Essays on Firm Heterogeneity and Entrepreneurship

by

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Essays on Firm Heterogeneity and Entrepreneurship

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Abstract

What determines firm heterogeneity? What are the consequences of this heterogeneity for the macroeconomy? Traditionally, economists have considered a representative firm as an approximation for reality. Although such a restriction can be useful to study some questions, in reality there is a great deal of heterogeneity in firm behavior. In this work, I look at different dimensions of heterogeneity in outcomes for firms, their sources and their implications for the macroeconomy.

In Chapter 1, I propose a rich general equilibrium model of entrepreneurship, where I allow both wage workers and unemployed to start firms. I show that in this framework, the lower opportunity cost of entrepreneurship for the unemployed induces the formation of lower quality firms relative to wage workers. Using a new confidential owner-employer-employee matched dataset from Canada I test these predictions by verifying that firms created by the unemployed are on average smaller and die faster. I test the mechanism behind this result, by verifying that workers are more responsive to wage changes in their decision to start a firm relative to the unemployed. Finally, I use this framework to evaluate the impact on the economy of a public policy that promotes entrepreneurship among the unemployed.

In the model presented in Chapter 2, we study the choice of an individual to start a firm as a function of their outside option as an unemployed and the implications for the efficient allocation in the economy. We show that by simply adding this additional margin to an otherwise standard general equilibrium theoretical framework, wage comparative statics become richer and the efficient allocation chosen by a benevolent social planner has a new interpretation. The chapter highlights the importance of modelling the entry margin into firm ownership in

determining firm heterogeneity as well as wage dynamics.

In the last chapter, we turn to the study of determinants of a firm's decision of which contract to offer a worker and the implications for wage dynamics and worker retention. We verify empirically that, due to a worker retention motive, match quality affects contract choice and wage cyclicality.

Lay Summary

How do we explain the differences we see in firm behaviour? Why are certains firms more productive than others? Why are some larger? Why are there differences in wages firms offer to workers? How does the answer to these questions affect our understanding of the labor market?

In this dissertation I provide evidence that making sense of an individual's decisions to start a firm can play a key role in our understanding of wage determination and firm dynamics. Furthermore, I show that these might have important implications for our understanding of the firm productivity distribution and the impact of public policy. Finally, I show that match quality is important in determining the contract a firm choses to offer a worker and as a result the wage.

Preface

Chapter 1 of this thesis ”*Unemployment, Entrepreneurship and Firm Outcomes*” is my original work. The empirical section of this chapter uses data from Statistics Canada’s Candian Employment Employee Dynamic Database (CEEDD).

The second chapter, ”*Entrepreneurship Outside options and Constrained Efficiency*”, is an unpublished working paper that I co-authored with Iain Snoddy. The authors contributed equally to the project overall.

In Chapter 3, ”*Match Quality, Contractual Sorting and Wage Cyclicality*” is an unpublished working paper that I co-authored with Giovanni Gallipoli and Yaniv Yedid-Levi. The authors contributed equally to the project overall.

Any views expressed in the thesis are mine alone and do not reflect the views of Statistics Canada or the Government of Canada.

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IV Instrumental Variable

OLS Ordinary Least Squares

UI Unemployment Insurance

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This work is dedicated to min älskling, Emilie.

Introduction

How do firms vary in behaviour, performance and how is this linked to the aggregate economy? The answer to these questions are crucial for our understanding of firm dynamics, wage formation and unemployment.

In this thesis I study different components of firm heterogeneity, investigate their sources and how they relate to the macroeconomy. One recurring and crucial component of the analysis will be the study of the individual's decision to start a firm. By investigating the sources of these decisions, we can investigate how changes in the economy for the household impact the firm productivity distribution. This channel has often been understudied. Most recent papers consider some form or another of a free entry condition. This condition can be stated as the assumption that there is an infinite amount of potential firms that will enter the market as long as the value of doing so is positive. On the other hand, in a framework with entrepreneurship this decision to operate as a firm will depend on the individual outside option to entrepreneurship.

In the first chapter entitled *Unemployment, Entrepreneurship and Firm Outcomes*, I investigate whether there are differences between firms created by unemployed individuals relative to otherwise identical employed individuals. I then show that these patterns are important for our understanding on whether policies promoting entrepreneuship among the unemployed are warranted strategies to promote job creation. This is relevant given the widespread usage of these policies across the world.

To shed light on these issues I propose a general equilibrium model of entrepreneurship that allows for different choices by the unemployed and the employed. In the model the only difference between the unemployed and wage worker is their outside option. Due to poorer outside options, the unemployed are less selective on which business projects to implement. In equilibrium, this implies that the unemployed are more likely to start a firm but conditional on doing so, hire fewer workers and are more likely to exit entrepreneurship relative to an individual who started a business (implemented a business project) from wage work.

These implications of the model hold in the data. I use firm closures to identify random assignments of an individual to unemployment (via layoff). I find that unemployment doubles the probability of an individual to start a firm, and conditional on starting, the individual hires 26% fewer workers and is 30% more likely to exit firm ownership. The data being used is composed of the entire universe of tax filers linked to privately owned incorporated firms in Canada. It improves on employer-employee datasets by having also the link between each firm and their corresponding owner.[1] This makes it fitting for studies in entrepreneurship. With an extension of the theory to a multi sector environment I derive the additional implication that higher wages decrease the entry rate into entrepreneurship of wage workers by more than that of unemployed. Wages represent the opportunity cost of entrepreneurship for wage workers but not for the unemployed. As a result, wage workers are more responsive to wage variation than the unemployed in their decision to open a firm. Using city wage variation and a Bartik style IV strategy for wages, I show that a 1% drop in wages increases by 3.2 percentage points the entry rate into entrepreneurship for wage workers and has no impact for laid off individuals.

Finally, I quantify the impact on the aggregate economy of a policy that subsidizes entry into entrepreneurship among the unemployed. The result is a 2.14% drop in average firm productivity and a 1% drop in the unemployment rate. The policy induces the creation of low productivity firms by the unemployed. With a larger mass of firms, the equilibrium cost of labour increases. This induces high productivity firms to hire fewer workers and wage workers to be more selective on business projects. This employment drop among high productivity firms offsets job

[1] The three most used employer-employee linked datasets, the LEHD for the US, the DADS for France and the LIAB for Germany, all lack information on individual owners of firms. With the exception of registry data from Sweden and Denmark, this is the first dataset to allow the tracking of all linkages between a firm and its employees and owners across time.

gains from firms created with the subsidy. The result is a shift in resources from high productivity firms created by wage workers to low productivity firms created by the unemployed.

In the second chapter entitled *Entrepreneurship, Outside options and Constrained Efficiency* we study a theoretical framework with search frictions in which the free entry condition is replaced by the decision of individuals to start a firm or not. We show that the outside options of a firm owner and a worker are now the same since both always have the option of reverting to unemployment. The corollary of this is that the direction of wage responses to shocks now becomes ambiguous. Wages are no longer necessarily increasing with the value of being unemployed. While a higher value of unemployment allows the worker to negotiate a higher wage, it also increases entrepreneurs' outside options. As a result, how wages respond to a higher value of unemployment now depends on which party has more bargaining power the worker or the entrepreneur. It follows that the bargaining parameter determining each party's bargaining strengh becomes crucial for wage response to exogenous shocks.

We next evaluate the conditions for which this economy attains the constrained efficiency allocation.[2] We find that the condition is the same as that found by Hosios ([Hosios, 1990] for an economy with search frictions but without entrepreneurs. But despite a similar condition the implication for the economy is now different. In particular, under this efficient allocation, dynamics of the model following a shock remain sensitive to the degree of friction in the labor market. In particular, wages do not necessarily exert a dampening effect in response to exogenous shocks.

In Chapter 3 entitled, *Match Quality, Contractual Sorting and Wage Cyclicality*, the focus shifts from firm formation to the contract choice by firms. We study the role of match quality for contractual arrangements, wage dynamics and workers retention. We develop a model in which profit maximizing firms offer a performance-based pay arrangement to retain workers with relatively high match-specific productivity. The key implications of our model hold in the data, where information about job histories and performance pay is available. We verify empirically that firms are more prone to offering performance pay based contracts to

[2]The constrained efficient allocation is defined as that which is equivalent to that chosen by a benevolent social planner constrained by all the market frictions of the decentralized economy.

workers for which match quality is higher. We also verify that wage cyclicality is coming from performance pay jobs, with those offering different contracts exhibiting no cyclicality. Finally we also show that match quality has a direct effect even after we control for contract choice and we relate our findings to the literature on occupation heterogeneity.

Chapter 1

Unemployment, Entrepreneurship and Firm Outcomes

1.1 Introduction

How does unemployment affect an individual's decision to open a firm and the outcomes of that firm relative to employment? The answer to this question is crucial for understanding the determinants of entrepreneurship, firm dynamics and the appropriate policies to promote job creation.

Across the world, countries have established policies to promote entrepreneurship among the unemployed.[1] Examples include the expenditure of 37.5 million euros by France in 2009 alone, with 40% of new businesses being started by the unemployed (Commission [2010]). In Germany in 2004, spending on these policies totalled 2.7 billion euros, representing 17.2% of expenditures in active labour market policies (Baumgartner and Caliendo [2007]). In the UK, such a policy has

[1] Policies vary from extended unemployment benefits to direct financial assistance and coaching in the startup process. Examples of such policies are the Back to Work Enterprise Allowance in Ireland and the Self-employment assistance program in the US, both of which allow individuals to keep welfare benefits while they start their own business. A list of policies across Europe, Australia, Canada and the US as well as coverage in the press are available upon request.

been responsible for the creation of nearly $2,000$ new businesses per month since its reintroduction in 2011 (Burn-Callender [2013]). In Canada in 2012, these policies cost 118 million Canadian dollars, representing 10% of expenditures in active labour market policies (CEIC [2014]).

Although there is a large literature on entrepreneurship and firm dynamics[2], the labour status of the potential entrepreneur has often been overlooked.[3] To analyze these issues, I propose a general equilibrium model of entrepreneurship that allows for different choices by the unemployed and the employed. In the framework, the only difference between the unemployed and wage workers is their outside option. As a result, the unemployed are less selective on which business projects they implement. In equilibrium, this implies that the unemployed are more likely to start a firm but, conditional on doing so, hire fewer workers and are more likely to exit entrepreneurship relative to an individual who started a business (implemented a business project) from wage work.

In the model, workers and unemployed draw business opportunities at a same rate ψ and from a same exogenous distribution F. Each business project is associated to an initial firm productivity. Upon drawing a business project each individual makes the endogenous decision to implement it or not. Individuals go into and out of unemployment from and to wage work at an exogenous rate.[4] Since there are no search frictions, at each instant an entrepreneur maximizes firm profits by choosing the optimal firm size given the productivity of the firm and the equilibrium wage rate.[5] However, the entrepreneur also faces a dynamic problem. Once the firm starts operating the productivity of the firm starts moving according to a brownian motion with a drift. This in turn results in a optimal stoppping problem for the entrepreneur, which gives us a optimal threshold productivity below which the

[2]Lucas Jr [1978], Holtz-Eakin et al. [1994], Fonseca et al. [2001], Hurst and Lusardi [2004], Cagetti et al. [2006], Quadrini [2000], Beaudry et al. [2011], Hamilton [2000] and Haltiwanger et al. [2013]

[3]The tradition in the literature on entrepreneurship has been to use models in which differences in outcomes arise due to differences in innate entrepreneurial ability of individuals. This chapter proposes a framework in which differences in outcomes between unemployed and employed individuals arise in the absence of ex-ante heterogeneity.

[4]This assumption is later relaxed in the extension of the model, in which the job finding rate becomes an equilibrium object determined by labor market tightness.

[5]Entrepreneurs hire from the pool of available workers, all individuals that did not open a firm and did not receive the exogenous unemployment shock.

individual exits entrepreneurship.

In equilibrium, because wage workers are more selective on which business projects to implement, conditional on entering, their initial productivity level is higher. This in turn translates to higher average size for firms created by wage workers. Furthermore, since the threshold below which an entrepreneur exits entrepreneurship is the same for wage worker and entrepreneur, this higher initial productivity also implies a lower average exit rate for firms created by wage workers.

These implications of the model hold in the data. The data being used is composed of the entire universe of tax filers linked to privately owned incorporated firms in Canada. It improves on employer-employee datasets by linking firms to their corresponding owner.[6] This makes it fitting for studies of entrepreneurship. I use firm closures to identify the random assignment of an individual to unemployment (via lay-offs). Furthermore, due to the use of individual fixed effects, I am using within individual variation. I find that unemployment doubles the probability of an individual to start a firm, and conditional on starting, an individual hires 26% fewer workers and is 30% more likely to exit firm ownership.

Next, I consider an extension of the model that adds congestion externalities in firm hiring to the baseline framework. This allows the job finding rate to become an equilibrium object. Using this extension, I quantify the impact on the aggregate economy of a policy that redistributes a share of total Unemployment Insurance (UI) income to those that are unemployed and starting a firm. In my numerical policy counterfactual, 5% of total UI income is redistributed to new entrepreneurs having entered from unemployment. This corresponds to an entrepreneur receiving 30% of her previous UI benefits during the first year of business. This is similar in magnitude to the subsidy program in British Columbia, Canada in which entrepreneurs entering from unemployment receive their full UI benefits for the first 38 weeks of a business operation.[7]

[6] The three most used employer-employee linked datasets, the Longitudinal Employer-Household Dynamics (LEHD) for the US, the Déclaration annuelle de données socials (DADS) for France and the Linked Employer-Employee-Data of the IAB (LIAB) for Germany, all lack information on individual owners of firms. With the exception of registry data from Sweden and Denmark, this is the first dataset to allow the tracking of all linkages between a firm and its employees and owners across time.

[7] For a period in which the average provincial unemployment rate is up to 8%, the total duration

The main metric for measuring the success of the policy is taken as its effect on job creation. The reason being that this is the most common argument for the use of such policies. The result is a 2.14% drop in average firm productivity and only a 1% drop in the unemployment rate.[8] The policy induces the creation of low productivity firms by the unemployed. This increases the share of firms created by the unemployed and decreases the share of firms created by the employed. With a larger mass of firms, the equilibrium cost of labour increases.[9] This induces firms to hire fewer workers. With higher wages, the value of being a worker increases and the value of being a business owner decreases for a given productivity level. As a consequence, the employed become more selective on which business projects to implement which further increases the share of firms created by the unemployed. Since, on average, the unemployed create lower productivity firms, average firm productivity drops. In the quantitative exercise, the employment drop among high productivity firms offsets job gains from firms created with the subsidy. The result is a shift in resources from high productivity firms created by the employed to low productivity firms created by the unemployed.

The model abstracts from learning. One possibility is to allow for learning during the entrepreneurial spell just like models on learning on the job (Jovanovic [1979]). In the presence of learning there might be gains associated to the government subsidizing entrepreneurship. However, to the extent that this information friction is not different between the unemployed and the wage worker, there should be no additional benefit of the government targetting the unemployed with the subsidy.

In the theoretical framework, the unemployed and wage workers are ex-ante identical. In that sense, I investigate the difference between firms created by unemployed and employed individuals that have the same level of innate ability. Al-

of unemployment insurance (UI) benefits is a maximum of 40 weeks. This implies that the program in British Columbia allows individuals to receive virtually the entirety of the UI benefits they were eligible for in that year.

[8] I focus on productivity and job creation, instead of welfare, as these are often the key variables targeted by policy makers.

[9] This increase in the "cost of labour" comes via a tighter market, that makes it harder to find workers, and a rise in wages. The model in the next section abstracts from congestion externalities but they are incorporated in the model used to evaluate policy, with the "cost of labour" for an entrepreneur being affected by the equilibrium wage as well as the tightness in the market.

though not the focus of this chapter, negative selection into unemployment should increase the differences in outcomes between unemployed and employed individuals. As a result, if negative selection were added to the model, the negative impacts of the policy would be amplified. It follows that the policy outcomes here can be thought of as lower bounds.[10]

Finally, an additional implication of the theory is that higher wages decrease the entry rate into entrepreneurship of the employed by more than that of the unemployed. Wages represent the opportunity cost of entrepreneurship for the employed but not for the unemployed. As a result, the employed are more responsive to wage variation than the unemployed in their decision to open a firm. With an extension of the theory to a multi-sector environment, I formally derive this additional implication and the Bartik style Instrumental Variable (IV) (Bartik [1993]) used to test it. Using region-wage variation and my instrumental variable strategy for wages, I show that a 1% drop in wages increases by 3.2 percentage points the entry rate into entrepreneurship for wage workers and has no impact for unemployed individuals.

While there are papers looking at the empirical relationship between unemployment and entrepreneurship (see Donovan [2014], Block and Wagner [2010] and Evans and Leighton [1989]), this is the first research to evaluate the impact of exogenous variation in unemployment. Using firm closures, I isolate the impact of unemployment on individual choice from the negative selection associated with unemployment.

Previous papers have investigated the impact of policies that subsidize entrepreneurship among the unemployed (see Caliendo and Künn [2011], Baumgartner and Caliendo [2007] and Hombert et al. [2014]), but the interplay between the decision of the wage worker and the unemployed to open a firm has not been studied before. Here, I show that these margins are important for the crowding out effects of the policy via a redistribution of resources from firms created by wage

[10] In the model, I abstract from credit constraints. Since workers are more likely to start higher productivity firms, adding capital and borrowing constraints to the model would imply that, conditional on wealth, workers are more likely to be liquidity constrained relative to the unemployed. Therefore, it is not obvious why the unemployed would be differentially more liquidity constrained and more misallocated relative to wage workers when it comes to entrepreneurship. This argument is consistent with Karaivanov and Yindok [2015] who find that, although ”involuntary” entrepreneurs have lower average wealth, they are less likely to be credit constrained. I leave such an extension for future work.

workers towards firms created by the unemployed.

This chapter also relates to the development literature looking at subsistence entrepreneurship in developing economies. The measure of involuntary entrepreneurship is often ad-hoc, such as self-employed with no employees (Earle and Sakova [2000]) and de Mel et al. [2008]) or education (Poschke [2013]). Here, instead of concentrating on the notion of involuntary entrepreneurship, I focus on the role of involuntary unemployment for entrepreneurial outcomes. Karaivanov and Yindok [2015] also evaluate the importance of involuntary unemployment but concentrate on its interplay with credit frictions in partial equilibrium. Here, instead, I consider a general equilibrium framework without credit frictions.

This chapter also links to papers showing that firms started in recessions are smaller (Sedláček and Sterk [2017] and Moreira [2015]) by providing microeconomic evidence that laid-off individuals create smaller firms.

The structure of the chapter is the following. Section 2 develops the baseline model. Section 3 describes the data and the empirical differences between firms created by the unemployed and wage workers. Section 4 develops the multi-sector model extension, the new additional testable implication and presents the results in the data. Section 5 develops the model extension with congestion externalities, explains the calibration and reports the policy counterfactual result. Section 6 concludes.

1.2 Model

In this section, I propose a theoretical framework to shed light on the interaction between individual decisions to open a business and the differences in outcomes between firms created by ex-ante homogeneous individuals and the implications for the labour market. In particular, the model generates predictions concerning differences in outcomes between firms created by the unemployed and the employed (hereafter, wage workers). In equilibrium, due to a higher value of being employed, W, relative to being unemployed, U, workers are more selective about which business projects to implement. As a result, despite ex-ante homogeneity among individuals, ex-post, firms created by the unemployed are different from those created by wage workers. In the next section I test these implications in the data. [11]

The population in the economy is of measure 1. At each instant an individual is in one of three states : business ownership, unemployed or employed.

The economy can be thought of as being composed of two islands : on one island a Walrasian market exists, with a unique wage that equates the supply and demand of workers. Demand is made up by all the jobs created by the individual business owners on that island. Supply is made up of all individuals on the Walrasian island who do not operate a firm. A second island is composed of the unemployed, who can transition to the Walrasian island by becoming a worker at a fixed exogenous rate, or alternatively, by deciding to operate a business opportunity. [12]

Workers can either be forced to move to the unemployment island by an exogenous shock or decide to operate a business opportunity and become a business owner. Business owners decide at each instant whether or not they should continue to operate their firm or transition to the unemployment island. Business opportunities arrive at a constant rate ψ, which is the same for both workers and the unemployed.

1.2.1 Static Profit Optimization

Let Z be the productivity of the firm, then, define $z \equiv log(Z)$. Conditional on firm survival, at each instant business owners maximize their profits. Production is given by $y = e^z n^\alpha$ where n is the number of employees. The static profit maximization problem for a firm is

$$\pi^*(z) \equiv \max_n e^z n^\alpha - wn. \tag{1.1}$$

The firm problem above implies

$$\pi^*(z) = (1-\alpha)(\frac{\alpha}{w})^{\frac{\alpha}{1-\alpha}} e^{\frac{z}{1-\alpha}}. \tag{1.2}$$

[11] An assumption is that there is no market for business opportunities. Wage workers are unable to trade with unemployed individuals opportunities they do not desire.

[12] This version of the model ignores the general equilibrium effects of the entrepreneurship margin on the rate at which the unemployed can become employed. In the section considering counterfactual policy scenarios, I develop a simple extension of the model that endogenizes this transition rate.

1.2.2 Dynamic Problem of the Business Owner

Although the profit maximization problem at any point in time is static, the entrepreneur faces a dynamic problem, which is whether or not they should continue to operate. If the firm is shut down, the individual has to pay a cost of χ and becomes unemployed with value U.

Once firm production starts, Z follows a geometric Brownian Motion with drift $\mu < 0$ and variance parameter σ

$$dZ(t) = (\mu + \frac{\sigma^2}{2})Z(t)dt + \sigma Z(t)d\Omega(t) \tag{1.3}$$

where $\Omega(t)$ is a standard Brownian Motion. Then,

$$dz(t) = \mu dt + \sigma d\Omega(t). \tag{1.4}$$

It follows that entrepreneurs face the following optimal stopping problem :

$$rJ(z) = \pi^*(z) + \mu J'(z) + \frac{\sigma^2}{2}J''(z) \quad \text{if} \quad z \geq \hat{z} \tag{1.5}$$

$$J(z) = U - \chi \quad \text{if} \quad z \leq \hat{z} \tag{1.6}$$

$$J'(\hat{z}) = 0. \tag{1.7}$$

Where μ is assumed to be negative, otherwise there would be an accumulation of firms that never exit the market. $\hat{z}$ is the productivity threshold chosen by the entrepreneur below which it is optimal to shut down the firm and exit entrepreneurship.

The cost of shutting down, χ, makes the algebra tractable by guaranteeing that the expressions for the distributions of both types are of the same functional form, with the only difference coming from the difference in selection of projects upon entry, $\underline{z}_u$ versus $\underline{z}_w$, and the unemployment to employment transition rate, f, versus the employment to unemployment transition rate, s.

1.2.3 Problems of the Unemployed and the Wage worker

Once unemployed, an individual receives a flow payment of bw, where $b < 1$ and w is the equilibrium wage. At rate f, the unemployed transitions to the Walrasian island as a wage worker. At exogenous rate ψ a business opportunity is drawn. Business opportunities are drawn from a distribution F. Let F be exponential of shape β.[13] For integrals to be well defined, assume $\beta > \frac{1}{1-\alpha}$ and $\frac{-2\mu}{\sigma^2} > \frac{1}{1-\alpha}$. In equilibrium we must have $W > U$, otherwise the individual would choose to remain on the unemployment island and markets would not clear on the Walrasian island. This is a direct consequence of the assumption that individuals at any moment can choose not to work.

If the productivity of the potential firm is sufficiently high, the individual makes the choice to become a business owner and receives $J(z)$. It follows the value function of the unemployed individual can be written as

$$rU = bw + f(W-U) + \psi \int_{\underline{z}_u} (J(z) - U) dF(z) \tag{1.8}$$

where $\underline{z}_u$ is the threshold productivity above which the unemployed individual decides to implement the business project.[14]

Once employed, an individual receives flow payment w, the equilibrium wage. At exogenous rate s, the person transitions onto the unemployment island and receives value U. At rate ψ, the same as for the unemployed, a business opportunity is drawn. If the opportunity is sufficiently productive, in other words, if z is high enough, the wage worker enters business ownership with value $J(z)$. The value function of the employed can be written as

$$rW = w + s(U-W) + \psi \int_{\underline{z}_w} (J(z) - W) dF(z) \tag{1.9}$$

where $\underline{z}_w$ is the threshold productivity above which the working individual decides

[13] Note that F is defined over $z \equiv log(Z)$, as such, to assume F is exponential is equivalent to defining a distribution G defined over Z, from which individuals draw from, where G is Pareto of scale 1 and shape β.

[14] The event in which the unemployed individual obtains a job and a business opportunity simultaneously is measure zero.

to implement the business project.[15] $\underline{z}_u$ and $\underline{z}_w$ are defined by

$$J(\underline{z}_u) = U \tag{1.10}$$

$$J(\underline{z}_w) = W. \tag{1.11}$$

The rate and distribution from which unemployed and employed workers receive business opportunities are the same. If they were allowed to be different, given the closer contact of wage workers with the labour market and currently operating firms, the arrival rate would be higher and the distribution shifted to the right for the employed. This would only reinforce the predictions of the model that firms created by employed individuals should last longer and hire more.[16]

1.2.4 Market Clearing

Let η be the measure of business owners in the population, u the measure of unemployed and $n(z,w)$ the optimal number of employees for a business owner with a firm of productivity z facing wage w. Then market clearing is determined by

$$(1-u-\eta) = \int n(z,w)\Lambda(z)dz. \tag{1.12}$$

The equilibrium wage is linked to the average marginal product of labor as this is in turn linked to the distribution of projects implemented, $\Lambda(z)$.

In frameworks such as these, where all jobs are being created by firms operated by individuals of that economy, demand and supply are tightly linked beyond the price mechanism. Supply and demand are jointly determined by individuals' choices over which side of the market to operate in. This is due to the fact that both job creators and workers come from the same pool. It follows that, beyond the general equilibrium price effect, anything that affects the supply of labor, directly affects labor demand and vice versa, since they are co-determined by the individual's decision to open a business or not.

[15] The event in which the worker is placed on the unemployment island and receives an opportunity simultaneously is measure zero.

[16] This choice of a similar distribution and rate of arrival of business projects is also motivated by the fact that when taking the model to the data, we explicitly control for the characteristics of the previous employer of the individual which controls partially for any learning mechanisms.

1.2.5 Equilibrium Measure of Unemployed Individuals

To close the model, we need the law of motion of the measure of unemployed in the economy, which is given by

$$\dot{u} = s(1-u-\eta) - fu - \psi(1-F(\underline{z}_u))u + E \tag{1.13}$$

and the law of motion of the measure of firms/business owners,

$$\dot{\eta} = \psi(1-F(\underline{z}_u))u + \psi(1-F(\underline{z}_w))(1-u-\eta) - E \tag{1.14}$$

where u is the measure of unemployed individuals, η the measure of business owners and E the measure of individuals exiting business ownership. Setting equations 1.13 and 1.14 to zero and replacing the expression for E in equation 1.13 gives

$$u = \frac{(s+\psi(1-F(\underline{z}_w)))(1-\eta)}{f+s+\psi(1-F(\underline{z}_w))}. \tag{1.15}$$

1.2.6 Characterizing the Equilibrium

Proposition 1. *The solution to the firm's optimal stopping problem implies*

$$J(z) = \frac{B}{r - \frac{\mu}{1-\alpha} - \frac{\sigma^2}{2}(\frac{1}{1-\alpha})^2}(e^{\frac{z}{1-\alpha}} + \frac{1}{a(1-\alpha)}e^{-a(z-\hat{z})+\frac{\hat{z}}{1-\alpha}}) \tag{1.16}$$

where

$$B \equiv (1-\alpha)(\frac{\alpha}{w})^{\frac{\alpha}{1-\alpha}} \tag{1.17}$$

$$a = \frac{\mu+\sqrt{\mu^2+2r\sigma^2}}{\sigma^2} > 0. \tag{1.18}$$

Unsurprisingly, the value function of the business owner $J(z)$ is increasing in productivity for the range of values for which the business operates $z \in [\hat{z}, \infty[$. [17] Let $\Lambda^w(z)$ denote the measure of business owners operating a business project of

[17] To see this note that

$$\frac{\partial^2 J(z)}{\partial z^2} = C(\frac{1}{1-\alpha})(\frac{e^{\frac{z}{1-\alpha}}}{1-\alpha} + ae^{-a(z-\hat{z})+\frac{\hat{z}}{1-\alpha}}) > 0 \tag{1.19}$$

productivity z that were employed when they received the current business opportunity. Let $\Lambda^u(z)$ be the measure of business owners operating a business project of productivity z that were unemployed at the moment they received the current business opportunity.[18]

Proposition 2. *For all $i \in \{u, w\}$, the measure of business owners running a firm of productivity z is given by*

- *For $z \in [\hat{z}, \underline{z}_i]$*
$$\Lambda^i(z) = \Lambda^i_1(z) = \frac{M^i}{-\mu}(1 - e^{\frac{2\mu}{\sigma^2}(z-\hat{z})}) \tag{1.23}$$

- *For $z \in]\underline{z}_i, \infty[$*

$$\Lambda^i(z) = \Lambda^i_2(z) = \frac{\beta M^i \frac{\sigma^2}{-2\mu} e^{\frac{2\mu}{\sigma^2}(z-\underline{z}_i)}}{(\mu + \frac{\sigma^2\beta}{2})} - \frac{M^i}{-\mu} e^{\frac{2\mu}{\sigma^2}(z-\hat{z})} - \frac{M^i e^{-\beta z}}{e^{-\beta \underline{z}_i}(\mu + \frac{\sigma^2}{2}\beta)} \tag{1.24}$$

where
$$M^i = \psi u e^{-\beta \underline{z}_u} \quad \text{if} \quad i = u \tag{1.25}$$

$$M^i = \psi(1 - u - \eta) e^{-\beta \underline{z}_w} \quad \text{if} \quad i = w. \tag{1.26}$$

Corollary 2.1. *The measure of business owners, η, and the fraction that were unemployed when they entered entrepreneurship, $\frac{\eta^u}{\eta}$, are given by*

$$\eta = \frac{\psi(1-\eta)}{s + f + \psi e^{-\beta \underline{z}_w}} [A_u(s + \psi e^{-\beta \underline{z}_w}) e^{-\beta \underline{z}_u} + A_w f e^{-\beta \underline{z}_w}] \tag{1.27}$$

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and for $z = \hat{z}$
$$\frac{\partial J(z)}{\partial z} = 0. \tag{1.20}$$
This implies for $z \geq \hat{z}$,
$$\frac{\partial J(z)}{\partial z} \geq 0. \tag{1.21}$$

[18] In other words, this is equivalent to saying that $\Lambda^w(z)$ and $\Lambda^u(z)$ are defined such that
$$\int_{\hat{z}} \Lambda^u(z) dz + \int_{\hat{z}} \Lambda^w(z) dz + u + e = 1 \tag{1.22}$$
where e is the measure of workers.

$$\frac{\eta^u}{\eta} = \frac{A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u}}{A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u} + A_w f e^{-\beta \underline{z}_w}} \tag{1.28}$$

where

$$A_i = [\frac{1+\beta(\underline{z}_i - \hat{z})}{-\mu\beta}] \quad for \quad i \in \{u, w\}. \tag{1.29}$$

We are now ready to define a Stationary competitive equilibrium.

Definition 1. *A Stationary competitive equilibrium is defined by* $\underline{z}_u, \underline{z}_w, w, \eta, \eta^u, \Lambda^u(z), \Lambda^w(z), u$ *such that*

- $W > U$
- $J(\underline{z}_w) = W$
- $J(\underline{z}_u) = U$
- $J(\hat{z}) = J(\underline{z}_u) - \chi$
- *The expression for* $J(z)$ *is given by Proposition 1*
- *The expression for* $\Lambda^u(z)$ *and* $\Lambda^w(z)$ *are given by Proposition 2*
- u *is given by*
$$u = \frac{(s+\psi(1-F(\underline{z}_w)))(1-\eta)}{f+s+\psi(1-F(\underline{z}_w))} \tag{1.30}$$
- η *and* η^u *are defined by Corollary 2.1*
- $$w = \alpha[\frac{1}{(1-u-\eta)}(\int_{\hat{z}} e^{\frac{z}{1-\alpha}}\Lambda^u(z)dz + \int_{\hat{z}} e^{\frac{z}{1-\alpha}}\Lambda^w(z)dz)]^{1-\alpha} \tag{1.31}$$

The first condition states that the value of being an employed worker is higher than the value of unemployment. Otherwise, no individual would ever choose to transition to wage work and markets would not clear. The second and third conditions guarantee that individuals' decisions to open a business are optimal and the last condition comes from market clearing. The next proposition states that the equilibrium can be characterized by a system of 4 equations and 4 unknowns.

Proposition 3. *A Stationary equilibrium can be characterized by* 4 *variables* $(w, \hat{z}, \underline{z}_u, \underline{z}_w)$ *and* 4 *equations :*

- $$rJ(\underline{z}_u) = bw + f(J(\underline{z}_w) - J(\underline{z}_u)) + \psi \int_{\underline{z}_u} (J(z) - J(\underline{z}_u))dF(z) \tag{1.32}$$

- $$rJ(\underline{z}_w) = w + s(J(\underline{z}_u) - J(\underline{z}_w)) + \psi \int_{\underline{z}_w} (J(z) - J(\underline{z}_w))dF(z) \tag{1.33}$$

- $$J(\hat{z}) = J(\underline{z}_u) - \chi \tag{1.34}$$

- $$w = \alpha[\frac{1}{(1-u-\eta)}(\int_{\hat{z}} e^{\frac{z}{1-\alpha}} \Lambda^u(z)dz + \int_{\hat{z}} e^{\frac{z}{1-\alpha}} \Lambda^w(z)dz)]^{1-\alpha} \tag{1.35}$$

where $J(z)$ *is given by Proposition 1 and* $\Lambda^u(z), \Lambda^w(z)$ *are given by Proposition 2*

Now I turn to examining the key proposition arising from the model, which generates the patterns documented in the data. It states that in equilibrium, wage workers are more selective about which business opportunities to implement. The necessary and sufficient condition for it to hold is simply that the income received while unemployed is lower than that received while employed. Were it not the case, the equilibrium would not exist as markets would not clear.

Proposition 4. *In equilibrium,* $\underline{z}_w > \underline{z}_u \Leftrightarrow b < 1$

The following corollaries result from the difference in selection on business projects between unemployed and wage workers.

Corollary 4.1. *In equilibrium, businesses created by employed workers have a lower exit rate than those created by unemployed individuals.*

Corollary 4.1 results from business owners exiting at the same threshold while having different levels of selection in the entry into business creation.

Corollary 4.2. *In equilibrium, businesses created by employed workers have a higher firm size and larger profits relative to those created by unemployed individuals.*

Corollary 4.2 is a direct consequence of the fact that both profits and firm size are monotonically increasing in productivity.

Corollary 4.3. *In equilibrium, the entry rate into business ownership of unemployed individuals is higher than that of employed workers.*

Finally, as it is often the case with selection mechanisms, an increased average productivity is associated with a lower entry rate.

The theory predicts that even when we compare ex-ante identical individuals, we should observe differences in outcomes for entrepreneurs that were unemployed when they opened their firm versus those that were working. In the next section, I test the following predictions

- Firms created when the individual was unemployed, are smaller.
- If the firm was started during a period of unemployment, the entrepreneur is more likely to exit entrepreneurship.
- Being unemployed makes an individual more likely to enter firm ownership.

1.3 Empirical Section

1.3.1 Data and Measurement

The data used for the empirical analysis is the Canadian Employer-Employee Dynamics Database (CEEDD). It contains the entire universe of Canadian tax filers, and privately owned incorporated firms. The dataset links employees to firms and firms to their corresponding owners across space and time. This is achieved by linking individual tax information (T1 files, individual tax returns), with linked employer-employee information (T4 files)[19] and firm ownership and structure information (T2 files).[20] The data is annual and is available from 2001 to 2010. This constitutes an advantage relative to employer-employee firm population data from the US, since this firm-level data does not allow the researcher to identify the owners of the firm.

The data is annual with information on all employers and any businesses an individual owned in a given year. Using this database, I can examine the characteristics of both the business owner and the firm. I concentrate on firms that contribute to job creation by hiring employees. This is done by focusing on employers instead of self-employed individuals.

Business owners are identified as individuals present in the schedule 50 files from the T2 that have employees. Wage workers are identified as those who are not entrepreneurs and report a positive employment income on their T4. I use the information in the T1 files to control for characteristics such as gender, age and marital status. For more information on the data see the Data Appendix.

The linkage between each firm and its corresponding owner is only available for privately owned incorporated firms. Incorporated firms have two key characteristics which correspond closely to how economists typically think about firms : limited liability and separate legal identity. Furthermore, there is a growing literature showing that incorporated firms tend to be larger and that they are more likely to contribute to aggregate employment.[21] There is also evidence that there is little transition from unincorporated to incorporated status.[22] These facts, highlight

[19]According to Canadian law, each employer must file a T4 file for each of her employees. The equivalent in the US is the W-2, Wage and Tax Statement. In this form, the employer identifies herself, identifies the employee and reports the labour earnings of the employee.

[20]T2 forms are the Canadian Corporate Income Tax forms. In the T2 files there is the schedule 50 in which each corporation must list all owners with at least 10% of ownership. This allows me to link each firm to individual entrepreneurs. The equivalent in the US to the schedule 50 of the T2 form is the schedule G of 1120 form (Corporate Income Tax Form in the US)

[21] Glover and Short [2010] document that incorporated entrepreneurs operate larger businesses, accumulate more wealth, and are on average more productive than unincorporated entrepreneurs. Chandler [1977] and Harris [2000] argue that over time the incorporated business structure was created with the explicit goal of fostering investment in large, long gestation, innovative and risky activities.

[22] Levine and Rubinstein [2017] show that there is little transition from unincorporated to incorporated status. They also show that the observed earnings increase for incorporated business owners does not take place before opening the business, indicating that incorporation is not just a result of

how incorporated firms with employees are the most appropriate measure of firms to consider if we are interested in the interplay between entrepreneurship and the aggregate economy.[23]

For the remainder of the paper, the empirical definition of an entrepreneur is an owner and founder of a privately owned incorporated firm with employees.

1.3.2 Identification

Exogeneity in the State of Unemployment

To identify differences in firms exclusively due to differences in outside options, we need to focus on episodes of random assignment of an individual to unemployment. The question then is how to identify these involuntary transitions to unemployment in the data. One possibility is to identify those unemployed based on whether they received any unemployment insurance during the year. However, such an approach faces endogeneity issues since those who do not expect to be unemployed for long will not take up the benefits. An alternative would be to consider individuals that did not work for the entire year, but that would restrict the analysis to individuals with low labour market attachment.

Instead, I follow an approach inspired in the literature on the effect on employment and earnings of mass layoffs and plant closures.[24] In particular, I identify laid-off individuals as those that lose their job due to a firm closure. Namely, I consider individuals who worked for a firm last year that does not exist this year. I compare this group to the benchmark group of individuals that worked for a firm

higher earnings, rather, people choose the firm structure based on their planned business activity. The authors demonstrate how the often cited puzzle, that entrepreneurs earn less than they would have as salaried workers, is no longer true once we consider incorporated business owners. Together with other patterns of income dynamics and observable characteristics of owners, the authors highlight how incorporated businesses are closer to firms in traditional macro models.

