Power Loss Estimation in LLC MOSFETs:
A Time Interval Analysis

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF APPLIED SCIENCE
in
The Faculty of Graduate and Postdoctoral Studies
(Electrical & Computer Engineering)

THE UNIVERSITY OF BRITISH COLUMBIA
(Vancouver)
December 2018
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Power Loss Estimation in LLC MOSFETs: A Time Interval Analysis

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Abstract

LLC resonant converters have become a mainstream topology in DC/DC power conversion applications such as electric vehicle charging, renewable energy generation and low power energy conversion. This converter presents advantages such as the soft-switching of active devices, which reduces power losses in the energy conversion process, allowing for a more efficient operation when compared to hard-switched topologies.

Losses in the MOSFETs of the inverting and rectifying stages of this converter should be accurately determined so to allow for proper heat management design and thermal dissipation. However, determination of losses through simulation can be challenging due to the significant difference between time constants of electrical and thermal phenomena. Moreover, the information presented by the datasheet of power electronic devices is often limited to select operating points, which may compromise the accuracy of power loss estimation.

In order to overcome the limitations imposed by the datasheet, a detailed characterization of the main loss mechanisms in the operation of LLC MOSFETs is presented. To avoid the time-consuming and computationally-intensive process of simulation, steady-state time-domain expressions for the converter are developed, based on the electrical behavior of the topology. These equations, based on the Time Interval Analysis, are able to predict key electrical and thermal behaviours based on circuit design considerations and operating conditions, being easily implementable in software such as MATLAB or MS Excel.

As verified by simulation and experimental results, estimation of losses using the proposed method is considerably more precise than using the well-established yet oversimplified...
Abstract

First-Harmonic Approximation (FHA). In the inverting stage, the observed error in loss determination is reduced from an average of 19%, using FHA, to 2.8% using the proposed method. When it comes to the rectification portion of the circuit, the reduction in error observed is from 12% to 2%. Such improvement in power loss estimation before the converter is built is fundamental for the design of an agile and cost-effective thermal management approach which guarantees the integrity and reliability of the power electronics device.
Lay Summary

In order to power and operate electronic devices, such as laptops, cell phones and electric vehicles, different power conversion circuits are necessary. These rely on power electronic components, which, even though highly efficient, still produce heat due to power dissipation. One of the many circuits for these applications and also for renewable energy generation is the LLC Resonant Converter, which is popular because of its increased efficiency.

Nevertheless, the power electronic components in this circuit still produce heat, which, if not properly dissipated, may damage the converter. This work develops a tool which can be used in common programs such as MS Excel for determining the amount of heat generated by the different power electronic components in the LLC Resonant Converter before the physical setup is built. This saves development time and costs, while allowing for the optimal heat dissipation technique to be implemented, which results in a lighter and smaller product.
Preface

This work is based on research performed at the Electrical and Computer Engineering Department at the University of British Columbia by Ettore Frederico Scabeni Glitz, under the supervision of Dr. Martin Ordonez.

A version of Chapter 2 has been presented and published at the IEEE Applied Power Electronics Conference and Exposition (APEC) 2018 [1]. A version of Chapters 2 and 3 has been submitted as two different manuscripts for two different IEEE Transactions papers.

As first author of the above-mentioned publications and work, the author of this thesis developed the theoretical concepts and wrote the manuscripts, receiving advice and technical guidance from Dr. Martin Ordonez. The author developed simulations and experimental platforms, receiving contributions from Dr. Ordonez’s research team. In particular, the Ph.D. student Jhih-Da Hsu assisted the author in the development of some experimental validations presented in Chapter 3, lending his experimental platform for measurements.
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<tr>
<td>BJT</td>
<td>Bipolar Junction Transistor</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>EMI</td>
<td>Electromagnetic Interference</td>
</tr>
<tr>
<td>FHA</td>
<td>First-Harmonic Approximation</td>
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<td>MOSFET</td>
<td>Metal-Oxide-Semiconductor Field-Effect Transistor</td>
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<td>RCE</td>
<td>Rectifier Current Equations based on TIA</td>
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<td>RFI</td>
<td>Radio-Frequency Interference</td>
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<td>RTD</td>
<td>Resistance Temperature Detectors</td>
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<tr>
<td>SR</td>
<td>Synchronous Rectification</td>
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<tr>
<td>TIA</td>
<td>Time Interval Analysis</td>
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<tr>
<td>ZCS</td>
<td>Zero-Current Switching</td>
</tr>
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<td>ZVS</td>
<td>Zero-Voltage Switching</td>
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List of Symbols

$\Delta P_{\text{loss}}$ Difference Between Power Losses

$A$ Parameter for calculating the TIA. $A = \frac{t_1}{\sqrt{L_r C_r}}$

$B$ Parameter for calculating the TIA. $B = \frac{1}{4f_{\text{sw}} \sqrt{L_r C_r}}$

$C_{DS}$ Drain-Source Capacitance

$C_{GD}$ Gate-Drain Capacitance

$C_{GS}$ Gate-Source Capacitance

$C_{\text{iss}}$ Input Capacitance

$C_{\text{oss}}$ Output Capacitance

$C_r$ Series Resonant Capacitance

$C_{r\text{ss}}$ Reverse Transfer Capacitance

$D$ Duty Cycle

$E_{\text{cond}}$ Conduction Energy

$E_{\text{off}}$ Turn-off Energy

$E_{\text{on}}$ Turn-on Energy

$f_{\text{res}}$ Resonant Frequency

$f_{\text{sw}}$ Switching Frequency

$I_0$ Parameter for calculating the TIA. $I_0 = i_{\text{tank}}(0)$

$I_{Lm0}$ Parameter for calculating the TIA. $I_{Lm0} = i_{Lm}(0)$

$I_1$ Parameter for calculating the TIA. $I_1 = i_{\text{tank}}(t_1)$

$I_{Lm1}$ Parameter for calculating the TIA. $I_{Lm1} = i_{Lm}(t_1)$
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<th>Symbol</th>
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<td>$R_{\text{gate}}$</td>
<td>Gate Resistance</td>
</tr>
<tr>
<td>$I_{\text{ch}}$</td>
<td>Channel Current</td>
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<td>$I_D$</td>
<td>Drain Current</td>
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<td>$I_{D(\text{on})}$</td>
<td>Drain Current when the MOSFET is on</td>
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<td>$i_D(t)$</td>
<td>Drain Current as a function of Time</td>
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<td>$i_{L_m}(t)$</td>
<td>LLC Magnetizing Inductance Current</td>
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<td>$I_{\text{load}}$</td>
<td>Output Load Current</td>
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<tr>
<td>$I_{\text{nom}}$</td>
<td>Nominal Load Current</td>
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<td>$I_{\text{off}}$</td>
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<td>$I_{\text{sw}}$</td>
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<td>$i_{\text{tank}}(t)$</td>
<td>LLC Resonant Tank Current</td>
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<td>LLC Transformer Primary Current</td>
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<td>$K_n$</td>
<td>Constant Parameters</td>
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<tr>
<td>$L_m$</td>
<td>Magnetizing Inductance</td>
</tr>
<tr>
<td>$L_r$</td>
<td>Series Resonant Inductance</td>
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<tr>
<td>$m$</td>
<td>Inductance Ratio</td>
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<td>$n$</td>
<td>Transformer Turns Ratio</td>
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<td>$P_{\text{cond}}$</td>
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<td>$P_{\text{diode}}$</td>
<td>Body Diode Power Losses</td>
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<tr>
<td>$P_{\text{in}}$</td>
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<td>$P_{\text{load}}$</td>
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<td>$P_{\text{loss}}$</td>
<td>Power Losses</td>
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<tr>
<td>$P_{\text{nom}}$</td>
<td>Nominal Power</td>
</tr>
<tr>
<td>$P_{\text{sw}}$</td>
<td>Switching Losses</td>
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List of Symbols

\( R_{DS(on)} \)  On-State Resistance
\( R_{eq} \)  Equivalent Resistance
\( R_{load} \)  Output Load Resistance
\( R_{th_{ca}} \)  Thermal Resistance between Case and Ambient
\( t \)  Time
\( T \)  Parameter for calculating the TIA. \( T = \frac{1}{2T_{sw}} - t_1 \sqrt{(L_r + L_m)C_r} \)
\( t_\sigma \)  Dead Time
\( t_0 \)  Initial Time
\( t_1 \)  Parameter for calculating the TIA. Represents the time at which \( v_{pr}(t) \) changes from its initial state.
\( T_{amb} \)  Ambient Temperature
\( T_{case} \)  Case Temperature
\( t_{d(off)} \)  Turn-off delay due to SR Operation
\( t_{d(on)} \)  Turn-on delay due to SR Operation
\( t_{delay(off)} \)  Turn-off delay due to MOSFET physical characteristics
\( t_{delay(on)} \)  Turn-on delay due to MOSFET physical characteristics
\( t_fi \)  Current Fall Time
\( t_{fv} \)  Voltage Fall Time
\( T_j \)  Junction Temperature
\( t_{on} \)  On-Time
\( t_{ri} \)  Current Rise Time
\( t_{rv} \)  Voltage Rise Time
\( V_0 \)  Parameter for calculating the TIA. \( V_0 = v_C(0) \)
\( V_1 \)  Parameter for calculating the TIA. \( V_1 = v_C(t_1) \)
\( v_C(t) \)  LLC Series Resonant Capacitance Voltage
\( V_{diode} \)  Body Diode Forward Voltage Drop
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<tr>
<td>$V_{dr}$</td>
<td>Driver Voltage Source</td>
</tr>
<tr>
<td>$V_{DS(\text{off})}$</td>
<td>Drain-Source Off-State Voltage</td>
</tr>
<tr>
<td>$V_{DS(\text{on})}$</td>
<td>Drain-Source On-State Voltage</td>
</tr>
<tr>
<td>$V_{DS}$</td>
<td>Drain-Source Voltage</td>
</tr>
<tr>
<td>$V_{GG}$</td>
<td>Nominal Driver Voltage Value</td>
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<td>$V_{sw}$</td>
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Acknowledgements

I would like to offer my deepest gratitude to Dr. Martin Ordonez, who granted me the opportunity of joining his research team. His leadership and dedication have pushed me further than I would ever expect to go, achieving more than I thought I ever could. I would also like to thank my labmates, without whom this work would have been impossible. Thanks for all the help and for making our lab a home away from home.

Kudos to UBC staff, who keep the place running smoothly, and to all instructors I have ever had the pleasure of interacting with: your care for education and your passion for teaching has inspired me time and again.

Last but definitely not least, I am forever grateful for my friends and family. No matter where you are, you will always have a special place in my heart. I will never be able to thank you enough for all the support you gave me, the small victories we celebrated, and the times you told me it was going to be OK. Thanks for being part of my life, and know I will always love you.
Chapter 1

Introduction

1.1 Motivation

With the intent of achieving sustainable development, the United Nation lists different global goals which include access to clean and affordable energy, and collaborative work towards combating climate change and its impacts \[2\]. In order to achieve the proposed milestones, expansion of energy generation from renewable sources and technological development towards the improvement of energy conversion efficiency is a must.

Power electronics plays a fundamental role in energy conversion, required for applications such as renewable energy generation, energy storage systems and operation of loads such as electric vehicles and electronic appliances. In fact, a considerable portion of the loads and storage systems of the electrical network consists of DC components, as can be observed in Fig. 1.1. These devices often rely on DC/DC converters, which can be accomplished with many topologies, among which the LLC resonant converter. This particular circuit configuration presents favorable characteristics which allow for increased efficiency, and in turn reduce the size and weight of converters \[3, 4\].

In the design process of energy converters, including the aforementioned LLC resonant converter, power switches play a major role. These devices, while very efficient, still produce power losses \((P_{\text{loss}})\) when in operation, which in turn generates heat, as depicted in Fig. 1.1. As a result, the efficiency of the system is reduced, and concerns regarding the operation of these converters arise, since their temperature must be kept at reasonable levels so to avoid
1.1. Motivation

In order to allow for proper heat dissipation and thus reasonable operating temperatures, an adequate thermal management must be designed for the power switches of the converters. However, current information for estimating $P_{\text{loss}}$ in such components is very limited, and prediction of losses in the design stage, before the converter is built, is compromised, which increases cost and development time [1]. While multiple design and control strategies for LLC converters can be found in the literature, there are opportunities for investigating and analyzing $P_{\text{loss}}$ of this circuit with increased precision and in a timely fashion.

In this work, an improved characterization of the main power loss mechanisms in MOS-FETs of the LLC resonant converters is performed, which increases the accuracy with which $P_{\text{loss}}$ are estimated. In addition, a time-domain method for determining $P_{\text{loss}}$ is presented,
which replaces oversimplified and inaccurate methods, and comes as an alternative for time-
consuming and computationally expensive simulation approaches. Key parameters of the
converter are analyzed, including the behaviour of $P_{\text{loss}}$, and the developed method is veri-
fied through simulation and experimental results.

1.2 Literature Review

In the design process of a power electronics converter, the determination of operating con-
ditions, electrical component capabilities and thermal management of active components
requires the adequate prediction of the behaviour of the topology. The $P_{\text{loss}}$ determination,
in particular, is fundamental for the design of a thermal management approach that will
ensure the durability of the equipment and safety of the operator [5, 6].

While resonant DC/DC converters present advantages that favor the efficiency of these
topologies due to the soft-switching of active components [7–9], thus generating smaller losses
and heat, these devices still require an appropriate thermal management design, which relies
on the adequate determination of $P_{\text{loss}}$.

The following paragraphs briefly introduce the LLC resonant converter, exploring its
advantages and applications. The modelling of different $P_{\text{loss}}$ mechanisms in power MOSFETs
is also briefly discussed. This work presents an analysis of the main loss mechanisms for the
MOSFETs of the LLC converter, as well as a time-domain analysis of this topology with
applications in $P_{\text{loss}}$ assessment.

1.2.1 LLC Resonant Converter

DC/DC power conversion has become a fundamental component in a variety of applications,
ranging from renewable energy generation [10, 11] and storage systems [12, 13] to electrical
loads [14–16], as shown in Fig. 1.1. In order to increase conversion efficiency and power
1.2. Literature Review

density, resonant DC/DC converters have become increasingly popular, since they allow for soft-switching of active components [7–9]. These techniques, known as zero-voltage switching (ZVS) and zero-current switching (ZCS), reduce switching losses and EMI, allowing for increased operating frequencies [17, 18].

LLC resonant converters, in particular, present advantages over other series and parallel resonant topologies, such as wider output regulation with both step-up and step-down operation [19]. Because of its operating characteristics, this topology is especially interesting for applications such as electric vehicle battery charging [20–24], renewable energy systems [25–27] and diverse low power applications [28–30]. Because of the high efficiency, reduced EMI, and improved regulation capabilities, this topology is of special interest among the various converters studied in power electronics.

Figure 1.2 shows the LLC resonant converter. As can be observed, this topology presents an inverting and a rectifying stage, separated by a resonant tank which often presents electrical decoupling through a transformer. Employing the transformer is desirable since it allows for changes in the magnitude of the concerned voltages and currents, as well as integration of magnetic components such as the magnetizing inductance ($L_m$) [9].

While the inverting stage is often implemented with MOSFETs, the rectification can be accomplished with either diodes, as shown in Fig. 1.2(a) or MOSFETs, shown in Fig. 1.2(b). In order to further improve the efficiency of the topology, synchronous rectification (SR) using active switches can be achieved with different control structures, ranging from complex control schemes which rely on measurements of circuit parameters such as MOSFET drain current ($I_D$) [31–36] or drain-source voltage ($V_{DS}$) [37, 38] to simple control algorithms with reduced sensing requirements [39–43].
1.2. Literature Review

Figure 1.2: Stages of the LLC resonant converter with a) Diode rectification and b) MOSFET rectification using Synchronous Rectification

1.2.2 Power Losses in LLC MOSFETs

In the operation of power MOSFETs, two main loss mechanisms arise: switching losses ($P_{\text{sw}}$), which occur during turn-on and turn-off of the device, and conduction losses ($P_{\text{cond}}$), which take place due to the resistive behaviour of the MOSFET when it is conducting [44]. Because the MOSFETs of both inverting and rectifying stages in the LLC resonant converter operate with soft-switching [45], $P_{\text{sw}}$ in this topology are reduced [46]. As a result, $P_{\text{cond}}$ are dominant in the active devices of this converter [9]. Thus, the proper modelling of the on-state resistance ($R_{\text{DS(on)}}$) which generates this loss mechanism is fundamental for an accurate estimation of $P_{\text{loss}}$ in LLC MOSFETs [1].

