ADVANCED MEMS INERTIAL SENSORS:
SIMSCAPE MODELLING AND APPLICATIONS IN SLIDING MODE CONTROL

by

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**Simscape modelling and applications in Sliding Mode Control**

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Abstract

As the technology behind Microelectromechanical Systems (MEMS) continues to progress, more complex microsystems evolve, in turn demanding more sophisticated and efficient macromodels that can be interfaced with the electronic subsystems. The thesis proposes a hybrid simulation approach that combines signal flow models in MATLAB/Simulink with more accurate and reusable energy flow reduced-order macromodels in Simscape to allow the exploration of nonlinear operating modes and bi-directional electro-mechanical coupling in MEMS devices. The physical system modelling capabilities of Simscape behaviour modelling language are used to develop and test a novel MEMS library containing parameterized fundamental building blocks (area and gap-varying MEMS capacitors and displacement stoppers). Simulations of the library models compare favourably with both analytical results and structural finite element analyses performed in COMSOL Multiphysics.

The thesis also aims at designing all-digital control and read-out of MEMS accelerometers and gyroscopes, simulated in the Simulink+Simscape hybrid environment. The research reports on the design and simulation of two distinct microsystems, a single-axis MEMS accelerometer and gyroscope, based on devices fabricated in a custom 50um SOI technology. Both microsystems use Sliding Mode Control (SMC) for digital feedback control. Motion cancelling SMC techniques are applied for the MEMS accelerometer and the sensing mode of the MEMS gyroscope, while a bandpass SMC scheme was designed for the driven mode of the MEMS gyroscope to track...
perturbations in its mechanical resonance frequency and maintain an optimum electrostatic actuation (similar to a digital PLL).

In the case of the accelerometer, the digital feedback has attenuated the motion of the proof-mass by 93.2%, while the output bitstream has captured the input quantity (external acceleration) with a steady-state error of 0.06%. Similarly, the motion induced in the sensing mode by an external angular rate was attenuated by 100 times. The output bitstream was subjected to a novel digital synchronous demodulation technique to reconstruct the input angular rate with an error of just 0.16%. The bandpass SMC was able to track and electrically actuate the drive mode to the mechanical resonant frequency with a steady-state error of 0.034%, and a tracking delay of 110ms.
Lay Summary

Micro electro-mechanical systems (MEMS) based accelerometers and gyroscopes have shown remarkable growth in recent years in applications such as inertial navigation systems, seismic measurements, crash detection and airbag control, motion sensing in smartphones and cameras etc. The growing sophistication of MEMS sensors demands accurate modelling of the microsystem.

The thesis proposes a novel library of MEMS components in Simscape to perform system-level co-simulation of MEMS-based microsystems. The library allows the simulation of both linear and extremely non-linear behaviour of MEMS structures, as well as their bidirectional electro-mechanical interactions. The Simscape library is intended to be readily available for MEMS researchers to expand it and use it for the design and simulation of complex microsystems. The thesis uses the proposed MEMS library to explore novel approaches for all-digital control and read-out of MEMS accelerometers and gyroscopes, thereby validating the performance of the library.
Preface

The work, hereby, presented was performed in Advanced Microsystems Laboratory at The University of British Columbia, Vancouver. I was the lead investigator in the research and responsible for major areas of conceptualization, modelling, simulation, analysis and manuscript composition.

A version of Chapter 4, ‘Double-digital control and read-out of MEMS gyroscope using Sliding Mode Control using energy flow based modelling components in Simscape’ has been published [51] in 2019, 10th IEEE annual Information technology, Electronics & Mobile Communication Conference (IEMCON). I was an investigator and co-author of the research work and responsible for areas of concept development and manuscript composition. The concepts of bitstream demodulation and band-pass sliding mode control were implemented by Mr. Jinhao Lu and Ms. Ruolan Ye. Dr. Edmond Cretu was the supervisory author on the project and was involved in conceptualization and manuscript preparation.
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List of Abbreviations

**DaCM** - Displacement-amplifying Compliant Mechanism

**DARPA** - Defense Advanced Research Project Agency

**DOF** - Degrees Of Freedom

**DRIE** - Deep Reactive Ion Etching

**IMUs** - Inertial Measurement Units

**MEMS** - Microelectromechanical systems

**PLL** - Phase-locked loop

**UV-LIGA** - Ultraviolet Lithographie, Galvanoformung, Abformung

**BP-SMC** - Bandpass Sliding Mode Control

**CAD** - Computer-Aided Design

**DCO** - Digitally controlled oscillator

**FEA** - Finite Element Analysis

**LPF** - Low pass filter
LP-SMC - Low-Pass Sliding Mode Control

MOR - Model Order Reduction

PDM - Pulse Density Modulator

SFU-MUMPS - Simon Fraser University - Multi-User MEMS Process

SLS - System-Level Simulations

SMC - Sliding Mode Control

SPICE - Simulation Program with Integrated Circuit Emphasis

VSC - Variable Structure Control
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Dedication

To my family …

அம்மா, அப்பா!

"செந்த பிறந்த நாளில் வல்லாற்றவியான கொஞ்சக்கூடா
ஒமையாய் எண்ணென்ற புனிதம்" - 69 ஆம் வாரத்தில்

நாசாவும் எனக்கு தேவை, அப்பா. பிறல்லை மேலும் வருக, மாம்மா வருக கூறிக்கொள்ளார்.
Chapter 1: Introduction

The term “inertial sensors” is used to refer to the sensors that help characterize the dynamic behaviour of an object by measuring the inertial forces acting on the object. The most common measurands are the linear acceleration and angular rate obtained by using accelerometer and gyroscope sensors, respectively. MEMS accelerometers and gyroscopes are often integrated with magnetometers in Inertial Measurement Units (IMUs) for complete monitoring of the dynamic behaviours of objects or parts of the body. Among their many applications are Inertial Navigation Systems for aerial and terrestrial vehicles, traffic collision, wearable applications [1] etc. Among the most common alternatives, a MEMS accelerometer is used to measure the linear acceleration of the subject [2], while a MEMS vibratory gyroscope makes use of its modes coupling to estimate the angular velocity [3].

1.1 MEMS inertial sensors market research

The term Microelectromechanical Systems (MEMS) first appeared in a proposal to the U.S. Defense Advanced Research Project Agency (DARPA) in the year 1986 [4]. Since then, the development of the MEMS industry has been prolific. Global MEMS/Microsystems Markets and Opportunities anticipates that the international MEMS market would reach 26.8 billion by 2020. According to Statista [5], in the year 2015, revenues from the global MEMS sensor market reached 9.6 billion U.S. dollars, out of which MEMS accelerometers and gyroscopes represented 23.5%, higher than all other categories of sensors, as highlighted in Figure 1.1.
Figure 1.1 Market revenue share in percentage [2]. The red circles highlight the contributions of accelerometers and gyroscopes to global revenue.

The reason for the MEMS accelerometers and gyroscopes to contribute so much to the global MEMS market revenues, as illustrated by Figure 1.1, is due to their intensive use in a wide range of applications such as human activity monitoring, inertial navigation systems, vibrational and seismic measurements, traffic collision systems, crash detection and airbag control, motion sensing
in smartphones and cameras, virtual reality and gaming interfaces etc. all these applications require monitoring the short-term or long-term dynamic behaviour of the object of interest [6].

1.2 MEMS accelerometers

According to [6], the first-ever micro-machined silicon accelerometer was designed at Stanford University in 1979. Since then, not only did the micro-fabrication technologies evolve [7]–[12] but also the applications of such devices [13]–[20]. For instance, R Gholamzadeh et al., in [7] propose a sequential and pulsed Deep Reactive Ion Etching (DRIE) technology for fabricating a half-bridge MEMS accelerometer. The authors of [8] have reported having achieved higher sensitivities with MEMS accelerometers via a UV-LIGA fabrication process that effectively decouples stiffness of beams and increase in mass. In [9], the amplitude of displacement of the proof mass has been augmented using Displacement-amplifying Compliant Mechanism (DaCM). The amplification factor at the output of the mechanism was reported to be 11 times greater than the actual displacement. Several such works have contributed to the innovations that are being made on the design standpoint of the device to enhance its sensitivity [10]–[12].

1.2.1 Principle of operation and modelling of MEMS accelerometer

Accelerometers are devices that measure the linear acceleration of the object of interest. Accelerometers have a seismic mass supported by beams anchored on either side, as shown in Figure 1.2. The air within the device enclosure offers mechanical damping to the movement of the proof mass. Hence, the resultant mass-spring-damper (open loop) system can be expressed using the typical second differential equation, as explained in (1) where \( m, b \) and \( k \) denote the mass,
damping coefficient of the medium within the enclosure and the spring constant of the beams, respectively. The displacement of the proof mass is represented using the variable $x$, while the external input acceleration, by $a$.

$$F_{\text{inertial}} + F_{\text{damping}} + F_{\text{spring}} = F_{\text{external}}$$

$$m\ddot{x} + b\dot{x} + kx = ma \tag{1}$$

From (1), the mechanical transfer function of accelerometers can be derived as in (2). However, the transfer function is highly linearized and does not provide any information on the physical structure, electrodes, capacitances, etc., of the device.

$$\frac{X(s)}{A(s)} = \frac{1}{s^2 + \frac{b}{m}s + \frac{k}{m}} \tag{2}$$

Figure 1.2 MEMS capacitive accelerometer (a) Basic physical components of a MEMS accelerometer (b) Single-axis MEMS accelerometer designed in Solidworks 2018
The proof mass has attached comb-like electrodes on its sides, such that the electrodes move with the proof mass under the influence of the external acceleration. These electrodes, along with a similar set of fixed electrodes on the structure, form readout capacitors. When the device experiences acceleration, the proof mass and the attached movable electrodes are free to move, thereby changing the capacitance between movable and immovable electrodes. One alternative is that the proof mass motion changes the gap distance between the capacitor plates, resulting in a change in the capacitance $\Delta C$ between the electrodes. With the help of a capacitive read-out circuit, the MEMS accelerometer maps, therefore, the $\Delta C$ to the external acceleration.

1.3 MEMS Gyroscopes

Gyroscopes measure the angular rate of a rotating object. Compared to accelerometers, gyroscopes have a more complex architecture and the principle of operation. The MEMS device measures the angular velocity of the object attached to by using the Coriolis force (3).

$$F_{\text{coriolis}} = 2m \vec{V} \times \vec{\Omega}$$  (3)
The Coriolis force, $F_{\text{coriolis}}$, as seen in (3), is proportional to the vector product of linear velocity, $V$ and the angular rate of the object, $\Omega$. Any moving object on a rotating frame experiences a Coriolis acceleration, which causes an apparent deflection of the object in the non-inertial frame of reference. In the case of a MEMS vibratory gyroscope, wherein the seismic mass is driven to vibrate along a specific axis (driving axis), the Coriolis acceleration acting on the mass is linearly proportional to the angular rate of rotation along an axis orthogonal to the driving axis, called the sensing axis. Thus, the angular rate is computed by measuring the Coriolis acceleration, as long as that the drive velocity, $V$ is known. According to the method described, the gyroscope is required to drive the proof mass into oscillations with a given velocity, $V$, and at the mechanical resonant
frequency, $f_r$ (to maximize the amplitude of the oscillations). This, carried out by a capacitive comb drive design, is referred to as the driven mode of the device. The sensing mode, using similar capacitive sensing electrodes, detects the corresponding motion of the same mass along an orthogonal axis to later calculate the angular rate from (3). Like accelerometers, gyroscopes can be represented through reduced-order macro models using mass-spring-damper systems, but with a total of 4 degrees of freedom (DOF’s), as opposed to accelerometer’s 2 DOF’s. The simplified model of a MEMS capacitive gyroscope and its layout structure are shown in Figure 1.3.

1.4 Need for closed-loop control in MEMS inertial sensors

With the expanding market size of MEMS inertial sensors, more pressure is being laid on the resolution and life of the devices. However, technological limitations in the microfabrication processes deter the development of high-performance inertial sensors. This engenders uncertainty in the device’s mechanical properties and also limits the dynamic range. The introduction of closed-loop control in the inertial sensors helps alleviate the said shortcomings. Owing to the advantages of digital control, such as simplified on-chip electronics and less power consumption, digital control schemes are being widely researched [21]–[24].

Inertial sensors are employed for a variety of applications involving a wide range of operating temperatures. The temperature fluctuations affect the spring constants of the beams, which in turn modify the resonant frequencies of the device. While minor variations in the mechanical resonant frequency do not influence the performance of accelerometers, the gyroscopes are resonant devices in which the driven-mode proof mass essentially has to be driven at its resonant frequency. Electrical actuation at resonance is critical not only because of the optimum electrical-mechanical
energy transfer into the driven mode (resulting in higher oscillation amplitudes), but also because the energy transfer between the mechanical driven and sensing vibration modes is optimum when the two modal frequencies match. Operating a gyroscope at any other frequency would lead to significant efficiency losses. Hence, MEMS gyroscopes are almost always operated in closed-loop for actuating the driven mode, where the control mechanism tracks the fundamental frequency and adjusts the driving frequency to match it.

Gyroscopes, in their sensing mode, operate analogously to MEMS accelerometers: the sensing mode capacitors sense the Coriolis acceleration acting along that axis. The role of closed-loop control in the accelerometers and the sensing modes of gyroscopes is to cancel the motion caused by the external acceleration, in order to linearize the measurement and to extend the dynamic range of sensing. Furthermore, numerous MEMS devices, to improve their performance and sensitivities, are operated on the verge of their instability in the so-called pull-in mode. For instance, MEMS pull-in accelerometers and RF electrostatic switches and actuators are devices that operate at their pull-in. Pull-in is the phenomenon of the collapse of the system stability when the linear spring resistance offered by the beams is overpowered by the highly non-linear electrostatic force. With the negative feedback control, the proof mass may be stabilized on the critical pull-in stability border to improve the sensitivity of the sensing mode (in gyroscopes) or accelerometer.
1.4.1 Digital Phase Locked-Loops and Sigma-Delta Modulation

Several control alternatives are used in gyroscopes’ driven mode to ensure that the proof mass is driven at its mechanical resonant frequency, even when environmental factors perturb it. Phase-locked loop (PLL) techniques are among the most widely applied solutions [25]–[27]. Digital PLL’s have gained popularity with their superior robustness, flexibility, compared with their analog implementation. However, digital PLL’s are still complicated and expensive.