[23]Another reason to focus on incorporated firms with employees is Canadian corporte law. In Canada there are significant tax advantages for incorporating as a higher earner. So to exclude from my analysis high-earning workers that incorporate exclusively due to tax purposes, I focus on incorporated firms with employees.

[24]In the seminal papers of Jacobson et al. [1993] and Couch and Placzek [2010], the authors document significant drops in earnings for displaced workers. Farber [2017] and Song and von Wachter [2014] complement these results by further documenting the drop in employment probability after displacement.

last year that is still operating this year. These displaced individuals are almost certainly involuntarily in that state. Focusing on the involuntarily out of work is an added benefit. Even if I could see all the unemployed in the data, I would be worried about using them since some are people who quit their job in order to begin the steps of opening their firm. For the remainder of the paper I refer to the individuals, for which their employer shut down, as displaced/laid off workers and those that did not have their employer shut down, as employed workers. [25]

Note that among the individuals that were employed by a firm that still exists this year, we have both individuals that remained employed since last year as well as individuals that had spells of unemployment of less than a year. In other words, the employed workers group is contaminated by individuals that were fired and had unemployment spells of less than a year. These individuals are likely to be negatively selected in overall ability relative to displaced individuals. To resolve this issue, I use within-individual variation when testing the model predictions. This is done by estimating fixed effects regressions.[26]

This implies that I will be comparing between moments when the individual was displaced to moments when the individual remained employed. This is a valid source of variation if displacement shocks due to firm closure are random over the life cycle.[27] We might be worried that individuals are laid off when they were already in a downward trend in total income or earnings.[28] To verify this is not a concern, I consider individual fixed effect specifications with the pre-displacement shock income/earnings on the left hand side and the displacement shock on the right hand side :

$$ln(w_{i,t-j}) = \mathbb{1}\{\text{Prev U}\}_{i,t} + u_i + v_{i,t} \tag{1.36}$$

$$ln(y_{i,t-j}) = \mathbb{1}\{\text{Prev U}\}_{i,t} + \kappa_i + \varepsilon_{i,t} \tag{1.37}$$

where $ln(w_{i,t-j})$ is log of total annual earnings at year $t-j$, $ln(y_{i,t-j})$ is total tax-

[25] This choice of identifying displaced workers is also a result of having only annual frequency data. Since I cannot observe spells smaller than 1 year of unemployment, I adopt the strategy of using firm closures to proxy for individuals that are unemployed for exogenous reasons.

[26] Readers interested in the results without fixed effects can refer to section A.3.1 of the Appendix.

[27] This is equivalent to the parallel trend restriction for validity of difference in difference estimators.

[28] This issue would arise if worker-specific productivity is time varying and firms shut down because many of their workers got hit by a low worker-specific productivity shock.

able income at year $t-j$ and $\mathbb{1}\{\text{Prev U}\}_{i,t}$ is a dummy taking value 1 if the individual was laid off at t and 0 if remained working (with $j \in \{2,3,4\}$).[29] The intuition is that if displacement shocks happen randomly in an individual's life cycle, it should be uncorrelated to pre-shock observables.

Table 1.1: Tests for Randomness of displacement shock

	$ln(w_{i,t-2})$	$ln(w_{i,t-3})$	$ln(w_{i,t-4})$	$ln(y_{i,t-2})$	$ln(y_{i,t-3})$	$ln(y_{i,t-4})$
$\mathbb{1}\{\text{Prev U}\}_{i,t}$	-0.0174	-0.01047	-0.0014	-0.0199	-0.0198	-0.0024
	(0.0021)	(0.0031)	(0.0035)	(0.0018)	(0.0026)	(0.0029)
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	13187561	11265230	9500351	13584834	11739333	9969862

Notes: Fixed effects regressions to check randommess of displacement shock. Column (1) regresses annual labor income from 2 periods ago ($w_{i,t-2}$) on whether recevied displacement shock in current period ($\mathbb{1}\{\text{Prev U}\}_{i,t}$). Column (2) regresses annual labor income from 3 periods ago ($w_{i,t-3}$) on whether recevied displacement shock in current period (t). Column (3) regresses annual labor income from 4 periods ago ($w_{i,t-4}$) on whether recevied displacement shock in current period (t). Only includes men 25 to 54 years old. Standard errors are clustered at the individual level. Columns (4), (5) and (6) reports results for similar regressions but with dependant variables $y_{i,t-2}$, $y_{i,t-3}$ and $y_{i,t-4}$, respectively.

In the first Column we see that displacement shocks due to firm closure are associated to a 1.74% smaller annual labor earnings two periods before. In Column 2 we see that these shocks are associated to a 1.05% smaller annual labor earnings three periods before. These differences are small indicating that these shocks are not associated to particular moments in the life cycle with unusually high or low earnings. It is worth noting that significance is likely coming from the large sample size which makes even such small coefficients significant. Finally, once we look at Column 3 we see that these displacement shocks are not associated to any difference in annual labor earnings 4 periods ago. Columns 4,5 and 6 show a similar pattern for past annual income.[30]

[29] I do not consider $j = 1$ because in the data the shock happens somewhere in the interval $[t-1,t]$. In particular, if we see a firm in $t-1$ and that firm no longer present at t, it is unclear if the firm died at $t-1$ or at t. For that reason we might expect to see lower $t-1$ income and $t-1$ earnings for the displaced, since for certain cases individuals will have been displaced at $t-1$.

[30] I have also verified that a similar pattern holds for lagged values of other observables such as

1.3.3 Descriptive Statistics

Table 1.2 gives the summary statistics for firms operated by the entrepreneurs in the data. Each observation is an entrepreneur operating an incorporated firm with employees in a given period of time. Looking at the first row, we see that the firms used in our analysis are on average young (≈ 2 years old). This is due to the fact that firms used in our analysis must be observable in their first year of operation. Then, looking at the second column, it is clear that the firms used in the analysis are on average small (≈ 6 employees). However, the average hides variation in firm size as seen by the standard deviation of 19. As is common in firm datasets, the firm distribution is such that the majority of firms are small but there are a few extremely large firms that account for most of employment.

Table 1.2: Summary Statistics Firms

	Mean	Std Dev	Number obs
Firm Age (in years)	2.0753	1.9777	450,502
Number of Employees	5.8736	19.4214	450,502

Notes: Summary statistics for privately owned incorporated firms with employees for which first year of operation is observable in the sample. Each observation is an entrepreneur with a firm in a given year. Includes only male entrepreneurs between 25 to 54 years of age.

In Table 1.3, I report summary statistics for individuals that last year worked for an employer that no longer operates in the current year (laid-off workers) and those who remain employed (not laid-off). Each observation is an individual in a given year. The first two rows report statistics for age (38.48 versus 37.23) and marriage rates (0.58 versus 0.51). In the third row, I report the average size of the last year employer for these individuals (laid-off, 233 versus not laid-off, 301). The averages for both groups indicate that most individuals in the dataset are employed by large firms despite the fact that the majority of firms are small (See Table 1.2).

age and marital status. When I do so, I find a coefficient on $\mathbb{1}\{\text{Prev U}\}_{i,t}$ of -0.008 for age at $t-2$, indicating a difference of less than a month between moments where individuals receive the displacement shocks and moments they don't. For marital status at $t-2$, I find a coefficient of -0.0075, indicating the difference in the likelihood of being married is less than 1%.

Table 1.3: Summary Statistics Individuals

	Not laid off			Laid off		
	Mean	Std Dev	# obs	Mean	Std Dev	# obs
Age	38.48	8.55	$15,651,346$	37.23	8.51	$284,807$
Marital Status	0.58	0.49	$15,651,346$	0.51	0.5	$284,807$
Size employer	233.94	875.86	$15,651,346$	301.19	868.93	$284,807$

Notes: Summary statistics for individuals that last year worked for a privately owned incorporated firm that this year shut down (laid off) and this year did not (not laid off). Includes only men between 25 to 54 years of age. Age is the age of the individual, marital status is a dummy taking value 1 if the individual is married and 0 otherwise. The size of employer is the number of employees of the employer of the individual.

1.3.4 Main Empirical Results

In this section I verify that the differences in performance of businesses created by laid-off versus employed workers are consistent with the predictions of the theory. The analysis focuses on men between 25 and 54 years of age. Consistent with the model, my two measures of performance are firm number of employees (hereafter, firm size) and the exit rate for entrepreneurs.[31]

The first outcome of interest is the number of employees hired by firms created by employed workers compared to those that were displaced. To account for observable characteristics, I control for the business owner's age, marital status, industry, province of residence, the year the business started and a quadratic in the age of the business. To control for the possibility of learning from the previous employer, I control for the number of employees and the industry of the previous employer.[32]

[31]This choice of sampling restrictions is made to narrow my focus on individuals with relatively high labour force attachment. All results in this section are robust to using both men and women aged 18 to 65 years old.

[32]If firms created by the employed are better than those created by the unemployed, as employees

Denote by $y_{i,t}$ the number of employees of a firm owned by individual i in period t. This variable can be expressed as a function of firm characteristics, observable characteristics of the owner, including whether the owner was laid off when the firm was started, and unobservable factors. Consider the following specification:

$$log(y_{i,t}) = \beta_{1,1} + M_{i,t}\gamma_{1,1} + X_{i,t}\gamma_{1,2} + L_{i,t}\gamma_{1,3} + \beta_{1,2}\mathbb{1}\{\text{Prev U}\}_{i,s} + T_t\gamma_{1,4} + u_i + \varepsilon_{i,t} \tag{1.38}$$

where $M_{i,t}$ are characteristics of the firm (firm age, start year and industry), $X_{i,t}$ is a matrix containing all observable characteristics of the owner (age group dummies, gender, marital status and province of residence), $L_{i,t}$ are characteristics of the individual's last employer (industry and number of employees), T_t are year dummies, u_i is the set of unobservable individual characteristics affecting firm performance such as innate ability and $\mathbb{1}\{\text{Prev U}\}_{i,s}$ is a dummy indicating if the individual was laid off when the business was started. This equation is estimated using a linear fixed effects model. $\beta_{1,2}$ gives us the estimated difference in number of employees between firms created by laid-off individuals versus those created by employed workers. The prediction of the model is that $\beta_{1,2} < 0$.

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learn from their previous employer, we should expect a close relationship between firm size and industry of the previous employer and the size and industry of the current firm of the entrepreneur.

Table 1.4: Log number of employees

	Baseline	Control GE shocks
$\mathbb{1}\{\text{Prev U}\}_{i,s}$	-0.257***	-0.256***
	(0.042)	(0.042)
Fixed Effects	Yes	Yes
Interaction of region year dummies	No	Yes
Observations	450,502	450,502

Notes: Fixed effects regressions of log number of employees in the firm on a dummy indicating if the business was started by an individual who was laid off ($\mathbb{1}\{\text{Prev U}\}_{i,s}$). Other controls include age-group dummies, and dummies for marital status, province of residence, start year of business, current year, 2-digit industry, 2-digit industry of prior employer, as well as the log number of employees working for the previous employer. Includes men aged 25 to 54 years old. Column (1) presents the results for the baseline specification. Column (2) presents the results once controlling for local labor market shocks.

Table 1.4 shows that firms created by individuals when they have been displaced ($\mathbb{1}\{\text{Prev U}\}_{i,s} = 1$) tend to be around 25% smaller relative to firms created by the same individuals when they are working. Column 1 shows results for the baseline specification and Column 2 shows results when controlling for each economic region and year pair.

The second measure of differences in firm performance is business survival. Let $z_{i,t}$ denote the choice of an entrepreneur which takes value 1 if the individual chooses to exit firm ownership and 0 otherwise. Using a fixed-effects linear probability model, the choice of an entrepreneur to exit entrepreneurship is a function of owner demographic characteristics, $X_{i,t}$, characteristics of the firm, $M_{i,t}$, characteristics of the previous employer, $L_{i,t}$, current year, T_t, whether the owner was displaced or not prior to entering entrepreneurship, $\mathbb{1}\{\text{Prev U}\}_{i,s}$ and unobserved characteristics ζ_i : [33]

$$z_{i,t} = \beta_{2,1} + M_{i,t}\gamma_{2,1} + X_{i,t}\gamma_{2,2} + L_{i,t}\gamma_{2,3} + \beta_{2,2}\mathbb{1}\{\text{Prev U}\}_{i,s} + T_t\gamma_{2,4} + \zeta_i + \upsilon_{i,t}. \tag{1.39}$$

[33] The definition of matrices $X_{i,t}$, $M_{i,t}$, $L_{i,t}$ and $T_{i,t}$ are the same as in the previous regression.

$\beta_{2,2}$ in the equation above represents the difference in the probability of exiting entrepreneurship for business owners that were displaced by firm closure when they started their business. The prediction of the model is that $\beta_{2,2} > 0$.

Table 1.5: Exit Probability

	Baseline	Control GE shocks
$\mathbb{1}\{\text{Prev U}\}_{i,s}$	0.017**	0.017**
	(0.007)	(0.0065)
Ratio of probabilities	1.3	1.3
Fixed Effects	Yes	Yes
Interaction of region year dummies	No	Yes
Baseline Exit Probability	0.055	0.055
Observations	341,214	341,214

Notes: Fixed effects regressions of the indicator for entrepreneurship exit on a dummy indicating if current business was started by the individual when laid off ($\mathbb{1}\{\text{Prev U}\}_{i,s}$). Other controls include age-group dummies, and dummies for marital status, province of residence, start year of business, current year, 2-digit industry, 2-digit industry of prior employer, as well as the log number of employees working for the previous employer. Includes men aged 25 to 54 years old. Column (1) presents the results for the baseline specification. Column (2) presents the results once controlling for local labor market shocks.

Table 1.5 shows that firm ownership spells end sooner when an individual starts a firm after displacement ($\mathbb{1}\{\text{Prev U}\}_{i,s} = 1$) relative to firm ownership spells started when employed ($\mathbb{1}\{\text{Prev U}\}_{i,s} = 0$). [34] In particular, it implies that the exit rate out of entrepreneurship for individuals that were displaced when they started the business is 30% larger relative to the exit rate for the same individuals who

[34] The number of observations is smaller for the regression of the exit of entrepreneurs because in that case I need at least two lags of the current observation to include it in the regressions. Consider the example of a firm that exited after its first year. To include the owner i of the firm in year t, we must see him for the current period t, the period prior, $t-1$, to determine he was an entrepreneur before and the period before that, $t-2$, to see if he started his business after involuntary loss of work or not. For the firm size regression, on the other hand, all that is required is to observe the individual in the current period t and in the previous period, $t-1$, to see if the firm was started following an episode of firm closure.

were working when they started their firm.[35] Column 1 shows the results for the baseline specification and column 2 shows the results when we add controls for each pair of economic region and year to control for aggregate shocks at the local labor market level.

Next, I verify whether there are significant differences in the likelihood of opening a firm when an individual is laid off (via firm closure) relative to when working. Let $d_{i,t}$ denote the choice of an individual who does not own a firm, this variable takes value 1 if the individual chooses to open a firm, and 0 otherwise. Using a fixed-effects linear probability specification, the probability of an individual choosing to open a firm is a function of owner demographic characteristics $X_{i,t}$, characteristics of the previous employer, $L_{i,t}$, the current year, T_t, whether the individual was displaced or not prior to entering entrepreneurship, $\mathbb{1}\{\text{Prev U}\}_{i,t}$, and unobserved characteristics η_i :

$$d_{i,t} = \beta_{3,1} + X_{i,t}\gamma_{3,1} + L_{i,t}\gamma_{3,2} + \beta_{3,2}\mathbb{1}\{\text{Prev U}\}_{i,t} + T_t\gamma_{3,4} + \eta_i + \nu_{i,t}. \tag{1.40}$$

$\beta_{3,2}$ in the equation above represents the difference in the probability of entering entrepreneurship for displaced versus working individuals. The prediction of the model is that $\beta_{3,2} > 0$. Table 1.6 shows that when individuals are displaced ($\mathbb{1}\{\text{Prev U}\}_{i,t} = 1$), they are 93% more likely to start a firm.[36] In particular the results imply that the entry probability into firm ownership doubles when an individual is displaced via firm closure.[37] Column 1 shows the results for the baseline specification, Column 2 and Column 3 show the results are robust to excluding individuals that in the prior year were already incorporated without employees and individuals that in the prior year had some unincorporated self-employment income.

[35] 30% comes from 0.017/0.055.

[36] These results are consistent with the findings of Evans and Leighton [1989], which state that the unemployed are more likely to become self-employed.

[37] The number of observations in the entry regression is not the same as in Table 1.3 of summary statistics for individuals, because I exclude individuals that started a firm by buying a share in an already existing firm.

Table 1.6: Entry Probability

	Baseline	Robust 1	Robust 2
$\mathbb{1}\{\text{Prev U}\}_{i,t}$	0.0054***	0.0054***	0.005***
	(0.0002)	(0.0003)	(0.0002)
Fixed Effects	Yes	Yes	Yes
Exclude if prior year already incorporated	No	Yes	No
Exclude if prior year self-emp income>0	No	No	Yes
Ratio of probabilities	1.93	1.93	1.93
Baseline Entry Probability	0.0058	0.0058	0.0058
Observations	15,928,932	15,873,979	15,658,403

Notes: Fixed effects regressions of the indicator for entry into firm ownership on the dummy indicating if the individual was laid off ($\mathbb{1}\{\text{Prev U}\}_{i,t}$). Other controls include age-group dummies, and dummies for marital status, province of residence, current year, 2-digit industry of prior employer, as well as the log number of employees working for the previous employer. Includes men aged 25 to 54 years old. Column (1) presents the results for the baseline specification. Column (2) presents the results we exclude individuals that in the prior year were already incorporated. Column (3) presents the results once we exclude individuals that in the prior year already some positive self-employment income.

The patterns documented in the data are consistent with the predictions of the model in the previous section.[38]

- When laid off, conditional on opening a firm, an individual hires 25.7% fewer workers relative to when opening a firm while employed.
- When laid off, conditional on opening a firm, an individual is 30% more likely to exit firm ownership, relative to when opening a firm while employed.
- Being laid off doubles the probability of opening a firm for an individual.

[38]Note that the regressions in this section all make use of fixed effects, hence, I am comparing observably identical individuals, who appear in different states, as are considered in the theoretical section.

One concern is that, if firms created after a lay-off tend to be the first firms an individual creates, the results might be capturing learning-by-doing. In particular, individuals might be learning how to be an entrepreneur when they start a firm after a lay-off, subsequently, upon entering from employment they create more productive firms. In Section A.1.2 of the Appendix, I show that the differences in size and exit rate persist once I control for an individual's total years in the sample as a business owner before the current entrepreneur spell.[39] These results are evidence that learning-by-doing cannot explain the differences in firms created by an individual when laid off, relative to when working for somebody else.[40]

1.4 Additional Model Implication

In this section I present an additional implication of my theoretical model. It is formally derived from an extension of the baseline model to a multi-sector economy.[41] Details of this extension are provided in the Appendix. This implication is closely linked to the differential selection between unemployed and wage workers.

Proposition 5. *An increase in the wage decreases the entry rate into entrepreneurship among the wage workers by more than that of the unemployed.*[42]

To understand the different channels through which wages affect the selection into entrepreneurship, let us consider two economies, one with larger wages relative to the other. A higher economy-wide wage w increases the cost of hiring other workers, decreasing the incentives to open a firm for both working and laid off-individuals. This translates into higher selection among both laid-off and working individuals. But for a worker, a higher wage also represents a higher opportunity cost of entrepreneurship.[43] As a result, the worker's response to the higher wage is

[39] The exact controls I use are discussed in the Appendix.

[40] This is not to say that learning-by-doing does not play a role in a firm's outcomes. This only highlights that it cannot explain the differences in firms created by individuals after a lay-off versus while working for somebody else.

[41] This additional testable implication can also be derived using the baseline model without multiple sectors and is available upon request from the author. The main added value of the multiple sector framework is to derive a valid instrument for wages to test the prediction.

[42] See Section A.1.3 of Appendix for proof of Proposition.

[43] This effect of the wage is also present for the unemployed due to the non-zero probability of transitioning to wage work. But this effect for the unemployed is discounted and so, is weaker.

larger than that of a laid-off individual. This differential selection response translates into a differential in entry rate responses to wage changes.

In the Appendix, I show that from the model extension with multiple sectors I can derive the following expression for the entry rate into entrepreneurship in an economy c for wage workers w and the unemployed u.

Corollary 5.1. *The average entry rate for wage workers in an economy c, $ER_{c,w}$ and that of unemployed individuals $ER_{c,u}$ can be expressed as*

$$ER_{c,w} = \beta_{0,w} + \beta_{1,w} log(w_c) + \upsilon_{c,w} \quad \textit{for wage workers} \tag{1.41}$$

$$ER_{c,u} = \beta_{0,u} + \beta_{1,u} log(w_c) + \upsilon_{c,u} \quad \textit{for unemployed individuals} \tag{1.42}$$

Combining both into one specification gives

$$ER_{c,n,t} = \alpha_0 + \beta_1 log(w_{c,t}) + \beta_2 \mathbb{1}\{\text{Prev U}\}_{c,t,n} log(w_{c,t}) + \alpha_2 \mathbb{1}\{\text{Prev U}\}_{c,t,n} + \mu_{c,t} \tag{1.43}$$

where $n = 1$ if the individual is laid off and $n = 0$ if he is working and $\mathbb{1}\{\text{Prev U}\}_{c,t,n}$ is an indicator for $n = 1$ or $n = 0$. I have added the time subscripts since the data is over different years. The prediction of the theory is that $\beta_1 < 0$ and $\beta_2 > 0$.

1.4.1 Identification

For my identification strategy, I use variation across different local labour markets within the national economy. Individuals belong to a local labour market based on their economic regions of residence.[44] The strategy is to then verify if the entry rate into entrepreneurship in a particular region c, in year t responds differently to wages for unemployed versus employed individuals.[45]

In practice, there might be reasons to believe that certain regions have a more pro-business attitude across all years. As a result, the entry rate in these regions should be higher for all years, pushing up labour demand and raising wages. This region

[44]Economic regions in Canada correspond closely to commuting zones in the US : there are 76 in total.

[45]Cells for which the number of displaced or employed workers of privately incorporated firms in a given economic region year pair is smaller than 20 observations are excluded from the analysis.

specific time-invariant component would create a positive correlation between the entry rate and wages. To address this concern I include region dummies, $\mathbb{1}\{c\}$. Similarly, there might be years in which the Canadian economy was doing well and entry into entrepreneurship was high, pushing wages higher, which would again bias our results. To address these concerns I include year dummies, $\mathbb{1}\{t\}$. And finally, there might be years in which, due to government policy, it was particularly more advantageous to start a firm as a worker than as a laid-off individual. This would bias the difference in responses between the two groups to a similar wage movement. To control for that variation, I include year dummies interacted with the dummy $\mathbb{1}\{\text{Prev U}\}_{c,t,n}$, indicating whether or not referring to laid off or wage workers. My final specification is

$$\begin{aligned} ER_{c,t,n} &= \xi_0 + \xi_1 log(w_{c,t}) + \xi_2 log(w_{c,t}) \mathbb{1}\{\text{Prev U}\}_{c,t,n} \\ &+ \xi_3 \mathbb{1}\{\text{Prev U}\}_{i,t} + \mathbb{1}\{c\}\xi_4 + \mathbb{1}\{t\}\xi_5 + \mathbb{1}\{t\} \cdot \mathbb{1}\{\text{Prev U}\}_{c,t,n}\xi_6 + \varepsilon_{c,t,n} \end{aligned} \tag{1.44}$$

where $\mathbb{1}\{c\}$ are dummies for regions and $\mathbb{1}\{t\}$ are dummies for years. The theory predicts that $\xi_1 < 0$ and $\xi_2 > 0$.

Using the Frisch-Waugh-Lovell Theorem, we know that the estimates of ξ_1 and ξ_2 are the same as those obtained from the specification

$$\widehat{ER}_{c,t,n} = \xi_0 + \xi_1 \widehat{log(w_{c,t})} + \xi_2 log(w_{c,t}) \widehat{\mathbb{1}\{\text{Prev}} \text{ U}\}_{c,t,n} + nu_{c,t} \tag{1.45}$$

where $\widehat{x} = x - (\hat{\xi}_3 \mathbb{1}\{\text{Prev U}\}_{c,t,n} + \mathbb{1}\{\text{Prev U}\}_{c,t,n} \cdot \mathbb{1}\{t\}\hat{\xi}_4 + \mathbb{1}\{c\}\hat{\xi}_5 + \mathbb{1}\{t\}\hat{\xi}_6)$ and $(\hat{\xi}_3, \hat{\xi}_4, \hat{\xi}_5, \hat{\xi}_6)$ are obtained by regressing x on $\mathbb{1}\{\text{Prev U}\}_{c,t,n}$, $\mathbb{1}\{\text{Prev U}\}_{c,t,n} \cdot \mathbb{1}\{t\}$, $\mathbb{1}\{c\}$ and $\mathbb{1}\{t\}$. For region level wages, it amounts to correcting for region and year specific averages :

$$\widehat{w}_{c,t} = w_{c,t} - \sum_t^T w_{c,t} - \sum_c^C w_{c,t}. \tag{1.46}$$

This result highlights how identification comes from comparing wage growth across regions.

1.4.2 Exogeneity

Despite the use of these additional dummies in regions and years to clean up the variation being used, there is still reason to expect that Ordinary Least Squares (OLS) estimates are biased. This is due to the presence of region-year specific demand shocks in the error term. We expect an OLS specification to be biased by a positive relationship between wages and the entry rate into entrepreneurship.[46]

To address this problem, I use an instrumental variable strategy that exploits the variation in wages due to differences in industrial composition across cities. The instrument I used was first proposed by Beaudry et al. [2012].[47] In particular, the instrument for $log(w_{c,t})$ is

$$IV_{c,t} = \sum_{\forall i} \kappa_{c,i,1} log(w^N_{i,t}) \tag{1.47}$$

where i stands for industry, $\kappa_{c,i,1}$ is the first sample year employment share of industry i in region c and $log(w^N_{i,t})$ is the wage for industry i at the national level at year t. This term is correlated to $w_{c,t}$ due to across city variation in industrial composition.[48] The intuition is that regions with a higher concentration of high-paying industries in the past have larger region-wide wages.[49]

The instrument relies on the traditional assumptions used for Bartik instruments. It requires region-wide demand shocks to be uncorrelated with the industrial composition of the region in the first year of the sample.[50] One concern is allowing for mobility of individuals across regions. Section A.1.4 of the Appendix shows how allowing for imperfect mobility across regions does not change our empirical specification.

[46] Demand shocks are understood here as any shocks that induce more job creation by firms. One example is a TFP shock.

[47] The authors derive the instrument from a model in which industry spillovers arise from Nash bargaining over wages.

[48] Variation in the vector of $\kappa_{c,i,1}$.

[49] See Section A.1.5 of the Appendix for full details on how this instrument and the main explanatory variable of interest, $w_{c,t}$, are constructed in the data.

[50] See Section A.1.3 of Appendix for formal conditions on the model structure to guarantee validity of the instrument.

1.4.3 Results

Column 1 of Table 1.7 indicates that when we ignore endogeneity, we get a positive relationship between wages and the entry rate for both employed (0.002) and laid-off individuals ($0.002 + 0.016$) as predicted by the theory. This is consistent with the intuition that the endogeneity is being caused by demand shocks. Looking at the IV results in column 2 of Table 1.7, we see that the positive relationship between wages and the entry rate into entrepreneurship goes from positive to negative for wage workers (0.002 to -0.032) and from positive to zero for laid-off individuals (0.018 to $-0.032 + 0.032$).

The results in column 2 indicate that, consistent with the model, the entry rate into entrepreneurship of wage workers is more responsive to wages than the entry rate into entrepreneurship of laid-off individuals. In particular, a 1% drop in wages increases by 3.2 percentage points the entry rate into entrepreneurship of wage workers and has no impact on laid-off individuals. This differential is due to the role of wages as an opportunity cost to entrepreneurship for wage workers. Finally, note that the first stage is strong as indicated by the F-statistic in column 2, row 6.

Table 1.7: Additional Implication Results

	OLS	IV
$log(w_{c,t})$	0.002	-0.032^{**}
	(0.0077)	(0.016)
$log(w_{c,t}) \cdot \mathbb{1}\{\text{Prev U}\}_{c,t,n}$	0.016^{***}	0.032^{***}
	(0.0044)	(0.005)
City Dummies	Yes	Yes
Year Dummies	Yes	Yes
Year Dummies X $\mathbb{1}\{\text{Prev U}\}_{c,t,n}$	Yes	Yes
Kleibergen-Paap rk Wald F Statistic		100.92
Observations	1357	1357

Notes: Linear regression with $ER_{c,n,t}$, the entry rate into firm ownership, as dependent variable. The main explanatory variables are $log(w_{c,t})$, log of wages at the city c and year t level, and $log(w_{c,t}) \cdot \mathbb{1}\{\text{Prev U}\}_{c,t,n}$, the interaction between $log(w_{c,t})$ and $\mathbb{1}\{\text{Prev U}\}_{c,t,n}$, an indicator taking value 1 if referring to the laid off and 0 if referring to employed individuals.

1.5 Policy Counterfactuals

In this section I evaluate the impact on job creation of transferring a share of total unemployment insurance income in the economy to unemployed individuals that start businesses.

The theoretical framework does not model explicit frictions that rationalize policies promoting entrepreneurship. One way of generating welfare gains from these policies is to introduce liquidity constraints associated with startup costs. Such an addition would limit the tractability of the model, without adding to the main message of the paper, that policies subsidizing the unemployed affect the allocation of resources across firms. For this reason, I leave such an extension for future work and take as given that governments implement these policies. I focus on the impact that these policies have on the selection margins of the unemployed and wage worker as well as the resulting effect on the firm productivity distribution and on job creation.

Until now, the model has disregarded the general equilibrium effects of the entrepreneurship margin on the job finding rate, which has been assumed exogenous and equal to f. Yet, to understand the impact of a policy on the unemployment rate, it is crucial to allow the job finding rate to be an equilibrium object. For this reason, I propose a simple extension of the benchmark model presented in Section 2 which allows for the entrepreneurship margin to affect the job finding probability via general equilibrium.[51]

A tractable way to do that is to assume that firms managed by an entrepreneur do not directly hire labour. Instead, they buy an intermediate good y. This intermediate good is produced with labour in a one-to-one fashion.[52] The entrepreneur takes the price of the intermediate good ρ as given and proceeds as before, deciding how many intermediate goods to use (static problem) and when to stop producing (dynamic problem). The only difference is that entrepreneurs, instead of hiring labour directly, buy intermediate goods y from intermediate goods producers that face search frictions. [53]

I assume the existence of a large set of intermediate goods producers, each of which can decide to post a vacancy at any point in time.[54] The flow cost of posting a vacancy for intermediate goods producers is a fraction c of the equilibrium wage w. When an intermediate goods producer finds a worker, it begins production and obtains a flow return of $\rho - w$. Job vacancies and unemployed workers match according to a constant returns to scale matching function given by $Kv^{\gamma}u^{1-\gamma}$, where u is the measure of unemployed and v the measure of vacancies. The rate at which the unemployed find jobs is given by $p(\theta)$ where $\theta \equiv \frac{v}{u}$. The value function of the unemployed, U, is now defined by

$$rU = bw + p(\theta)(W - U) + \psi \int_{\underline{z}_u} (J(z) - U)dF(z). \tag{1.48}$$

[51] In addition, the presence of congestion externalities makes wages less flexible and is important in determining the magnitude of the impact of the policy.

[52] One can think of that as an intermediate sector that must transform workers so they can be employed by the entrepreneurs. The intermediate good is then just ”transformed labour”.

[53] This way of introducing search frictions follows closely Beaudry et al. [2014], who also include search frictions in a model of entrepreneurship using an intermediate goods sector.

[54] This means that for the intermediate goods sector, firms do not come from the same pool as workers, the unemployed and entrepreneurs.

Let s be the rate at which matches exogenously break up, then the value function of the worker, W, is as before. Wages are determined by Nash Bargaining,

$$\phi(W-U) = (1-\phi)(F-V) \tag{1.49}$$

where F is the value of a filled vacancy and V of an unfilled vacancy in the intermediate sector. The price of intermediate goods ρ is determined by market clearing

$$(1-u-\eta) = \int_{\hat{z}} n(z,\rho)\Lambda(z)dz \tag{1.50}$$

where η is the measure of entrepreneurs, $n(z,\rho)$ the optimal number of intermediate goods to hire for a firm of productivity z facing price ρ, and $\Lambda(z)$ is the measure of firms of productivity z. The full solution of this extension is in Supplemental Appendix II.

1.5.1 Calibration

I calibrate the model to the aggregate economy using my full population data. I consider an annual frequency. r is set to 4.5%. α, the curvature of the production function of the entrepreneur, is equal to the aggregate labour share, and, as such, is set to $\frac{2}{3}$. Remember the matching function is of the form $m(u,v) = Ku^{\gamma}v^{1-\gamma}$. I follow Shimer [2005] in setting γ equal to 0.72. Still following Shimer [2005], I set ϕ, the Nash Bargaining parameter, equal to γ. The rate at which workers transition to unemployment s is taken from Hobijn and Şahin [2009].[55] For the cost of posting a vacancy, I note that as in Shimer [2005], the model allows a normalization. From the free entry condition and the expressions for the value of an unfilled and a filled vacancy I arrive at[56] :

$$\frac{cw}{q(\theta)} = \frac{\rho - w}{r+s} \Rightarrow \theta = \left(\frac{cw(r+s)}{(\rho - w)k}\right)^{-\frac{1}{\gamma}} \tag{1.51}$$

Equation 1.51 implies that doubling c and multiplying k by a factor of $2^{1-\gamma}$ divides θ by half and doubles the rate at which intermediate good firms contact workers,

[55] The authors estimate the rate at which employed individuals transition to non-work for twenty-seven OECD countries, including Canada.

[56] (See Supplemental Appendix II for expressions)

$q(\theta)$, but does not affect the rate at which workers find jobs, $p(\theta)$. It follows that we can normalize θ. I follow Shimer [2005] and choose c so as to normalize θ to 1. Section A.3 of the Appendix contains the results for an alternative calibration in which I follow Hagedorn and Manovskii [2008] in setting the cost of posting a vacancy to 4.5% of the equilibrium wage, $c = 0.045$. The results are robust to this alternative calibration. The replacement rate for the unemployed, b is set to 0.6.

For μ and σ, the parameters governing the evolution of productivity of entrepreneur owned firms, I use the average growth rate in firm size conditional on positive growth and the tail parameter of the ergodic distribution. In Section A.1.6 of the Appendix I state and prove the formal theorem relating these moments.

Finally, to make the model consistent with the patterns in the data, I choose β, the shape parameter of the exogenous distribution business opportunities are drawn from, χ, the cost of shutting down and K, the scale parameter of the matching function, to match the differences in the entry rate between the unemployed and workers and the differences in size and exit between the firms created by both groups.

The value of χ in the calibration, 0.268, represents a cost equivalent to 6% of average firm revenue. This is consistent with World Bank data (Ease of Doing Business Statistics) for which the cost of resolving firm insolvency for Canada is estimated at 7% of the debtor's estate. Finally, ψ is shown to not matter in the impact for the policy in the economy. In Table A.3.1 of section A.3 of the Appendix I show that the results are robust to changing the values for ψ. In the baseline calibration I choose $\psi = 24$, corresponding to an average arrival time for business projects of $\frac{1}{2}$ a month. See Table A.2.1 in the Appendix for a complete list of parameter and sources/targets used. The model is highly tractable with clear intuition this comes at a cost of making it inadequate for tests of external fit of the model. Consistent with this, I focus on internal fit of the model for which all moments are shown to be matched in Table A.2.1 of the Appendix.

1.5.2 Policy Analysis

In this section I use a calibrated version of the model with search frictions and a version of the baseline model to evaluate the impact of a policy that subsidizes entry into entrepreneurship among the unemployed. The calibration for the baseline model follows the calibration described for the model extension with the only additional caveat that the rate at which the unemployed become workers (f) is set to match the job finding rate in the model extension and is kept at that same value once I evaluate the impact of policy.

I consider a policy that takes 5% of total unemployment insurance (UI) income and redistributes it to any unemployed individual that makes the decision to start a firm. The entrepreneurship subsidy policy corresponds to entrepreneurs that were unemployed when starting their firm receiving 30% of their previously received UI benefits during their first year of business. This is less than, but comparable in magnitude to, the subsidy program in British Columbia in which entrepreneurs entering from unemployment remain eligible to their full UI benefits for the first 38 weeks of operating the business.[57]

The main metric for measuring the success of the policy is taken as its effect on job creation. The reason being that this is the most common argument for the use of such policies. The question asked here is to what extent can this policy generate higher job creation and what is the associated cost, in the form of lower productivity.

In Table 1.8 Column 1, we see that the effect of the policy in the benchmark model is a drop in average firm productivity , $E(z)$ (-3%)), a small drop in the unemployment rate (-1%) and an increase in wages (1.29%). Despite the relative lack of movement in the unemployment rate, there is an important change in the composition of firms. This reallocation can be seen with the change in the number of jobs created by wage workers, (-6.39%), and of jobs in firms started by the unemployed, (14.49%). The new equilibrium is one in which more resources are being used by firms created by the unemployed (low productivity) at the expense of less being used by firms created by wage workers (high productivity). Consistent with this, average firm exit rate increases.

The subsidy policy makes entrepreneurship relatively more attractive to the

[57]For the year 2016, given an average unemployment rate below 8%, residents of the province were entitled to a maximum of 40 weeks of employment insurance. This means that an unemployed that applied to receive the subsidy is entitled to virtually the entirety of the benefits he was already in British Columbia, Canada.

unemployed. Hence, their level of selectivity decreases, prompting a rise in the mass of firms in the economy (via more low productivity firms). The increase in low productivity firms decreases average firm productivity. The rise in the number of firms increases labour demand which in turns puts upward pressure on wages. The rise in wages decreases the value of being an entrepreneur and increases the value of being a wage worker. As a result of these two forces, the wage worker becomes more selective on which business projects to implement.[58] This further increases the share of firms created by the unemployed.

[58]Note that for the wage worker all that has changed in the world with the policy is that wages are higher. In the new equilibrium with the policy the value of being an entrepreneur is higher for the unemployed and lower for the wage worker.

