Design Optimization of a Mechatronic Device in the Presence of Quantitative and Qualitative Design Criteria and Multiple Objectives

by

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Abstract

Mechatronic devices and multi-domain (multi-physics) systems are widely used in modern industry and other engineering applications. Mechatronic engineering focuses on developing a design solution that integrates multiple domains, particularly electrical and mechanical systems. For a successful product, these systems require to be accurate, fast, reliable, flexible, minimalist, easy to use and cost effective. Such design demands are diverse, can interact with each other, and might be characterized quantitatively, qualitatively, or both. This might require different scales, units, and physical representations between multiple criteria or objectives. Interacting criteria or objectives might be conflicting, e.g., improving one requirement might deteriorate another requirement. This requires reaching a compromise between objectives, a trade-off decision. The present dissertation addresses the multi-objective design optimization problem that involves quantitative and qualitative design criteria and objectives, in a mechatronic system. The methods developed in this thesis are applied to the design of a wearable sleep monitoring system. For the benefit of that application, a design optimization framework is proposed for sensor placement on a human body to improve the wearability and reliability of a monitoring system that contains the sensors. The developed framework assists the designer in selecting the type and location of the sensors, and the pertinent wiring. The framework uses fuzzy sets and numbers to reduce the subjectivity that arises with qualitative criteria. To describe the qualitative objective comfort, fuzzy measures and the Choquet integral are used, particularly for combining multiple criteria and handling model interactions. Furthermore, fuzzy measures with the Choquet integral and the decision-making method VIKOR are introduced to make a relatively less subjective trade-off decision between conflicting objectives. Finally, a comparison is made between an improved VIKOR method and a fuzzy measure and Choquet integral approach, related to their optimization trade-off decisions. This study leads to a synthesis of all presented results and concludes that the proposed methods provide comparable results and are effective strategies for trade-off decisions. In this manner, the present investigation significantly contributes to the development of a more effective approach for solving a multi-objective design problem with quantitative and qualitative design criteria.
Lay Summary

The design of a product or device typically requires the consideration of multiple design criteria. Such a design must be accurate, reliable, esthetically appealing and cost effective to be acceptable for the consumer. Some of these objectives can be represented as numerical functions or quantities, while others may be “qualitative” and only be described over linguistic terms. When considering or improving multiple design objectives at the same time, conflicts could arise. For example, the improvement of one objective might make another one worse. This dissertation seeks to improve a multi-objective design of a mechatronic system, by considering both qualitative and quantitative design criteria simultaneously, by incorporating not only numerical functions but also linguistic methods, in a less subjective manner. Furthermore, two methods are proposed to make a more effective trade-off decision when design objectives are conflicting. In this manner, the present thesis contributes to designing a mechatronic product more effectively.
Preface

This thesis is original work completed by Lucas Falch in the Industrial Automation Laboratory (IAL) at The University of British Columbia, Vancouver campus, under direct supervision and guidance of Dr. Clarence W. de Silva, Professor of Mechanical Engineering, The University of British Columbia. Dr. de Silva proposed and supervised the overall research project, acquired funding and resources for the project, suggested the topic of the thesis, suggested concepts and methodologies in addressing problems in the topic, provided research facilities, continuously supervised the progress of the research, and revised the thesis presentation. This thesis includes two published manuscripts, and in addition two other manuscripts submitted. Because parts of the thesis have been published individually, there is some cross-over among different chapters, in particular in the introductory sections.

Chronologically, the first paper (Chapter 2) has been published in the Sensors Journal by authors Lucas Falch (first author) and Clarence W. de Silva. Lucas Falch was responsible for all major areas of concept formulation, algorithm development and contribution to the manuscript. Clarence W. de Silva was the supervisory author on this work and was involved throughout the project in concept revisions and manuscript composition.

The second paper (Chapter 3) has been published and presented at the 2018 IEEE 9th Annual Information Technology, Electronics and Mobile Communication Conference (IEMCON) by authors Lucas Falch (first author) and Clarence W. de Silva. Clarence W. de Silva suggested the concept of fuzzy measures, which Lucas Falch applied as a decision making framework for a design optimization problem. Lucas Falch was responsible for major areas of concept formulation, algorithm development and contribution to the manuscript. Clarence W. de Silva was the supervisory author and was involved throughout the work in concept revisions, guidance, and manuscript composition.

The third paper (Chapter 4) has been submitted to an appropriate journal by authors Lucas Falch (first author) and Clarence W. de Silva. Lucas Falch was responsible for all major areas of concept formation, questionnaire formulation, execution of questionnaire study, algorithm development, experiment validation, as well as manuscript composition. Clarence W. de Silva was the supervisory author and was involved throughout the work in concept revisions, guidance and editing the manuscript.

Chapter 5 has been submitted to an appropriate journal by authors Lucas Falch (first
author) and Clarence W. de Silva. Lucas Falch discovered the shortcomings in an existing decision making method, improved them and used the improved decision making method for comparison with the fuzzy measure approach. Clarence W. de Silva provided scientific guidance and suggestions, and is the supervisory author.

List of publications from this thesis:

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Chapter 3:

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Chapter 4:
L. Falch and C. W. de Silva, “Incorporating the Qualitative Variable Comfort into the Design of a Wearable Body Sensor System”

Chapter 5:
L. Falch and C. W. de Silva, “Decision Making in a Multi-objective Design Problem”
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# List of Acronyms and Abbreviations

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<td>AASM</td>
<td>American Academy of Sleep Medicine</td>
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<tr>
<td>AHP</td>
<td>Analytic Hierarchy Process</td>
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<tr>
<td>CAD</td>
<td>Computer Aided Design</td>
</tr>
<tr>
<td>CB</td>
<td>Carbon Black</td>
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<tr>
<td>CFP</td>
<td>Contention Free Period</td>
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<tr>
<td>CS</td>
<td>Candidate Site</td>
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<tr>
<td>CSA</td>
<td>Central Sleep Apnea</td>
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<tr>
<td>ECG</td>
<td>Electrocardiogram</td>
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<tr>
<td>EEG</td>
<td>Electroencephalography</td>
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<tr>
<td>ELECTRE</td>
<td>ELimination Et Choix Traduisant la REalité (Elimination and Choice Expressing Reality)</td>
</tr>
<tr>
<td>EMG</td>
<td>Electromyography</td>
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<tr>
<td>EOG</td>
<td>Electrooculography</td>
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<tr>
<td>FAHP</td>
<td>Fuzzy Analytic Hierarchy Process</td>
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<tr>
<td>FDA</td>
<td>Food and Drug Administration</td>
</tr>
<tr>
<td>GRA</td>
<td>Gray Relation Analysis</td>
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<tr>
<td>GTS</td>
<td>Guaranteed Time Slot</td>
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<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
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<tr>
<td>IP</td>
<td>Inactive Period</td>
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<tr>
<td>LS</td>
<td>Least-Squares</td>
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<td>MAVT</td>
<td>Multi-Attribute Value Theory</td>
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<td>MCDA</td>
<td>Multi-Criteria Decision Aiding</td>
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<td>MR</td>
<td>Marichal and Roubens method</td>
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<td>MSLT</td>
<td>Multiple Sleep Latency Tests</td>
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<td>MST</td>
<td>Minimum Spanning Tree</td>
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<td>NRMSD</td>
<td>Normalized Root Mean Square Deviation</td>
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<td>NSGA</td>
<td>Non-dominated Sorting Genetic Algorithm</td>
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<td>OSA</td>
<td>Obstructive Sleep Apnea</td>
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<tr>
<td><strong>PROMETHEE</strong></td>
<td>Preference Ranking Organization Method for Enrichment of Evaluations</td>
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<td><strong>PSG</strong></td>
<td>Polysomnography</td>
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<td><strong>QFD</strong></td>
<td>Quality Function Deployment</td>
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<td><strong>REM</strong></td>
<td>Rapid Eye Movement</td>
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<tr>
<td><strong>RMSD</strong></td>
<td>Root Mean Square Deviation</td>
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<tr>
<td><strong>SPEA</strong></td>
<td>Strength Pareto Evolutionary Algorithm</td>
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<tr>
<td><strong>TDMA</strong></td>
<td>Time Division Multiple Access</td>
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<tr>
<td><strong>TOPSIS</strong></td>
<td>Technique for Order of Preference by Similarity to Ideal Solutions</td>
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<td><strong>VIKOR</strong></td>
<td>from Serbian: VIseKriterijumska Optimizacija I Kompromisno Resenje, meaning: Multi-criteria Optimization and Compromise Solution</td>
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<tr>
<td><strong>WBSN</strong></td>
<td>Wireless Body Sensor Network</td>
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Dedication

To my friends and family, who were always there for me –

and the mountains that gave me the best views after a hard climb.
Chapter 1

Introduction

This dissertation is based on two published papers (Chapters 2 and 3) and two paper submitted (Chapter 4 and 5). The present introductory chapter provides the motivation of the research and rationale, organization of the dissertation, and some relevant background material. In the format of a complete paper, each chapter includes a detailed introduction to the specific issue that it addresses, pertinent developments, results, and a discussion.

1.1 Motivation

Every engineering device is assessed by its properties, characteristics and the functional purpose. A mechatronic device, in particular, contains both mechanical and electrical parts, and its development requires the involvement of different fields of knowledge, such as controls and computer engineering, mechanics and electronics [1]. Traditionally, electro-mechanical systems were designed or selected separately in a sequential manner and combined with other components, hardware and software in later steps. In a mechatronic approach, the complete electro-mechanical system is handled as a whole in an integrated manner that incorporates multiple domains (e.g., electrical, mechanical, thermal, and fluid). A multi-domain (or, multi-physics) device requires a mechatronic approach, which is integrated, unified, systematic and unique. This is the extended and more generalized definition for a mechatronic device, as proposed by de Silva [2], which goes beyond the previous definition involving only electro-mechanical systems an integrated design. In this definition, “unified” means similar or analogous techniques to handle different domains, using the corresponding across-variables and through-variables. Next, “unique” result is achieved through optimization of the development according the pertinent performance indices and constraints. It can be argued that, a mechatronic product that is developed through this approach will be more efficient and cost effective, precise, reliable, flexible, functional and mechanically less complex than a non-mechatronic product with a comparable level development effort and expenditure.

A mechatronic system consist of many different types of interconnected components and elements. Various properties and characteristics have to be considered during the design process
of the device. Product design can be a time-consuming, complex, and costly task. Many design tools, such as Computer Aided Design (CAD) and simulation software, are available to reduce the time to bring a product to the market. Properties and design specifications of a device are diverse and have different characteristics. A “unique” engineering design is realized through design optimization, which may use a mathematical formulation of the design problem to support the selection of the “optimal” (or “unique”) design from many alternatives. The design optimization procedure involves various types of objective functions, which may incorporate such attributes as performance, quality, cost, speed, ease of operation, safety, and environmental impact of the designed product or system, which involve both quantitative and qualitative aspects, in general. The involvement of these various properties will generally require many decisions, which suggests that the task of design can be viewed as a decision-making process [3]. A design optimization problem involves design variables, describing the design alternatives; design objectives, which are functional combinations of variables to be maximized or minimized; and constraints that must be satisfied by the acceptable alternatives. When solving a mechatronic design problem that incorporates a number of objectives, multi-disciplinary design optimization allows the designer to consider all relevant factors simultaneously to improve the design and reduce the time and cost of the design cycle. Furthermore, the solution to a multidisciplinary (or, mechatronic) design optimization problem will have to consider interactions between different aspects and domains, and is therefore more desirable than the optimization of each aspect individually [4]. Also, limits of the objectives (constraints) are not necessarily known, particularly when qualitative objectives are treated as constraints. Therefore, it is desirable to optimize the design objectives simultaneously, using a multi-objective optimization formulation. Multi-objective design optimization was initially developed for and is still widely used in aerospace engineering, such as aircraft and spacecraft design [5–7] in view of the complex and multi-domain nature of the design of such systems. DeWispelare [8] presented in his PhD research a process algorithms for multiobjective optimization theory and multiple attribute utility theory with application in a defense system problem, specifically for electronic retrofit of warfare aircraft. Multi-objective design optimization has been extended to a number of fields, including automobile design [9] and building design [10].

The optimization of multiple-objective functions often requires a trade-off, because an improvement in one objective might downgrade another one. Designing a new mechatronic device sometimes requires the construction of a few models or prototypes to gain knowledge and experience. However, each model or prototype might have its advantages and disadvantages, making it difficult to chose the optimal design. A simple way to make the selection might be to count the number of advantages and pick the one with the highest count. However, advantages can be of qualitative nature and they can be as important, if not more than, the quantitative advantages. Qualitative objectives in a design process are often not known in the beginning. An example of a mechatronic device for which qualitative design criteria are important is a healthcare monitoring device. Kasabach et al. [11] sought to design an accurate and comfort-
able wearable body monitoring system. To find a good mounting location on the human body, they created three prototypes and tested them on subjects to select the best location. This is a reasonable approach; however, resources are not always available to build several prototypes and conduct a study. Instead, an analytical approach is desirable, and a tool that facilitates optimal design decisions would save time and money. Kasabach et al. [11] make their design decision for the device location based on arguments rather than through an efficient and analytical study. This leads to subjective decisions, does not consider possible differences in importance of arguments, and is almost impossible to validate. Therefore, a more powerful and analytical approach is needed to incorporate qualitative design objectives as well as quantitative and make less subjective design decisions.

In this backdrop, the main challenges associated with the design of a mechatronic product include: (i) Incorporating qualitative design criteria (e.g., the level of comfort of a wearable body sensor network) into the design process, (ii) Evaluation of a design trade-off by considering multiple design objectives.

When considering multiple design objectives, the design space contains several possible alternatives. The best alternative would be the one that has the highest satisfactory value in each criterion, but in reality an alternative in which all criteria dominate may not exist. Solutions in a multi-objective optimization problem that are not dominated by other solutions are called Pareto optimal solutions [12]. Determination of non-dominated solutions reduces the set of possible design alternatives, and a decision that considers various importance values for criteria is required to choose one of them. Out of this set of solutions, the one with the highest satisfaction value for the overall design requires a decision by an expert or the designer. A more effective multi-criteria decision making method that incorporates the importance and interactions between criteria in a non-subjective way is a helpful tool for designing a mechatronic device.

The present research addresses the development, implementation, and evaluation of a practical and effective approach that incorporates qualitative design objectives directly into the design process that also has quantitative design objectives. The developed methods within this research are applied to the design of a portable sleep monitoring system. The following sections present an overview and the importance of a wearable sleep monitoring system and indicate the need to design a highly functional, comfortable and reliable sleep monitoring system.

1.2 Wearable Sleep Monitoring System

Poor-quality sleep over a long period of time can lead to adverse effects such as hypertension, cardiovascular pathologies, obesity and diabetes, and it can affect the immune system [13]. Often, poor-quality sleep is caused by sleep disorders, which affect nearly 27% of the world population [14]. Lack of sleep can harm the economy due to the resulting loss of productivity. The costs are estimated to be about $18 billion globally [15]. There are numerous methods to evaluate sleep in a clinical laboratory. These include overnight Polysomnography (PSG),
multiple sleep latency tests (MSLT), and Video-PSG. However, a clinical environment is costly to maintain because of the necessary facilities and staff. Long delay in getting an appointment at a sleep clinic is another concern. Furthermore, the sleep condition at a clinic, typically, is quite different from that at the familiar home environment. Hence, the monitored results may not be quite representative. Consequently, there is a high interest in developing portable devices that can perform comprehensive monitoring of sleep in a home environment. The existing home monitoring devices do not record adequate data (in both type and duration) to properly diagnose sleep apnea and other sleep disorders. A comprehensive yet portable sleep monitoring device will cater to a large portion of the population at a lower cost.

1.2.1 Polysomnography

Polysomnography (PSG) is the current gold standard in sleep monitoring and takes place in a clinical environment. It is an effective, precise, and comprehensive sleep monitoring method to diagnose diverse sleep disorders like obstructive sleep apnea (OSA), central sleep apnea (CSA) and insomnia. OSA is a disease where the upper airway is obstructed during sleep [16]. The throat muscles relax and close the pharynx and as a result the airway is partially or completely blocked. OSA is usually associated with a reduction in the blood oxygen and causes awakening from sleep. In CSA the breathing intermittently stops and starts during sleep and this occurs because the brain does not send proper signals to the muscles that control the breathing [17]. This is different from OSA and can cause heart failure and structural damage to the central nervous system. Therefore it is important to diagnose such sleep disorders to perform appropriate treatment. Figure 1.1 shows a typical PSG setup with a range of sensors to diagnose the mentioned sleep diseases.

With PSG it is possible to detect different sleep stages like REM sleep and non-REM sleep, and record periodic limb movements during sleep, like restless legs syndrome [18]. While PSG
provides extensive information, the unfamiliar sleep environment and the single night monitoring can lead to unacceptable results. Many people have variations in their daily sleep or different symptoms over several nights, weeks or months, which might be related to caffeine, alcohol consumption, medication, stress or daily exercising. These conditions cannot be analyzed and diagnosed with just a single night of sleep monitoring. Further drawbacks of PSG include, for example, the non-standardized protocols in laboratories, the clinical conditions of equipment and personnel, and the long duration of monitoring and analysis and the related costs and other hardships. These shortcomings can be improved with comprehensive, yet portable sleep monitoring devices.

1.2.2 Portable Sleep Monitoring Systems

The American Academy of Sleep Medicine (AASM) divides portable monitoring devices into four types [17]:

- Type 1: Full attended polysomnography (≥ 7 channels) in a laboratory setting
- Type 2: Full unattended polysomnography (≥ 7 channels)
- Type 3: Modified portable sleep apnea testing (minimum 4 channels)
- Type 4: Continuous single or dual bio parameter recording, usually using oximetry as one of the parameters

Mostly, type 3 devices are available at present. The existing portable monitoring devices are only suitable for people with a high probability of moderate to severe OSA. However, with more sensors and improved technology of portable devices, sleep monitoring for the home environment could be made comparable to clinical sleep monitoring. With portable monitoring devices, more people can be monitored at lower cost, because there is no need for special facilities with all the sophisticated equipment and staff. However, current portable monitoring devices do not contain all the needed sensors for comprehensive PSG. A comprehensive sleep monitoring device must record at least the airflow, respiratory effort, blood oximetry, electrocardiography, electroencephalography (EEG), electrooculography (EOG) and electromyography (EMG). Non-invasive sensors such as cameras are not suitable to detect the mentioned sleep diseases such as OSA or CSA. Cameras would only monitor the sleeping behavior, but it is not possible to detect OSA or CSA by using them. According to the current AASM standards, a certified sleep specialist or a person that satisfies the criteria for sleep medicine examination must evaluate the raw data from the monitoring process and make a statement about an eventual sleep disorder [19]. The first portable monitoring test was published in 1994, which promised to reduce the unacceptable delays in accessing PSG. However, its acceptance and success were limited.

A portable monitoring device is accepted as a diagnostic tool only if it follows the AASM-accredited criteria, the United States Food and Drug Administration (FDA) guidelines, and ISO 13485 for Europe and Canada. The certification pathways in the United States at present are through the American Board of Sleep Medicine and the American Board of Medical Specialties.
The currently used technologies in portable sleep monitoring devices include:

- **Electroencephalography (EEG):** Records the electrical activity of the brain through electrodes on the scalp.
- **Electrooculogram (EOG):** Records the cornea-retinal potential between front and back of the eye, detecting eye movements.
- **Electromyogram (EMG) for chin:** Measures the electrical activity of the muscles to detect bruxism.
- **Electromyogram (EMG) for leg:** Measures the electrical activity of the muscles, to detect movement related disorders.
- **Airflow signals:** Measure the flow of air through the nose and/or mouth. Such sensors as pressure sensor and thermistor may be used.
- **Respiratory effort (RE) signals:** Record the movement of the chest and the abdomen due to respiration, using elastics band with a strain sensor.
- **Oxygen saturation:** Measures the concentration of oxygen in the blood.
- **Body position:** The position of the body during sleep (spine, lateral recumbent, etc.)
- **Electrocardiogram (ECG):** Records the electrical activity of the heart using electrodes.

Figure 1.2 shows the arrangement of the listed sensors on a human.

![Figure 1.2: The figure shows the necessary sleep monitoring sensors and their location on a human body.](image)

The demand for portable home sleep monitoring devices is high. Currently there are just a few such devices on the market and many of them do not fulfill the requirements for accurate medical monitoring. With most of the portable monitoring devices, it is only possible to diagnose specific sleep disorders. The sensors that are missing in current portable monitoring systems include Electroencephalography (EEG) and Electrooculography (EOG).
1.2.3 EEG/EOG Monitoring

An important sensor system that is not commonly available in the present home-based sleep monitoring devices is Electroencephalography (EEG). EEG records the change in human brain signals [20]. For monitoring the eye movement for sleep stage detection, Electrooculography (EOG) measures the electrical potential between the cornea and the retina.

Evaluating sleep stages is one part in a sleep study and can be precisely done with the analysis of brain wave patterns. Observing the EEG signals in sleep monitoring requires only a portion of the electrodes used in clinical applications. A minimum of three EEG channels is required to classify sleep stages. Section 2.5 provides further details on EEG/EOG in portable sleep monitoring.

1.3 Related Work

1.3.1 Dealing with Qualitative Measures

Many methods are available to assign importance weightings to design variables or factors. A common technique to compute the importance of a qualitative criterion or a linguistic variable is by using an analytic hierarchy process (AHP) [21]. AHP provides a comprehensive and rational framework for structuring a decision by creating a comparison matrix where alternatives are compared with respect to criteria. Decision makers or experts give ratings of importance, and by using the principal right eigenvector (e.g., [22]) or the geometric mean of rows (e.g., [23]) of the matrix, utility values for all the criteria are determined.

In order to reduce the subjectivity of the design decision in a multi-objective optimization algorithm, a group of researchers [22] used a fuzzy analytic hierarchy process (FAHP). They used triangular fuzzy numbers, α-cuts and the lower and upper bounds of the α-cut to quantify the importance weights determined by AHP.

Another method for dealing with qualitative measures is given in the study of Babbar et al. [24], where they considered qualitative and quantitative criteria to select a set of suppliers. They determined the quantity of an order to reduce costs, and considered environmental impacts as well. To assign weights and thereby rank the suppliers, they developed a quality function deployment (QFD) model. QFD is based on the opinion of decision makers and is therefore sensitive to the number of experts used in the study and their level of experience. In their approach, three experts rated a product based on such customer considerations as price and quality. Babbar, et al. [24] used trapezoidal fuzzy numbers to reduce subjectivity in the rantings by experts. The overall weight of each product property is computed using fuzzy arithmetic. With this method, experts rated if a supplier can satisfy customer needs and specific predefined characteristics of the supplier. The fuzzy weights as determined through expert ratings were quantified using the mean of the four trapezoid fuzzy parameters and were normalized within the range 0 and 1. They formulated a multi-objective optimization problem that was based on
the previously mentioned supplier weights and on quantitative parameters such as the unit cost of a product. The problem was solved using three different methods: (i) weighted sum method, (ii) weighted-constraint method, and (iii) distance method.

A more standardized technique to quantify qualitative design criteria is quantification theory type 1, which was developed in the 50’s by Hayashi [25]. It makes use of linear regression and is expressed as:

\[ Y_k = \sum_{i=1}^{m} \sum_{j=1}^{n} \beta_{ij} x_{ij} + \epsilon_k \]  

(1.1)

where \( Y_k \) is the observed value or the response, usually rated by volunteers or experts; \( \beta_{ij} \) is the parameter of the model or the weight of a level or category; and \( x_{ij} \) is the level or the category variable (dummy). The least square algorithm minimizes the prediction error \( \epsilon_k \).

Hsiao et al. [23] applied quantification theory type 1 to measure the perception of homepages. Human perceptions are thought to be non-quantifiable, subjective and affect-based, and are difficult to objectively measure using conventional methods. They used qualitative factors like proximity and similarity to describe human perception. Diverse homepage samples were evaluated by volunteers over a set of linguistic variables. Ratings given by experts or decision makers are highly subjective. To reduce subjectivity in the evaluation of homepage design, Hsiao et al. [23] used triangular fuzzy membership functions with seven linguistic importance judgments of qualitative factors. To quantify the linguistic importance judgments of qualitative factors, they used \( \alpha \)-cuts and center-of-gravity method. Fuzzy entropy was then used as a measure for human perception. A higher fuzzy entropy indicated a higher human perception of a single homepage. Fuzzy entropy indicated the fuzziness of the fuzzy set. In this manner, their approach addressed and weakened the subjectivity of human-based decisions.