Bandpass $\Sigma\Delta$ control techniques have replaced the former control scheme for the digital control of MEMS gyroscopes [28], [29]. The authors of [28] have successfully demonstrated the digital integration of MEMS gyroscope with unconstrained band-pass $\Sigma\Delta$ control on both the drive (primary) and the sense (secondary) modes and several works on first to sixth order $\Sigma\Delta$ control has been demonstrated to work efficiently on MEMS accelerometers [30]. However, the $\Sigma\Delta$ control usually oversimplifies the device model to a linear system, and the design methodology for implementing the control is relatively intricate.

In the present work, PLL or bandpass $\Sigma\Delta$ approaches are replaced by a sliding mode control (SMC) technique [31] in the digital control of MEMS accelerometers and gyroscopes. SMC is a type of Variable Structure Control (VSC). Emelyanov et al. were the first to report VSC in the 1950s in the Soviet Union [32]. VSC is a discontinuous non-linear control that alters the dynamics of the system by exercising a high frequency switching control. Like the name suggests, the scheme switches from one stable continuous ‘structure’ to another, on the whole being discontinuous.
1.4.2 Theory of Sliding Mode Control

Sliding mode control (SMC) is a type of VSC in which the system states are driven along the desired trajectory called the sliding surface. The operation of SMC can be broken down into two stages:

- Selection of sliding surface
- Binary feedback action to maintain the system on the sliding surface

![Figure 1.4 The action of Sliding Mode Control for cancelling a mechanical displacement](image)

---

*Figure 1.4 The action of Sliding Mode Control for cancelling a mechanical displacement*
SMC can be used to maintain the proof mass at its original position, say \( d_0 \), irrespective of the external force excitation acting on the system, provided that the closed-loop action is fast enough and the binary correction forces have sufficient amplitude. Therefore, in this case, the sliding surface becomes the nominal gap position, i.e. \( x(t) = d_0 \), where \( x(t) \) is the variable describing the instantaneous position of the proof mass. Proceeding to the second step of the SMC operation, the feedback force (electrostatic in this case) is computed to maintain the position of the proof mass, as shown in Figure 3.1. The green line in the figure represents the intended sliding surface, while orange denotes \( x(t) \). The binary feedback actuation force, \( F_{act} \) tends to bring the instantaneous position to the sliding surface through high-speed switching.

1.4.3 Chattering

SMC, besides its simplicity, has several noticeable features, such as robustness, low sensitivity to uncertain parameters and a straightforward digital implementation [24],[33], [34]. In this method, a high-frequency actuation force tries to balance, through the average value of its switching action, any external input signal that generates a perturbation. As previously reported in [33], the technique has been applied to a closed-loop MEMS accelerometer, with the generated switching bitstream being used as a direct digital output. Whether or not external forces act on gyroscopes, there are always stray internal phenomena like inertia in mechanical parts and residual charges on the capacitors, that knock the system states off the sliding surface.
SMC is in constant action to prevent this from happening. The resultant switching of the monitored output variable (e.g. displacement) below and above the sliding surface takes the form of what is called ‘chattering’. Although in some applications, chattering is disadvantageous on account of the loss of energy and injection of noise, researchers have later learned to exploit the chatter. For instance, Sarraf et al., have developed a novel sensing concept where the switching bitstream was used to extract the external input acceleration [33].

1.5 Need for modelling and simulation of MEMS devices

Mathematical modelling of systems is being performed in various areas of engineering and natural sciences. Modelling of a system helps in explaining the behaviour of the system and its components and also in predicting their response in different environments and conditions. Owing to the complexity of the real-life processes/systems, numerical simulations (also called computer simulations) have replaced manual solutions to the mathematical models. The first large-scale employment of numerical simulations was to simulate a nuclear detonation during the Manhattan Project in World War II. Following the advent of Microelectromechanical systems (MEMS) after 1986 [4], extensive research has been done on modelling MEMS devices [35], [36] each of them having their own merits and demerits. The authors of the chapter [35] have investigated the challenges in modelling MEMS devices and surveyed the different methods and techniques used for modelling. The authors Dwivedi and Khanna in [36] have developed a novel analytical modelling of MEMS capacitive accelerometers for fully-implantable hearing aids.
Due to complications such as many dimensional details, multiple degrees of freedoms and the heterogeneous nature of the systems, where more energy domains are coupled, a technique called Model Order Reduction (MOR) had evolved. The MOR technique is employed to alleviate the computational burden in numerical simulations of the system’s mathematical model. Prof. Dr. Jan G. Korvink has expressed in [37], “MOR has the potential to completely revolutionize scientific computation.”. The authors Bechtold, Schrag and Feng in their book [37] have delivered a valuable introduction to physical system modelling and MOR in the field of MEMS.

As the discipline of MEMS continues to mature, more complex and diverse Multiphysics microsystems evolve. MEMS inertial sensors, such as accelerometers and gyroscopes, have complex and sometimes highly non-linear interactions between inertial and electric excitations, electric fields between the capacitors, thermal and fluidic interaction with the medium, mechanical stresses in the beams etc. Therefore, numerical simulation of the mechanical components of the device and circuit-level simulation (SPICE) of the read-out and control electronics, when performed separately in different software environments, would not be sufficient to mimic the system behaviour as a whole. Thus, a co-simulation of the multidomain interactions in the system, called system-level simulations (SLS) is vital for the accurate modelling and prediction of the system behaviour. The authors of [38]–[41] have explored and validated several methods of performing SLS of MEMS inertial sensors. Specifically, in [39]–[41], various approaches to model and simulate MEMS capacitive accelerometers have been investigated.
1.5.1 Why acausal modelling (energy flow-based mode) is better than the causal model (information flow-based model)

Information flow-based modelling (block diagram models) portrays the mathematical behaviour behind system dynamics, and have high levels of abstraction. This type of modelling is characterized by components (or blocks) that signify the causal relationship within the system. In other words, these blocks and connectors have inputs, outputs, variables or states and directionality of operation. Causal models are efficient at expressing explicit computation steps. Hence, block diagram modelling is suitable for describing the control system assembly. While causal models are presently used to build very large systems (due to their higher abstraction level), they are less reusable than acausal models when it comes to describing plant behaviour. Various elements (sub-systems) in acausal models are linked through physical connectors involving physical quantities (like electrical or mechanical signals). Acausal models are a physically-oriented modelling approach. The inputs and outputs need not be defined explicitly, and the model assumes direction of operation from the underlying defining equations of the components, which can describe the plant realistically. Although acausal components (or connectors) are more elaborate than the causal blocks, the former is versatile and can be reused for different physical analyses without significant alterations. Another major advantage to acausal (here, energy flow-based) modelling is that the physical connectors prevent logical errors during the creation of the physical models.
1.6 Motivations for the research

This research work has a two-tiered purpose. Firstly, this work is motivated by the industry needs for high-performance MEMS inertial sensors that are controlled by digital hardware, offer a low-cost direct digital readout and are robust while operating in a changing environment. Almost 90% of electronics in modern devices like smartphones are digital. An ideal sensor is that which can be easily integrated with ambient electronics. Besides, digital systems are cheaper and more robust than their analog counterparts. Thus, industries need all-digital MEMS inertial sensors.

Therefore, the thesis has delivered an all-digital control and read-out of the MEMS accelerometer and gyroscope using Sliding Mode Control and Bitstream demodulation techniques. Innovations have been made in the method of implementing SMC for the drive mode and the accelerometer and the sense mode of gyroscope, and in the implementation of fully-digital readout using Bitstream Demodulation.

Secondly, in due course of the research, the simulation of the proposed digital control system and the read-out signal processing, along with the device became indispensable, before the entire system was tested in hardware. The author also identified similar needs in various other projects involving MEMS design and system-level modelling. Hence, the thesis allocates substantial attention to developing a new library of MEMS components in Simscape, modelling significant non-linearities particular to MEMS devices.
The dissertation presents a comprehensive account of various MEMS modelling aspects, from small signal to large signal models, including the simulation of the stability loss in MEMS structures. The implementation of all-digital control and read-out of the MEMS accelerometer and gyroscope is validated using the above-mentioned MEMS library in Simscape. Hence, the second part of the thesis has been motivated by the need to implement the primary function of the thesis.

1.7 Thesis organization

The organization of the dissertation is as follows:

Chapter 1, the current chapter, after reporting the commercial and research review of MEMS inertial sensors, clearly states the motivations behind the thesis. The chapter introduces the readers to the working of MEMS accelerometers and gyroscopes.

Chapter 2 presents the challenges in modelling MEMS devices and explains the implementation of a novel MEMS components library in Simscape®, a behavioural modelling language suitable for energy flow based modelling. Also, the chapter gives detailed instructions on how to build MEMS devices from the library by taking the case study of a MEMS accelerometer. The novel MEMS library of components, implementing reduced-order macro models of elementary MEMS structures, was validated for both small and large-signal regime through comparative simulations of the MEMS accelerometer through detailed finite element analysis software using Comsol Multiphysics, Matlab/Simulink equivalent blocks (Information-flow method), and analytical computations using simplified differential equations.
Chapter 3 and Chapter 4 address the system-level modelling of a MEMS accelerometers and a MEMS gyroscope, respectively, using hybrid modelling that combines Matlab/Simulink and the novel Simscape library. The presented model efficiently accounts for the bi-directional energy flow of the multi-domain system. Moreover, chapter 4 reports a novel methodology to implement double-digital control and readout of MEMS gyroscope using Band-Pass Sliding Mode Control (BP-SMC) and Bitstream demodulation.

The final chapter, chapter 5 (Conclusions and Future work), summarizes the work presented in the dissertation and provides further directions for this work. The thesis concludes by recounting the main aspects of the work done by the author and the research team, and the new perspectives that the present research work has opened in the field.
Chapter 2: Simscape library for physical system modelling of inertial MEMS devices

In this chapter, a new standard library dedicated to MEMS components in Matlab/Simscape has been introduced and characterized. The following section establishes the need to create a new library of MEMS components in Simscape and then discusses the challenges in designing such libraries. The chapter proceeds to characterize the non-linearities particular to MEMS capacitive devices. After reviewing the methodology to model the said non-linear traits, the chapter demonstrates how the system-level simulation (SLS) of a MEMS accelerometer can be performed using the Simscape library components. Furthermore, the functioning of the MEMS library components is successfully validated by comparing its results with Finite Element Simulations in Comsol Multiphysics 5.3 and signal flow based simulations in MATLAB/Simulink.

2.1 Need for a MEMS library in Simscape

Microelectromechanical devices, as the name suggests, benefit from a highly intricate and non-linear coupling between the electrical and mechanical domains. At the microscale, this coupling leads to a distinct phenomenon that gives rise to loss of stability, called pull-in [42]. Naturally, control systems, including robust and adaptive feedback mechanisms, were conceived to prevent the tipping of device stability toward collapse [33], [43], [44]. Sometimes, like in [24], [45], control schemes are used to improve the sensitivity of the device. For example, a digital control scheme called Sliding Mode Control has been implemented on a micro-accelerometer [45] and resonator [24] to operate the devices on the verge of their instabilities to boost their sensitivities.
Besides, microfabrication processes are subjected to all kinds of manufacturing glitches just as much as any other production assembly, if not more, due to inherent limitations and variabilities of the micromanufacturing processes. As a result, read-out circuits were not only utilized to convert non-electrical quantities into electrical or vice-versa, but also to compensate for minor structural imprecisions. Thus, a MEMS device generally will consist of the physical structure on the chip along with a wide range of control and readout circuits. Evidently, a co-simulation of the physical device with the readout and control circuitries is equally or, if possible, more crucial than the simulation of the physical structure alone.

Chapter 2 demonstrates the energy flow-based modelling using hybrid MATLAB/Simulink and Simscape interface and simulator. The critical difference between signal flow-based modelling and energy flow-based has been explored using various test cases. The results from both ways of modelling are compared with Finite Element methods to validate the claim that energy flow-based modelling is more advantageous than the other two when it comes to performing system-level simulations on non-linear systems. Finite Element Simulations are highly complicated and time-consuming for running real-time multi-physics simulations along with control feedback and read-out circuit.
2.1.1 Bi-directional energy flow

The interplay between different domains becomes significant in systems like MEMS, where more than one discipline is involved. MEMS are characterized by the bi-directional interaction between electrical and mechanical domains. For example, in MEMS capacitive accelerometers, the external mechanical acceleration moves the internal capacitor plates within the device, thereby causing a change in the capacitance [46]. This quantity is used as the measurand of the accelerometer. This signifies the energy flow from the mechanical to the electrical domain.

In most practical cases, in order to protect the device stability against external unduly forces and to improve sensitivity, negative feedback is employed. This feedback usually takes the form of the electric voltage difference applied across the parallel plates capacitors. This means that now, the energy flow is reversed: from the electrical to the mechanical domain. It is now imperative that the interaction between the domains must be accounted for an accurate simulation of the coupled energy domains. To deal with this kind of unified behaviour, we adopt the across-through formalization, an alternative to the effort-flow formalism firstly described by Paynter in 1961. The analogous variables and their relationship in the electrical and the translational mechanical domains are presented in Table 2.1. Since the topology of the electrical and mechanical domain components remain unchanged in the force-current analogy, it has been chosen to model the bidirectional energy flow diports in this thesis.
<table>
<thead>
<tr>
<th>Translational Mechanical domain</th>
<th>Electrical domain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Force – Current)</td>
</tr>
<tr>
<td>Force</td>
<td>Current</td>
</tr>
<tr>
<td>Velocity</td>
<td>Voltage</td>
</tr>
<tr>
<td>Displacement</td>
<td>Flux</td>
</tr>
<tr>
<td>Mass, $m$</td>
<td>$C$</td>
</tr>
<tr>
<td>Spring constant, $K$</td>
<td>$1/L$</td>
</tr>
<tr>
<td>Damper, $B$</td>
<td>$1/R$</td>
</tr>
</tbody>
</table>

Table 2.1 Mechanical-Electrical variables analogy. The force-current analogy used in this work has been highlighted in green.