Table 1.8: Policy outcomes

	Benchmark Model	Model Extension
	(1)	(2)
$\Delta E[z]$	-3%	-2.14%
Δ Unemployment Rate (% change)	-1%	-1.11%
Δ Wage	1.29%	0.65%
Δ Labor Market Tightness (θ)	–	2.35%
Δ Jobs by Firms created by Unemployed	14.49%	7.12%
Δ Jobs by Firms created by Workers	-6.39%	-7.1%
Δ Average Firm Exit Rate (% change)	55.42%	36.37%

Notes: Outcome of policies that make a share of total UI benefits income conditional on the unemployed opening a firm. $\Delta E[z]$ is the percentage change in the average firm productivity, Δ Jobs by firms created by workers is the percentage change in the measure of jobs associated to firms created by wage workers, Δ Unemployment is the percentage change in the unemployment rate. The policy takes 5% of total unemployment insurance (UI) income and redistributes it to any unemployed individual that makes the decision to start a firm. The entrepreneurship subsidy policy corresponds to entrepreneurs that were unemployed when starting their firm receiving 30% of their previously received UI benefits during their first year of business.

In Table 1.8 Column 2 we see that the effect of the policy in the model extension is almost identical for average firm productivity, $E(z)$ (-2.14%) and unemployment (-1.11%). The key difference in the mechanism lies in the response of wages to the shock (1.29% versus 0.65%) and its contribution to the general equilibrium effect.

After the drop in selectivity among the unemployed and the corresponding increase in the number of firms, the price of intermediate good increases. This prompts more intermediate good firms to post vacancies, which in turn increases labor market tightness.

The increase in labor market tightness has a direct and indirect general equilibrium effect. The direct effect is to increase the job finding rate, making wage work more attractive relative to entrepreneurship. Together with the increase in the price of intermediate goods, it increases workers' selectivity. The indirect effect is the rise in the worker's threat point during wage bargaining. As a result, workers bargain higher wages, further increasing the value of wage work relative to entrepreneurship. The indirect effect complements the direct effect further increasing worker selectivity.

Since wages are determined via Nash Bargaining rather than supply and demand the responsiveness of wages is smaller in the model extension with search frictions. But the total effect on aggregates ends up being similar because with search frictions the model gets one more margin of adjustment, labor market tightness. In contrast, for the benchmark model, all of the general equilibrium adjustment can only happen via prices. The implication is a much smaller wage increase in the model with search frictions.

Note that, despite the increase in job finding rate in the model extension and its absence in the benchmark, both models deliver a same change in the unemployment rate. This is achieved by a larger inflow into the pool of unemployed in the model extension relative to the benchmark model. This happens via a larger increase in the firm failure rate in the model extension (55.42%) relative to the benchmark model (36.37%).

I conclude that, in the context of my model, the policy has close to no impact on the unemployment rate while decreasing average firm productivity and reallocating resources from high to low productivity firms. The results also highlight the importance of general equilibrium effects. In particular, the channel of these general equilibrium effects will depend on the labor market structure. Note that, although I abstract from negative selection into unemployment on worker ability, adding this margin would only strengthen the results presented here. [59]

1.6 Conclusion

I study the differences between firms created by unemployed individuals relative to otherwise identical employed individuals. I show that these differences are important for our understanding of policies that promote entrepreneurship among the

[59] This is conditional on worker and entrepreneurial ability being positively correlated.

unemployed to fight unemployment.

I develop a general equilibrium model of endogenous business ownership. In this framework, the only difference between unemployed and employed individuals is their outside option. In equilibrium, due to poorer outside options, the unemployed are more likely to open a firm, but conditional on doing so, generate smaller firms that shut down sooner. I test these implications using a novel confidential dataset with the universe of Canadian tax filers. I use firm closures to identify random assignments of an individual to unemployment. I find that unemployment induces a doubling of the probability to start a business, and conditional on doing so, an individual hires 26% fewer workers and is 30% more likely to exit entrepreneurship. Finally, I use the data facts to discipline a numerical version of the model. I evaluate the impact of a policy that subsidizes entry into entrepreneurship among the unemployed. The result is a drop in average productivity despite little movement in the unemployment rate. Furthermore, the policy induces the creation of low productivity firms that crowd out resources from high productivity firms.

Chapter 2

Entrepreneurship, Outside options and Constrained Efficiency

2.1 Introduction

Understanding the process and choices that drive the creation of new firms and hence spur employment is crucial to a complete understanding of employment, productivity growth, wages, vacancy creation and a host of other labor market variables. The focus of this contribution is on the entrepreneurship margin. We make a theoretical contribution to the literature on firm creation by placing the decision of individuals to create a firm inside the search and matching framework. Jobs are created by ex-ante identical individuals who face a choice between entrepreneurship and wage work. By modelling the start-up decision as an endogenous choice in this manner we forgo the inclusion of the typical free entry condition to close the model as in Mortensen and Pissarides [1994]. Instead the model is closed by a condition whereby entrepreneurs are indifferent between remaining unemployed or creating a business, conditional on their productivity draw.

Modelling firm creation in this manner implies an interesting distinction between our framework and the baseline search model. For instance, in the standard search framework wages are unambiguously increasing in the value of unemployment. In contrast, the direction, and not just the magnitude of this relationship is dependent on the Nash bargaining parameter in our framework. For threshold values of the bargaining parameter the slope of the wage can become negative, or indeed flat. The intuition underlying this result is that firms are created by individuals whose outside option is to search for wage work through rejoining the pool of unemployed workers. Due to this, the outside option value for both firms and workers in this model is the value of unemployment. This contrasts with exogenous models of firm creation where the outside option value for the firm is the value of an unfilled vacancy. As a result the equilibrium wage equation in our framework includes an additional term coming from the firm side. This relationship also appears counter-intuitive: for high values of workers bargaining power wages are negatively related to the value of unemployment.

Additionally, the inclusion of endogenous firm creation here implies the existence of an additional externality in the model, in addition to the standard congestion and thick margin externalities, which we refer to as the 'job-creation margin'. This margin arises from the endogeneous choice to search for a business idea or a job. If the labor market is tight then individuals will prefer to search for wage work and hence entrepreneurs will be more selective on which business ideas they implement. However, those deciding between searching for a job or a business venture do not take into account the effect of their choice on the search process of other potential entrepreneurs or other job seekers. Their choices also affect the choices of entrepreneurs currently operating in the market. This effect again operates through changes in the entrepreneurs outside-option term, which is the value of unemployment.

Given the inclusion of this additional externality and the distinct difference in the wage function, it is not ex-ante clear what form the efficient solution to the model will take. In solving for the planners problem we find that the solution is identical to that of the Hosios condition for the standard search framework: externalities are balanced when agents bargaining power is equated to the elasticity of the matching function. However, this socially efficient solution does not pin down a clear direction for the wage, and hence a clear direction of adjustment to equilibrium. The dynamics of the model following a shock remain sensitive to the size of the elasticity parameter. In particular, wages do not necessarily exert a dampening effect in response to exogenous shocks.

This paper contributes to the theoretical literature evaluating constrained efficiency in search theoretic models of the labor market. The Hosios rule [Hosios, 1990] states that a standard search model a là Pissarides [2000] is constrained efficient when the Nash bargaining parameter is equal to the elasticity of the matching function. Literature in this area has sought to examine the set of conditions under which the Hosios rule gives the socially efficient outcome[1] or generalizes the Hosios rule to alternative environments[2].

This paper also relates to the literature on the individual choice between working and opening a business. The empirical literature is vast as exemplified by the seminal papers of Hamilton [2000] and Quadrini [2000] as well as more recent re-

[1] see Albrecht et al. [2010], Gavrel [2011]

[2] see Acemoglu and Shimer [1999], Julien et al. [2016]

search such as that of Humphries [2016] and Poschke [2013]. Finally entrepreneurship is important to the extent that the extensive margin of firm creation is important for the macroeconomy. In that respect there is evidence on the importance of the firm creation process for persistence in firm outcomes (Sedláček and Sterk [2017], Moreira [2015]), wealth inequality (Quadrini [2000] and Cagetti et al. [2006]) and the importance of young firms for job creation (Haltiwanger et al. [2013]). This paper contributes with a richer theoretical framework to investigate these decisions of individuals to open a business.

The remainder of this paper is structured as follows: In section 2 we present our theoretical model, the dynamics of which are discussed in section 3. In section 4 we discuss the Hosios condition for efficiency and we present concluding remarks in section 5.

2.2 Model

At a given point in time an individual can be one of four types; a worker, an entrepreneur, a searcher for paid work, or a searcher for a business idea. Individuals search for a 'business idea' while unemployed only. Each business idea represents the productivity level of the firm and is modelled as an exogenous productivity draw, ε. To maintain tractability we assume there is no direct entry into entrepreneurship from wage work[3], and that there is no recall of productivity draws. Labor market tightness, defined in the standard manner, includes as the unemployed only those seeking wage work. It is worth noting however that this does not

[3] This would require a separate threshold rule for each realized wage. The comparative statics of the model in that case becomes more complex. For every shock that increases the selection on business projects among unemployed, there would be a corresponding decrease in selection among the workers.

mean that entrepreneurship only affects market tightness on the vacancy side. The size of the unemployed pool is in part the result of the endogenous choice between searching for a job or an idea.

A choice is made by all unemployed workers to either accept a job, or implement a business idea. If the unemployed chooses to search for business idea, he or she receives one at rate ψ from distribution $F(\varepsilon)$, which he or she then chooses whether to implement or not. Individuals finding sufficiently productive business ideas post a vacancy next period, in which case they receive the value of a unfilled vacancy of productivity ε, $V(\varepsilon)$ and lose the value of being a searcher U. If the unemployed decide to search for a job, they receive one at rate $p(\theta)$, in which case they draw a job from the endogenous firm productivity distribution $\mu(\varepsilon)$.[4] Upon finding an employer of productivity ε, the worker receives the value of being a worker in a firm of productivity ε, $W(\varepsilon)$. From the assumptions above it follows that the value function of a searcher U is given by

$$rU = b + \max(p(\theta)\int(W(\varepsilon^*) - U)\mu(\varepsilon^*)d\varepsilon^*, \psi\int_{\underline{\varepsilon}}(V(\varepsilon^*) - U)dF(\varepsilon^*)) \tag{2.1}$$

As all individuals are ex-ante identical, in equilibrium individuals will be indifferent between searching for a job or for a business idea.

The productivity threshold, below which no entrepreneurs will implement their business idea is characterized by:

$$V(\underline{\varepsilon}) = U \tag{2.2}$$

[4]This distribution is a equilibrium object that depends on which business opportunities individuals choose to implement.

The cost of posting a vacancy is given by c, $q(\theta)$ defines the firm match probability. Let $J(\varepsilon)$ denote the value of a filled vacancy of productivity ε. The value functions for a vacant job ($V(\varepsilon)$) is standard and given by

$$rV(\varepsilon) = -c + q(\theta)(J(\varepsilon) - V(\varepsilon)). \tag{2.3}$$

When working for an employer of productivity ε the worker receives wage $w(\varepsilon)$. Firms experience exogenous destruction shocks at rate λ. The value of being a worker, $W(\varepsilon)$, is given by

$$rW(\varepsilon) = w(\varepsilon) + \lambda(U - W(\varepsilon)). \tag{2.4}$$

The value of a filled job ($J(\varepsilon)$) takes into account that at exogenous rate λ the firm is destroyed and the entrepreneur transitions back to unemployment

$$rJ(\varepsilon) = \varepsilon - w(\varepsilon) + \lambda(U - J(\varepsilon)). \tag{2.5}$$

Wages are formed through Nash bargaining. The threat point in the bargaining process for both workers and entrepreneurs is the value of unemployment, U. If either party chooses to walk away from the match then the firm will shut down. Equilibrium in this model is characterized by the set of equations

1. $V(\underline{\varepsilon}) = U$
2. $\beta(W(\varepsilon) - U) = (1-\beta)(J(\varepsilon) - U)$
3. $W(\varepsilon) > U \quad \forall \varepsilon > \underline{\varepsilon}$
4. $p(\theta)\int(W(\varepsilon^*) - U)\mu(\varepsilon^*)d\varepsilon^* = \psi \int_{\underline{\varepsilon}}(V(\varepsilon^*) - U)dF(\varepsilon^*)$

where the last equation ensures individuals are indifferent between searching for a job or a business idea.

The remainder of our analysis considers the economy in steady state. From steady state equations we derive a tight, negative relationship between the threshold productivity and market tightness which we refer to as the Job Creation Curve,

$$p(\theta) = \psi(1 - F(\underline{\varepsilon})) \tag{2.6}$$

Furthermore, using the entrepreneur's value functions combined with the derived endogenous productivity distribution we derive a second curve showing a positive relationship between tightness and threshold productivity, which we refer to as the Entrepreneurship curve.

Theorem 6. *The indifference condition determining $\underline{\varepsilon}$ implies the following positive relationship between $\underline{\varepsilon}$ and θ.*

$$\frac{b(r+\lambda) + \beta\psi\int_{\underline{\varepsilon}}(\varepsilon^*)dF(\varepsilon^*)}{r(r+\lambda+2\psi\beta(1-F(\underline{\varepsilon})))} = \frac{-c(r+\lambda)+q(\theta)[(\underline{\varepsilon})(1-\beta)]}{r(r+\lambda+2(1-\beta)q(\theta))} \tag{2.7}$$

The intuition underlying this curve is that a tighter labor market implies a greater benefit to seeking wage work. To remain indifferent, the benefits to entrepreneurship must be high for potential entrants, which translates into a higher threshold productivity, $\underline{\varepsilon}$. The interaction of the Entrepreneurship curve and the Job Creation curve pins down an equilibrium pair of $(\theta, \underline{\varepsilon})$.

Finally, wages in equilibrium are given by

$$w(\varepsilon) = \beta\varepsilon + (1-2\beta)rU, \tag{2.8}$$

which differs from the traditional search and matching model due to the inclusion of an additional $-\beta rU$ term coming from the outside option term of the entrepreneur. The direction of fluctuations in the wage due to movements in the value of unemployment are determined by the size of the bargaining parameter. Wages are increasing in the value of unemployment, U, for $\beta < \frac{1}{2}$ and decreasing for $\beta > \frac{1}{2}$.

2.3 Model Exploration

In this section we present some comparative statics outlining the underlying mechanisms present in the model. From equation (2.8) it is quite clear that the direction of wage movements is dependent upon the value of the bargaining parameter. Intuitively two effects are operating here. Firstly, an increase in U pushes wages upward as the threat point of job seekers has increased. Secondly, failure becomes less costly to the entrepreneur as the outside option to firm closure is higher. As a result, wages are negatively related to U when workers have a greater share of bargaining power. The intuition behind this seemingly counter-intuitive result is that when β is high a greater weight is placed upon the decline in the cost of failure to the entrepreneur than on the cost of job loss to the worker. The reverse is true if $\beta < \frac{1}{2}$. However, even when the direction of wage movements are unknown the following result holds true:

Theorem 7. *An increase in the value of unemployment income, b leads to a higher value of threshold productivity, $\underline{\varepsilon}$, and a lower value of market tightness, θ, in equilibrium, implying an increase in average firm productivity.*

This means that even if wages fall in response to an increase in the flow value of unemployment, net job creation will still fall. The intuition for this result is simply that a higher b provides greater insurance to potential entrepreneurs and as a result those searching for ideas become more selective on the business ideas they implement. This raises average productivity and decreases net job creation as captured by an increase in the threshold productivity $\underline{\varepsilon}$. Furthermore regardless of how wages are affected there will be a decline in market tightness. This results from lower net vacancy creation. The implication of our model therefore, is that there is a trade-off between entrepreneur quality and quantity. If a greater social safety net is provided for entrepreneurs then there will be a fall in the rate of entrepreneurship but an increase in the average quality of entrepreneurs. Furthermore it is interesting to note that this relationship holds regardless of how wages adjust. In particular it is possible to have an increase in aggregate productivity with flat or even decreasing wages. This contrasts with Mortensen and Pissarides [1994] where wages are the key channel through which the flow value of unemployment affects vacancy creation.

Similarly, without knowing the direction of wage movements it is possible to pinpoint a relationship between the other endogenous variables and the flow cost of posting a vacancy.

Theorem 8. *An increase in the cost of posting a vacancy, c, leads to a higher value of threshold productivity, $\underline{\varepsilon}$, and a lower value of market tightness θ in equilibrium, implying an increase in average firm productivity.*

An increase to the flow cost c leads to a shift in the entrepreneurship curve, while the job creation curve remains unchanged. In a model with exogenous firms and a free entry condition, the value of unemployment is affected only indirectly through a change in the firm productivity distribution. In our framework there is an additional effect whereby the value of a vacancy enters the value of unemployment through the agents choice to become an entrepreneur. In both models the value of unemployment falls as a result. The wage effect serves to dampen the employment response in the case of exogenous firms, but in our model the employment response is exacerbated if wages are negatively related to the value of unemployment.

2.4 The Constrained Efficient Solution: Deriving the Hosios Condition

We solve the planners problem for the model to derive the constrained efficient solution which is summarised by the following theorem:

Theorem 9. *The competitive equilibrium allocation is constrained efficient when* $\beta = 1 - \alpha$.

The condition under which the model is at a social optimum, is the same as the original Hosios condition [Hosios, 1990], whereby externalities are fully balanced when the bargaining parameter is equal to the elasticity of the matching function. Given the unique relationship the bargaining parameter plays in our model in determining the direction of the wage response to changes in the value of unemployment, it is worth exploring this result somewhat.

In particular, the constrained efficient wage equation now takes the form:

$$w(\varepsilon) = (1-\alpha)\varepsilon + (2\alpha - 1)rU \tag{2.9}$$

Wages are declining in the value of unemployment for values of α lower than $\frac{1}{2}$ and strictly non-negative otherwise. A high value of α implies that workers match probability is sensitive to market tightness, relative to that of firms.

Consider the case in which $\alpha > \frac{1}{2}$. Holding wages constant, an increase in market tightness (θ) has three effects. First, the value of job search increases due to a rise in $p(\theta)$ holding constant the value of unemployment (U) and the threshold productivity ($\underline{\varepsilon}$). We refer to this as the *job search effect*. Secondly, for entrepreneurs, the likelihood of matching with a worker falls ($q(\theta)$) but by a lesser amount than the increase in $p(\theta)$, again fixing U and $\underline{\varepsilon}$. This we call the *worker finding effect*.

Thirdly, the *outside option effect* comes from fluctuations in the value of unemployment (U) which affects entrepreneurs via the outside option channel. Note that via the outside option channel (changes in U), entrepreneurs are affected by changes in $p(\theta)$, holding changes in $q(\theta)$ constant. This contrasts with more standard search models where changes in U and $p(\theta)$ affect firms only indirectly via wages once we hold $q(\theta)$ constant.

This channel, where the decision to enter entrepreneurship is dependent on the value of unemployment and the value of wage work introduces novel externalities. The first being that when agents choose to search for a business idea they do not take into account the impact of this decision on the choice between job creation and wage work for other individuals. Secondly, they do not take into account how their choice affects the outside-option value of entrepreneurs currently operating in the market. These externalities arise from the presence of the *job search effect* and the *outside option effect*, respectively, both not present in a model with exogenous firms and free entry.

In the absence of wage effects, the value of unemployment will rise, and due to the indifference condition on entry (equation 2.2), the value of $\underline{\varepsilon}$ increases. A rise in $\underline{\varepsilon}$ decreases firm creation and increases average firm productivity. Under the constrained efficient allocation ($\beta = 1 - \alpha$), as from equation (2.8), a larger share of productivity accrues to the firm, and hence a smaller increase in $\underline{\varepsilon}$ maintains the equality. However, assuming that $\alpha > \frac{1}{2}$, wages are increasing in the value of unemployment, which further increases the gains to job search, exacerbating the rise in $\underline{\varepsilon}$ required to make the individual indifferent. As a result, the constrained efficient allocation generates less firm creation (higher $\underline{\varepsilon}$) relative to the allocation where wages do not adjust. In other words, compared to the socially efficient allocation, there is excessive firm entry coming from an increase in the value of unemployment, despite the rise in $q(\theta)$.

The intuition is that with $\alpha > \frac{1}{2}$, $p(\theta)$ is more responsive to changes in θ relative to $q(\theta)$. As a result, for a rise in θ, the *job search* and the *outside option effects* are quite large while the *worker finding effect* is small. Since the *job search* and the *outside option effect* pull $\underline{\varepsilon}$ in opposite directions the overall result is a small response of $\underline{\varepsilon}$ to a rise in θ. Therefore, to attain the constrained efficient allocation prices have to move so as to generate more selection than would happen otherwise. This is achieved through wage adjustment where the bargaining parameter is such that wages are increasing in the value of unemployment.

If we consider the opposing case where α is less than one half a similar logic applies. An increase in tightness, θ, generates a large response of $q(\theta)$ and a small response of $p(\theta)$. As a result, the *job search* and *outside option effects* are small while the *worker finding effect* is large. The overall response of $\underline{\varepsilon}$ to the change in θ is larger than would be induced in a constrained efficient allocation. Therefore, to attain efficiency, prices must adjust to reduce selection on productivity. This is achieved when wages are decreasing in the value of unemployment. The same is true if we consider a fall in θ. The wage response will mitigate the increased entry to entrepreneurship when $\alpha < \frac{1}{2}$ and exacerbate entry when $\alpha > \frac{1}{2}$. If the slope of the wage equation with respect to U was α rather than $1-2\alpha$ then this mitigating effect on $\underline{\varepsilon}$ would not operate when $\alpha < \frac{1}{2}$.

Therefore, this model includes a range of values over which the unemployment response to a shock is more severe, and a region where it is lessened. This is driven by the ambiguity in the direction of the wage response arising from entrepreneurs and workers sharing a threat point in the bargaining process. In a search model with exogenous firm creation and a free entry condition a shock that increases tightness, θ, always feeds into a larger wage, w, via a increase in the value of unemployment, U, which partially offsets the increase in θ by decreasing the incentives for firms to hire. The wage response here has a mitigation effect like in standard models without entrepreneurs and with a free entry condition when $\alpha > \frac{1}{2}$ but has a amplification effect when $\alpha < \frac{1}{2}$. The constrained efficient solution therefore balances traditional search externalities with additional externalities arising from the dependency of the gains to firm creation on the value of unemployment. In particular the social planner weighs the additional effects on currently operating entrepreneurs and those deciding between entrepreneurship or wage work, of the endogenous choice of agents to search for a job or an entrepreneurial venture.

2.5 Conclusion

We forgo of the traditional free entry condition by proposing a more realistic framework in which individuals are constantly making the decision whether or not to open a firm. We endogenize firm entry as a dynamic entry process where both workers and business owners are drawn from the same population. We do so by allowing ex-ante individuals to search for either a job or an idea. In the stylized framework considered here the threat point of the entrepreneur and the worker become one and the same. This generates a novel interplay between the bargaining parameter and the direction of wage changes to any feasible shock.

In deriving the planners solution to the model we find that unemployment effects are either muted or intensified in response to a shock at the social optimum. This mechanism operates through wages and contrasts with the standard search framework where wages serve to only dampen unemployment fluctuations. This result is particularly interesting given that the Hosios condition takes the same form.

Chapter 3

Match Quality, Contractual Sorting and Wage Cyclicality

3.1 Introduction

Compensation arrangements influence the evolution of workers' wages. In this chapter we examine how profit maximizing firms choose pay arrangements depending on worker-firm match quality, and provide evidence that such arrangements help shape both wage dynamics and employment durations.

We begin by developing a simple model of worker pay based on match quality and worker retention considerations. Our main theoretical result is that firms retain workers in high quality matches by offering compensation that is linked to the performance (production outcome) of the match. Moreover, as production is influenced by an aggregate cyclical component, the model implies that the wage of workers in performance-pay jobs should be more sensitive to cyclical fluctuations.

In the second part of the chapter we bring these theoretical predictions to the data. We use detailed information from the NLSY79 to characterize work histories, and resort to specific questions regarding the form of compensation to distinguish between jobs with and without performance pay components. We construct measures of match quality and, following an established literature, we use the unemployment rate as a proxy for business cycle conditions. Our results provide empirical support for the three main theoretical predictions of the model. First, there is a clear positive relationship between match quality and the prevalence of jobs with performance pay. Second, match quality has a direct effect on wages, after controlling for the adoption of performance pay. Third, wages in performance pay jobs exhibit significant sensitivity to cyclical conditions, while wages in jobs with no performance pay components do not. Given our focus on worker retention motives, we also provide evidence that job durations are significantly higher when performance-based pay is adopted.

We relate our results to the growing literature on occupation heterogeneity and show that variation in the way workers are compensated in different occupations is intimately linked to match quality. In fact, we argue that this simple observation can go a long way towards understanding some of the observed differences in the cyclicality of wages, match-specific productivity and job durations across occupations. To this purpose we show that jobs in "cognitive" occupations exhibit higher match quality, are more likely to include performance pay components, have more cyclical wages and last longer.

Our study naturally brings together two branches of the literature on pay arrangements and wage dynamics. The first looks at the choice of compensation mechanisms and their effects on wages.[1] Our theoretical analysis is especially re-

[1] A detailed overview of the vast, and growing, literature on personnel and human resource man-

lated to the work of Oyer (2004), who was the first to argue that firms may tie employees' pay to firm performance in order to closely match employees' compensation to their outside options. Our theoretical analysis shows that this retention motive becomes extremely salient in the presence of match-specific heterogeneity, leading to interesting patterns of contractual sorting and wage dynamics.

Some of our empirical findings confirm those by Lemieux et al. (2009, 2012), and Makridis [2014]. These studies show that performance pay jobs are concentrated at the upper end of the wage distribution, where most jobs entail relatively high skills and labor returns.[2]

Finally, our results on the cyclicality of wages directly relate to the empirical literature going back to the work of Bils [1985] on the effect of aggregate labor market conditions on employees' wages. This line of research uses the unemployment rate as a proxy for business cycle conditions. One of the most recent contributions in this broad area (Hagedorn and Manovskii, 2013) proposes a theory-based approach to the measurement of match quality, and we adopt this method to generate empirical proxies for match quality.

Our findings highlight the role of aggregate labor market conditions for wages. The idea that contracts play a role in determining the cyclicality of wages is not a new one (see for example the original contribution by (Beaudry and DiNardo, 1991). Unlike previous research, however, we focus on the theoretical and empirical linkages between match-specific productivity, pay arrangements and wage cyclicality. By explicitly studying the contract choice of a firm in the presence of heterogeneous match qualities, we closely follow the approach used in organiza-

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agement is presented in Lazear and Oyer [2012].

[2]These studies do not explicitly incorporate match quality in the analysis.

tion and personnel economics. In this way we provide novel evidence supporting the view that firms use profit-sharing to retain well-matched workers, and this retention motive helps shape both wage dynamics and job durations.

The remainder of the chapter is organized as follows. The model and the theoretical predictions are discussed in Section 2. Section 3 describes the empirical specification and its relation to the model, as well as the measurement of match quality and performance pay. Empirical results and various robustness checks are overviewed in Section 4. Section 5 concludes.

3.2 A Simple Model of Worker Pay

In what follows we study the problem of a firm that has to decide how to compensate workers, given (i) time-varying aggregate conditions and (ii) match-specific productivity. To simplify the analysis we consider a stylized model with ex-ante identical risk neutral firms and workers. The model highlights the importance of worker retention considerations, as in Weitzman [1984] and Oyer [2004].

Production. A firm-worker pair produces output using production technology

$$y = Pm, \quad m \in [m^{min}, m^{max}] \tag{3.1}$$

where P is an aggregate (economy-wide) state variable, while m is a match-specific productivity component, assuming values between $m^{min} > 0$ and $m^{max} < \infty$. The aggregate state is either high (P_H), or low (P_L), where $P_H > P_L$. The match-specific productivity component is drawn once and persists throughout the life of the match.

Timing. We assume that, for all new matches, the first production period is used to learn about match quality. Only at the end of this initial period, after production takes place, match quality m is revealed to the firm and the worker.

To attract a new worker the firm commits to pay some given wage in the initial (learning) period even though match quality is unknown ex-ante. We assume that this wage is a function of the aggregate state P and of the idiosyncratic match quality m in the worker's *previous* job. Specifically, we assume that the wage paid during the learning period is equal to $a(P)m$ and posit that (i) it is increasing in the aggregate state ($a'(P) > 0$); and (ii) that workers compensation is strictly bounded from above by the total value of output in the current match ($a(P) < P$). In the context of our model the firm's commitment to pay $a(P)m$ clearly defines the value of each worker's outside option.[3] The assumptions we make about $a(P)$ imply that workers have better outside options during high productivity periods, when the aggregate state is $P = P_H$.

At the end of the initial period the new match specific productivity is revealed and the firm offers an employment contract to workers.[4] A surviving match lasts for up to two more periods, denoted as 1 and 2. We assume that $P_1 = P_H$ with certainty, while $P_2 = P_H$ with probability q and $P_2 = P_L$ with probability $(1-q)$.[5]

Some workers might separate from the firm after the initial learning period. This happens when a sufficiently low match quality is revealed. The ex-ante participation constraint of a worker at the start of the period after learning about match quality is

[3]For simplicity we consider the unemployed state as a job with a latent non-zero m value.

[4]Profits or losses incurred during the initial learning period are sunk and the firm does not take them into account when making a new contract offer. This means that the realization of the aggregate state during the learning period has no effect on the contract offer.

[5]In Appendix Section A.6 we show that the same qualitative results hold if the state in the initial period is low ($P_1 = P_L$).

$$w_1(m|P_H) + E(w_2(m)) \geq a(P_H)m + [qa(P_H) + (1-q)a(P_L)]E(m),$$

where w_1 and w_2 are the wages in period 1 and 2, respectively, and $E(m)$ is the expected match quality for a worker who decides to leave at the end of the learning period. We show in Appendix A.5.2 that this participation constraint is satisfied for workers who draw match quality m larger than $E(m)$. If m is below $E(m)$ the constraint may be violated. If so, a separation occurs and the worker moves to a different employer, starting a new learning period.

Contractual arrangements. After the learning period, and conditional on match quality, the firm chooses an arrangement to maximize expected profits over the remaining two periods. In what follows we characterize the optimal contract offered by the firm to the workers who did not quit after the learning period. By choosing to remain in the match these workers commit to remain with the same firm in period 1. However they still have the opportunity to find a new job that will pay $a(P)m$ in the following period.

At the beginning of period 1 the firm offers a contract that specifies a wage for period 1 and a state-contingent compensation for period 2 that guarantees the worker's continuous employment (that is, it satisfies the participation constraints). We posit that the firm can offer one of three alternative pay arrangements to the worker. The three arrangements represent very diverse allocations of cyclical risk between worker and firm, encompassing the extreme cases in which either the firm or the worker carry all cyclical risk. The possible pay arrangements are:

1. A fixed wage contract that guarantees the worker's participation (continuous

employment within the firm). To retain the worker under this contract the firm must offer a fixed wage that equals the highest possible outside option conditional on x,

$$w(m) = a(P_H)m, \quad \forall P. \tag{3.2}$$

This arrangement guarantees worker retention in both periods. The firm subsidizes the worker in bad aggregate states and carries all the production risk.

2. A wage equal to the the worker's outside option, which we call the "spot market" wage. This is a rolling period-by-period arrangement that stipulates that the wage is changed to match the start-of-period outside option of the worker as follows,

$$w(m) = \begin{cases} a(P_H)m & \text{if} \quad P = P_H \\ a(P_L)m & \text{if} \quad P = P_L. \end{cases} \tag{3.3}$$

If the wage is changed between the two periods, there is a fixed (adjustment) cost $T > 0$ paid by the firm.

3. A performance pay arrangement that stipulates that the worker compensation is a combination of a fixed wage $\hat{w}(m)$ and a fraction b [6] of the match surplus Pm:

$$w(m) = \begin{cases} \hat{w}(m) + bP_H m & \text{if} \quad P = P_H, \\ \hat{w}(m) + bP_L m & \text{if} \quad P = P_L. \end{cases} \tag{3.4}$$

[6] We impose $b \leq 1$. Otherwise, the worker would be able to leverage production risk.

We assume that the firm has to pay a variable cost $K(m) = \kappa(m^{max} - m) \geq 0$ to implement performance pay. The cost $K(m)$ is lower when quality m is higher, indicating that workers in better matches are easier to monitor. In Appendix A.5.3 we derive additional results under the assumption of fixed costs of implementing performance pay contracts.[7]

3.2.1 Participation Constraints and Performance Pay Contracts

To guarantee worker retention each of these contracts must satisfy the workers' participation constraints in period 2, requiring that wage w during that period is at least as high as the available outside option. When aggregate productivity is high the constraint is

$$a(P_H)m \leq w(m). \tag{3.5}$$

Similarly, the constraint for low productivity periods is

$$a(P_L)m \leq w(m). \tag{3.6}$$

Both the period-by-period and the fixed wage contractual arrangements trivially satisfy these constraints. For performance pay contracts, however, the firm's offered wage schedule must exhibit parameter values $\hat{w}(m)$ and b such that the contract maximizes expected profits when either one (good times) or both (good and bad times) participation constraints bind. As in Oyer [2004], we consider these cases separately.

[7] As we show below, two types of performance pay contract are possible, depending on parameter values. One type of contract entails a single binding participation constraint (SPC), the other features a double participation constraint (DPC). For SPC contracts to be implemented by the firm, one needs the additional requirement that $\kappa > (1-q)[(a(P_H) - a(P_L)) - (P_H - P_L)]$. However, no such requirement is necessary for DPC contracts.

Case 1: A single binding constraint. If the retention constraint is only binding in good times (SPC, 'single participation constraint') we have,

$$\begin{aligned} E[\pi^{SPC}] = \max_b &(1+q)(P_H m - \hat{w}(m) - bP_H m) \\ &+ (1-q)(P_L m - \hat{w}(m) - bP_L m) - \kappa(m^{max} - m) \end{aligned} \tag{3.7}$$

$$\text{s.t.:} \quad a(P_H)m = \hat{w}(m) + bP_H m$$

After rearranging the constraint, substituting $\hat{w}(m)$ in the objective and deriving the first order condition with respect to b, one obtains

$$\frac{\partial E[\pi^{SPC}]}{\partial b} = (1-q)(P_H - P_L)m > 0 \tag{3.8}$$

Since, by assumption, match quality is not negative, the optimal contract is at a corner solution,[8]

$$\begin{aligned} b &= 1 \\ \hat{w}(m) &= (a(P_H) - P_H)m. \end{aligned} \tag{3.9}$$

Given the maintained assumption that $a(P_H) < P_H$, it follows that $\hat{w}(m) < 0$. Therefore, in the case of a single binding constraint, one can interpret the pay contract as an arrangement in which the worker effectively pays upfront to "buy" the job from the firm. The wage is:

$$w(m) = (a(P_H) - P_H)m + Pm. \tag{3.10}$$

[8] We also posit that the worker cannot leverage production risk. That is, b is bounded from above at 1.

Under the SPC contract participation is guaranteed in the bad state if $P_H - P_L \leq a(P_H) - a(P_L)$. One can show that, in this case, the "L" constraint holds (even though it does not necessarily bind), implying that firms are able to retain workers in both high and low productivity periods.[9]

Case 2: Two binding constraints. If the participation constraint is binding in both good and bad times (DPC, 'double participation constraint'), it must be the case that

$$\begin{aligned} a(P_H)m &= \hat{w}(m) + bP_H m \\ a(P_L)m &= \hat{w}(m) + bP_L m. \end{aligned}$$

The solution for b is derived by subtracting the "L" constraint from the "H" constraint and rearranging, which results in

$$b = \frac{a(P_H) - a(P_L)}{P_H - P_L} \tag{3.11}$$

and

$$\hat{w}(m) = \left[a(P_H) - P_H \frac{a(P_H) - a(P_L)}{P_H - P_L} \right] m. \tag{3.12}$$

Performance Pay Contracts: DPC or SPC? The discussion above suggests that the set of feasible performance pay contracts crucially depends on the ratio $\frac{\Delta a(P)}{\Delta P}$, which relates the cyclical gap in outside offers (numerator) to changes in cyclical productivity (denominator).

[9] To see this, substitute the optimal contract into the "L" constraint to obtain:

$$a(P_L)m < (a(P_H) - P_H)m + P_L m.$$

Specifically, if $[a(P_H) - (a(P_L)] = [P_H - P_L]$ then the two contracts are identical and feature $b = 1$, with both participation constraints binding. If $[a(P_H) - (a(P_L)] > [P_H - P_L]$ it is feasible to have a performance pay contract entailing only one binding participation constraint (SPC), where the other constraint holds but does not bind. Under this contractual arrangement the worker carries all production risk. Finally, if $[a(P_H) - (a(P_L)] < [P_H - P_L]$, the performance pay contract must feature two binding participation constraints (DPC) and cyclical production risk is carried by both worker and firm.[10]

In what follows we show that a firm will offer performance pay contracts to workers when match quality is sufficiently high (high m). This is true whether the ratio $\frac{\Delta a(P)}{\Delta P}$ is greater or less than one.

3.2.2 Contract Choice and Wage Cyclicality

The behavior of wages, both cross-sectionally and over time, is intimately related to the type of contractual arrangement offered by the firm. Moreover, as we make clear in the following section, match quality plays a key role in determining which contract is offered to workers. This 'contractual sorting' based on match quality has important consequences for wage dynamics, as different contractual arrangements exhibit different cyclical properties.

Which Contract is Offered by the Firm?

Given high aggregate productivity in period 1, we compare the expected profits that firms achieve (over period 1 and 2) by offering each of the three contractual

[10] To see this note that $[a(P_H) - a(P_L)] < [P_H - P_L]$ implies that $b < 1$ under a DPC contract. Note that, as we do for SPC contracts, we do not allow for DPC contracts with $b > 1$, as this would imply that workers can leverage the production risk.

arrangements: fixed wage, spot, or a performance pay contract. We first consider the case of $\frac{\Delta a(P)}{\Delta P} < 1$, in which the feasible performance pay contract is DPC; then we examine the case of $\frac{\Delta a(P)}{\Delta P} > 1$, when SPC is the feasible performance pay contract. We conduct pairwise comparisons between any two contracts and show that a simple threshold rule, based on match quality m, determines the contract offered by the firm. Finally, we rank these thresholds and show that performance pay contracts are consistently preferred for sufficiently high levels of match quality m.

Match-quality thresholds with DPC performance pay contracts. In what follows we derive the match-quality thresholds that identify which contract is preferred in pairwise comparisons. Substituting the wage functions for the three possible contracts (DPC performance pay, spot, fixed wage) we can write firms' expected profits as,

$$\text{DPC:} \quad E\left[\pi^{DPC}\right] = (1+q)\left(P_H - a(P_H)\right)m + (1-q)\left(P_L - a(P_L)\right)m - \kappa(m^{max} - m)$$

$$\text{SPOT:} \quad E\left[\pi^{SPOT}\right] = (1+q)\left(P_H - a(P_H)\right)m + (1-q)\left(P_L - a(P_L)\right)m - (1-q)T$$

$$\text{FW:} \quad E\left[\pi^{FW}\right] = (1+q)P_H m + (1-q)P_L m - 2a(P_H)m.$$

By pairwise comparison of expected profits, one can characterize the threshold conditions that describe the contractual choice of the firm. We do this in Proposition (10).[11]

Proposition 10. *If $\frac{\Delta a(P)}{\Delta P} < 1$, the contract choice of the firm is described by the following threshold rule.*

[11] In Appendix A.5.3 we show that a modified version of Proposition (10) holds also when $K(m) = K, \forall m$.

