrivals.

Figure 4.2 depicts the average number of arriving flights and passengers along the 24-hour daily window with 10-minute intervals over the sample period. Peak hours at ICN includes morning and late afternoon. It can be observed that LCCs use slots in unfavorable periods, such as early morning and late evening, to avoid the peak periods during which many full-service carriers (FSCs) arrive.
Table 4.2 summarizes the operational characteristics of the public transport (bus and train) from ICN to the three destinations (Daejeon, Daegu, and Busan). It shows that trains have significantly higher speed compared to buses, but buses are scheduled at a much higher frequency. As a result, passengers who choose buses experience a shorter average waiting time compared to those choosing trains. Typically, an air passenger has to trade-off between the in-vehicle travel time and waiting time when choosing the ground transportation mode. Due to the speed advantage and higher operating cost of high-speed trains, bus fares are cheaper. The train fares and bus fares at ICN are fixed and are regulated by the Ministry of Land, Infrastructure and Transportation. Thus, we will not include train and bus fares as decision variables in our study.

### Table 4.2 Operational characteristics of the public transport from ICN to three destinations

<table>
<thead>
<tr>
<th></th>
<th>Earliest</th>
<th>Latest</th>
<th>Daily Frequency</th>
<th>Travel Time</th>
<th>Fare</th>
<th>Avg. Ridership</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bus</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daejeon</td>
<td>06:00</td>
<td>23:30</td>
<td>57</td>
<td>3hr20min</td>
<td>$21</td>
<td>23</td>
</tr>
<tr>
<td>Daegu</td>
<td>06:20</td>
<td>23:30</td>
<td>42</td>
<td>3hr50min</td>
<td>$32</td>
<td>11</td>
</tr>
<tr>
<td>Busan</td>
<td>07:00</td>
<td>23:30</td>
<td>18</td>
<td>5hr10min</td>
<td>$38</td>
<td>9</td>
</tr>
<tr>
<td><strong>Train</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daejeon</td>
<td>06:55</td>
<td>20:38</td>
<td>7</td>
<td>1hr50min</td>
<td>$32</td>
<td>22</td>
</tr>
<tr>
<td>Daegu</td>
<td>06:55</td>
<td>20:38</td>
<td>7</td>
<td>2hr40min</td>
<td>$49</td>
<td>49</td>
</tr>
<tr>
<td>Busan</td>
<td>06:55</td>
<td>20:38</td>
<td>7</td>
<td>3hr30min</td>
<td>$64</td>
<td>28</td>
</tr>
</tbody>
</table>

Note: The $ in this chapter refers to US dollar. Avg. Ridership refers to the average number of passengers per bus or per train.

### 4.4 Estimation

#### 4.4.1 Structural estimation model

Assume passengers arriving at the airport maximize an indirect utility function of the following form when choosing a ground transportation mode to their final destinations:
\[ U_{imjd} = V_m + \theta_1 TT_{mjd} + \theta_2 WT_{imjd} + \theta_3 P_{md} + \theta_4 X_i + \epsilon_{imjd}, \] (4.1)

where the notations are explained as follows:

- \( i \): arriving flight;
- \( m \): airport ground transportation mode;
- \( j \): scheduled time of the airport ground transportation mode;
- \( d \): final destination;

- \( U_{imjd} \): indirect utility of an air passenger arriving on flight \( i \) choosing transport mode \( m \) at the scheduled time \( j \) to destination \( d \);
- \( V_m \): constant utility of choosing transport mode \( m \);
- \( TT_{mjd} \): in-vehicle travel time by mode \( m \) at scheduled time \( j \) to destination \( d \);
- \( WT_{imjd} \): waiting time of an air passenger arriving on flight \( i \) choosing transport mode \( m \) at the scheduled time \( j \) to destination \( d \);
- \( P_{md} \): ticket price by transport mode \( m \) to destination \( d \) (all trains or buses have the same price regardless of their departure times);
- \( X_i \): characteristics of arriving flight \( i \), including flight type (FSC or LCC), flight distance, region of the origin airport, and whether the flight is operated by a Korean airline;
- \( \epsilon_{imjd} \): unobserved idiosyncratic utility shock;
- \( \Theta \equiv (V_m, \theta_1, \theta_2, \theta_3, \theta_4) \): parameters to estimate.

I focus on two public transport modes (e.g., buses and trains) at the airport. Thus, private transport, such as private cars and taxi, is considered as an outside option, with its utility normalized to zero. By assuming \( \epsilon_{imjd} \) is independent and follows the Type-I Extreme Value distribution, the probability of a passenger arriving on flight \( i \) choosing transport mode \( m \) at scheduled time \( j \) to destination \( d \), defined as \( S_{imjd} \), admits a closed form solution by the classical multinomial logit model (McFadden et al., 1973),

\[
S_{imjd} = \frac{\exp(V_m + \theta_1 TT_{mjd} + \theta_2 WT_{imjd} + \theta_3 P_{md} + \theta_4 X_i)}{1 + \sum_{u=bus,train} \exp(V_u + \theta_1 TT_{ujd} + \theta_2 WT_{iujd} + \theta_3 P_{ud} + \theta_4 X_i}).
\] (4.2)

The estimated ridership of transport mode \( m \) scheduled at time \( j \) to destination \( d \), defined as \( \hat{Q}_{mjd} \).
can be estimated as:

\[ \hat{Q}_{mjd}(\Theta) = \sum_{i \in I(m,j,d)} A_i r_d S_{imjd}, \]  

(4.3)

where

- \( A_i \): the number of passengers on flight \( i \);
- \( r_d \): the share of passengers on each flight whose final destination is \( d \);  
- \( I(m,j,d) \): the set of arriving flights whose passengers can take transport mode \( m \) scheduled at time \( j \) to destination \( d \);

To compute the model coefficients \( \Theta \), I apply the non-linear least squares estimation that minimizes the sum of squared errors between the estimated ridership \( \hat{Q}_{mjd}(\Theta) \) and the actual ridership \( Q_{mjd} \) of transport mode \( m \) to destination \( d \) scheduled at time \( j \):

\[ \min_{\Theta} M_n(\Theta) = \min_{\Theta} \frac{1}{n} \sum_{m,j,d} (\hat{Q}_{mjd}(\Theta) - Q_{mjd})^2, \]  

(4.4)

where \( n \) denotes the total number of buses and trains to all destinations in the study period. The main idea of this method is to convert unobservable individual’s choice probability to ridership forecasts. I seek for coefficient estimators that lead to forecasts with the minimal least squared error.

Figure 4.3 illustrates how I compute the forecasted ridership \( \hat{Q}_{mjd}(\Theta) \). Consider an airport with two public transport modes (e.g., buses and trains), with fixed daily schedules. Compared to the departure times of buses and trains, the flight arrivals are much more frequent, as manifested in Figure 4.3. For one particular flight, we assume that its passengers will choose the next departing bus or train to their destinations, or neither of them. Thus, the captured market, which is denoted as \( I(m,j,d) \) in Eq. (4.3), of each bus/train can be defined as all the flights whose passengers are able to catch this bus/train, taking into account the time it takes from landing to reach the bus/train platform. In other words, a scheduled bus/train departing before a passenger can arrive at the

---

47 Based on the surveys that ICN conducted, on each flight, there are 2%, 5%, and 9% of passengers whose final destination is Daejeon, Daegu, and Busan, respectively.
bus/train platform is not in the choice set of the passenger. For all the observations, the seating capacity of bus/train is sufficient to accommodate for the observed ridership. Therefore, it is assumed in the structural estimation model that passengers will always take the next available bus/train.

A passenger’s waiting time is calculated as the duration between the arrival time at the bus/train platform and the departure time of the next available bus/train. As flights arrive at ICN can land at two different terminals: the main terminal and the concourse, it takes different times from the two terminals to the bus/train platform. A survey conducted by ICN indicates that a passenger arrives at the main terminal (the concourse, respectively) on average takes 47 minutes (62 minutes, respectively) after landing to reach the bus platform, and 62 minutes (77 minutes, respectively) to reach the train platform. The duration takes into account the time spent on baggage claim, customs and immigration, and walking. Figure 4.4 illustrates how waiting time are calculated for passengers arriving at different terminals. Therefore, the waiting time is flight-specific, depending on the flight arrival time, the landing terminal, and the bus or train schedules.

---

48 The seating capacity of intercity buses at ICN is 40 to 60. The maximum bus ridership in the sample data is 33. The high-speed train at ICN sells standing tickets.
Figure 4.4 Calculation of passengers’ waiting time

Passengers’ preferences for public transport modes (i.e., bus and train) and private transport modes (i.e., outside options) are differentiated by controlling for the flight characteristics, the variables $X_i$ in Eq. (4.1). Specifically, I control for the region of the departure airport for each arriving flight. It is conjectured that passengers from different regions may have different preferences over airport ground transport modes. In addition, passengers may also be differentiated by the types of airlines. For example, passengers flying LCCs are likely to be leisure travelers, and may have different preference compared with passengers flying FSCs. In addition, passengers flying Korean airlines are likely to be Koreans, which may result in different choice behaviors compared with foreign passengers. The flight distance is also controlled. Table 4.3 displays the summary statistics of the variables included in the estimation.

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49 Korean airlines include Korean Air (KE), Asiana Airlines (OZ), Jeju Air (7C), Jin Air (LJ), Air Seoul (RS), T’way Air (TW), and Eastar Jet (ZE).
Table 4.3 Summary statistics of the variables used in the estimation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Destination</th>
<th>Obs.</th>
<th>Mean</th>
<th>Std.</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-vehicle travel time by train (hr)</td>
<td>Daejeon</td>
<td>56</td>
<td>1.907</td>
<td>0.071</td>
<td>1.816</td>
<td>2.050</td>
</tr>
<tr>
<td></td>
<td>Daegu</td>
<td>56</td>
<td>2.690</td>
<td>0.074</td>
<td>2.583</td>
<td>2.783</td>
</tr>
<tr>
<td></td>
<td>Busan</td>
<td>56</td>
<td>3.591</td>
<td>0.077</td>
<td>3.450</td>
<td>3.700</td>
</tr>
<tr>
<td>In-vehicle travel time by bus (hr)</td>
<td>Daejeon</td>
<td>456</td>
<td>3.333</td>
<td>0</td>
<td>3.333</td>
<td>3.333</td>
</tr>
<tr>
<td></td>
<td>Daegu</td>
<td>336</td>
<td>3.853</td>
<td>0.252</td>
<td>3.666</td>
<td>4.666</td>
</tr>
<tr>
<td></td>
<td>Busan</td>
<td>144</td>
<td>5.111</td>
<td>0.157</td>
<td>5.000</td>
<td>5.333</td>
</tr>
<tr>
<td>Waiting time by train (hr)</td>
<td>Daejeon</td>
<td>3623</td>
<td>1.358</td>
<td>1.132</td>
<td>0</td>
<td>6.917</td>
</tr>
<tr>
<td></td>
<td>Daegu</td>
<td>3623</td>
<td>1.358</td>
<td>1.132</td>
<td>0</td>
<td>6.917</td>
</tr>
<tr>
<td></td>
<td>Busan</td>
<td>3623</td>
<td>1.358</td>
<td>1.132</td>
<td>0</td>
<td>6.917</td>
</tr>
<tr>
<td>Waiting time by bus (hr)</td>
<td>Daejeon</td>
<td>3623</td>
<td>0.292</td>
<td>0.747</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Daegu</td>
<td>3623</td>
<td>0.368</td>
<td>0.798</td>
<td>0</td>
<td>6.333</td>
</tr>
<tr>
<td></td>
<td>Busan</td>
<td>3623</td>
<td>0.747</td>
<td>0.921</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Fare by train ($)</td>
<td>Daejeon</td>
<td>56</td>
<td>31.946</td>
<td>0</td>
<td>31.946</td>
<td>31.946</td>
</tr>
<tr>
<td></td>
<td>Daegu</td>
<td>56</td>
<td>49.380</td>
<td>0</td>
<td>49.380</td>
<td>49.380</td>
</tr>
<tr>
<td></td>
<td>Busan</td>
<td>56</td>
<td>63.805</td>
<td>0</td>
<td>63.805</td>
<td>63.805</td>
</tr>
<tr>
<td>Fare by bus ($)</td>
<td>Daejeon</td>
<td>456</td>
<td>20.620</td>
<td>0</td>
<td>20.620</td>
<td>20.620</td>
</tr>
<tr>
<td></td>
<td>Daegu</td>
<td>336</td>
<td>31.850</td>
<td>0</td>
<td>31.850</td>
<td>31.850</td>
</tr>
<tr>
<td></td>
<td>Busan</td>
<td>144</td>
<td>37.550</td>
<td>0</td>
<td>37.550</td>
<td>37.550</td>
</tr>
<tr>
<td>Flight distance (1000 km)</td>
<td></td>
<td>3623</td>
<td>2.960</td>
<td>2.815</td>
<td>0.262</td>
<td>12.095</td>
</tr>
<tr>
<td>Korean airlines</td>
<td></td>
<td>3623</td>
<td>0.674</td>
<td>0.469</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>LCC</td>
<td></td>
<td>3623</td>
<td>0.301</td>
<td>0.459</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Southeast Asia</td>
<td></td>
<td>3623</td>
<td>0.238</td>
<td>0.426</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Europe</td>
<td></td>
<td>3623</td>
<td>0.055</td>
<td>0.228</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>America</td>
<td></td>
<td>3623</td>
<td>0.075</td>
<td>0.263</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Middle East and Eastern Africa</td>
<td></td>
<td>3623</td>
<td>0.012</td>
<td>0.107</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Southwest Pacific</td>
<td></td>
<td>3623</td>
<td>0.036</td>
<td>0.186</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Mainland China</td>
<td></td>
<td>3623</td>
<td>0.229</td>
<td>0.420</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Japan</td>
<td></td>
<td>3623</td>
<td>0.208</td>
<td>0.406</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Other Northeast Asia</td>
<td></td>
<td>3623</td>
<td>0.133</td>
<td>0.339</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>South and Central Asia</td>
<td></td>
<td>3623</td>
<td>0.015</td>
<td>0.120</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: Obs. stands for the number of observations. Std. stands for standard deviation.
Since the information on the number of business or leisure passengers on each arrival flight is not available, I treat passengers on the same flight as homogenous choice makers. However, if the number of passengers traveling by first/business/economy class on each flight is observable, the model can be extended to capture that heterogeneity among passengers on the same flight.

This estimation model only captures the demand side, while treating supply side as fixed. It is possible that train and bus operators can compete or coordinate in schedules, which may result in biased estimation. However, this is not the case in the current context. Figure C.1 in Appendix C plots the bus and train departure times from ICN. Bus schedule features high frequency and balanced distribution (around every 20min to Daejeon, every 25min to Daegu, and every 1hr to Busan). During the period (e.g., at noon time) when there are more trains, the bus operator allocated neither more buses (to compete) nor fewer buses (to coordinate) with the train operation. Thus, it can be justified from the data that the bus and train schedules are exogenous.

### 4.4.2 Estimation results

Table 4.4 shows the estimation results with several specifications. Specification 1 includes all the variables described in Section 4.4.1. We can see that the in-vehicle travel time, waiting time, and fare all have negative and statistically significant impacts on passengers’ choice of public transport modes. The value of travel time (VOTT) and the value of waiting time (VOWT) can be estimated by using the compensation variation approach. Specifically, a passenger is willing to pay $22.29 to reduce the in-vehicle travel time by one hour, and $147.28 to reduce waiting time by one hour. It indicates that, on average, passengers incur about 6.6 times higher cost of waiting than in-vehicle traveling. This seems reasonable as it could be more stressful waiting at the platform than being onboard. The VOWT estimated in this study can be significantly higher than the VOWT for public transit in intra-city travels, because passengers experience the waiting time in this study after they have taken a flight and they are doing inter-city travel from the airport. For instance, Mohring et

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50 Compensation variation approach calculates the willingness-to-pay or monetary value of a particular variable by measuring the change in price to keep a passenger’s utility constant, when a particular variable varies. In other words, it calculates the marginal rates of substitution between attributes and money.
al. (1987) estimate that the VOTT is about two to three times that of time in transit for intra-city travel. Note that 98.5% of the flights arriving at ICN are international flights, which may involve onerous security and customs screening and the inconvenience of carrying luggage.

The estimated VOTT and VOWT seem to be comparable to the existing empirical findings. Specifically, the estimated VOTT can be benchmarked with the value of access time (VOAT) to the airport or the value of airborne time. Koster et al. (2011) estimate the VOAT to be 40.05 euro/hour for business passengers and 30.02 euro/hour for leisure passengers. Tam et al. (2011) estimate that the value of travel time to access the Hong Kong International Airport is 16.2$/hr for business passengers and 6.6$/hr for leisure passengers. The US DOT (Department of Transportation) also issues guidance about air passengers’ value of time (VOT), and it suggests that the value is between 36.1 $/hr and 63.2 $/hr as of 2016 (DOT, 2016). Schumer and Maloney (2008) suggest the VOT for airborne of US air passengers to be $37.6/hour. For the VOWT, there are, however, very few direct comparisons. It could be sensible to benchmark with the value of flight delay in that these two could be similar in nature from a passenger’s perspective. For example, Yan and Winston (2014) estimate airport delay value to be $104 /hour. Landau et al. (2016) estimate the flight delay cost in the US to be $123.3/hour for leisure passengers and $286.3/hour for business passengers. In Table 4.5, I summarize the VOT estimates in recent literature.

In addition, $v_n$ is estimated to capture the effect of passengers’ preference between trains (or buses) and the private options. The estimation indicates that train is preferred to bus ceteris paribus. The

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51 VOAT has been usually estimated to be considerably higher than the value of time for daily commuting trips, because travelers see the increasing travel time as an increase in risk to miss their flight (Koster et al. 2011).

52 This overall VOT does not differentiate different segments of the airline trip, for example, ground access to airport, check-in time, waiting time in terminal, and airborne time etc.

53 Waiting time is a type of schedule delay, which is often defined as the difference between a desired arrival time and an actual arrival time (e.g., at the workplace) in road congestion literature (i.e., daily commuting trips). Commuters can incur different costs for arriving early and arriving late. It is commonly assumed in the bottleneck models that the value of in-vehicle travel time is larger than the schedule delay for early arrival (Arnott et al., 1990; 1993; Silva et al., 2016), which is also supported by empirical evidence (Small, 1982). Tseng and Verhoef (2008) empirically show that values of travel time savings and values of schedule delay (both early and late arrivals) vary by time of day.
variable Korean airlines shows a negative sign, and the effect is statistically significant. Since passengers flying Korean airlines are likely to be Koreans, this result may indicate that Koreans are less likely to take public transport at the airport for intercity travels than foreigners. It also makes sense because Koreans may have family members to pick them up at the airport, and driving can also be a viable option for them, because they are more familiar with the road conditions. The rest of the variables, such as the LCC, the departure region, and the flight distance are not statistically significant in Specification 1.

As a robustness check, I test a few other specifications. Specification 2 excludes the region variables, Korean airlines, and LCC variable. Specification 3 excludes the region variables, flight distance, and Korean airlines. Specification 4 only keeps the three key variables (i.e., travel time, waiting time, and fare). Flight distance has a statistically significant negative coefficient in Specification 2, suggesting that the longer the flight distance, the less likely the passengers are to take public transport. LCC has a statistically significant positive coefficient in Specification 2, suggesting that passengers traveling by LCCs are more likely to take public transport at the airport than those traveling by FSCs. The three key variables (i.e., travel time, waiting time, and fare) are statistically significant at 0.1% level in all four specifications. Thus, the estimation results are robust in terms of the effects of travel time, waiting time, and fare.
Table 4.4 Estimation results

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Specification 1</th>
<th>Specification 2</th>
<th>Specification 3</th>
<th>Specification 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1.055)</td>
<td>(0.598)</td>
<td>(0.532)</td>
<td>(0.501)</td>
</tr>
<tr>
<td>Train constant</td>
<td>8.737***</td>
<td>8.160***</td>
<td>7.360***</td>
<td>7.578***</td>
</tr>
<tr>
<td></td>
<td>(1.286)</td>
<td>(0.850)</td>
<td>(0.780)</td>
<td>(0.723)</td>
</tr>
<tr>
<td>In-vehicle travel time (hr)</td>
<td>-1.291***</td>
<td>-1.247***</td>
<td>-1.246***</td>
<td>-1.237***</td>
</tr>
<tr>
<td></td>
<td>(0.137)</td>
<td>(0.127)</td>
<td>(0.124)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>Waiting time (hr)</td>
<td>-8.533***</td>
<td>-8.341***</td>
<td>-7.829***</td>
<td>-7.921***</td>
</tr>
<tr>
<td></td>
<td>(1.363)</td>
<td>(1.170)</td>
<td>(1.124)</td>
<td>(1.067)</td>
</tr>
<tr>
<td>Fare ($100)</td>
<td>-5.794***</td>
<td>-5.783***</td>
<td>-5.323***</td>
<td>-5.536***</td>
</tr>
<tr>
<td></td>
<td>(1.377)</td>
<td>(1.253)</td>
<td>(1.238)</td>
<td>(1.217)</td>
</tr>
<tr>
<td>Flight distance (1000km)</td>
<td>-0.121</td>
<td>-0.084*</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(0.336)</td>
<td>(0.041)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Korean airlines</td>
<td>-0.561*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.248)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LCC</td>
<td>0.606</td>
<td></td>
<td>0.511**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.423)</td>
<td></td>
<td>(0.196)</td>
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</tr>
<tr>
<td>Southeast Asia</td>
<td>0.005</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>(0.759)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>-0.338</td>
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</tr>
<tr>
<td></td>
<td>(2.562)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>America</td>
<td>0.521</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(3.113)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Middle East and Eastern Africa</td>
<td>0.961</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.122)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Southwest Pacific</td>
<td>-0.496</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>(0.764)</td>
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<td></td>
</tr>
<tr>
<td>Japan</td>
<td>-0.019</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.576)</td>
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<td></td>
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</tr>
<tr>
<td>Other Northeast Asia</td>
<td>-0.405</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.453)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>South and Central Asia</td>
<td>-0.176</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.621)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VOTT ($/hr)</td>
<td>22.29</td>
<td>21.56</td>
<td>23.40</td>
<td>22.35</td>
</tr>
<tr>
<td>VOWT ($/hr)</td>
<td>147.28</td>
<td>144.24</td>
<td>147.06</td>
<td>143.07</td>
</tr>
</tbody>
</table>

Note: (1) The region Mainland China is normalized to zero; (2) Standard errors are in parentheses; (3) *, **, *** represent that the estimated effect is significant at 5%, 1%, and 0.1%-levels, respectively.
Table 4.5 Estimation of value of time in recent literature

<table>
<thead>
<tr>
<th>Study</th>
<th>Data</th>
<th>Method</th>
<th>Estimation of VOT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Context: airport access mode choice</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harvey (1986)</td>
<td>RP survey conducted in the Bay Area in 1980</td>
<td>MNL</td>
<td>VOAT: 39.92-45.75 $/hr (business); 14.97-33.03 $/hr (leisure)</td>
</tr>
<tr>
<td>Tam et al. (2008)</td>
<td>SP survey conducted in Hong Kong in 2004 and 2005</td>
<td>MNL</td>
<td>VOAT: 0.1-0.25 $/min</td>
</tr>
<tr>
<td>Koster et al. (2011)</td>
<td>SP survey conducted in the Netherlands</td>
<td>mixed logit</td>
<td>VOAT: 40.05 Euro/hr (business); 30.02 Euro/hr (leisure) VSDE: 51.84 Euro/hr (business); 38.86 Euro/hr (leisure) VSDL: 194.51 Euro/hr (business); 145.82 Euro/hr (leisure)</td>
</tr>
<tr>
<td>Tam et al. (2011)</td>
<td>SP and RP data collected in Hong Kong</td>
<td>MNL</td>
<td>VOAT (RP value): 0.27 $/min (business); $0.11 $/min (leisure) VOAT (SP value): 0.25 $/min (business); $0.08 $/min (leisure)</td>
</tr>
<tr>
<td><strong>Context: airport and destination choice; airport and access mode choice; airport and airline choice</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Furuichi and Koppelman (1994)</td>
<td>RP survey of international air travellers departing from Japan in 1989</td>
<td>nested logit</td>
<td>VOAT: 72.6 $/hr (business); 41.1 $/hr (leisure) VOLHT: 35.7 $/hr (business); 51.2 $/hr (leisure)</td>
</tr>
<tr>
<td>Pels et al. (2003)</td>
<td>SP survey conducted in August and October 1995 in Bay Area</td>
<td>nested logit</td>
<td>VOAT (August sample): 2.9 $/min (business); 1.58 $/min (leisure) VOAT (October sample): 1.97 $/min (business); 1.6 $/min (leisure)</td>
</tr>
<tr>
<td>Loo et al. (2008)</td>
<td>SP survey collected in Hong Kong International Airport in 2003</td>
<td>MNL</td>
<td>VOAT: 7.61 $/min</td>
</tr>
<tr>
<td><strong>Context: airline industry</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yan and Winston (2014)</td>
<td>DB1B database</td>
<td>GMM</td>
<td>VOLHT: 33 $/hr (business); 16 $/hr (leisure); 24 $/hr (overall) VOAQ: 144 $/hr (business); 71 $/hr (leisure); 104 $/hr (overall)</td>
</tr>
<tr>
<td>Forbes (2008)</td>
<td>DB1A database, and ASQP database</td>
<td>Linear regression</td>
<td>VOLHT: 1.42 $/min</td>
</tr>
<tr>
<td>Landau et al. (2016)</td>
<td>SP survey</td>
<td>Estimated from experiment</td>
<td>VOAT: 18.60 $/hr (business); 16.95 $/hr (leisure) VOLHT: 51.01 $/hr (business); 34.91 $/hr (leisure) VOAQ: 286.32 $/hr (business); 123.30 $/hr (leisure)</td>
</tr>
<tr>
<td><strong>Context: intercity travel mode choice (air, rail)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fu et al. (2014)</td>
<td>2005 Japan Transportation Statistical Survey, and MIDT database</td>
<td>GMM</td>
<td>Value of intercity travel time: 42 $/hr</td>
</tr>
</tbody>
</table>

Note: RP: revealed preference; SP: stated preference; VOAT: value of access time; VSDE: value of schedule delay early; VSDL: value of schedule delay late; MNL: multinomial logit; VOLHT: value of line-haul time (or value of airborne time), which is the time from departure airport to destination; VOAD: value of airport delay; GMM: generalized method of moments; ASQP: Airline Service Quality Performance; MIDT: Marketing Information Data Transfer.
Next, I quantify the effect of waiting time on the market share of public transport. I use the estimation results from Specification 4 in Table 4.4, thus abstracting away the effects of flight characteristics on passengers’ mode choice. Using Eq. (4.2) and plugging in the estimated parameters ($\Theta$) given in Specification 4 and the average traveling time and fare information given in Table 4.2, I calculate and plot in Figure 4.5 how the market share of public transport changes with waiting time.