2.1.2 Large signal analysis: Electrostatic Pull-in

An electrostatically actuated pair of capacitor parallel plates is depicted in Figure 2.1 to explain the notion of pull-in. While the electrode at position $B$ is rigidly fixed, the one at position $A$ is connected to the fixtures (anchor) with the help of beams (modelled as springs with spring constant, $k$). The total initial distance between the two electrodes is $d_0$, and a damping coefficient $b$ is associated with the surrounding medium, even though it is not depicted in Figure 2.1.
Figure 2.1 Simplified illustration of gap-varying capacitors

Electrostatic force can be varied by either varying the potential difference between the plates or the distance between them or by a combination of the two cases, as illustrated in (4).

\[ F_e = \frac{1}{2} \left( \frac{\varepsilon A n V_{bias}^2}{(d_0 - x)^2} \right) \]  

(4)

As explained by (4), \( F_e \) is proportional to the square of the voltage between the plates, \( V_{bias} \). The variable \( A \) represents the area of the electrodes, \( n \) is the total number of electrode pairs and \( x \) is the distance travelled by the proof mass. Based on the dynamics leading to the loss of stability, pull-in can be categorized into static pull-in and dynamic pull-in. As their names suggest, static pull-in occurs when the bias (potential difference between the two electrodes) increases slowly (quasi-
static variations), to become high enough for the resulting electrostatic force, $F_e$ to not be compensated anymore by the spring force, $F_{spring}$.

$$F_{spring} = kx$$  \hspace{1cm} (5)

In dynamic pull-in, the loss of stability involves as well dynamic forces like the damping and the inertia of the proof mass, leading to more complex behaviour. The equilibrium between two forces in the system, in the case of static pull-in analysis, the electrostatic force and the spring force has been presented in Figure 2.2 as per the equation in (6).

$$F_e = F_{spring} \Rightarrow \frac{1}{2} \left( \epsilon A n V_{bias} \right)^2 = kx$$  \hspace{1cm} (6)

Figure 2.2 Electrostatic and spring forces versus displacement curve for asymmetric pull-in
The electrostatic force being highly non-linear intersects the linear curve of the spring force at 3 points: A, B and C. As seen from the image, there are two equilibrium positions in the range \( x \in (0, d_0) \). By giving small perturbations to the displacement of the movable electrode around the two equilibrium points, we could identify the stable and unstable equilibrium states among them. A slight increase in the displacement \( (x << \frac{2}{3} d_0) \) from A would result in the more significant spring force pulling the mobile electrode back to A. A perturbation in the opposite direction would result in the bigger electrostatic force pulling the electrode to A. Hence, all small perturbations around point A will result in the net restoring force acting toward the stable equilibrium point A.

On the other hand, a small increase in displacement around point B would give rise to the net restoring force, dominated by the highly non-linear regime of the electrostatic force, pulling the moving electrode away from the equilibrium and toward the fixed electrode. Hence, point B is an unstable equilibrium. Thus, it can be deduced that when the movable plate reached two-third of the total gap distance, the electrostatic force exceeds the spring force, causing complete loss of stability.

The governing equation, being a third-order polynomial equivalence, there is a third solution at point C. This point, although being stable, is unattainable because of the physical structure of the plates. Point C is beyond \( d_0 \), indicating that the movable electrode should be on the other side of the fixed plate. In the regular operation of gap-varying comb-drive devices, the electrodes do not cross over to the flip side of its adjacent electrodes.
2.2 Introduction to MATLAB/Simulink

Simulink is a product of Mathworks that provides an interactive, graphical environment for block-diagram based modelling, simulation and analysis of dynamical systems. Simulink is mostly used with MATLAB, combining textual and graphical programming to model physical systems. Simulink provides a wide-ranging library of components, as presented in Figure 2.3, to be used to build block diagram models of real-time systems based on information flow. The elements of the Simulink library can be ‘dragged and dropped’ in the model window.

Simulink enables the users to perform several functionalities such as Event-based modelling, physical system modelling, model-based design verification and validation in embedded systems, and real-time testing. Although signal flow based modelling such as block diagrams and bond graphs are well suited for the concept development of Multiphysics systems, they prove to be inadequate for prototype development [47]. The author of [47] has recounted several examples where physical system modelling at an early stage of prototype development could have prevented some costly mistakes.
2.3 Introduction to Simscape Toolbox and Simscape Language

Simscape is a simulation platform included in the Mathworks suite that allows physical system modelling based on the energy flow paradigm. Therefore, instead of modelling an electric motor with its transfer function, it can be performed by assembling the motor’s fundamental components into an equivalent physical network. The physical component-based models in the Simscape environment can be incorporated with the signal-flow based modelling in Simulink to conduct a
co-simulation of the complete system-level model. The components of the present Simscape library are displayed in Figure 2.4.

![Components of Simscape library](image)

**Figure 2.4 Components of Simscape library**

It is to be noted from Figure 2.4 that components that describe the phenomena described in sections 2.1.1 and 2.1.2 are not present in the Simscape library. In other words, with the pre-defined Simscape parts and the signal flow components in the Simulink library, it is not possible to perform the system-level simulation of MEMS devices. Nevertheless, Simscape allows text-based implementation of custom components using the MATLAB based Simscape Language, in which the new library containing MEMS components was developed.
2.4 Methodology for designing Electromechanical diports in Simscape Language

In capacitive MEMS devices, the electromechanical interfacing takes place in the capacitor plates. The energy flow from the electrical to the mechanical domain can occur in capacitive actuators, where the plates are actuated by varying the electrostatic force in between the plates. Conversely, in capacitive sensors, the energy flow from the mechanical to the electrical domain takes place by varying the capacitance (electrical quantity) between the plates through the changes in mechanical quantities like the gap distance between the plates or the total area of action. Generally, MEMS uses two types of such capacitive diports, namely the gap-varying capacitors and the area-varying capacitors. It is known that the capacitance of parallel conducting plates separated by a dielectric (in this case, air), depends on the overlapping area, $A$ and the gap distance, $d_0$ between the plates. The working principle and the governing equations of capacitance and the resulting electrostatic force of the two types of capacitor arrangement (gap-varying and area-varying) are depicted in Figure 2.5. The notations, $k$ and $x$ were already explained in Figure 2.2.
Figure 2.5 Simplified geometry and governing equations of gap-varying and area-varying capacitors, respectively. The variable quantity, $x$ and the overlapping length, $l$ have been depicted in red colour for better discernment.

The relation between the across-through variables for the gap-varying and the area-varying scenarios are discussed and derived in Appendix A. The unified expression consisting of the electrical and mechanical quantities are shown in Table 2.2, along with their functions. The across variables are voltage, $u$ and velocity, $v$, while the through variables are current, $i$ and force, $f$ in Table 2.2. The parameter $l_0$ in the table refers to the length of the area-varying electrodes.
<table>
<thead>
<tr>
<th>S. No</th>
<th>Component name / Functions</th>
<th>Equations</th>
</tr>
</thead>
</table>
| 1.    | **Gap-varying capacitors (moving towards):** Embodies a pair of gap-varying fixed & movable capacitor plates in which one is moved towards the other. | Charge conservation equation: $$i = uv \frac{C_0}{d_0 \left(1 - \frac{x}{d_0}\right)^2} + C \frac{du}{dt}$$  
Energy conservation equation  $$f = -\frac{1}{2}u^2 \frac{C_0}{d_0 \left(1 - \frac{x}{d_0}\right)^2}$$ |
| 2.    | **Gap-varying capacitors (moving away):** Embodies a pair of gap-varying fixed & movable capacitor plates in which one is moved away from the other. | Charge conservation equation: $$i = -uv \frac{C_0}{d_0 \left(1 + \frac{x}{d_0}\right)^2} + C \frac{du}{dt}$$  
Energy conservation equation  $$f = \frac{1}{2}u^2 \frac{C_0}{d_0 \left(1 + \frac{x}{d_0}\right)^2}$$ |
| 3.    | **Area-varying capacitors (moving towards):** Embodies a pair of area-varying capacitor plates in which the movable capacitor moves towards the fixed plate | Charge conservation equation: $$i = \frac{2C_0uv}{l_0} + 2C_0u' + \frac{2C_0xu'}{l_0}$$  
Energy conservation equation  $$f = -\frac{C_0u^2}{l_0}$$ |
| 4.    | **Area-varying capacitors (moving away):** Embodies a pair of area-varying capacitor plates in which the movable capacitor moves towards the fixed plate | Charge conservation equation: $$i = \frac{-2C_0uv}{l_0} + 2C_0u' - \frac{2C_0xu'}{l_0}$$  
Energy conservation equation  $$f = \frac{C_0u^2}{l_0}$$ |

Table 2.2 List of components describing the bi-directional energy flow in MEMS devices, along with their functions and governing equations.
The first two components listed in Table 2.2 (Gap-varying capacitors – moving towards and Gap-varying capacitors – moving away), both represent gap-varying capacitors. However, their governing equations have a difference in signs. This is because the models do not dictate the geometrical ‘direction’ of the movable electrodes’ motion. The direction is only expressed through the sign of ‘displacement’ variable, $x$ in the model derivations and equations. In other words, it is impossible to say whether the proof mass is going left or right, only whether it moves towards the fixed electrode or away from it. Therefore, $-x$ signifies that the movable electrode is shifting towards the fixed electrode while $+x$ denotes the alternative motion.

Hence, a pair of gap-varying and area-varying capacitors were developed, one of the pair signifying motion towards the fixed electrode and the other, away from it. For instance, to build a set of say, gap-varying capacitors, it is essential to connect the pair (towards and away) of gap-varying capacitors (S.no 1 and 2, listed in Table 2.2) in parallel to adequately model the physical system.

The algorithm for building the components in Simscape Language is presented in Figure 2.6.
An image of the diports and their interface is shown in Figure 2.7. In the custom diports, parameters such as the nominal gap distance, $d_0$ and the nominal capacitance, $C_0$ are received from the users. The diport, in addition to the two ports (or four terminals) for electromechanical interfacing, also contains a terminal that outputs the change in capacitance to provide an easy interface to Simulink.
MEMS devices, like any other instrument, experience mechanical shock due to transportation and use. Mainly, since inertial sensors are used in vehicles and devices that are in rapid motion, they are more prone to mechanical shocks that can potentially damage the device’s structural integrity. Among these reasons are the internal factors as well. Evidently, pull-in can be detrimental in the sense that it can cause snapping of the electrodes and may lead to the device breakdown.

Consequently, existing devices employ a simple mechanical stopper to prevent any collision of suspended structures. A stopper is an anchored structure, generally bulkier than the movable electrodes, fixed at a certain distance from the proof mass. The proof mass, under shock, may collide with the stopper but is prevented from the collision with electrodes, averting short circuit and other fatal collapses of the structural integrity. There have been many investigations on the
designs and materials for stoppers [18]. However, from the Finite Element Simulations’ point of view, stoppers are embodied only as boundary conditions.

In this work, the effects of pull-in have been modelled using a non-linear spring component that behaves like a stopper, thus also preventing the imminent short-circuit of the device, should the pull-in occur. The action of the stopper has been elucidated using a non-linear spring in Figure 2.8, which has a linear region with the user-desired spring constant for the displacement that is within two-thirds of the gap distance $d_{stop}$. Beyond $d_{stop}$, the spring exercises a saturation on the position of the proof-mass. The algorithm behind the non-linear spring component has been explained using Figure 2.9.

![Figure 2.8 The action of non-linear spring component to simulate large-signal behaviour](image)
Figure 2.9 Algorithm flowchart for designing the non-linear spring component in Simscape Language

An image of the stopper component with its block parameters is shown in Figure 2.10. In the custom stopper component, parameters such as the spring constant and stopper position are got from the users. The component has two ports, very similar to a built-in mechanical spring block in Simscape.
2.6 System-level simulation using the hybrid simulator (Simulink + Simscape)

As established in section 2.2, the mechanical parts of a MEMS capacitive accelerometer have been modelled as a mass-spring-damper system. It is to be noted that in order to model the pull-in phenomenon in the device, the regular spring component was replaced with the newly designed non-linear spring block from the proposed MEMS library. After the translational grounding of the mechanical circuit, the two terminals have been interfaced with the $+M$ and $-M$ ports of the electromechanical diport block, as shown in Figure 2.8.

In the design, only the actuation capacitors are area-varying while the biasing and the sensing capacitors are gap-varying electrodes. This means that the appropriate diport components (gap-varying and area-varying) were used for different sets of capacitors (sensing, biasing and actuation capacitors). The biasing component was connected to a voltage source that may be excited for biased scenarios and inactive otherwise. The instantaneous capacitance, $C(t)$ is directly obtained
from the diport models. Therefore, we have the provision to interface a read-out circuit with the sensing capacitors.

The actuation capacitors are interfaced with the control circuitry (only in the case of closed-loop). External force or acceleration from Simulink can be fed to the mechanical circuit as an ideal force source. While Simscape performs energy-flow via the electromechanical diports, the information-flow from Simulink can be interfaced using blocks called “S-PS Converter” and “PS-S Converter”, in which PS stands for Physical Signals and S stands for Simulink, as shown in Figure 2.11.

The structural design of the device will be explained in the next chapter in section 3.1. However, the bias electrodes were designed such that when excited, the proof mass shall be attracted to the positive x-axis (assuming that the axis of the device operation is x-axis). Hence, by biasing the component ‘gap-varying capacitor - moving towards’, (highlighted in orange in Figure 2.11), the proof mass is automatically biased and attracted to the positive x-axis.
Figure 2.11 MEMS accelerometer in Simscape using the proposed MEMS library a) Mechanical circuit; b) Electromechanical diport and electrical circuit. The biasing capacitor is highlighted in orange.
2.7 Results and Discussions

The system-level simulation of the MEMS capacitive accelerometer has been carried out in two phases. In order to test the performance of the proposed components’ models in linear operating conditions, the small-signal analysis in section 2.5.1 compares the output from the Simscape model with the analytical computation results and the results from Finite Element simulation in COMSOL Multiphysics.

To establish the reliability of the proposed library during system instabilities, section 2.4.2 tests the device model in its non-linear region by performing the large-signal analysis.

2.7.1 Small signal analysis: operation in the linear region

The open-loop accelerometer was simulated using both energy-based macro-modelling (Simscape+Simulink), and through finite element analysis (Comsol Multiphysics). External mechanical accelerations from 10 mg to 5 g were applied to the models, and their resultant displacements (of the proof mass) were detected. Further, the results from physical network modelling and FEA are compared with analytical computations in Table 2.2.
<table>
<thead>
<tr>
<th>$A_{ext}$ (g)</th>
<th>$F_{ext}$ (N)</th>
<th>Displacement, $x(t)$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(9.8 m $A_{ext}$)</td>
<td>FEA</td>
</tr>
<tr>
<td>0.01</td>
<td>8.82E-09</td>
<td>1.4817E-10</td>
</tr>
<tr>
<td>0.05</td>
<td>4.41E-08</td>
<td>7.4083E-10</td>
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<tr>
<td>0.1</td>
<td>8.82E-08</td>
<td>1.4817E-09</td>
</tr>
<tr>
<td>0.5</td>
<td>4.41E-07</td>
<td>7.4083E-09</td>
</tr>
<tr>
<td>1</td>
<td>8.82E-07</td>
<td>1.4817E-08</td>
</tr>
<tr>
<td>5</td>
<td>4.41E-06</td>
<td>7.4083E-08</td>
</tr>
</tbody>
</table>

Table 2.3 Small-signal analysis: Proof mass displacements comparison between FEA, energy-based modelling and analytical computation

From the results tabulated in Table 2.2, the relative displacement error between the energy-based modelling and Finite element simulation ranges from -0.022% to 0.004% and that between the energy-based modelling and analytical calculation is -0.022% to 0.003%. However, for the relative error to be as low as the above-said percentages, the magnitude of the displacement error is in the order of $10^{-14}$ m. Therefore, the MEMS library components employed in the simulation were able to achieve results as reliable as Finite Element Simulations (in Comsol Multiphysics) and the analytical computation.