Mohsenzadeh et al. [26] presented a method to rate appliances based on a predicted electricity usage costs and the occupant comfort in the household. A fuzzy-logic technique was used to model these two objectives. Their goal was to minimize the utility cost and maximize the comfort. A normal distribution curve, a rectangular curve, or a trapezoidal curve, was used depending on the appliance in order to model the comfort profile of the device.

A different approach to make a model by capturing subjective and qualitative measures, was developed by Grabisch et al. [27]. They used fuzzy measures and integrals to model discomfort for subjects sitting for a long period of time (such as when driving a car). Subjects filled out a questionnaire to express their experiences during an experiment that used different car seats under various conditions. Based on this approach, an overall comfort value was determined and fuzzy measures were identified by minimizing the squared error of the overall comfort. However, inferring the overall comfort value from a questionnaire is subjective and requires a prototype design to conduct the experiment. Despite these concerns, it is a common approach and has been applied by various researchers (e.g., [28], [29], [30]).
1.3.2 Design Optimization of Product Style

Other applications that require the quantification of qualitative variables is in the field of product style design. To assess and select a product design Hsiao et al. [31] collected product styles and used morphological analysis theory to form different styles for various parts of a coffee maker. To obtain a score for each part (casing, water compartment, etc.) and category (funneled, spheroid, etc.) the researchers applied quantification theory type 1. Specifically, they used a questionnaire, in which the subjects rated 26 representative samples based on seven linguistic variables. In order to obtain an optimal design solution for a coffee maker, they applied a genetic algorithm. The fitness function for the optimization process was formed by the weight of each linguistic variable and the individual score for the styles. The method used in their work requires a generation of representative designs. This is possible when a model representation (such as a CAD drawing) exists. However, it considers qualitative design criteria only based on the appearance of the product and is highly dependent on the survey group. A similar approach to optimize the form and shape of a product was used by Shieh et al. [22], where the main difference lies with regard to the optimization method. Hsiao et al. [31] created a single objective function with a score for each category and a weighting for the linguistic descriptive variables, whereas Shieh et al. [22] created one objective function for each linguistic variable and applied a multi-objective optimization algorithm. Their algorithm [22] was applied to the form design of a car. In describing and judging design variables, they used factor analysis to pick the main factors such as modern, rounded, and simple. A questionnaire was created where 60 participants rated 27 sample designs using a scoring scale from 1 to 7 based on linguistic factors. Each linguistic factor was represented in an objective function, and they used the multi-objective optimization algorithm NSGA-II in order to optimize the mentioned factors (modern, rounded, etc.). The multi-objective optimization algorithm delivered the non-dominated solutions. To select a solution and make a final design decision, they applied fuzzy analytic hierarchy process (FAHP). This provided a weight for each factor, which gave a performance value as the product of each Pareto solution vector and the weight. The highest performance value represented the final and optimal solution.

Xue et al. [32] presented an approach to develop a product design using Gray Relational Analysis (GRA). GRA defines situations with no information as black and those with perfect information as white, whereas information in between is classified as gray. The input and output in their approach were linguistic variables that were determined based on fuzzy theory. Morphological analysis extracted the product from the design elements of experimental samples. Subjects rated these samples on a seven point scale, which provided a numerical database of the product image to construct fuzzy rules. In their work, a train seat design served as a case study, and Computer Aided Design (CAD) was used to build 3-D models.

The study of Brintrup et al. [33] emphasizes the importance of combining qualitative and quantitative criteria in a design process. They developed an interactive genetic algorithm for multiple objectives and tested it on the design of an office chair. Their method was based on
multi-criteria decision making and user-interactive evolutionary optimization techniques. For interactive evolutionary optimization, subjective evaluation by humans was integrated into the fitness function. This implied that the designer acted as a qualitative fitness function within the genetic algorithm and gave ratings for qualitative objectives. Therefore, the human himself was not a model, but was incorporated into the algorithm and produced subjective input into the design. This publication presented three methods: (i) a parallel, (ii) a sequential, and (iii) a multi-objective interactive genetic algorithm. In (i), the offspring population was created from the qualitative and quantitative fitness functions in parallel by migrating them. In (ii), an initial qualitative run created the parent population for the sequential quantitative run. Method (iii) was based on the usual non-domination techniques and used elitism to combine parent and offspring populations before sorting them for non-domination and creating a set of solutions. The connection between the qualitative and quantitative aspects lied within the optimization algorithm itself, and the fitness function for qualitative objectives was the design expert. This step induced human evaluation after each run into the process which adds subjectivity, might be inconsistent and vary depending on the person or experience.

1.3.3 Design Optimization of Wearable Body Sensor Network

Designing a wearable body sensor network requires the consideration of multiple factors such as comfort. Even a well-functioning and reliable wearable body sensor network might not be accepted by the end user, if it is uncomfortable or distracting. Comfort is a highly subjective and qualitative measure. It can be a physical sensation, a psychological state, or both simultaneously. Several authors have previously tried to optimize comfort or wearability. Anliker et al. [34] sought to optimize multiple objectives of a wearable architecture. Their work presents a systematic way to choose and attach devices to a human body. The design space included battery life, system functionality, and wearability. For battery life they used the average power consumption of the system, and considered a variety of computing devices. A specific computing device was able to perform different tasks like image encoding and zip decoding at various body locations. For each task they used an average computing power in order to optimize the battery life. In addition, the power consumption for connecting the devices was considered, since it varied depending on the connection channel (USB, I2C, Bluetooth, etc.). To quantify the functionality of the system, they used the execution and communication delay between devices. The wearability objective was based on the weight vectors assigned to different body locations. The wearability factors also depended on the size, weight, and wired or wireless connections. The weights given for the wearability factors were not explained in their paper, and it was left for treatment through ergonomic research and social acceptability to determine these weights. The underlying design challenge was addressed by considering a multi-objective optimization problem and using the evolutionary SPEA2 algorithm. It created a Pareto front with a set of trade-off solutions.

In a follow-up paper [35], the authors additionally considered sensors that could be attached
to a human body. The sensor selection was based on the trade-off parameters: power consumption and recognition performance. Recognition quality was quantified using a theory based on mutual information. Mutual information is a measure of the information that a sensor (e.g., accelerometer, air pressure sensor) provides about a recognition class such as the user activities. It was computed using experimental data, and joint probability functions were estimated. Anliker et al. [35] used a genetic algorithm to derive a set of Pareto optimal system configurations and optimize the two objective functions.

Other researchers in the field of wearable sensor systems, such as Tabares et al. [36], attempted to optimize the same objective functions, wearability and functionality, as mentioned before ([34], [35]). Here, they considered state-of-the-art platforms in order to build their objective functions. Their product was an ergonomic clothing article with an integrated electrocardiogram system. The design variables were $x_1 =$ skin contact quality, $x_2 =$ size, $x_3 =$ weight, $x_4 =$ fluid repellency and $x_5 =$ magnetic protection. They formulated a simple objective function $f$ for functionality, as:

$$f = \left( x_1^2 + x_2 + x_3 + x_4 + x_5 \right) / 5$$

(1.2)

The impact of each design variable (quadratic or linear) was based on the intuition and available information. They did not perform actual quantification of the design variables. Results were expressed as percentages; for example, the skin contact quality = 69.45% or size = 33.03%. Such values may be interpreted as importance factors. However, because of the intuitive nature of the objective function and seemingly random choice of a solution from the Pareto front, the solution presents some caveats.

In a subsequent paper, the same researchers [37] used a different approach to address some of these caveats and attempted to model the objective functions through multivariate regression. They studied a linear regression model and a nonlinear regression model with real and categorical values. Their specific application was an electrocardiogram system. The researchers made 160 measurements from 12 volunteers for various body locations, with different rotation angles and pressure levels. The volunteers provided information on the device comfort for each measurement. To gain information about the functionality, they correlated their measured signal with a reference signal. A higher correlation meant higher functionality. The conflicting objective functions wearability and functionality were fitted using regression and then optimized through a genetic algorithm and represented as a Pareto set (non-dominated solutions). This approach helped to select a good location for the rotation and the pressure level of an electrocardiogram system. Yet, in designing a wearable system or a mechatronic device in general, it might not be always possible to conduct experiments at first. Even if an experiment produces representable results, different types of parameters such as different electrodes can skew the outcome.

In the field of powered lower-limb prostheses design, Sahoo et al. [38] optimized the power consumption of the input motor by maximizing the ratio of maximum knee torque to max-
mum torque for a gait cycle. Their goal was to achieve low power consumption, low weight, fast response and compactness for the prostheses. Laschowski et al. [39] reviewed the electromechanical design and optimization of lower-limb devices and discussed design optimization objectives such as electrical energy regeneration, mechanical power transmission, electromagnetic machine, electrical drive, mass and moment of inertia and energy storage. Considered objectives were quantitative aspects and they did not consider qualitative objectives such as comfort, which also plays a role in lower-limb prostheses.

1.4 Research Issues

The following section highlights the questions addressed in this thesis. In particular it addresses how it contributes to solving a design problem, motivated by the design optimization of wearable sleep monitoring systems (Section 1.2).

Design problems often times have a range of objectives to be considered. Focusing on the field of medical devices, a wearable device does not only have to function properly, it also has to be comfortable to wear, should not distract the person while performing daily tasks, and should consume little power. These properties are diverse, some of them are quantitative or quantifiable and others are rather qualitative. The difficulties that come with such design properties are different scales and units between objectives. These objectives need to be considered simultaneously, taking into account existing interactions. Multiple criteria or objectives can be conflicting and by improving one, another one might degrade, which leads to the situation in which a trade-off decision needs to be made. Qualitative criteria or objectives are subjective, as well as the trade-off decision that is necessary to pick one final design. Taking all this into account is a challenging task for the design and development of a mechatronic device. A strategy is required to perform such a complex design task. This thesis applies the developed concepts to a wearable body sensor network, specifically a sleep monitoring device. It thus contributes to the team effort in developing such a device within our research group. However, developed concepts can of course be applied on a much broader scale which is reflected in the following research questions:

- How to include and handle qualitative design requirements?
- How to incorporate quantitative and qualitative design criteria or objectives simultaneously and handle different scales and units?
- How to reduce subjectivity associated with qualitative criteria or objectives?
- How to make a trade-off decision, when criteria or objectives are conflicting?

1.5 Contributions and Thesis Overview

By addressing the indicated research issue, the present thesis makes important contributions to the state of the art of the focus area. The main contributions and an overview of the present
dissertation are given next.

1. Chapter 2 presents a design optimization framework for the sensor placement on a human body with specific application to an EEG/EOG device for sleep monitoring. The quantitative optimization method uses pre-chosen qualitative metrics to improve wearability and reliability and assists the designer in the selection of type, location and wiring of the devices. Fuzzy weights express the qualitative metrics. In the application of EEG/EOG monitoring, the approach provides solutions for various types of electrodes instead of just one. The design space is reduced through this approach. A simple decision making method with equal importance for both objectives is presented as an illustrative example in the case study.

2. In the present work, two decision making methods are improved and introduced to mechatronic design optimization. One method is based on fuzzy measures and the Choquet integral with the ability to model interactions, as developed in Chapter 3.

3. How to describe the qualitative design objective comfort among multiple criteria is demonstrated in Chapter 4. Criteria describing comfort are combined with fuzzy measures and the Choquet integral. The improvement and comparison between two fuzzy measure determination techniques is another important contribution of the dissertation. The outcome of this work is a comfort model for a wearable body sensor network, which is determined through a questionnaire study and validated using test samples. This makes the determination of the qualitative objective comfort less subjective.

4. The information needed to make a trade-off decision is based on preferences of some alternatives. As the second decision making approach that is developed in the present work, an existing method with clear shortcomings is improved, to overcome these deficiencies, in Chapter 5.

5. Finally, the two introduced methods are compared with each other in Chapter 6. The developed decision making methods reduce subjectivity in the design trade-off selection.
Chapter 2

Optimization of Multiple Design Objectives with Qualitative and Quantitative Criteria


2.1 Summary

This chapter presents an approach to optimize multiple design objectives that have qualitative and quantitative design variables, with specific application for a wearable body sensor system. The methodology incorporates a way to group the qualitative and quantitative design variables and the design objectives that are present in the problem and the establishment of the domains that exist in such a design problem. Design indices are the objective functions that represent the qualitative design goals. The design space is systematically reduced thereby making it easier to decide on an optimal design solution, specifically to pick a good solution from the non-dominated solutions. A technique of fuzzy logic is used to assign weights to the subjective and non-crisp design criteria. With the method presented in this chapter it is illustrated how to design a wearable body sensor network with respect to comfort and reliability. Specifically, the design of an EEG/EOG monitoring system for sleep monitoring is considered, where the presented approach provides a systematic way to find an appropriate number and type of components, their locations, and how they are connected and arranged.
2.2 Introduction

A wearable body sensor system is assessed through their features, properties, characteristics, and the functional purpose. All these aspects have to be considered during the design process of the system. However, the system properties and design specifications can be quite diverse and have different characteristics. For example, the weight, size and the cost of the device are typically represented by real numbers, whereas functionality, comfort, complexity and aesthetics are complex attributes that are rather subjective and of qualitative nature. An experienced designer might be able to handle both qualitative and quantitative design objectives in a meaningful way in the design process. Even with many years of design experience, however, a human designer may not be able to directly and completely apply the previous experience for a new device in a different field. Therefore, a systematic and algorithmic approach to incorporate qualitative and quantitative design parameters and optimize multiple design objectives is beneficial, particularly with regard to the efficiency, effectiveness, flexibility, speed, and cost of the design. It will enable the industries to design reliable and well-functioning devices in a systematic manner. This is a key focus of the present research.

Researchers have sought to optimize a variety of devices with respect to qualitative and quantitative aspects. To assess and select a product design Hsiao et al. [31] collected product styles and used morphological analysis theory to form different styles for various parts of a coffee maker. To obtain a score for each part (casing, water compartment, etc.,) and category (funneled, spheroid, etc.,) the researchers applied quantification theory type 1: They used a questionnaire, where subjects rated 26 representative samples based on seven linguistic variables. In quantification theory type 1, the degree of influence of the target variable on each category of items is computed by multiple regression. They used Analytic Hierarchy Process (AHP) to compute the importance of each linguistic variable. In AHP, as developed by Saaty [21], a decision maker judges the importance by pairwise comparison of each factor. The relative scale measurement is presented as a matrix and the weights are calculated through the method of normalization of the geometric mean of rows. To obtain an optimal design solution for a coffee maker they applied a genetic algorithm. The fitness function for the optimization process was formed by the weight of each linguistic variable and the individual score for the styles. The method used in their work required a generation of representative designs. This is possible when a model representation (such as a CAD drawing) exists. However, it considered qualitative design criteria only based on the appearance of the product and was highly dependent on the survey group. The method did not consider other objectives such as functionality, reliability, and power consumption.

A similar approach to optimize the form and shape of a product is used by Shieh et al. [22], where the main difference lies with regard to the optimization method. Hsiao et al. [31] created a single objective function with a score for each category and a weighting for the linguistic descriptive variables whereas Shieh et al. [22] created one objective function for each linguistic variable and applied a multi-objective optimization algorithm. Their algorithm [22] was applied
to the form design of a car. Through morphological analysis they selected representative design variables with the help of experts. To describe and judge these design variables they used factor analysis to pick the main factors such as modern, rounded, and simple. A questionnaire was created where 60 participants rated 27 sample designs using a scoring scale from 1 to 7 based on linguistic factors. To quantify and build a model, the researchers used quantification theory analysis 1. Each linguistic factor was represented in an objective function, and they used the multi-objective optimization algorithm NSGA-II in order to optimize the mentioned factors (modern, rounded, etc.). The multi-objective optimization algorithm delivered the non-dominated Pareto solutions. In order to select a solution and make a final design decision, they applied fuzzy analytic hierarchy process (FAHP). Because AHP is subjective, the researchers used triangular fuzzy membership functions to judge the comparison. This provided a weight for each factor. The highest utility value computed by the weighted sum was their optimal solution.

Brintrup et al. [33] developed an interactive genetic algorithm for multiple objectives with qualitative and quantitative criteria for a chair design. Their method was based on multi-criteria decision making and user-interactive evolutionary optimization techniques. For interactive evolutionary optimization, subjective evaluation by humans was integrated into the fitness function. This implied that the designer acted as a qualitative fitness function within the genetic algorithm and gave ratings for qualitative objectives. Therefore, the human themselves was not a model, but incorporated into the algorithm and produced subjective input into the design. Their paper presented three methods: (i) a parallel, (ii) a sequential, and (iii) a multi-objective interactive genetic algorithm. In (i), the offspring population was created from the qualitative and quantitative fitness functions in parallel by migrating them. In (ii), an initial qualitative run created the parent population for the sequential quantitative run. Method (iii) was based on the usual non-domination techniques and used elitism to combine parent and offspring populations before sorting them for non-domination and creating a set of solutions. The connection between the qualitative and quantitative aspects was within the optimization algorithm itself, and the fitness function for qualitative objectives was the design expert, which required human evaluation after each run, which is subjective. The human evaluation might even be inconsistent and vary depending on the group/person or experience.

The work in [23] applied quantification theory type 1 to measure the perception of homepages. Quantification theory type 1 has been developed in the 50’s by Hayashi [25] and that work made use of linear regression. Human perceptions are thought to be non-quantifiable, subjective and affect-based experiences, and are difficult to objectively measure using conventional methods. Their paper used triangular fuzzy membership functions with seven linguistic importance judgments of qualitative factors. They applied the method to homepage design and used qualitative factors like proximity and similarity. Diverse homepage samples were evaluated by volunteers using a set of linguistic variables. In order to quantify each factor, they used $\alpha$-cuts and the center-of-gravity method. Fuzzy entropy was then used as a measure for
human perception. Fuzzy entropy indicates the fuzziness of a fuzzy set. In this manner, their approach addressed and weakened the subjectivity of human-based decisions. A higher fuzzy entropy indicated a greater human perception of a single homepage. Other applications that require the quantification of qualitative variables is in the design of the product style.

Mohsenzadeh et al. [26] presented a method to rate appliances based on predicted electricity usage costs and the occupant comfort in the household. A fuzzy-logic technique was used to model these two objectives. Their goal was to minimize the utility cost and maximize the comfort. A normal distribution curve, a rectangular curve, or a trapezoidal curve, was used depending on the appliance in order to model the comfort profile of the device.

Babbar et al. [24] considered qualitative and quantitative criteria to select a set of suppliers, determine the quantity of an order to reduce costs, and consider environmental impact. To assign weights and thereby rank the suppliers, they developed a quality function deployment (QFD) model. QFD is based on the opinion of decision makers and is therefore sensitive to the used number of experts and their level of experience. In their approach, three experts rated a product based on such customer considerations as price and quality. The rating scale was a trapezoidal fuzzy number, and the rating was weighted depending on the expertise of the decision maker. The weight was also a trapezoid fuzzy number. The overall weight of each product property was computed using fuzzy arithmetic. With this method, experts rate if a supplier can satisfy customer needs and specific predefined characteristics of the supplier. The fuzzy weights as determined through expert ratings were quantified using the mean of the four trapezoid fuzzy parameters and were normalized within the range 0 and 1. They formulated a multi-objective optimization problem that was based on the previously mentioned supplier weights and on such quantitative parameters as the unit cost of a product. The problem was solved using three different methods: (i) weighted sum method, (ii) weighted-constraint method, and (iii) distance method. An underlying issue in the existing studies was the lack of a clear definition for “qualitative” design objectives and the weight assignment for qualitative design variables.

In the field of wearable body sensor systems, Anliker et al. [34] sought to optimize multiple objectives of a wearable architecture. Their work presented a systematic way to choose and attach devices to a human body. The design space included battery life, system functionality, and wearability. For battery life they used the average power consumption of the system, and considered a variety of computing devices. A specific computing device was able to perform different tasks like image encoding and zip decode at various body locations. For each task they used an average computing power in order to optimize the battery life. Also, the power consumption for connecting the devices was considered, since it varied depending on the connection channel (USB, I2C, Bluetooth, etc.,). To quantify the functionality of the system they used the execution and communication delay between devices. The wearability objective was based on the weight vectors assigned to different body locations. The wearability factors also depend on the size, weight, and wired or wireless connections. The weights given for the wearability
factors were not explained in their paper, and it was left for treatment through ergonomic re-
search and social acceptability. The underlying design challenge was addressed by considering a
multi-objective optimization problem and using the evolutionary SPEA2 algorithm. It created
a Pareto front with a set of trade-off solutions.

In a follow-up paper [35] the authors additionally considered sensors that could be attached
to a human body. The sensor selection was based on the trade-off parameters: power consump-
tion and recognition performance. Recognition quality was quantified using a theory based on
mutual information. Mutual information is a measure of the information that a sensor (e.g.,
accelerometer, air pressure sensor) provides about a recognition class such as user activities.
Mutual information was computed using experimental data, and joint probability functions
were estimated. They used a genetic algorithm to derive a set of Pareto optimal system con-
figurations and optimized the two objective functions.

Other researchers in the field of wearable sensor systems, such as Tabares et al. [36], at-
ttempted to optimize the same objective functions, wearability and functionality, as mentioned
before ([34], [35]). Here, they considered state-of-the-art platforms in order to build their
objective functions. Their product was an ergonomic clothing article with an integrated elec-
trocardiogram system. The impact of each design variable was mainly based on intuition. They
did not perform actual quantification of the design variables. Because of the intuitive nature
of the objective function and seemingly random choice of a solution from the Pareto front, the
same researchers [37] used a different approach to address some of these caveats and attempted
to model the objective functions through multivariate regression. They studied a linear and a
nonlinear regression model with real and categorical values. Their specific application was an
electrocardiogram system. The researchers made 160 measurements from 12 volunteers for var-
ious body locations, with different rotation angles and pressure levels. The volunteers provided
information on the device comfort for each measurement. To gain information about the func-
tionality, they correlated their measured signal with a reference signal. A higher correlation
meant higher functionality. The conflicting objective functions wearability and functionality
were fitted using regression and then optimized through a genetic algorithm and represented as
a Pareto set (non-dominated solutions). This approach helped to select a good location for the
rotation and the pressure level of an electrocardiogram system. Yet, in designing a wearable
system or a mechatronic device in general, it might not be always possible to conduct experi-
ments. Even if an experiment produces representable results, different types of parameters such
as different electrodes can skew the outcome.

The literature here, which deal with optimization of a mechatronic design, does not consider
both qualitative and quantitative design variables. To improve the time and cost of designing
and developing a good product, the production of prototypes during the design process must
be minimized. Therefore, it is important to include qualitative design aspects such as comfort
as well in the design process, to quickly attain an optimal and user-accepted design. In order to
incorporate subjectivity in the optimization process with respect to qualitative variables, fuzzy
logic is used in the present work to represent the associated non-crisp boundaries. Further, to satisfy several aspects of the design, multiple objectives are considered in solving the design optimization problem. The literature reviewed here show some shortcomings and hence call for combined approaches in order to tackle a variety of optimization problems and efficiently design products. All these are considerations in the present research.

2.3 Design Variables

The first key step of designing a mechatronic device is the identification of the relevant design variables/parameters and objective functions. Typically, more than one design variable may be needed to define a design objective, and more than one design objective may be needed as well. In the design process, the designer needs to define the objectives of the designed system, which depend on the application of the system. Functionality is an important design criterion, but in case of a wearable body sensor system, comfort also plays an important role. Even a highly functional and quality product might not be accepted by the customer, if the comfort level of the product is not adequate. Therefore, it is necessary to include such qualitative objectives within the set of design objectives. The next step consists of characterizing the design variables by sorting them into relevant domains. The following list shows examples of domains where a design variable is quantitative or qualitative:

- **Quantitative**
  - $\in \mathbb{R}$ Real numbers (cost, weight, lifetime, ...)
  - $\in \mathbb{Z}$ Integer numbers (number of components, ...)
  - $\in \mathbb{FD}$ Finite domain (design possibilities, component locations, ...)
  - $\in \mathbb{P}$ Probability distribution (failure rate, probability of fault/malfunction, ...)

- **Qualitative**
  - $\in \mathbb{QS}$ Sortable in decreasing or increasing order of value (comfort, drive-ability, ...)
  - $\in \mathbb{NO}$ Subjective and with no order of value (aesthetics, appearance, ...)