Figure 4.5 has the following implications. First, if passengers would experience the same waiting time for the intercity bus and high-speed train, the market share of the train will dominate that of the bus. This indicates that passengers strongly prefer high-speed trains even though its fare is more expensive. The distance between ICN and the three destinations can be considered as medium-haul or long-haul, because even going to the closest destination (Daejeon) takes almost 3.5hrs by bus and 2hrs by train. Thus, it is not surprising that a faster mode is preferred. Second, the market share of both bus and train decreases rapidly with waiting time. When the waiting time reaches 1hr, the market share of both bus and train becomes very small. This again highlights the strong negative effect of waiting time on using public transport. The average waiting time by bus is around 18min to Daejeon, 22min to Daegu and 45min to Busan, while the average waiting time by train is around 1hr20min to the three destinations (See Table 4.3 for the summary statistics). This explains the low train ridership problem, which the train operator has been struggling with. It also appears in Figure 4.5 that train passengers are more sensitive to waiting time than bus passengers, given the stronger effect of waiting time on train market share. Third, Figure 4.5 also illustrates that the market share of both public modes to Busan becomes very small when waiting time exceeds around 0.5hr, while this number is around 1hr for Daejeon and 0.75hr for Daegu. This indicates passengers traveling to Daejeon has more tolerance of waiting time, while passengers traveling to Busan has the least tolerance of waiting time. Passengers going to Daejeon incur shorter in-vehicle traveling time than to Daegu or Busan, which may explain their higher tolerance for waiting time. Fourth, other travel options dominate the market for passengers traveling to Busan, even in the extreme case where the waiting time for public transport is zero. This is reasonable considering the long distance to Busan, which makes public transport difficult to compete with private alternatives. Public transport going to Daejeon and Daegu can capture
certain market share as long as the waiting time is not too long.

Figure 4.5 Waiting time and market share of airport ground transport modes

4.5 Counterfactual analysis

In this section, I perform counterfactual analysis to see if adjusting train schedules would increase its ridership. I formulate the objective as to maximize the total ridership on trains for the following reasons. First, this project is motivated by the relatively low ridership of trains. Second, compared to bus or private options, train is more environmental friendly, and thus should be encouraged as a travel mode. Third, buses are more frequent and evenly scheduled during the day. For instance, buses to Daejeon, Daegu and Busan depart roughly every 20 min, 25 min, and 1 hr on each day.

4.5.1 A nonlinear optimization formulation

One potential approach is to use discrete optimization by searching for an optimal solution in a finite set of possible train schedules. For example, I can break one day into 10-minute intervals and denote the number of potential slots for scheduling trains as $N$. Suppose we would like to schedule $K$ trains in a day, the set of schedules contains a total of $\binom{N}{K} = \frac{N!}{K!(N-K)!}$ possibilities. The objective-function value (e.g., train ridership) of each possible schedule can be evaluated to choose the optimal one. Assume that train operates from 6:00AM to 10:00PM. With 10-minute intervals
Suppose we would like to schedule $K = 7$ trains per day, the number of combinations is $\binom{N}{K} = 1.28 \times 10^{10}$. Problem instances with such a size cannot be computed in real time. I thus propose a nonlinear optimization program, which imposes less restrictions (such as the train operating time) and is much less computationally demanding. Denote the arrival rate of airport passengers as $\lambda_a(t)$, where $a = 1, 2$ indicates the two terminals at ICN, and $t$ $(0 \leq t \leq 24)$ denotes the time of the day. I divide one day’s time horizon into $6 \times 24 = 144$ ten-minute slots. The passenger arrival rate in each ten-minute slot is assumed to be constant, with the value $\lambda_a(t)$ sampled from historical data. As a result, $\lambda_a(t)$ is a piecewise-constant function. A similar approach has been used in (de Palma and Lindsey, 2001). We index the trains by $k = 1, 2, ..., K$, according to the order of their departure times. Let $k(t)$ denote the index of the first train that departs after $t$. Let $y_k$ denote the departure time of the $k^{th}$ train. Let $B_d(t)$ denote the departure time of the next bus to destination $d$ that departs after $t$. Let $S_d^{trn}(t, B_d(t), y_{k(t)})$ denote the probability of passengers arriving at time $t$ that would take train to destination $d$, with the next bus to destination $d$ departing at time $B_d(t)$, and the next train departing at time $y_{k(t)}$. Unlike buses that go directly to destination $d$ without any stop, passengers going to different destinations take the same train, because the trains that depart from ICN will stop first at Daejeon, next at Daegu, and finally reach Busan. I thus suppress the destination subscript $d$ in $y_k$. The bus departure times $(B_d(t))$ are fixed, while the train schedule $y_k, k = 1, ..., K$ are the decision variables. The function $S_d^{trn}$ also depends on the fares and travel times. Since fares and travel times of buses and trains is time invariant, I exclude them from the arguments of function $S_d^{trn}$. In addition, since flight characteristics $X_i$ includes many coefficients, I do not consider their impact on passenger choice to make the optimization problem tractable. I use the estimated parameters in Specification 4 of Table 4.4 for counterfactual analysis.

The objective function is the total ridership on all trains, which has the following expression:

$$F(y_1, y_2, ..., y_K) := \sum_{a=1,2} \sum_d \int_0^{24} \lambda_a(t) S_d^{trn}(t, B_d(t), y_{k(t)}) \, dt,$$

where 0 and 24 denotes the starting and ending time of a day (i.e., 00:00 and 24:00), respectively.
\( S^{trn}_d(t, B_d(t), y_{k(t)}) \) can be computed as:

\[
S^{trn}_d(t, B_d(t), y_{k(t)}) = r_d \exp(\hat{V}_{trn} + \hat{\theta}_1 T T_{d, trn} + \hat{\theta}_2 (y_{k(t)} - t) + \hat{\theta}_3 P_{d, trn})/(1 + \exp(\hat{V}_{trn} + \hat{\theta}_1 T T_{d, trn} + \hat{\theta}_2 (y_{k(t)} - t) + \hat{\theta}_3 P_{d, trn}) + \exp(\hat{V}_{bus} + \hat{\theta}_1 T T_{d, bus} + \hat{\theta}_2 (B_{d(t)} - t) + \hat{\theta}_3 P_{d, bus})),
\]

where the parameters \( \hat{V}_{trn}, \hat{V}_{bus}, \hat{\theta}_1, \hat{\theta}_2, \text{ and } \hat{\theta}_3 \) were estimated in Section 4.4.2 (See Table 4.4).

Therefore, the integral in Eq. (4.5) \( \int_0^{24} \lambda_a(t) S^{trn}_d(t, B_d(t), y_{k(t)}) dt \) gives the total number of passengers from terminal \( a \) that choose trains to go destination \( d \). The nonlinear program can be formulated as:

\[
\max_{y_1, y_2, \ldots, y_K} F(y_1, y_2, \ldots, y_K),
\]

\[
s. t. \ 0 \leq y_1 < y_2 < \cdots < y_K \leq 24.
\]

### 4.5.2 Comparison of the optimal solution and benchmark solutions

By solving the nonlinear program in Eq. (4.7), I obtain an optimal train schedule, which is described by the departure times of the \( K \) trains, \((y^*_k)_{k=1, \ldots,K}\). To evaluate its performance, I compare the total ridership on trains corresponding to two benchmark solutions: (1) the actual schedule \((y_k)\); (2) a simple heuristic schedule called “load-balancing” schedule \( y^{lb}_k \). I show that the current schedule has the worst performance because it did not even consider the passengers’ intra-day arrival pattern to the airport. However, even if one considers the arrival pattern, a simple heuristic method that tries to allocate more trains in the busy period will make limited improvement. To achieve the optimal performance, one needs to analyze the passenger choice behavior and take advantage of that information in designing the train schedule.
Figure 4.6 Arrival rate at the train platform and train departure times

Figure 4.6 depicts the actual schedule, the load-balancing schedule, and the optimal schedule, with
the aggregate arrival rate at the train platform as the background. Note that the arrival time at the train platform is calculated by summing up the flights’ arrival time and the transfer time from the arriving terminal to the train platform. As shown in Figure 4.6(a), there are two peak periods for arriving passengers: between 07:00 and 09:00, and between 16:00 and 20:00. Under the existing train schedules, two trains depart at around noon (12:07 and 12:50) and one train departs in early morning (06:55). Those times are obviously not the peak periods. In fact, the actual train schedule was designed without even using the data of the arrival flights.

If we have the passenger arrival pattern, a simple approach is to allocate the seven trains to balance their load. Specifically, we assume that the first and the last train departs at 7:00 and 21:00\textsuperscript{54}, and allocate the rest five trains in a way that the area under the arrival-rate curve between two adjacent trains are all equal. Intuitively, this method assumes that passengers choose trains with a constant probability, so allocating trains in this way will result in balanced load. Figure 4.6(b) depicts the load-balancing schedule ($y_k^{lb}$) in comparison to the arrival rates throughout a day. Compared to the current schedule, the load-balancing schedule has allocated more trains during the peak hours (e.g., 07:00-09:00 and 16:00-20:00).

Finally, the optimal schedule was depicted in Figure 4.6(c). The optimal schedule differs from the load-balancing schedule by considering the effect of waiting times on passengers’ choice and the competition from the buses. It schedules more trains between 16:00 and 20:00 while skipping the less busy period from 09:00 to 12:00.

| Table 4.6 Ridership per train to each destination under different schedules |
|-----------------|-------|-------|-------|-------|
|                 | Daejeon | Daegu | Busan | Total |
| Actual schedule | 22     | 49    | 28    | 99    |
| Load-balancing schedule | 24     | 53    | 31    | 108   |
| Proposed schedule      | 30     | 63    | 37    | 130   |

\textsuperscript{54} I choose to fix the departure times of the first and the last train, because otherwise this schedule will allocate trains in mid-night and results in even worse performance.
Table 4.6 summarizes the average train ridership to the three different destination cities under the three different schedules. The train ridership under the current schedule is the actual average ridership. The train ridership under the load-balancing schedule and the proposed schedule is the predicted average ridership. We can see that the proposed schedule increases ridership by 31.3% over the actual schedule and by 20.4% over the load-balancing schedule in terms of total train ridership. Since the bus schedules are more evenly distributed within a day, changing to the proposed train schedule would not affect bus ridership. Thus, the increase in train ridership mainly comes from other private alternatives (such as taxi or renting a car).

I next estimate the changes in consumer surplus (CS) and revenue. If a passenger arrives at time \( t \) and goes to destination \( d \) using transport mode \( m = \text{trn, bus} \), his/her expected utility is given by

\[
U_{md}(t) = V_m + \hat{\theta}_1 TT_{md} + \hat{\theta}_2 WT_{md}(t) + \hat{\theta}_3 P_{md}, \tag{4.8}
\]

where \( WT_{md}(t) := y_{k(t)} - t \) if \( m = \text{trn} \), and \( WT_{md}(t) := B_{d}(t) - t \) if \( m = \text{bus} \). The expected welfare of that customer can be estimated using the well-known log-sum formula for calculating the expected CS under multinomial logit model (McFadden, 1981; Small and Rosen, 1981; Hausman et al., 1995). That is:

\[
CS(d, t) = \frac{1}{|\theta_3|} \log \left( \sum_m \exp(U_{md}(t)) \right) = \frac{1}{|\theta_3|} \log \left( 1 + \sum_{m=\text{trn, bus}} \exp(V_m + \theta_1 TT_{md} + \theta_2 WT_{md}(t) + \theta_3 P_{md}) \right). \tag{4.9}
\]

Note that \( \theta_3 \) is the price coefficient specified in Eq. (4.1). Dividing by \( \theta_3 \) will convert the CS to monetary value. Given arrival rate \( \lambda_a(t) \), the total CS under train schedule \( (y_k) \) can be estimated as

\[
CS(y_1, y_2, ..., y_K) = \sum_{a=1,2} \sum_d \int_0^{24} \lambda_a(t) CS(d, t) dt, \tag{4.10}
\]

where \( CS(d, t) \) is given in Eq. (4.9). The total revenues collected by the train company can be estimated using the fare information in Table 4.2. Table 4.7 summarize the incremental train
revenue and expected CS from schedule change. Changing to the load-balancing schedule from the actual schedule increases revenue by $3,164, and increase CS by $2,100 per day; while changing to the proposed schedule from the actual schedule increases revenue by $10,626, and increases CS by $5,701 per day.

Table 4.7 Increase in train revenue and CS from schedule change

<table>
<thead>
<tr>
<th>Schedule change</th>
<th>Increase in revenue</th>
<th>Increase in CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>From Actual</td>
<td>To Load-Balancing</td>
<td>3,164</td>
</tr>
<tr>
<td>Load-Balancing</td>
<td>Proposed</td>
<td>7,462</td>
</tr>
<tr>
<td>Actual</td>
<td>Proposed</td>
<td>10,626</td>
</tr>
</tbody>
</table>

Unit: $ per day

4.5.3 Implementation of the proposed schedule

It is desirable to show that the proposed train schedule is not costly for practical implementation because it does not require extra trains. To demonstrate that, I use departure times of trains from the opposite direction (i.e., from Busan to ICN) to show that for both the actual and the proposed departure times at ICN, only six trains are needed to complete all the roundtrips between ICN and the three cities. It is illustrated in Figure 4.7. Schedules that marked by the same color are operated by the same train. We can see that it takes the same number of trains (six) to carry out the actual schedule and the proposed schedule, while at least ten trains are currently commuting between ICN and the three cities.
4.5.4 The effects of increasing or decreasing train frequency

In this section, I investigate the effects of increasing or decreasing train frequency. The analysis has important policy implications in improving train scheduling at the airport, especially when the
The train operator has been facing low ridership. I first increase the number of trains per day departing from ICN. By solving the optimization problem in Eq. (4.7) with $K = 8, \ldots, 15$, respectively, I obtain the optimal schedules which are relegated in Table C.1. Figure 4.8 plots the estimated daily train ridership by destination under the actual train schedule and under the optimized train schedule with daily frequency varying from 7 to 15. It shows that the daily ridership to the three destinations increases with service frequency, which is unsurprising, but the curves tend to flatten out when more services are added (e.g., for $K = 13$ to $15$), indicating decreasing returns.

![Figure 4.8 Daily train ridership comparison with increasing train frequency](image)

**Figure 4.8 Daily train ridership comparison with increasing train frequency**

Note: 7A denotes the actual schedule with 7 trains per day, 7 denotes the optimized schedule with 7 trains, and the same applies to other x-axis labels. The same applies to Figure 4.10, Figure 4.12, and Figure 4.13.

Using the CS estimation in Eq. (4.10) and the fare information in Table 4.2, I can estimate the revenue and CS under different schedules. Figure 4.9 shows the changes in daily ridership, revenue, and CS, when train schedule is changed from the actual schedule to the optimized schedule with 7 trains per day, with 8 trains per day, and so on, till with 15 trains per day. For example, if the train operator increases the frequency to 15 trains per day, the daily ridership would increase by 1,042, revenue would increase by $51,610 per day, and CS would increase by $28,990 per day. However, the estimation did not take into account the effect on passengers going to the airport, and the
practical implementation of the schedule with more services (e.g., the need to coordinate train schedules both to and from the airport, and the availability of the fleet of trains).

(a) Effect of increasing train frequency on daily ridership

(b) Effect of increasing train frequency on revenue and CS

Figure 4.9 Effects of increasing train frequency

Note: 7At07 denotes the change from the actual schedule to the optimized schedule with 7 trains, and the same applies to other x-axis labels. The same applies to Figure 4.11.
The estimation provides insights for the train operator to determine whether more services should be added by comparing the incremental operating cost associated with more services and the estimated incremental revenue and/or CS. The objective of the train operator can be pure profit maximization, in which case only the revenue matters for decision, or the operator may care about public interest, in which case both the revenue and CS would be considered.

Since Korail has temporarily suspended high-speed train operations between ICN and Busan in March 2018, I also conduct counterfactual analysis of reducing daily train frequency to 6 till to 0 to see if scaling down the train services is economically sensible. By solving the optimization problem in Eq. (4.7) with \( K = 6, \ldots, 1 \), respectively, I obtain the optimal schedules which are relegated in Table C.2. Figure 4.10 plots the estimated daily train ridership by destination under the actual train schedule and under the optimized train schedule with daily frequency varying from 6 to 1. It shows that the optimized schedule with 6 daily services has higher ridership than the actual schedule, which indicates that the actual schedule cannot even outperform the schedule with one fewer service. The optimized schedule with 5 daily services has similar ridership as the actual schedule, indicating that the performance of the actual schedule is only equivalent to the optimal schedule with 2 fewer services. When we further reduce services, it is not surprising to see that the total daily ridership decreases.

![Figure 4.10 Daily train ridership comparison with decreasing train frequency](image-url)
Figure 4.11 shows the changes in daily ridership, revenue, and CS, when train schedule is changed from the actual schedule to the optimized schedule with 6 trains per day, with 5 trains per day, and so on, until no service is provided. Specifically, cancelling the train operation between ICN and Busan would result in an estimated revenue loss of $34,279 and an estimated CS loss of $18,610.
per day. The train operator, depending on its objective, has to trade off the savings in operational costs and the losses in revenue and CS in order to decide whether cancelling the service is optimal.

The impacts on bus ridership from increasing and decreasing train frequency are plotted in Figure 4.12 and Figure 4.13, respectively. Increasing train frequency generally reduces bus ridership, but the effect is small. Specifically, increasing train frequency from the current 7 trains per day to 15 trains per day reduces bus ridership by 2.6 per bus (or $2.6 \times 57 \approx 148$ per day) to Daejeon, 1.8 per bus (or $1.8 \times 42 \approx 76$ per day) to Daegu, and 0.5 per bus (or $0.5 \times 18 = 9$ per day) to Busan. This impact is equivalent to a loss of bus revenue per day of $148 \times 21 = 3,108$ to Daejeon, $76 \times 32 = 2,432$ to Daegu, and $9 \times 38 = 342$ to Busan. The scale in the reduction of bus ridership and revenue is small compared with the scale in the increase of train ridership and revenue (see Figure 4.9), indicating that the increase in train ridership from more frequency primarily comes from other private alternatives.

Reducing train frequency generally increases bus ridership, but the effect is also small. Specifically, canceling train service increases bus ridership by 2.1 per bus (or $2.1 \times 57 \approx 120$ per day) to Daejeon, 1.9 per bus (or $1.9 \times 42 \approx 80$ per day) to Daegu, and 0.3 per bus (or $0.3 \times 18 \approx 5$ per day) to Busan, which is equivalent to a gain in bus revenue per day of $120 \times 21 = 2,520$ to Daejeon, $80 \times 32 = 2,560$ to Daegu, and $5 \times 38 = 190$ to Busan. The relatively small
effect on bus from changing train frequency may also be attributed to the fact that buses are much more frequent and evenly spaced by time of day.

![Figure 4.13 Changes in bus ridership with decreasing train frequency](image)

### 4.6 Concluding remarks

This study proposed a structural estimation method to identify factors affecting passengers’ choice of airport ground transport modes using aggregate ridership data. Essentially, this approach converted unobservable choice probabilities into aggregate counts which can be compared to the observables. The model coefficients could be computed by the nonlinear least-squares estimation that minimizes the distance between the estimated and observed ridership for each scheduled public transport service. The estimation results suggest that passengers who fly domestic airlines or have experienced longer waiting times are less likely to take public transport, while passengers who fly low-cost airline are more likely to take public transport. In addition, the effect of waiting time on the disutility of choosing public transport is much stronger than the effect of in-vehicle travel time. The estimation results deliver important information for understanding airport passengers’ choice behaviors and allow airport managers to optimize public transport schedules to achieve higher efficiency.

Based on the estimation results, counterfactual analysis was conducted to estimate what ridership
of airport ground transport services would be if the schedules were adjusted. I formulate a nonlinear program in which the objective is to optimize train ridership. I show that the current train schedule is not efficiently matched with the intraday demand, and could be substantially improved to achieve higher ridership. In particular, I demonstrate that by adjusting the current schedule to the proposed one, the ridership could be increased by 31.3%, and that the implementation of the proposed schedule is practically feasible because it does not require adding extra trains. By changing to the proposed train schedule, the estimated increase in consumer surplus and the revenue for the train operator would be $5,701 and $10,626 per day, respectively. Since the bus schedules are more evenly spaced by time of day, changing to the proposed train schedule would not affect bus ridership. Thus, the increase in train ridership mainly comes from other private alternatives (such as taxi or renting a car). I also estimate the monetary gains or losses of increasing or decreasing train frequency in consumer surplus and the train operator’s revenue. The estimates can provide insights for the train operator to determine the optimal train frequency.

However, the optimization procedure to solve the train schedule is deterministic in the sense that the flight arrival times and the number of passengers on a flight are treated as deterministic based on historical data. Future research may model the stochastic flight arrival process by considering the probability distribution of flight delays, and formulate a stochastic program to solve the optimal schedule. However, the estimation results from the structural discrete choice model will still be needed in predicting a passenger’s choice. Although this study focuses on a specific example of transportation scheduling, the general framework can be applied to other scheduling problems in which the scheduled service time may affect the incoming customer demand. Before solving the scheduling problem, a key step is to analyze and understand the customers’ choice in response to the given schedule.
Chapter 5: Conclusion

This dissertation addresses three important issues in transportation and logistics, and provides insights for more efficiently operating and managing transportation and logistics systems. Specifically, I investigate: (1) the effects of airlines and high-speed rail (HSR) cooperation in a region with multiple airports, (2) the optimal timing and scale of seaport capacity and protection investments under the uncertainty of climate change, and (3) passenger choice estimation and schedule optimization of airport ground transportation modes using aggregate data. The dissertation contributes to the existing literature by providing theoretical modeling and empirical analysis to a few important problems that have been overlooked or not well studied. In this chapter, I discuss the policy implications, the future research directions, and the generalized insights of the dissertation.

The first essay (Chapter 2) explores the conditions under which a specific air-rail cooperation mechanism (where the airlines offer intermodal passengers free HSR tickets, and the HSR operator shares revenue with the airlines) can be feasible, and the corresponding market outcome. It finds that when the HSR is more profit-oriented, the air-rail cooperation is less likely to be feasible. This implies that the objective of the HSR plays an essential role. In addition, when the revenue sharing amount is negotiated between the airline and the HSR, the resultant equilibrium depends on the bargaining power of the two parties. However, such a negotiated equilibrium may not lead to the greatest welfare. In such cases, market intervention might be needed to reach a cooperation that is closest to the social optimum. Given some limitations of the study, there are several directions for future research. First, it is interesting to investigate a network structure where airlines and HSR can compete and cooperate in both price and schedule. The implications of such co-opetition between airlines and HSR on air-rail intermodality and market outcome might be different from the results of this essay. Second, a revenue sharing mechanism involving three parties (airport, airlines, and HSR) can also be an important avenue for future investigation. Airports can have various ownership types: public, private, or partial public/private, and it may result in different conditions for achieving cooperation with the HSR, especially when taking into account the airport-airline vertical structure. Third, the inter-airlines competition within one airport or across
two airports in the multiple-airport regions can also be considered. Last, data can be collected to empirically verify the model results.

The second essay (Chapter 3) provides a general framework on how seaports can better adapt to climate change by dynamically determining their protection and capacity investments, as well as user charges, given uncertainty about the probability and magnitude of climate-related disasters. This essay highlights the importance of considering the two types of investment together, especially for seaports that are expanding capacity to accommodate growing demand. We also explore when a port prefers to invest in capacity and protection early on, and when it prefers to hold off and wait for better information. The results offer managerial insights for ports and their stakeholders to develop appropriate capacity and adaptation management strategies to climate change. However, this study also suggests several directions for future research. First, the model omitted a number of considerations such as demand uncertainty, depreciation of capacity and protection infrastructure, and disaster-related damage to protection, as well as competition among ports. The timing of the model could be refined to allow shippers to respond to events more quickly than ports. Second, this essay assumes in the dynamic model that any damage gets fully repaired at the end of period 1. However, in extreme disasters, the damage incurred in period 1 might not be fully repaired, and it may be cost-effective to abandon a port or relocate it to a less vulnerable site. Thus, the option of relocation can be considered in a port’s investment decisions. Third, this essay implicitly assumes that period 2 inherits the capacity and protection decisions made in period 1. However, it is possible that new technology could be installed following a disaster so that capacity and/or protection are more productive or effective in period 2 than period 1. Thus, future research should incorporate the consideration on whether technology will change and how long the protection investments will last. Fourth, economies or diseconomies of scale in protection investment could be introduced. Economies of scope in investment may also be relevant since it may be cost-effective to invest in capacity and protection at the same time to minimize the disruption to a port’s operations. Finally, since ports located in different regions can face vastly different local and regional climate-change threats, a numerical case study of a real world problem is necessary to explain how the model can be practically applied.
The third essay (Chapter 4) proposes a new structural estimation approach which uses aggregate data to estimate passengers’ mode choice of airport ground transportation. Unlike existing studies, this approach does not require individual air travelers’ choice data. Essentially, this approach converts unobservable individual choice probabilities into aggregate counts which can be compared to the observables. This new approach is applied to Incheon International Airport (ICN), which is connected to the Southern part of South Korea by intercity buses and high-speed trains. I estimate the value of in-vehicle travel time and the value of waiting time, and show that passengers incur much higher disutility from waiting than in-vehicle traveling. Using the estimated information for airport passengers’ choice behaviors, I formulate a nonlinear program to solve the train scheduling problem, and quantify the effects of adding and reducing train frequency. The estimation results deliver important managerial insights and allow airport managers to more efficiently manage the airport ground transportation system. However, the optimization procedure to solve the train schedule is deterministic in the sense that the flight arrival times and the number of passengers on a flight are treated as deterministic based on historical data. Future research may model the stochastic nature of the flight arrival process and the number of passengers onboard. Specifically, historical data on the actual flight arrival times and the scheduled flight arrival times can be used to generate an airline-specific or a flight-specific probability distribution of flight delays. A probability distribution of the number of passengers onboard can also be generated using historical data. A stochastic program can thus be formulated to solve the optimal schedule of airport ground transportation.