The simulation time taken for the simulation of the open-loop accelerometer using finite element methods (in COMSOL Multiphysics), block-diagram modelling in MATLAB/Simulink and energy-based modelling in the hybrid Simulink+Simscape simulator are tabulated in Table 2.4. Note that even though the energy-based modelling is slower than transfer function modelling, it is
much quicker than Finite Element Simulations. This is reasonable since FEA considers several degrees of freedom to compute the solution.

<table>
<thead>
<tr>
<th></th>
<th>Finite Element Method in COMSOL Multiphysics</th>
<th>Energy-based modelling</th>
<th>Signal flow-based modelling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation time (seconds)</td>
<td>21</td>
<td>6.08</td>
<td>3.625</td>
</tr>
<tr>
<td>(With coarse mesh)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.4 Simulation time comparison between different methods of simulation of open-loop non-linear systems

2.7.2 Large-signal analysis: Operation at the pull-in

The simulations in this section aim to check the reliability of the stopper action of the non-linear spring component.

Firstly, as depicted in Figure 2.9, the device was subjected to an external acceleration of 0.5 g, while the bias electrodes were excited with the pull-in voltage (25V, DC). The purpose of this experiment is to witness the non-linear region (wherein the interaction between the electrostatic force and the spring force transpires) before the proof mass motion is arrested by the stopper.
Figure 2.12 Non-linear region in the operation of the open-loop accelerometer, biased at the pull-in.

The stopper action can be simulated using the saturation block in Simulink information flow modelling. However, biasing a capacitor cannot be implemented per se. Electrostatic force due to that voltage need to be established as a function of displacement an

### 2.7.2.1 Hysteresis in the proof mass displacement due to the bias voltage

This experiment is performed by biasing the device, asymmetrically, with a periodic bias voltage of maximum magnitude slightly higher than the pull-in voltage. When there is no external acceleration on the device, it is seen that until reaching the vertical line A (in Figure 2.11), the proof mass is subjected to an electrostatic force that is proportional to the square of the applied bias voltage. Upon reaching the pull-in voltage (line A), the proof mass is actuated by electrostatic forces that cannot be compensated anymore by the restoring spring force. The stopper prevents the collapse of the system by saturating the displacement at 2.33 um.
However, the proof mass is observed to be under the undue influence of the electrostatic force even after the bias voltage reduces to less than 25V (at line B). Similarly, lines C and D represent the mark of application and removal of bias voltage higher than the pull-in voltage, but with the opposite polarity. Since, irrespective of the polarity of the voltage, the electrostatic force is attractive, both the positive and negative half cycles of the bias voltage engender displacements along the same direction.

![Image](image.png)

**Figure 2.13** Dependence of the proof mass displacements on the actuation voltage (zero input acceleration).

The above-discussed hysteresis behaviour was confirmed by plotting the displacement versus bias voltage graph in Figure 2.14. Notice that the full plot, including the positive and negative polarities of the bias voltage (plotted in the inset of Figure 2.14), though symmetrical, is always to the positive side of the displacement.
Figure 2.14 Periodic excitation with no external acceleration on the device.

2.8 Conclusions

This chapter begins by revealing the need for energy flow-based modelling and the need for Simscape library dedicated to MEMS components. After explaining various non-linear phenomena of MEMS inertial sensors, the chapter describes the challenges in and methodology for building the MEMS library. The newly developed library components are summarized in Table 2.5. The performance and accuracy of the library components have been corroborated by performing small-signal and large-signal analysis of the MEMS accelerometer in the hybrid simulator (Simulink+Simscape).
<table>
<thead>
<tr>
<th>S. No</th>
<th>Component name</th>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Gap-varying capacitors (moving towards)</td>
<td>Embodies a pair of gap-varying fixed &amp; movable capacitor plates in which one is moved towards the other.</td>
</tr>
<tr>
<td>2.</td>
<td>Gap-varying capacitors (moving away)</td>
<td>Embodies a pair of gap-varying fixed &amp; movable capacitor plates in which one is moved away from the other.</td>
</tr>
<tr>
<td>3.</td>
<td>Area-varying capacitors (moving towards)</td>
<td>Embodies a pair of area-varying capacitor plates in which the movable capacitor moves towards the fixed plate</td>
</tr>
<tr>
<td>4.</td>
<td>Area-varying capacitors (moving away)</td>
<td>Embodies a pair of area-varying capacitor plates in which the movable capacitor moves towards the fixed plate</td>
</tr>
<tr>
<td>5.</td>
<td>Stopper</td>
<td>Models a non-linear spring that blocks any displacements beyond the stopper distance, set by the users</td>
</tr>
</tbody>
</table>

Table 2.5 List of newly developed MEMS library components in Simscape

It has been established that in the small-signal analysis, the proposed model was able to emulate the accuracy level of Finite Element Simulations with a maximum error in the order of $10^{-14}$ m. The FEA in COMSOL Multiphysics took 3.5 times more time than the simulation in the hybrid simulator (Simulink + Simscape), while Simulink took only 0.6 times that time. Although acausal simulation takes almost twice the time, several phenomena such as the bi-directional energy
interaction and the behaviour of the system near pull-in are taken into account. For instance, various tests performed on the stopper component (non-linear spring block) in section 2.5.2 demonstrates the hysteresis of the proof mass displacement with respect to the bias voltage.

By validations from the small-signal and large-signal analysis performed on the library components, the system-level modelling using the library will be integrated with digital control and read-out in the following chapters.
Chapter 3: Fully digital MEMS accelerometer using Sliding Mode Control

MEMS accelerometers are generally operated in closed-loop to improve the linearity of measurement and expand the dynamical range of sensing. Besides, many MEMS devices, such as the pull-in accelerometer, are operated on the verge of stability to improve the sensitivity of the device. In this chapter, we have demonstrated the closed-loop control of a single-axis MEMS capacitive accelerometer using the proposed MEMS library components in Matlab/Simscape. The working principles of MEMS capacitive accelerometers have been visited in sections 1.2.1.

Firstly, the design specifications were discussed, followed by the modal analysis of the device in Finite Element software, COMSOL Multiphysics. The single-axis accelerometer with closed-loop SMC feedback has been built using the proposed new Simscape components. The results have been validated against the manual computations and information-flow based modelling in Matlab/Simulink.

3.1 MEMS structure design

A single-axis MEMS capacitive accelerometer was built using Solidworks 2018 following the design rules presented in Table 3.1. The device, shown in Figure 3.1, was fabricated using Simon Fraser University’s Multi-user MEMS process (SFU-MUMPS). As seen from the figure, the proof mass has many etch holes to release the suspended structural layer. In spite of the minimum gap limited by the fabrication technology to 2 um, the minimum gap in the design was made to be 3.5 um to remain on the safer side.
<table>
<thead>
<tr>
<th>Rule/Specifications</th>
<th>Limit of dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum line width</td>
<td>3 um</td>
</tr>
<tr>
<td>Minimum gap</td>
<td>2 um</td>
</tr>
<tr>
<td>Maximum gap between structures</td>
<td>20 um</td>
</tr>
<tr>
<td>Maximum length of any released structure</td>
<td>1000 um</td>
</tr>
<tr>
<td>Maximum length to width ratio for beams</td>
<td>100:1</td>
</tr>
<tr>
<td>Etch hole dimension</td>
<td>3 um X 3 um</td>
</tr>
<tr>
<td>Lateral distance between etch holes</td>
<td>10 um</td>
</tr>
<tr>
<td>Device layer thickness (height)</td>
<td>50 um</td>
</tr>
<tr>
<td>Oxide layer thickness (height)</td>
<td>2 um</td>
</tr>
<tr>
<td>Blanket metal thickness</td>
<td>50 nm Cr + 250 nm Au</td>
</tr>
</tbody>
</table>

Table 3.1 Design rules and specifications for fabrication in SFU – Multi-user MEMS process (SFU-MUMPS)
A MEMS accelerometer was constructed in compliance with the SFU-MUMPs technology, with the parameters presented in Table 3.2. Further, Table 3.3 renders the assumed constants and some related parameters such as the nominal capacitance, pull-in voltage, etc., which were computed using known equations (7-9).
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbols</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>m</td>
<td>90</td>
<td>ug</td>
</tr>
<tr>
<td>Resonant frequency</td>
<td>( f_r )</td>
<td>4092.5</td>
<td>Hz</td>
</tr>
<tr>
<td>Spring constant</td>
<td>k</td>
<td>59.53</td>
<td>N/m</td>
</tr>
<tr>
<td>Length of actuation electrodes</td>
<td>( L_a )</td>
<td>180</td>
<td>um</td>
</tr>
<tr>
<td>Number of actuation electrodes</td>
<td>( n_a )</td>
<td>36</td>
<td>-</td>
</tr>
<tr>
<td>Length of bias electrodes</td>
<td>( L_b )</td>
<td>125</td>
<td>um</td>
</tr>
<tr>
<td>Number of bias electrodes</td>
<td>( n_b )</td>
<td>22</td>
<td>-</td>
</tr>
<tr>
<td>Length of sensing electrodes</td>
<td>( L_s )</td>
<td>125</td>
<td>um</td>
</tr>
<tr>
<td>Number of sensing electrodes</td>
<td>( n_s )</td>
<td>28</td>
<td>-</td>
</tr>
<tr>
<td>Gap distance between electrodes</td>
<td>( d_0 )</td>
<td>3.5</td>
<td>um</td>
</tr>
<tr>
<td>Width of electrodes</td>
<td>w</td>
<td>3.5</td>
<td>um</td>
</tr>
</tbody>
</table>

Table 3.2 Physical dimensions of the accelerometer submitted for micro-fabrication

\[
C_0 = \frac{\varepsilon An}{d_0} \quad (7)
\]

\[
V_{pi} = \left( \frac{8kd_0^2}{27Cb_0} \right)^{1/2} \quad (8)
\]

50
\[ d_{stop} = \frac{2}{3} d_0 \]  

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbols</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permittivity</td>
<td>( \varepsilon )</td>
<td>8.854e-12</td>
<td>F/m</td>
</tr>
<tr>
<td>Nominal actuation capacitance</td>
<td>( C_{a0} )</td>
<td>0.82</td>
<td>pF</td>
</tr>
<tr>
<td>Nominal bias capacitance</td>
<td>( C_{b0} )</td>
<td>0.35</td>
<td>pF</td>
</tr>
<tr>
<td>Nominal sensing capacitance</td>
<td>( C_{s0} )</td>
<td>0.44</td>
<td>pF</td>
</tr>
<tr>
<td>Pull-in voltage</td>
<td>( V_{pi} )</td>
<td>24.92</td>
<td>V</td>
</tr>
<tr>
<td>Pull-in distance</td>
<td>( d_{stop} )</td>
<td>2.33</td>
<td>um</td>
</tr>
</tbody>
</table>

Table 3.3 Assumed and computed quantities

3.2 Design validation through Finite Element Simulations

The 3D CAD model of the device was imported into Comsol Multiphysics 5.3 got finite element analyses. Firstly, using the modal analysis, the next three eigenmodes following the first resonant mode were plotted in Figure 3.2. Since the second eigenfrequency is at 20 kHz (approximately six times the first resonant mode), significantly far away from the first mode (at 4000Hz), it is safe to declare that the accelerometer design would be immune to cross-axial perturbations and that the cross-talks are minimized.
Figure 3.2 Eigenmodes computed in Comsol Multiphysics 5.3
3.3 Information flow-based linear modelling in Matlab/Simulink

Capacitive accelerometer generally consists of a proof mass suspended by beams anchored to a stationary structure. Hence, from a dynamic viewpoint, a simplified model is that of a second-order mass-spring-damper system. The (open-loop) system dynamics can be condensed using the second-order differential equation (10).

\[ F_{\text{inertial}} + F_{\text{damping}} + F_{\text{spring}} = F_{\text{external}} \]
\[ m\ddot{x} + b\dot{x} + kx = ma \]  

\[ (10) \]

---

**Figure 3.3** Implementation of open-loop linearized MEMS accelerometer using information flow-based modelling in Matlab/Simulink.

The resulting transfer function (10) can be implemented in Matlab/Simulink in two ways: Using block diagrams and using transfer function block, both methods effectively having the same functionality (in the case of a linear system approximation). Figure 3.3 illustrates both ways of implementation in Matlab/Simulink using signal flow-based modelling.
3.3.1 Implementation of Sliding Mode Control in Matlab/Simulink

Figure 3.4 Implementation of Sliding Mode Control for a linearized transfer function of the accelerometer in Matlab/Simulink

Figure 3.4 is the screen-capture of the author’s implementation of the SMC loop using signal-flow based modelling in Matlab/Simulink R2018b. The output (proof mass displacement) from the transfer function block is digitized by the Clock and Sample & Hold blocks. Later, the comparator performs a simple digital switching, based on the sign of the proof-mass displacement.