Information about the $R_{\text{DS(on)}}$ of MOSFETs is often provided by their datasheet as a function of the junction temperature of the device ($T_j$) for a given fixed $I_D$ and gate-source
1.2. Literature Review

While many design tools are available for LLC resonant converters, such as controllers [51–54], SR driving schemes [31–43] and design considerations for improved performance [55–58], there are still opportunities for exploring accurate $P_{\text{loss}}$ estimation techniques in LLC MOSFETs. The $P_{\text{loss}}$ in LLC MOSFETs generate heat, which has a much lower time scale when compared to electrical behaviour [59]. This means that thermal phenomena require a substantially larger time to reach steady-state when compared to electrical phenomena, even though both events take place simultaneously in the operation of high-frequency circuits such as the LLC resonant topology. Because of the substantial difference between time scales of electrical and thermal phenomena, and since power electronics circuits operate at high frequencies, simulation of $P_{\text{loss}}$ in such converters can be a computationally expensive and time consuming process. As an alternative, steady-state time-domain equations can be employed for the determination of the electric behavior of the converter, and the $P_{\text{loss}}$ can be estimated from the concerned $I_D$ and rectified current $I_{SR}$ expressions. While First-Harmonic Approximation (FHA) is the established method for estimating the time-domain behavior of LLC resonant converters, it does not provide accurate results under certain operating conditions.
1.3 Contribution of the Work

Steady-state analysis of different resonant converters such as series [61–63], parallel [64, 65] and others [66–69] have been presented in the literature, including LLC resonant converters [70–75]. However, the time-domain models for the LLC converter either do not consider the impact of different loading conditions [70–72] or they do not present time-domain expressions which can be easily applied to different operating conditions, loading configurations or design parameters [73–75]. As a result, determination of $P_{\text{loss}}$ employing existing time-domain expressions can be a challenging process, which has not yet been investigated in the literature. Moreover, in-depth analysis of $P_{\text{loss}}$ using SR as a function of design considerations and control strategies has not been thoroughly discussed yet.

1.3 Contribution of the Work

With the intent of providing engineers with a design tool for fast and accurate $P_{\text{loss}}$ prediction in LLC MOSFETs which can be easily implemented, the following topics are explored:

- First, a method for performing a detailed characterization of $P_{\text{cond}}$, $P_{\text{sw}}$ and $P_{\text{diode}}$ in MOSFETs is presented. The resulting polynomial expressions which better represent the behaviour of the device under different values of $T_j$, $V_{GS}$, $V_{DS}$ and $I_D$ improve the determination of $P_{\text{loss}}$ in LLC MOSFETs, since they capture the peculiarities observed in the device under different operating conditions.

- Second, time-domain expressions for the LLC resonant converter are derived based on the electrical behaviour of the topology when operating under different conditions and design parameters. While basic time-domain expressions are available in the literature, these either do not consider the impact of different loading conditions [70–72] or they do not present time-domain expressions which can be easily applied to different operating conditions, loading configurations or design parameters [73–75]. The developed
1.3. Contribution of the Work

equations based on the Time Interval Analysis allow for the determination of key electrical values and can be extended to the calculation of $P_{\text{loss}}$ in LLC MOSFETs, both in the inverting and rectifying stages. This allows for a considerably faster alternative to simulation, and to more precise estimations when compared to the well-established but oversimplified FHA.

- Third, a simple calorimetric method for determining $P_{\text{loss}}$ in an operating MOSFET is briefly investigated, which allows for the verification of the developed $P_{\text{loss}}$ estimation method using TIA. Since electrical measurements introduce disturbances to the behaviour of the circuit and at high frequencies EMI/RFI pose challenges to the measurement of electrical quantities, $P_{\text{loss}}$ estimation using electrical measurements is not adequate [76]. While measurements using a calorimeter have been discussed in the literature [76, 77], these do not allow for enough granularity so to enable the determination of $P_{\text{loss}}$ in a single switch as the method discussed in this work.

- Finally, an analysis of the behaviour of the converter and the developed time-domain expressions is presented. The implication of variations in circuit design parameters in circuit behaviour are analyzed and discussed with the intent of improving design practices.

The integration of the detailed characterization performed with the time-domain expressions developed allows for the accurate and fast determination of $P_{\text{loss}}$ in LLC MOSFETs of both inverting and rectifying stages. The calorimetric method investigated verifies the estimated $P_{\text{loss}}$ from calculation with significant accuracy when compared with $P_{\text{loss}}$ determined using FHA.
1.4 Thesis Outline

This work is organized and presented as follows:

- In Chapter 2, the main power loss mechanisms of LLC MOSFETs are discussed, and a detailed characterization of the device is presented. The resulting polynomial expressions for $R_{DS(on)}$, $E_{off}$ and $V_{diode}$ allow for a more detailed determination of $P_{cond}$, $P_{sw}$ and $P_{diode}$, which greatly contributes to the determination of $P_{loss}$ as circuit parameters are modified.

- In Chapter 3, time-domain equations for the LLC Resonant converter are developed based on the Time Interval Analysis. These equations offer greater insight on the behaviour of the topology when compared to the well-established yet oversimplified First-Harmonic Approximation. In addition, an analysis of $P_{loss}$ in the inverting and rectifying MOSFETs of the converter is performed. Moreover, studies regarding the impact of design parameters and operating conditions on $P_{loss}$ are presented, which offer insights on the expected behaviour of the topology as modifications are performed.

- Finally, in Chapter 4 a summary and conclusions of the work are presented, along with ideas for future research.
Power MOSFETs are fundamental components in any power electronics topology. These devices act as switches, working either as a short circuit when turned on, or as an open circuit when off. While very efficient, these components still produce heat when in operation. The two main power loss ($P_{\text{loss}}$) mechanisms for power MOSFETs are switching losses ($P_{\text{sw}}$) and conduction losses ($P_{\text{cond}}$).

The $P_{\text{sw}}$ can be divided into two categories: turn-on and turn-off losses. Whenever the switch is turned on, current starts flowing through its drain ($I_D$), while the blocking voltage between drain and source ($V_{DS}$) starts to subside, as shown in the simplified diagram on Fig. 2.1. The turn-on energy ($E_{\text{on}}$) is the area below the $V_{DS}$ and $I_D$ curves, and can be determined as:

![Figure 2.1: Waveforms for MOSFET switching showing the turn-on and turn-off energy, as well as the energy dissipated during conduction](image-url)
Chapter 2. Power Loss Characterization

\[ E_{on} = \frac{1}{2} V_{DS(off)} I_{D(on)} (t_{ri} + t_{fu}) \]  

(2.1)

When the switch is turned off, \( V_{DS} \) starts to increase while \( I_D \) diminishes, as shown in Fig. 2.1. Once again, the area below \( V_{DS} \) and \( I_D \) represents the turn-off energy \( E_{off} \):

\[ E_{off} = \frac{1}{2} V_{DS(off)} I_{D(on)} (t_{rv} + t_{fi}) \]  

(2.2)

Every time the switch is turned on or off, energy is released in the form of heat. The \( P_{sw} \) can then be obtained as follows:

\[ P_{sw} = f_{sw} (E_{on} + E_{off}) \]  

(2.3)

where \( f_{sw} \) is the switching frequency of the device.

The \( P_{cond} \), on the other hand, arise due to the resistive behaviour of the MOSFET when it is on. When the device is fully on, it does not behave as a perfect short-circuit but instead as a variable resistor \( R_{DS(on)} \). This resistance depends on the junction temperature of the device \( (T_j) \), \( I_D \) and the voltage magnitude that is applied to the gate with respect to the source of the device \( (V_{GS}) \). The \( P_{cond} \) can then be determined by using the Joule-Lenz law:

\[ P_{cond} = R_{DS(on)} I_{D(on)}^2 \]  

(2.4)

Graphically, the area below \( V_{DS} \) and \( I_D \) while the MOSFET is on represents the conduction energy \( (E_{cond}) \), as can be observed in Fig. 2.1.

One of the techniques that exist for reducing \( P_{sw} \) is the soft-switching of the MOSFETs. This condition can be achieved in some topologies such as resonant converters, including the LLC converter. It consists on turning the MOSFETs on when \( V_{DS} \) is near zero, which consists on zero-voltage switching (ZVS); or turning the devices off when \( I_D \) is near zero,
which consists on zero-current switching (ZCS). Under these conditions, the area under the 
$V_{DS}$ and $I_D$ curves will be significantly reduced, which will diminish $P_{sw}$ considerably. On 
the other hand, there are limited ways of reducing $P_{cond}$ in a topology. In addition, limited 
studies have been conducted on $P_{cond}$, since most of the literature focuses on modeling $P_{sw}$ of 
the device. In the following pages, a thorough analysis of $P_{loss}$ will be performed, focusing on 
the behaviour of $R_{DS(on)}$. A method for more accurately determining $P_{cond}$ will be studied, 
which is verified by experimental results and produces improved results when compared to 
information provided by the datasheet of the device. In addition, a detailed characterization 
for turn-off losses and body diode conduction losses will be presented, since these loss 
mechanisms are present in the inverting MOSFETs of the LLC Resonant Converter.

2.1 Physical Aspects of Power MOSFETs

In order to comprehend how $P_{loss}$ behaves and changes as a function of different parameters, 
it is important to look into the physical aspects of a power MOSFET. The following pages 
briefly explain how this device is constructed and operates, and how changes in $T_J$, $I_D$, $V_{GS}$ 
and $V_{DS}$ can affect $P_{loss}$. This portion of the thesis (Section 2.1) was based on select parts 
of [44].

2.1.1 Structural Characteristics

Semiconductors used in power electronics are often constructed with doped silicon. If the 
intrinsic semiconductor is doped with donor impurities such as phosphorus and arsenic, an 
n-type semiconductor is formed, where electrons are the majority carriers and holes are the 
minority carriers. If the semiconductor is doped with acceptor impurities instead, such as 
boron or gallium, a p-type semiconductor is formed, where holes are the majority carriers 
and electrons are the minority carriers. Differences in doping levels are often indicated with
2.1. Physical Aspects of Power MOSFETs

+ or - signs. For instance, an n+ layer is more heavily doped than a n− layer, yet they are still both n-type semiconductors.

Enhancement-mode n-channel MOSFETs are comprised by a n+pn−n+ structure, as shown in Fig. 2.2(a). The n+ layers connected to the drain and source are usually doped at around $10^{19}$ cm$^{-3}$. The p-layer is called the body of the MOSFET and is where the conducting channel is formed. This layer is often doped at around $10^{16}$ cm$^{-3}$. The n− layer is called the drift region, and determines the breakdown voltage of the device. It is usually doped at around $10^{14}$ cm$^{-3}$, which is substantially lower than the levels used for the n+ layers.

The gate of the MOSFET is isolated from the body by a layer of silicon dioxide (SiO$_2$), which is called gate oxide. When a positive voltage is applied to the gate with respect to the source, the silicon surface beneath the gate oxide is converted into an n-type channel, connecting drain with source and allowing for current to flow through the device. This ability to modify the conductivity type of semiconductors with an applied voltage is called “field effect”.

However, because of the physical characteristics of the device, a parasitic BJT is formed between the drain and source of the MOSFET. In order to prevent this parasitic device from ever turning on, the body is shorted to the source. As a result, a parasitic diode is formed between drain and source of the MOSFET, which is called body diode. Whenever a negative voltage

![Diagram](image_url)

Figure 2.2: a) Schematic for the enhancement-mode n-channel MOSFET, b) Components of $R_{DS(on)}$ through the channel, c) Bottleneck effect in the inversion layer due to $I_D$
2.1. Physical Aspects of Power MOSFETs

$I_D$ is applied to the MOSFET, this parasitic diode conducts and produces a forward voltage drop ($V_{diode}$).

When a small $V_{GS}$ is applied to the device, a depletion region is formed between the gate oxide and the silicon located right underneath it. The positive charges induced in the gate create an electric field which repels the holes of the body, which are the majority carriers. This exposes the negatively-charged acceptors present in the silicon. As $V_{GS}$ is increased, the electric field generated increases as well, and the depletion layer formed grows in thickness, since free electrons are attracted and holes are repelled. Eventually, the density of free electrons is high enough to become highly conductive, and a layer is formed with the same properties of an n-type semiconductor. This conducting layer, called inversion layer, allows for the flow of current between drain and source of the device.

2.1.2 Origin of the On-State Resistance

The $R_{DS(on)}$ of a MOSFET has different components, as shown in Fig. 2.2(b). Since applying larger values of $V_{GS}$ increases the thickness of the inversion layer, the resistive component of this portion reduces as larger voltages are applied. As a result, the overall $R_{DS(on)}$ is inversely proportional to the $V_{GS}$ applied. When a current flows through the channel, a voltage drop is generated, which “strangles” the inversion layer, creating a bottleneck effect, as observed in Fig. 2.2(c). Therefore, increasing $I_D$ results in increased values of $R_{DS(on)}$. Another factor that influences the behaviour of $R_{DS(on)}$ is the operating temperature of the switch, namely $T_j$. At higher values of $T_j$, the charge carriers collide more often with semiconductor lattices due to atomic vibrations. This phenomena reduces carrier mobility, which in turn is translated as an increase in $R_{DS(on)}$.

In summary, it is observed that $R_{DS(on)}$ increases as $T_j$ or $I_D$ increase, or when $V_{GS}$ decreases. However, as pointed out previously in the literature review, in most of the cases only the dependance of $R_{DS(on)}$ with $T_j$ is taken into account. Doing so may result in reduced
accuracy for $P_{\text{loss}}$ determination, as will be seen in the following pages.

### 2.1.3 Body Diode Forward Voltage Drop

As mentioned previously, the constructive pattern of power MOSFETs leads to the formation of a parasitic diode between the drain and source of the device. Once current flows from source to drain, a forward voltage drop ($V_{\text{diode}}$) will be generated due to this body diode, which will in turn generate power losses ($P_{\text{diode}}$).

This $V_{\text{diode}}$ originates because of the pn junction of diodes, where a potential barrier is originated from the electric field generated by the diffusion of majority carriers from one side of the junction to the other. Once the device is forward biased, the height of the potential barrier is reduced, since the positive charge applied to the p layer repels the holes and the negative charge which is applied to the n layer repels the electrons. Eventually, the electric field generated from the diffusion of the majority carriers cannot counteract the charge carrier motion and a net flow of current can be established [78]. Because at high temperatures the mobility of major carriers is decreased, the contact potential is reduced and $V_{\text{diode}}$ becomes smaller.

### 2.1.4 Switching Characteristics in Power MOSFETs

When turning a MOSFET on or off, it is necessary to move the charges to or from the stray capacitances of the device, shown in Fig. 2.3(a). These capacitor values change significantly with the applied voltage, as can be observed in Fig. 2.3(c), where $C_{\text{iss}}$ is the input capacitance, defined as $C_{\text{iss}} = C_{\text{GD}} + C_{\text{GS}}$, $C_{\text{oss}}$ is the output capacitance, defined as $C_{\text{oss}} = C_{\text{GD}} + C_{\text{DS}}$ and $C_{\text{rss}}$ is the reverse transfer capacitance, defined as $C_{\text{rss}} = C_{\text{GD}}$ [80]. In order to study the switching transient of a MOSFET, it is possible to perform an analysis using the test circuit depicted in Fig. 2.3(b), where the free-wheeling diode is assumed to be ideal, as well
2.1. Physical Aspects of Power MOSFETs

![Figure 2.3](image1.png)

Figure 2.3: a) Stray capacitances in a MOSFET; b) Circuit used for studying switching transients; c) Input, output and reverse transfer capacitances of a MOSFET.

As the current source $I_{sw}$ and the voltage sources $V_{dr}$ and $V_{sw}$.