Some design variables like the monetary cost, size, and weight are quantitative measures and can be represented by real numbers. Number of components; e.g., number of screws and number of IMUs, are integer values. However, these design variables are also indicators of qualitative measures. The comfort level of a wearable product depends on the number of components in the product, but also on the location where a component such as a sensor is attached to the body of the wearer. Possible locations to place sensors are finite and are an important criterion in optimizing the user comfort (some locations on the human body are less sensitive than others). Some design criteria may be representable as probability functions.

Qualitative measures may be grouped into two categories, namely qualitative sortable (QS) and non-sortable (NO). For example, if comfort is related to pain, it is a qualitative aspect since
humans have different comfort zones and thresholds of pain tolerance. However, the ratings of locations/devices with respect to the user comfort can be sorted based on the anatomy. However, aesthetics or appearance may depend on an individual’s taste or level of maturity. It might be possible to rate devices or the style of a product for a specific group of people having similar tastes or backgrounds, but the resulting ratings would be biased. When including non-sortable design aspects in the design process it is important to factor in the characteristics of the group of people who use the device. The present work considers sortable design aspects only and does not focus on a specific group of people. However, qualitative design criteria are still subjective, and in order to deal with subjectivity and the lack of crisp boundaries for qualitative measures, the use of fuzzy set theory is appropriate.

2.4 Fuzzy Sets and Fuzzy Numbers

To approximate subjective and qualitative knowledge, Zadeh [40] introduced the concept of fuzzy sets and related it to fuzzy logic, in the mid-1960s. The theory has been widely used to deal with subjectivity and qualitative measures, and applied to many practical situations. A fuzzy set is defined by its membership function, which indicates the grade (degree) of membership of an element (member) within the set. The membership value is defined in the interval [0, 1], and the membership function is expressed as $\mu_M : X \rightarrow [0, 1]$ [41, 42]. For analytical purposes, a fuzzy set may be represented by a group of crisp sets called $\alpha$-cuts [42]. The $\alpha$-cut of a fuzzy set is the crisp set that includes all the members of the fuzzy set whose membership values are greater than or equal to $\alpha$. If the elements of this crisp set are in the interval $[m, n]$, then the corresponding $\alpha$-cut can be expressed as $M_\alpha = [m, n]$.

Any fuzzy set can be completely defined by its set of $\alpha$-cuts [42, 43] and therefore it is possible to limit arithmetic operations to its interval, provided that they are continuous membership functions. It is clear from the above definition that the sum and the multiplication of $\alpha$-cuts are given by:

$$M_\alpha + N_\alpha = [m_1, m_2] + [n_1, n_2] = [m_1 + n_1, m_2 + n_2]$$

$$M_\alpha N_\alpha = [m_1, m_2] \cdot [n_1, n_2] = [\min(m_1n_1, m_1n_2, m_2n_1, m_2n_2), \max(m_1n_1, m_1n_2, m_2n_1, m_2n_2)]$$

(2.1)

(2.2)

Here $M$ and $N$ are two fuzzy sets, and $M_\alpha$ and $N_\alpha$ are their respective $\alpha$-cuts. Obviously, the result of the $\alpha$-cut summation is a crisp set because the $\alpha$-cuts themselves are crisp sets.

The defuzzification of a fuzzy set may be done using the center-of-gravity method. Specifically:

$$\mu_M(x) = \frac{\int x \cdot \mu(x)dx}{\int \mu(x)dx}$$

(2.3)
Further details on fuzzy logic and soft computing are found in [41], [42] and [43].

2.5 EEG/EOG Body Sensors in Sleep Monitoring

The methods investigated and developed in the next sections will be applied to the design of an Electroencephalography (EEG) and Electrooculography (EOG) device for sleep monitoring.

As mentioned in Chapter 1 Electroencephalography (EEG) records the change in human brain signals [20]. Observing EEG signals in sleep monitoring does not require all the electrodes that are used in clinical applications. A minimum of three EEG channels is required to classify sleep stages [44]. The analysis of brain wave patterns is an important step of evaluating sleep stages. Electrooculography (EOG) is also important in the detection of sleep stages. EOG measures the electrical potential between cornea and retina. In the process, electrodes record vertical eye movements (such as blinking) and horizontal eye movements simultaneously.

The American Academy of Sleep Medicine (AASM) manual [17] suggests different electrode mounting locations and combinations for sleep monitoring. Four acceptable versions and locations of the placement of EEG and EOG electrodes are listed below and shown in Figure 2.1.

- **EEG**
  - Version1: F4-M1, C4-M1, O2-M1
  - Version2: F3-M2, C3-M2, O1-M2
  - Version3: Fz-Cz, Cz-Oz, C4-M1
  - Version4: Fpz-Cz, Cz/C3-O1, C4/C3-M2

- **EOG**
  - E1-M1/M2/Fpz
  - E2-M1/M2/Fpz

In the figure, circles indicate EOG electrodes. Electrode locations marked in red represent version 1 of the acceptable positions for EEG monitoring in a sleep study [17]. Green, blue and purple correspond to version 2, 3 and 4, respectively. A combination of the four versions is also possible. Specifically, combining the four versions of electrode arrays will give 15 possible electrode arrangements.
Figure 2.1: Possible electrode locations for EEG and EOG in sleep monitoring.

2.6 Multi-objective Design Optimization for EEG/EOG Monitoring

This section demonstrates the design optimization of an EEG/EOG system for sleep monitoring. The goal here is to find comfortable and reliable locations in a human body for EEG/EOG monitoring. For different locations, type of devices and connection channels, it is possible to assign linguistic fuzzy numbers such as those shown in Table 2.1. Here, comfort and reliability are rated ranging from unreliable/uncomfortable to reliable/comfortable, in five categories, and intervals of fuzzy numbers are assigned for each linguistic variable. The membership function is triangular (Figure 2.2). The assignment of a fuzzy membership function reduces the subjectivity and, after defuzzification, a numerical value for the reliability and comfort of the objectives is provided.

Table 2.1: Fuzzy weights for linguistic variables

<table>
<thead>
<tr>
<th>Linguistic variable</th>
<th>Interval of triangular fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unreliable (UR)</td>
<td>[0, 0.333]</td>
</tr>
<tr>
<td>Uncomfortable (U)</td>
<td></td>
</tr>
<tr>
<td>Moderate Unreliable (MUR)</td>
<td>[0.167, 0.5]</td>
</tr>
<tr>
<td>Moderate Uncomfortable (MU)</td>
<td>[0.333, 0.667]</td>
</tr>
<tr>
<td>Moderate (M)</td>
<td>[0.333, 0.667]</td>
</tr>
<tr>
<td>Moderate Reliable (MR)</td>
<td>[0.5, 0.833]</td>
</tr>
<tr>
<td>Moderate Comfortable (MC)</td>
<td>[0.667, 1]</td>
</tr>
<tr>
<td>Reliable (R)</td>
<td>[0.667, 1]</td>
</tr>
<tr>
<td>Comfortable (C)</td>
<td></td>
</tr>
</tbody>
</table>
The possible locations and the combinations of the connections for the electrodes represent an extensive design space. Therefore, an approach to find an optimal design is needed. The fuzzy weightings that characterize each location for comfort and reliability are given in Table 2.2.

**Table 2.2:** Fuzzy weight assignment for electrode locations. Column 1 presents the electrode locations according to the 10-20 EEG system (see Figure 2.1). $x_i$ are binary variables for placing an electrode at that particular location. The last two columns are the fuzzy weights assigned to each electrode location.

<table>
<thead>
<tr>
<th>Electrode</th>
<th>Design variable</th>
<th>Fuzzy weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>$x_1$</td>
<td>C</td>
</tr>
<tr>
<td>E2</td>
<td>$x_2$</td>
<td>C</td>
</tr>
<tr>
<td>Fpz</td>
<td>$x_3$</td>
<td>C</td>
</tr>
<tr>
<td>F3</td>
<td>$x_4$</td>
<td>C</td>
</tr>
<tr>
<td>Fz</td>
<td>$x_5$</td>
<td>MC</td>
</tr>
<tr>
<td>F4</td>
<td>$x_6$</td>
<td>C</td>
</tr>
<tr>
<td>M1</td>
<td>$x_7$</td>
<td>U</td>
</tr>
<tr>
<td>C3</td>
<td>$x_8$</td>
<td>MU</td>
</tr>
<tr>
<td>Cz</td>
<td>$x_9$</td>
<td>M</td>
</tr>
<tr>
<td>C4</td>
<td>$x_{10}$</td>
<td>MU</td>
</tr>
<tr>
<td>M2</td>
<td>$x_{11}$</td>
<td>U</td>
</tr>
<tr>
<td>O1</td>
<td>$x_{12}$</td>
<td>U</td>
</tr>
<tr>
<td>O2</td>
<td>$x_{13}$</td>
<td>U</td>
</tr>
<tr>
<td>O2</td>
<td>$x_{14}$</td>
<td>U</td>
</tr>
</tbody>
</table>

The assigned weights are intuitive and therefore subjective, yet they are derived by considering the human anatomy and sleeping behavior. The majority of people sleep on the side [45] and in order to breathe while sleeping, the forehead and the upper part of the head should not touch the pillow. Considering this aspect, it is more comfortable and reliable to place electrodes on the forehead, and the lack of hair at such locations is an added advantage. Furthermore, the back or the side of the head is more likely to touch the pillow and the additional applied pressure through the attached electrodes causes further discomfort. These areas are also considered more unreliable, since movements at night can lead to detachment of the electrodes. However, to assign fuzzy weights more precisely and less intuitively, a survey or questionnaire is suggested. For demonstration purposes of the theory, these intuitively assigned weights are employed.
2.7 Design Optimization Using Linear Programming and Minimum Spanning Tree (MST)

This section presents an approach to design a wearable body sensor system by considering two qualitative objective functions: wearability and reliability. Previous approaches (e.g., [34], [36], [37]) attempted to model wearability and reliability as two separate objective functions. In their approach, they encountered problems in the combination of design variables having different units and incomparable magnitudes. The approach presented in this chapter, however, overcomes this problem by finding an appropriate number of devices with respect to the two specific objective functions. Afterwards a minimum spanning tree algorithm is applied, where the inputs are the combined weightings of the two objective functions and the connection length of the electrodes. This approach ensures that the electrodes are associated with the smallest possible weightings. In order to optimize wearability, the number of devices attached to the body should be minimized. On the other hand, by increasing the number of devices, the reliability can improve, as well as the functionality. The latter aspect is not considered in the present work. An increase in the number of components would lead to degraded reliability when the components are connected in series. However, here only parallel connections (backup/redundant devices) are considered. Each device has its own connection, but the wires can be in the same cable channel. The present method that addresses the described problem is detailed now.

Wearability: \[
\min \# \text{ devices} = \sum_{i=1}^{n} (\hat{c}t) t_i x_i \tag{2.4}
\]

Reliability: \[
\max \# \text{ devices} = \sum_{i=1}^{n} (\hat{r}t) t_i x_i \tag{2.5}
\]

\[
\min \text{ tree of graph } G \text{ with edges } c_{ij} \tag{2.6}
\]

subject to \(x_i \in \{0, 1\}\)
\(t \in \{\text{available devices}\}\)
\(c_{ij} \in \{0, 1\}\)
\(c_{ij} \leq x_i \cdot x_j, \quad \forall ij \text{ connections} \tag{2.7}\)
\(c_{ij} = \begin{cases} 
1 & \text{if } (\hat{c}r)_{ij} \leq \frac{\max(\hat{c}r) - \min(\hat{c}r)}{2} \\
0 & \text{otherwise} \end{cases} \tag{2.8}\)

Here \(x_i\) are the locations at which a device/sensor can be placed, as represented by a binary value (1 for present and 0 for absent); \(n\) is the number of locations that are available to place a device/sensor. The available type of devices/sensors to be placed at particular locations is denoted by \(t\). Each device and location gets a fuzzy weighting (see Table 2.2) depending on the corresponding comfort and reliability. The combined weighting of comfort/reliability and the type of device, \((\hat{c}t)\) and \((\hat{r}t)\) is computed using fuzzy arithmetic (equation (2.2)) and defuzzified by the center-of-gravity method, as given by equation (2.3).
The selected locations \( x_i \), which are suitable for mounting a device or sensor, are the nodes in a graph \( G \) \([46]\) and \( c_{ij} \) denotes the possible connections (edges in graph \( G \)). If a location \( x_i \) or \( x_j \) is not selected, \( c_{ij} \) does not exist, as expressed in equation (2.7). The weightings \( \hat{c}_{ij} \) in graph \( G \) are computed using wearability and reliability fuzzy weightings by equation (2.2) and (2.3) and also the length \( l_{ij} \) to connect the nodes. To optimize wearability and reliability, the required length to connect devices must be minimized, and comfortable and reliable connections should be preferred (equation (2.6)) as the minimum spanning tree of graph \( G \). It is necessary to supply devices/sensors. It is possible to have a single power supply for each sensor/device (all \( c_{ij} = 0 \)), which will make the system more reliable, because sensors are connected independently. On the other hand, a common power supply, which is the connection of several devices/sensors \( (c_{ij} \neq 0) \) would make the system more comfortable. Therefore, not every connection \( c_{ij} \) in the minimum spanning tree will be chosen. This is implemented using equation (2.8). The present work considers a connection as valid if the weighting of the connection lies in the middle of the minimum and maximum of all possible weightings of the minimum spanning tree, because the present work assigns equal importance for wearability and reliability. Hence, equation (2.8) is only true for this case. The proposed approach is applied to design an EEG/EOG monitoring system for sleep monitoring, as presented next.

### 2.7.1 EEG/EOG Monitoring Design Optimization Using Linear Programing and MST

This section presents the design optimization of a monitoring system that has the sensing capabilities of EEG and EOG.

Acceptable versions and locations of the EEG and EOG electrodes are listed in Section 2.5 and shown in Figure 2.1. They take into consideration that at minimum 3 EEG electrodes are required to classify sleep stages, as suggested by the American Academy of Sleep Medicine (AASM) \([17]\).

Fuzzy weightings characterizing comfortable and reliable locations are given in Table 2.2, in the previous section. This approach also considers the type of electrodes. The fuzzy weights for three different types of electrodes are listed in Table 2.3. The weight assignment is intuitive and based on personal experience.

<table>
<thead>
<tr>
<th>Type of electrode</th>
<th>Fuzzy weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comfort</td>
<td>Reliability</td>
</tr>
<tr>
<td>(A) Capacitive</td>
<td>M</td>
</tr>
<tr>
<td>(B) Spiky contact</td>
<td>U</td>
</tr>
<tr>
<td>(C) Microneedle</td>
<td>C</td>
</tr>
</tbody>
</table>

In total, there are 14 locations for electrodes. The resulting multi-objective optimization problem aiming to find a comfortable and reliable number of electrodes may be expressed as
Wearability: \( \min \# \text{ electrodes} = \sum_{i=1}^{n} (\hat{ct}) t x_i \) \hspace{1cm} (2.9)

Reliability: \( \max \# \text{ electrodes} = \sum_{i=1}^{n} (\hat{rt}) t x_i \) \hspace{1cm} (2.10)

subject to \( x_i \in \{0, 1\} \)

\[
t \in \{\text{available electrodes}\} \\
x_1 + x_2 \geq 2 \\
x_6 + x_7 + x_{10} + x_{14} \geq 4 - M(1 - y_1) \hspace{1cm} (2.11) \\
x_4 + x_8 + x_{12} + x_{11} \geq 4 - M(1 - y_2) \hspace{1cm} (2.12) \\
x_5 + x_7 + x_9 + x_{10} + x_{13} \geq 5 - M(1 - y_3) \hspace{1cm} (2.13) \\
x_3 + x_8 + x_9 + x_{11} + x_{12} \geq 5 - M(1 - y_4) \hspace{1cm} (2.14) \\
y_1 + y_2 + y_3 + y_4 \geq 1 \hspace{1cm} (2.15) \\
y_i \in \{0, 1\}, M \text{ is chosen sufficiently large} \hspace{1cm} (2.16) \\
\]

Equations (2.9) and (2.10) are the two objective functions that have to be optimized. Electrodes \( x_1 \) and \( x_2 \) have to be a part of the system in order to form an EOG-monitoring system. This is specified in the constraint equation (2.11). Equations (2.12) to (2.15) are the possible combinations, for a sleep disorder EEG-monitoring system (specified in the AASM manual [17]). Equation (2.16) ensures that at least one possible EEG-combination is selected. The minimization of the wiring length and choosing comfortable and reliable connections is implemented using the minimum spanning tree algorithm and described in equation (2.6). The wiring length and the fuzzy weightings for each connection are presented in Table 2.4. The wiring length is chosen based on the 10/20 EEG positioning system and average head size dimensions [47].

2.7.2 Results of an EEG/EOG Monitoring Design

The approach proposed in the previous section optimizes the objective functions simultaneously. The optimization of multiple-criteria requires a trade-off between the objective functions. However, in the case of two objective functions, there is a set of solutions that optimize both objective functions simultaneously, which are called non-dominated solutions or Pareto optimal solutions [48]. In order to obtain the non-dominated solutions, it is common to use an evolutionary algorithm (e.g., NSGA-II [49], SPEA2 [50]). In the present research, the chosen algorithm has to handle binary variables and linear constraints. However, in the EEG/EOG design, the design space is sufficiently small to go through all the possibilities without a dedicated algorithm. Therefore, the present research does not have to deal with local minima. The resulting Pareto front is shown in Figure 2.3.
Table 2.4: Comparison matrix with given length and fuzzy weightings for electrode connection.

<table>
<thead>
<tr>
<th></th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
<th>x6</th>
<th>x7</th>
<th>x8</th>
<th>x9</th>
<th>x10</th>
<th>x11</th>
<th>x12</th>
<th>x13</th>
<th>x14</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>L [cm]</td>
<td>C</td>
<td>R</td>
<td>5.9</td>
<td>5.9</td>
<td>3.8</td>
<td>5.5</td>
<td>5.5</td>
<td>2.3</td>
<td>3.8</td>
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Finding a suitable number of electrodes and acceptable locations to place the electrodes requires a trade-off between reliability and wearability. Fewer electrodes lead to improved wearability, but will degrade the reliability, because additional electrodes are considered as backup electrodes (parallel connections). However, after finding the non-dominated solutions, the decision space is reduced and it is then easier to pick a suitable solution.
Figure 2.3: Pareto front for electrodes with trade-off solutions of non-reliability and non-wearability. The red circle indicates the chosen trade-off solution. Each solution on the Pareto front corresponds to a particular EEG/EOG electrode arrangement with reliability and wearability values according to equations (2.9) and (2.10).

The Pareto front is divided into three parts as highlighted in Figure 2.3. Each part corresponds to a particular electrode type. Microneedles are known to be the most comfortable electrodes. Hence, solutions with microneedles are in the Pareto front located with low non-wearability but high non-reliability. It is observed that capacitive electrodes are actually dominated by microneedle electrodes, because the microneedle is better or same in both objectives, namely comfort and reliability (see Table 2.3). However, mounting capacitive electrodes at locations with different wearability and reliability weightings can make them also non-dominant. This is due to the nonlinearity of fuzzy relations, which would not be possible to model with conventional weight multiplication.

Indeed, the Pareto front is a reduced design space, yet a decision is needed to pick a solution out of the Pareto front. Note that Figure 2.3 presents the combinations of electrode locations that should be used to monitor sleep at the highest level of wearability and reliability. However, a trade-off is required. One way to achieve this is to count how often a combination of electrode locations is selected for non-dominance. Four combinations of electrode locations are non-dominant solutions for all three types of electrodes. This means these combinations are comfortable and reliable locations for all types of electrodes and rather independent on the type of electrode. In the current implementation it was only possible to select one type for all locations, and mixed types of electrodes for various locations was not feasible (e.g., a mix of type (A) and type (C) electrodes). The next step is to select one solution out of the four combinations of electrode locations. Therefore, how often a location itself is part of the non-dominated solutions is counted (Figure 2.4). Locations chosen more often for non-dominance are both, comfortable and reliable. The electrode locations with most comfortable and reliable
locations from Figure 2.4 are selected as the final solution.

\[
\begin{align*}
&x_1, x_2, x_3, x_4, x_5, x_{11}, x_{14}, x_8, x_9, x_{13} \\
&x_1, x_2, x_3, x_4, x_5, x_{11}, x_8, x_9, x_{12}, x_{14}, x_{13} \\
&x_1, x_2, x_3, x_4, x_5, x_{11}, x_{14}, x_8, x_9, x_{13} \\
&x_1, x_2, x_3, x_4, x_5, x_{10}, x_{11}, x_{14}, x_8, x_9, x_{12}, x_{13}
\end{align*}
\]

**Figure 2.4:** The bars in the bar plots indicate number of times a location is part of individual non-dominated solutions.

Equal importance of wearability and reliability leads to a solution in the middle of the Pareto front and therefore to electrode type (A)-capacitive. The selected solution from the Pareto front in Figure 2.3 is marked by a red dot. The mounting locations of electrodes in the design of a wearable and reliable EEG/EOG monitoring system are \(x_1, x_2, x_3, x_4, x_5, x_7, x_{10}, x_{11}\) and \(x_{14}\) (Figure 2.5).

After selecting an adequate amount of electrodes and appropriate locations, it is necessary to find the most suitable way to supply them with power and read the EEG/EOG signals. The applied minimum spanning tree algorithm in equation (2.6) provides a spanning tree whose sum of edge weights is as small as possible. However, to strike a good trade-off between having a common power and signal connection and possibly supply electrodes individually results in the solution of connecting electrodes \(x_1, x_2, x_3, x_4, x_5\) and \(x_6\) and supply them together (implemented using equation (2.8)). Electrodes \(x_{10}\) and \(x_{14}\) are connected and electrode \(x_7\) and electrode \(x_{11}\) are individually supplied (see Figure 2.5). The reason for this is that more supply connections make the device more uncomfortable, while they create improved independence between electrodes and make the system more reliable. If one electrode goes off and is not connected with another one, it is more likely that it does not affect the others. This strikes a good trade-off between connecting electrodes to have a common supply connection or supply them separately. The final EEG/EOG design is presented in Figure 2.5. Comfortable and reliable connections in the front are connected through a cable channel and uncomfortable and particularly unreliable connections get their own supply connection. Hence, it makes sense for the two reference electrodes \(M1\) and \(M2\) to get higher independence. In the present work it is
assumed that wearability and reliability have the same level of importance. If one or the other objective is more important for the design, a different solution has to be picked from the Pareto front.

Figure 2.5: An EEG/EOG design. Electrodes on the forehead are connected in parallel and have a common supply connection.

2.8 Conclusion

The approach proposed in this chapter provided a systematic way to design a body sensor system that is comfortable and reliable wearable. A fuzzy technique was used to reduce the subjectivity of the assigned weights. The approach was applied to EEG/EOG-monitoring in sleep disorder evaluation, and an optimized result for an EEG/EOG monitoring system was presented. The methodology provides a tool that will assist a designer in the selection of the type of devices/sensors and good body locations to attach a device/sensor, and in deciding how to connect and supply them.

The qualitative objective functions in this work were wearability and reliability. To optimize these two objective functions, the design variables (number of devices) were considered as fitness functions. Thus design variables with various magnitudes and units (dimensions) need not be combined into one fitness function. The design space is reduced by keeping a non-dominant solution, thereby easing the design decisions. In the process, combinations of electrode locations and how often a location was selected for non-dominance were counted. Some aspects of this study were predefined for simplicity but could theoretically be adapted depending on the application. It was assumed that the two objective functions had the same level of importance. If one or the other is more important in the design, a solution further left or right on the Pareto front would result, thereby changing the final design. For the final design, only
components/electrodes of the same type were feasible solutions. A mix of electrodes was not considered. This would increase the design space and it would take a longer time to go through all possible design solutions on a standard personal computer. Also, memory issues have to be addressed for memory allocation operations. Therefore, a more sophisticated algorithm, such as an evolutionary algorithm would be suitable for optimization in relatively large design spaces. The algorithm must deal with binary variables and linear and nonlinear constraints. Finally, a sensitivity analysis for the weight assignment and the \( \alpha \)-value for the quantification of the fuzzy values could help to select a solution from the Pareto front.

