Physical Layer Security for Jamming Signal Aided Multi-Antenna Communication Systems

by

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Abstract

Physical layer security is a promising technique for information security. In multi-antenna systems, secure transmission designs are of crucial importance for physical layer security. In this thesis, our research focuses on the secure transmission designs for jamming signal aided multi-antenna systems.

We first investigate robust secure transmission designs for a cooperative jamming aided multiple-input-single-output (MISO) system. Two scenarios are considered: (a) eavesdroppers' channel state information (ECSI) is available and (b) ECSI is unavailable. In scenario (a), a quality-of-service (QoS)-based design is considered to minimize the worst case signal-to-interference-and-noise ratio at the eavesdroppers. A secrecy rate based design is also studied. In scenario (b), a QoS-based design is considered to maximize the power of jamming signals, and the secrecy rate based design is not applicable. We propose an algorithm for each design problem through semidefinite relaxation. Our analysis shows that, even though the beamforming scheme in our designs is fixed as single stream beamforming, this does not cause any loss of optimality.

We further consider secure transmission designs for a simultaneous wireless information and power transfer enabled MISO heterogeneous cellular network. In the considered system, all base stations (BSs) send confidential messages to information receivers and energy signals to energy receivers (ERs). BSs collaboratively utilize the energy signals as jamming signals to cripple ERs' interceptions. We propose a sum logarithmic secrecy rate maximization design problem. To tackle the design problem, we propose a semidefinite relaxation and successive convex approximation based centralized secure transmission design algorithm. Moreover, an alternating direction method of multipliers based distributed secure transmission design

is also proposed. Simulation results demonstrate the effectiveness of the proposed algorithms.

Secure transmission is designed for a cooperative jamming aided MISO non-orthogonal multiple access system. We consider an outage constrained secure transmission design problem under the assumption that only statistical ECSI is available. In the design problem, the minimum probabilistic secrecy rate is maximized. Based on 0-norm and the path-following approach, we develop an algorithm to find a solution to the design problem. Simulation results demonstrate that the secrecy outage probability can be guaranteed through the proposed secure transmission design algorithm.

Lay Summary

Physical layer security, which exploits the physical characteristics of wireless channels, is a promising technique for information security. In multi-antenna communication systems, secure transmission designs are of crucial importance for physical layer security. By designing the information transmission schemes, we can degrade the reception of eavesdroppers while at the same time guaranteeing the reception of legitimate receivers. Moreover, besides designing the information transmission schemes, the physical layer security performance of multi-antenna systems can be further enhanced by sending designed jamming signals to confuse eavesdroppers. In this thesis, our research focuses on the secure transmission designs for three different jamming signal aided multi-antenna systems. Our analysis and simulation results reveal the effectiveness of the proposed secure transmission designs.

Preface

A list of my publications at The University of British Columbia is provided in the following.

Refereed Journal Publications

J1. **Hui Ma**, Julian Cheng, and Xianfu Wang, "Proportional fair secrecy beamforming for MISO heterogeneous cellular networks with wireless information and power Transfer," *IEEE Transactions on Communications*, vol. 67, no. 8, pp. 5659-5673, Aug. 2019.

J2. **Hui Ma**, Julian Cheng, Xianfu Wang, and Piming Ma,"Robust MISO beamforming with cooperative jamming for secure transmission from perspectives of QoS and secrecy rate," *IEEE Transactions on Communications*, vol. 66, no. 2, pp. 767-780, Feb. 2018.

J3. **Hui Ma**, Haijun Zhang, Xianfu Wang, and Julian Cheng, "Backhaul-aware user association and resource allocation for massive MIMO enabled HetNets," *IEEE Communications Letters*, vol. 21, no. 12, pp. 2710–2713, Dec. 2017.

J4. Adebola Olutayo, **Hui Ma**, Julian Cheng, and Jonathan Holzman, "Level Crossing Rate and Average Fade Duration for the Beaulieu-Xie Fading Model," *IEEE Wireless Communications Letters*, vol. 6, no. 3, pp. 326–329, June 2017.

Refereed Conference Publications

C1. **Hui Ma**, Julian Cheng, and Xianfu Wang, "Secrecy beamforming for SWIPT MISO heterogeneous cellular networks," *Proceedings of the 29th Biennial Symposium on Communications (BSC)*, Toronto, Canada, June 6-7, 2018. Winner of 2018 Canadian Society of Information Theory (CSIT) Best Paper Award.

C2. **Hui Ma**, Julian Cheng, Xianfu Wang, and Piming Ma, "Cooperative jamming aided robust beamforming for MISO channels With unknown eavesdroppers," in *Proceedings of IEEE Global Communications Conference (GLOBECOM)*, Singapore, Dec. 4-8, 2017.

Table of Contents

List of Tables

List of Figures

List of Acronyms

Acronyms	Definitions
ADMM	Alternating Direction Method of Multipliers
AN	Artificial Noise
BS	Base Station
CJ	Cooperative Jamming
CSCG	Circularly Symmetric Complex Gaussian
CSI	Channel State Information
ECSI	Eavesdroppers' Channel State Information
EH	Energy Harvesting
ER	Energy Receiver
EVD	Eigenvalue decomposition
FBS	Femtocell Base Station
FU	Femtocell User
HCN	Heterogeneous cellular network
IR	Information Receiver
KKT	Karush–Kuhn–Tucker
LCSI	Legitimate receiver's Channel State Information
MBS	Macrocell Base Station
MC	Multi-Carrier
MISO	Multiple-Input Single-Output

MUI	Mutual Interference
MU	Macrocell User
NOMA	Non-Orthogonal Multiple Access
QoS	Quality-of-Service
RF	Radio Frequency
SCA	Successive Convex Approximation
SDP	Semidefinite Programming
SDR	Semidefinite Relaxation
SC	Single-Carrier
SIC	Successive Interference Cancellation
SINR	Signal-to-Interference-and-Noise Ratio
SISO	Single-Input Single-Output
SWIPT	Simultaneous Wireless Information and Power Transfer
ZF	Zero-Forcing

List of Symbols

Symbols	Definitions
$\arg\max\{\cdot\}$	Points of the domain of the function at which the function values are maximized
$\mathrm{diag}(\cdot)$	The function to put matrices or scalars into a block diagonal matrix
$\dim(\cdot)$	The dimension of a subspace
$E[\cdot]$	The statistical expectation
$\ker(\cdot)$	The null space of a matrix
$\log_2(\cdot)$	The log function with base 2
$\ln(\cdot)$	The log function with base e
$\max\{\cdot\}$	The maximum value of a function
$\min\{\cdot\}$	The minimum value of a function
$\mathrm{Prob}(\cdot)$	The probability of an event
$\mathrm{rank}(\cdot)$	The rank of a matrix
s.t.	Subject to
$\mathrm{tr}(\cdot)$	The trace operator
$\mathrm{vec}(\cdot)$	The function to put the real and imaginary parts of the entries which are in the upper triangular part of a Hermitian matrix into a vector by a certain order. The order is established as follows. Suppose the entry at the l-th row

	and the n-th column of the Hermitian matrix is represented by $a_{l,n}$. Since $a_{1,1}$ is on the main diagonal, it does not have imaginary part. The real part of $a_{1,1}$ is placed at the first position of the vector. The real part of $a_{1,2}$ is placed at the second position of the vector; the imaginary part of $a_{1,2}$ is placed at the third position and so forth.
$\mathbf{0}$	A null matrix with suitable dimension
$\mathbf{I}$	An identity matrix with suitable dimension
$\mathbf{A} \succ \mathbf{B}$	$\mathbf{A} - \mathbf{B}$ is positive definite
$\mathbf{A} \succeq \mathbf{B}$	$\mathbf{A} - \mathbf{B}$ is positive semidefinite
$\partial \mathbf{A} / \partial a$	The entry wise partial derivative with respect to a
$\lvert \cdot \rvert$	The modulus
$(\cdot)^T$	The transpose operator
$(\cdot)^H$	The conjugate transpose operator
$\lVert \cdot \rVert$	The Euclidean norm
$\lVert \cdot \rVert_0$	The 0-norm
$\lVert \cdot \rVert_1$	The 1-norm
$\lVert \cdot \rVert_\infty$	The infinity norm

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Chapter 1

Introduction

1.1 Background and Motivation

Information security is traditionally provided by cryptographic encryption which is significantly challenged by the open nature of wireless communication in areas such as secret key distribution and management [1], [2]. Different from cryptographic encryption which works in the application layer of communication networks, physical layer security aims at providing information security by employing the physical layer characteristics. Therefore, physical layer security, which can be seen as an alternative or a complement to cryptographic encryption, has attracted increasing attention. This research area was initiated by Wyner. He introduced and studied the basic model of wiretap channel where there is one legitimate receiver's single-input-single-output (SISO) channel and one eavesdropper's SISO channel [3]. Wyner claimed that information can be reliably transmitted to the legitimate receiver at a positive rate and achieve perfect secrecy from the eavesdropper when the eavesdropper's channel is a degraded version of the legitimate receiver's channel. This rate is defined as the secrecy rate. To make physical layer secrecy viable in SISO wiretap channels, we usually require the legitimate receiver's channel to be better than the eavesdroppers'. However, this may not always be possible in a practical wireless environment. With the help of multiple antennas, the dependence on channel conditions can be greatly alleviated. By designing the multi-antenna transmission strategies, we can degrade the reception of eavesdroppers while at the same time guaranteeing the reception of legitimate receivers. Recently, considerable research in physical layer security has focused on multi-antenna transmission.

In multi-antenna communication systems, beamforming has been proven to be an effective transmit strategy to concentrate the information signal and control the power leakage [4–10]. Consequently, optimizing the beamforming schemes (i.e., optimizing the the information covariance matrices[1]) is an effective approach in the secure transmission designs for multi-antenna systems [11–16]. In [13, 14], general rank information covariance matrix[2] optimization problems were studied for secure transmission. However, to implement beamforming with general rank information covariance matrix requires the support of multiple data streams, which is expensive. Therefore, some search works focused on beamforming with rank-one information covariance matrix, i.e., single stream beamfoming, which is the most implementable beamforming scheme. In [15], the existence of rank-one information covariance matrix solutions was discussed. While in [16], the beamforming scheme at the information transmitter was chosen as single stream beamforming and the single stream beamforming vector was optimized.

Besides optimizing the beamforming schemes, the security performance of multi-antenna systems can be further enhanced by sending jamming signals to confuse eavesdroppers through the scheme cooperative jamming (CJ) or artificial noise (AN), in which jamming signals are generated by external helpers or integrated with the information signals sent by the information transmitters. In addition, in many system setups where eavesdroppers are also energy harvesters, jamming signals which take charge of power transfer must be considered. Consequently, the secure transmission designs for jamming signal aided multi-antenna systems have attracted much research attention. Even though there have been many investigations on secure transmission designs for jamming signal aided multi-antenna systems, research gaps still exist in this area, which motivates new research efforts.

[1]The information covariance matrix refers to the covariance matrix of the information signal vector transmitted to a legitimate receiver.

[2]The general rank information covariance matrix refers to the information covariance matrix with uncertain rank.

1.2 Literature Review

In the area of secure transmission designs for jamming signal aided multi-antenna systems, considerable research focused on the conventional multi-antenna systems[3] [17–28]. In these works, it is commonly assumed that both the legitimate receiver's channel state information (LCSI) and eavesdroppers' channel state information (ECSI) are available. Under this assumption, it is popular to consider secrecy rate based secure transmission designs, which aim at perfectly secret information transmission. In [17–19], secure transmission designs for the maximum achievable secrecy rate were studied with the help of perfect channel state information (CSI). While, in [20, 21], robust secrecy rate maximization design problems were formulated and solved for AN aided multiple-input-single-output (MISO) systems with imperfect CSI. In [22], a similar robust secure transmission design was studied for CJ-aided MISO system. Nevertheless, to achieve perfectly secret information transmission, secrecy rate based secure transmission designs have to incorporate complicated secrecy coding techniques [3, 29]. Consequently, secure transmission designs for the scenario where both LCSI and ECSI are available were also considered from the perspective of quality-of-service (QoS)[4] [22, 24–26]. In these designs, the signal-to-interference-and-noise ratios (SINRs) at eavesdroppers are controlled so that the eavesdroppers can only retrieve a small amount of information from their observations. For such a security strategy, secrecy coding is not required. Moreover, the secrecy rate based secure transmission designs cannot satisfy the data rate demand which exceeds the maximum achievable secrecy rate. By contrast, the QoS based secure transmission designs avoid this drawback at the expense of limited information leakage. In [22, 24], robust QoS based secure transmission designs were considered for CJ aided MISO systems. Most previous works on robust secure transmission designs for CJ aided conventional MISO systems focused on optimizing the general rank information covariance matrices and seldom considered single stream beamforming. However,

[3]The conventional multi-antenna systems refer to the systems in which the multi-antenna technique is not integrated with other techniques, such as SWIPT and NOMA.

[4]QoS refers to SINR and these two words are used equivalently in this thesis.

single stream beamforming is the most implementable beamforming scheme. This motivates us to consider QoS based and secrecy rate based robust secure transmission designs for CJ aided conventional MISO channels, where the single stream beamforming vector and the jamming covariance matrix[5] are jointly optimized with imperfect LCSI and ECSI. However, it is impractical to assume ECSI is available when the eavesdroppers are purely passive. Without ECSI, neither the secrecy rate nor the SINRs at the eavesdroppers can be calculated. Therefore, we further consider a QoS based robust secure transmission design with unknown ECSI for the CJ aided conventional MISO system, where the QoS of the legitimate receiver is guaranteed at first, and then the transmit power of jamming signals is maximized for crippling eavesdroppers' interceptions. This work follows the approaches in [17, 27, 28]. But, to our best knowledge, the research on robust secure transmission designs with unknown ECSI for CJ aided MISO systems is still missing.

In recent years, the explosive increase of traffic demands causes fast growth of the energy consumption in mobile devices. The limited energy supplies (batteries) can hardly satisfy all the energy requirements of mobile devices. Consequently, simultaneous wireless information and power transfer (SWIPT), which enables mobile devices to harvest energy from ambient radio frequency (RF) signals, has been proposed as a promising approach to address the energy-limited issue [30–32]. However, in the SWIPT enabled systems, information can be easily wiretapped by the energy receivers (ERs), because, to harvest energy, ERs are located near to the base stations (BSs). Therefore, considerable research focused on secure transmission designs for SWIPT enabled systems [33], [34]. Since the energy-carrying signals in SWIPT enabled systems can also play the role of jamming signals to cripple the possible interception of ERs, in the secure transmission designs for SWIPT enabled systems, the covariance matrices of energy signals can also be optimized for security [35–40].

The majority works on the secure transmission designs having SWIPT considered single-cell systems and assumed that there exists only one legiti-

[5]The jamming covariance matrix refers to the covariance matrix of jamming signals.

mate information receiver (IR) [41–47]. In [46], a secrecy rate maximization design probelm was studied for a MISO system. In the design probelm, the single stream beamforming vector and the jamming covariance matrix were jointly optimized with the help of perfect CSI. In [47], under the assumption that only statistical ECSI is available, an outage constrained secure transmission design was investigated. Besides, there are several secure transmission schemes that were designed for SWIPT enabled broadcast single-cell systems [48, 49].

Most recently, secure transmission designs having SWIPT were considered for heterogeneous cellular networks (HCNs) [50, 51]. HCN, in which the small cells are deployed over the macrocell coverage area to improve the throughput and coverage, is one of the core characteristics of 5G cellular networks [52, 53]. In [50, 51], secrecy rate maximization design problems were investigated for MISO systems where BSs collaboratively utilize the energy-carrying signals as jamming signals to cripple the eavesdropping capacities of ERs under the assumption that there is only one IR that should be protected from eavesdropping in an HCN. However, this assumption is too ideal in practice. This motivates us to consider the scenario where there are more than one IRs that require protection against eavesdropping. Moreover, most of the secure transmission designs for SWIPT enabled HCNs assume that BSs are connected with a central control unit that requires global CSI and optimizes the transmission schemes in a centralized manner. However, the centralized secure transmission designs are not applicable if the central control unit does not exist. Therefore, it is also worthy to develop distributed designs, where the optimization of transmission schemes does not rely on the central control unit but is performed at each BS. However, the distributed secure transmission designs for SWIPT enabled HCNs are seldom investigated. Therefore, in our work, both centralized and distributed secure transmission designs are studied.

Driven by the rapid increase of wireless devices and wide usage of mobile Internet, non-orthogonal multiple access (NOMA) has emerged as a promising technique for the next generation mobile communication systems. [54–61]. Compared with the conventional orthogonal multiple access scheme,

the main advantage of NOMA is that it can simultaneously serves multiple users on one subcarrier to increase the system throughput [62]. For the users scheduled on the same subcarrier, their signals are superimposed by using different power levels and successive interference cancellation (SIC) is installed at the users for mitigating the mutual interference (MUI) imposed by using non-orthogonal resources [63].

In NOMA system, security is a major concern. Numerous efforts have been invested into conceiving security for NOMA systems [64–69]. In [67, 68], secrecy rate based secure transmission designs were investigated for jamming signal aided MISO single-carrier (SC) NOMA system. In [69], a secure transmission design was studied for a jamming signal aided MISO multi-carrier (MC) NOMA system. However, most existing works on secure transmission designs for jamming signal aided MISO NOMA systems were developed with perfect ECSI or estimated ECSI. The scenario, where only statistical ECSI is available, has not been considered, which motivates us to investigate secure transmission designs for jamming signal aided MISO NOMA systems based on statistical ECSI.

1.3 Thesis Outline and Contributions

In this thesis, we present the research work conducted on the following three topics:

- Robust MISO Beamforming With CJ for Secure Transmission
- Secure Transmission for SWIPT enabled MISO HCNs
- Outage Constrained Secure Transmission for CJ Aided MISO MC NOMA

The summary and contributions of each chapter are as follows.

Chapter 1 presents background knowledge on the secure transmission designs for jamming signal aided multi-antenna systems. In addition, this chapter provides a detailed literature review related to the rest of the thesis.

Chapter 2 presents the required technical background for the entire thesis. We first introduce the beamforming schemes. We then present background knowledge on SWIPT and NOMA which are usually integrated with the multi-antenna technique. Finally, we briefly introduce the concept of convex optimization, which will be the mathematical tool in our research.

In Chapter 3, we investigate robust secure transmission designs for a CJ aided MISO system having multiple eavesdroppers from both the perspectives of QoS and secrecy rate. Different from the previous works on robust secure transmission designs for CJ aided MISO systems, the beamforming scheme is single stream beamforming in our designs. We assume that the CSI is imperfect and the CSI error are norm bounded. Therefore, the robust secure transmission design problems are formulated by the worst-case approach. We consider two scenarios: (a) ECSI is available, which is possible when eavesdroppers[6] are participating users but attempt to access unauthorized services [20, 23, 25]; and (b) ECSI is unavailable, which occurs when eavesdroppers are purely passive. For scenario (a), we first consider a QoS based robust secure transmission design problem where the worst-case SINR at eavesdroppers is minimized under the QoS constraint of the legitimate receiver. After that, the secrecy rate based transmission design is studied where the worst-case secrecy rate is maximized. For scenario (b), the secrecy rate based design is not applicable and we consider a QoS based robust secure transmission design to provide security. Our strategy here is to maximize the power of jamming signals while ensuring the SINR at the legitimate receiver. To solve the non-convex design problems, we use the semidefinite relaxation (SDR) approach to obtain the general rank information covariance matrix based counterparts [70]. We demonstrate that rank-one optimal information covariance matrix solutions can always be found in the SDR problems and similar results do not exist in previous works on robust secure transmission designs for CJ aided MISO systems.

In Chapter 4, we consider an HCN where there exist one macrocell base station (MBS) and multiple femtocell base stations (FBSs) in co-channel de-

[6] All the eavesdroppers considered in this thesis are assumed not to send malicious jamming signals to attack the reception of legitimate receivers.

ployment. The MBS and FBSs are equipped with multiple antennas while femtocell users (FUs) and macrocell users (MUs) are equipped with single antenna. MUs and FUs are IRs. All BSs transmit different confidential messages to their connecting IRs. In the meanwhile, there are multiple ERs in the HCN which harvest energy from the energy signals transmitted by BSs. The ERs have the potential to eavesdrop. Thus, BSs collaboratively utilize the energy signals sent to ERs as jamming signals to cripple ERs' interception capabilities. Different from most previous works on secure transmission designs for SWIPT enabled HCNs, we assume that all IRs should be protected from eavesdropping. In order to achieve a good balance between system secrecy throughput and fairness among IRs, we adopt the proportional fairness scheme [71], i.e., we introduce the logarithm utility function. Therefore, we propose a sum logarithmic secrecy rate maximization secure transmission design problem under the energy harvesting (EH) constraints for ERs. To solve the sum logarithmic secrecy rate maximization design problem, we use SDR to relax the outer products of the single stream beamforming vectors to be general rank information covariance matrices. We prove that the SDR is tight. The tight SDR reveals that, compared with beamforming with general rank information covariance matrix, utilizing single stream beamforming to transmit confidential information message to each IR does not cause any loss of performance optimality. Then, we deal with the SDR problem by proposing a successive convex approximation (SCA) process and develop an SCA-based centralized secure transmission design algorithm. Since the distributed secure transmission designs for SWIPT enabled HCNs are seldom investigated, we also propose a distributed secure transmission design algorithm.

In Chapter 5, we extend our study on secure transmission designs to a CJ aided MISO MC NOMA system. Different from the previous works on secure transmission designs for MISO NOMA systems, we consider a scenario where only statistical ECSI is available. Therefore, an outage constrained secure transmission design problem is proposed. In the design problem, the single stream beamforming vectors, the jamming covariance matrices and the subcarrier allocation policy are jointly optimized to maximize the minimum

probabilistic secrecy rate. Since the optimization of the subcarrier allocation policy is considered, the design problem is a mixed integer programming problem, which is challenging to deal with. In order to find a solution to the design problem, an algorithm is developed based on 0-norm and the path-following approach.

Chapter 6 summarizes the entire thesis and lists our contributions. In addition, some future works related to our current research are also suggested.

Chapter 2

Background on Secure Transmission Designs for Jamming Signal Aided Multi-antenna Systems

In this chapter, we first introduce the beamforming schemes. We then present background knowledge on SWIPT and NOMA which are usually integrated with the multi-antenna technique. Finally, we briefly introduce the convex optimization.

2.1 Beamforming Schemes

Consider a conventional MISO system, where a BS (Alice) transmits information to a legitimate receiver (Bob) and K eavesdroppers (Eves) wiretap. Let $\mathbf{x}$ denotes the information signal vector. The received signals at Bob and Eves can be, respectively, expressed as

$$y_b = \mathbf{h}_b^H \mathbf{x} + n_b \tag{2.1}$$

and

$$y_{e,k} = \mathbf{h}_{e,k}^H \mathbf{x} + n_{e,k},\ k = 1, \ldots, K \tag{2.2}$$

where $\mathbf{h}_b$ and $\mathbf{h}_{e,k}$ denote the channel vectors from Alice to Bob and the kth Eve, and n_b and $n_{e,k}$ represent the zero mean circularly symmetric complex Gaussian (CSCG) noises with variances σ_b^2 and $\sigma_{e,k}^2$ respectively. Assume $\mathbf{x}$

is a zero mean vector. Based on (2.1) and (2.2), the SINRs at Bob and the k-th Eve can be, respectively, expressed as

$$SINR_b = \frac{\mathbf{h}_b^H \mathbf{Q}_x \mathbf{h}_b}{\sigma_b^2} \tag{2.3}$$

and

$$SINR_{e,k} = \frac{\mathbf{h}_{e,k}^H \mathbf{Q}_x \mathbf{h}_{e,k}}{\sigma_{e,k}^2} \tag{2.4}$$

where $\mathbf{Q}_x = \mathrm{E}\left[\mathbf{x}\mathbf{x}^H\right]$ is the information covariance matrix.

In secure transmission designs, it is often to optimize the information covariance matrix $\mathbf{Q}_x$ without considering its rank, i.e., optimize the general rank information covariance matrix. However, to implement beamforming with general rank information covariance matrix requires the support of multiple data streams, which is expensive. Therefore, some search works focused on beamforming with rank-one information covariance matrix, i.e., single stream beamfoming, which is the most implementable linear precoding scheme. In single stream beamfoming, the information signal vector takes the form $\mathbf{x} = \mathbf{w}x$ and the information covariance matrix takes the form $\mathbf{Q}_x = \mathbf{w}\mathbf{w}^H$, where $\mathbf{w}$ is the single stream beamforming vector and x represents the single data stream with unit power. Generally, compared with beamforming with general rank information covariance matrix, fixing the beamforming scheme to be single stream beamforming will cause some performance loss. However, in some scenarios, the single stream beamforming vector can perform as well as the general rank information covariance matrix.

2.2 Simultaneous Wireless Information and Power Transfer

The explosive increase of traffic demands causes fast growth of the energy consumption in mobile devices. The limited energy supplies (batteries) can hardly satisfy all the energy requirements of mobile devices. However, the

pace of enhancing the battery capacity is slow over the past decades [72]. Consequently, the EH technology which enables mobile devices to scavenge energy from the environment has attracted much recent research attention [73–76]. The natural energy sources, such as solar power, wind and tide, are climate or location dependent and cannot always be available for portable wireless devices in practice [77]. On the contrary, SWIPT, which enables mobile devices to harvest energy from ambient RF signals, can provide cost-effective and perpetual power supplies for mobile devices. Therefore, SWIPT has been considered as a promising approach to address the energy-limited issue [30–32].

In the early study of SWIPT systems, a commonly used EH model is the linear model. Let P^{ER} denote the received RF power at an ER. Then, by following the linear model, the harvested power at the ER is

$$\Lambda = \xi P_{ER}$$

where ξ represents the EH efficiency. However, in practice, EH circuits usually result in a non-linear end-to-end wireless power transfer [78]. Hence, the linear EH model cannot properly model the power dependent EH efficiency. Most recently, a practical non-linear EH model was proposed in [79]. In this EH model, the harvested power at the ER can be give by

$$\Lambda = \frac{\Upsilon_{ER} - S\Omega}{1 - \Omega} \tag{2.5}$$

where $\Omega = 1/\left(1 + \exp(\eta o)\right)$, $\Upsilon_{ER} = S/(1 + \exp(\eta o - \eta P_{ER}))$, and where S, η and o are constants related to the detailed EH circuit specifications such as the resistance, capacitance and diode turn-on voltage.

2.3 Non-Orthogonal Multiple Access

NOMA is a promising technique to improve spectral efficiency and to provide massive connectivity. In NOMA systems, multiple users can be served on one subcarrier for increasing the system throughput. SIC is ap-

plied in NOMA. Through SIC, strong users can nullify the MUIs caused the weak users.

Consider a MISO SC NOMA system, where a BS transmits information to two users m and n. The signal vector transmitted by the BS can be given by

$$\mathbf{x} = \mathbf{x}_m + \mathbf{x}_n \tag{2.6}$$

where $\mathbf{x}_m$ and $\mathbf{x}_n$ are the information signal vectors intended for Bob m and Bob n. Thus, the received signals at Bob m and Bob n on can be, respectively, expressed as

$$y_m = \mathbf{h}_m^H \mathbf{x} + n_m \tag{2.7}$$

and

$$y_n = \mathbf{h}_n^H \mathbf{x} + n_n \tag{2.8}$$

where $\mathbf{h}_m$ and $\mathbf{h}_n$ denote the channel vectors from the BS to user m and user n and $n_m \sim \mathcal{CN}(0, \sigma_m^2)$, $n_n \sim \mathcal{CN}(0, \sigma_n^2)$ are the Gaussian noises at user m and user n. Suppose user m is the weak user and user m is the strong user. Then, user m directly decodes its own signal and user n nullifies the MUI caused by user m through performing SIC before decoding its own message. Assume $\mathbf{x}_m$ and $\mathbf{x}_n$ are zero mean vectors. The SINRs at user m and n can be, respectively, given by

$$SINR_m = \frac{\mathbf{h}_m^H \mathbf{Q}_m \mathbf{h}_m}{\mathbf{h}_m^H \mathbf{Q}_n \mathbf{h}_m + \sigma_m^2} \tag{2.9}$$

and

$$SINR_n = \frac{\mathbf{h}_n^H \mathbf{Q}_n \mathbf{h}_n}{\sigma_n^2} \tag{2.10}$$

where $\mathbf{Q}_m = \mathrm{E}\left[\mathbf{x}_m \mathbf{x}_m^H\right]$ and $\mathbf{Q}_n = \mathrm{E}\left[\mathbf{x}_n \mathbf{x}_n^H\right]$ are the information covariance matrices for user m and user n.

2.4 Convex Optimization

In secure transmission designs, we aim at optimizing the information covariance matrices and the jamming covariance matrices to improve the security performance of considered systems. Convex optimization is an effective mathematical tool to solve the secure transmission design problems [80, 81].

Let us consider a standard form optimization problem:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\ \text{s.t.} \quad & f_i(\mathbf{x}) \leq 0, i = 1, 2, ..., m \\ & h_i(\mathbf{x}) = 0, i = 1, 2, ..., p. \end{aligned} \tag{2.11}$$

Equation (2.11) describes the problem of finding an $\mathbf{x}$ that minimizes $f_0(\mathbf{x})$ among all $\mathbf{x}$ that satisfy the conditions $f_i(\mathbf{x}) \leq 0$, $i = 1, 2, ..., m$ and $h_i(\mathbf{x}) = 0$, $i = 1, 2, ..., p$. We call vector $\mathbf{x}$ the optimization variable and $f_0(\mathbf{x})$ the objective function. Functions $f_i(\mathbf{x})$ and $h_i(\mathbf{x})$ are called the constraint functions. $f_i(\mathbf{x}) \leq 0$ and $h_i(\mathbf{x}) = 0$ are called the inequality and equality constraints, respectively. The set of points for which the objective and all constraint functions are defined,

$$\mathcal{D} = \bigcap_{i=0}^{m} \mathbf{dom}\ f_i \cap \bigcap_{i=1}^{p} \mathbf{dom}\ h_i, \tag{2.12}$$

is called the domain of problem (2.11). A point $\mathbf{x} \in \mathcal{D}$ is feasible if it satisfies the constraints $f_i(\mathbf{x}) \leq 0, i = 1, 2, ..., m$ and $h_i(\mathbf{x}) = 0, i = 1, 2, ..., p$. Problem (2.11) is said to be feasible if there exists at least one feasible point, and infeasible otherwise. The set of all feasible points is called the feasible set. The optimal value of problem (2.11) is the minimum value of the objective function over the feasible set. A point $\mathbf{x}^*$ is an optimal solution to problem (2.11), if $\mathbf{x}^*$ is feasible and can minimize the objective function.

The convexity of this problem can be verified by the following conditions [81]. First, the objective function $f_0(\mathbf{x})$ should be convex. Second, the inequality constraint functions $f_i(\mathbf{x}), i = 1, 2, ..., m$, should be convex. Third,

the equality constraint functions $h_i(\mathbf{x}), i = 1, 2, ..., p$ should be affine[7]. Once problem (2.15) is convex, we can find an optimal solution x^* to it by using standard algorithms from convex optimization theory [81], e.g, the interior point method. On the other hand, if problem (2.11) is non-convex, the solutions that we can obtain are usually suboptimal[8].

The Lagrange dual problem associated with problem (2.11) can be given by

$$\begin{aligned} \max_{\boldsymbol{\lambda},\mathbf{v}} \;\; & g(\boldsymbol{\lambda}, \mathbf{v}) \\ \text{s.t.} \;\; & \lambda_i \geq 0, i = 1, 2, ..., m \end{aligned} \tag{2.13}$$

where $g(\boldsymbol{\lambda}, \mathbf{v}) = \min_{\mathbf{x}\in\mathcal{D}} \left(f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{p} v_i h_i(\mathbf{x}) \right)$, vectors $\boldsymbol{\lambda}$ and $\mathbf{v}$ are called the dual variables, and λ_i and v_i are the elements in $\boldsymbol{\lambda}$ and $\mathbf{v}$. Let p^* and q^* denote the optimal values of the primal problem (2.11) and the dual problem (2.13) respectively. If the equality $p^* = q^*$ holds, then we say that strong duality holds for problems (2.11) and (2.13). Suppose the primal problem (2.11) is a convex optimization problem with differentiable objective and constraint functions and strong duality holds for it and its dual problem. Then, the Karush–Kuhn–Tucker (KKT) conditions

$$\begin{aligned} f_i(\mathbf{x}^*) &\leq 0, i = 1, ..., m, \\ h_i(\mathbf{x}^*) &= 0, i = 1, ..., p, \\ \lambda_i^* &\geq 0, i = 1, ..., m, \\ \lambda_i^* f_i(\mathbf{x}^*) &= 0, i = 1, ..., m, \\ \nabla f_0(\mathbf{x}^*) + \sum_{i=1}^{m} \lambda_i^* \nabla f_i(\mathbf{x}^*) + \sum_{i=1}^{p} v_i^* \nabla h_i(\mathbf{x}^*) &= 0, \end{aligned} \tag{2.14}$$

are necessary and sufficient for $\mathbf{x}^*$ and $\{\boldsymbol{\lambda}^*, \mathbf{v}^*\}$ where $\{\boldsymbol{\lambda}^*, \mathbf{v}^*\}$ is an optimal solution to the dual problem (2.13).

For further details on convex optimization, the reader can refer to [81, 82]

[7] For the definitions of affine functions and convex functions, the reader can refer to [81, pp. 36, 67].

[8] A suboptimal solution to problem (2.11) is a solution which is feasible but may not minimize the objective function.

and the references therein.

2.5 Summary

This chapter presented the required technical background for the entire thesis. We first introduced the beamforming schemes. We then presented background knowledge on SWIPT and NOMA. Finally, convex optimization was briefly introduced.

Chapter 3

Robust MISO Beamforming With CJ for Secure Transmission

In this chapter, we investigate robust QoS-based and secrecy rate based secure transmission designs for a MISO system having multiple eavesdroppers and a cooperative jammer. Two scenarios are considered: (a) ECSI is available and (b) ECSI is unavailable. In scenario (a), a QoS-based design is considered to minimize the worst case SINR at the eavesdroppers and to guarantee the QoS of the legitimate receiver. A secrecy rate-based design is also studied where the worst case secrecy rate is maximized. In scenario (b), a QoS-based design is considered to maximize the power of jamming signals under the QoS constraint of the legitimate receiver, and the secrecy rate-based design is not applicable. In all these designs, we jointly optimize the single stream beamforming vector and the jamming covariance matrix. As the optimization problems are non-convex, we propose an algorithm for each problem through SDR. Our analysis shows that, even though the beamforming scheme in our designs is single stream beamforming rather than beamforming with general rank information covariance, this does not cause any loss of optimality.

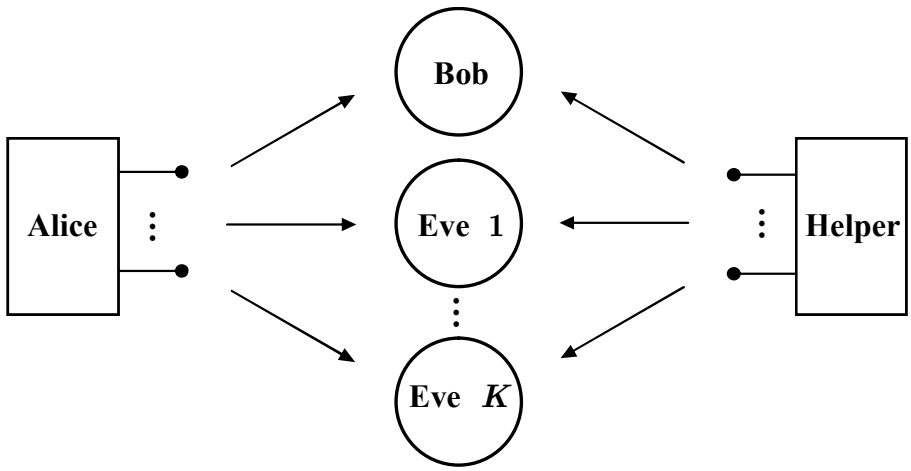

Figure 3.1: System model of secure transmission with K eavesdroppers.

3.1 System Model and Robust Transmission Designs Goals

We consider a CJ aided MISO system in Fig. 3.1 where a source node (Alice) sends information to a legitimate receiver (Bob) and a Helper sends jamming signals to confuse K eavesdroppers (Eves). We assume Alice and the Helper are equipped, respectively, with N_a and N_h transmit antennas, while both Bob and Eves have single antenna.

Denote the information signal vector transmitted from Alice and the jamming vector generated by the Helper as $\mathbf{x} \in \mathbb{C}^{N_a}$ and $\mathbf{z} \in \mathbb{C}^{N_h}$ respectively. The received signals at Bob and Eves can be expressed as

$$y_b = \mathbf{h}_b^H \mathbf{x} + \mathbf{g}_b^H \mathbf{z} + n_b \tag{3.1}$$

and

$$y_{e,k} = \mathbf{h}_{e,k}^H \mathbf{x} + \mathbf{g}_{e,k}^H \mathbf{z} + n_{e,k},\ k = 1, \ldots, K \tag{3.2}$$

respectively, where the vectors $\mathbf{h}_b, \mathbf{h}_{e,k}$ denote the channels from Alice to Bob and the kth Eve; the vectors $\mathbf{g}_b, \mathbf{g}_{e,k}$ denote the channels from the Helper

to Bob and the kth Eve. All the channels are assumed to be quasi-static flat fading channels. In (3.1) and (3.2), the jamming vector $\mathbf{z}$ follows a zero mean CSCG distribution, i.e, $\mathbf{z} \sim \mathcal{CN}(0, \mathbf{Q}_z)$ where $\mathbf{Q}_z \succeq \mathbf{0}$ is the jamming covariance matrix; $n_b \sim \mathcal{CN}(0, \sigma_b^2)$ and $n_{e,k} \sim \mathcal{CN}(0, \sigma_{e,k}^2)$ represent the zero mean Gaussian noises. In the robust secure transmission designs in this work, we consider single stream beamforming. Therefore, the information signal vector takes the form of $\mathbf{x} = \mathbf{w}x$ where the column vector $\mathbf{w}$ is the single stream beamforming vector and $x \sim \mathcal{CN}(0, 1)$ is the information bearing signal intended for Bob. Therefore, the SINRs at Bob and the k-th Eve can be, respectively, expressed as

$$SINR_b(\mathbf{w}, \mathbf{Q}_z) = \frac{\mathbf{h}_b^H \mathbf{w}\mathbf{w}^H \mathbf{h}_b}{\mathbf{g}_b^H \mathbf{Q}_z \mathbf{g}_b + \sigma_b^2} \tag{3.3}$$

and

$$SINR_{e,k}(\mathbf{w}, \mathbf{Q}_z) = \frac{\mathbf{h}_{e,k}^H \mathbf{w}\mathbf{w}^H \mathbf{h}_{e,k}}{\mathbf{g}_{e,k}^H \mathbf{Q}_z \mathbf{g}_{e,k} + \sigma_{e,k}^2}. \tag{3.4}$$

Since the CSI error is inevitable, the channel vectors $\mathbf{h}_b, \mathbf{h}_{e,k}, \mathbf{g}_b, \mathbf{g}_{e,k}$ cannot be perfectly known. In addition, for Eve's channels $\mathbf{h}_{e,k}, \mathbf{g}_{e,k}$, it is even possible that no channel knowledge can be obtained. Consequently, we consider the following two scenarios in this work.

1) Scenario (a): ECSI is available. This scenario arises when Eves are participating users of the system but attempt to access unauthorized services [20, 23, 25]. In this scenario, imperfect estimates of both Bob's and Eves' channels are available at Alice and the Helper. To describe the imperfect CSI, we have

$$\begin{aligned} \mathbf{h}_b &= \tilde{\mathbf{h}}_b + \mathbf{e}_{h,b}, \mathbf{g}_b = \tilde{\mathbf{g}}_b + \mathbf{e}_{g,b}, \\ \mathbf{h}_{e,k} &= \tilde{\mathbf{h}}_{e,k} + \mathbf{e}_{h,e,k}, \mathbf{g}_{e,k} = \tilde{\mathbf{g}}_{e,k} + \mathbf{e}_{g,e,k}, \ k = 1, \ldots, K \end{aligned} \tag{3.5}$$

where $\tilde{\mathbf{h}}_b$, $\tilde{\mathbf{g}}_b$, $\tilde{\mathbf{h}}_{e,k}$ and $\tilde{\mathbf{g}}_{e,k}$ are the estimates for each channel and $\mathbf{e}_{h,b}$, $\mathbf{e}_{g,b}$, $\mathbf{e}_{h,e,k}$ and $\mathbf{e}_{g,e,k}$ represent the corresponding CSI errors. We assume that the CSI errors are bounded by spheres, i.e., they lie in the bounded sets as

follows [22, 23, 83],

$$\varepsilon_{h,b} = \{\mathbf{e}_{h,b} : \ \|\mathbf{e}_{h,b}\|^2 \leq \xi_{h,b}^2\}, \quad \varepsilon_{g,b} = \{\mathbf{e}_{g,b} : \ \|\mathbf{e}_{g,b}\|^2 \leq \xi_{g,b}^2\},$$

$$\varepsilon_{h,e,k} = \{\mathbf{e}_{h,e,k} : \ \|\mathbf{e}_{h,e,k}\|^2 \leq \xi_{h,e,k}^2\}, \varepsilon_{g,e,k} = \{\mathbf{e}_{g,e,k} : \ \|\mathbf{e}_{g,e,k}\|^2 \leq \xi_{g,e,k}^2\}$$

where $\xi_{h,b}$, $\xi_{g,b}$, $\xi_{h,e,k}$ and $\xi_{g,e,k}$ are known radius of the uncertainty regions. In addition, we further define sets $\mathcal{H}_e = \{\tilde{\mathbf{h}}_{e,1}, \ldots, \tilde{\mathbf{h}}_{e,K}\}$, $\mathcal{G}_e = \{\tilde{\mathbf{g}}_{e,1}, \ldots, \tilde{\mathbf{g}}_{e,K}\}$, $\mathcal{N}_e = \{\sigma_{e,1}^2, \ldots, \sigma_{e,K}^2\}$, $\mathcal{B}_{h,e} = \{\xi_{h,e,1}^2, \ldots, \xi_{h,e,K}^2\}$ and $\mathcal{B}_{g,e} = \{\xi_{g,e,1}^2, \ldots, \xi_{g,e,K}^2\}$.

In this scenario, we first study a QoS based secure transmission design problem with the goal to minimize Eves' worst-case SINR while guaranteeing Bob's worst-case SINR[9] no less than a target value. Then, a secrecy rate based secure transmission design problem is considered where the worst-case secrecy rate is maximized with constrained power.

2) Scenario (b): ECSI is unavailable. This scenario occurs when Eves are passive and not part of the legitimate system. In this scenario, Alice and the Helper can only know the estimates of Bob's channels, $\tilde{\mathbf{h}}_b$ and $\tilde{\mathbf{g}}_b$, and the CSI error uncertainty radius, $\xi_{h,b}$ and $\xi_{g,b}$, but have no knowledge of Eves' channels.

Since there is no information about Eves' channels, secrecy rate is not achievable and the best approach for information security is to allocate as much power as possible to transmit the jamming signals while ensuring the QoS of Bob [27]. Thus, a robust QoS based secure transmission design is studied for the CJ aided MISO system to maximize the transmit power of jamming signals with the QoS requirement of Bob.

Note that all the secure transmission design problems in this work are investigated under individual power constraints. However, our results can be easily extended to global power constraint cases. We comment that, if the more general ellipsoidally bounded CSI error model [27, 84] is used instead of the spherically bounded one, the methods and conclusions in this work are still applicable.

[9] Eves' worst-case SINR refers to the largest SINR at Eves with CSI errors, and Bob's worst-case SINR refers to the smallest SINR at Bob with CSI errors.

3.2 Robust QoS Based Transmission Design for Scenario (a)

Based on (3.3)-(3.5), we can formulate the robust QoS based design problem as

$$\begin{aligned}\text{P-QoS-A: } \min_{\mathbf{w},\mathbf{Q}_z} \max_{\substack{\mathbf{e}_{h,e,k}\in\varepsilon_{h,e,k}\\ \mathbf{e}_{g,e,k}\in\varepsilon_{g,e,k}\\ k=1,\ldots,K}} & SINR_{e,k}(\mathbf{w},\mathbf{Q}_z)\\ \text{s.t. } \min_{\substack{\mathbf{e}_{h,b}\in\varepsilon_{h,b}\\ \mathbf{e}_{g,b}\in\varepsilon_{g,b}}} & SINR_b(\mathbf{w},\mathbf{Q}_z)\geq\gamma_b\\ & \|\mathbf{w}\|^2\leq P_s,\ \mathrm{tr}(\mathbf{Q}_z)\leq P_c,\ \mathbf{Q}_z\succeq\mathbf{0}\end{aligned}$$

where γ_b is the minimum SINR requirement at Bob, P_s and P_c are the power constraints for Alice and the Helper respectively. To make the design problem meaningful, we must have $\gamma_b > 0$ and $P_s > 0$.

It is natural to ask under what condition the robust design problem P-QoS-A is feasible. Motivated by [25, Lemma 1], the following proposition provides a verifiable criterion.

Proposition 3.1. *Assume that*

$$\|\tilde{\mathbf{h}}_b\|\geq\xi_{h,b}, \tag{3.6}$$

$$\|\tilde{\mathbf{h}}_b\|^2(\|\tilde{\mathbf{h}}_b\|-\xi_{h,b})^2\geq\gamma_b(\xi_{g,b}^2+\sigma_b^2), \tag{3.7}$$

$$\|\tilde{\mathbf{h}}_b\|^2\leq P_s \text{ and } (N_h-1)\leq P_c. \tag{3.8}$$

Then the robust design problem P-QoS-A is feasible.

Proof: See Appendix A.

In the following, we will discuss how to solve P-QoS-A under the assumption that it is feasible. In fact, the infeasible case implies that the given parameters are too stringent. In this case, we should raise the power constraints P_s and P_c or reduce the minimum SINR requirement γ_b to make P-QoS-A feasible.