When turning a MOSFET on, there are four distinct stages, as shown in Fig. 2.4. Each one of these stages has a different equivalent circuit, as shown in Fig. 2.5. When $t = 0$, the ideal voltage source $V_{dr}$ is changed from zero to $V_{GG}$. Current flows through the gate of the MOSFET, and the voltage between gate and source starts to rise. There is no current flowing between drain and source, so the current being applied by $I_{sw}$ flows through the free-wheeling

![Figure 2.4](image2.png)

Figure 2.4: Key waveforms for MOSFET turn-on
2.1. Physical Aspects of Power MOSFETs

Figure 2.5: Circuit used for studying switching transients a) During $t_{\text{delay}}$, b) During $t_{\text{ri}}$, c) During $t_{fV}$ and d) During $t_{\text{on}}$

diode, which is assumed to be ideal. As a result, the voltage being applied to the drain of the MOSFET remains constant at $V_{sw}$. This period lasts for an amount of time called $t_{\text{delay(on)}}$, until $V_{GS}$ reaches the threshold voltage of the device ($V_{GS(\text{th})}$). The equivalent circuit during this time period can be observed in Fig. 2.5(a).

Once $V_{GS}$ reaches $V_{GS(\text{th})}$, current starts to flow from drain to source of the device. However, the transfer of current from the free-wheeling diode to the channel of the device is not instantaneous, and part of the current still flows through the free-wheeling diode during this period of time. As a result, the voltage applied to the drain of the MOSFET is still $V_{sw}$. During this period of time, which lasts for an amount of time called $t_{\text{ri}}$, the voltage $V_{GS}$
2.2. Detailed MOSFET Characterization

continues to rise. The period ends when \( I_{ch} = I_{sw} \), and is presented by the equivalent circuit shown in Fig. 2.5(b).

Once the channel is conducting \( I_{sw} \), the free-wheeling diode is no longer playing a role in the equivalent circuit of this period, which is shown in Fig. 2.5(c). During this period, which lasts for an amount of time called \( t_{fv} \), the voltage between drain and source starts to drop, since the diode is no longer conducting and clamping the voltage of the drain to \( V_{sw} \). During this period of time, \( V_{GS} \) remains relatively constant at a certain value \( V_{GS(mp)} \), under a phenomenon commonly referred to the Miller Plateau region [81]. This period lasts until \( V_{DS} \) reaches its final on-state value \( (V_{DS(on)}) \), which will depend on \( R_{DS(on)} \).

The last period which occurs when a MOSFET is turned on is the on-time \( t_{on} \). During this period, the device is conducting all of \( I_{sw} \), and \( V_{DS} \) can be obtained by multiplying \( R_{DS(on)} \) and \( I_{sw} \). During this time period \( V_{GS} \) rises until it reaches \( V_{GG} \), and it lasts until the driver turns off. The equivalent circuit of this time period can be observed in Fig. 2.5(d).

During turn-off, the inverse sequence of events that occurred during turn-on takes place, with \( V_{DS} \) rising to \( V_{sw} \) before \( I_{ch} \) starts to cease conducting. Instantaneous power losses occur during both turn-on and turn-off, whenever the multiplication of \( V_{DS} \) and \( I_{ch} \) is different from zero.

2.2 Detailed MOSFET Characterization

In order to better understand how different phenomena related to \( P_{loss} \) in a MOSFET behave as key parameters of the device change, it is possible to perform a detailed characterization of the switch, and analyze how \( P_{cond} \), \( P_{sw} \) and \( P_{diode} \) are affected by parameters such as \( T_j \), \( I_D \), \( V_{DS} \) and \( V_{GS} \). This detailed characterization has the potential of improving the determination of \( P_{loss} \) when the switch is employed in different topologies, such as the LLC resonant converter.
2.2. Detailed MOSFET Characterization

2.2.1 Conduction Loss Characterization

In order to study the behaviour of $P_{\text{cond}}$ as operating conditions of a circuit change, it is necessary to comprehend how $R_{DS(on)}$ is affected by changes in $T_j$, $I_D$ and $V_{GS}$. The information available from datasheets for $R_{DS(on)}$ is often provided for a constant predetermined $I_D$ and $V_{GS}$, as shown in Fig. 2.6(a). This limited information may not be adequate for all applications, since there are topologies where $I_D$ changes substantially during operation, such as in the case of the LLC resonant converter. In addition, different gate driver voltages must be accounted for.

Therefore, it is necessary to obtain a characterization of $R_{DS(on)}$ which accounts not only for different values of $T_j$ but also takes into account different magnitudes of $V_{GS}$ and different values and polarities of $I_D$. In order to do so, a detailed characterization can be performed, as shown in Fig. 2.6(b). The device is turned on with a constant and known voltage $V_{GS}$. Afterwards, a constant and known DC current is applied to it, and precision instruments are used to record $V_{DS}$. In addition, thermocouples are employed to monitor the case temperature of the component. The power losses of the device can be obtained by multiplying $I_D$ and $V_{DS}$, and $T_j$ can be determined by using the simplified thermal model of a MOSFET, shown in Fig. 2.6(c). Subsequent changes in $I_D$, $V_{GS}$ and on the operating

![Diagram](a) $R_{DS(on)} = f(T_j)$

![Diagram](b) Test circuit for the detailed characterization of $R_{DS(on)}$

![Diagram](c) Simplified thermal model of a MOSFET.

Figure 2.6: a) Datasheet $R_{DS(on)} = f(T_j)$ for a constant $I_D$ and $V_{GS}$ [79], b) Test circuit for the detailed characterization of $R_{DS(on)}$, c) Simplified thermal model of a MOSFET.
2.2. Detailed MOSFET Characterization

temperature with the assistance of a thermal chamber allow for the determination of $R_{DS(on)}$ under different operating conditions.

With the measurements obtained, it is possible to determine a polynomial expression that represents the observed behaviour of $R_{DS(on)}$ as a function of the concerned variables. In order to verify the proposed method, a characterization of a sample MOSFET was carried out. The MOSFET IPZ60R040C7 was considered, since it is employed in industrial applications for power conversion. Measurements under different operating conditions were performed, and a 3rd order polynomial expression for $R_{DS(on)}$ as a function of $T_j$, $I_D$ and $V_{GS}$ was obtained. The equation shows a good least-squares fit for the considered points, while keeping a reasonably low order. This equation can be translated into a family of surfaces, and is shown in Fig. 2.7.

From the plots, it is possible to observe that changes in $V_{GS}$ were not as impactful in $R_{DS(on)}$ as the other parameters for the considered range of values. As expected, lower values of $T_j$ resulted in smaller $R_{DS(on)}$. Interestingly, the behaviour of $I_D$ becomes rather peculiar when a negative $I_D$ is applied to the device. This is due to the fact that the body diode of

![Figure 2.7](image-url)

Figure 2.7: a) Family of surfaces of $R_{DS(on)}$ as a function of $V_{GS}$ and $I_D$ for different values of $T_j$. b) Family of surfaces of $R_{DS(on)}$ as a function of $V_{GS}$ and $T_j$ for different values of $I_D$. Both figures show that $R_{DS(on)}$ changes significantly with $T_j$ and $I_D$. 

2.2. Detailed MOSFET Characterization

the MOSFET starts conducting in parallel with the channel of the device, so the resulting equivalent channel resistance is smaller than what would be originally expected. This becomes clear in Fig. 2.7(b) when comparing the obtained values for $R_{DS(on)}$ for -20 A with the other scenarios. Since changes in $V_{GS}$ are not as impactful as variations in the other considered parameters, it is possible to obtain a single surface that represents the behaviour of the $R_{DS(on)}$ as a function of $T_j$ and $I_D$ for a constant value of $V_{GS}$, as shown in Fig. 2.8.

2.2.2 Diode Loss Characterization

In LLC resonant converters, in order to obtain the soft-switching of the inverter MOSFETs, it is necessary for the body diode to conduct during the dead time. As a result, losses arise due to the $V_{diode}$ of the device. This parameter can be characterized in a similar fashion to that employed for $R_{DS(on)}$, with the difference that the device is kept off with zero voltage applied between gate and source. In addition, the polarity of the current of the test circuit shown in Fig. 2.7(b) is inverted. Because $V_{GS}$ is set to zero, the only parameters that influence $V_{diode}$ are $I_D$ and $T_j$. 

Figure 2.8: Surface of $R_{DS(on)}$ for $V_{GS} = 8$ V
2.2. Detailed MOSFET Characterization

After the required measurements are performed in $V_{\text{diode}}$ of the body diode of the MOSFET IPZ60R040C7, it is possible to obtain a 3rd order polynomial expression for this parameter as a function of $I_D$ and $T_j$. This equation can be translated into an operating surface, shown in Fig. 2.9. From the figure, it is possible to observe that lower values of $T_j$ and higher $I_D$ result in higher $V_{\text{diode}}$, which is in accordance with the expected behaviour of this component.

While reverse recovery losses are another source of heat dissipation that results from the operation of diodes, including the body diode, this loss mechanism does not occur during regular operation of the LLC resonant converter. As long as the operation of the converter is stable, proper dead times are considered, and the converter operates outside of the capacitive region, reverse recovery losses are not a concern \[82, 83\]. This is because the body diode will stop conducting as the channel is formed, and $I_D$ will instead circulate through the channel of the device.

2.2.3 Switching Loss Characterization

As mentioned in the Introduction, one of the advantages of LLC resonant converters is the soft-switching of the inverter MOSFETs during turn-on. As a result, only turn-off losses
2.2. Detailed MOSFET Characterization

play a role during regular operation of the converter. In order to properly determine $P_{loss}$ of this topology, this loss mechanism must be appropriately characterized. To obtain a detailed characterization of $P_{sw}$, it is possible to employ a simple test circuit with the device under test (DUT). Under different testing conditions, different values of $P_{loss}$ are observed, and a model of losses can be extracted.

However, the measurement of losses considering the difference between input and output power is not possible, since other elements in the circuit will be producing losses, such as the inductor and the capacitor. Measurement of current and voltage waveforms right across the switch is also not recommended due to the high-frequency switching of the device, since EMI/RFI pose challenges to the measurement of electrical quantities. In addition, electrical measurements introduce disturbances to the electrical behavior of the circuit, which compromise the quality of the power loss measurement, and the skewing of signals must be accounted for. Measurement of temperature using RTDs or thermocouples is also not possible since these rely on the measurement of electrical parameters for determining temperature. As an alternative, calorimetric methods can be employed to measure the rise in temperature which results from power dissipation.

The test circuit used for the determination of $P_{sw}$ in the MOSFETs which were later on employed in the LLC resonant converter (IPZ60R040C7) can be visualized in Fig. 2.10(a). It consists of a topology in which the input voltage and loading condition can be easily changed. This allows for easy variations of $V_{DS}$ and $I_D$, which are two parameters which substantially influence $P_{sw}$ of the converter. In addition to these, the $T_j$ was also considered in the analysis as one of the components which could affect this loss mechanism. While other parameters such as $V_{GS}$ and gate resistance ($R_{gate}$) are known to alter $P_{sw}$, these were kept constant and at the same values which were subsequently used for operation of the LLC resonant converter.

To isolate turn-off losses, the device under test, represented by the switch S1, is placed
2.2. Detailed MOSFET Characterization

in parallel with the switch S2. The gating signal used for both MOSFETs is shown in Fig. 2.10(b), along with key waveforms of the test circuit. By turning S2 on a couple nanoseconds before S1, the voltage across S1 is brought down before the device turns on, which eliminates turn-on losses. After S1 is turned completely on, S2 is turned off, and S1 conducts the entirety of the current during the majority of the on-time. After the on time is over, S1 is turned off, and the diode D1 starts conducting. As a result, all the loss mechanisms which are present in the switching process of S1 are accounted for, including ringing and the effect of stray elements. In Fig. 2.10(b), it is possible to observe that the gate of S2 is activated at the time instant $\alpha$. This causes $V_{DS}$ to fall until it reaches the on-state value which is close to zero, at the time instant $\beta$. After this has happened, S1 is turned on ($\gamma$) and the switch S2 is turned off ($\delta$). When turning off S1 ($\epsilon$), the switch is subjected to turn-off losses, since both $V_{DS}$ and $I_D$ are present until the switching action is over ($\zeta$).

The operation of the converter with different values of ambient temperature ($T_{amb}$), input voltage ($V_{in}$) and output load ($I_{load}$) allows for the determination of different operating temperatures of the MOSFET S1. The case temperature ($T_{case}$) is measured using a thermal camera, and compared with the ambient temperature which surrounds the converter, as

Figure 2.10: a) Test circuit employed for characterizing $E_{off}$ and b) Waveforms of the test circuit.
shown in Fig. 2.11(a). Changes in the aforementioned parameters allow for measurements of the device under different values of $T_j$, $I_D$ and $V_{DS}$.

The relationship between $T_{case}$ and $P_{loss}$ is given by $T_{case} = P_{loss} R_{th_{ca}} + T_{amb}$, derived from the thermal equivalent circuit of the device shown in Fig. 2.6(c). The thermal resistance $R_{th_{ca}}$ can be obtained by applying a constant known value of current through the device under test, and measuring $P_{loss}$ with precision instruments, as well as $T_{case}$ and $T_{amb}$ with a thermal camera, as shown in Fig. 2.11(b). By changing the applied current, it is possible to obtain different temperature and power levels, and a linear relationship between $T_{case} - T_{amb}$ and $P_{loss}$ is obtained, as Fig. 2.11(c) shows. This allows for the determination of $P_{loss}$ of a device based on its operating temperature, regardless of the frequency it operates, in a minimally invasive fashion. It is important to note that the operating conditions such as air flow and measurement positions must be maintained the same for the relationship to remain valid. Since the duty cycle ($D$) of the switch is known, $P_{cond}$ can be extracted from $P_{loss}$, and $P_{sw}$ can be divided by the switching frequency $f_{sw}$ so the turn-off energy $E_{off}$ can be obtained for each testing point.

As a result, a 3rd order expression for $E_{off} = f(T_j, I_D, V_{DS})$ is obtained, which translates into different families of operating surfaces, shown in Fig. 2.12. It is possible to observe

![Figure 2.11: a) Thermal view of the MOSFETs under different operating temperatures; b) Circuit employed for the determination of the relationship between operating temperatures and $P_{loss}$; c) Linear relationship between $T_{case} - T_{amb}$ and $P_{loss}$.](image-url)
2.2. Detailed MOSFET Characterization

Figure 2.12: a) Family of surfaces of $E_{off}$ as a function of $T_j$ and $I_D$ for different values of $V_{DS}$. b) Family of surfaces of $E_{off}$ as a function of $V_{DS}$ and $T_j$ for different values of $I_D$.

From the surfaces that $T_j$ does not significantly impact on the observed $E_{off}$, unlike what is observed in the case of the characterization of $R_{DS(on)}$. As expected, an increase in $I_D$ or $V_{DS}$ represent higher $E_{off}$ values, and increased $P_{sw}$.
Chapter 3

Time Interval Analysis of LLC Resonant Converters

As discussed in the Introduction, DC/DC power conversion has become a fundamental component in a variety of applications ranging from high to low power applications. One of the most popular topologies for DC/DC conversion is the LLC resonant converter, since it presents wide output voltage regulation with step-up and step-down capabilities. In addition, EMI/RFI is reduced in this topology because of the soft-switching of the MOSFETs, which also reduce switching losses ($P_{sw}$) in the devices.