The next chapter presents a more sophisticated approach to make a design decision. The method proposed here will be compared with other methods in Chapter 6.
Chapter 3

Design Optimization Using Fuzzy Sets and Fuzzy Measures

This chapter includes the research that was published and presented at the *IEMCON conference* (L. Falch and, C. W. de Silva, “Fuzzy Techniques to Reduce Subjectivity and Combine Qualitative and Quantitative Criteria in a Multi-Objective Design Problem”, *IEEE 9th Annual Information Technology, Electronics and Mobile Communication Conference (IEMCON)*, Vancouver, BC, Canada, 2018, pp. 42-48. doi:10.1109/IEMCON.2018.8614996)

3.1 Summary

The approach in the last chapter assisted the designer in reducing the design space to select a solution quite easily. However, the final design decision could not deal with different importance values of the objectives and was not able to consider interactions. This chapter uses fuzzy sets and fuzzy numbers to make a less subjective design decision when multiple objectives and qualitative criteria are involved. Through fuzzy measures and the Choquet integral it is possible to combine multiple qualitative and quantitative criteria into a single numerical value and make a proper design decision. The fuzzy measures, indicators of the importance of single criteria are determined using a linear program optimization method. The input for the linear program is only a preference order of some of the alternatives. With the verified fuzzy measures it is then possible to compute a single numerical value for the remaining alternatives. This results in an order of preferred design constructions and a recommendation for an optimized design. The theory is applied to EEG/EOG (electroencephalography/electrooculography) electrode placement for sleep monitoring by incorporating the criteria: comfort, reliability and power consumption.
3.2 Introduction

By considering multiple design criteria, the decision maker, expert or designer has to strike a trade-off to satisfy multiple objectives. The present chapter seeks to reduce the subjectivity and combine qualitative and quantitative objectives in a design problem by incorporating fuzzy sets, fuzzy numbers, fuzzy measures and fuzzy integrals. The theory is applied to design an EEG/EOG sensory device for sleep monitoring. The goal is to design a comfortable and reliable EEG/EOG monitoring system with low power consumption. These three objectives are partly qualitative and partly quantitative, and also subjective to some degree. It may not be possible to optimize them concurrently, and an acceptable trade-off for different design criteria is required.

Many researchers have sought to optimize or make decisions about a design using qualitative and quantitative design criteria. In the field of wearable body sensor systems, Anliker et al. [34] attempted a systematic way to choose and attach devices to a human body. Their design criteria included battery life, system functionality and wearability. To quantify these criteria, they used the average power consumption for battery life and the execution and communication delay between devices for functionality. For wearability they assigned intuitive weights to different body locations, which is highly subjective. To solve the design problem, the researchers applied the multi-objective evolutionary SPEA2 algorithm, which creates a set of solutions. Choosing the final design out of the solution set required a trade-off between the objectives battery life, functionality and wearability. The trade-off decision in their paper had no mathematical reason and was rather intuitive.

Also in the field of wearable body sensor systems (e.g., [36], [37]), researchers sought to design a wearable electrocardiogram (ECG) system with high functionality. One approach was rather intuitive and the other one was based on multivariate regression with real and categorical variables. To apply the technique of regression, the researchers made measurements of volunteers by attaching electrocardiogram electrodes at different body locations, in various rotation angles of the electrodes and with different pressure levels. In this process, the volunteers provided information about their comfort level. With the collected data the researchers created a model using a regression method and subsequently optimized the model with a multi-objective optimization algorithm, which was a set of solutions. The trade-off decision between the two objectives was executed by the researchers.

A standard technique to quantify qualitative design criteria, such as the appearance or the comfort of a product, in the field of wearable sensor systems, is quantification theory type 1 [25]. It makes use of linear regression, and volunteers or experts give weights to categories or criteria. This technique was for example applied to evaluate the design of a coffee maker, in [31] or to assess the design appearance of a car, in [22].

A common technique to compute the importance of qualitative criteria or linguistic variables is presented in the form of analytic hierarchy process (AHP) [21]. A decision maker or expert judges the importance of criteria by pairwise comparison of the criterion. The relative scale measurement is presented as a matrix and the weights are calculated through the method of
normalization of the geometric mean of rows (e.g., [23]) or eigenvectors (e.g., [22]).

Fuzzy logic or fuzzy numbers have been used to reduce subjective evaluation of decision makers. Hsiao and Chou [23] for example tried to make a model for human perception in homepage design. They used triangular fuzzy membership functions with linguistic importance judgments of qualitative factors. Volunteers evaluated a range of homepages and assigned linguistic weights addressing given design criteria. To quantify the linguistic importance judgments, the authors used $\alpha$-cuts and the center-of-gravity method. Their approach addressed and weakened the subjectivity of human-based decisions.

To reduce subjectivity in AHP, researchers in [22] used the fuzzy analytic hierarchy process (FAHP). They used triangular fuzzy numbers, $\alpha$-cuts and the lower and upper bounds of the $\alpha$-cuts to quantify the importance weights determined by AHP. FAHP was then applied to make a design decision in a multi-objective design problem.

Grabisch et al. [27] used a different approach to model and capture subjective and qualitative measures. With fuzzy measures and integrals they modeled the discomfort levels of subjects sitting in a car seat for a long period of time. Under various conditions while sitting on different car seats, subjects expressed their feelings in a questionnaire and provided an overall comfort value. To compute the fuzzy measures for particular criteria, they minimized the squared error of the overall comfort value.

Generally speaking, the overall satisfaction of all given criteria for a particular design is hardly possible. Oftentimes, it requires a questionnaire and/or a prototype to find satisfaction values.

The present chapter uses fuzzy sets and fuzzy numbers to reduce the subjectivity of qualitative judgments and fuzzy measures and integrals to make a more objective decision about the importance of multiple design criteria. This does not require questionnaires or prototypes and aims to achieve the most acceptable design solution.

### 3.3 Multi-Criteria Decision Aiding (MCDA)

Generally, it is not possible to measure a qualitative variable using a measuring device. The present work uses Multi-Criteria Decision Aiding (MCDA) to combine multiple objectives with qualitative design criteria and determine a ranking of preferred design solutions. Multiple criteria are represented by $N = \{1, ..., n\}$ as the set of criteria. Elements of a set of alternatives $A = \{a, b, c, ...\}$ are associated with a utility $x^a = (x_1^a, ..., x_n^a) \in \mathbb{R}^n$, where $x_i^a$ is the numerical value of $a$ related to criterion $i$, with $x_i^a \in \mathcal{X}_i \subseteq \mathbb{R}, i = 1, ..., n$. Note that all $x_i^a$ use a common scale ($\mathcal{X}_i = \mathcal{X} \forall i$).

To rank alternatives, or more specifically make a decision of which qualitative variables are a better choice than others, single criteria are combined into one numerical value using an aggregation operation, specifically the aggregation function [51]. Arguably the most popular method to aggregate criteria is the weighted arithmetic mean (weighted sum). However, many disadvantages of the weighted sum are known, such as the inability to model dependencies
between criteria or its sensitivity to extreme values [52]. In Multi-Attribute Value Theory (MAVT) [53], alternative $a$ is preferred over alternative $b$ ($a \succ b$), if their aggregated values computed using the aggregation function $F : X^n \rightarrow \mathbb{R}$ with $F(x^a) = F(x^a_1, ..., x^a_n)$ fulfill:

$$a \succ b \iff F(x^a) > F(x^b), \quad \forall a, b \in A$$  \hfill (3.1)

In order to model possible interactions between criteria and overcome the drawbacks of the weighted sum, a more generalized aggregation operator is necessary. The introduction of capacities by Choquet [54] or a similar concept proposed by Sugeno [55] as “fuzzy measures” or later by Schmeidler [56] as “non-additive measures” help to deal with interactions.

### 3.3.1 Capacities/Fuzzy Measures

A fuzzy measure [55] on $N$ is a mapping of $\mu : 2^N \rightarrow [0, 1]$ with the following conditions:

i) $\mu(\emptyset) = 0$, $\mu(N) = 1$ (boundary condition)

ii) $S \subseteq T \subseteq N$ implies $\mu(S) \leq \mu(T)$ (monotonicity)

A fuzzy measure assigns a weight to each subset of criteria. The mapping $\mu(S)$, with $S \subseteq N$ may be interpreted as the extensive weightings of criteria. For example, if $S = \{i, j\}$, the fuzzy measures $\mu(\{i, j\})$ would describe the combined weighting of criteria $i$ and $j$. An aggregation operator that uses capacities (fuzzy measures) is the Choquet integral [54], which is described next.

### 3.3.2 Choquet Integral as an Aggregation Operator

If there are no interactions between criteria, then $\mu(S \cup T) = \mu(S) + \mu(T)$, for any $S, T \subseteq N$ such that $S \cap T = \emptyset$. Then the fuzzy measures are said to be additive and consequently the fuzzy measures coincide with the weights of the weighted sum. However, if the fuzzy measures are non-additive then the weighted sum can be extended to the Choquet integral, which can evaluate $2^N$ values $\mu(S)$, with $S \subseteq N$.

$$C_\mu(a) = y^a_1 \mu(N) + \sum_{i=2}^{n}(y^a_i - y^a_{i-1})\mu(\{i, ..., n\})$$  \hfill (3.2)

where $y^a_i$ are the rearranged utility values $x^a_i$ of the criteria $a$, such that $y^a_1 \leq ... \leq y^a_n$. The complexity of fuzzy measures makes their use challenging, because $n$ criteria require $2^n$ coefficients in $[0, 1]$ to define the fuzzy measure $\mu$ of every subset. Even a fuzzy measure that considers the combined weighting of three criteria, is already challenging to interpret. Furthermore, if this information needs to be provided by a decision maker or expert, it is hardly possible to address more than two criteria. To reduce the complexity of this power set ($2^n$ fuzzy measures), Grabisch [57] introduced $k$-additive fuzzy measures. In order to explain
the concept of \( k\)-additive fuzzy measures, it is useful to introduce the Möbius representation of fuzzy measure \( \mu \). The Möbius representation is a function \( m : 2^N \rightarrow \mathbb{R} \) and is defined as:

\[
\mu(S) = \sum_{T \subseteq S} m(T)
\]  

(3.3)

where the Möbius representation \( m(S) \) is obtained from \( \mu(S) \) as follows:

\[
m(S) = \sum_{T \subseteq S} (-1)^{|S|-|T|} \mu(T)
\]  

(3.4)

The following equation shows the Choquet integral in terms of the Möbius representation:

\[
C_m(a) = \sum_{T \subseteq N} m(T) \min_{i \in T} x_i^a
\]  

(3.5)

A fuzzy measure is \( k\)-additive, if for \(|T| > k\), \( m(T) = 0 \), with \( T \subseteq N \). Because of the mentioned complexity of \( 2^n \) fuzzy measures, the present study only considers \( 2\)-additive measures, which represents the most common scenario in practical applications. This only requires \( n + \binom{n}{2} = \frac{n(n+1)}{2} \) coefficients. A \( 2\)-additive fuzzy measure or capacity \( \mu \) for a set \( S \subseteq N \) is specified with respect to the Möbius representation as follows:

\[
\mu(S) = \sum_{i \in S} m(\{i\}) + \sum_{\{i,j\} \subseteq S} m(\{i,j\}), \quad \forall S \subseteq N
\]  

(3.6)

For the \( 2\)-additive case the Choquet integral expressed in terms of the Möbius representation becomes [57]

\[
C_m(a) = \sum_{i \in N} m(\{i\}) x_i^a + \sum_{\{i,j\} \subseteq N} m(\{i,j\}) \min(x_i^a, x_j^a)
\]  

(3.7)

One disadvantage of the Choquet integral is that every criterion has to have the same scale. To resolve this problem the utility values are normalized as follows:

\[
x_i^a = \frac{x_i^a - \min(x_i)}{\max(x_i) - \min(x_i)}
\]  

(3.8)

The importance of a criterion does not only depend on one single fuzzy measure \( \mu(\{i\}) \) or \( m(\{i\}) \), but also on each contribution of each coalition of criteria. The “importance” index introduced in cooperative game theory facilitates the understanding of the meaning of the numerical value of a fuzzy measure. The importance index, also known as the Shapley index [58] indicates the overall importance of a criterion \( i \in N \), not only represented by the fuzzy measure \( \mu(\{i\}) \), but also by all \( \mu(S) \) with \( i \in S \). A criterion \( i \) is classified as important whenever \( i \) enters a group of criteria \( T \) and the difference \( \mu(T \cup i) - \mu(T) \) is high. The Shapley index \( \phi_{Sh}(\{i\}) \) is a weighted average of the difference over all possible \( T \subseteq N \setminus \{i\} \). In the \( 2\)-additive
The Shapley index is described as

$$
\phi_{Sh}({i}) = m({i}) + \frac{1}{2} \sum_{j \in N \setminus \{i\}} m({i, j})
$$

(3.9)

To illustrate the degree of interaction of a pair of criteria \{i, j\} ⊆ N an interaction index helps to elucidate the interaction between i and j. In the 2-additive Möbius representation the interaction \(I({i, j})\) is simply:

$$
I({i, j}) = m({i, j})
$$

(3.10)

If \(I({i, j})\) is positive then there is a positive interaction or synergy between i and j, which means that the satisfaction of both criteria is more valuable than the satisfaction of them separately. If it is negative it means that the criteria overlap and the satisfaction of both is not better than the satisfaction of one of them. A value of zero means that there is no interaction between criteria i and j.

To illustrate the concept of the Shapley index consider three workers \(x_1, x_2\) and \(x_3\). Suppose that their fuzzy measures, representing the worker’s productivity, are as follows:

- \(m({x_1}) = 0.3, m({x_2}) = 0.2, m({x_3}) = 0.3\)
- \(m({x_1, x_2}) = 0.6, m({x_1, x_3}) = -0.2, m({x_2, x_3}) = -0.2\)

By looking at the fuzzy measures of single criterion one can see, that the workers \(x_1\) and \(x_3\) have the same importance for a company when they work alone on a project. However, the combined fuzzy measure \(m({x_1, x_2}) = m({x_1}) + m({x_2})\) indicates that the worker \(x_1\) has a great relationship with the worker \(x_2\) and is therefore able to work as a team. The combined fuzzy measure \(m({x_1, x_3}) \leq m({x_1}) + m({x_3})\), indicates that they have a bad relationship and the worker \(x_3\) is not able to work in a team. Therefore, the worker \(x_1\) is more important for the company, which is represented by the Shapley values \(\phi_{Sh}({x_1}) = 0.5, \phi_{Sh}({x_2}) = 0.4, \phi_{Sh}({x_3}) = 0.1\).

As mentioned before, \(n + \binom{n}{2}\) parameters are required to compute the Choquet integral of a 2-additive fuzzy measure. These fuzzy measures have to be determined.

### 3.3.3 Determination of Fuzzy Measures

A decision maker or expert can decide the importance of a single criterion, but not an exact numerical value corresponding to it. Grabisch, Kojadinovic and Meyer [59] reviewed useful techniques to identify fuzzy measures that can achieve this. A commonly used method is based on the least-square approach. It requires the knowledge of the overall aggregated value of each alternative. It can be hard to evaluate this or may be completely unknown. Generally, the overall satisfaction of all given criteria for a particular design is hardly possible. Often, it requires a questionnaire and/or a prototype to obtain the satisfaction values. Therefore,
the least-squares approach is not always acceptable. The approach of Marichal and Roubens [60] based on linear programming is another possible way to determine fuzzy measures. Here, a decision maker or expert provides information on preferences, such as the preferences of alternatives or criteria. Here, a decision maker or expert ranks solutions (e.g., a preferred overall design) and also the preference of criteria. The model can be stated as follows:

\[
\max \ F_{MR}(\epsilon) = \epsilon \quad (3.11)
\]

subject to

\[
C_m(a) - C_m(b) \geq \epsilon + \delta \quad \text{if } a \succ_A b \quad (3.12)
\]

\[
\phi_{Sh}(\{i\}) - \phi_{Sh}(\{j\}) \geq \epsilon \quad \text{if } i \succ_N j \quad (3.13)
\]

\[
\sum_{i \in N} m(\{i\}) + \sum_{\{i,j\} \subseteq N} I(\{i,j\}) = 1 \quad (3.14)
\]

\[
m(\{i\}) + \sum_{j \in T} I(\{i,j\}) \geq 0, \forall i \in N, \forall T \subseteq N \setminus i \quad (3.15)
\]

\[
m(\{i\}) \in [0, 1], I(\{i,j\}) \in [-1,1], \epsilon \in [0,2] \quad (3.16)
\]

The principle of this approach is to maximize the minimal difference between the overall aggregated alternatives, where the alternatives have been ordered with respect to their preference by an expert or a decision maker. Equation (3.12) represents the rankings of the alternatives, which indicates that the aggregated value of alternative \( a \) is greater than the aggregated value of \( b \) (\( C_m(a) \geq C_m(b) \)). \( \delta \) in equation (3.12) guarantees that the difference between two aggregated values is at least \( \delta \). A larger \( \delta \) results in a smaller optimized \( \epsilon \). Once \( \delta \) exceeds a threshold value it can result in no solution at all. It is suggested to compare the solutions for different values of \( \delta \). Equation (3.13) specifies the preference of criterion \( i \) over \( j \) and makes criterion \( i \) more important than criterion \( j \). Equation (3.15) represents the monotonicity conditions, which are \( (2^{n-1} - 1) \cdot n \) constraints \( (2^{n-1}, \forall T \subseteq N \setminus i; -1 \text{ for } \emptyset; \text{ times } n, \forall i \in N) \). Equation (3.16) are the boundary conditions.

In the next sections, the theory of fuzzy measures and integrals is applied to the design optimization of an EEG/EOG device for sleep monitoring with respect to comfort, reliability and power consumption. In order to get the utility values for each electrode location, as presented previously, the assignment of fuzzy weights is necessary. The relevant fuzzy weights for comfort and reliability are listed in Section 2.6, Table 2.2.

### 3.4 Classification of Utility Values

There are four possible ways to place electrodes for EEG/EOG monitoring (see Section 2.5). Combinations of these four are possible as well. In order to estimate the overall comfort and reliability of each version, the present work uses fuzzy arithmetic (Section 2.4). The level of discomfort increases when more electrodes are placed on the head. Placing electrodes at locations with high comfort (such as the forehead) minimally impacts the overall discomfort (see
Table 2.1). In order to measure EEG/EOG signals, each electrode is connected with a reference electrode (Section 2.5). Therefore, electrodes are connected in parallel; the reliability increases when more electrodes are used because of the increased redundancy. The power consumption increases with each additional electrode. Since the electrodes are connected in parallel with a reference electrode, each electrode (except the reference electrode) creates a voltage drop and hence increases the power consumption. Simply, the number of electrodes, excluding the reference electrodes, is proportional to the power consumption. With this information and the theory of fuzzy arithmetic and defuzzification, utility values for the criteria: comfort, reliability and power consumption can be determined for 15 possible electrode placements (Table 3.1). In order to compare the three criteria and aggregate them using the Choquet integral, a normalization between 0 and 1 is performed (see parentheses in Table 3.1).

Table 3.1: Utility values for discomfort, reliability and power consumption. The numbers in brackets are the normalized utility values.

<table>
<thead>
<tr>
<th>Id</th>
<th>Version</th>
<th>Discomfort</th>
<th>Reliability</th>
<th>Power consumption (# of electrodes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2.83 (0.0714)</td>
<td>3.5 (1)</td>
<td>5 (0)</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2.83 (0.0714)</td>
<td>3.5 (1)</td>
<td>5 (0)</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2.83 (0.0716)</td>
<td>3.83 (0.9189)</td>
<td>5 (0)</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>2.49 (0)</td>
<td>3.50 (0.9999)</td>
<td>5 (0)</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>5.33 (0.6071)</td>
<td>5.33 (0.5542)</td>
<td>8 (0.4286)</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>4.50 (0.4287)</td>
<td>5 (0.6351)</td>
<td>7 (0.2857)</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>4.99 (0.5357)</td>
<td>5.33 (0.5541)</td>
<td>9 (0.5714)</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>5.33 (0.6072)</td>
<td>5.66 (0.4728)</td>
<td>9 (0.5714)</td>
</tr>
<tr>
<td>9</td>
<td>24</td>
<td>3.49 (0.2143)</td>
<td>4.83 (0.6754)</td>
<td>7 (0.2857)</td>
</tr>
<tr>
<td>10</td>
<td>34</td>
<td>4.50 (0.4287)</td>
<td>5.16 (0.5945)</td>
<td>8 (0.4286)</td>
</tr>
<tr>
<td>11</td>
<td>123</td>
<td>7.00 (0.9644)</td>
<td>6.83 (0.1893)</td>
<td>11 (0.8571)</td>
</tr>
<tr>
<td>12</td>
<td>124</td>
<td>5.99 (0.7500)</td>
<td>6.66 (0.2297)</td>
<td>9 (0.5714)</td>
</tr>
<tr>
<td>13</td>
<td>134</td>
<td>6.16 (0.7858)</td>
<td>6.33 (0.3107)</td>
<td>10 (0.7143)</td>
</tr>
<tr>
<td>14</td>
<td>234</td>
<td>6.33 (0.8216)</td>
<td>6.66 (0.2297)</td>
<td>9 (0.5714)</td>
</tr>
<tr>
<td>15</td>
<td>1234</td>
<td>7.16 (1)</td>
<td>7.61 (0)</td>
<td>12 (1)</td>
</tr>
</tbody>
</table>

3.5 Identifications of Fuzzy Measures

Table 3.1 shows that the optimization of the three criteria: comfort, reliability and power consumption requires a trade-off. There are 15 possible ways to place the electrodes for EEG/EOG monitoring. For example, version 14 is the combination of version 1 and 4. Version 4 has the lowest discomfort value and also the lowest power consumption. However, the highest reliability is achieved by version 1234. For the described approach to identify fuzzy measures (Section 3.3.3) a preference of some versions over others is needed. A decision maker or expert decides, for example, that version 1 combined with version 4 (version 14) is preferred over version 4 and version 4 is preferred over version 3. This can be described as a linear programming inequality constraint, as in equation 3.12:

- version 14 $\succ$ version 4 $\succ$ version 3

The designer can furthermore decide that comfort and reliability are more important than power.
consumption and comfort is more important than reliability. These relations are implemented as inequality constraints as in equation 3.13:

- comfort $\succ$ power consumption
- reliability $\succ$ power consumption
- comfort $\succ$ reliability

The MATLAB function `linprog` determines the fuzzy measures for the preferred alternatives and criteria. Figure 3.1 shows the evolution of the fuzzy measures for comfort, reliability and power consumption as a function of $\delta$. The $\delta$ value changes in a range, and the linear program finds a solution to maximize $\epsilon$. As stated before, the fuzzy measure for comfort appears as the most preferred fuzzy measure $\mu$, followed by reliability and power consumption.

![Figure 3.1: Sensitivity of the $\delta$ value. The plot shows the fuzzy measures as a function of $\delta$. A too high $\delta$ value will violate the preference rankings (equation (3.12)).](image)

A high $\delta$ value makes the linear program infeasible. Therefore, the final solution will be determined with a $\delta$ value of 0.005, because a smaller $\delta$ makes $\epsilon$ bigger, and the linear program is still feasible. The determined fuzzy measures and Shapley indices are presented in Table 3.2.

<table>
<thead>
<tr>
<th>$\mu({i})$</th>
<th>Comfort</th>
<th>Reliability</th>
<th>Power consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{Sh}({i})$</td>
<td>0.3172</td>
<td>0.2989</td>
<td>0.7531</td>
</tr>
</tbody>
</table>

The order of importance of the fuzzy measures $\mu$ coincides with the linear program constraints. However, the fuzzy measures $\mu$ do not necessarily reflect the importance of the criteria as described earlier. The importance of a single criterion is given by the Shapley index and therefore power consumption is the most important criterion, followed by comfort and reliability as the least important ones. The importance values are derived from the preferred order of alternatives (version 14 $\succ$ version 4 $\succ$ version 3). Table 3.3 presents the interaction indices between the criteria.
Table 3.3: Interaction indices between the three criteria.

<table>
<thead>
<tr>
<th></th>
<th>Comfort</th>
<th>Reliability</th>
<th>Power consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comfort</td>
<td>—</td>
<td>0.3839</td>
<td>-0.5612</td>
</tr>
<tr>
<td>Reliability</td>
<td>0.3839</td>
<td>—</td>
<td>-0.5612</td>
</tr>
<tr>
<td>Power consumption</td>
<td>-0.5612</td>
<td>-0.5612</td>
<td>—</td>
</tr>
</tbody>
</table>

There is a positive interaction between reliability and comfort. The negative interaction between comfort and power consumption indicates that the satisfaction of both criteria is not better than satisfying one of them. The negative interaction also means that there is an overlap between these two criteria, which is consistent with the results in Table 3.1. The normalized utility values of discomfort and power consumption tend to increase concurrently. The overlap between these two criteria can be explained by the fact that on increasing the number of electrodes, the power consumption increases as well as the discomfort. The determined fuzzy measures are the parameters required to compute the Choquet integral.