The optimization problem described in P-QoS-A is non-convex; therefore, we relax it by using the SDR approach at first. According to SDR, we consider a general rank information covariance matrix based design problem

$$\min_{\mathbf{Q}_x,\mathbf{Q}_z,\gamma_e} \gamma_e \tag{3.9a}$$

$$\text{s.t.} \max_{\substack{\mathbf{e}_{h,e,k}\in\varepsilon_{h,e,k}\\ \mathbf{e}_{g,e,k}\in\varepsilon_{g,e,k}\\ k=1,\ldots,K}} \frac{\mathbf{h}_{e,k}^H\mathbf{Q}_x\mathbf{h}_{e,k}}{\mathbf{g}_{e,k}^H\mathbf{Q}_z\mathbf{g}_{e,k}+\sigma_{e,k}^2} \leq \gamma_e \tag{3.9b}$$

$$\min_{\substack{\mathbf{e}_{h,b}\in\varepsilon_{h,b}\\ \mathbf{e}_{g,b}\in\varepsilon_{g,b}}} \frac{\mathbf{h}_b^H\mathbf{Q}_x\mathbf{h}_b}{\mathbf{g}_b^H\mathbf{Q}_z\mathbf{g}_b+\sigma_b^2} \geq \gamma_b \tag{3.9c}$$

$$\text{tr}(\mathbf{Q}_x) \leq P_s,\ \text{tr}(\mathbf{Q}_z) \leq P_c,\ \mathbf{Q}_x \succeq \mathbf{0},\ \mathbf{Q}_z \succeq \mathbf{0}, \gamma_e \geq 0 \tag{3.9d}$$

where $\mathbf{Q}_x$ can be explained as the information covariance matrix. It is obvious if $\mathbf{Q}_x$ is restricted to be rank-one, the optimization problem described in (3.9) is equivalent to that of P-QoS-A. Since P-QoS-A is feasible, problem (3.9) must be feasible. Problem (3.9) is also non-convex, because both the constraints (3.9b) and (3.9c) are non-convex.

By introducing the slack variables v, ϱ, $u_1,\ldots,u_K$ and $z_1,\ldots,z_K$, we now rewrite (3.9) as

$$\min_{\substack{\mathbf{Q}_x,\mathbf{Q}_z,v,\varrho,\gamma_e\\ u_1,\ldots,u_K,z_1,\ldots,z_K}} \gamma_e$$

$$\text{s.t. } \mathbf{h}_b^H\mathbf{Q}_x\mathbf{h}_b \geq v, \forall \mathbf{e}_{h,b}\in\varepsilon_{h,b} \tag{3.10a}$$

$$\mathbf{g}_b^H\mathbf{Q}_z\mathbf{g}_b+\sigma_b^2 \leq \varrho, \forall \mathbf{e}_{g,b}\in\varepsilon_{g,b} \tag{3.10b}$$

$$v \geq \gamma_b\varrho,\ v\geq 0,\ \varrho\geq 0$$

$$\mathbf{h}_{e,k}^H\mathbf{Q}_x\mathbf{h}_{e,k} \leq u_k,\ \forall \mathbf{e}_{h,e,k}\in\varepsilon_{h,e,k} \tag{3.10c}$$

$$\mathbf{g}_{e,k}^H\mathbf{Q}_z\mathbf{g}_{e,k}+\sigma_{e,k}^2 \geq z_k,\ \forall \mathbf{e}_{g,e,k}\in\varepsilon_{g,e,k} \tag{3.10d}$$

$$u_k \leq z_k\gamma_e,\ u_k\geq 0,\ z_k\geq 0,\ \gamma_e\geq 0,\ k=1,\ldots,K$$

$$\text{tr}(\mathbf{Q}_x) \leq P_s,\ \text{tr}(\mathbf{Q}_z) \leq P_c,\ \mathbf{Q}_x \succeq \mathbf{0}, \mathbf{Q}_z \succeq \mathbf{0}.$$

Through the S-procedure (see [81, pp. 655] or [85]), we know (3.10a)-(3.10d) are equivalent to

$$T_{h,b}(\mathbf{Q}_x, \psi, v) = \begin{pmatrix} \psi\mathbf{I} + \mathbf{Q}_x & \mathbf{Q}_x\tilde{\mathbf{h}}_b \\ \tilde{\mathbf{h}}_b^H\mathbf{Q}_x & \tilde{\mathbf{h}}_b^H\mathbf{Q}_x\tilde{\mathbf{h}}_b - v - \psi\xi_{h,b}^2 \end{pmatrix} \succeq \mathbf{0}, \psi \geq 0 \tag{3.11}$$

$$T_{g,b}(\mathbf{Q}_z, \alpha, \varrho, \sigma_b^2) = \begin{pmatrix} \alpha\mathbf{I} - \mathbf{Q}_z & -\mathbf{Q}_z\tilde{\mathbf{g}}_b \\ -\tilde{\mathbf{g}}_b^H\mathbf{Q}_z & \varrho - \sigma_b^2 - \tilde{\mathbf{g}}_b^H\mathbf{Q}_z\tilde{\mathbf{g}}_b - \alpha\xi_{g,b}^2 \end{pmatrix} \succeq \mathbf{0}, \alpha \geq 0 \tag{3.12}$$

$$T_{h,e,k}(\mathbf{Q}_x, \varphi_k, u_k) = \begin{pmatrix} \varphi_k\mathbf{I} - \mathbf{Q}_x & -\mathbf{Q}_x\tilde{\mathbf{h}}_{e,k} \\ -\tilde{\mathbf{h}}_{e,k}^H\mathbf{Q}_x & u_k - \tilde{\mathbf{h}}_{e,k}^H\mathbf{Q}_x\tilde{\mathbf{h}}_{e,k} - \varphi_k\xi_{h,e,k}^2 \end{pmatrix} \succeq \mathbf{0}, \varphi_k \geq 0 \tag{3.13}$$

$$T_{g,e,k}(\mathbf{Q}_z, \beta_k, z_k, \sigma_{e,k}^2) = \begin{pmatrix} \beta_k\mathbf{I} + \mathbf{Q}_z & \mathbf{Q}_z\tilde{\mathbf{g}}_{e,k} \\ \tilde{\mathbf{g}}_{e,k}^H\mathbf{Q}_z & \tilde{\mathbf{g}}_{e,k}^H\mathbf{Q}_z\tilde{\mathbf{g}}_{e,k} - z_k + \sigma_{e,k}^2 - \beta_k\xi_{g,e,k}^2 \end{pmatrix} \succeq \mathbf{0}, \beta_k \geq 0. \tag{3.14}$$

Replacing (3.10a)-(3.10d) with (3.11)-(3.14), we convert problem (3.10) into

$$\min_{\substack{\mathbf{Q}_x,\mathbf{Q}_z,v,t,\psi,\alpha,\gamma_e,u_1,\ldots,u_K \\ z_1,\ldots,z_K,\varphi_1,\ldots,\varphi_K,\beta_1,\ldots,\beta_K}} \gamma_e \tag{3.15a}$$

$$\text{s.t. } (3.11) - (3.14), v \geq \gamma_b\varrho,\ v \geq 0,\ \varrho \geq 0 \tag{3.15b}$$

$$u_k \leq \gamma_e z_k, u_k \geq 0,\ z_k \geq 0,\ \gamma_e \geq 0, k = 1, \ldots, K \tag{3.15c}$$

$$\text{tr}(\mathbf{Q}_x) \leq P_s,\ \text{tr}(\mathbf{Q}_z) \leq P_c,\ \mathbf{Q}_x \succeq \mathbf{0}, \mathbf{Q}_z \succeq \mathbf{0}. \tag{3.15d}$$

The feasibility problem of (3.15) for fixed γ_e, i.e.,

$$\min_{\substack{\mathbf{Q}_x,\mathbf{Q}_z,v,t,\psi,\alpha,u_1,\ldots,u_K \\ z_1,\ldots,z_K,\varphi_1,\ldots,\varphi_K,\beta_1,\ldots,\beta_K}} 0 \text{ s.t. } (3.15b) - (3.15d), \tag{3.16}$$

is convex.

Because problem (3.9) is feasible and (3.15) is equivalent to (3.9), we can

always find a sufficient large value $\bar{\gamma}_e$ such that problem (3.16) is feasible when $\gamma_e = \bar{\gamma}_e$. Let γ_e^* denote the optimal value of (3.15), we have $\gamma_e^* \leq \bar{\gamma}_e$. According to (3.15c), we know $0 \leq \gamma_e^*$. Thus, γ_e^* is included in the finite interval $[0, \bar{\gamma}_e]$. In addition, we know the feasibility problem (3.16) is convex. Therefore, problem (3.15) can be solved by the bisection method [81, pp. 144-146], in which γ_e^* is searched over $[0, \bar{\gamma}_e]$ by solving the feasibility problem (3.16) for different values of γ_e.

Through the quasi-convex problem in (3.15), problem (3.9) can be solved, but the rank profile of the optimal solutions cannot be guaranteed, which will be demonstrated in our simulation. To solve the original design problem P-QoS-A, we need to further discuss how to obtain a rank-one solution to (3.9).

Consider the following optimization problem[10]

$$\min_{\mathbf{Q}_x, \mathbf{Q}_z, \theta_b, \theta_e} f(\theta_b, \theta_e) \tag{3.17a}$$

$$\text{s.t.} \max_{\substack{\mathbf{e}_{h,e,k} \in \varepsilon_{h,e,k} \\ \mathbf{e}_{g,e,k} \in \varepsilon_{g,e,k} \\ k=1,\ldots,K}} \frac{\mathbf{h}_{e,k}^H \mathbf{Q}_x \mathbf{h}_{e,k}}{\mathbf{g}_{e,k}^H \mathbf{Q}_z \mathbf{g}_{e,k} + \sigma_{e,k}^2} \leq \theta_e \tag{3.17b}$$

$$\min_{\substack{\mathbf{e}_{h,b} \in \varepsilon_{h,b} \\ \mathbf{e}_{g,b} \in \varepsilon_{g,b}}} \frac{\mathbf{h}_b^H \mathbf{Q}_x \mathbf{h}_b}{\mathbf{g}_b^H \mathbf{Q}_z \mathbf{g}_b + \sigma_b^2} \geq \theta_b \tag{3.17c}$$

$$\text{tr}(\mathbf{Q}_x) \leq P_s,\ \text{tr}(\mathbf{Q}_z) \leq P_c,\ \mathbf{Q}_x \succeq \mathbf{0},\ \mathbf{Q}_z \succeq \mathbf{0}, \theta_b \geq 0, \theta_e \geq 0 \tag{3.17d}$$

where $f(\theta_b, \theta_e)$ is a $\mathbb{R}^2 \to \mathbb{R}$ function of variables θ_b and θ_e. For fixed θ_b and θ_e which are feasible in problem (3.17) and $\theta_b > 0$, we have the following lemma and theorem.

Lemma 3.1. *Define* $\mathbf{F} = \text{diag}\{ T_{h,b}(\mathbf{Q}_x, \psi, v),\ T_{g,b}(\mathbf{Q}_z, \alpha, \varrho, \sigma_b^2),\ T_{h,e,k}(\mathbf{Q}_x, \varphi_k, u_k)|_{k=1,\ldots,K},\ T_{g,e,k}\ (\mathbf{Q}_z,\ \beta_k, z_k, \sigma_{e,k}^2)|_{k=1,\ldots,K}, \mathbf{Q}_x, \mathbf{Q}_z, P_c - \text{tr}(\mathbf{Q}_z), v - \theta_b \varrho, \theta_e z_k - u_k|_{k=1,\ldots,K},\ \psi,\ \alpha,\ \varphi_k|_{k=1,\ldots,K},\ \beta_k|_{k=1,\ldots,K}\}$ *and consider the fol-*

[10] We consider Lemma 3.1 and Theorem 3.1 for problem (3.17) rather than dealing with problem (3.9) (or its equivalent problem (3.15)) directly. In this way, Lemma 3.1 and Theorem 3.1 take general forms; therefore, we can use them in Section 3.3 to make the presentation more concise.

lowing convex SDP problem

$$\begin{aligned} &\min_{\mathbf{s}} \sum_{i=1}^{n_{\mathbf{s}}} c_i s_i \\ &s.t.\ \mathbf{F}_0 + \sum_{i=1}^{n_{\mathbf{s}}} s_i \mathbf{F}_i \succeq \mathbf{0} \end{aligned} \tag{3.18}$$

where s_i *is the* i*-th element in the* $n_{\mathbf{s}}$ *dimensional vector* $\mathbf{s}=[\mathrm{vec}(\mathbf{Q}_x), \mathrm{vec}(\mathbf{Q}_z), v, \varrho, \psi, \alpha, u_1, \ldots, u_K, z_1, \ldots, z_K, \varphi_1, \ldots, \varphi_K, \beta_1, \ldots, \beta_K]$*, matrix* $\mathbf{F}_i = \partial \mathbf{F}/\partial s_i$*,* $i = 1, \ldots, n_{\mathbf{s}}$*,* $\mathbf{F}_0 = \mathbf{F} - \sum_{i=1}^{n_{\mathbf{s}}} s_i \mathbf{F}_i$ *and* c_i *is defined as follows. We have* $c_i = 1$ *when* s_i *corresponds to a diagonal entry of matrix* $\mathbf{Q}_x$*;* $c_i = 0$ *otherwise. Then,*

(i) *Let* $\mathbf{s}^*=[\mathrm{vec}(\mathbf{Q}_x^*),\ \mathrm{vec}(\mathbf{Q}_z^*),\ v^*,\ \varrho^*,\ \psi^*,\ \alpha^*,\ u_1^*, \ldots,\ u_K^*,\ z_1^*, \ldots,\ z_K^*,\ \varphi_1^*, \ldots,\ \varphi_K^*,\ \beta_1^*, \ldots,\ \beta_K^*]$ *be an optimal solution to* (3.18)*. Then,* $\mathbf{Q}_x^*$ *and* $\mathbf{Q}_z^*$*, together with the fixed* θ_b *and* θ_e*, must be in the feasible set of* (3.17)*.*

(ii) *Strong duality holds for* (3.18)*.*

Proof: See Appendix B.

With the help of Lemma 3.1, we have Theorem 3.1.

Theorem 3.1. *For the fixed* θ_b *and* θ_e *which are feasible in problem* (3.17) *and the corresponding optimal solution* $\mathbf{s}^*$ *to* (3.18)*, if* $\theta_b > 0$*, we have* $\mathrm{rank}(\mathbf{Q}_x^*) = 1$*.*

Proof: See Appendix C.

Based on Lemma 3.1 and Theorem 3.1, we have the following proposition for P-QoS-A.

Proposition 3.2. *Let* $\theta_b = \gamma_b$ *and* $\theta_e = \gamma_e^*$*. Then for the corresponding optimal solution* $\mathbf{s}^*$ *to* (3.18)*, we have* $\{\mathbf{Q}_x^*,\ \mathbf{Q}_z^*,\ \gamma_e^*\}$ *as an optimal solution to* (3.9) *with* $\mathrm{rank}(\mathbf{Q}_x^*) = 1$*, and we have* $\{\mathbf{w}^*,\ \mathbf{Q}_z^*\}$ *as an optimal solution to the original design problem P-QoS-A, where the optimal single stream beamforming vector* $\mathbf{w}^*$ *is decomposed from* $\mathbf{Q}_x^*$*, i.e.,* $\mathbf{Q}_x^* = \mathbf{w}^*(\mathbf{w}^*)^H$*.*

Proof: Since (3.15) is equivalent to (3.9), γ_e^* is also the optimal value of (3.9). Thus, by letting $\theta_b = \gamma_b$ and $\theta_e = \gamma_e^*$, it is easy to see $\theta_b > 0$; and θ_b and θ_e must be feasible in (3.17). Then, we can observe that, for any feasible point $\{\mathbf{Q}_x, \mathbf{Q}_z, \gamma_b, \gamma_e^*\}$ in (3.17), $\{\mathbf{Q}_x, \mathbf{Q}_z, \gamma_e^*\}$ is an optimal solution to (3.9). Thus, by applying Lemma 3.1(i), we know $\{\mathbf{Q}_x^*, \mathbf{Q}_z^*, \gamma_e^*\}$ must be an optimal solution to (3.9). Furthermore, because of Theorem 3.1, there must be $\text{rank}(\mathbf{Q}_x^*) = 1$. Since (3.9) is an SDR of P-QoS-A, $\{\mathbf{w}^*, \mathbf{Q}_z^*\}$ must be an optimal solution to P-QoS-A.

Remark: The existence of rank-one optimal $\mathbf{Q}_x$ *in* (3.9) *means that the general rank information covariance matrix based design problem* (3.9) *is a tight relaxation of the single stream beamforming vector based original design problem P-QoS-A. Therefore, single stream beamforming is the optimal beamforming scheme.*

Based on Proposition 3.2, the details of solving P-QoS-A are summarized in Algorithm 1.

Algorithm 1: Algorithm of robust QoS based transmission design for scenario(a)

Input: $\tilde{\mathbf{h}}_b$, $\mathcal{H}_e$, $\tilde{\mathbf{g}}_b$, $\mathcal{G}_e$, σ_b^2, $\mathcal{N}_e$, γ_b, $\xi_{h,b}^2$, $\xi_{g,b}^2$, $\mathcal{B}_{h,e}$, $\mathcal{B}_{g,e}$, P_s, P_c.

1. Use bisection method to solve the quasi-convex problem (3.15) to obtain an optimal solution $\{\bar{\mathbf{Q}}_x^*, \bar{\mathbf{Q}}_z^*, \bar{v}^*, \bar{\varrho}^*, \bar{\psi}^*, \bar{\alpha}^*, \gamma_e^*, \bar{u}_1^*, \ldots, \bar{u}_K^*, \bar{z}_1^*, \ldots, \bar{z}_K^*, \bar{\varphi}_1^*, \ldots, \bar{\varphi}_K^*, \bar{\beta}_1^*, \ldots, \bar{\beta}_K^*\}$.
2. If $\text{rank}(\bar{\mathbf{Q}}_x^*) = 1$, let $\mathbf{Q}_x^* = \bar{\mathbf{Q}}_x^*$ $\mathbf{Q}_z^* = \bar{\mathbf{Q}}_z^*$ and go to step 4. Otherwise, go to step 3.
3. Solve the SDP problem (3.18) and obtain $\mathbf{Q}_x^*$ and $\mathbf{Q}_z^*$ which is contained in solution $\mathbf{s}^*$.
4. Obtain $\mathbf{w}^*$ by performing eigenvalue decomposition (EVD) for $\mathbf{Q}_x^*$.

Output: $\mathbf{w}^*$, $\mathbf{Q}_z^*$.

3.3 Robust Secrecy Rate Based Transmission Design for Scenario (a)

The worst-case secrecy rate maximization design problem described in Section 3.1 can be mathematically expressed as

$$\text{P-SRM-A:} \max_{\mathbf{w},\mathbf{Q}_z} \min_{\substack{\mathbf{e}_{h,b}\in\varepsilon_{h,b},\mathbf{e}_{g,b}\in\varepsilon_{g,b}\\ \mathbf{e}_{h,e,k}\in\varepsilon_{h,e,k},\mathbf{e}_{g,e,k}\in\varepsilon_{g,e,k}\\ k=1,\ldots,K}} \begin{array}{l}\log_2(1+SINR_b(\mathbf{w},\mathbf{Q}_z))\\ -\log_2(1+SINR_{e,k}(\mathbf{w},\mathbf{Q}_z))\end{array}$$
$$\text{s.t. } \|\mathbf{w}\|^2 \le P_s, \mathrm{tr}(\mathbf{Q}_z) \le P_c, \mathbf{Q}_z \succeq \mathbf{0}.$$

Note that the optimal value R^+ of problem P-SRM-A must be greater or equal to 0, because $\{\mathbf{w}=\mathbf{0}, \mathbf{Q}_z=\mathbf{0}\}$ exits in the feasible set.

To solve P-SRM-A, we first use the SDR approach to relax it into

$$\max_{\mathbf{Q}_x,\mathbf{Q}_z} \min_{\substack{\mathbf{e}_{h,b}\in\varepsilon_{h,b}\\ \mathbf{e}_{g,b}\in\varepsilon_{g,b}}} \log_2\left(1+\frac{\mathbf{h}_b^H\mathbf{Q}_x\mathbf{h}_b}{\mathbf{g}_b^H\mathbf{Q}_z\mathbf{g}_b+\sigma_b^2}\right) - \max_{\substack{\mathbf{e}_{h,e,k}\in\varepsilon_{h,e,k}\\ \mathbf{e}_{g,e,k}\in\varepsilon_{g,e,k}\\ k=1,\ldots,K}} \log_2\left(1+\frac{\mathbf{h}_{e,k}^H\mathbf{Q}_x\mathbf{h}_{e,k}}{\mathbf{g}_{e,k}^H\mathbf{Q}_z\mathbf{g}_{e,k}+\sigma_{e,k}^2}\right) \tag{3.19}$$
$$\text{s.t. } \mathrm{tr}(\mathbf{Q}_x) \le P_s,\ \mathrm{tr}(\mathbf{Q}_z) \le P_c,\ \mathbf{Q}_x \succeq \mathbf{0},\ \mathbf{Q}_z \succeq \mathbf{0}.$$

Problem (3.19) can be further rewritten as

$$\max_{\mathbf{Q}_x,\mathbf{Q}_z,\phi_b,\phi_e} \log_2(1+\phi_b) - \log_2(1+\phi_e) \tag{3.20a}$$
$$\text{s.t.} \max_{\substack{\mathbf{e}_{h,e,k}\in\varepsilon_{h,e,k}\\ \mathbf{e}_{g,e,k}\in\varepsilon_{g,e,k}\\ k=1,\ldots,K}} \frac{\mathbf{h}_{e,k}^H\mathbf{Q}_x\mathbf{h}_{e,k}}{\mathbf{g}_{e,k}^H\mathbf{Q}_z\mathbf{g}_{e,k}+\sigma_{e,k}^2} \le \phi_e \tag{3.20b}$$
$$\min_{\substack{\mathbf{e}_{h,b}\in\varepsilon_{h,b}\\ \mathbf{e}_{g,b}\in\varepsilon_{g,b}}} \frac{\mathbf{h}_b^H\mathbf{Q}_x\mathbf{h}_b}{\mathbf{g}_b^H\mathbf{Q}_z\mathbf{g}_b+\sigma_b^2} \ge \phi_b \tag{3.20c}$$
$$\mathrm{tr}(\mathbf{Q}_x) \le P_s, \mathrm{tr}(\mathbf{Q}_z) \le P_c,\ \mathbf{Q}_x \succeq \mathbf{0},\ \mathbf{Q}_z \succeq \mathbf{0}, \phi_b \ge 0, \phi_e \ge 0. \tag{3.20d}$$

Even though (3.20) is a relaxation of the original design problem P-SRM-

A, it can be shown that this relaxation is tight and P-SRM-A can thus be solved.

Proposition 3.3. *Let ϖ^+ denote the optimal value of* (3.20) *and assume* $\{\mathbf{Q}_x^+,\ \mathbf{Q}_z^+,\ \phi_b^+,\ \phi_e^+\}$ *is an optimal solution to it. If $\varpi^+ > 0$, we have* $\text{rank}(\mathbf{Q}_x^+) = 1$ *with* $\mathbf{Q}_x^+ = \mathbf{w}^+(\mathbf{w}^+)^H$*, and* $\{\mathbf{w}^+, \mathbf{Q}_z^+\}$ *is an optimal solution to P-SRM-A. Otherwise,* $\varpi^+ = 0$ *and* $\{\mathbf{w}^+ = \mathbf{0}, \mathbf{Q}_z^+ = \mathbf{0}\}$ *is an optimal solution to P-SRM-A.*

Proof: See Appendix D.

Remark: Proposition 3.3 implies that the single stream beamforming vector based design P-SRM-A must perform as well as the general rank information covariance matrix based design (3.20) *in terms of maximizing the secrecy rate, which gives a theoretical justification for using single stream beamforming.*

In the following, we will focus on solving problem (3.20). Our method to solve (3.20) is to divide it into two layers. The outer layer is an one-dimensional optimization problem and the inner layer is a sequence of convex SDPs.

In the inner layer, we consider solving problem (3.20) with fixed ϕ_e. By the Charnes-Cooper method [86], we have the following proposition.

Proposition 3.4. *When ϕ_e is fixed, let $\varpi(\phi_e)$ denote the optimal value of problem* (3.20) *and assume* $\{\tilde{\mathbf{Q}}_x(\phi_e),\ \tilde{\mathbf{Q}}_z(\phi_e),\ \tilde{\phi}_b(\phi_e)\}$ *is an optimal solution to it. Consider a convex SDP problem*

$$\begin{aligned}
\chi(\phi_e) = \max_{\substack{\mathbf{X}_\eta, \mathbf{Z}_\eta, \eta, \phi_b, \psi, \alpha, u_{\eta,1}, \ldots, u_{\eta,K} \\ z_{\eta,1}, \ldots, z_{\eta,K}, \varphi_1, \ldots, \varphi_K, \beta_1, \ldots, \beta_K}} & \log_2(1+\phi_b) - \log_2(1+\phi_e) \\
s.t.\ & T_{h,b}(\mathbf{X}_\eta, \psi, \phi_b) \succeq \mathbf{0}, T_{g,b}(\mathbf{Z}_\eta, \alpha, 1, \sigma_b^2\eta) \succeq \mathbf{0}, \psi \geq 0, \alpha \geq 0 \\
& T_{h,e,k}(\mathbf{X}_\eta, \varphi_k, u_{\eta,k}) \succeq \mathbf{0}, T_{g,e,k}(\mathbf{Z}_\eta, \beta_k, z_{\eta,k}, \sigma_{e,k}^2\eta) \succeq \mathbf{0} \\
& \varphi_k \geq 0, \beta_k \geq 0, u_{\eta,k} \geq 0, z_{\eta,k} \geq 0, u_{\eta,k} \leq \phi_e z_{\eta,k}, k = 1, \ldots, K \\
& \text{tr}(\mathbf{X}_\eta) \leq P_s\eta,\ \text{tr}(\mathbf{Z}_\eta) \leq P_c\eta,\ \mathbf{X}_\eta \succeq \mathbf{0},\ \mathbf{Z}_\eta \succeq \mathbf{0}, \eta \geq 0, \phi_b \geq 0
\end{aligned} \tag{3.21}$$

where $\chi(\phi_e)$ denotes the optimal value of (3.21)*. If $\chi(\phi_e) > 0$, there must be $\chi(\phi_e) = \varpi(\phi_e)$. Moreover, for $\tilde{\mathbf{X}}_\eta(\phi_e)$, $\tilde{\mathbf{Z}}_\eta(\phi_e)$ and $\tilde{\eta}(\phi_e)$ which are involved in an optimal solution to* (3.21)*, we have $\tilde{\mathbf{Q}}_x(\phi_e) = \tilde{\mathbf{X}}_\eta(\phi_e)/\tilde{\eta}(\phi_e)$, $\tilde{\mathbf{Q}}_z(\phi_e) = \tilde{\mathbf{Z}}_\eta(\phi_e)/\tilde{\eta}(\phi_e)$. Otherwise, $\varpi(\phi_e) \leq \chi(\phi_e) \leq 0$.*

Proof: See Appendix E.

In the outer layer, we first note that

$$\phi_e \leq \max_{\substack{\mathbf{Q}_x \succeq \mathbf{0}, \mathbf{Q}_z \succeq \mathbf{0} \\ \mathrm{tr}(\mathbf{Q}_x) \leq P_s, \mathrm{tr}(\mathbf{Q}_z) \leq P_c}} \min_{\substack{\mathbf{e}_{h,b} \in \varepsilon_{h,b} \\ \mathbf{e}_{g,b} \in \varepsilon_{g,b}}} \frac{\mathbf{h}_b^H \mathbf{Q}_x \mathbf{h}_b}{\mathbf{g}_b^H \mathbf{Q}_z \mathbf{g}_b + \sigma_b^2} \tag{3.22a}$$

$$\leq \max_{\mathbf{Q}_x \succeq \mathbf{0}, \mathrm{tr}(\mathbf{Q}_x) \leq P_s} \min_{\mathbf{e}_{h,b} \in \varepsilon_{h,b}} \frac{\mathbf{h}_b^H \mathbf{Q}_x \mathbf{h}_b}{\sigma_b^2} \tag{3.22b}$$

where (3.22a) holds because there must be $\varpi(\phi_e) \leq \chi(\phi_e) < 0$ otherwise. Assuming the value of the optimization problem in (3.22b) is denoted by $\bar{\phi}_e$, by the similar manner as that in Section 3.2, we can see $\bar{\phi}_e$ can be calculated through

$$\max_{\mathbf{Q}_x, \phi_e, \psi} \phi_e \text{ s.t. } T_{h,b}(\mathbf{Q}_x, \psi, \phi_e \sigma_b^2) \succeq \mathbf{0}, \mathbf{Q}_x \succeq \mathbf{0}, \mathrm{tr}(\mathbf{Q}_x) \leq P_s, \psi \geq 0, \phi_e \geq 0 \tag{3.23}$$

which is convex. Then, we solve (3.20) by

$$\begin{aligned} &\max_{\phi_e} \chi(\phi_e) \\ &\text{s.t. } 0 \leq \phi_e \leq \bar{\phi}_e. \end{aligned} \tag{3.24}$$

The function $\chi(\phi_e)$ is not analytically tractable, but can be efficiently computed for every fixed ϕ_e by solving (3.21). Therefore, the one-dimensional optimization problem (3.24) can be handled by any line search method, e.g., uniform sampling or the golden search [87].

Remark: It can be shown that problem (3.20) *can be simplified into problem* (3.9) *if ϕ_b is fixed. Thus, it is natural to consider solving problem* (3.20) *on the basis of the discussion in Section III for solving (9), i.e., consider a two-layer method, which is similar to the proposed ϕ_e fixed method, with fixing ϕ_b in the inner layer. However, compared with this ϕ_b fixed method,*

the proposed ϕ_e fixed method is superior in computational complexity. Since solving the inner layer problem of the ϕ_e fixed method (i.e., the convex problem (3.21)*) is much more efficient than solving the inner layer problem of the ϕ_b fixed method (i.e., the quasi-convex problem* (3.15)*). On the other hand, while the ϕ_e fixed method has lower complexity in solving (20), it cannot be applied to problem (9) in Section III since γ_b is constant in (9).*

Suppose $\chi(\phi_e)$ is maximized at a point ϕ_e^* in (3.24). Through Proposition 3.4, if $\chi(\phi_e^*) > 0$, we have $\chi(\phi_e^*) = \varpi(\phi_e^*) = \varpi^+$ and $\mathbf{Q}_x^+ = \tilde{\mathbf{Q}}_x(\phi_e^*) = \tilde{\mathbf{X}}_\eta(\phi_e^*)/\tilde{\eta}(\phi_e^*)$, $\mathbf{Q}_z^+ = \tilde{\mathbf{Q}}_z(\phi_e^*) = \tilde{\mathbf{Z}}_\eta(\phi_e^*)/\tilde{\eta}(\phi_e^*)$. Otherwise, since $\varpi(\phi_e) \leq \chi(\phi_e) \leq \chi(\phi_e^*) \leq 0$, we know $\varpi^+ = 0$. Thus, by Proposition 3.3, problem P-SRM-A can be solved. More specifically, the steps for solving the original design problem P-SRM-A are described in Algorithm 2.

Algorithm 2: Algorithm of robust secrecy rate based transmission design for scenario(a)

Input: $\tilde{\mathbf{h}}_b$, $\mathcal{H}_e$, $\tilde{\mathbf{g}}_b$, $\mathcal{G}_e$, σ_b^2, $\mathcal{N}_e$, $\xi_{h,b}^2$, $\xi_{g,b}^2$, $\mathcal{B}_{h,e}$, $\mathcal{B}_{g,e}$, P_s, P_c.

1. Solve problem (3.23) and take the optimal value as $\bar{\phi}_e$.
2. Apply a suitable line search method to solve (3.24) and obtain ϕ_e^* by selecting specific ϕ_e over the interval $[0, \bar{\phi}_e]$ and calculating $\chi(\phi_e)$ through (3.21) iteratively.
3. If $\chi(\phi_e^*) > 0$, go to step 4. Otherwise, let $\mathbf{w}^+ = \mathbf{0}$, $\mathbf{Q}_z^+ = \mathbf{0}$ and go to **Output**.
4. Obtain $\tilde{\mathbf{X}}_\eta(\phi_e^*)$, $\tilde{\mathbf{Z}}_\eta(\phi_e^*)$, $\tilde{\eta}(\phi_e^*)$ by solving problem (3.21) with $\phi_e = \phi_e^*$.
5. Let $\mathbf{Q}_x^+ = \tilde{\mathbf{X}}_\eta(\phi_e^*)/\tilde{\eta}(\phi_e^*)$, $\mathbf{Q}_z^+ = \tilde{\mathbf{Z}}_\eta(\phi_e^*)/\tilde{\eta}(\phi_e^*)$.
6. Obtain $\mathbf{w}^+$ by performing EVD for $\mathbf{Q}_x^+$.

Output: $\mathbf{w}^+$, $\mathbf{Q}_z^+$.

3.4 Robust QoS Based Transmission Design for Scenario (b)

According to the discussion in Section 3.1, the QoS based transmission design problem for scenario (b) can be derived as

$$\begin{aligned}
\text{P-QoS-B: } & \max_{\mathbf{w},\mathbf{Q}_z} \ \mathrm{tr}(\mathbf{Q}_z) \\
& \text{s.t.} \min_{\substack{\mathbf{e}_{h,b}\in\varepsilon_{h,b}\\ \mathbf{e}_{g,b}\in\varepsilon_{g,b}}} SINR_b(\mathbf{w},\mathbf{Q}_z) \geq \gamma_b \\
& \qquad \|\mathbf{w}\|^2 \leq P_s, \mathrm{tr}(\mathbf{Q}_z) \leq P_c, \mathbf{Q}_z \succeq \mathbf{0}.
\end{aligned}$$

For the feasibility of P-QoS-B, because P-QoS-A and P-QoS-B have the same constraints, Proposition 3.1 can also be applied.

Similar to Section 3.2, the following discussion is based on the assumption that $P_s > 0$, $\gamma_b > 0$ and problem P-QoS-B is feasible.

The SDR problem of P-QoS-B is

$$\begin{aligned}
& \max_{\mathbf{Q}_x,\mathbf{Q}_z} \ \mathrm{tr}(\mathbf{Q}_z) \\
& \text{s.t.} \min_{\substack{\mathbf{e}_{h,b}\in\varepsilon_{h,b}\\ \mathbf{e}_{g,b}\in\varepsilon_{g,b}}} \frac{\mathbf{h}_b^H\mathbf{Q}_x\mathbf{h}_b}{\mathbf{g}_b^H\mathbf{Q}_z\mathbf{g}_b+\sigma_b^2} \geq \gamma_b \\
& \quad \mathrm{tr}(\mathbf{Q}_x) \leq P_s, \mathrm{tr}(\mathbf{Q}_z) \leq P_c, \mathbf{Q}_x \succeq \mathbf{0}, \mathbf{Q}_z \succeq \mathbf{0}.
\end{aligned} \tag{3.25}$$

Through the discussion for problem (3.9), we know (3.25) can be transformed into

$$\begin{aligned}
& \max_{\mathbf{Q}_x,\mathbf{Q}_z,\psi,\alpha,v,\varrho} \ \mathrm{tr}(\mathbf{Q}_z) \\
& \text{s.t. } T_{h,b}(\mathbf{Q}_x,\psi,v) \succeq \mathbf{0}, T_{g,b}(\mathbf{Q}_z,\alpha,\varrho,\sigma_b^2) \succeq \mathbf{0} \\
& \quad v \geq \gamma_b\varrho,\ \psi \geq 0, \alpha \geq 0, v \geq 0, \varrho \geq 0 \\
& \quad \mathrm{tr}(\mathbf{Q}_x) \leq P_s, \mathrm{tr}(\mathbf{Q}_z) \leq P_c,\ \mathbf{Q}_x \succeq \mathbf{0}, \mathbf{Q}_z \succeq \mathbf{0}
\end{aligned} \tag{3.26}$$

which is a convex SDP. The SDR problem (3.25) can be solved through (3.26). However, the rank profile of its optimal solutions cannot be guaran-

teed, which will be shown in our simulation. To solve P-QoS-B, we need a method to find out a rank-guaranteed solution in (3.26).

Proposition 3.5. *Suppose problem P-QoS-B is feasible. By letting* $\{\mathbf{Q}_x^-, \mathbf{Q}_z^-, \psi^-, \alpha^-, v^-, \varrho^-\}$ *denote an optimal solution to problem* (3.26)*, we construct a convex SDP problem as*

$$\min_{\mathbf{s}} \sum_{i=1}^{n_{\mathbf{s}}} c_i s_i \tag{3.27a}$$

$$s.t.\ \mathbf{G}_0 + \sum_{i=1}^{n_{\mathbf{s}}} s_i \mathbf{G}_i \succeq \mathbf{0}. \tag{3.27b}$$

Here s_i *is the* i*-th element in the* $n_{\mathbf{s}}$ *dimensional vector* $\mathbf{s} = [\mathrm{vec}(\mathbf{Q}_x), v, \psi]$*.* c_i *is 1 when* s_i *represents a diagonal entry of* $\mathbf{Q}_x$*; otherwise, it is 0. For matrices,* $\mathbf{G}_i = \partial \mathbf{G}/\partial s_i, i = 1, \ldots, n_{\mathbf{s}}$*,* $\mathbf{G}_0 = \mathbf{G} - \sum_{i=1}^{n_{\mathbf{s}}} s_i \mathbf{G}_i$ *and* $\mathbf{G} = \mathrm{diag}\{T_{h,b}(\mathbf{Q}_x, \psi, v), \mathbf{Q}_x, v - \gamma_b \varrho^-, \psi\}$*. For each optimal solution* $\ddot{\mathbf{s}} = [\mathrm{vec}(\ddot{\mathbf{Q}}_x), \ddot{v}, \ddot{\psi}]$ *to* (3.27)*, we have* $\{\ddot{\mathbf{Q}}_x, \mathbf{Q}_z^-, \ddot{\psi}, \alpha^-, \ddot{v}, \varrho^-\}$ *is an optimal solution to* (3.26) *and* $\ddot{\mathbf{Q}}_x$ *must be rank-one. Thus,* $\ddot{\mathbf{Q}}_x$ *can be decomposed as* $\ddot{\mathbf{w}}\ddot{\mathbf{w}}^H$ *and* $\{\ddot{\mathbf{w}}, \mathbf{Q}_z^-\}$ *is an optimal solution to P-QoS-B.*

Proof: See Appendix F.

Based on Proposition 3.5, the original design problem in P-QoS-B can be exactly solved as shown in Algorithm 3.

Algorithm 3: Algorithm of robust QoS based transmission design for scenario(b)

Input: $\tilde{\mathbf{h}}_b$, $\tilde{\mathbf{g}}_b$, σ_b^2, γ_b, $\xi_{h,b}^2$, $\xi_{g,b}^2$, P_s, P_c.

1. Obtain $\{\mathbf{Q}_x^-, \mathbf{Q}_z^-, \psi^-, \alpha^-, v^-, \varrho^-\}$ by solving problem (3.26).
2. If $\mathrm{rank}(\mathbf{Q}_x^-) = 1$, let $\ddot{\mathbf{Q}}_x = \mathbf{Q}_x^-$ and go to step 4. Otherwise, go to step 3.
3. Solve (3.27) and obtain $\ddot{\mathbf{Q}}_x$ which is contained in solution $\ddot{\mathbf{s}}$.
4. Obtain $\ddot{\mathbf{w}}$ by performing EVD for $\ddot{\mathbf{Q}}_x$.

Output: $\ddot{\mathbf{w}}$, $\mathbf{Q}_z^-$.

3.5 Simulation Results

In this section, we first demonstrate numerical results on the performance of the robust secrecy rate maximization design (SRMD) proposed in Section 3.3. Then, we present simulation experiments to illustrate the performance of the robust QoS based secure transmission designs for scenarios (a) and (b) (which are referred to as QoSD(a) and QoSD(b)). For comparison, we also examine the robust and non-robust individual power constrained SRMDs discussed in [22] and the relaxed counterparts of QoSD(a) and QoSD(b) (which are referred to as QoSD(a)-SDR and QoSD(b)-SDR), i.e., the general rank information covariance matrix based designs in (3.9) and (3.25) respectively.

For all the examples in this section, we assume Alice and the Helper both have four antennas, i.e., $N_a = 4$, $N_h = 4$. Elements of channel estimates are independent, zero mean complex Gaussian random variables with unit variance and the channel uncertainty are given by $\xi_{h,b}^2 = \xi_{g,b}^2 = \xi_b^2$ and $\xi_{h,e,k}^2 = \xi_{g,e,k}^2 = \xi_e^2$, $k = 1, ..., K$. The noise power at Bob and Eves is assumed to be unity, i.e., $\sigma_d^2 = \sigma_{e,k}^2 = 1$. For the power bounds for Alice and the Helper, we assume they are the same, $P_s = P_c$, and their sum $P = P_s + P_c$ is defined in dB. Convex optimization problems are solved by using CVX [88] and the number of trials for Monte Carlo simulations is $1,000$.

In Figure 3.2, we compare the impacts of increasing ξ_b^2 on the proposed SRMD and its counterparts. Because SRMDs in [22] is only for the one eavesdropper scenario, we assume that $K = 1$. Moreover, we set $P = 10$ dB and $\xi_e^2 = 0.3$ in this case. During the aggregation of channel uncertainty in LCSI, the secrecy rate gaps between the the proposed SRMD and the SRMDs in [22] are widen significantly. The reason behind this is that both the robust and non-robust SRMDs in [22] are based on zero-forcing and the zero-forcing approach is sensitive to LCSI errors. Figure 3.3 shows the optimal worst-case secrecy rates achieved by the proposed SRMD in multiple eavesdroppers cases as functions of P, with $\xi_b^2 = 0.2$ and $\xi_e^2 = 0.3$. We can observe that the increase of the number of eavesdroppers causes performance

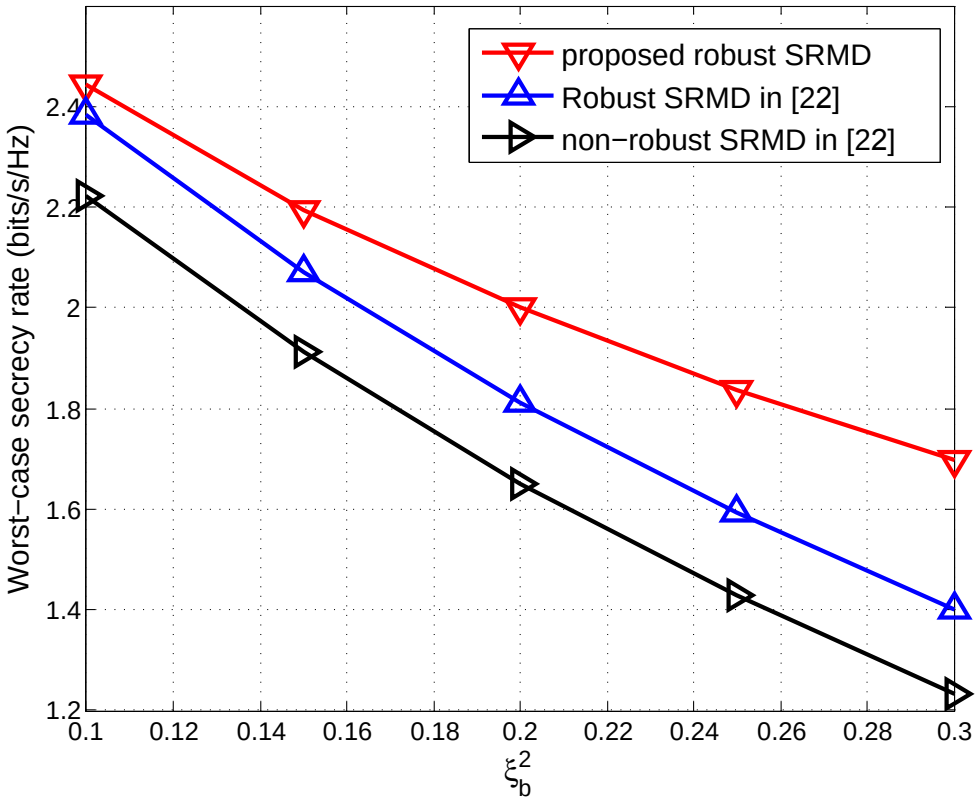

Figure 3.2: Average worst-case secrecy rates for proposed SRMD and SRMDs in existing work versus ξ_b^2 with $K = 1$.

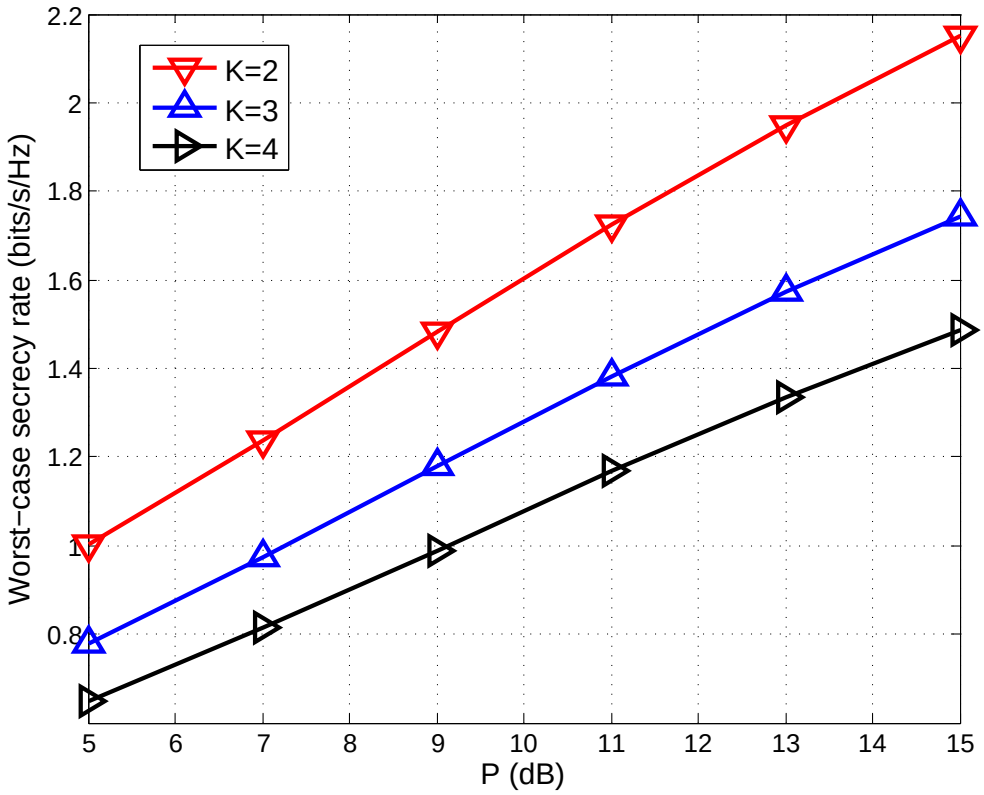

Figure 3.3: Average worst-case secrecy rates in multiple eavesdroppers scenarios for proposed SRMD versus P.

degradation, which can be overcome by increasing P.