However, the behaviour of this converter is highly non-linear, which renders its analysis and control a challenging task. Because of the complexity of the topology, a technique called First-Harmonic Approximation (FHA) has been developed for this converter, which aims to predict the concerned waveforms of the topology by modifying the inverting and rectifying stages shown in Fig. 1.2. The inverting stage is replaced by a sinusoidal voltage source with peak-to-peak amplitude of $\frac{4V_{in}}{\pi}$ and frequency equal to the switching frequency ($f_{sw}$). The rectifying stage is replaced by an equivalent resistance $R_{eq} = \frac{8R_{load}}{\pi^2}$, where $R_{load}$ is the load resistance of the LLC resonant converter. The equivalent resistor is then scaled accordingly by the transformer’s turns ratio, and connected in parallel with the magnetizing inductance ($L_m$). While this equivalent circuit gives insights about the operation of the converter, the obtained results are not precise, which may compromise the determination of important aspects such as power loss ($P_{loss}$) estimation in the switches of the topology.
3.1 Time Interval Equations of LLC Resonant Converters

As an alternative, time-domain equations for the topology have been developed in the literature, but these either do not consider the impact of different loading conditions, or they do not present time-domain expressions which can be easily applied to different operating conditions, loading configurations or design parameters. In this work, a more detailed analysis of the waveforms of the converter was performed, and a time-domain methodology was developed based on the behaviour of the topology, accounting for different operating conditions and design parameters. It produces results which are closer to those observed in experimental measurements, replacing the established yet oversimplified FHA. In addition, the developed method allows for the determination of $P_{\text{loss}}$ in the MOSFETs of this topology with increased precision, and in a timely fashion when compared to simulation results.

3.1 Time Interval Equations of LLC Resonant Converters

Steady-state time-domain equations for power converters allow for the behaviour of topologies to be studied under different operating conditions and design parameters without the need of using simulation software. This becomes especially interesting when phenomena with substantially different timescales are being investigated, such as the case of thermal and electrical behaviours. In such cases, simulation with significantly small timesteps must be carried on for a considerable amount of time, which can be computationally expensive. Employing time-domain expressions eliminates this problem, since the electrical behaviour is already determined at steady-state, and thermal characteristics can be assessed within a couple iterations.

In order to determine $P_{\text{loss}}$ in the inverting and rectifying stages of LLC resonant converters, precise time-domain expressions are necessary so to avoid time-consuming simulation. While FHA is the established tool for investigating the behaviour of this converter,
it oversimplifies the obtained waveforms to a degree that they are not useful for power loss
determination. This is because the current waveforms will be approximated by sinusoidal
signals, which result in inaccurate calculations for conduction losses ($P_{\text{cond}}$). In addition,
$P_{\text{sw}}$ in the MOSFETs of the inverting stage cannot be estimated properly since the turn-on
current ($I_{\text{on}}$) and turn-off current ($I_{\text{off}}$) are not properly calculated using FHA.

As an alternative, the Time Interval Analysis (TIA) developed allows for the determination
of time-domain equations which reflect the actual behaviour of the topology under different
operating conditions and design considerations, accounting for the non-sinusoidal characteristics presented by the waveforms of the converter. The equations are developed as function
of circuit parameters, such as transformer turns ratio ($n$), series resonant inductance ($L_r$)
and capacitance ($C_r$), magnetizing inductance ($L_m$) and inductance ratio ($m = \frac{L_m}{L_r}$), and
operating conditions such as switching frequency ($f_{\text{sw}}$). In addition, equations are provided
for parameters such as input voltage ($V_{\text{in}}$), output voltage ($V_{\text{out}}$) and load power ($P_{\text{load}}$), since
different combinations of these parameters can be considered for different applications. It is
important to notice that $P_{\text{load}}$ can also be expressed as a function of the load current ($I_{\text{load}}$),
since $P_{\text{load}} = V_{\text{out}}I_{\text{load}}$.

Consider the LLC resonant converter shown in Fig. 3.1(a). The circuit connected to the
primary of the transformer can be represented by that shown in Fig. 3.1(b). Based on this

![Figure 3.1: a) LLC Resonant Converter and b) Equivalent circuit for the primary of the LLC resonant converter](image-url)
3.1. Time Interval Equations of LLC Resonant Converters

circuit, analysis of inductor currents and capacitor voltages allows for the determination of accurate time-domain equations for LLC resonant converters, which are fundamental for the adequate calculation of MOSFET power losses.

3.1.1 Operation Above the Resonant Frequency

It is possible to observe from Fig. 3.2 that the operation of the converter can be divided into two main portions. The first one occurs between the moment \( v_{in}(t) \) goes from zero to \( V_{in} \) until the instant \( v_{pr}(t) \) goes from \(-nV_{out}\) to \( nV_{out} \). This instant when the polarity of \( v_{pr}(t) \) changes from \(-nV_{out}\) to \( nV_{out} \) has been named \( t_1 \), and is an unknown parameter which needs to be determined from the circuit parameters and operating conditions. The moment when \( v_{in}(t) \) goes from zero to \( V_{in} \) has been named \( t_0 = 0 \). The second period starts at \( t = t_1 \), and lasts until \( v_{in}(t) \) goes from \( V_{in} \) to zero. As a result, there are two main time intervals that need to be analyzed in the circuit: the first one is when \( t_0 < t < t_1 \) and the second one is

![Figure 3.2: Key waveforms for operation of the LLC resonant converter a) Above the resonant frequency, b) Below the resonant frequency and c) At the resonant frequency](image)
3.1. Time Interval Equations of LLC Resonant Converters

when \( t_1 < t < \frac{1}{2f_{sw}} \). As a result, two equivalent circuits can be analyzed for each of the time intervals, as shown in Fig. 3.3.

During the first time interval, it is possible to perform mesh analysis of the resulting circuit shown in Fig. 3.3(a):

\[
-V_{\text{in}} + L_r \frac{di_{\text{tank}}(t)}{dt} + v_C(t) - nV_{\text{out}} = 0 \tag{3.1}
\]

In addition, the relationship between the capacitor current and voltage is known in this situation:

\[
i_{\text{tank}}(t) = C_r \frac{dv_C(t)}{dt} \tag{3.2}
\]

Moreover, a constant voltage is being applied to the magnetizing inductance \( L_m \), so:

\[
-nV_{\text{out}} = L_m \frac{di_{\text{m}}(t)}{dt} \tag{3.3}
\]

By rearranging the terms of (3.1), (3.2) and (3.3) it is possible to obtain the system of equations that represents the behaviour of the converter when operating above \( f_{res} \) during the first time interval \( 0 < t < t_1 \):

\[
t_0 < t < t_1 \quad \quad t_1 < t < \frac{1}{2f_{sw}}
\]

Figure 3.3: a) Equivalent circuit for operation of the converter for \( f_{sw} > f_{res} \) and \( t_0 < t < t_1 \). b) Equivalent circuit for \( f_{sw} > f_{res} \) and \( t_1 < t < \frac{1}{2f_{sw}} \).
3.1. Time Interval Equations of LLC Resonant Converters

\[
\begin{aligned}
\left\{ 
&v_C(t) = V_{in} + nV_{out} - L_r \frac{di_{tank}(t)}{dt}, \\
&i_{tank}(t) = C_r \frac{dv_C(t)}{dt}, \\
&-nV_{out} = L_m \frac{dL_m(t)}{dt}
\end{aligned}
\] (3.4)

Taking the derivative of (3.1) with respect to time results in:

\[
\frac{dv_C(t)}{dt} = -L_r \frac{d^2i_{tank}(t)}{dt^2}
\] (3.5)

Replacing (3.5) into (3.2) gives:

\[
i_{tank}(t) = -C_r L_r \frac{d^2i_{tank}(t)}{dt^2}
\] (3.6)

Now, (3.6) is given only as a function of \(i_{tank}(t)\), and can be solved using ODE:

\[
i_{tank}(t) = K_1 \sin\left( \frac{t}{\sqrt{C_r L_r}} \right) + K_2 \cos\left( \frac{t}{\sqrt{C_r L_r}} \right)
\] (3.7)

where \(K_1\) and \(K_2\) are constant values. Taking the derivative of (3.2) with respect to time results in:

\[
\frac{di_{tank}(t)}{dt} = C_r \frac{d^2v_C(t)}{dt^2}
\] (3.8)

Replacing (3.8) into (3.1) gives:

\[-V_{in} + L_r C_r \frac{d^2v_C(t)}{dt^2} + v_C(t) - nV_{out} = 0
\] (3.9)

Now, (3.9) is given only as a function of \(v_C(t)\), and can be solved using ODE:

\[
v_C(t) = K_3 \sin\left( \frac{t}{\sqrt{C_r L_r}} \right) + K_4 \cos\left( \frac{t}{\sqrt{C_r L_r}} \right) + V_{in} + nV_{out}
\] (3.10)

where \(K_3\) and \(K_4\) are constant values.
The next step is to determine the values of $K_1$, $K_2$, $K_3$ and $K_4$ from (3.7) and (3.10). At $t_0 = 0$, the current $i_{\text{tank}}(t)$ assumes the value $I_0$, and $v_C(t)$ assumes the value $V_0$, as shown in Fig. 3.2. When $t = 0$, (3.7) becomes:

$$K_2 = I_0$$

(3.11)

As a result, (3.7) becomes:

$$i_{\text{tank}}(t) = K_1 \sin\left(\frac{t}{\sqrt{C_r L_r}}\right) + I_0 \cos\left(\frac{t}{\sqrt{C_r L_r}}\right)$$

(3.12)

When $t = 0$, (3.10) becomes:

$$K_4 = V_0 - V_{\text{in}} - nV_{\text{out}}$$

(3.13)

As a result, (3.10) becomes:

$$v_C(t) = K_3 \sin\left(\frac{t}{\sqrt{C_r L_r}}\right) + (V_0 - V_{\text{in}} - nV_{\text{out}}) \cos\left(\frac{t}{\sqrt{C_r L_r}}\right) + V_{\text{in}} + nV_{\text{out}}$$

(3.14)

In order to determine $K_1$, it is possible to take the derivative of (3.12) with respect to time, and replace it into (3.1):

$$- V_{\text{in}} + L_r \left( \frac{K_1}{\sqrt{C_r L_r}} \cos\left(\frac{t}{\sqrt{C_r L_r}}\right) - \frac{I_0}{\sqrt{C_r L_r}} \sin\left(\frac{t}{\sqrt{C_r L_r}}\right) \right) + v_C(t) - nV_{\text{out}} = 0$$

(3.15)

When $t = 0$, (3.15) becomes:

$$- V_{\text{in}} + L_r \frac{K_1}{\sqrt{C_r L_r}} + V_0 - nV_{\text{out}} = 0$$

(3.16)

This allows for $K_1$ to be determined:
3.1. Time Interval Equations of LLC Resonant Converters

\[ K_1 = \sqrt{\frac{C_r}{L_r}}(V_{in} + nV_{out} - V_0) \]  

(3.17)

Similarly, in order to determine \( K_3 \), it is possible to take the derivative of (3.14) with respect to time, and replace it into (3.2):

\[ i_{tank}(t) = C_r \left( \frac{K_3}{\sqrt{C_r L_r}} \cos\left( \frac{t}{\sqrt{C_r L_r}} \right) - \frac{V_0 - V_{in} - nV_{out}}{\sqrt{C_r L_r}} \sin\left( \frac{t}{\sqrt{C_r L_r}} \right) \right) \]  

(3.18)

When \( t = 0 \), (3.18) becomes:

\[ I_0 = C_r \frac{K_3}{\sqrt{C_r L_r}} \]  

(3.19)

This allows for \( K_3 \) to be determined:

\[ K_3 = I_0 \sqrt{\frac{L_r}{C_r}} \]  

(3.20)

Now that \( K_1, K_2, K_3 \) and \( K_4 \) have been determined, it is possible to obtain expressions for \( i_{tank}(t) \) and \( v_C(t) \) as a function of circuit parameters and operating conditions and the variables \( I_0 \) and \( V_0 \):

\[ i_{tank}(t) = \sqrt{\frac{C_r}{L_r}}(V_{in} + nV_{out} - V_0) \sin\left( \frac{t}{\sqrt{C_r L_r}} \right) + I_0 \cos\left( \frac{t}{\sqrt{C_r L_r}} \right) \]  

(3.21)

\[ v_C(t) = I_0 \sqrt{\frac{L_r}{C_r}} \sin\left( \frac{t}{\sqrt{C_r L_r}} \right) + (V_0 - V_{in} - nV_{out}) \cos\left( \frac{t}{\sqrt{C_r L_r}} \right) + V_{in} + nV_{out} \]  

(3.22)

It is also possible to solve (3.3) to obtain an expression for \( i_{Lm}(t) \)

\[ i_{Lm}(t) = -\frac{nV_{out}}{L_m} + K_5 \]  

(3.23)

The constant \( K_5 \) can be determined by considering that when \( t = 0 \), \( i_{Lm}(0) = I_{Lm0} \), so \( K_5 = I_{Lm0} \). As a result, the system of equations that regulate the behaviour of the converter
3.1. Time Interval Equations of LLC Resonant Converters

when it operates above \( f_{\text{res}} \) and during the first time interval can be obtained with (3.21), (3.22) and (3.23):

\[
\begin{align*}
  v_C^a(t) &= V_{\text{in}} + nV_{\text{out}} + I_0 \sqrt{\frac{L_r}{C_r}} \sin \left( \frac{t}{\sqrt{L_r C_r}} \right) + (V_0 - V_{\text{in}} - nV_{\text{out}}) \cos \left( \frac{t}{\sqrt{L_r C_r}} \right) \\
  i_{\text{tank}}^a(t) &= I_0 \cos \left( \frac{t}{\sqrt{L_r C_r}} \right) - (V_0 - V_{\text{in}} - nV_{\text{out}}) \sqrt{\frac{C_r}{L_r}} \sin \left( \frac{t}{\sqrt{L_r C_r}} \right) \\
  i_{L_m}^a(t) &= -\frac{nV_{\text{out}}}{L_m} + I_{L_m0}
\end{align*}
\]

where the superscript \( a \) indicates that these waveforms represent the behavior during the first interval, \( t_0 = 0 < t < t_1 \).

During the second time interval, which is comprised when \( t_1 < t < \frac{1}{2f_{\text{sw}}} \), the analysis follows a similar procedure as the one described previously. Considering Fig. 3.2, it is possible to observe that the only change that occurred in the circuit is that \( v_{pr}(t) \) is now \( V_{pr} \) instead of \(-V_{pr}\). As a result, the following expressions that describe the behavior of the circuit shown in Fig. 3.3(b) can be found:

\[
\begin{align*}
  v_C(t) &= V_{\text{in}} - nV_{\text{out}} - L_r \frac{d_i_{\text{tank}}(t - t_1)}{dt} \\
  i_{\text{tank}}(t) &= C_r \frac{dv_C(t - t_1)}{dt} \\
  nV_{\text{out}} &= L_m \frac{d_i_{L_m}(t - t_1)}{dt}
\end{align*}
\]

Because of the time reference considered, it is necessary to displace the waveforms in time by \( t_1 \). If \( v_C(t_1) = V_1 \), \( i_{\text{tank}}(t_1) = I_1 \) and \( i_{L_m}(t_1) = I_{L_m1} \), then the time-domain equations that describe the behavior of the converter can be obtained by following a similar procedure as to that described for the first time interval:
3.1 Time Interval Equations of LLC Resonant Converters

\[
\begin{align*}
    v_C^b(t) &= V_{in} - nV_{out} + I_1 \sqrt{\frac{L_r}{C_r}} \sin \left( \frac{t - t_1}{\sqrt{L_rC_r}} \right) + (V_1 - V_{in} + nV_{out}) \cos \left( \frac{t - t_1}{\sqrt{L_rC_r}} \right) \\
    i_{tank}^b(t) &= I_1 \cos \left( \frac{t - t_1}{\sqrt{L_rC_r}} \right) - (V_1 - V_{in} + nV_{out}) \sqrt{\frac{C_r}{L_r}} \sin \left( \frac{t - t_1}{\sqrt{L_rC_r}} \right) \\
    i_{Lm}^b(t) &= \frac{nV_{out}(t - t_1)}{L_m} + I_{Lm1}
\end{align*}
\]

(3.26)

where the superscript \( b \) indicates that these waveforms represent the behavior during the second interval, \( t_1 < t < \frac{1}{2f_{sw}} \).

Now, the equations that represent the behavior of the circuit are known for operation above the resonant frequency, and are shown in (3.24) and (3.26). The superscript \( a \) indicates that that set of waveforms is valid during the first time interval, \( 0 < t < t_1 \), while the superscript \( b \) indicates that those waveforms are valid during the second time interval, \( t_1 < t < \frac{1}{2f_{sw}} \).