Figure 3.5 Composition of comparator block with one-channel bitstream output in the signal flow-based modelling
The bitstream from the digital comparator is then used to adjust the electrostatic feedback acceleration, that is then subtracted from the original input acceleration in order to feed the error signal as the input to the transfer function block. The code within the ‘feedback acceleration’ block is presented in Figure 3.6.

```
function Aelec = fcn(x,bitstream)

m = 9.00e-08;  % Mass in kgs
fr = 4092.500;  % Resonant frequency in Hz - Obtained from comsol
wr = 2*pi*fr;  % Resonant frequency in rad/s
Q = 0.707;  % Quality factor
k = 59.5085;  % Spring constant in N/m
b = 0.0032;  % Damping coefficient
Cao = 8.19e-13;  % Nominal capacitance in Farads
do = 3.5e-6;  % Gap distance
Vext = (2*do*(50*m)/Cao)^(-0.5);
Vact = alpha*Vext;
Felec = (Cao*do*Vact^2)/(2*(do-x)^2);  % Feedback electrostatic force
Aelec = bitstream*Felec/m;  % Feedback electrostatic acceleration
```

Figure 3.6 Codes for the function block titled ‘Feedback Acceleration’ in Figure 3.4.
3.4 Energy flow-based modelling of closed-loop MEMS accelerometer with Sliding Mode Control in hybrid (Simulink + Simscape) simulator

In physical reality, SMC feedback excites the actuation electrodes such that the electrostatic actuation force always tends to cancel out the input mechanical force on the proof mass due to the external acceleration as shown in (11).

\[
\text{Actuation electrostatic force} = \begin{cases} 
+F_{\text{act}} & \text{When } x > x_{\text{threshold}} \\
-F_{\text{act}} & \text{When } x < -x_{\text{threshold}} \\
0 & \text{When } x_{\text{threshold}} > x > -x_{\text{threshold}} 
\end{cases} \tag{11}
\]

The expression for the electrostatic actuation force is given in (12). It is clear that electrostatic force, \( F_{\text{act}} \) is proportional to the square of the applied actuation voltage, \( V_{\text{act}} \).

\[
F_{\text{act}} = \frac{1}{2} \varepsilon A n V_{\text{act}}^2 \frac{d}{d_0} \tag{12}
\]

This implies that irrespective of the polarity of the voltage, the force is always attractive. Therefore, the actuation capacitors, placed on either side along the direction of motion, are excited only one at a time for the movement towards the capacitor. For instance, if the external force tries to push the proof mass to one side, the actuation force tends to attract it towards the other side, thereby maintaining a neutral position. The sliding mode control technique is very robust, so this method can be used to operate the accelerometer beyond the pull-in voltage, in order to improve the sensitivity of the system.
The two sets of actuation electrodes on either side of the proof mass can be found in Figure 3.1. Compared with the pure signal flow Simulink implementation, Simscape modelling can better model the physical system, including the bi-directional electromechanical coupling.

The block diagram in Figure 3.7 is the screen-capture of the author’s implementation of Sliding Mode architecture using energy-flow based modelling in the hybrid (Simulink and Simscape) simulator. The “capacitive accelerometer” sub-system in Figure 3.7 was be constructed using the energy-flow based modelling in Simscape, as demonstrated in Figure 2.8. The variable sensed capacitance, $C(t)$ from the Simscape, is digitized using the clock and Sample & Hold components of Simulink library.

![Figure 3.7 Implementation of Sliding Mode Control on MEMS accelerometer using energy flow-based modelling in the hybrid simulator (Simulink+Simscape). Forward information is represented in green while the SMC feedback is in red.](image)

57
Figure 3.8 Energy-based modelling of the MEMS accelerometer with SMC feedback actuation. This entire system is encompassed in the sub-system named ‘Accelerometer in Simscape’ in Figure 3.7.
The clock frequency was set to be 1MHz, which is at least 25 times the resonant frequency ($f_{\text{clk}} \gg f_r$) of the accelerometer. The digital $C(t)$ is then sent to a comparator block whose composition is shown in Figure 3.8. The comparator on the top compared $C(t)$ with 0. It outputs 1, if $C(t)>0$ or 0, if $C(t)<0$ (zero threshold)

![Comparator Block Diagram](image)

**Figure 3.9 Composition of comparator block with two-channel output bitstreams in the energy flow-based modelling**

The comparator block used during the information flow-based modelling in Simulink (in Figure 3.5) outputs a signal channel bitstream that is directly subtracted from the input acceleration to feed the error signal to the accelerometer. This is similar to the Simulink implementation shown in Figure 3.4. Nevertheless, the energy flow-based modelling of the MEMS accelerometer requires two-channel bitstreams for exciting the two sets of actuation electrodes on either side of the proof mass to provide the electrostatic force feedback. Simscape mimics the physical system structure.
more accurately by taking into account physical quantities, like the actuation voltage, rather than merely signal manipulation like generating and feeding error signals.

As seen from Figure 3.7, the two-channel bitstreams output (out port 1) from the comparator are provided as feedback inputs to the ‘Accelerometer in Simscape’ sub-system, independently. This digital feedback is reflected in Figure 3.8, in the input ports 3 and 4. Each of these channels (1s and 0s), multiplied by the actuation voltage, $V_{act}$ will control the actuation electrodes digitally by exciting them with 0 (OFF) or $V_{act}$ (ON). Thus the two-channel bitstreams activate the actuation capacitors on either side of the proof mass.

### 3.5 Results and discussions

#### 3.5.1 Motion cancellation using Sliding Mode Control

The operation of the MEMS accelerometer was tested using 3 cases: Open-loop, SMC modelling with information flow in MATLAB/Simulink and SMC modelling with energy flow in hybrid (Simulink+Simscape) environment. Figure 3.10 compares the proof-mass displacements in all three cases.
Figure 3.10 Motion cancellation using Sliding Mode Control using signal flow and energy flow-based modelling

From the Figure 3.10, it is observed that attenuation in closed-loop displacement was 91.9% of the open-loop displacement in Simulink modelling (signal flow), while it was 93.2% in hybrid (Simulink+Simscape) modelling (energy flow). The time taken by the simulators (Simulink for linear model and hybrid – Simulink+Simscape – for energy-based model) are listed in Table 3.4.

<table>
<thead>
<tr>
<th></th>
<th>Signal flow-based modelling in MATLAB/Simulink</th>
<th>Energy flow-based modelling in Simulink+Simscape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation time</td>
<td>43.5</td>
<td>50.4</td>
</tr>
</tbody>
</table>

Table 3.4 Simulation time comparison for closed-loop accelerometer with Sliding Mode Control, using information flow and energy flow-based models
Acausal modelling using the hybrid simulator takes 6.9 seconds more to simulate the model than the causal simulation in Simulink. It is natural since the solvers are different, with continuous communication between the two simulation environments (Simulink and Simscape). However, the signal flow itself in open-loop only took 3.2 seconds, according to Table 2.4, whereas the closed-loop simulation takes 43.5 seconds. Hence, it is safe to infer that the SMC loop is responsible for the delay in simulation time than the model of the accelerometer itself. It is owed to the digitization of the output signal from the accelerometer (displacement or sensed capacitance) with the clock period of 1 microsecond (frequency = 1MHz).

3.5.2 Signal reconstruction using low pass filter

Accelerometers are devices that measure the input acceleration. Hence, the reconstructed accelerations using signal flow and energy flow-based modelling were plotted respectively against the input acceleration in Figure 3.11. A periodic acceleration of 5g amplitude was the input applied in both cases. Both types of modelling cases (signal flow and energy flow) perform low-pass filtering of the corresponding bitstreams to reconstruct the input acceleration. Hence, the dissimilarity is not in the low-pass filtering of the digital output signal but rather in the way the bitstream has been generated.
Figure 3.11 Comparison of the reconstructed and input periodic acceleration from simulations using both methods of modelling techniques (signal flow and energy flow)

The absolute steady-state error and the relative error of the reconstructed acceleration with respect to the input acceleration were computed and tabulated in Table 3.5, according to (13) & (14). Note that the steady-state delay is a normal consequence of the low-pass filtering chain and can be improved by using better-performing filters.

\[
error(t) = \text{input acceleration} - \text{reconstructed acceleration} \tag{13}
\]

\[
\text{Relative error} = \frac{error(t)}{\text{input acceleration}} \times 100\% \tag{14}
\]
<table>
<thead>
<tr>
<th></th>
<th>Input acceleration</th>
<th>Signal flow-based modelling in MATLAB/Simulink</th>
<th>Energy flow-based modelling in Simulink+Simscape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute maximum value,</td>
<td>49</td>
<td>48.55</td>
<td>49.03</td>
</tr>
<tr>
<td>(ms$^{-2}$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steady-state error, (ms$^{-2}$)</td>
<td>-</td>
<td>0.45</td>
<td>-0.03</td>
</tr>
<tr>
<td>Relative error %</td>
<td>-</td>
<td>0.92%</td>
<td>-0.06%</td>
</tr>
<tr>
<td>Phase difference</td>
<td>-</td>
<td>2.2 rad (lag)</td>
<td>2.2 rad (lag)</td>
</tr>
</tbody>
</table>

Table 3.5 Reconstruction of input acceleration from the bitstreams generated by Sliding Mode Control loop implemented in MATLAB/Simulink and Simulink+Simscape

3.5.3 Performance of the biased closed-loop MEMS accelerometer

For a slow (ramp) increase in the input acceleration, the reconstructed (output) accelerations under different bias voltages are plotted in Figure 3.12. The input acceleration was increased linearly from 0g to 1g. Bias voltages of 0V, 10V, 20V and 25V (pull-in) were applied to the bias capacitors in the direction of application of the input acceleration. The slopes of the input acceleration versus output acceleration graph in Figure 3.12 denote the sensitivity of each of the cases. At 0V bias, the sensitivity of the mechanism, 1.005, is the sensitivity of the closed-loop accelerometer with the low-pass filtering.
Figure 3.12 Comparison of the sensitivity of closed-loop MEMS accelerometer modelled using the proposed Simscape MEMS components with different bias voltages

The sensitivities are observed to increase with the increase in the bias voltage to the device. Figure 3.12 not only proves that it is beneficial to operate the MEMS accelerometer at the pull-in to improve its sensitivity, but also that the proposed SMC loop has successfully balanced the proof mass at the pull-in voltage, even when a maximum of 1g acceleration has been applied in the same direction.

Figure 3.13 proves that the model can successfully balance the device at the pull-in voltage under the application of periodic input acceleration with the maximum amplitude of 5g. While the open-loop accelerometer pre-biased at pull-in is an unstable system, the closed-loop SMC system
operates, maintaining the MEMS accelerometer on the stability border, proving the robustness of the digital control technique.

**Figure 3.13** Closed-loop operation of the MEMS accelerometer, biased at the pull-in voltage

### 3.6 Conclusions

This chapter has shown the design and simulation of Sliding Mode Control for MEMS accelerometers, using both Simulink and Simscape. The chapter demonstrated the seamless operation of the single-axis MEMS accelerometer with digital SMC loop and the reconstruction of the input acceleration from the digital bitstream. The results obtained from implementing the Sliding Mode Control loop for the designed MEMS accelerometer are tabulated below in Table 3.6.
<table>
<thead>
<tr>
<th>Motion attenuation (relative to open-loop displacement)</th>
<th>Signal flow-based modelling in MATLAB/Simulink</th>
<th>Energy flow-based modelling in Simulink+Simscape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>91.9%</td>
<td>93.2%</td>
</tr>
<tr>
<td>Signal reconstruction steady-state relative error</td>
<td>0.92%</td>
<td>-0.06%</td>
</tr>
<tr>
<td>Signal reconstruction phase error</td>
<td>2.2 rad (lagging)</td>
<td>2.2 rad (lagging)</td>
</tr>
</tbody>
</table>

Table 3.6 Results summary of MEMS accelerometer with Sliding Mode Control loop

In addition the pull-in results prove that the sensitivity of the accelerometer increases with the bias voltage. At open-loop operation, biasing the device at the pull-in will lead to the loss of stability. Whereas, SMC loop acts as negative feedback and stabilizes the proof mass with biasing even beyond the pull-in voltage.

The chapter advocates the use of physical system modelling in hybrid simulator (Simulink+Simscape) for various reasons. Firstly, the energy flow-based modelling captures all the non-linear behaviour of the system, whereas the Simulink (signal flow)-based modelling incorporates higher levels of abstraction in the system by linearizing the system dynamics around the initial conditions \( (x=0) \). To model the biasing electrodes in information flow modelling, a non-linear function block has to be created for computing the bias electrostatic force as a function of instantaneous displacement. Secondly, this way of modelling bias electrodes is highly application-specific and not reusable. Nevertheless, the gap-varying and area-varying capacitor components
from the proposed MEMS library are versatile and adaptable to function as actuation, bias or sense electrodes (here, bias electrodes are gap-varying, to bias at the pull-in voltage). Thirdly, the feedback actuation is delivered directly to the actuation electrodes in the energy-based model Simscape+Simulink model. This represents the real-time operation of closed-loop devices, while the information flow-based modelling in Simulink models the feedback forces into signals and adds or subtracts with the input signal to feed the error signal into the system. This is only an overall approximation of the real-time system with high levels of linearization, which would not always be accurate. Hence, energy flow-based physical system modelling using the proposed Simscape MEMS library seems to be a better alternative to simulate non-linear systems, than the usual signal flow-based modelling.
Chapter 4: Double-digital control and read-out of MEMS gyroscope using Sliding Mode Control

MEMS vibratory gyroscopes rely on their operation on the coupling between two vibrating modes, the driving and the sensing mode. While the driving mode is responsible for driving the proof mass at the resonant frequency, the sensing mode (orthogonal to the driving mode) is actuated, through the Coriolis effect, only in the presence of an external angular rate that couples the two modes and transfers energy from one into another. Most of the reported works on MEMS gyroscopes have applied control on only one of these modes at a time [22], [48], [49], and the concept of a fully digital MEMS gyroscope has scarcely been reported [28], [50].

In this work, a fully digital MEMS gyroscope using sliding mode control has been designed and simulated [51]. Furthermore, the read-out for sensing the external angular rate signal has been made fully digital by applying an innovative synchronous demodulation technique at the bitstream level. Sliding mode control has been chosen as the digital control scheme, for the reasons mentioned in Chapter 1. In addition to reporting the two novelties, this chapter compares the implementation of the SMC feedback of a MEMS gyroscope using two approaches of modelling: signal flow in MATLAB/Simulink, and energy flow in the hybrid – Simulink+Simscape – environments.
4.1 MEMS gyroscope design

A MEMS single-axis gyroscope was designed in Solidworks 2018 to implement the proposed double-digital SMC scheme. The mask layout was developed by a member of the research team and submitted for fabrication in Simon Fraser University’s Multi-User MEMS Process (SFU-MUMP) technology featuring a 50um thick device layer, able to achieve larger inertial masses and interface capacitance values. Figure 4.1 shows the fabricated gyroscope under the microscope. The design rules and other design specifications particular to this technology have been mentioned in Table 2.2 (of Chapter II). The mode decoupling frame structure, highlighted in Figure 4.1, was adopted from [52]. The model was evaluated through Finite Element Analysis in ANSYS MULTIPHYSICS software. The design parameters have been summarized in Table 4.1.