Figure 3.4 represents the average worst-case SINRs at both Bob and Eves versus the channel uncertainty in ECSI for QoSD(a) and QoSD(a)-SDR with $K = 2$, $P = 10$dB, $\xi_b^2 = 0.2$ and $\gamma_b = 5$dB. We can observe that the performance of QoSD(a) coincides with that of QoSD(a)-SDR. This verifies our analysis that single stream beamforming is the best beamforming strategy. Compared with Figure 3.3, Figure 3.4 also shows that Alice can transmit information to Bob at a speed higher than the maximum achievable secrecy rate with limited SINR leakage to Eves. Thus, when the achievable secrecy rate cannot meet the data rate demand, QoS based transmission design can be used to help information security from physical layer by crippling eavesdroppers' interceptions. In Figure 3.5, we examine the probabilities of obtaining rank-one and non-rank-one optimal $\mathbf{Q}_x$ in QoSD(a)-SDR with the same parameters as those in Figure 3.4. It can be observed that the chance of directly obtaining rank-one optimal $\mathbf{Q}_x$ through QoSD(a)-SDR is no more than 60% and this chance decreases when ξ_e^2 is increased.

In Figure 3.6, the worst-case SINRs at both Bob and Eves[11] are plotted against increasing γ_b for QoSD(b) and QoSD(b)-SDR, when $\xi_b^2 = 0.2$ and $P = 10$dB. In addition, the probabilities of obtaining rank-one and non-rank-one optimal $\mathbf{Q}_x$ in QoSD(b)-SDR are demonstrated in Figure 3.7 for different values of γ_b. In Figure 3.6, we can see that, even though there is no knowledge about ECSI, the proposed design QoSD(b) can suppress the SINR at Eves and impose a 3dB gap with respect to the worst-case SINR at Bob. It can also be observed for QoSD(b)-SDR that the QoS provided at Bob exceeds the requirement while signals with higher SINR are leaked to Eves in comparison with QoSD(b). This can be explained by the fact that the non-unique-rank solutions in QoSD(b)-SDR usually maximize the power of jamming signals at the expense of enhancing the power of information transmission and non-unique-rank solutions are obtained in QoSD(b)-SDR with high probability, which is shown in Figure 3.7. Since the QoS requirement

[11] ECSI is unknown in QoSD(b) and QoSD(b)-SDR. To calculate the worst-case SINR at Eves, we randomly generate channel estimates for Eves ($\tilde{\mathbf{h}}_{e,k}$ and $\tilde{\mathbf{g}}_{e,k}$) and assume $\sigma_{e,k}^2 = 1$, $K = 2$, $\xi_e^2 = 0.3$.

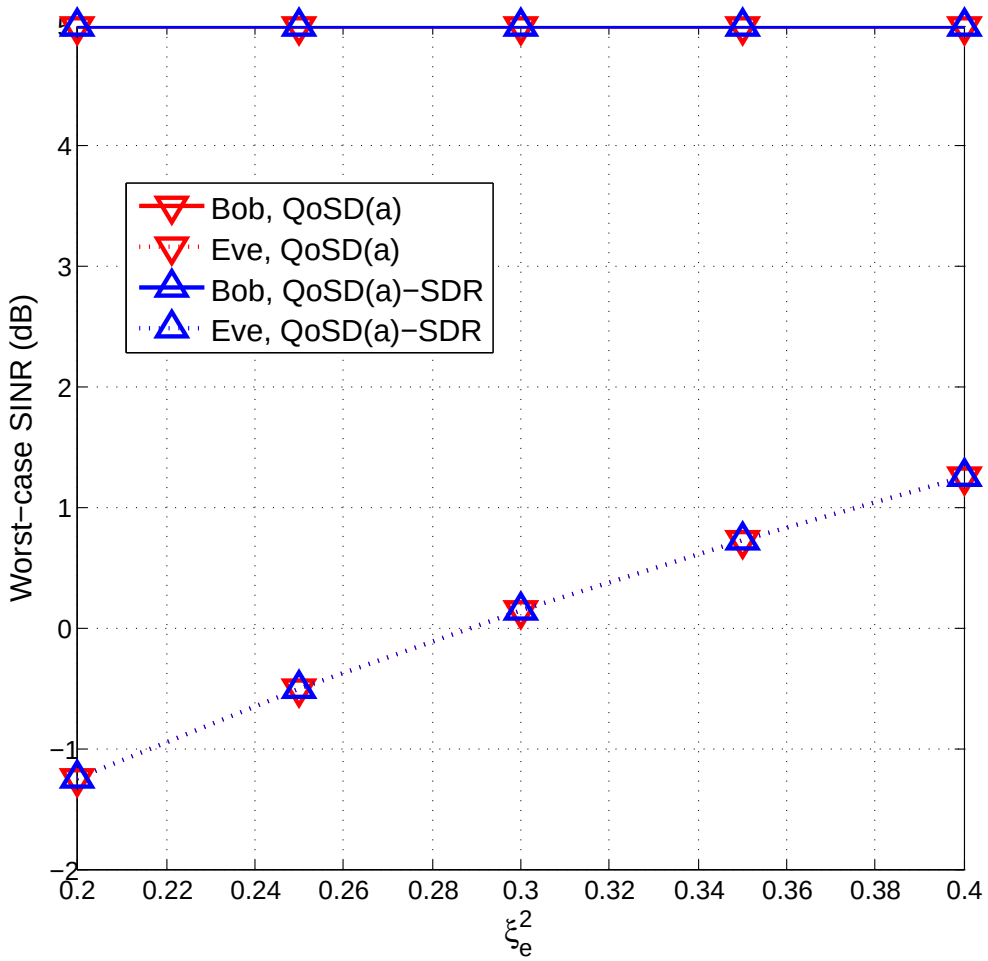

Figure 3.4: Average worst-case SINRs at Bob and Eves for QoSD(a) and QoSD(a)-SDR versus ξ_e^2.

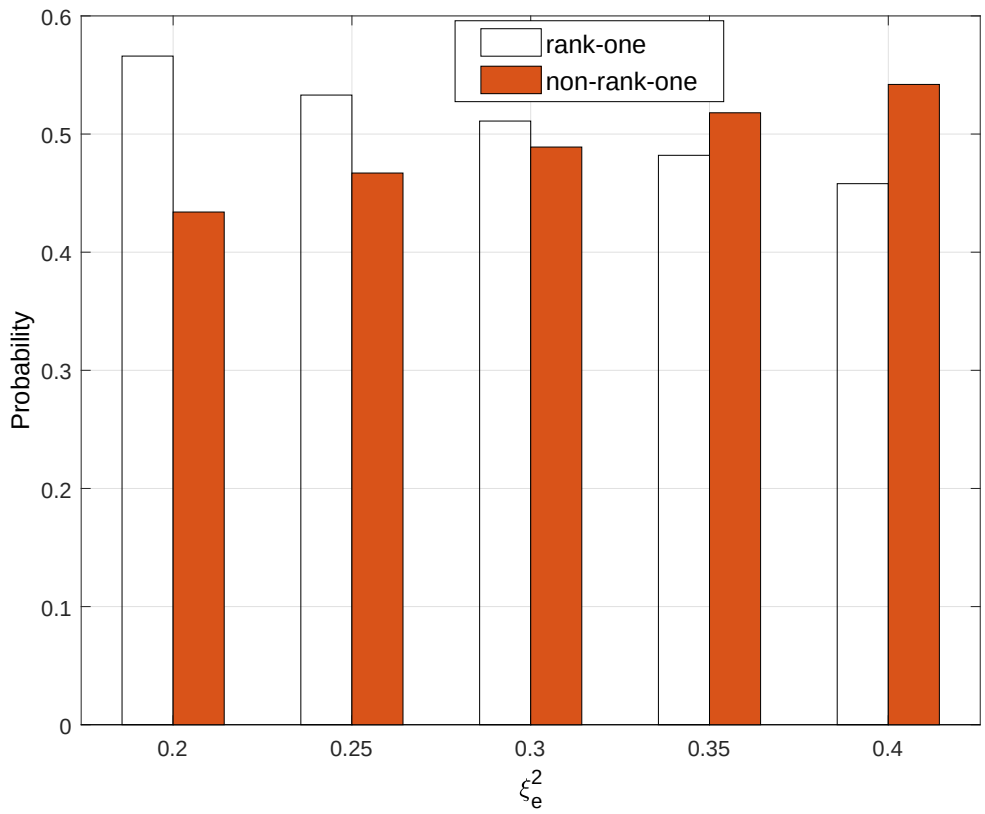

Figure 3.5: Probabilities of rank-one and non-rank-one optimal $\mathbf{Q}_x$ in QoSD(a)-SDR versus ξ_e^2.

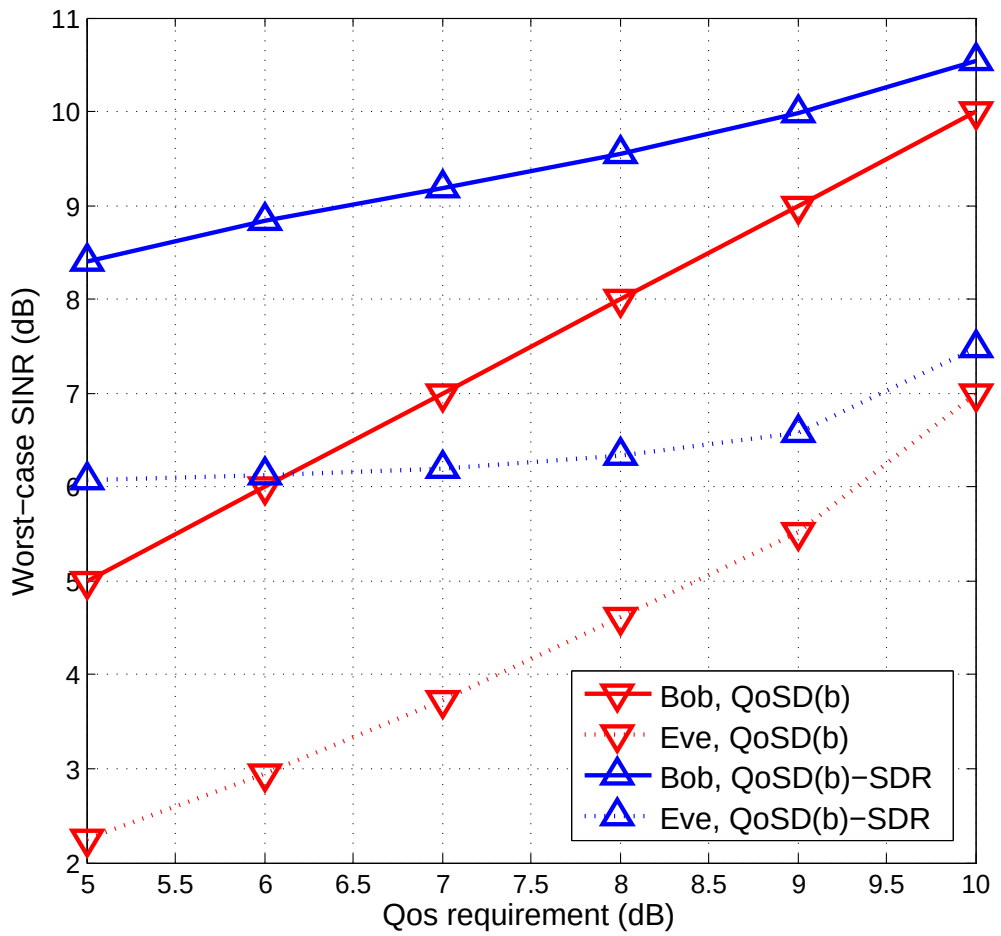

Figure 3.6: Average worst-case SINRs at Bob and Eves for QoSD(b) and QoSD(b)-SDR versus QoS requirement at Bob.

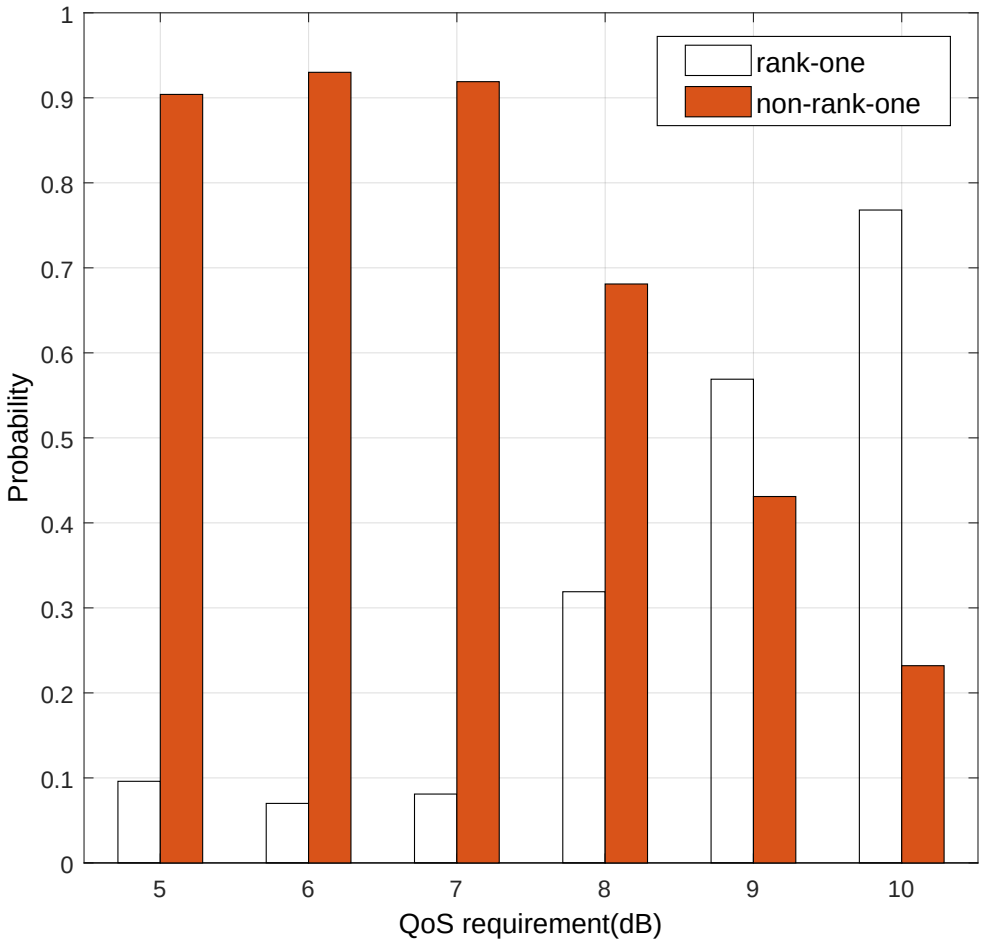

Figure 3.7: Probabilities of rank-one and non-rank-one optimal $\mathbf{Q}_x$ in QoSD(b)-SDR versus the QoS requirement at Bob.

is met by both QoSD(b) and QoSD(b)-SDR, QoSD(b) shows a performance improvement by suppressing the SINR at Eves.

3.6 Summary

We studied robust secure transmission designs from both QoS based and secrecy rate based perspectives for a CJ aided MISO system with norm bounded channel uncertainties. Two scenarios were considered: (a) ECSI is available and (b) ECSI is unavailable. For scenario (a), a QoS based design and a secrecy rate based design were investigated. For scenario (b), only the QoS based design was considered since the secrecy rate based design is not applicable. To solve the robust QoS based secure transmission design problem under scenario (a), we used the SDR approach [70] to relax the beamforming scheme to be beamforming with general rank information covariance matrix and constructed a convex SDP problem. We proved that the solution to the original design problem can be obtained through the constructed convex SDP problem. Moreover, our analysis showed that the relaxation is tight, which implies that single stream beamforming is the optimal beamforming scheme. For the robust secrecy rate maximization design problem under scenario (a), it was first relaxed by the SDR approach. We demonstrated that the optimal information covariance matrix in the SDR problem must be rank-one. Thus the relaxation was tight and the original design problem can be solved through the SDR problem. In addition, our proof reveals that the non-convex SDR problem can be handled via the help of one-dimensional line search and a sequence of convex SDPs which were built according to the Charnes-Cooper method. For the robust QoS based secure transmission design under scenario (b), the SDR approach was used at first. Then we constructed a new convex SDP problem which can assist us to find out an optimal solution to the original design problem. The simulation results revealed that, when the ECSI is available, the secrecy rate based design outperformed previously proposed designs which are based on general rank information covariance matrix optimization. The effectiveness and robustness of the QoS based designs were also demonstrated by the

simulation results.

Chapter 4

Secure Transmission for SWIPT enabled MISO HCNs

In this chapter, we consider a MISO HCN with one MBS and multiple FBSs in co-channel deployment. In the considered system, all BSs send confidential messages to IRs and energy signals to ERs. The ERs have the potential to wiretap the confidential messages. Consequently, by taking fairness into account, we propose a sum logarithmic secrecy rate maximization secure transmission design problem under the EH constraints for ERs. In the design problem, we also treat the energy signals sent to ERs as jamming signals that can cripple ERs' interception capabilities. The formulated design problem is nontrivial to solve due to the nonconvexity in the objective and the constraints. To tackle the design problem, a SDR and SCA based centralized secure transmission design algorithm is proposed. Our analysis reveals that using single-stream beamforming to transmit confidential messages does not cause any loss of optimality. Moreover, an alternating direction method of multipliers (ADMM) based distributed secure transmission design is also proposed.

4.1 System Model

We consider a MISO downlink HCN as shown in Figure 4.1 where there exist one T_M-antenna MBS and L T_F-antenna FBSs. We let $\mathcal{L}_F = \{1, \ldots, L\}$ denote the set of FBSs and let $\mathcal{L} = \{0\} \bigcup \mathcal{L}_F$ denote the set of all BSs,

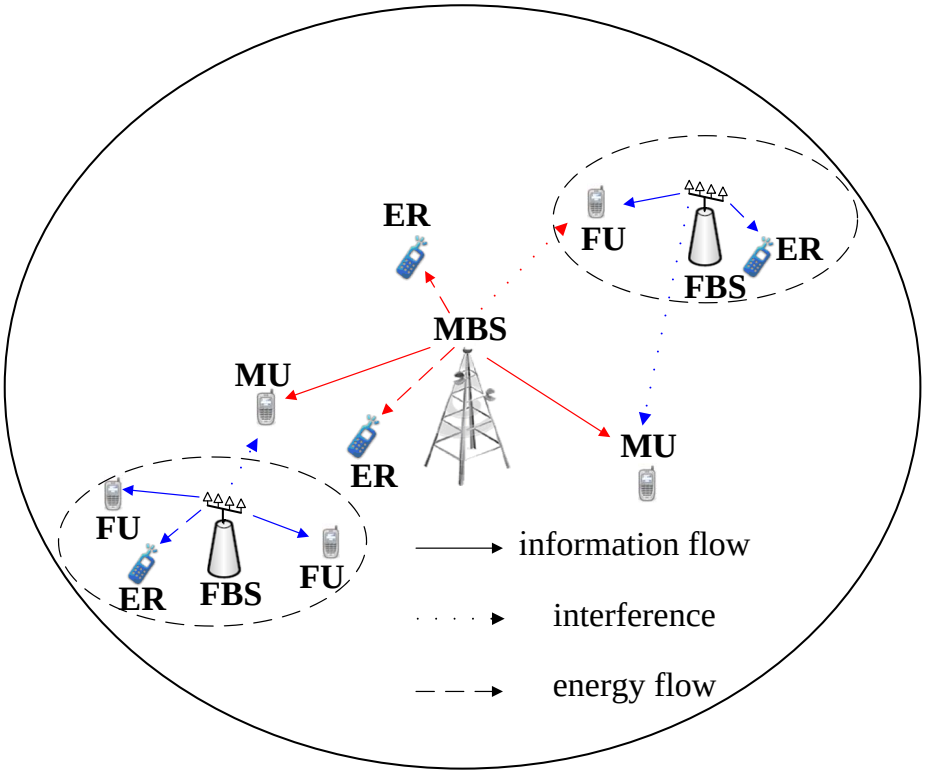

Figure 4.1: A two-tier HCN with wireless information and power transfer.

where the index 0 is introduced for the MBS. MUs and FUs are IRs. More specifically, the MBS provides information service to J_0 MUs and the l-th FBS provides information service to J_l FUs. Furthermore, there exist K ERs that harvest energy from MBS and FBSs. The ERs are assumed to be close to one BS in order to harvest enough energy. The MUs and FUs who are authorized to have access to information services are trustworthy, whereas the ERs may accidentally eavesdrop the messages for MUs and FUs. MUs, FUs and ERs are assumed to be equipped with single antenna. We let $\mathcal{J}_0 = \{1, \ldots, J_0\}$, $\mathcal{J}_l = \{1, \ldots, J_l\}$ and $\mathcal{K} = \{1, \ldots, K\}$ denote the sets of MUs, FUs in the l-th Femtocell and ERs respectively, and let MU_m, $\mathrm{FU}_{l,j}$ and ER_k represent the m-th MU, the j-th FU in the l-th Femtocell and

the k-th ER respectively. We assume single-stream beamforming at each BS for information transmission. In addition, without loss of generality, we assume the MBS and the l-th FBS assign γ_0 ($\gamma_0 \leq T_M$) and γ_l ($\gamma_l \leq T_F$) energy beams for ERs respectively. Therefore, the transmitted signals from the MBS and the l-th FBS can be respectively expressed as

$$\mathbf{x}_0 = \sum_{m \in \mathcal{J}_0} \mathbf{w}_{0,m} s_{0,m}^{MU} + \sum_{i=1}^{\gamma_0} \mathbf{q}_{0,i} s_{0,i}^{ER}, \tag{4.1}$$

$$\mathbf{x}_l = \sum_{j \in \mathcal{J}_l} \mathbf{w}_{l,j} s_{l,j}^{FU} + \sum_{i=1}^{\gamma_l} \mathbf{q}_{l,i} s_{l,i}^{ER}, \forall l \in \mathcal{L}_F \tag{4.2}$$

where $\mathbf{w}_{0,m} \in \mathbb{C}^{T_M \times 1}$ and $\mathbf{w}_{l,j} \in \mathbb{C}^{T_F \times 1}$ represent the single stream beamforming vectors for MU_m and $\mathrm{FU}_{l,j}$, respectively; $\mathbf{q}_{0,i} \in \mathbb{C}^{T_M \times 1}$ and $\mathbf{q}_{l,i} \in \mathbb{C}^{T_F \times 1}$ represent the i-th energy transmission vector at the MBS and the l-th FBS respectively; $s_{0,m}^{MU}$ and $s_{l,j}^{FU}$ denote the information-bearing signal intended for MU_m and $\mathrm{FU}_{l,j}$, while $s_{0,i}^{ER}$ and $s_{l,i}^{ER}$ denote the i-th energy-carrying signal sent by the MBS and the l-th FBS. It is assumed that $s_{0,m}^{MU}$ and $s_{l,j}^{FU}$ are independent and identically distributed (i.i.d.) CSCG random variables with zero mean and unit variance, i.e., $s_{0,m}^{MU} \sim \mathcal{CN}(0,1)$ and $s_{l,j}^{FU} \sim \mathcal{CN}(0,1)$. Furthermore, $s_{0,i}^{ER}$ and $s_{l,i}^{ER}$ can be arbitrary independent random signals, each having unit average power. Because of the possible interceptions of ERs, the MBS and the FBSs also collaboratively utilize the energy signals as jamming signals to cripple the eavesdropping capacities of ERs. We assume that $s_{0,i}^{ER}$ and $s_{l,i}^{ER}$ are i.i.d. CSCG random variables denoted by $s_{0,i}^{ER} \sim \mathcal{CN}(0,1)$ and $s_{l,i}^{ER} \sim \mathcal{CN}(0,1)$, since the worst-case noise distribution for the eavesdropping ERs is known to be Gaussian.

We assume a quasi-static fading environment. Denote $\mathbf{h}_{0,0,m} \in \mathbb{C}^{T_M \times 1}$, $\mathbf{h}_{0,u,j} \in \mathbb{C}^{T_M \times 1}$ and $\mathbf{g}_{0,k} \in \mathbb{C}^{T_M \times 1}$ as the channel vectors from MBS to MU_m, $\mathrm{FU}_{u,j}$ and ER_k respectively, $\mathbf{h}_{l,0,m} \in \mathbb{C}^{T_F \times 1}$, $\mathbf{h}_{l,u,j} \in \mathbb{C}^{T_F \times 1}$ and $\mathbf{g}_{l,k} \in \mathbb{C}^{T_F \times 1}$ as the channel vectors from the l-th FBS to MU_m, $\mathrm{FU}_{u,j}$ and ER_k respectively. Here, we assume ERs are active devices. Thus, MUs, FUs and ERs can communicate with all BSs in the uplink. When the HCN

operates under time division duplexing (TDD), BSs can obtain the CSI in the uplink transmission and then acquire the downlink CSI via channel reciprocity. As for the case where the HCN operates under frequency division duplexing (FDD), MUs, FUs and ERs can obtain the CSI from BSs in the downlink transmission and feedback the downlink CSI to BSs via uplink communication.

The received baseband signals at MU_m, $\mathrm{FU}_{l,j}$ and ER_k can be, respectively, given by

$$y_{0,m} = \sum_{l\in\mathcal{L}} \mathbf{h}_{l,0,m}^H \mathbf{x}_l + n_{0,m}, \forall m \in \mathcal{J}_0, \tag{4.3}$$

$$y_{l,j} = \sum_{u\in\mathcal{L}} \mathbf{h}_{u,l,j}^H \mathbf{x}_u + n_{l,j}, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}_F, \tag{4.4}$$

$$z_k = \sum_{l\in\mathcal{L}} \mathbf{g}_{l,k}^H \mathbf{x}_l + n_k, \forall k \in \mathcal{K} \tag{4.5}$$

where $n_{0,m} \sim \mathcal{CN}(0, \sigma_{0,m}^2)$, $n_{l,j} \sim \mathcal{CN}(0, \sigma_{l,j}^2)$ and $n_k \sim \mathcal{CN}(0, \sigma_k^2)$ are the i.i.d. Gaussian noises at MU_m, $\mathrm{FU}_{l,j}$ and ER_k respectively. In (4), $\mathbf{x}_u$ is the transmitted signal from BS u, whose definition follows (1) and (2).

According to (4.3) and (4.4), the SINR at MU_m and $\mathrm{FU}_{l,j}$ can be expressed as

$$SINR_{0,m} = \frac{\left|\mathbf{h}_{0,0,m}^H \mathbf{w}_{0,m}\right|^2}{\sum\limits_{u\in\mathcal{J}_0, u\neq m} \left|\mathbf{h}_{0,0,m}^H \mathbf{w}_{0,u}\right|^2 + \sum\limits_{u\in\mathcal{L}_F} \sum\limits_{i\in\mathcal{J}_u} \left|\mathbf{h}_{u,0,m}^H \mathbf{w}_{u,i}\right|^2 + \sum\limits_{u\in\mathcal{L}} \mathbf{h}_{u,0,m}^H \mathbf{Q}_u \mathbf{h}_{u,0,m} + \sigma_{0,m}^2}, \quad \forall m \in \mathcal{J}_0, \tag{4.6}$$

$$SINR_{l,j} = \frac{\left|\mathbf{h}_{l,l,j}^H \mathbf{w}_{l,j}\right|^2}{\sum\limits_{u\in\mathcal{J}_l, u\neq j}\left|\mathbf{h}_{l,l,j}^H \mathbf{w}_{l,u}\right|^2 + \sum\limits_{u\in\mathcal{L}, u\neq l}\sum\limits_{i\in\mathcal{J}_u}\left|\mathbf{h}_{u,l,j}^H \mathbf{w}_{u,i}\right|^2 + \sum\limits_{u\in\mathcal{L}} \mathbf{h}_{u,l,j}^H \mathbf{Q}_u \mathbf{h}_{u,l,j} + \sigma_{l,j}^2}, \quad \forall j \in \mathcal{J}_l, l \in \mathcal{L}_F \tag{4.7}$$

where $\mathbf{Q}_u = \sum_{i=1}^{\gamma_u} \mathbf{q}_{u,i}\mathbf{q}_{u,i}^H$ for $u \in \mathcal{L}$.

From (4.5), if ER_k acts as an eavesdropper who intends to decode the message for MU_m instead of harvesting energy, the SINR at ER_k can be expressed as

$$SINR_{0,m,k} = \frac{\left|\mathbf{g}_{0,k}^H \mathbf{w}_{0,m}\right|^2}{\sum\limits_{u\in\mathcal{J}_0, u\neq m}\left|\mathbf{g}_{0,k}^H \mathbf{w}_{0,u}\right|^2 + \sum\limits_{u\in\mathcal{L}_F}\sum\limits_{i\in\mathcal{J}_u}\left|\mathbf{g}_{u,k}^H \mathbf{w}_{u,i}\right|^2 + \sum\limits_{u\in\mathcal{L}} \mathbf{g}_{u,k}^H \mathbf{Q}_u \mathbf{g}_{u,k} + \sigma_k^2}, \quad \forall m \in \mathcal{J}_0, \forall k \in \mathcal{K}. \tag{4.8}$$

Similarly, the SINR at ER_k for eavesdropping $\mathrm{FU}_{l,j}$ is

$$SINR_{l,j,k} = \frac{\left|\mathbf{g}_{l,k}^H \mathbf{w}_{l,j}\right|^2}{\sum\limits_{u\in\mathcal{J}_l, u\neq j}\left|\mathbf{g}_{l,k}^H \mathbf{w}_{l,u}\right|^2 + \sum\limits_{u\in\mathcal{L}, u\neq l}\sum\limits_{i\in\mathcal{J}_u}\left|\mathbf{g}_{u,k}^H \mathbf{w}_{u,i}\right|^2 + \sum\limits_{u\in\mathcal{L}} \mathbf{g}_{u,k}^H \mathbf{Q}_u \mathbf{g}_{u,k} + \sigma_k^2}, \quad \forall j \in \mathcal{J}_l, l \in \mathcal{L}_F, \forall k \in \mathcal{K}. \tag{4.9}$$

The secrecy rates at MU_m and $\mathrm{FU}_{l,j}$ are thus given by

$$R_{0,m} = \max(0, \log_2(1 + SINR_{0,m}) - \max_{k\in\mathcal{K}} \log_2(1 + SINR_{0,m,k})), \tag{4.10}$$

$$R_{l,j} = \max(0, \log_2(1 + SINR_{l,j}) - \max_{k\in\mathcal{K}} \log_2(1 + SINR_{l,j,k})). \tag{4.11}$$

On the other hand, for wireless power transfer, owing to the broadcast

property of wireless channels, the energy carried by information and energy beams can be harvested by each ER. The practical non-linear EH model is adopted in this chapter and the harvested power Λ_k at ER_k can be written as [79]

$$\Lambda_k(P_k^{ER}) = \frac{\Upsilon_{ER,k} - S_k\Omega_k}{1-\Omega_k} \tag{4.12}$$

where $\Omega_k = 1/\left(1+\exp(\eta_k o_k)\right)$, $\Upsilon_{ER,k} = S_k/(1+\exp(\eta_k o_k - \eta_k P_k^{ER}))$, and where S_k, η_k and o_k are constants related to the detailed EH circuit specifications such as the resistance, capacitance and diode turn-on voltage, and

$$P_k^{ER} = \sum_{u\in\mathcal{L}}\sum_{i\in\mathcal{J}_u}\left|\mathbf{g}_{u,k}^H\mathbf{w}_{u,i}\right|^2 + \sum_{u\in\mathcal{L}}\mathbf{g}_{u,k}^H\mathbf{Q}_u\mathbf{g}_{u,k}$$

is the received RF power of ER_k.

4.2 Problem Formulation

Secrecy rate fairness among IRs cannot be guaranteed if we only focus on maximizing the total secrecy throughput of the network. In this chapter, we introduce the proportional fairness scheme through which a good balance between the network secrecy throughput and the fairness among MUs and FUs can be achieved. Thus, the summation of the logarithmic secrecy rates of IRs is maximized. By letting ϖ_k denote the EH threshold for ER_k, $P_0^{\max}$ and $P_l^{\max}$ denote the maximal transmit power for the MBS and the l-th FBS, the sum logarithmic secrecy rate maximization secure transmission design problem is formulated as follows

$$\max_{\mathbf{w}_{l,j},\mathbf{Q}_l,\forall j\in\mathcal{J}_l,\forall l\in\mathcal{L}} f(\mathbf{w}_{l,j},\mathbf{Q}_l,\forall j\in\mathcal{J}_l,\forall l\in\mathcal{L}) = \sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{J}_l}\ln(R_{l,j}) \tag{4.13a}$$

$$\text{s.t.}\ \Lambda_k(P_k^{ER}) \geq \varpi_k,\ \forall k\in\mathcal{K}, \tag{4.13b}$$

$$\sum_{j\in\mathcal{J}_l}\|\mathbf{w}_{l,j}\|^2 + \mathrm{tr}(\mathbf{Q}_l) \leq P_l^{\max}, \forall l\in\mathcal{L}, \tag{4.13c}$$

$$\mathbf{Q}_l \succeq \mathbf{0}, \forall l\in\mathcal{L}. \tag{4.13d}$$

Note that we optimize $\mathbf{Q}_l$ instead of $\mathbf{q}_{l,i}$ in (4.13). However, from the solution of $\mathbf{Q}_l$, the number of energy signals γ_l can be derived as $\gamma_l = \text{rank}(\mathbf{Q}_l)$ and the energy transmission vectors $\mathbf{q}_{l,i}$ can be obtained by the EVD of $\mathbf{Q}_l$.

The design problem (4.13) is not convex. We can observe from (4.6)-(4.11) that the secrecy rate at each IR is the difference between two non-convex functions, and thus it is non-convex with respect to $\mathbf{w}_{l,j}$ and $\mathbf{Q}_l$. Therefore, the objective function $f(\mathbf{w}_{l,j}, \mathbf{Q}_l, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L})$, which is the sum of the logarithmic secrecy rates, is non-convex. As for the constraints, the non-convexity of constraint (4.13b) causes the feasible set to be non-convex. The challenges in dealing with the design problem lie in its non-convexity.

4.3 Centralized Secure Transmission Design

In this section, we aim at developing a centralized secure transmission design algorithm, which should be implemented on a central control unit with global CSI, for the original design problem (4.13).

4.3.1 SDR and Relaxation Tightness

To make the original design problem (4.13) tractable, we define $\mathbf{H}_{l,u,j} = \mathbf{h}_{l,u,j}\mathbf{h}_{l,u,j}^H$, $\mathbf{G}_{l,k} = \mathbf{g}_{l,k}\mathbf{g}_{l,k}^H$ and $\mathbf{W}_{l,j} = \mathbf{w}_{l,j}\mathbf{w}_{l,j}^H$. Then, it follows that $\text{rank}(\mathbf{W}_{l,j}) = 1, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}$. By ignoring the rank-one constraints on $\mathbf{W}_{l,j}$'s, the SDR problem of (4.13) can be expressed as

$$\max_{\mathbf{W}_{l,j}, \mathbf{Q}_l, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}} \bar{f}(\mathbf{W}_{l,j}, \mathbf{Q}_l, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}) = \sum_{l \in \mathcal{L}} \sum_{j \in \mathcal{J}_l} \ln(\log_2\left(1 + \overline{SINR}_{l,j}\right) - \max_{k \in \mathcal{K}} \log_2(1 + \overline{SINR}_{l,j,k})) \tag{4.14a}$$

$$\text{s.t.} \sum_{u \in \mathcal{L}} \sum_{i \in \mathcal{J}_u} \text{tr}\left(\mathbf{G}_{u,k}\mathbf{W}_{u,i}\right) + \sum_{u \in \mathcal{L}} \text{tr}\left(\mathbf{G}_{u,k}\mathbf{Q}_u\right) \geq \check{\Lambda}_k(\varpi_k), \forall k \in \mathcal{K}, \tag{4.14b}$$

$$\sum_{j \in \mathcal{J}_l} \text{tr}\left(\mathbf{W}_{l,j}\right) + \text{tr}(\mathbf{Q}_l) \leq P_l^{\max}, \forall l \in \mathcal{L}, \tag{4.14c}$$

$$\mathbf{Q}_l \succeq \mathbf{0}, \mathbf{W}_{l,j} \succeq \mathbf{0}, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L} \tag{4.14d}$$

where

$$\overline{SINR}_{l,j} = \frac{\mathrm{tr}(\mathbf{H}_{l,l,j}\mathbf{W}_{l,j})}{\sum\limits_{u\in\mathcal{J}_l,u\neq j}\mathrm{tr}(\mathbf{H}_{l,l,j}\mathbf{W}_{l,u}) + \sum\limits_{u\in\mathcal{L},u\neq l}\sum\limits_{i\in\mathcal{J}_u}\mathrm{tr}(\mathbf{H}_{u,l,j}\mathbf{W}_{u,i}) + \sum\limits_{u\in\mathcal{L}}\mathrm{tr}(\mathbf{H}_{u,l,j}\mathbf{Q}_u) + \sigma_{l,j}^2},$$

$$\overline{SINR}_{l,j,k} = \frac{\mathrm{tr}(\mathbf{G}_{l,k}\mathbf{W}_{l,j})}{\sum\limits_{u\in\mathcal{J}_l,u\neq j}\mathrm{tr}(\mathbf{G}_{l,k}\mathbf{W}_{l,u}) + \sum\limits_{u\in\mathcal{L},u\neq l}\sum\limits_{i\in\mathcal{J}_u}\mathrm{tr}(\mathbf{G}_{u,k}\mathbf{W}_{u,i}) + \sum\limits_{u\in\mathcal{L}}\mathrm{tr}(\mathbf{G}_{u,k}\mathbf{Q}_u) + \sigma_k^2}.$$

In (4.14), $\check{\Lambda}_k(\cdot)$ is the inverse function of $\Lambda_k(\cdot)$; $\mathbf{W}_{l,j}$'s and $\mathbf{Q}_l$ can be explained as general rank information covariance matrices and energy covariance matrices respectively.

Due to the lack of rank constraints, the tightness of the SDR is worthy to be discussed. Let $\{\ddot{\mathbf{W}}_{l,j}, \ddot{\mathbf{Q}}_l, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}\}$ be a (sub)optimal solution[12] to (4.14) and define

$$\ddot{\Gamma}_{l,j} = \frac{\mathrm{tr}(\mathbf{H}_{l,l,j}\ddot{\mathbf{W}}_{l,j})}{\sum\limits_{u\in\mathcal{J}_l,u\neq j}\mathrm{tr}(\mathbf{H}_{l,l,j}\ddot{\mathbf{W}}_{l,u}) + \sum\limits_{u\in\mathcal{L},u\neq l}\sum\limits_{i\in\mathcal{J}_u}\mathrm{tr}(\mathbf{H}_{u,l,j}\ddot{\mathbf{W}}_{u,i}) + \sum\limits_{u\in\mathcal{L}}\mathrm{tr}(\mathbf{H}_{u,l,j}\ddot{\mathbf{Q}}_u) + \sigma_{l,j}^2}, \quad \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}$$

$$\ddot{\Gamma}_{l,j,k} = \frac{\mathrm{tr}(\mathbf{G}_{l,k}\ddot{\mathbf{W}}_{l,j})}{\sum\limits_{u\in\mathcal{J}_l,u\neq j}\mathrm{tr}(\mathbf{G}_{l,k}\ddot{\mathbf{W}}_{l,u}) + \sum\limits_{u\in\mathcal{L},u\neq l}\sum\limits_{i\in\mathcal{J}_u}\mathrm{tr}(\mathbf{G}_{u,k}\ddot{\mathbf{W}}_{u,i}) + \sum\limits_{u\in\mathcal{L}}\mathrm{tr}(\mathbf{G}_{u,k}\ddot{\mathbf{Q}}_u) + \sigma_k^2}, \quad \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}, \forall k \in \mathcal{K}$$

$$\mathcal{J}_{l,\nabla} = \{j|\,\mathrm{rank}(\ddot{\mathbf{W}}_{l,j}) > 1, j \in \mathcal{J}_l\}, \mathcal{J}_{l,\triangle} = \mathcal{J}_l - \mathcal{J}_{l,\nabla}, \forall l \in \mathcal{L},$$

[12]Since problem (4.14) is non-convex, the obtained solution is usually suboptimal.

$$\ddot{v}_{l,i,u,j} = \mathrm{tr}(\mathbf{H}_{l,u,j}\ddot{\mathbf{W}}_{l,i}), \forall i \in \mathcal{J}_l, \forall j \in \mathcal{J}_u, \forall l \in \mathcal{L}, \forall u \in \mathcal{L},$$

$$\ddot{I}_{l,u,j} = \sum_{i \in \mathcal{J}_{l,\nabla}} \ddot{v}_{l,i,u,j} + \mathrm{tr}(\mathbf{H}_{l,u,j}\ddot{\mathbf{Q}}_l), \forall u \in \mathcal{L}, \forall l \in \mathcal{L}, u \neq l, \forall j \in \mathcal{J}_u,$$

$$\ddot{w}_{l,j} = \sum_{i \in \mathcal{J}_{l,\nabla}} \ddot{v}_{l,i,l,j} + \mathrm{tr}(\mathbf{H}_{l,l,j}\ddot{\mathbf{Q}}_l), \forall j \in \mathcal{J}_{l,\triangle}, \forall l \in \mathcal{L},$$

$$\ddot{\mu}_{l,j,k} = \mathrm{tr}\left(\mathbf{G}_{l,k}\ddot{\mathbf{W}}_{l,j}\right), j \in \mathcal{J}_l, \forall l \in \mathcal{L}, \forall k \in \mathcal{K},$$

$$\ddot{y}_{l,k} = \sum_{j \in \mathcal{J}_{l,\nabla}} \ddot{\mu}_{l,j,k} + \mathrm{tr}(\mathbf{G}_{l,k}\ddot{\mathbf{Q}}_l), \forall k \in \mathcal{K}, \forall l \in \mathcal{L}.$$

Suppose $\mathcal{J}_l \neq \mathcal{J}_{l,\triangle}$. Then, we have the following proposition.

Proposition 4.1. *Consider the following convex optimization problem for BS $l \in \mathcal{L}$*

$$\min_{\mathbf{W}_{l,j}, \forall j \in \mathcal{J}_{l,\nabla}, \mathbf{Q}_l} \sum_{j \in \mathcal{J}_{l,\nabla}} \mathrm{tr}\left(\mathbf{W}_{l,j}\right) + \mathrm{tr}(\mathbf{Q}_l) \tag{4.15a}$$

$$s.t. \sum_{i \in \mathcal{J}_{l,\nabla}} \mathrm{tr}\left(\mathbf{H}_{l,l,j}\mathbf{W}_{l,i}\right) + \mathrm{tr}\left(\mathbf{H}_{l,l,j}\mathbf{Q}_l\right) \leq \ddot{w}_{l,j}, \forall j \in \mathcal{J}_{l,\triangle}, \tag{4.15b}$$

$$\begin{aligned}\frac{\mathrm{tr}\left(\mathbf{H}_{l,l,j}\mathbf{W}_{l,j}\right)}{\ddot{\Gamma}_{l,j}} \geq & \sum_{i \in \mathcal{J}_{l,\triangle}} \ddot{v}_{l,i,l,j} + \sum_{u \in \mathcal{J}_{l,\nabla}, u \neq j} \mathrm{tr}\left(\mathbf{H}_{l,l,j}\mathbf{W}_{l,u}\right) + \sum_{u \in \mathcal{L}, u \neq l} \ddot{I}_{u,l,j} \\ & + \sum_{u \in \mathcal{L}, u \neq l} \sum_{i \in \mathcal{J}_{u,\triangle}} \ddot{v}_{u,i,l,j} + \mathrm{tr}\left(\mathbf{H}_{l,l,j}\mathbf{Q}_l\right) + \sigma_{l,j}^2, \forall j \in \mathcal{J}_{l,\nabla},\end{aligned} \tag{4.15c}$$

$$\sum_{j \in \mathcal{J}_{l,\nabla}} \mathrm{tr}\left(\mathbf{G}_{l,k}\mathbf{W}_{l,j}\right) + \mathrm{tr}\left(\mathbf{G}_{l,k}\mathbf{Q}_l\right) \geq \ddot{y}_{l,k}, \forall k \in \mathcal{K}, \tag{4.15d}$$

$$\begin{aligned}\frac{\mathrm{tr}\left(\mathbf{G}_{l,k}\mathbf{W}_{l,j}\right)}{\ddot{\Gamma}_{l,j,k}} \leq & \sum_{i \in \mathcal{J}_{l,\triangle}} \ddot{\mu}_{l,i,k} + \sum_{u \in \mathcal{J}_{l,\nabla}, u \neq j} \mathrm{tr}\left(\mathbf{G}_{l,k}\mathbf{W}_{l,u}\right) + \sum_{u \in \mathcal{L}, u \neq l} \ddot{y}_{u,k} \\ & + \sum_{u \in \mathcal{L}, u \neq l} \sum_{i \in \mathcal{J}_{u,\triangle}} \ddot{\mu}_{u,i,k} + \mathrm{tr}\left(\mathbf{G}_{l,k}\mathbf{Q}_l\right) + \sigma_k^2, \forall j \in \mathcal{J}_{l,\nabla}, \forall k \in \mathcal{K},\end{aligned} \tag{4.15e}$$

$$\sum_{i \in \mathcal{J}_{l,\nabla}} \mathrm{tr}\left(\mathbf{H}_{l,u,j}\mathbf{W}_{l,i}\right) + \mathrm{tr}\left(\mathbf{H}_{l,u,j}\mathbf{Q}_l\right) \leq \ddot{I}_{l,u,j}, \forall j \in \mathcal{J}_u, \forall u \in \mathcal{L}, u \neq l, \tag{4.15f}$$

$$\mathbf{Q}_l \succeq \mathbf{0}, \mathbf{W}_{l,j} \succeq \mathbf{0}, \forall j \in \mathcal{J}_\nabla. \tag{4.15g}$$

i) Let $\{\mathbf{W}^*_{l,j}, \forall j \in \mathcal{J}_{l,\nabla}, \mathbf{Q}^*_l\}$ *denote an optimal solution to* (4.15)*. Then,* $\{\ddot{\mathbf{W}}_{l,i}, \mathbf{W}^*_{l,j}, \mathbf{Q}^*_l, \forall i \in \mathcal{J}_{l,\triangle}, \forall j \in \mathcal{J}_{l,\nabla}, \forall l \in \mathcal{L}\}$ *must perform no worse than* $\{\ddot{\mathbf{W}}_{l,j}, \ddot{\mathbf{Q}}_l, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}\}$ *in terms of maximizing the objective function in* (4.14)*.*

ii) Suppose $\hat{\mathcal{J}}_{l,\nabla} \neq \mathcal{J}_{l,\nabla}$ *where* $\hat{\mathcal{J}}_{l,\nabla} = \mathcal{J}_{l,\nabla} - \check{\mathcal{J}}_{l,\nabla}$ *and* $\check{\mathcal{J}}_{l,\nabla} = \{j | \operatorname{rank}(\mathbf{W}^*_j) > 1, j \in \mathcal{J}_{l,\nabla}\}$*. Let us further define*

$$\begin{aligned}\mathbf{A}^*_{l,j} = \mathbf{I} &+ \sum_{i\in\mathcal{J}_{l,\triangle}} \alpha^*_{l,i}\mathbf{H}_{l,l,i} + \sum_{u\in\mathcal{J}_{l,\nabla,u\neq j}} \beta^*_{l,u}\mathbf{H}_{l,l,u} - \frac{1}{\ddot{\Gamma}_{l,j}}\beta^*_{l,j}\mathbf{H}_{l,l,j} - \sum_{k\in\mathcal{K}} \rho^*_{l,k}\mathbf{G}_{l,k} \\ &- \sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{J}_{l,\nabla},u\neq j} \chi^*_{l,k,u}\mathbf{G}_{l,k} + \sum_{k\in\mathcal{K}} \frac{1}{\ddot{\Gamma}_{l,j,k}}\chi^*_{l,k,j}\mathbf{G}_{l,k} + \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_u} \lambda^*_{l,u,i}\mathbf{H}_{l,u,i},\end{aligned}$$

$$\begin{aligned}\mathbf{E}^*_{l,j} = \mathbf{I} &+ \sum_{i\in\mathcal{J}_{l,\triangle}} \alpha^*_{l,i}\mathbf{H}_{l,l,i} + \sum_{u\in\mathcal{J}_{l,\nabla,u\neq j}} \beta^*_{l,u}\mathbf{H}_{l,l,u} - \left(\frac{1}{\ddot{\Gamma}_{l,j}}\beta^*_{l,j} - 1\right)\mathbf{H}_{l,l,j} \\ &- \sum_{k\in\mathcal{K}} \rho^*_{l,k}\mathbf{G}_{l,k} - \sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{J}_{l,\nabla},u\neq j} \chi^*_{l,k,u}\mathbf{G}_{l,k} + \sum_{k\in\mathcal{K}} \frac{1}{\ddot{\Gamma}_{l,j,k}}\chi^*_{l,k,j}\mathbf{G}_{l,k} \\ &+ \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_u} \lambda^*_{l,u,i}\mathbf{H}_{l,u,i}\end{aligned}$$

where $\alpha^*_{l,j}$*,* $\beta^*_{l,j}$*,* $\rho^*_{l,k}$*,* $\chi^*_{l,k,j}$ *and* $\lambda^*_{l,u,j}$ *are the optimal solutions to the dual variables associated with constraints* (4.15b)-(4.15f) *in problem* (4.15)*. Then,* $\{\mathbf{W}^*_{l,j}, \forall j \in \hat{\mathcal{J}}_{l,\nabla}, \bar{\mathbf{W}}^*_{l,j}, \forall j \in \check{\mathcal{J}}_{l,\nabla}, \bar{\mathbf{Q}}^*_l\}$ *with* $\bar{\mathbf{W}}^*_{l,j} = \tau_{l,j}\boldsymbol{\psi}_{l,j}\boldsymbol{\psi}^H_{l,j}$*,* $\bar{\mathbf{Q}}^*_l = \mathbf{Q}^*_l + \sum_{j\in\check{\mathcal{J}}_{l,\nabla}} \left(\mathbf{W}^*_{l,j} - \tau_{l,j}\boldsymbol{\psi}_{l,j}\boldsymbol{\psi}^H_{l,j}\right)$ *is also an optimal solution to* (4.15)*, where* $\{\boldsymbol{\psi}_{l,j}, \boldsymbol{\Theta}_{l,j}\}$ *is the orthogonal basis for the null space of* $\mathbf{A}^*_{l,j}$*,* $\boldsymbol{\Theta}_{l,j}$ *is the orthogonal basis for the null space of* $\mathbf{E}^*_{l,j}$ *and* $\tau_{l,j} = \boldsymbol{\psi}^H_{l,j}\mathbf{W}^*_{l,j}\boldsymbol{\psi}_{l,j}$*.*

Proof: See Appendix G.