Transportation and logistics is one of the key components of the modern economy. A well-managed transportation and logistics sector has direct benefits to the people, the businesses, the environment, and the overall local, regional and national economy. This dissertation demonstrates the necessity to develop new models, decision-making processes, and managerial insights for transportation systems that are traditionally standalone but now increasingly intertwined (i.e., air and rail multimodal transportation). It is also essential to develop the digital infrastructure and the cooperative mindset to facilitate data sharing among different transport operators. In addition, climate change will become a real challenge to the existing transportation infrastructure, because it is vulnerable to extreme weather events. This dissertation also demonstrates the necessity to
enhance the resilience and adaptability of the vulnerable transportation infrastructure (i.e., seaports) to climate-driven weather phenomena.

Apart from the three topics explored in this dissertation, there are many research questions to address in urban transportation. For example, there has been rapid progress in both technologies (such as autonomous driving and electric vehicles) and business models (such as ride-sharing and sharing-economy). It is essential to understand the comprehensive effects of these new technologies and business models on travel demand, land use, and road congestion in order to help policy makers plan, design and operate a transportation and logistics system in a more efficient, greener, and sustainable way.
Bibliography


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Appendices

Appendix A  Appendix for Chapter 2

A.1  Summary of equilibrium and boundary values in basic model

The equilibrium prices and quantities with competing airlines in the benchmark case (no air-HSR cooperation) are obtained in the below equations, where superscript $BD$ stands for benchmark and duopoly (two competing airlines):

\[
p_{1BD} = \frac{(2\lambda + 1)((2 - \beta)\alpha_1 + (P_{HSR} + t)\beta) - \alpha_1\beta^2}{4 - \beta^2 + 8\lambda},
\]

\[
p_{2BD} = \frac{\beta^2(P_{HSR} + t) + 2(2\lambda + 1)(\alpha_1 - P_{HSR} - t) - \alpha_1\beta(1 + \beta)}{4 - \beta^2 + 8\lambda},
\]

\[
q_{1BD} = \frac{(2\lambda + 1)((2 - \beta)\alpha_1 + (P_{HSR} + t)\beta) - \alpha_1\beta^2}{(4 - \beta^2 + 8\lambda)(1 - \beta^2 + 2\lambda)},
\]

\[
q_{TBD} = \frac{(1 + 2\lambda)(\beta^2(P_{HSR} + t) + 2(2\lambda + 1)(\alpha_1 - P_{HSR} - t) - \alpha_1\beta(1 + \beta)\beta^3(2 - 3\omega))}{(4 - \beta^2 + 8\lambda)(1 - \beta^2 + 2\lambda)}.
\]

The equilibrium prices and quantities with competing airlines in the case of air-HSR cooperation are obtained in the below equations, where superscript $RD$ stands for revenue sharing and duopoly:

\[
\delta_{RD} = P_{HSR} + ((4(2\lambda + 1)^2 - 3\beta^2(2\lambda + 1))(\alpha_1 - t)(1 - 3\omega) + ((\alpha_1 - t)\beta^4 - 4\alpha_1\beta(2\lambda + 1))(1 - 2\omega) + \alpha_1\beta^3(2 - 3\omega)) / ((3\omega + 1)(1 + 2\lambda)(4 - 3\beta^2 + 8\lambda) + 2\beta^4\omega),
\]

\[
P_{1RD} = (2\alpha_1(2\lambda + 1)^2(1 + 3\omega) - 2\alpha_1\beta^2(4 - \beta^2 + 8\lambda)\omega - \beta(2 - \beta^2 + 4\lambda)(1 + 2\lambda)(\alpha_1 - t) - \alpha_1\beta^2(1 + 2\lambda)) / ((3\omega + 1)(1 + 2\lambda)(4 - 3\beta^2 + 8\lambda) + 2\beta^4\omega),
\]

\[
P_{2RD} = (\alpha_1\beta(1 + 2\lambda)(1 - 7\omega) + (t - \alpha_1)\beta^2(1 + 2\lambda)(1 + 9\omega) - (2(\beta^4 + 6) + 48\lambda(\lambda + 1))(t - \alpha_1) - 2\alpha_1\beta^3\omega)) / ((3\omega + 1)(1 + 2\lambda)(4 - 3\beta^2 + 8\lambda) + 2\beta^4\omega),
\]

\[
q_{1RD} = (2\alpha_1(2\lambda + 1)^2(1 + 3\omega) - \alpha_1\beta^2(1 + 2\lambda)(1 + 8\omega) + 2\alpha_1\beta^4\omega + \beta(2 - \beta^2 + 4\lambda)(1 + 2\lambda)(t - \alpha_1)) / ((1 - \beta^2 + 2\lambda)((3\omega + 1)(1 + 2\lambda)(4 - 3\beta^2 + 8\lambda) + 2\beta^4\omega)).
\]
\[ q_{T}^{RD} = (1 + 2\lambda)((2 - \beta^2 + 4\lambda)^2(\alpha_1 - t) - \alpha_1\beta(1 + 2\lambda)(3 - \omega) + \alpha_1\beta^3(2 - \omega)) / ((1 - \beta^2 + 2\lambda)((3\omega + 1)(1 + 2\lambda)(4 - 3\beta^2 + 8\lambda) + 2\beta^4\omega)). \]

The equilibrium prices and quantities with collusive airlines in the benchmark case are obtained in the below equations, where superscript BM stands for benchmark and monopoly (i.e., collusive airlines):

\[ p_{1}^{BM} = \frac{\alpha_1}{2}, \]
\[ p_{2}^{BM} = \frac{1}{2}(\alpha_1 - P_{HSR} - t), \]
\[ q_{1}^{BM} = \frac{\alpha_1(1 - \beta) + (P_{HSR} + t)\beta}{2(1 - \beta^2 + 2\lambda)}, \]
\[ q_{T}^{BM} = \frac{(2\lambda + 1)(\alpha_1 - t - P_{HSR}) - \alpha_1\beta}{2(1 - \beta^2 + 2\lambda)}. \]

The equilibrium prices and quantities with collusive airlines in the case of air-HSR cooperation are obtained in the below equations, where superscript RM stands for revenue sharing and monopoly (i.e., collusive airlines):

\[ \delta^{RM} = P_{HSR} + \frac{((1 + 2\lambda)(\alpha_1 - t) - \alpha_1\beta)(1 - 3\omega)}{(1 + 2\lambda)(1 + 3\omega)}, \]
\[ p_{1}^{RM} = \frac{\alpha_1}{2}, \]
\[ p_{2}^{RM} = \frac{\alpha_1\beta(1 - 3\omega) - 6(1 + 2\lambda)(t - \alpha_1)\omega}{2(1 + 2\lambda)(1 + 3\omega)}, \]
\[ q_{1}^{RM} = \frac{\alpha_1(1 + 2\lambda)(1 + 3\omega) + \alpha_1\beta^2(1 - 3\omega) + 2(t - \alpha_1)\beta(1 + 2\lambda)}{2(1 - \beta^2 + 2\lambda)(1 + 2\lambda)(1 + 3\omega)}, \]
\[ q_{T}^{RM} = \frac{(\alpha_1 - t)(1 + 2\lambda) - \alpha_1\beta}{(1 - \beta^2 + 2\lambda)(1 + 3\omega)}. \]
With competing airlines, the conditions for revenue sharing are \( 0 \leq \omega < \bar{\omega}_D \) and 
\( (t \leq \bar{t}^D \text{ or } (t > \bar{t}^D \text{ and } P_{HSR} > \bar{P}_{HSR}^D) ) \) or 
\( \bar{\omega}_D \leq \omega \leq 1 \text{ and } P_{HSR} > \bar{P}_{HSR}^D \), where the boundary values \( \bar{\omega}_D, \bar{t}^D, \) and \( \bar{P}_{HSR}^D \) are given in the below equations:

\[
\bar{\omega}_D = \frac{(1-\beta)(2+\beta)^2 + 2\lambda(8(1+\lambda) - \beta(4+3\beta))}{(1-\beta)(2+\beta)(3-2\beta) + 2\lambda(24(1+\lambda) - \beta(8+9\beta))},
\]
\[
\bar{t}^D = \frac{\alpha_1(4-3\beta^2 + 8\lambda)(1+2\lambda)(1-3\omega) - \alpha_1\beta(4-\beta^3 + 8\lambda)(1-2\omega) + \alpha_1\beta^3(2-3\omega)}{(4-3\beta^2 + 8\lambda)(1+2\lambda)(1-3\omega) + \beta^4(1-2\omega)},
\]
\[
\bar{P}_{HSR}^D = \frac{(t - \alpha_1)(4-3\beta^2 + 8\lambda)(1+2\lambda)(1-3\omega) + \beta(4\alpha_1(1+2\lambda) + (t - \alpha_1)\beta^3)(1-2\omega) - \alpha_1\beta^3(2-3\omega))}{((4-3\beta^2 + 8\lambda)(1+2\lambda)(1+3\omega) + 2\beta^4\omega)}.
\]

With collusive airlines, the conditions for revenue sharing are \( 0 \leq \omega < 1/3 \) or \( (1/3 \leq \omega \leq 1 \text{ and } P_{HSR} > \bar{P}_{HSR}^M) \), where the boundary value \( \bar{P}_{HSR}^M \) is given in the below equation:

\[
\bar{P}_{HSR}^M = \frac{(t - \alpha_1)(1+2\lambda) + \alpha_1\beta(1-3\omega)}{(1+2\lambda)(1+3\omega)}.
\]

The boundary values in Section 2.3.2 (i.e., \( \omega_D^D \) in Figure 2.2 and \( \bar{t}^D \) in Table 2.2) are given in the below equations:

\[
\omega_D^D = \frac{4(4 - 5\beta^2 + 8\lambda)(1+2\lambda) + 5\beta^4}{32(1-\beta^2 + 2\lambda)(1+2\lambda) + 7\beta^4},
\]
\[
\bar{t}^D = \frac{\alpha_1((1-\beta)(2+\beta)(12 - 4\beta - 11\beta^2 + 2\beta^3 + 2\beta^4) + 2\lambda(72(1+2\lambda) - 20\beta(2+3\beta)(1+\lambda) + 19\beta^3 + 13\beta^4 + 96\lambda^2))}{(2-\beta^2 + 4\lambda)(12 - 9\beta^2 + 2\beta^4 + 48\lambda - 18\beta^2\lambda + 48\lambda^2)}.
\]

### A.2 Proofs of the Propositions

**Proof of Proposition 1**:

The condition for revenue sharing is \( \pi_2^{RD} > \pi_2^{BD} \) and \( \omega\pi_{HSR}^{RD} + (1 - \omega)SW_{RD}^{RD} > \omega\pi_{HSR}^{BD} + (1 - \omega)SW_{BD}^{RD} \) with competing airlines and \( \pi_1^{RM} + \pi_2^{RM} > \pi_1^{BM} + \pi_2^{BM} \) and \( \omega\pi_{HSR}^{RM} + (1 - \omega)SW_{RM}^{RM} > \omega\pi_{HSR}^{BM} + (1 - \omega)SW_{BM}^{RM} \) with collusive airlines.
With competing airlines,

\[
(\omega \pi_{HR}^{RD} + (1 - \omega)SW^{RD}) - (\omega \pi_{HR}^{BD} + (1 - \omega)SW^{BD})
= (1 + 2\lambda)A^2 / (2(4 - \beta^2 + 8\lambda)^2(1 - \beta^2 + 2\lambda)(4 - 3\beta^2 + 16\lambda - 6\beta^2 \lambda
+ 16\lambda^2 + 12\omega - 9\beta^2 \omega + 2\beta^4 \omega + 48\lambda \omega - 18\beta^2 \lambda \omega + 48\lambda^2 \omega)),
\]

\[
\pi_2^{RD} - \pi_2^{BD} = (2 - \beta^2 + 4\lambda)(1 + 2\lambda)AB / ((4 - \beta^2 + 8\lambda)^2(1 - \beta^2
+ 2\lambda)(4 - 3\beta^2 + 16\lambda - 6\beta^2 \lambda + 16\lambda^2 + 12\omega - 9\beta^2 \omega + 2\beta^4 \omega + 48\lambda \omega
- 18\beta^2 \lambda \omega + 48\lambda^2 \omega)^2),
\]

\[
\delta^{RD} = A/(4 - 3\beta^2 + 16\lambda - 6\beta^2 \lambda + 16\lambda^2 + 12\omega - 9\beta^2 \omega + 2\beta^4 \omega + 48\lambda \omega - 18\beta^2 \lambda \omega
+ 48\lambda^2 \omega),
\]

where

\[
A = (4P_{HSR} - 4t + 4\alpha_1 - 4\alpha_1 \beta - 3P_{HSR}^2 + 3t\beta^2 - 3\alpha_1 \beta^2 + 2\alpha_1 \beta^3 - t\beta^4 + \alpha_1 \beta^4
+ 16P_{HSR} \lambda - 16t \lambda + 16\alpha_1 \lambda - 8\alpha_1 \beta \lambda - 6P_{HSR} \beta^2 \lambda + 6t \beta^2 \lambda - 6\alpha_1 \beta^2 \lambda
+ 16P_{HSR} \lambda^2 - 16t \lambda^2 + 16\alpha_1 \lambda^2 + 12P_{HSR} \omega + 12t \omega - 12\alpha_1 \omega + 8\alpha_1 \beta \omega
- 9P_{HSR} \beta^2 \omega - 9t \beta^2 \omega + 9\alpha_1 \beta^2 \omega - 3\alpha_1 \beta^3 \omega + 2P_{HSR} \beta^4 \omega + 2t \beta^4 \omega - 2\alpha_1 \beta^4 \omega
+ 48P_{HSR} \lambda \omega + 48t \lambda \omega - 48\alpha_1 \lambda \omega + 16\alpha_1 \beta \lambda \omega - 18P_{HSR} \beta^2 \lambda \omega - 18t \beta^2 \lambda \omega
+ 18\alpha_1 \beta^2 \lambda \omega + 48P_{HSR} \lambda^2 \omega + 48t \lambda^2 \omega - 48\alpha_1 \lambda^2 \omega),
\]
\[ B = (-8P_{HSR} - 24t + 24\alpha_1 - 16\alpha_1\beta + 10P_{HSR}\beta^2 + 30t\beta^2 - 30\alpha_1\beta^2 + 14\alpha_1\beta^3 - 3P_{HSR}\beta^4 - 11t\beta^4 + 11\alpha_1\beta^4 - 2\alpha_1\beta^5 + t\beta^6 - \alpha_1\beta^6 - 48P_{HSR}\lambda - 144t\lambda + 144\alpha_1\lambda - 64\alpha_1\lambda\beta + 40P_{HSR}\beta^2\lambda + 120t\beta^2\lambda - 120\alpha_1\beta^2\lambda + 28\alpha_1\beta^3\lambda - 6P_{HSR}\beta^4\lambda - 22t\beta^4\lambda + 22\alpha_1\beta^4\lambda - 96P_{HSR}\lambda^2 - 288t\lambda^2 + 288\alpha_1\lambda^2 - 64\alpha_1\lambda^2 + 40P_{HSR}\beta^2\lambda^2 + 120t\beta^2\lambda^2 - 120\alpha_1\beta^2\lambda^2 - 64P_{HSR}\lambda^3 - 192t\lambda^3 + 192\alpha_1\lambda^3 - 24P_{HSR}\omega - 24t\omega + 24\alpha_1\omega - 8\alpha_1\beta\omega + 30P_{HSR}\beta^2\omega + 30t\beta^2\omega - 30\alpha_1\beta^2\omega + 4\alpha_1\beta^2\omega - 3P_{HSR}\beta^4\omega - 13t\beta^4\omega + 13\alpha_1\beta^4\omega - \alpha_1\beta^5\omega + 2P_{HSR}\beta^6\omega + 2t\beta^6\omega - 2\alpha_1\beta^6\omega - 144P_{HSR}\lambda\omega - 144t\lambda\omega + 144\alpha_1\lambda\omega - 32\alpha_1\lambda\omega + 120P_{HSR}\beta^2\omega \lambda + 120t\beta^2\omega \lambda - 120\alpha_1\beta^2\omega \lambda + 8\alpha_1\beta^3\omega \lambda - 26P_{HSR}\beta^4\omega \lambda - 26t\beta^4\omega \lambda + 26\alpha_1\beta^4\omega \lambda - 288P_{HSR}\lambda^2\omega - 288t\lambda^2\omega + 288\alpha_1\lambda^2\omega - 32\alpha_1\beta^2\lambda^2\omega + 120P_{HSR}\beta^2\lambda^2\omega + 120t\beta^2\lambda^2\omega - 120\alpha_1\beta^2\lambda^2\omega - 192P_{HSR}\lambda^3\omega - 192t\lambda^3\omega + 192\alpha_1\lambda^3\omega). \]

It is obvious to see that \((\omega\pi_{HSR}^{RD} + (1 - \omega)SW^{RD}) - (\omega\pi_{HSR}^{RD} + (1 - \omega)SW^{BD}) > 0\) given \(0 < \beta < 1\), \(0 \leq \omega \leq 1\), and \(\lambda > 0\), indicating that HSR is always better off from such cooperation.

With the help of Wolfram Mathematica 11, it can be shown that \(B > 0\) by imposing nonnegative equilibrium traffic (i.e., \(q_T^H \geq 0\) and \(q_T^B \geq 0\)). The condition for \(\pi_2^{RD} - \pi_2^{BD} > 0\) is thus to have \(A > 0\), which is equivalent to \(\delta^{RD} > 0\).

Given that the parameters are in the range that ensures nonnegative traffic, using Wolfram Mathematica 11, we can derive the condition to achieve \(A > 0\) : \(\{ 0 \leq \omega < \tilde{\omega}^D \) and \([ t \leq \tilde{t}^D \) or \(( t > \tilde{t}^D \) and \(P_{HSR} > \tilde{P}_{HSR}^D ) \} \) or \(\{ \tilde{\omega}^D \leq \omega \leq 1 \) and \(P_{HSR} > \tilde{P}_{HSR}^D \}\). The boundary values \((\tilde{\omega}^D, \tilde{t}^D\) and \(\tilde{P}_{HSR}^D\)) are specified in Appendix A.1.

With collusive airlines,

\[ (\omega\pi_{HSR}^{RM} + (1 - \omega)SW^{RM}) - (\omega\pi_{HSR}^{BM} + (1 - \omega)SW^{BM}) = \frac{C^2}{8(1 - \beta^2 + 2\lambda)(1 + 2\lambda)(1 + 3\omega)}. \]
\[(\pi_1^{RM} + \pi_2^{RM}) - (\pi_1^{BM} + \pi_2^{BM}) = \frac{CD}{4(1 - \beta^2 + 2\lambda)(1 + 2\lambda)(1 + 3\omega)^2},\]

\[\delta^{RM} = \frac{C}{(1 + 2\lambda)(1 + 3\omega)},\]

where

\[C = P_{HSR} - t + \alpha_1 - \alpha_1\beta + 2P_{HSR}\lambda - 2t\lambda + 2\alpha_1\lambda + 3P_{HSR}\omega + 3t\omega - 3\alpha_1\omega + 3\alpha_1\beta\omega + 6P_{HSR}\lambda\omega + 6t\lambda\omega - 6\alpha_1\lambda\omega,\]

\[D = 3\alpha_1 - P_{HSR} - 3t - 3\alpha_1\beta - 2P_{HSR}\lambda - 6t\lambda + 6\alpha_1\lambda - 3P_{HSR}\omega - 3t\omega + 3\alpha_1\omega - 3\alpha_1\beta\omega - 6P_{HSR}\lambda\omega - 6t\lambda\omega + 6\alpha_1\lambda\omega.\]

It is obvious to see that \((\omega\pi_1^{RM} + (1 - \omega)SW^{RM}) - (\omega\pi_1^{BM} + (1 - \omega)SW^{BM}) > 0\). Similarly, we can show that \(D > 0\) by imposing \(q_T^{RM} > 0\) and \(q_T^{BM} > 0\). The condition for \((\pi_1^{RM} + \pi_2^{RM}) - (\pi_1^{BM} + \pi_2^{BM}) > 0\) is thus to have \(C > 0\), which is equivalent to \(\delta^{RM} > 0\).

Given that the parameters are in the range that ensures nonnegative traffic, using Wolfram Mathematica 11, we can derive the condition to achieve \(C > 0\): \(\{0 \leq \omega \leq 1/3\}\) or \(\{1/3 < \omega \leq 1\text{ and } P_{HSR} > \bar{p}_{HSR}^M\}\). The boundary value \(\bar{p}_{HSR}^M\) is specified in Appendix A.1.

Comparing the conditions when HSR is welfare-oriented and profit-oriented (i.e., Figure 2.2 and Figure 2.3), we can reach Proposition 1.

**Proof of Proposition 2:**

\[\delta^{RM} - \delta^{RD} = (\beta(\alpha_1\beta^2 + t\beta^3 - \alpha_1\beta^3 + 2\alpha_1\beta^2\lambda + 2t\beta^3\lambda - 2\alpha_1\beta^3\lambda + 4\alpha_1\omega - 3\alpha_1\beta^2\omega - t\beta^3\omega + \alpha_1\beta^3\omega - 2\alpha_1\beta^4\omega + 16\alpha_1\lambda\omega - 6\alpha_1\beta^2\lambda\omega - 2t\beta^3\lambda\omega + 2\alpha_1\beta^3\lambda\omega + 16\alpha_1\lambda^2\omega + 12\alpha_1\omega^2 - 18\alpha_1\beta^2\omega^2 + 6\alpha_1\beta^4\omega^2 + 48\alpha_1\lambda\omega^2 - 36\alpha_1\beta^2\lambda\omega^2 + 48\alpha_1\lambda^2\omega^2)) / ((1 + 2\lambda)(1 + 3\omega)(4 - 3\beta^2 + 16\lambda - 6\beta^2\lambda + 16\lambda^2 + 12\omega - 9\beta^2\omega + 2\beta^4\omega + 48\lambda\omega - 18\beta^2\lambda\omega + 48\lambda^2\omega)).\]

Using Wolfram Mathematica 11, we get the result that \(\delta^{RM} - \delta^{RD} > 0\), given nonnegative traffic. Thus, Proposition 2 is proved.
Proof of Proposition 3:

With competing airlines,

\[
\frac{\partial P_{H,SR}}{\partial \lambda} = (-2\beta(16\alpha_1(1 - \beta^2) + 8t\beta^3(1 - \omega) - 8\alpha_1\beta^3(1 - \omega) + 6\alpha_1\beta^4 - 3t\beta^5 + 3\alpha_1\beta^5
+ 64\alpha_1\lambda - 32\alpha_1\beta^2\lambda + 16t\beta^3\lambda(1 - \omega) - 16\alpha_1\beta^3\lambda + 64\alpha_1\lambda^2 + 16\alpha_1\omega
- 24\alpha_1\beta^2\omega + 16\alpha_1\lambda^2 + 64\alpha_1\lambda\omega - 48\alpha_1\beta^2\lambda\omega
+ 16\alpha_1\beta^3\lambda\omega + 64\alpha_1\lambda^2\omega - 96\alpha_1\omega^2 + 72\alpha_1\beta^2\omega^2 - 11\alpha_1\beta^4\omega^2 - 384\alpha_1\lambda(1 + \lambda)\omega^2 + 144\alpha_1\beta^2\lambda\omega^2)/(4 - \beta^2 - 3\alpha_1\beta^4 - 16\alpha_1\lambda + 16\alpha_1\omega - 9\beta^2\omega
+ 2\beta^4\omega + 48\lambda\omega - 18\beta^2\lambda\omega + 48\lambda^2\omega)^2.
\]

\[
\frac{\partial \tilde{t}_{DS}}{\partial \lambda} = 2\alpha_1\beta(16(1 - \beta^2) + 2\beta^4 + 64\lambda(1 + \lambda) - 32\beta^2\lambda - 80\omega + 72\beta^2\omega(1 - \omega) - 11\beta^4\omega(1 - \omega) - 320\lambda\omega + 144\beta^2\lambda\omega(1 - \omega) - 320\lambda^2\omega + 96\omega^2 + 384\lambda(1 + \lambda)\omega^2)/(-4 + 3\beta^2 - \beta^4 - 16\lambda + 6\beta^2\lambda - 16\lambda^2 + 12\omega - 9\beta^2\omega + 2\beta^4\omega
+ 48\lambda\omega - 18\beta^2\lambda\omega + 48\lambda^2\omega)^2.
\]

Using Wolfram Mathematica 11, we derive that with competing airlines, \(\frac{\partial \tilde{t}_{DS}}{\partial \lambda} > 0\) and \(\frac{\partial P_{H,SR}}{\partial \lambda} < 0\) if \(0 \leq \omega < \omega^D\), and \(\frac{\partial P_{H,SR}}{\partial \lambda} > 0\) if \(\omega^D \leq \omega < 1\). However, \(\frac{\partial P_{H,SR}}{\partial \lambda} \leq 0\) if \(\omega^D \leq \omega < \omega^D\), where \(\omega^D\) is specified in Appendix A.1. With collusive airlines, \(\frac{\partial P_{H,SR}}{\partial \lambda} = \frac{2\alpha_1\beta(-1 + 3\omega)}{(1 + 2\lambda)(1 + 3\omega)} > 0\), when \(\frac{1}{3} < \omega < 1\). Thus, Proposition 3 is proved.