Figure 4.1 Fabricated single-axis MEMS gyroscope under a microscope
<table>
<thead>
<tr>
<th>Design Parameters</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (m)</td>
<td>kg</td>
<td>8.44E-8</td>
</tr>
<tr>
<td>Spring constant (kd, ks)</td>
<td>N/m</td>
<td>104.42</td>
</tr>
<tr>
<td>Natural frequency (fd, fs)</td>
<td>kHz</td>
<td>5.60</td>
</tr>
<tr>
<td>Nominal capacitance (Co)</td>
<td>pF</td>
<td>0.496</td>
</tr>
</tbody>
</table>

Table 4.1 Single-axis MEMS gyroscope design parameters

4.2 Gyroscope Microsystem

The architecture of the entire gyroscope microsystem is explained in Figure 4.2. The driving circuit actuates the proof mass along the driven axis, ideally with a harmonic electrostatic force having a frequency coinciding to the resonant frequency of the driving mode (for optimum electromechanical energy transfer). When the entire device is subject to an external angular rate, a Coriolis force is generated along a perpendicular axis (called the sensing axis), according to (3). The Coriolis force, $F_{coriolis}$ is proportional to the vector product of linear velocity, $V$ and the angular rate of the object, $\Omega$. This Coriolis force acts as the input excitation for the sensing vibration mode.
Figure 4.2 Double digital sliding mode control implementation in MEMS gyroscope. The parts in blue colour show the driven mode, while the orange and green parts represent the sensing mode subsystems. Orange illustrates the motion cancellation control, while green describes the process of signal reconstruction.

While in the driven mode a bandpass sliding mode control (BP-SMC) was used to drive the proof mass to oscillate at its resonant frequency, a standard (lowpass) sliding mode control (LP-SMC) was used in the sensing mode to cancel the motion of the proof mass along the sensing mode axis.

The motion cancellation provides advantages in terms of linearity and dynamic range and allows a design with reduced gap distance for capacitive sensing, thus improving the readout sensitivity. The bitstream generated in the SMC loop for sensing mode vibration cancellation embeds the information related to the Coriolis force, and the external angular rate can be extracted through digital synchronous demodulation of the bitstream. Figure 4.3 shows the implementation of the entire gyroscope with the driving and the sensing modes, along with their control and read-out circuits.
4.3 Sense mode sub-system

In contrast with the driving mode where SMC was used to drive the proof mass at the resonant frequency, similar to a tracking phase-locked loop scheme, SMC takes the role of cancelling the motion of the proof mass, in the sensing mode. This motion cancellation provides many advantages, for instance, because the proof mass would not move much, the gap distance between the movable and fixed electrodes can be reduced, thus improving the nominal capacitance. System dynamics are linearized by not allowing the proof mass to experience high displacements. Attenuating the motion of the proof mass would improve the linearity in the sense mode modelling and also increase the dynamic range of the sensing. Significant research has been performed on pull-in accelerometers, resonators and gyroscopes that operate on the verge of their instabilities (at pull-in). On the border of stability, any small perturbation to the system can knock it out of equilibrium. However, this also means that the system is highly sensitive to any input at the border
of its stability. Hence, to improve the device performance, pull-in inertial sensors are operated at or slightly beyond their pull-in but being prevented from collapse by negative feedback.

4.3.1 Methodology for modelling motion cancellation using information flow and energy flow-based modelling

The Coriolis acceleration, from the drive mode, is the input acceleration to the sense mode. The implementation of Sliding Mode Control for the sensing mode of the MEMS gyroscope is very similar to that of the MEMS accelerometer (both with the signal flow in MATLAB/Simulink and energy flow in hybrid Simulink+Simscape simulator), as already explained in detail in sections 3.3 and 3.4. Figures 4.4 and 4.5 illustrate the sensing mode SMC sub-system in MATLAB/Simulink and hybrid (Simulink+Simscape) simulator, respectively.

![Signal flow-based model of the sensing mode sub-system in MATLAB/Simulink](image)

Figure 4.4 Signal flow-based model of the sensing mode sub-system in MATLAB/Simulink
Signal flow implementation of the Sliding Mode Control loop in MATLAB/Simulink was integrated with the two models shown in figures 4.4 and 4.5. The displacement signal measured by the sensing mechanical block was first sent to a range comparator that introduced a tunable dead zone to remain unresponsive to the noise in the system. In other words, if the displacement is small enough to lie within the dead zone, no actuation occurs. Since only the direction of the displacement is taken into consideration, the comparator block outputs a 3-level digital signal (−1,0,1) describing only the instantaneous position of the proof mass relative to its original position (at $d_0$), at every clock cycle. This bitstream output is used as a switch to excite the left-right actuation electrodes with the voltage of $V_{act}$ according to (11).

The range comparison and generation of bitstreams occur inside the block titled ‘Bitstream divider’ in figure 4.3. This bitstream was used to reconstruct the input angular rate using synchronous demodulation, as explained in section 4.3.2.
4.3.1.1 Motion cancellation results

A step angular rate of 300 rad/s with an initial value of 100 rad/s is applied. The graph in Figure 4.6 describes the step response of the sensing mode. The displacements of the closed-loop sensing mode and the open-loop system have been compared, to illustrate the effectiveness of the feedback action for the motion cancellation.

Figure 4.6 The response of the sensing mode to a step variation in the input angular rate for both of the modelling approaches: Information flow (in MATLAB/Simulink) and Energy flow (in hybrid Simulink+Simscape simulator)
It was observed that the motion cancellation using SMC was able to reduce the displacement by about two orders of magnitude. Besides, the performance of the set-up was comparable in both information flow and energy flow-based modelling. The closed-loop peak displacement of the hybrid simulator was found to be only 0.125 nm higher than the peak displacement with the Simulink model.

4.3.2 Signal reconstruction

Low pass filtering of the bitstream generated by SMC is enough to reconstruct the input signal for MEMS accelerometers. However, gyroscopes are resonant devices; the interaction between the two modes (driving and sensing modes) and the electrical and mechanical domains occur at the resonant frequency. Consequently, the low pass filter cannot be employed to reconstruct Coriolis acceleration in MEMS gyroscopes, as it would filter out useful signals. Therefore, this section introduces a technique called bitstream synchronous demodulation, using the digital output of the sensing mode SMC to reconstruct the input angular rate.

Synchronous demodulation is a standard signal reconstruction method employed in various applications. Ignoring quadrature error, non-proportional damping and the aniso-elasticity, the sensing mode displacement dynamics can be expressed in (15) where the instantaneous displacement is given by \( x(t) \) and \( \omega_n \) is the driving frequency, matching (in the ideal case) the mechanical eigenfrequency of the sensing mode.

\[
x(t) = X(t) \cos(\omega t)
\]  

(15)
In (15), the term \(X(t) \cos \omega t\), is the function representing the displacement of the proof mass, where \(X(t)\) is the instantaneous magnitude of displacement, modulated by the external angular rate. An ‘auxiliary’ term, \(\cos \omega t\), which is of the same frequency as and in phase with the displacement of the proof mass is introduced in order to facilitate the demodulation in (16). Figure 4.7 explains the algorithm behind the process of bitstream demodulation.

\[
X \cos(\omega t) \cos(\omega t) = \frac{X(1 + \cos(2\omega t))}{2}
\]

(16)

Figure 4.7 Process flow of signal reconstruction using synchronous demodulation
4.3.2.1 Bitstream demodulation

This process of synchronous demodulation is implemented by a digital multiplier (here, bitstream demodulator). The displacement function is fed to the bitstream multiplier block. The auxiliary term is required to be a bitstream to enable signal multiplication with the bitstream displacement data. For that reason, a pulse density modulator (PDM) with a phase lag of $\frac{\pi}{2}$ to the driving force (used for the driven mode) is used as an auxiliary term generator. Consequently, a separate PLL subsystem is not necessary for auxiliary signal generation. This sine wave bitstream signal is multiplied with the direction bitstream for synchronous demodulation, using 1-bit XOR operations [53].

The operation of the bitstream multiplier block can be explained by the progression in (17), where $L$ is the averaging length, and $x(i)$ and $y(j)$ are the two bitstream inputs. $x(i)$ can be regarded as direction bitstream, $y(j)$ being the auxiliary PDM bitstream. Instead of averaging the input then multiplying the two resultant analog signals, the two inputs are multiplied on bitstream level and then averaged. The output of the bitstream product is $(L^2 + 1)$ level data. For better hardware efficiency, this can be stored as $(L+1)$ bit checksum instead of decimals. This $(L+1)$ bit checksum is then passed through a digital low pass filter to obtain the angular rate by attenuating the double frequency term. The output of LPF is scaled according to system characteristics and a factor of $L^2$ to compensate for the checksum simplification.

\[
\left[\frac{1}{L} \sum_{i=n-L+1}^{n} x(i)\right] \left[\frac{1}{L} \sum_{j=n-L+1}^{n} y(j)\right] \frac{1}{L^2} \sum_{i,j=n-L+1}^{n} x(i)y(j)
\]  

(17)
4.3.2.2 Results and discussions

The proposed methodology of signal reconstruction was simulated using both Matlab/Simulink and hybrid (Simulink+Simscape) models. The microsystem was excited with a step external angular rate of 150 rad/s. Figure 4.8 compares the external angular rate with the reconstructed signals obtained using both ways of modelling. The inset graph in the figure shows the reconstruction of the input angular rate before the system achieves steady-state.

![Step response](image)

**Figure 4.8 Step response of MEMS gyroscope sensing mode with angular rate reconstruction**

For the MATLAB/Simulink model (information flow), the static error was observed as 0.2 rad/s, while for the modelling in the hybrid simulator, using Simulink and Simscape, the error was 0.24 rad/s. This trend substantiates that the demonstrated digital signal reconstruction system was efficient.
Figure 4.9 The sensitivity of the reconstructed angular rate

Figure 4.9 plots the output (reconstructed) angular rates from Simulink and hybrid simulator models against the input angular rate, so that the slope gives the sensitivity of the sensing mode subsystem and the signal reconstruction. It was found that the sensitivity of the energy-based modelling (0.85) is comparable with the sensitivity from the signal flow-based modelling (0.95). Note that the signal reconstruction structure was completely performed using information flow-based modelling in Simulink, for both the cases. Hence, the difference arises only in the bitstreams from the sense modes of each method of modelling.
4.4 SMC in the driving mode

4.4.1 Methodology for implementing driving mode SMC

The function of the driving system is to set and maintain the proof mass in oscillation at the resonant frequency corresponding to the driven eigenmode. Nevertheless, changes in external factors like the operating temperature engender variations in the internal structural elements such as the spring constant. Consequently, the resonant frequency of the device, being dependent on its spring constant, becomes a variable factor. During such times, the driving frequency will not match the eigenfrequency of the device, in the absence of a mechanism for tracking the changes in the mechanical resonance frequency. In this work, band-pass Sliding Mode Control (BP-SMC) is employed to match the driving frequency with the resonant frequency, irrespective of potential deviations in the resonant frequency. The authors of [51] implemented a linearized block diagram based modelling in Matlab/Simulink, as shown in Figure 4.10. The implementation of the drive mode using the hybrid simulation paradigm is explained by Figure 4.11. The frequency-tracking algorithm is explained using the flowchart in Figure 4.12

![Diagram](image-url)  
**Figure 4.10 Implementation of drive-mode in Matlab/Simulink using signal flow-based modelling [51]**
Figure 4.11 Implementation of drive-mode with SMC feedback actuation in Simscape using energy flow-based modelling. The electrical domain components are represented in blue, while the mechanical domain components in green colour.
Figure 4.12 Algorithm of Band-Pass Sliding Mode Control frequency tracking in drive-mode sub-system

The SMC feedback control tracks the changes in the mechanical resonant frequency at each sampling time by checking the phase relation between the electrical actuation and the output displacement signals. The phase difference signal is analyzed to determine whether the actual driving frequency is higher or lower than the current resonant frequency.

4.4.1.1 XOR phase detector

The phase detector block contains an XOR gate which identifies the phase relation between actuation and displacement signals in a binary form. From the general Bode plot of a general second-order system, the output (here, the displacement $x(t)$ of the proof mass) lags the input (here, the electrical actuation force) by $90^\circ$ when the harmonic actuation matches the mechanical resonance. Using this phase difference, depending on whether it is lagging or leading the actuation
force, the phase-detector block decides whether the driving frequency needs to be decreased or increased respectively. Figure 4.13 shows the signal flow within the XOR phase detector block.

![Figure 4.13 Implementation of XOR phase detector block in Simulink [51]](image)

The output binary signal from the XOR gate is sampled and fed to a moving average filter. Ideally, the averaged output is 0.5 when the system is driven at the resonant frequency. Therefore, the averaged output is compared with 0.5 to determine whether the driving frequency is slower or faster than the resonant frequency.

### 4.4.1.2 Digitally controlled oscillator (DCO)

The moving average output from the phase detector (0 to 1) is converted to a 3-level digital signal (-1,0,1) in the intermediate “switch” block, which determines the sign of the $\Delta \omega$ in (18).

$$
\omega_k = \omega_{(k-1)} + \Delta \omega ; \quad \omega_0 = \sqrt{\frac{k_d}{m}}
$$

(18)

$$
\varphi_k = \varphi_{(k-1)} + \omega_k \cdot \Delta t ; \quad (\varphi_0 = 0)
$$

(19)
A constant loop gain defines the absolute value of $\Delta \omega$, whose magnitude affects the lock-in range and the tracking speed of the feedback system. The state-space representation of the resonant frequency tracking is shown by (18) and (19), where, $k_d$ is the designed spring constant of the driving mode, and the initial value of the driving frequency, $\omega_0$ is set to be the designed natural frequency. According to the adjusted driving frequency, the DCO generates a continuous sinusoidal wave by updating the phase, $\varphi$ of its generated signal according to (19), then converts the signal into binary digital output (actuation bitstream) which drives the proof mass.

The DCO is responsible for generating the required electrostatic force to drive the proof mass’s oscillations. The phase of the generated actuation force is recorded and updated at each sampling time to ensure the continuity of the signal. Consequently, the generated wave is smoother so that less disorder is involved even though the driving frequency varies frequently.

4.4.2 Simulation and Results

4.4.2.1 Response time and static error computation with step response

The objective of this simulation is to compute the tracking delay and the steady-state tracking error of the driven mode SMC loop. For this purpose, a step-change in the mechanical resonant frequency was generated by adjusting the spring constant according to the step function in (20), where the variable $k(t)$, denoting instantaneous spring constant, is static in the time range $[0.1, \infty)$.

$$
k(t) = \begin{cases} 
  k_d & \text{for } t < 0.1s \\
  1.05 k_d & \text{for } t \geq 0.1s 
\end{cases} \quad (20)
$$
Figure 4.14 Automatic tracking of the optimal driving frequency for a step variation of the mechanical resonance frequency.