3.6 Final Design Stage for EEG/EOG Electrode Placement

Table 3.4 presents the sorted Choquet integral values and an order of the preferred versions to attach EEG/EOG electrodes in sleep monitoring. The Choquet integral was computed with the determined fuzzy measures.

Table 3.4: Sorted Choquet integral values and the corresponding electrode placements.

<table>
<thead>
<tr>
<th>Index</th>
<th>$C_{\mu}(f)$</th>
<th>Version</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4815</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>0.5248</td>
<td>34</td>
</tr>
<tr>
<td>3</td>
<td>0.5407</td>
<td>124</td>
</tr>
<tr>
<td>4</td>
<td>0.5531</td>
<td>23</td>
</tr>
<tr>
<td>5</td>
<td>0.5561</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>0.5794</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>0.5835</td>
<td>234</td>
</tr>
<tr>
<td>8</td>
<td>0.5947</td>
<td>134</td>
</tr>
<tr>
<td>9</td>
<td>0.5978</td>
<td>1234</td>
</tr>
<tr>
<td>10</td>
<td>0.6027</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>0.6285</td>
<td>13</td>
</tr>
<tr>
<td>12</td>
<td>0.6496</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>0.6496</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>0.6526</td>
<td>123</td>
</tr>
<tr>
<td>15</td>
<td>0.6563</td>
<td>12</td>
</tr>
</tbody>
</table>

Version 24 is the most preferred solution, followed by version 34. Note that the solution depends on the preference of some versions. However, the advantage of this approach is that some experience or limited preferences can help in the determination of the fuzzy measures and the order or preference of all other unknown options. This leads to less subjective decision making.
3.7 Conclusion

The approach presented in this chapter used fuzzy sets and fuzzy numbers to reduce subjectivity of qualitative measures. Fuzzy arithmetic and the center-of-gravity method quantified the qualitative criteria. To combine different criteria with distinct weightings of importance, the present work used fuzzy measures and the Choquet integral. Fuzzy measures take into account the interaction between criteria and the Choquet integral also considers non-linearities in the aggregation process.

The theory was applied to the design of an EEG/EOG device for sleep monitoring. With the proposed approach it was possible to combine multiple objectives with quantitative and qualitative criteria and reduce subjectivity in the decision making process to find an appropriate design.

However, the fuzzy weight assignment for comfort and reliability was still somewhat intuitive. The use of fuzzy techniques admittedly reduced subjectivity, but for a more precise solution the use of a questionnaire would be desirable. The ranking of alternatives over others was carried out as an example case. In a real application, an expert would be required to provide the ranking. Furthermore, the design space of 15 alternatives was rather small. A bigger design space would require a multi-objective optimization algorithm to determine the Pareto optimal solutions first. The non-dominant solutions could then be considered as a preselection, and further decision making could be handled using the proposed theory. The proposed theory however, will be compared with another decision making method in Chapter 6. The next chapter applies the theory of fuzzy measures to determine comfortable locations on the human body. With the help of a questionnaire, the objective comfort used in the present and the previous chapters can be described more meaningfully.
Chapter 4

Incorporating Comfort into the Design

This chapter is based on a paper that has been submitted to an appropriate journal.

4.1 Summary

To demonstrate the approach developed in Chapter 2 and 3, the formulation of qualitative objectives was based on rather intuitive aspects. This chapter presents an approach to represent a qualitative objective in a systematic and analytic way in the design of a wearable device. This is particularly useful when designing a wearable mechatronic device that incorporates sensors, where qualitative design objectives play a significant role in the consumer appeal and success of the product. An approach is developed in which the qualitative objective is represented using multiple criteria. Those criteria are then combined using fuzzy measures and the Choquet integral for making design decisions. The particular qualitative variable that is considered in this chapter is the comfort of a wearable body sensor system. The presented approach evaluates the nature and importance of the criteria that describe comfort in a wearable body sensor system and determines the most suitable locations where the hardware should be mounted, by taking into consideration those criteria. The fuzzy measures are determined through two different methods. One method uses a least squares algorithm and the other determines fuzzy measures using a preference ranking of alternatives. Preference rankings are locations, ordered from the most comfortable to the least comfortable. This order of locations can be replicated with the determined fuzzy measures and Choquet integral, to build a comfort model. An error describing the inaccuracy in the order of preferred to non-preferred alternatives is introduced. The comfort model established in this work is validated using a training set and test set. A comparison between the two methods that determine the fuzzy measures is given.
4.2 Introduction

This chapter addresses the design of a wearable body sensor system. For the design of such a system one needs to consider qualitative design aspects as well, such as comfort. The optimization of devices with quantitative and qualitative optimization parameters is a rather complex and challenging process, because qualitative objectives can be highly subjective and difficult to describe in a mathematical way. An analytical formulation, however, is required in order to integrate qualitative objectives with quantitative objectives. Even for an experienced designer it is difficult to simultaneously include quantitative and qualitative design criteria into the design process in a systematic and reliable manner. This chapter presents an analytical approach to incorporate a qualitative design objective into the design process, in a less subjective manner. It shows how to systematically deal with the qualitative objective of comfort. It describes the qualitative design objective using multiple criteria and combines them by using fuzzy measures and integrals. Two methods to determine the fuzzy measures are developed and compared. The introduction of an error that describes the mismatch of preference rankings of alternatives facilitates this comparison.

In the design of various devices and products it is important to include qualitative design objectives as well in the design process. Qualitative variables cannot be quantified by absolute numerical values and depend on considerations such as intuition, subjectivity, experience, expertise and other “fuzzy” attributes. Incorporation of qualitative variables into the design process will require methodologies that are not used in traditional, quantitative design. In particular, qualitative variables may be represented using weights or descriptive terms. A common approach that deals with qualitative variables is the analytic hierarchy process (AHP) [21]. AHP provides a comprehensive and rational framework for structuring a decision by creating a comparison matrix where alternatives are compared with respect to some criteria. Decision makers or experts give ratings of importance, and using the principal right eigenvector of the comparison matrix, utility values for all criteria are determined. For the final decision \((|A| + 1) \cdot n(n-1)/2\) individual values have to be considered, where \(n\) as number of criteria and \(|A|\) as the number of alternatives. In the field of product design, Hsiao et al. [31] or Shieh et al. [22] have used AHP to improve the shape and style of a coffee maker and a car, respectively, by evaluating linguistic variables. The linguistic values have been assigned by subjects via a questionnaire. Azeez et al. [61] applied AHP for selecting a configuration of a transmission system for a winch, and thereby configure the design of a gearbox.

In the field of wearable devices Anliker et al. [34] have attempted to systematically choose and attach devices to desirable locations of the human body. To represent the qualitative objective of wearability, they assigned intuitive weighting vectors to different body locations. The weight assignment itself was highly subjective and they have pointed to ergonomic research and social acceptability considerations for further investigation of wearability.

Other work in the field of body sensor systems (e.g., [36], [37]), have attempted to optimize the design of an electrocardiogram (ECG) with respect to comfort and functionality. Their first
attempt to do so, has been rather intuitive, and a follow up paper used multivariate regression with real and categorical variables for this purpose. To apply regression analysis, the researchers have collected measurements by attaching electrocardiogram electrodes to volunteers, and have used a questionnaire to gain insight into functionality and comfort.

Grabisch [27] modeled comfort with fuzzy measures and integrals to represent the discomfort of subjects sitting in a car seat for long periods of time. Subjects expressed their comfort level under various conditions by filling out a questionnaire. The fuzzy measures, which describe the importance and interactions among criteria to some extent, were then identified by minimizing the squared error of the overall comfort value.

A recent paper in the field of product form design also used the technique of fuzzy measures and integrals [62] to optimize the design of a vase. They used $\lambda$-fuzzy measures and determined them by assigning numerical values through a decision maker. This of course increased the subjectivity and lead to a somewhat non-reproducible result.

Our previous work [63] used fuzzy weights as qualitative measurements for body worn sensors. The method was based on linear programming and considered wearability and reliability. The appropriate hardware locations were then selected based on constraints. That investigation also provided information on how to attach and wire wearable devices. A case study applied the approach to an electroencephalogram (EEG)/electrooculography (EOG) system for sleep monitoring.

Fuzzy measures and integrals have been used in our previous work [64] to make a more sophisticated design decision. The considered objective functions were comfort, reliability and power consumption. The method was again applied to an EEG/EOG design for sleep monitoring. The comfort and reliability values were rather intuitively assigned using fuzzy weights, and the power consumption had a proportional relation to the number of attached electrodes.

An important qualitative design objective in the field of wearable body sensor systems is comfort. Kasabach et al. [11] have attempted to design an accurate and comfortable body sensor system. To determine a good mounting location on the human body, they developed three prototypes and tested them on subjects to decide on the mounting locations. This is a reasonable approach; however, resources are not always available to build prototypes and conduct experimental studies. Alternatively, a software tool that facilitates optimal design decisions will save time and money. Furthermore, Kasabach et al. [11], made their design decision for the device location based on human reasoning. This results in a subjective decision, does not consider relative importance of the arguments, and it is difficult to validate a particular decision. The existing wearable body sensor systems have not included comfort as an objective in a mathematical or analytical way into the design process. Past research has only evaluated comfort through questionnaires, using an experimental prototype or an already manufactured device [29], [30].

The present chapter revisits the qualitative criterion comfort, which is highly subjective and depends on many factors. In fact, Knight et al. [29] identified 92 comfort terms to
describe a wearable body sensor system and they reduced them into 6 main groups (emotions, attachment, harm, perceived change, movement, and anxiety). Many design variables such as attachment methods, materials, and so on, are challenging to incorporate into a comfort model. Therefore, the present study does not seek to determine the most comfortable setup for the sensors themselves, but rather find a location at which the device can be most comfortably attached. In order to do that, this research study describes comfort using multiple criteria such as pressure pain threshold, motion impedance, social acceptability, touch sensitivity, and pain sensitivity. Data sets and utility values are available to describe these criteria. Specifically, the paper by Zeagler [65] summarizes the literature over the past 20 years that considers these criteria. However, up to this point it is unclear how important these criteria are, how they interact with each other, and how they can describe comfort for a wearable body sensor system in an analytical way. The present chapter considers these aspects of designing a wearable device.

4.3 Comfort as a Design Objective

Producing a successful product that achieves wide acceptance from the consumer requires the consideration of the locations where various hardware components are placed. Here, comfort is described using multiple criteria. Comfort is a very broad term, and depending on the application different criteria may be considered. In order to compare the comfort of device locations, various aspects such as the type of attachment, the material used to attach hardware and the attachment force have to be considered. The considered attachment for the comfort model that is used here is a belt wrapped around a specific body location. The criteria and the corresponding data that describe comfort at various body locations have been summarized by Zeagler [65], on considering functional, technical and social aspects of body locations for wearable technology in the past 20 years. The present paper uses criteria and definitions given by Zeagler [65]. Some pertinent terminology is given now.

- **Proxemics (Prox)** is the perception of the self-size of a human. The self-size is the maximum distance a device can extend from the body while it is still naturally considered as part of the human body, through self-size cognition [66].
- **Pressure Pain Threshold (PPT)** is the amount of weight or pressure that can be applied before it is perceived as discomfort. It is defined by Weber-Fechner’s law [67], which states that a sensation is proportional to the logarithm of the stimulus.
- **Motion Impedance (MI)** describes the opposite of the ability to move freely [65]. Regions that are not restricted by a mounted device provide the least level of body motion impedance.
- **Social Acceptability (SA)** areas on the body are associated with sexual sensation or excretion of body waste [65].
- **Touch Sensitivity** is perceived through mechanoreceptors, which are sensors in the skin that respond to mechanical pressure or distortion [68].
• **Pain Sensitivity** Webster’s on-line dictionary [69] defines comfort as a “state of being relaxed and feeling no pain.” Several studies address pain and touch sensations (e.g., [70], [71]) and give information about the body locations that are sensitive to varying degree.

• **Tissue Volume** Amount of tissue and/or muscles at a particular body location. On engaging the tissue, such as a muscle or the lungs, the volume increases and therefore the attachment would feel tighter. For the present study, the muscle volume was collected from the past work [72], [73], [74] and [75]. The following are the tissues/muscles used for this criterion.

  – Forearm: brachioradialis, pronator teres, flexor carpi radialis, flexor carpi ulnaris, palmaris longus, flexor digitorum superficcius, extensor carpi radialis brevis, extensor carpi radialis longus, extensor digitorum communis, extensor carpi ulnaris
  – Upper Arm: biceps femoris long head, semimembranosus, brachialis, biceps brachii, triceps brachii long head, triceps brachii
  – Shoulder: deltoid, subscapularis
  – Chest: pectoralis major sternocostal, pectoralis major, lung, heart, rhomboid major, trapezius, infraspinatus, subscapularis, supraspinatus, rhomboid minor
  – Abdomen: external obliques, rectus abdominus, internal obliques, jejunum, colon, latissimus dorsi
  – Hip: gluteus maximus, gluteus medius, gluteus minimus, iliopsoas
  – Thigh: vastus intermedius, adductor longus, sartorius, adductor magnus, quadriceps femoris, biceps femoris, vastus lateralis, vastus medialis, rectus femoris, sartorius, semitendinosus, semimembranosus
  – Calf: soleus, medial gastrocnemius, lateral gastrocnemius, tibialis anterior
  – Locations such as the wrist or the ankle are assumed to have no tissue volume.

The data used in this research are presented in Table 4.1. The values are normalized between 0 and 1. The reason to do so is because of the Choquet integral and is explained in Section 3.3.2 and the corresponding operation is given by equation (3.8). A value of 0 means it is the worst location to attach hardware in terms of the specific criterion. A value of 1 for tissue volume means there is no tissue at all and it is therefore ranked highest in terms of the tissue volume.

The degree of comfort does not necessarily increase additively between two criteria. Therefore, interactions are considered using the theory of fuzzy measures and integrals to develop a model for the qualitative objective comfort. It is then possible to decide on the most comfortable location and get a better understanding of the criteria that play a role in designing a comfortable wearable body sensor system.
### Table 4.1: Normalized utility values

<table>
<thead>
<tr>
<th>Location</th>
<th>Prox</th>
<th>PPT</th>
<th>MI</th>
<th>SA</th>
<th>Touch</th>
<th>Pain</th>
<th>Tissue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Wrist</td>
<td>0.000</td>
<td>0.013</td>
<td>0.361</td>
<td>1.000</td>
<td>0.391</td>
<td>0.409</td>
<td>1.000</td>
</tr>
<tr>
<td>2 Forearm</td>
<td>0.045</td>
<td>0.013</td>
<td>1.000</td>
<td>1.000</td>
<td>0.435</td>
<td>0.455</td>
<td>0.960</td>
</tr>
<tr>
<td>3 Elbow</td>
<td>0.636</td>
<td>0.000</td>
<td>0.023</td>
<td>1.000</td>
<td>0.609</td>
<td>0.682</td>
<td>1.000</td>
</tr>
<tr>
<td>4 Upper Arm</td>
<td>0.455</td>
<td>0.093</td>
<td>0.776</td>
<td>1.000</td>
<td>0.696</td>
<td>0.773</td>
<td>0.859</td>
</tr>
<tr>
<td>5 Shoulder</td>
<td>0.636</td>
<td>0.360</td>
<td>0.361</td>
<td>1.000</td>
<td>0.696</td>
<td>0.091</td>
<td>0.980</td>
</tr>
<tr>
<td>6 Forehead</td>
<td>0.045</td>
<td>0.000</td>
<td>0.877</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>7 Chest</td>
<td>0.455</td>
<td>0.360</td>
<td>0.530</td>
<td>1.000</td>
<td>0.739</td>
<td>0.818</td>
<td>0.249</td>
</tr>
<tr>
<td>8 Abdomen</td>
<td>0.455</td>
<td>0.573</td>
<td>0.639</td>
<td>0.500</td>
<td>0.913</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>9 Hip</td>
<td>1.000</td>
<td>1.000</td>
<td>0.598</td>
<td>0.000</td>
<td>0.478</td>
<td>0.636</td>
<td>0.962</td>
</tr>
<tr>
<td>10 Upper Thigh</td>
<td>0.455</td>
<td>0.360</td>
<td>0.868</td>
<td>0.500</td>
<td>0.565</td>
<td>0.818</td>
<td>0.677</td>
</tr>
<tr>
<td>11 Lower Thigh</td>
<td>0.455</td>
<td>0.053</td>
<td>0.708</td>
<td>1.000</td>
<td>0.565</td>
<td>0.818</td>
<td>0.707</td>
</tr>
<tr>
<td>12 Knee</td>
<td>0.455</td>
<td>0.573</td>
<td>0.000</td>
<td>1.000</td>
<td>0.522</td>
<td>0.682</td>
<td>1.000</td>
</tr>
<tr>
<td>13 Calf</td>
<td>0.182</td>
<td>0.053</td>
<td>0.986</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.878</td>
</tr>
<tr>
<td>14 Ankle</td>
<td>0.182</td>
<td>0.173</td>
<td>0.562</td>
<td>1.000</td>
<td>0.870</td>
<td>0.955</td>
<td>1.000</td>
</tr>
</tbody>
</table>

One reason that the technique of fuzzy measures is suitable in the present methodology is the monotonicity condition. Through monotonicity it is guaranteed that if an alternative, in this case a location, is the best in each criterion, (e.g., lowest motion impedance, lowest touch, pain sensitivity), another alternative cannot be preferred. Hence, it is ensured that the used criteria describe the objective. The theory of fuzzy measures and the Choquet integral was described in the previous chapter in Sections 3.3.1 and 3.3.2. However, the determination of fuzzy measures is extended here and it is compared with a least squares approach.

### 4.4 Determination of Fuzzy Measures

As mentioned in the previous chapter, a commonly used method is based on the least-square approach. It requires the knowledge of the overall aggregated value $d^a$ of each alternative. In this chapter, the comfort value for each location is used for this purpose. The data from a survey questionnaire are used to compute this value. The least squares approach may be formulated as follows:

$$
\min_m \frac{1}{2} \sum_A ||C_m(a) - d^a||_2^2
$$

subject to

$$BoundMonCond.$$

$BoundMonCond.$ are the same boundary and monotonicity conditions as in Section 3.3.3:

$$
\sum_{i \in N} m(\{i\}) + \sum_{\{i, j\} \subseteq N} I(\{i, j\}) = 1
$$

$$
m(\{i\}) + \sum_{j \in T} I(\{i, j\}) \geq 0, \forall i \in N, \forall T \subseteq N \setminus i
$$

$$
m(\{i\}) \in [0, 1], I(\{i, j\}) \in [-1, 1]
$$

The explanation of the constraints is provided in Section 3.3.3.

The approach of Marichal and Roubens [60] based on linear programming, which is also used
in the previous chapter, is improved here. A decision maker or expert provides information on preferences, such as the preferences of alternatives or criteria. As before, the necessary data for the computation of the preferences are again provided through a survey questionnaire. The two approaches of determining the fuzzy measures differ as follows: The least square approach seeks to minimize the sum of errors between $C_m(a)$ and $d^a$, whereas Marichal and Roubens approach tries to accomplish the same preference ranking as from the questionnaire (ordering of locations from the most comfortable to the least comfortable). The linear programming problem is extended to a mixed integer linear program:

$$\max \mathcal{F}_{MR}(\epsilon) = \epsilon + \sum_{k=1}^{p} w_k \cdot y_k \quad (4.5)$$

subject to

$$C_m(a) - C_m(b) \geq \epsilon - M(1 - y_k) + \delta \quad \text{if } a \succ_A b \quad (4.6)$$

$$\phi_{Sh}(\{i\}) - \phi_{Sh}(\{j\}) \geq \epsilon \quad \text{if } i \succ_N j \quad (4.7)$$

$BoundMonCond.$

$p$ number of rankings, $M$ is positive large number

$$y_k \in \{0, 1\}, \epsilon \in [0, 2] \quad (4.8)$$

Equation (4.6) represents the rankings of the alternatives, which indicates that the aggregated value of alternative $a$ is greater than the aggregated value of $b$ ($C_m(a) \geq C_m(b)$). Sometimes it is not possible to fulfill all rankings of alternatives with the given data, and then the linear program would not be able to find a solution with the given constraints. Therefore, the binary values $y_k$ are introduced to achieve as many rankings as possible. Similar overall comfort values for locations, get a low ranking $w_k$ because this ranking does not necessarily has to be accepted, and the ranking can be switched. $\delta$ in equation (4.6) guarantees that the difference between two aggregated comfort values is at least $\delta$. The linear program maximizes the difference ($\epsilon$) between the two aggregated values. This will make sure that there is clear distinction between the two alternatives. Equation (4.7) specifies the preference of criterion $i$ over $j$ and makes criterion $i$ more important than criterion $j$.

### 4.5 Modeling Error

In the least squares parameter estimation, the residual is used to describe the difference between the overall determined model value and the actual data value. However, when a questionnaire is used to collect data, the actual overall data value might not be precise. Also, in making a decision on the most comfortable location, the ranking is more important than the overall numerical comfort value. To accomplish this, this study introduces a ranking error. Through the questionnaire data, a ranking of locations from the most comfortable to the least comfortable is established. By determining the fuzzy measures, the Choquet integral can be computed. The
ranking error describes how far an alternative (in this case a location) moved from its original ranked position. The root mean square ranking deviation is defined as,

\[
RMSD_{\text{rank}} = \sqrt{\frac{\sum_{A}(\hat{i} - i)^2}{|A|}}
\]  

(4.9)

where, \( \hat{i} \) is the index of the ranked alternative established through fuzzy measures, \( i \) is the index of an alternative determined through the questionnaire data and \( |A| \) is the cardinality of the set of alternatives \( A \). For a fair comparison the RMSD is normalized, by dividing the maximum possible index an alternative can move from one position to another minus the minimum position an alternative can move. Here, the maximum position is \( |A| - 1 \) and the minimum possible is 0. Therefore,

\[
NRMSD_{\text{rank}} = \frac{RMSD_{\text{rank}}}{|A| - 1}
\]  

(4.10)

Additionally, the study uses the standard root mean square deviation \( RMSD \) of the residuals.

### 4.6 Comfort Model for a Wearable Body Sensor System

To determine a comfort model with low subjectivity, the present work first conducts a survey (using a questionnaire) and then determines the fuzzy measures.

#### 4.6.1 Comfort Questionnaire

The data for the overall comfort value has been determined through a questionnaire. To compare the individual locations with each other in a fair manner, the type of attachment and the attachment force must be the same. A belt, more specifically a blood pressure cuff, has been used for the experiment. The blood pressure cuff is a belt that can be inflated with air. The belt is attached to the 14 body locations listed in Table 4.1 and then inflated to equal pressure (20 mmHg). For wider body locations such as the chest, the blood pressure belt was extended as appropriate. 20 males participated in the experiment and gave a comfort rating from 1 corresponding to very comfortable to 5, which corresponds to not comfortable, for each body location. The outcome was normalized between 0 and 1. The results are shown in Figure 4.1. The blue boxes in the plot describe the middle 50% of all the values for that location. The red horizontal line represents the median and the red plus signs show the outliers. The lines extending above and below each box are the whiskers (see box-and-whisker plot for more details).
4.6.2 Determination of Comfort Model

To obtain the comfort model, the fuzzy measures are determined via the least squares ($LS$) method and the Marichal and Roubens [60] ($MR$) method. All seven criteria are used. To check if the criteria can generally model comfort, we check the following condition:

$$a \succ b \rightarrow \exists i \in N : x_i^a > x_i^b$$  \hspace{1cm} (4.11)

This means, if an alternative $a$ (location) ranked better (more comfortable) than an alternative $b$, then there has to be at least one utility value $x_i^a$ bigger than the utility value of the same criterion of alternative $b$. If this is not the case the monotonicity condition described in Section 3.3.1 is violated and the given criteria do not describe comfort. Potentially, more criteria are necessary to describe comfort.

By optimizing the objective $\mathcal{F}_{MR}(\epsilon)$ of $MR$ method two preference rankings cannot be fulfilled, which can be seen from the binary variable $y_k$. The violated preference rankings are, location 2 more comfortable than location 13, and location 11 more comfortable than location 4. However, Figure 4.1 shows that the mean of the location 2 and 13, and locations 4 and 11 are similar (within 8% and 1%, respectively).