Proof of Proposition 4:

With competing airlines,
\[
\frac{\partial \delta^{RD}}{\partial \omega} = (-4 + \beta^2 - 8\lambda)(-24t + 24\alpha_1 - 20\alpha_1\beta + 30t\beta^2 - 30\alpha_1\beta^2 + 19\alpha_1\beta^3 - 13t\beta^4 \\
+ 13\alpha_1\beta^4 - 4\alpha_1\beta^5 + 2t\beta^6 - 2\alpha_1\beta^6 - 144t\lambda + 144\alpha_1\lambda - 80\alpha_1\beta\lambda + 120t\beta^2\lambda \\
- 120\alpha_1\beta^2\lambda + 38\alpha_1\beta^3\lambda - 26t\beta^4\lambda + 26\alpha_1\beta^4\lambda - 288t\lambda^2 + 288\alpha_1\lambda^2 \\
- 80\alpha_1\beta\lambda^2 + 120t\beta^2\lambda^2 - 120\alpha_1\beta^2\lambda^2 - 192t\lambda^3 + 192\alpha_1\lambda^3) / (4 - 3\beta^2 \\
+ 16\lambda - 6\beta^2\lambda + 16\lambda^2 + 12\omega - 9\beta^2\omega + 2\beta^4\omega + 48\lambda\omega - 18\beta^2\lambda\omega \\
+ 48\lambda^2\omega)^2.
\]

Using Wolfram Mathematica 11, we derive that \(\frac{\partial \delta^{RD}}{\partial \omega} > 0\) if \(t > \hat{t}^D\); \(\frac{\partial \delta^{RD}}{\partial \omega} < 0\) if \(0 < t < \hat{t}^D\), where \(\hat{t}^D\) is specified in Appendix A.1. With collusive airlines,

\[
\frac{\partial \delta^{RM}}{\partial \omega} = \frac{6(t - \alpha_1 + \alpha_1\beta + 2t\lambda - 2\alpha_1\lambda)}{(1 + 2\lambda)(1 + 3\omega)^2}.
\]

We derive that \(\frac{\partial \delta^{RM}}{\partial \omega} < 0\), given nonnegative equilibrium traffic (i.e., \(q^{RM}_T \geq 0\) and \(q^{BM}_T \geq 0\)). Therefore, Proposition 4 is proved.

A.3 Summary of equilibrium when airlines decide revenue sharing amount

Suppose the revenue sharing amount \(\delta\) is determined by the airlines who propose the cooperation. With competing airlines, given the demand function and the best-response functions of \(P_1\) and \(P_2\) derived in Eq. (2.5) and Eq. (2.6), we can have the HSR’s objective function \(\omega \pi^{RD}_{HSR}(\delta) + (1 - \omega)SW^{RD}(\delta)\) and airline 2’s objective function \(\pi^{RD}_2(\delta)\), both as functions of \(\delta\). Next, we check the concavity of their objective functions, by deriving the second derivative with respect to \(\delta\). As shown in the below equations where the subscript \(\delta\delta\) denotes the second derivative, the HSR’s objective function is concave in \(\delta\), while the airline’s is convex in \(\delta\):

\[
[\omega \pi^{RD}_{HSR}(\delta) + (1 - \omega)SW^{RD}(\delta)]_{\delta\delta} = -\frac{(1 + 2\lambda)(2\beta^4\omega + (1 + 2\lambda)(1 + 3\omega)(4(1 + 2\lambda) - 3\beta^2))}{(4 - \beta^2 + 8\lambda)^2(1 - \beta^2 + 2\lambda)} < 0,
\]

\[
[\pi^{RD}_2(\delta)]_{\delta\delta} = \frac{2(2 - \beta^2 + 4\lambda)^2(1 + 2\lambda)}{(4 - \beta^2 + 8\lambda)^2(1 - \beta^2 + 2\lambda)} > 0.
\]
With collusive airlines, we show that the same result holds:

\[
\omega \pi_{HSR}^M(\delta) + (1 - \omega)SW^M(\delta)]_{\delta\delta} = -\frac{(1 + 2\lambda)(1 + 3\omega)}{4(1 - \beta^2 + 2\lambda)} < 0,
\]

\[
[\pi_1^M(\delta) + \pi_2^M(\delta)]_{\delta\delta} = \frac{2(2 - \beta^2 + 4\lambda)^2(1 + 2\lambda)}{(4 - \beta^2 + 8\lambda)^2(1 - \beta^2 + 2\lambda)} > 0.
\]

The equilibrium \(\delta\) under competing airlines and collusive airlines with air-proposing are given respectively by \(\delta^{RD-A} = 2\delta^{RD-H}\) and \(\delta^{RM-A} = 2\delta^{RM-H}\), where the superscripts “A” and “H” denote the cases where air and HSR proposes \(\delta\), \(\delta^{RD-H}\) and \(\delta^{RM-H}\) are expressed in Appendix A.1. Air-HSR cooperation can only be achieved when \(\delta^{HSR} > 0\), which gives the relationship \(\underline{\delta} < 0 < \delta^{HSR} < \delta^{Air}\), \(^{55}\) where \(\underline{\delta}\) denotes the minimizer of the airlines’ objective function.

The equilibrium prices and quantities with competing airlines under air-proposing are given in the below equations:

\[
P_{1}^{RD-A} = (P_{HSR}\beta(1 + 2\lambda)(3\beta^2(1 + 2\lambda)(1 + 3\omega) - 4(1 + 2\lambda)^2(1 + 3\omega) - 2\beta^4\omega)
+ \alpha_1(8(1 + 2\lambda)^3(1 + 3\omega) - \beta^4(1 + 2\lambda)(1 - 19\omega) - 2\beta^2(1 + 2\lambda)^2(1 + 23\omega)
- 2\beta^6\omega) + (t - \alpha_1)\beta(1 + 2\lambda)(12(1 + 2\lambda)^2 + 2\beta^4 - 9\beta^2(1 + 2\lambda))(1
- \omega))/((4 - \beta^2 + 8\lambda)(2\beta^4\omega + (1 + 2\lambda)(4 - 3\beta^2 + 8\lambda)(1 + 3\omega))),
\]

\[
P_{2}^{RD-A} = (2P_{HSR}(1 + 2\lambda)((3\beta^2 - 4(1 + 2\lambda))(1 + 2\lambda)(1 + 3\omega) - 2\beta^4\omega) - \alpha_1\beta(2\beta^4\omega
- 4(1 + 2\lambda)^2(3 - 11\omega) + \beta^2(1 + 2\lambda)(5 - 21\omega)) + (t - \alpha_1)(2\beta^6\omega
+ 8(1 + 2\lambda)^3(1 - 9\omega) + \beta^4(1 + 2\lambda)(1 - 21\omega) - 2\beta^2(1 + 2\lambda)^2(1
- 33\omega)))/((4 - \beta^2 + 8\lambda)(2\beta^4\omega + (1 + 2\lambda)(4 - 3\beta^2 + 8\lambda)(1 + 3\omega))),
\]

---

\(^{55}\) This holds for both collusive airlines and competing airlines, as illustrated in Figure 2.4.
\[ q_{1}^{RD-A} = (P_{HSR} \beta (1 + 2 \lambda)(3 \beta^2 (1 + 2 \lambda)(1 + 3 \omega) - 4(1 + 2 \lambda)^2 (1 + 3 \omega) - 2 \beta^4 \omega) \\
+ \alpha_1 (8(1 + 2 \lambda)^3 (1 + 3 \omega) - \beta^4 (1 + 2 \lambda)(1 - 19 \omega) - 2(\beta + 2 \beta \lambda)^2 (1 + 23 \omega) \\
- 2 \beta^6 \omega) + (t - \alpha_1) \beta (12(1 + 2 \lambda)^3 + \beta^4 (2 + 4 \lambda) - 9(\beta + 2 \beta \lambda)^2)(1 \\
- \omega))/((4 - \beta^2 + 8 \lambda)(1 - \beta^2 + 2 \lambda)(2 \beta^4 \omega + (1 + 2 \lambda)(4 - 3 \beta^2 + 8 \lambda)(1 \\
+ 3 \omega))), \]

\[ q_{T}^{RD-A} = (1 + 2 \lambda)(\alpha_1 \beta (19 \beta^2 (1 + 2 \lambda) - 20(1 + 2 \lambda)^2 - 4 \beta^4)(1 - \omega) + P_{HSR}(2 - \beta^2 \\
+ 4 \lambda)(2 \beta^4 \omega - (3 \beta^2 - 4(1 + 2 \lambda))(1 + 2 \lambda)(1 + 3 \omega)) + (t - \alpha_1)(2 \beta^6 \\
- 13 \beta^4 (1 + 2 \lambda) - 24(1 + 2 \lambda)^3 + 30 \beta^2(1 + 2 \lambda)^2)(1 - \omega))/((4 - \beta^2 + 8 \lambda)(1 \\
- \beta^2 + 2 \lambda)(2 \beta^4 \omega + (1 + 2 \lambda)(4 - 3 \beta^2 + 8 \lambda)(1 + 3 \omega))). \]

The equilibrium prices and quantities with collusive airlines under air-proposing are given in the below equations:

\[ p_{1}^{RM-A} = \alpha_1/2, \]

\[ p_{2}^{RM-A} = \frac{(t - \alpha_1)(1 - 9 \omega) - P_{HSR}(1 + 3 \omega))(1 + 2 \lambda) + 2 \beta \alpha_1 (1 - 3 \omega)}{2(1 + 2 \lambda)(1 + 3 \omega)}, \]

\[ q_{1}^{RM-A} = \frac{(\alpha_1 - P_{HSR} \beta)(1 + 2 \lambda)(1 + 3 \omega) + 2 \alpha_1 \beta^2 (1 - 3 \omega) + 3(t - \alpha_1) \beta(1 + 2 \lambda)(1 - \omega)}{2(1 - \beta^2 + 2 \lambda)(1 + 2 \lambda)(1 + 3 \omega)}, \]

\[ q_{T}^{RM-A} = \frac{P_{HSR}(1 + 2 \lambda)(1 + 3 \omega) - (3 \alpha_1 \beta + 3(t - \alpha_1)(1 + 2 \lambda))(1 - \omega)}{2(1 - \beta^2 + 2 \lambda)(1 + 3 \omega)}. \]

The minimizer of the airlines’ objective function as shown in Figure 2.4 as \( \delta \) for competition airlines and collusive airlines are given in the below equations, respectively:

\[ \delta^{RD} = \frac{(2 - \beta^2 + 4 \lambda)(P_{HSR} + t - \alpha_1) + \alpha_1 \beta}{2 - \beta^2 + 4 \lambda} < 0, \]

\[ \delta^{RM} = \frac{(2 \lambda + 1)(P_{HSR} + t - \alpha_1) + \alpha_1 \beta}{1 + 2 \lambda} < 0. \]
A.4 Nash bargaining between the HSR and the airlines

In this Section, we briefly discuss how to use a Nash bargaining model to quantify the effect of bargaining power. Considering $\delta$ to be determined by Nash bargaining in which airlines and the HSR each have an exogenous bargaining power to negotiate $\delta$, we modify the three-stage game described in Section 2.2. In the second stage, the airlines and the HSR bargain for $\delta$, and the objective function can be formulated as a Nash Bargaining Product (e.g., Kalai and Smordinsky, 1975; Binmore et al., 1986; Baron and Berman 2014; Yang et al., 2015; Wan et al., 2016) as expressed in the below equation:

$$\max_{\delta} (\pi_A(\delta) - \pi_A^0)^a (\Pi_{HSR}(\delta) - \Pi_{HSR}^0)^{1-a},$$

where $a \in [0,1]$ indicates the exogenous bargaining power of the airlines and $1-a$ the bargaining power of the HSR. $\pi_A$ and $\Pi_{HSR}$ are the objective functions of the airlines and the HSR, respectively. $\pi_A^0$ and $\Pi_{HSR}^0$ indicate threat points (which, in our setting, is the objective achieved in benchmark). Since the first-order condition gives an expression of $\delta^3$, the analytical solution is complicated. However, by setting the threat points to zero and by only considering pure welfare/profit-maximizing HSR, we can show analytically that there is a threshold of the airline’s bargaining power, above which the HSR is not willing to collaborate. In such a case, the airlines with a higher bargaining power will compromise to this threshold value so that the HSR is indifferent between cooperation and no cooperation. But even with a compromise in bargaining power, the airlines can still be better off than no revenue sharing.

A.5 Numerical analysis of endogenous HSR price and economies of traffic density

To account for endogenous HSR price, the local HSR market should be taken into account. Since the HSR is the only option in the OD market 1-2, the utility function of passengers is just $U = \alpha_{HSR} q_{HSR} - \frac{1}{2} (q_{HSR})^2 - q_{HSR} P_{HSR}$, which gives the linear demand function $q_{HSR} = \alpha_{HSR} - P_{HSR}$, where $\alpha_{HSR}$ is the market size and $q_{HSR}$ is the local HSR traffic. In addition, it is well observed that the airline industry exhibits significant economies of traffic density (e.g., Caves et al., 1984; Brueckner et al., 1992; Brueckner and Spiller, 1994), meaning that the unit operating cost falls with more traffic. Following Brueckner and Spiller (1991), Zhang (1996), Brueckner (2001),
Bilotkach (2007), Jiang and Zhang (2014), and Xia and Zhang (2016), we adopt the following total cost function for airlines: \( c_A = q - \rho q^2 \), where \( q \) is the airlines’ traffic and \( \rho \) is the extent of density economies. We assume that the HSR incurs constant marginal cost \( c_R \).56

In the benchmark case, the airlines and the HSR maximize their objective functions simultaneously. With competing airlines, the objective function of airline 1 is \( \max_{P_1} P_1 q_1 - (q_1 - \rho q_1^2) \), while that of airline 2 is \( \max_{P_2} P_2 q_T - (q_T - \rho q_T^2) \). With collusive airlines, the objective function is \( \max_{\{P_1, P_2\}} P_1 q_1 - (q_1 - \rho q_1^2) + P_2 q_T - (q_T - \rho q_T^2) \). The objective function of the HSR is \( \max_{\{P_{HSR}\}} \omega (P_{HSR} - c_R)(q_{HSR} + q_T) + (1 - \omega)SW \). In the revenue sharing case, airlines in the first stage determine whether to initiate the cooperation; in the second stage, the HSR decides the price and revenue sharing amount: \( \max_{\{P_{HSR}, \delta\}} \omega ((P_{HSR} - \delta - c_R)q_T + (P_{HSR} - c_R)q_{HSR}) + (1 - \omega)SW \); in the last stage, airlines decide airfares. The objective function is \( \max_{\{P_2\}} P_2 - P_3 + \delta)q_T - (q_T - \rho q_T^2) \) for airline 2 under competing airlines, and \( \max_{\{P_1, P_2\}} P_1 q_1 - (q_1 - \rho q_1^2) + (P_2 - P_3 + \delta)q_T - (q_T - \rho q_T^2) \) under collusive airlines.

We chose the Beijing-Guangzhou market for the numerical analysis. This is an ideal market for our calibration for two reasons. First, this OD pair has a long distance of about 2,300 km, such that the HSR is not that competitive to the airlines in providing direct inter-city service. The HSR linkage between Beijing and Shijiazhuang is only 1.5-hour ride, making it feasible to travel to Guangzhou via Shijiazhuang. Second, there has been a survey done by Li and Sheng (2016) to provide us with the necessary parameters for numerical simulation. Li and Sheng conducted a stated preference (SP) survey for passengers on four city pairs in the Beijing-Guangzhou corridor. They investigate passengers’ mode choice among air transport, HSR, and air-HSR intermodal services with Shijiazhuang as a possible transfer point. Table A.1 collates the parameter values for our numerical analysis. The detailed calibration procedure is discussed in Appendix A.6.

56 Constant returns to scale has been identified in railway cost functions (See Table 1 in Oum and Waters (1996)).
Table A.1 Parameter values for numerical analysis

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
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<tr>
<td>$\alpha_1$</td>
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<tr>
<td>$\beta$</td>
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<tr>
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<tr>
<td>$\alpha_{HSR}$</td>
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</table>

We first investigate how the local HSR market size $\alpha_{HSR}$ will affect the conditions for revenue sharing. Figure A.1 plots the areas of revenue sharing under different combinations of $\alpha_{HSR}$ and $\omega$. It shows that, when the HSR is sufficiently welfare maximizing (i.e., $\omega$ is small), revenue sharing can always be achieved, but when the HSR tends to be profit-maximizing (i.e., $\omega$ is large), revenue sharing can be achieved only when $\alpha_{HSR}$ is large enough, consistent with Proposition 1. In addition, the comparison between competing airlines and collusive airlines indicates that revenue sharing is more likely to be achieved with collusive airlines, in line with Proposition 2(i).

![Figure A.1 The effect of $\alpha_{HSR}$ and $\omega$ on the conditions for revenue sharing](image)

Note: the shaded area indicates revenue sharing can be reached.
We next investigate how $\alpha_{HSR}$ and $\omega$ affect the revenue sharing amount $\delta$. Figure A.2 plots the revenue sharing amount under competing airlines $\delta^{RD}$, under collusive airlines $\delta^{RM}$, and their difference $\delta^{RM} - \delta^{RD}$. We can see that the revenue sharing amount $\delta$ is higher when the HSR is more welfare-oriented (i.e., smaller $\omega$) or when the market size $\alpha_{HSR}$ is larger, which is applicable to both collusive and competing airlines. This result is consistent with Proposition 4. In addition, the $\delta$ under collusive airlines is higher than the $\delta$ under competing airlines, verifying Proposition 2(ii).

![Figure A.2 The effect of $\alpha_{HSR}$ and $\omega$ on $\delta$](image)

To investigate the effect of the congestion parameter $\lambda$ on the conditions for revenue sharing, we plot Figure A.3. The result indicates that when the HSR cares sufficiently about social welfare, revenue sharing can always be achieved regardless of the congestion parameter, but when the HSR profit matters, revenue sharing can only be achieved when congestion is not severe, verifying Proposition 3.

![Figure A.3 Effect of congestion parameter](image)

Note: we fix $\alpha_{HSR} = 9072$, relax $\lambda$, and the shaded area indicates revenue sharing can be reached.
Last, we examine the effect of airline economies of traffic density in airline industry. Figure A.4 shows that the stronger the economies of traffic density, the less welfare-oriented the HSR has to be to achieve revenue sharing, indicating economies of traffic density enhance the possibility for revenue sharing.

Figure A.4 The effect of airline economies of traffic density

Note: we fix $\alpha_{\text{HSR}} = 9072$, relax $\rho$, and the shaded area indicates revenue sharing can be reached.

Figure A.3 and Figure A.4 both again imply that revenue sharing is more likely to be achieved under collusive airlines than under competing airlines. Overall, when taking into account endogenous HSR price and airline economies of traffic density, our analytical results in Section 2.3 do not change.

A.6 Parameter calibration for the numerical analysis

We focus on the Beijing-Guangzhou OD market (2,298 km). Based on the survey in Li and Sheng (2016), the modal split of each transport service in Beijing-Guangzhou market is: air 41%, HSR 23% and air-HSR 36%. The air-HSR intermodal service means taking the Beijing-Shijiazhuang segment by HSR and the Shijiazhuang-Guangzhou segment by air. We first calibrate the market size of Beijing-Guangzhou market (i.e., the parameter $\alpha_1$) in the model. We examined the schedule of direct flights from Beijing to Guangzhou on September 5, 2018 (Wednesday), and recorded the aircraft type of each flight. This date could be more representative of the general flight pattern as it is in the middle of the working days. The direct flight schedule distribution is shown in Table
A.2. Specifically, there are 24 flights in total. By identifying the seating capacity from Swan and Adler (2006) and King (2007) and assuming 80% load factor, we estimated the daily total passengers traveling by direct flight from Beijing to Guangzhou. The market size can be calculated as $\alpha_1 = 5,329/41\% = 12,997$. The daily air-HSR traffic can also be estimated: $q_T = 12,997 \times 36\% = 4,679$.

<table>
<thead>
<tr>
<th>Aircraft types</th>
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<th>Seating capacity</th>
<th>Source</th>
<th>Total passengers (80% load factor)</th>
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<td>120</td>
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<td>A380</td>
<td>2</td>
<td>550</td>
<td>King (2007)</td>
<td>880</td>
</tr>
<tr>
<td>A321</td>
<td>3</td>
<td>183</td>
<td>Swan and Adler (2006)</td>
<td>439</td>
</tr>
<tr>
<td>B737</td>
<td>1</td>
<td>162</td>
<td>Swan and Adler (2006)</td>
<td>130</td>
</tr>
<tr>
<td>B777</td>
<td>2</td>
<td>385</td>
<td>Swan and Adler (2006)</td>
<td>616</td>
</tr>
<tr>
<td><strong>Sum: 24</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>Sum: 5,329</strong></td>
</tr>
</tbody>
</table>

Note: The actual seating capacity for the listed aircraft types may vary, depending on how airlines arrange seats among first class, business class, premium economy class, and economy class.

Next, we estimate the parameter $t$ based on the value-of-time estimated in Li and Sheng (2016). The HSR travel time from Beijing to Shijiazhuang is 75 min (see Table 6 in Li and Sheng (2016)); the net access time of air-HSR mode to air mode is 20 min (see Table 4 in Li and Sheng (2016)); the connection time between Shijiazhuang HSR station and Shijiazhuang Airport (SJW) is 5 min (see Table 6 in Li and Sheng (2016)). Given the estimated value-of-time (see Table 3 in Li and Sheng (2016)), we can calculate the parameter $t = 111.5 \times (75/60) + 117.3 \times (20/60) + 140.3 \times (5/60) = 190$ RMB.

We then estimate $\beta$ and $\lambda$. We first calculate the cross elasticity of direct air demand with respect to air-HSR price based on the utility specification in Li and Sheng (2016): $\varepsilon_{q_A, P_{AH}} = \frac{\partial S_A}{\partial P_{AH}} \frac{P_{AH}}{S_A}$, where $S_A = \exp(U_A)/\left(\exp(U_A) + \exp(U_H) + \exp(U_{AH})\right)$ and $U_A, U_H, $ and $U_{AH}$ are the utility.
specified for air, HSR and air-HSR modes, respectively. Using the estimates in Table 2 of Li and Sheng (2016) and the survey result in Tables 1 and 4 of Li and Sheng (2016), we can obtain that $\varepsilon_{q_A,P_{AH}} = 0.345$. The cross elasticity of direct air demand with respect to air-HSR price in our model is calculated as: $\varepsilon_{q_1,(P_2+P_{HSR})} = \frac{\beta(-(P_3+t-\alpha_1)\beta(1+2\lambda)+\alpha_1(-2+\beta^2-4\lambda))}{(1+2\lambda)(\alpha_1\beta+(P_3+t-\alpha_1)(2-\beta^2+4\lambda))}$. Equating the cross elasticity ($\varepsilon_{q_1,(P_2+P_{HSR})} = 0.345$) and the equilibrium traffic ($q_1^{BD} = 5329$ and $q_1^{TD} = 4679$), we can thus obtain $\beta = 0.3$ and $\lambda = 0.07$.\(^{57}\)

Last, we estimate the size of the local HSR market $\alpha_{HSR}$. The number of high-speed trains running from Beijing to Shijiazhuang per day is 84 (with the majority of the trains going to Guangzhou as the final destination)\(^{58}\), and each train carries around 540 seats. Since this capacity is deployed to the Beijing-Guangzhou line, we take the discount factor 0.2 to account for the capacity deployed to Beijing-Shijiazhuang. Thus, $\alpha_{HSR} = 9,072$. Since the HSR operating cost is not publicly available, assumption has to be made. We take $c_R = 100$ RMB/person in the simulation. Robustness checks have also been done by varying the discount factor for $\alpha_{HSR}$ and $c_R$, and the results do not significantly change.

---

\(^{57}\) In our model, we abstract away the alternative of direct HSR in the OD market 1-3. Nevertheless, HSR is less competitive than direct air and air-HSR modes in Beijing-Guangzhou. Shijiazhuang city is considered as the transfer point but the option of free HSR ticket offered by Spring Airlines is not mentioned in the stated preference survey of Li and Sheng (2016). Thus, the estimated traffic more resembles the equilibrium of no revenue sharing and competing airlines, which we used to calibrate the parameters.

\(^{58}\) The information is retrieved from www.12306.cn (in Chinese) for the day September 5, 2018 (Wednesday).
Appendix B  Appendix for Chapter 3

B.1 One-period model and private port

Properties of the demand function \( q (\tau, K, I; x, \theta, \delta, m, a, b) \).

Let \( \Delta = \frac{\partial p}{\partial q} - \frac{\partial g}{\partial q} = -b - \frac{\delta}{K} < 0 \). Totally differentiating the equilibrium condition \( \rho(q) = p(q) \) with respect to \( \tau, K, I \) and re-arranging, we obtain:

\[
\frac{\partial q}{\partial \tau} = \frac{1}{\Delta} < 0, \quad \frac{\partial q}{\partial K} = \frac{1}{\Delta} (\frac{\partial g}{\partial K} + x \frac{\partial f}{\partial K}) > 0, \quad \frac{\partial q}{\partial I} = \frac{1}{\Delta} x \frac{\partial f}{\partial I} > 0.
\]

Increasing capacity has two effects on shippers’ demand: it relieves congestion (\( \frac{\partial g}{\partial K} < 0 \)) but exposes shippers to more damage in case of a disaster (\( x \frac{\partial f}{\partial K} > 0 \)). Eq. (3.5) assures that the congestion effect dominates so that \( \frac{\partial q}{\partial K} > 0 \).