While these equations are represented as a function of design parameters such as \( L_r, C_r, L_m \) and \( n \), there are also parameters which are unknown, such as \( I_0, I_1, V_0, V_1, I_{Lm0}, I_{Lm1} \) and \( t_1 \). In addition, the value of \( V_{in} \) or \( V_{out} \) may not be known for a certain operating condition. In order to determine the value of the unknown parameters, certain electrical properties of the circuit must be considered.

First, it is known that at \( t = t_1 \) there is no current flowing to the transformer connected to the primary of the circuit, so \( i_{tank}(t_1) = i_{Lm}(t_1) = I_1 = I_{Lm1} \). This implies that at \( t = t_1 \), \( i_{Lm}^a(t) \) from (3.24) becomes:

\[
I_{Lm0} = I_1 + \frac{nV_{out}t_1}{L_m}
\]

(3.27)

In addition, as can be observed in Fig. 3.2, the average value of \( i_{Lm}(t) \) is zero over a switching period. As a result, \( i_{Lm}(0) = -i_{Lm}(\frac{1}{2f_{sw}}) \), since the switching cycles are symmetric and a duty cycle of \( D = 0.5 \) is assumed. This implies that \( i_{Lm}(\frac{1}{2f_{sw}}) = -I_{Lm0} \). Thus, at
3.1. Time Interval Equations of LLC Resonant Converters

\[ t = \frac{1}{2f_{sw}}, \quad i_{L_m}^b(t) \] from (3.26) becomes:

\[ I_{L_m0} = \frac{nV_{out} \left( t_1 - \frac{1}{2f_{sw}} \right)}{L_m} - I_1 \] (3.28)

Equating (3.27) and (3.28) allows for the determination of \( I_1 \) as a function of circuit parameters, operating conditions and \( V_{out} \):

\[ I_1 = -\frac{nV_{out}}{4L_m f_{sw}} \] (3.29)

Replacing (3.29) into (3.27) allows for the determination of \( I_{L_m0} \):

\[ I_{L_m0} = \frac{nV_{out}}{4L_m f_{sw}} (4f_{sw} t_1 - 1) \] (3.30)

Assuming that \( A = \frac{t_1}{\sqrt{L_r C_r}} \) and \( B = \frac{1}{4f_{sw} \sqrt{L_r C_r}} \), (3.30) becomes:

\[ I_{L_m0} = \frac{nV_{out}}{4L_m f_{sw}} \left( \frac{A}{B} - 1 \right) \] (3.31)

Replacing \( I_1 \) from (3.29) in \( v_C^b(t) \) from (3.26) gives:

\[ v_C^b(t) = V_{in} - nV_{out} - \frac{nV_{out}}{4L_m f_{sw}} \sqrt{\frac{L_r}{C_r}} \sin \left( \frac{t - t_1}{\sqrt{L_r C_r}} \right) + (V_1 - V_{in} + nV_{out}) \cos \left( \frac{t - t_1}{\sqrt{L_r C_r}} \right) \] (3.32)

Replacing \( I_1 \) (3.29) in \( i_{tank}^b(t) \) from (3.26) gives:

\[ i_{tank}^b(t) = -\frac{nV_{out}}{4L_m f_{sw}} \cos \left( \frac{t - t_1}{\sqrt{L_r C_r}} \right) - (V_1 - V_{in} + nV_{out}) \sqrt{\frac{C_r}{L_r}} \sin \left( \frac{t - t_1}{\sqrt{L_r C_r}} \right) \] (3.33)

Because the voltage in the series resonant capacitor cannot change instantaneously, \( v_C^a(t_1) \) from (3.24) must be equal to \( v_C^b(t_1) \) from (3.32). That is:

\[ V_1 = V_{in} + nV_{out} + I_0 \sqrt{\frac{L_r}{C_r}} \sin(A) + (V_0 - V_{in} - nV_{out}) \cos(A) \] (3.34)
where \( A = \frac{t_1}{\sqrt{L_rC_r}} \). Similarly, the tank current cannot change suddenly because of the resonant inductance, so \( i_{\text{tank}}^a(t_1) \) from (3.24) must be equal to \( i_{\text{tank}}^b(t_1) \) from (3.33). That is:

\[
- \frac{nV_{\text{out}}}{4L_mf_{\text{sw}}} = I_0 \cos(A) - (V_0 - V_{\text{in}} - nV_{\text{out}}) \sqrt{\frac{C_r}{L_r}} \sin(A) \tag{3.35}
\]

Isolating \( I_0 \) in (3.35) gives:

\[
I_0 = (V_0 - V_{\text{in}} - nV_{\text{out}}) \sqrt{\frac{C_r}{L_r}} \tan(A) - \frac{nV_{\text{out}}}{4L_mf_{\text{sw}} \sec(A)} \tag{3.36}
\]

Replacing \( I_0 \) in (3.34) gives:

\[
V_1 = V_{\text{in}} + nV_{\text{out}} - \frac{nV_{\text{out}} \sin(2A)}{8L_mf_{\text{sw}}} \sqrt{\frac{L_r}{C_r}} + (V_0 - V_{\text{in}} - nV_{\text{out}}) \sec(A) \tag{3.37}
\]

Another property that can be explored is related to \( v_C(t) \). Because the topology being considered consists of the half-bridge inverter, and since a duty cycle \( D = 0.5 \) is assumed, \( v_C^b(\frac{1}{2f_{\text{sw}}}) = -V_0 + V_{\text{in}} \). This property can be visualized in Fig. 3.2 and through simulation of the circuit. Equating \( v_C^b(\frac{1}{2f_{\text{sw}}}) \) from (3.26) to \( -V_0 + V_{\text{in}} \) gives:

\[
-V_0 + V_{\text{in}} = V_{\text{in}} - nV_{\text{out}} + I_1 \sqrt{\frac{L_r}{C_r}} \sin \left( \frac{\frac{1}{2f_{\text{sw}}} - t_1}{\sqrt{L_rC_r}} \right) + (V_1 - V_{\text{in}} + nV_{\text{out}}) \cos \left( \frac{\frac{1}{2f_{\text{sw}}} - t_1}{\sqrt{L_rC_r}} \right) \tag{3.38}
\]

Replacing \( I_1 \) from (3.29), \( V_1 \) from (3.37), considering that \( A = \frac{t_1}{\sqrt{L_rC_r}} \) and \( B = \frac{1}{4f_{\text{sw}} \sqrt{L_rC_r}} \), and simplifying the resulting equation gives:

\[
V_0 = V_{\text{in}} + nV_{\text{out}} (1 - 2 \cos(A)) + \frac{B}{m} nV_{\text{out}} \sin(2B) + \cos(A)(2nV_{\text{out}} \cos(A) - V_{\text{in}}) \frac{2 \cos(B) \cos(A - B)}{2 \cos(B) \cos(A - B)} \tag{3.39}
\]

Replacing \( V_0 \) from (3.39) into (3.36) gives:
3.1. Time Interval Equations of LLC Resonant Converters

\[ I_0 = -\sqrt{\frac{C_r}{L_r}} \frac{2B}{m} nV_{out} \cos^2(B) + \sin(A)(2nV_{out} \cos(A - 2B) + V_{in})}{2 \cos(B) \cos(A - B)} \] (3.40)

Replacing \( V_0 \) from (3.39) into (3.37) gives:

\[ V_1 = V_{in} - nV_{out} - \frac{V_{in} - 2nV_{out} \cos(A)}{2 \cos(B) \cos(A - B)} - \frac{B}{m} nV_{out} \tan(A - B) \] (3.41)

In order to obtain the relationship between input and output voltage, it is possible to explore a characteristic of the current that flows through the resonant tank \( i_{tank}(t) \): when \( t = \frac{1}{2f_{sw}} \), \( i_{tank}(\frac{1}{2f_{sw}}) = -I_0 \), since the average current of the inductor is zero within a switching cycle. Therefore:

\[ -I_0 = I_1 \cos \left( \frac{1}{2f_{sw}} - t_1 \right) - (V_1 - V_{in} + nV_{out}) \sqrt{\frac{C_r}{L_r}} \sin \left( \frac{1}{2f_{sw}} - t_1 \right) \] (3.42)

Replacing \( I_0 \) from (3.40), \( I_1 \) from (3.29), \( V_1 \) from (3.41), considering that \( A = \frac{t_1}{\sqrt{L_r C_r}} \) and \( B = \frac{1}{4f_{sw} \sqrt{L_r C_r}} \), and simplifying the resulting equation gives:

\[ \frac{V_{out}}{V_{in}} = -\frac{1}{2n} \frac{B}{m} \frac{\sin(A - B)}{\cos(B) + \sin(B)} \] (3.43)

Together, the equations for \( I_1 \) and \( I_{L_m1} \) from (3.29), \( V_1 \) from (3.41), \( I_{L_m0} \) from (3.31), \( I_0 \) from (3.40), \( V_0 \) from (3.39) and the relationship between \( V_{in} \) and \( V_{out} \) from (3.43) result in the system of equations presented in (3.44):
3.1. Time Interval Equations of LLC Resonant Converters

\[
\begin{aligned}
I_{L,m1} &= I_1 = -\frac{nV_{out}}{4L_{m}f_{sw}} \\
V_1 &= V_{in} - nV_{out} - \frac{V_{in} - 2nV_{out} \cos(A)}{2 \cos(B) \cos(A - B)} - \frac{B}{m} nV_{out} \tan(A - B) \\
I_{L,m0} &= \frac{nV_{out}}{4L_{m}f_{sw}} \left( \frac{A}{B} - 1 \right) \\
I_0 &= -\sqrt{\frac{C_r}{L_r} \frac{2B}{m} nV_{out} \cos^2(B) + \sin(A) (2nV_{out} \cos(A - 2B) + V_{in})}{2 \cos(B) \cos(A - B)} \\
V_0 &= V_{in} + nV_{out} (1 - 2 \cos(A)) + \frac{B}{m} nV_{out} \sin(2B) + \cos(A) (2nV_{out} \cos(A) - V_{in})}{2 \cos(B) \cos(A - B)} \\
V_{out} &= \frac{1}{2n} \frac{B}{m} \sin(A - B) \\
V_{in} &= -\frac{1}{2n} \frac{B}{m} \cos(B) + \sin(B)
\end{aligned}
\]

where \( A = \frac{t_1}{\sqrt{L_rC_r}} \) and \( B = \frac{1}{4f_{sw}\sqrt{L_rC_r}} \). If both \( V_{in} \) and \( V_{out} \) are known, \( t_1 \) can be calculated from the equations presented. Otherwise, conservation of energy can be used to determine this last unknown parameter: the input energy, found using the time-domain equations developed, is equal to the output power being delivered to the load, which is a required parameter to determine the unknown variables. This last equation can be determined as follows:

\[
P_{in} = V_{in} f_{sw} \left( \int_0^{t_1} i_{tank}^a(t) dt + \int_{t_1}^{\frac{1}{f_{sw}}} i_{tank}^b(t) dt \right) = P_{load}
\]

(3.45)

It is important to notice that \( P_{load} \) can be expressed in terms of \( I_{out}V_{out} \), which grants greater flexibility in the calculation of the time-domain expressions. Solving (3.45) and replacing the unknown variables from (3.44) results in (3.46):

\[
P_{in} = 4C_r f_{sw} V_{in}^2 \sin(A - B) \sin(\frac{A}{2} - B) \sin(\frac{A}{2}) = P_{load}
\]

(3.46)

where \( A = \frac{t_1}{\sqrt{L_rC_r}} \) and \( B = \frac{1}{4f_{sw}\sqrt{L_rC_r}} \).
3.1. Time Interval Equations of LLC Resonant Converters

3.1.2 Operation Below the Resonant Frequency

From Fig. 3.2 it is possible to observe once again that the operation of the converter can be divided into two main portions. The first one occurs between the moment $v_{in}(t)$ goes from zero to $V_{in}$ until the instant $v_{pr}(t)$ is no longer clamped to $nV_{out}$. This instant, which occurs when the primary of the circuit is disconnected from the secondary, and current stops flowing to the transformer, has been named $t_1$, and is an unknown parameter which needs to be determined from the circuit parameters and operating conditions. The moment when $v_{in}(t)$ goes from zero to $V_{in}$ has been named $t_0 = 0$. The second period starts at $t = t_1$, and lasts until $v_{in}(t)$ goes from $V_{in}$ to zero. As a result, there are two main time intervals that need to be analyzed in the circuit: the first one is when $t_0 < t < t_1$ and the second one is when $t_1 < t < \frac{1}{2f_{sw}}$. Two equivalent circuits can be analyzed for each of the time intervals, as shown in Fig. 3.4.

During the first time interval, it is possible to perform mesh analysis of the resulting circuit shown in Fig. 3.4(a), and the process for obtaining the equations is fairly similar to that developed previously for operation above the resonant frequency. The resulting time-domain expressions for this first time interval are:

$$t_0 < t < t_1$$

$$t_1 < t < \frac{1}{2f_{sw}}$$

Figure 3.4: a) Equivalent circuit for operation of the converter for $f_{sw} < f_{res}$ and $t_0 < t < t_1$. b) Equivalent circuit for $f_{sw} < f_{res}$ and $t_1 < t < \frac{1}{2f_{sw}}$. 
3.1. Time Interval Equations of LLC Resonant Converters

\[
\begin{align*}
\left\{ \begin{array}{l}
  v_C^a(t) = V_{in} - nV_{out} + I_0 \sqrt{\frac{L_r}{C_r}} \sin \left( \frac{t}{\sqrt{L_rC_r}} \right) + (V_0 - V_{in}) \cos \left( \frac{t}{\sqrt{L_rC_r}} \right) \\
  i_{tank}^a(t) = I_0 \cos \left( \frac{t}{\sqrt{L_rC_r}} \right) - (V_0 - V_{in}) \sqrt{\frac{C_r}{L_r}} \sin \left( \frac{t}{\sqrt{L_rC_r}} \right) \\
  i_{Lm}^a(t) = \frac{nV_{out} t}{L_m} + I_{Lm0}
\end{array} \right.
\end{align*}
\] (3.47)

where the superscript \( a \) indicates that these equations are valid during the first operating interval. During the second time interval, there is no current flowing to the secondary of the circuit, so \( i_{tank}(t) = i_{Lm}(t) \), as shown in Fig. 3.4(b). As a result, since the same current flows through both inductors, the voltage drop across them is \( (L_r + L_m) \frac{di_{tank}(t)}{dt} \). By following a similar process as that described previously, it is possible to obtain the time-domain expressions that regulate the behavior of the converter during the second time interval:

\[
\begin{align*}
\left\{ \begin{array}{l}
  v_C^b(t) = V_{in} + I_1 \sqrt{\frac{L_r + L_m}{C_r}} \sin \left( \frac{t - t_1}{\sqrt{(L_r + L_m)C_r}} \right) + (V_1 - V_{in}) \cos \left( \frac{t - t_1}{\sqrt{(L_r + L_m)C_r}} \right) \\
  i_{tank}^b(t) = I_1 \cos \left( \frac{t - t_1}{\sqrt{(L_r + L_m)C_r}} \right) - (V_1 - V_{in}) \sqrt{\frac{C_r}{(L_r + L_m)}} \sin \left( \frac{t - t_1}{\sqrt{(L_r + L_m)C_r}} \right) \\
  i_{Lm}^b(t) = i_{tank}^b(t)
\end{array} \right.
\end{align*}
\] (3.48)

where the superscript \( b \) indicates that these equations are valid for the second time interval.

While these equations are represented as a function of design parameters such as \( L_r, C_r, L_m \) and \( n \), there are also parameters which are unknown, such as \( I_0, I_1, V_0, V_1, I_{Lm0}, I_{Lm1} \) and \( t_1 \). In addition, the value of \( V_{in} \) or \( V_{out} \) may not be known for a certain operating condition. In order to determine the value of the unknown parameters, certain electrical properties of the circuit must be considered.