From Proposition 4.1, it can be observed that the SDR problem in (4.14) is a tight relaxation of the original design problem (4.13). If $\operatorname{rank}(\ddot{\mathbf{W}}_{l,j}) = 1$ for all $\ddot{\mathbf{W}}_{l,j}$'s, the tightness of (4.14) is obvious and the corresponding beamforming vector $\ddot{\mathbf{w}}_{l,j}$ can be obtained by EVD. On the other hand, if there exist $\ddot{\mathbf{W}}_{l,j}$'s with $\operatorname{rank}(\ddot{\mathbf{W}}_{l,j}) > 1$, we can always obtain a solution with

rank-one information covariance matrices, i.e., $\{\ddot{\mathbf{W}}_{l,i}, \mathbf{W}^*_{l,u}, \bar{\mathbf{W}}^*_{l,j}, \bar{\mathbf{Q}}^*_l, \forall i \in \mathcal{J}_{l,\triangle}, \forall u \in \hat{\mathcal{J}}_{l,\nabla}, \forall j \in \check{\mathcal{J}}_{l,\nabla}, \forall l \in \mathcal{L}\}$, which performs no worse than $\{\ddot{\mathbf{W}}_{l,j}, \ddot{\mathbf{Q}}_l, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}\}$ in terms of maximizing problem (4.14). Thus, the relaxation tightness can be verified and we can obtain single stream beamforming vectors $\{\ddot{\mathbf{w}}_{l,i}, \mathbf{w}^*_{l,u}, \bar{\mathbf{w}}^*_{l,j}, \forall i \in \mathcal{J}_{l,\triangle}, \forall u \in \hat{\mathcal{J}}_{l,\nabla}, \forall j \in \check{\mathcal{J}}_{l,\nabla}, \forall l \in \mathcal{L}\}$ such that $f(\ddot{\mathbf{w}}_{l,i}, \mathbf{w}^*_{l,u}, \bar{\mathbf{w}}^*_{l,j}, \bar{\mathbf{Q}}^*_l, \forall i \in \mathcal{J}_{l,\triangle}, \forall u \in \hat{\mathcal{J}}_{l,\nabla}, \forall j \in \check{\mathcal{J}}_{l,\nabla}, \forall l \in \mathcal{L}) \geq \bar{f}(\ddot{\mathbf{W}}_{l,j}, \ddot{\mathbf{Q}}_l, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L})$, which reveals that, compared with beamforming with general rank information covariance matrix, utilizing single-stream beamforming to transmit information message to each IR does not cause any loss of performance optimality.

4.3.2 SCA Processes

In order to handle the SDR problem (4.14), we can first introduce relax variables $a_{l,j}$, $b_{l,j}$, c_k and $d_{l,j,k}$ and $r_{l,j}$ to rewrite problem (4.14) into

$$\max_{\mathbf{W}_{l,j}, \mathbf{Q}_l, r_{l,j}, a_{l,j}, b_{l,j}, c_k, d_{l,j,k}, \forall k \in \mathcal{K}, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}} \sum_{l \in \mathcal{L}} \sum_{j \in \mathcal{J}_l} \ln(r_{l,j}) \tag{4.16a}$$

$$\text{s.t. } a_{l,j} - b_{l,j} - c_k + d_{l,j,k} \geq r_{l,j}, \forall k \in \mathcal{K}, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}, \tag{4.16b}$$

$$\sum_{u \in \mathcal{L}} \sum_{i \in \mathcal{J}_u} \text{tr}(\mathbf{H}_{u,l,j} \mathbf{W}_{u,i}) + \sum_{u \in \mathcal{L}} \text{tr}(\mathbf{H}_{u,l,j} \mathbf{Q}_u) + \sigma^2_{l,j} \geq 2^{a_{l,j}}, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}, \tag{4.16c}$$

$$\sum_{u \in \mathcal{J}_l, u \neq j} \text{tr}(\mathbf{H}_{l,l,j} \mathbf{W}_{l,u}) + \sum_{u \in \mathcal{L}, u \neq l} \sum_{i \in \mathcal{J}_u} \text{tr}(\mathbf{H}_{u,l,j} \mathbf{W}_{u,i}) + \sum_{u \in \mathcal{L}} \text{tr}(\mathbf{H}_{u,l,j} \mathbf{Q}_u) + \sigma^2_{l,j} \leq 2^{b_{l,j}}, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}, \tag{4.16d}$$

$$\sum_{u \in \mathcal{L}} \sum_{i \in \mathcal{J}_u} \text{tr}(\mathbf{G}_{u,k} \mathbf{W}_{u,i}) + \sum_{u \in \mathcal{L}} \text{tr}(\mathbf{G}_{u,k} \mathbf{Q}_u) + \sigma^2_k \leq 2^{c_k}, \forall k \in \mathcal{K}, \tag{4.16e}$$

$$\sum_{u \in \mathcal{J}_l, u \neq j} \text{tr}(\mathbf{G}_{l,k} \mathbf{W}_{l,u}) + \sum_{u \in \mathcal{L}, u \neq l} \sum_{i \in \mathcal{J}_u} \text{tr}(\mathbf{G}_{u,k} \mathbf{W}_{u,i}) + \sum_{u \in \mathcal{L}} \text{tr}(\mathbf{G}_{u,k} \mathbf{Q}_u) + \sigma^2_k \geq 2^{d_{l,j,k}}, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}, \forall k \in \mathcal{K} \tag{4.16f}$$

(4.14b) − (4.14d).

Problem (4.16) can be handled by an SCA process, in which convex approximation problems are iteratively solved. More specifically, the convex approximation problem at the $(\kappa+1)$-th iteration is given as

$$\max_{\mathbf{W}_{l,j},\mathbf{Q}_l,r_{l,j},a_{l,j},b_{l,j},c_k,d_{l,j,k},\forall k\in\mathcal{K},\forall j\in\mathcal{J}_l,\forall l\in\mathcal{L}} \sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{J}_l}\ln(r_{l,j}) \tag{4.17a}$$

$$\text{s.t.}\ (4.16\text{b}), (4.16\text{c}), (4.16\text{f}), (4.14\text{b}) - (4.14\text{d}),$$

$$\sum_{u\in\mathcal{J}_l,u\neq j}\mathrm{tr}(\mathbf{H}_{l,l,j}\mathbf{W}_{l,u}) + \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_u}\mathrm{tr}(\mathbf{H}_{u,l,j}\mathbf{W}_{u,i}) + \sum_{u\in\mathcal{L}}\mathrm{tr}(\mathbf{H}_{u,l,j}\mathbf{Q}_u) + \sigma_{l,j}^2 \leq 2^{b_{l,j}^{(\kappa)}}(\ln(2)b_{l,j} - \ln(2)b_{l,j}^{(\kappa)} + 1), \forall j\in\mathcal{J}_l, \forall l\in\mathcal{L}, \tag{4.17b}$$

$$\sum_{u\in\mathcal{L}}\sum_{i\in\mathcal{J}_u}\mathrm{tr}(\mathbf{G}_{u,k}\mathbf{W}_{u,i}) + \sum_{u\in\mathcal{L}}\mathrm{tr}(\mathbf{G}_{u,k}\mathbf{Q}_u) + \sigma_k^2 \leq 2^{c_k^{(\kappa)}}(\ln(2)c_k - \ln(2)c_k^{(\kappa)} + 1), \forall k\in\mathcal{K}, \tag{4.17c}$$

where $b_{l,j}^{(\kappa)}$ and $c_k^{(\kappa)}$ are the convex approximation problem construction parameters and can be obtained through the convex approximation problem at the κ-th iteration. Let $\{\hat{\mathbf{W}}_{l,j}^{(\kappa+1)}, \hat{\mathbf{Q}}_l^{(\kappa+1)}, \hat{r}_{l,j}^{(\kappa+1)}, \hat{a}_{l,j}^{(\kappa+1)}, \hat{b}_{l,j}^{(\kappa+1)}, \hat{c}_k^{(\kappa+1)}, \hat{d}_{l,j,k}^{(\kappa+1)}\}$ be an optimal solution to (4.17). Then,

$$\begin{aligned} b_{l,j}^{(\kappa+1)} &= \log_2\left(2^{b_{l,j}^{(\kappa)}}(\ln(2)\hat{b}_{l,j}^{(\kappa+1)} - \ln(2)b_{l,j}^{(\kappa)} + 1)\right), \\ c_k^{(\kappa+1)} &= \log_2\left(2^{c_k^{(\kappa)}}(\ln(2)\hat{c}_k^{(\kappa+1)} - \ln(2)c_k^{(\kappa)} + 1)\right) \end{aligned} \tag{4.18}$$

can be used to construct the convex approximation problem in the $(\kappa+2)$-th iteration. By solving the convex approximation problem at the convergence of the aforementioned SCA process, we can obtain a (sub)optimal solution to problem (4.16).

Note that, according to the discussion in [51], we can build another SCA process to handle problem (4.16) by constructing the convex approximation

problem in the $(\kappa+2)$-th iteration through

$$b_{l,j}^{(\kappa+1)} = \hat{b}_{l,j}^{(\kappa+1)}, c_k^{(\kappa+1)} = \hat{c}_k^{(\kappa+1)}. \tag{4.19}$$

The SCA process based on (4.19) can be regarded as an extension of the SCA process discussed in [51]. However, the proposed SCA process, which is based on (4.18), converges faster than the SCA process extended from [51], which is verified in the simulation.

Initialization for the SCA processes: Initialization influences the convergence of the SCA processes. Initial points that are not feasible in problem (4.16) usually lead to infeasible convex approximation problems in the first SCA iteration, so that the convergence of SCA processes cannot be guaranteed. For the convergence of SCA processes, it is necessary to find feasible initial points for the initialization of the SCA processes. However, the existence of non-convex constraints in problem (4.16) makes it difficult to find feasible initial points by solving the feasibility problem of (4.16), which motivates the following discussion. For convenience, we let $\mathcal{N}_l = \{1, ..., N_l\}$ denote the set of ERs which are located close to BS l, and let $\mathrm{ER}_{l,n}$ represent the n-th ER which is near to the l-th BS. Assuming $\mathrm{ER}_{l,n}$ and ER_k refer to the same ER, we define $\sigma_{ER,l,n}^2 = \sigma_k^2$, $\bar{P}_{l,n}^{ER} = \check{\Lambda}_k(\varpi_k)$ and $\mathbf{G}_{u,l,n} = \mathbf{G}_{u,k}$.

Consider the following convex feasibility problem for BS l

$$
\begin{aligned}
&\min_{\mathbf{W}_{l,j},\forall j\in J_l,\mathbf{Q}_l} \quad 0 \\
&\text{s.t.}\ \mathrm{tr}(\mathbf{H}_{l,l,j}\mathbf{W}_{l,j}) \geq \sum_{i\in\mathcal{J}_l,i\neq j} \mathrm{tr}(\mathbf{H}_{l,l,j}\mathbf{W}_{l,i}) + \mathrm{tr}(\mathbf{H}_{l,l,j}\mathbf{Q}_l) + \sum_{u\in\mathcal{L},u\neq l} I_{u,l,j} + \sigma_{l,j}^2, \\
&\qquad\qquad \forall j\in\mathcal{J}_l, \\
&\sum_{i\in J_l} \mathrm{tr}(\mathbf{H}_{l,u,j}\mathbf{W}_{l,i}) + \mathrm{tr}(\mathbf{H}_{l,u,j}\mathbf{Q}_l) \leq I_{l,u,j}, \forall j\in\mathcal{J}_u, \forall u\in\mathcal{L}, u\neq l, \\
&\mathrm{tr}(\mathbf{G}_{l,l,n}\mathbf{W}_{l,j}) \leq \sum_{i\in J_l,i\neq j} \mathrm{tr}(\mathbf{G}_{l,l,n}\mathbf{W}_{l,i}) + \mathrm{tr}(\mathbf{G}_{l,l,n}\mathbf{Q}_l) + \sigma_{ER,l,n}^2, \\
&\qquad\qquad \forall j\in\mathcal{J}_l, \forall n\in\mathcal{N}_l, \\
&\mathrm{tr}(\mathbf{G}_{l,u,n}\mathbf{W}_{l,j}) \leq \sum_{i\in J_l,i\neq j} \mathrm{tr}(\mathbf{G}_{l,u,n}\mathbf{W}_{l,i}) + \mathrm{tr}(\mathbf{G}_{l,u,n}\mathbf{Q}_l) + \bar{P}_{u,n}^{ER} + \sigma_{ER,u,n}^2, \\
&\qquad\qquad \forall j\in\mathcal{J}_l, \forall n\in\mathcal{N}_u, \forall u\in\mathcal{L}, u\neq l, \\
&\sum_{i\in J_l} \mathrm{tr}(\mathbf{G}_{l,l,n}\mathbf{W}_{l,i}) + \mathrm{tr}(\mathbf{G}_{l,l,n}\mathbf{Q}_l) \geq \bar{P}_{l,n}^{ER}, \forall n\in\mathcal{N}_l \\
&\sum_{j\in\mathcal{J}_l} \mathrm{tr}\left(\mathbf{W}_{l,j}\right) + \mathrm{tr}(\mathbf{Q}_l) \leq P_l^{\max}, \\
&\mathbf{Q}_l \succeq \mathbf{0}, \mathbf{W}_{l,j} \succeq \mathbf{0}, \forall j\in\mathcal{J}_l,
\end{aligned}
\tag{4.20}
$$

where $I_{l,u,j} = \frac{P_u^{\max}\|\mathbf{h}_{u,u,j}\|^2}{\nu_{u,j}J_uL}$ is the threshold for the interference from BS l to $\mathrm{FU}_{u,j}$ and $\nu_{u,j}$ is the feasibility adjustment parameter. Let $\{\mathbf{W}_{l,j}^{(0)}, \forall j\in\mathcal{J}_l, \mathbf{Q}_l^{(0)}\}$ denote a solution to the feasibility problem (4.20). It can be observed that $\{\mathbf{W}_{l,j}^{(0)}, \mathbf{Q}_l^{(0)}, \forall j\in\mathcal{J}_l, \forall l\in\mathcal{L}\}$ is feasible in (4.14). Because problem (4.14) is equivalent to (4.16), the feasible initial points $\{b_{l,j}^{(0)}, c_k^{(0)}\}$ for the SCA processes can be obtained through $\{\mathbf{W}_{l,j}^{(0)}, \mathbf{Q}_l^{(0)}, \forall j\in\mathcal{J}_l, \forall l\in\mathcal{L}\}$ by setting the corresponding constraints in (4.16) to be equality.

According to the above discussion, we propose the centralized secure transmission design algorithm as follows.

Proposition 4.2. *The SCA process in Algorithm* 4 *generates a sequence* $\{\hat{\mathbf{W}}_{l,j}^{(\kappa+1)},\ \hat{\mathbf{Q}}_l^{(\kappa+1)},\ \hat{r}_{l,j}^{(\kappa+1)},\ \hat{a}_{l,j}^{(\kappa+1)}, \hat{b}_{l,j}^{(\kappa+1)}, \hat{c}_k^{(\kappa+1)}, \hat{d}_{l,j,k}^{(\kappa+1)}\}$ *which achieves im-*

Algorithm 4: Algorithm of centralized secure transmission design

1. Initialize $\kappa := 0$ and set $b_{l,j}^{(0)}, c_k^{(0)}, \forall j \in \mathcal{J}_l, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}$.
2. **Repeat**
3. Solve (4.17) for $\{\hat{\mathbf{W}}_{l,j}^{(\kappa+1)}, \hat{\mathbf{Q}}_l^{(\kappa+1)}, \hat{r}_{l,j}^{(\kappa+1)}, \hat{a}_{l,j}^{(\kappa+1)}, \hat{b}_{l,j}^{(\kappa+1)}, \hat{c}_k^{(\kappa+1)}, \hat{d}_{l,j,k}^{(\kappa+1)}\}$
4. Calculate $b_{l,j}^{(\kappa+1)}$ and $c_k^{(\kappa+1)}$ through (4.18).
5. Set $\kappa := \kappa + 1$.
6. **Until** convergence of the objective in (4.16).
7. Set $\ddot{\mathbf{W}}_{l,j} := \hat{\mathbf{W}}_{l,j}^{(\bar{\kappa})}, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}$ and $\ddot{\mathbf{Q}}_l := \hat{\mathbf{Q}}_l^{(\bar{\kappa})}, \forall l \in \mathcal{L}$ where $\bar{\kappa}$ is the iteration index at which the SCA process converges.
8. Generate rank-one information covariance matrices and the corresponding energy covariance matrices for problem (4.14) through Proposition 4.1.
9. Obtain the (sub)optimal single stream beamforming vectors and energy transmission vectors for the secure transmission design problem (4.13) through EVD.
10. **End**

proved objective values in (4.16) *with increasing* κ*. Thus, as* κ *grows, the SCA process converges to a KKT point.*

Proof: See Appendix H.

As mentioned in the beginning of this section, the centralized secure transmission design, which is summarized in Algorithm 4, should be implemented on a central control unit with global CSI. Therefore, each BS is required to inform the central control unit of their local complex-valued CSI. In details, the MBS should send $2T_M(\sum_{l\in\mathcal{L}} J_l + K)$ real values to the central control unit for its CSI and an FBS should send $2T_F(\sum_{l\in\mathcal{L}} J_l + K)$ real values. On the other hand, after the (sub)optimal beamforming vectors are obtained, the central control unit needs to feed back the corresponding beamforming vectors to each BS. In this stage, the central control unit should send $2J_0T_M + 2T_M^2$ real values to the MBS and $2J_lT_F + 2T_F^2$ real values to FBS l. Consequently, the amount of information exchange for the proposed centralized secure transmission design is $\Xi_C = 2T_M(J_0 + \sum_{l\in\mathcal{L}} J_l + K + T_M) + 2T_F(-J_0 + (L+1)\sum_{l\in\mathcal{L}} J_l + LK + LT_F)$. Moreover, accord-

ing to the discussion for SCA processes, we can obtain anther centralized secure transmission design which can be seen as an extension from [51]. In the secure transmission design extended from [51], a solution to the original design problem (13) is found through the SCA process extended from [51]. The secure transmission design extended from [45] is also required to be implemented on a central control unit with global CSI. Therefore, the amount of information exchange for the design extended from [51] is Ξ_C as well.

4.4 Distributed Secure Transmission Design

In this section, we develop a distributed secure transmission design based on Algorithm 4, which enables each BS to optimize its single stream beamforming vectors and energy covariance matrix through its local CSI. Algorithm 4 includes three main parts: the first part is initialization; the second part is to iteratively solve (4.17); the third part is to obtain the single stream beamforming vector and the corresponding energy transmission vectors. It can be observed from the initialization problem (4.20) and Proposition 4.1 that the first and the third parts can be implemented distributedly at each BS. However, problem (4.17) cannot be directly solved in a distributed manner. Therefore, in this section, we focus on proposing a distributed approach to solve (4.17).

For decomposition, we rewrite problem (4.17) by introducing interference control variables $\theta_{l,uj}$, $\eta_{l,uj}$, $\eta'_{l,uj}$, $\vartheta_{l,k}$, $\zeta_{l,k}$ and $\zeta'_{l,uk}$ as follows

$$\max_{\substack{\mathbf{W}_{l,j},\mathbf{Q}_l,r_{l,j},a_{l,j},b_{l,j},c_{l,k},d_{l,j,k},\forall k\in\mathcal{K},\forall j\in\mathcal{J}_l,\forall l\in\mathcal{L}\\ \eta_{l,uj},\eta'_{l,uj},\zeta_{l,k},\zeta'_{l,uk},\theta_{l,uj},\vartheta_{l,k},\forall k\in\mathcal{K},\forall j\in\mathcal{J}_u,\forall l\in\mathcal{L},\forall u\in\mathcal{L},l\neq u}} \sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{J}_l}\ln(r_{l,j}) \tag{4.21a}$$

$$\text{s.t.}\eta_{l,uj}=\sum_{i\in\mathcal{J}_l}\text{tr}(\mathbf{H}_{l,u,j}\mathbf{W}_{l,i})+\text{tr}(\mathbf{H}_{l,u,j}\mathbf{Q}_l),\forall l\in\mathcal{L},\forall j\in\mathcal{J}_u,\forall u\in\mathcal{L},u\neq l, \tag{4.21b}$$

$$\zeta_{l,k}=\sum_{i\in\mathcal{J}_l}\text{tr}(\mathbf{G}_{l,k}\mathbf{W}_{l,i})+\text{tr}(\mathbf{G}_{l,k}\mathbf{Q}_l),\forall l\in\mathcal{L},\forall k\in\mathcal{K}, \tag{4.21c}$$

$$a_{l,j}-b_{l,j}-c_{l,k}+d_{l,j,k}\geq r_{l,j},\forall k\in\mathcal{K},\forall j\in\mathcal{J}_l,\forall l\in\mathcal{L}, \tag{4.21d}$$

$$\sum_{i\in\mathcal{J}_l}\mathrm{tr}(\mathbf{H}_{l,l,j}\mathbf{W}_{l,i})+\mathrm{tr}(\mathbf{H}_{l,l,j}\mathbf{Q}_l)+\sum_{u\in\mathcal{L},u\neq l}\eta'_{u,lj}+\sigma^2_{l,j}\geq 2^{a_{l,j}},\forall j\in\mathcal{J}_l,\forall l\in\mathcal{L}, \tag{4.21e}$$

$$\sum_{i\in\mathcal{J}_l,i\neq j}\mathrm{tr}(\mathbf{H}_{l,l,j}\mathbf{W}_{l,i})+\mathrm{tr}(\mathbf{H}_{l,l,j}\mathbf{Q}_l)+\sum_{u\in\mathcal{L},u\neq l}\eta'_{u,lj}+\sigma^2_{l,j}\leq 2^{b^{(\kappa)}_{l,j}}(\ln(2)b_{l,j}-\ln(2)b^{(\kappa)}_{l,j}+1),\forall j\in\mathcal{J}_l,\forall l\in\mathcal{L}, \tag{4.21f}$$

$$\sum_{i\in\mathcal{J}_l}\mathrm{tr}(\mathbf{G}_{l,k}\mathbf{W}_{l,i})+\mathrm{tr}(\mathbf{G}_{l,k}\mathbf{Q}_l)+\sum_{u\in\mathcal{L},u\neq l}\zeta'_{u,lk}+\sigma^2_k\leq 2^{c^{(\kappa)}_k}(\ln(2)c_{l,k}-\ln(2)c^{(\kappa)}_k+1),\forall l\in\mathcal{L},\forall k\in\mathcal{K}, \tag{4.21g}$$

$$\sum_{i\in\mathcal{J}_l,i\neq j}\mathrm{tr}(\mathbf{G}_{l,k}\mathbf{W}_{l,i})+\mathrm{tr}(\mathbf{G}_{l,k}\mathbf{Q}_l)+\sum_{u\in\mathcal{L},u\neq l}\zeta'_{u,lk}+\sigma^2_k\geq 2^{d_{l,j,k}},\quad \forall j\in\mathcal{J}_l,\forall l\in\mathcal{L},\forall k\in\mathcal{K}, \tag{4.21h}$$

$$\sum_{i\in\mathcal{J}_l}\mathrm{tr}(\mathbf{G}_{l,k}\mathbf{W}_{l,i})+\mathrm{tr}(\mathbf{G}_{l,k}\mathbf{Q}_l)+\sum_{u\in\mathcal{L},u\neq l}\zeta'_{u,lk}\geq\check{\Lambda}_k(\varpi_k),\ \forall l\in\mathcal{L},\forall k\in\mathcal{K}, \tag{4.21i}$$

$$\eta_{l,uj}=\eta'_{l,uj}=\theta_{l,uj},\forall l\in\mathcal{L},\forall j\in\mathcal{J}_u,\forall u\in\mathcal{L},l\neq u, \tag{4.21j}$$

$$\zeta_{l,k}=\zeta'_{l,uk}=\vartheta_{l,k},\forall l\in\mathcal{L},\forall k\in\mathcal{K},\forall u\in\mathcal{L},l\neq u, \tag{4.21k}$$

$$(4.14c)-(4.14d).$$

In problem (4.21), $\eta_{l,uj}$ and $\zeta_{l,k}$ are local variables at BS l. Variables $\eta'_{l,uj}$ and $\zeta'_{l,uk}$ are copies of $\eta_{l,uj}$ and $\zeta_{l,k}$ stored locally at BS u. In addition, $\theta_{l,uj}$, which is stored at both BSs l and u, and $\vartheta_{l,k}$, which is stored at all BSs, are also copies of $\eta_{l,uj}$ and $\zeta_{l,k}$ respectively. Then, we can stack all the local variables at BS l in (4.21) into a row vector $\mathbf{s}_l=[\bar{\mathrm{vec}}(\mathbf{W}_{l,j}),\bar{\mathrm{vec}}(\mathbf{Q}_l),r_{l,j},a_{l,j},b_{l,j},c_{l,k},d_{l,j,k},\forall k\in\mathcal{K},\forall j\in\mathcal{J}_l,\boldsymbol{\eta}_l^T,\boldsymbol{\zeta}_l^T]$ and define the set $\mathcal{S}_l^{(\kappa+1)}=\{\mathbf{s}_l|(4.21b),(4.21c),(4.21d),(4.21e),(4.21f),(4.21g),(4.21h),(4.21i),(4.14c),(4.14d)\}$ where $\bar{\mathrm{vec}}(\cdot)$ maps the variables in a Hermitian matrix into a row vector, $\boldsymbol{\eta}_l=[\eta_{l,(0)(1)},...,\eta_{l,(l-1)(J_{l-1})},\eta_{l,(l+1)(1)},...,\eta_{l,(L)(J_L)},\eta'_{0,(l)(1)},...,\eta'_{l-1,(l)(J_l)},\eta'_{l+1,(l)(1)},...,\eta'_{L,(l)(J_l)}]^T$ and $\boldsymbol{\zeta}_l=[\zeta_{l,1},...,\zeta_{l,K},\zeta'_{0,(l)(1)},...,\zeta'_{l-1,(l)(K)},\zeta'_{l+1,(l)(1)},...,\zeta'_{L,(l)(K)}]^T$. Similarly, we denote $\boldsymbol{\theta}_l=[\theta_{l,(0)(1)},...,\theta_{l,(l-1)(J_{l-1})},\theta_{l,(l+1)(1)},...,\theta_{l,(L)(J_L)},\theta_{0,(l)(1)},...,\theta_{l-1,(l)(J_l)},\theta_{l+1,(l)(1)},...,\theta_{L,(l)(J_l)}]^T$ and $\boldsymbol{\vartheta}_l=[\vartheta_{l,1},...,\vartheta_{l,K},\vartheta_{0,1},...,\vartheta_{l-1,K},\vartheta_{l+1,1},...,\vartheta_{L,K}]^T$. Based on these definitions, problem (4.21) can be equiva-

lently transformed into

$$\max_{\mathbf{s}_l,\boldsymbol{\theta}_l,\boldsymbol{\vartheta}_l,\forall l\in\mathcal{L}} \sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{J}_l} \ln(r_{l,j}) \tag{4.22a}$$

$$\text{s.t.}\ \mathbf{s}_l \in \mathcal{S}_l^{(\kappa+1)}, \forall l \in \mathcal{L}, \tag{4.22b}$$

$$\boldsymbol{\eta}_l = \boldsymbol{\theta}_l, \forall l \in \mathcal{L}, \tag{4.22c}$$

$$\boldsymbol{\zeta}_l = \boldsymbol{\vartheta}_l, \forall l \in \mathcal{L}. \tag{4.22d}$$

In the following, we adopt the ADMM approach [82] to solve problem (4.22) in a distributed manner.

The augmented Lagrangian function of (4.22) can be expressed as

$$\begin{aligned} L_D^{(\kappa+1)}(\mathbf{s}_l,\boldsymbol{\theta}_l,\boldsymbol{\vartheta}_l;\boldsymbol{\xi}_l,\boldsymbol{\omega}_l,\forall l\in\mathcal{L}) = \sum_{l\in\mathcal{L}}\Big(\sum_{j\in\mathcal{J}_l}\ln(r_{l,j}) + \boldsymbol{\xi}_l^T(\boldsymbol{\eta}_l-\boldsymbol{\theta}_l) \\ + \boldsymbol{\omega}_l^T(\boldsymbol{\zeta}_l-\boldsymbol{\vartheta}_l) - \frac{c}{2}\|\boldsymbol{\eta}_l-\boldsymbol{\theta}_l\|^2 - \frac{c}{2}\|\boldsymbol{\zeta}_l-\boldsymbol{\vartheta}_l\|^2\Big) \end{aligned} \tag{4.23}$$

where $\boldsymbol{\xi}_l$ and $\boldsymbol{\omega}_l$, locally stored at BS l, are the Lagrange multipliers associated with (4.22c) and (4.22d) and c ($c > 0$) is the penalty parameter. In the ADMM procedure, BS l iteratively updates the variables $\mathbf{s}_l$, $\boldsymbol{\theta}_l$, $\boldsymbol{\vartheta}_l$, $\boldsymbol{\xi}_l$ and $\boldsymbol{\omega}_l$ in the augmented Lagrangian function (4.23) to attain the optimal solution to (4.22). The details of the ADMM procedure are described as follows.

4.4.1 Primal Variables Update

In the $(\iota+1)$-th iteration of the ADMM procedure, the primal variables $\boldsymbol{\theta}_l$ and $\boldsymbol{\vartheta}_l$ are updated at first. More specifically, we solve

$$\max_{\boldsymbol{\theta}_l,\boldsymbol{\vartheta}_l,\forall l\in\mathcal{L}} L_D^{(\kappa+1)}(\mathbf{s}_l^{(\iota)},\boldsymbol{\theta}_l,\boldsymbol{\vartheta}_l;\boldsymbol{\xi}_l^{(\iota)},\boldsymbol{\omega}_l^{(\iota)},\forall l\in\mathcal{L})$$

to obtain $\boldsymbol{\theta}_l^{(\iota+1)}$ and $\boldsymbol{\vartheta}_l^{(\iota+1)}$, i.e.,

$$\{\boldsymbol{\theta}_l^{(\iota+1)}, \forall l \in \mathcal{L}\} = \underset{\boldsymbol{\theta}_l, \forall l \in \mathcal{L}}{\operatorname{argmax}} \sum_{\forall l \in \mathcal{L}} \left((\boldsymbol{\xi}_l^{(\iota)})^T (\boldsymbol{\eta}_l^{(\iota)} - \boldsymbol{\theta}_l) - \frac{c}{2} \left\| \boldsymbol{\eta}_l^{(\iota)} - \boldsymbol{\theta}_l \right\|^2 \right), \quad (4.24)$$

$$\{\boldsymbol{\vartheta}_l^{(\iota+1)}, \forall l \in \mathcal{L}\} = \underset{\boldsymbol{\vartheta}_l, \forall l \in \mathcal{L}}{\operatorname{argmax}} \sum_{\forall l \in \mathcal{L}} \left((\boldsymbol{\omega}_l^{(\iota)})^T (\boldsymbol{\zeta}_l^{(\iota)} - \boldsymbol{\vartheta}_l) - \frac{c}{2} \left\| \boldsymbol{\zeta}_l^{(\iota)} - \boldsymbol{\vartheta}_l \right\|^2 \right). \quad (4.25)$$

Problems (4.24) and (4.25) are unconstrained quadratic programming and enjoy analytical solutions as follows

$$\theta_{l,uj}^{(\iota+1)} = \frac{\eta_{l,uj}^{(\iota)} + \eta_{l,uj}^{'(\iota)}}{2} - \frac{\xi_{l,uj}^{(\iota)} + \xi_{l,uj}^{'(\iota)}}{2c}, \quad (4.26)$$

$$\vartheta_{l,k}^{(\iota+1)} = \frac{\sum_{u \in \mathcal{L}} \left(-\omega_{l,uk}^{'(\iota)} + c\zeta_{l,uk}^{'(\iota)} \right)}{(L+1)c} \quad (4.27)$$

where $\zeta_{l,lk}^{'(\iota)} = \zeta_{l,k}^{(\iota)}$, $\omega_{l,lk}^{'(\iota)} = \omega_{l,k}^{(\iota)}$ and $\xi_{l,uj}$, $\xi'_{l,uj}$, $\omega_{l,k}$ and $\omega'_{l,uk}$ are dual variables associated with the primal variables $\eta_{l,uj}$, $\eta'_{l,uj}$, $\zeta_{l,k}$ and $\zeta'_{l,uk}$. Eqs. (4.26) and (4.27) can be calculated at each BS so that the updated primal variables $\boldsymbol{\theta}_l^{(\iota+1)}$ and $\boldsymbol{\vartheta}_l^{(\iota+1)}$ are available for BS l to update the other variables. Note that, for BS l, $\theta_{l,uj}^{(\iota+1)}$, $\theta_{u,lj}^{(\iota+1)}$ and $\vartheta_{l,k}^{(\iota+1)}$ are calculated after $\frac{\eta_{l,uj}^{'(\iota)}}{2} - \frac{\xi_{l,uj}^{'(\iota)}}{2c}$, $\frac{\eta_{u,lj}^{(\iota)}}{2} - \frac{\xi_{u,lj}^{(\iota)}}{2c}$ and $-\omega_{l,uk}^{'(\iota)} + c\zeta_{l,uk}^{'(\iota)}$ are gathered from BS $u \neq l$.

Then, the primal variable $\mathbf{s}_l$ is updated via

$$\{\mathbf{s}_l^{(\iota+1)}, \forall l \in \mathcal{L}\} = \underset{\mathbf{s}_l \in \mathcal{S}_l^{(\kappa+1)}, \forall l \in \mathcal{L}}{\operatorname{argmax}} L_D^{(\kappa+1)}(\mathbf{s}_l, \boldsymbol{\theta}_l^{(\iota+1)}, \boldsymbol{\vartheta}_l^{(\iota+1)}; \boldsymbol{\xi}_l^{(\iota)}, \boldsymbol{\omega}_l^{(\iota)}, \forall l \in \mathcal{L}). \quad (4.28)$$

Since the objective function is decomposable, problem (4.28) can be solved

in parallel at each BS. The decomposed subproblem at BS l is

$$\mathbf{s}_l^{(\iota+1)} = \underset{\mathbf{s}_l \in \mathcal{S}_l^{(\kappa+1)}}{\operatorname{argmax}} \sum_{j \in \mathcal{J}_l} \ln(r_{l,j}) + (\boldsymbol{\xi}_l^{(\iota)})^T (\boldsymbol{\eta}_l - \boldsymbol{\theta}_l^{(\iota+1)}) + (\boldsymbol{\omega}_l^{(\iota)})^T (\boldsymbol{\zeta}_l - \boldsymbol{\vartheta}_l^{(\iota+1)}) - \frac{c}{2} \left\| \boldsymbol{\eta}_l - \boldsymbol{\theta}_l^{(\iota+1)} \right\|^2 - \frac{c}{2} \left\| \boldsymbol{\zeta}_l - \boldsymbol{\vartheta}_l{}^{(\iota+1)} \right\|^2 . \tag{4.29}$$

Note that (4.29) is a convex optimization problem, and thus it can be solved. Also note that all the parameters in (4.29) are locally known at BS l. Therefore, the update of $\mathbf{s}_l$'s does not require information exchange.

4.4.2 Dual Variables Update

In the end of the $(\iota+1)$-th iteration of the ADMM procedure, the dual variables can be updated as

$$\boldsymbol{\xi}_l^{(\iota+1)} = \boldsymbol{\xi}_l^{(\iota)} - c(\boldsymbol{\eta}_l^{(\iota+1)} - \boldsymbol{\theta}_l^{(\iota+1)}), \forall l \in \mathcal{L}, \tag{4.30}$$

$$\boldsymbol{\omega}_l^{(\iota+1)} = \boldsymbol{\omega}_l^{(\iota)} - c(\boldsymbol{\zeta}_l^{(\iota+1)} - \boldsymbol{\vartheta}_l^{(\iota+1)}), \forall l \in \mathcal{L}. \tag{4.31}$$

Similar to the update of $\mathbf{s}_l$'s, BS l can update the dual variables without requiring the information from the other BSs.

Since the ADMM procedure meets the properties listed in [82, Proposition 7.4.1], it must converge to an optimal solution to problem (4.22). Thus, an optimal solution to problem (4.17) can be found through the equivalence between problems (4.22) and (4.17). According to the above discussion, we propose Algorithm 5 that can be used to solve problem (4.17) in a distributed manner. Note that, we can implement Algorithm 4 in the distributed manner by using Algorithm 5 to solve problem (4.17), i.e., we obtain the distributed secure transmission design by combining Algorithm 4 and Algorithm 5. Since the optimality of problem (4.17) can be achieved by Algorithm 5 at each iteration of the SCA process in Algorithm 4, the convergence of the distributed design, which is the combination of Algorithm 4 and Algorithm 5, can be guaranteed based on Proposition 4.2.

Algorithm 5: Distributed algorithm for solving problem (4.17)

1. Initialize $\iota := 0$, set $\boldsymbol{\xi}_l^{(0)}$, $\boldsymbol{\omega}_l^{(0)}$ by zeros and set $\boldsymbol{\eta}_l^{(0)}$, $\boldsymbol{\zeta}_l^{(0)}$ through (4.21b) and (4.21c) by using the solution to (4.16) in the κ-th iteration of the SCA process.
2. **Repeat**
3. **for** $l \in \mathcal{L}$
4. BS l updates $\boldsymbol{\theta}_l^{(\iota+1)}$ and $\boldsymbol{\vartheta}_l^{(\iota+1)}$ by (4.26) and (4.27).
5. BS l updates $\mathbf{s}_l^{(\iota+1)}$ by (4.29).
6. BS l updates $\boldsymbol{\xi}_l^{(\iota+1)}$ and $\boldsymbol{\omega}_l^{(\iota+1)}$ by (4.30) and (4.31).
7. **end for** $l \in \mathcal{L}$
8. $\iota := \iota + 1$,
9. **Until** $\left\|\boldsymbol{\eta}_l^{(\iota)} - \boldsymbol{\theta}_l^{(\iota)}\right\|$ and $\left\|\boldsymbol{\zeta}_l^{(\iota)} - \boldsymbol{\vartheta}_l^{(\iota)}\right\|$ are small enough for each $l \in \mathcal{L}$.
10. Obtain an optimal solution to (4.17).
11. **End**

According to the discussion for the ADMM procedure, in Algorithm 5, step 4 requires BS u to send $\frac{\eta_{l,uj}^{'(\iota)}}{2} - \frac{\xi_{l,uj}^{'(\iota)}}{2c}$, $\frac{\eta_{u,lj}^{(\iota)}}{2} - \frac{\xi_{u,lj}^{(\iota)}}{2c}$ and $-\omega_{l,uk}^{'(\iota)} + c\zeta_{l,uk}^{'(\iota)}$ to BS l, which means BS u should send $J_u + J_l + K$ real values to BS l. Therefore, the amount of information exchange in each iteration of Algorithm 5 is $\Xi_D = 2L\sum_{l\in\mathcal{L}} J_l + KL(L+1)$. Comparing Ξ_D with Ξ_C, we can find that Ξ_D is not related to the antenna numbers T_M and T_F. Thus, when BSs are equipped with a large number of antennas, Ξ_D will only account for a small proportion of Ξ_C and the distributed secure transmission design can consume less information exchange than the centralized ones.

4.5 Simulation Results

In this section, we present numerical results to illustrate the performance of the proposed centralized secure transmission design (i.e., Algorithm 4) and distributed secure transmission design (i.e., combining Algorithm 4 and Algorithm 5 together, which is referred to as Algorithm 4&5). We consider the scenario where two Femtocells, each with a radius of 30 m, are overlaid with a Macrocell with a radius of 150 m, as illustrated in Figure 4.2.

Table 4.1: Simulation parameters setting

Parameters	Values
Number of antennas at MBS T_M	10
Maximal transmit power of MBS $P_0^{\max}$	40 dBm
Maximal transmit power at each FBS $P^{\max}$	36 dBm
EH parameters $\{S_k, \eta_k, o_k\}$	$\{0.024, 150, 0.014\}$ [89]
White noise power σ^2	-50 dBm
Penalty parameter c	0.2

Two MUs, two FUs and one FU are randomly distributed in the Macrocell, Femtocell 1 and Femtocell 2. In the meanwhile, we assume two ERs are randomly located in a circular region centered at the MBS with a radius of 20 m and one ER is scattered randomly in each circular region centered at an FBS with a radius of 15 m. All channel coefficients are modeled as i.i.d Rayleigh fading, i.e., the channel vectors $\mathbf{h}_{l,u,j}$ and $\mathbf{g}_{l,k}$ follow the distribution $\mathcal{CN}(\mathbf{0}, P_L\mathbf{I})$ where P_L represents the pathloss. We assume that the path loss model is $29.5 + 22.5\log_{10}(d)$ where d denotes the distance between one IR or one ER to the corresponding BS. As for the noise power, we set $\sigma_{l,j}^2 = \sigma_k^2 = \sigma^2, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}, \forall k \in \mathcal{K}$. In addition, we assume $P_l^{\max} = P^{\max}, \forall l \in \mathcal{L}_F$, $\varpi_k = \varpi, \forall k \in \mathcal{K}$ and all ERs have the same EH parameters. The parameter value setting is provided in Table I.

In Figure 4.3, we study the convergence of the SCA processes in Algorithm 4 and Algorithm 4&5 by setting the antenna number at FBSs, T_F, as 6 and ERs' EH threshold ϖ as 20 uW. The convergence of the SCA process extended from [51] is also studied. As can be observed from Figure 4.3, the SCA process in Algorithm 4 converges four iterations faster than the SCA process extended from [51]. Thus, the SCA process in Algorithm 4 is more efficient than its counterpart, which attributes to the acceleration step (4.18). In addition, the performances of the SCA processes in Algorithm 4 and Algorithm 4&5 coincide, which verifies that the optimality of problem (4.17) can be achieved by Algorithm 5.