1. The firm prefers a performance pay contract over a spot market contract if

$$m \geq \frac{\kappa m^{max} - T(1-q)}{\kappa} \equiv \underline{m}_1. \tag{3.13}$$

2. The firm prefers a performance pay contract over a fixed wage contract if

$$m \geq \frac{\kappa m^{max}}{\kappa + (1-q)(a(P_H) - a(P_L))} \equiv \underline{m}_2. \tag{3.14}$$

3. The firm prefers a spot contract over a fixed wage contract if

$$m \geq \frac{T}{a(P_H) - a(P_L)} \equiv \underline{m}_3. \tag{3.15}$$

Proofs are in Appendix Section A.5.2.

The firm's contract choice outlined in Proposition 10 has a simple interpretation. The threshold $\underline{m}_3$ is a function of adjustment costs in period 2. Under fixed wages there are no adjustment costs, but the firm subsidizes ('overpays') the worker relative to a spot contract if aggregate productivity is lower in period 2. On the other hand, under the spot contract, lowering the wage in period 2 entails a fixed cost T. This cost-benefit tradeoff varies with match quality, and is reflected in different contract choices for different match qualities. A similar intuition applies to threshold $\underline{m}_2$: a fixed wage contract features a subsidy to the worker in bad times, but implementing a performance pay contract entails a cost $K(m)$.[12]

Crucially, these thresholds can be ordered, as outlined in Corollary 10.1.

[12] We note that performance pay and spot contracts exhibit the same wages; only differences in the implementation costs differentiate the profits that accrue to the firm from each of these contracts.

Corollary 10.1. *If the adjustment cost T is sufficiently small, then $\underline{m}_1 \geq \underline{m}_2 \geq \underline{m}_3$ and the following holds:*

- $\forall m \geq \underline{m}_1$*, the firm offers a performance pay contract;*
- $\forall m \in [\underline{m}_3, \underline{m}_1[$*, the firm offers a spot contract;*
- $\forall m < \underline{m}_3$*, the firm offers a fixed wage contract.*

Otherwise, if T is not small enough, $\underline{m}_3 > \underline{m}_2 > \underline{m}_1$ and the contractual choice of the firm is:

- $\forall m \geq \underline{m}_2$*, the firm offers a performance pay contract;*
- $\forall m < \underline{m}_2$*, the firm offers a fixed wage contract.*

These results suggest that profits grow relatively faster with match quality if firms offer performance pay contracts. It follows that there exists a match quality above which performance pay contracts deliver higher profits than other contracts.[13] By the same logic, for sufficiently low match quality, revenues do not cover the implementation costs of performance pay and spot contracts. As a result, fixed wages become the most profitable pay arrangement in lower productivity matches.[14] Finally, whether or not spot contracts are ever implemented, depends on whether the cost of implementing the contract, T, is sufficiently low. An immediate implication of these findings is that matches with relatively high productivity should adopt a performance pay contract. On the other hand, jobs with low match quality are more likely to adopt a fixed wage arrangement. Wage cyclicality is

[13] We posit that match quality can take values high enough for this to happen.

[14] This result is conditional on the subsidy given to the worker in a fixed wage contract, Δa, not being too big. To see this, note that if $\Delta a \to \infty$, then $\underline{m}_2 \to 0$ and $\underline{m}_3 \to 0$, which implies fixed wages are never implemented.

affected by the contract choice in an obvious way as spot and performance pay arrangements imply pro-cyclical wages while fixed pay contracts do not. As a result of contract choice, there exists a relationship between match quality and wage cyclicality.

Next, we turn to the case in which $\frac{\Delta a(P)}{\Delta P} > 1$ and SPC contracts are feasible, and we show that the same qualitative conclusions can be drawn.

Match-quality thresholds with SPC performance pay contracts. Substituting the wage functions for the three possible contracts (SPC performance pay, spot, fixed wage) we can write the firm's expected profits as,

$$\text{SPC:} \quad E\left[\pi^{SPC}\right] = 2\left(P_H - a(P_H)\right)m - \kappa(m^{max} - m).$$

$$\text{SPOT:} \quad E\left[\pi^{SPOT}\right] = (1+q)\left(P_H - a(P_H)\right)m + (1-q)\left(P_L - a(P_L)\right)m - (1-q)T$$

$$\text{FW:} \quad E\left[\pi^{FW}\right] = (1+q)P_H m + (1-q)P_L m - 2a(P_H)m.$$

Proceeding as before, we show that the firm's contract choice follows a simple threshold rule.

Proposition 11. *If $\frac{\Delta a(P)}{\Delta P} > 1$, the contract choice of the firm is described by the following threshold rule.*

1. *The firm prefers a performance pay contract over a spot market contract if*

$$m \geq \frac{\kappa m^{max} - T(1-q)}{\kappa - (1-q)[(a(P_H) - a(P_L)) - (P_H - P_L)]} \equiv \underline{m}_4. \tag{3.16}$$

2. *The firm prefers a performance pay contract over a fixed wage contract if*

$$m \geq \frac{\kappa m^{max}}{\kappa + (1-q)(P_H - P_L)} \equiv \underline{m}_5. \tag{3.17}$$

3. *The firm prefers a spot contract over a fixed wage contract if*

$$m \geq \frac{T}{a(P_H) - a(P_L)} \equiv \underline{m}_6. \tag{3.18}$$

The intuition for the results in Proposition 11 also relates to the varying costs and benefits for different match-specific productivities. The threshold $\underline{m}_6$ is exactly the same as $\underline{m}_3$ in the DPC case and has the same interpretation. Similarly, the intuition for $\underline{m}_5$ is the same as the one we discussed for $\underline{m}_2$: a fixed wage contract 'overpays' workers in bad states of the world but has no implementation costs. In contrast, a performance pay contract entails a cost $K(m)$ but does not subsidize workers. Finally, performance pay is preferred to spot contracts for high enough match quality m because profits grow faster with m under performance pay arrangements, which explains Corollary 11.1.

Corollary 11.1. *If the adjustment cost T is sufficiently small, then $\underline{m}_4 \geq \underline{m}_5 \geq \underline{m}_6 >$ and the following holds:*

- $\forall m \geq \underline{m}_4$, *the firm offers a performance pay contract;*
- $\forall m \in [\underline{m}_6, \underline{m}_4[$, *the firm offers a spot contract;*
- $\forall m < \underline{m}_6$, *the firm offers a fixed wage contract.*

Otherwise, if T is not small enough, $\underline{m}_6 > \underline{m}_5 > \underline{m}_4$ and the contractual choice of the firm is:

- $\forall m \geq \underline{m}_5$, *firms offer a performance pay contract*
- $\forall m < \underline{m}_5$, *firms offers a fixed wage contract*

Brief discussion and empirical implications. Propositions 10 and 11, and their corollaries, suggest that high productivity matches are more likely to adopt performance pay contracts, exhibit higher pay and have more cyclical wages.

Our stylized model describes the firm's retention problem over a fictitious three-periods interval, while real work relationships often extend over long horizons. Given enough time, new information may accrue and perturb the original arrangements, possibly leading to renegotiations and separations, about which the model is silent. However, if the contractual sorting implied by heterogeneous match quality is in fact due to retention motives, one might expect that different contracts have different implications for job durations. We explicitly examine this hypothesis in the empirical analysis.

3.3 Data and Measurement

Our model of pay highlights the relationship between match quality and contract choice. Empirically linking contractual sorting, wage cyclicality and match quality poses several measurement issues. To identify the effects of match-specific heterogeneity on contractual arrangements and wage dynamics one needs to: (i) establish an empirical counterpart of the wage process and control for possible confounding effects; (ii) outline a procedure to approximate match quality using data; (iii) identify jobs in which pay is linked to output through some form of performance-related arrangement.

In this section we describe the key features of our empirical approach. We proceed sequentially. First, we outline the empirical counterpart of the theoretical wage processes. Second, we show how match quality proxies can be constructed using information about labor market tightness. Third, we describe data sources and highlight how theory guides the data organization. Finally, we discuss how we can identify jobs featuring performance-related pay.

3.3.1 Empirical Wage Processes

One can show that the empirical counterparts of the different pay arrangements examined above can all be nested within one general wage representation. This wage representation is obtained through simple log-linear approximations. We begin by noting that, in addition to the specific mechanism outlined in the theoretical section, wages obviously are affected by other individual and job characteristics. Hence, allowing for an additively separable vector of characteristics X, the following proposition holds.

Proposition 12. *Let workers be paid according to one of the four possible contractual arrangements (DPC, SPC, FW, or Spot). Assume that: (a) X_t is a log additive component to the wage that captures observable worker characteristics; (b) z_{ijt} is an approximation error. Then the conditional expectation of the wage, under any of the contracts, can be generally represented as*

$$E[\log(w_{ijt})|P_t, m_i j, X_t] = \beta_0 + \beta_1 \log(m_{ij}) + \beta_2 \log(P_t) + \beta_3 \log(X_t) + E[z_{ijt}] \tag{3.19}$$

where i identifies a worker, j identifies a job, t denotes the time period and $E[z_{ijt}]$ is the expectation of the unobserved residual implied by the approximation error. In the case of a fixed wage contract $\beta_2 = 0$, while $\beta_2 > 0$ for other contracts. Under all contracts $\beta_1 > 0$.

The proof is obtained by log-linearization of the various wage functions. Details are in Appendix A.7.

We consider a simple representation of the unobserved residual productivity z_{ijt}. Specifically, we assume that z_{ijt} consists of an individual fixed effect a_i and an i.i.d. shock η_{ijt}. In our empirical specification we explicitly account for observable heterogeneity, for time effects and for worker fixed effects. As a result, the empirical specification for the wage processes is

$$log(w_{ijt}) = \beta_0 + \beta_1 log(m_{ij}) + \beta_2 log(P_t) + \beta_3 log(V_{ijt}) + z_{ijt}, \tag{3.20}$$

with $\beta_2 = 0$ in the case of a fixed wage contract.

Following Bils [1985], and a large subsequent literature, we focus on the sensitivity of wages to fluctuations in aggregate unemployment to capture wage cyclicality.

The theoretical analysis suggests that match quality plays a key role for the cross-sectional distribution of wages and their cyclicality. Match quality influences wages directly and through contractual sorting effects. In particular, wage sensitivity to contemporaneous aggregate conditions depends on the type of pay arrangement in place and, therefore, on match quality. In the next section we describe how we approximate match-specific quality.

3.3.2 Measuring Match Quality

The match quality proxies are constructed following the approach of Hagedorn and Manovskii [2013] and build on the idea that changes in labor market tightness have a direct bearing on the match quality distribution. The two proxies (respectively denoted as q^{eh} and q^{hm}) rely on the assumption that the number of offers a worker receives is positively correlated with match quality. If an employed worker receives a job offer and accepts it, then it must be the case that match quality has a good chance of being weakly improved. Similarly, if a worker receives a job offer and rejects it, then current match quality is more likely to be preferable to the alternative. Hence a worker who receives many offers has, on average, better match quality, whether these offers were accepted or rejected. The basic empirical challenge is how to measure the number of offers a worker receives. The reasoning above suggests that labor market tightness, measured before and during a particular job, conveys information about the number of offers. As an example consider a worker i employed in the same job between periods T_{begin} and T_{end}, with $T_{end} > T_{begin}$. If the sum of labor market tightness between T_{begin} and T_{end} is high, and we observe i staying at her job, then i received and rejected relatively many job offers. Therefore i's job must have high match quality. Following this logic, the variable $q^{hm}_{i,j}$ is defined as

$$q^{hm} = \sum_{t=T_{begin}}^{T_{end}} \left(\frac{V_t}{U_t} \right), \tag{3.21}$$

where V_t is an index of vacancies and U_t is the unemployment rate in period t.

The same line of reasoning implies that match quality in the current job is also sensitive to market tightness during employment periods preceding the current job. In the example above suppose that worker i had a different job prior to the current one. Moreover, while working on the previous job the labor market was tight and she received many offers. The fact that she received many offers before accepting the current job suggests that the quality of the current match is likely to be relatively high. Hence past labor market tightness conveys information about current match quality. The variable $q^{eh}_{i,j}$ is meant to capture past labor market conditions and is defined as,

$$q^{eh} = \sum_{t=T_1}^{T_{begin}} \left(\frac{V_t}{U_t} \right), \tag{3.22}$$

where $T_1 < T_{begin}$ denotes the first period of the employment cycle, that is, the first period of work after involuntary unemployment.[15]

3.3.3 Data on Work Histories

The data source for wages is the National Longitudinal Survey of Youth (NLSY79). We construct the (weekly) job history for each worker and identify an observation as the wage of a worker at the current job.[16] We construct the current unemployment rate using the seasonally adjusted unemployment series from the Current Population Survey (CPS). We use the Composite Help Wanted Index constructed by Barnichon [2010] as a measure of vacancies. Details about data are in Appendix A.5.1. All of the analysis focuses on men between 25 to 55 years old.

Key to the analysis is the concept of employment cycles. An employment cycle is defined as a continuous spell of employment, possibly entailing a sequence of jobs and employers. The cycle begins in the period when the worker transitions from non-employment to employment, and ends when the worker transitions back to involuntary non-employment.[17]

To measure individual employment cycles, and job spells within each cycle, we follow Wolpin [1992], Barlevy [2008], and Hagedorn and Manovskii [2013]. At

[15]The interval between T_1 and T_{end} must not be interrupted by involuntary unemployment spells, as this would make it hard to argue for sequential on-the-job renegotiations.

[16]For each week we define the 'main job' as the one with the highest mode of reported hours worked. Past research focuses on male workers. For comparability we follow this convention.

[17]As in Barlevy [2008] and Hagedorn and Manovskii [2013] a separation is considered voluntary if (i) the worker reports a quit, rather than a layoff; and (ii) the interval between the end of the previous job and the beginning of the next is shorter than 8 weeks. Employment cycles may include short periods of non-employment.

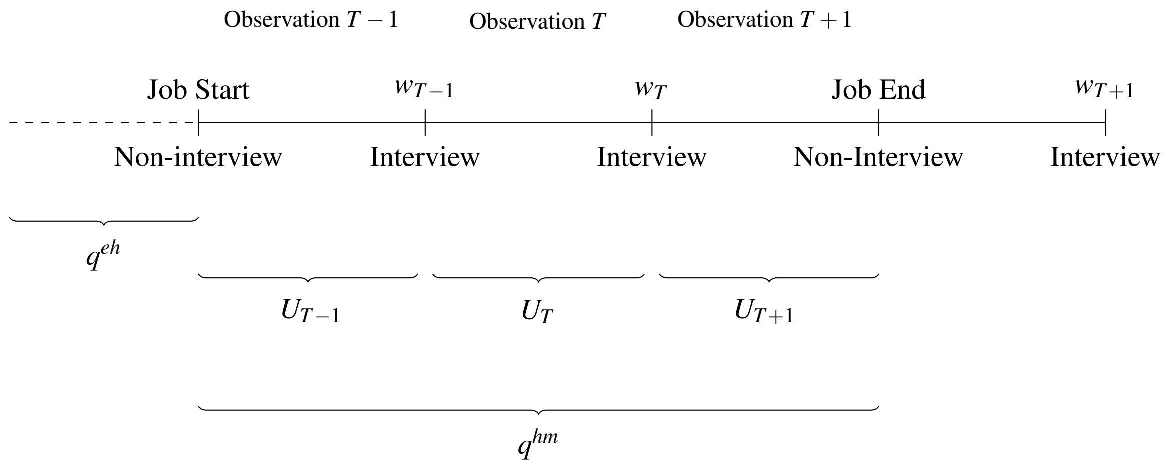
each interview date the NLSY provides a complete description of jobs held since the last interview, including start and stop dates (week), wage, hours worked, and occupation. In addition one can link employers across interviews and identify a job as a worker's spell with a given employer.

In the NLSY79 the information related to a specific job is only recorded once per interview. Therefore wage changes within a job are recorded only if an individual works at the same job for a period covered by two or more interviews, implying that within-job wage variation is identified using jobs that extend over at least two NLSY interview dates. If a job appeared for the first time in the year T interview, and again in the year $T+1$ interview, then this job counts as two observations within the same employment cycle. Each observation is a wage-job pair. The wage refers to a job that was active at any time between the current and the previous interview date. Thus we view an observation (a wage-job pair) as the wage prevailing over the period between two successive interviews while employed at a particular job, or in any subset of that period during which the job was active.

For illustration consider the example in Figure 3.1. A worker is interviewed at date $T-2$, begins to work for a specific employer between $T-2$ and $T-1$, is interviewed again at $T-1$, T, and $T+1$, but eventually stops working for this employer at some point between T and $T+1$. Given this sequence of events, we use the wage w_{T-1}, recorded during the first interview, as the wage applying to the period between the start of the job and $T-1$. Similarly, we use the wage w_T for the period between $T-1$ and T, and the wage w_{T+1} for the period between T and the end of the job.

Partitioning the data into employment cycles and job spells allows us to construct the match quality proxies described in Section 3.3.2. We use data on aggre-

Figure 3.1: Employment Cycles: an Example.



gate vacancies and unemployment to calculate tightness ratios $\frac{V_t}{U_t}$ and define: (i) q^{eh} as the sum of tightness ratios from the beginning of the employment cycle to the period preceding the start of the current job; (ii) q^{hm} as the sum of market tightness ratios during a job spell. The latter captures past, current and future tightness over the current job spell and reflects the expected match quality of that particular job.

Next, we assign to each observation a contemporaneous unemployment rate, measured as the average unemployment recorded over the period in which a job is active between consecutive interview dates. Figure 3.1 illustrates how match quality proxies and unemployment rates are assigned to different observations w_{T-1}, w_T and w_{T+1}: q^{eh} is the sum of labor market tightness from the start of the employment cycle until the start of the current job; q^{hm} is the sum of labor market tightness from the start to the end of the current job. A different contemporaneous unemployment rate applies to each relevant time interval.

3.3.4 Performance Pay in the NLSY79

The NLSY79 reports partial information about performance pay for the years 1988 to 1990, 1996, 1998 and 2000. For years $1988 - 1990$ individuals were asked whether, in their most current job, earnings were partly based on performance. For years $1996, 1998, 2000$, individuals were asked for each of their jobs if earnings featured any of the following types of compensation: piece rate, commission, bonuses, stock options and/or tips. Therefore in $1996, 1998, 2000$, for each job-individual pair we generate a binary variable indicating if that particular type of compensation was used in determining the pay received for that job. A performance pay observation is then a job-year-individual triplet for which one of following conditions is satisfied:

- The year is $1988, 1989$ or 1990, and the individual reports being paid based on performance;
- The year is $1996, 1998$ or 2000 and the individual reports having earnings based on at least one among tips, commission, bonuses or piece rate.
- It is a job-year-individual triplet pertaining to a job/individual pair that satisfies one of the above two conditions for at least one of the interviews. This imposes the restriction that the performance pay status is constant within a job, adding observations for the years in which the performance pay variables are not available.

3.4 Empirical Results

In this section we report our main empirical findings. Specifically, we present results documenting that (i) a significant relationship exists between match quality and contractual arrangements; (ii) contractual arrangements play a key role in determining wage cyclicality; (iii) employment durations vary with contractual arrangements (and match quality) as predicted by theory; (iv) occupations that exhibit higher average match quality tend to adopt performance-pay more frequently; wages in such occupations appear to be more cyclical, as predicted by our model. Finally, we discuss some extensions and robustness checks.

3.4.1 Match Quality and Performance Pay Adoption

An immediate implication of our theoretical analysis is that firms offer different pay arrangements depending on match quality. Corollaries (10.1) and (11.1)) imply that high quality matches should exhibit a higher adoption of performance-related pay schemes.

Given the information available in our sample, we can directly estimate the empirical relationship linking each job's PPJ status to its match quality proxies. We do this by using a set of Logit models. The unit of observation for this analysis is the job-worker pair, with the dependent variable being a binary indicator for whether the job uses any performance related compensation and the key right-hand side variables being measures of match quality. We estimate a fixed effect specification to control for worker unobserved heterogeneity and restrict the sample to men between ages 25 and 55.[18] We also control for a variety of observable job-worker characteristics.[19]

In Table 3.1 we report the results of this analysis for three alternative specifica-

[18]The sampling restrictions implicit in the fixed-effect Logit estimator imply that our sample only includes workers who are observed at least once in both PPJ and non-PPJ, at different points in time.

[19]We include controls for year, geographic and SMSA region, job tenure with current employer, work experience, industry, marital status, education, age (maximum in the employment spell), union status.

tions in which we control for each measure of match quality, both separately and together.

Table 3.1: Performance Pay and Match Quality: Fixed Effects Logits

	Specification		
Variables	(1)	(2)	(3)
$log(q^{eh})$	18.4***	-	19.9***
	[7.68]	-	[7.73]
$log(q^{hm})$	-	52.9***	54.6***
	-	[1.84]	[1.85]
Observations	2,028	2,058	2,028

Note a. The notation $\ln q^x$, with $x = \{hm, eh\}$, denotes the natural logarithm of the sum of market tightness

Note b. Estimated coefficients and associated standard errors are multiplied by 100. All standard errors are clustered by observation start-date and end-date. Results are robust to clustering by individual. Significance: *** 1%, ** 5%, * 10%.

Note c. The sample includes male workers between age 25 and 55. We include controls for year, job tenure with current employer, work experience, geographic and SMSA region, industry, marital status, education, age (maximum in the employment spell), union status.

The results clearly indicate the presence of a significant, and sizeable, relationship between match quality and performance pay adoption. Both proxies of match quality are highly significant, and the magnitudes of their effects remain unchanged when they are both included.

To gauge the magnitude of the match quality effects we compute the change in the probability of being PPJ implied by a one standard deviation increase in match quality. To this purpose, we generate a random subsample of worker-job pairs such that each worker is sampled only once, and use it to measure the baseline probability that an individual-job pair exhibits performance pay. This exercise returns an average probability of 38.7%. Then, we perturb each individual match quality and make it larger by one standard deviation. This results in an average likelihood of PPJ equal to 54.6%. Hence, our results suggest that a one-standard-deviation change in match quality is associated to an increase of over 40% in the probability of being in a performance pay job. Replicating this analysis for the median probability of PPJ suggests an increase from a baseline value of 26.4% to 44.5%. These are large effects, and clearly indicate that match quality and performance pay are strongly associated. We confirm the robustness of this association in Section 3.4.5.

As we discuss below, this strong association between match quality and contractual choice has important implications for wage cyclicality and job durations.

3.4.2 Match Quality and Wage Cyclicality

A second, crucial implication of our theoretical analysis is that selection into different contractual arrangements has an indirect effect on the cyclicality of wages. As mentioned above, we follow an extensive literature and measure the cyclicality of wages with respect to labor market conditions by gauging wage responses to aggregate unemployment.

We use the baseline (log-linearized) approximation derived in Section 3.3.1 to estimate how the sensitivity of log wages depends on the current unemployment rate, and on both match quality proxies. The unit of observation for this analysis is the wage observed for a job-worker pair at a point in time. We use a fixed effect specification and, as before, also control for a full set of observable job and worker characteristics.[20] The model suggests that there should be a direct effect of match quality proxies on wages. Moreover, as shown above, match quality also has a strong, indirect effect on wages by determining the contractual arrangement in the job-worker relationship. This contractual selection effect has a variety of testable

[20] We control for all variables used in the linear probability model. For workers we use current age, rather than maximum age, to allow for within job age profiles.

implications. Namely, we use our general empirical specification (equation 3.20, derived in Section 3.3.1) to test the following theoretical predictions:

(i) Do performance pay jobs (PPJ) exhibit positive cyclicality?

(ii) Is any cyclicality detected among non-PPJ?[21]

(iii) Does match-quality have a direct effect on wages after controlling for PPJ status?

We begin by documenting the properties of the pooled sample of jobs (both PPJ and non-PPJ). Table (3.2) reports results from the analysis of such pooled data. The first column reports results for a specification in which wages depend on unemployment, without controlling for match quality (this is the kind of regression originally suggested by Bils, 1985). In the second column we add controls for match quality as well as cyclical responses to the unemployment rate. In the third column we extend the model by allowing for different cyclical responses depending on PPJ status.

Results suggest that match quality has a direct effect (level shift) on wages, as predicted by the model and illustrated in Section 3.3.1. The sensitivity of wages to cyclical unemployment is however similar with or without quality controls, with a gradient of roughly 1.6%. Yet, our results also indicate that all the cyclical sensitivity of wages is due to PPJ status: column 3 shows that only wages in performance-pay jobs exhibit cyclical responses to the unemployment rate. Moreover, these responses are much stronger than in the pooled sample. A 1% increase in the unemployment rate is associated to a 3% decrease in average wages for PPJ, and to no significant wage change in non-PPJ.

[21] Such cyclicality could occur if the cost T of implementing spot contracts is sufficiently small that firms offer them to a large enough share of workers.

Table 3.2: Pooled wage regression

Dependent Variable: Log Wage			
Variables	(1) (Bils specification)	(2) (add match quality)	(3) (add match quality)
U	-0.0164***	-0.0167***	-0.004
	[0.0043]	[0.0042]	[0.005]
$log(q^{eh})$	-	7.59***	7.47***
	-	[0.66]	[0.66]
$log(q^{hm})$	-	6.81***	6.70***
	-	[0.66]	[0.68]
$U \cdot PPJ$	-	-	-0.0298***
	-	-	[0.0064]
Observations	17.995	17,434	17,434
R-squared	0.642	0.646	0.646

Note a. The notation $\ln q^x$, with $x = \{hm, eh\}$, denotes the natural logarithm of the sum of market tightness. The explanatory variable $U \cdot PPJ$ is the interaction between current unemployment rate and an indicator function taking value equal to one if the job includes performance-related compensation.

Note b. Estimated coefficients for $\ln q^{eh}$ and $\ln q^{hm}$, and associated standard errors, are multiplied by 100. All standard errors are clustered by observation start-date and end-date. Results are robust to clustering by individual. Significance: *** 1%, ** 5%, * 10%.

Note c. The sample includes male workers between age 25 and 55. We include controls for year, job tenure with current employer, work experience, geographic and SMSA region, industry, marital status, education, age and union status.

Taken together, these results are consistent with the view that match quality has a strong indirect effect on pay by selecting workers into different contractual arrangements, indirectly affecting wage cyclicality. To explicitly test this hypothesis, we perform the same analysis separately on PPJ and non-PPJ jobs. This allows to flexibly control for observables in the two groups. Table (3.3) reports estimation results for different PPJ status.

The findings confirm that strong and significant wage cyclicality is present in jobs where performance-related pay is used. In fact, the magnitudes of the cyclical response of PPJ wages is almost identical to the one estimated from the pooled

Table 3.3: Wage regressions: PPJ vs non-PPJ.

Variables	(1) $PPJ = 1$ (Bils specification)	(2) $PPJ = 0$ (Bils specification)	(3) $PPJ = 1$ (add match quality)	(4) $PPJ = 0$ (add match quality)
U	-0.0283***	-0.0089	-0.0282***	-0.0096
	[0.0056]	[0.0063]	[0.0056]	[0.0064]
$\ln q^{eh}$	-	-	9.88***	6.12***
			[1.43]	[0.974]
$\ln q^{hm}$	-	-	8.79***	5.94***
			[1.50]	[0.892]
Observations	7,280	10,715	7,065	10,369
R-squared	0.719	0.613	0.723	0.614

Note a. The notation $\ln q^x$, with $x = \{hm, eh\}$, denotes the natural logarithm of the sum of labour market tightness

Note b. Estimated coefficients for $\ln q^{eh}$ and $\ln q^{hm}$, and associated standard errors, are multiplied by 100 for $\ln q^x$. All standard errors are clustered by observation start-date and end-date. Results are robust to clustering by individual. Significance: *** 1%, ** 5%, * 10%.

Note c. The sample includes male workers between age 25 and 55. We include controls for year, geographic and SMSA region, industry, marital status, education, age and union status.

sample (-.0282 vs -0.0298 in column 3 of Table 3.2). As before, wages seem not to respond to cyclical unemployment in jobs with no performance related pay. When we test for the significance of the difference between the cyclical gradient of PPJ and non-PPJ we reject the null hypothesis of equal coefficients at the 5% confidence level.

These results document that match quality has a direct effect on wages even after we control for contractual arrangements (PPJ status). The match quality effect is positive as expected in all cases. Hence, higher match quality is associated to higher wages and, on average, to stronger cyclical sensitivity.

3.4.3 Evidence from Occupation Groups

As highlighted in our discussion of match quality, we expect tighter labour markets to be associated to a higher frequency of job offers to workers, which in turn translates into higher average match quality.

This line of reasoning has an interesting implication: the adoption of performance pay should be more widespread in occupations which are in high demand. The reason for this is that retention considerations (participation constraints) induce firms to use variable compensation as a way to keep workers when they are most in demand. This argument suggests that employee profit-sharing or other forms of performance-related pay should be relatively more attractive in occupations which are in strong demand. This is clearly the case of cognitive and non-routine jobs over the past few decades, as documented for example by Autor and Dorn [2013] and Cortes et al. [2015].

In this section we document that occupations that are in higher demand exhibit larger frequency of performance pay jobs and better match quality.

Table 3.4 reports two important dimensions of heterogeneity across occupation groups: (i) the relative frequency of PPJ; (ii) the relative share of above-median match qualities. Cognitive occupations have a considerably higher occurrence of both PPJ and of above-median match quality, when compared to manual occupations. A similar, but less marked difference, is present when comparing non-routine and routine occupations.

These differences are highly significant and lend direct support to the view that, especially in cognitive occupations, stronger demand is associated to relatively higher match qualities and more frequent recourse to performance pay. Of

Table 3.4: Occupation heterogeneity: share of jobs with (i) above median match quality and (ii) performance pay, by occupation group.

	Occupation Groups			
	COG	MAN	NR	R
Share PPJ	40.7%	23.91%	33.15%	29.21%
	(49.13%)	(42.66%)	(47.09%)	(45.48%)
q^{eh} above median	54.54%	46.82%	51.93%	48.67%
	(49.8%)	(49.9%)	(50%)	(50%)
q^{hm} above median	57.67%	43.57%	55.41%	45.21%
	(49.42%)	(49.59%)	(49.72%)	(49.78%)
Observations	2,433	3,475	2,413	3,495

Note a. Top panel: share of jobs with performance pay arrangements (Share PPJ) for coarse occupation groups: cognitive vs manual jobs (COG vs MAN); routine vs non-routine jobs (R vs NR). Standard deviations in parentheses (also as shares).

Note b. Bottom panel: share of jobs with match quality above the unconditional median for coarse occupation groups: cognitive vs manual jobs (COG vs MAN); routine vs non-routine jobs (R vs NR). First line based on q^{eh} match quality proxy; second line based on q^{hm} match quality proxy.

course, contractual sorting across occupation might have direct effects on the cyclicality of wages in different occupations, an implication that we investigate in the next section.

Wage Cyclicality across Occupations

Our theory suggests that compensation arrangements are key for the sensitivity of wages to current unemployment. For this reason we re-estimate the general wage specification (equation 3.20) for different occupation groups. To retain reasonably large, and comparable, sample sizes we focus on broad occupation categories (cognitive vs manual jobs; non-routine vs routine jobs).

Columns (1) and (2) in Table 3.5 report results obtained for, respectively, the samples of cognitive (Cog) and manual (Man) occupations. While we detect positive, strong and significant responses of wages to current unemployment in cog-

Table 3.5: Wage Regressions: Cyclicality by Occupation Group.

	Dependent Variable: Log Wage			
	(1)	(2)	(3)	(4)
Variables	COG	MAN	NR	R
U	-0.0245**	-0.0041	-0.0204*	-0.0098
	[0.0110]	[0.0059]	[0.0118]	[0.0066]
$\ln q^{eh}$	5.43***	6.51***	4.49***	5.57***
	[1.34]	[1.0]	[1.50]	[0.943]
$\ln q^{hm}$	6.68***	8.16***	7.44***	6.60***
	[1.23]	[0.842]	[1.34]	[0.860]
Observations	7,495	6,123	6,978	6,640
R-squared	0.611	0.705	0.650	0.709

Note a. The notation $\ln q^x$, with $x = \{hm, eh\}$, denotes the natural logarithm of the sum of labour market tightness

Note b. Estimated coefficients and associated standard errors are multiplied by 100 for $\ln q^x$. All standard errors are clustered by observation start-date and end-date. Results are robust to clustering by individual. Significance: *** 1%, ** 5%, * 10%.

Note c. The sample includes male workers between age 25 and 55. We include controls for year, geographic and SMSA region, industry, marital status, education, age and union status.

nitive occupations, no significant effect can be detected for manual jobs. Columns (3) and (4) in Table 3.5 report results of the same regressions for an alternative occupation grouping in which jobs are split between non-routine (NR) and routine occupations (R). For non-routine jobs we find weaker evidence (at the 10% confidence level) of wage cyclicality. Results for routine jobs indicate that current labor market conditions have no detectable effect on wages.

Taking stock of all these results, we conclude that there are visible discrepancies in the wage-unemployment relationship across occupation groups. In manual and routine jobs the current labor market conditions (as captured by the current unemployment rate) have no gradient on wages. However we find evidence that wages in cognitive occupations are strongly cyclical, while non-routine jobs exhibit a somewhat weaker and less significant cyclicality. To the extent that match quality is higher, and performance pay more widespread, among cognitive and non-routine occupations, these results offer further evidence that contractual sorting may have an important role in determining the cyclical behavior of wages.

3.4.4 Performance Pay and Job Durations

Our model highlights the role of worker retention for the adoption of performance pay. However, given its stylized nature, it has no direct implications for the duration of jobs, as all pay arrangements satisfy the participation constraints when a contract is offered. Nonetheless, if the retention motive is, in fact, one of the main reasons for introducing performance-related pay, one might suppose that a relationship exists between PPJ and job durations. We examine this possibility by checking whether: (i) job durations are higher in PPJ than in non-PPJ; (ii) job durations are higher in occupations in higher demand.

These relationships are fairly easy to test using job histories from the NLSY79, as we can construct the duration of each worker's tenure with a given employer. In Table (3.6) we report the mean and standard deviation of job durations for different groups in our NLSY79 sample. We find that all duration differences are well above one year (five quarters or more).[22] All differences (PPJ vs. non PPJ, cognitive vs. manual, routine vs. non-routine) are extremely significant at levels well below 1%.

These findings confirm that PPJ jobs, or occupations in higher demand (in which PPJ is more prevalent), exhibit higher job durations. Hence they provide direct evidence that the adoption of alternative contractual arrangements is closely linked to retention outcomes.

[22]Durations in Table (3.6) refer to a sample of workers with relatively strong labor market attachment and are higher than durations for the overall population.

Table 3.6: Summary statistics of job durations in different occupation groups.

	Mean	S.D.	Observations
PPJ=1	26.4	27.7	2,738
PPJ=0	18.4	23.3	5,823
COG	22.8	24.3	2,492
MAN	17.4	21.8	3,570
NR	22.3	24.3	2,460
R	17.8	21.9	3,602

Job durations are measured in quarters. Cog = cognitive, MAN = manual, NR = non-routine, R = routine. Unit of observation is a job/year pair.

3.4.5 Extensions and Robustness

In what follows we replicate the analysis for some alternative specifications to gauge the robustness of our findings. First, we verify that the key predictions of the model, and baseline empirical results, are robust to the inclusion of working women in our samples. Second, we estimate a simple linear probability model linking PPJ status to match quality proxies, and show that a positive relationship continues to hold. Third, we document that the main result about wage cyclicality remains intact even when we use GDP variation, rather than unemployment, to proxy for cyclical conditions. Finally, we split workers into different education groups to assess whether the cyclicality of wages across education groups lines up with the relative frequency of PPJ across these groups.

Extending the sample to include women. Our baseline results are based on a sample of male workers. This restriction was introduced to facilitate comparisons to previous work on the cyclicality of wages. In what follows we extend the sample by adding women. We maintain all the sampling restrictions described in Section 3.3.3 and Appendix A.5.1, which guarantee a sample with fairly strong labor market attachment.

We begin by replicating the Logit analysis linking PPJ status to match quality proxies. Table (3.7) shows that also in the expanded sample there exists a strong, positive and significant relationship between probability of being in a performance pay job and match quality. Both men and women exhibit an increased likelihood of performance-related pay when match quality is higher. Magnitudes are broadly comparable to the ones estimated for the sample on male workers and reported in Table (3.1).

Table 3.7: Performance Pay and Match Quality: Fixed Effects Logits (men and women)

			Specification
Variables	(1)	(2)	(3)
$log(q^{eh})$	14.6***	-	15.7***
	[5.55]	-	[5.59]
$log(q^{hm})$	-	67.3***	66.0***
	-	[1.36]	[1.37]
Observations	3,635	3,691	3,635

Note a. The notation $\ln q^x$, with $x = \{hm, eh\}$, denotes the natural logarithm of the sum of market tightness

Note b. Estimated coefficients and associated standard errors are multiplied by 100. All standard errors are clustered by observation start-date and end-date. Results are robust to clustering by individual. Significance: *** 1%, ** 5%, * 10%.

Note c. The sample includes female and male workers between age 25 and 55. We include controls for year, geographic and SMSA region, industry, marital status, education, age and union status.

Next, having verified the significance of this positive relationship, we move on to replicate the wage cyclicality analysis presented in Tables (3.2-3.3) using the extended sample. Table (3.8) reports the regression results for a fixed effect specification based on the pooled sample of all jobs, whether PPJ or not. Then, Table (3.9) shows the estimation results when the estimator is run separately in PPJ and non-PPJ jobs.

Table 3.8: Pooled wage regression (men and women)

Dependent Variable: Log Wage			
Variables	(1) (Bils specification)	(2) (add match quality)	(3) (add match quality)
U	-0.0120***	-0.0121***	-0.0026
	[0.0045]	[0.0044]	[0.0051]
$log(q^{eh})$	-	6.15***	6.06***
	-	[0.56]	[0.509]
$log(q^{hm})$	-	6.62***	6.44***
	-	[0.47]	[0.483]
$U \cdot PPJ$	-	-	-0.0298***
	-	-	[0.0064]
Observations	34,050	33,043	33,043
R-squared	0.625	0.627	0.627

Note a. The notation $\ln q^x$, with $x = \{hm, eh\}$, denotes the natural logarithm of the sum of market tightness. The explanatory variable $U \cdot PPJ$ is the interaction between current unemployment rate and an indicator function taking value equal to one if the job includes performance-related compensation.

Note b. Estimated coefficients for $\ln q^{eh}$ and $\ln q^{hm}$, and associated standard errors, are multiplied by 100. All standard errors are clustered by observation start-date and end-date. Results are robust to clustering by individual. Significance: *** 1%, ** 5%, * 10%.

Note c. The sample includes female and male workers between age 25 and 55. We include controls for year, job tenure with current employer, work experience, geographic and SMSA region, industry, marital status, education, age and union status.

While cyclicality is slightly less pronounced, all these robustness checks confirm the baseline findings. The cyclical responses of wages in PPJ are highly significant, whether we pool all observations or split them by PPJ status. In contrast, no evidence of cyclicality is detected for non-PPJ. These findings provide further support to the theoretical model's predictions.