It is straightforward to derive comparative statics results for other exogenous variables:

\[
\frac{\partial q}{\partial x} = \frac{1}{\Delta} f < 0, \quad \frac{\partial q}{\partial \theta} = x \frac{\partial f}{\partial \theta} > 0, \quad \frac{\partial q}{\partial \delta} = \frac{1}{\Delta} \frac{\partial g}{\partial \delta} < 0,
\]

\[
\frac{\partial q}{\partial m} = \frac{1}{\Delta} x \frac{\partial f}{\partial m} < 0, \quad \frac{\partial q}{\partial a} = -\frac{1}{\Delta} \frac{\partial p}{\partial a} > 0, \quad \frac{\partial q}{\partial b} = -\frac{1}{\Delta} \frac{\partial p}{\partial b} < 0.
\]

The effects of capacity and protection on optimal pricing in partial equilibrium.

By the implicit function theorem,

\[
\frac{\partial \tau^*}{\partial K} = -\frac{\Pi_{\tau k}}{\Pi_{\tau \tau}}, \quad \frac{\partial \tau^*}{\partial I} = -\frac{\Pi_{\tau I}}{\Pi_{\tau \tau}},
\]

where \( \Pi_{\tau k} = \frac{\partial q}{\partial K} + (\tau - c) \frac{\partial^2 q}{\partial \tau \partial K} \), and \( \Pi_{\tau I} = \frac{\partial q}{\partial I} + (\tau - c) \frac{\partial^2 q}{\partial \tau \partial I} \).

Since \( \frac{\partial^2 q}{\partial \tau \partial K} = \frac{1}{\Delta} > 0, \frac{\partial^2 q}{\partial \tau \partial I} < 0 \), we obtain:

\[
\Pi_{\tau k} = \frac{1}{\Delta} x \frac{\partial f}{\partial K} + \frac{\partial g}{\partial K} + (\tau - c) \frac{\partial q}{\partial \tau} \frac{\partial^2 q}{\partial \tau \partial K} \bigg|_{\tau^*} = \frac{1}{\Delta} x \frac{\partial f}{\partial K} < 0,
\]

where the last equality follows from Eq. (3.12) and the congestion function in Eq. (3.1), which gives \( \frac{\partial g}{\partial K} + (-q) \frac{\partial^2 q}{\partial K \partial q} = 0 \). Thus, \( \frac{\partial \tau^*}{\partial K} < 0 \).
We can further show that \( \frac{\partial^2 q}{\partial \tau \partial l} = 0 \), which gives \( \Pi_{\tau l} = \frac{\partial q}{\partial l} > 0 \). Thus, \( \frac{\partial \tau^*}{\partial l} > 0 \).

The effects of parameters on optimal pricing in partial equilibrium.

Applying the implicit function theorem,

\[
\frac{\partial \tau^*}{\partial x} = -\frac{\Pi_{\tau x}}{\Pi_{\tau\tau}} = -\frac{\frac{\partial q}{\partial x} + \frac{\partial^2 q}{\partial \tau \partial x} (\tau - c)}{\Pi_{\tau \tau}} = -\frac{\frac{\partial q}{\partial x}}{\Pi_{\tau \tau}} < 0,
\]

where the last equality follows because \( \frac{\partial^2 q}{\partial \tau \partial x} = 0 \).

Similarly,

\[
\frac{\partial \tau^*}{\partial \theta} = -\frac{\frac{\partial q}{\partial \theta}}{\Pi_{\tau \theta}} > 0, \quad \frac{\partial \tau^*}{\partial m} = -\frac{\frac{\partial q}{\partial m}}{\Pi_{\tau \tau}} < 0, \quad \frac{\partial \tau^*}{\partial a} = -\frac{\frac{\partial q}{\partial a}}{\Pi_{\tau \tau}} > 0, \quad \frac{\partial \tau^*}{\partial c} = -\frac{\frac{\partial q}{\partial \tau}}{\Pi_{\tau \tau}} > 0.
\]

Next,

\[
\Pi_{\tau \delta} = \frac{\partial q}{\partial \delta} + \frac{\partial^2 q}{\partial \tau \partial \delta} (\tau - c) = \frac{1}{\Delta} \frac{\partial g}{\partial \delta} + \frac{1}{\Delta} \frac{\partial^2 g}{\partial \delta \partial q} \frac{\partial q}{\partial \tau} (\tau - c) = \frac{1}{\Delta} (\frac{\partial g}{\partial \delta} + \frac{\partial^2 g}{\partial \delta \partial q} q) = 0,
\]

where the second last equality follows from the optimal private charge in Eq. (3.12), and the last equality follows from Eq. (3.1). Thus,

\[
\frac{\partial \tau^*}{\partial \delta} = -\frac{\Pi_{\tau \delta}}{\Pi_{\tau \tau}} = 0.
\]

For the demand slope parameter, \( b \):

\[
\Pi_{\tau b} = \frac{\partial q}{\partial b} + \frac{\partial^2 q}{\partial \tau \partial b} (\tau - c) = -\frac{1}{\Delta} \frac{\partial p}{\partial b} + \frac{1}{\Delta} \frac{\partial q}{\partial \tau} (\tau - c) = \frac{1}{\Delta} (q + \frac{\partial q}{\partial \tau} (\tau - c)) = 0.
\]

Thus,

\[
\frac{\partial \tau^*}{\partial b} = -\frac{\Pi_{\tau b}}{\Pi_{\tau \tau}} = 0.
\]

In addition,

\[
\frac{\partial \tau^*}{\partial \eta} = 0, \quad \frac{\partial \tau^*}{\partial M} = 0, \quad \frac{\partial \tau^*}{\partial c_k} = 0, \quad \frac{\partial \tau^*}{\partial c_l} = 0.
\]

Second-order condition for capacity.
The SOC for capacity is $\Pi_{kk} = (\tau - c) \frac{\partial^2 q}{\partial K^2} - x \frac{\partial^2 D}{\partial K^2}$, the sign of which depends on two second-order derivatives $\frac{\partial^2 q}{\partial K^2}$ and $\frac{\partial^2 D}{\partial K^2}$.

$$\frac{\partial^2 q}{\partial K^2} = \frac{1}{\Delta^2} \left( \frac{\partial^2 g}{\partial K^2} + x \frac{\partial^2 f}{\partial K^2} \right) \Delta + 2 \frac{\partial^2 g}{\partial K \partial q} \left( \frac{\partial g}{\partial K} + x \frac{\partial f}{\partial K} \right) < 0,$$

where the second equality follows because $-\frac{\partial^2 q}{\partial K^2} + 2 \frac{\partial^2 g}{\partial K \partial q} \frac{\partial q}{\partial K} = 0$ as a result of Eq. (3.1), and the inequality follows because $\frac{\partial^2 f}{\partial K^2} > 0$ as assumed in Eq. (3.3).

$$\frac{\partial^2 D}{\partial K^2} = 2 \frac{\partial F}{\partial K} + \frac{\partial^2 F}{\partial K^2} K > 0,$$

where the inequality follows because $\frac{\partial^2 F}{\partial K^2} > 0$, as assumed in Eq. (3.8).

Given $\frac{\partial^2 q}{\partial K^2} < 0$ and $\frac{\partial^2 D}{\partial K^2} > 0$, $\Pi_{kk} = (\tau - c) \frac{\partial^2 q}{\partial K^2} - x \frac{\partial^2 D}{\partial K^2} < 0$ follows directly.

**The effects of fee and protection on optimal capacity in partial equilibrium.**

For the fee:

$$\frac{\partial K^*}{\partial \tau} = -\frac{\Pi_{kt}}{\Pi_{kk}},$$

where $\Pi_{kt} = \frac{\partial q}{\partial K} + (\tau - c) \frac{\partial^2 q}{\partial K \partial \tau}$. As shown above, $\Pi_{tk \mid \tau} < 0$. Thus,

$$\left. \frac{\partial K^*}{\partial \tau} \right|_{\tau^*} < 0.$$

For protection,

$$\frac{\partial K^*}{\partial l} = -\frac{\Pi_{kl}}{\Pi_{kk}},$$

where $\Pi_{kl} = (\tau - c) \frac{\partial^2 q}{\partial K \partial l} - x \frac{\partial^2 D}{\partial K \partial l}$.

The first cross-partial derivative is

$$\frac{\partial^2 q}{\partial K \partial l} = \frac{1}{\Delta} \left( x \frac{\partial^2 f}{\partial K \partial l} + \frac{\partial^2 g}{\partial K \partial q} \frac{\partial q}{\partial l} \right) > 0,$$
where the inequality follows from Eq. (3.1), Eq. (3.3), and Eq. (3.6). The second cross-partial derivative is
\[ \frac{\partial^2 D}{\partial K \partial I} = \frac{\partial^2 F}{\partial K \partial I} + \frac{\partial F}{\partial I} < 0, \]
where the inequality follows from the assumptions in Eq. (3.8). Together, the two inequalities imply
\[ \Pi_{kI} > 0. \]

The effects of parameters on optimal capacity in partial equilibrium.

\[ \frac{\partial K^*}{\partial x} = -\frac{\Pi_{kx}}{\Pi_{kk}} = -\frac{1}{\Pi_{\tau \tau}} \left( \frac{\partial^2 q}{\partial K \partial x} (\tau - c) - \frac{\partial D}{\partial K} \right) < 0, \]
where the inequality follows because \( \frac{\partial^2 q}{\partial K \partial x} = \frac{1}{\Delta} \left( \frac{\partial^2 g}{\partial K \partial q} \frac{\partial q}{\partial x} + \frac{\partial f}{\partial K} \right) < 0 \) and \( \frac{\partial D}{\partial K} > 0 \).

\[ \frac{\partial K^*}{\partial \theta} = -\frac{\Pi_{k\theta}}{\Pi_{kk}} = -\frac{1}{\Pi_{kk}} \left( \frac{\partial^2 q}{\partial K \partial \theta} (\tau - c) \right) > 0, \]
where the inequality follows because \( \frac{\partial^2 q}{\partial K \partial \theta} = \frac{1}{\Delta} \left( x \frac{\partial^2 f}{\partial K \partial \theta} + \frac{\partial^2 g}{\partial K \partial q} \frac{\partial q}{\partial \theta} \right) > 0 \) given \( \frac{\partial^2 g}{\partial K \partial q} < 0, \frac{\partial q}{\partial \theta} > 0 \), and \( \frac{\partial^2 f}{\partial K \partial \theta} < 0 \) as per Eq. (3.4).

\[ \frac{\partial K^*}{\partial m} = -\frac{\Pi_{km}}{\Pi_{kk}} = -\frac{1}{\Pi_{kk}} \left( \frac{\partial^2 q}{\partial K \partial m} (\tau - c) \right) < 0, \]
where the inequality follows because \( \frac{\partial^2 q}{\partial K \partial m} = \frac{1}{\Delta} \left( x \frac{\partial^2 f}{\partial K \partial m} + \frac{\partial^2 g}{\partial K \partial q} \frac{\partial q}{\partial m} \right) < 0 \) given \( \frac{\partial^2 f}{\partial K \partial m} > 0 \) as per Eq. (3.4).

\[ \frac{\partial K^*}{\partial \eta} = -\frac{\Pi_{k\eta}}{\Pi_{kk}} = -\frac{1}{\Pi_{kk}} \left( -x \frac{\partial^2 D}{\partial K \partial \eta} \right) > 0, \]
where the inequality follows because \( \frac{\partial^2 D}{\partial K \partial \eta} = \frac{\partial^2 F}{\partial K \partial \eta} K + \frac{\partial F}{\partial \eta} < 0 \) given \( \frac{\partial^2 F}{\partial K \partial \eta} < 0 \) as per Eq. (3.9).

\[ \frac{\partial K^*}{\partial M} = -\frac{\Pi_{km}}{\Pi_{kk}} = -\frac{1}{\Pi_{kk}} \left( -x \frac{\partial^2 D}{\partial K \partial M} \right) < 0, \]
where the inequality follows because \( \frac{\partial^2 D}{\partial K \partial M} = \frac{\partial^2 F}{\partial K \partial M} K + \frac{\partial F}{\partial M} > 0 \), given \( \frac{\partial^2 F}{\partial K \partial M} > 0 \) and \( \frac{\partial F}{\partial M} > 0 \) as per Eq. (3.9).
\[
\frac{\partial K^*}{\partial a} = -\frac{\Pi_{ka}}{\Pi_{kk}} = -\frac{1}{\Pi_{kk}} \left( \frac{\partial^2 q}{\partial K \partial a} (\tau - c) \right) > 0,
\]
where the inequality follows because \( \frac{\partial^2 q}{\partial K \partial a} = \frac{1}{\Delta} \frac{\partial^2 g}{\partial K \partial q} \frac{\partial q}{\partial a} > 0. \)

\[
\frac{\partial K^*}{\partial b} = -\frac{\Pi_{kb}}{\Pi_{kk}} = -\frac{1}{\Pi_{kk}} \left( \frac{\partial^2 q}{\partial K \partial b} (\tau - c) \right) < 0,
\]
where the inequality follows because \( \frac{\partial^2 q}{\partial K \partial b} = \frac{1}{\Delta} \left( \frac{\partial^2 g}{\partial K \partial q} \frac{\partial q}{\partial b} + \frac{\partial q}{\partial K} \right) < 0. \)

Three more derivatives are easily signed:

\[
\frac{\partial K^*}{\partial c} = -\frac{\Pi_{kc}}{\Pi_{kk}} = -\frac{1}{\Pi_{kk}} \left( -\frac{\partial q}{\partial K} \right) < 0, \quad \frac{\partial K^*}{\partial c_k} = -\frac{\Pi_{kck}}{\Pi_{kk}} = -\frac{1}{\Pi_{kk}} < 0, \quad \frac{\partial K^*}{\partial c_I} = 0.
\]

Finally,

\[
\frac{\partial K^*}{\partial \delta} = -\frac{\Pi_{k\delta}}{\Pi_{kk}} = -\frac{1}{\Pi_{kk}} \left( \frac{\partial^2 q}{\partial K \partial \delta} (\tau - c) \right) \leq 0.
\]

The derivative \( \frac{\partial^2 q}{\partial K \partial \delta} \) cannot be signed because \( \frac{\partial^2 g}{\partial K \partial \delta} < 0, \) while

\[
\frac{\partial^2 q}{\partial K \partial \delta} > 0 \quad \text{and} \quad \frac{\partial^2 g}{\partial K \partial \delta} > 0.
\]

**Second-order condition for protection.**

The SOC for protection is

\[
\Pi_{II} = (\tau - c) \frac{\partial^2 q}{\partial I^2} - \Delta \frac{\partial^2 D}{\partial I^2}.
\]

Given the assumptions in Eq. (3.3) and Eq. (3.8), it is straightforward to show that

\[
\frac{\partial^2 q}{\partial I^2} = \frac{1}{\Delta} x \frac{\partial^2 f}{\partial I^2} < 0, \quad \frac{\partial^2 D}{\partial I^2} = \frac{\partial^2 F}{\partial I^2} K > 0.
\]

Thus, \( \Pi_{II} < 0. \)

**The effects of fee and capacity on optimal protection in partial equilibrium.**

We have shown that \( \Pi_{III} > 0 \) and \( \Pi_{II} > 0. \) Hence

\[
\frac{\partial I^*}{\partial \tau} = -\frac{\Pi_{III}}{\Pi_{II}} > 0, \quad \frac{\partial I^*}{\partial K} = -\frac{\Pi_{III}}{\Pi_{II}} > 0.
\]
The effects of parameters on optimal protection in partial equilibrium.

\[
\frac{\partial l^*}{\partial x} = -\frac{\Pi_{lx}}{\Pi_{II}} = -\frac{1}{\Pi_{II}} \left( (\tau - c) \frac{\partial^2 q}{\partial l \partial x} - \frac{\partial D}{\partial l} \right) > 0,
\]

where the inequality follows because \( \frac{\partial^2 q}{\partial l \partial x} = \frac{1}{\Delta} \frac{\partial f}{\partial l} > 0 \) and \( \frac{\partial D}{\partial l} < 0 \).

\[
\frac{\partial l^*}{\partial \theta} = -\frac{\Pi_{l\theta}}{\Pi_{II}} = -\frac{1}{\Pi_{II}} \left( (\tau - c) \frac{\partial^2 q}{\partial l \partial \theta} \right),
\]

where \( \frac{\partial^2 q}{\partial l \partial \theta} = \frac{1}{\Delta} \chi \frac{\partial^2 f}{\partial l \partial \theta} \). Since it is not obvious whether the marginal benefit of protection increases or decreases with more effective protection, the sign of \( \frac{\partial^2 q}{\partial l \partial \theta} \) is a priori unclear. Thus \( \frac{\partial l^*}{\partial \theta} \) can be non-monotone. The same is true of \( \frac{\partial l^*}{\partial \eta} \).

\[
\frac{\partial l^*}{\partial m} = -\frac{\Pi_{lm}}{\Pi_{II}} = -\frac{1}{\Pi_{II}} \left( (\tau - c) \frac{\partial^2 q}{\partial l \partial m} \right) > 0,
\]

where the inequality follows because \( \frac{\partial^2 q}{\partial l \partial m} = \frac{1}{\Delta} x \frac{\partial^2 f}{\partial l \partial m} > 0 \) given \( \frac{\partial^2 f}{\partial l \partial m} < 0 \) as per Eq. (3.4).

\[
\frac{\partial l^*}{\partial M} = -\frac{\Pi_{IM}}{\Pi_{II}} = -\frac{1}{\Pi_{II}} \left( -x \frac{\partial^2 D}{\partial l \partial M} \right) > 0,
\]

where the inequality follows because \( \frac{\partial^2 D}{\partial l \partial M} = K \frac{\partial^2 F}{\partial l \partial M} < 0 \) as per Eq. (3.9).

\[
\frac{\partial l^*}{\partial a} = -\frac{\Pi_{la}}{\Pi_{II}} = -\frac{1}{\Pi_{II}} \left( (\tau - c) \frac{\partial^2 q}{\partial l \partial a} \right) = 0,
\]

where the equality follows because \( \frac{\partial^2 q}{\partial l \partial a} = 0 \).

\[
\frac{\partial l^*}{\partial b} = -\frac{\Pi_{lb}}{\Pi_{II}} = -\frac{1}{\Pi_{II}} \left( (\tau - c) \frac{\partial^2 q}{\partial l \partial b} \right) < 0,
\]

where the inequality follows because \( \frac{\partial^2 q}{\partial l \partial b} = \frac{1}{\Delta} x \frac{\partial f}{\partial l} < 0 \).

We also have:

\[
\frac{\partial l^*}{\partial c} = -\frac{\Pi_{lc}}{\Pi_{II}} = -\frac{1}{\Pi_{II}} \left( -\frac{\partial q}{\partial l} \right) < 0, \quad \frac{\partial l^*}{\partial c_t} = -\frac{\Pi_{lct}}{\Pi_{II}} = -\frac{1}{\Pi_{II}} < 0, \quad \frac{\partial l^*}{\partial c_k} = 0.
\]

Finally,

\[
\frac{\partial l^*}{\partial \delta} = -\frac{\Pi_{l\delta}}{\Pi_{II}} = -\frac{1}{\Pi_{II}} \left( (\tau - c) \frac{\partial^2 q}{\partial l \partial \delta} \right) < 0,
\]

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where the equality follows because \( \frac{\partial^2 q}{\partial t \partial \delta} = \frac{1}{\Delta} x \frac{\partial f}{\partial I} \frac{\partial^2 q}{\partial q \partial \delta} < 0. \)

**The effects of parameters on output in partial equilibrium.**

*Only \( \tau \) is the decision variable.*

\[
\frac{dq^*}{dx} = \frac{\partial q}{\partial x} + \frac{\partial q}{\partial t} \frac{\partial \tau^*}{\partial x} = \frac{\partial q}{\partial x} + \frac{\partial q}{\partial t} \left( -\frac{\partial q}{\partial x} \frac{\Pi_{\tau t}}{2} \right) = \frac{1}{2} \frac{\partial q}{\partial x} < 0,
\]

where the last equality follows because \( \Pi_{\tau t} = 2 \frac{\partial q}{\partial \tau} \).

 Similarly, we can show that

\[
\frac{dq^*}{dm} = \frac{1}{2} \frac{\partial q}{\partial m} < 0, \quad \frac{dq^*}{d\theta} = \frac{1}{2} \frac{\partial q}{\partial \theta} > 0, \quad \frac{dq^*}{da} = \frac{1}{2} \frac{\partial q}{\partial a} > 0.
\]

Since \( \frac{\partial \tau^*}{\partial b} = 0 \) and \( \frac{\partial \tau^*}{\partial \delta} = 0 \),

\[
\frac{dq^*}{db} = \frac{\partial q}{\partial b} + \frac{\partial q}{\partial \tau} \frac{\partial \tau^*}{\partial b} = \frac{\partial q}{\partial b} < 0, \quad \frac{dq^*}{d\delta} = \frac{\partial q}{\partial \delta} + \frac{\partial q}{\partial \tau} \frac{\partial \tau^*}{\partial \delta} = \frac{\partial q}{\partial \delta} < 0.
\]

Next,

\[
\frac{dq^*}{d\tau} = \frac{\partial q}{\partial \tau} + \frac{\partial q}{\partial \tau} \frac{\partial \tau^*}{\partial \tau} = 0,
\]

where the last equality follows because \( \frac{\partial q}{\partial M} = 0 \) and \( \frac{\partial \tau^*}{\partial M} = 0 \).

Similarly, we can show that

\[
\frac{dq^*}{d\eta} = 0, \quad \frac{dq^*}{dc_k} = 0, \quad \frac{dq^*}{dc_l} = 0.
\]

Last, since \( \frac{\partial q}{\partial c} = 0 \):

\[
\frac{dq^*}{dc} = \frac{\partial q}{\partial \tau} \frac{\partial \tau^*}{\partial c} < 0.
\]

*Only \( K \) is the decision variable.*

\[
\frac{dq^*}{dx} = \frac{\partial q}{\partial x} + \frac{\partial q}{\partial K} \frac{\partial K^*}{\partial x} < 0,
\]

where the inequality follows because \( \frac{\partial q}{\partial x} < 0, \frac{\partial q}{\partial K} > 0 \) and \( \frac{\partial K^*}{\partial x} < 0. \)
Similarly, we can show that

\[
\frac{dq^*}{dm} = \frac{\partial q}{\partial m} + \frac{\partial q}{\partial K} \frac{\partial K^*}{\partial m} < 0, \quad \frac{dq^*}{dM} = \frac{\partial q}{\partial M} + \frac{\partial q}{\partial K} \frac{\partial K^*}{\partial M} < 0, \quad \frac{dq^*}{d\theta} = \frac{\partial q}{\partial \theta} + \frac{\partial q}{\partial K} \frac{\partial K^*}{\partial \theta} > 0, \\
\frac{dq^*}{d\eta} = \frac{\partial q}{\partial \eta} + \frac{\partial q}{\partial K} \frac{\partial K^*}{\partial \eta} > 0, \quad \frac{dq^*}{da} = \frac{\partial q}{\partial a} + \frac{\partial q}{\partial K} \frac{\partial K^*}{\partial a} > 0, \quad \frac{dq^*}{db} = \frac{\partial q}{\partial b} + \frac{\partial q}{\partial K} \frac{\partial K^*}{\partial b} < 0, \\
\frac{dq^*}{dc} = \frac{\partial q}{\partial c} + \frac{\partial q}{\partial K} \frac{\partial K^*}{\partial c} < 0, \quad \frac{dq^*}{dc_k} = \frac{\partial q}{\partial c_k} + \frac{\partial q}{\partial K} \frac{\partial K^*}{\partial c_k} < 0, \quad \frac{dq^*}{dc_I} = \frac{\partial q}{\partial c_I} + \frac{\partial q}{\partial K} \frac{\partial K^*}{\partial c_I} = 0.
\]

Since \(\frac{\partial K^*}{\partial \delta}\) cannot be signed, \(\frac{dq^*}{d\delta}\) cannot be signed either.

Only \(I\) is the decision variable.

\[
\frac{dq^*}{dx} = \frac{\partial q}{\partial x} + \frac{\partial q}{\partial l^*} \frac{\partial l^*}{\partial x}, \quad \frac{dq^*}{dm} = \frac{\partial q}{\partial m} + \frac{\partial q}{\partial l^*} \frac{\partial l^*}{\partial m}.
\]

Since both \(x\) and \(m\) reduce demand, but increase optimal protection (which increases demand in turn), the effect of \(x\) and \(m\) on output is uncertain.

Due to the ambiguity in \(\frac{\partial l^*}{\partial \theta}\) and \(\frac{\partial l^*}{\partial \eta}\), the effect of \(\theta\) and \(\eta\) on output cannot be determined.

\[
\frac{dq^*}{dM} = \frac{\partial q}{\partial M} + \frac{\partial q}{\partial l} \frac{\partial l^*}{\partial M} > 0, \quad \frac{dq^*}{da} = \frac{\partial q}{\partial a} + \frac{\partial q}{\partial l} \frac{\partial l^*}{\partial a} > 0, \quad \frac{dq^*}{db} = \frac{\partial q}{\partial b} + \frac{\partial q}{\partial l} \frac{\partial l^*}{\partial b} < 0, \\
\frac{dq^*}{d\delta} = \frac{\partial q}{\partial \delta} + \frac{\partial q}{\partial l} \frac{\partial l^*}{\partial \delta} < 0, \quad \frac{dq^*}{dc} = \frac{\partial q}{\partial c} + \frac{\partial q}{\partial l} \frac{\partial l^*}{\partial c} < 0, \quad \frac{dq^*}{dc_k} = \frac{\partial q}{\partial c_k} + \frac{\partial q}{\partial l} \frac{\partial l^*}{\partial c_k} = 0, \\
\frac{dq^*}{dc_I} = \frac{\partial q}{\partial c_I} + \frac{\partial q}{\partial l} \frac{\partial l^*}{\partial c_I} < 0.
\]
Full-equilibrium analysis with general disaster functions

Table B.1 Comparative statics in full equilibrium for private port: General disaster functions

<table>
<thead>
<tr>
<th></th>
<th>Charge (τ)</th>
<th>Capacity (K)</th>
<th>Protection (I)</th>
<th>Output (q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate and damage parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>x (disaster probability)</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>m (disaster intensity to shippers)</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>M (disaster intensity to port)</td>
<td>+</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>θ (Protection effectiveness to shippers)</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>η (Protection effectiveness to port)</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Demand parameters</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>a (additive demand shocks)</td>
<td>?</td>
<td>?</td>
<td>+</td>
<td>?</td>
</tr>
<tr>
<td>b (multiplicative demand shocks)</td>
<td>?</td>
<td>-</td>
<td>-</td>
<td>?</td>
</tr>
<tr>
<td>Cost parameters</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>δ (congestion cost to shippers)</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>c (port operating cost)</td>
<td>?</td>
<td>-</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>c_k (cost of capacity investment)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>c_I (cost of protection investment)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>?</td>
</tr>
</tbody>
</table>

B.2 One-period model and public port

Show SOCs are satisfied.