Table 4.2 provides the importance values $\phi_{Sh}(\{i\})$ for each criterion. The importance of Touch sensitivity is 0, and therefore does not contribute to describing comfort when attaching
a belt to a body. As usual, the fuzzy measures are determined by leaving out touch sensitivity. Also, proximics has 0 importance for the LS approach and a really small level of importance with the MR method. The criteria for the model are reduced and the corresponding normalized root mean square deviations (NRMSD) are represented in Figure 4.2.

**Table 4.2:** Shapley index of MR method and LS method.

<table>
<thead>
<tr>
<th>Prox, PPT, MI, SA, Touch, Pain, Tissue</th>
<th>Prox, PPT, MI, SA, Tiss</th>
<th>Prox, MI, SA, P, Tiss</th>
<th>MI, SA, P, Tiss</th>
<th>MI, P, Tiss</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_{Sh}({i})_{MR} )</td>
<td>0.023</td>
<td>0.013</td>
<td>0.596</td>
<td>0.142</td>
</tr>
<tr>
<td>( \phi_{Sh}({i})_{LS} )</td>
<td>0.000</td>
<td>0.041</td>
<td>0.612</td>
<td>0.047</td>
</tr>
</tbody>
</table>

**Figure 4.2:** Normalized root mean square deviation (NRMSD) for various criteria.

The NRMSD of the MR method is bigger than that of the LS method. However, the NRMSD_{rank} is smaller for the MR method. This is because the MR method attempts to keep the order of the alternatives, in this case the order of most comfortable location to the least comfortable location. By reducing the number of unimportant criteria, determined through the Shapley index, the NRMSD does not become too large until only 3 criteria are left. By examining the NRMSD, it is seen that the criteria Motion Impedance (MI), Social Acceptability (SA), Pain Sensitivity (P) and Tissue Volume (Tiss) are the determined criteria that describe comfort most accurately. Table 4.3 and Table 4.4 list the fuzzy measures and interaction indices, respectively. Generally, there is low interaction between the criteria.

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With the fuzzy measures and the Choquet integral it is now possible to compute the comfort values for a wearable body sensor system that attaches a device using a belt. The Shapley value shows that motion impedance (MI) is the most important criterion when designing a comfortable wearable body sensor system. Also, the high correlation between MI and the overall comfort value of the questionnaire demonstrates the importance of this criterion. Unimportance of proxemetics also makes sense, because the blood pressure cuff used in the survey using the questionnaire was still part of the perceived human self-size (see Section 4.3).

Table 4.3: Shapley indices of MR and LS methods.

<table>
<thead>
<tr>
<th></th>
<th>MI</th>
<th>SA</th>
<th>Pain</th>
<th>Tissue</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m{1}) MR</td>
<td>0.7694</td>
<td>0.1999</td>
<td>0.0704</td>
<td>0.1669</td>
</tr>
<tr>
<td>(\phi_{Sh}({1})) MR</td>
<td>0.6399</td>
<td>0.1262</td>
<td>0.1243</td>
<td>0.1096</td>
</tr>
<tr>
<td>(m{1}) LS</td>
<td>0.4599</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0536</td>
</tr>
<tr>
<td>(\phi_{Sh}({1})) LS</td>
<td>0.6448</td>
<td>0.0583</td>
<td>0.1366</td>
<td>0.1603</td>
</tr>
</tbody>
</table>

Table 4.4: Interaction indices between criteria for MR method and LS method.

<table>
<thead>
<tr>
<th></th>
<th>MI</th>
<th>SA</th>
<th>Pain</th>
<th>Tissue</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I({i,j})) MI MR</td>
<td>—</td>
<td>-0.199</td>
<td>0.108</td>
<td>-0.167</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MI LS</td>
<td>—</td>
<td>0.000</td>
<td>0.273</td>
</tr>
<tr>
<td>SA</td>
<td>MR</td>
<td>-0.199</td>
<td>—</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>LS</td>
<td>0.0000</td>
<td>—</td>
<td>0.000</td>
</tr>
<tr>
<td>Pain</td>
<td>MR</td>
<td>0.108</td>
<td>0.000</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>LS</td>
<td>0.273</td>
<td>0.000</td>
<td>—</td>
</tr>
<tr>
<td>Tissue</td>
<td>MR</td>
<td>-0.167</td>
<td>0.052</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>LS</td>
<td>0.097</td>
<td>0.117</td>
<td>0.000</td>
</tr>
</tbody>
</table>

4.6.3 Validation of the Comfort Model

For validating the fuzzy measures and the model, only 10 locations are used to determine the fuzzy measures and compute the overall comfort value of the rest of the locations through the fuzzy measures and the Choquet integral. Table 4.5 presents the Shapley indices computed over the 10 locations. It is seen that the Shapley index of the LS approach hardly changes, and the change in the MR approach is slightly larger.

Table 4.5: Shapley indices of the MR method and LS method.

<table>
<thead>
<tr>
<th></th>
<th>MI</th>
<th>SA</th>
<th>Pain</th>
<th>Tissue</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_{Sh}({i})) MR</td>
<td>0.5627</td>
<td>0.0885</td>
<td>0.1663</td>
<td>0.1825</td>
</tr>
<tr>
<td></td>
<td>(\phi_{Sh}({i})) LS</td>
<td>0.6399</td>
<td>0.0637</td>
<td>0.1400</td>
</tr>
</tbody>
</table>

Table 4.6 lists the normalized root mean square deviation (\(NRMSD\)). The column “Prev” is the error previously computed (Figure 4.2 third bar: MI, SA, P, Tiss). Column “Test” describes the \(NRMSD\) only computed with the locations not used to compute the fuzzy measures. The “Total” column lists the \(NRMSD\) computed with all 14 locations. Compared to the previous error (“Prev” column), the \(NRMSD\) hardly changes. This confirms a valid description of the qualitative objective comfort via the computed fuzzy measures and therefore the model.
Table 4.6: Normalized root mean square deviation (NRMSD).

<table>
<thead>
<tr>
<th></th>
<th>Prev</th>
<th>Test</th>
<th>Total</th>
<th></th>
<th>Prev</th>
<th>Test</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRMSD − MR</td>
<td>0.187</td>
<td>0.270</td>
<td>0.229</td>
<td>NRMSD_rank − MR</td>
<td>0.050</td>
<td>0.168</td>
<td>0.096</td>
</tr>
<tr>
<td>NRMSD − LS</td>
<td>0.087</td>
<td>0.075</td>
<td>0.088</td>
<td>NRMSD_rank − LS</td>
<td>0.087</td>
<td>0.067</td>
<td>0.087</td>
</tr>
</tbody>
</table>

Table 4.7 lists the locations sorted from the most comfortable to the least comfortable. The outcome of the questionnaire would suggest forearm as the most comfortable location. However, through the criteria describing comfort and the determined fuzzy measures the calf would be the most comfortable location to mount hardware using a belt. This is absolutely possible, since the outcome of the survey questionnaire (Figure 4.1) shows a similar mean for forearm (location #2) and calf (location #13). Similarly, this is true for other locations that do not correspond to the questionnaire data. Therefore, backing up the data through a questionnaire with a model makes it less subjective, and reduces uncertainties in the questionnaire study. Furthermore, the model will help in the design process, because it gives a deeper understanding as to which criteria are more important than others and where improvements can be made.

Table 4.7: Locations sorted from the most comfortable to the least comfortable.

<table>
<thead>
<tr>
<th>Questionnaire</th>
<th>MR</th>
<th>LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forearm</td>
<td>Calf</td>
<td>Calf</td>
</tr>
<tr>
<td>Calf</td>
<td>Forearm</td>
<td>Forearm</td>
</tr>
<tr>
<td>Upper Thigh</td>
<td>Upper Thigh</td>
<td>Upper Arm</td>
</tr>
<tr>
<td>Lower Thigh</td>
<td>Lower Thigh</td>
<td>Upper Thigh</td>
</tr>
<tr>
<td>Upper Arm</td>
<td>Ankle</td>
<td>Forehead</td>
</tr>
<tr>
<td>Ankle</td>
<td>Wrist</td>
<td>Wrist</td>
</tr>
<tr>
<td>Forehead</td>
<td>Abdomen</td>
<td>Abdomen</td>
</tr>
<tr>
<td>Wrist</td>
<td>Hip</td>
<td>Hip</td>
</tr>
<tr>
<td>Abdomen</td>
<td>Shoulder</td>
<td>Chest</td>
</tr>
<tr>
<td>Hip</td>
<td>Shoulder</td>
<td>Shoulder</td>
</tr>
<tr>
<td>Shoulder</td>
<td>Elbow</td>
<td>Elbow</td>
</tr>
<tr>
<td>Chest</td>
<td>Knee</td>
<td>Knee</td>
</tr>
</tbody>
</table>

Finally, this paragraph compares the two methods for determining the fuzzy measures. The method of Marichal and Roubens requires only the ranking of alternatives (locations). It might be the case that no solution for the given alternative ranking could be found through the linear program. However, a solution that describes the ranking as accurately as possible can be found with the introduced binary variable (see Section 4.4). This converts the linear program into a mixed integer program. If it is not possible to find a solution for the provided ranking, then some preferences would be violated and the binary variable would become 0 in the mixed integer program. For example, if one preference ranking at the end (e.g., least comfortable) is violated it can happen that it moves all the way to the front (most comfortable). In order to prevent this, it must be specified that, for example, an alternative in the front is preferred over every single alternative that comes after. For example, $c \succ a \succ b$, automatically specifies that $c \succ b$. However, if in the algorithm $c \succ a$ cannot be fulfilled, the constraint $c \succ b$ also does not hold and $b$ can be preferred over $c$, even though it would be possible to keep the order of $c \succ b$. 

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Consequently, instead of \((n - 1)\) preference rankings, \(n \cdot (n - 1)/2\) preference rankings have to be fed into the algorithm. This vastly increases the number of constraints and the algorithm will become slow if it has to deal with a high number of alternatives. However, if the overall value is not known, from the ranking of the alternatives it is possible to still determine the fuzzy measures. The least squares approach can definitively handle more alternatives with a much shorter computing time and is also easier to implement. However, the approach needs an overall numerical value, and simple ranking is not sufficient.

4.7 Discussion and Conclusion

This chapter presented a systematic approach on how to analytically and mathematically handle a qualitative design objective. Specifically, it used fuzzy measures and the Choquet integral to describe comfort. The determined fuzzy measures were evaluated through training data and test data. A modeling error was introduced that described the error of the preference order of alternatives. Uncertainties in the outcome of a survey using a questionnaire could be reduced using the presented method, because the objective was described through multiple criteria based on data sets. The reduction of uncertainties also reduced the subjectivity that came with qualitative variables. For the design of a product it is beneficial to have a better understanding of a qualitative objective and which criteria really matter to make the necessary changes to the design. In contrast to a machine learning approach, this method does not need a huge amount of data, and through an importance index (Shapley index), it is easy to interpret the relevance of variable criteria. The present chapter provided much insight into comfort, specifically addressing wearable body sensor systems and attaching hardware through a belt.

The developed method did not consider uncertainties in the utility values themselves. For example, the utility value for the criteria tissue volume was created through the muscle and the tissue volume of a human body. Future work is planning to handle these uncertainties by using fuzzy sets and fuzzy numbers for the utility values themselves. In order to do so, the Choquet integral must be fuzzyfied in order to operate with fuzzy sets and fuzzy numbers. For doing so, the fuzzy measure determination methods (Least Square and Marichal and Roubens) have to be able to handle fuzzy sets and fuzzy numbers as well. The method should then be applied to another qualitative design objective to increase its validity. The objective comfort, as considered in the present chapter, will be included in a multi-objective design decision problem in the next chapter.
Chapter 5

Decision Making in Multi-objective Design

This chapter is based on a paper that has been submitted to an appropriate journal.

5.1 Summary

The objective function comfort determined in Chapter 4 is used as one of the objectives in the design problem of the present chapter. Chapter 3 introduced fuzzy measures and the Choquet integral as decision making method for situations with multiple design objectives. For comparison, the present chapter introduces another decision making method.

For the design process of mechatronic devices, which typically incorporate mechanical and electrical domains, it is often necessary to consider multiple objectives/criteria. The design problem can then be formulated as a multi-objective optimization problem. Multiple objectives can be conflicting and to pick a design solution a trade-off between those is required. A good trade-off is important for a successful product. Different decision making methods are available aiming towards a successful design trade-off; a commonly used way is the VIKOR method (from Serbian: VIseKriterijumska Optimizacija I Kompromisno Resenje, meaning: Multi-criteria Optimization and Compromise Solution). This chapter focuses on the aspects of this method and reveals some weaknesses. Then, a different normalization method is introduced that overcomes these weaknesses. Next, a minimum weight margin is established that gives information about the stability of a design solution that is generated by the VIKOR method. The weight margin is helpful for elucidating the decision maker’s uncertainty in the original weight assignment. The modified VIKOR method is then applied to the design of a wearable body sensor network and the design of an EEG electrode. The two design examples show the strength of the new modified VIKOR method of the present chapter, which resolves the shortcomings of the original VIKOR method.
5.2 Introduction

One challenging and subjective task in a multi-objective design problem is the final design decision. Most multi-objective optimization problems do not have a single solution, because the objectives can be conflicting. Hence, a trade-off between objectives is unavoidable in the selection of the final design. Multi-objective optimization identifies non-dominated solutions. A solution is called non-dominated or Pareto optimal if none of the objective functions can be improved in value without degrading some of the other objective values \[12\]. The trade-off between objectives should be a good compromise and should follow a clear strategy. There are numerous methods to select a final solution out of the non-dominated solutions. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) originally developed by Hwang et al. \[76\] is based on the concept that the compromise solution should have the shortest distance from the ideal solution. Other common decision making methods are ELECTRE \[77\], an outranking method which depends on comparison of pairs of alternatives; PROMETHEE \[78\] that helps decision makers to find the alternative that best suits their goal and the VIKOR \[79\] method. Opricovic developed the basic idea of the VIKOR method in his PhD dissertation and a first application was published one year later in 1980 \[80\]. The VIKOR method combines the maximum “group utility” and a minimum individual regret of criteria. Opricovic’s paper in 2004 \[79\] contributed to the recognition of the importance of the VIKOR method. Since then the VIKOR method has been used in numerous decision making applications across a range of fields. For example, Jahan et al. \[81\] used the VIKOR method for selecting materials with different properties in biomedical implants, where implant materials should have similar characteristics to the human tissues. Kiani et al. \[82\] applied the VIKOR method in the field of civil engineering to select the best repair material for concrete structures. They used three different methods to assign the importance weights to the individual criteria within the VIKOR method and performed a sensitivity analysis for the weight assignment. Garcia-Segura et al. \[83\] used analytic hierarchy process (AHP) for the weight determination of individual criteria within the VIKOR method. Ren et al. \[84\] suggested a group decision making method to determine the weights in the VIKOR method. Fei et al. \[85\] used the VIKOR method for supplier selection and introduced Dempster-Shafer evidence theory for handling the uncertain weight assignment by the decision maker.

This chapter first demonstrates some shortcomings of the VIKOR method and resolves them by introducing a modified version of the VIKOR method. The modified version provides a weight margin, which informs the decision maker how sensitive the provided weights are to the determined compromise solution. The modified VIKOR method is then applied to two design examples. The examples demonstrate the advantage of the new method over the original VIKOR method. The procedure of the original VIKOR method is presented in the next section.
5.3 VIKOR Method

5.3.1 VIKOR Procedure

The VIKOR method is a multi-criteria decision making method, which has been developed for multi-criteria/objective optimization [79]. The criteria or objectives in an optimization problem usually conflict with each other. Alternatives that are not dominated by other alternatives are the non-dominated solutions or Pareto optimal solutions [86]. The VIKOR method enables ranking of these conflicting alternatives and selecting a good (compromise) alternative out of the Pareto optimal solutions. The $L_p$-metric is used as an aggregating function and element $a^i$ is part of a set of alternatives $A = \{a^1, a^2, ..., a^m\}$. It is assigned with a utility value $x^i = (x^i_1, ..., x^i_n) \in \mathbb{R}^n$, where $n$ is the number of criteria which are represented by the set $N = \{1, ..., n\}$.

The steps of the traditional VIKOR method are as follows [79, 87]:

i) Determine the best $x_j^*$ and the worst $x_j^-$ values of all criteria $j \in N$

\[
x_j^* = \max_{i} x_{ij}^i, \quad x_j^- = \min_{i} x_{ij}^i,
\]

if the $j$-th objective represents a benefit;

\[
x_j^* = \min_{i} x_{ij}^i, \quad x_j^- = \max_{i} x_{ij}^i,
\]

if the $j$-th function represents a cost.

ii) Compute the values $S_i$ and $R_i$, $i = 1, ..., m$, by relations

\[
S_i = \sum_{j=1}^{n} w_j (x_j^* - x_j^i) / (x_j^* - x_j^-)
\]

\[
R_i = \max_j \{w_j (x_j^* - x_j^i) / (x_j^* - x_j^-)\},
\]

where $w_j$ are the weights of criteria/objectives, expressing the decision maker’s preference. The solution obtained by $S_i$ is the maximum group utility and the solution obtained by $R_i$ is the minimum individual regret of the “opponent”.

iii) Compute the values $Q_i$, $i = 1, ..., m$ using the relation

\[
Q_i = \nu (S_i^* - S_i) / (S_i^* - S_i^-) + (1 - \nu) (R_i^* - R_i) / (R_i^- - R_i^*),
\]

where $S_i^* = \min_i S_i^i, S_i^- = \max_i S_i^i, R_i^* = \min_i R_i^i, R_i^- = \max_i R_i^i$; $\nu$ is the weight for the strategy of maximum group utility, and $(1 - \nu)$ is the weight of the individual regret.

iv) Rank the alternatives by sorting the values of $S, R$ and $Q$. The results provide three ranking lists.

v) The minimum value of the sorted $Q$ is the proposed compromise solution of alternative $a^* = a^{(1)}$ if the following conditions (C1 and C2) are satisfied:

\[
Q(a^{(2)}) - Q(a^{(1)}) \geq DQ , \quad \text{where} \quad a^{(2)} \text{ is the alternative ranked on second position in the ranking list } Q \text{ and } DQ = 1/(m - 1)
\]
C2 Acceptable stability in decision making: Alternative \( a^{(1)} \) also has to be best ranked by \( S \) or/and \( R \).

If one of the conditions is not satisfied, then a set of compromise solution is proposed:

- Alternatives \( a^{(1)} \) and \( a^{(2)} \) if only condition C2 is not satisfied or
- Alternatives \( a^{(1)}, a^{(2)}, \ldots, a^{(M)} \) if condition C1 is not satisfied; \( a^{(M)} \) is determined by the relation \( Q(a^{(M)}) - Q(a^{(1)}) < DQ \) for maximum \( M \) (the positions of alternatives are “in closeness”)

vi) Determine a weight stability interval \([w_j^L, w_j^U]\) for each \( j \)-th criterion/objective, separately, with the initial given weights from the decision maker. The weight of each criterion is increased or decreased from the initial value \( w_j \) by \( \lambda \) (\( w'_j = \lambda \cdot w_j \)). The weights are normalized so that \( \sum_{j=1}^{n} w_j = 1 \). The other weights keep their initial ratios: \( w'_k = \varphi w_k \), \( k \neq j \), \( k = 1, \ldots, n \). \( \varphi(\lambda) \) is obtained by the equation \( \lambda w_j + \varphi \sum_{k \neq j} w_k = 1 \) and can therefore be transformed into: \( \varphi = (1 - \lambda w_j) / (1 - w_j) \). The parameter \( \lambda \) can vary between 0 \( \leq \lambda \leq 1/w_j \). The VIKOR method is applied with different values of parameter \( \lambda \) (searching) to find the interval \( \lambda_1 \leq \lambda \leq \lambda_2 \), where the same compromise solution is obtained. The weight stability interval for the \( j \)-th criterion is: \( w_j^L \leq w'_j \leq w_j^U \), where \( w_j^L = \lambda_1 w_j \) and \( w_j^U = \lambda_2 w_j \).

The weight stability intervals are determined for each criterion \( j = 1, \ldots, n \), with the given initial values of weights.

5.3.2 Shortcomings

VIKOR is a helpful decision making tool, given a problem with conflicting alternatives and non-commensurable (having different units or non-comparable magnitudes) criteria. The method aims to select a solution that is the closest to the ideal one. The next few examples demonstrate some shortcomings of the VIKOR method. As Example 1 (Table 5.1 from Huang et al. [88]), consider two criteria and three alternatives:

<table>
<thead>
<tr>
<th>( a )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>WS</th>
<th>( S )</th>
<th>( R )</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^1 )</td>
<td>0.8</td>
<td>0.5</td>
<td>0.65</td>
<td>0.375</td>
<td>0.375</td>
<td>0.000</td>
</tr>
<tr>
<td>( a^2 )</td>
<td>0.6</td>
<td>0.8</td>
<td>0.70</td>
<td>0.500</td>
<td>0.500</td>
<td>0.667</td>
</tr>
<tr>
<td>( a^3 )</td>
<td>0.7</td>
<td>0.4</td>
<td>0.55</td>
<td>0.750</td>
<td>0.500</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The preference ranking based on the \( Q \)-value is \( a^1 \succ a^2 \succ a^3 \). \( S \) is computed using a weighted sum, where the utility values are normalized between 0 and 1. The highest value corresponds to 0 and the lowest to 1. Normalization is required so that non-commensurable
different units and non-comparable magnitudes) criteria can be aggregated together. However, in this example, the criteria are already on a common scale, and computing the weighted sum results in a preference ranking of $a^2 \succ a^1 \succ a^3$. This is not consistent with the ranking of $S$, which is the same as the ranking of $Q$. As second example consider (Table 5.2):

Table 5.2: Example 2, where $a_i$ are design alternatives, $x_i$ are the objective functions, $S$ is the maximum group utility in the VIKOR method, $R$ is the minimum individual regret in the VIKOR method, and $Q$ is the aggregated value of $S$ and $R$ in the VIKOR method.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$S$</th>
<th>$R$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^1$</td>
<td>90</td>
<td>10</td>
<td>0.50</td>
<td>0.50</td>
<td>0.5</td>
</tr>
<tr>
<td>$a^2$</td>
<td>85</td>
<td>15</td>
<td>0.72</td>
<td>0.47</td>
<td>0.5</td>
</tr>
<tr>
<td>$a^3$</td>
<td>80</td>
<td>90</td>
<td>0.50</td>
<td>0.50</td>
<td>0.5</td>
</tr>
</tbody>
</table>

In this example the VIKOR method does not deliver a solution at all, because all $Q$ values are the same. However, one can see that alternative $a^3$ has a high value in both criteria $x_1^3$ and $x_2^3$ and should be preferred over the alternatives $a^1$ and $a^2$. The problem lies again in normalizing the maximum value to 0 and the minimum value to 1. As third example, consider the case where, there is a 4-th alternative $a^4$ with $x_1^4 = 10$ and $x_2^4 = 95$. Then the alternative $a^3$ with the lowest $Q$ value, would be the clear favorite with $Q(a^1) = 0.99$, $Q(a^2) = 0.97$, $Q(a^3) = 0.00$ and $Q(a^4) = 0.99$. One problem in the normalization method when optimizing a benefit, is that a minimum value is not always available, as in Example 2.

Applying the VIKOR method to either the entire set of alternatives or only the non-dominated alternatives should not change the outcome of the decision. However, with the current normalization, this is not necessarily the case (see Section 5.5.1, Figure 5.3). This is possible because the minimum value, if the objective is a benefit, or the maximum value, if the objective is a cost, in the normalization is different when using all alternatives or the non-dominated set.

Also, in a multi-objective design problem it might happen that a constraint is included (see Section 5.5.2, Figure 5.5). One of the multiple objectives is maximized and has to be bigger than a specific value in order to realize an acceptable design. The solution determined through the VIKOR method might change, depending on the inclusion of the constraint into the multi-objective optimization algorithm prior to the decision making or later to the Pareto optimal solutions. This is because when maximizing, the minimum that is used for the normalization procedure changes. Hence, the method can only be applied to selected cases.

The conditions for an acceptable advantage in the original VIKOR method are not established on a mathematical basis but are rather intuitive (see Section 5.3.1 point v). In a multi-objective optimization algorithm it is possible to specify the number of solutions on the Pareto front. This is necessary if the objectives are continuous functions, because then there are theoretically infinite Pareto solutions. The normalization will fit more alternatives between 0 and 1 and the distances between the $Q^i$ values get smaller as well as the $DQ$ value. However,
a multi-objective optimization algorithm might not spread the solutions evenly on the Pareto front. Also, if there is a discontinued Pareto front and one part has only a few alternatives and the other part has much more, then using the number of alternatives to judge an acceptable advantage might not make sense.