Table 3.9: Wage regressions: PPJ vs non-PPJ (men and women)

Variables	(1) $PPJ = 1$ (Bils specification)	(2) $PPJ = 0$ (Bils specification)	(3) $PPJ = 1$ (add match quality)	(4) $PPJ = 0$ (add match quality)
U	-0.0187***	-0.0093	-0.0201***	-0.0092
	[0.0044]	[0.0065]	[0.0043]	[0.0066]
$\ln q^{eh}$	-	-	8.82***	4.54***
			[1.18]	[0.734]
$\ln q^{hm}$	-	-	9.04***	5.47***
			[1.25]	[0.59]
Observations	12,002	22,048	11,588	21,455
R-squared	0.72	0.593	0.723	0.592

Note a. The notation $\ln q^x$, with $x = \{hm, eh\}$, denotes the natural logarithm of the sum of labour market tightness

Note b. Estimated coefficients for $\ln q^{eh}$ and $\ln q^{hm}$, and associated standard errors, are multiplied by 100 for $\ln q^x$. All standard errors are clustered by observation start-date and end-date. Results are robust to clustering by individual. Significance: *** 1%, ** 5%, * 10%.

Note c. The sample includes female and male workers between age 25 and 55. We include controls for year, geographic and SMSA region, industry, marital status, education, age and union status.

Performance pay and match quality: a linear probability model. The linear probability specification provides a simple and relatively unrestrcited test of the statistical relationship between PPJ and match quality proxies. As for the Logit analysis, we estimate a fixed effect specification to control for additively separable heterogeneity and control for a variety of observable characteristics.

The findings confirm that match quality and PPJ are positively and significantly linked. A ten percent increase in the q^{eh} match quality proxy is associated to an average thirty percent increase in the prevalence of performance-related pay. The effect is even stronger for the q^{hm} measure of match quality: in this case a ten percent increase in match quality is associated to a sixty percent change in the prevalence of performance pay. Interestingly, including both measures of match quality in the right-hand side of the linear probability model does not change their gradient or significance, suggesting that both measures capture relevant and independent aspects of match quality. When both measures are included, a ten percentage points change in match quality is associated to a doubling of the probability that performance pay is adopted.

Table 3.10: Performance Pay and Match Quality: Linear Probability Regressions

	Dependent Variable: Performance Pay Indicator		
Variables	(1)	(2)	(3)
$log(q^{eh})$	2.89**	-	3.09***
	[1.13]	-	[1.13]
$log(q^{hm})$	-	6.00***	6.33***
	-	[2.04]	[2.03]
Observations	4,704	4,810	4,704
R-squared	0.630	0.632	0.631

Note a. The notation $\ln q^x$, with $x = \{hm, eh\}$, denotes the natural logarithm of the sum of market tightness

Note b. Estimated coefficients and associated standard errors are multiplied by 100. All standard errors are clustered by observation start-date and end-date. Results are robust to clustering by individual. Significance: *** 1%, ** 5%, * 10%.

Note c. The sample includes male workers between age 25 and 55. We include controls for year, job tenure with current employer, work experience, geographic and SMSA region, industry, marital status, education, age (maximum in the employment spell), union status.

Using GDP to gauge cyclicality. In our baseline specification we follow the literature and estimate the cyclical responsiveness of wages to unemployment. Here we verify the robustness of our results to using GDP as an alternative measure of cyclicality. Specifically, we approximate cyclical fluctuations using the log deviations of quarterly GDP from its linear trend.

Our findings suggest that the key results about wage cyclicality and performance-related pay remain intact. Column (1) of Table (3.11) shows that the GDP gradient is positive and significant only when interacted with the PPJ dummy, indicating that only wages for PPJ=1 exhibit cyclical fluctuations. In columns (2) and (3) we replicate the analysis separately for $PPJ = 1$ and $PPJ = 0$. We find that only performance pay jobs exhibit cyclical responses to GDP fluctuations, just as we did when using unemployment rate to approximate for cyclical labor market conditions. A 1% upward deviation of GDP from trend is associated to a 1.3% increase in wages.[23]

[23] The magnitude of the cyclical wage responses in performance-pay jobs is in fact comparable to the one estimated using the unemployment rate. Assuming that an extra 1% of GDP is associated with a decline in the aggregate unemployment rate of between 0.3% and 0.5%, a back of the envelope calculation (and our estimates in Table 3.3) suggest that a 1% deviation of GDP from trend should be associated to a wage change between 0.85% and 1.4%.

Table 3.11: Wage regressions using GDP as a cyclical proxy.

	Dependent Variable: Log Wage		
	(1)	(2)	(3)
Variables	All	$PPJ = 1$	$PPJ = 0$
GDP	0.158	1.33***	-0.00514
	[0.253]	[0.279]	[0.298]
$GDP \cdot PPJ$	0.797**	-	-
	[0.348]	-	-
$log(q^{eh})$	6.61**	8.67***	5.90***
	[0.678]	[1.50]	[0.893]
$log(q^{hm})$	7.53***	9.81***	6.16***
	[0.667]	[1.43]	[0.972]
Observations	17,434	7,065	10,369
R-squared	0.646	0.723	0.614

Note a. The notation $\ln q^x$, with $x = \{hm, eh\}$, denotes the natural logarithm of the sum of market tightness

Note b. Estimated coefficients and associated standard errors are multiplied by 100. All standard errors are clustered by observation start-date and end-date. Results are robust to clustering by individual. Significance: *** 1%, ** 5%, * 10%.

Note c. The sample includes male workers between age 25 and 55. We include controls for year, job tenure with current employer, work experience, geographic and SMSA region, industry, marital status, education, age (maximum in the employment spell), union status.

Evidence from Education Groups. Next, we split workers into three groups (high school dropouts, high school graduates including those with some college, and college graduates) and document significant differences in the prevalence of performance pay across different education groups. As shown in Table (3.12) the prevalence of performance-related pay is higher among more educated workers.

Table 3.12: Proportion of performance pay jobs (PPJ) by education group.

	COL	HSG	HSD
Share PPJ	43.56%	30.66%	25.04%
	(49.6%)	(46.11%)	(43.33%)
q^{eh} above median	39.43%	35.46%	31.28%
	(48.88%)	(47.85%)	(46.37%)
q^{hm} above median	41.47%	34.24%	30.07%
	(49.27%)	(47.46%)	(45.86%)
Observations	2,011	3,832	2,564

Note a. Top panel: share of jobs with performance pay arrangements (Share PPJ) for coarse education groups: college versus high school graduates versus high school dropouts (COL vs HSG vs HSD). Standard deviations in parentheses (also as shares).

Note b. Bottom panel: share of jobs with match quality above the unconditional median for coarse education groups: college versus high school graduates versus high school dropouts (COL vs HSG vs HSD). First line based on q^{eh} match quality proxy; second line based on q^{hm} match quality proxy.

Table 3.13: Wage Regressions: Cyclicality by Education Group.

	Dependent Variable: Log Wage		
	(1)	(2)	(3)
Variables	HSD	HSG	CG
U	0.001	-0.0084	-0.0266***
	[0.0125]	[0.0055]	[0.0103]
$\ln q^{eh}$	5.55***	7.01***	6.57***
	[1.74]	[0.792]	[1.34]
$\ln q^{hm}$	8.88***	6.75***	5.84***
	[1.91]	[0.80]	[1.25]
Observations	1,884	9,367	6,183
R-squared	0.666	0.652	0.572

Note a. The notation $\ln q^x$, with $x = \{hm, eh\}$, denotes the natural logarithm of the sum of labour market tightness

Note b. Estimated coefficients and associated standard errors are multiplied by 100 for $\ln q^x$. All standard errors are clustered by observation start-date and end-date. Results are robust to clustering by individual. Significance: *** 1%, ** 5%, * 10%.

Note c. We exclude women and individuals with less than 25 years old.

When we re-estimate our wage specification for different education groups, results (in Table 3.13) suggest that patterns by education mirror those found for occupations. While wages of workers with no college degrees appear to be insensitive to aggregate labor market fluctuations, those for college grads respond strongly and significantly. In fact, both the sign and magnitude of the responses for college-graduates are similar to those estimated for workers in cognitive occupations or in performance pay jobs.

3.5 Conclusions

Heterogeneity in match-specific productivity has been the object of much attention in recent theoretical and applied studies of labor markets. This chapter investigates the implications of match quality heterogeneity for the choice of pay arrangements, and examines how differences in these arrangements influence wage dynamics and workers' retention.

Several interesting and empirically relevant implications become apparent when one explicitly considers the heterogeneity of contractual arrangements. Our theoretical and empirical results clearly point towards a strong association between match-specific productivity, pay arrangements, and wage cyclicality. We provide evidence that employers tend to adopt performance-based pay when match quality is higher. In turn, this is associated to better retention and longer job durations.

We also find that this type of contractual sorting has implications for wage cyclicality: wages in jobs with higher match quality exhibit significant and sizeable responses to aggregate cyclical conditions whereas lower match quality jobs exhibit no such cyclicality.

These findings have implications for the behavior of wages across occupations. Retention considerations should induce firms to use variable compensation as a way to retain workers whenever they are most in demand. Hence, employee profit-sharing (or other forms of performance-related pay) should be relatively more attractive in occupations which are in high demand. We are able to document that jobs in cognitive occupations exhibit strong wage cyclicality and longer durations, while routine and non-cognitive jobs do not. These features are consistent with better average match quality and higher prevalence of performance pay jobs, observations that we are able to confirm using micro data.

Conclusion

In this thesis, I have studied the sources and consequences of heterogeneity in firms outcomes.

In Chapter 1, I study the differences in firm outcomes between firms created by unemployed, versus employed, individuals. I start by developing a general equilibrium model of entrepreneurship in which both unemployed and wage workers make the endogenous decision to start or not a firm. With this rich yet tractable framework, I derive key predictions which I test in the data. In particular, I use firm closures to identify random assingment of an individual to unemployment. I find unemployment doubles the probability of an individual to start a firm, and conditional on starting, the individual hires 26% fewer workers and is 30% more likely to exit firm ownership. These patterns are completely consistent with the predictions of the model.

This is the first general equilibrium model with both unemployed and wage workers making the endogenous decision to start a firm or not. This is also the first research to evaluate the casual effect of unemployment on firm ownership decisions and firm performance. This is possible thanks to the novel data being used. It is composed of the entire universe of tax filers linked to privately owned incorporated firms in Canada. It improves on employer-employee datasets by also having the link between each firm and their corresponding owner.

With an extension of the theory to a multi sector environment I derive the additional implication that higher wages decrease the entry rate into entrepreneurship of wage workers by more than that of unemployed. Using city wage variation and a Bartik style IV strategy for wages, I show that a 1% drop in wages increases by 3.2 percentage points the entry rate into entrepreneurship for wage workers and has no impact for laid off individuals. Such heterogeneity in entry into entrepreneurship has never been documented before.

Finally, I use a numerical version of my general equilibrium model, disciplined by the data, to evaluate the impact of policies that subsidize entrepreneurship among the unemployed. I find that the effect of these policies is to decrease average firm productivity and reallocate ressources to low productivity firms.

In the second chapter, we analyze an extension of the standard search and matching framework in which individuals make the endogenous decision to either look for a business project or a job. I find that the direction of wage responses to aggregate shocks becomes dependant on which party has the most bargaining power (the entrepreneur or the worker). I also find that the condition for constrained efficient allocation of the decentralized market is unchanged although the interpretation changes. This is the first paper to analyze the implications for contrained efficiency and wage dynamics of adding entrepreneurship to a search and matching framework.

In Chapter 3 entitled, *Match Quality, Contractual Sorting and Wage Cyclicality*, the focus shifts from firm formation to the contract choice by firms. We study the role of match quality for contractual arrangements, wage dynamics and workers retention. We develop a model in which profit maximizing firms offer a performance-based pay arrangement to retain workers with relatively high match-specific productivity. The key implications of our model hold in the data, where information about job histories and performance pay is available. We verify empirically that firms are more prone to offering performance pay based contracts to workers for which match quality is higher. We also verify that wage cyclicality is coming from performance pay jobs, with those offering different contracts exhibiting no cyclicality. Finally we also show that match quality has a direct effect even after we control for contract choice and we relate our findings to the literature on occupation heterogeneity.

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Appendix A

Supporting Materials

A.1 Appendix to Chapter 1

A.1.1 Proofs Benchmark Model

For proofs and characterization of model with multiple sectors see Supplemental Appendix to the paper.

Proof of Proposition 1.

We know that $J(z)$ is equal to U, $\forall z \leq \hat{z}$. We need to find the value of $J(z)$ for $z \geq \hat{z}$.

Define

$$B \equiv (1-\alpha)(\frac{\alpha}{w})^{\frac{\alpha}{1-\alpha}} \tag{A.1}$$

Guess that $J(z)$ will be of the form $Ce^{\frac{z}{1-\alpha}} + Ge^{-az}$ for $z \geq \hat{z}$. Imposing the $J'(\hat{z}) = 0$ condition

$$aGe^{-a\hat{z}_i} = C(\frac{1}{1-\alpha})e^{\frac{1}{1-\alpha}\hat{z}_i} \tag{A.2}$$

$$G = \frac{C}{a(1-\alpha)}e^{\hat{z}_i(\frac{1}{1-\alpha}+a)} \tag{A.3}$$

Then

$$rCe^{\frac{z}{1-\alpha}} + rGe^{-az} = Be^{\frac{z}{1-\alpha}} + \frac{\mu}{1-\alpha}Ce^{\frac{z}{1-\alpha}} - \mu Gae^{-az} + \frac{\sigma^2}{2}(\frac{1}{1-\alpha})^2Ce^{\frac{z}{1-\alpha}} + \frac{\sigma^2}{2}Ga^2e^{-az} \tag{A.4}$$

Then solving gives C defined by

$$rC = B + \frac{\mu}{1-\alpha}C + \frac{\sigma^2}{2}(\frac{1}{1-\alpha})^2C \tag{A.5}$$

$$C = \frac{B}{r - \frac{\mu}{1-\alpha} - \frac{\sigma^2}{2}\frac{1}{(1-\alpha)^2}} \tag{A.6}$$

and a defined by (condition to guarantee $rG = -\mu Ga + \frac{\sigma^2}{2}Ga^2$)

$$r = -\mu a + \frac{\sigma^2}{2}a^2 \tag{A.7}$$

Choosing the positive root[1]

$$a = \frac{\mu + \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2} > 0 \tag{A.11}$$

Which then implies

$$J(z) = C(e^{\frac{z}{1-\alpha}} + \frac{1}{a(1-\alpha)} e^{-a(z-\hat{z}) + \frac{\hat{z}}{1-\alpha}}) \tag{A.12}$$

$$J(z) = \frac{B}{r - \frac{\mu}{1-\alpha} - \frac{\sigma^2}{2}(\frac{1}{1-\alpha})^2}(e^{\frac{z}{1-\alpha}} + \frac{1}{(1-\alpha)} e^{-a(z-\hat{z}) + \theta\hat{z}}) \tag{A.13}$$

Proof of Proposition 2.

Solving generic KFE. The solution below is the same for both types of business owners (i.e., $i = u, w$)

Let $\hat{z}$ be the point at which firms exit and $\underline{z}_i$ the point in which firms enter, with $\underline{z}_i > \hat{z}$. Let $\Lambda(z)$ denote the endogenous pdf and M the measure of entrants. For type u ($i = u$), M is equal to $\psi u e^{-\beta \underline{z}_u}$ and for type w ($i = w$) M is equal to $\psi(1-u-\eta)e^{-\beta \underline{z}_w}$

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[1] Choosing the positive root makes sense, or else for parameters values that satisfy $|a| > \frac{1}{1-\alpha}$

$$\lim_{z \to \infty} J(z) = -\infty \tag{A.8}$$

to see this first note that

$$\frac{\partial J(z)}{\partial z} = C(\frac{1}{1-\alpha})(e^{\frac{z}{1-\alpha}} - e^{-a(z-\hat{z}) + \frac{\hat{z}}{1-\alpha}}) \tag{A.9}$$

It follows that if $a < 0$ and $|a| > \frac{1}{1-\alpha}$, $\exists z_o$, s.t.: $\forall z > z_o$

$$\frac{\partial J(z)}{\partial z} < 0 \quad \forall z > z_o \tag{A.10}$$

Finally, let for $[\underline{z}_i, \infty[$

$$\Lambda^i(z) = \Lambda_2^i(z) \tag{A.14}$$

and for $]\hat{z}, \underline{z}_i]$

$$\Lambda^i(z) = \Lambda_1^i(z) \tag{A.15}$$

Then for $[\underline{z}_i, \infty[$

$$\frac{\partial \Lambda_2^i(z)}{\partial t} = -\mu \frac{\partial \Lambda_2^i(z)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 \Lambda_2^i(z)}{\partial z^2} + M^i \frac{\beta e^{-\beta z}}{e^{-\beta \underline{z}_i}} = 0 \tag{A.16}$$

for $]\hat{z}, \underline{z}_i]$

$$\frac{\partial \Lambda_1^i(z)}{\partial t} = -\mu \frac{\partial \Lambda_1^i(z)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 \Lambda_1^i(z)}{\partial z^2} = 0 \tag{A.17}$$

The four boundary conditions are

1. $\int_{\underline{z}_i}^{\infty} \Lambda^i(z) dz < \infty$
2. $\Lambda_1^i(\underline{z}_i) = \Lambda_2^i(\underline{z}_i)$
3. $\frac{\partial \Lambda_1^i(\underline{z}_i)}{\partial z} = \frac{\partial \Lambda_2^i(\underline{z}_i)}{\partial z}$
4. $\Lambda_1^i(\hat{z}) = 0$

Guess

$$\Lambda_1^i(z) = k_1^1 + k_2^1 e^{\frac{2\mu}{\sigma^2} z} \tag{A.18}$$

and

$$\Lambda_2^i(z) = k_1^2 + k_2^2 e^{\frac{2\mu}{\sigma^2} z} - \frac{M^i e^{-\beta z}}{e^{-\beta \underline{z}_i}(\mu + \frac{\sigma^2}{2}\beta)} \tag{A.19}$$

From $\int_{\underline{z}_i}^{\infty} \Lambda^i(z)dz < \infty$ we get

$$k_1^2 = 0 \tag{A.20}$$

From $\Lambda_1^i(\underline{z}_i) = \Lambda_2^i(\underline{z}_i)$ we get

$$k_2^2 = \frac{M^i}{\mu + \frac{\sigma^2}{2}\beta} e^{\frac{-2\mu}{\sigma^2}\underline{z}_i} + k_1^1 e^{\frac{-2\mu}{\sigma^2}\underline{z}_i} + k_2^1 \tag{A.21}$$

From $\frac{\partial \Lambda_1^i(\underline{z}_i)}{\partial z} = \frac{\partial \Lambda_2^i(\underline{z}_i)}{\partial z}$ we get

$$k_2^2 = k_2^1 - \frac{\beta M^i \frac{\sigma^2}{2\mu} e^{\frac{-2\mu}{\sigma^2}\underline{z}_i}}{(\mu + \frac{\sigma^2\beta}{2})} \tag{A.22}$$

Equating equations $(A.21)$ and $(A.22)$

$$k_1^1 = \frac{-M^i}{\mu} \tag{A.23}$$

This implies

$$\Lambda_1^i(z) = \frac{M^i}{-\mu} + k_2^1 e^{\frac{2\mu}{\sigma^2}z} \tag{A.24}$$

Now using $\Lambda_1^i(\hat{z}) = 0$ we get

$$k_2^1 = \frac{M^i}{\mu} e^{\frac{-2\mu}{\sigma^2}\hat{z}} \tag{A.25}$$

It follows

$$\Lambda_1^i(z) = \frac{M^i}{-\mu}(1 - e^{\frac{2\mu}{\sigma^2}(z-\hat{z})}) \tag{A.26}$$

and

$$k_2^2 = \frac{M^i}{\mu} e^{\frac{-2\mu}{\sigma^2}\hat{z}} - \frac{\beta M^i \frac{\sigma^2}{2\mu} e^{\frac{-2\mu}{\sigma^2}\underline{z}_i}}{(\mu + \frac{\sigma^2}{2}\beta)} \tag{A.27}$$

which implies

$$\Lambda_2^i(z) = \frac{\beta M^i \frac{\sigma^2}{-2\mu} e^{\frac{2\mu}{\sigma^2}(z-\underline{z}_i)}}{(\mu + \frac{\sigma^2\beta}{2})} - \frac{M^i}{-\mu} e^{\frac{2\mu}{\sigma^2}(z-\hat{z})} - \frac{M^i e^{-\beta z}}{e^{-\beta \underline{z}_i}(\mu + \frac{\sigma^2}{2}\beta)} \tag{A.28}$$

Proof of Corollary 2.1.

It then follows

$$\eta^i = \int_{\hat{z}}^{\underline{z}_i} \Lambda_1^i(z) dz + \int_{\underline{z}_i}^{\infty} \Lambda_2^i(z) dz \tag{A.29}$$

Note that

$$\int_{\underline{z}_i} \Lambda_2^i(z) dz = \frac{-M^i \sigma^2}{2\mu^2} e^{\frac{2\mu}{\sigma^2}(\underline{z}_i-\hat{z})} + \frac{\beta M^i (\frac{\sigma^2}{2\mu})^2}{\mu + \frac{\sigma^2}{2}\beta} - \frac{M^i}{\mu\beta + \frac{\sigma^2}{2}\beta^2} \tag{A.30}$$

and

$$\int_{\hat{z}}^{\underline{z}_i} \Lambda_1^i(z) dz = \frac{M^i}{-\mu}[\underline{z}_i - \hat{z} + \frac{\sigma^2}{-2\mu}(e^{\frac{2\mu}{\sigma^2}(\underline{z}_i-\hat{z})} - 1)] \tag{A.31}$$

Which then implies

$$\eta^i = \frac{M^i}{\mu + \frac{\sigma^2\beta}{2}}[\beta(\frac{\sigma^2}{2\mu})^2 - \frac{1}{\beta}] + \frac{M^i}{-\mu}(\underline{z}_i - \hat{z}) - \frac{M\sigma^2}{2\mu^2} \tag{A.32}$$

$$\eta^i = \frac{M^i}{\mu + \frac{\sigma^2}{2}\beta}[\beta(\frac{\sigma^2}{2\mu})^2 - \frac{1}{\beta} - \frac{\sigma^2}{2\mu^2}(\mu + \frac{\sigma^2}{2}\beta)] + \frac{M^i}{-\mu}(\underline{z}_i - \hat{z}) \tag{A.33}$$

$$\eta^i = \frac{M^i}{\mu + \frac{\sigma^2}{2}\beta}[\frac{\beta\sigma^4}{4\mu^2} - \frac{1}{\beta} - \frac{\sigma^2}{2\mu} - \frac{\sigma^4\beta}{4\mu^2}] + \frac{M^i}{-\mu}(\underline{z}_i - \hat{z}) \tag{A.34}$$

$$\eta^i = \frac{M^i}{\mu + \frac{\sigma^2}{2}\beta}[-\frac{\mu + \frac{\sigma^2}{2}\beta}{\mu\beta}] + \frac{M^i}{-\mu}(\underline{z}_i - \hat{z}) \tag{A.35}$$

$$\eta^i = \frac{M^i}{-\mu\beta} + \frac{M^i}{-\mu}(\underline{z}_i - \hat{z}) = \frac{M^i}{-\mu}[\frac{1+\beta(\underline{z}_i - \hat{z})}{\beta}] \tag{A.36}$$

Now using the fact that $M^u = \psi u e^{-\beta \underline{z}_u}$ and $M^w = \psi(1-u-\eta)e^{-\beta \underline{z}_w}$ we get

$$\eta^u = A_u \psi u e^{-\beta \underline{z}_u} \tag{A.37}$$

and

$$\eta^w = A_w \psi(1-u-\eta)e^{-\beta \underline{z}_w} \tag{A.38}$$

which implies

$$\eta = A_u \psi u e^{-\beta \underline{z}_u} + A_w \psi(1-u-\eta)e^{-\beta \underline{z}_w} \tag{A.39}$$

Now using $u = \frac{(s+\psi(1-F(\underline{z}_w)))(1-\eta)}{s+f+\psi(1-F(\underline{z}_w))}$ and $1-u-\eta = \frac{f(1-\eta)}{s+f+\psi(1-F(\underline{z}_w))}$

$$\eta = \frac{\psi(1-\eta)}{s+f+\psi e^{-\beta \underline{z}_w}}[A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u} + A_w f e^{-\beta \underline{z}_w}] \tag{A.40}$$

$$\eta = \frac{\psi[A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u} + A_w f e^{-\beta \underline{z}_w}]}{s+f+\psi e^{-\beta \underline{z}_w} + \psi[A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u} + A_w f e^{-\beta \underline{z}_w}]} \tag{A.41}$$

It follows

$$1-\eta = \frac{s+f+\psi e^{-\beta \underline{z}_w}}{s+f+\psi e^{-\beta \underline{z}_w} + \psi[A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u} + A_w f e^{-\beta \underline{z}_w}]} \tag{A.42}$$

which implies

$$\eta^u = \frac{A_u(s+\psi e^{-\beta \underline{z}_w})\psi e^{-\beta \underline{z}_u}}{s+f+\psi e^{-\beta \underline{z}_w}+\psi[A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u}+A_w f e^{-\beta \underline{z}_w}]} \tag{A.43}$$

$$\frac{\eta^u}{\eta} = \frac{A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u}}{A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u}+A_w f e^{-\beta \underline{z}_w}} \tag{A.44}$$

Proof of Proposition 4.

In equilibrium $W > U$, otherwise all wage workers would exit wage work to go to the unemployment island and markets would not clear in the Walrasian market. Here I present a formal proof showing that if $b < 1 \Leftrightarrow W > U$. Note that rW and rU can be rewritten as

$$rW = w + f(W-U) + \psi \int (\max(J(z),W) - W)dF(z) \tag{A.45}$$

$$rU = bw + s(U-W) + \psi \int (\max(J(z),U) - U)dF(z) \tag{A.46}$$

This implies

$$(r+\psi+f+s)(W-U) = w(1-b) + \psi \int \max(J(z),W)dF(z) - \psi \int \max(J(z),U)dF(z) \tag{A.47}$$

First prove $b < 1 \Rightarrow W > U$. Using the equation A.47 above :

$$w(1-b) = \psi(W-U) + (r+f+s)(W-U) - (\psi \int_{\underline{z}_w} J(z)dF(z) + \psi \int^{\underline{z}_w} W dF(z)$$

$$-\psi \int_{\underline{z}_u} J(z)dF(z) - \psi \int^{\underline{z}_u} U dF(z)) + \psi \int^{\underline{z}_u} W dF(z) - \psi \int^{\underline{z}_u} W dF(z) \quad \text{(A.48)}$$

$$0 < w(1-b) = \psi(W-U) + (r+f+s)(W-U) + \psi \int_{\underline{z}_u}^{\underline{z}_w} (J(z)-W)dF(z) - \psi \int^{\underline{z}_u} (W-U)dF(z)$$

$$< (r+f+s)(W-U) + \psi(W-U) - \psi(W-U)F(\underline{z}_u) \quad \text{(A.49)}$$

where the last inequality follows from the fact that $J(z) < W$ for $\underline{z}_u < z < \underline{z}_w$. It follows that $b < 1 \Rightarrow W > U$.

Now to prove that $W > U \Rightarrow b < 1$ start by

$$w(1-b) = \psi(W-U) + (r+f+s)(W-U) + \psi \int_{\underline{z}_u}^{\underline{z}_w} (J(z)-W)dF(z) - \psi \int^{\underline{z}_u} (W-U)dF(z)$$

$$+ \psi \int_{\underline{z}_u}^{\underline{z}_w} U dF(z) - \psi \int_{\underline{z}_u}^{\underline{z}_w} U dF(z) \quad \text{(A.50)}$$

$$w(1-b) = \psi(W-U) + (r+f+s)(W-U) + \psi \int_{\underline{z}_u}^{\underline{z}_w} (J(z)-U)dF(z) - \psi \int^{\underline{z}_u} (W-U)dF(z)$$

$$- \int_{\underline{z}_u}^{\underline{z}_w} (W-U) \quad \text{(A.51)}$$

$$w(1-b) = \psi(W-U)+(r+f+s)(W-U)+\psi\int_{\underline{z}_u}^{\underline{z}_w}(J(z)-U)dF(z)-\psi\int^{\underline{z}_w}(W-U)dF(z) \tag{A.52}$$

$$w(1-b) = \psi(W-U)(1-F(\underline{z}_w))+(r+f+s)(W-U)+\psi\int_{\underline{z}_u}^{\underline{z}_w}(J(z)-U)dF(z) \tag{A.53}$$

Note that $J(z) > U$ for $\underline{z}_u < z < \underline{z}_w$, which implies $W > U \Rightarrow b < 1$. It follows that $b < 1 \Leftrightarrow W > U$.

The result that $\underline{z}_w > \underline{z}_u$ then just follows.

Proof of Corollary 4.1.

Note that in steady state the flow of exiting firms of each type is equal to M^i.[2] Letting ER^i denote Exit Rate for type i, we have

$$(ER^i)^{-1} = [\frac{1+\beta(\underline{z}_i-\hat{z})}{-\mu\beta}] \tag{A.54}$$

$$(ER^w)^{-1} > (ER^u)^{-1} \Rightarrow ER^u > ER^w \tag{A.55}$$

where the first inequality follows from $\underline{z}_w > \underline{z}_u$

It follows that in the steady state equilibrium the exit rate is higher for firms of type u.

Proof of Corollary 4.2.

The expression for optimal firm size is given by

$$n(z,w) = (\frac{\alpha}{w})^{\frac{1}{1-\alpha}}e^{\frac{z}{1-\alpha}} \tag{A.56}$$

[2]In steady state, the flow of firms exiting a group has to be equal to the flow entering.

It follows average size for type i, where $i \in \{u,w\}$

$$\int_{\hat{z}} n(z,w)\frac{\Lambda^i(z)}{\eta^i}dz = (\frac{\alpha}{w})^{\frac{1}{1-\alpha}}\frac{M^i}{\eta^i}\int_{\hat{z}} e^{\frac{z}{1-\alpha}}\frac{\Lambda^i(z)}{M^i}dz \tag{A.57}$$

Note that

$$\frac{\Lambda^i(z)}{M^i} \quad \text{does not depend on } M^i \tag{A.58}$$

Now concentrate on the term

$$\int_{\hat{z}} e^{\frac{z}{1-\alpha}}\frac{\Lambda^i(z)}{M^i}dz = \int_{\hat{z}}^{\underline{z}} e^{\frac{z}{1-\alpha}}\frac{\Lambda_1^i(z)}{M^i}dz + \int_{\underline{z}} e^{\frac{z}{1-\alpha}}\frac{\Lambda_2^i(z)}{M^i}dz \tag{A.59}$$

Taking derivative with respect to $\underline{z}_i$ gives

$$\frac{\partial \int_{\hat{z}} e^{\frac{z}{1-\alpha}}\frac{\Lambda^i(z)}{M^i}dz}{\partial \underline{z}_i} = \frac{e^{\frac{\underline{z}_i}{1-\alpha}}}{M^i}(\Lambda_1^i(\underline{z}_i) - \Lambda_2^i(\underline{z}_i)) + \int_{\hat{z}}^{\underline{z}_i}\frac{e^{\frac{z}{1-\alpha}}}{M^i}\frac{\partial \Lambda_1^i(z)}{\partial \underline{z}_i}dz + \int_{\underline{z}_i}\frac{e^{\frac{z}{1-\alpha}}}{M^i}\frac{\partial \Lambda_2^i(z)}{\partial \underline{z}_i}dz \tag{A.60}$$

Using the expressions for $\Lambda_1^i(z)$ and $\Lambda_2^i(z)$ note that

$$(\Lambda_1^i(\underline{z}_i) - \Lambda_2^i(\underline{z}_i)) = [\frac{1}{-\mu} + \frac{-\beta\frac{\sigma^2}{-2\mu}+1}{\mu+\frac{\sigma^2}{2}\beta}] = [\frac{1}{-\mu} - \frac{(\mu+\frac{\sigma^2}{2}\beta)}{-\mu(\mu+\frac{\sigma^2}{2}\beta)}] = 0 \tag{A.61}$$

and that

$$\frac{\partial \Lambda_1^i(z)}{\partial \underline{z}} = 0 \tag{A.62}$$

Replacing this back in equation (A.60)

$$\frac{\partial \int_{\hat{z}} e^{\frac{z}{1-\alpha}}\frac{\Lambda^i(z)}{M^i}dz}{\partial \underline{z}_i} = \int_{\underline{z}_i}\frac{e^{\frac{z}{1-\alpha}}}{M^i}\frac{\partial \Lambda_2^i(z)}{\partial \underline{z}_i}dz = \int_{\underline{z}_i} e^{\frac{z}{1-\alpha}}\frac{\beta[e^{\frac{2\mu}{\sigma^2}(z-\underline{z}_i)} - e^{-\beta(z-\underline{z}_i)}]}{\mu+\frac{\sigma^2}{2}\beta}dz \tag{A.63}$$

$$= \frac{\beta(1-\alpha)}{\mu + \frac{\sigma^2}{2}\beta}((\frac{-\sigma^2(1-\alpha)}{2\mu(1-\alpha)+\sigma^2}) - \frac{(1-\alpha)}{\beta(1-\alpha)-1})e^{\frac{z_i}{1-\alpha}} \tag{A.64}$$

$$= \frac{\beta(1-\alpha)}{\mu + \frac{\sigma^2\beta}{2}}[\frac{-2(1-\alpha)^2[\mu + \frac{\sigma^2\beta}{2}]}{(2\mu(1-\alpha)+\sigma^2)(\beta(1-\alpha)-1)}]e^{\frac{z_i}{1-\alpha}} \tag{A.65}$$

$$= \frac{2\beta(1-\alpha)^3}{(-(2\mu(1-\alpha)+\sigma^2))(\beta(1-\alpha)-1)}e^{\frac{z_i}{1-\alpha}} > 0 \tag{A.66}$$

The positive sign follows from the assumption that $-\mu > \frac{\sigma^2}{2(1-\alpha)}$

Finally, to complete the proof note that

$$\frac{M^i}{\eta^i} = (ER)^{-1} \quad \text{where } ER \text{ stands for Exit Rate} \tag{A.67}$$

With the proof that the Exit Rate is higher for type I individuals than type II, it follows that

$$\frac{M^w}{\eta^w} > \frac{M^u}{\eta^u} \tag{A.68}$$

Then letting $E[n]_i$ denote average size for type i. With abuse of notation let $\Lambda^i(z, \underline{z}_j)$ represent the function $\Lambda^i(z)$ replacing $\underline{z}_i$ by $\underline{z}_j$, similarly for the measure $\eta^i(\underline{z}_j)$. Then using $\underline{z}_w > \underline{z}_u$,

$$\begin{aligned} E[n]_w &= \int_{\hat{z}} n(z,w)\frac{\Lambda^w(z,\underline{z}_w)}{\eta^w(\underline{z}_w)}dz > \int_{\hat{z}} n(z,w)\frac{\Lambda^w(z,\underline{z}_u)}{\eta^w(\underline{z}_u)}dz \\ &= (\frac{\alpha}{w})^{\frac{1}{1-\alpha}}\frac{M^w}{\eta^w(\underline{z}_u)}\int_{\hat{z}} e^{\frac{\rho z}{1-\rho}}\frac{\Lambda^w(z,\underline{z}_u)}{M^w}dz > \int_{\hat{z}} n(z,w)\frac{\Lambda^u(z,\underline{z}_u)}{\eta^u(\underline{z}_u)}dz = E[n]_u \end{aligned} \tag{A.69}$$

where the first inequality follows from

$$\frac{\partial \int_{\hat{z}} e^{\frac{\rho z}{1-\rho}}\frac{\Lambda^i(z)}{M^i}dz}{\partial \underline{z}_i} > 0 \tag{A.70}$$

and the second from

$$\frac{M^w}{\eta^w} > \frac{M^u}{\eta^u} \tag{A.71}$$

Now to see the result for profits note that

$$E[\pi]_i = (1-\alpha)(\frac{w}{\alpha})E[n]_i \tag{A.72}$$

Proof of Corollary 4.3.

$\underline{z}_w > \underline{z}_u \Rightarrow \psi(1-F(\underline{z}_u)) > \psi(1-F(\underline{z}_w))$.

A.1.2 Controlling for learning by doing mechanism

In this section, I show that the differences in size and exit rate between firms created by an individual when laid off relative to working for somebody else cannot be explained by a learning by doing story. In particular, one concern is that these differences might be driven by individuals first starting a firm when laid off, during which they acquire entrepreneurial skills. After that experience, upon entering during wage work, individuals would generate more productive firms due to their accumulated experience as an entrepreneur. To show that such mechanism cannot rationalize the differences in size and exit rate, I rerun the benchmark regressions with additional controls for the total experience an individual had accumulated as a business owner upon starting the current firm. The control I use is a quadratic in total years I observe the individual as an entrepreneur prior to this current firm spell interacted with dummies for current year. The interaction with years is to control for the fact that the value of entrepreneurial skills might vary with the business cycle.

Table A.1.1: Log number of employees

Dependant variable	Log # employees	Dummy for exit
$\mathbb{1}\{\text{Prev U}\}_{i,s}$	-0.2894***	0.013*
	(0.0419)	(0.0069)
Fixed Effects	Yes	Yes
Controls for entrepreneurial experience	Yes	Yes
Ratio of probabilities	N/A	1.24
Baseline Exit Probability	0.055	0.055
Observations	450,502	341,241

Notes: Column (1) reports results for fixed effects regression of log number of employees of current business on dummy indicating if the current business was started by the individual when laid off ($\mathbb{1}\{\text{Prev U}\}_{i,s}$). Column (2) reports results for fixed effect regression of exit dummy (taking value 1 if individual exits firm ownership and 0 otherwise) on ($\mathbb{1}\{\text{Prev U}\}_{i,s}$). Other controls include dummies in age groups, marital status, province of residence, year business started, current year, 2 digit NAICS industry code for current business, 2 digit NAICS industry code for the last employer, log number of employees for the last employer and total years individuals observed as a business owner prior to current entrepreneur spell interacted with current year. Only includes men 25 to 54 years old.

A.1.3 Model with multiple sectors and testable prediction

Model description

The baseline theoretical framework is useful in its clarity to understand exactly how the selection mechanism operates. But in reality an economy is composed of different sectors each with a different labour productivity and wage. Since for each sector the opportunity cost of entering entrepreneurship is different, this has implications for individual decisions to open a business. Furthermore, the model with multiple sectors is useful in motivating the instrument I choose when I test the additional prediction of the theory.