In the following, we examine the secrecy throughput and fairness perfor-

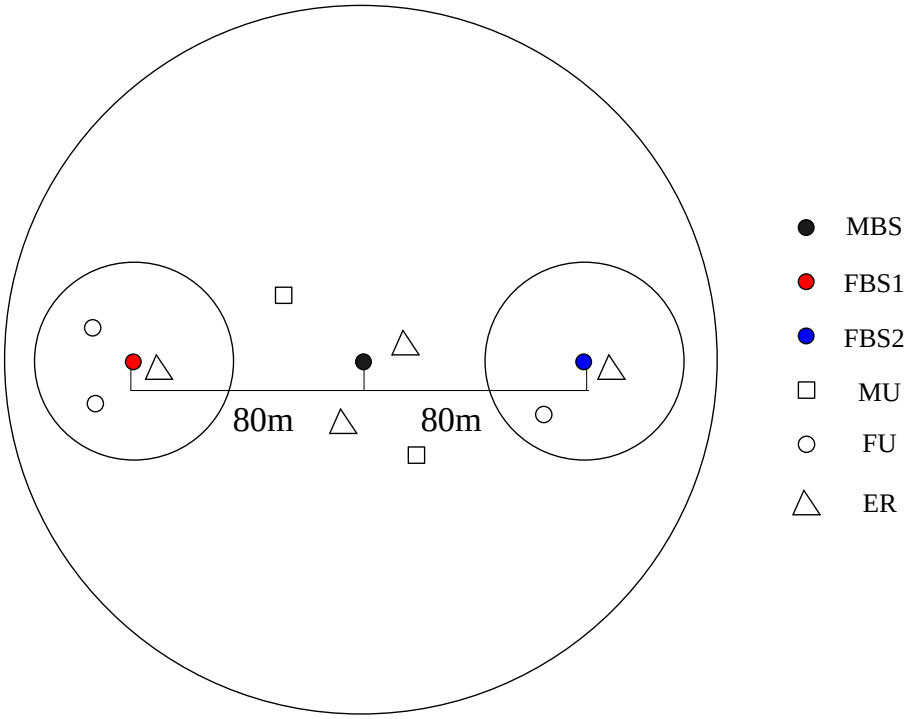

Figure 4.2: Illustration of the simulation scenario.

mances of Algorithm 4 and Algorithm 4&5 through Monte Carlo simulations. The secrecy throughput refers to the sum of secrecy rates at all IRs in the network. To measure the fairness between secrecy rates for different IRs, we adopt the *Jain's fairness index*

$$(\sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{J}_l}R_{l,j})^2/((\sum_{l\in\mathcal{L}}J_l)\sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{J}_l}R_{l,j}^2).$$

The value of the fairness index lies in the interval $[1/J, 1]$, where J represents the number of all IRs, and the larger value means more balanced secrecy rates among IRs [90].

Figure 4.4 and Figure 4.5 demonstrate the secrecy throughputs and the fairness achieved by Algorithm 4 and Algorithm 4&5 as functions of the EH

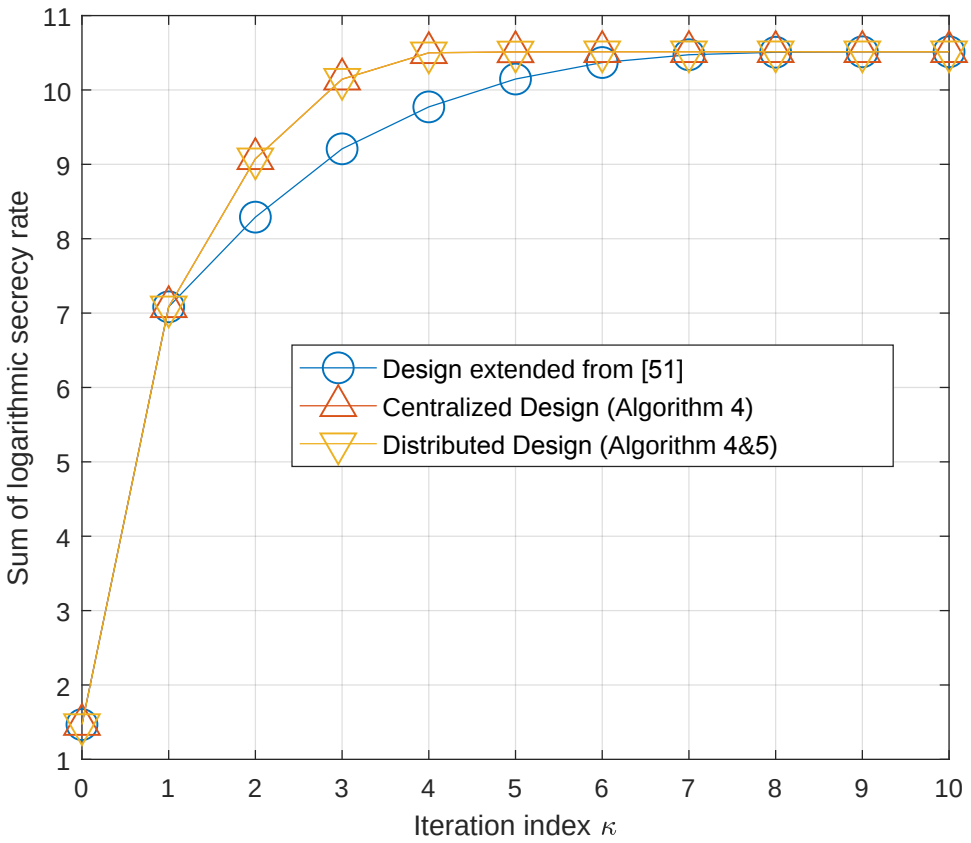

Figure 4.3: Convergence of the SCA processes in Algorithm 4 and Algorithm 4&5 and the SCA process extended from [51] with $T_F = 6$ and $\varpi = 20$ uW.

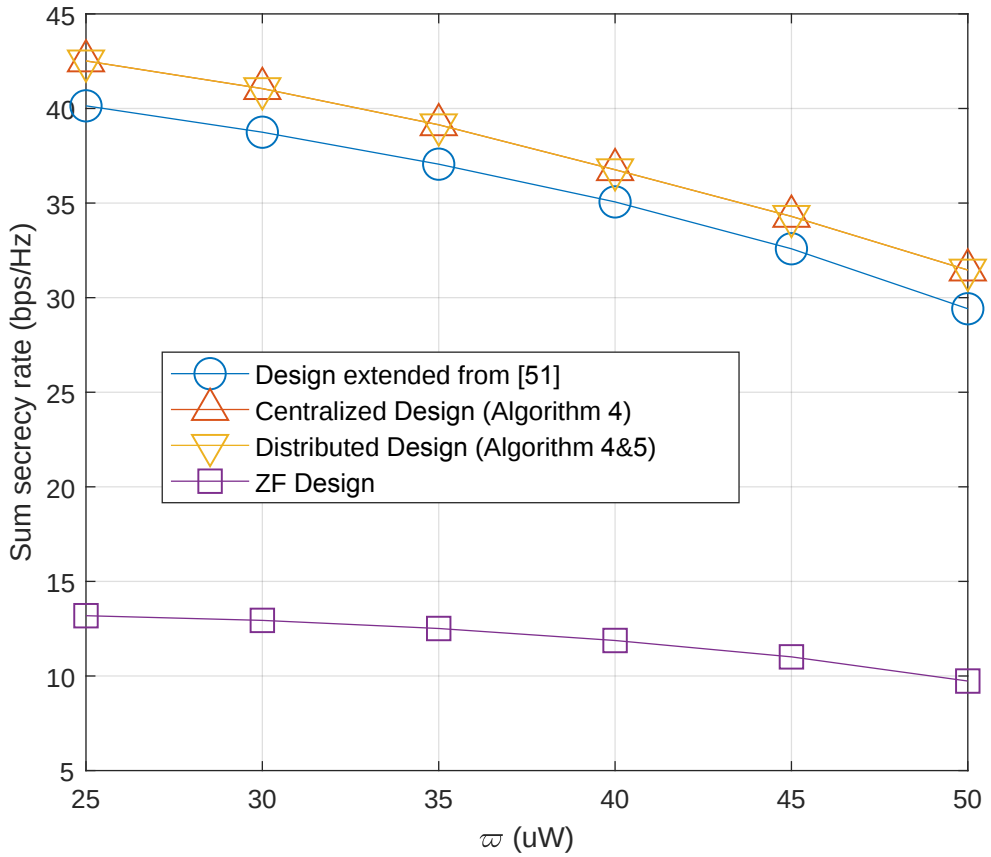

Figure 4.4: Secrecy throughputs achieved by Algorithm 4, Algorithm 4&5, the design extended from [51] and the ZF design versus ϖ with $T_F = 6$.

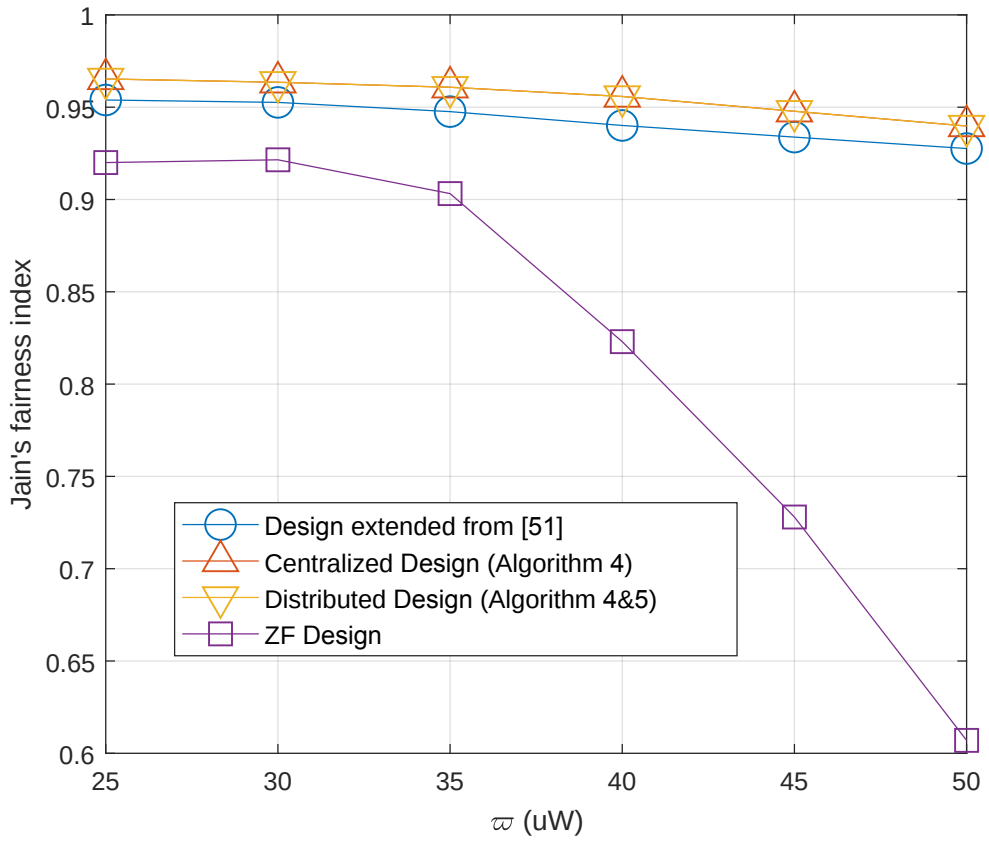

Figure 4.5: Jain's fairness indexes achieved by Algorithm 4, Algorithm 4&5, the design extended from [51] and the ZF design versus ϖ with $T_F = 6$.

threshold ϖ with $T_F = 6$. For comparison, the extension of the zero-forcing (ZF) secure transmission design discussed in [91] is considered as well. In the ZF scheme, each BS controls its interferences to all non-serving IRs to be less than the threshold $I_{l,u,j}$. The MBS eliminates its information leakage to all ERs and each FBS eliminates its information leakage to the ERs which are close to itself or to the MBS. In addition, we also compare the proposed designs with the secure transmission design extended from [51]. We set the maximal iteration number of SCA processes, I_{SCA}, to be 5. From Figure 4.4, we can observe that Algorithm 4 and Algorithm 4&5 outperform the ZF secure transmission design in terms of secrecy throughput. As can be seen from Figure 4.5, when the EH threshold ϖ is greater than 35 uW, the fairness performance of the ZF secure transmission design degrades significantly. In contrast, Algorithm 4 and Algorithm 4&5 are relatively insensitive to the change of ϖ.

In Figure 4.6 and Figure 4.7, the secrecy throughputs and fairness achieved by Algorithm 4, Algorithm 4&5, the secure transmission design extended from [51] and the ZF secure transmission design are plotted against T_F with $\varpi = 20$ uW and $I_{SCA} = 5$. From Figure 4.4 to Figure 4.7, it can be seen that the design extended from [51] performs worse than Algorithm 4 and Algorithm 4&5. The performance losses of the secure transmission design extended from [51] are caused by the slow convergence of the SCA process extended from [51]. Moreover, from Figure 4.5 and Figure 4.7, we can observe that, no matter how the parameter setting is changed, both Algorithm 4 and Algorithm 4&5 can provide high level fairness among IRs, which verifies the effectiveness of the proportional fairness.

4.6 Summary

In this chapter, we considered a MISO HCN where there exist one MBS and multiple FBSs in co-channel deployment. All BSs transmit different confidential messages to their connecting IRs and energy signals to ERs. By considering the risk that the information for IRs may be eavesdropped by ERs and the fairness between among IRs, we proposed a sum logarithmic

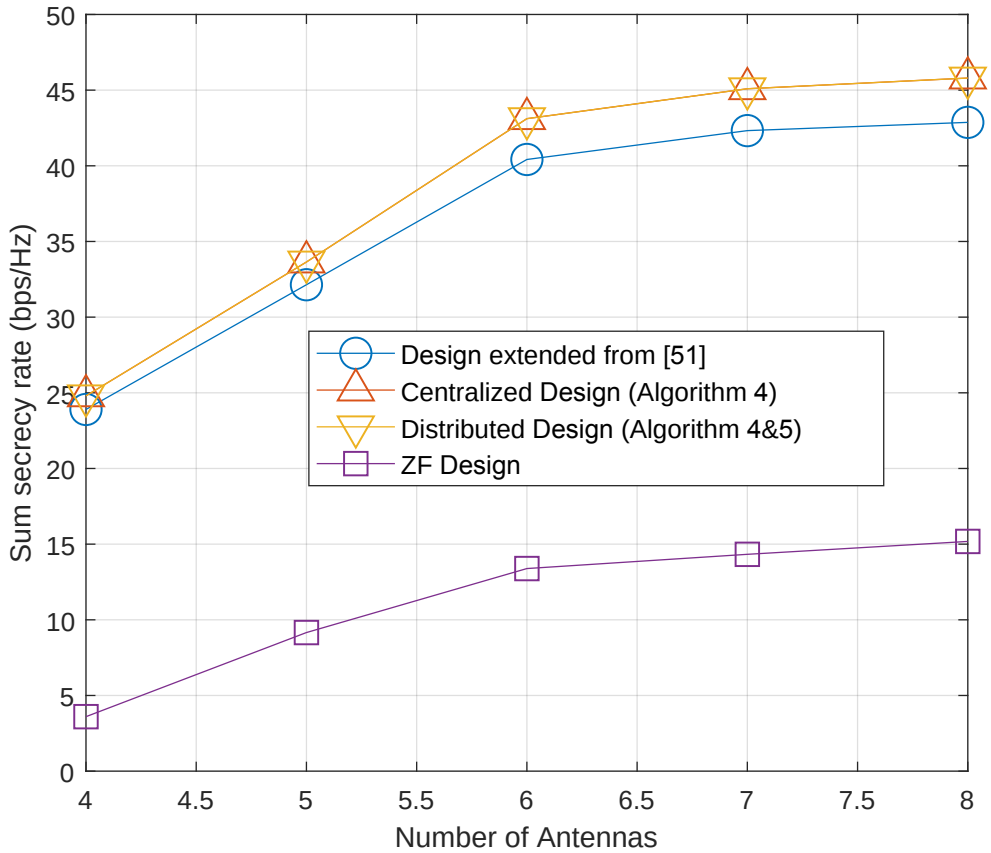

Figure 4.6: Secrecy throughputs achieved by Algorithm 4, Algorithm 4&5, the design extended from [51] and the ZF design versus T_F with $\varpi = 20$ uW.

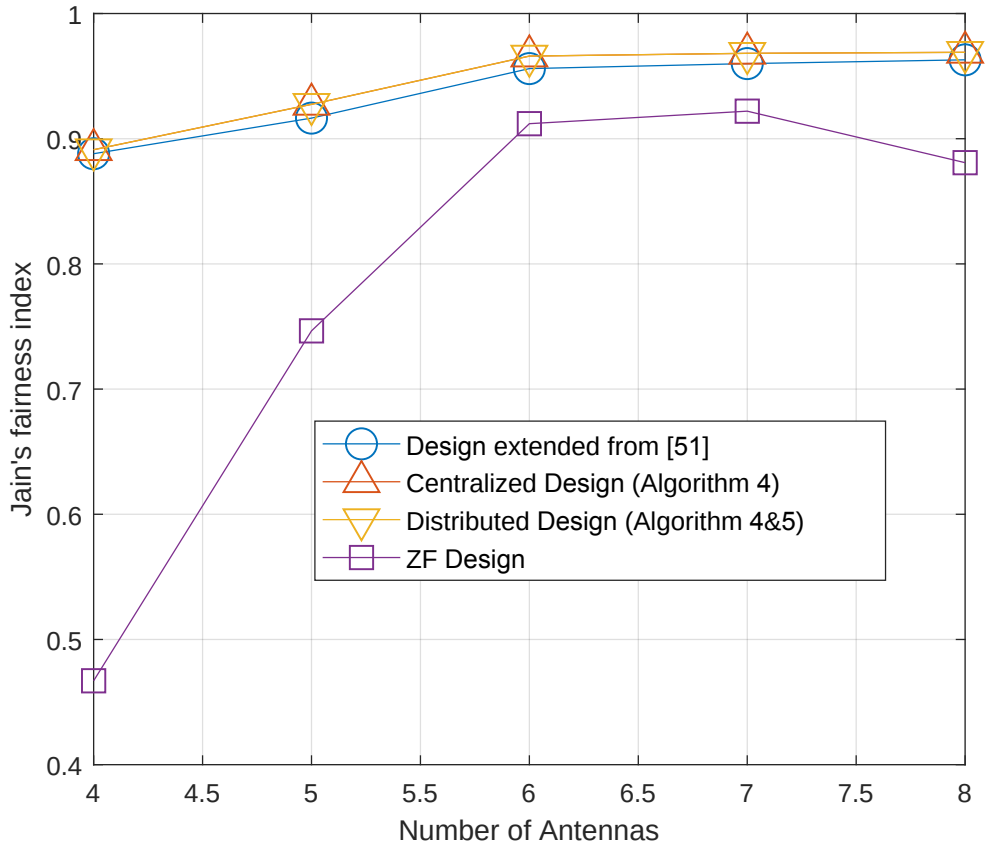

Figure 4.7: Jain's fairness indices achieved by Algorithm 4, Algorithm 4&5, the design extended from [51] and the ZF design versus T_F with $\varpi = 20$ uW.

secrecy rate maximization secure transmission design problem under the EH constraints for ERs. To deal with the sum logarithmic secrecy rate maximization secure transmission design problem, we used SDR to relax the outer products of the single stream beamforming vectors to be general rank information covariance matrices. We proved that the SDR is tight. The tight SDR revealed that, compared with beamforming having general rank information covariance matrix, utilizing single stream beamforming to transmit confidential information message to each IR does not cause any loss of performance optimality. This finding provided a theoretical justification for using single stream beamforming. Then, we dealt with the SDR problem by proposing an SCA process and developed an SCA-based centralized secure transmission design algorithm. We proved that the proposed SCA process converges to a KKT point of the SDR problem. The SDR problem can also be dealt by an extension of the SCA process discussed in [51]. Nevertheless, compared with the SCA process extended from [51], the proposed one converges faster. We also proposed a distributed secure transmission design. In the distributed secure transmission design, the ADMM was employed, and the sum logarithmic secrecy rate maximization design problem was decomposed so that secure transmission schemes can be designed in a distributed manner. Simulation results demonstrated the effectiveness of the proposed centralized and distributed designs.

Chapter 5

Outage Constrained Secure Transmission for CJ Aided MISO MC NOMA

In this chapter, we consider a CJ aided MISO NOMA system. Under the assumption that only statistical ECSI is available, we study an outage constrained secure transmission design problem. In the design problem, the single stream beamforming vector, the covariance matrix of jamming signals and the subcarrier allocation policy are jointly optimized to maximize the minimum probabilistic secrecy rate. Since the optimization of the subcarrier allocation policy is considered, the design problem is a mixed integer programming problem, which is challenging to deal with. In order to find a solution to the design problem, we develop an algorithm based on 0-norm and the path-following approach.

5.1 System Model and Problem Formulation

5.1.1 MISO NOMA System Model

We consider a CJ aided downlink MISO NOMA system shown as Figure 5.1. In the system, a BS (Alice) sends information to M legitimate users (Bobs) through I orthogonal subcarriers. We assume that each Bob can only be scheduled on one subcarrier and each subcarrier can be assigned to a pair of Bobs. One of the two Bobs in a pair, which directly decodes its own signal, is referred to as the weak Bob; while another one, which cancels the MUI by performing SIC before decoding its own message, is referred to as

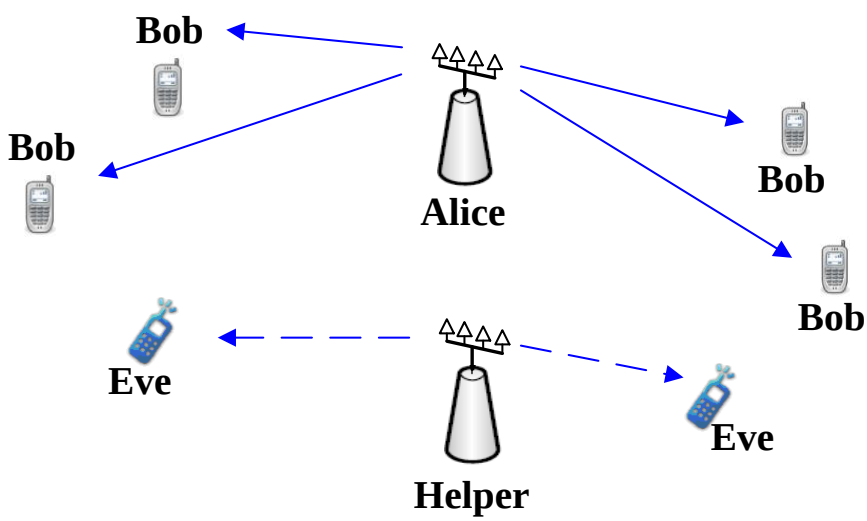

Figure 5.1: A MISO NOMA system with CJ.

the strong Bob. The information intended for Bobs suffers from the risk of being wiretapped by K eavesdroppers (Eves) and a Helper sends jamming signals to confuse K Eves for secure communication. We assume Alice and the Helper have N_a and N_h transmit antennas; while Bobs and Eves are equipped with single antenna. We let $\mathcal{M} = \{1, \ldots, M\}$, $\mathcal{K} = \{1, \ldots, K\}$ and $\mathcal{I} = \{1, \ldots, I\}$ denote the sets of Bobs, Eves and subcarriers respectively. We assume that all the channels in the system are flat quasi-static fading channels which are invariant during a scheduling time slot.

In each scheduling time slot, Alice transmits different confidential information bearing signals to two Bobs on each subcarrier. More specifically, assuming that Bob $m \in \mathcal{M}$, which is weak, and Bob $n \in \mathcal{M}$, which is strong, are scheduled on subcarrier $i \in \mathcal{I}$ in a given time slot, the signal vector transmitted by Alice on subcarrier i can be given by

$$\mathbf{x}_{m,n,i} = \mathbf{w}_{m,n,i}^{[m]} x_m + \mathbf{w}_{m,n,i}^{[n]} x_n \tag{5.1}$$

where $x_m \in \mathbb{C}$ and $x_n \in \mathbb{C}$ are the CSCG distributed information bearing signals intended for Bob m and Bob n with $x_m \sim \mathcal{CN}(0,1)$ and $x_n \sim \mathcal{CN}(0,1)$, and $\mathbf{w}_{m,n,i}^{[m]} \in \mathbb{C}^{N_a}$ and $\mathbf{w}_{m,n,i}^{[n]} \in \mathbb{C}^{N_a}$ are the corresponding single stream beamforming vectors. Simultaneously, the Helper generates jamming vector $\mathbf{z}_i \in \mathbb{C}^{N_h}$ to cripple Eves' interception. The jamming vector $\mathbf{z}_i$ follows a zero mean CSCG distribution, i.e, $\mathbf{z}_i \sim \mathcal{CN}(0, \mathbf{Q}_i)$ where $\mathbf{Q}_i \succeq \mathbf{0}$ is the covariance matrix of $\mathbf{z}_i$. Therefore, the received signals at Bob m and Bob n on subcarrier i can be, respectively, expressed as

$$y_{b,m,i} = \mathbf{h}_{b,m,i}^H \mathbf{w}_{m,n,i}^{[m]} x_m + \mathbf{h}_{b,m,i}^H \mathbf{w}_{m,n,i}^{[n]} x_n + \mathbf{g}_{b,m,i}^H \mathbf{z}_i + n_{b,m,i} \tag{5.2}$$

and

$$y_{b,n,i} = \mathbf{h}_{b,n,i}^H \mathbf{w}_{m,n,i}^{[m]} x_m + \mathbf{h}_{b,n,i}^H \mathbf{w}_{m,n,i}^{[n]} x_n + \mathbf{g}_{b,n,i}^H \mathbf{z}_i + n_{b,n,i} \tag{5.3}$$

where $\mathbf{h}_{b,m,i} \in \mathbb{C}^{N_a}$ and $\mathbf{h}_{b,n,i} \in \mathbb{C}^{N_a}$ denote the channel vectors from Alice to Bob m and Bob n on subcarrier i, $\mathbf{g}_{b,m,i} \in \mathbb{C}^{N_h}$ and $\mathbf{g}_{b,n,i} \in \mathbb{C}^{N_h}$ denote the channel vectors from the Helper to Bob m and Bob n on subcarrier i, and $n_{b,m,i} \sim \mathcal{CN}(0, \sigma_{b,m,i}^2)$, $n_{b,n,i} \sim \mathcal{CN}(0, \sigma_{b,n,i}^2)$ are the Gaussian noises at Bob m and Bob n on subcarrier i. Bob m, which is weak, directly decodes its own signal. According to (5.2), the SINR at Bob m can be expressed as

$$SINR_{m,n,i}^{[m]} = \frac{\left|\mathbf{h}_{b,m,i}^H \mathbf{w}_{m,n,i}^{[m]}\right|^2}{\left|\mathbf{h}_{b,m,i}^H \mathbf{w}_{m,n,i}^{[n]}\right|^2 + \mathbf{g}_{b,m,i}^H \mathbf{Q}_i \mathbf{g}_{b,m,i} + \sigma_{b,m,i}^2}. \tag{5.4}$$

In contrast, Bob n, which is strong, first performs SIC to remove the MUI caused by the signal intended for Bob m before decoding its own message. As a result, the SINR at Bob n is

$$SINR_{m,n,i}^{[n]} = \frac{\left|\mathbf{h}_{b,n,i}^H \mathbf{w}_{m,n,i}^{[n]}\right|^2}{\mathbf{g}_{b,n,i}^H \mathbf{Q}_i \mathbf{g}_{b,n,i} + \sigma_{b,n,i}^2}. \tag{5.5}$$

However, SIC is not always possible. To ensure successful SIC, the date rate to Bob m should not exceed Bob n's decoding capacity for the information

intended to Bob m [61]. Therefore the data rate to Bob m can be given by

$$R_{m,n,i}^{[m]} = \min\{\log_2\left(1 + SINR_{m,n,i}^{[m]}\right), \log_2\left(1 + SINR_{m,n,i}^{[m,n]}\right)\} \tag{5.6}$$

where

$$SINR_{m,n,i}^{[m,n]} = \frac{\left|\mathbf{h}_{b,n,i}^H \mathbf{w}_{m,n,i}^{[m]}\right|^2}{\left|\mathbf{h}_{b,n,i}^H \mathbf{w}_{m,n,i}^{[n]}\right|^2 + \mathbf{g}_{b,n,i}^H \mathbf{Q}_i \mathbf{g}_{b,n,i} + \sigma_{b,n,i}^2} \tag{5.7}$$

is the SINR of the signal intended for Bob m at Bob n.

As for the data rate to Bob n, based on (5.5), it can be expressed as

$$R_{m,n,i}^{[n]} = \log_2\left(1 + SINR_{m,n,i}^{[n]}\right). \tag{5.8}$$

While the information for Bob m and Bob n is transmitted, Eves attempt to wiretap. The received signal at Eve $k \in \mathcal{K}$ on subcarrier i can be expressed as

$$y_{e,k,i} = \mathbf{h}_{e,k,i}^H \mathbf{w}_{m,n,i}^{[m]} x_m + \mathbf{h}_{e,k,i}^H \mathbf{w}_{m,n,i}^{[n]} x_n + \mathbf{g}_{e,k,i}^H \mathbf{z}_i + n_{e,k,i} \tag{5.9}$$

where $\mathbf{h}_{e,k,i}$ denotes the channel vector from Alice to Eve k on subcarrier i, $\mathbf{g}_{e,k,i}$ denotes the channel vector from the Helper to Eve k on subcarrier i, and $n_{e,k,i} \sim \mathcal{CN}(0, \sigma_{e,k,i}^2)$ is the Gaussian noise at Eve k on subcarrier i. To guarantee communication security, we consider the worst-case eavesdropping scenario where Eves have strong interception capabilities. In particular, we assume Eves can cancel the MUI before decoding the information of the desired Bob on each subcarrier [92–94]. Thus, the capacities of Eve k in eavesdropping Bob m and Bob n on subcarrier i can be given by

$$C_{m,n,k,i}^{[m]} = \log_2\left(1 + \frac{\left|\mathbf{h}_{e,k,i}^H \mathbf{w}_{m,n,i}^{[m]}\right|^2}{\mathbf{g}_{e,k,i}^H \mathbf{Q}_i \mathbf{g}_{e,k,i} + \sigma_{e,k,i}^2}\right) \tag{5.10}$$

and

$$C_{m,n,k,i}^{[n]} = \log_2\left(1 + \frac{\left|\mathbf{h}_{e,k,i}^H \mathbf{w}_{m,n,i}^{[n]}\right|^2}{\mathbf{g}_{e,k,i}^H \mathbf{Q}_i \mathbf{g}_{e,k,i} + \sigma_{e,k,i}^2}\right) \tag{5.11}$$

respectively.

We let $s_{m,n,i} \in \{0,1\}$ indicate the subcarrier allocation policy. Specifically, $s_{m,n,i} = 1$ if Bob m, which is weak, and Bob n, which is strong, are scheduled on subcarrier i and $s_{m,n,i} = 0$ if another scheduling policy is executed. Then, based on (5.6), (5.8),(5.10) and (5.11), the secrecy rate at Bob m can be expressed as

$$\begin{aligned} R_m = \max\Bigg\{0,\ \sum_{i\in\mathcal{I}}\Bigg(\sum_{l\in\mathcal{M},l\neq m} s_{m,l,i}\left(R^{[m]}_{m,l,i} - \max_{k\in\mathcal{K}} C^{[m]}_{m,l,k,i}\right) \\ + \sum_{l\in\mathcal{M},l\neq m} s_{l,m,i}\left(R^{[m]}_{l,m,i} - \max_{k\in\mathcal{K}} C^{[m]}_{l,m,k,i}\right)\Bigg)\Bigg\}. \end{aligned} \tag{5.12}$$

Secure transmission design for improving the secrecy rate at each Bob relies on the CSI. Since Alice and the Helper can always learn Bobs' CSI through Bobs' feedback, we assume Alice and the Helper have full CSI of Bob's channels. For Eves, we assume they are not part of the legitimate system and only the channel distributions of Eves' channels, i.e., $\mathbf{h}_{e,k,i} \sim \mathcal{CN}(\mathbf{0}, \bar{h}_{e,k,i}\mathbf{I})$ and $\mathbf{g}_{e,k,i} \sim \mathcal{CN}(\mathbf{0}, \bar{g}_{e,k,i}\mathbf{I})$, are known at Alice and the Helper [95], where $\bar{h}_{e,k,i}$ and $\bar{g}_{e,k,i}$ are deterministic quantities which usually depend on the distances from transmitters to the k-th Eve.

5.1.2 Problem Formulation

The goal of our secure transmission design is to optimize jointly the single stream beamforming vectors, jamming covariance matrices and the subcarrier allocation policy to improve the secure transmission performance of the considered MISO NOMA system. Since only statistical CSI is available for Eves' channels, secrecy outage probability is considered in our secure transmission design. For fairness, in our design, we maximize the minimum probabilistic secrecy rate at Bobs. By letting P_s and P_c denote the power constraints for Alice and the Helper, the minimum probabilistic secrecy rate

maximization problem can be formulated as

$$\max_{\mathbf{w}_{m,n,i}^{[m]},\mathbf{w}_{m,n,i}^{[n]},\mathbf{Q}_i,r_m,s_{m,n,i},\forall m\in\mathcal{M},\forall n\in\mathcal{M},n\neq m,\forall i\in\mathcal{I}} \quad \min_{m\in\mathcal{M}} r_m \tag{5.13a}$$

$$\text{s.t.}\ \text{Prob}\left(R_m \leq r_m\right) \leq \varepsilon_m,\ \forall m \in \mathcal{M}, \tag{5.13b}$$

$$r_m \geq 0, \forall m \in \mathcal{M}, \tag{5.13c}$$

$$\sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{M},n\neq m} \sum_{i\in\mathcal{I}} s_{m,n,i} \left(\left\| \mathbf{w}_{m,n,i}^{[m]} \right\|^2 + \left\| \mathbf{w}_{m,n,i}^{[n]} \right\|^2 \right) \leq P_s, \tag{5.13d}$$

$$\sum_{i\in\mathcal{I}} \text{tr}(\mathbf{Q}_i) \leq P_c, \tag{5.13e}$$

$$\mathbf{Q}_i \succeq \mathbf{0},\ \forall i \in \mathcal{I}, \tag{5.13f}$$

$$\sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{M},n\neq m} s_{m,n,i} \leq 1,\ \forall i \in \mathcal{I}, \tag{5.13g}$$

$$\sum_{i\in\mathcal{I}} \sum_{l\in\mathcal{M},l\neq m} (s_{m,l,i} + s_{l,m,i}) \leq 1,\ \forall m \in \mathcal{M}, \tag{5.13h}$$

$$s_{m,n,i} = \{0,1\},\ \forall n \in \mathcal{M}, n \neq m, \forall i \in \mathcal{I}, \forall m \in \mathcal{M} \tag{5.13i}$$

where r_m is the probabilistic secrecy rate at which information can be transmitted to Bob m with secrecy outage probability no greater than the predefined secrecy outage tolerance ε_m. Constraint (5.13b) is to constrain the secrecy outage probability; constraint (5.13c) demonstrates that the probabilistic secrecy rate at each Bob should be no less than zero; constraint (5.13d) and (5.13e) limit the transmit power of Alice and the transmit power of the Helper respectively; constraint (5.13f) ensures that the jamming covariance matrix on subcarrier i is Hermitian positive semidefinite; constraints (5.13g) and (5.13i) are imposed since each subcarrier can be assigned to at most one pair of Bobs; while constraints (5.13h) and (5.13i) ensure that each Bob can be scheduled on at most one subcarrier.

5.2 Outage Constrained Secure Transmission Design

Because of constraints (5.13g)-(5.13i), the design Problem (5.13) is mixed-integer programming and mathematically intractable. Moreover, constraint (5.13b), which is not in a closed form, is not convex. Therefore, even through exhaustive search, which cannot finish within polynomial time, it is still difficult to find an optimal solution to (5.13). In this section, we aim at solving problem (5.13) suboptimally.

Because of (5.13c), constraint (5.13b) can be equivalently transformed into

$$\begin{aligned}\mathrm{Prob}\Bigg(&\sum_{i\in\mathcal{I}}\sum_{l\in\mathcal{M},l\neq m}\Big(s_{m,l,i}R^{[m]}_{m,l,i}+s_{l,m,i}R^{[m]}_{l,m,i}\Big)-\\&\sum_{i\in\mathcal{I}}\sum_{l\in\mathcal{M},l\neq m}\Big(s_{m,l,i}\max_{k\in\mathcal{K}}C^{[m]}_{m,l,k,i}+s_{l,m,i}\max_{k\in\mathcal{K}}C^{[m]}_{l,m,k,i}\Big)\leq r_m\Bigg)\leq\varepsilon_m,\\&\forall m\in\mathcal{M}.\end{aligned}\tag{5.14}$$

Based on (5.14), by introducing slack variables $\Gamma_{e,m}$ and r, we rewrite the original design problem (5.13) as

$$\max_{\mathbf{w}^{[m]}_{m,n,i},\mathbf{w}^{[n]}_{m,n,i},\mathbf{Q}_i,r_m,\Gamma_{e,m},s_{m,n,i},r,\forall m\in\mathcal{M},\forall n\in\mathcal{M},n\neq m,\forall i\in\mathcal{I}}\quad r\tag{5.15a}$$

$$\begin{aligned}\text{s.t. }\mathrm{Prob}\Bigg(&\sum_{i\in\mathcal{I}}\sum_{l\in\mathcal{M},l\neq m}\Big(s_{m,l,i}\max_{k\in\mathcal{K}}C^{[m]}_{m,l,k,i}+s_{l,m,i}\max_{k\in\mathcal{K}}C^{[m]}_{l,m,k,i}\Big)\\&\geq\log_2(1+\Gamma_{e,m})\Bigg)\leq\varepsilon_m,\forall m\in\mathcal{M},\end{aligned}\tag{5.15b}$$

$$\sum_{i\in\mathcal{I}}\sum_{l\in\mathcal{M},l\neq m}\Big(s_{m,l,i}R^{[m]}_{m,l,i}+s_{l,m,i}R^{[m]}_{l,m,i}\Big)-\log_2(1+\Gamma_{e,m})\geq r_m,\forall m\in\mathcal{M},\tag{5.15c}$$

$$\Gamma_{e,m} \geq 0, \forall m \in \mathcal{M}, \tag{5.15d}$$

$$r_m \geq r, \forall m \in \mathcal{M}, \tag{5.15e}$$

$$(5.13c) - (5.13i).$$

The probabilistic constraint (5.15b) is still complicated. Nevertheless, due to constraints (5.13h) and (5.13i), we know that there is at most one active term in the summation

$$\sum_{i\in\mathcal{I}} \sum_{l\in\mathcal{M}, l\neq m} \left(s_{m,l,i} \max_{k\in\mathcal{K}} C^{[m]}_{m,l,k,i} + s_{l,m,i} \max_{k\in\mathcal{K}} C^{[m]}_{l,m,k,i} \right)$$

in constraint (5.15b). Therefore, constraint (5.15b) can be replaced by less complicated constraints and problem (5.15) can be equivalently rewritten as

$$\max_{\mathbf{w}^{[m]}_{m,n,i}, \mathbf{w}^{[n]}_{m,n,i}, \mathbf{Q}_i, r_m, \Gamma_{e,m}, s_{m,n,i}, r, \forall m\in\mathcal{M}, \forall n\in\mathcal{M}, n\neq m, \forall i\in\mathcal{I}} \quad r \tag{5.16a}$$

$$\text{s.t. } s_{m,l,i}\text{Prob}\left(\max_{k\in\mathcal{K}} C^{[m]}_{m,l,k,i} \geq \log_2(1+\Gamma_{e,m}) \right) \leq \varepsilon_m, \quad \forall l \in \mathcal{M}, l \neq m, \forall i \in \mathcal{I}, \forall m \in \mathcal{M}, \tag{5.16b}$$

$$s_{l,m,i}\text{Prob}\left(\max_{k\in\mathcal{K}} C^{[m]}_{l,m,k,i} \geq \log_2(1+\Gamma_{e,m}) \right) \leq \varepsilon_m, \quad \forall l \in \mathcal{M}, l \neq m, \forall i \in \mathcal{I}, \forall m \in \mathcal{M}, \tag{5.16c}$$

$$(5.15c) - (5.15e), (5.13c) - (5.13i).$$

However, problem (5.16) is still intractable, because the integer variable $s_{m,n,i}$ exists in all the constraints related to the continuous variables $\mathbf{w}^{[m]}_{m,n,i}$, $\mathbf{w}^{[n]}_{m,n,i}$ and the probabilistic constraints (5.16b) and (5.16c) are not in closed form. To resolve these issues, we first rewrite problem (5.16) as the following equivalent 0-norm optimization problem

$$\max_{\mathbf{w}^{[m]}_{m,n,i}, \mathbf{w}^{[n]}_{m,n,i}, \mathbf{Q}_i, r_m, \Gamma_{e,m}, p_{m,n,i}, r, \forall m\in\mathcal{M}, \forall n\in\mathcal{M}, n\neq m, \forall i\in\mathcal{I}} \quad r \tag{5.17a}$$

$$\text{s.t. Prob}\left(\max_{k\in\mathcal{K}} C_{m,l,k,i}^{[m]} \geq \log_2(1+\Gamma_{e,m})\right) \leq \varepsilon_m,$$
$$\forall l \in \mathcal{M}, l \neq m, \forall i \in \mathcal{I}, \forall m \in \mathcal{M}, \quad (5.17b)$$

$$\text{Prob}\left(\max_{k\in\mathcal{K}} C_{l,m,k,i}^{[m]} \geq \log_2(1+\Gamma_{e,m})\right) \leq \varepsilon_m,$$
$$\forall l \in \mathcal{M}, l \neq m, \forall i \in \mathcal{I}, \forall m \in \mathcal{M}, \quad (5.17c)$$

$$\sum_{i\in\mathcal{I}} \sum_{l\in\mathcal{M},l\neq m} \left(R_{m,l,i}^{[m]} + R_{l,m,i}^{[m]}\right) - \log_2(1+\Gamma_{e,m}) \geq r_m, \forall m \in \mathcal{M}, \quad (5.17d)$$

$$\Gamma_{e,m} \geq 0, \forall m \in \mathcal{M}, \quad (5.17e)$$

$$r_m \geq r, \forall m \in \mathcal{M}, \quad (5.17f)$$

$$r_m \geq 0, \forall m \in \mathcal{M}, \quad (5.17g)$$

$$\sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{M},n\neq m} \sum_{i\in\mathcal{I}} p_{m,n,i} \leq P_s, \quad (5.17h)$$

$$\left\|\mathbf{w}_{m,n,i}^{[m]}\right\|^2 + \left\|\mathbf{w}_{m,n,i}^{[n]}\right\|^2 \leq p_{m,n,i}, \forall l \in \mathcal{M}, l \neq m, \forall i \in \mathcal{I}, \forall m \in \mathcal{M}, \quad (5.17i)$$

$$\sum_{i\in\mathcal{I}} \text{tr}(\mathbf{Q}_i) \leq P_c, \quad (5.17j)$$

$$\mathbf{Q}_i \succeq \mathbf{0}, \forall i \in \mathcal{I}, \quad (5.17k)$$

$$\|\mathbf{p}_{c,i}\|_0 \leq 1 \, \forall i \in \mathcal{I}, \quad (5.17l)$$

$$\|\mathbf{p}_{b,m}\|_0 \leq 1, \forall m \in \mathcal{M}, \quad (5.17m)$$

where $p_{m,n,i}$ is the power control slack variable for the Bob pair $\{m,n\}$ on subcarrier i, $\mathbf{p}_{c,i} = [p_{1,2,i}, ..., p_{1,M,i}, p_{2,1,i}, ..., p_{M-1,M,i}, p_{M,1,i}, ..., p_{M,M-1,i}]^T$ and $\mathbf{p}_{b,m} = [\sum_{n\in\mathcal{M},n\neq m} (p_{m,n,1} + p_{n,m,1}), ..., \sum_{n\in\mathcal{M},n\neq m} (p_{m,n,I} + p_{n,m,I})]^T$. In problem (5.17), constraints (5.17i) and (5.17l) demonstrate that each subcarrier can be assigned to at most one pair of Bobs, and (5.17i) and (5.17m) guarantee that each Bob can be served on at most one subcarrier. We note that constraint (5.17g) can be neglected in problem (5.17), since the optimum to (5.17) will not change without (5.17g). In addition, for constraint

(5.17b), we have

$$\text{Prob}\left(\max_{k\in\mathcal{K}} C_{m,l,k,i}^{[m]} \geq \log_2(1+\Gamma_{e,m})\right) \leq \varepsilon_m \tag{5.18a}$$

$$\Leftrightarrow$$

$$\text{Prob}\left(\max_{k\in\mathcal{K}} C_{m,l,k,i}^{[m]} \leq \log_2(1+\Gamma_{e,m})\right) \geq 1-\varepsilon_m \tag{5.18b}$$

$$\Leftrightarrow$$

$$\text{Prob}\left(C_{m,l,1,i}^{[m]} \leq \log_2(1+\Gamma_{e,m}), ..., C_{m,l,K,i}^{[m]} \leq \log_2(1+\Gamma_{e,m})\right) \geq 1-\varepsilon_m \tag{5.18c}$$

$$\Leftrightarrow$$

$$\prod_{k\in\mathcal{K}} \text{Prob}\left(\frac{\left|\mathbf{h}_{e,k,i}^H \mathbf{w}_{m,l,i}^{[m]}\right|^2}{\mathbf{g}_{e,k,i}^H \mathbf{Q}_i \mathbf{g}_{e,k,i} + \sigma_{e,k,i}^2} \leq \Gamma_{e,m}\right) \geq 1-\varepsilon_m \tag{5.18d}$$

where the equivalence between (5.18c) and (5.18d) holds because channels to Eves are independent to each other. Similarly, for constraint (5.17c), we have

$$\begin{aligned}
&\text{Prob}\left(\max_{k\in\mathcal{K}} C_{l,m,k,i}^{[m]} \geq \log_2(1+\Gamma_{e,m})\right) \leq \varepsilon_m \\
&\Leftrightarrow \\
&\prod_{k\in\mathcal{K}} \text{Prob}\left(\frac{\left|\mathbf{h}_{e,k,i}^H \mathbf{w}_{l,m,i}^{[m]}\right|^2}{\mathbf{g}_{e,k,i}^H \mathbf{Q}_i \mathbf{g}_{e,k,i} + \sigma_{e,k,i}^2} \leq \Gamma_{e,m}\right) \geq 1-\varepsilon_m.
\end{aligned} \tag{5.19}$$