First, it is known that at \( t = t_1 \) there is no current flowing to the transformer connected to the primary of the circuit, so \( i_{tank}(t_1) = i_{Lm}(t_1) = I_1 = I_{Lm1} \). In addition, unlike the scenario...
for operation above the \( f_{\text{res}} \), there is no current flowing to the secondary of the circuit at \( t = 0 \). As a result, \( i_{\text{tank}}(0) = i_{\text{Lm}}(0) = I_0 = I_{\text{Lm}0} \). This implies that at \( t = t_1 \), \( i_{\text{Lm}}^2(t) \) from (3.47) becomes:

\[
I_0 = I_1 - \frac{nV_{\text{out}}t_1}{L_m} \tag{3.49}
\]

Because the voltage in the series resonant capacitor cannot change instantaneously, \( v_C^a(t_1) \) from (3.47) must be equal to \( v_C^b(t_1) \) from (3.48). That is:

\[
V_1 = V_{\text{in}} - nV_{\text{out}} + I_0 \sqrt{\frac{L_r}{C_r}} \sin \left( \frac{t_1}{\sqrt{L_r C_r}} \right) + (V_0 - V_{\text{in}} + nV_{\text{out}}) \cos \left( \frac{t_1}{\sqrt{L_r C_r}} \right) \tag{3.50}
\]

Replacing \( I_0 \) from (3.49) into (3.50), and assuming that \( A = \frac{t_1}{\sqrt{L_r C_r}} \) gives:

\[
V_1 = V_{\text{in}} - nV_{\text{out}} + \left( I_1 - \frac{nV_{\text{out}}t_1}{L_m} \right) \sqrt{\frac{L_r}{C_r}} \sin(A) + (V_0 - V_{\text{in}} + nV_{\text{out}}) \cos(A) \tag{3.51}
\]

Similarly, the tank current cannot change suddenly because of the resonant inductance, so \( i_{\text{tank}}^a(t_1) \) from (3.47) must be equal to \( i_{\text{tank}}^b(t_1) \) from (3.48). That is:

\[
I_1 = I_0 \cos \left( \frac{t_1}{\sqrt{L_r C_r}} \right) - (V_0 - V_{\text{in}} + nV_{\text{out}}) \sqrt{\frac{C_r}{L_r}} \sin \left( \frac{t_1}{\sqrt{L_r C_r}} \right) \tag{3.52}
\]

Replacing \( I_0 \) from (3.49) into (3.52), and assuming that \( A = \frac{t_1}{\sqrt{L_r C_r}} \) gives:

\[
I_1 = -\frac{nV_{\text{out}}t_1 \cos(A)}{L_m(1 - \cos(A))} - (V_0 - V_{\text{in}} + nV_{\text{out}}) \sqrt{\frac{C_r}{L_r}} \cot \left( \frac{A}{2} \right) \tag{3.53}
\]

Replacing \( I_1 \) from (3.53) into (3.49) gives:

\[
I_0 = -\frac{nV_{\text{out}}t_1}{L_m(1 - \cos(A))} - (V_0 - V_{\text{in}} + nV_{\text{out}}) \sqrt{\frac{C_r}{L_r}} \cot \left( \frac{A}{2} \right) \tag{3.54}
\]

Replacing \( I_1 \) from (3.53) into (3.51) gives:
3.1. Time Interval Equations of LLC Resonant Converters

\[ V_1 = 2V_{in} - 2nV_{out} - V_0 - \sqrt{\frac{L_r}{C_r}} \frac{nV_{out}t_1 \cot \left( \frac{A}{2} \right)}{L_m} \]  (3.55)

Another property that can be explored is related to \( v_C(t) \). Because the topology being considered consists of the half-bridge inverter, and since a duty cycle \( D = 0.5 \) is assumed, \( v_C^b(\frac{1}{2f_{sw}}) = -V_0 + V_{in} \). This property can be visualized in Fig. 3.2 and through simulation of the circuit. Equating \( v_C^b(\frac{1}{2f_{sw}}) \) from (3.48) to \( -V_0 + V_{in} \) gives:

\[ -V_0 = I_1 \sqrt{\frac{L_r + L_m}{C_r}} \sin \left( \frac{\frac{1}{2f_{sw}} - t_1}{\sqrt{(L_r + L_m)C_r}} \right) + (V_1 - V_{in}) \cos \left( \frac{\frac{1}{2f_{sw}} - t_1}{\sqrt{(L_r + L_m)C_r}} \right) \]  (3.56)

Replacing \( I_1 \) from (3.53), \( V_1 \) from (3.55), considering that \( A = \frac{t_1}{\sqrt{L_rC_r}} \) and \( T = \frac{\frac{1}{2f_{sw}} - t_1}{\sqrt{(L_r + L_m)C_r}} \), and simplifying the resulting equation gives:

\[ V_0 = V_{in} - nV_{out} + \frac{\frac{A}{m} nV_{out}(\cos(T) \sin(A) - \sqrt{m + \frac{1}{4}} \sin(T) \cos(A))}{(\cos(A) - 1)(\cos(T) - 1) - \sqrt{m + \frac{1}{4}} \sin(A) \sin(T)} \]

\[ + \frac{(V_{in} - nV_{out}(\cos(T) + 1))(\cos(A) - 1)}{(\cos(A) - 1)(\cos(T) - 1) - \sqrt{m + \frac{1}{4}} \sin(A) \sin(T)} \]  (3.57)

Replacing \( V_0 \) from (3.57) into (3.53) gives:

\[ I_1 = \sqrt{\frac{C_r}{L_r}} \frac{\frac{A}{m} nV_{out}(\cos(T) + \cos(A)) + \sin(A)(nV_{out}(1 + \cos(T)) - V_{in})}{\sqrt{m + \frac{1}{4}} \sin(T) \sin(A) - (\cos(T) - 1)(\cos(A) - 1)} \]  (3.58)

Replacing \( V_0 \) from (3.57) into (3.54) gives:

\[ I_0 = \sqrt{\frac{C_r}{L_r}} \frac{\frac{A}{m} nV_{out}(\cos(T) \cos(A) - \sqrt{m + \frac{1}{4}} \sin(T) \sin(A) + 1) + \sin(A)(nV_{out}(1 + \cos(T)) - V_{in})}{\sqrt{m + \frac{1}{4}} \sin(T) \sin(A) - (\cos(T) - 1)(\cos(A) - 1)} \]  (3.59)

Replacing \( V_0 \) from (3.57) into (3.55) gives:
3.1. Time Interval Equations of LLC Resonant Converters

\[ V_1 = V_{in} - nV_{out} + \frac{A}{m} nV_{out}(\sin(A) - \sqrt{m + 1}\sin(T)) + (V_{in} - nV_{out}(\cos(T) + 1))(\cos(A) - 1) \]
\[ \frac{\sqrt{m + 1}\sin(A)}{\sqrt{m + 1}\sin(A) - (\cos(T) - 1)(\cos(A) - 1)} \]

(3.60)

In order to obtain the relationship between input and output voltage, it is possible to explore a characteristic of the current that flows through the resonant tank \(i_{tank}(t)\): when \(t = \frac{1}{2f_{sw}}\), \(i_{tank}^b(\frac{1}{2f_{sw}}) = -I_0\), since the average current of the inductor is zero within a switching cycle. Therefore:

\[-I_0 = I_1 \cos \left( \frac{1}{2f_{sw}} - t_1 \sqrt{L_r + L_m} \right) - (V_1 - V_{in}) \sqrt{\frac{C_r}{L_r + L_m}} \sin \left( \frac{1}{2f_{sw}} - t_1 \sqrt{L_r + L_m} \right) \]

(3.61)

Replacing \(I_0\) from (3.59), \(I_1\) from (3.58), \(V_1\) from (3.60), considering that \(A = \frac{t_1}{\sqrt{L_r C_r}}\) and \(T = \frac{1}{2f_{sw}} - t_1 \sqrt{L_r + L_m} C_r\), and simplifying the resulting equation gives:

\[ \frac{V_{out}}{V_{in}} = \frac{2m}{n} + \cot \left( \frac{A}{2} \right) - \sqrt{m + 1} \tan \left( \frac{T}{2} \right) \]

(3.62)

Together, the equations for \(I_1\) and \(I_{Lm1}\) from (3.58), \(V_1\) from (3.60), \(I_0\) and \(I_{Lm0}\) from (3.59), \(V_0\) from (3.57) and the relationship between \(V_{in}\) and \(V_{out}\) from (3.62) result in a system of equations which define the unknown parameters for operation of the converter below \(f_{res}\):
3.1. Time Interval Equations of LLC Resonant Converters

\[
\begin{align*}
I_{L_{m1}} &= I_1 \\
I_{L_{m0}} &= I_0 \\
I_1 &= \sqrt{\frac{C_r}{L_r}} \frac{\Delta n V_{out} \cos(T) + \cos(A)}{\sqrt{m + 1} \sin(T) \sin(A) - (\cos(T) - 1)(\cos(A) - 1)} \\
V_1 &= V_{in} - n V_{out} + \frac{\Delta n V_{out} \sin(A) - \sqrt{m + 1} \sin(T)}{\sqrt{m + 1} \sin(T) \sin(A) - (\cos(T) - 1)(\cos(A) - 1)} \\
I_0 &= \sqrt{\frac{C_r}{L_r}} \frac{\Delta n V_{out} \cos(T) \cos(A) - \sqrt{m + 1} \sin(T) \sin(A) + 1 + \sin(A)(n V_{out} (1 + \cos(T)) - V_{in})}{\sqrt{m + 1} \sin(T) \sin(A) - (\cos(T) - 1)(\cos(A) - 1)} \\
V_0 &= V_{in} - n V_{out} + \frac{\Delta n V_{out} \cos(T) \sin(A) + \sqrt{m + 1} \sin(T) \cos(A)}{(\cos(A) - 1)(\cos(T) - 1) - \sqrt{m + 1} \sin(A) \sin(T)} \\
V_{out} &= V_{in} + \frac{m}{A} + \cot\left(\frac{A}{2}\right) - \sqrt{m + 1} \tan\left(\frac{T}{2}\right)
\end{align*}
\]

(3.63)

where \( A = \frac{t_1}{L_r C_r} \) and \( T = \frac{1}{L_r C_r (L_r + L_m C_r)} \). If both \( V_{in} \) and \( V_{out} \) are known, \( t_1 \) can be calculated from the equations presented. Otherwise, conservation of energy can be used to determine this last unknown parameter: the input energy, found using the time-domain equations developed, is equal to the output power being delivered to the load, which is a required parameter to determine the unknown variables. This last equation can be determined as follows:

\[
P_{in} = V_{in} f_{sw} \left( \int_0^{t_1} i_{tank}^a(t) dt + \int_{t_1}^{1/f_{sw}} i_{tank}^b(t) dt \right) = P_{load}
\]

(3.64)

It is important to notice that \( P_{load} \) can be expressed in terms of \( I_{out} V_{out} \), which grants greater flexibility in the calculation of the time-domain expressions. Solving (3.64) and replacing the unknown variables from (3.63) results in the following expression:
3.1. Time Interval Equations of LLC Resonant Converters

\[ P_{in} = C_r f_{sw} V_{in}^2 \left( \sqrt{m + 1} - \tan \left( \frac{T}{2} \right) \tan \left( \frac{A}{2} \right) \right) \left( \frac{A}{2} \cot \left( \frac{A}{2} \right) - 1 \right) \]

\[
\left( \frac{A}{m} \frac{\cos(A) + \cos(T) - 2 \cos(A) \cos(T) - 2}{\sin(T)(\cos(A) - 1)} - \sqrt{m + 1} \frac{\cos(A) + \cos(T) + 2 \cos(A) \cos(T) + 2}{\sin(A)(\cos(T) + 1)} \right) \\
+ \frac{1 + \frac{A}{2} \cot \left( \frac{A}{2} \right) + m}{\tan \left( \frac{A}{2} \right) \tan \left( \frac{A}{2m + 1} \right)} + \tan \left( \frac{T}{2} \right) \left( \tan \left( \frac{A}{2} \right) + A \right) - \sqrt{m + 1} \left( 2 \frac{A}{2} \cot \left( \frac{A}{2} \right) \right) \right)^{-1} = P_{load}
\]

(3.65)

where \( A = \frac{t_1}{\sqrt{L_r C_r}} \) and \( T = \frac{\frac{1}{2f_{sw}} - t_1}{\sqrt{(L_r + L_m)C_r}} \). Since the equation presented in (3.65) is rather bulky and difficult to read in its present form, it can be split into different parts, as follows:

\[ P_{in} = C_r f_{sw} V_{in}^2 \frac{\alpha}{\gamma + \beta} = P_{load} \]

(3.66)

where:

\[
\begin{align*}
\alpha &= \left( \sqrt{m + 1} - \tan \left( \frac{T}{2} \right) \tan \left( \frac{A}{2} \right) \right) \left( \frac{A}{2} \cot \left( \frac{A}{2} \right) - 1 \right) \\
\beta &= \frac{A}{m} \frac{\cos(A) + \cos(T) - 2 \cos(A) \cos(T) - 2}{\sin(T)(\cos(A) - 1)} - \sqrt{m + 1} \frac{\cos(A) + \cos(T) + 2 \cos(A) \cos(T) + 2}{\sin(A)(\cos(T) + 1)} \\
\gamma &= \frac{1 + \frac{A}{2} \cot \left( \frac{A}{2} \right) + m}{\tan \left( \frac{A}{2} \right) \tan \left( \frac{A}{2m + 1} \right)} + \tan \left( \frac{T}{2} \right) \left( \tan \left( \frac{A}{2} \right) + A \right) - \sqrt{m + 1} \left( 2 \frac{A}{2} \cot \left( \frac{A}{2} \right) \right)
\end{align*}
\]

(3.67)

where \( A = \frac{t_1}{\sqrt{L_r C_r}} \) and \( T = \frac{\frac{1}{2f_{sw}} - t_1}{\sqrt{(L_r + L_m)C_r}} \).

3.1.3 Operation at the Resonant Frequency

It is possible to observe from Fig. 3.2 that the operation of the converter at resonance is comprised of a single period of time. This period goes from the moment \( v_{in}(t) \) goes from zero to \( V_{in} \) until the instant \( v_{in}(t) \) goes from \( V_{in} \) to zero. As a result, there is a single period to be analyzed in the circuit: \( t_0 < t < \frac{1}{2f_{sw}} \), and one single equivalent circuit, shown in Fig. 3.5.

Under this specific operating condition, \( f_{sw} = f_{res} = \frac{1}{2\pi\sqrt{L_r C_r}} \), so the time interval to be considered is \( t_0 < t < \pi\sqrt{L_r C_r} \).
3.1. Time Interval Equations of LLC Resonant Converters

\[ t_0 < t < \frac{1}{2f_{sw}} \]

Figure 3.5: Equivalent circuit for operation of the converter at the resonant frequency for \( t_0 < t < \frac{1}{2f_{sw}} \).