The determination of the weight stability margin in Section 5.3.1 point vi, will follow an increase/decrease of one weight and an equal decrease/increase in all the other weights for the upper/lower bound, respectively. This however is only one way of how the weights can change. It is also possible that some weights are unchanged or assume any other possible increase/decrease of weights. The only condition that needs to be satisfied is, if one weight is increasing/decreasing, at least one weight must decrease/increase, because the sum of weights must add up to 1 (\( \sum w_j = 1 \)). Choosing weights out of the weight margin presented by Opricovic et al. [87] does not guarantee that the suggested solution will still be the most preferred one after using weights out of the weight margin. It only applies to one specific case.

The research in the present chapter proposes a modified VIKOR method, which addresses above issues extending and generalizing the applicability of the originally proposed method. The improved VIKOR method is presented in the next section.

### 5.4 Modified VIKOR-method

When maximizing multiple objectives (benefits) and determining the non-dominated or Pareto optimal solutions, the maximum value of each criterion is always part of the Pareto optimal solution. When minimizing an objective (cost) the utility values can be modified by taking the negative of that particular criterion (\( \min x_j = \max(-x_j) \)). Because the maximum (benefit) or minimum (cost) value is the important factor in an optimization problem, it is suggested to use the \( L_\infty \)-norm or maximum norm:

\[
||x_j||_\infty = \max\{|x_j^1|, |x_j^2|, \ldots, |x_j^m|\}
\] (5.1)

#### 5.4.1 Modified Procedure

The modified VIKOR method is stated as follows:

\[
S^i = \sum_{j=1}^{n} w_j \frac{x_j^i}{||x_j||_\infty}
\] (5.2)

\[
R^i = \min \left\{ w_j \frac{x_j^i}{||x_j||_\infty} \right\}, j = 1, \ldots, n
\] (5.3)

\[
Q^i = \nu \frac{S^i}{||S||_\infty} + (1 - \nu) \frac{R^i}{||R||_\infty}
\] (5.4)

Here, \( S \) and \( R \) are not on a common scale. Specifically, \( R \) is smaller than \( S \), because it selects the weighted utility value of just one criterion. Therefore, \( S \) and \( R \) are normalized with
the $L^\infty$-norm. $Q^i$ is the aggregated value and is sorted in the decreasing order. The highest value of $Q^i$ is the most preferred solution that is determined by the present modified VIKOR method.

The present research also proposes a modified weight margin in addition to the preferred alternative. The weight margin will inform the decision maker how sensitive the preferred solution is to given weights, or in other words, how much they can change the weights so that the preferred alternative will still be chosen.

### 5.4.2 Modified VIKOR Weight Margins

The extended VIKOR method of Opricovic et al. [87] determines a weight stability as mentioned in Section 5.3 point vi. A more helpful weight margin is to provide a margin, in which any combination of weights will deliver the same compromise solution. More specifically, some weights are increased while some are decreased and some might not change at all. The smallest weight margin guarantees no change in the solution. The sum of all weights however, must add up to 1 ($\sum w_j = 1$). A weight can increase minimally, when only one weight decreases and all others increase. For example, consider three criteria with three weights:

i) $\varphi w_1 + \lambda w_2 + \lambda w_3 = 1$

ii) $\lambda w_1 + \varphi w_2 + \lambda w_3 = 1$

iii) $\lambda w_1 + \lambda w_2 + \varphi w_3 = 1$

Here $\lambda$ represents an increase in weight and $\varphi$ a decrease in weight to find the upper bound. $w_3$ can then be minimally increased, if all the other weights except one are also increased. This is because, $w_3$ has to “share” the increase with the other weights. In this example there are two possible ways to increase $w_3$ and at least decrease one. The minimum increase in $w_3$ will be the upper bound for the chosen alternative still to be described as the most preferred one. The algorithm that determines the upper bound for each $w_j$ is presented now.
\[
\begin{align*}
\text{max} \quad & \lambda_k, \quad k = 1, \ldots, n \\
\text{subject to} \quad & Q_{\text{new}}(a^{(1)}) > Q_{\text{new}}(a^{(2)}) \\
& : \\
& Q_{\text{new}}(a^{(i)}) > Q_{\text{new}}(a^{(m)}) \quad (5.6) \\
& Q_{\text{new}}(a^{(i)}) = \nu \frac{S_{\text{new}}^i}{\|S_{\text{new}}\|_{\infty}} + (1 - \nu) \frac{R_{\text{new}}^i}{\|R_{\text{new}}\|_{\infty}} \quad (5.7) \\
& S_{\text{new}}^i = \sum_{j=1}^{n} w_{\text{new}}^j \frac{x_j^i}{\|x_j\|_{\infty}}, \quad R_{\text{new}}^i = \min \left\{ w_{\text{new}}^j \frac{x_j^i}{\|x_j\|_{\infty}} \right\} \quad (5.8) \\
& w_{\text{new}} = \begin{cases} \\
\lambda_k w_j, & j = 1, \ldots, n, j \neq k \\
\varphi_k w_k \end{cases} \quad (5.9) \\
& \varphi_k w_k + \lambda_k \sum_{j=1}^{n} w_j = 1, \ j \neq k \quad (5.10) \\
& \varphi_k = \frac{1 - \lambda_k (1 - w_k)}{w_k} \quad (5.11) \\
& 1 \leq \lambda_k \leq \frac{1}{\sum_{j=1}^{n} w_j}, \ j \neq k \quad (5.12)
\end{align*}
\]

An upper bound that can guarantee that the solution will not change, independent of the weight increase (e.g., increase two, decrease one, increase one, decrease two) is the minimum of all determined upper bounds:

\[
\begin{align*}
\text{w}_{j}^{\text{Upper}} = \min \{ \lambda_k w_j \}, \ k = 1, \ldots, n, k \neq j \quad (5.14)
\end{align*}
\]

The lower bound is determined by minimizing \( \lambda_k \) in equation (5.5) and using the maximum in equation (5.14). The boundary condition in equation (5.13) changes to \( 0 \leq \lambda_k \leq 1 \).

The presented algorithm can be solved as an optimization problem with nonlinear constraints or by increasing/decreasing \( \lambda_k \) by a specific step size until one of the constraints (5.6) to (5.7) is violated.

The importance weights assigned by a decision maker are subjective and the provided numerical value might not be exact. Thus, slight variations of the provided weights are possible. If the determined weight margin is sufficiently large, a decision maker can be more confident about the solution provided by the VIKOR method.

### 5.4.3 Weight Margin Using a Range of Objectives Values

In the case of a continuous objective function, it is possible to increase the number of alternatives on the Pareto front. This on the other hand decreases the weight margin. To still decide if the weight margin is sufficient, this section introduces a weight margin in which the objectives should lie within a certain range. More specifically, this section provides a weight margin in
which the objectives will not change more than a specific amount. The following steps determine
the weight margin.

i) Compute $Q^i$ with initial weights.

ii) Find $Q^*$, which is the highest value of $Q^i$, and the corresponding utility values $x^i_j$ of the
objectives.

iii) Compute or specify the range describing how much the objective values $x^i_j$ can change.

iv) Find the alternatives, where the utility values are not within the specified range.

v) Compute the weight margin as presented in Section 5.4.2 without the alternatives falling
into the range identified in iv. Thus, these alternatives are omitted in the constraints ((5.6)
to (5.7)).

5.5 Design Optimization Examples
The following sections apply the modified VIKOR method to some design examples. The first
element applies the VIKOR method to the design of a wireless sleep monitoring system.

5.5.1 Design of a Wireless Sleep Monitoring System
In order to monitor the sleep quality and diagnose sleep disorders, portable sleep monitoring
devices should include the following measurements [19]:

- Electroencephalography (EEG), records the change in human brain signals [20].
- Electrooculography (EOG) measures the electrical potential between cornea and retina, which is important to detect sleep stages.
- Measuring the airflow through the nose, is important to score apnea events.
- Respiratory effort is measured through the chest movement.
- Electrocardiogram (ECG) displays the heart rate and determines the heart’s electrical activity.
- Pulse-Oximetry records the level of oxygen in the blood.
- Electromyography (EMG) measures muscle activity and helps determine sleep stages. It also detects leg movements.

More information on the present topic and the functioning of a sleep monitoring system can be found in Section 1.2 or the American Academy of Sleep Medicine (AASM) manual [17].
A wireless sleep monitoring system or in general a wireless body sensor network (WBSN) connects various medical sensors and appliances that are located on the human body. The set of sensor nodes is denoted by $S$. Sensor nodes transmit their data to a base station or relay node. Potential locations for a base station or relay node are indicated as Candidate Sites ($z_j$). The locations for the sensors $S$ are predetermined. In this example, the objectives are to find a location for the base station/coordinator that is comfortable (objective 1), has low energy consumption (objective 2), and has low signal interference (objective 3). Figure 5.1 shows possible candidate sites for a base station and the necessary sensors for a wireless sleep monitoring system. In total 14 candidate sites are considered.

![Sensor nodes and candidate sites for a wireless sleep monitoring system](image)

**Figure 5.1:** Sensor nodes $i$ and candidate sites $z_j$ for a wireless sleep monitoring system.

The comfort values for each of these locations have been determined in Chapter 4. The determination of the energy consumption for each possible candidate site is described next.

**Energy Consumption**

A WBSN needs to record data for a determined amount of time, using the least possible energy. Less energy consumption also leads to a smaller sized WBSN, because needed battery for data
transmission can have a smaller overall size. Consequently, it also makes it more comfortable for the person wearing the device. Therefore, it is desirable to design a WBSN with low energy consumption.

A widely used channel access method in WBSNs is time-division multiple access (TDMA) governed by the IEEE 802.15.4 and IEEE 802.15.6 standards [89]. A TDMA frame structure is shown in Figure 5.2.

<table>
<thead>
<tr>
<th>GTS1</th>
<th>GTS2</th>
<th>————</th>
<th>GTSn</th>
<th>Inactive Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 5.2:** TDMA frame structure.

A super-frame has a Contention Free Period (CFP) and an Inactive Period (IP). Each sensor node has its Guaranteed Time Slot (GTS) in a super-frame. All sensors turn off their radios in the inactive period to save energy. In one super-frame, each sensor transmits data with a specific transmission power \( P_i, i \in S \) and transmission time \( T_i, i \in S \) to a candidate site \( j \in CS \). The energy consumption for one sensor \( i \) in one super-frame is \( P_i \cdot T_i \). Over the lifetime of the WBSN each sensor sends data \( L_i/T_{frame} \) times, where \( L_i \) is the lifetime of a sensor \( i \) and \( T_{frame} \) the duration of one super-frame. The optimization model of Zhou et al. [90] and Minhas et al. [91] is adopted here, with the difference that the previous work optimize the lifetime of the WBSN, and the base station is at a predefined position. In their approach the sensors themselves can vary in position. For a WBSN design however, lifetime is a design requirement and a function is limited to a specified amount of time, i.e., in the lifetime, the energy consumption should be as small as possible. The following model minimizes the energy consumption and determines the transmission power and transmission time for each sensor node \( i \).

\[
\min_{P,T,z} \sum_{i \in S} P_i \cdot T_i \cdot \frac{L_i}{T_{frame}} \quad (5.15)
\]

subject to

\[
\sum_{i \in S} T_i \leq T_{frame} \quad (5.16)
\]

\[
\sum_{j \in CS} T_i \cdot r_{ij} \cdot z_j \geq x_i \cdot T_{frame}, \quad \forall i \in S \quad (5.17)
\]

\[
r_{ij} = W \cdot \log_2 \left( 1 + \frac{P_i |h_{ij}|^2}{N_j} \right) \quad (5.18)
\]

\[
PL_{ij} = PL(d_0) + 10 \cdot n \cdot \log_{10} \left( \frac{d_{ij}}{d_0} \right) \quad (5.19)
\]

\[
P_{min} \leq P_i \leq P_{max} \quad (5.20)
\]

\[
\sum_{j \in CS} z_j = 1, \quad z_j \in \{0, 1\} \quad (5.21)
\]
Equation 5.15 gives the total WBSN’s energy consumption to be optimized. The transmission time of all sensors cannot be bigger than the duration of one super-frame (equation 5.16). \( r_{ij} \) in equation 5.18 is the channel capacity and the upper bound of information transmitted between sensor \( i \) and base station \( j \). \( N_j \) is the power of noise, \( W \) is the bandwidth, \( |h_{ij}|^2 \) is the channel gain between node \( i \) and \( j \). For the channel, the present study only considers the path loss \( PL \) (equation 5.19), because the transmitter and receiver gain are not dependent on the base station’s location. \( n \) in equation 5.19 is the path loss exponent, \( d_0 \) is the reference distance, \( PL(d_0) \) is the path loss at the reference location and \( d_{ij} \) is the distance from sensor node \( i \) to candidate site \( j \). Constraint 5.17 states that the generated data \( x_i \) of a sensor node \( i \) in one super-frame have to be smaller than the channel capacity for guaranteed time slot \( T_i \). 

\( z_j \) are the possible Candidate Sites (CS) for the location of a base station. The transmission power of each sensor must lie within \( P_{min} \) and \( P_{max} \) (equation 5.20). The energy consumption \( P_i \cdot T_i \cdot L_i / T_{frame} \) is basically the battery capacity that is needed for one sensor to transmit data. The described model is validated in Zhou et al. [90] and Minhas et al. [91].

Reusens et al. [92] examine an on-body wireless channel and the present work uses their data as follows. The path loss at the reference location determined by Reusens et al. [92] is \( PL(d_0) = 35.2 \) dB, with a reference distance of \( d_0 = 10 \) cm and the path loss exponent of \( n = 3.11 \). The power of noise \( N_j \) in equation 5.18 is \( -174 \) dBm/Hz. The present work uses a bandwidth of 0.3 MHz and considers an average data generation speed of 40 kbps for each sensor. The duration of a super-frame \( T_{frame} \) is 400 ms. The minimum transmission power is \( P_{min} = -15 \) dBm and the maximum transmission power is \( P_{max} = 0 \) dBm. Table 5.3 summarizes the notations and the numerical values used for the design optimization of a wireless sleep monitoring system.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>Sensor node set</td>
<td>7 sensors</td>
</tr>
<tr>
<td>( CS )</td>
<td>Candidate site set</td>
<td>14 CSs</td>
</tr>
<tr>
<td>( L_i )</td>
<td>Lifetime of sensor ( i )</td>
<td>10 h</td>
</tr>
<tr>
<td>( T_{frame} )</td>
<td>Duration of super-frame</td>
<td>400 ms</td>
</tr>
<tr>
<td>( N_j )</td>
<td>The power of noise</td>
<td>( -174 ) dBm/Hz</td>
</tr>
<tr>
<td>( x_i )</td>
<td>Data generation rate for sensor node ( i )</td>
<td>40 kbps</td>
</tr>
<tr>
<td>( W )</td>
<td>Bandwidth</td>
<td>0.3 MHz</td>
</tr>
<tr>
<td>( P_{min} )</td>
<td>Minimum transmission power</td>
<td>( -15 ) dBm</td>
</tr>
<tr>
<td>( P_{max} )</td>
<td>Maximum transmission power</td>
<td>0 dBm</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>Reference distance</td>
<td>10 cm</td>
</tr>
<tr>
<td>( n )</td>
<td>Path loss exponent</td>
<td>3.11</td>
</tr>
<tr>
<td>( PL(d_0) )</td>
<td>Path loss at reference location</td>
<td>35.2 dB</td>
</tr>
</tbody>
</table>

Although the energy consumption model is available in literature, it still had to be modified for the present application. Also the parameters such as the distances \( d_{ij} \) from the sensor nodes to the candidate sites are specific to the present application. The optimization problem stated above is not solved using a mixed integer program, but it is optimized for each candidate site \( z_j \) separately. It minimizes the total energy consumption and thereby determines the transmission time and the transmission power for each sensor node.
Another feature considered next to the comfort and the energy consumption is the signal interference, as discussed next.

**Signal Interference**

Networking from the body sensor system to an external station through wireless frequencies can cause signal interference. Zeagler [65] describes areas with the least possibility of signal interference by the mass of the body. The data provided in Zeagler’s paper [65] directly relate to this problem and can be used with no further changes.

**Utility Values**

The utility values of the three objective functions are summarized in Table 5.4. The comfort value and the signal interference are already normalized between 0 and 1.

**Table 5.4:** Three objectives and utility values for the design of a wireless sleep monitoring system (The maximum utility value for each objective is highlighted in bold).

<table>
<thead>
<tr>
<th>Location Number</th>
<th>Location</th>
<th>Comfort Value</th>
<th>Total Energy Consumption [J]</th>
<th>Signal Interference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wrist</td>
<td>0.6125</td>
<td>20.0042</td>
<td>0.3333</td>
</tr>
<tr>
<td>2</td>
<td>Forearm</td>
<td><strong>0.9333</strong></td>
<td>19.7945</td>
<td>0.3333</td>
</tr>
<tr>
<td>3</td>
<td>Elbow</td>
<td>0.2083</td>
<td>19.4097</td>
<td>0.3333</td>
</tr>
<tr>
<td>4</td>
<td>Upper Arm</td>
<td>0.7333</td>
<td>19.2829</td>
<td><strong>0.0000</strong></td>
</tr>
<tr>
<td>5</td>
<td>Shoulder</td>
<td>0.3208</td>
<td>18.8608</td>
<td><strong>0.0000</strong></td>
</tr>
<tr>
<td>6</td>
<td>Forehead</td>
<td>0.6208</td>
<td>18.1051</td>
<td>0.3333</td>
</tr>
<tr>
<td>7</td>
<td>Chest</td>
<td>0.2542</td>
<td>18.4125</td>
<td>1.0000</td>
</tr>
<tr>
<td>8</td>
<td>Abdomen</td>
<td>0.5583</td>
<td>19.0510</td>
<td>1.0000</td>
</tr>
<tr>
<td>9</td>
<td>Hip</td>
<td>0.5417</td>
<td>19.6804</td>
<td>0.6667</td>
</tr>
<tr>
<td>10</td>
<td>Upper Thigh</td>
<td>0.8167</td>
<td>19.8450</td>
<td>0.8333</td>
</tr>
<tr>
<td>11</td>
<td>Lower Thigh</td>
<td>0.7458</td>
<td>20.7547</td>
<td>0.8333</td>
</tr>
<tr>
<td>12</td>
<td>Knee</td>
<td>0.1083</td>
<td>20.5040</td>
<td>0.8333</td>
</tr>
<tr>
<td>13</td>
<td>Calf</td>
<td>0.8542</td>
<td>20.4927</td>
<td>0.8333</td>
</tr>
<tr>
<td>14</td>
<td>Ankle</td>
<td>0.6542</td>
<td>21.2553</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Still, out of all 14 alternatives, it might be unclear to the decision maker as to which alternative (location) should be chosen in order to satisfy all the objectives in the best possible manner and to make a minor trade-off.

**Design Decision using the modified VIKOR Method**

This section describes finding the most preferred design solution with the modified VIKOR method and comparing it with the original VIKOR method. No commercial software of the VIKOR method is available at present and therefore the original and modified VIKOR method are implemented in Matlab. It starts with the assumption that the decision maker places equal importance on all objectives \(w_1 = w_2 = w_3 = 1/3\). The weight for maximum group utility and minimum regret is \(\nu = 0.5\). In Figure 5.3 the three objective functions comfort, energy consumption and signal interference are assigned to one of the axes. Black crosses present all the alternatives. Blue dots are the non-dominated solutions. Red stars are the solutions determined using the original VIKOR method, and the green square solution is the one determined using
the modified VIKOR method.

Figure 5.3: Green square: most preferred solution determined over the modified VIKOR method; Red star: determined through the original VIKOR method. The usage of all alternatives or only the non-dominated solutions change slightly in the original VIKOR method.

Figure 5.3 shows that the conditions specified by the original VIKOR method in Section 5.3 provide a different solution, depending on the inclusion of all or only non-dominated alternatives. The original VIKOR method suggests two compromise solutions (see Figure 5.3 subplot a) when using all alternatives. However, when using only the non-dominated solutions, the number of alternatives decreases. This is used as a condition in the original VIKOR method, and the method provides only one alternative as the compromise solution. This undermines the conditions used for the original VIKOR method.

Choosing a weight $w_j$ within the lower interval in Table 5.5 guarantees that the preferred alternative determined using the modified VIKOR method will remain the most preferred alternative no matter which direction the weights are changed, and therefore stabilizes the solution. The weight margin states, how much the weight can be changed in general; for example, $w_1$ can be changed by 23%. Thus, the margin describes the stability of the solution.
Table 5.5: Modified VIKOR weight margin for comfort ($w_1$), energy consumption ($w_2$) and signal interference ($w_3$).

<table>
<thead>
<tr>
<th></th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{lower}$</td>
<td>0.1723</td>
<td>0.2600</td>
<td>0.2600</td>
</tr>
<tr>
<td>$w_{upper}$</td>
<td>0.4031</td>
<td>0.4031</td>
<td>0.4693</td>
</tr>
<tr>
<td>Margin</td>
<td>0.2308</td>
<td>0.1431</td>
<td>0.2093</td>
</tr>
</tbody>
</table>

The most preferred compromise solution with equal weights for the three objectives in this example is the location number 4 (the upper arm). This means the upper arm is a good candidate site to mount a coordinator or base station and it has a good trade-off with respect to comfort, energy consumption and signal interference.

5.5.2 Design of an EEG Electrode

Electroencephalography (EEG) records the change in the human brain signals [20]. EEG monitoring is an important way to detect sleep apnea (see Section 1.2). The present example considers the design of an EEG electrode. The EEG electrode should be flexible to adapt to the shape of the head, which means it should have low stiffness, but at the same time should be durable over a long lifetime. The impedance of the electrode-electrolyte interface should be as small as possible (<10 kΩ). The electrode’s width is 0.0127 m, its length is 0.01 m and the thickness is 0.003 m. The next sections describe the three objectives used in this design optimization problem.

Conductivity:

The conductivity depends on the material characteristics. Different types of polymers are typically used in EEG manufacturing [93, 94]. To increase the conductivity of the polymer, carbon black can be added. Although the change in conductivity for variable amounts of carbon black has not been tested in depth, the present research makes an assumption about their relationship in order to demonstrate the modified VIKOR method. Specifically, a sigmoid function is assumed, implying that the conductivity initially increases and saturates after a particular amount of carbon black.

\[ \frac{a_i}{1 + e^{-b_i(CB-c_i)}} \]  \hspace{1em} (5.22)

Here, $a_i$, $b_i$ and $c_i$ are parameters for different polymers, and $CB$ is the amount of carbon black added to the polymer.

Durability:

Durability also depends on the polymer and the added amount of carbon black. More carbon black results in a harder electrode. Durability or toughness can be determined by integrating the stress-strain curve. Toughness is defined as the energy of mechanical deformation per
unit volume prior to failure. Similar to the above assumptions, the present work makes an
assumption about the physical behavior, for the purpose of testing introduced methodology, as
follows:

\[ \frac{1}{2} \epsilon_{f_i}^2 E_i (1 + CB^{1/4}) \]  (5.23)

Here, \( \epsilon_{f_i} \) is the strain at failure and \( E_i \) is the Young’s modulus of polymer \( i \).

**Stiffness:**

In order to make the EEG electrode attachment as comfortable as possible it should be flexible,
i.e., the stiffness should be small. The stiffness depends on the cross sectional area, the length
of the electrode and the Young’s modulus. The Young’s modulus changes when carbon black
is added. For the stiffness, the present analysis assumes the following relation:

\[ \frac{wt}{l} E_i (1 + CB^{1/3}) \]  (5.24)

where, \( w \) is the width of the electrode, \( t \) is the thickness of the electrode, \( l \) is the length,
and \( E_i \) is the Young’s modulus of polymer \( i \).

Table 5.6 lists 4 assumed polymers with different parameters for the formulated functions.
These functions and parameters are used for the multi-objective optimization of an EEG elec-
trode.