Based on (5.18) and (5.19), by introducing slack variables $\varepsilon_{m,l,k,i}^{[m]}$ and $\varepsilon_{l,m,k,i}^{[m]}$, we can equivalently transform problem (5.17) into

$$\max_{\substack{\mathbf{w}_{m,n,i}^{[m]},\mathbf{w}_{m,n,i}^{[n]},\mathbf{Q}_i,r_m,\Gamma_{e,m},p_{m,n,i},r,\varepsilon_{m,n,k,i}^{[m]},\varepsilon_{n,m,k,i}^{[m]} \\ \forall k\in\mathcal{K},\forall m\in\mathcal{M},\forall n\in\mathcal{M},n\neq m,\forall i\in\mathcal{I}}} \quad r \tag{5.20a}$$

$$\text{s.t. Prob}\left(\frac{\left|\mathbf{h}_{e,k,i}^H\mathbf{w}_{m,l,i}^{[m]}\right|^2}{\mathbf{g}_{e,k,i}^H\mathbf{Q}_i\mathbf{g}_{e,k,i}+\sigma_{e,k,i}^2}\leq\Gamma_{e,m}\right)\geq 1-\varepsilon_{m,l,k,i}^{[m]},$$
$$\forall k\in\mathcal{K},\forall l\in\mathcal{M},l\neq m,\forall i\in\mathcal{I},\forall m\in\mathcal{M},\quad(5.20b)$$
$$\text{Prob}\left(\frac{\left|\mathbf{h}_{e,k,i}^H\mathbf{w}_{l,m,i}^{[m]}\right|^2}{\mathbf{g}_{e,k,i}^H\mathbf{Q}_i\mathbf{g}_{e,k,i}+\sigma_{e,k,i}^2}\leq\Gamma_{e,m}\right)\geq 1-\varepsilon_{l,m,k,i}^{[m]},$$
$$\forall k\in\mathcal{K},\forall l\in\mathcal{M},l\neq m,\forall i\in\mathcal{I},\forall m\in\mathcal{M},\quad(5.20c)$$
$$\prod_k\left(1-\varepsilon_{m,l,k,i}^{[m]}\right)\geq 1-\varepsilon_m,\ \ \forall l\in\mathcal{M},l\neq m,\forall i\in\mathcal{I},\forall m\in\mathcal{M},\quad(5.20d)$$
$$\prod_k\left(1-\varepsilon_{l,m,k,i}^{[m]}\right)\geq 1-\varepsilon_m,\ \ \forall l\in\mathcal{M},l\neq m,\forall i\in\mathcal{I},\forall m\in\mathcal{M},\quad(5.20e)$$
$$0\leq\varepsilon_{m,l,k,i}^{[m]}\leq 1,\ \forall k\in\mathcal{K},\forall l\in\mathcal{M},l\neq m,\forall i\in\mathcal{I},\forall m\in\mathcal{M},\quad(5.20f)$$
$$0\leq\varepsilon_{l,m,k,i}^{[m]}\leq 1,\ \forall k\in\mathcal{K},\forall l\in\mathcal{M},l\neq m,\forall i\in\mathcal{I},\forall m\in\mathcal{M},\quad(5.20g)$$
$$(5.17d)-(5.17f),(5.17h)-(5.17m).$$

Constraints (5.20b) and (5.20c) are not in closed form. Fortunately, for constraint (5.20b), when $\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2\neq 0$, we have

$$\text{Prob}\left(\frac{\left|\mathbf{h}_{e,k,i}^H\mathbf{w}_{m,l,i}^{[m]}\right|^2}{\mathbf{g}_{e,k,i}^H\mathbf{Q}_i\mathbf{g}_{e,k,i}+\sigma_{e,k,i}^2}\leq\Gamma_{e,m}\right)\geq 1-\varepsilon_{m,l,k,i}^{[m]}$$
$$\Leftrightarrow\quad(5.21)$$
$$\ln\left(\varepsilon_{m,l,k,i}^{[m]}\right)+\frac{\Gamma_{e,m}\sigma_{e,k,i}^2}{\bar{h}_{e,k,i}\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2}+\sum_{j\in\mathcal{J}}\ln\left(\frac{\Gamma_{e,m}\bar{g}_{e,k,i}\lambda_{i,j}}{\bar{h}_{e,k,i}\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2}+1\right)\geq 0$$

where $\lambda_{i,j}$ is the j-th smallest eigenvalue of the matrix $\mathbf{Q}_i$ with $j\in\mathcal{J}=\{1,...,N_h\}$. Details of the derivation of (5.21) can be found in Appendix I. By considering the possibility that $\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2=0$, we replace constraint

(5.20b) by

$$\ln\left(\varepsilon_{m,l,k,i}^{[m]}\right)+\frac{\Gamma_{e,m}\sigma_{e,k,i}^{2}}{\bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^{2}+\xi_{k,i}\right)}+\sum_{j\in\mathcal{J}}\ln\left(\frac{\Gamma_{e,m}\bar{g}_{e,k,i}\lambda_{i,j}}{\bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^{2}+\xi_{k,i}\right)}+1\right)\geq 0,\quad \forall k\in\mathcal{K},\forall l\in\mathcal{M},l\neq m,\forall i\in\mathcal{I},\forall m\in\mathcal{M} \tag{5.22}$$

where $\xi_{k,i}$ is a predefined variable with $0<\xi_{k,i}\ll\sigma_{e,k,i}^{2}$. When $\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^{2}=0$, owing to $\xi_{k,i}\ll\sigma_{e,k,i}^{2}$, constraint (5.22) allows $\varepsilon_{m,l,k,i}^{[m]}$ to achieve a small value which is close to 0. Therefore, constraint (5.20d) can always be satisfied. On the other hand, when $\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^{2}\neq 0$, since $0<\xi_{k,i}$, we have

$$\begin{aligned}&\ln\left(\varepsilon_{m,l,k,i}^{[m]}\right)+\frac{\Gamma_{e,m}\sigma_{e,k,i}^{2}}{\bar{h}_{e,k,i}\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^{2}}+\sum_{j\in\mathcal{J}}\ln\left(\frac{\Gamma_{e,m}\bar{g}_{e,k,i}\lambda_{i,j}}{\bar{h}_{e,k,i}\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^{2}}+1\right)\geq\\&\ln\left(\varepsilon_{m,l,k,i}^{[m]}\right)+\frac{\Gamma_{e,m}\sigma_{e,k,i}^{2}}{\bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^{2}+\xi_{k,i}\right)}\\&+\sum_{j\in\mathcal{J}}\ln\left(\frac{\Gamma_{e,m}\bar{g}_{e,k,i}\lambda_{i,j}}{\bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^{2}+\xi_{k,i}\right)}+1\right).\end{aligned} \tag{5.23}$$

Through (5.21) and(5.23), we know the point that satisfies constraint (5.22) must satisfy constraint (5.20b). Consequently, constraint (5.22) is a safe approximation for constraint (5.20b). Similarly, constraint (5.20c) can also be replaced. By rewritting constraints (5.20d) and (5.20e), we obtain the

following optimization problem

$$\max_{\substack{\mathbf{w}_{m,n,i}^{[m]},\mathbf{w}_{m,n,i}^{[n]},\mathbf{Q}_i,r_m,\Gamma_{e,m},p_{m,n,i},r,\varepsilon_{m,n,k,i}^{[m]},\varepsilon_{n,m,k,i}^{[m]},\lambda_{i,j}\\ \forall j\in\mathcal{J},\forall k\in\mathcal{K},\forall m\in\mathcal{M},\forall n\in\mathcal{M},n\neq m,\forall i\in\mathcal{I}}} \quad r \tag{5.24a}$$

$$\text{s.t. } \ln\left(\varepsilon_{l,m,k,i}^{[m]}\right)+\frac{\Gamma_{e,m}\sigma_{e,k,i}^2}{\bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{l,m,i}^{[m]}\right\|^2+\xi_{k,i}\right)} + \sum_{j\in\mathcal{J}}\ln\left(\frac{\Gamma_{e,m}\bar{g}_{e,k,i}\lambda_{i,j}}{\bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{l,m,i}^{[m]}\right\|^2+\xi_{k,i}\right)}+1\right)\geq 0,$$

$$\forall k\in\mathcal{K},\forall l\in\mathcal{M},l\neq m,\forall i\in\mathcal{I},\forall m\in\mathcal{M}, \tag{5.24b}$$

$$\sum_{k\in\mathcal{K}}\ln\left(1-\varepsilon_{m,l,k,i}^{[m]}\right)\geq\ln(1-\varepsilon_m),\ \ \forall l\in\mathcal{M},l\neq m,\forall i\in\mathcal{I},\forall m\in\mathcal{M}, \tag{5.24c}$$

$$\sum_{k\in\mathcal{K}}\ln\left(1-\varepsilon_{l,m,k,i}^{[m]}\right)\geq\ln(1-\varepsilon_m),\ \ \forall l\in\mathcal{M},l\neq m,\forall i\in\mathcal{I},\forall m\in\mathcal{M}, \tag{5.24d}$$

$$\text{eig}_j\left(\mathbf{Q}_i\right)=\lambda_{i,j},\forall j\in\mathcal{J},\forall i\in\mathcal{I}, \tag{5.24e}$$

$$\lambda_{i,j}\geq 0,\forall j\in\mathcal{J},\forall i\in\mathcal{I}, \tag{5.24f}$$

$$(5.22),(5.20\text{f}),(5.20\text{g}),(5.17\text{d})-(5.17\text{f}),(5.17\text{h})-(5.17\text{m})$$

where function $\text{eig}_j(\cdot)$ returns the j-th smallest eigenvalue of a matrix and constraints (5.24e) and (5.24f) are imposed since $\lambda_{i,j}$ is the j-th smallest eigenvalue of the matrix $\mathbf{Q}_i$. Since constraints (5.22) and (5.24b) are safe approximations for constraints (5.20b) and (5.20c), a suboptimal solution to problem (5.20) can be found by solving (5.24).

In problem (5.24), all the constraints are in closed form. However, problem (5.24) is non-convex, since there exist non-convex constraints. In order to find a solution to problem (5.24), we first represent the 0-norm constraint (5.17l) in the equivalent form

$$\|\mathbf{p}_{c,i}\|_1-\|\mathbf{p}_{c,i}\|_\infty\leq 0,\forall i\in\mathcal{I}. \tag{5.25}$$

Similarly, the 0-norm constraint (5.17m) can be expressed in the following form

$$\|\mathbf{p}_{b,m}\|_1 - \|\mathbf{p}_{b,m}\|_\infty \leq 0, \forall m \in \mathcal{M}. \tag{5.26}$$

Based on (5.25) and (5.26), by applying $\text{eig}_j(\mathbf{Q}_i) = \text{Seig}_j(\mathbf{Q}_i) - \text{Seig}_{j-1}(\mathbf{Q}_i)$ and relaxing constraint (5.24e), we obtain the following optimization problem

$$\max_{\substack{\mathbf{w}_{m,n,i}^{[m]}, \mathbf{w}_{m,n,i}^{[n]}, \mathbf{Q}_i, r_m, \Gamma_{e,m}, p_{m,n,i}, r, \varepsilon_{m,n,k,i}^{[m]}, \varepsilon_{n,m,k,i}^{[m]}, \lambda_{i,j} \\ \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall n \in \mathcal{M}, n \neq m, \forall i \in \mathcal{I}}} \quad r \tag{5.27a}$$

$$\text{s.t.} \text{Seig}_1(\mathbf{Q}_i) \geq \lambda_{i,1}, \forall i \in \mathcal{I}, \tag{5.27b}$$

$$\text{Seig}_j(\mathbf{Q}_i) - \text{Seig}_{j-1}(\mathbf{Q}_i) \geq \lambda_{i,j}, \forall j \in \mathcal{J}, j > 1, \forall i \in \mathcal{I}, \tag{5.27c}$$

$$(5.25), (5.26), (5.24\text{b}) - (5.24\text{d}), (5.24\text{f}), (5.22), (5.20\text{f}), (5.20\text{g}),$$
$$(5.17\text{d}) - (5.17\text{f}), (5.17\text{h}) - (5.17\text{k})$$

where function $\text{Seig}_j(\cdot)$ returns the sum of j smallest eigenvalues of a matrix. Suppose $\{\bar{\mathbf{w}}_{m,n,i}^{[m]}, \bar{\mathbf{w}}_{m,n,i}^{[n]}, \bar{\mathbf{Q}}_i, \bar{r}_m, \bar{\Gamma}_{e,m}, \bar{p}_{m,n,i}, \bar{r}, \bar{\varepsilon}_{m,n,k,i}^{[m]}, \bar{\varepsilon}_{n,m,k,i}^{[m]}, \bar{\lambda}_{i,j}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall n \in \mathcal{M}, n \neq m, \forall i \in \mathcal{I}\}$ is a solution to problem (5.27). By letting, $\dot{\lambda}_{i,j} = \text{eig}_j(\bar{\mathbf{Q}}_i)$, we can obtain a solution to problem (5.24), i.e., $\{\bar{\mathbf{w}}_{m,n,i}^{[m]}, \bar{\mathbf{w}}_{m,n,i}^{[n]}, \bar{\mathbf{Q}}_i, \bar{r}_m, \bar{\Gamma}_{e,m}, \bar{p}_{m,n,i}, \bar{r}, \bar{\varepsilon}_{m,n,k,i}^{[m]}, \bar{\varepsilon}_{n,m,k,i}^{[m]}, \dot{\lambda}_{i,j} \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall n \in \mathcal{M}, n \neq m, \forall i \in \mathcal{I}\}$. Then, by following [69], we reformulate problem (5.27) into the following penalty form

$$\begin{aligned} &\max_{\substack{\mathbf{w}_{m,n,i}^{[m]}, \mathbf{w}_{m,n,i}^{[n]}, \mathbf{Q}_i, r_m, \Gamma_{e,m}, p_{m,n,i}, r, \varepsilon_{m,n,k,i}^{[m]}, \varepsilon_{n,m,k,i}^{[m]}, \lambda_{i,j} \\ \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall n \in \mathcal{M}, n \neq m, \forall i \in \mathcal{I}}} \quad r - U \\ &\text{s.t.} (5.27\text{b}), (5.27\text{c}), (5.24\text{b}) - (5.24\text{d}), (5.24\text{f}), (5.22), (5.20\text{f}), (5.20\text{g}), \\ &\quad (5.17\text{d}) - (5.17\text{f}), (5.17\text{h}) - (5.17\text{k}) \end{aligned} \tag{5.28}$$

where $U = \rho(\sum_{i \in \mathcal{I}} (\|\mathbf{p}_{c,i}\|_1 - \|\mathbf{p}_{c,i}\|_\infty) + \sum_{m \in \mathcal{M}} (\|\mathbf{p}_{b,m}\|_1 - \|\mathbf{p}_{b,m}\|_\infty))$ and $\rho \gg 0$ acts as a penalty coefficient. According to [69, 96], problems (5.28) and (5.27) are equivalent for $\rho \gg 0$. Problem (5.28) is non-convex, but we can find a solution to it through the path-following approach where con-

vex approximation problems are iteratively solved [97]. In the following, we elaborate on a path-following iteration process for finding a solution to (5.28).

Let $\{\mathbf{w}_{m,n,i}^{[m](\kappa)}, \mathbf{w}_{m,n,i}^{[n](\kappa)}, \mathbf{Q}_i^{(\kappa)}, r_m^{(\kappa)}, \Gamma_{e,m}^{(\kappa)}, p_{m,n,i}^{(\kappa)}, r^{(\kappa)}, \varepsilon_{m,n,k,i}^{[m](\kappa)}, \varepsilon_{n,m,k,i}^{[m](\kappa)}, \lambda_{i,j}^{(\kappa)}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall n \in \mathcal{M}, n \neq m, \forall i \in \mathcal{I}\}$ denote an optimal solution obtained from the (κ)-th iteration of the path-following iteration process. Based on the optimal solution obtained from the (κ)-th iteration, we can build the convex approximation problem of (5.28) for the $(\kappa+1)$ -th iteration.

Since x^2/y is convex for $y > 0$, we have

$$\frac{x^2}{y} \geq \frac{2\bar{x}x}{\bar{y}} - \frac{\bar{x}^2 y}{\bar{y}^2}$$

at point $\{\bar{x}, \bar{y}\}$ with $y > 0$ and $\bar{y} > 0$. For constraint (5.22), by letting $x = \sigma_{e,k,i}\sqrt{\Gamma_{e,m}}$, $y = \bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2 + \xi_{k,i}\right)$, $\bar{x} = \sigma_{e,k,i}\sqrt{\Gamma_{e,m}^{(\kappa)}}$ and $\bar{y} = \bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{m,l,i}^{[m](\kappa)}\right\|^2 + \xi_{k,i}\right)$, we know

$$\begin{aligned}
\frac{\Gamma_{e,m}\sigma_{e,k,i}^2}{\bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2 + \xi_{k,i}\right)} &= f_{k,i}^{[1]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}) \\
&\geq \bar{f}_{k,i}^{[1]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}, \Gamma_{e,m}^{(\kappa)}, \mathbf{w}_{m,l,i}^{[m](\kappa)}) \\
&= \frac{2\sigma_{e,k,i}^2\sqrt{\Gamma_{e,m}^{(\kappa)}}\sqrt{\Gamma_{e,m}}}{\bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{m,l,i}^{[m](\kappa)}\right\|^2 + \xi_{k,i}\right)} \\
&\quad - \frac{\sigma_{e,k,i}^2\Gamma_{e,m}^{(\kappa)}\left(\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2 + \xi_{k,i}\right)}{\bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{m,l,i}^{[m](\kappa)}\right\|^2 + \xi_{k,i}\right)^2}.
\end{aligned} \tag{5.29}$$

In addition, from the discussion in [98], we have

$$\ln(1 + \frac{1}{xy}) \geq \ln(1 + \frac{1}{\bar{x}\bar{y}}) + \frac{1}{1 + \bar{x}\bar{y}}(2 - \frac{x}{\bar{x}} - \frac{y}{\bar{y}}) \tag{5.30}$$

at $\{\bar{x}, \bar{y}\}$ for $x > 0$, $y > 0$, $\bar{x} > 0$ and $\bar{y} > 0$. In order to utilize (5.30), we replace constraint (5.24f) by

$$\lambda_{i,j} \geq \ddot{\lambda}_{i,j}, \forall j \in \mathcal{J}, \forall i \in \mathcal{I} \tag{5.31}$$

where $\ddot{\lambda}_{i,j}$ is a predefined positive variable close to 0. Note that the point which satisfies (5.31) must satisfy (5.24f). Then, for constraint (5.22), by letting $x = \frac{\left(\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2 + \xi_{k,i}\right)}{\Gamma_{e,m}}, y = \frac{\bar{h}_{e,k,i}}{\bar{g}_{e,k,i}\lambda_{i,j}}$, $\bar{x} = \frac{\left(\left\|\mathbf{w}_{m,l,i}^{[m](\kappa)}\right\|^2 + \xi_{k,i}\right)}{\Gamma_{e,m}^{(\kappa)}}, \bar{y} = \frac{\bar{h}_{e,k,i}}{\bar{g}_{e,k,i}\lambda_{i,j}^{(\kappa)}}$, we can obtain

$$\begin{aligned}
\ln\left(\frac{\Gamma_{e,m}\bar{g}_{e,k,i}\lambda_{i,j}}{\bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2 + \xi_{k,i}\right)} + 1\right) &= f_{k,i}^{[2]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}, \lambda_{i,j}) \\
&\geq \bar{f}_{k,i}^{[2]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}, \lambda_{i,j}, \Gamma_{e,m}^{(\kappa)}, \mathbf{w}_{m,l,i}^{[m](\kappa)}, \lambda_{i,j}^{(\kappa)}) \\
&= \ln\left(\frac{\Gamma_{e,m}^{(\kappa)}\bar{g}_{e,k,i}\lambda_{i,j}^{(\kappa)}}{\bar{h}_{e,k,i}\left(\left\|\mathbf{w}_{m,l,i}^{[m](\kappa)}\right\|^2 + \xi_{k,i}\right)} + 1\right) \\
&\quad + \frac{\Gamma_{e,m}^{(\kappa)}\bar{g}_{e,k,i}\lambda_{i,j}^{(\kappa)}}{\Gamma_{e,m}^{(\kappa)}\bar{g}_{e,k,i}\lambda_{i,j}^{(\kappa)} + \left(\left\|\mathbf{w}_{m,l,i}^{[m](\kappa)}\right\|^2 + \xi_{k,i}\right)\bar{h}_{e,k,i}} \\
&\quad \times \left(2 - \frac{\left(\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2 + \xi_{k,i}\right)}{\Gamma_{e,m}} \frac{\Gamma_{e,m}^{(\kappa)}}{\left(\left\|\mathbf{w}_{m,l,i}^{[m](\kappa)}\right\|^2 + \xi_{k,i}\right)} - \frac{\lambda_{i,j}^{(\kappa)}}{\lambda_{i,j}}\right).
\end{aligned} \tag{5.32}$$

Since $f_{k,i}^{[1]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}) \geq \bar{f}_{k,i}^{[1]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}, \Gamma_{e,m}^{(\kappa)}, \mathbf{w}_{m,l,i}^{[m](\kappa)})$, $f_{k,i}^{[2]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}, \lambda_{i,j}) \geq \bar{f}_{k,i}^{[2]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}, \lambda_{i,j}, \Gamma_{e,m}^{(\kappa)}, \mathbf{w}_{m,l,i}^{[m](\kappa)}, \lambda_{i,j}^{(\kappa)})$ and $\bar{f}_{k,i}^{[1]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}, \Gamma_{e,m}^{(\kappa)}, \mathbf{w}_{m,l,i}^{[m](\kappa)})$ and $\bar{f}_{k,i}^{[2]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}, \lambda_{i,j}, \Gamma_{e,m}^{(\kappa)}, \mathbf{w}_{m,l,i}^{[m](\kappa)}, \lambda_{i,j}^{(\kappa)})$ are concave functions, a safe

and convex approximation of constraint (5.22) can be given by

$$\begin{aligned}\ln\left(\varepsilon_{m,l,k,i}^{[m]}\right) &+ \bar{f}_{k,i}^{[1]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}, \Gamma_{e,m}^{(\kappa)}, \mathbf{w}_{m,l,i}^{[m](\kappa)}) \\ &+ \sum_{j\in\mathcal{J}} \bar{f}_{k,i}^{[2]}(\Gamma_{e,m}, \mathbf{w}_{m,l,i}^{[m]}, \lambda_{i,j}, \Gamma_{e,m}^{(\kappa)}, \mathbf{w}_{m,l,i}^{[m](\kappa)}, \lambda_{i,j}^{(\kappa)}) \geq 0, \\ &\forall k \in \mathcal{K}, \forall l \in \mathcal{M}, l \neq m, \forall i \in \mathcal{I}, \forall m \in \mathcal{M}.\end{aligned} \tag{5.33}$$

Similarly, constraint (5.24b) can be approximated by the following convex constraint:

$$\begin{aligned}\ln\left(\varepsilon_{l,m,k,i}^{[m]}\right) &+ \bar{f}_{k,i}^{[1]}(\Gamma_{e,m}, \mathbf{w}_{l,m,i}^{[m]}, \Gamma_{e,m}^{(\kappa)}, \mathbf{w}_{l,m,i}^{[m](\kappa)}) \\ &+ \sum_{j\in\mathcal{J}} \bar{f}_{k,i}^{[2]}(\Gamma_{e,m}, \mathbf{w}_{l,m,i}^{[m]}, \lambda_{i,j}, \Gamma_{e,m}^{(\kappa)}, \mathbf{w}_{l,m,i}^{[m](\kappa)}, \lambda_{i,j}^{(\kappa)}) \geq 0, \\ &\forall k \in \mathcal{K}, \forall l \in \mathcal{M}, l \neq m, \forall i \in \mathcal{I}, \forall m \in \mathcal{M}.\end{aligned} \tag{5.34}$$

According to [97], we have

$$\begin{aligned}\log_2\left(1 + \frac{|x|^2}{y}\right) &\geq \log_2(1 + \frac{|\bar{x}|^2}{\bar{y}}) + 2\frac{\Re(\bar{x}^H(x-\bar{x}))}{\ln(2)\bar{y}} \\ &- \frac{1}{\ln(2)}\left(\frac{1}{\bar{y}} - \frac{1}{\bar{y}+|\bar{x}|^2}\right)(y + |x|^2 - \bar{y} - |\bar{x}|^2)\end{aligned}$$

for $x \in \mathbb{C}$, $y > 0$, $\bar{x} \in \mathbb{C}$, $\bar{y} > 0$. By letting $x = \mathbf{h}_{b,m,i}^H \mathbf{w}_{m,l,i}^{[m]}$, $\bar{x} = \mathbf{h}_{b,m,i}^H \mathbf{w}_{m,l,i}^{[m](\kappa)}$, $y = \underline{R}_{m,l,i}^{[m]} = \left|\mathbf{h}_{b,m,i}^H \mathbf{w}_{m,l,i}^{[l]}\right|^2 + \mathbf{g}_{b,m,i}^H \mathbf{Q}_i \mathbf{g}_{b,m,i} + \sigma_{b,m,i}^2$ and $\bar{y} = \underline{R}_{m,l,i}^{[m](\kappa)} = \left|\mathbf{h}_{b,m,i}^H \mathbf{w}_{m,l,i}^{[l](\kappa)}\right|^2 + \mathbf{g}_{b,m,i}^H \mathbf{Q}_i^{(\kappa)} \mathbf{g}_{b,m,i} + \sigma_{b,m,i}^2$, for constraint (5.17d), we

have

$$\begin{aligned}\log_2\left(1+SINR_{m,l,i}^{[m]}\right) &\geq \bar{f}_{m,i}^{[3]}\left(\mathbf{w}_{m,l,i}^{[m]},\mathbf{w}_{m,l,i}^{[l]},\mathbf{Q}_i,\mathbf{w}_{m,l,i}^{[m](\kappa)},\mathbf{w}_{m,l,i}^{[l](\kappa)},\mathbf{Q}_i^{(\kappa)}\right)\\ &= \log_2\left(1+\frac{\left|\mathbf{h}_{b,m,i}^H\mathbf{w}_{m,l,i}^{[m](\kappa)}\right|^2}{\underline{R}_{m,l,i}^{[m](\kappa)}}\right)\\ &+2\frac{\Re\left(\left(\mathbf{w}_{m,l,i}^{[m](\kappa)}\right)^H\mathbf{h}_{b,m,i}\mathbf{h}_{b,m,i}^H\left(\mathbf{w}_{m,l,i}^{[m]}-\mathbf{w}_{m,l,i}^{[m](\kappa)}\right)\right)}{\ln(2)\underline{R}_{m,l,i}^{[m](\kappa)}}\\ &-\frac{1}{\ln(2)}\left(\frac{1}{\underline{R}_{m,l,i}^{[m](\kappa)}}-\frac{1}{\underline{R}_{m,l,i}^{[m](\kappa)}+\left|\mathbf{h}_{b,m,i}^H\mathbf{w}_{m,l,i}^{[m](\kappa)}\right|^2}\right)\\ &\times\left(\underline{R}_{m,l,i}^{[m]}+\left|\mathbf{h}_{b,m,i}^H\mathbf{w}_{m,l,i}^{[m]}\right|^2-\underline{R}_{m,l,i}^{[m](\kappa)}-\left|\mathbf{h}_{b,m,i}^H\mathbf{w}_{m,l,i}^{[m](\kappa)}\right|^2\right).\end{aligned} \tag{5.35}$$

Similarly, we can further obtain the following inequalities for constraint (5.17d):

$$\begin{aligned}\log_2\left(1+SINR_{m,l,i}^{[m,l]}\right) &\geq \bar{f}_{l,i}^{[4]}\left(\mathbf{w}_{m,l,i}^{[m]},\mathbf{w}_{m,l,i}^{[l]},\mathbf{Q}_i,\mathbf{w}_{m,l,i}^{[m](\kappa)},\mathbf{w}_{m,l,i}^{[l](\kappa)},\mathbf{Q}_i^{(\kappa)}\right)\\ &= \log_2\left(1+\frac{\left|\mathbf{h}_{b,l,i}^H\mathbf{w}_{m,l,i}^{[m](\kappa)}\right|^2}{\underline{R}_{m,l,i}^{[m,l](\kappa)}}\right)\\ &+2\frac{\Re\left(\left(\mathbf{w}_{m,l,i}^{[m](\kappa)}\right)^H\mathbf{h}_{b,l,i}\mathbf{h}_{b,l,i}^H\left(\mathbf{w}_{m,l,i}^{[m]}-\mathbf{w}_{m,l,i}^{[m](\kappa)}\right)\right)}{\ln(2)\underline{R}_{m,l,i}^{[m,l](\kappa)}}\\ &-\frac{1}{\ln(2)}\left(\frac{1}{\underline{R}_{m,l,i}^{[m,l](\kappa)}}-\frac{1}{\underline{R}_{m,l,i}^{[m,l](\kappa)}+\left|\mathbf{h}_{b,l,i}^H\mathbf{w}_{m,l,i}^{[m](\kappa)}\right|^2}\right)\\ &\times\left(\underline{R}_{m,l,i}^{[m,l]}+\left|\mathbf{h}_{b,l,i}^H\mathbf{w}_{m,l,i}^{[m]}\right|^2-\underline{R}_{m,l,i}^{[m,l](\kappa)}-\left|\mathbf{h}_{b,l,i}^H\mathbf{w}_{m,l,i}^{[m](\kappa)}\right|^2\right),\end{aligned} \tag{5.36}$$

$$
\begin{aligned}
R_{l,m,i}^{[m]} &\geq \bar{f}_{m,i}^{[5]}\left(\mathbf{w}_{l,m,i}^{[m]}, \mathbf{Q}_i, \mathbf{w}_{l,m,i}^{[m](\kappa)}, \mathbf{Q}_i^{(\kappa)}\right) \\
&= \log_2\left(1+\frac{\left|\mathbf{h}_{b,m,i}^H \mathbf{w}_{l,m,i}^{[m](\kappa)}\right|^2}{\underline{R}_{l,m,i}^{[m](\kappa)}}\right) \\
&+ 2\frac{\Re\left(\left(\mathbf{w}_{l,m,i}^{[m](\kappa)}\right)^H \mathbf{h}_{b,m,i}\mathbf{h}_{b,m,i}^H\left(\mathbf{w}_{l,m,i}^{[m]}-\mathbf{w}_{l,m,i}^{[m](\kappa)}\right)\right)}{\ln(2)\underline{R}_{l,m,i}^{[m](\kappa)}} \\
&- \frac{1}{\ln(2)}\left(\frac{1}{\underline{R}_{l,m,i}^{[m](\kappa)}}-\frac{1}{\underline{R}_{l,m,i}^{[m](\kappa)}+\left|\mathbf{h}_{b,m,i}^H\mathbf{w}_{l,m,i}^{[m](\kappa)}\right|^2}\right) \\
&\times\left(\underline{R}_{l,m,i}^{[m]}+\left|\mathbf{h}_{b,m,i}^H\mathbf{w}_{l,m,i}^{[m]}\right|^2-\underline{R}_{l,m,i}^{[m](\kappa)}-\left|\mathbf{h}_{b,m,i}^H\mathbf{w}_{l,m,i}^{[m](\kappa)}\right|^2\right)
\end{aligned}
\tag{5.37}
$$

where $\underline{R}_{m,l,i}^{[m,l]} = \left|\mathbf{h}_{b,l,i}^H\mathbf{w}_{m,l,i}^{[l]}\right|^2 + \mathbf{g}_{b,l,i}^H\mathbf{Q}_i\mathbf{g}_{b,l,i} + \sigma_{b,l,i}^2$, $\underline{R}_{m,l,i}^{[m,l](\kappa)} = \left|\mathbf{h}_{b,l,i}^H\mathbf{w}_{m,l,i}^{[l](\kappa)}\right|^2 + \mathbf{g}_{b,l,i}^H\mathbf{Q}_i^{(\kappa)}\mathbf{g}_{b,l,i} + \sigma_{b,l,i}^2$, $\underline{R}_{l,m,i}^{[m]} = \mathbf{g}_{b,m,i}^H\mathbf{Q}_i\mathbf{g}_{b,m,i} + \sigma_{b,m,i}^2$ and $\underline{R}_{l,m,i}^{[m](\kappa)} = \mathbf{g}_{b,m,i}^H\mathbf{Q}_i^{(\kappa)}\mathbf{g}_{b,m,i} + \sigma_{b,m,i}^2$. Moreover, since $-\log_2(1+\Gamma_{e,m})$ is a convex function, we have

$$
-\log_2\left(1+\Gamma_{e,m}\right) \geq -\log_2\left(1+\Gamma_{e,m}^{(\kappa)}\right) - \frac{1}{\left(1+\Gamma_{e,m}^{(\kappa)}\right)\ln(2)}\left(\Gamma_{e,m}-\Gamma_{e,m}^{(\kappa)}\right). \tag{5.38}
$$

Through (5.35)-(5.38), we know constraint (5.17d) can be approximated by

$$
\begin{aligned}
&\sum_{i\in\mathcal{I}}\sum_{l\in\mathcal{M},l\neq m}\Big(\min\Big\{\bar{f}_{m,i}^{[3]}\left(\mathbf{w}_{m,l,i}^{[m]}, \mathbf{w}_{m,l,i}^{[l]}, \mathbf{Q}_i, \mathbf{w}_{m,l,i}^{[m](\kappa)}, \mathbf{w}_{m,l,i}^{[l](\kappa)}, \mathbf{Q}_i^{(\kappa)}\right), \\
&\bar{f}_{l,i}^{[4]}\left(\mathbf{w}_{m,l,i}^{[m]}, \mathbf{w}_{m,l,i}^{[l]}, \mathbf{Q}_i, \mathbf{w}_{m,l,i}^{[m](\kappa)}, \mathbf{w}_{m,l,i}^{[l](\kappa)}, \mathbf{Q}_i^{(\kappa)}\right)\Big\} + \bar{f}_{m,i}^{[5]}\left(\mathbf{w}_{l,m,i}^{[m]}, \mathbf{Q}_i, \mathbf{w}_{l,m,i}^{[m](\kappa)}, \mathbf{Q}_i^{(\kappa)}\right)\Big) \\
&-\log_2\left(1+\Gamma_{e,m}^{(\kappa)}\right) - \frac{1}{\left(1+\Gamma_{e,m}^{(\kappa)}\right)\ln(2)}\left(\Gamma_{e,m}-\Gamma_{e,m}^{(\kappa)}\right) \geq r_m, \forall m\in\mathcal{M}.
\end{aligned}
\tag{5.39}
$$

Note that the point wise minimum of concave functions is concave. Thus, constraint (5.39) is convex.

Constraint (5.27c) is non-convex. $\mathrm{Seig}_j\left(\mathbf{Q}_i\right) - \mathrm{Seig}_{j-1}\left(\mathbf{Q}_i\right)$ is the differ-

ence of concave functions. Through the property of subgradient, we have

$$\mathrm{Seig}_{j-1}\left(\mathbf{Q}_i\right) \leq \mathrm{Seig}_{j-1}\left(\mathbf{Q}_i^{(\kappa)}\right) + \mathrm{tr}\left(\mathbf{\Omega}_{j-1}^{(\kappa)}\left(\mathbf{\Omega}_{j-1}^{(\kappa)}\right)^H\left(\mathbf{Q}_i - \mathbf{Q}_i^{(\kappa)}\right)\right) \quad (5.40)$$

where $\mathbf{\Omega}_{j-1}^{(\kappa)}$ is composed of eigenvectors of $\mathbf{Q}_i^{(\kappa)}$ corresponding to the $j-1$ smallest eigenvalues. From (5.40), we can obtain a convex approximation of constraint (5.27c) as

$$\mathrm{Seig}_{j}\left(\mathbf{Q}_i\right) - \mathrm{Seig}_{j-1}\left(\mathbf{Q}_i^{(\kappa)}\right) - \mathrm{tr}\left(\mathbf{\Omega}_{j-1}^{(\kappa)}\left(\mathbf{\Omega}_{j-1}^{(\kappa)}\right)^H\left(\mathbf{Q}_i - \mathbf{Q}_i^{(\kappa)}\right)\right) \geq \lambda_{i,j}, \quad \forall j \in \mathcal{J}, j > 1, \forall i \in \mathcal{I}. \quad (5.41)$$

For the objective function, we have

$$\left\|\mathbf{p}_{c,i}\right\|_\infty \geq \left\|\mathbf{p}_{c,i}^{(\kappa)}\right\|_\infty + \partial\left\|\mathbf{p}_{c,i}^{(\kappa)}\right\|_\infty^T\left(\mathbf{p}_{c,i} - \mathbf{p}_{c,i}^{(\kappa)}\right) \quad (5.42)$$

and

$$\left\|\mathbf{p}_{b,m}\right\|_\infty \geq \left\|\mathbf{p}_{b,m}^{(\kappa)}\right\|_\infty + \partial\left\|\mathbf{p}_{b,m}^{(\kappa)}\right\|_\infty^T\left(\mathbf{p}_{b,m} - \mathbf{p}_{b,m}^{(\kappa)}\right) \quad (5.43)$$

where $\partial\left\|\cdot\right\|_\infty$ represents a subgradient of $\left\|\cdot\right\|_\infty$, $\mathbf{p}_{c,i}^{(\kappa)} = [p_{1,2,i}^{(\kappa)}, ..., p_{1,M,i}^{(\kappa)}, p_{2,1,i}^{(\kappa)}, ..., p_{M-1,M,i}^{(\kappa)}, p_{M,1,i}^{(\kappa)}, ..., p_{M,M-1,i}^{(\kappa)}]^T$ and $\mathbf{p}_{b,m}^{(\kappa)} = [\sum_{n\in\mathcal{M},n\neq m}\left(p_{m,n,1}^{(\kappa)} + p_{n,m,1}^{(\kappa)}\right), ..., \sum_{n\in\mathcal{M},n\neq m}\left(p_{m,n,I}^{(\kappa)} + p_{n,m,I}{}^{(\kappa)}\right)]^T$. The subgradient $\partial\left\|\cdot\right\|_\infty$ at a vector $\mathbf{a} = [a_1, ..., a_V]^T$ can be given by

$$\partial\left\|\mathbf{a}\right\|_\infty = \underset{\boldsymbol{\theta}}{\mathrm{argmax}}\left\{\sum_{v=1}^{V}\theta_v a_v : \sum_{v=1}^{V}|\theta_v| = 1, -1 \leq \theta_v \leq 1, \forall 1 \leq v \leq V\right\},$$

where $\boldsymbol{\theta} = [\theta_1, ..., \theta_V]^T$.

Based on (5.42) and (5.43), the objective function can be approximated by the following concave function

$$r - U^{(\kappa)} \quad (5.44)$$

where $U^{(\kappa)} = \rho\Bigg(\sum_{i\in\mathcal{I}}\Bigg(\|\mathbf{p}_{c,i}\|_1 - \left\|\mathbf{p}_{c,i}^{(\kappa)}\right\|_\infty - \partial\left\|\mathbf{p}_{c,i}^{(\kappa)}\right\|_\infty^T\left(\mathbf{p}_{c,i} - \mathbf{p}_{c,i}^{(\kappa)}\right)\Bigg) + \sum_{m\in\mathcal{M}}\Bigg(\|\mathbf{p}_{b,m}\|_1 - \left\|\mathbf{p}_{b,m}^{(\kappa)}\right\|_\infty - \partial\left\|\mathbf{p}_{b,m}^{(\kappa)}\right\|_\infty^T\left(\mathbf{p}_{b,m} - \mathbf{p}_{b,m}^{(\kappa)}\right)\Bigg)\Bigg)$.

Through (5.33) (5.34) (5.39) and (5.41), the convex approximation for problem (5.28) in the $(\kappa+1)$-th iteration can be give by

$$\begin{aligned}
&\max_{\substack{\mathbf{w}_{m,n,i}^{[m]},\mathbf{w}_{m,n,i}^{[n]},\mathbf{Q}_i,r_m,\Gamma_{e,m},p_{m,n,i},r,\varepsilon_{m,n,k,i}^{[m]},\varepsilon_{n,m,k,i}^{[m]},\lambda_{i,j}\\ \forall j\in\mathcal{J},\forall k\in\mathcal{K},\forall m\in\mathcal{M},\forall n\in\mathcal{M},n\neq m,\forall i\in\mathcal{I}}} \quad r - U^{(\kappa)}\\
&\text{s.t.}(5.41),(5.39),(5.34),(5.33),(5.31),(5.27\text{b}),(5.24\text{c}),(5.24\text{d}),(5.20\text{f}),(5.20\text{g}),\\
&\quad (5.17\text{e}),(5.17\text{f}),(5.17\text{h})-(5.17\text{k}).
\end{aligned} \tag{5.45}$$

and the obtained optimal solution to problem (5.45) can be used to build the convex approximation problem in the next iteration.

According to the path-following approach, we can find a solution to problem (5.28) by solving the convex approximation problem at the convergence of the aforementioned path-following iteration process.

Based on the discussion above, we propose the following secure transmission design algorithm.

The convergence of the path-following iteration process in Algorithm 6 can be guaranteed and similar proof can be found in [97].

5.3 Simulation Results

In this section, we present numerical results on the performance of the proposed outage constrained secure transmission design (i.e., Algorithm 6). We consider a scenario where there are six Bobs and two Eves. For all the examples in this section, we assume that the available frequency band is divided into 3 orthogonal subcarriers for NOMA. We assume that Alice and the Helper both have N antennas, i.e., $N_a = N_h = N$. The channel distributions are set as $\mathbf{h}_{b,m,i} \sim \mathcal{CN}(\mathbf{0},\mathbf{I})$, $\mathbf{g}_{b,m,i} \sim \mathcal{CN}(\mathbf{0},\mathbf{I})$, $\mathbf{h}_{e,k,i} \sim \mathcal{CN}(\mathbf{0},0.3\mathbf{I})$ and $\mathbf{g}_{e,k,i} \sim \mathcal{CN}(\mathbf{0},0.3\mathbf{I})$. The noise power at Bobs and Eves is assumed to

Algorithm 6: Algorithm of outage constrained secure transmission design

1. Initialize $\kappa := 0$ and $\{\mathbf{w}_{m,n,i}^{[m](0)}, \mathbf{w}_{m,n,i}^{[n](0)}, \mathbf{Q}_i^{(0)}, r_m^{(0)}, \Gamma_{e,m}^{(0)}, p_{m,n,i}^{(0)}, r^{(0)}, \varepsilon_{m,n,k,i}^{[m](0)}, \varepsilon_{n,m,k,i}^{[m](0)}, \lambda_{i,j}^{(0)}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall n \in \mathcal{M}, n \neq m, \forall i \in \mathcal{I}\}$.
2. **Repeat**
3. Solve (5.45) to obtain an optimal solution $\{\mathbf{w}_{m,n,i}^{[m](\kappa+1)}, \mathbf{w}_{m,n,i}^{[n](\kappa+1)}, \mathbf{Q}_i^{(\kappa+1)}, r_m^{(\kappa+1)}, \Gamma_{e,m}^{(\kappa+1)}, p_{m,n,i}^{(\kappa+1)}, r^{(\kappa+1)}, \varepsilon_{m,n,k,i}^{[m](\kappa+1)}, \varepsilon_{n,m,k,i}^{[m](\kappa+1)}, \lambda_{i,j}^{(\kappa+1)}, \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, \forall m \in \mathcal{M}, \forall n \in \mathcal{M}, n \neq m, \forall i \in \mathcal{I}\}$
4. Set $\kappa := \kappa + 1$.
5. **Until** convergence of the objective in (5.28).
6. Apply $\{\mathbf{w}_{m,n,i}^{[m](\bar{\kappa})}, \mathbf{w}_{m,n,i}^{[n](\bar{\kappa})}, \mathbf{Q}_i^{(\bar{\kappa})}, \forall m \in \mathcal{M}, \forall n \in \mathcal{M}, n \neq m, \forall i \in \mathcal{I}\}$ for the secure transmission and $\{p_{m,n,i}^{(\bar{\kappa})}, \forall m \in \mathcal{M}, \forall n \in \mathcal{M}, n \neq m, \forall i \in \mathcal{I}\}$ for subcarrier allocation policy where $\bar{\kappa}$ is the iteration index at which the path-following iteration process converges.
7. **End**

be unity, i.e., $\sigma_{b,m,i}^2 = \sigma_{e,k,i}^2 = 1$. For the power bounds for Alice and the Helper, we assume they are the same, $P_s = P_c$, and $P = P_s = P_c$ is defined in dB. For the secrecy outage tolerance, we assume $\varepsilon_m = \varepsilon$.

In Figure 5.2, we study the convergence of the path-following process in Algorithm 6 with different values of P by setting $N = 6$ and $\varepsilon = 0.15$. As observed from Figure 5.2, no matter how P is changed, the objective value achieved in problem (5.28) increases monotonically and the path-following process in Algorithm 6 can converge efficiently in 14 iterations.

Figure 5.3 demonstrates the secrecy outage probabilities at Bobs for different values of P with $N = 6$ and $\varepsilon = 0.15$. The probabilities are calculated through 1000 realizations of Eves' channels. From Figure 5.3, we can see that for all values of P, the secrecy outage probabilities appear at Bobs are less than $\varepsilon = 0.15$, which verifies that the proposed outage constrained secure transmission design can effectively control the secrecy outage probabilities.

In Figure 5.4, we investigate the average secrecy throughput (i.e., the av-

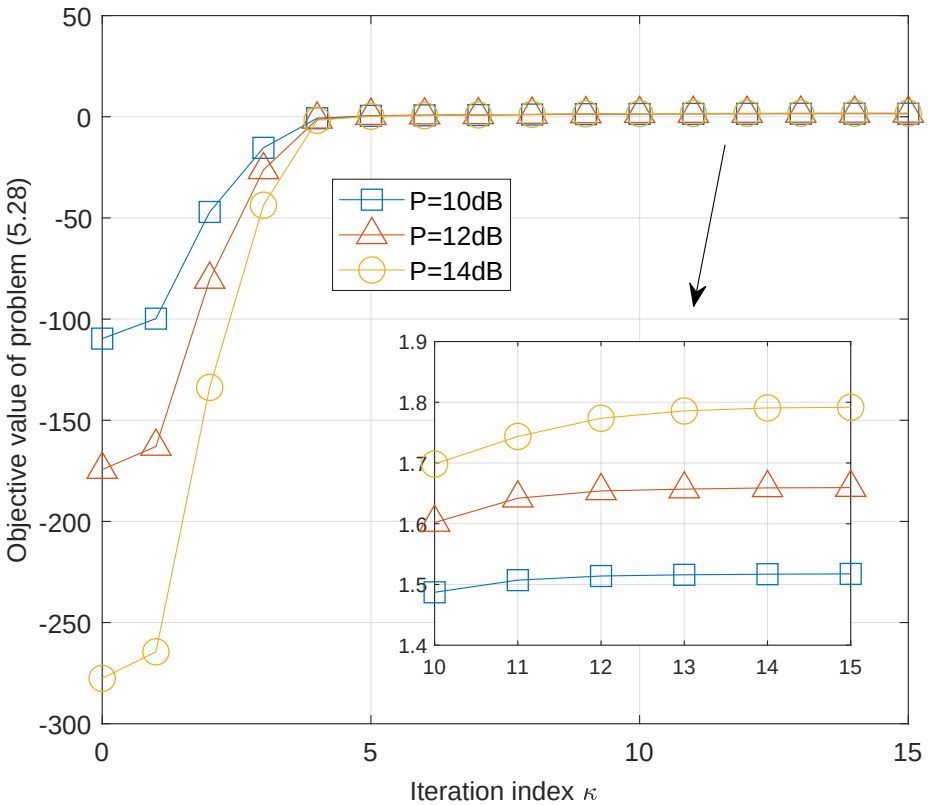

Figure 5.2: Convergence of the path-following process in Algorithm 6 for different values of P with $N = 6$ and $\varepsilon = 0.15$.