SOC for τ:

\[ W_{ττ} = \frac{∂q}{∂τ}\left(2 - \frac{∂q}{∂τ} \frac{∂ρ}{∂q}\right) = \frac{∂q}{∂τ}\left(2 - \frac{1}{Δ} \frac{∂ρ}{∂q}\right) < 0, \]

where the inequality follows because \( \frac{1}{Δ} \frac{∂ρ}{∂q} = \frac{b}{b + δ_K} < 1 \), and \( \frac{∂q}{∂τ} < 0 \).

SOC for K:

\[ W_{kk} = \left(τ - c - q \frac{∂ρ}{∂q}\right) \frac{∂^2 q}{∂K^2} < 0 - x \frac{∂^2 D}{∂K^2} > 0 - \frac{∂ρ}{∂q} \left(\frac{∂q}{∂K}\right)^2. \]
Plug in \( \frac{\partial^2 q}{\partial K^2} = \frac{1}{\Delta^2} \left( \frac{\partial^2 f}{\partial q} + x \frac{\partial^2 f}{\partial K^2} \Delta + 2x \frac{\partial^2 g}{\partial q} \frac{\partial f}{\partial K} \right) \) and \( \frac{\partial q}{\partial K} = \frac{1}{\Delta} \left( \frac{\partial g}{\partial K} + x \frac{\partial f}{\partial K} \right) \) and rearrange to get

\[
W_{kk} = \frac{1}{\Delta^2} \frac{\partial \rho}{\partial q} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 g}{\partial K^2} - \left( \frac{\partial g}{\partial K} \right)^2 \right) + \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{1}{\Delta^2} \left( x \frac{\partial^2 f}{\partial K^2} \Delta + 2x \frac{\partial^2 g}{\partial q} \frac{\partial f}{\partial K} \right) \]

\[
- \frac{1}{\Delta^2} x \frac{\partial f}{\partial K} \frac{\partial \rho}{\partial q} \left( 2 \frac{\partial g}{\partial K} + x \frac{\partial f}{\partial K} \right) - x \frac{\partial^2 D}{\partial K^2} > 0
\]

where the first inequality follows because \( \frac{\partial^2 f}{\partial K^2} > 0 \) and \( \frac{\partial f}{\partial K} > 0 \) from the assumptions in Eq. (3.3) and \( \frac{\partial g}{\partial K} < 0 \), the second inequality follows because \( \frac{\partial g}{\partial K} + x \frac{\partial f}{\partial K} < 0 \) from Eq. (3.5) and \( \frac{\partial g}{\partial K} < 0 \), and the last inequality follows because \( \frac{\partial f}{\partial K} > 0 \) and \( \frac{\partial^2 f}{\partial K^2} > 0 \) from the assumptions in Eq. (3.8), which gives \( \frac{\partial^2 D}{\partial K^2} = 2 \frac{\partial f}{\partial K} + K \frac{\partial^2 f}{\partial K^2} > 0 \).

Now we only need to show \( \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial^2 g}{\partial K^2} - \left( \frac{\partial g}{\partial K} \right)^2 > 0 \). Plug in the functional form of \( g \) in Eq. (3.1), we get

\[
\frac{\partial^2 g}{\partial K^2} - \left( \frac{\partial g}{\partial K} \right)^2 = \frac{\delta}{K^3} \left( 2(\tau - c) + q (2b - \delta/K) \right),
\]

which cannot be signed directly. We thus evaluate this expression at the optimal \( \tau^w \), which gives

\[
\left. \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \right|_{\tau^w} = -q \frac{\partial q}{\partial \tau} = -q \Delta. \text{ Thus,}
\]

\[
\left. \left( \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial^2 g}{\partial K^2} - \left( \frac{\partial g}{\partial K} \right)^2 \right) \right|_{\tau^w} = \frac{\delta q^2}{K^3} (2b + \delta/K) > 0,
\]

which gives \( W_{kk} |_{\tau^w} < 0 \). Even when \( W_{kk} \) is not evaluated at \( \tau^w \), it is still plausible that \( W_{kk} < 0 \), because most of the terms in the expression of \( W_{kk} \) are negative. Another sufficient condition for \( W_{kk} < 0 \) is \( K > \frac{\delta}{2b} \) (\( K \) is sufficiently large).
SOC for I:

\[ W_{II} = \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial^2 q}{\partial I^2} \mid_{<0} - x \frac{\partial^2 D}{\partial I^2} \frac{\partial q}{\partial I} \left( \frac{\partial q}{\partial I} \right)^2 = \left( \tau - c - q \frac{\partial^2 q}{\partial I^2} - x \frac{\partial^2 D}{\partial I^2} \frac{\partial q}{\partial I} \left( \frac{\partial q}{\partial I} \right)^2 \right) \] 

where \( \frac{\partial^2 q}{\partial I^2} = \frac{1}{\Delta} x \frac{\partial^2 f}{\partial I^2} \) and \( \frac{\partial q}{\partial I} = \frac{1}{\Delta} x \frac{\partial f}{\partial I} \). From the demand system \( \rho(q) = p(q) \), we have

\[ a - b q = \tau + \delta \frac{q}{K} + x f, \]

which gives the demand function \( q = \frac{a - \tau - x f}{b + \delta / K} \). Substituting \( q \) in \( W_{II} \) and rearranging, we have

\[ W_{II} = \frac{\delta / K}{-\Delta} (\tau - c) \frac{\partial^2 q}{\partial I^2} \mid_{<0} - x \frac{\partial^2 D}{\partial I^2} \frac{\partial q}{\partial I} \left( \frac{\partial q}{\partial I} \right)^2 + b \frac{1}{-\Delta} \frac{\partial^2 f}{\partial I^2} (a - c - x f) + x \left( \frac{\partial f}{\partial I} \right)^2. \]

Substituting in the expression of \( \frac{\partial q}{\partial I} \) and \( \frac{\partial^2 q}{\partial I^2} \) and rearranging, we have

\[ W_{II} = \frac{\delta / K}{-\Delta} (\tau - c) \frac{\partial^2 q}{\partial I^2} \mid_{<0} - x \frac{\partial^2 D}{\partial I^2} \frac{\partial q}{\partial I} \left( \frac{\partial q}{\partial I} \right)^2 + b \frac{x}{\Delta^2} \left( \frac{\partial^2 f}{\partial I^2} (a - c - x f) + x \left( \frac{\partial f}{\partial I} \right)^2 \right). \]

Thus, to determine the sign of \( W_{II} \), we only need to determine the sign of \((- \frac{\partial^2 f}{\partial I^2} (a - c - x f) + x \left( \frac{\partial f}{\partial I} \right)^2 \). Since \(- \frac{\partial^2 f}{\partial I^2} (a - c - x f) < 0 \) and \(x \left( \frac{\partial f}{\partial I} \right)^2 > 0\), we need to impose a specific functional form of \( f \) in order to sign this expression. For comparison, we use the specific functional form of \( f \) in Eq. (3.21). We have \( \frac{\partial^2 f}{\partial I^2} = \frac{\theta^2}{K^2} f \) and \( \left( \frac{\partial f}{\partial I} \right)^2 = \frac{\theta^2}{K^2} f^2 \). Thus,

\[ W_{II} = \frac{\delta / K}{-\Delta} (\tau - c) \frac{\partial^2 q}{\partial I^2} \mid_{<0} - x \frac{\partial^2 D}{\partial I^2} \frac{\partial q}{\partial I} \left( \frac{\partial q}{\partial I} \right)^2 - b \frac{1}{\Delta^2} \frac{\theta^2}{K^2} x f (a - c - 2 x f). \]

Thus, we need to show \((a - c - 2 x f) > 0\).

Since \( \frac{\partial q}{\partial k} + x \frac{\partial f}{\partial K} < 0 \), with the specific functional form, we have \( q = \frac{a - \tau - x f}{b + \delta / k} > 0 \), which gives

\[ a > \tau + x f + x f \frac{1}{K} + x f \frac{b}{\delta I}. \]

Thus,
\[(a - c - 2xf) > (\tau - c) + xf(l\theta/K - 1) + xf \frac{b}{\theta}l\theta > 0,\]

where the last inequality follows because of the assumption \(\frac{\partial^2 f}{\partial \theta \partial l} < 0\). With the specific disaster function in Eq. (3.21), \(\frac{\partial^2 f}{\partial \theta \partial l} = \frac{\partial f}{k^3} (K - \theta l) < 0\), which gives \(l\theta/K - 1 > 0\).

Therefore, using the specific disaster function, we can show \(W_{II} < 0\). Without the specific disaster function, the sufficient condition for \(W_{II} < 0\) is \(-\frac{\partial^2 f}{\partial l^2} (a - c - xf) + x \left(\frac{\partial f}{\partial l}\right)^2 < 0\).

### The effects of capacity and protection on optimal pricing in partial equilibrium.

In the following analysis, we use the second-order conditions \(W_{tt} < 0\), \(W_{kk} < 0\), and \(W_{ll} < 0\), established above.

\[
\frac{\partial \tau^w}{\partial K} = -\frac{W_{tk}}{W_{tt}} = -\frac{1}{W_{tt}} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial \tau \partial K} + \frac{\partial q}{\partial K} \left( 1 - \frac{\partial \rho}{\partial \theta} \frac{\partial q}{\partial \tau} \right) \right)
\]

\[
= -\frac{1}{W_{tt}} \left( \frac{1}{\Delta^2} \right) \left( x \frac{\partial f}{\partial K} + \frac{\partial g}{\partial K} \frac{\partial \rho}{\partial q} \right) < 0,
\]

where the last equality follows by plugging in the expressions for \(\frac{\partial^2 q}{\partial \tau \partial K}\) and \(\frac{\partial q}{\partial K}\).

\[
\frac{\partial \tau^w}{\partial I} = -\frac{W_{tl}}{W_{tt}} = -\frac{1}{W_{tt}} \frac{\partial q}{\partial I} \left( 1 - \frac{1}{\Delta \frac{\partial q}{\partial \theta}} \right) > 0,
\]

where the inequality follows because \(\frac{1}{\Delta \frac{\partial q}{\partial \theta}} < 1\).

### The effects of parameters on optimal pricing in partial equilibrium.

For the following parameters, the derivations are straightforward:

\[
\frac{\partial \tau^w}{\partial x} = -\frac{1}{W_{tt}} \frac{\partial q}{\partial x} \left( 1 - \frac{1}{\Delta \frac{\partial q}{\partial \theta}} \right) < 0, \quad \frac{\partial \tau^w}{\partial m} = -\frac{1}{W_{tt}} \frac{\partial q}{\partial m} \left( 1 - \frac{1}{\Delta \frac{\partial q}{\partial \theta}} \right) < 0,
\]

\[
\frac{\partial \tau^w}{\partial \theta} = -\frac{1}{W_{tt}} \frac{\partial q}{\partial \theta} \left( 1 - \frac{1}{\Delta \frac{\partial q}{\partial \theta}} \right) > 0, \quad \frac{\partial \tau^w}{\partial a} = -\frac{1}{W_{tt}} \frac{\partial q}{\partial a} \left( 1 - \frac{1}{\Delta \frac{\partial q}{\partial \theta}} \right) < 0,
\]

\[
\frac{\partial \tau^w}{\partial M} = 0, \quad \frac{\partial \tau^w}{\partial \eta} = 0, \quad \frac{\partial \tau^w}{\partial c} = -\frac{1}{W_{tt}} \left( \frac{\partial q}{\partial \tau} \right) > 0, \quad \frac{\partial \tau^w}{\partial c_k} = 0, \quad \frac{\partial \tau^w}{\partial c_i} = 0.
\]
For parameter $b$,
\[
\frac{\partial \tau^w}{\partial b} = -\frac{W_{tb}}{W_{\tau \tau}} < 0,
\]
where the inequality follows because
\[
W_{tb}|_{\tau^w} = \frac{\partial q}{\partial b} + \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial^2 q}{\partial \tau \partial b} - \frac{\partial q}{\partial q} \frac{\partial (q \frac{\partial \rho}{\partial q})}{\partial \tau} = \frac{\partial q}{\partial \tau} \left( 1 - \frac{1}{\Lambda} \frac{\partial \rho}{\partial q} \right) < 0,
\]
where the second equality follows because at $\tau^w$, \( \frac{\partial q}{\partial b} + \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial^2 q}{\partial \tau \partial b} = \frac{\partial q}{\partial b} + \frac{q}{\partial q} \frac{\partial^2 q}{\partial \tau \partial b} = 0 \).

For parameter $\delta$,
\[
\frac{\partial \tau^w}{\partial \delta} = -\frac{W_{t\delta}}{W_{\tau \tau}} > 0,
\]
where the inequality follows because
\[
W_{t\delta}|_{\tau^w} = \frac{\partial q}{\partial \delta} + \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial^2 q}{\partial \tau \partial \delta} = \frac{\partial q}{\partial \tau} \frac{\partial q}{\partial \delta} > 0,
\]
where the second equality follows because at $\tau^w$, \( \frac{\partial q}{\partial \delta} + \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial^2 q}{\partial \tau \partial \delta} = 0 \).

The effects of fee and protection on optimal capacity in partial equilibrium.

\[
\frac{\partial K^w}{\partial \tau} = -\frac{W_{k\tau}}{W_{kk}} = -\frac{1}{W_{kk}} \left( \frac{\partial q}{\partial K} \left( 1 - \frac{1}{\Delta} \frac{\partial \rho}{\partial q} \right) + \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial^2 q}{\partial K \partial \tau} \right).
\]

Since the sign is ambiguous, we evaluate $\frac{\partial K^w}{\partial \tau}$ at the optimal $\tau^w$.

\[
\left. \frac{\partial K^w}{\partial \tau} \right|_{\tau^w} = -\frac{1}{W_{kk}} \left( \frac{\partial q}{\partial K} \left( 1 - \frac{1}{\Delta} \frac{\partial \rho}{\partial q} \right) + \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial^2 q}{\partial K \partial \tau} \right) = -\frac{1}{W_{kk}} \left( \frac{\partial q}{\partial K} \left( 1 - \frac{1}{\Delta} \frac{\partial \rho}{\partial q} \right) \right) < 0,
\]
where the last equality follows because $\frac{\partial q}{\partial K} + \frac{\partial^2 q}{\partial q \partial K} = 0$, and the inequality follows directly.

\[
\frac{\partial K^w}{\partial l} = -\frac{W_{kl}}{W_{kk}} = -\frac{1}{W_{kk}} \left( \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial^2 q}{\partial K \partial l} - \frac{\partial q}{\partial K} \frac{\partial q}{\partial l} \frac{\partial q}{\partial K} - x \frac{\partial^2 D}{\partial K \partial l} \right) > 0,
\]
where the inequality follows because $\frac{\partial^2 F}{\partial K \partial i} < 0$ gives $\frac{\partial^2 q}{\partial K \partial i} > 0$ and $\frac{\partial^2 F}{\partial K \partial i} < 0$ gives $\frac{\partial^2 D}{\partial K \partial i} < 0$.

The effects of parameters on optimal capacity in partial equilibrium.

$$\frac{\partial K^w}{\partial x} = -\frac{1}{W_{kk}} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial K \partial x} - \frac{\partial q}{\partial x} \frac{\partial \rho}{\partial q} \frac{\partial q}{\partial K} \right) < 0,$$

where the inequality follows because $\frac{\partial^2 q}{\partial K \partial x} = \frac{1}{\Delta} \left( \frac{\partial^2 q}{\partial K \partial q} \frac{\partial q}{\partial x} + \frac{\partial q}{\partial x} \frac{\partial q}{\partial K} \right) < 0$, and $\frac{\partial q}{\partial x} \frac{\partial \rho}{\partial q} \frac{\partial q}{\partial K} > 0$ and $\frac{\partial D}{\partial K} > 0$.

$$\frac{\partial K^w}{\partial m} = -\frac{1}{W_{kk}} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial K \partial m} - \frac{\partial q}{\partial m} \frac{\partial q}{\partial K} \right) < 0,$$

$$\frac{\partial K^w}{\partial \theta} = -\frac{1}{W_{kk}} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial K \partial \theta} - \frac{\partial q}{\partial \theta} \frac{\partial q}{\partial K} \right) > 0,$$

$$\frac{\partial K^w}{\partial a} = -\frac{1}{W_{kk}} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial K \partial a} - \frac{\partial q}{\partial a} \frac{\partial q}{\partial K} \right) > 0,$$

where the inequalities follow because we show in Appendix B.1 that $\frac{\partial^2 q}{\partial K \partial x} < 0$, $\frac{\partial^2 D}{\partial K \partial M} > 0$, $\frac{\partial^2 q}{\partial K \partial \eta} > 0$, and $\frac{\partial^2 q}{\partial K \partial a} > 0$.

$$\frac{\partial^2 q}{\partial K \partial \theta} > 0, \frac{\partial^2 D}{\partial K \partial \eta} < 0, \text{ and } \frac{\partial^2 q}{\partial K \partial a} > 0.$$

$$\frac{\partial K^w}{\partial b} = -\frac{1}{W_{kk}} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial K \partial b} - \frac{\partial q}{\partial b} \frac{\partial \rho}{\partial q} \frac{\partial q}{\partial K} \right)$$

$$= -\frac{1}{W_{kk}} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial K \partial b} - \frac{\partial q}{\partial b} \frac{\partial \rho}{\partial q} \left( 1 - \frac{1}{\Delta} \frac{\partial \rho}{\partial q} \right) \right).$$

Since $\frac{\partial^2 q}{\partial K \partial b} < 0$ but $1 - \frac{1}{\Delta} \frac{\partial \rho}{\partial q} > 0$, we cannot sign $\frac{\partial K^w}{\partial b}$ directly. We thus evaluate $\frac{\partial K^w}{\partial b}$ at the optimal $\tau^w$:

$$\frac{\partial K^w}{\partial b} \bigg|_{\tau^w} = -\frac{1}{W_{kk}} (-q) \left( \Delta \frac{\partial^2 q}{\partial K \partial b} + \frac{\partial q}{\partial K} \left( 1 - \frac{1}{\Delta} \frac{\partial \rho}{\partial q} \right) \right) = -\frac{1}{W_{kk}} (-q) \frac{1}{\Delta} \left( \frac{\partial^2 q}{\partial K \partial q} + \frac{\partial q}{\partial K} \frac{\partial \rho}{\partial q} \right) < 0.$$

$$\frac{\partial K^w}{\partial \delta} = -\frac{1}{W_{kk}} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial K \partial \delta} - \frac{\partial q}{\partial \delta} \frac{\partial \rho}{\partial q} \frac{\partial q}{\partial K} \right).$$

Since $\frac{\partial^2 q}{\partial K \partial \delta}$ cannot be signed, $\frac{\partial K^w}{\partial \delta}$ cannot be signed either.
\[
\frac{\partial K^w}{\partial c} = -\frac{1}{W_{kk}} \left( \frac{\partial q}{\partial q} \right) < 0, \quad \frac{\partial K^w}{\partial c_k} = -\frac{1}{W_{kk}} (-1), \quad \frac{\partial K^w}{\partial c_I} = 0.
\]

The effects of fee and capacity on optimal protection in partial equilibrium.

It is straightforward to show that
\[
\frac{\partial I^w}{\partial \tau} = -\frac{W_{I\tau}}{W_{ll}} = -\frac{1}{W_{ll}} \frac{\partial q}{\partial I} \left( 1 - \frac{1}{\Delta \frac{\partial q}{\partial I}} \right) > 0.
\]

Since we have shown \( W_{kl} > 0 \),
\[
\frac{\partial I^w}{\partial K} = -\frac{W_{Ih}}{W_{ll}} > 0.
\]

The effects of parameters on optimal protection in partial equilibrium.

\[
\frac{\partial I^w}{\partial x} = -\frac{W_{Ix}}{W_{ll}} = -\frac{1}{W_{ll}} \left[ \left( \tau - c - q \frac{\partial \rho}{\partial q} \right) \frac{\partial^2 q}{\partial I \partial x} - \frac{\partial q}{\partial x} \frac{\partial q}{\partial I} - \frac{\partial D}{\partial I} \right] \]
\[
= -\frac{1}{W_{ll}} \left[ \frac{1}{\Delta \frac{\partial I}{\partial I}} \left( \tau - c - \frac{\partial \rho}{\partial q} q \left( 1 + \frac{\chi \partial q}{\partial x} \right) \right) - \frac{\partial D}{\partial I} \right] \]
\[
= -\frac{1}{W_{ll}} \left[ -\frac{1}{\Delta^2 \frac{\partial I}{\partial I}} \left( \frac{\delta}{K} (\tau - c) + b(a - c - 2xf) \right) - \frac{\partial D}{\partial I} \right] > 0,
\]

where the inequality follows because \((a - c - 2xf) > 0\) holds using specific disaster functions.

With general disaster functions, a sufficient condition for \( \frac{\partial I^w}{\partial x} > 0 \) is \( a - c - 2xf > 0 \) (i.e., demand is sufficiently large).

Since we did not impose any relationship between \( \frac{\partial^2 f}{\partial I \partial m} \frac{\partial f}{\partial m} \) and \( \frac{\partial f}{\partial I} \), we need to use specific disaster functions to evaluate \( \frac{\partial I^w}{\partial m} \).
\[
\frac{\partial I^w}{\partial m} = -\frac{W_{tx}}{W_{tl}} = -\frac{1}{W_{tl}} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial l \partial m} - \frac{\partial q}{\partial m} \frac{\partial \rho}{\partial q} \frac{\partial q}{\partial l} \right)
\]
\[
= -\frac{1}{W_{tl}} \frac{1}{\Delta^x} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 f}{\partial l \partial m} - \frac{\partial f}{\partial m} \frac{\partial \rho}{\partial q} \frac{\partial f}{\partial l} \frac{1}{\Delta^x} \right)
\]
\[
= -\frac{1}{W_{tl}} \left( \frac{1}{\Delta^2 m} \frac{\partial f}{\partial l} \right) \left( \delta (\tau - c) + b(a - c - 2xf) \right) > 0.
\]

Due to the potential non-monotone effect of protection effectiveness \( \theta \) and \( \eta \), we cannot sign \( \frac{\partial I^w}{\partial \theta} \) and \( \frac{\partial I^w}{\partial \eta} \).

\[
\frac{\partial I^w}{\partial M} = -\frac{1}{W_{tl}} \left( -x \frac{\partial^2 D}{\partial l \partial M} \right) > 0, \quad \frac{\partial I^w}{\partial a} = -\frac{1}{W_{tl}} \left( -\frac{\partial q}{\partial a} \frac{\partial \rho}{\partial q} \frac{\partial q}{\partial l} \right) > 0.
\]

\[
\frac{\partial I^w}{\partial b} = -\frac{1}{W_{tl}} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial l \partial b} - \frac{\partial q}{\partial l} \left( \frac{\partial q}{\partial b} \frac{\partial \rho}{\partial q} + q \frac{\partial^2 \rho}{\partial q \partial b} \right) \right)
\]
\[
= -\frac{1}{W_{tl}} \frac{1}{\Delta^x} \frac{\partial f}{\partial l} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{1}{\Delta} + q \left( 1 - \frac{1}{\Delta} \frac{\partial \rho}{\partial q} \right) \right) = -\frac{1}{W_{tl}} \frac{1}{\Delta^2} \frac{\partial f}{\partial l} q \frac{\partial \rho}{\partial q} < 0,
\]
where the last equality follows by evaluating \( \frac{\partial I^w}{\partial b} \) at optimal pricing \( \tau^w \).

\[
\frac{\partial I^w}{\partial \delta} = -\frac{1}{W_{tl}} \left( (\tau - c - q \frac{\partial \rho}{\partial q}) \frac{\partial^2 q}{\partial l \partial \delta} - \frac{\partial q}{\partial l} \frac{\partial q}{\partial \delta} \frac{\partial \rho}{\partial q} \right) < 0,
\]
where the inequality follows because \( \frac{\partial^2 q}{\partial l \partial \delta} < 0 \) and \( \frac{\partial q}{\partial l} \frac{\partial \rho}{\partial q} \frac{\partial \delta}{\partial q} > 0 \).

\[
\frac{\partial I^w}{\partial c} = \frac{1}{W_{tl}} \frac{\partial q}{\partial l} < 0, \quad \frac{\partial I^w}{\partial c_k} = 0, \quad \frac{\partial I^w}{\partial c_l} = \frac{1}{W_{tl}} < 0.
\]

The effects of parameters on output in partial equilibrium.