From the step response in Figure 4.14, where the variations of desired and adjusted driving frequencies are plotted against time, the settling time was found to be 92 ms for the signal flow model and 110 ms for the energy flow-based model. The error between the theoretical resonance frequency and the adjusted driving frequency was calculated according to (21). The steady-state tracking error was found to be less than 3 rad/s for both the models and the relative error given by (22) was less than 0.0083%.

\[ error(t) = \omega_{\text{adjusted}}(t) - \sqrt{\frac{k(t)}{m}} \]  \hspace{1cm} (21)

Relative error = \[ \frac{error(t)}{\sqrt{\frac{k(t)}{m}}} \times 100\% \]  \hspace{1cm} (22)
4.4.2.2 Sensitivity and static error computation with ramp response

In this step response, a step jump of more than 10,000 rad/s in the resonant frequency was produced instantaneously. However, in practical systems, this level of temperature fluctuations cannot be generated spontaneously. Since a step-change in the resonant frequency cannot be produced naturally by adjusting the operating temperature, the following experiment demonstrates the driving frequency tracking by a ramp variation in the mechanical resonant frequency. Thus, the spring constant was modelled as a ramp function, which increases from 99.2% of \( k_d \) to 100.8% of \( k_d \) in the time frame specified in Figure 4.15, where \( k_d \) is the static spring constant based on the design specifications.

According to the simulation result in Figure 4.15, the BP-SMC method effectively adjusts the driving frequency to follow the change in the spring constant as the relative steady-state error ranges from 0.03% - 0.06%, depending on the model. Figure 4.16 plots the driving frequencies of the two models against the mechanical resonant frequency to obtain its slope (sensitivity). The results of the two models in the driven mode are summarized in Table 4.2.
Figure 4.15  Automatic tracking of the optimal driving frequency for a ramp variation of the mechanical resonance frequency.

Figure 4.16  Automatic tracking of the optimal driving frequency for a ramp variation of the mechanical resonance frequency.
Signal flow-based modelling in MATLAB/Simulink | Energy flow-based modelling in Simulink+Simscape
---|---
Range of steady-state error in frequency tracking (rad/s) | +20 to -22 | +12 to -10
Relative steady-state error (%) | 0.062 | 0.034
Sensitivity | 1.002 | 1.0002

Table 4.2 Result summary comparing the performance of the two models based on the tracking of the resonant frequency in the driven mode

It can be observed from Table 4.2 that the sensitivity of the hybrid model is 0.18% less than that of the signal flow-based model. However, the steady-state error of the hybrid model is almost half of that of the Simulink model. Hence, not only does the signal reconstruction perform efficiently, but the energy flow-based model works competently as well, and in fact, being more realistic than the signal flow-based model.
4.5 Conclusion & Future Work

The chapter achieves the preliminary objective of implementing an all-digital MEMS gyroscope using sliding mode control for the driven and the sensing modes, and the novel bitstream demodulation for the reconstruction of the input angular rate. The information flow-based implementation of Sliding Mode Control as a substitute for PLL in the drive mode, and for the motion attenuation in the sense mode was put forth in [51] by the members of the research group. The application of synchronous demodulation on bitstreams (the technique of bitstream demodulation) was also an innovative technique [51]. The implementation and validation of the sensing mode’s signal reconstruction and motion cancellation, along with the driven mode’s resonant frequency tracking are the pivotal efforts in building a fully digital MEMS gyroscope.

The proposed control system protects the device and its operation from instabilities in its external environment. Furthermore, the system remains robust despite the prevalence of internal disturbances, such as disturbances in resonant frequency.

However, [51] puts forth the ideas only using MATLAB/Simulink (block diagram) based modelling. In this chapter, in addition to shedding light on the above-discussed novelties of [51] using Simulink modelling, the concepts were implemented also using physical system modelling in the hybrid paradigm (Simulink+Simscape). The published results in [51] have been compared to the results obtained from the energy-based modelling under the same test conditions.
The sense mode SMC was highly efficient in attenuating the motion of the proof mass under the influence of the external acceleration. The closed-loop displacements in both cases were about 100 times less than the open-loop displacements. This is, however, not surprising as the similar performance was achieved in motion cancellation of the MEMS accelerometer in Chapter 3.

Synchronous demodulation on the bitstreams was performed in signal flow, for both Simulink and the hybrid Simulink+Simscape modelling. Hence the difference arises with the gyroscope device model itself and its resulting bitstream from motion cancellation (just like the MEMS accelerometer). The steady-state error in signal reconstruction is 0.2 rad/s for causal model and 0.24 rad/s for the acausal model. This slight discrepancy of 0.04 rad/s in the steady-state error of the two models is almost insignificant when compared to the input angular rate of 150 rad/s.

In the driven mode control loop, the steady-state error in the acausal model was only 0.034% while that in the causal model was 0.062%. The sensitivity of the resonant frequency tracking, illustrated in Figure 4.16 shows that both the models are performing efficiently. Substantial delays in tracking are mostly because of the digital control structure itself, rather than the device model.

Therefore, it is safe to establish that signal flow and energy flow-based models are almost equally competent when it comes to accuracy. However, in terms of simulation time, the signal flow model beats the energy flow model by a few seconds. But both the models are majorly delayed due to the control mechanism; Hence, the delay caused due to the choice of model is almost insignificant.
Chapter 5: Conclusion

This research has been motivated by two levels of goals. Firstly, the MEMS industry’s demand for digital control and read-out of MEMS sensors has driven the thesis into focusing on the full-digital implementation of MEMS read-out and control. Industries prefer digital electronics because of its low-cost, robustness, inherent immunity to noise injection. Additionally, the fact that 90% of consumer electronics are digital makes it imperative for the sensors to be better interfaced with digital electronics. Hence, the primary aim of the research was to deliver the all-digital control and read-out of MEMS accelerometers and gyroscopes. Seamless operation of the MEMS accelerometer and gyroscope were demonstrated with closed-loop digital control and read-out in Chapters 3 and 4, respectively.

However, the research had an indispensable need for a proper simulation tool to model MEMS components, their non-linearities and bi-directional energy coupling between energy domains (aspects in which the pure causal information flow approach suffers), maintaining nevertheless a straightforward implementation and short simulation time. Therefore, the thesis has emphasized developing different MEMS components in Simscape Language as well, which could be easily integrated with Simulink’s signal flow-based modelling. The implementation of all-digital control and read-out of MEMS accelerometers and gyroscopes was validated using the proposed MEMS library in a hybrid simulation environment using both Simscape and Simulink. Hence, the second objective of the thesis has been motivated by the need to implement the primary function of the thesis.
5.1 Need for system-level modelling using hybrid (signal and energy flow-based modelling) simulator for MEMS inertial sensors and resonant devices

MEMS inertial sensors and resonant devices inherently accommodate complex non-linear multidomain interactions in the form of inertial excitations, mechanical stresses in the beams, electrostatic fields, thermal noise and damping (viscous) forces of the medium. Moreover, MEMS fabrication processes (microfabrication) are exposed to manufacturing anomalies, possibly due to the devices’ microscopic topographies. The ensuing structural imperfections and uncertainties in the device parameters are partially compensated by suitably designing the read-out and control circuitry of the device. For that reason, numerical simulations of the mechanical components of the device and circuit-level simulation (SPICE) of the read-out and control electronics, when performed separately in different software environments, would not suffice to describe the system behaviour aptly. A co-simulation of the multidomain interactions in the system is essential for the accurate modelling of the system behaviour. The authors of [47] debate on the merits and demerits of different types of modelling for microsystem devices.

At present, Finite Element Methods (FEM), network-based macro-modelling and signal flow-based modelling are predominantly used to model MEMS inertial devices. The information-flow (signal-flow) based simulations have high levels of abstraction, use mostly linearized models, which cannot account for several non-linear phenomena inherently present in MEMS devices. SPICE-based macro-modelling of microsystems is challenging due to the limited inventory of primitive components to describe the physics of the system. Microsystems pose complexities that are, at best, time-consuming to solve using Finite Element Simulations. FEM considers more than
enough degrees of freedom for microsystems simulations. For any simulator to consume reasonable runtime, while not compromising accuracy, it should consider only the number of degrees of freedom required to capture the essential system dynamics. Besides, FEM is not well-suited for solving circuits.

Identifying the applicable multi-physics manifestations in a system and its non-linearities is highly challenging and also is application-specific. More than often, combinations of modelling techniques are necessary to create behavioural models for system-level co-simulations.

Both the methods of modelling (causal or information flow modelling) and (acausal or energy flow modelling) are employed in this work for different purposes. Table 5.1 enumerates the advantages and disadvantages of the two types of modelling stated above, based upon which their roles in this work were determined.
Causal models (Signal flow) | Acausal models (Energy flow)
--- | ---
High levels of abstractions using input-output system dynamics | Enables physical system modelling by taking into account the bi-directional energy flow between interconnected components in the system
Components or blocks express the causality within the plant. Need to declare inputs, outputs and direction of operation | Need not explicitly define inputs, outputs or directionality. The model assumes directionality from the applied excitation conditions.
Application-specific causal components | More versatile and reusable acausal components
Efficient in dealing with explicit computations | Physical network models realistically describe the systems using unified governing equations

Table 5.1 Comparison between the causal and acausal methods of modelling

Hence, causal models in Simulink were used in the thesis for modelling control system structures and acausal models for describing the physical network. Energy flow-based modelling has been performed using hybrid MATLAB/Simulink and Simscape interface and simulator.
5.2 Energy flow-based modelling of MEMS inertial sensors

This thesis identifies two significant phenomena to be accounted for during the modelling of MEMS inertial sensors and resonant devices. Chapter 2 explains the bi-directional energy flow between the electrical and mechanical domains in MEMS capacitive accelerometers and gyroscopes. In microscale, this non-linear coupling may also lead to the loss of stability, through electrostatic pull-in. With proper feedback control, MEMS devices can take advantage of the effect of electrostatic forces and can be operated on the border of their stability, to gain improved sensitivities because, in that region, even the slightest input perturbation would be amplified by the positive feedback created by the electrostatic field and lead to a much larger output signal.

Hence, chapter 2 modelled the discussed phenomena (bi-directional energy diports and pull-in) and encoded their constitutive laws into Simscape components. Simscape allows physical system modelling of components and creating customized components through Matlab-based Simscape Language. Table 5.2 recapitulates the newly developed components of Simscape MEMS library.
<table>
<thead>
<tr>
<th>Component name</th>
<th>Component interface</th>
<th>Functions</th>
<th>Equations</th>
</tr>
</thead>
</table>
| Gap-varying capacitors (moving towards) | ![Gap-varying capacitor (towards)](image) | Embodies a pair of gap-varying fixed & movable capacitor plates in which one is moved towards the other. | Charge conservation equation: 
\[ i = uv \frac{C_0}{d_0 \left( 1 - \frac{x}{d_0} \right)} + C \frac{du}{dt} \]
Energy conservation equation 
\[ f = -\frac{1}{2} u^2 \frac{C_0}{d_0 \left( 1 - \frac{x}{d_0} \right)^2} \] |
| Gap-varying capacitors (moving away) | ![Gap-varying capacitor (away)](image) | Embodies a pair of gap-varying fixed & movable capacitor plates in which one is moved away from the other. | Charge conservation equation: 
\[ i = -uv \frac{C_0}{d_0 \left( 1 + \frac{x}{d_0} \right)^2} + C \frac{du}{dt} \]
Energy conservation equation 
\[ f = \frac{1}{2} u^2 \frac{C_0}{d_0 \left( 1 + \frac{x}{d_0} \right)^2} \] |
| Area-varying capacitors (moving towards) | ![Area-varying capacitor (towards)](image) | Embodies a pair of area-varying capacitor plates in which the movable capacitor moves towards the fixed plate | Charge conservation equation: 
\[ i = \frac{2C_0 u v}{l_0} + 2C_0 u' + \frac{2C_0 xu'}{l_0} \]
Energy conservation equation 
\[ f = -\frac{C_0 u^2}{l_0} \] |
Area-varying capacitors (moving away)

Embody a pair of area-varying capacitor plates in which the movable capacitor moves towards the fixed plate

Charge conservation equation:
\[ i = \frac{-2C_0 u v}{l_0} + 2C_0 u' - \frac{2C_0 x u'}{l_0} \]

Energy conservation equation
\[ f = \frac{C_0 u^2}{l_0} \]

Stopper

Table 5.2 List of the newly developed components of the MEMS library in Simscape

In order to test the proposed components, a system-level simulation of a MEMS accelerometer was performed in two cases: linear region (small-signal analysis) and pull-in region (large-signal behaviour). The chapter gives detailed instructions on how to construct the accelerometer using the newly developed components in Simscape and to interface the Simscape components with the Simulink environment for a combined simulation with the hybrid simulator. For various input accelerations, the proof mass displacements were measured using the multi-domain simulation in the hybrid simulation platform, and it was corroborated by the results from Finite Element Simulations in COMSOL Multiphysics and using analytical computation. Table 5.3 presents the abridged results of the small-signal analysis.
The relative error between the system-level modelling in Simulink and Simscape, and Finite Element Analysis in COMSOL Multiphysics

-0.022% to 0.004%

The relative error between the system-level modelling in Simulink and Simscape, and numerical calculation

-0.022% to 0.003%

<table>
<thead>
<tr>
<th>Simulation time in seconds</th>
<th>Finite Element Simulations</th>
<th>Signal flow in Simulink</th>
<th>Co-simulation in Simulink+Simscape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21</td>
<td>3.625</td>
<td>6.08</td>
</tr>
</tbody>
</table>

Table 5.3 Results of small-signal analysis of the MEMS accelerometer modelled in hybrid (Simulink+Simscape) simulation environments

Even though the time for co-simulation in Simscape+Simulink is twice as long as the signal-flow simulation in Simulink, the hybrid model was able to address all the non-linearities. The performance of the proposed model library was attested by testing the MEMS accelerometer's large-signal behaviour. Besides, the hysteresis produced by the stopper component (non-linear spring block) from the proposed MEMS library further verifies that the energy flow-based model has mimicked the real-time MEMS device. This arises because when the proof mass hits the stopper (placed at \( \frac{2}{3} d_0 \)), the electrostatic force is highly non-linear and overpowers the spring force, hence preventing a linear relation of bias voltage and proof mass displacement. This kind of behaviour cannot be simulated straightforwardly in the causal block-diagram approach taken by Simulink alone. Even if this could be performed in Finite Element software, it leads to a complex setup and simulation convergence issues.
5.3 CASE STUDY: System-level Modeling of MEMS accelerometer with Sliding Mode Control

Chapter 3 demonstrates the system-level modelling of the closed-loop MEMS accelerometers using the proposed MEMS library components. Firstly, the chapter starts with the design and mask layout of a MEMS accelerometer in a custom microfabrication technology. The design rules, parameters, assumed and computed quantities were defined. Instructions on modelling of Sliding Mode Control in Simulink and integrating it with energy-based components of the accelerometer were presented.