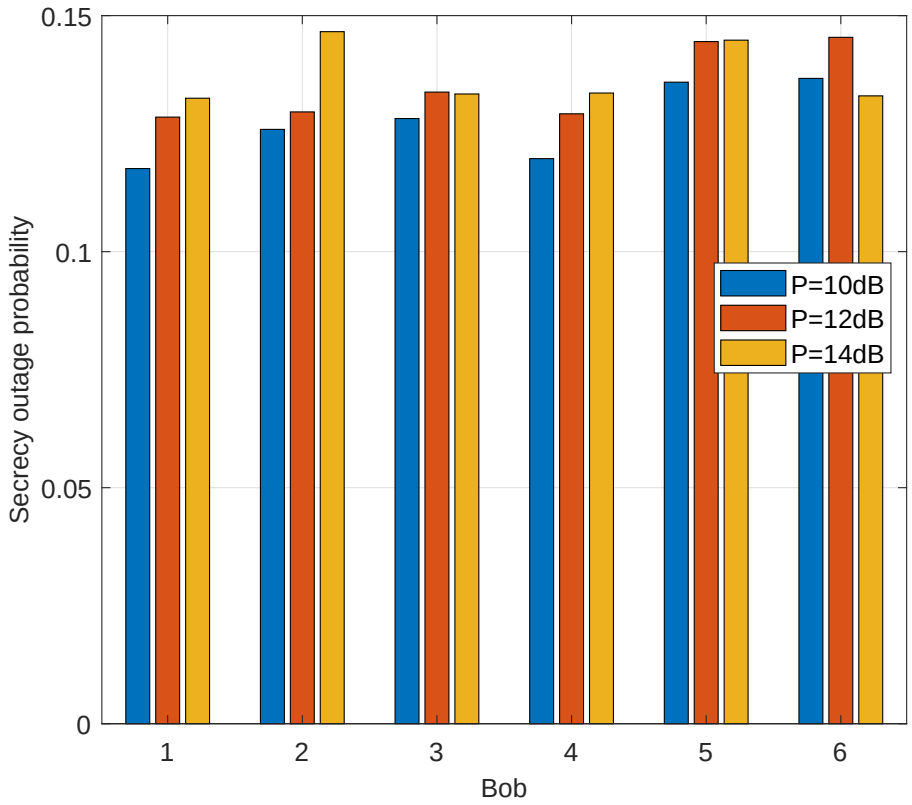

Figure 5.3: Secrecy outage probabilities at Bobs for different values of P with $N = 6$ and $\varepsilon = 0.15$.

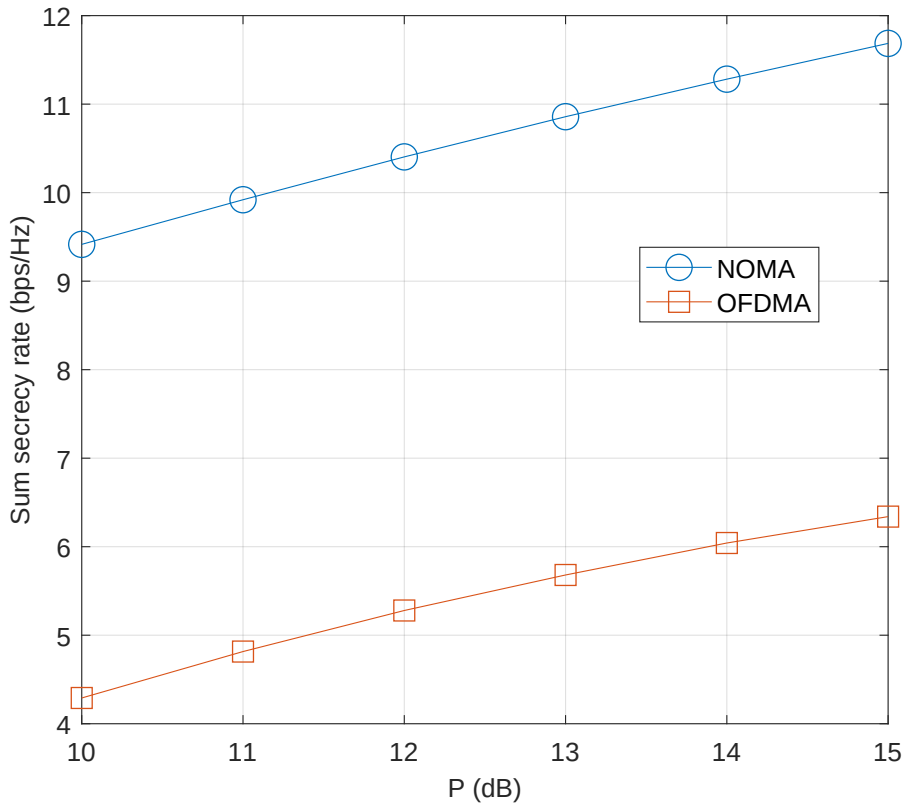

Figure 5.4: Average sum secrecy rates achieved by NOMA and OFDMA versus P with $N = 6$ and $\varepsilon = 0.15$.

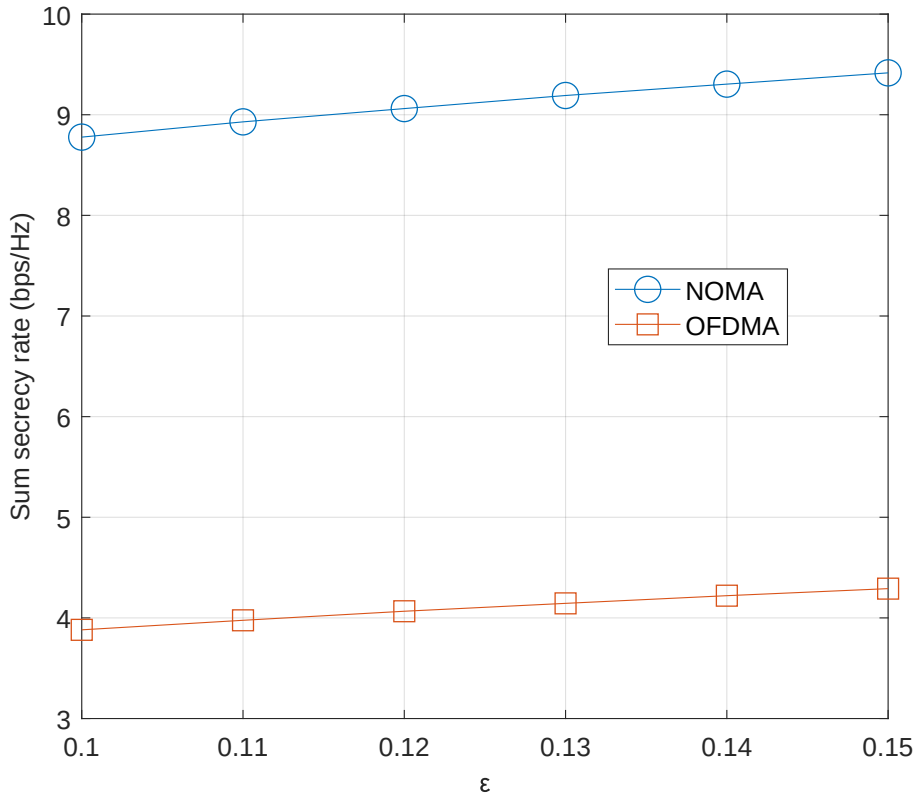

Figure 5.5: Average sum secrecy rates achieved by NOMA and OFDMA versus ε with $N = 6$ and $P = 10$dB.

erage sum secrecy rate) achieved by the proposed secure transmission design versus the maximal transmit power P with $N = 6$ and $\varepsilon = 0.15$. For comparison, an orthogonal frequency-division multiple access (OFDMA) based scheme is also considered. The OFDMA scheme is obtained by the methods similar to those used for developing Algorithm 6 which is for NOMA. In the OFDMA scheme, each subcarrier can only be assigned to serve one Bob and the bandwidth for each subcarrier is a half of the one for each subcarrier in NOMA. From Figure 5.4, we can observe that, compared with the OFDMA, NOMA can significantly improve the secrecy throughput.

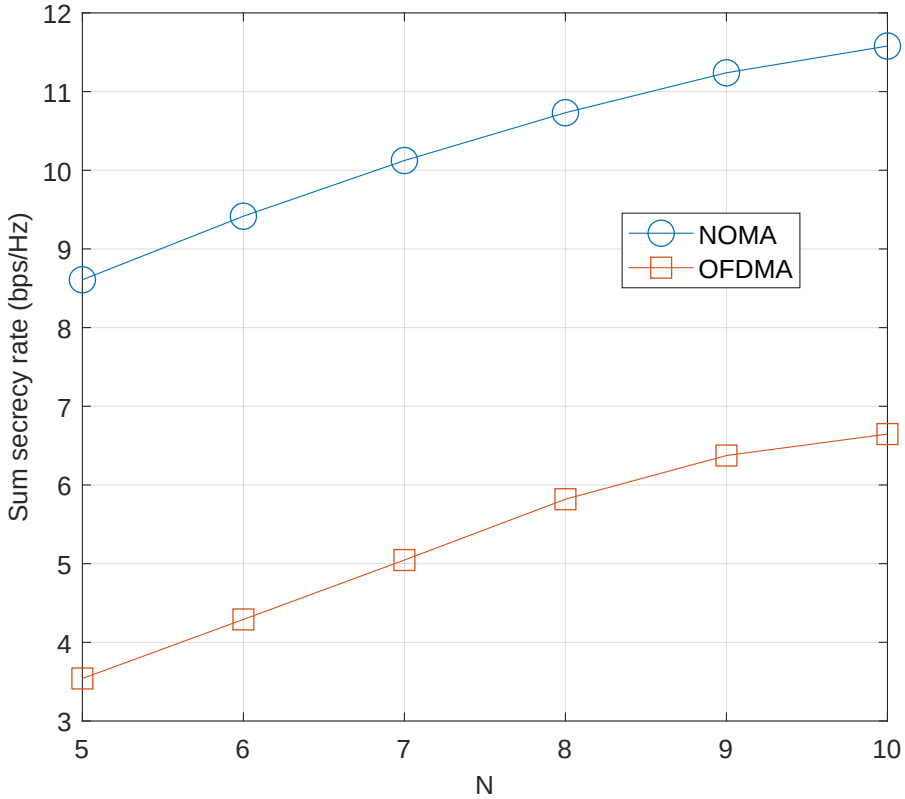

Figure 5.6: Average sum secrecy rates achieved by NOMA and OFDMA versus N with $P = 10$dB and $\varepsilon = 0.15$.

Figure 5.5 represents the average secrecy throughputs achieved by the proposed NOMA based design and the OFDMA based scheme against ε with $N = 6$ and $P = 10$dB. We can observe that the impacts of the secrecy outage tolerance ε on the secrecy throughput performances of the proposed NOMA based design and the OFDMA based scheme are slight.

Figure 5.6 demonstrates the average secrecy throughputs achieved by the proposed NOMA based design and the OFDMA based scheme as functions of the antenna number N with $P = 10$dB and $\varepsilon = 0.15$. Through Figure 5.4 to Figure 5.6, it can be seen that, compared with loosening the secrecy outage

probability requirements, allowing more transmit power is a more effective approach to improve the secrecy throughput performances of NOMA and OFDMA. In addition, adding antennas is more effective than providing more transmit power, especially when the antenna number is small.

5.4 Summary

In this chapter, we considered an outage constrained secure transmission design problem for a CJ aided MISO MC NOMA system. In the outage constrained secure transmission design problem, the single stream beamforming vector, the covariance matrix of jamming signals and the subcarrier allocation policy were jointly optimized for maximizing the minimum probabilistic secrecy rate. In order to find a solution to the design problem, we first converted it into a 0-norm optimization problem. Then, we replaced the outage constraints by analytic approximations. By applying the path-following approach, a secure transmission design algorithm was proposed through which a solution to the original design problem can be found. Simulation results demonstrated the effectiveness of the proposed secure transmission design.

Chapter 6

Conclusions

In this chapter, we summarize the contributions of this thesis. Also, we present several potential future research topics that are related to our accomplished work.

6.1 Summary of Contributions

In this thesis, we investigated secure transmission designs for jamming signal aided multi-antenna systems. Here we summarize the results obtained in each chapter.

Contribution 1: We studied robust secure transmission designs from both perspectives of QoS and secrecy rate for a CJ aided MISO system with norm bounded channel uncertainties. Two scenarios were considered: (a) ECSI is available and (b) ECSI is unavailable. For scenario (a), a QoS based design and a secrecy rate based design were investigated. For scenario (b), only the QoS based design was considered since the secrecy rate based design is not applicable. In all the designs, we optimized the single stream beamforming vector and the jamming covariance matrix under individual power constraints. To solve the non-convex design problems, we used the SDR approach to obtain the general rank information covariance matrix based counterparts. We demonstrated that rank-one optimal information covariance matrix solutions can always be found in the SDR problems and similar results do not exist in previous works on robust secure transmission designs for CJ aided MISO systems. The simulation results revealed that, when the ECSI is available, the secrecy rate based design outperforms previously proposed designs which were based on general rank information covariance matrix optimization. The effectiveness and robustness of the QoS based

designs were also demonstrated by the simulation results.

Contribution 2: We considered a SWIPT enabled MISO HCN where there exist one MBS and multiple FBSs in co-channel deployment and proposed a sum logarithmic secrecy rate maximization secure transmission design problem with EH constraints for ERs. To solve the proposed design problem, we use SDR technique to relax the outer products of the single stream beamforming vectors to be general rank information covariance matrices. Our analysis revealed that, compared with beamforming having general rank information covariance matrix, using single-stream beamforming to transmit confidential messages does not cause any loss of optimality. This finding provided a theoretical justification for using single stream beamforming. Then, we dealt with the SDR problem by proposing an SCA process and developed an SCA-based centralized secure transmission design algorithm. Simulation results revealed that, compared with the SCA process extended from [51], the proposed one converges faster. Moreover, a distributed design algorithm was also proposed. Simulation results demonstrated that the distributed algorithm can perform as well as the centralized one.

Contribution 3: We considered an outage constrained secure transmission design problem for a CJ aided MISO MC NOMA system under the assumption that only statistical ECSI is available. In the design problem, the single stream beamforming vector, the covariance matrix of jamming signals and the subcarrier allocation policy were jointly optimized for maximizing the minimum probabilistic secrecy rate. In order to find a solution to the design problem, we first converted it into a 0-norm optimization problem. Then, we replaced the outage constraints by analytic approximations. By applying the path-following approach, an secure transmission design algorithm was proposed through which a solution to the original design problem can be found. Simulation results demonstrated that the secrecy outage probability can be guaranteed through the proposed secure transmission design. In addition, simulation results showed that our proposed secure transmission design for NOMA can achieve higher secrecy throughput than that of OFDMA systems.

6.2 Future Works

In Chapter 3, we studied robust secure transmission designs for a CJ aided MISO system by considering that the CSI errors are bounded. However, besides the bounded CSI error model, the statistical CSI error model is also important. The bounded CSI error model is applicable for the case where the CSI errors are dominantly caused by quantization error and the statistical CSI error model is suitable for the case where the CSI errors are dominantly caused by estimation error. Therefore, robust secure transmission designs for CJ aided MISO systems having statistical CSI errors are worthy to be investigated in the future.

In Chapter 4, we considered secure transmission designs for a SWIPT enabled MISO HCN with the help of perfect CSI. However, in practice, the CSI is hard to be perfectly known. Therefore, the secure transmission designs for SWIPT enabled MISO HCNs with imperfect CSI are interesting future research topics.

In Chapter 5, we considered an outage constrained secure transmission design problem for a CJ aided MISO MC NOMA system under the assumption that only statistical ECSI is available. Studying outage constrained secure transmission designs for MISO NOMA systems having estimated CSI and statistical CSI error is a possible future research direction.

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Appendix

Appendix A

Proof of Proposition 3.1

Take

$$\mathbf{w} = \tilde{\mathbf{h}}_b \text{ and } \mathbf{Q}_z = \mathbf{I} - \frac{\tilde{\mathbf{g}}_\mathbf{b}\tilde{\mathbf{g}}_\mathbf{b}^H}{\|\tilde{\mathbf{g}}_\mathbf{b}\|^2}. \tag{A.1}$$

Due to (3.6), it can be seen that

$$\begin{aligned}\min_{\mathbf{e}_{h,b}\in\varepsilon_{h,b}} (\tilde{\mathbf{h}}_b + \mathbf{e}_{h,b})^H \mathbf{w}\mathbf{w}^H (\tilde{\mathbf{h}}_b + \mathbf{e}_{h,b}) &= \min_{\mathbf{e}_{h,b}\in\varepsilon_{h,b}} |\tilde{\mathbf{h}}_b^H \tilde{\mathbf{h}}_b + \mathbf{e}_{h,b}^H \tilde{\mathbf{h}}_b|^2 \\ &\geq (\|\tilde{\mathbf{h}}_b\|^2 - \xi_{h,b}\|\tilde{\mathbf{h}}_b\|)^2 \\ &= (\|\tilde{\mathbf{h}}_b\| - \xi_{h,b})^2 \|\tilde{\mathbf{h}}_b\|^2.\end{aligned} \tag{A.2}$$

Then, we employ $\mathbf{Q}_z\tilde{\mathbf{g}}_\mathbf{b} = 0$, $\tilde{\mathbf{g}}_\mathbf{b}^H\mathbf{Q}_z = 0$ and $\|\mathbf{Q}_z\| = 1$ to have

$$\begin{aligned}\max_{\mathbf{e}_{g,b}\in\varepsilon_{g,b}} (\tilde{\mathbf{g}}_\mathbf{b} + \mathbf{e}_{g,b})^H \mathbf{Q}_z (\tilde{\mathbf{g}}_\mathbf{b} + \mathbf{e}_{g,b}) + \sigma_b^2 &= \max_{\mathbf{e}_{g,b}\in\varepsilon_{g,b}} (\tilde{\mathbf{g}}_\mathbf{b} + \mathbf{e}_{g,b})^H \mathbf{Q}_z \mathbf{e}_{g,b} + \sigma_b^2 \\ &= \max_{\mathbf{e}_{g,b}\in\varepsilon_{g,b}} \mathbf{e}_{g,b}^H \mathbf{Q}_z \mathbf{e}_{g,b} + \sigma_b^2 \\ &\leq \max_{\mathbf{e}_{g,b}\in\varepsilon_{g,b}} \|\mathbf{e}_{g,b}\|^2 + \sigma_b^2 = \xi_{g,b}^2 + \sigma_b^2\end{aligned} \tag{A.3}$$

which, together with (3.7) and (A.2), leads to

$$\begin{aligned}\min_{\mathbf{e}_{h,b}\in\varepsilon_{h,b},\mathbf{e}_{g,b}\in\varepsilon_{h,b}} \frac{(\tilde{\mathbf{h}}_b + \mathbf{e}_{h,b})^H \mathbf{w}\mathbf{w}^H (\tilde{\mathbf{h}}_b + \mathbf{e}_{h,b})}{(\tilde{\mathbf{g}}_\mathbf{b} + \mathbf{e}_{g,b})^H \mathbf{Q}_z (\tilde{\mathbf{g}}_\mathbf{b} + \mathbf{e}_{g,b}) + \sigma_b^2} &\geq \frac{(\|\tilde{\mathbf{h}}_b\| - \xi_{h,b}\|)^2 \|\tilde{\mathbf{h}}_b\|^2}{\xi_{g,b}^2 + \sigma_b^2} \\ &\geq \gamma_b.\end{aligned} \tag{A.4}$$

Furthermore, according to the assumption (3.8), it is easy to have

$$\|\mathbf{w}\|^2 = \left\|\tilde{\mathbf{h}}_b\right\|^2 \leq P_s \tag{A.5}$$

and

$$\mathrm{tr}(\mathbf{Q}_z) = N_h - \mathrm{tr}\left(\frac{\tilde{\mathbf{g}}_\mathbf{b}\tilde{\mathbf{g}}_\mathbf{b}^H}{\|\tilde{\mathbf{g}}_\mathbf{b}\|^2}\right) = N_h - \mathrm{tr}\left(\frac{\tilde{\mathbf{g}}_\mathbf{b}^H\tilde{\mathbf{g}}_\mathbf{b}}{\|\tilde{\mathbf{g}}_\mathbf{b}\|^2}\right) = N_h - 1 \leq P_c. \tag{A.6}$$

Because of (A.4)-(A.6) and $\mathbf{Q}_z \succeq \mathbf{0}$, $\{\mathbf{w}, \mathbf{Q}_z\}$ given by (A.1) is a feasible point and problem P-QoS-A is feasible.

Appendix B

Proof of Lemma 3.1

Recall that we have $\text{diag}\{\mathbf{\Omega}, \mathbf{\Upsilon}\} \succeq \mathbf{0}$ if and only if $\mathbf{\Omega} \succeq \mathbf{0}$ and $\mathbf{\Upsilon} \succeq \mathbf{0}$. Therefore, $\mathbf{F} = \mathbf{F}_0 + \sum_{i=1}^{n_{\mathbf{s}}} s_i \mathbf{F}_i \succeq \mathbf{0}$ is equivalent to

$$(3.11) - (3.14), v \geq \theta_b \varrho \tag{B.1a}$$

$$u_k \leq \theta_e z_k, k = 1, \ldots, K \tag{B.1b}$$

$$\text{tr}(\mathbf{Q}_z) \leq P_c, \mathbf{Q}_x \succeq \mathbf{0}, \mathbf{Q}_z \succeq \mathbf{0}. \tag{B.1c}$$

Furthermore, it can be observed that $\sum_{i=1}^{n_{\mathbf{s}}} c_i s_i = \text{tr}(\mathbf{Q}_x)$. Consequently, problem in (3.18) can be equivalently expressed as

$$\begin{aligned} &\min_{\substack{\mathbf{Q}_x, \mathbf{Q}_z, v, \varrho, \psi, \alpha, u_1, \ldots, u_K \\ z_1, \ldots, z_K, \varphi_1, \ldots, \varphi_K, \beta_1, \ldots, \beta_K}} \text{tr}(\mathbf{Q}_x) \\ &\text{s.t. (B.1a)} - \text{(B.1c)}. \end{aligned} \tag{B.2}$$

Since (3.12) implies (3.10b), in problem (B.2) there must be $\varrho \geq 0$. Hence $v \geq \theta_b \varrho$ forces $v \geq 0$. Similarly, we also have $u_k, z_k \geq 0$ in (B.2). Therefore, by replacing (3.11)-(3.14) with their equivalent counterparts, problem (B.2) can be rewritten as

$$\begin{aligned} &\min_{\substack{\mathbf{Q}_x, \mathbf{Q}_z, v, \varrho, \\ u_1, \ldots, u_K, z_1, \ldots, z_K}} \text{tr}(\mathbf{Q}_x) \\ &\text{s.t. (3.10a)} - \text{(3.10d)}, v \geq \theta_b \varrho,\ v \geq 0, \varrho \geq 0 \\ &\quad u_k \leq \theta_e z_k, u_k \geq 0, z_k \geq 0, k = 1, \ldots, K \\ &\quad \text{tr}(\mathbf{Q}_z) \leq P_c, \mathbf{Q}_x \succeq \mathbf{0}, \mathbf{Q}_z \succeq \mathbf{0} \end{aligned} \tag{B.3}$$

which can be further converted into

$$\begin{aligned} &\min_{\mathbf{Q}_x,\mathbf{Q}_z} \operatorname{tr}(\mathbf{Q}_x) \\ &\text{s.t. } (3.17\text{b}), (3.17\text{c}), (\text{B.1c}). \end{aligned} \tag{B.4}$$

Due to the equivalent relationship between (3.18) and (B.4), $\{\mathbf{Q}_x^*, \mathbf{Q}_z^*\}$ is an optimal solution of (B.4). Since problem (3.17) is feasible for the fixed θ_b and θ_e, there exists a feasible point in (B.4) such that $\operatorname{tr}(\mathbf{Q}_x) \leq P_s$. Thus we have $\operatorname{tr}(\mathbf{Q}_x^*) \leq P_s$, from which we can see $\{\mathbf{Q}_x^*, \mathbf{Q}_z^*, \theta_b, \theta_e\}$ lies in the feasible set of (3.17).

Because $\mathbf{s}^*$ is optimal to (3.18), $\{\mathbf{Q}_x^*,\ \mathbf{Q}_z^*,\ v^*,\ \varrho^*,\ \psi^*,\ \alpha^*,\ u_1^*,\ldots,\ u_K^*,\ z_1^*,\ldots,\ z_K^*,\ \varphi_1^*,\ldots,\ \varphi_K^*,\ \beta_1^*,\ldots,\ \beta_K^*\}$ is an optimal solution to (B.2). Thus, v^*, ϱ^*, u_k^* and z_k^* must be greater or equal to 0. Through the inequalities $\operatorname{tr}(\mathbf{Q}_x^*) \leq P_s$, $\operatorname{tr}(\mathbf{Q}_z^*) \leq P_c$, we know that $\mathbf{Q}_x^*$ and $\mathbf{Q}_z^*$ are in bounded sets. Together this with $v \geq \theta_b \varrho$ and (3.10a) which is equivalent to (3.11), we can show that v^* and ϱ^* lie in bounded sets. Then, using (3.12), we see that α^* is in a bounded set. Because of (3.11), ψ^* lies in a bounded set. By (3.10d), the equivalent counterpart of (3.14), z_k^* lies in a bounded set; consequently, by $u_k \leq \theta_e z_k$, u_k^* is bounded. The boundedness of φ_k^* and β_k^* follows from (3.13) and (3.14). Therefore $\mathbf{s}^*$ must be in a bounded set and the set of optimal solutions of (3.18) is bounded. Then, we consider the dual problem of (3.18)

$$\begin{aligned} &\max_{\mathbf{E}} -\operatorname{tr}(\mathbf{F}_0\mathbf{E}) \\ &\text{s.t. } \operatorname{tr}(\mathbf{F}_i\mathbf{E}) = c_i, i = 1,\ldots,n_{\mathbf{s}}, \mathbf{E} \succeq \mathbf{0} \end{aligned} \tag{B.5}$$

where matrix $\mathbf{E}$ is the Lagrangian multiplier corresponding to the constraint of (3.18). Suppose that (B.5) is not strictly feasible, i.e. $\nexists\, \mathbf{E}^0$ s.t. $\operatorname{tr}(\mathbf{F}_i\mathbf{E}^0) = c_i, i = 1,\ldots,n_{\mathbf{s}}, \mathbf{E}^0 \succ \mathbf{0}$. Then, we know the set $\Theta = \{\mathbf{h} | \operatorname{tr}(\mathbf{F}_i\mathbf{E}^0) = h_i; \forall\, \mathbf{E}^0 \succ \mathbf{0}, \text{or}\, \mathbf{E}^0 = \mathbf{0}\}$ and the set $\Phi = \{\mathbf{c}\}$ are disjoint, where $\mathbf{h}$ and $\mathbf{c}$ are column vectors composed of h_i and c_i respectively. Because Θ is a convex cone and Φ is also convex, by the separating hyperplane theorem [81], a vector $\mathbf{d} \neq \mathbf{0}$ can be found to make $\mathbf{h}^T\mathbf{d} \geq 0$, $\forall \mathbf{h} \in \Theta$ and $\mathbf{c}^T\mathbf{d} \leq 0$. From

$\mathbf{h}^T\mathbf{d} \geq 0$, $\forall \mathbf{h} \in \Theta$, we have $\sum_{i=1}^{n_\mathbf{s}} d_i \mathrm{tr}(\mathbf{F}_i\mathbf{E}^0) = \mathrm{tr}(\mathbf{E}^0 \sum_{i=1}^{n_\mathbf{s}} d_i\mathbf{F}_i) \geq 0, \forall \mathbf{E}^0 \succ \mathbf{0}$, where d_i is the i-th entry in the vector $\mathbf{d}$. This implies $\sum_{i=1}^{n_\mathbf{s}} d_i\mathbf{F}_i \succeq \mathbf{0}$. Then consider $\mathbf{y} = \mathbf{s}^* + \tau\mathbf{d}$, $\tau > 0$. $\mathbf{y}$ is feasible in (3.18) since $\mathbf{F}_0 + \sum_{i=1}^{n_\mathbf{s}} (s_i^* + \tau d_i)\mathbf{F}_i \succeq \mathbf{0}$. If $\mathbf{c}^T\mathbf{d} < 0$, then $\mathbf{c}^T\mathbf{y} < \mathbf{c}^T\mathbf{s}^*$ which contradicts with the statement $\mathbf{s}^*$ is the optimality in (3.18). Otherwise, if $\mathbf{c}^T\mathbf{d} = 0$, then we know a ray $\{\mathbf{y} = \mathbf{s}^* + \tau\mathbf{d} | \forall \tau > 0\}$ lies in the set of optimal solutions of (3.18) which is bounded. This, of course, is a paradox. Therefore, problem (B.5) is strictly feasible. Since (3.18) and (B.5) are self-dual, by using Slater's condition, we conclude that strong duality holds for them.

Appendix C

Proof of Theorem 3.1

Lemma 3.1 (ii) implies $\mathbf{s}^*$ must satisfy the KKT conditions [81, pp. 267] showing below

$$\mathbf{F}_0 + \sum_{i=1}^{n_{\mathbf{s}}} s_i^* \mathbf{F}_i \succeq \mathbf{0}, \tag{C.1}$$

$$\mathbf{E}^* \succeq \mathbf{0}, \tag{C.2}$$

$$\mathrm{tr}(\mathbf{E}^*(\mathbf{F}_0 + \sum_{i=1}^{n_{\mathbf{s}}} s_i^* \mathbf{F}_i)) = 0, \tag{C.3}$$

$$\mathrm{tr}(\mathbf{F}_i \mathbf{E}^*) = c_i, i = 1, \ldots, n_{\mathbf{s}} \tag{C.4}$$

where $\mathbf{E}^*$ is an optimal dual variable.

Since $\mathbf{F}_0 + \sum_{i=1}^{n_{\mathbf{s}}} s_i^* \mathbf{F}_i$ is positive semidefinite, the diagonal matrices or scalars of it must be positive semidefinite or nonnegative. Similarly, the diagonal matrices and scalars of $\mathbf{E}^*$ are also positive semidefinite and nonnegative respectively. Without loss of generality, we let the diagonal matrices or scalars of $\mathbf{E}^*$ be sequentially denoted by $\mathbf{A}^*$, $\mathbf{C}^*$, $\mathbf{B}_1^*, \ldots, \mathbf{B}_K^*$, $\mathbf{D}_1^*, \ldots, \mathbf{D}_K^*$, $\mathbf{Y}^*$, $\mathbf{Z}^*$, $\lambda_{P_c}^*$, λ_1^*, $\lambda_{2,1}^*, \ldots, \lambda_{2,K}^*$, λ_ψ^*,λ_α^*, $\lambda_{\varphi_1}^*, \ldots, \lambda_{\varphi_K}^*$,$\lambda_{\beta_1}^*, \ldots, \lambda_{\beta_K}^*$, where $\mathbf{A}^* \in \mathbb{S}^{(N_a+1)\times(N_a+1)}$, $\mathbf{C}^* \in \mathbb{S}^{(N_h+1)\times(N_h+1)}$, $\mathbf{B}_k^* \in \mathbb{S}^{(N_a+1)\times(N_a+1)}, k = 1, \ldots, K$, $\mathbf{D}_k^* \in \mathbb{S}^{(N_h+1)\times(N_h+1)}, k = 1, \ldots, K$, $\mathbf{Y}^* \in \mathbb{S}^{N_a \times N_a}$, $\mathbf{Z}^* \in \mathbb{S}^{N_h \times N_h}$, $\mathbb{S}^{n\times n}$ denotes the set of positive semidefinite $n \times n$ matrices and the other

variables are nonnegative scalars. Then, according to (C.3), we have

$$\begin{aligned}\mathrm{tr}(\mathbf{E}^*(\mathbf{F}_0+\sum_{i=1}^{n_{\mathbf{s}}} s_i^*\mathbf{F}_i)) &= \mathrm{tr}(\mathbf{A}^* T_{h,b}(\mathbf{Q}_x^*,\psi^*,v^*)) + \mathrm{tr}(\mathbf{C}^* T_{g,b}(\mathbf{Q}_z^*,\alpha^*,\varrho^*,\sigma_b^2)) \\ &+ \sum_{k=1}^{K}\mathrm{tr}(\mathbf{B}_k^* T_{h,e,k}(\mathbf{Q}_x^*,\varphi_k^*,u_k^*)) \\ &+ \sum_{k=1}^{K}\mathrm{tr}(\mathbf{D}_k^* T_{g,e,k}(\mathbf{Q}_z^*,\beta_k^*,z_k^*,\sigma_{e,k}^2)) \\ &+ \mathrm{tr}(\mathbf{Y}^*\mathbf{Q}_x^*) + \mathrm{tr}(\mathbf{Z}^*\mathbf{Q}_z^*) + \lambda_{P_c}^*(P_c - \mathrm{tr}(\mathbf{Q}_z^*)) \\ &+ \lambda_1^*(v^* - \theta_b\varrho^*) + \sum_{k=1}^{K}\lambda_{2,k}^*(\theta_e z_k^* - u_k^*) + \lambda_\psi^*\psi^* + \lambda_\alpha^*\alpha^* \\ &+ \sum_{k=1}^{K}\lambda_{\varphi_k}^*\varphi_k^* + \sum_{k=1}^{K}\lambda_{\beta_k}^*\beta_k^* = 0.\end{aligned} \tag{C.5}$$

Consider that if $\mathbf{\Omega} \succeq \mathbf{0}$, $\mathbf{\Upsilon} \succeq \mathbf{0}$ then $\mathrm{tr}(\mathbf{\Omega\Upsilon}) \geq 0$. From (C.5), it can be known that

$$\mathrm{tr}(\mathbf{A}^* T_{h,b}(\mathbf{Q}_x^*,\psi^*,v^*)) = 0, \tag{C.6}$$

$$\mathrm{tr}(\mathbf{Y}^*\mathbf{Q}_x^*) = 0. \tag{C.7}$$

Recall that, when $\mathbf{\Omega} \succeq \mathbf{0}$ and $\mathbf{\Upsilon} \succeq \mathbf{0}$, $\mathrm{tr}(\mathbf{\Omega\Upsilon}) = \mathbf{0}$ is equivalent to $\mathbf{\Omega\Upsilon} = \mathbf{0}$. Hence, eqs. (C.6)-(C.7) can be re-written as

$$\mathbf{A}^* T_{h,b}(\mathbf{Q}_x^*,\psi^*,v^*) = \mathbf{0}, \tag{C.8}$$

$$\mathbf{Y}^*\mathbf{Q}_x^* = \mathbf{0}. \tag{C.9}$$

Let $q_{l,n}^x$ denote the entry at the l-th row and n-th column of matrix $\mathbf{Q}_x$ with $l \leq n$. When $l = n$, $q_{l,n}^x$ is on the diagonal of $\mathbf{Q}_x$ and the KKT condition in (C.4) which is corresponding to $q_{l,n}^x$ can be denoted by $\mathrm{tr}(\mathbf{F}_{q_{l,n}^x}\mathbf{E}^*) = c_{q_{l,n}^x}$. Suppose $\mathrm{tr}(\mathbf{F}_{q_{l,n}^x}\mathbf{E}^*)$ is in the l-th position of the diagonal of a matrix which is on the left side of an equation (left matrix) and $c_{q_{l,n}^x}$ is in the l-th position of the diagonal of a matrix which is on the right side of an equation (right

matrix). When $l < n$, let $q_{l,n}^{x,R}$ and $q_{l,n}^{x,I}$ be the real part and imaginary part of $q_{l,n}^{x}$ respectively. The KKT condition in (C.4) which are associated with $q_{l,n}^{x,R}$ and $q_{l,n}^{x,I}$ can be denoted as $\mathrm{tr}(\mathbf{F}_{q_{l,n}^{x,R}}\mathbf{E}^*) = c_{q_{l,n}^{x,R}}$ and $\mathrm{tr}(\mathbf{F}_{q_{l,n}^{x,I}}\mathbf{E}^*) = c_{q_{l,n}^{x,I}}$. Let $j = \sqrt{-1}$. Suppose $(\mathrm{tr}(\mathbf{F}_{q_{l,n}^{x,R}}\mathbf{E}^*) + \mathrm{tr}(\mathbf{F}_{q_{l,n}^{x,I}}\mathbf{E}^*)j)/2$ is at the l-th row and n-th column of the left matrix; $(\mathrm{tr}(\mathbf{F}_{q_{l,n}^{x,R}}\mathbf{E}^*) + \mathrm{tr}(\mathbf{F}_{q_{l,n}^{x,I}}\mathbf{E}^*/j))/2$ is at the n-th row and l-th column of the left matrix; $(c_{q_{l,n}^{x,R}} + c_{q_{l,n}^{x,I}}j)/2$ is at the l-th row and n-th column of the right matrix; and $(c_{q_{l,n}^{x,R}} + c_{q_{l,n}^{x,I}}/j)/2$ is at the n-th row and l-th column of the right matrix. Then all such conditions form a matrix equation

$$\mathbf{Y}^* + (\mathbf{I}, \tilde{\mathbf{h}}_b)\mathbf{A}^*(\mathbf{I}, \tilde{\mathbf{h}}_b)^H - \sum_{k=1}^{K}(\mathbf{I}, \tilde{\mathbf{h}}_{e,k})\mathbf{B}_k^*(\mathbf{I}, \tilde{\mathbf{h}}_{e,k})^H = \mathbf{I}. \tag{C.10}$$

According to (C.1), we know $T_{g,b}(\mathbf{Q}_z^*, \alpha^*, \varrho^*, \sigma_b^2) \succeq \mathbf{0}$ which leads to $\varrho^* \geq \sigma_b^2 + \tilde{\mathbf{g}}_b^H\mathbf{Q}_z^*\tilde{\mathbf{g}}_b + \alpha\xi_{g,b}^2 > 0$. As $\theta_b > 0$, we have $v^* \geq \theta_b\varrho^* > 0$. Then assume $\psi^* = 0$, we have

$$\begin{aligned}&(-\tilde{\mathbf{h}}_b^H, 1)T_{h,b}(\mathbf{Q}_x^*, \psi^*, v^*)(-\tilde{\mathbf{h}}_b^H, 1)^H \\ &= (-\tilde{\mathbf{h}}_b^H, 1)\begin{pmatrix} \mathbf{Q}_x^* & \mathbf{Q}_x^*\tilde{\mathbf{h}}_b \\ \tilde{\mathbf{h}}_b^H\mathbf{Q}_x^* & \tilde{\mathbf{h}}_b^H\mathbf{Q}_x^*\tilde{\mathbf{h}}_b - v^* \end{pmatrix}(-\tilde{\mathbf{h}}_b^H, 1)^H = -v^* < 0.\end{aligned} \tag{C.11}$$

This contradicts that $T_{h,b}(\mathbf{Q}_x^*, \psi^*, v^*) \succeq \mathbf{0}$. Consequently, $\psi^* > 0$, which leads to $\psi^*\mathbf{I} + \mathbf{Q}_x^* \succ \mathbf{0}$. This implies that $\mathrm{rank}(T_{h,b}(\mathbf{Q}_x^*, \psi^*, v^*)) \geq N_a$ according to (3.11). Because of (C.8), $T_{h,b}(\mathbf{Q}_x^*, \psi^*, v^*)\ (\mathbf{A}^*)^H = \mathbf{0}$. The column space of $(\mathbf{A}^*)^H$ lies in the null space of $T_{h,b}(\mathbf{Q}_x^*, \psi^*, v^*)$. Then

$$\begin{aligned}\mathrm{rank}(\mathbf{A}^*) &= \mathrm{rank}(\mathbf{A}^{*H}) \\ &\leq \dim(\ker(T_{h,b}(\mathbf{Q}_x^*, \psi^*, v^*))) = N_a + 1 - \mathrm{rank}(T_{h,b}(\mathbf{Q}_x^*, \psi^*, v^*)) \\ &\leq N_a + 1 - N_a = 1.\end{aligned} \tag{C.12}$$

Multiplying (C.10) by $\mathbf{Q}_x^*$ and using (C.9), we have

$$\left(\mathbf{I} + \sum_{k=1}^{K}(\mathbf{I}, \tilde{\mathbf{h}}_{e,k})\mathbf{B}_k^*(\mathbf{I}, \tilde{\mathbf{h}}_{e,k})^H\right)\mathbf{Q}_x^* = (\mathbf{I}, \tilde{\mathbf{h}}_b)\mathbf{A}^*(\mathbf{I}, \tilde{\mathbf{h}}_b)^H\mathbf{Q}_x^*. \tag{C.13}$$

As $\left(\mathbf{I} + \sum_{k=1}^{K} (\mathbf{I}, \tilde{\mathbf{h}}_{e,k})\mathbf{B}_k^*(\mathbf{I}, \tilde{\mathbf{h}}_{e,k})^H\right)$ is positive definite, we must have

$$\begin{aligned}
\operatorname{rank}(\mathbf{Q}_x^*) &= \operatorname{rank}\left(\left(\mathbf{I} + \sum_{k=1}^{K}\left(\mathbf{I}, \tilde{\mathbf{h}}_{e,k}\right)\mathbf{B}_k^*\left(\mathbf{I}, \tilde{\mathbf{h}}_{e,k}\right)^H\right)\mathbf{Q}_x^*\right) \\
&= \operatorname{rank}\left(\left(\mathbf{I}, \tilde{\mathbf{h}}_b\right)\mathbf{A}^*\left(\mathbf{I}, \tilde{\mathbf{h}}_b\right)^H\mathbf{Q}_x^*\right) \\
&\leq \operatorname{rank}(\mathbf{A}^*) \leq 1.
\end{aligned} \tag{C.14}$$

Assume $\mathbf{Q}_x^* = \mathbf{0}$, then

$$T_{h,b}(\mathbf{Q}_x^*, \psi^*, v^*) = \begin{pmatrix} \psi^*\mathbf{I} & \mathbf{0} \\ \mathbf{0} & -v^* - \psi^*\xi_{h,b}^2 \end{pmatrix} \not\succeq \mathbf{0} \tag{C.15}$$

because $v^* > 0$. This contradicts the fact $T_{h,b}(\mathbf{Q}_x^*, \psi^*, v^*) \succeq \mathbf{0}$. So $\operatorname{rank}(\mathbf{Q}_x^*) = 1$.

Appendix D

Proof of Proposition 3.3

We first prove the conclusion which is under the condition $\varpi^+ > 0$. When we let $\theta_b = \phi_b^+$ and $\theta_e = \phi_e^+$, $\{\mathbf{Q}_x^+, \mathbf{Q}_z^+\}$ is feasible in problem (B.4). Assume $\{\mathbf{Q}_x^+, \mathbf{Q}_z^+\}$ is not optimal. Then, for an optimal solution $\{\bar{\mathbf{Q}}_x^+, \bar{\mathbf{Q}}_z^+\}$ to (B.4), there must be $\mathrm{tr}(\bar{\mathbf{Q}}_x^+) < \mathrm{tr}(\mathbf{Q}_x^+) \leq P_s$. Thus, we can see $\{\bar{\mathbf{Q}}_x^+, \bar{\mathbf{Q}}_z^+, \phi_b^+, \phi_e^+\}$ satisfies all the constraints in (3.20) and becomes an optimal solution to (3.20). Consequently, $\{\bar{\mathbf{Q}}_x^+, \bar{\mathbf{Q}}_z^+\}$ is an optimal solution to (3.19). By denoting $\{\bar{\mathbf{e}}_{h,b}^+, \bar{\mathbf{e}}_{g,b}^+, \bar{\mathbf{e}}_{h,e,k}^+|_{k=1,\ldots,K}, \bar{\mathbf{e}}_{g,e,k}^+|_{k=1,\ldots,K}, k^+\}$ as an optimal solution of the objective function in (3.19) when $\mathbf{Q}_x = \bar{\mathbf{Q}}_x^+$, $\mathbf{Q}_z = \bar{\mathbf{Q}}_z^+$ and letting

$$a^+ = (\tilde{\mathbf{h}}_b + \bar{\mathbf{e}}_{h,b}^+)^H \bar{\mathbf{Q}}_x^+ (\tilde{\mathbf{h}}_b + \bar{\mathbf{e}}_{h,b}^+),$$

$$b^+ = (\tilde{\mathbf{g}}_b + \bar{\mathbf{e}}_{g,b}^+)^H \bar{\mathbf{Q}}_z^+ (\tilde{\mathbf{g}}_b + \bar{\mathbf{e}}_{g,b}^+) + \sigma_b^2$$

$$c_k^+ = (\tilde{\mathbf{h}}_{e,k} + \bar{\mathbf{e}}_{h,e,k}^+)^H \bar{\mathbf{Q}}_x^+ (\tilde{\mathbf{h}}_{e,k} + \bar{\mathbf{e}}_{h,e,k}^+),$$

$$d_k^+ = (\tilde{\mathbf{g}}_{e,k} + \bar{\mathbf{e}}_{g,e,k}^+)^H \bar{\mathbf{Q}}_z^+ (\tilde{\mathbf{g}}_{e,k} + \bar{\mathbf{e}}_{g,e,k}^+) + \sigma_{e,k}^2,$$

we can express the optimal value of problem (3.19) as $r^+ = \log_2((1 + a^+/b^+)/(1 + c_{k^+}^+/d_{k^+}^+))$. Owing to $\mathrm{tr}(\bar{\mathbf{Q}}_x^+) < P_s$, there must be some $\rho > 1$ such that $\mathrm{tr}(\rho\bar{\mathbf{Q}}_x^+) \leq P_s$. Therefore, $\{\rho\bar{\mathbf{Q}}_x^+, \bar{\mathbf{Q}}_z^+\}$ is feasible in (3.19). For $\{\rho\bar{\mathbf{Q}}_x^+, \bar{\mathbf{Q}}_z^+\}$, because of $\log_2((1 + \rho a^+/b^+)/(1 + \rho c_{k^+}^+/d_{k^+}^+)) \leq \log_2((1 + \rho a^+/b^+)/(1 + \rho c_k^+/d_k^+))\ \forall 1 \leq k \leq K$ and $k \neq k^+$, we can see $\{\bar{\mathbf{e}}_{h,b}^+, \bar{\mathbf{e}}_{g,b}^+, \bar{\mathbf{e}}_{h,e,k}^+|_{k=1,\ldots,K}, \bar{\mathbf{e}}_{g,e,k}^+|_{k=1,\ldots,K}, k^+\}$ is still optimal in the objective function of (3.19). Thus, when $\mathbf{Q}_x = \rho\bar{\mathbf{Q}}_x^+$, $\mathbf{Q}_z = \bar{\mathbf{Q}}_z^+$, the value of the objective function of (3.19) is $r_\rho = \log_2((1 + \rho a^+/b^+)/(1 + \rho c_{k^+}^+/d_{k^+}^+))$. Since $\varpi^+ > 0$ and problem (3.19) is equivalent to (3.20), we have $a^+ d_{k^+}^+ > b^+ c_{k^+}^+$ from $r^+ > 0$. Thus, the function $f(x) = \log_2((1 + a^+x/b^+)/(1 + c_{k^+}^+x/d_{k^+}^+))$ is strictly increasing. Through $f(x)$ and $\rho > 1$, we have $r_\rho > r^+$, which leads to a

contradiction. Therefore $\{\mathbf{Q}_x^+, \mathbf{Q}_z^+\}$ is an optimal solution to problem (B.4) for $\theta_b = \phi_b^+$ and $\theta_e = \phi_e^+$. Since $\{\mathbf{Q}_x^+, \mathbf{Q}_z^+, \phi_b^+, \phi_e^+\}$ is optimal in (3.20), we must have $\theta_b = \phi_b^+$ and $\theta_e = \phi_e^+$ are feasible in (3.17). In addition, because of $\varpi^+ > 0$, we know $\theta_b = \phi_b^+ > 0$. By applying the equivalent relationship between (B.4) and (3.18) and Theorem 3.1, we can see $\text{rank}(\mathbf{Q}_x^+) = 1$. Thus, $\mathbf{Q}_x^+$ can be decomposed as $\mathbf{Q}_x^+ = \mathbf{w}^+(\mathbf{w}^+)^H$ and $\{\mathbf{w}^+, \mathbf{Q}_z^+\}$ is an optimal solution to P-SRM-A.