With this in mind I consider a small extension of the previous framework, in which now there are C economies each with I industries an individual can work on. What characterizes an industry is the amount of efficiency units a worker is endowed with. All workers in each economy c have the same endowment of efficiency units across industries. Entrepreneurs in this scenario choose the optimal amount of efficiency units to hire and pay a same wage per efficiency unit across industries. Conditional on transitioning to the working island as a worker, the unemployed transition to work at industry i at rate Ω_i. It follows the problem of the unemployed individual can be summarized by

$$rU^c = bw_c\zeta_c + f(\sum_{\forall i}\Omega_i W_i^c - U^c) + \psi\int_{z_u^c}(\sum_{\forall i}\Omega_i J^c(z) - U^c)dF(z) \qquad \text{(A.73)}$$

where w_c is the unique equilibrium wage, ζ_c is an economy-wide efficiency unit for workers in economy c, $bw_c\zeta_c$ is the income of the unemployed individual, W_i^c is the value of being a worker at industry i at economy c. Note that since the value of unemployment U^c and the wage per efficiency unit w^c are the same across industries in a same economy c, then, conditional on z, every entrepreneur is indifferent over which industry to operate in. With this in mind, I consider an equilibrium in which the transition rate of an entrepreneur to industry i is also given by rate Ω_i.

The value function for a worker in industry $i \in I$ is given by

$$rW_i^c = w_c \nu_{c,i} \zeta_c + s(U^c - W_i^c) + \psi \int_{\underline{z}_{w,i}^c} (\sum_{\forall i} \Omega_i J^c(z) - W_i^c) dF(z) \tag{A.74}$$

where $\nu_{c,i}$ is the relative amount of efficiency units a worker is endowed for industry i at economy c and ζ_c is the economy-wide efficiency unit endowment for workers in economy c, where $E[log(\zeta)] \equiv \sum_i \Omega_i log(\zeta) = K$ is time invariant. Let $\sum_i \Omega_i log(\nu_{c,i}) = 0$. Now define the economy level wage as w_c and the average industry, economy level wage as $w_{c,i} \equiv w_c \nu_{c,i}$.

In equilibrium,

$$J^c(\underline{z}_{w,i}^c) = W_i^c \quad \text{and} \quad J^c(\underline{z}_u^c) = U^c \tag{A.75}$$

As in the previous theoretical framework we get the result of differences in performance between firms created by employed versus unemployed individuals.[3]

Proposition 13. *In a multi-sector model of endogenous entrepreneurship, firms created by employed individuals have on average more employees, higher profits and lower exit rates. Furthermore, unemployed individuals are more likely to enter entrepreneurship relative to workers of all industries.*

The Proposition below highlights a prediction that lies at the heart of the selection mechanism.

Proposition 14.

$$log(\underline{z}_{w,i}^c) = \xi_0^w + \xi_1^w log(w_{c,i}) + \xi_2^w log(w_c) + \xi_3^w log(\varepsilon_{c.i}) + \xi_4^{w_c} log(\zeta_c) \tag{A.76}$$

[3]See Supplemental Appendix I for full characterization of the model with multiple sectors.

$$log(\underline{z}_u^c) = \xi_0^u + \xi_1^u log(w_c) + \xi_4^u log(\zeta_c) \tag{A.77}$$

where $\xi_1^u > 0, \xi_1^w > 0$ and $\xi_2^w > 0$, furthermore, let $E[\underline{z}_{w,i}^c]$ be the average threshold productivity for wage workers across industries in economy c then

$$E(log(\underline{z}_{w,i}^c)) = \xi_0^w + \Lambda^w log(w_c) + \xi_3^w log(\zeta_c) \tag{A.78}$$

where $\Lambda^w = \xi_1^w + \xi_2^w > \xi_1^u$

The corollary below formally relates the entry rate into firm ownership of both wage workers and laid off individuals to region-wide wages.

Corollary 14.1. *The average entry rate for wage workers in an economy/region c, $ER_{c,w}$ and that of unemployed individuals $ER_{c,u}$ can both be expressed as*

$$ER_{c,w} = \beta_{0,w} + \beta_{1,w} log(w_c) + \upsilon_{c,w} \quad \textit{for employed workers} \tag{A.79}$$

where υ_w is linear function of $log(\zeta)$

$$ER_{c,u} = \beta_{0,u} + \beta_{1,u} log(w_c) + \upsilon_{c,u} \quad \textit{for not working individuals} \tag{A.80}$$

where υ_u is linear function of $log(\zeta)$ and $\beta_{1,w} < \beta_{1,u} \leq 0$.

Now combining both into one specification we get

$$ER_{c,n,t} = \alpha_0 + \beta_1 log(w_{c,t}) + \beta_2 \mathbb{1}\{\text{Prev U}\}_{c,t,n} log(w_{c,t}) + \alpha_2 \mathbb{1}\{\text{Prev U}\}_{c,t,n} + \mu_{c,t} \tag{A.81}$$

where μ_c is a function of $log(\zeta_c)$, $n=1$ if the individual is involuntarily unemployed and $n=0$ if he is working and $\mathbb{1}\{\text{Prev U}\}_{c,t,n}$ is an indicator for whether the individual was involuntarily unemployed $n=1$ or was working $n=0$. I have added the time subscripts since the data is over different time periods. The prediction of the theory is that $\beta_1 < 0$ and $\beta_2 > 0$.

The following theorem gives a linearized expression for past industrial composition as a function of the shocks in the model that will be useful in the discussion of the validity of the instrument.

Proposition 15.

$$\kappa_{c,i,1} = \frac{\Omega_i\Gamma_0}{\Gamma_0+\Gamma_2 K} + \frac{\Omega_i\Gamma_1}{\Gamma_0+\Gamma_2 K} log(\nu_{c,i,1}) + \frac{\Omega_i\Gamma_2}{\Gamma_0+\Gamma_2 K} log(\zeta_{c,1}) \tag{A.82}$$

From the discussion on identification we concluded that the level of variation being used is that of changes across regions. It follows that, using the result of Proposition 15, for consistency we need

$$\text{plim}_{C,I\to\infty}\frac{1}{C}\frac{1}{I}\sum_{c=1}^{C}\widehat{log(\zeta_{c,t})}\sum_{\forall i} log(w_{i,t}^{N})\left(\frac{\Omega_i\Gamma_0}{\Gamma_0+\Gamma_2 K} + \frac{\Omega_i\Gamma_1}{\Gamma_0+\Gamma_2 K} log(\nu_{c,i,1}) + \frac{\Omega_i\Gamma_2}{\Gamma_0+\Gamma_2 K} log(\zeta_{c,1})\right) \tag{A.83}$$

where c stands for region, i for industry, C for total number of cities and I total number of industries.

$$= \text{plim}_{I\to\infty}\frac{1}{I}\sum_{\forall i} log(w_{i,t}^{N})\Omega_i \text{plim}_{C\to\infty}\frac{1}{C}\sum_{c=1}^{C}\Big(\frac{\Gamma_0}{\Gamma_0+\Gamma_2 K} + \frac{\Gamma_1}{\Gamma_0+\Gamma_2 K} log(\nu_{c,i,1}) + \frac{\Gamma_2}{\Gamma_0+\Gamma_2 K} log(\zeta_{c,1})\Big)\widehat{log(\zeta_{c,t})}. \tag{A.84}$$

In other words, the validity of the instrument is guaranteed as long as

1. $\widehat{log(\zeta_{c,t})}$ is uncorrelated with $log(\zeta_{c,1})$.

2. The distribution of $log(\nu_{c,i,1})$ is uncorrelated with $\widehat{log(\zeta_{c,t})}$.

The first requirement is that region-wide comparative advantage in labour efficiency $log(\zeta)$ follows a process of the form

$$log(\zeta_{c,t}) = \gamma_c + \gamma_t + \sigma_{c,t} \tag{A.85}$$

where $\sigma_{c,t} = \sum_{j=2}^{t} \nu_{i,j}$, with $\nu_{i,t}$ iid. This amounts to having the component of region-wide comparative advantage that varies across cities and time ($\sigma_{c,t}$) to be limited in its serial correlation. It must be that eventually past shocks to $\sigma_{c,t}$ no longer influence its current value.[4] Note that this is a much weaker restriction than imposing $log(\sigma_{c,t})$ to be independent across time and even weaker to assuming $log(\zeta_{c,t})$ is independent across time.

Intuitively, the second condition states that for validity of the instrument we need the first year industry comparative advantage distribution of a region to be uncorrelated with region-wide demand shocks at the current period. In other words, the fact that a city had a comparative advantage in a particular industry initially should not impact later in the future the region-wide demand shock it receives.

Proofs

Proof of Proposition 13.

See Proofs of Corollary 19.1, 19.2, 19.3 in Supplemental Appendix I.

[4]This property is typical but not limited to moving average processes.

Proof of Proposition 14.

I log linearize $(\underline{z}_u, \underline{z}_{w,i}, \hat{z}, w, w_i, \zeta, \varepsilon_i)$ around $(log(z^*), log(z^*), log(z^*), w^*, w^*, log(1), log(1))$ for the expressions of the value function of the unemployed individual rU and for the employed individual rW. Starting by rU

$$r\gamma_0 + r\gamma_1 log(\underline{z}_u) + r\gamma_2 log(\hat{z}) = \phi_0^u + \phi_1^u log(w) + \phi_2^u log(\zeta) + \\ f(\gamma_1 \sum_{\forall i} \Omega_i log(\underline{z}_{w,i})) - f\gamma_1 log(\underline{z}_u) + \alpha_0 - \alpha_1 log(\underline{z}_u) - \alpha_2 log(\hat{z}) \quad \text{(A.86)}$$

Now using

$$\frac{r}{a} \approx 0 \Rightarrow r\gamma_2 \approx 0 \quad \text{and} \quad \frac{a+\theta}{a+\beta} \approx 0 \Rightarrow \alpha_2 \approx 0 \quad \text{(A.87)}$$

Rearranging gives

$$log(\underline{z}_u) = \frac{\phi_0^u + \alpha_0}{(r+f)\gamma_1 + \alpha_1} + \frac{\phi_1^u log(w)}{(r+f)\gamma_1 + \alpha_1} + \frac{f\gamma_1}{(r+f)\gamma_1 + \alpha_1} \sum_{\forall i} \Omega_i log(\underline{z}_{w,i}) + \frac{\phi_2^u log(\zeta)}{(r+f)\gamma_1 + \alpha_1} \quad \text{(A.88)}$$

Doing the same procedure for rW and rearranging gives

$$log(\underline{z}_{w,i}) = \frac{\phi_0^w + \alpha_0}{(r+s)\gamma_1 + \alpha_1} + \frac{\phi_1^w log(w_i)}{(r+s)\gamma_1 + \alpha_1} + \frac{s\gamma_1 log(\underline{z}_u)}{(r+s)\gamma_1 + \alpha_1} + \frac{\phi_3^w log(\zeta)}{(r+s)\gamma_1 + \alpha_1} \quad \text{(A.89)}$$

Now using equation (A.89) to sum over all $log(\underline{z}_{w,i})$ gives[5]

$$\sum_{\forall i} \Omega_i log(\underline{z}_{w,i}) = A_1 + \frac{\sum_{\forall i} \Omega_i \phi_1^w ln(w_i)}{(r+s)\gamma_1 + \alpha_1} + \frac{s\gamma_1 log(\underline{z}_u)}{(r+s)\gamma_1 + \alpha_1} \quad \text{(A.90)}$$

[5]Remember that $E[log(\nu_{c,i})] \equiv \sum_{\forall i} \Omega_i log(\nu_{c,i}) = 0$ and $\sum_{\forall i} \Omega_i log(\zeta) = K$ (constant)

Now replace this sum back in equation (A.88) to get

$$
\begin{aligned}
log(\underline{z}_u) = \frac{\phi_0^u + \alpha_0}{(r+f)\gamma_1 + \alpha_1} + \frac{\phi_1^u log(w)}{(r+f)\gamma_1 + \alpha_1} + \frac{f\gamma_1 \phi_1^w ln(w)}{((r+s)\gamma_1 + \alpha_1)((r+f)\gamma_1 + \alpha_1)} \\
+ \frac{f\gamma_1 s\gamma_1}{((r+s)\gamma_1 + \alpha_1)((r+f)\gamma_1 + \alpha_1)} log(\underline{z}_u) + \frac{A_1 f\gamma_1}{(r+f)\gamma_1 + \alpha_1} + \frac{\phi_2^u log(\zeta)}{(r+f)\gamma_1 + \alpha_1}
\end{aligned} \tag{A.91}
$$

Rearranging gives

$$
log(\underline{z}_u) = \xi_0^u + \frac{(\phi_1^u((r+s)\gamma_1 + \alpha_1) + f\gamma_1\phi_1^w)}{((r+f+s)\gamma_1 + \alpha_1)(r\gamma_1 + \alpha_1)} log(w) + \xi_2^u log(\zeta) \tag{A.92}
$$

$$
log(\underline{z}_u) = \xi_0^u + \xi_1^u ln(w) + \xi_2^u log(\zeta) \tag{A.93}
$$

Now replace this final expression of $log(\underline{z}_u)$ into $log(\underline{z}_{w,i})$ to get

$$
\begin{aligned}
log(\underline{z}_{w,i}) = \xi_0^w + \frac{\phi_1^w ln(w_i)}{(r+s)\gamma_1 + \alpha_1} + \frac{s\gamma_1(\phi_1^u((r+s)\gamma_1 + \alpha_1) + f\gamma_1\phi_1^w)}{((r+s)\gamma_1 + \alpha_1)[((r+f+s)\gamma_1 + \alpha_1)(r\gamma_1 + \alpha_1)]} ln(w) \\
+ \frac{s\gamma_1 \xi_2^u log(\zeta)}{(r+s)\gamma_1 + \alpha_1} + \frac{\phi_3^w log(\zeta)}{(r+s)\gamma_1 + \alpha_1}
\end{aligned} \tag{A.94}
$$

$$
log(\underline{z}_{w,i}) = \xi_0^w + \xi_1^w ln(w_i) + \xi_2^w ln(w) + \xi_3^w log(\zeta) \tag{A.95}
$$

Now taking an average over all industries gives[6]

$$
E_i(log(\underline{z}_{w,i})) \equiv \sum_{\forall i} \Omega_i log(\underline{z}_{w,i}) = \xi_0^w + (\xi_1^w + \xi_2^w) ln(w) + \xi_3^w log(\zeta) \tag{A.96}
$$

[6] Using the fact that $\sum_{\forall i} log(\nu_{c,i}) = 0$

Finally, note that

$$\phi_1^w > \phi_1^u \Rightarrow (\phi_1^w - \phi_1^u)(r\gamma_1 + s\gamma_1 + \alpha_1) + f\gamma_1\phi_1^w - f\gamma_1\phi_1^w > 0 \tag{A.97}$$

Passing $-f\gamma_1\phi_1^w - \phi_1^u(r\gamma_1 + s\gamma_1 + \alpha_1)$ to the other side, dividing both sides by $(r\gamma_1 + f\gamma_1 + \alpha_1 + s\gamma_1)((r+s)\gamma_1 + \alpha_1)$, and using the fact that $\frac{(r\gamma_1+\alpha_1)}{(r\gamma_1+\alpha_1)} = 1$

$$\Rightarrow \frac{\phi_1^w}{(r+s)\gamma_1 + \alpha_1} > \frac{(r\gamma_1 + \alpha_1)(\phi_1^u((r+s)\gamma_1 + \alpha_1) + f\gamma_1\phi_1^w)}{((r+s)\gamma_1 + \alpha_1)((r+f+s)\gamma_1 + \alpha_1)(r\gamma_1 + \alpha_1)} \tag{A.98}$$

Now add $\frac{s\gamma_1(\phi_1^u((r+s)\gamma_1+\alpha_1)+f\gamma_1\phi_1^w)}{((r+s)\gamma_1+\alpha_1)[((r+f)\gamma_1+\alpha_1)(r\gamma_1+\alpha_1)+s\gamma_1(r\gamma_1+\alpha_1)]}$ to both sides

$$\Rightarrow (\xi_1^w + \xi_2^w) \equiv \frac{\phi_1^w}{(r+s)\gamma_1 + \alpha_1} + \frac{s\gamma_1(\phi_1^u((r+s)\gamma_1 + \alpha_1) + f\gamma_1\phi_1^w)}{((r+s)\gamma_1 + \alpha_1)[((r+f+s)\gamma_1 + \alpha_1)(r\gamma_1 + \alpha_1)]}$$
$$> \frac{\phi_1^u((r+s)\gamma_1 + \alpha_1) + f\gamma_1\phi_1^w}{((r+f+s)\gamma_1 + \alpha_1)(r\gamma_1 + \alpha_1)} \equiv \xi_1^u \tag{A.99}$$

Proof of Proposition 15.

In the supplemental appendix I show that equilibrium employment at an industry (E_i) is

$$E_{i,c,1} = \frac{f\Omega_i u_{c,1}}{\psi(1 - F(\underline{z}_{w,i,c,1})) + s}, \quad \forall i \in I. \tag{A.100}$$

Then $\kappa_{c,i,1}$ is equal to

$$\kappa_{c,i,1} \equiv \frac{E_{i,c,1}}{E_{c,1}} = \Omega_i(\psi(1 - F(\underline{z}_{w,i,c,1})) + s)^{-1}(\sum_j \Omega_j(\psi(1 - F(\underline{z}_{w,j,c,1})) + s)^{-1})^{-1} \tag{A.101}$$

Now linearize $(\psi(1-F(\underline{z}_{w,i,c,1}))+s)^{-1}$ with respect to $log(\underline{z}_{w,i,c,1})$ to get

$$\kappa_{c,i,1} = \Omega_i(\rho_0+\rho_1 log(\underline{z}_{w,i,c,1}))(\sum_j \Omega_j(\rho_0+\rho_1 log(\underline{z}_{w,j,c,1})))^{-1} \tag{A.102}$$

Now remember that $log(\underline{z}_{w,i,c,1})$ can be written as a function of $log(w_{c,1})$, $log(w_{c,i,1})$ and $log(\zeta_{c,1})$. Furthermore from market clearing we can linearize both $log(w_{c,1})$ and $log(w_{c,i,1})$ with respect to $log(\zeta_{c,1})$ and $log(\nu_{c,i,1})$ around $(K,0)$.[7] Then replacing $log(\underline{z}_{w,j,c,1})$ by its expression with only $log(\nu_{i,c,1})$ and $log(\zeta_{c,1})$ we get

$$\kappa_{c,i,1} = \frac{\Omega_i(\Gamma_0+\Gamma_1 log(\nu_{c,i,1})+\Gamma_2 log(\zeta_{c,1}))}{(\sum_j \Omega_j(\Gamma_0+\Gamma_1 log(\nu_{c,i,1})+\Gamma_2 log(\zeta_{c,1})))} \tag{A.103}$$

Now using $\sum_j \Omega_j log(\nu_{c,i,1}) = 0$ and $\sum_j \Omega_j log(\zeta_{c,1}) = K, \forall t$ gives the result.

A.1.4 Robustness of Testable Prediction to allow for Worker Mobility

In this Appendix section I discuss the implications of allowing for mobility of unemployed individuals across local labour markets. Let μ_1 represent the rate at which the unemployed has the opportunity to change local labour market. When doing so the individual chooses the city (economic region) that gives the highest utility. It follows that the problem of the unemployed at region c can be rewritten as

$$(r+\mu_1)U^{c,t} = bw_{c,t} + \psi \int_{\underline{z}_u^{c,t}} (\sum_{\forall i} \Omega_i J^{c,t}(z) - U^{c,t}) dF(z) + \mu_1 \max_c U^{c,t} + f(\sum_{\forall i} \Omega_i W_i^{c,t} - U^{c,t}) \tag{A.104}$$

Note that the term $\max_c U^{c,t}$ is a city invariant time effect. It follows that after linearizing we get the same expressions for $\underline{z}_u$ and $\underline{z}_w$ as a function of wages w as

[7] Remember that $\sum_{\forall i} \Omega_i log(\nu_{c,i,1}) = 0$ and $\sum_{\forall i} \Omega_i \zeta_{c,t} = K$

in Section A.1.3 of the Appendix except now with an additional constant for $\underline{z}_u$. As a result the empirical specification for testing the model is unchanged.

A.1.5 Details on Instrument and Wage Measure

In this section I describe how I construct my economic region/year wage measure $log(w_{c,t})$ and the instrument used in the wage regression $\sum_{\forall i} \kappa_{c,i,1} log(w^N_{i,t})$. The definition of local labour market is always an economic region, and the industry category used is always 3 digit NAICS industry classifications. Below let p denote individual p in the sample.

For each year t I run the following regression :

$$log(\text{annual worker earnings})_{p,t} = X_{p,t}\gamma_{4,1} + \sum_{\forall y} \gamma_{4,y} \mathbb{1}\{year = y\} + \sum_{\forall c} \gamma_{4,c} \mathbb{1}\{region = c\}$$
$$+ \sum_{\forall y} \sum_{\forall c} \gamma_{4,c,y} \mathbb{1}\{region = c \cap year = y\} + \varepsilon_{i,t}$$

where $X_{p,t}$ are dummies in age, gender, country of birth and 3 digit NAICS industry code. The wage measure is

$$log(w_{c,t}) = \sum_{\forall y} \hat{\gamma}_{4,y} \mathbb{1}\{year = y\} + \sum_{\forall c} \hat{\gamma}_{4,c} \mathbb{1}\{region = c\} + \sum_{\forall y} \sum_{\forall c} \hat{\gamma}_{4,c,y} \mathbb{1}\{region = c \quad \cap \quad year = y\}$$

Now for constructing the instrument I first estimate the national industry premia for each industry $log(w^N_{i,t})$. For each year t I run the following regression :

$$log(\text{annual worker earnings})_{p,t} = Z_{p,t}\gamma_{5,1} + \sum_{\forall y}\gamma_{5,y}\mathbb{1}\{year = y\} + \sum_{\forall I}\gamma_{5,I}\mathbb{1}\{industry = I\}$$
$$+ \sum_{\forall y}\sum_{\forall I}\gamma_{5,I,y}\mathbb{1}\{industry = I \cap year = y\} + \varepsilon_{i,t}$$
$$\text{where} \quad Z_{p,t} \quad \text{are dummies in age, gender, country of birth and city.}$$

Then the national industry premium is

$$log(w^N_{i,t}) = \sum_{\forall y}\hat{\gamma}_{5,y}\mathbb{1}\{year = y\} + \sum_{\forall I}\hat{\gamma}_{5,I}\mathbb{1}\{industry = I\}$$
$$+ \sum_{\forall y}\sum_{\forall I}\hat{\gamma}_{5,I,y}\mathbb{1}\{industry = I \cap year = y\}$$

Finally, the employment share of a particular industry i, in region c, at the first year of the sample, is calculated as

$$\kappa_{c,i,1} = \frac{\text{Total employment in industry } i \text{ at region } c \text{ at year 2001}}{\text{Total employment at region } c \text{ at year 2001}} \tag{A.105}$$

A.1.6 Proofs Calibration section

In this section I go over the formal theorem that allows me to pin down μ and σ in the data, where μ and σ are the two parameters governing how the productivity of an entrepreneur owned firm evolves once the firm start operating.

Proposition 16. *Let δ be the shape of the size distribution of the entire population* *of firms, then*

$$E[\Delta log(n_{i,t})|\Delta log(n_{i,t}) > 0] = \frac{\mu}{1-\alpha} + \frac{\sigma}{1-\alpha}\lambda(\frac{-\mu}{\sigma}) \quad and \quad \frac{-2\mu}{\sigma^2} = \frac{\delta+1}{1-\alpha}$$

where $\lambda(.)$ is the Inverse Mils Ratio.

$E[\Delta log(n_{i,t})|\Delta log(n_{i,t}) > 0]$ and δ are computed using firms of all ages.

Proof of Proposition 16.

Note that the expression for $dz(t)$ can be approximated as

$$z_{i,t} = z_{i,t-1} + \mu + \sigma\varepsilon_{i,t} \tag{A.106}$$

with

$$\varepsilon_{i,t} \sim N(0,1) \tag{A.107}$$

Replacing $z_{i,t}$ by its expression with firm size $n_{i,t}$

$$log(n_{i,t}) = log(n_{i,t-1}) + \frac{\mu}{1-\alpha} + \frac{\sigma}{1-\alpha}\varepsilon_{i,t} \tag{A.108}$$

It follows

$$\Delta log(n_{i,t}) = \frac{\mu}{1-\alpha} + \frac{\sigma}{1-\alpha}\varepsilon_{i,t} \tag{A.109}$$

Now let m be $E[\Delta log(n_{i,t})|\Delta log(n_{i,t}) > 0]$ it follows,[8]

$$m \equiv E[\Delta log(n_{i,t})|\Delta log(n_{i,t}) > 0] = \frac{\mu}{1-\alpha} + \frac{\sigma}{1-\alpha}E[\varepsilon|\Delta log(n_{i,t}) > 0] \quad \text{(A.110)}$$

$$m \equiv \frac{\mu}{1-\alpha} + \frac{\sigma}{1-\alpha}E[\varepsilon|\varepsilon > \frac{-\mu}{\sigma}] \quad \text{(A.111)}$$

$$m = \frac{\mu}{1-\alpha} + \frac{\sigma}{1-\alpha}\lambda(\frac{-\mu}{\sigma}) \quad \text{(A.112)}$$

where $\lambda(.)$ is the Inverse Mils Ratio.

Now note that for large enough z the distribution of type j, where $j \in \{u, w\}$ will be given by[9]

$$\Lambda^j(z) = \Lambda^j_2(z) = e^{\frac{2\mu}{\sigma^2}z}M^j[(\frac{\beta\frac{\sigma^2}{2\mu}e^{\frac{-2\mu \underline{z}_j}{\sigma^2}}}{\mu+\frac{\sigma^2}{2}\beta} - \frac{e^{\frac{-2\mu\hat{z}}{\sigma^2}}}{-\mu}) - \frac{e^{-(\beta+\frac{2\mu}{\sigma^2})z}}{e^{-\beta\underline{z}_j}(\mu+\frac{\sigma^2}{2}\beta)}] \quad \text{(A.113)}$$

Using

$$e^z = n^{1-\alpha}\frac{w}{\alpha} \quad \text{(A.114)}$$

$$\Lambda^j(n(z,w)) = \Lambda^j_2(n(z,w)) = (\frac{w}{\alpha})^{\frac{2\mu}{\sigma^2}}n^{\frac{(1-\alpha)2\mu}{\sigma^2}}M^j[(\frac{\beta\frac{\sigma^2}{2\mu}e^{\frac{-2\mu \underline{z}_j}{\sigma^2}}}{\mu+\frac{\sigma^2}{2}\beta} - \frac{e^{\frac{-2\mu\hat{z}}{\sigma^2}}}{-\mu}) - \frac{n^{-(1-\alpha)(\beta+\frac{2\mu}{\sigma^2})}(\frac{w}{\alpha})^{(\beta+\frac{2\mu}{\sigma^2})}}{e^{-\beta\underline{z}_j}(\mu+\frac{\sigma^2}{2}\beta)}] \quad \text{(A.115)}$$

[8] Note that taking the unconditional expectation and comparing it to the mean in the data would be wrong since the observed population of firms is a selected group among those that survived, i.e., $log(n) > log(n(\hat{z}, w))$. On the other hand, note that conditional on $log(n_{i,t-1})$ being observed, conditioning on $log(n_{i,t}) > log(n_{i,t-1})$ is stronger than $log(n_{i,t}) > log(n(\hat{z}, w))$. To see this note that $log(n_{i,t-1})$ observed means $log(n_{i,t-1}) > log(n(\hat{z}, w))$. It follows that once I condition on positive growth and adjust the expectation accordingly I don't need to adjust for selection.

[9] More precisely, for $z \geq \max\{\underline{z}_1, \underline{z}_2\}$

which implies

$$n^{-\frac{(1-\alpha)2\mu}{\sigma^2}}\Lambda^j(n) = (\frac{w}{\alpha})^{\frac{2\mu}{\sigma^2}} M^j[(\frac{\beta\frac{\sigma^2}{2\mu}e^{\frac{-2\mu z_j}{\sigma^2}}}{\mu+\frac{\sigma^2}{2}\beta} - \frac{e^{\frac{-2\mu\hat{z}}{\sigma^2}}}{-\mu}) - \frac{n^{-(1-\alpha)(\beta+\frac{2\mu}{\sigma^2})}(\frac{w}{\alpha})^{(\beta+\frac{2\mu}{\sigma^2})}}{e^{-\beta z_j}(\mu+\frac{\sigma^2}{2}\beta)}] \tag{A.116}$$

Now summing over all j

$$n^{-\frac{(1-\alpha)2\mu}{\sigma^2}}\Lambda(n) = (\frac{w}{\alpha})^{\frac{2\mu}{\sigma^2}} \sum_j M^j[(\frac{\beta\frac{\sigma^2}{2\mu}e^{\frac{-2\mu z_j}{\sigma^2}}}{\mu+\frac{\sigma^2}{2}\beta} - \frac{e^{\frac{-2\mu\hat{z}}{\sigma^2}}}{-\mu}) - \frac{n^{-(1-\alpha)(\beta+\frac{2\mu}{\sigma^2})}(\frac{w}{\alpha})^{(\beta+\frac{2\mu}{\sigma^2})}}{e^{-\beta z_j}(\mu+\frac{\sigma^2}{2}\beta)}] \tag{A.117}$$

Now assume $\beta \geq -\frac{2\mu}{\sigma^2}$

$$\lim_{n\to\infty} n^{-\frac{(1-\alpha)2\mu}{\sigma^2}}\Lambda(n) = (\frac{w}{\alpha})^{\frac{2\mu}{\sigma^2}} \sum_j M^j[(\frac{\beta\frac{\sigma^2}{2\mu}e^{\frac{-2\mu z_j}{\sigma^2}}}{\mu+\frac{\sigma^2}{2}\beta} - \frac{e^{\frac{-2\mu\hat{z}}{\sigma^2}}}{-\mu})] < \infty \tag{A.118}$$

It follows that for large enough $n, \Lambda(n)$ decays at speed given by $n^{\frac{2\mu(1-\alpha)}{\sigma^2}}$ $\forall i$. It follows that for a large enough firm size, the firm size distribution will be Pareto of tail parameter x,

$$x = \frac{-2\mu(1-\alpha)}{\sigma^2} - 1 \tag{A.119}$$

It follows that given x and α, μ and σ can be pinned down by the following two equations

$$\frac{-2\mu}{\sigma^2} = \frac{1+x}{1-\alpha} \tag{A.120}$$

and

$$E[\Delta log(n_{i,t})|\Delta log(n_{i,t}) > 0] = \frac{\mu}{1-\alpha} + \frac{\sigma}{1-\alpha}\lambda(\frac{-\mu}{\sigma}) \tag{A.121}$$

where $\lambda(.)$ is the Inverse Mils Ratio. $E[\Delta log(n_{i,t})|\Delta log(n_{i,t}) > 0]$ and x are estimated in the CEED data.

A.2 Calibration

The following table lists the whole set of parameters and the proceedures to chose each parameter value. For details see main body of the paper.

Table A.2.1: Calibration

Parameter	value at annual frequency	Source/Target
β	8.32	$Entry_u/Entry_w$
μ	-0.11	$E[\Delta log(n)\vert\Delta log(n) > 0]$ in data
σ	0.186	Shape of size distribution of all firms in data
r	4.5%	-
α	2/3	Average aggregate labour share
K	0.4	$E[log(n^w)] - E[log(n^u)]$
s	0.214	Hobijn and Şahin [2009]
b	0.6	Replacement rate for unemployed
χ	0.268	$Exit_u/Exit_w$
c	0.562	Normalize θ to 1 as in Shimer [2005]
ϕ	0.72	Shimer [2005]
γ	0.72	Shimer [2005]
ψ	24	Consider robustness to different values

Notes: Calibration Table. $Entry_u/Entry_w$ is the ratio of entry rate into entrepreneurship between the unemployed and wage workers. $E[log(n^w)] - E[log(n^u)]$ is the difference in average number of employees between firms created by workers versus the unemployed. $Exit_u/Exit_w$ is the ratio of exit rates out of entrepreneurship between entrepreneurs that were unemployed when they started their business and those that were working when they started their firm.

A.3 Alternative Calibration

In Table A.3.1 I show that the impact of the policy in the aggregate economy is robust to changing the value of the rate at which individuals receives business projects, ψ.

Table A.3.1: Model Extension - Different ψ values

ψ values	12	24	36
E[arrival time of projects]	1 month	$\frac{1}{2}$ month	$\frac{1}{3}$ month
$\Delta E[z]$	-2.10%	-2.14%	-2.16%
Δ Unemployment Rate (% change)	-1.11%	-1.11%	-1.11%
Δ Wage	0.62%	0.61%	0.60%
Δ Labor Market Tightness (θ)	2.30%	2.35%	2.38%
Δ Jobs by Firms created by Unemployed	7.22%	7.12%	7.06%
Δ Jobs by Firms created by Workers	-7.13%	-7.10%	-7.08%
Δ Firm Exit Rate (% change)	35.27%	36.38%	37.07%

Notes: Outcome of policies that make a share of total UI benefits income conditional on the unemployed opening a firm. $\Delta E[z]$ is the percentage change in the average firm productivity, Δ Jobs by firms created by workers is the percentage change in the measure of jobs associated to firms created by wage workers, Δ Unemployment is the percentage change in the unemployment rate. Results are shown for different values of ψ. *psi* is the rate at which individuals receive business projects. E[arrival time of projects] is the expected arrival time of a business project in the economy given the ψ value chosen.

Next we go over the impact of the counterfactual policies of subsidizing or taxing unemployed starting a business with an alternative calibration strategy for the cost of posting a vacancy c. I follow Hagedorn and Manovskii [2008] in setting the cost of posting of a vacancy to 4.5% of the equilibrium wage ($c = 0.045$). All other parameters are chosen in the same manner as in the benchmark calibration.

Table A.3.2: Policy outcomes

	Baseline Calibration	Robustness - c
	(1)	(2)
$\Delta E[z]$	-2.14%	-2.14%
Δ Unemployment Rate (% change)	-1.11%	-1.11%
Δ Wage	0.61%	0.61%
Δ Labor Market Tightness (θ)	2.35%	2.35%
Δ Jobs by Firms created by Unemployed	7.12%	7.12%
Δ Jobs by Firms created by Workers	-7.1%	-7.1%
Δ Average Firm Exit Rate (percent change)	36.38%	36.38%

Notes: Outcome of policies that make a share of total UI benefits income conditional on the unemployed opening a firm. $\Delta E[z]$ is the percentage change in the average firm productivity, Δ Jobs by firms created by workers is the percentage change in the measure of jobs associated to firms created by wage workers, Δ Unemployment is the percentage change in the unemployment rate. First column presents results for baseline calibration. Second column shows robustness to calibration of the cost of posting a vacancy, c. In particular, I follow Hagedorn and Manovskii [2008] in setting the cost of posting of a vacancy to 4.5% of the equilibrium wage ($c = 0.045$). All other parameters are chosen in the same manner as in the benchmark calibration.

A.3.1 Firms created by Laid off versus not Laid-off individuals (without Fixed Effects)

Before proceeding to the results without fixed effects, recall that the baseline group compared to the displaced individuals are all individuals that were employed in the previous year by a firm that in the current year continues to exist. This implies that the group of entrepreneurs tagged as having entered from wage work also includes individuals that were employed in the prior year to opening a firm and had an unemployment spell in between the job and the start of a firm. As a result, this group includes individuals that started a firm after being fired as long as the spell of unemployment was shorter than a year. Individuals who are fired are likely to be a negatively selected group of the population. This negative selection becomes particularly important if individuals fired are more likely to start a firm than individuals that never lost their job.

The result is that, without fixed effects, we capture some of this negative selection that offsets the differences between laid off and employed individuals. Consistent with this concern, the results in Column (1) and (2) of Table A.3.3 indicate that once we do not control for individual fixed effects the differences in firm size between laid off and not laid-off individuals disappears and the difference in exit rates decreases and flips sign.

Table A.3.3: Log number of employees

Dependant variable	log # employees	Exit dummy
	(1)	(2)
$\mathbb{1}\{\text{Prev U}\}_{i,s}$	0.012	-0.0064^{***}
	(0.013)	(0.007)
Fixed Effects	No	No
Baseline Exit Probability	0.055	0.055
Ratio of probabilities	Not applicable	0.9
Observations	450,502	341,214

Notes: Column (1) reports results for regression without fixed effects of log number of employees of the current business on a dummy indicating if the current business was started by an individual when laid off ($\mathbb{1}\{\text{Prev U}\}_{i,s}$). Column (2) reports results for regression without fixed effects of dummy for exit (takes value 1 if individual exits entrepreneurship and 0 otherwise) on ($\mathbb{1}\{\text{Prev U}\}_{i,s}$). Regression on Column (2) only includes individuals that last year were running a business. Other controls include dummies in age groups, marital status, province of residence, year business started, current year, 2 digit industry code for current business, 2 digit industry code for the last employer, log number of employees for the last employer. Only includes men 25 to 54 years old. Without fixed effects, we do not control for the fact that the group of not laid-off individuals includes individuals that were fired and are likely to be negatively selected in ability. This negative selection among the individuals that were fired and started a firm offsets the differences between firms created by the employed versus laid off individuals. Standard errors are clustered at the individual level.

A.3.2 Data Appendix

In this section I describe the components of the dataset being used. The construction of the data was done by Statistics Canada and not by the author.

The dataset is a combination of information from three types of tax forms in Canada. The first is the T1 form, which is just the individual tax return form.[10] From there, we get demographic information, age and marital status, total annual income of the individual and total labour earnings of the individual. The second is the T4 form. This is a form that every employer must file for each of its employees.[11] These files give us information for each individual the firms for which they worked for and their labour earnings in that tax year. The final tax files come from the schedule 50 of the T2 form. According to Canadian law, incorporated firms must list all owners that have at least 10% ownership. These files allow me to link each firm to individual entrepreneurs.[12] Together these files allow me to link each individual to a firm they are working on or to a firm they own.

The last step is matching all these incorporated firms to firms present in the Longitudinal Employment Analysis Program (LEAP) Dataset. This dataset contains the entire universe of firms with employees in Canada, whether incorporated or not. From this dataset, I get a measure of the number of employees for each firm (ALU, average labour unit). The matching with the LEAP dataset allows us to construct a time consistent firm identifier that takes into account mergers and splitting of a same firm in multiple ones.[13]

[10]The equivalent in the United States is the 1040A form.

[11]The equivalent in the US is the W-2, Wage and Tax Statement.

[12]The equivalent in the US to the schedule 50 of the T2 form is the schedule G of 1120 form (Corporate Income Tax Form in the US). The only difference is that under US law, a corporation only needs to list owners that own at least 20% of the firm.

[13]To identify a same firm the LEAP dataset uses a strategy entitled ”labour tracking”. If a firm A

A.3.3 Supplemental Appendix I : Solving for Multi-Industry Economy model.