Similarly to the previous scenarios, the time-domain expressions that govern the behavior of the topology are presented in (3.68). The derivation process for these equations is similar to that presented previously, and when \( t = 0 \), \( v_C(0) = V_0 \), \( i_{tank}(0) = I_0 \) and \( i_{Lm}(0) = I_{Lm0} \).

\[
\begin{align*}
\left\{ 
&v_C(t) = V_{in} - nV_{out} + I_0 \sqrt{\frac{L_r}{C_r}} \sin \left( \frac{t}{\sqrt{L_r C_r}} \right) + (V_0 - V_{in} + nV_{out}) \cos \left( \frac{t}{\sqrt{L_r C_r}} \right) \\
&i_{tank}(t) = I_0 \cos \left( \frac{t}{\sqrt{L_r C_r}} \right) - (V_0 - V_{in} + nV_{out}) \sqrt{\frac{C_r}{L_r}} \sin \left( \frac{t}{\sqrt{L_r C_r}} \right) \\
&i_{Lm}(t) = \frac{nV_{out} t}{L_m} + I_{Lm0}
\end{align*}
\] (3.68)

Under this specific operating condition, there is no current flowing to the secondary of the circuit when \( t = 0 \), so \( i_{tank}(0) = i_{Lm}(0) = I_0 = I_{Lm0} \). In addition, it is known that the same happens when \( t = \pi \sqrt{L_r C_r} \), and as this instant \( i_{Lm}(\pi \sqrt{L_r C_r}) = -I_{Lm0} \), since \( i_{Lm}(t) \) is symmetrical with an average value of zero. As a result:

\[
I_0 = -\frac{nV_{out} \pi \sqrt{L_r C_r}}{2L_m}
\] (3.69)

Similarly to the previous cases, the topology being considered consists of the half-bridge inverter, and since a duty cycle \( D = 0.5 \) is assumed, \( v_C(\pi \sqrt{L_r C_r}) = -V_0 + V_{in} \). Thus, equating \( v_C(\pi \sqrt{L_r C_r}) \) from (3.68) to \( -V_0 + V_{in} \) gives:
\[ \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{2n} \]  \hspace{1cm} (3.70)

In order to determine \( V_0 \), conservation of energy needs to be considered. That is:

\[ P_{\text{in}} = V_{\text{in}} f_{sw}\int_{0}^{1/f_{sw}} i_{\text{tank}}(t) = P_{\text{load}} \]  \hspace{1cm} (3.71)

Developing (3.71), and knowing that \( f_{sw} = f_{\text{res}} = \frac{1}{2\pi\sqrt{L_r C_r}} \) gives:

\[ V_0 = \frac{V_{\text{in}}}{2} - \pi \sqrt{\frac{L_r P_{\text{load}}}{C_r V_{\text{in}}}} \]  \hspace{1cm} (3.72)

As a result, (3.69), (3.70) and (3.72) define the unknown parameters of the circuit when operating at \( f_{\text{res}} \):

\[
\begin{align*}
I_{L_{\text{m0}}} & = I_0 = -\frac{n V_{\text{out}} \pi \sqrt{L_r C_r}}{2L_m} \\
V_0 & = \frac{V_{\text{in}}}{2} - \pi \sqrt{\frac{L_r P_{\text{load}}}{C_r V_{\text{in}}}} \\
\frac{V_{\text{out}}}{V_{\text{in}}} & = \frac{1}{2n}
\end{align*}
\]  \hspace{1cm} (3.73)

### 3.1.4 Experimental Validation of the Obtained Waveforms

In order to verify that the developed equations represent the actual behaviour observed by the circuit, especially when it comes to the current waveforms that generate \( P_{\text{loss}} \), it is possible to operate an LLC converter under the three different scenarios considered: below, at and above the resonant frequency. The topology was built considering the parameters shown in Table 3.1.

**Operation Above the Resonant Frequency**

Selected experimental waveforms for this operating condition are shown in Fig. 3.6, as well as the calculated waveforms using TIA and FHA. When operating above \( f_{\text{res}} \), the \( v_{pr}(t) \) is
3.1. Time Interval Equations of LLC Resonant Converters

Table 3.1: LLC design parameters for $P_{loss}$ analysis in inverter MOSFETs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Input Voltage $V_{in}$</th>
<th>Nominal Power $P_{nom}$</th>
<th>Nominal Load Current $I_{nom}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>100 V</td>
<td>500 W</td>
<td>10 A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Magnetizing Inductance $L_m$</th>
<th>Resonant Inductance $L_r$</th>
<th>Resonant Capacitance $C_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>19.7 $\mu$H</td>
<td>15.2 $\mu$H</td>
<td>192 nF</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Resonant Frequency $f_{res}$</th>
<th>Switching Frequency $f_{sw}$</th>
<th>Transformer Turns Ratio $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>93 kHz</td>
<td>80-120 kHz</td>
<td>1</td>
</tr>
</tbody>
</table>

The resulting current $i_{tank}(t)$ is also negative at this instant, so the MOSFET is turned on under ZVS conditions. While the circulating current is seemingly sinusoidal, it is possible to observe that this approximation does not truly represent the behaviour of the circuit. More importantly, it is possible to note that the turn-on and turn-off currents shown in $i_{D_1}(t)$ are not appropriately determined if a

Figure 3.6: Experimental waveforms for operation of the LLC resonant converter above the $f_{res}$ show good correlation with calculated TIA waveforms.
3.1. Time Interval Equations of LLC Resonant Converters

A sinusoidal waveform is considered, which will underestimate $P_{sw}$ of the topology. In addition, $P_{cond}$ determination using FHA is likely to be underestimated due to the simplifications imposed by the method.

Comparing the obtained experimental waveforms with the calculated ones, it is clear that not only the shape but also the presented magnitudes are closely related to those obtained using TIA. FHA, on the other hand, fails to capture the peculiarities of the waveforms, especially for the case of $i_{D1}(t)$, which is fundamental for the determination of $P_{loss}$ of the inverting MOSFETs.

Operation Below the Resonant Frequency

The experimental waveforms for operation of the converter below $f_{res}$ are shown in Fig. 3.7, as well as the calculated waveforms using TIA and FHA. When operating below $f_{res}$, the primary of the circuit becomes disconnected from the secondary during part of the switching cycle, as can be observed in the waveforms of $v_{pr}(t)$. This effect is not observed when using

Figure 3.7: Experimental waveforms for operation of the LLC resonant converter below the $f_{res}$ show good correlation with calculated TIA waveforms.
3.1. Time Interval Equations of LLC Resonant Converters

FHA, which results in inaccurate waveforms for the topology. Similarly to what is observed for operation above $f_{res}$, during turn-on the $i_{D1}(t)$ is negative, so the MOSFET is turned on under ZVS conditions.

Under this operating condition, the $i_{tank}(t)$ does not closely resemble a sinusoidal waveform, thus FHA cannot adequately trace the expected waveform. As a result, the tracing using a sinusoidal approximation results in underestimated currents, which will result in reduced $P_{cond}$ and $P_{sw}$ estimations.

When comparing the obtained experimental waveforms with the calculated ones, it is possible to observe that both shape and magnitudes are closely correlated to those calculated using TIA. As expected, using FHA does not represent adequately the behaviour of the circuit, and $P_{loss}$ estimation using this method should be compromised.

Operation At the Resonant Frequency

Experimental and calculated waveforms for operation of the converter when $f_{sw} = f_{res}$ can be observed in Fig. 3.8. Under this specific operating condition, the shapes of the waveforms for $v_{in}(t)$ and $v_{pr}(t)$ are aligned, and the current waveforms more closely correlate to a sinusoidal waveform. Once again, at turn-on the $i_{D1}(t)$ is negative, which results in soft-switching of the inverter MOSFET. As observed in Fig. 3.8, while the shape of the waveform is similar for TIA and FHA, the magnitude of the waveform as determined by FHA does not correspond to the experimental measurements as closely as that calculated using TIA. As a result, the calculation of $P_{loss}$ is compromised.
3.2 Power Loss Estimation of LLC MOSFETs in the Inverting Stage

Using the equations developed in Section 3.1, it is possible to calculate $P_{loss}$ in the inverter MOSFETs of LLC resonant converters. These equations account for the peculiarities observed in the waveforms shown in Fig. 3.2, and do not result on purely sinusoidal waveforms as would be obtained from FHA. As a result, $P_{loss}$ assessment using TIA provides increased precision with measurement and simulation results when compared to the oversimplified FHA.

The MOSFETs in the inverting stage of the LLC resonant converter are driven in complementary mode. While certain control and regulation techniques employ varying $D$, often operation of this converter is realized with $D = 0.5$. The current that flows through these MOSFETs is derived from $i_{tank}(t)$, as can be observed in Fig. 3.9.
3.2. Power Loss Estimation of LLC MOSFETs in the Inverting Stage

Figure 3.9: Waveforms for $I_D$ which flows through the inverter MOSFETs for operation a) Above the resonant frequency, b) Below the resonant frequency and c) At the resonant frequency.

For operation above and below $f_{res}$, $i_{D1}(t)$ is determined as:

$$
i_{D1}(t) = \begin{cases} i^a_{tank}(t), & \text{for } 0 < t < t_1 \\ i^b_{tank}(t), & \text{for } t_1 < t < \frac{1}{2f_{sw}} \\ 0, & \text{for } \frac{1}{2f_{sw}} < t < \frac{1}{f_{sw}} \end{cases}$$ (3.74)

where $i^a_{tank}(t)$ and $i^b_{tank}(t)$ are defined in (3.24) and in (3.26) for operation above $f_{res}$, and in (3.47) and (3.48) for operation below $f_{res}$. For operation at the resonant frequency, $i_{D1}(t)$ is determined as:

$$
i_{D}(t) = \begin{cases} i^a_{tank}(t), & \text{for } 0 < t < \frac{1}{2f_{sw}} \\ 0, & \text{for } \frac{1}{2f_{sw}} < t < \frac{1}{f_{sw}} \end{cases}$$ (3.75)

where $i^a_{tank}(t)$ is defined in (3.68). Once the equations for $i_{D1}(t)$ have been determined, it is possible to calculate the $P_{cond}$ of the device through integration:

$$P_{cond} = f_{sw} \int_{0}^{\frac{1}{2f_{sw}}} i^2_{D}(t)R_{DS(on)}dt$$ (3.76)

where $R_{DS(on)}$ can be determined from a detailed characterization, such as that presented in Chapter 2, or with information from the datasheet of the device. Because of the dependency
of this parameter with $T_j$, calculation of $P_{\text{loss}}$ and subsequent update of $T_j$ must be performed, and an iterative process must be employed until the $T_j$ value converges.

In this topology, $P_{\text{sw}}$ are minimized due to the soft-switching of the devices as long as the converter operates outside of the capacitive region. This grants soft-switching of the inverter MOSFETs during turn-on, as can be seen in Fig. 3.9. In addition, during turn-off the current value is reduced, especially when operating close to $f_{\text{res}}$. Nevertheless, $P_{\text{sw}}$ should not be neglected, since it can have a considerable contribution in $P_{\text{loss}}$, especially when the converter operates away from $f_{\text{res}}$ and under light-loading conditions, as Fig. 3.10 shows.

In order to calculate $P_{\text{sw}}$, it is necessary to determine the value of $I_{D_1}$ when the turn-off action occurs. This can be done with ease using (3.74) and (3.75), since $I_{\text{off}} = i_D \left( \frac{1}{2f_{\text{sw}}} \right)$. Because $V_{DS} = V_{\text{in}}$ in LLC resonant converters, it is possible to determine the turn-off energy by using information either from a detailed characterization, such as the one performed in Chapter 2, or by considering approaches which rely on the datasheet of the device, such as [84], which is based on gate charging. While this loss mechanism does not vary considerably with changes in $T_j$, it is still necessary to determine this parameter through an iterative process.

The $P_{\text{diode}}$ which occurs due to the conduction losses of the body diode during dead time

![Figure 3.10: Power loss assessment for the LLC inverting MOSFETs under different operating conditions showing greater contribution of $P_{\text{cond}}$ to total $P_{\text{loss}}$.](image)
3.2. Power Loss Estimation of LLC MOSFETs in the Inverting Stage

can also be determined, based on the modeling of $V_{\text{diode}}$ which may come either from a detailed characterization such as the one presented in Chapter 2, or from datasheet information:

$$P_{\text{diode}} = f_{sw} \int_{\frac{1}{2}f_{sw}-t_{\sigma}}^{\frac{1}{2}f_{sw}} i_{D1}(t) V_{\text{diode}} dt \quad (3.77)$$

where $t_{\sigma}$ is the dead time being considered for operation of the topology. Once again, since $V_{\text{diode}}$ varies with $T_j$, an iteration process must be considered. By accounting all the loss mechanisms presented, it is possible to determine the total $P_{\text{loss}}$ of the switching device:

$$P_{\text{loss}} = P_{\text{cond}} + P_{\text{sw}} + P_{\text{diode}} \quad (3.78)$$

3.2.1 Comparison with Simulation Results

In order to verify the accuracy of the developed method, the calculated $P_{\text{loss}}$ can be compared with simulation results. In addition, calculation of $P_{\text{loss}}$ can be obtained by using waveforms from FHA, and also compared with simulation results. PLECS by Plexim was employed as the simulation software due to its ability to integrate electrical and thermal behaviours with ease. The calculation using Time Interval Analysis was implemented in MS Excel using VBA, and allowed for calculation 50 times faster than the implementation using simulation software. Figure 3.11 shows the obtained $P_{\text{loss}}$ estimations using TIA and FHA, as well as the error observed between calculation and simulation results.

For the considered operating conditions shown in Fig. 3.11, an average error of 0.22% for $P_{\text{loss}}$ estimation was obtained when using TIA, with more than 97% of the calculations presenting an error smaller than 2%. The well-established yet oversimplified FHA, on the other hand, presented an average error of 10.7%, with errors of up to 60% for $P_{\text{loss}}$ estimation.
3.2. Power Loss Estimation of LLC MOSFETs in the Inverting Stage

3.2.2 Comparison with Experimental Results

In order to validate the simulated and calculated $P_{\text{loss}}$ obtained for the inverting MOSFETs, it is necessary to determine this parameter experimentally. However, as mentioned in Chapter 2, electrical measurements introduce disturbances to the behaviour of electrical circuits, and at high frequencies EMI/RFI pose challenges to the measurement of electrical quantities. As a result, the determination of $P_{\text{loss}}$ using electrical probes is not adequate.

As an alternative, the temperature rise due to the heat dissipated caused by $P_{\text{loss}}$ can be measured directly using a thermal camera, in a process that can be easily implemented and does not affect the electrical behaviour of the topology. This process, which is illustrated in Fig. 3.12, consists of two stages: calibration and measurement, and is identical to that employed for the characterization of $P_{\text{sw}}$ presented in Chapter 2.

The first step consists on determining the relationship between measured temperatures and $P_{\text{loss}}$. Based on Fig. 2.6(c), the relationship between $T_{\text{case}} - T_{\text{amb}}$ and $P_{\text{loss}}$ is a constant value $R_{\text{th,ca}}$. In order to determine this magnitude, a constant DC current can be applied to the MOSFET, which is kept turned on with $D = 1$. By using precision instruments, $P_{\text{loss}}$ can be calculated by multiplying $V_{DS}$ and $I_D$. The temperature difference can then be
3.2. Power Loss Estimation of LLC MOSFETs in the Inverting Stage

Calibration Stage

![Diagram of calibration stage]

Measurement Stage

![Diagram of measurement stage]

Figure 3.12: Measurement of $P_{\text{loss}}$ using calorimetry: first, a relationship between $P_{\text{loss}}$ and operating temperatures is obtained, which allows for the subsequent determination of $P_{\text{loss}}$ based on temperature readings.

determined by using the readings from the thermal camera. Subsequent changes in the value of the applied $I_D$ allow for the tracing of the linear relationship, as shown in Fig. 3.12.

Once this relationship is obtained, the LLC MOSFET can be operated as intended, with high $f_{\text{sw}}$ and $D = 0.5$. By reading the $T_{\text{case}} - T_{\text{amb}}$ using the thermal camera, it is then possible to assess the value of $P_{\text{loss}}$ which corresponds to the observed temperature rise. This allows for the determination of $P_{\text{loss}}$ without the need of interfering with the operating topology at high switching frequencies. In addition, it provides information regarding $P_{\text{loss}}$ of a single device instead of the whole circuit, as would be obtained by comparing input and output power.

Select measurements of $P_{\text{loss}}$ are shown in Fig. 3.13 when the LLC resonant converter op-
3.2. Power Loss Estimation of LLC MOSFETs in the Inverting Stage

Figure 3.13: Measured Power Losses in LLC inverting MOSFETs for a constant $P_{\text{load}} = 0.7P_{\text{nom}}$ for operation a) Below and at the resonant frequency and b) Above the resonant frequency.

erates under different $f_{\text{sw}}$ and at constant $P_{\text{load}}$. In addition, $P_{\text{loss}}$ estimations using different methods are provided. Both green and orange bars represent calculations using TIA, while red bars show estimated losses using FHA. The detailed characterization of $R_{DS(on)}$, $E_{\text{off}}$, and $V_{\text{diode}}$ developed in Chapter 2 for the calculation of $P_{\text{cond}}$, $P_{\text{sw}}$ and $P_{\text{diode}}$ are employed for the estimations represented by the green bars. On the other hand, the modelling provided by the datasheet for $R_{DS(on)}$ is used in both scenarios represented by the orange and red bars. Moreover, the contribution of $P_{\text{cond}}$, $P_{\text{sw}}$ and $P_{\text{diode}}$ is indicated by different colour tones.