Only \( \tau \) is the decision variable.

\[
\frac{dq^w}{dx} = \frac{\partial q}{\partial x} + \frac{\partial q}{\partial \tau} \frac{\partial \tau^w}{\partial x} = \frac{\partial q}{\partial x} + \frac{\partial q}{\partial \tau} \left( -\frac{W_{tx}}{W_{tt}} \right) = \frac{\partial q}{\partial x} + \frac{\partial q}{\partial \tau} \left( -\frac{\partial q}{\partial x} \left( 1 - \frac{1}{\Delta} \frac{\partial \rho}{\partial q} \right) \right) = \frac{\partial q}{\partial x} \left( 2 \frac{\partial \rho}{\partial q} \right) < 0.
\]

Similarly, we can show that
\[
\frac{dq^w}{dm} = \left(\frac{\partial q}{\partial m}\right) < 0, \quad \frac{dq^w}{d\theta} = \left(\frac{\partial q}{\partial \theta}\right) > 0, \quad \frac{dq^w}{da} = \left(\frac{\partial q}{\partial a}\right) > 0,
\]
\[
\frac{dq^w}{db} = \left(\frac{\partial q}{\partial b}\right) < 0, \quad \frac{2\partial q}{d\delta} = \left(\frac{\partial q}{\partial \delta}\right) < 0.
\]

Since \( M \) and \( \eta \) do not affect demand and \( \tau^w \), \( \frac{dq^w}{dM} = 0 \) and \( \frac{dq^w}{d\eta} = 0 \).
\[
\frac{dq^w}{dc} = \frac{\partial q}{\partial c} < 0, \quad \frac{dq^w}{dc_k} = 0, \quad \frac{dq^w}{dc_I} = 0.
\]

Only \( K \) is the decision variable.

Since parameters \( x, m, \theta, a, \) and \( b \) have the same qualitative (i.e., sign) effects on demand \( q \) and optimal capacity \( K^w \), their effects on output are the same as their effect on \( q \) or \( K^w \). Since parameters \( M, \eta, c, \) and \( c_k \) do not affect demand, their effects on output are the same as their effect on \( K^w \). Intuitively, the effect of \( c_I \) on output is 0, and the effect of \( \delta \) on output is ambiguous.

Only \( I \) is the decision variable.
\[
\frac{dq^w}{dx} = \frac{\partial q}{\partial x} + \frac{\partial q}{\partial l} \frac{\partial l^w}{\partial x}, \quad \frac{dq^w}{dm} = \frac{\partial q}{\partial m} + \frac{\partial q}{\partial l} \frac{\partial l^w}{\partial m}.
\]

Since \( x \) and \( m \) reduces demand, but increase optimal protection (which in turn can increase demand), the net effect of \( x \) and \( m \) on output are ambiguous. Due to the ambiguity of the effects of protection effectiveness \( \theta \) and \( \eta \) on optimal protection \( l^w \), their effects on the output are also ambiguous. Since parameters \( a, b, \) and \( \delta \) have the same qualitative effects on demand \( q \) and optimal protection \( l^w \), their effects on output are qualitatively the same as their effects on \( q \) or \( l^w \). Since parameters \( M, c, \) and \( c_I \) do not affect demand, their effects on output are qualitatively the same as their effects on \( l^w \). Intuitively, the effect of \( c_k \) on output is 0.
Compare the effects of parameters on public and private port fees in partial equilibrium.

\[
\frac{\partial \tau^*}{\partial K} - \frac{\partial \tau^w}{\partial K} = \left( -\frac{\Pi_{tk}}{\Pi_{tt}} \right) - \left( -\frac{W_{tk}}{W_{tt}} \right) = \left( -\frac{1}{\Delta} \frac{\partial f}{\partial K} \right) - \left( -\frac{1}{\Delta^2} \left( x \frac{\partial f}{\partial K} + q \frac{\partial g}{\partial q K} \frac{\partial g}{\partial q} \right) \right) \frac{\partial q}{\partial \tau} \left( 2 - \frac{1}{\Delta} \frac{\partial g}{\partial q} \right)
\]

\[
= \frac{1}{2} \left( 2 - \frac{1}{\Delta} \frac{\partial g}{\partial q} \right) \left( x \frac{\partial f}{\partial K} + 2 \frac{\partial g}{\partial K} \right) > 0,
\]

where the inequality follows because \( x \frac{\partial f}{\partial K} + 2 \frac{\partial g}{\partial K} < 0. \)

\[
\frac{\partial \tau^*}{\partial I} - \frac{\partial \tau^w}{\partial I} = \left( -\frac{\Pi_{ti}}{\Pi_{tt}} \right) - \left( -\frac{W_{ti}}{W_{tt}} \right) = \left( -\frac{\partial q}{\partial I} \frac{\partial f}{\partial I} \left( 1 - \frac{1}{\Delta} \frac{\partial g}{\partial q} \right) \right) - \left( -\frac{\partial q}{\partial I} \frac{\partial g}{\partial I} \left( 2 - \frac{1}{\Delta} \frac{\partial g}{\partial q} \right) \right) \frac{\partial q}{\partial I} \left( 2 - \frac{1}{\Delta} \frac{\partial g}{\partial q} \right) > 0.
\]

Similarly, we can show that

\[
\frac{\partial \tau^*}{\partial x} - \frac{\partial \tau^w}{\partial x} = \frac{\partial q}{\partial x} \left( -\frac{\partial g}{\partial q} \right) \left( 2 - \frac{1}{\Delta} \frac{\partial g}{\partial q} \right) < 0,
\]

\[
\frac{\partial \tau^*}{\partial m} - \frac{\partial \tau^w}{\partial m} = \frac{\partial q}{\partial m} \left( -\frac{\partial g}{\partial q} \right) \left( 2 - \frac{1}{\Delta} \frac{\partial g}{\partial q} \right) < 0,
\]

\[
\frac{\partial \tau^*}{\partial \theta} - \frac{\partial \tau^w}{\partial \theta} = \frac{\partial q}{\partial \theta} \left( -\frac{\partial g}{\partial q} \right) \left( 2 - \frac{1}{\Delta} \frac{\partial g}{\partial q} \right) > 0,
\]

\[
\frac{\partial \tau^*}{\partial a} - \frac{\partial \tau^w}{\partial a} = \frac{\partial q}{\partial a} \left( -\frac{\partial g}{\partial q} \right) \left( 2 - \frac{1}{\Delta} \frac{\partial g}{\partial q} \right) > 0.
\]

For parameter \( b \) and \( \delta \), since \( \frac{\partial \tau^*}{\partial b} = 0 \) and \( \frac{\partial \tau^*}{\partial \delta} = 0, \)

\[
\frac{\partial \tau^*}{\partial b} - \frac{\partial \tau^w}{\partial b} = -\frac{\partial \tau^w}{\partial b} > 0,
\]

\[
\frac{\partial \tau^*}{\partial \delta} - \frac{\partial \tau^w}{\partial \delta} = -\frac{\partial \tau^w}{\partial \delta} < 0.
\]

Parameters \( M, \eta, c_k, \) and \( c_l \) do not affect fees for public or private ports because these parameters only affect the port’s cost, not demand.

Last,

\[
\frac{\partial \tau^*}{\partial c} - \frac{\partial \tau^w}{\partial c} = \frac{-\frac{\partial g}{\partial q} \frac{1}{\Delta}}{2 \left( 2 - \frac{1}{\Delta} \frac{\partial g}{\partial q} \right)} < 0.
\]
Note that the partial-equilibrium comparison of optimal protection for public and private port is conducted using the specific disaster functions, whereas the partial-equilibrium comparison of optimal capacity is analytically intractable even using the specific disaster functions.

Proof of Proposition 3.

Compare full equilibrium $K^*(\tau^*, I^*)$ and $K^w(\tau^w, I^w)$.

We know $K^*(\tau, I) < K^w(\tau, I)$. Thus, $K^*(\tau^w, I) < K^w(\tau^w, I)$. Next, we compare $K^*(\tau^*, I)$ and $K^*(\tau^w, I)$. Since $\tau^*(K^*, I) > \tau^w(K^*, I)$, we have $K^*(\tau^*, I) < K^*(\tau^w, I)$. Thus, $K^*(\tau^*, I) < K^w(\tau^w, I)$, which gives $K^*(\tau^*, I^w) < K^w(\tau^w, I^w)$. Next, we compare $K^*(\tau^*, I^w)$ and $K^*(\tau^*, I^*)$. Since $I^w(\tau^*, K^*) > I^*(\tau^*, K^*)$, we have $K^*(\tau^*, I^w) > K^*(\tau^*, I^*)$. Thus, $K^*(\tau^*, I^*) < K^w(\tau^w, I^w)$.

Fix $\tau$, compare equilibrium $I^*(\tau, K^*)$ and $I^w(\tau, K^w)$.

We know $I^*(\tau, K) < I^w(\tau, K)$. Thus, $I^*(\tau, K^w) < I^w(\tau, K^w)$. Next, we compare $I^*(\tau, K^w)$ and $I^*(\tau, K^*)$. Since $K^w(\tau, I^t) > K^*(\tau, I^*)$, we have $I^*(\tau, K^w) > I^*(\tau, K^*)$. Thus, $I^*(\tau, K^*) < I^w(\tau, K^w)$. We thus have $I^*(\tau^w, K^*) < I^w(\tau^w, K^w)$. Next, we compare $I^*(\tau^w, K^*)$ and $I^*(\tau^*, K^*)$. Since $\tau^w(K^*, I^*) < \tau^*(K^*, I^*)$, we have $I^*(\tau^w, K^*) < I^*(\tau^*, K^*)$. Now we have shown that $I^w(\tau^w, K^w) > I^*(\tau^w, K^*)$ and $I^*(\tau^w, K^*) < I^*(\tau^*, K^*)$. Thus, we are unable to compare $I^w(\tau^w, K^w)$ and $I^*(\tau^*, K^*)$. But we show that $I^*(\tau, K^*) < I^w(\tau, K^w)$. Thus, if $\tau$ is fixed, we are able to compare the protection in full equilibrium in the sense that both capacity and protection are at equilibrium.

Fix $I$, compare equilibrium $\tau^*(K^*, I)$ and $\tau^w(K^w, I)$.

We know that $\tau^*(K, I) > \tau^w(K, I)$. Thus, $\tau^*(K^*, I) > \tau^w(K^*, I)$. Next, we compare $\tau^*(K^w, I)$ and $\tau^*(K^*, I)$. Since $K^w(\tau^*, I) > K^*(\tau^*, I)$, we have $\tau^*(K^w, I) < \tau^*(K^*, I)$. Thus, $\tau^*(K^*, I) > \tau^w(K^w, I)$. Thus, we have $\tau^*(K^*, I^w) > \tau^w(K^w, I^w)$. Next, we compare $\tau^*(K^*, I^w)$ and $\tau^*(K^*, I^*)$. Since $I^w(\tau^*, K^*) > I^*(\tau^*, K^*)$, we have $\tau^*(K^*, I^w) > \tau^*(K^*, I^*)$. Now we have shown that $\tau^*(K^*, I^w) > \tau^w(K^w, I^w)$ and $\tau^*(K^*, I^w) > \tau^*(K^*, I^*)$. Thus, we are unable to compare $\tau^w(K^w, I^w)$ and $\tau^*(K^*, I^*)$. But we show that $\tau^*(K^*, I) > \tau^w(K^w, I)$. Thus, if $I$ is fixed, we are
able to compare the port charge in full equilibrium in the sense that both port charge and capacity are at equilibrium.

B.3 Two-period model and private port

Proof of Proposition 4.

First, prove that the first column in Table 3.6 (Cases 1, 5, 9, 13) can be conditionally ruled out.

Proof by contradiction:

Suppose $(K_1, K_{2L}, K_{2H}, l_1, l_{2L}, l_{2H}, \tau_1, \tau_{2L}, \tau_{2H})$ is the optimal solution. We know that $l_{2L} > l_1$ and $l_{2H} > l_1$. We don’t place any binding restrictions on capacity ($K_{2L} \geq K_1$ and $K_{2H} \geq K_1$).

Let Sol1 = $(K_1, K_{2L}, K_{2H}, l_1, l_{2L}, l_{2H}, \tau_1, \tau_{2L}, \tau_{2H})$. Now we propose another solution Sol2 = $(K_1, K_{2L}, K_{2H}, l'_1 = \min\{l_{2L}, l_{2H}\}, l_{2L}, l_{2H}, \tau_1, \tau_{2L}, \tau_{2H})$. Everything in Sol1 is the same as in Sol2, except that we increase $l'_1$ in Sol2 to $\min\{l_{2L}, l_{2H}\}$. Thus, $l'_1 > l_1$. We need to show that Sol2 dominates Sol1.

Let

$$
\pi_{2H}(\text{Sol2}) - \pi_{2H}(\text{Sol1}) = c_{l2}(l'_1 - l_1) > 0.
$$

$$
\pi_{2L}(\text{Sol2}) - \pi_{2L}(\text{Sol1}) = c_{l2}(l'_1 - l_1) > 0.
$$

Thus, in both period 2H and 2L, we save building cost $c_{l2}(l'_1 - l_1)$ under Sol2. Next, let’s look at period 1.

Let

$$
\pi_1(\text{Sol2}) - \pi_1(\text{Sol1}) = (\tau_1 - c_1)\left(q_1(l'_1) - q_1(l_1)\right) - x_1\left(D_1(l'_1) - D_1(l_1)\right) - c_{l1}(l'_1 - l_1).
$$

Now, let’s examine the port’s objective function in both periods.

$$
\Pi(\text{Sol2}) - \Pi(\text{Sol1})
$$

$$
= (\tau_1 - c_1)(q_1(l'_1) - q_1(l_1)) - x_1\left(D_1(l'_1) - D_1(l_1)\right) + (\beta c_{l2} - c_{l1})(l'_1 - l_1).
$$

Since $q_1(l'_1) - q_1(l_1) > 0$ because $\frac{\partial q}{\partial l} > 0$, and $D_1(l'_1) - D_1(l_1) < 0$ because $\frac{\partial D}{\partial l} < 0$, a sufficient condition for $\Pi(\text{Sol2}) - \Pi(\text{Sol1}) > 0$ is $\beta c_{l2} - c_{l1} \geq 0$.

Thus, if the present-value cost of protection is no lower in period 2 than period 1, it is not optimal for the port to withhold protection in period 1.
Second, prove that Cases 11, 12, 15 in Table 3.6 can be ruled out.

In this proof, we treat port fees as optimal functions of capacity and protection that satisfy the FOCs in Eq. (3.42).

We rule out Case 11 in Table 3.6 first. Thus, we need to show that $K_{2H} > K_{2L} = K_1$ and $I_{2L} > I_{2H} = I_1$ is not optimal.

Since $(K_{2H}, I_{2H})$ is the optimal solution, we have

$$
\pi_{2H}(K_{2H}, I_{2H}) \geq \pi_{2H}(K_{2L}, I_{2L}),
$$

because otherwise the port could have switched to $(K_{2L}, I_{2L})$ in period 2H without changing decisions in period 1 and 2L. To rule out Case 11, we need to show:

$$
\pi_{2L}(K_{2H}, I_{2H}) > \pi_{2L}(K_{2L}, I_{2L}),
$$

or to show:

$$
\pi_{2L}(K_{2H}, I_{2H}) - \pi_{2L}(K_{2L}, I_{2L}) > \pi_{2H}(K_{2H}, I_{2H}) - \pi_{2H}(K_{2L}, I_{2L}) \geq 0.
$$

Since

$$
\pi_{2s}(K_{2H}, I_{2H}) - \pi_{2s}(K_{2L}, I_{2L}) = \int_{I_{2L}}^{I_{2H}} \frac{\partial \pi_{2s}(K_{2L}, I)}{\partial I} dI + \int_{K_{2L}}^{K_{2H}} \frac{\partial \pi_{2s}(K, I_{2H})}{\partial K} dK,
$$

we need to show:

$$
\int_{I_{2L}}^{I_{2H}} \frac{\partial \pi_{2L}(K_{2L}, I)}{\partial I} dI + \int_{K_{2L}}^{K_{2H}} \frac{\partial \pi_{2L}(K_{2H}, I_{2H})}{\partial K} dK > \int_{I_{2L}}^{I_{2H}} \frac{\partial \pi_{2H}(K_{2L}, I)}{\partial I} dI + \int_{K_{2L}}^{K_{2H}} \frac{\partial \pi_{2H}(K_{2H}, I_{2H})}{\partial K} dK,
$$

or equivalently:

$$
\int_{K_{2L}}^{K_{2H}} \left( \frac{\partial \pi_{2L}(K, I_{2H})}{\partial K} - \frac{\partial \pi_{2H}(K, I_{2H})}{\partial K} \right) dK + \int_{I_{2L}}^{I_{2H}} \left( \frac{\partial \pi_{2L}(K_{2L}, I)}{\partial I} - \frac{\partial \pi_{2H}(K_{2L}, I)}{\partial I} \right) dI > 0.
$$

We know that $K_{2H} > K_{2L}$ and $I_{2L} > I_{2H}$. Thus, if

$$
\frac{\partial \pi_{2L}(K, I_{2H})}{\partial K} > \frac{\partial \pi_{2H}(K, I_{2H})}{\partial K},
$$

and

$$
\frac{\partial \pi_{2L}(K_{2L}, I)}{\partial I} < \frac{\partial \pi_{2H}(K_{2L}, I)}{\partial I},
$$

we have
then Case 11 can be ruled out.

Thus, we need to show that fixing \( I_{2s} \) the function \( \frac{\partial \pi_{2s}(K_{2s}, \tau_{2s}(K_{2s}))}{\partial K_{2s}} \) decreases in disaster probability \( x_{2s} \), and that fixing \( K_{2s} \) the function \( \frac{\partial \pi_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s}} \) increases in disaster probability \( x_{2s} \). Since

\[
\frac{\partial \pi_{2s}(K_{2s}, \tau_{2s}(K_{2s}))}{\partial K_{2s}} = (\tau_{2s}(K_{2s}) - c_2) \frac{\partial q_{2s}}{\partial K_{2s}} - x_{2s} \frac{\partial D_{2s}}{\partial K_{2s}} - c_{k2},
\]

we have

\[
\frac{\partial^2 \pi_{2s}(K_{2s}, \tau_{2s}(K_{2s}))}{\partial K_{2s} \partial x_{2s}} = (\tau_{2s}(K_{2s}) - c_2) \left( \frac{\partial^2 q_{2s}}{\partial K_{2s}^2 \partial x_{2s}} + \frac{\partial^2 q_{2s}}{\partial K_{2s} \partial \tau_{2s} \partial x_{2s}} \right) + \frac{\partial \tau_{2s}}{\partial x_{2s}} \frac{\partial q_{2s}}{\partial K_{2s}} - \frac{\partial D_{2s}}{\partial K_{2s}}.
\]

Rearranging the terms by substituting in \( \frac{\partial^2 q_{2s}}{\partial K_{2s} \partial x_{2s}} = \frac{1}{\Delta_{2s}} \left( \frac{\partial^2 q_{2s}}{\partial K_{2s} \partial q_{2s}} \frac{\partial q_{2s}}{\partial x_{2s}} + \frac{\partial f_{2s}}{\partial K_{2s}} \right), \frac{\partial^2 q_{2s}}{\partial \tau_{2s} \partial x_{2s}} = -\frac{\partial q_{2s}}{\partial \tau_{2s}}, \) and \( (\tau_{2s}(K_{2s}) - c_2) = -\frac{q_{2s}}{\partial \tau_{2s}} \), we can obtain

\[
\frac{\partial^2 \pi_{2s}(K, \tau_{2s}(K))}{\partial K \partial x_{2s}} = -\frac{1}{2 \Delta_{2s}} \left( 2 \frac{\partial q_{2s}}{\partial K_{2s}} x_{2s} - \frac{\partial f_{2s}}{\partial K_{2s}} \right) q_{2s}, \quad \frac{\partial f_{2s}}{\partial K_{2s}} > 0, \quad \frac{\partial D_{2s}}{\partial K_{2s}} < 0.
\]

Since

\[
\frac{\partial \pi_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s}} = (\tau_{2s}(I_{2s}) - c_2) \frac{\partial q_{2s}}{\partial I_{2s}} - x_{2s} \frac{\partial D_{2s}}{\partial I_{2s}} - c_{l2},
\]

we have

\[
\frac{\partial^2 \pi_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s} \partial x_{2s}} = (\tau_{2s}(I_{2s}) - c_2) \left( \frac{\partial^2 q_{2s}}{\partial I_{2s}^2 \partial x_{2s}} + \frac{\partial^2 q_{2s}}{\partial I_{2s} \partial \tau_{2s} \partial x_{2s}} \right) + \frac{\partial \tau_{2s}}{\partial x_{2s}} \frac{\partial q_{2s}}{\partial I_{2s}} - \frac{\partial D_{2s}}{\partial I_{2s}}.
\]

Rearranging terms, we obtain

\[
\frac{\partial^2 \pi_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s} \partial x_{2s}} = -\frac{\partial f_{2s}}{\partial I_{2s}} \left( \frac{1}{2} \frac{\partial q_{2s}}{\partial x_{2s}} x_{2s} + q_{2s} \right) - \frac{\partial D_{2s}}{\partial I_{2s}}.
\]
Thus, we need to determine the sign of \( \frac{1}{2} x_{2s} \frac{\partial q_{2s}}{\partial x_{2s}} + q_{2s} \).

From the demand system in period 2s: \( a_2 - b_2 q_{2s} = \tau_{2s} + \delta_2 \frac{q_{2s}}{K_{2s}} + x_{2sf_{2s}} \), the demand function is

\[
q_{2s} = \frac{1}{b_2 + \delta_2/K_{2s}}(a_2 - \tau_{2s} - x_{2sf_{2s}}).
\]

The optimal private charge in Eq. (3.12) gives the optimal \( \tau_{2s} \) as:

\[
\tau_{2s}^* = \frac{1}{2}(a_2 + c_2 - x_{2sf_{2s}}).
\]

Thus, demand evaluated at optimal \( \tau_{2s} \) is

\[
q_{2s}\big|_{\tau_{2s}^*} = \frac{1}{2b_2 + \delta_2/K_{2s}}(a_2 - c_2 - x_{2sf_{2s}}).
\]

Since \( \frac{\partial q_{2s}}{\partial x_{2s}} = \frac{-f_{2s}}{b_2 + \delta_2/K_{2s}} \),

\[
\frac{1}{2} x_{2s} \frac{\partial q_{2s}}{\partial x_{2s}} + q_{2s} = \frac{1}{2(b_2 + \delta_2/K_{2s})}(a_2 - c_2 - 2x_{2sf_{2s}}).
\]

Therefore, if \( a_2 - c_2 - 2x_{2sf_{2s}} \geq 0 \) (i.e., demand is sufficiently large), we have

\[
\frac{\partial^2 \pi_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s} \partial x_{2s}} > 0.
\]

Note that in Appendix B.2 (the SOC for \( l \)), we show that \( a_2 - c_2 - 2x_{2sf_{2s}} > 0 \) holds using specific disaster functions. But with general disaster functions, we require demand (i.e., \( a_2 \)) to be sufficiently large.

To rule out Case 12 in Table 3.6, we need to show that \( K_{2H} > K_{2L} = K_1 \) and \( I_{2H} = I_{2L} = I_1 \) is not optimal. Using the same argument as above, we only need to show \( \frac{\partial^2 \pi_{2s}(K, \tau_{2s}(K))}{\partial K \partial x_{2s}} \) < 0, which we have proved above. To rule out Case 15 in Table 3.6, we need to show \( K_{2H} = K_{2L} = K_1 \) and \( I_{2L} > I_{2H} = I_1 \) is not optimal. Using the same argument, we only need to show \( \frac{\partial^2 \pi_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s} \partial x_{2s}} > 0 \), which holds if \( a_2 - c_2 - 2x_{2sf_{2}} > 0 \).

*Third, prove that Cases 3, 4 in the Table 3.6 can be ruled out.*
To rule out Case 3, we need to show $K_{2H} > K_1, K_{2L} > K_1$, and $l_{2L} > I_{2H} = I_1$ is not optimal. We again treat port fees as optimal functions of capacity and protection in this proof. There are three cases for adding capacity in both periods 2H and 2L: (1). $K_{2H} > K_{2L} > K_1$; (2). $K_{2H} = K_{2L} > K_1$; (3). $K_{2L} > K_{2H} > K_1$.

The case $K_{2H} > K_{2L} > K_1$ and $l_{2L} > I_{2H} = I_1$, and the case $K_{2H} = K_{2L} > K_1$ and $l_{2L} > I_{2H} = I_1$ can be ruled out using the same integral approach as above. Thus, we only need to show $K_{2L} > K_{2H} > K_1$ and $l_{2L} > I_{2H} = I_1$ is not optimal.

Suppose there is an optimal solution $\text{Sol1} = (K_1, K_{2L}, K_{2H}, l_1, l_{2L}, l_{2H})$ that satisfies $K_{2L} > K_{2H} > K_1$ and $l_{2L} > I_{2H} = I_1$. We want to show another solution $\text{Sol2} = (K'_1 = K_{2H}, K_{2L}, K_{2H}, I_1, l_{2L}, l_{2H})$ dominates $\text{Sol1}$. In this alternative solution $\text{Sol2}$, we raise capacity in period 1 to $K_{2H}$. Thus, we want to show $\Pi(\text{Sol2}) > \Pi(\text{Sol1})$ or equivalently to show $\pi_1(\text{Sol2}) \geq \pi_1(\text{Sol1})$, $\pi_2L(\text{Sol2}) \geq \pi_2L(\text{Sol1})$, and $\pi_2H(\text{Sol2}) \geq \pi_2H(\text{Sol1})$, with at least one strict inequality. For clarification, the arguments of the function $\pi_i$ for period $i = 1, 2H, 2L$ are written $\pi_i(K_i, I_i; K_{-i}, I_{-i})$, where $-i$ indicates the period other than $i$.

Since $(K_{2H}, l_{2H}; K_1, I_1, K_{2L}, l_{2L})$ is the optimal solution in period 2H, it must be that

$$\pi_{2H}(K_{2H}, l_{2H}; K_1, I_1, K_{2L}, l_{2L}) \geq \pi_{2H}(K_{2H} = K_1, l_{2H}; K_1, I_1, K_{2L}, l_{2L}),$$

which indicates that changing $K_{2H}$ to $K_1$ doesn’t benefit the port.