This section solves for the multi-sector model economy presented in the paper. It starts by the full characterization of the model. Solving the model then allows the proof of differential performance between firms created by not working versus working individuals. (Proposition 13 in Paper)

$$rU = bw\zeta + f(\sum_{\forall j \in I} \Omega_j W_j - U) + \psi \int_{\underline{z}_u} (\sum_{\forall j \in I} \Omega_j J(z) - U) dF(z) \tag{A.122}$$

$$rW_i = w\nu_i\zeta + s(U - W_i) + \psi \int_{\underline{z}_{w,i}} (\sum_{\forall j \in I} \Omega_j J(z) - W_i) dF(z) \tag{A.123}$$

where ν_i is the relative efficiency units a worker is endowed for industry i and ζ is an economy-wide efficiency unit endowment, where $E[log(\zeta)] \equiv \sum_i \Omega_i log(\zeta) = K$ is time invariant. Let $\nu_i = \overline{\nu}_i \varepsilon_i$, where $\sum\limits_i \Omega_i log(\varepsilon_i) = 0$, $\sum\limits_i \Omega_i log(\overline{\nu}_i) = 0$ and $\overline{\nu}$ is time invariant. Now define the economy level wage as w and the average industry level wage as $w_i \equiv w\overline{\nu}_i$.

$$\sum_{\forall j \in I} \Omega_j J(\underline{z}_u) = U \tag{A.124}$$

$$\sum_{\forall j \in I} \Omega_j J(\underline{z}_{w,i}) = W_i \tag{A.125}$$

splits into firm B and firm C but continues to do the exact same business as before, the method marks firms B and C with the identifier of firm A, since firm B and C together have the same industry and workforce as A. This is important since for all purposes, nothing has changed except for the official naming of the company that now are two firms, even though the owners and employees are the same. For more details see the Statistics Canada website.

Firm static decision :

$$\pi^*(z) = \max_n zn^\alpha - wn \tag{A.126}$$

which implies

$$\pi^*(z) = (1-\alpha)(\frac{\alpha}{w})^{\frac{\alpha}{1-\alpha}} e^{\frac{z}{1-\alpha}} \tag{A.127}$$

Once a business starts operating, Z follows a geometric Brownian Motion with drift $\mu < 0$ and variance parameter σ.

$$dZ(t) = (\mu + \frac{\sigma^2}{2})Z(t)dt + \sigma Z(t)d\Omega(t) \tag{A.128}$$

Where $\Omega(t)$ is a standard Brownian Motion. Then it follows

$$dz(t) = \mu dt + \sigma d\Omega(t) \tag{A.129}$$

It follows entrepreneurs face the following stopping problem

$$rJ(z) = \pi^*(z) + \mu J'(z) + \frac{\sigma^2}{2} J''(z) \quad \text{if} \quad z \geq \hat{z} \tag{A.130}$$

$$J(z) = U - \chi \quad \text{if} \quad z \leq \hat{z} \tag{A.131}$$

$$J'(\hat{z}) = 0 \tag{A.132}$$

where χ is a cost of shutting down.

μ is assumed to be negative otherwise there would be an accumulation of firms that never exit the market. The cost of shutting down g makes the algebra tractable by guaranteeing the expressions for the distributions of both types will be identical with the only difference coming from the difference in thresholds $\underline{z}_u$ versus $\underline{z}_{w,i}$

and the unemployment to employment transition rate versus the employment to unemployment transition rate, i.e., f and s.

Market Clearing

$$\sum_{\forall i} E_i \zeta \nu_i = \int n(z,w)\Lambda(z)dz \tag{A.133}$$

Proposition 17. *The solution to the firm's optimal stopping problem implies*

$$J(z) = \frac{B}{r - \frac{\mu}{1-\alpha} - \frac{\sigma^2}{2}\frac{1}{(1-\alpha)^2}}\left(e^{\frac{z}{1-\alpha}} + \frac{1}{a(1-\alpha)}e^{-a(z-\hat{z})+\frac{\hat{z}}{1-\alpha}}\right) \tag{A.134}$$

where

$$B \equiv (1-\alpha)\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} \tag{A.135}$$

$$a = \frac{\mu + \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2} > 0 \tag{A.136}$$

Not surprisingly, the value function of the business owner is increasing in productivity for the range of values for which the business operates $z \in [\hat{z}, \infty[$. [14]

Let $\Lambda^w_{i,j}(z)$ denote the measure of individuals operating a firm of productivity z in industry j, that were workers in industry i else prior to opening the firm and $\Lambda^u_i(z)$ the measure with productivity z that entered from unemployment into industry i.

---

[14] To see this note that

$$\frac{\partial^2 J(z)}{\partial z^2} = C\left(\frac{1}{1-\alpha}\right)\left(\frac{e^{\frac{z}{1-\alpha}}}{1-\alpha} + ae^{-a(z-\hat{z})+\frac{\hat{z}}{1-\alpha}}\right) > 0 \tag{A.137}$$

and for $z = \hat{z}$

$$\frac{\partial J(z)}{\partial z} = 0. \tag{A.138}$$

This implies for $z \geq \hat{z}$,

$$\frac{\partial J(z)}{\partial z} \geq 0. \tag{A.139}$$

Proposition 18. *For all $d \in \{u, w\}$, the measure of business owners of productivity z will be given by,*

- *For $z \in [\hat{z}, \underline{z}_{d,i}]$*

$$\Lambda^d_{i,j}(z) = \Lambda^d_{i,j,1}(z) = \frac{M^d_{i,j}}{-\mu}(1 - e^{\frac{2\mu}{\sigma^2}(z-\hat{z})}) \tag{A.140}$$

- *For $z \in]\underline{z}_{d,i}, \infty[$*

$$\Lambda^d_{i,j}(z) = \Lambda^d_{i,j,2}(z) = \frac{\beta M^d_{i,j} \frac{\sigma^2}{-2\mu} e^{\frac{2\mu}{\sigma^2}(z-\underline{z}_{d,i})}}{(\mu + \frac{\sigma^2 \beta}{2})} - \frac{M^d_{i,j}}{-\mu} e^{\frac{2\mu}{\sigma^2}(z-\hat{z}_i)} - \frac{M^d_{i,j} e^{-\beta z}}{e^{-\beta \underline{z}_{d,i}}(\mu + \frac{\sigma^2}{2}\beta)} \tag{A.141}$$

where

$$M^d_{i,j} = \psi \Omega_i u e^{-\beta \underline{z}_u} \quad \text{if} \quad d = u \tag{A.142}$$

$$M^d_{i,j} = \psi \Omega_j E_i e^{-\beta \underline{z}_{w,i}} \quad \text{if} \quad d = w \tag{A.143}$$

We are now ready to define a Stationary competitive equilibrium

Definition 2. *A Stationary competitive equilibrium is defined by a set of $\underline{z}_u, \underline{z}_{w,i}, w_i, E_i, \eta^u_i, \eta^w_{i,j}, \Lambda^u_i(z)$, $\Lambda^w_{i,j}(z), u$, $\forall (i, j) \in IxI$ such that*

- $W_i > U, \quad \forall i \in I$
- $\sum_{\forall j \in I} \Omega_j J(\underline{z}_{w,i}) = W_i, \quad \forall i \in I$
- $\sum_{\forall j \in I} \Omega_j J(\underline{z}_u) = U$
- $J(\hat{z}) = U - \chi, \quad \forall i \in I$
- *The expression for $J(z)$ is given by Proposition 17*

- *The expression for $\Lambda_i^u(z)$ and $\Lambda_{i,j}^w(z)$ are given by Proposition 18*

- *E_i is given by*

$$E_i = \frac{f\Omega_i u}{\psi(1-F(\underline{z}_{w,i}))+s}, \quad \forall i \in I \tag{A.144}$$

- *u is given by*

$$u = \frac{1}{1+\psi A_u e^{-\beta \underline{z}_u} + f\sum_{\forall i \in I} \frac{s\Omega_i[1+A_{w,i}(1-F(\underline{z}_{w,i}))]}{s+\psi(1-F(\underline{z}_{w,i}))}} \tag{A.145}$$

where

$$A_u = \frac{1+\beta(\underline{z}_u - \hat{z})}{-\mu\beta} \tag{A.146}$$

and

$$A_{w,i} = \frac{1+\beta(\underline{z}_{w,i} - \hat{z})}{-\mu\beta} \tag{A.147}$$

- *η_j^u is given by*

$$\eta_j^u = \psi A_u \Omega_j u e^{-\beta \underline{z}_u} \tag{A.148}$$

- *$\Lambda_{i,j}^w(z)$ is given by*

$$\Lambda_{i,j}^w = \psi A_{w,i} \Omega_j E_i e^{-\beta \underline{z}_{w,i}} \tag{A.149}$$

-

$$w = \alpha[(\frac{1}{\sum_{\forall i} E_i \zeta \nu_i})(\sum_{\forall i}\int_{\hat{z}} e^{\frac{z}{1-\alpha}} \Lambda_i^u(z)dz + \sum_{\forall i}\sum_{\forall j}\int_{\hat{z}} e^{\frac{z}{1-\alpha}} \Lambda_{i,j}^w(z)dz)]^{1-\alpha} \tag{A.150}$$

The first condition states that the value of being a wage worker is higher than the value of being unemployed. Otherwise, no individual would ever choose to transition to wage work and markets would not clear. The second and third guarantee that individuals' decisions to open a business are optimal and the last just comes from market clearing.

Next we are ready to go over the main theorem that will subsequently generate all the patterns that were documented in the data. It states that in equilibrium wage workers are more selective on which business opportunities to implement. The necessary and sufficient condition for it is simply that the income received as unemployed is lower than that received as a worker. Note that were it not the case the equilibrium would not exist as markets would not clear.

Proposition 19. *In equilibrium,* $\underline{z}_{w,i} > \underline{z}_u \Leftrightarrow b < 1 \quad \forall i \in I$

The next corollaries are all a result of the difference in selection directly relating to the patterns documented empirically. The first states that businesses created by wage workers have a smaller exit rate. This comes from the combination of all business owners exiting at a same threshold while having different levels of selection upon entry between the two types.

Corollary 19.1. *In equilibrium businesses created by wage workers have a lower exit rate than those created by unemployed*

The next corollary states that firms created by wage workers have higher profits and more employees. This is a direct consequence of the fact that both profits and firm size are monotonically increasing in productivity.[15]

Corollary 19.2. *In equilibrium, businesses created by wage workers, on average, have higher firm size and profits.*

[15]The result that fixing aggregates, the number of employees of a firm matches one to one with productivity is a direct consequence of the absence of frictions in the hiring and firing process of firms.

Finally, as it is often the case with selection mechanisms an increased average productivity is associated to a lower entry rate. It follows that in equilibrium the rate at which wage workers enter will be lower than that of unemployed.

Corollary 19.3. *In equilibrium the entry rate into business ownership of the unemployed is higher than that of salary workers.*

Proof of Proposition 17.

We know that it is equal to $U \quad \forall z \leq \hat{z}$. We need to find the value of $J(z)$ for $z \geq \hat{z}$. The proof just follows from the proof in Proposition 1.[16]

Proof of Proposition 18.

Solving generic KFE. The solution below is the same for both types of business owners (i.e., $d = u, w$). Let j refer to the industry the individual entered and i the industry the individual came from. Since the income received when unemployed is independent of the individual's work history, the origin of all unemployed that become entrepreneurs is always the same.[17] With abuse of notation denote $\Lambda^d_{i,j}$ as the measure of firms created by d type where d indicates whether a worker ($d = w$) or an unemployed ($d = u$), that entered into industry j and, if $d = w$, the owner came from industry i.

Let $\hat{z}$ be the point at which firms exit and $\underline{z}_{d,i}$ the point in which firms enter, with $\underline{z}_{d,i} > \hat{z}_i$. Let $\Lambda(z)^d_{i,j}$ denote the endogenous pdf and $M^d_{i,j}$ the measure of entrants. For type u ($d = u$), $M^d_{i,j}$ is equal to $\psi \Omega_j u e^{-\beta \underline{z}_u}$ and for type w ($d = w$) $M^d_{i,j}$ is equal

[16] Conditional on a wage, the problem for the entrepreneur is exactly as in the model with just one sector.

[17] In other words, there is only one type of unemployment an individual can be in. In contrast, there are many different types of wage work an individual can be in.

to $\psi E_i \Omega_j e^{-\beta \underline{z}_{w,i}}$

Finally, let for $[\underline{z}_{d,i}, \infty[$

$$\Lambda^d_{i,j}(z) = \Lambda^d_{i,j,2}(z) \tag{A.151}$$

and for $]\hat{z}, \underline{z}_{d,i}]$

$$\Lambda^d_{i,j}(z) = \Lambda^d_{i,j,1}(z) \tag{A.152}$$

Then for $[\underline{z}_{d,i}, \infty[$

$$\frac{\partial \Lambda^d_{i,j,2}(z)}{\partial t} = -\mu \frac{\partial \Lambda^d_{i,j,2}(z)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 \Lambda^d_{i,j,2}(z)}{\partial z^2} + M^d_{i,j} \frac{\beta e^{-\beta z}}{e^{-\beta \underline{z}_{d,i}}} = 0 \tag{A.153}$$

for $]\hat{z}_i, \underline{z}_{d,i}]$

$$\frac{\partial \Lambda^d_{i,j,1}(z)}{\partial t} = -\mu \frac{\partial \Lambda^d_{i,j,1}(z)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 \Lambda^d_{i,j,1}(z)}{\partial z^2} = 0 \tag{A.154}$$

The four boundary conditions are

1. $\int_{\underline{z}_{d,i}}^{\infty} \Lambda^d_{i,j}(z) dz < \infty$
2. $\Lambda^d_{i,j,1}(\underline{z}_{d,i}) = \Lambda^d_{i,j,2}(\underline{z}_{d,i})$
3. $\frac{\partial \Lambda^d_{i,j,1}(\underline{z}_{d,i})}{\partial z} = \frac{\partial \Lambda^d_{i,j,2}(\underline{z}_{d,i})}{\partial z}$
4. $\Lambda^d_{i,j,1}(\hat{z}_i) = 0$

Now, to avoid cumbersome notation drop the subscript (i, j).

Then, the proof just follows the same steps as the proof for Proposition 2.

Proof of Proposition 19.

In equilibrium $W^i > U$ otherwise, $\forall i \in I$ such $U > W^i$ all workers in that industry would choose unemployment over employment in that industry and that industry would cease to exist.

Proof of Corollary 19.1.

Note that in steady state the flow of exiting firms of each type is equal to $M^d_{i,j}$.[18] Letting ER^d_i denote Exit Rate for type d , where $d = \{u,w\}$ having entered from industry i if $d = w$ we will have

$$(ER^d_i)^{-1} = [\frac{1+\beta(\underline{z}_{d,i} - \hat{z})}{-\mu\beta}] \tag{A.155}$$

$$(ER^w_i)^{-1} > (ER^u)^{-1} \Rightarrow ER^u > ER^w_i \quad \forall i \in I \tag{A.156}$$

where the first inequality follows from $\underline{z}_{w,i} > \underline{z}_u \quad \forall i \in I$

It follows that in the steady state equilibrium the exit rate is higher for firms of type u.

Proof of Corollary 19.2.

The expression for optimal firm size is given by

$$n(z,w) = (\frac{\alpha}{w})^{\frac{1}{1-\alpha}} e^{\frac{z}{1-\alpha}} \tag{A.157}$$

It follows average size for type (d,i,j), where $d \in \{u,w\}$, j the industry the

[18] This comes from the fact that, in steady state, the flow of firms exiting a group has to be equal to the flow entering.

individual entered and i representing the industry of origin when $d = w$

$$\int_{\hat{z}} n(z,w)\frac{\Lambda^d_{i,j}(z)}{\eta^d_{i,j}}dz = (\frac{\alpha}{w})^{\frac{1}{1-\alpha}}\frac{M^d_{i,j}}{\eta^d_{i,j}}\int_{\hat{z}} e^{\frac{z}{1-\alpha}}\frac{\Lambda^d_{i,j}(z)}{M^d_{i,j}}dz \tag{A.158}$$

Now to avoid heavy notation drop the subscripts (i, j) but remember all of the proof is done for a particular (i, j) group. Then the rest of the proof just follows the proof of corollary 4.2.

Proof of Corollary 19.3.

To see that entry is higher for the unemployed, just note that

$$\underline{z}_{w,i} > \underline{z}_u \Rightarrow \psi(1 - F(\underline{z}_u)) > \psi(1 - F(\underline{z}_{w,i})), \forall i \tag{A.159}$$

A.3.4 Supplemental Appendix II : Solving for model with search frictions

To get search frictions assume there is an intermediate goods sector which transforms individuals l into labour units y used by entrepreneurs. The intermediate goods sector has free entry condition ($V = 0$)

$$rV = -cw + q(\theta)(F - V) \tag{A.160}$$

$$rF = \rho - w + \lambda(V - F) \tag{A.161}$$

$$V = 0 \tag{A.162}$$

Wage determined by Nash Bargaining

$$\phi(W-U) = (1-\phi)(F-V) \tag{A.163}$$

Problem of the unemployed

$$rU = bw + p(\theta)(W-U) + \psi\int_{\underline{z}_u}(J(z)-U)dF(z) \tag{A.164}$$

$$rW = w + s(U-W) + \psi\int_{\underline{z}_w}(J(z)-W)dF(z) \tag{A.165}$$

Problem of the entrepreneur is as before (Optimal Stopping Problem)

$$rJ(z) = \pi^*(z) + \mu J'(z) + \frac{\sigma^2}{2}J''(z) \quad \text{if} \quad z \geq \hat{z} \tag{A.166}$$

$$J(z) = U - \chi \quad \text{if} \quad z \leq \hat{z} \tag{A.167}$$

$$J'(\hat{z}) = 0 \tag{A.168}$$

and optimal quantity n of intermediate good y to purchase solves

$$\pi^*(z) \equiv \max_n e^z n^\alpha - \rho n \tag{A.169}$$

where ρ is determined by

$$(1-u-\eta) = \int_{\hat{z}} n(z,p)\Lambda(z)dz \tag{A.170}$$

Note that total production of intermediate goods is just equal to the measure of workers $(1-u-\eta)$ and the demand is the total demand due to entrepreneurs.

To see a relationship between w and ρ use Equations (A.160), (A.161) and (A.162) to get

$$w = \frac{\rho q(\theta)}{c(r+s)+q(\theta)} \tag{A.171}$$

Using the Nash Bargaining condition (Equation (A.163)) and Equations (A.162) and (A.160)

$$\frac{cw}{q(\theta)} = \frac{\phi}{1-\phi}(W-U) \tag{A.172}$$

Now replace w by its expression

$$\frac{c\rho}{c(r+s)+q(\theta)} = \frac{\phi}{1-\phi}(W-U) \tag{A.173}$$

which pins down θ for a given value of W and U. Finally, ρ is given by market clearing in the intermediate goods sector

$$1-u-\eta = \int_{\hat{z}} n(z,\rho)\Lambda(z)dz \tag{A.174}$$

where u is the measure of unemployed, η the measure of entrepreneurs, $n(z,\rho)$ the optimal amount of transformed labour to hire for a given productivity z and price ρ and $\Lambda(z)$ is the distribution of firm productivity.

Solving for optimal profits gives

$$\pi^*(z) = (1-\alpha)(\frac{\alpha}{\rho})^{\frac{\alpha}{1-\alpha}} e^{\frac{z}{1-\alpha}} \tag{A.175}$$

Replacing ρ by its expression as a function of w and θ we get

$$\pi^*(z) = (1-\alpha)(\frac{\alpha}{\frac{w(c(r+\lambda)+q(\theta))}{q(\theta)}})^{\frac{\alpha}{1-\alpha}} e^{\frac{z}{1-\alpha}} \tag{A.176}$$

The cost of an intermediate goods unit for entrepreneurs is a function of wages individuals receive and tightness in the market θ. Note that

$$\frac{\partial \pi^*(z)}{\partial \theta} < 0 \tag{A.177}$$

and

$$\frac{\partial \pi^*(z)}{\partial w} < 0 \tag{A.178}$$

Characterizing the Equilibrium

Proposition 20. *The solution to the firm's optimal stopping problem implies*

$$J(z) = \frac{B}{r - \frac{\mu}{1-\alpha} - \frac{\sigma^2}{2}\frac{1}{(1-\alpha)^2}}\left(e^{\frac{z}{1-\alpha}} + \frac{1}{a(1-\alpha)}e^{-a(z-\hat{z})+\frac{\hat{z}}{1-\alpha}}\right) \tag{A.179}$$

where

$$B \equiv (1-\alpha)\left(\frac{\alpha}{\rho}\right)^{\frac{\alpha}{1-\alpha}} \tag{A.180}$$

$$a = \frac{\mu + \sqrt{\mu^2 + 2r\sigma^2}}{\sigma^2} > 0 \tag{A.181}$$

Not surprisingly, the value function of the business owner $J(z)$ is increasing in productivity for the range of values for which the business operates $z \in [\hat{z}, \infty[$. [19]

[19] To see this note that

$$\frac{\partial^2 J(z)}{\partial z^2} = C\left(\frac{1}{1-\alpha}\right)\left(\frac{e^{\frac{z}{1-\alpha}}}{1-\alpha} + ae^{-a(z-\hat{z})+\frac{\hat{z}}{1-\alpha}}\right) > 0 \tag{A.182}$$

and for $z = \hat{z}$

$$\frac{\partial J(z)}{\partial z} = 0. \tag{A.183}$$

This implies for $z \geq \hat{z}$,

$$\frac{\partial J(z)}{\partial z} \geq 0. \tag{A.184}$$

Let $\Lambda^w(z)$ denote the measure of business owners operating a business project of productivity z that were employed by somebody else when they received the current business opportunity and $\Lambda^u(z)$ the measure of business owners operating a business project of productivity z that were not working at the moment they received the current business opportunity.[20]

Proposition 21. *For all $i \in \{u,w\}$, the measure of business owners running a firm of productivity z is given by,*

- *For $z \in [\hat{z}, \underline{z}_i]$*

$$\Lambda^i(z) = \Lambda_1^i(z) = \frac{M^i}{-\mu}(1 - e^{\frac{2\mu}{\sigma^2}(z-\hat{z})}) \tag{A.186}$$

- *For $z \in]\underline{z}_i, \infty[$*

$$\Lambda^i(z) = \Lambda_2^i(z) = \frac{\beta M^i \frac{\sigma^2}{-2\mu} e^{\frac{2\mu}{\sigma^2}(z-\underline{z}_i)}}{(\mu + \frac{\sigma^2 \beta}{2})} - \frac{M^i}{-\mu} e^{\frac{2\mu}{\sigma^2}(z-\hat{z})} - \frac{M^i e^{-\beta z}}{e^{-\beta \underline{z}_i}(\mu + \frac{\sigma^2}{2}\beta)} \tag{A.187}$$

where

$$M^i = \psi u e^{-\beta \underline{z}_u} \quad \text{if} \quad i = u \tag{A.188}$$

$$M^i = \psi(1 - u - \eta) e^{-\beta \underline{z}_w} \quad \text{if} \quad i = w \tag{A.189}$$

Corollary 21.1. *The measure of business owners, η, and the fraction that were not*

[20] In other words, this amount to saying that $\Lambda^w(z)$ and $\Lambda^u(z)$ are defined such that

$$\int_{\hat{z}} \Lambda^u(z)dz + \int_{\hat{z}} \Lambda^w(z)dz + u + e = 1 \tag{A.185}$$

where e is the measure of workers.

working prior to entering entrepreneurship, $\frac{\eta^u}{\eta}$*, are given by :*

$$\eta = \frac{\psi(1-\eta)}{s+f+\psi e^{-\beta \underline{z}_w}}[A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u} + A_w f e^{-\beta \underline{z}_w}] \tag{A.190}$$

$$\frac{\eta^u}{\eta} = \frac{A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u}}{A_u(s+\psi e^{-\beta \underline{z}_w})e^{-\beta \underline{z}_u} + A_w f e^{-\beta \underline{z}_w}} \tag{A.191}$$

where

$$A_i = [\frac{1+\beta(\underline{z}_i - \hat{z})}{-\mu\beta}]. \tag{A.192}$$

We are now ready to define a Stationary competitive equilibrium

Definition 3. *A Stationary competitive equilibrium is defined by* $\underline{z}_u, \underline{z}_w, \rho, \eta, \eta^u, \Lambda^u(z), \Lambda^w(z), u, \theta, w$ *such that*

- $W > U$
- $J(\underline{z}_w) = W$
- $J(\underline{z}_u) = U$
- $J(\hat{z}) = J(\underline{z}_u) - g$
- *The expression for* $J(z)$ *is given by Proposition 20*
- *The expression for* $\Lambda^u(z)$ *and* $\Lambda^w(z)$ *are given by Proposition 21*
- *u is given by*

$$u = \frac{(s+\psi(1-F(\underline{z}_w)))(1-\eta)}{p(\theta)+s+\psi(1-F(\underline{z}_w))} \tag{A.193}$$

- *η and η^u are defined by corollary 21.1*

- $$\rho = \alpha[\frac{1}{(1-u-\eta)}(\int_{\hat{z}} e^{\frac{z}{1-\alpha}}\Lambda^u(z)dz + \int_{\hat{z}} e^{\frac{z}{1-\alpha}}\Lambda^w(z)dz)]^{1-\alpha} \quad \text{(A.194)}$$

- $$\frac{c\rho}{c(r+s)+q(\theta)} = \frac{\phi}{1-\phi}(W-U) \quad \text{(A.195)}$$

- $$w = \frac{\rho q(\theta)}{c(r+s)+q(\theta)} \quad \text{(A.196)}$$

The first condition states that the value of being a employed worker is higher than the value of not working. Otherwise, no individual would ever choose to transition to wage work and markets would not clear. The second and third guarantee that individuals' decisions to open a business are optimal and the last three just come from market clearing in the final good sector, determination of tightness in the intermediate goods sector and wage determination via Nash Bargaining. Next, I summarize that the equilibrium can be characterized by a system of 5 equations and 5 unknowns.

A Stationary equilibrium can be characterized by 5 variables $(\theta, \rho, \hat{z}, \underline{z}_u, \underline{z}_w)$ and 4 equations

- $$rJ(\underline{z}_u) = bw + f(J(\underline{z}_w) - J(\underline{z}_u)) + \psi \int_{\underline{z}_u} (J(z) - J(\underline{z}_u))dF(z) \quad \text{(A.197)}$$

- $$rJ(\underline{z}_w) = w + s(J(\underline{z}_u) - J(\underline{z}_w)) + \psi \int_{\underline{z}_w} (J(z) - J(\underline{z}_w))dF(z) \tag{A.198}$$

- $$J(\hat{z}) = J(\underline{z}_u) - \chi \tag{A.199}$$

- $$\rho = \alpha[\frac{1}{(1-u-\eta)} \int_{\hat{z}} e^{\frac{z}{1-\alpha}} \Lambda^u(z)dz + \int_{\hat{z}} e^{\frac{z}{1-\alpha}} \Lambda^w(z)dz]^{1-\alpha} \tag{A.200}$$

-

- $$\frac{c\rho}{c(r+s)+q(\theta)} = \frac{\phi}{1-\phi}(J(\underline{z}_w) - J(\underline{z}_u)) \tag{A.201}$$

where $J(z)$ is given by Proposition 20 and $\Lambda^u(z), \Lambda^w(z)$ are given by Proposition 21

We are ready to go over the main theorem that subsequently generates all the patterns that were documented in the data. It states that in equilibrium wage workers are more selective on which business opportunities to implement. The necessary and sufficient condition for it is simply that the income received while not working is lower than that received as a worker. Were it not the case the equilibrium would not exist as markets would not clear.

Proposition 22. *In equilibrium,* $\underline{z}_w > \underline{z}_u \Leftrightarrow b < 1$

The next corollaries are all a result of the difference in selection directly relating to the patterns documented empirically.

Corollary 22.1. *In equilibrium, businesses created by employed workers have a lower exit rate than those created by not working individuals.*

Corollary 22.1 is a result of the combination of all business owners exiting at the same threshold while having different levels of selection upon entry between the two types.

Corollary 22.2. *In equilibrium, businesses created by employed workers, on average, have higher firm size and profits relative to those created by not working individuals.*

Corollary 22.2 is a direct consequence of the fact that both profits and firm size are monotonically increasing in productivity.

Corollary 22.3. *In equilibrium, the entry rate into business ownership of not working individuals is higher than that of employed workers.*

Finally, as it is often the case with selection mechanisms an increased average productivity is associated to a lower entry rate.

It follows that this stylized model is capable of capturing the differences in businesses created by not working individuals versus employed workers in the data. The next section derives a testable prediction from the theory and tests it in the data.

Proof of Proposition 20.

We know that it is equal to $U \quad \forall z \leq \hat{z}$. We need to find the value of $J(z)$ for $z \geq \hat{z}$. As in the benchmark model conditional on a price for the input of the entrepreneur (w before, and now ρ), the optimal stopping problem is the same. It follows, the proof just follows from the proof in Proposition 1.[21]

Proof of Proposition 21.

Solving generic KFE. The solution below is the same for both types of business owners (i.e., $i = u, w$)

Let $\hat{z}$ be the point at which firms exit and $\underline{z}_i$ the point in which firms enter, with $\underline{z}_i > \hat{z}$. Let $\Lambda(z)$ denote the endogenous pdf and M the measure of entrants. For type u ($i = u$), M is equal to $\psi u e^{-\beta \underline{z}_u}$ and for type w ($i = w$) M is equal to $\psi(1-u-\eta)e^{-\beta \underline{z}_w}$

Finally, for $[\underline{z}_i, \infty[$

$$\Lambda^i(z) = \Lambda^i_2(z) \tag{A.202}$$

and for $]\hat{z}, \underline{z}_i]$

$$\Lambda^i(z) = \Lambda^i_1(z) \tag{A.203}$$

Then for $[\underline{z}_i, \infty[$

$$\frac{\partial \Lambda^i_2(z)}{\partial t} = -\mu \frac{\partial \Lambda^i_2(z)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 \Lambda^i_2(z)}{\partial z^2} + M^i \frac{\beta e^{-\beta z}}{e^{-\beta \underline{z}_i}} = 0 \tag{A.204}$$

for $]\hat{z}, \underline{z}_i]$

$$\frac{\partial \Lambda^i_1(z)}{\partial t} = -\mu \frac{\partial \Lambda^i_1(z)}{\partial z} + \frac{\sigma^2}{2} \frac{\partial^2 \Lambda^i_1(z)}{\partial z^2} = 0 \tag{A.205}$$

The four boundary conditions are

1. $\int_{\underline{z}_i}^{\infty} \Lambda^i(z)dz < \infty$

[21] Conditional on a wage, the problem for the entrepreneur is exactly as in the model with just one sector.

2. $\Lambda_1^i(\underline{z}_i) = \Lambda_2^i(\underline{z}_i)$

3. $\frac{\partial \Lambda_1^i(\underline{z}_i)}{\partial z} = \frac{\partial \Lambda_2^i(\underline{z}_i)}{\partial z}$

4. $\Lambda_1^i(\hat{z}) = 0$

The proof then just follows the same steps as the proof for Proposition 2.

Proof of Corollary 21.1.

The steps of this proof just follow the steps of the proof of corollary 2.1.

Proof of Proposition 22.

The only difference between the value functions of U and W in the framework with search frictions relative to the benchmark model is that the exogenous transition rate from U to W, f, is replaced by an equilibrium object $p(\theta)$. But from the point of view of the individual making the decision to open a firm or not, the transition rate from W to U is taken as given.

It follows that for this proof we can just follow the same steps as the proof for Proposition 4, except that I replace f by $p(\theta)$.

Proof of Corollary 22.1.

The proof of this corollary just follows the proof of corollary 4.1.

Proof of Corollary 22.2.

The expression for optimal firm size is given by

$$n(z,w) = (\frac{\alpha}{\rho})^{\frac{1}{1-\alpha}} e^{\frac{z}{1-\alpha}} \tag{A.206}$$

The only difference relative to the model without search frictions is that the cost of one input for the entrepreneur was w and here it is ρ. But other than that the expression is identical. It follows that the proof of this corollary just follows the proof of corollary 4.2.

Proof of Corollary 22.3.

$\underline{z}_w > \underline{z}_u \Rightarrow \psi(1-F(\underline{z}_u)) > \psi(1-F(\underline{z}_w))$.

A.4 Appendix to Chapter 2

Proof of Theorem 6. To get this expression first evaluate $V(\varepsilon)$ at $\varepsilon = \underline{\varepsilon}$ and using $U = V(\underline{\varepsilon})$ to obtain the following expression for U

$$U = \frac{-c(r+\lambda) + q(\theta)(1-\beta)(\underline{\varepsilon})}{r(r+2q(\theta)(1-\beta)+\lambda)}$$

Now using the condition that individuals must be indifferent between searching for a job or for an idea we know that

$$rU = b + p(\theta)\int (W(\varepsilon^*) - U)d\mu(\varepsilon^*)$$

Replacing $W(\varepsilon^*) - U = \frac{w-rU}{r+\lambda} = \frac{\beta(\varepsilon)-2\beta rU}{r+\lambda}$, $\mu(\varepsilon) = \frac{f(\varepsilon)}{1-F(\varepsilon)}$ and $p(\theta) = \psi(1 - F(\underline{\varepsilon}))$ gives

$$rU = b + \psi\int_{\underline{\varepsilon}} \frac{\beta(\varepsilon) - 2\beta rU}{r+\lambda} f(\varepsilon)d\varepsilon$$

$$U = \frac{b(r+\lambda) + \beta\psi\int_{\underline{\varepsilon}}(\varepsilon^*)f(\varepsilon^*)d\varepsilon^*}{r(r+\lambda+2\beta\psi(1-F(\underline{\varepsilon})))}$$

Setting both expressions for U equal we get the desired expression. This expression can also be written as

$$\begin{aligned}(r+\lambda+2\psi\beta(1-F(\underline{\varepsilon})))&(-c(r+\lambda)+q(\theta)(z+\underline{\varepsilon})(1-\beta)) \\ &= (r+\lambda+2(1-\beta)q(\theta))(b(r+\lambda)+\beta\psi\int_{\underline{\varepsilon}}(\varepsilon^*)dF(\varepsilon^*))\end{aligned}$$

To see the expression implies a positive relationship between θ and $\underline{\varepsilon}$ totally differentiate with respect to both getting

$$[-\beta\psi(r+\lambda)f(\underline{\varepsilon})\underline{\varepsilon}-\beta\psi 2(1-\beta)q(\theta)\underline{\varepsilon}f(\underline{\varepsilon})-f(\underline{\varepsilon})c2\beta(r+\lambda)$$
$$-q(\theta)(1-\beta)(r+\lambda)-2\beta(1-F(\underline{\varepsilon}))q(\theta)(1-\beta)+\beta\psi 2(1-\beta)q(\theta)\underline{\varepsilon}f(\underline{\varepsilon})]d\underline{\varepsilon}$$
$$=[q'(\theta)(1-\beta)(r+\lambda)[\underline{\varepsilon}-2b]+2\beta(1-\beta)q'(\theta)\psi\int_{\underline{\varepsilon}}(\varepsilon^*-\underline{\varepsilon})dF(\varepsilon^*)]d\theta$$

Note that $\underline{\varepsilon}-2b>0$ in equilibrium otherwise for the marginal entrepreneur both parties would be better if the match separated. This would contradict with the individuals initial decision to become an entrepreneur.

It follows that both sides of the expression above are negative implying a positive relationship between θ and $\underline{\varepsilon}$. □

Proof of Theorem 7. To show the result, totally differentiate the Entrepreneurship equation with respect to $\underline{\varepsilon}$ and b holding θ constant to obtain

$$[-\beta\psi(r+\lambda)f(\underline{\varepsilon})\underline{\varepsilon}-f(\underline{\varepsilon})c2\beta(r+\lambda)-q(\theta)(1-\beta)(r+\lambda)-2\beta(1-F(\underline{\varepsilon}))q(\theta)(1-\beta)]d\underline{\varepsilon}$$
$$=-(r+\lambda+2(1-\beta)q(\theta))(r+\lambda)db$$

From there we see that $\underline{\varepsilon}$ will increase for all θ levels. Since the Job Creation curve does not shift, it follows that θ will decrease and $\underline{\varepsilon}$ will increase. To see that aggregate productivity increases remember that

$$\mu(\varepsilon)=\frac{f(\varepsilon)}{1-F(\underline{\varepsilon})}$$

It follows aggregate productivity can be written as

$$\int_{\underline{\varepsilon}} \frac{\varepsilon f(\varepsilon)}{1-F(\underline{\varepsilon})} dz$$

$\square$

Proof of Theorem 8. To show the result, totally differentiate the Entrepreneurship equation with respect to $\underline{\varepsilon}$ and c holding θ constant to obtain

$$[-\beta\psi(r+\lambda)f(\underline{\varepsilon})\underline{\varepsilon} - f(\underline{\varepsilon})c2\beta(r+\lambda) - q(\theta)(1-\beta)(r+\lambda) - 2\beta(1-F(\underline{\varepsilon}))q(\theta)(1-\beta)]d\underline{\varepsilon}$$

$$= -(r+\lambda+2\beta\psi(1-F(\underline{\varepsilon})))(r+\lambda)dc$$

From there we see that $\underline{\varepsilon}$ will increase for all θ levels. Since the Job Creation curve does not shift, it follows that θ will decrease and $\underline{\varepsilon}$ will increase. To see that aggregate productivity increases remember that

$$\mu(\varepsilon) = \frac{f(\varepsilon)}{1-F(\underline{\varepsilon})}$$

It follows aggregate productivity can be written as

$$\int_{\underline{\varepsilon}} \frac{\varepsilon f(\varepsilon)}{1-F(\underline{\varepsilon})} dz$$

Since $\underline{\varepsilon}$ increases with c, average firm productivity will increase following the shock.

$\square$

Proof of Theorem 9. The law of motion for match surplus of average productivity

$$rS(\overline{\varepsilon}) = \frac{\xi\underline{\varepsilon}}{\xi-1} - b - ((\delta\int_{\underline{\varepsilon}} p(\theta)\beta S(\varepsilon)\frac{v(\varepsilon)}{v}d\varepsilon) + (1-\delta)(\psi\int_{\underline{\varepsilon}}(V(\varepsilon)-U)dF(\varepsilon))) - \lambda S(\overline{\varepsilon}) + \dot{S}(\overline{\varepsilon}) \tag{A.207}$$

where $\frac{\xi\varepsilon}{\xi-1} = \int_{\underline{\varepsilon}} \frac{\varepsilon f(\varepsilon)}{1-F(\underline{\varepsilon})}d\varepsilon$.