The SMC loop acts as negative feedback to cancel the motion of the proof mass by consistently producing bitstream(s) which are utilized to reconstruct the input signal (acceleration), using a second-order low-pass filter. The chapter delves more into the working principle of the control loop. Table 5.4 summarizes the results obtained in Chapter 3.
Table 5.4 Results from open-loop and closed-loop (SMC) accelerometer simulated in signal flow and energy flow-based modelling

<table>
<thead>
<tr>
<th></th>
<th>Signal flow-based modelling in MATLAB/Simulink</th>
<th>Energy flow-based modelling in Simulink+Simscape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proof mass displacement for 5g input acceleration (Open-loop displacement = 74.11 nm)</td>
<td>5.99 nm</td>
<td>5.04 nm</td>
</tr>
<tr>
<td>Motion attenuation (relative to open-loop displacement)</td>
<td>91.9%</td>
<td>93.2%</td>
</tr>
<tr>
<td>Signal reconstruction steady-state relative error</td>
<td>0.92%</td>
<td>-0.06%</td>
</tr>
<tr>
<td>Signal reconstruction phase error</td>
<td>2.2 rad (lagging)</td>
<td>2.2 rad (lagging)</td>
</tr>
<tr>
<td>Simulation time (s)</td>
<td>43.5</td>
<td>50.4</td>
</tr>
</tbody>
</table>

According to the results in Table 5.4, the energy-based co-simulation of the closed-loop accelerometer took 15.86% more time than the signal flow simulation, but the co-simulation of the open-loop accelerometer took 50% more time than its Simulink counterpart. This divergence in simulation time is due to the delay from the SMC loop structure, which remained the same for both the models. The comparator input quantities (displacements and sensed capacitances) were digitized using a clock with a period of 1 microsecond.
This compromise in simulation time is a small price to pay when the model could address all non-linearities of the device. For example, section 3.5.3 shows that the sensitivity of the accelerometer increases with increasing bias voltage, with the sensitivity being the highest at the pull-in voltage. The result of the same has been summarized in Table 5.5.

<table>
<thead>
<tr>
<th>Bias voltage, V</th>
<th>Sensitivity (slope)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.005</td>
</tr>
<tr>
<td>10</td>
<td>11.7</td>
</tr>
<tr>
<td>20</td>
<td>42.6</td>
</tr>
<tr>
<td>25 (Pull-in voltage)</td>
<td>64.3</td>
</tr>
</tbody>
</table>

Table 5.5 Summary of the performance of the closed-loop accelerometer excited with different bias voltages

5.4 CASE STUDY: System-level Modeling of double-digital MEMS gyroscope with Sliding Mode Control

The author of the thesis along with three co-authors has successfully demonstrated and published the concept of all-digital control and read-out of MEMS gyroscope in [51]. The implementation of SMC loops in the driven and sensing modes and application of the synchronous demodulation technique on the bitstreams (bitstream demodulation) from the sense mode SMC loop for input signal reconstruction were modelled using signal flow in MATLAB/Simulink in [51]. The proposed model was proven to protect the microsystem from the fluctuations in its operating environment. The system also gained robustness from immunity to internal disturbances such as variations in its mechanical resonant frequency. This is performed by applying SMC loop that
tracks the resonant frequency, much like a digital PLL system. Unlike the SMC loop in the driven mode, which was used to maintain a particular frequency (resonance), the SMC loop in the sense mode was used to cancel the motion of the proof mass, just like in MEMS accelerometers (Chapter 3). Furthermore, the bitstream from the sense mode SMC was utilized to extract information about the input acceleration by employing the technique of synchronous demodulation. The work in [51] concludes by demonstrating a seamless operation of the closed-loop MEMS gyroscope with a double-digital SMC loop with the driven mode resonant frequency tracking error of 0.008%, sense mode motion attenuation of up to 100 times the open-loop displacements.

However, [51] has implemented the double-digital control and read-out structure in MATLAB/Simulink using causal blocks (block diagram models). Chapter 4 presents the novel concepts in [51] and also implement those concepts using physical system modelling in the hybrid paradigm.

Chapter 4 provides a concise introduction to the need for closed-loop dual-mode control in MEMS gyroscopes. It gives comprehensive instructions on how to model the MEMS gyroscope using the novel MEMS components in Simscape. The implementation of the sense mode SMC is very similar to that of the MEMS accelerometer already discussed under Chapter 3. Hence, it was no surprise that both the models were able to attenuate the sense mode displacements by 100 times.
Table 5.6 Result summary of Signal reconstruction using bitstream demodulation and resonant frequency tracking in the driven mode

The results for the bitstream demodulation and the driven mode frequency tracking are listed in Table 5.6. The results from both models are comparable to each other. On account of the hybrid model offering more advantages in terms of non-linear physical system modelling, modelling the device using acausal modelling would be more beneficial than information flow, which incorporates high levels of abstraction.
5.5 Future scope

The design and validation of energy flow-based acausal components in the MEMS library is the pivotal effort in enabling the physical system modelling of MEMS devices. The work so far has furnished two types of electromechanical diports (area-varying and gap-varying electrodes) and a non-linear spring block (to function as a mechanical stopper). However, the work is intended as a prototype of what the future holds for energy-based acausal modelling of MEMS devices. More experiments can be dedicated to testing these components (and future components to be included in the library) under different test inputs. The author is interested in modelling the phenomenon of symmetric pull-in, for advanced high-sensitivity MEMS devices.

As for the hardware experiments, several single-axis MEMS accelerometers and gyroscopes have been custom fabricated in 50um SOI technology (SFU-MUMPS), out of which two devices are shown in Figures 3.1 and 4.1. After characterizing the devices’ parameters, work would be underway for implementing the SMC loop in both accelerometer and gyroscope. Some accelerometers have equally spaced, gap-varying bias electrodes to facilitate symmetric pull-in, which shall be tested at last. The sensitivity of the accelerometers would be tested by biasing the device at the pull-in voltage. Modelling of the SMC unit is likely to be performed on FPGA.
Bibliography


Appendices

Appendix A : Unified equations of gap-varying and area-varying capacitive electrodes

The relation between the across-through variables in the mechanical domain and electrical domain and the unifying equations are presented for gap varying and area-varying electrodes. Figure A.1 shows the two types of capacitor (gap-varying and area-varying) arrangements possible while constructing MEMS inertial sensors.

Figure A. 1 Gap-varying and area-varying capacitive electrodes in MEMS inertial sensors

\[ C = \frac{\varepsilon n A}{d_0 \pm x} \]

\[ F_e = \frac{1}{2} CV_{\text{bias}}^2 = \frac{1}{2} \left( \frac{\varepsilon n A V_{\text{bias}}^2}{(d_0 \pm x)^2} \right) \]

\[ C = \frac{\varepsilon n (h \pm l)}{d_0} \]

\[ F_e = \frac{1}{2} \frac{en h V_{\text{bias}}^2}{d_0^2} \times \pm l \]
In Figure A.1, $d_0$ represents the nominal gap distance between the electrodes and $C_0$ represents the nominal capacitance. The across and through variables and block parameters used in the MEMS library components are defined in Table A.1.

<table>
<thead>
<tr>
<th>Variables or parameters</th>
<th>Notations</th>
<th>Variables or parameters</th>
<th>Notations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current (through)</td>
<td>$i$</td>
<td>Force (through)</td>
<td>$f$</td>
</tr>
<tr>
<td>Current when the movable electrode moves towards and away from the fixed electrode</td>
<td>$i^+ \text{ and } i^-$</td>
<td>Current when the movable electrode moves towards and away from the fixed electrode</td>
<td>$f^+ \text{ and } f^-$</td>
</tr>
<tr>
<td>Voltage (Across)</td>
<td>$u$</td>
<td>Velocity (Across)</td>
<td>$v$</td>
</tr>
<tr>
<td>Charge</td>
<td>$Q_c$</td>
<td>Nominal gap distance</td>
<td>$d_0$</td>
</tr>
<tr>
<td>Nominal capacitance</td>
<td>$C_0$</td>
<td>Overlap length of electrodes</td>
<td>$l$</td>
</tr>
<tr>
<td>Capacitance when the movable electrode moves towards and away from the fixed electrode</td>
<td>$C^+ \text{ and } C^-$</td>
<td>Displacement of the movable electrode towards and away from fixed electrode (Gap-varying)</td>
<td>$\pm x$</td>
</tr>
</tbody>
</table>

Table A.1 List of across and through variables and parameters of the MEMS components
A.1 Unified equation of gap-varying capacitive electrodes

The changes in capacitance caused by the movement of the movable electrode are derived below. The variables, $C^+$ and $C^-$ denote the capacitances when the movable electrodes move towards and away from the fixed electrode, respectively, $A$ is the total surface area of the electrode interfaces, and $n$ is the total number of such electrode pairs.

$$C^+ = \frac{eA}{d_o - x} - n = \frac{C_0}{d_0(1 - \frac{x}{d_o})}$$

Therefore, $\frac{dC^+(x)}{dx} = \frac{nC_0}{d_0(1 - \frac{x}{d_o})^2}$

$$C^- = \frac{eA}{d_o + x} - n = \frac{C_0}{d_0(1 + \frac{x}{d_o})}$$

Therefore, $\frac{dC^-(x)}{dx} = \frac{nC_0}{d_0(1 + \frac{x}{d_o})^2}$

The laws of conservation of charge and energy are considered to obtain the governing equation, unifying the electrical and mechanical domains. According to the law of conservation of electric charge, (A.2) computes the instantaneous current, $i$ by differentiating the charge on the capacitors, $Q_c$.

$$i = \frac{dQ_c}{dt} = \nu \frac{dC(x)}{dx} + u'C(x)$$

(A.2)

From the set of equations in (A.1), the currents, $i^+$ and $i^-$ are given by (A.3)

$$i^+(t) = \frac{\nu C_0}{d_0(1 - \frac{x}{d_o})^2} + \frac{u'C_0}{1 - \frac{x}{d_o}}$$

and

$$i^-(t) = \frac{-\nu C_0}{d_0(1 + \frac{x}{d_o})^2} + \frac{u'C_0}{1 + \frac{x}{d_o}}$$

(A.3)
Similarly, according to law of conservation of energy below, the forces, \( f^+ \) and \( f^- \) are given by (A.4)

\[
\frac{1}{2}C(x)2uu' + \frac{1}{2}u^2V \frac{dC(x)}{dx} = \frac{1}{2}u^2V \frac{dC(x)}{dx} + uu'C(x) + fv
\]

\[
f = -\frac{1}{2}u^2 \frac{dC(x)}{dx}
\]

\[
f_1(t) = \frac{-u^2C_o}{2d_o(1-x/d_o)^2} \quad \text{and} \quad f_2(t) = \frac{u^2C_o}{2d_o(1+x/d_o)^2}
\] (A.4)

### A.2 Unified equation of area-varying capacitive electrodes

The derivation of the governing equation for the area-varying electrodes is very similar to that of the gap-varying electrodes. However, the motion of the movable electrode is not perpendicular to the fixed capacitor plate (like the gap-varying arrangement), but instead the movable electrode moves parallel to the latter plate. Therefore, the change in capacitance is caused by the variation in the overlap length, \( l \) (in turn, surface area, \( A \)) rather than the difference in the gap distance, \( d_o \).

The change in capacitance of the plates is explained by the equations below, where \( C_1^+ \) and \( C_2^+ \) are the capacitance experienced by the movable plate because of the fixed plates on either side of it when the plate moves such that the overlap length increases. Similarly, \( C_1^- \) and \( C_2^- \) are the capacitance on either side of the plate when it moves such that the overlap length decreases.
\[C_1^* = C_2^* = \frac{eh}{d} (l + x)\]
\[= C_0 + \frac{C_0}{l} x(t)\]
\[C_1^- = C_2^- = \frac{eh}{d} (l - x)\]
\[= C_0 - \frac{C_0}{l} x(t)\]

The total capacitances when the electrode moves are described by (A.5)

\[
C^+(x) = 2C_0 + \frac{2C_0}{w} x(t) \frac{dC^+(x)}{dx} = \frac{2C_0}{w} \\
C^-(x) = 2C_0 - \frac{2C_0}{w} x(t) \frac{dC^-(x)}{dx} = -\frac{2C_0}{w}
\]

(A.5)

Analogous to the previous case, the charge conservation and energy conservation laws are utilized to come up with the unified equations for current and force (across variables) in terms of voltage and velocity (through variables). The set of equations in (A.6) was obtained from substituting the expression for capacitances in the law of conservation of charge.

\[
i^+(t) = \frac{dQ_+}{dt} = uv \frac{dC(x)}{dx} + u'C(x) = \frac{2C_0 u v}{w} + 2C_0 \frac{du}{dt} + \frac{2C_0 x}{w} \frac{du}{dt}
\]
\[
i^-(t) = \frac{dQ_-}{dt} = uv \frac{dC(x)}{dx} + u'C(x) = -\frac{2C_0 u v}{w} + 2C_0 \frac{du}{dt} - \frac{2C_0 x}{w} \frac{du}{dt}
\]

(A.6)

By substituting the capacitances expression in the energy conservation equation, the final equation for force is obtained in (A.7).
\[
\frac{d}{dt}\left(\frac{1}{2}C^+(x)u^2\right) = u\dot{u}^+(t) + \frac{1}{2}\left(2\frac{C_0}{w}u^2v + 4C_0u \frac{du}{dt} + \frac{4C_0}{w}xu \frac{du}{dt}\right)
\]
\[
= \frac{2C_0}{w}u^2v + 2C_0u \frac{du}{dt} + \frac{2C_0}{w}xu \frac{du}{dt} + f^+\dot{f}^+(t)
\]
\[
= -\frac{C_0}{w}u^2
\]

\[
\frac{d}{dt}\left(\frac{1}{2}C^-(x)u^2\right) = u\dot{u}^-(t) + \frac{1}{2}\left(-2\frac{C_0}{w}u^2v + 4C_0u \frac{du}{dt} - \frac{4C_0}{w}xu \frac{du}{dt}\right)
\]
\[
= -\frac{2C_0}{w}u^2v + 2C_0u \frac{du}{dt} - \frac{2C_0}{w}xu \frac{du}{dt} + f^-\dot{f}^-(t)
\]
\[
= +\frac{C_0}{w}u^2
\]

\[f^+(t) = -\frac{C_0}{w}u^2 \quad \text{and} \quad f^-(t) = +\frac{C_0}{w}u^2 \quad (A.7)\]

Therefore, the sets of equations in (A.3) and (A.4) together represent the governing equations for gap-varying arrangement while the equations in (A.5) and (A.7) represent the unified equations for the area-varying mechanism.