Since

$$\pi_{2H}(K_{2H}, l_{2H}; K_1, I_1, K_{2L}, l_{2L}) = (\tau_{2H}(K_{2H}, l_{2H}) - c_2)q_{2H}(K_{2H}, l_{2H}) - x_{2H}D_2(K_{2H}, l_{2H}) - c_{k2}(K_{2H} - K_1) - c_{l2}(l_{2H} - l_1),$$

and

$$\pi_{2H}(K_{2H} = K_1, l_{2H}; K_1, I_1, K_{2L}, l_{2L}) = (\tau_{2H}(K_1, l_{2H}) - c_2)q_{2H}(K_1, l_{2H}) - x_{2H}D_2(K_1, l_{2H}) - c_{k2}(K_1 - K_1) - c_{l2}(l_{2H} - l_1).$$
we have
\[ \pi_{2H}(K_{2H}, I_{2H}; K_1, I_1, K_{2L}, I_{2L}) - \pi_{2H}(K_{2H} = K_1, I_{2H}; K_1, I_1, K_{2L}, I_{2L}) \]
\[ = (\tau_{2H}(K_{2H}, I_{2H}) - c_2)q_{2H}(K_{2H}, I_{2H}) - (\tau_{2H}(K_{1}, I_{2H}) - c_2)q_{2H}(K_1, I_{2H}) \]
\[ - x_{2H}(D_2(K_{2H}, I_{2H}) - D_2(K_1, I_{2H})) - c_{k2}(K_{2H} - K_1) \geq 0. \]

Next, derive \( \pi_1(sol2) - \pi_1(sol1) \):
\[ \pi_1(sol2) - \pi_1(sol1) = \pi_1(K_1 = K_{2H}, I_1; K_{2H}, I_{2H}, K_{2L}, I_{2L}) - \pi_1(K_1, I_1; K_{2H}, I_{2H}, K_{2L}, I_{2L}) \]
\[ = (\tau_1(K_{2H}, I_1) - c_1)q_1(K_{2H}, I_1) - (\tau_1(K_1, I_1) - c_1)q_1(K_1, I_1) \]
\[ - x_1(D_1(K_{2H}, I_1) - D_1(K_1, I_1)) - c_{k1}(K_{2H} - K_1) \]
\[ = (\tau_1(K_{2H}, I_{2H}) - c_1)q_1(K_{2H}, I_{2H}) - (\tau_1(K_1, I_{2H}) - c_1)q_1(K_1, I_{2H}) \]
\[ - x_1(D_1(K_{2H}, I_{2H}) - D_1(K_1, I_{2H})) - c_{k1}(K_{2H} - K_1), \]
where the last equality follows because \( I_1 = I_{2H} \).

Suppose all the parameters (except the disaster probability) in the two periods are the same:
\[ c_1 = c_2, c_{i1} = c_{i2}, c_{k1} = c_{k2}, \theta_1 = \theta_2, \eta_1 = \eta_2, m_1 = m_2, M_1 = M_2, a_1 = a_2, b_1 = b_2. \]
To show \( \pi_1(sol2) - \pi_1(sol1) > 0 \), we need to show \( \pi_1(sol2) - \pi_1(sol1) > \pi_{2H}(K_{2H}, I_{2H}; K_1, I_1, K_{2L}, I_{2L}) - \pi_{2H}(K_{2H} = K_1, I_{2H}; K_1, I_1, K_{2L}, I_{2L}) \) \( \geq 0 \). Compare the expressions of \( \pi_{2H}(K_{2H}, I_{2H}; K_1, I_1, K_{2L}, I_{2L}) - \pi_{2H}(K_{2H} = K_1, I_{2H}; K_1, I_1, K_{2L}, I_{2L}) \) and \( \pi_1(sol2) - \pi_1(sol1) \), it is equivalent to show:
\[ (\tau_1(K_{2H}, I_{2H}) - c_1)q_1(K_{2H}, I_{2H}) - (\tau_1(K_1, I_{2H}) - c_1)q_1(K_1, I_{2H}) \]
\[ > (\tau_{2H}(K_{2H}, I_{2H}) - c_2)q_{2H}(K_{2H}, I_{2H}) - (\tau_{2H}(K_1, I_{2H}) - c_2)q_{2H}(K_1, I_{2H}), \]
or equivalently
\[ \int_{K_1}^{K_{2H}} \left( \frac{\partial(\tau_1(K, I_{2H}) - c_1)q_1}{\partial K} - \frac{\partial(\tau_{2H}(K, I_{2H}) - c_2)q_{2H}}{\partial K} \right) dK > 0. \]
Since all parameters in both periods are the same, the only difference is that \( (\tau_1(K_{2H}, I_{2H}) - c_1)q_1 \) is evaluated at \( x_1 \) and \( (\tau_{2H}(K, I_{2H}) - c_2)q_{2H} \) is evaluated at \( x_{2H} \).

Since \( x_1 < x_{2H} \) (the climate deteriorates), we have shown that fixing \( I_{2H}, \frac{\partial^2 \pi_{2s}(K, x_{2s}(K))}{\partial K \partial x_{2s}} < 0 \) holds.

Thus,
\[ \frac{\partial (\tau_1(K, I_{2H}) - c_2)q_1}{\partial K} > \frac{\partial (\tau_2(K, I_{2H}) - c_2)q_{2H}}{\partial K}, \]

which gives

\[ \pi_1(sol2) > \pi_1(sol1). \]

Also, \( \pi_{2H}(sol2) > \pi_{2H}(sol1) \) and \( \pi_{2L}(sol2) > \pi_{2L}(sol1) \), because \( \pi_{2H}(sol2) - \pi_{2H}(sol1) = \pi_{2L}(sol2) - \pi_{2L}(sol1) = c_{k2}(K_{2H} - K_1) > 0 \). Thus, \( \Pi(Sol2) > \Pi(Sol1) \), which indicates Sol1 can’t be optimal.

To rule out Case 4, we need to show \( K_{2H} > I_1, K_{2L} > I_1, \) and \( I_{2L} = I_{2H} = I_1 \) is not optimal. With the same argument above, we only need to show \( K_{2L} > K_{2H} > K_1 \) and \( I_{2L} = I_{2H} = I_1 \) is not optimal. The proof is the same as above (since we did not use \( I_{2L} \) in the above proof).

**Additional figures from Section 3.4.4.**

- **Figure B.3.1** Varying the demand slope in period 2, \( b_2 \) (private port)

- **Figure B.3.2** Varying the demand constant with \( a_1 = a_2 \) (private port)
Figure B.3.3 Varying the demand intercept with $b_1 = b_2$ (private port)

Figure B.3.4 Varying the effectiveness of protection to shippers in period 2, $\theta_2$ (private port)

Figure B.3.5 Varying the effectiveness of protection to the port in period 2, $\eta_2$ (private port)
In Table 3.2, we show that $\frac{d\tau^*}{d\eta} < 0$. As stated in the note for Table 3.2, we impose the assumptions in Eqns. (3.3), (3.4), (3.5), (3.8), and (3.9) in signing the comparative statics. A sufficient condition for $\frac{d\tau^*}{d\eta} < 0$ is $\eta I > K$, which holds given the assumption $\frac{\partial^2 F}{\partial \delta \partial \ell} = \frac{\eta(K-\eta I)}{I^2} F < 0$ in Eq. (3.8). As shown in Figure B.3.6, $\eta I > K$ does not hold when $\eta_1/\eta_2$ is very small ($\eta_1 = \eta_2 \leq 0.9$), and indeed $\frac{d\tau^*}{d\eta} > 0$. But for a wide range of parameter values ($\eta_1 = \eta_2 > 0.9$), $\eta I > K$ holds, which gives the consistent results as in Table 3.2.

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Figure B.3.6 Varying the effectiveness of protection to port with $\eta_1 = \eta_2$ (private port) 59

Figure B.3.7 Varying the congestion cost to shippers in period 2, $\delta_2$ (private port)

Figure B.3.8 Varying the disaster cost to shippers in period 2, $m_2$ (private port)
Figure B.3.9 Varying the disaster cost to shippers with $m_1 = m_2$ (private port)

Figure B.3.10 Varying the disaster cost to port in period 2, $M_2$ (private port)

Figure B.3.11 Varying the disaster cost to port with $M_1 = M_2$ (private port)
B.4 Two-period model and public port

Proof of Proposition 4.

First, prove that the first column in Table 3.6 can be conditionally ruled out.

We prove by contradiction following the same argument of a private port.
Suppose $\text{Sol1} = (K_1, K_{2L}, K_{2H}, l_1, l_{2L}, l_{2H}, \tau_1, \tau_{2L}, \tau_{2H})$ with $l_{2L} > l_1$ and $l_{2H} > l_1$ is the optimal solution. We show another solution $\text{Sol2} = (K_1, K_{2L}, K_{2H}, l_1', l_{2L}, l_{2H}, \tau_1, \tau_{2L}, \tau_{2H})$, where $l_1' = \min\{l_{2L}, l_{2H}\}$ dominates $\text{Sol1}$ (i.e., $W(\text{Sol2}) > W(\text{Sol1})$). In both period 2H and 2L, the port saves building cost $c_{l_2}(l_1' - l_1)$ under $\text{Sol2}$: $w_{2H}(\text{Sol2}) - w_{2H}(\text{Sol1}) = w_{2L}(\text{Sol2}) - w_{2L}(\text{Sol1}) = c_{l_2}(l_1' - l_1) > 0$. Next, let’s look at period 1.

In both periods, $W(\text{Sol2}) - W(\text{Sol1})$

\[
= (\tau_1 - c_1) (q_1(l_1') - q_1(l_1)) + \frac{1}{2} b_1 (q_1(l_1')^2 - q_1(l_1)^2) - x_1 (D_1(l_1') - D_1(l_1))
\]

$- c_{l_1}(l_1' - l_1)$. 

Thus, a sufficient condition for $W(\text{Sol2}) - W(\text{Sol1}) > 0$ is $\beta c_{l_2} - c_{l_1} \geq 0$.

Second, prove that Cases 11, 12, 15 in Table 3.6 can be ruled out.

Following the same argument for private port, we need to show

\[
\frac{\partial w_{2L}(K, l_{2H})}{\partial K} > \frac{\partial w_{2H}(K, l_{2H})}{\partial K},
\]

and

\[
\frac{\partial w_{2L}(K_{2L}, l)}{\partial l} < \frac{\partial w_{2H}(K_{2L}, l)}{\partial l}.
\]

Thus, we need to show that fixing $l_{2s}$ the function $\frac{\partial w_{2s}(K_{2s}, \tau_{2s}(K_{2s}))}{\partial K_{2s}}$ decreases in disaster probability $x_{2s}$, and that fixing $K_{2s}$ the function $\frac{\partial w_{2s}(l_{2s}, \tau_{2s}(l_{2s}))}{\partial l_{2s}}$ increases in disaster probability $x_{2s}$. Since

\[
\frac{\partial w_{2s}(K_{2s}, \tau_{2s}(K_{2s}))}{\partial K_{2s}} = (\tau_{2s}(K_{2s}) - c_2 - \frac{\partial \rho_{2s}}{\partial q_{2s}} q_{2s}) \frac{\partial q_{2s}}{\partial K_{2s}} - x_{2s} \frac{\partial D_{2s}}{\partial K_{2s}} - c_{k2},
\]

we have

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\[
\frac{\partial^2 w_{2s}(K_{2s}, \tau_{2s}(K_{2s}))}{\partial K_{2s} \partial x_{2s}} = \left( \tau_{2s}(K_{2s}) - c_2 - \frac{\partial \rho_{2s}}{\partial q_{2s}} q_{2s} \right) \left( \frac{\partial^2 q_{2s}}{\partial K_{2s} \partial x_{2s}} + \frac{\partial^2 q_{2s}}{\partial K_{2s} \partial \tau_{2s}} \frac{\partial \tau_{2s}}{\partial x_{2s}} \right) \\
+ \left( \frac{\partial \tau_{2s}}{\partial x_{2s}} - \frac{\partial \rho_{2s}}{\partial q_{2s}} \left( \frac{\partial q_{2s}}{\partial x_{2s}} + \frac{\partial q_{2s}}{\partial \tau_{2s}} \frac{\partial \tau_{2s}}{\partial x_{2s}} \right) \right) \frac{\partial q_{2s}}{\partial K_{2s}} - \frac{\partial D_{2s}}{\partial K_{2s}}.
\]

Rearranging the terms, we obtain
\[
\frac{\partial^2 w_{2s}(K, \tau_{2s}(K))}{\partial K \partial x_{2s}} = -\frac{\partial q_{2s}}{\partial x_{2s}} + \frac{1}{2} \frac{\partial \rho_{2s}}{\partial q_{2s}} \left( 2 \frac{\partial g_{2s}}{\partial K_{2s}} + x_{2s} \frac{\partial f_{2s}}{\partial K_{2s}} \right) < 0
\]
\[
- \frac{\partial q_{2s}}{\partial x_{2s}} + \frac{1}{2} \frac{\partial \rho_{2s}}{\partial q_{2s}} \left( 2 \frac{\partial g_{2s}}{\partial K_{2s}} + x_{2s} \frac{\partial f_{2s}}{\partial K_{2s}} \right) \frac{\partial q_{2s}}{\partial K_{2s}} + \frac{\partial D_{2s}}{\partial K_{2s}} < 0.
\]

Since
\[
\frac{\partial w_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s}} = \left( \tau_{2s}(I_{2s}) - c_2 - \frac{\partial \rho_{2s}}{\partial q_{2s}} q_{2s} \right) \frac{\partial q_{2s}}{\partial I_{2s}} - x_{2s} \frac{\partial D_{2s}}{\partial I_{2s}} - c_{12},
\]
we have
\[
\frac{\partial^2 w_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s} \partial x_{2s}} = \left( \tau_{2s}(I_{2s}) - c_2 - \frac{\partial \rho_{2s}}{\partial q_{2s}} q_{2s} \right) \frac{\partial^2 q_{2s}}{\partial I_{2s} \partial x_{2s}} + \left( \frac{\partial \tau_{2s}}{\partial x_{2s}} - \frac{\partial \rho_{2s}}{\partial q_{2s}} \left( \frac{\partial q_{2s}}{\partial x_{2s}} + \frac{\partial q_{2s}}{\partial \tau_{2s}} \frac{\partial \tau_{2s}}{\partial x_{2s}} \right) \right) \frac{\partial q_{2s}}{\partial I_{2s}} - \frac{\partial D_{2s}}{\partial I_{2s}}.
\]

Rearranging terms, we obtain
\[
\frac{\partial^2 w_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s} \partial x_{2s}} = -\frac{\partial f_{2s}}{\partial I_{2s}} \left( \frac{x_{2s} \frac{\partial q_{2s}}{\partial x_{2s}} + q_{2s}}{2 - \frac{1}{\Delta_{2s}} \frac{\partial \rho_{2s}}{\partial q_{2s}}} \right) \frac{\partial D_{2s}}{\partial I_{2s}} < 0.
\]

Thus, we need to determine the sign of\(\frac{x_{2s} \frac{\partial q_{2s}}{\partial x_{2s}} + q_{2s}}{2 - \frac{1}{\Delta_{2s}} \frac{\partial \rho_{2s}}{\partial q_{2s}}}\). The optimal public charge in Eq. (3.28)
gives \(\tau_{2s}^w\) as:
\[
\tau_{2s}^w = \frac{\delta_2}{b_2 K_{2s} + 2 \delta_2} \left( a_2 + c_2 - x_{2s} \frac{\partial f_{2s}}{\partial I_{2s}} + \frac{b_2 K_{2s}}{\delta_2} \frac{\partial \rho_{2s}}{\partial q_{2s}} \right).
\]
Thus, the demand evaluated at optimal \( \tau_{2s} \) is

\[ q_{2s}^{\tau_{2s}} = \frac{1}{b_2 + 2\delta_2/K_{2s}}(a_2 - c_2 - x_{2s}f_{2s}). \]

Thus,

\[ \frac{x_{2s} \partial q_{2s}}{2 - \frac{1}{\Delta_{2s}} \partial q_{2s}} + q_{2s} = \frac{1}{b_2 + 2\delta_2/K_{2s}}(a_2 - c_2 - 2x_{2s}f_{2s}). \]

Therefore, if \( a_2 - c_2 - 2x_{2s}f_{2s} \geq 0 \) (i.e., demand is sufficiently large), we have

\[ \frac{\partial^2 w_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s} \partial x_{2s}} > 0. \]

Note that this condition is the same as for a private port \( \frac{\partial^2 \pi_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s} \partial x_{2s}} > 0. \)

Since we show that \( \frac{\partial^2 \pi_{2s}(K, \tau_{2s}(K))}{\partial K \partial x_{2s}} < 0 \) and \( \frac{\partial^2 \pi_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s} \partial x_{2s}} > 0 \), these two conditions rule out Case 11 in Table 3.6. Since \( \frac{\partial^2 \pi_{2s}(I_{2s}, \tau_{2s}(I_{2s}))}{\partial I_{2s} \partial x_{2s}} > 0 \), Case 15 can be ruled out.

Third, prove that Cases 3, 4 in the Table 3.6 can be ruled out.

Following the same argument as for a private port, we only need to show \( K_{2L} > K_{2H} > K_1 \) and \( I_{2L} > I_{2H} = I_1 \) is not optimal. Suppose the optimal solution \( Sol1 = (K_1, K_{2L}, K_{2H}, I_1, I_{2L}, I_{2H}) \) satisfies \( K_{2L} > K_{2H} > K_1 \) and \( I_{2L} > I_{2H} = I_1 \). Thus, we want to show \( W(Sol2) > W(Sol1) \), where \( Sol2 = (K_1 = K_{2H}, K_{2L}, K_{2H}, I_1, I_{2L}, I_{2H}) \), or equivalently to show \( w_1(Sol2) \geq w_1(Sol1) \), \( w_{2L}(Sol2) \geq w_{2L}(Sol1) \), and \( w_{2H}(Sol2) \geq w_{2H}(Sol1) \), with at least one strict inequality.

For clarification, the arguments of the function \( \pi_i \) for period \( i = 1, 2H, 2L \) are written \( \pi_i(K_i, I_i; K_{-i}, I_{-i}) \), where \(-i\) indicates the period other than \(i\).

Since \( (K_{2H}, I_{2H}; K_1, I_1, K_{2L}, I_{2L}) \) is the optimal solution in period \( 2H \), we have
\begin{align*}
w_2(H(K_2H, I_2H; K_1, I_1, K_2L, I_2L)) - w_2(H(K_2H = K_1, I_2H; K_1, I_1, K_2L, I_2L)) \\
= (\tau_2(H(K_2H, I_2H) - c_2)q_2(H(K_2H, I_2H) - (\tau_2(H(K_1, I_2H) - c_2)q_2(H(K_1, I_2H) \\
- x_2(H(D_2(K_2H, I_2H) - D_2(K_1, I_2H)) - c_{k_2}(K_2H - K_1) + \frac{1}{2} b_2(q_2(H(K_2H, I_2H)^2 \\
- q_2(H(K_1, I_2H)^2) \geq 0,
\end{align*}
which indicates that changing $K_{2H}$ to $K_1$ doesn’t benefit the port in period $2H$.

Next, derive $w_1(sol2) - w_1(sol1)$:
\begin{align*}
w_1(sol2) - w_1(sol1) \\
= (\tau_1(K_2H, I_2H) - c_1)q_1(K_2H, I_2H) - (\tau_1(K_1, I_2H) - c_1)q_1(K_1, I_2H) \\
- x_1(D_1(K_2H, I_2H) - D_1(K_1, I_2H)) - c_{k_1}(K_2H - K_1) + \frac{1}{2} b_1(q_1(K_2H, I_2H)^2 \\
- q_1(K_1, I_2H)^2),
\end{align*}
where the equality follows because $I_1 = I_{2H}$.

Suppose all the parameters (except the disaster probability) in the two periods are the same, To show $w_1(sol2) - w_1(sol1) > 0$ , we need to show $w_1(sol2) - w_1(sol1) > w_2(H(K_2H, I_2H; K_1, I_1, K_2L, I_2L)) - w_2(H(K_2H = K_1, I_2H; K_1, I_1, K_2L, I_2L)) \geq 0$, which is equivalent to showing:
\begin{align*}
\int_{K_1}^{K_2H} \frac{\partial}{\partial K} ((\tau_1(K, I_2H) - c_1)q_1(K, I_2H) + \frac{1}{2} b_1(q_1(K, I_2H)^2 \\
- \frac{\partial}{\partial K} ((\tau_2(K, I_2H) - c_2)q_2(H(K, I_2H) + \frac{1}{2} b_2(q_2(H(K, I_2H)^2)) dK > 0,
\end{align*}
which holds because we have shown that $\frac{\partial^2 w_2\left(H_2(K, I_{2H}(K))\right)}{\partial K \partial x_{2H}} < 0$ when fixing $I_{2H}$. Thus,
\begin{align*}
w_1(sol2) > w_1(sol1).
\end{align*}
Also, $w_{2H}(sol2) - w_{2H}(sol1) = w_{2L}(sol2) - w_{2L}(sol1) = c_{k_2}(K_{2H} - K_1) > 0$.

Thus, $\Pi(sol2) > \Pi(sol1)$, which indicates Sol1 can’t be optimal. Thus, Case 4 can also be ruled out accordingly.
Numerical analysis for public port.

Figure B.4.1 Varying the probability of disaster $x^H_2$, with $x_1 = x^L_1 = 0.2$ (public port)

Figure B.4.2 Varying the probability of disaster $x^L_2$, with $x_1 = 0.5, x^H_2 = 0.8$ (public port)

Figure B.4.3 Varying the probability of disaster $x_1$, with $x^L_2 = x_1; x^H_2 = x_1 + 0.3$ (public port)
Figure B.4.4 Varying the demand intercept in period 2, $a_2$ (public port)

Figure B.4.5 Varying the cost of capacity in period 2, $c_{k2}$ (public port)

Figure B.4.6 Varying the cost of protection in period 2, $c_{i2}$ (public port)
In Table 3.4, we show that $\frac{d\tau_i}{d\theta} < 0$. A necessary and sufficient condition for $\frac{d\tau_i}{d\theta} < 0$ is $\theta I > K$, which holds given the assumption $\frac{d^2f}{d\theta^2} = \frac{\theta(K-\theta)}{K^3} f < 0$ in Eq. (3.3). As shown in Figure D.4.7, $\theta I > K$ does not hold when $\theta_1/\theta_2$ is small ($\theta_1 = \theta_2 \leq 1$) for period 1 and 2L, and indeed $\frac{d\tau_i}{d\theta} > 0$. But for a wide range of parameter values ($\theta_1 = \theta_2 > 1$), $\theta I > K$ holds, which yields the results in Table 4.4.
Figure B.4.10 Varying the demand constant with $a_1 = a_2$ (public port)

Figure B.4.11 Varying the demand intercept with $b_1 = b_2$ (public port)

Figure B.4.12 Varying the effectiveness of protection to shippers in period 2, $\theta_2$ (public port)
Figure B.4.13 Varying the effectiveness of protection to the port in period 2, $\eta_2$ (public port)

Figure B.4.14 Varying the effectiveness of protection to port with $\eta_1 = \eta_2$ (public port)

Figure B.4.15 Varying the congestion cost to shippers in period 2, $\delta_2$ (public port)
Figure B.4.16 Varying the disaster cost to shippers in period 2, $m_2$ (public port)

Figure B.4.17 Varying the disaster cost to shippers with $m_1 = m_2$ (public port)

Figure B.4.18 Varying the disaster cost to port in period 2, $M_2$ (public port)
Figure B.4.19 Varying the disaster cost to port with $M_1 = M_2$ (public port)

Figure B.4.20 Varying the cost of capacity with $c_{k1} = c_{k2}$ (public port)

Figure B.4.21 Varying the cost of protection with $c_{i1} = c_{i2}$ (public port)

61 In Table 4.4, we show that $\frac{d\tau^w}{dc_I} > 0$. A sufficient condition for $\frac{d\tau^w}{dc_I} > 0$ is $\theta l > K$, which holds given the assumption $\frac{d\tau^2}{dk1} < 0$ in Eq. (3.3). However, when $c_I$ is large ($c_{i1} = c_{i2} > 0.3$), protection investment drops substantially, which may lead to $\theta l < K$. Thus, the result $\frac{d\tau^w}{dc_I} > 0$ may not hold because condition $\frac{d\tau^2}{dk1} < 0$ is violated. Note that when $c_{i1} = c_{i2} = 0.5$, the protection investment reduces to 0 in period 1 and 2L. But we show in the numerical analysis that $\frac{d\tau^2}{dk1} < 0$ holds for a wide range of parameter combinations.
Figure B.4.22 Varying the discount factor $\beta$ (public port)
Appendix C  Appendix for Chapter 4

(a) Bus departure times to Daejeon

(b) Bus departure times to Daegu

(c) Bus departure times to Busan
Table C.1 Optimal schedules with increasing train frequency

<table>
<thead>
<tr>
<th>Actual schedule</th>
<th>Optimal schedules under different train frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>7A</td>
<td>7</td>
</tr>
<tr>
<td>09:10</td>
<td>08:35</td>
</tr>
<tr>
<td>16:30</td>
<td>18:20</td>
</tr>
</tbody>
</table>

Note: 7A denotes the actual train schedule with 7 trains per day; 7 denotes the optimized schedule with 7 trains, and the same applies to other columns.
<table>
<thead>
<tr>
<th>Actual schedule</th>
<th>Optimal schedules under different train frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>7A</td>
<td>6</td>
</tr>
<tr>
<td>06:55</td>
<td>07:25</td>
</tr>
<tr>
<td>09:10</td>
<td>08:35</td>
</tr>
<tr>
<td>12:07</td>
<td>16:15</td>
</tr>
<tr>
<td>12:50</td>
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<tr>
<td>16:30</td>
<td>19:50</td>
</tr>
<tr>
<td>18:35</td>
<td>21:20</td>
</tr>
<tr>
<td>20:38</td>
<td></td>
</tr>
</tbody>
</table>

Note: 7A denotes the actual train schedule with 7 trains per day; 6 denotes the optimized schedule with 6 trains, and the same applies to other columns.