$\int_{\underline{\varepsilon}} p(\theta)\beta S(\varepsilon)\frac{v(\varepsilon)}{v}d\varepsilon$ is the private opportunity cost of it unemployed always transition back to wage work from unemployment. Note that $\beta S(\varepsilon) = W(\varepsilon) - U$. Similarly, $\int_{\underline{\varepsilon}}(V(\varepsilon)-U)dF(\varepsilon))$ is the private opportunity cost if unemployed always transition to business ownership from unemployment. δ is the fraction of unemployed that search for a job. In equilibrium, individuals are indifferent whether to search for a job or a business opportunity and so we can rewrite the condition above as

$$rS(\overline{\varepsilon}) = \frac{\xi\underline{\varepsilon}}{\xi-1} - b - (\int_{\underline{\varepsilon}} p(\theta)\beta S(\varepsilon)\frac{v(\varepsilon)}{v}d\varepsilon) - \lambda S(\overline{\varepsilon}) + \dot{S}(\overline{\varepsilon}) \tag{A.208}$$

Finally using the fact that $S(\varepsilon)$ is linear in ε, we can rewrite it as

$$rS(\overline{\varepsilon}) = \frac{\xi\underline{\varepsilon}}{\xi-1} - b - [\lambda + p(\theta)\beta]S(\overline{\varepsilon})) + \dot{S}(\overline{\varepsilon}) \tag{A.209}$$

From the Job Creation Curve,using the expression for $rV(\underline{\varepsilon})$

$$J(\underline{\varepsilon}) = \frac{rU+c}{q(\theta)} + U \tag{A.210}$$

which gives

$$c = q(\theta)(1-\beta)S(\underline{\varepsilon}) - rU \tag{A.211}$$

The social planner maximizes total welfare

$$\max_{v} \int_0^\infty e^{-rt}[\frac{\xi \underline{\varepsilon}(\theta)}{\xi - 1}(1-u-v) + ub - cv]dt \tag{A.212}$$

$$s.t.: \dot{u} = \lambda(1-u-v) - (\frac{v}{\frac{u}{2}})^\alpha u \tag{A.213}$$

where $\theta \equiv \frac{v}{\frac{u}{2}}$

Setting up the hamiltonean and taking FOCs (using $\underline{\varepsilon} = \psi^{\frac{1}{\xi}} \theta^{-\frac{\alpha}{\xi}}$) gives

$$c = \frac{\xi}{\xi - 1}(-\frac{\alpha}{\xi})\psi^{\frac{1}{\xi}} \theta^{-\frac{\alpha}{\xi}-1} \frac{1}{\frac{u}{2}}(1-u-v) - \frac{\xi \underline{\varepsilon}}{\xi - 1} - \mu_1[\lambda + p'(\theta)] \quad \text{(optimality for v)} \tag{A.214}$$

and

$$r\mu_1 - \dot{\mu}_1 = \frac{-\xi}{\xi - 1}\underline{\varepsilon} + b - \lambda\mu_1 - (1-\alpha)(\frac{v}{\frac{u}{2}})^\alpha \quad \text{(optimality for u)} \tag{A.215}$$

Now define $\pi = -\mu_1$, and rewrite both conditions as

$$r\pi = \frac{\xi}{\xi - 1}\underline{\varepsilon} - b - \pi[\lambda + (1-\alpha)p(\theta)] + \dot{\pi} \tag{A.216}$$

and

$$c = \frac{\xi}{\xi - 1}(\frac{-\underline{\varepsilon}(1-u)}{v}) + \pi[\lambda + p'(\theta)] \tag{A.217}$$

For a given value of π the two equations together with the expressions for $S(\overline{\varepsilon})$ and $\underline{\varepsilon} = \psi^{\frac{1}{\xi}} \theta^{-\frac{\alpha}{\xi}}$ allow us to solve for θ and β. π is the additional benefit of increasing marginally the measure of non-unemployed in the economy. Note that in equilibrium $\pi = S(\overline{\varepsilon})$.

Then comparing equation $(A.216)$ and equation $(A.209)$ gives

$$[\lambda + p(\theta)\beta] = [\lambda + (1-\alpha)p(\theta)] \tag{A.218}$$

which implies

$$\beta = (1-\alpha) \tag{A.219}$$

$\square$

A.4.1 Deriving the Endogenous Productivity Distribution

The law of motion for the measure of vacancies of a particular productivity ε is

$$\dot{v}(\varepsilon) = \psi(1-\gamma)uf(\varepsilon) - q(\theta)v(\varepsilon) \quad \forall\varepsilon \geq \underline{\varepsilon}$$

$$v(\varepsilon) = 0 \quad \forall\varepsilon < \underline{\varepsilon}$$

Setting the expression above to zero implies

$$q(\theta)v(\varepsilon) = \psi(1-\gamma)uf(\varepsilon)$$

Integrating with respect to ε and dividing the integrated expression in the equation above yields

$$\mu(\varepsilon) = \frac{v(\varepsilon)}{v} = \frac{f(\varepsilon)}{1-F(\underline{\varepsilon})}$$

This equation states that the distribution of enacted ideas, or firm productivity in the economy, is fully characterized by the underlying distribution of feasible ideas and the threshold rule for entering entrepreneurship.

A.4.2 Deriving the Job Creation Curve

For the remainder of our analysis we will consider the economy to be in steady state. The law of motion of the measure of jobs in the economy is given by

$$\dot{n} = p(\theta)\gamma u - \lambda n$$

The law of motion of the measure of total entrepreneurs in this economy is given by

$$\dot{e} = \psi(1 - F(\underline{\varepsilon}))(1-\gamma)u - \lambda n$$

Setting these two to zero implies that

$$p(\theta) = \frac{\psi(1 - F(\underline{\varepsilon}))(1-\gamma)}{\gamma}$$

The law of motion of the total unemployed looking for a job in this economy is

$$\dot{u}^{w} = \gamma 2\lambda n - p(\theta)\gamma u$$

and that for those looking for an idea is

$$\dot{u}^{v} = (1-\gamma)2\lambda n - (1-\gamma)u\psi(1 - F(\underline{\varepsilon}))$$

Setting these two to zero implies that

$$p(\theta) = \psi(1 - F(\underline{\varepsilon}))$$

A.5 Appendix to Chapter 3

A.5.1 Data

In this section we describe the data sources, as well as how we construct work histories and other relevant variables.

Data Sources

The main data source is the National Longitudinal Survey of Youth (NLSY79). The NLSY79 is a nationally representative sample of individuals aged 14 to 22 in 1979. The sample period is 1979 to 2010, which makes the maximum age in the sample equal to 53. The NLSY79 consists of three samples: a main representative sample, a military sample, and a supplemental sample designed to over-represent minorities. We only use the main representative sample. Throughout the baseline analysis we focus on males 25 year or older. To gauge robustness we also extend the sample to women who satisfy the sampling restrictions.

Observations for which the reported stop date of the job precedes the reported start date, as well as jobs that last less than 4 weeks, are dropped. Following Hagedorn and Manovskii [2013] we impose some basic sampling restrictions: (i) all observations for which the reported hours worked are below 15 hours are excluded; (ii) the education variable is forced to be non-decreasing over the life cycle. Wages are deflated using the CPI. Following Barlevy [2008] we only consider observations with reported hourly wages above $\$0.10$ and below $\$1,000$. Only observations for individuals that have completed a long-term transition to full time labor market attachment are used in the analysis. As in Yamaguchi [2010], an individual is considered to have made this transition starting from the first employment cycle that lasts 6 or more quarters. Finally, for each job we assign the mode of hours worked as the relevant value for that job. The reorganized NLSY79 data consists of 34,860 job-wage observations, for a sample of 5,712 individuals. Not all of these observations can be used in the estimation because some control variables may be missing in certain years.

Jobs and Employment Cycles

We define each job as one subset of an employment cycle during which the employer does not change. Each wage observation in the NLSY79 is linked to a measure of the current unemployment rate. To construct the current unemployment rates, we use the seasonally adjusted unemployment series from the Current Population Survey (CPS). We use the Composite Help Wanted Index constructed by Barnichon [2010] as a measure of vacancies.[22] We use the crosswalk provided by Autor and Dorn [2013] to link Census occupation codes with Dorn's 'standardized' occupation codes.[23] We classify occupations into four categories: non-routine cognitive, non-routine manual, routine cognitive, and routine manual.[24] Furthermore, as in Yamaguchi [2012], if a worker reports having the same job between period t and $t+2$, with occupation i in year t, occupation B in year $t+1$, and again occupation i in $t+2$, then we assume that occupation B is misclassified and we correct it to be A. To minimize the effects of other coding errors, we follow Neal [1998] and Pavan [2006] and disregard observations that report a change in occupation within a job (during a spell with the same employer). Industry codes are aggregated up to 15 major categories to make them comparable over time. In order to reduce the

[22]https://sites.google.com/site/regisbarnichon/research.
[23]David Dorn's crosswalks are available at http://www.cemfi.es/dorn/data.htm.
[24]This classification replicates the one presented in Cortes and Gallipoli [2014], Table A.1.

effects of industry coding error, and similar to the treatment of occupations, we only consider observations for which there are no industry changes within the job.

A.5.2 Proofs

Proofs for Model Section

Proof of Proposition 10. **Derivation of $\underline{m}_1$:**

$$E[\pi^{DPC}] = (1+q)(P_H - a(P_H))m + (1-q)(P_L - a(P_L))m - \kappa(m^{max} - m)$$

$$\geq (1+q)(P_H - a(P_H))m + (1-q)(P_L - a(P_L))m - (1-q)T = E[\pi^{spot}]$$

$$\Rightarrow$$

$$-\kappa(m^{max} - m) \geq -(1-q)T$$

Rearrange to have:

$$m > \frac{\kappa m^{max} - T(1-q)}{\kappa} \equiv \underline{m}_1 \tag{A.220}$$

Derivation of $\underline{m}_2$:

$$E[\pi^{DPC}] = (1+q)(P_H - a(P_H))m + (1-q)(P_L - a(P_L))m - \kappa(m^{max} - m)$$

$$\geq (1+q)P_H m + (1-q)P_L m - 2a(P_H)m = E[\pi^{FW}]$$

$$\Rightarrow$$

$$2a(P_H)m - (1+q)a(P_H)m - (1-q)a(P_L)m + \kappa m \geq \kappa m^{max}$$

$$m\left[(1-q)\left(a(P_H) - a(P_L)\right) + \kappa\right] \geq \kappa m^{max}$$

Rearrange to have:

$$m \geq \frac{\kappa m^{max}}{\kappa + (1-q)(a(P_H) - a(P_L))} \equiv \underline{m}_2 \tag{A.221}$$

Derivation of $\underline{m}_3$:

$$E[\pi^{spot}] = (1+q)\,(P_H - a(P_H))\,m + (1-q)\,(P_L - a(P_L))\,m - (1-q)T$$
$$\geq (1+q)P_H m + (1-q)P_L m - 2a(P_H)m = E[\pi^{FW}]$$

Rearrange to have m on the left hand side:

$$m \geq \frac{T}{a(P_H) - a(P_L)} \equiv \underline{m}_3 \tag{A.222}$$

Now for the second part of the proposition :

$\underline{m}_1 \geq \underline{m}_2$ iff

$$\frac{\kappa m^{max} - T(1-q)}{\kappa} > \frac{\kappa m^{max}}{\kappa + (1-q)(a(P_H) - a(P_L))}$$

which implies

$$\kappa^2 m^{max} - \kappa T(1-q) + (1-q)(a(P_H)-a(P_L))\kappa m^{max} - T(1-q)^2(a(P_H)-a(P_L)) \geq \kappa^2 m^{max}$$

$$-\kappa T(1-q) + (1-q)(a(P_H)-a(P_L))\kappa m^{max} - T(1-q)^2(a(P_H)-a(P_L)) \geq 0$$

$$-\kappa T + (a(P_H)-a(P_L))\kappa m^{max} - T(1-q)(a(P_H)-a(P_L)) \geq 0$$

$$\frac{(a(P_H)-a(P_L))\kappa m^{max}}{\kappa+(1-q)(a(P_H)-a(P_L))} \geq T \tag{A.223}$$

$\underline{m}_2 \geq \underline{m}_3$, iff

$$\frac{\kappa m^{max}}{\kappa+(1-q)(a(P_H)-a(P_L))} \geq \frac{T}{a(P_H)-a(P_L)}$$

which implies

$$\frac{(a(P_H)-a(P_L))\kappa m^{max}}{\kappa+(1-q)(a(P_H)-a(P_L))} \geq T \tag{A.224}$$

It follows the above thresholds are ordered according to

- If $T \leq \frac{(a(P_H)-a(P_L))\kappa m^{max}}{\kappa+(1-q)(a(P_H)-a(P_L))}$, then $\underline{m}_1 \geq \underline{m}_2 \geq \underline{m}_3$
- If $T > \frac{(a(P_H)-a(P_L))\kappa m^{max}}{\kappa+(1-q)(a(P_H)-a(P_L))}$, then $\underline{m}_3 > \underline{m}_2 > \underline{m}_1$.

□

Proof of Corollary 10.1. Proposition 1 implies

(a) **For sufficiently low T**: If $T \leq \frac{(a(P_H)-a(P_L))\kappa m^{max}}{\kappa+(1-q)(a(P_H)-a(P_L))}$ then:

1. If $m \geq \underline{m}_1$ then the firm offers a performance pay contract. In this range a DPC contract is preferable over both FW and SPOT.

2. If $\underline{m}_3 \leq m < \underline{m}_1$ then the firm offers a SPOT contract. In this range SPOT is preferable over both DPC and FW.

3. If $m < \underline{m}_3$ then the firm offers a FW contract.

(b) **For sufficiently high T:** If $T > \frac{(a(P_H)-a(P_L))\kappa m^{max}}{\kappa+(1-q)(a(P_H)-a(P_L))}$ then:

1. If $m \geq \underline{m}_2$ then the firm offers a DPC contract. In this range DPC is preferable to FW by definition of the threshold $\underline{m}_2$ and it is also preferable to SPOT because $m > \underline{m}_1$.

2. If $m < \underline{m}_2$ then the firm offers a FW contract. In this range FW is preferable to DPC by definition of the threshold $\underline{m}_2$, and it also preferable to SPOT because $m < \underline{m}_3$.

$\square$

Proof of Proposition 11. **Derivation of $\underline{m}_4$:**

$$E[\pi^{SPC}] = 2(P_H - a(P_H))m - \kappa(m^{max} - m)$$

$$\geq (1+q)(P_H - a(P_H))m + (1-q)(P_L - a(P_L))m - T(1-q) = E[\pi^{spot}]$$

$$\Rightarrow$$

$$(1-q)(P_H - a(P_H))m - (1-q)(P_L - a(P_L))m + \kappa m \geq \kappa m^{max} - T(1-q)$$

$$m\left[(1-q)\left(P_H - P_L - (a(P_H) - a(P_L))\right) + \kappa\right] \geq \kappa m^{max} - T(1-q)$$

Rearrange to have :

$$m > \frac{\kappa m^{max} - T(1-q)}{\kappa - (1-q)[(a(P_H) - a(P_L)) - (P_H - P_L)]} \equiv \underline{m}_4 \tag{A.225}$$

Derivation of $\underline{m}_5$:

$$E[\pi^{SPC}] = 2(P_H - a(P_H))m - \kappa(m^{max} - m) \geq (1+q)P_H m + (1-q)P_L m - 2a(P_H)m = E[\pi^{FW}]$$

$$2P_H m - \kappa(m^{max} - m) \geq (1+q)P_H m + (1-q)P_L m$$

$$(1-q)(P_H - P_L)m + \kappa m \geq \kappa m^{max}$$

Rearrange to have :

$$m \geq \frac{\kappa m^{max}}{(1-q)(P_H - P_L) + \kappa} \equiv \underline{m}_5 \tag{A.226}$$

Derivation of $\underline{m}_6$:

$$E[\pi^{spot}] = (1+q)(P_H - a(P_H))m + (1-q)(P_L - a(P_L))m - (1-q)T$$

$$\geq (1+q)P_H m + (1-q)P_L m - 2a(P_H)m = E[\pi^{FW}]$$

Rearrange to have m on the left hand side:

$$m \geq \frac{T}{a(P_H) - a(P_L)} \equiv \underline{m}_6 \tag{A.227}$$

Now for the second part of the proposition :

$\underline{m}_4 \geq \underline{m}_5$ iff

$$\frac{\kappa m^{max} - T(1-q)}{\kappa - (1-q)(a(P_H) - a(P_L) - (P_H - P_L))} > \frac{\kappa m^{max}}{\kappa + (1-q)(P_H - P_L)}$$

which implies

$$\kappa^2 m^{max} - \kappa T(1-q) + (1-q)(P_H - P_L)(\kappa m^{max} - T(1-q))$$
$$\geq \kappa^2 m^{max} - (1-q)((a(P_H) - a(P_L)) - (P_H - P_L))\kappa m^{max}$$

$$-\kappa T(1-q) + (1-q)(P_H - P_L)\kappa m^{max} - T(1-q)^2 (P_H - P_L)$$
$$\geq -(1-q)(a(P_H) - a(P_L))\kappa m^{max} + (1-q)(P_H - P_L)\kappa m^{max}$$

$$-\kappa T(1-q) - T(1-q)^2 (P_H - P_L) \geq -(1-q)(a(P_H) - a(P_L))\kappa m^{max}$$

$$\kappa T + T(1-q)(P_H - P_L) \leq (a(P_H) - a(P_L))\kappa m^{max}$$

$$T \leq \frac{\kappa m^{max}(a(P_H) - a(P_L))}{\kappa + (1-q)(P_H - P_L)} \tag{A.228}$$

$\underline{m}_5 > \underline{m}_6$, iff

$$\frac{\kappa m^{max}}{\kappa + (1-q)(P_H - P_L)} > \frac{T}{a(P_H) - a(P_L)}$$

which implies

$$T \leq \frac{\kappa m^{max}(a(P_H) - a(P_L))}{\kappa + (1-q)(P_H - P_L)} \tag{A.229}$$

It follows the above thresholds are ordered according to

- If $T \leq \frac{\kappa m^{max}(a(P_H)-a(P_L))}{\kappa+(1-q)(P_H-P_L)}$, then $\underline{m}_4 \geq \underline{m}_5 \geq \underline{m}_6$
- If $T > \frac{\kappa m^{max}(a(P_H)-a(P_L))}{\kappa+(1-q)(P_H-P_L)}$, then $\underline{m}_6 > \underline{m}_5 > \underline{m}_4$.

□

Proof of Corollary 11.1. Proposition 2 implies

(a) **For sufficiently low T**: If $T \leq \frac{\kappa m^{max}(a(P_H)-a(P_L))}{\kappa+(1-q)(P_H-P_L)}$ then:

1. If $m \geq \underline{m}_4$ then the firm offers a performance pay contract. In this range a SPC contract is preferable over both FW and SPOT.
2. If $\underline{m}_6 \leq m < \underline{m}_4$ then the firm offers a SPOT contract. In this range SPOT is preferable over both DPC and FW.
3. If $m < \underline{m}_6$ then the firm offers a FW contract.

(b) **For sufficiently high T:** If $T > \frac{\kappa m^{max}(a(P_H)-a(P_L))}{\kappa+(1-q)(P_H-P_L)}$ then:

1. If $m \geq \underline{m}_5$ then the firm offers a SPC contract. In this range SPC is preferable to FW by definition of the threshold $\underline{m}_5$ and it is also preferable to SPOT because $m > \underline{m}_4$.

2. If $m < \underline{m}_5$ then the firm offers a FW contract. In this range FW is preferable to SPC by definition of the threshold $\underline{m}_5$, and it also preferable to SPOT because $m < \underline{m}_6$.

□

Period 1 participation constraint (after learning period)

In the main text we explain that an ex-ante participation constraint must hold for workers who choose to stay with their employer:

$$w_1(m|P_H) + E(w_2(m)) \geq a(P_H)m + [qa(P_H) + (1-q)a(P_L)]\, E(m)$$

Fixed wage contract: in this case $w_1(m) = w_2(m) = a(P_H)m$. Therefore:

$$\begin{aligned}
&2a(P_H)m \geq a(P_H)m + [qa(P_H) + (1-q)a(P_L)]\, E(m) \\
&a(P_H)m \geq [qa(P_H) + (1-q)a(P_L)]\, E(m) \\
&m \geq \frac{[qa(P_H) + (1-q)a(P_L)]}{a(P_H)} E(m)
\end{aligned}$$

Since $\frac{[qa(P_H)+(1-q)a(P_L)]}{a(P_H)} < 1$ it implies that for any $m > E(m)$ the match does not separate.

Spot contract: in this case $w_1(m) = a(P_H)m$ and $E(w_2(m)) = qa(P_H)m + (1-q)a(P_L)m$. Therefore:

$$\begin{aligned}
&a(P_H)m + qa(P_H)m + (1-q)a(P_L)m \geq a(P_H)m + [qa(P_H) + (1-q)a(P_L)]\, E(m) \\
&[qa(P_H) + (1-q)a(P_L)]\, m \geq [qa(P_H) + (1-q)a(P_L)]\, E(m) \\
&m \geq E(m)
\end{aligned}$$

Which trivially implies that under spot contract matches survive if $m > E(m)$.

SPC: in this case equation (12) implies that the wages are $w_1(m) = a(P_H)m$ and $E(w_2(m)) = (a(P_H) - P_H)m + qP_Hm + (1-q)P_Lm$. Substitute:

$$a(P_H)m + (a(P_H) - P_H)m + qP_Hm + (1-q)P_Lm \geq a(P_H)m + [qa(P_H) + (1-q)a(P_L)]\,E(m)$$

$$a(P_H)m + (1-q)\,[P_L - P_H]\,m \geq [qa(P_H) + (1-q)a(P_L)]\,E(m)$$

$$\frac{m}{E(m)} > \frac{qa(P_H) + (1-q)a(P_L)}{a(P_H) + (1-q)\,[P_L - P_H]}$$

Note that the last condition implies a threshold for m such that matches do not separate. In addition, it can be shown that given the assumption that $a(P_H) - a(P_L) \geq P_H - P_L$, which is required for SPC, the right hand side of this condition is smaller than 1. Therefore, it must be that the threshold is lower than $E(m)$ and therefore every match with $m > E(m)$ does not separate before period 1. To see this, check the conditions such that the right hand side is smaller than 1:

$$\frac{qa(P_H) + (1-q)a(P_L)}{a(P_H) + (1-q)\,[P_L - P_H]} < 1$$

$$qa(P_H) + (1-q)a(P_L) < a(P_H) + (1-q)\,[P_L - P_H]$$

$$0 < (1-q)\,[a(P_H) - a(P_L) - (P_H - P_L)]$$

$$P_H - P_L < a(P_H) - a(P_L)$$

DPC : in this case the wages are identical as to those in a spot contract. The results just follow for the results for **Spot contract** described above.

A.5.3 Results with $K(m) = K, \forall m$

The firm has the choice between the following contracts and corresponding expected profits :

$$\text{DPC:} \quad E\left[\pi^{DPC}\right] = (1+q)\left(P_H - a(P_H)\right)m + (1-q)\left(P_L - a(P_L)\right)m - K.$$

$$\text{SPOT:} \quad E\left[\pi^{SPOT}\right] = (1+q)\left(P_H - a(P_H)\right)m + (1-q)\left(P_L - a(P_L)\right)m - (1-q)T$$

$$\text{FW:} \quad E\left[\pi^{FW}\right] = (1+q)P_H m + (1-q)P_L m - 2a(P_H)m.$$

After comparing expected profits, one can characterize the threshold conditions. We do this in Proposition (23).

Proposition 23. *The firm decides which contract to offer depending on observed match-quality.*

1. *The firm prefers a performance pay contract over a fixed wage contract if*

$$m \geq \frac{K}{(1-q)a(P_H) - a(P_L)} \equiv \underline{m}_2 \tag{A.230}$$

2. *The firm prefers a spot contract over a fixed wage contract if*

$$m \geq \frac{T}{a(P_H) - a(P_L)} \equiv \underline{m}_3 \tag{A.231}$$

3. *The firm prefers a performance pay contract over a spot market contract if*

$$T(1-q) > K \tag{A.232}$$

From the above set of thresholds we can see that whether or not firms offer spot or performance pay (DPC) contracts depends crucially on the costs T and K of each contract. These costs cannot be observed in the data. However, independent of these costs, we get the result that for low enough match quality m only fixed wage contracts will be implemented.[25] Hence, the conclusion from this section is that while for low match quality values m only fixed wage contracts are implemented, for high enough m employers could offer either performance pay or spot contracts. Secondly, it also tells us that performance pay contracts should exhibit more wage cyclicality since jobs not paid according to performance include both spot and fixed wages. [26]

A.6 An alternative assumption on period 1 aggregate productivity: $P_1 = P_L$

In what follows we consider our model and the empirical implications when the state of the world at the $t = 1$ is low, $P_1 = P_L$. We follow the same steps as described as in the main text. We start by solving for the optimal choice of b, then perform the pairwise comparisons between contracts, and rank the range of match quality for which we should observe different types of contracts.

SPC

The optimal choice of b is given by

$$\max_b \{q(P_H m - \hat{w}(m) - bP_H m) + (2-q)(P_L m - \hat{w}(m) - bP_L m) - \kappa(m^{max} - m)\} \tag{A.233}$$

[25]To see this note that, independant of the values of K and T, the thresholds imply that for $m \leq \min(\underline{m}_2, \underline{m}_3)$ fixed wages are implemented by employers.

[26]This is true given that wage cyclclality is identical between wages determined by (DPC) contracts and spot wages.

subject to

$$a(P_H)m = \hat{w}(m) + bP_H m \tag{A.234}$$

Now using $\hat{w}(m) = a(P_H)m - bP_H m$ and replacing it in the maximization problem gives

$$\max_b \{q(P_H m - a(P_H)m + bP_H m - bP_H m) + (2-q)(P_L m - a(P_H)m + bP_H m - bP_L m)\} - \kappa(m^{max} - m) \tag{A.235}$$

Taking first order condition gives

$$(2-q)(P_H - P_L)m > 0 \tag{A.236}$$

which implies $b = 1$ and $\hat{w}(m) = a(P_H)m - P_H m$. So it follows that

$$E[\pi^{SPC}] = q(P_H m - a(P_H)m) + (2-q)(-a(P_H)m + P_H m) - \kappa(m^{max} - m) \tag{A.237}$$

$$E[\pi^{SPC}] = 2(P_H - a(P_H))m - \kappa(m^{max} - m) \tag{A.238}$$

DPC

The optimal choice of b is given

$$a(P_H)m = \hat{w}(m) + bP_H m \tag{A.239}$$

$$a(P_L)m = \hat{w}(m) + bP_L m \tag{A.240}$$

Subtracting one equation from the other gives

$$b = \frac{a(P_H) - a(P_L)}{P_H - P_L} \tag{A.241}$$

and replacing b back into the H constraint gives

$$\hat{w}(m) = [a(P_H) - P_H \frac{a(P_H) - a(P_L)}{P_H - P_L}] \tag{A.242}$$

It follows that

$$E[\pi^{DPC}] = q[P_H - a(P_H)]m + (2-q)[P_L - a(P_L)]m - \kappa(m^{max} - m) \tag{A.243}$$

Spot

$$E[\pi^{Spot}] = q(P_H - a(P_H))m - Tq + (2-q)(P_L - a(P_L))m \tag{A.244}$$

Fixed Wages

$$E[\pi^{FW}] = q(P_H - a(P_H))m + (2-q)(P_L - a(P_H))m = (qP_H + (2-q)P_L)m - 2a(P_H)m \tag{A.245}$$

Deriving Cutoff Conditions We start by considering the case where $a(P_H) - a(P_L) < P_H - P_L$. Recall this is the case for which **DPC** is feasible and **SPC** is not. Then we proceed to the case where **SPC** is feasible and **DPC** is not.

1st Case : $a(P_H) - a(P_L) < P_H - P_L$

DPC is preferred to **Spot** if

$$q(P_H - a(P_H))m + (2-q)(P_L - a(P_L))m - \kappa(m^{max} - m) > q(P_H - a(P_H))m + (2-q)(P_L - a(P_L))m - Tq \quad (A.246)$$

which simplifies to

$$Tq > \kappa(m^{max} - m) \quad (A.247)$$

$$m > \frac{\kappa m^{max} - Tq}{\kappa} \equiv \underline{m}_1 \quad (A.248)$$

DPC is preferred to **FW** if

$$q(P_H - a(P_H))m + (2-q)(P_L - a(P_L))m - \kappa(m^{max} - m) > q(P_H - a(P_H))m + (2-q)(P_L - a(P_H))m \quad (A.249)$$

which simplifies to

$$-qa(P_H)m-(2-q)a(P_L)m-\kappa(m^{max}-m)>-2a(P_H)m \tag{A.250}$$

$$(2-q)(a(P_H)-a(P_L))m>\kappa(m^{max}-m) \tag{A.251}$$

$$m>\frac{\kappa m^{max}}{(2-q)(a(P_H)-a(P_L))+\kappa}\equiv \underline{m}_2 \tag{A.252}$$

Spot is preferred to **FW** if

$$\begin{aligned} &q(P_H-a(P_H))m+(2-q)(P_L-a(P_L))m-Tq \\ &\qquad > q(P_H-a(P_H))m+(2-q)(P_L-a(P_H))m \end{aligned} \tag{A.253}$$

which simplifies to

$$-qa(P_H)m-(2-q)a(P_L)m-Tq>-2a(P_H)m \tag{A.254}$$

$$m>\frac{Tq}{(2-q)(a(P_H)-a(P_L))}\equiv \underline{m}_3 \tag{A.255}$$

Ordering of the thresholds

We have $\underline{m}_1>\underline{m}_2$ iff

$$\frac{\kappa m^{max}-Tq}{\kappa}>\frac{\kappa m^{max}}{(2-q)(a(P_H)-a(P_L))+\kappa} \tag{A.256}$$

which implies

$$(2-q)(a(P_H)-a(P_L))\kappa m^{max} > Tq(\kappa+(2-q)(a(P_H)-a(P_L)) \tag{A.257}$$

$$\frac{\kappa m^{max}(2-q)(a(P_H)-a(P_L))}{\kappa+(2-q)(a(P_H)-a(P_L))} > Tq \tag{A.258}$$

and we have $\underline{m}_2 > \underline{m}_3$ iff

$$\frac{\kappa m^{max}}{(2-q)(a(P_H)-a(P_L))+\kappa} > \frac{Tq}{(2-q)(a(P_H)-a(P_L))} \tag{A.259}$$

which implies

$$\frac{\kappa m^{max}(2-q)(a(P_H)-a(P_L))}{(2-q)(a(P_H)-a(P_L))+\kappa} > Tq \tag{A.260}$$

It follows the two possible cases are

1. $\frac{\kappa m^{max}(2-q)(a(P_H)-a(P_L))}{(2-q)(a(P_H)-a(P_L))+\kappa} > Tq$, which implies $\underline{m}_1 > \underline{m}_2 > \underline{m}_3$
2. $\frac{\kappa m^{max}(2-q)(a(P_H)-a(P_L))}{(2-q)(a(P_H)-a(P_L))+\kappa} \leq Tq$, which implies $\underline{m}_3 > \underline{m}_2 > \underline{m}_1$

For $\frac{\kappa m^{max}(2-q)(a(P_H)-a(P_L))}{(2-q)(a(P_H)-a(P_L))+\kappa} > Tq$, we obtain

- $\forall m$ such that $m > \underline{m}_1$, *DPC* is implemented
- $\forall m$ such that $m \in [\underline{m}_3, \underline{m}_1]$, *Spot* is implemented
- $\forall m$ such that $m < \underline{m}_3$, *FW* is implemented.

For $\frac{\kappa m^{max}(2-q)(a(P_H)-a(P_L))}{(2-q)(a(P_H)-a(P_L))+\kappa} < Tq$, we obtain

- $\forall m$ such that $m > \underline{m}_2$, DPC is implemented.
- $\forall m$ such that $m \leq \underline{m}_2$, FW is implemented.

2nd Case : $a(P_H) - a(P_L) > P_H - P_L$

SPC is preferred to **Spot** if

$$2(P_H - a(P_H))m - \kappa(m^{max} - m) > q(P_H - a(P_H))m + (2-q)(P_L - a(P_L))m - Tq \tag{A.261}$$

which implies

$$2(P_H - a(P_H))m - \kappa(m^{max} - m) > q(P_H - a(P_H))m + (2-q)(P_L - a(P_L)) - Tq \tag{A.262}$$

$$Tq - \kappa(m^{max} - m) > (2-q)[(a(P_H) - a(P_L)) - (P_H - P_L)]m \tag{A.263}$$

$$(\kappa - (2-q)[(a(P_H) - a(P_L)) - (P_H - P_L)])m > \kappa m^{max} - Tq \tag{A.264}$$

$$m > \frac{\kappa m^{max} - Tq}{\kappa - (2-q)[(a(P_H) - a(P_L)) - (P_H - P_L)]} \equiv \underline{m}_4 \tag{A.265}$$

SPC is preferred to **FW** if

$$2(P_H - a(P_H))m - \kappa(m^{max} - m) > q(P_H - a(P_H))m + (2-q)(P_L - a(P_H))m \tag{A.266}$$

which implies

$$(2-q)(P_H - P_L)m + \kappa m > \kappa m^{max} \tag{A.267}$$

$$m > \frac{\kappa m^{max}}{\kappa + (2-q)(P_H - P_L)} \equiv \underline{m}_5 \tag{A.268}$$

Ordering of the thresholds

We have $\underline{m}_4 > \underline{m}_5$ iff

$$\frac{\kappa m^{max} - Tq}{\kappa - (2-q)[(a(P_H) - a(P_L)) - (P_H - P_L)]} > \frac{\kappa m^{max}}{\kappa + (2-q)(P_H - P_L)} \tag{A.269}$$

which implies

$$\begin{aligned} &- \kappa Tq - Tq(2-q)(P_H - P_L) + \kappa m^{max}(2-q)(P_H - P_L) \\ &\qquad > -(2-q)[(a(P_H) - a(P_L)) - (P_H - P_L)]\kappa m^{max} \end{aligned} \tag{A.270}$$

$$\begin{aligned} &\kappa m^{max}(2-q)[(a(P_H) - a(P_L)) - (P_H - P_L) + (P_H - P_L)] \\ &\qquad > \kappa Tq + Tq(2-q)(P_H - P_L) \end{aligned} \tag{A.271}$$

$$\frac{\kappa m^{max}(2-q)}{\kappa + (2-q)(P_H - P_L)} > Tq \tag{A.272}$$

and we have $\underline{m}_5 > \underline{m}_3$ iff

$$\frac{\kappa m^{max}}{\kappa + (2-q)(P_H - P_L)} > \frac{Tq}{(2-q)(a(P_H) - a(P_L))} \tag{A.273}$$

which implies

$$\frac{\kappa m^{max}(2-q)(a(P_H) - a(P_L))}{\kappa + (2-q)(P_H - P_L)} > Tq \tag{A.274}$$

It follows the two possible cases are

1. $\frac{\kappa m^{max}(2-q)(a(P_H)-a(P_L))}{\kappa+(2-q)(P_H-P_L)} > Tq$, which implies $\underline{m}_4 > \underline{m}_5 > \underline{m}_3$
2. $\frac{\kappa m^{max}(2-q)(a(P_H)-a(P_L))}{\kappa+(2-q)(P_H-P_L)} \leq Tq$, which implies $\underline{m}_3 > \underline{m}_5 > \underline{m}_4$

For $\frac{\kappa m^{max}(2-q)(a(P_H)-a(P_L))}{\kappa+(2-q)(P_H-P_L)} > Tq$, we obtain

- $\forall m$ such that $m > \underline{m}_4$, *SPC* is implemented
- $\forall m$ such that $m \in [\underline{m}_3, \underline{m}_4]$, *Spot* is implemented
- $\forall m$ such that $m < \underline{m}_3$, *FW* is implemented.

For $\frac{\kappa m^{max}(2-q)(a(P_H)-a(P_L))}{\kappa+(2-q)(P_H-P_L)} \leq Tq$, we obtain

- $\forall m$ such that $m > \underline{m}_3$, *DPC* is implemented.
- $\forall m$ such that $m \leq \underline{m}_3$, *FW* is implemented.

A.7 Proof for Empirical Wage Processes section

Proof of Proposition 12. Proof. Log-linearize (w,m,P,X) around (w^*,m^*,P^*,X^*) for the *SPC* and spot contract wage expressions and (w,m,X) around (w^*,m^*,X^*) for the fixed wage contract, where

$$P^* = \frac{P_h+P_l}{2} \quad w^* = E[w], \quad m^* = E[m], \quad X^* = E[X] \tag{A.275}$$

Log-linearization results in:

1. For SPC : $w^*(log(w)-log(w^*)) = (\frac{P_h+P_l}{2}+a(P_h)-P_h)m^*(log(m)-log(m^*))+ P^*m^*(log(P)-log(P^*))+X^*\gamma(log(X)-log(X^*))$

2. For Fixed wage : $w^*(log(w)-log(w^*)) = a(P_h)m^*(log(m)-log(m^*))+ X^*\gamma(log(X)-log(X^*))$

3. For Spot : $w^*(log(w)-log(w^*)) = \frac{da(P)}{dP}|_{P=P^*}P^*m^*(log(P)-log(P^*))+ a(P^*)m^*(log(m)-log(m^*))+X^*\gamma(log(X)-log(X^*))$

4. For DPC : $w^*(log(w)-log(w^*)) = \frac{da(P)}{dP}|_{P=P^*}P^*m^*(log(P)-log(P^*))+a(P^*)m^*(log(m)- log(m^*))+X^*\gamma(log(X)-log(X^*))$

After rearranging, and keeping only $log(w)$ on the left hand side, we obtain:

1. For SPC : $log(w) = \frac{-(log(X^*)-log(w^*)w^*+log(m^*)+log(P^*))}{w^*}+\frac{(\frac{P_h+P_l}{2}+a(P_h)-P_h)m^*}{w^*}log(m)+ \frac{P^*m^*}{w^*}log(P)+\frac{X^*\gamma}{w^*}log(X)$

2. For Fixed wage : $log(w) = \frac{-(log(m^*)+log(X^*)-log(w^*)w^*)}{w^*}+\frac{a(P_h)m^*}{w^*}log(m)+\frac{X^*\gamma}{w^*}log(X)$

3. For Spot : $log(w) = \frac{-(log(P^*)+log(m^*)+log(X^*)-log(w^*)w^*)}{w^*}+\frac{a(P^*)m^*}{w^*}log(m)+\frac{\frac{da(P)}{dP}|_{P=P^*}P^*m^*}{w^*}log(P)+ \frac{X^*\gamma}{w^*}log(X)$

4. For DPC : $log(w) = \frac{-(log(P^*)+log(m^*)+log(X^*)-log(w^*)w^*)}{w^*} + \frac{a(P^*)m^*}{w^*}log(m) + \frac{\frac{da(P)}{dP}|_{P=P^*}P^*m^*}{w^*}log(P) + \frac{X^*\gamma}{w^*}log(X)$

Denote the by β_1 and β_2 the coefficients multiplying $\log(m)$ and $\log(P)$, respectively. Then:

1. $\beta_1^{DPC} > 0, \beta_1^{SPC} > 0, \beta_1^{FW} > 0, \beta_1^{Spot} > 0$

2. $\beta_2^{DPC} > 0, \beta_2^{SPC} > 0, \beta_2^{Spot} > 0$ and $\beta_2^{FW} = 0$

In particular, to see that $\beta_2^{SPC} > 0$, note that

$$
\begin{aligned}
a(P_h) - a(P_l) \quad & > P_h - P_l \\
\Rightarrow \qquad & a(P_h) > P_h - P_l \\
\Rightarrow \qquad & \frac{P_h + P_l}{2} > P_l > P_h - a(P_h) \\
\Rightarrow \qquad & \frac{P_h + P_l}{2} + a(P_h) - P_h > 0
\end{aligned}
$$

where $a(P_h) - a(P_l) > P_h - P_l$ is just the necessary condition for the *SPC* contract to be feasible.

$\square$