Problem (3.20) is a relaxation of P-SRM-A, which implies $\varpi^+ \geq R^+$. Since $\{\mathbf{Q}_x = \mathbf{0}, \mathbf{Q}_z = \mathbf{0}, \phi_b = 0, \phi_e = 0\}$ lies in the feasible set of (3.20), we have $\varpi^+ \geq 0$. If ϖ^+ is not greater than 0, there must be $R^+ \leq \varpi^+ = 0$. Together with $R^+ \geq 0$, we know $R^+ = 0$ and $\{\mathbf{w}^+ = \mathbf{0}, \mathbf{Q}_z^+ = \mathbf{0}\}$ is an optimal solution to P-SRM-A.

Appendix E

Proof of Proposition 3.4

By introducing the slack variables u_k and z_k, constraint (3.20b) can be reformulated as

$$\mathbf{h}_{e,k}^H \mathbf{Q}_x \mathbf{h}_{e,k} \leq u_k,\ \forall \mathbf{e}_{h,e,k} \in \varepsilon_{h,e,k} \tag{E.1}$$

$$\mathbf{g}_{e,k}^H \mathbf{Q}_z \mathbf{g}_{e,k} + \sigma_{e,k}^2 \geq z_k,\ \forall \mathbf{e}_{g,e,k} \in \varepsilon_{g,e,k} \tag{E.2}$$

$$u_k \leq z_k \phi_e,\ u_k \geq 0, z_k \geq 0,\ k = 1, \ldots, K. \tag{E.3}$$

Then, according to the Charnes-Cooper method, we let $\mathbf{X}_\eta = \mathbf{Q}_x \eta$, $\mathbf{Z}_\eta = \mathbf{Q}_z \eta$, $u_{\eta,k} = u_k \eta$, $z_{\eta,k} = z_k \eta$ and

$$\eta = \frac{1}{\max_{\mathbf{e}_{g,b} \in \varepsilon_{g,b}} \mathbf{g}_b^H \mathbf{Q}_z \mathbf{g}_b + \sigma_b^2}. \tag{E.4}$$

Through (E.1)-(E.4), we have another form of (3.20) for fixed ϕ_e

$$\max_{\substack{\mathbf{X}_\eta, \mathbf{Z}_\eta, \eta, \phi_b, u_{\eta,1}, \ldots, u_{\eta,K} \\ z_{\eta,1}, \ldots, z_{\eta,K}}} \log_2(1 + \phi_b) - \log_2(1 + \phi_e) \tag{E.5a}$$

$$\text{s.t.} \mathbf{h}_{e,k}^H \mathbf{X}_\eta \mathbf{h}_{e,k} \leq u_{\eta,k},\ \forall \mathbf{e}_{h,e,k} \in \varepsilon_{h,e,k} \tag{E.5b}$$

$$\mathbf{g}_{e,k}^H \mathbf{Z}_\eta \mathbf{g}_{e,k} + \sigma_{e,k}^2 \eta \geq z_{\eta,k},\ \forall \mathbf{e}_{g,e,k} \in \varepsilon_{g,e,k} \tag{E.5c}$$

$$u_{\eta,k} \leq z_{\eta,k} \phi_e,\ u_{\eta,k} \geq 0, z_{\eta,k} \geq 0,\ k = 1, \ldots, K \tag{E.5d}$$

$$\min_{\mathbf{e}_{h,b} \in \varepsilon_{h,b}} \mathbf{h}_b^H \mathbf{X}_\eta \mathbf{h}_b \geq \phi_b \tag{E.5e}$$

$$\max_{\mathbf{e}_{g,b} \in \varepsilon_{g,b}} \mathbf{g}_b^H \mathbf{Z}_\eta \mathbf{g}_b + \sigma_b^2 \eta = 1 \tag{E.5f}$$

$$\text{tr}(\mathbf{X}_\eta) \leq P_s \eta,\ \text{tr}(\mathbf{Z}_\eta) \leq P_c \eta,\ \mathbf{X}_\eta \succeq \mathbf{0},\ \mathbf{Z}_\eta \succeq \mathbf{0}, \eta > 0, \phi_b \geq 0. \tag{E.5g}$$

Then, consider a convex semidefinite programming (SDP) problem

$$\begin{aligned}
&\max_{\substack{\mathbf{X}_\eta,\mathbf{Z}_\eta,\eta,\phi_b,u_{\eta,1},\ldots,u_{\eta,K}\\ z_{\eta,1},\ldots,z_{\eta,K}}} \log_2(1+\phi_b)-\log_2(1+\phi_e)\\
&\text{s.t.(E.5b)}-\text{(E.5e)}\\
&\quad \max_{\mathbf{e}_{g,b}\in\varepsilon_{g,b}} \mathbf{g}_b^H\mathbf{Z}_\eta\mathbf{g}_b+\sigma_b^2\eta\le 1\\
&\quad \mathrm{tr}(\mathbf{X}_\eta)\le P_s\eta,\ \mathrm{tr}(\mathbf{Z}_\eta)\le P_c\eta,\ \mathbf{X}_\eta\succeq\mathbf{0},\ \mathbf{Z}_\eta\succeq\mathbf{0},\eta\ge 0,\phi_b\ge 0.
\end{aligned} \tag{E.6}$$

which can be equivalently rewritten as (3.21) by the S-procedure. Obviously, the optimal values of (E.5) and (E.6) are $\varpi(\phi_e)$ and $\chi(\phi_e)$, respectively. Let $\{\mathbf{X}_\eta^*,\mathbf{Z}_\eta^*,\eta^*,\phi_b^*,u_{\eta,1}^*,\ldots,u_{\eta,K}^*,z_{\eta,1}^*,\ldots,z_{\eta,K}^*\}$ denote an optimal solution to (E.6). Assume $\chi(\phi_e)>0$. Because of $\chi(\phi_e)=\log_2(1+\phi_b^*)-\log_2(1+\phi_e)$, we have $\phi_b^*>0$. Suppose $\max_{\mathbf{e}_{g,b}\in\varepsilon_{g,b}}\mathbf{g}_b^H\ \mathbf{Z}_\eta^*\mathbf{g}_b+\sigma_b^2\eta^*<1$. Then there must be some $\kappa>1$ which can make $\{\kappa\mathbf{X}_\eta^*,\kappa\mathbf{Z}_\eta^*,\kappa\eta^*,\ \kappa\phi_b^*,\kappa_{\eta,1}^*,\ldots,\kappa u_{\eta,K}^*,\kappa z_{\eta,1}^*,\ldots,\kappa z_{\eta,K}^*\}$ be feasible in (E.6). However, due to $\phi_b^*>0$, we know $\log_2(1+\kappa\phi_b^*)-\log_2(1+\phi_e)>\chi(\phi_e)=\log_2(1+\phi_b^*)-\log_2(1+\phi_e)$, which is a contradiction. Therefore, $\mathbf{Z}_\eta^*$ and η^* must satisfy (E.5f). In addition, we have $\eta^*>0$. Otherwise, constraints (E.5e), $\mathrm{tr}(\mathbf{X}_\eta)\le P_s\eta$ and $\mathbf{X}_\eta\succeq\mathbf{0}$ cannot be met at the same time. Therefore, eqs. (E.5) and (E.6) are equivalent, through which we have $\varpi(\phi_e)=\chi(\phi_e)$ and (3.21) is equivalent to (E.5). Since (E.5) is converted from (3.20) through the Charnes-Cooper method, we have $\tilde{\mathbf{Q}}_x(\phi_e)=\tilde{\mathbf{X}}_\eta(\phi_e)/\tilde{\eta}(\phi_e)$, $\tilde{\mathbf{Q}}_z(\phi_e)=\tilde{\mathbf{Z}}_\eta(\phi_e)/\tilde{\eta}(\phi_e)$. Because (E.6) is a relaxation of (E.5), there must be $\varpi(\phi_e)\le\chi(\phi_e)$. Thus, when $\chi(\phi_e)\le 0$, we conclude that $\varpi(\phi_e)\le\chi(\phi_e)\le 0$.

Appendix F

Proof of Proposition 3.5

Because $\ddot{\mathbf{s}} = [\mathrm{vec}(\ddot{\mathbf{Q}}_x), \ddot{v}, \ddot{\psi}]$ is optimal in (3.27), $\ddot{\mathbf{s}}$ must satisfy the constraint (3.27b). Due to the properties of semidefinite matrix, we know

$$T_{h,b}(\ddot{\mathbf{Q}}_x, \ddot{\psi}, \ddot{v}) \succeq \mathbf{0}, \ddot{\mathbf{Q}}_x \succeq \mathbf{0}, \ddot{v} \geq \gamma_b \varrho^-, \ddot{\psi} \geq 0. \tag{F.1}$$

The assumption that $\{\mathbf{Q}_x^-,\ \mathbf{Q}_z^-,\ \psi^-, \alpha^-, v^-, \varrho^-\}$ is an optimal solution to (3.26) implies

$$T_{g,b}(\mathbf{Q}_z^-, \alpha^-, \varrho^-, \sigma_b^2) \succeq \mathbf{0}, \alpha^- \geq 0, \varrho^- > 0, \mathrm{tr}(\mathbf{Q}_z^-) \leq P_c,\ \mathbf{Q}_z^- \succeq \mathbf{0} \tag{F.2}$$

where $\varrho^- > 0$ is owing to $T_{g,b}(\mathbf{Q}_z^-, \alpha^-, \varrho^-, \sigma_b^2) \succeq \mathbf{0}$. Furthermore, it can be also known that $\mathbf{Q}_x^-$ is feasible in (3.27). Therefore, there must be

$$\mathrm{tr}(\ddot{\mathbf{Q}}_x) \leq \mathrm{tr}(\mathbf{Q}_x^-) \leq P_s. \tag{F.3}$$

From $\ddot{v} \geq \gamma_b \varrho^-$, $\varrho^- > 0$ and $\gamma_b > 0$, we have $\ddot{v} > 0$ which, together with (F.1), (F.2) and (F.3), leads to the result that $\{\ddot{\mathbf{Q}}_x,\ \mathbf{Q}_z^-,\ \ddot{\psi}, \alpha^-, \ddot{v}, \varrho^-\}$ satisfies the constraints of (3.26) and is an optimal solution to it.

The inequality in (F.3) implies the boundedness of $\ddot{\mathbf{Q}}_x$. Because of $T_{h,b}(\ddot{\mathbf{Q}}_x, \ddot{\psi}, \ddot{v}) \succeq \mathbf{0}$, $\ddot{v}$ and $\ddot{\psi}$ must be bounded. Therefore, $\ddot{\mathbf{s}}$ lies in a bounded set and strong duality can be proved to be valid for (3.27) through the method similar to the one used in Appendix A.

The remaining part of this proof follows the logic line in Appendix B and we briefly describe it. Through the KKT conditions that $\ddot{\mathbf{s}}$ must satisfy, we can obtain the following useful equations

$$\ddot{\mathbf{A}} T_{h,b}(\ddot{\mathbf{Q}}_x, \ddot{\psi}, \ddot{v}) = \mathbf{0} \tag{F.4}$$

$$\ddot{\mathbf{Y}}\ddot{\mathbf{Q}}_x = \mathbf{0} \tag{F.5}$$

$$\mathbf{I} - (\mathbf{I}, \tilde{\mathbf{h}}_b)\ddot{\mathbf{A}}(\mathbf{I}, \tilde{\mathbf{h}}_b)^H = \ddot{\mathbf{Y}} \tag{F.6}$$

where $\ddot{\mathbf{A}}$ and $\ddot{\mathbf{Y}}$ are positive semidefinite optimal dual variables. Because of $\ddot{v} > 0$, we have $\ddot{\psi} > 0$. Otherwise, $(-\tilde{\mathbf{h}}_b^H, 1)T_{h,b}(\ddot{\mathbf{Q}}_x, \ddot{\psi}, \ddot{v})(-\tilde{\mathbf{h}}_b^H, 1)^H = -\ddot{v} < 0$ contradicts $T_{h,b}(\ddot{\mathbf{Q}}_x, \ddot{\psi}, \ddot{v}) \succeq \mathbf{0}$. Thus, we know $\text{rank}(T_{h,b}(\ddot{\mathbf{Q}}_x, \ddot{\psi}, \ddot{v})) \geq N_a$. This, together with (F.4), leads to $\text{rank}(\ddot{\mathbf{A}}) \leq 1$. Then, by multiplying both sides of equation (F.6) by $\ddot{\mathbf{Q}}_x$ and employing (F.5), we can see $\text{rank}(\ddot{\mathbf{Q}}_x) = \text{rank}((\mathbf{I}, \tilde{\mathbf{h}}_b)\ddot{\mathbf{A}}(\mathbf{I}, \tilde{\mathbf{h}}_b)^H\ddot{\mathbf{Q}}_x) \leq \text{rank}(\ddot{\mathbf{A}}) \leq 1$. Since $T_{h,b}(\ddot{\mathbf{Q}}_x, \ddot{\psi}, \ddot{v}) \succeq \mathbf{0}$ forces $\ddot{\mathbf{Q}}_x \neq \mathbf{0}$, we have $\text{rank}(\ddot{\mathbf{Q}}_x) = 1$ and $\ddot{\mathbf{Q}}_x$ can be decomposed into $\ddot{\mathbf{w}}\ddot{\mathbf{w}}^H$. Since $\{\ddot{\mathbf{Q}}_x, \mathbf{Q}_z^-, \ddot{\psi}, \alpha^-, \ddot{v}, \varrho^-\}$ is optimal to (3.26) and (3.26) is equivalent to the SDR of P-QoS-B, we see $\{\ddot{\mathbf{w}}, \mathbf{Q}_z^-\}$ is an optimal solution to the original design problem P-QoS-B.

Appendix G

Proof of Proposition 4.1

Part. I

It can be easily observed that $\sum_{l\in\mathcal{L}}(\ddot{y}_{l,k}+\sum_{j\in\mathcal{J}_{l,\triangle}}\ddot{u}_{l,j,k})\geq\check{\Lambda}_k(\varpi_k),\forall k\in\mathcal{K}$. Since $\{\mathbf{W}^*_{l,j},\forall j\in\mathcal{J}_{l,\nabla},\ \mathbf{Q}^*_l\}$ is optimal to (4.15), according to (4.15d), we have $y^*_{l,k}=\sum\limits_{j\in\mathcal{J}_{l,\nabla}}\mathrm{tr}\left(\mathbf{G}_{l,k}\mathbf{W}^*_{l,j}\right)+\mathrm{tr}\left(\mathbf{G}_{l,k}\mathbf{Q}^*_l\right)\geq\ddot{y}_{l,k},\ \forall k\in\mathcal{K}$. Therefore, $\sum_{l\in\mathcal{L}}(y^*_{l,k}+\sum_{j\in\mathcal{J}_{l,\triangle}}\ddot{u}_{l,j,k})\geq\check{\Lambda}_k(\varpi_k),\forall k\in\mathcal{K}$, which implies that $\{\ddot{\mathbf{W}}_{l,i},\mathbf{W}^*_{l,j},\mathbf{Q}^*_l,\forall i\in\mathcal{J}_{l,\triangle},\forall j\in\mathcal{J}_{l,\nabla},\forall l\in\mathcal{L}\}$ satisfies the constraints (4.14b). Moreover, the feasibility of $\{\ddot{\mathbf{W}}_{l,j},\ddot{\mathbf{Q}}_l,\forall j\in\mathcal{J}_l,\forall l\in\mathcal{L}\}$ in (4.14) leads to

$$\sum_{j\in\mathcal{J}_l}\mathrm{tr}\left(\ddot{\mathbf{W}}_{l,j}\right)+\mathrm{tr}(\ddot{\mathbf{Q}}_l)\leq P_l^{\max},\forall l\in\mathcal{L}. \tag{G.1}$$

The feasibility of $\{\ddot{\mathbf{W}}_{l,j},\forall j\in\mathcal{J}_{l,\nabla},\ddot{\mathbf{Q}}_l\}$ and the optimality of $\{\mathbf{W}^*_{l,j},\forall j\in\mathcal{J}_{l,\nabla},\ \mathbf{Q}^*_l\}$ in (4.15) lead to

$$\sum_{j\in\mathcal{J}_{l,\nabla}}\mathrm{tr}\left(\mathbf{W}^*_{l,j}\right)+\mathrm{tr}(\mathbf{Q}^*_l)\leq\sum_{j\in\mathcal{J}_{l,\nabla}}\mathrm{tr}\left(\ddot{\mathbf{W}}_{l,j}\right)+\mathrm{tr}(\ddot{\mathbf{Q}}_l) \tag{G.2}$$

which holds for BS l. Combining (G.1) and (G.2), we have

$$\sum_{j\in\mathcal{J}_{l,\triangle}}\mathrm{tr}\left(\ddot{\mathbf{W}}_{l,j}\right)+\sum_{j\in\mathcal{J}_{l,\nabla}}\mathrm{tr}\left(\mathbf{W}^*_{l,j}\right)+\mathrm{tr}(\mathbf{Q}^*_l)\leq P_l^{\max},\forall l\in\mathcal{L}$$

i.e., $\{\ddot{\mathbf{W}}_{l,i},\mathbf{W}^*_{l,j},\mathbf{Q}^*_l,\forall i\in\mathcal{J}_{l,\triangle},\forall j\in\mathcal{J}_{l,\nabla},\forall l\in\mathcal{L}\}$ satisfies the constraint (4.14c). Since constraint (4.14d) is also satisfied by $\{\ddot{\mathbf{W}}_{l,i},\mathbf{W}^*_{l,j},\mathbf{Q}^*_l,\forall i\in\mathcal{J}_{l,\triangle},\forall j\in\mathcal{J}_{l,\nabla},\forall l\in\mathcal{L}\}$, we know it is feasible in (4.14).

Furthermore, from constraints (4.15b)-(4.15f), we can obtain

$$\begin{aligned}\frac{\operatorname{tr}\left(\mathbf{H}_{l,l,j}\mathbf{W}_{l,j}^{*}\right)}{\ddot{\Gamma}_{l,j}} \geq & \sum_{i\in\mathcal{J}_{l,\triangle}} \operatorname{tr}\left(\mathbf{H}_{l,l,j}\ddot{\mathbf{W}}_{l,i}\right) + \sum_{u\in\mathcal{J}_{l,\nabla},u\neq j} \operatorname{tr}\left(\mathbf{H}_{l,l,j}\mathbf{W}_{l,u}^{*}\right) \\ & + \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_{u,\nabla}} \operatorname{tr}\left(\mathbf{H}_{u,l,j}\mathbf{W}_{u,i}^{*}\right) + \sum_{u\in\mathcal{L},u\neq l} \operatorname{tr}\left(\mathbf{H}_{u,l,j}\mathbf{Q}_{u}^{*}\right) \\ & + \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_{u,\triangle}} \operatorname{tr}\left(\mathbf{H}_{u,l,j}\ddot{\mathbf{W}}_{u,i}\right) + \operatorname{tr}\left(\mathbf{H}_{l,l,j}\mathbf{Q}_{l}^{*}\right) + \sigma_{l,j}^{2}, \\ & \forall j\in\mathcal{J}_{l,\nabla}, \forall l\in\mathcal{L},\end{aligned}$$

$$\begin{aligned}\frac{\operatorname{tr}\left(\mathbf{H}_{l,l,j}\ddot{\mathbf{W}}_{l,j}\right)}{\ddot{\Gamma}_{l,j}} \geq & \sum_{i\in\mathcal{J}_{l,\triangle},i\neq j} \operatorname{tr}\left(\mathbf{H}_{l,l,j}\ddot{\mathbf{W}}_{l,i}\right) + \sum_{u\in\mathcal{J}_{l,\nabla}} \operatorname{tr}\left(\mathbf{H}_{l,l,j}\mathbf{W}_{l,u}^{*}\right) \\ & + \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_{u,\nabla}} \operatorname{tr}\left(\mathbf{H}_{u,l,j}\mathbf{W}_{u,i}^{*}\right) + \sum_{u\in\mathcal{L},u\neq l} \operatorname{tr}\left(\mathbf{H}_{u,l,j}\mathbf{Q}_{u}^{*}\right) \\ & + \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_{u,\triangle}} \operatorname{tr}\left(\mathbf{H}_{u,l,j}\ddot{\mathbf{W}}_{u,i}\right) + \operatorname{tr}\left(\mathbf{H}_{l,l,j}\mathbf{Q}_{l}^{*}\right) + \sigma_{l,j}^{2}, \\ & \forall j\in\mathcal{J}_{l,\triangle}, \forall l\in\mathcal{L},\end{aligned}$$

$$\begin{aligned}\frac{\operatorname{tr}\left(\mathbf{G}_{l,k}\mathbf{W}_{l,j}^{*}\right)}{\ddot{\Gamma}_{l,j,k}} \leq & \sum_{i\in\mathcal{J}_{l,\triangle}} \operatorname{tr}\left(\mathbf{G}_{l,k}\ddot{\mathbf{W}}_{l,i}\right) + \sum_{u\in\mathcal{J}_{l,\nabla},u\neq j} \operatorname{tr}\left(\mathbf{G}_{l,k}\mathbf{W}_{l,u}^{*}\right) \\ & + \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_{u,\nabla}} \operatorname{tr}\left(\mathbf{G}_{u,k}\mathbf{W}_{u,i}^{*}\right) + \sum_{u\in\mathcal{L},u\neq l} \operatorname{tr}\left(\mathbf{G}_{u,k}\mathbf{Q}_{u}^{*}\right) \\ & + \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_{u,\triangle}} \operatorname{tr}\left(\mathbf{G}_{u,k}\ddot{\mathbf{W}}_{u,i}\right) + \operatorname{tr}\left(\mathbf{G}_{l,k}\mathbf{Q}_{l}^{*}\right) + \sigma_{k}^{2}, \\ & \forall j\in\mathcal{J}_{l,\nabla}, \forall k\in\mathcal{K}, \forall l\in\mathcal{L},\end{aligned}$$

$$\frac{\operatorname{tr}\left(\mathbf{G}_{l,k}\ddot{\mathbf{W}}_{l,j}\right)}{\ddot{\Gamma}_{l,j,k}} \leq \sum_{i\in\mathcal{J}_{l,\triangle},i\neq j} \operatorname{tr}\left(\mathbf{G}_{l,k}\ddot{\mathbf{W}}_{l,i}\right) + \sum_{u\in\mathcal{J}_{l,\nabla}} \operatorname{tr}\left(\mathbf{G}_{l,k}\mathbf{W}^*_{l,u}\right)$$
$$+ \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_{u,\nabla}} \operatorname{tr}\left(\mathbf{G}_{u,k}\mathbf{W}^*_{u,i}\right) + \sum_{u\in\mathcal{L},u\neq l} \operatorname{tr}\left(\mathbf{G}_{u,k}\mathbf{Q}^*_{u}\right)$$
$$+ \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_{u,\triangle}} \operatorname{tr}\left(\mathbf{G}_{u,k}\ddot{\mathbf{W}}_{u,i}\right) + \operatorname{tr}\left(\mathbf{G}_{l,k}\mathbf{Q}^*_{l}\right) + \sigma_k^2,$$
$$\forall j \in \mathcal{J}_{l,\triangle}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}.$$

Since

$$\bar{f}(\ddot{\mathbf{W}}_{l,j}, \ddot{\mathbf{Q}}_l, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L}) = \sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{J}_l} \ln(\log_2(1+\ddot{\Gamma}_{l,j}) - \max_{k\in\mathcal{K}} \log_2(1+\ddot{\Gamma}_{l,j,k}))$$

we can see that $\bar{f}(\ddot{\mathbf{W}}_{l,i}, \mathbf{W}^*_{l,j}, \mathbf{Q}^*_l, \forall i \in \mathcal{J}_{l,\triangle}, \forall j \in \mathcal{J}_{l,\nabla}, \forall l \in \mathcal{L}) \geq \bar{f}(\ddot{\mathbf{W}}_{l,j}, \ddot{\mathbf{Q}}_l, \forall j \in \mathcal{J}_l, \forall l \in \mathcal{L})$, which completes the proof for conclusion in i).

Part. II

In this part, we follow the logic similar to the one in [46] and [49] to prove the conclusion in ii).

The Lagrangian of problem (4.15) can be expressed as

$$L_l(\mathbf{W}_{l,j}, \mathbf{Q}_l; \alpha_{l,j}, \beta_{l,j}, \rho_{l,k}, \chi_{l,k,j}, \lambda_{l,u,j}) = \sum_{j\in\mathcal{J}_{l,\nabla}} \operatorname{tr}(\mathbf{A}_{l,j}\mathbf{W}_{l,j}) + \operatorname{tr}(\mathbf{B}_l\mathbf{Q}_l) + \varsigma \tag{G.3}$$

where

$$\mathbf{A}_{l,j} = \mathbf{I} + \sum_{i\in\mathcal{J}_{l,\triangle}} \alpha_{l,i}\mathbf{H}_{l,l,i} + \sum_{u\in\mathcal{J}_{l,\nabla},u\neq j} \beta_{l,u}\mathbf{H}_{l,l,u} - \frac{1}{\ddot{\Gamma}_{l,j}}\beta_{l,j}\mathbf{H}_{l,l,j} - \sum_{k\in\mathcal{K}} \rho_{l,k}\mathbf{G}_{l,k}$$
$$- \sum_{k\in\mathcal{K}}\sum_{u\in\mathcal{J}_{l,\nabla},u\neq j} \chi_{l,k,u}\mathbf{G}_{l,k} + \sum_{k\in\mathcal{K}} \frac{1}{\ddot{\Gamma}_{l,j,k}}\chi_{l,k,j}\mathbf{G}_{l,k} + \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_u} \lambda_{l,u,i}\mathbf{H}_{l,u,i},$$

$$\mathbf{B}_l = \mathbf{I} + \sum_{i\in\mathcal{J}_{l,\triangle}} \alpha_{l,i}\mathbf{H}_{l,l,i} + \sum_{i\in\mathcal{J}_{l,\nabla}} \beta_{l,i}\mathbf{H}_{l,l,i} - \sum_{k\in\mathcal{K}} \rho_{l,k}\mathbf{G}_{l,k}$$
$$- \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{J}_{l,\nabla}} \chi_{l,k,i}\mathbf{G}_{l,k} + \sum_{u\in\mathcal{L},u\neq l}\sum_{i\in\mathcal{J}_u} \lambda_{l,u,i}\mathbf{H}_{l,u,i}, \tag{G.4}$$

and ς is the sum of terms that are not related to $\mathbf{W}_{l,j}$'s and $\mathbf{Q}_l$, and $\alpha_{l,j}$,

$\beta_{l,j}$, $\rho_{l,k}$, $\chi_{l,k,j}$ and $\lambda_{l,u,j}$ are dual variables. To ensure that the Lagrangian (G.3) is bounded below such that the dual function is finite, there must be $\mathbf{A}_{l,j} \succeq \mathbf{0}, \forall j \in \mathcal{J}_{l,\nabla}$. Then, we have $\mathbf{A}^*_{l,j} \succeq \mathbf{0}, \forall j \in \mathcal{J}_{l,\nabla}$. Moreover, to reach the dual function for $\alpha^*_{l,j}$, $\beta^*_{l,j}$, $\rho^*_{l,k}$, $\chi^*_{l,k,j}$, and $\lambda^*_{l,u,j}$, we know $\mathrm{tr}(\mathbf{A}^*_{l,j}\mathbf{W}^*_{l,j}) = 0, \forall j \in \mathcal{J}_{l,\nabla}$, which implies $\mathbf{A}^*_{l,j}\mathbf{W}^*_{l,j} = \mathbf{0}, \forall j \in \mathcal{J}_{l,\nabla}$.

In the following, we focus on the case $j \in \check{\mathcal{J}}_{l,\nabla}$. Through the definition of $\mathbf{A}^*_{l,j}$ and $\mathbf{E}^*_{l,j}$, it is easy to see

$$\mathbf{A}^*_{l,j} = \mathbf{E}^*_{l,j} - \mathbf{H}_{l,l,j}. \tag{G.5}$$

Define

$$T = \begin{cases} T_M & \text{if } l = 0, \\ T_F & \text{if } l \in \mathcal{L}_F. \end{cases}$$

Suppose $\mathrm{rank}(\mathbf{E}^*_{l,j}) = T$. Then, we know from (G.5) that $\mathrm{rank}(\mathbf{A}^*_{l,j}) \geq T-1$. Because of $\mathbf{A}^*_{l,j}\mathbf{W}^*_{l,j} = \mathbf{0}$, we can conclude $\mathrm{rank}(\mathbf{W}^*_{l,j}) \leq 1$, which contradicts the assumption $\mathrm{rank}(\mathbf{W}^*_{l,j}) > 1$. Thus, $\mathrm{rank}(\mathbf{E}^*_{l,j}) < T$ and the dimension of the null space of $\mathbf{E}^*_{l,j}$ is $T - \mathrm{rank}(\mathbf{E}^*_{l,j})$. Let $\boldsymbol{\theta}_{l,j,\pi}$ denote the πth column of $\boldsymbol{\Theta}_{l,j}$, where $\boldsymbol{\Theta}_{l,j} \in \mathbb{C}^{T\times(T-\mathrm{rank}(\mathbf{E}^*_{l,j}))}$. Then, we have $\boldsymbol{\theta}^H_{l,j,\pi}\mathbf{A}^*_{l,j}\boldsymbol{\theta}_{l,j,\pi} = \boldsymbol{\theta}^H_{l,j,\pi}\left(\mathbf{E}^*_{l,j} - \mathbf{H}_{l,l,j}\right)\boldsymbol{\theta}_{l,j,\pi} = -\left|\mathbf{h}^H_{l,l,j}\boldsymbol{\theta}_{l,j,\pi}\right|^2$. Since $\mathbf{A}^*_{l,j} \succeq \mathbf{0}$, we can deduce that $\mathbf{h}^H_{l,l,j}\boldsymbol{\theta}_{l,j,\pi} = 0$, i.e., $\mathbf{H}_{l,l,j}\boldsymbol{\Theta}_{l,j} = \mathbf{0}$. Consequently, there exists $\mathbf{A}^*_j\boldsymbol{\Theta}_{l,j} = \mathbf{0}$, which means the $T - \mathrm{rank}(\mathbf{E}^*_{l,j})$ orthogonal column vectors of $\boldsymbol{\Theta}_{l,j}$ lie in the null space of $\mathbf{A}^*_{l,j}$. In addition, according to (G.5), we know $\mathrm{rank}(\mathbf{A}^*_{l,j}) \geq \mathrm{rank}(\mathbf{E}^*_{l,j}) - 1$, which leads to $T - \mathrm{rank}(\mathbf{E}^*_{l,j}) \leq \varphi = T - \mathrm{rank}(\mathbf{A}^*_{l,j}) \leq T - \mathrm{rank}(\mathbf{E}^*_{l,j}) + 1$ where φ denotes the dimension of the null space of $\mathbf{A}^*_{l,j}$. By exploiting [46, Prop. 4.1], it can be shown that $\varphi = T - \mathrm{rank}(\mathbf{E}^*_{l,j}) + 1$ and $\mathbf{W}^*_{l,j}$ can be expressed as $\mathbf{W}^*_{l,j} = \sum_{\pi=1}^{T-\mathrm{rank}(\mathbf{E}^*_{l,j})} \gamma_{l,j,\pi}\boldsymbol{\theta}_{l,j,\pi}\boldsymbol{\theta}^H_{l,j,\pi} + \tau_{l,j}\boldsymbol{\psi}_{l,j}\boldsymbol{\psi}^H_{l,j}$ where $\gamma_{l,j,\pi} \geq 0$ and $\tau_{l,j} > 0$.

Since $\mathbf{H}_{l,l,j}\boldsymbol{\Theta}_{l,j} = \mathbf{0}$, it is easy to check that $\{\mathbf{W}^*_{l,j}, \forall j \in \hat{\mathcal{J}}_{l,\nabla}, \bar{\mathbf{W}}^*_{l,j}, \forall j \in \check{\mathcal{J}}_{l,\nabla}, \bar{\mathbf{Q}}^*_l\}$ not only satisfies all the constraints in problem (4.15), but also achieves the same optimum value as $\{\mathbf{W}^*_{l,j}, \forall j \in \mathcal{J}_{l,\nabla}, \mathbf{Q}^*_l\}$, which completes the proof.

Appendix H

Proof of Proposition 4.2

From (4.18), we have $2^{b_{l,j}^{(\kappa+1)}} = 2^{b_{l,j}^{(\kappa)}}(\ln(2)\hat{b}_{l,j}^{(\kappa+1)} - \ln(2)b_{l,j}^{(\kappa)} + 1)$ and $2^{c_k^{(\kappa+1)}} = 2^{c_k^{(\kappa)}}(\ln(2)\hat{c}_k^{(\kappa+1)} - \ln(2)c_k^{(\kappa)} + 1)$. Because of $2^x \geq 2^{\bar{x}}(\ln(2)x - \ln(2)\bar{x} + 1)$, we can further have $\hat{b}_{l,j}^{(\kappa+1)} \geq b_{l,j}^{(\kappa+1)}$ and $\hat{c}_k^{(\kappa+1)} \geq c_k^{(\kappa+1)}$. Consequently, $2^{b_{l,j}^{(\kappa+1)}} \leq 2^{b_{l,j}^{(\kappa+1)}}(\ln(2)\hat{b}_{l,j}^{(\kappa+1)} - \ln(2)b_{l,j}^{(\kappa+1)} + 1)$ and $2^{c_k^{(\kappa+1)}} \leq 2^{c_k^{(\kappa+1)}}(\ln(2)\hat{c}_k^{(\kappa+1)} - \ln(2)c_k^{(\kappa+1)} + 1)$, which implies

$$\begin{aligned} &2^{b_{l,j}^{(\kappa)}}(\ln(2)\hat{b}_{l,j}^{(\kappa+1)} - \ln(2)b_{l,j}^{(\kappa)} + 1) \leq 2^{b_{l,j}^{(\kappa+1)}}(\ln(2)\hat{b}_{l,j}^{(\kappa+1)} - \ln(2)b_{l,j}^{(\kappa+1)} + 1), \\ &2^{c_k^{(\kappa)}}(\ln(2)\hat{c}_k^{(\kappa+1)} - \ln(2)c_k^{(\kappa)} + 1) \leq 2^{c_k^{(\kappa+1)}}(\ln(2)\hat{c}_k^{(\kappa+1)} - \ln(2)c_k^{(\kappa+1)} + 1). \end{aligned} \tag{H.1}$$

Since $\{\hat{\mathbf{W}}_{l,j}^{(\kappa+1)},\ \hat{\mathbf{Q}}_l^{(\kappa+1)},\ \hat{r}_{l,j}^{(\kappa+1)},\ \hat{a}_{l,j}^{(\kappa+1)}, \hat{b}_{l,j}^{(\kappa+1)}, \hat{c}_k^{(\kappa+1)}, \hat{d}_{l,j,k}^{(\kappa+1)}\}$ is an optimal solution to (4.17), by combining (4.17b), (4.17c) and (H.1), we have

$$\begin{aligned} &\sum_{u\in\mathcal{J}_l, u\neq j} \mathrm{tr}(\mathbf{H}_{l,l,j}\hat{\mathbf{W}}_{l,u}^{(\kappa+1)}) + \sum_{u\in\mathcal{L}, u\neq l}\sum_{i\in\mathcal{J}_u} \mathrm{tr}(\mathbf{H}_{u,l,j}\hat{\mathbf{W}}_{u,i}^{(\kappa+1)}) \\ &\qquad + \sum_{u\in\mathcal{L}} \mathrm{tr}(\mathbf{H}_{u,l,j}\hat{\mathbf{Q}}_u^{(\kappa+1)}) + \sigma_{l,j}^2 \leq 2^{b_{l,j}^{(\kappa+1)}}(\ln(2)\hat{b}_{l,j}^{(\kappa+1)} - \ln(2)b_{l,j}^{(\kappa+1)} + 1), \end{aligned} \tag{H.2a}$$

$$\begin{aligned} &\sum_{u\in\mathcal{L}}\sum_{i\in\mathcal{J}_u} \mathrm{tr}(\mathbf{G}_{u,k}\hat{\mathbf{W}}_{u,i}^{(\kappa+1)}) + \sum_{u\in\mathcal{L}} \mathrm{tr}(\mathbf{G}_{u,k}\hat{\mathbf{Q}}_u^{(\kappa+1)}) + \sigma_k^2 \\ &\qquad\qquad \leq 2^{c_k^{(\kappa+1)}}(\ln(2)\hat{c}_k^{(\kappa+1)} - \ln(2)c_k^{(\kappa+1)} + 1). \end{aligned} \tag{H.2b}$$

From (H.2), we see that $\{\hat{\mathbf{W}}_{l,j}^{(\kappa+1)}, \hat{\mathbf{Q}}_l^{(\kappa+1)}, \hat{r}_{l,j}^{(\kappa+1)}, \hat{a}_{l,j}^{(\kappa+1)}, \hat{b}_{l,j}^{(\kappa+1)}, \hat{c}_k^{(\kappa+1)}, \hat{d}_{l,j,k}^{(\kappa+1)}\}$ is feasible to the optimization problem (4.17) at the $(\kappa+2)$th iteration. Thus, it immediately holds that $\sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{J}_l} \ln(\hat{r}_{l,j}^{(\kappa+1)}) \leq \sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{J}_l} \ln(\hat{r}_{l,j}^{(\kappa+2)})$. In other

words, the SCA process in Algorithm 1 yields a non-decreasing sequence of objective values in (4.16). Because of constraints (4.14c), problem (4.16) is bounded, and thus the SCA process in Algorithm 1 is guaranteed to converge. Note that the solution to (4.17) must satisfy the KKT conditions of (4.17) at each iteration. Moreover, the KKT conditions of (4.17) are identical to those of (4.16) at the convergence [99, Theorem 1]. Consequently, the sequence $\{\hat{\mathbf{W}}_{l,j}^{(\kappa+1)}, \hat{\mathbf{Q}}_{l}^{(\kappa+1)}, \hat{r}_{l,j}^{(\kappa+1)}, \hat{a}_{l,j}^{(\kappa+1)}, \hat{b}_{l,j}^{(\kappa+1)}, \hat{c}_{k}^{(\kappa+1)}, \hat{d}_{l,j,k}^{(\kappa+1)}\}$ converges to a KKT point of (4.16) when κ increases.

Appendix I

The Derivation of (5.21)

Since $\mathbf{Q}_i$ is a Hermitian matrix, through EVD, we have

$$\mathbf{Q}_i = \mathbf{U}_i \mathbf{\Lambda}_i \mathbf{U}_i^H$$

where $\mathbf{\Lambda}_i = \text{diag}(\lambda_{i,1}, ..., \lambda_{i,N_h})$ and $\mathbf{U}_i$ is composed of the eigenvectors of $\mathbf{Q}_i$ with $\mathbf{U}_i^H \mathbf{U}_i = \mathbf{I}$. Therefore, $\mathbf{g}_{e,k,i}^H \mathbf{Q}_i \mathbf{g}_{e,k,i}$ can be written as

$$\mathbf{g}_{e,k,i}^H \mathbf{Q}_i \mathbf{g}_{e,k,i} = \mathbf{g}_{e,k,i}^H \mathbf{U}_i \mathbf{\Lambda}_i \mathbf{U}_i^H \mathbf{g}_{e,k,i}.$$

By letting $\mathbf{a}_{e,k,i} = (\mathbf{\Lambda}_i^{1/2})^H \mathbf{U}_i^H \mathbf{g}_{e,k,i}$, we have $\mathbf{g}_{e,k,i}^H \mathbf{Q}_i \mathbf{g}_{e,k,i} = \mathbf{a}_{e,k,i}^H \mathbf{a}_{e,k,i}$, which leads to

$$\frac{\left|\mathbf{h}_{e,k,i}^H \mathbf{w}_{m,l,i}^{[m]}\right|^2}{\mathbf{g}_{e,k,i}^H \mathbf{Q}_i \mathbf{g}_{e,k,i} + \sigma_{e,k,i}^2} = \frac{\left|\mathbf{h}_{e,k,i}^H \mathbf{w}_{m,l,i}^{[m]}\right|^2}{\sum\limits_{j \in \mathcal{J}} \left|a_{e,k,i}^{(j)}\right|^2 + \sigma_{e,k,i}^2} \tag{I.1}$$

where $a_{e,k,i}^{(j)}$ denotes the j-th element in the vector $\mathbf{a}_{e,k,i}$. Through (I.1), we have

$$\begin{aligned}
&\text{Prob}\left(\frac{\left|\mathbf{h}_{e,k,i}^H \mathbf{w}_{m,l,i}^{[m]}\right|^2}{\mathbf{g}_{e,k,i}^H \mathbf{Q}_i \mathbf{g}_{e,k,i} + \sigma_{e,k,i}^2} \leq \Gamma_{e,m}\right) \\
&= \text{Prob}\left(\frac{\left|\mathbf{h}_{e,k,i}^H \mathbf{w}_{m,l,i}^{[m]}\right|^2}{\sum\limits_{j \in \mathcal{J}} \left|a_{e,k,i}^{(j)}\right|^2 + \sigma_{e,k,i}^2} \leq \Gamma_{e,m}\right)
\end{aligned}$$

$$= \text{Prob}\left(\left|\mathbf{h}_{e,k,i}^{H}\mathbf{w}_{m,l,i}^{[m]}\right|^2 \leq \Gamma_{e,m}\sigma_{e,k,i}^2 + \sum_{j\in\mathcal{J}}\Gamma_{e,m}\left|a_{e,k,i}^{(j)}\right|^2\right).$$

Since $\mathbf{h}_{e,k,i} \sim \mathcal{CN}(\mathbf{0}, \bar{h}_{e,k,i}\mathbf{I})$, we know $\left|\mathbf{h}_{e,k,i}^{H}\mathbf{w}_{m,l,i}^{[m]}\right|^2$ follows an exponential distribution with mean $\bar{h}_{e,k,i}\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2$. Moreover, since $\mathbf{g}_{e,k,i} \sim \mathcal{CN}(\mathbf{0}, \bar{g}_{e,k,i}\mathbf{I})$, we know $\mathbf{a}_{e,k,i} \sim \mathcal{CN}(\mathbf{0}, \bar{g}_{e,k,i}\mathbf{\Lambda}_i)$. Thus, $\left|a_{e,k,i}^{(1)}\right|^2, ..., \left|a_{e,k,i}^{(N_h)}\right|^2$ are independent and exponentially distributed with $\text{E}\left[\left|a_{e,k,i}^{(j)}\right|^2\right] = \bar{g}_{e,k,i}\lambda_{i,j}$.

Suppose $z_0, ..., z_N$ are independent exponentially distributed random variables having $\text{E}[z_j] = \bar{z}_j$. According to [100], we can obtain

$$\text{Prob}(z_0 \leq c + \sum_{j=1}^{N} z_j) = 1 - e^{-\frac{c}{\bar{z}_0}} \prod_{j=1}^{N} \frac{1}{\frac{\bar{z}_j}{\bar{z}_0} + 1} \tag{I.2}$$

where c is a constant. From (I.2), it can be seen that

$$\begin{aligned} &\text{Prob}\left(\left|\mathbf{h}_{e,k,i}^{H}\mathbf{w}_{m,l,i}^{[m]}\right|^2 \leq \Gamma_{e,m}\sigma_{e,k,i}^2 + \sum_{j\in\mathcal{J}}\Gamma_{e,m}\left|a_{e,k,i}^{(j)}\right|^2\right) \\ &= 1 - e^{-\frac{\Gamma_{e,m}\sigma_{e,k,i}^2}{\bar{h}_{e,k,i}\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2}} \prod_{j\in\mathcal{J}} \frac{1}{\frac{\Gamma_{e,m}\bar{g}_{e,k,i}\lambda_{i,j}}{\bar{h}_{e,k,i}\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2} + 1}. \end{aligned} \tag{I.3}$$

Consequently, we have

$$\text{Prob}\left(\frac{\left|\mathbf{h}_{e,k,i}^{H}\mathbf{w}_{m,l,i}^{[m]}\right|^2}{\mathbf{g}_{e,k,i}^{H}\mathbf{Q}_i\mathbf{g}_{e,k,i} + \sigma_{e,k,i}^2} \leq \Gamma_{e,m}\right) \geq 1 - \varepsilon_{m,l,k,i}^{[m]}$$

$$\Leftrightarrow$$

$$1 - e^{-\frac{\Gamma_{e,m}\sigma_{e,k,i}^2}{\bar{h}_{e,k,i}\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2}} \prod_{j} \frac{1}{\frac{\Gamma_{e,m}\bar{g}_{e,k,i}\lambda_{i,j}}{\bar{h}_{e,k,i}\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2} + 1} \geq 1 - \varepsilon_{m,l,k,i}^{[m]}$$

$\Leftrightarrow$

$$\ln\left(\varepsilon_{m,l,k,i}^{[m]}\right) + \frac{\Gamma_{e,m}\sigma_{e,k,i}^2}{\bar{h}_{e,k,i}\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2} + \sum_{j\in\mathcal{J}} \ln\left(\frac{\Gamma_{e,m}\bar{g}_{e,k,i}\lambda_{i,j}}{\bar{h}_{e,k,i}\left\|\mathbf{w}_{m,l,i}^{[m]}\right\|^2} + 1\right) \geq 0.$$