**Table 5.6:** Parameters in the multi-objective optimization of EEG electrode (four types
of polymer are considered).

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_i )</td>
<td>0.0049</td>
<td>0.0054</td>
<td>0.0051</td>
<td>0.005</td>
</tr>
<tr>
<td>( b_i )</td>
<td>0.9023</td>
<td>2.5937</td>
<td>1.965</td>
<td>1.122</td>
</tr>
<tr>
<td>( c_i )</td>
<td>26</td>
<td>18</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>( \epsilon_{f_i} )</td>
<td>0.18</td>
<td>0.11</td>
<td>0.21</td>
<td>0.09</td>
</tr>
<tr>
<td>( E_i )</td>
<td>130.90</td>
<td>1062.73</td>
<td>560.96</td>
<td>2129.01</td>
</tr>
</tbody>
</table>

**Multi-objective Optimization and Design Decision**

The three objective functions are optimized simultaneously with a multi-objective evolutionary
algorithm [95]. The resulting Pareto front of polymer 1 is plotted in Figure 5.4. All objectives
have the same importance value with \( w_1 = w_2 = w_3 = 1/3 \) and \( \nu = 0.5 \). The original
and modified VIKOR method provide the same most preferred solution. The original VIKOR
method however, provides multiple compromise solutions due to the conditions in Section 5.3.1
point v.

To record brain waves with the electrode, the impedance must be \(< 10 \text{kΩ} \), which with
the electrode dimensions, corresponds to a conductivity greater than \( 2.4 \times 10^{-3} \text{S/m} \). This
constraint can be directly applied to the Pareto front or added as a constraint in the multi-
objective optimization algorithm, leading to a reduced number of possible alternatives from
Figure 5.4 compared with Figure 5.5.

Figure 5.4: Non-dominated solutions of polymer 1 with marked preferred design solution of original and modified VIKOR method.

Now the preferred solution determined by the original VIKOR method moves down. Because only a conductivity of $2.4 \times 10^{-3} \text{ S/m}$ is necessary in order to design a well-functioning EEG electrode, it would be sufficient to choose a solution with the minimum value of conductivity in the constraint Pareto front. On examining the Pareto front in more detail, one can see that by increasing conductivity, flexibility does not decrease up to a certain point when the curve falls off. The same is true for durability, and therefore, the trade-off made by increasing the conductivity with respect to the other objectives is rather small. The chosen solution by the modified VIKOR method therefore intuitively makes sense. However, the chosen compromise solution by the original VIKOR method results in a large trade-off that greatly affects flexibility and durability.
This again shows that, on introducing a constraint into the design problem, the solution in the modified VIKOR method does not change, and demonstrates its advantage over the original method. Any combination out of the weight margin presented in Table 5.7 will result in the same compromise solution provided by the modified VIKOR method.

**Table 5.7:** Weight margins for conductivity ($w_1$), durability ($w_2$) and flexibility ($w_3$)

<table>
<thead>
<tr>
<th></th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{\text{lower}}$</td>
<td>0.2422</td>
<td>0.2422</td>
<td>0.1584</td>
</tr>
<tr>
<td>$w_{\text{upper}}$</td>
<td>0.5000</td>
<td>0.4347</td>
<td>0.4347</td>
</tr>
<tr>
<td>Margin</td>
<td>0.2578</td>
<td>0.1925</td>
<td>0.2763</td>
</tr>
</tbody>
</table>

A bigger weight margin means the solution is more stable and a decision maker can be more confident about the provided solution.

Figure 5.6 presents the Pareto front of the four different polymers. The more complex a Pareto front gets, the more challenging it is to find a good compromise solution. This demonstrates the need for a method that helps a designer to choose one solution.

Again, equal weights are used to determine a compromise solution. The different curves on the overall Pareto front represent different types of material (Table 5.6).
The modified VIKOR method chooses the same solution as before. The original VIKOR method chooses the same polymer but a different solution. This is because the durability of one of the polymers that was added to the design space can be much higher, but at the same time the flexibility is also much lower for the same material. However, as discussed before, it makes sense to choose the solution provided by the modified VIKOR method, because going down the trade-off in this part of the Pareto front increases drastically, for flexibility.

Figure 5.6 presents the compromise solutions in green square, where the objectives can change within a predefined range (in this case $10\%$). Therefore, the weight margins for the increased range of alternatives are determined according to Section 5.4.3. The weight margins presented in Table 5.8, in which the objectives can change within 10% (range of alternatives), is bigger than that for a single alternative (Table 5.8 one alternative).

<table>
<thead>
<tr>
<th>$w_{\text{lower}}^j$</th>
<th>One alternative</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of alternatives</td>
<td>0.2379</td>
<td>0.2379</td>
<td>0.0921</td>
<td></td>
</tr>
<tr>
<td>$w_{\text{upper}}^j$</td>
<td>One alternative</td>
<td>0.5000</td>
<td>0.4862</td>
<td>0.4862</td>
</tr>
<tr>
<td>Range of alternatives</td>
<td>0.5000</td>
<td>0.4862</td>
<td>0.4862</td>
<td></td>
</tr>
<tr>
<td>Margin</td>
<td>One alternative</td>
<td>0.2621</td>
<td>0.2483</td>
<td>0.3942</td>
</tr>
<tr>
<td>Range of alternatives</td>
<td>0.2621</td>
<td>0.2483</td>
<td>0.4129</td>
<td></td>
</tr>
</tbody>
</table>

The difference in this case is rather small however (only $w_3$ has a higher margin). One reason
for this is that the upper weight margin for $w_1$ is already at its maximum. For example, when decreasing one of the weights except $w_1$ and two weights are increasing at the same time (either $w_1$ and $w_2$ or $w_1$ and $w_3$), then the highest possible increase is 0.5. That is, if one goes down to 0 from $1/3$, the other ones can only increase by $1/6$. This weight margin however, is the worst case scenario, and by providing this value it is guaranteed that by choosing a weight within the weight margin the same compromise solution would be picked. Clearly, there are many more possibilities how to change the weights (e.g., one weight unchanged, one increased, one decreased; one increased, two decreased). Therefore, it makes sense to provide the lowest weight margin, as opposed to the highest possible weight margin (Opricovic et al. [87]).

5.6 Conclusion

This chapter revealed some shortcomings of the VIKOR decision making method and proposed improvements to the originally proposed method. With the use of the $L^\infty$-norm, a design decision is not affected by the inclusion of only the reduced set of non-dominated solutions versus a complete set of alternatives. The solution also does not change when introducing a design constraint into a multi-objective design problem. Within an introduced minimum weight margin, it guarantees that the final design solution will not change on changing the weights in any direction. This is an important information for the decision maker, because they might be uncertain in their weight assignment, and the present approach therefore assesses the stability of the solution. If the decision maker’s uncertainty lies within the weight margin they can be confident about the provided compromise solution. The shortcomings of the original VIKOR method were demonstrated and it was shown that the conditions for a stable solution provided by the original VIKOR method were rather intuitive.

The modified VIKOR method was applied to two design examples. The first design example was applied to an existing problem that had been addressed in part in Chapter 2 of the thesis. In that chapter a solution was selected based on intuition while acknowledging that a more sophisticated approach would be desirable. The present chapter revisited the problem. The application of the modified VIKOR method helped to objectively select a body location for a wearable body sensor network while taking into account the objectives comfort, energy consumption and signal interference. The second design example concerned the design of an EEG electrode with high conductivity, durability and flexibility.

The weight assignment to individual objectives/criteria was illustrated in these examples. Techniques such as AHP or group decision making methods as in [84] could be used for further improving the developed method, to less subjectively assign the weight importance values to objectives. The next chapter will compare the VIKOR method improved in the present chapter with the methods used in Chapter 2 and 3.
Chapter 6

Comparison of Decision Making Methods

Chapter 2 provided an approach to include qualitative objectives in a less subjective way. When considering multiple-objectives, some of the objectives might be conflicting and a trade-off is inevitable. The technique proposed in Chapter 2 to make a trade-off decision or in other words to pick a solution out of the Pareto front was rather simple. Therefore, a more refined strategy to make this design decision was introduced in Chapter 3. Since the qualitative objective comfort used in Chapter 2 and Chapter 3 was rather intuitively formulated, Chapter 4 further investigated this objective. For enhanced validation and further comparison, Chapter 5 introduced another decision making method. The next section compares the technique developed in Chapter 2 with the more effective technique proposed in Chapter 5.

6.1 Comparison of Chapter 2 Decision Making Method with VIKOR

The method used in Chapter 2 chose a solution based on the number of times locations were picked as Pareto optimal in the multi-objective optimization algorithm. This was a rather intuitive way to make a trade-off. In addition, the method was applied only to the example in Chapter 2. Therefore, the trade-off decision is compared now with the more effective and broadly applicable VIKOR method, which was developed in Chapter 5. Figure 6.1 presents the result from the method used in Chapter 2 and the VIKOR approach. As can be seen from the figure, the VIKOR method and also the modified VIKOR method proposed in Chapter 5 select a slightly different solution on the Pareto front. Importance values are the same for both objectives. Since the threshold value used in Chapter 2 to select a solution was rather intuitive, it is suggested to use the solution provided by the VIKOR method. Chapter 3 uses the technique of fuzzy measures and the Choquet integral to improve the trade-off decision technique of Chapter 2. The next section will compare the decision making technique of fuzzy measures and Choquet integral used in Chapter 3 with the method proposed in Chapter 5.
6.2 An Example Comparison of Fuzzy Measures/Choquet Integral with VIKOR

The VIKOR method introduced in Chapter 5 is based on the maximum “group utility” and “minimal regret.” The Choquet integral in terms of the Möbius representation and the VIKOR method are similar in taking the “minimum regret”. The Choquet integral however, takes the “minimum regret” of the whole power set of criteria/objectives. For easier comparison the VIKOR method and the Choquet integral in terms of the Möbius representation are presented here, again.

\[
C_m(a^i) = \sum_{T \subseteq N} m(T) \min_{j \in T} x^i_j \tag{6.1}
\]

\[
S^i = \sum_{j=1}^{n} w_j \frac{x^i_j}{||x^i_j||_{\infty}}, \; j = 1, \ldots, n \tag{6.2}
\]

\[
R^i = \min \left\{ w_j \frac{x^i_j}{||x^i_j||_{\infty}} \right\}, \; j = 1, \ldots, n \tag{6.3}
\]

\[
Q^i = \nu \frac{S^i}{||S||_{\infty}} + (1 - \nu) \frac{R^i}{||R||_{\infty}} \tag{6.4}
\]

Making it more specific, considering only three criteria and omitting index \(i\) of alternatives, the Choquet integral and VIKOR method appear as follows:
\[ C_m(a) = m_1 x_1 + m_2 x_2 + m_3 x_3 + m_{12} \min\{x_1, x_2\} + m_{13} \min\{x_1, x_3\} + m_{23} \min\{x_2, x_3\} + m_{123} \min\{x_1, x_2, x_3\} \] (6.5)

\[ Q = \frac{\nu}{||S||_{\infty}} (w_1 x_1 + w_2 x_2 + w_3 x_3) + \frac{1-\nu}{||R||_{\infty}} \min\{w_1 x_1, w_2 x_2, w_3 x_3\} \] (6.6)

Here, \( m_j \) are the fuzzy measures in Möbius representation and \( w_j \) are the weights for single criteria. As seen in the equations above, the first part of the Chouet integral and the first part of the VIKOR aggregation are essentially the same with an additional weighting of the weighted sum for the VIKOR method. The Choquet integral takes the minimum regret of each combination of criteria, whereas the VIKOR method takes the minimum regret of all criteria. The Choquet integral uses an importance value (fuzzy measure) for each “minimum regret” without a weighting applied to the criteria. By setting fuzzy measures \( m_T \) with \(|T| < |N| \) to zero, only the “minimum regret” of all criteria is considered in the Choquet integral.

To compare the technique of fuzzy measures/Choquet integral with the VIKOR method of Chapter 5, the objectives and utility values from Chapter 3 (see Table 3.1) are used. The fuzzy measures are determined using the algorithm in Chapter 3 with the following preference rankings:

- version 4 \( \succ \) version 3; version 24 \( \succ \) version 14; version 124 \( \succ \) version 1234
- comfort \( \succ \) reliability \( \succ \) power consumption

Table 6.1 shows the fuzzy measures and Shapley index through these preferences.

| Table 6.1: Fuzzy measures \( m \) in Möbius representation and Shapley indices \( \phi_{Sh} \) |
|---------------------------------|----------------|----------------|
| \( m(\{i\}) \)                 | Comfort        | Reliability    | Power consumption |
| 0.4534                         | 0.0            | 0.4534         |
| \( \phi_{Sh}(\{i\}) \)        | 0.5000         | 0.2733         | 0.2267            |
| Comfort                        |                | 0.5466         | -0.4534           |
| Reliability                    | 0.5466         |                | 0.0000            |
| Power consumption              | -0.4534        | 0.0000         | -               |

The Shapley values are then used as importance weightings for the VIKOR method with a \( \nu \) value of 0.5.

Figure 6.2 shows that the preferred alternative determined through fuzzy measures and Choquet integral coincide with the VIKOR method. This may be due the similarities of the two methods.
6.3 Another Comparison of Fuzzy Measures/Choquet Integral with VIKOR

This section compares fuzzy measures/Choquet integral with VIKOR using the example in Chapter 5 (for utility values see Table 5.4). This example uses equal importance values for all objectives \( w_1 = w_2 = w_3 = 1/3 \) in the VIKOR method and equal weightings for the 2-additive fuzzy measures \( m_1 = m_2 = m_3 = m_{12} = m_{13} = m_{23} = 1/6 \). The fuzzy measures/Choquet integral approach shows the same preferred alternative as the VIKOR method (Figure 6.3). This demonstrates the similarities between the two methods again. However, the ranking error defined by equations 4.9 and 4.10 in Chapter 4 shows that the order of ranked alternatives differs slightly \( \text{NRMSD}_{\text{rank}} = 0.1007 \).

By further comparing the two methods, one can see that the VIKOR method cannot model interactions. Consider the following example:

<table>
<thead>
<tr>
<th>( A )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>26</td>
<td>30</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>30</td>
<td>26</td>
</tr>
</tbody>
</table>
Using the VIKOR method it is not possible to prefer an alternative with a high utility value in either of the two criteria over an alternative with two medium utility values \((a^1 \succ a^2\) and \(a^3 \succ a^2\)). This is because the \(\nu\) value weighting the maximum group utility \((S^i)\) and minimum individual regret \((R^i)\) is a positive number. More specifically:

\[
Q^1 = \frac{\nu}{||S_\infty||} w_1 26 + w_2 30 + \frac{1 - \nu}{||R_\infty||} \min\{w_1 26, w_2 30\} \\
Q^2 = \frac{\nu}{||S_\infty||} w_1 28 + w_2 28 + \frac{1 - \nu}{||R_\infty||} \min\{w_1 28, w_2 28\} \\
Q^3 = \frac{\nu}{||S_\infty||} w_1 30 + w_2 26 + \frac{1 - \nu}{||R_\infty||} \min\{w_1 30, w_2 26\}
\]

There is no \(w_1, w_2\) so that \(Q^1 > Q^2\) and \(Q^3 > Q^2\). In contrast, the Choquet integral with fuzzy measures:

\[
C_m(a^1) = m_1 26 + m_2 30 + m_{12} \min\{26, 30\} \\
C_m(a^2) = m_1 28 + m_2 28 + m_{12} \min\{28, 28\} \\
C_m(a^3) = m_1 30 + m_2 26 + m_{12} \min\{30, 26\}
\]

and for example, \(m_1 = m_2 = 1; m_{12} = -1\) provides a Choquet integral of \(C_m(a^1) = 30, C_m(a^2) = 28, C_m(a^3) = 30\) and therefore \(a^1 \succ a^2\) and \(a^3 \succ a^2\).

It is important to note that the VIKOR method can become confusing when a decision
maker increases the weight in one objective and observes the opposite effect of what he/she expects. This is the case in Table 6.2, where by increasing the weight of \( w_1 \) alternative \( a^3 \) should get the highest \( Q^1 \) value, because it also has the highest value in \( x_1 = 30 \). On the other hand, it is the case when observing the \( S^i \) values. However, because the minimum regret of \( a^1 \) is higher than those in the other alternatives, \( a^1 \) will be preferred. This might lead to misinterpretation. An increase in the importance value \( \nu \) makes \( R^i \) less important and \( a^3 \) will be the most preferred solution.

Table 6.2: Utility values of the VIKOR method for two different weightings.

<table>
<thead>
<tr>
<th>A</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( S^i )</th>
<th>( R^i )</th>
<th>( Q^i )</th>
<th>( S^i )</th>
<th>( R^i )</th>
<th>( Q^i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^1 )</td>
<td>26</td>
<td>30</td>
<td>0.933</td>
<td>0.433</td>
<td>0.964</td>
<td>0.880</td>
<td>0.100</td>
<td><strong>0.946</strong></td>
</tr>
<tr>
<td>( a^2 )</td>
<td>28</td>
<td>28</td>
<td>0.933</td>
<td>0.467</td>
<td><strong>1.000</strong></td>
<td>0.933</td>
<td>0.093</td>
<td>0.940</td>
</tr>
<tr>
<td>( a^3 )</td>
<td>30</td>
<td>26</td>
<td>0.933</td>
<td>0.433</td>
<td>0.964</td>
<td>0.987</td>
<td>0.087</td>
<td>0.933</td>
</tr>
</tbody>
</table>

The advantage of the VIKOR method lies in its simplicity. A decision maker or expert only has to provide a linear set of weightings, whereas for the Choquet integral \( 2^n - 2 \) fuzzy measures or in case of the 2-additive case \( n(n + 1)/2 \) fuzzy measures have to be provided or determined.

6.4 Conclusion

This chapter compared the VIKOR method with the theory of fuzzy measures and Choquet integral approach and demonstrated the advantages and disadvantages. The two decision making methods provided comparable results. Both methods used similar aggregation functions, although the concepts behind them were different. The VIKOR method’s concept is based on maximum group utility and individual regret, whereas the main purpose of the fuzzy measure and Choquet integral approach is the consideration of interactions. Solely fuzzy measures in the Möbius representation show similarities among their methods of aggregation. The VIKOR method is simpler to apply, because only a linear set of weights is necessary. However, preference rankings that are more complicated can be modeled with the fuzzy measure approach. The application itself determines the use of the appropriate decision making method. Chapters 3 and 4 of this dissertation provided methods to determine fuzzy measures; however, additional information is needed. Techniques such as AHP or group decision making methods as in [84] can be used to assign the importance values in the VIKOR method. The next chapter summarizes and concludes the present dissertation and presents possible future work.
Chapter 7

Discussion and Conclusion

This thesis covered several aspects of the design optimization of a mechatronic system or device. The methods developed in this thesis were mainly applied to a wearable sleep monitoring system. In the design of such a system, multiple features and objectives need to be considered. Some of these features are of quantitative nature, while others are qualitative. When considering multiple conflicting objectives, a trade-off is required. Decision making methods help to make better trade-offs depending on the application and preferences. However, it involves subjectivity, it depends on the decision maker providing the importance values and also on the group of people the product is designed for (customers). This leads to different trade-offs, and a validation for the best trade-off method is hardly possible. If the design trade-off decision was satisfactory, still it would not be clear if it was the best trade-off decision. In addition, it is challenging to validate a wrong trade-off decision because the decision making framework could have led to the wrong decision or the assigned weights within the decision making framework. Furthermore, how to measure a good trade-off decision is not quite clear. One measure might be the profit a company makes with the designed product; however, that also depends on the current economic situation and therefore it is uncertain if another trade-off would have provided a higher profit. However, a methodology that includes design preferences helps to make trade-off decisions and it will potentially lead to a better design. While a full validation of the decision making method might not be possible and the present research is limited to make improvements and comparing them, following a systematic validation strategy is more desirable than choosing a random solution. The methods developed in the present dissertation help the designer in the design process. First the design space is reduced to a set of non-dominated solutions by including quantitative and qualitative aspects. Qualitative objectives are modeled using multiple criteria and the criteria describing the qualitative objective are aggregated to one representative value. The criteria itself can be quantitative or qualitative. The introduced and enhanced decision making methods lead to less subjective trade-off decisions when the objectives are conflicting.
7.1 Summary

This thesis significantly contributes to resolving the question of how to include qualitative criteria in a less subjective way. In Chapter 2, this was done by using fuzzy sets and fuzzy numbers. The defuzzified fuzzy numbers were the numerical representation of the qualitative criteria used in Chapter 2. That chapter optimized comfort and wearability simultaneously. It presented a systematic way to design a comfortable and reliable body sensor system. The method presented was then applied to a potential design of EEG/EOG for sleep monitoring. However, Chapter 2 did not provide the most comfortable and reliable EEG/EOG system, but rather implemented a method that would assist a designer in the selection of the type of devices, location on the body and how the devices are wired.

In Chapter 3 the same exemplary utility values were used to make a more effective design decision, by using the technique of fuzzy measures and Choquet integral.

To obtain more realistic values for the objective functions, Chapter 4 developed a comfort model for a wearable body sensor network based on fuzzy measures and the Choquet integral. With fuzzy measures it is possible to model interactions between objectives. This ability of interaction modeling described the qualitative objective comfort in a refined way, by using multiple criteria to describe comfort. The comfort model was validated through a training set and test set with the help of a comfort questionnaire study. The solutions discovered in Chapter 4 showed how to incorporate a qualitative objective into a multi-objective design problem with quantitative and qualitative design criteria. When designing a wearable body sensor system it is helpful to know the importance of comfort and the criteria for describing it, in order to better adjust the design.

Chapter 5 described an improved version of the VIKOR decision making method in which a sensitivity analysis in terms of a stability weight margin extended the original VIKOR method. The improved method was then applied to two design examples in the field of wearable body sensor networks, focusing on a portable sleep monitoring system.

Chapter 6 provided a comparison of the improved VIKOR method to the decision making methods enhanced in the previous chapters.

7.2 Possible Future Work

This dissertation advanced the methods to incorporate qualitative criteria or objectives in a less subjective way into a multi-objective design process for a mechatronic system, where it is necessary to make a trade-off decision. The focus application was a comprehensive sensor system for sleep monitoring in the familiar home environment. Since none of the sensor systems in the sleep monitoring project are yet ready for testing, the methods developed in the present dissertation were applied to potential design optimization problems of a wearable sleep monitoring system.

In a later design stage of the project, these methods can be applied to the actual sleep
monitoring system that is currently developed in the sleep monitoring research group at The University of British Columbia. The methods are also applicable to other design optimization problems of wearable sensor technology. Developed methodologies are therefore of great importance in a variety of fields. This section indicates several next steps to follow the work of the present thesis:

1. The method that addresses modeling of qualitative objectives (used in Chapter 4) can be further improved by making the utility values for criteria describing the objective to fuzzy numbers. This is because the utility values are not precise physical quantities, and also because the overall aggregated value determined through a questionnaire is not an exact numerical value. Therefore, it could help to treat these utility values as fuzzy numbers. In addition, the criteria chosen to represent comfort can be better described through analytical models (e.g., mathematical model for motion impedance). Consequently, the model depends less on a questionnaire study. A further expansion of the presented study includes an additional qualitative objective such as complexity, which can be used to further validate the concepts developed in the present thesis to handle qualitative objectives. The developed methods can be further applied to different design optimization areas, such as automotive design. Furthermore, techniques such as AHP or group decision making methods, as in [84], can be used to determine the importance weightings in the VIKOR method. To deal with uncertainties in the weight assignment procedure, the Dempster-Shafer evidence theory introduced by Fei et al. [85] may be applied, which will improve the VIKOR method.

2. An application of the VIKOR method was presented in Chapter 5 of the thesis. Preliminary results were obtained in analyzing material properties as the input for the proposed methodology. Studies regarding relations between carbon black and different polymers within the sleep monitoring research will facilitate applications of the proposed method there and evaluate an improved trade-off when optimizing for flexibility, durability and conductivity. The VIKOR method applied to the EEG electrode design example can be applied, when the relationship between carbon black and different polymer is known, for flexibility, durability and conductivity.

3. Further validation of the decision making methods developed in this dissertation (Chapters 3 and 5) is desirable. It may be carried out by choosing multiple trade-off solutions out of the same Pareto-front and manufacturing multiple prototypes for the same product purpose. A group of people in different categories (job, country, gender, etc.,) may rate these products from best to worst and provide a reason for their rating. If the given reason coincides with a high importance value for the same criterion in the decision making method, it strengthens the decision making of the developed method, and vice versa. By going back to the same people who provided the input earlier can partly validate the decision. It will enable further evaluation of the developed decision making methods.
and will provide more insight into their strengths and weaknesses. There is a need for conceptual and operational validation of the developed methodology through application in real world problems.
References


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