In these measurements, it is possible to observe that employing TIA for the determination of $P_{\text{loss}}$ yields in results which are closely related to the measured values, with an error of no more than $7\%$. In addition, for this presented scenario, it is possible to notice that the maximum error is reduced from $7\%$ to $5\%$ when the detailed characterization of the device is employed. On the other hand, using FHA results in increased error for $P_{\text{loss}}$ estimation,
3.2. Power Loss Estimation of LLC MOSFETs in the Inverting Stage

resulting in an accuracy of up to 35%. Thus, TIA is presented as a useful tool for inverting MOSFETs loss estimation, either using a detailed characterization or datasheet modelling of MOSFET parameters for $P_{\text{loss}}$ calculation.

Figure 3.14 shows additional measurements obtained at different $f_{\text{sw}}$ for slightly higher values of $P_{\text{load}}$ when compared with those presented in Fig. 3.13. In this case, operation of the converter at $f_{\text{sw}} = 1.1f_{\text{res}}$ was not considered since the resulting $P_{\text{loss}}$ was substantial for the thermal management employed during the measurements. It is interesting to observe that the results obtained in Fig. 3.13 and Fig. 3.14 are highly consistent, with similar errors being observed by using the different modelling approaches.

Measurements considering different $I_{\text{load}}$ under a constant $f_{\text{sw}}$ are represented in Fig. 3.15. Similarly to the scenarios reported previously, TIA provides a better estimation of $P_{\text{loss}}$ than FHA, especially when the detailed characterization of the switching device is employed. The error observed using FHA is reduced in this scenario since these measurements were taken...
3.2. Power Loss Estimation of LLC MOSFETs in the Inverting Stage

Figure 3.15: Measured Power Losses in LLC inverting MOSFETs for a constant $f_{sw}$ under different $I_{load}$ considering a) Lighter loading conditions and b) Heavier loading conditions when the circuit operated at $f_{res}$, which is when the current waveforms more closely resemble a sinusoidal signal, as can be observed in Fig. 3.8. It is interesting to observe that in this situation that the detailed characterization developed in Chapter 2 significantly contributes to improving the accuracy with which $P_{loss}$ is assessed.

Measurements considering the same $I_{load}$ under different extremes of $f_{sw}$ are represented in Fig. 3.16. In these measurements, it is possible to observe that FHA provides a considerably less accurate estimation of $P_{loss}$ especially when the converter operates at low $f_{sw}$. This is due to the poor estimation of $I_{D1}(t)$ using this method, which can be clearly observed in Fig. 3.7. For operation at high $f_{sw}$, similar levels of accuracy are observed between the methods that employ the datasheet characterization of the device, while the detailed characterization performed yields greater accuracy for $P_{loss}$ estimation.

In all considered scenarios, it is possible to observe that the $P_{loss}$ using FHA is under-
3.2. Power Loss Estimation of LLC MOSFETs in the Inverting Stage

Estimated when compared with the experimental measurements. This becomes clear when a comparison between the obtained $P_{\text{loss}}$ using the different methods is normalized with respect to the measured $P_{\text{loss}}$, as shown in Fig. 3.17, where the testing conditions for each case are available in Table 3.2. The underestimation of $P_{\text{loss}}$ using FHA is in accordance with what was observed in the waveforms presented in Fig. 3.6, Fig. 3.7 and Fig. 3.8, and can be especially challenging for the design of a thermal management approach for the device, since underestimation of losses may result in overheating of the switches. In addition, it is possible to observe that $P_{\text{cond}}$ are dominant over $P_{\text{sw}}$ and $P_{\text{diode}}$ due to the soft-switching of the MOSFETs. In fact, considering the presented measurements, $P_{\text{cond}}$ accounts for 86.8% of the total $P_{\text{loss}}$, while $P_{\text{sw}}$ and $P_{\text{diode}}$ are responsible for 8.5 and 4.7% of the total $P_{\text{loss}}$, respectively.

Figure 3.16: Measured Power Losses in LLC inverting MOSFETs under the same $I_{\text{load}}$ for different values of $f_{\text{sw}}$
### 3.2. Power Loss Estimation of LLC MOSFETs in the Inverting Stage

![Figure 3.17](image_url): $P_{\text{loss}}/P_{\text{loss(measured)}}$ in inverting MOSFETs under different conditions

Table 3.2: Testing conditions for LLC inverting MOSFETs $P_{\text{loss}}$ analysis

<table>
<thead>
<tr>
<th>Testing condition</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{sw}}$</td>
<td>$0.9f_{\text{res}}$</td>
<td>$0.95f_{\text{res}}$</td>
<td>$f_{\text{res}}$</td>
<td>$1.05f_{\text{res}}$</td>
</tr>
<tr>
<td>Load</td>
<td>$0.7P_{\text{nom}}$</td>
<td>$0.7P_{\text{nom}}$</td>
<td>$0.7P_{\text{nom}}$</td>
<td>$0.7P_{\text{nom}}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Testing condition</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{sw}}$</td>
<td>$1.1f_{\text{res}}$</td>
<td>$0.9f_{\text{res}}$</td>
<td>$0.95f_{\text{res}}$</td>
<td>$f_{\text{res}}$</td>
</tr>
<tr>
<td>Load</td>
<td>$0.7P_{\text{nom}}$</td>
<td>$0.76P_{\text{nom}}$</td>
<td>$0.76P_{\text{nom}}$</td>
<td>$0.76P_{\text{nom}}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Testing condition</th>
<th>i</th>
<th>j</th>
<th>k</th>
<th>l</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{sw}}$</td>
<td>$1.05f_{\text{res}}$</td>
<td>$0.9f_{\text{res}}$</td>
<td>$1.2f_{\text{res}}$</td>
<td>$f_{\text{res}}$</td>
</tr>
<tr>
<td>Load</td>
<td>$0.76P_{\text{nom}}$</td>
<td>$0.5I_{\text{nom}}$</td>
<td>$0.5I_{\text{nom}}$</td>
<td>$0.6I_{\text{nom}}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Testing condition</th>
<th>m</th>
<th>n</th>
<th>o</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{sw}}$</td>
<td>$f_{\text{res}}$</td>
<td>$f_{\text{res}}$</td>
<td>$f_{\text{res}}$</td>
<td>$f_{\text{res}}$</td>
</tr>
<tr>
<td>Load</td>
<td>$0.7I_{\text{nom}}$</td>
<td>$0.8I_{\text{nom}}$</td>
<td>$0.9I_{\text{nom}}$</td>
<td>$I_{\text{nom}}$</td>
</tr>
</tbody>
</table>
3.3 Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

In order to improve the efficiency of LLC resonant converters, the rectifying stage of the topology can be implemented with active switches instead of diodes through a process called synchronous rectification (SR). The SR current \( i_{SR}(t) \) which generates \( P_{loss} \) in SR MOSFETs consists on the rectified current that flows through the secondary of the transformer of the topology during half of the switching period. Similarly to \( i_D(t) \) in the case of the inverter MOSFETs, the \( i_{SR}(t) \) equations can be determined based on the relationship between \( f_{sw} \) and \( f_{res} \).

3.3.1 Rectifier Current Equations

In order to estimate \( P_{loss} \) of the SR MOSFETs, it is necessary to determine \( i_{SR}(t) \). This can be performed by using the equations developed in Section 3.1, which then allows for the Rectifier Current Equations (RCE) to be calculated. For operation above \( f_{res} \), \( i_{SR}(t) \) is determined as:

\[
\begin{align*}
    i_{SR}(t) &= n \left( i^b_{tank}(t + t_\phi) - i^b_{Lm}(t + t_\phi) \right), & \text{for } 0 < t < t_\phi \\
    i_{SR}(t) &= n \left( -i^a_{tank}(t - t_\phi) + i^a_{Lm}(t - t_\phi) \right), & \text{for } t_\phi < t < \frac{1}{2f_{sw}} \\
    i_{SR}(t) &= 0, & \text{for } \frac{1}{2f_{sw}} < t < \frac{1}{f_{sw}}
\end{align*}
\]  

(3.79)

where \( i^a_{tank}(t) \) and \( i^a_{Lm}(t) \) are defined in (3.24), \( i^b_{tank}(t) \) and \( i^b_{Lm}(t) \) are defined in (3.26), and \( t_\phi = \frac{1}{2f_{sw}} - t_1 \). The graphical representation of the \( i_{SR}(t) \) waveforms can be observed in Fig. 3.18. For operation below \( f_{res} \), \( i_{SR}(t) \) is determined as:
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

Figure 3.18: Theoretical waveforms for $i_{SR}(t)$ for operation a) Above the resonant frequency, b) Below the resonant frequency and c) At the resonant frequency.

\[
\begin{align*}
& i_{SR}(t) = n \left( i^a_{tank}(t) - i^a_{L_m}(t) \right), \quad \text{for } 0 < t < t_1 \\
& i_{SR}(t) = 0, \quad \text{for } t_1 < t < \frac{1}{f_{sw}}
\end{align*}
\] (3.80)

where $i^a_{tank}(t)$ and $i^a_{L_m}(t)$ are defined in (3.47). For operation at the resonant frequency, $i_{SR}(t)$ is determined as:

\[
\begin{align*}
& i_{SR}(t) = n \left( i^a_{tank}(t) - i^a_{L_m}(t) \right), \quad \text{for } 0 < t < \frac{1}{2f_{sw}} \\
& i_{SR}(t) = 0, \quad \text{for } \frac{1}{2f_{sw}} < t < \frac{1}{f_{sw}}
\end{align*}
\] (3.81)

where $i^a_{tank}(t)$ and $i^a_{L_m}(t)$ are defined in (3.68).

In order to validate the developed equations, it is possible to operate a converter under different $f_{sw}$ and loading conditions, and compare the experimental waveforms with those obtained using simulation and RCE. The topology was built considering the parameters shown in Table 3.3.

Selected operating conditions are shown in Fig. 3.19. It is possible to observe from
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

Table 3.3: LLC design parameters for $P_{\text{loss}}$ analysis in rectifier MOSFETs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Input Voltage $V_{in}$</th>
<th>Nominal Power $P_{\text{nom}}$</th>
<th>Nominal Load Current $I_{\text{nom}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>400 V</td>
<td>650 W</td>
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<th>Resonant Inductance $L_r$</th>
<th>Resonant Capacitance $C_r$</th>
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<td>37.7 $\mu$H</td>
<td>18.8 nF</td>
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</table>

<table>
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<th>Parameter</th>
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<th>Switching Frequency $f_{\text{sw}}$</th>
<th>Transformer Turns Ratio $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
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<td>150-250 kHz</td>
<td>8.1</td>
</tr>
</tbody>
</table>

the figure a close correlation between the obtained experimental, simulated and calculated results, with the peak of $i_{SR}(t)$ presenting a maximum difference of no more than 1% between simulated and calculated waveforms, and of less than 2.5% between calculated waveforms using RCE and experimental measurements. From the figure it is also possible to observe that the waveforms are not symmetrical with respect to their peak value: in all presented scenarios, the rising slope of the first half of the waveform is considerably less steep than that of the falling slope of the second half of the waveform. Another interesting observation is that when the converter operates below $f_{\text{res}}$, the current is zero for longer than half of the switching period. This is in accordance with (3.80), and is a result of the fact that during part of the switching period the secondary of the circuit is disconnected from the primary. As a result, no current flows through the SR MOSFETs, and $I_{\text{load}}$ is maintained by the load capacitor in the output of the circuit.

3.3.2 Accounting for Turn-on and Turn-off Delays

Once the RCE equations have been determined, it is possible to calculate the $P_{\text{loss}}$ of the device through integration. However, unlike the inverting MOSFETs, the driving signal of the synchronous rectification must be determined with a control algorithm, in order to avoid
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

Figure 3.19: Experimental $i_{SR}(t)$ waveforms for different $f_{sw}$ and $I_{load}$

shoot-through or reverse power transfer. In addition, the conduction period of the MOSFET is not necessarily half of the switching period, and the turn-on and turn-off times are not synchronized with those of the inverting MOSFETs. As a result, the device is turned on with a time delay ($t_{d(on)}$) and turned off prematurely by another amount of time ($t_{d(off)}$).
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

During these periods, the body diode of the MOSFET conducts instead of the channel of the device, generating a voltage drop ($V_{\text{diode}}$) which results in increased $P_{\text{loss}}$. For operation of the converter above and at $f_{\text{res}}$, the $P_{\text{loss}}$ can be determined as:

$$P_{\text{loss}} = f_{\text{sw}} \left( \int_0^{t_d(\text{on})} i_{\text{SR}} V_{\text{diode}} dt + \int_{t_d(\text{on})}^{t_{\frac{1}{2}f_{\text{sw}}}-t_d(\text{off})} i_{\text{SR}}^2 R_{DS(\text{on})} dt + \int_{t_{\frac{1}{2}f_{\text{sw}}}-t_d(\text{off})}^{\frac{1}{2}f_{\text{sw}}} i_{\text{SR}} V_{\text{diode}} dt \right)$$

(3.82)

where $i_{\text{SR}}(t)$ is obtained from (3.79) for $f_{\text{sw}} > f_{\text{res}}$ and from (3.81) for $f_{\text{sw}} = f_{\text{res}}$. The expression for $P_{\text{loss}}$ for the converter operating below $f_{\text{res}}$ can be determined as follows:

$$P_{\text{loss}} = f_{\text{sw}} \left( \int_0^{t_d(\text{on})} i_{\text{SR}} V_{\text{diode}} dt + \int_{t_d(\text{on})}^{t_{\frac{1}{2}f_{\text{sw}}}-t_d(\text{off})} i_{\text{SR}}^2 R_{DS(\text{on})} dt + \int_{t_{\frac{1}{2}f_{\text{sw}}}-t_d(\text{off})}^{\frac{1}{2}f_{\text{sw}}} i_{\text{SR}} V_{\text{diode}} dt \right)$$

(3.83)

where the expression for $i_{\text{SR}}(t)$ is determined by (3.80).

As can be observed in Fig. 3.19, the RCE waveforms are not symmetric, presenting a steeper slope during their second half. As a result, the contribution of $t_d(\text{on})$ and $t_d(\text{off})$ in $P_{\text{loss}}$ is not the same, since losses are proportional to the circulating current. In order to visualize this effect, it is possible to calculate $P_{\text{loss}}$ under different operating frequencies for a constant $I_{\text{load}}$, considering the effects of different values of $t_d(\text{on})$ and $t_d(\text{off})$ separately. The graphical representation of this analysis is shown in Fig. 3.20(a). As expected, large $t_d$ result in increased $P_{\text{loss}}$, since the body diode of the MOSFET conducts for longer periods of time. It can also be observed that, for the same operating condition, presenting a certain $t_d(\text{off})$ results in more $P_{\text{loss}}$ than presenting the same amount of $t_d(\text{on})$. In reality, both $t_d$ will be present, but from this analysis it becomes clear that increased effort must be made to reduce $t_d(\text{off})$ as much as possible. This can be obtained with improved and more sophisticated SR control strategies. Another interesting conclusion that can be drawn from Fig. 3.20(a) is that
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

![Figure 3.20: a) P_{loss} for different f_{sw} and t_{d} b) I_{on/off} for different f_{sw} and t_{d}](image)

operation above \( f_{res} \) is more prone to increased \( P_{loss} \) as \( t_{d(\text{off})} \) increases. This can be justified by the steep slope observed in the waveform shapes observed in Fig. 3.19 for operation above the resonant frequency.

Because of the asymmetry of the RCE waveforms, the turn-on currents (\( I_{on} \)) and turn-off currents (\( I_{off} \)) will present different values for the same \( t_{d} \) under the same operating condition. This effect can be observed in Fig. 3.20(b). As expected, \( I_{off} \) presents larger values due to the steeper slope of the second half of the waveforms. This effect is in accordance with the observations presented previously, and once again indicates the importance of an effective SR control strategy that reduces \( t_{d(\text{off})} \).

These effects cannot be observed if FHA is used to trace the expected waveforms of the circuit, since it produces sinusoidal waveforms which are symmetric by nature. In addition, complications arise for operation of the converter below \( f_{res} \), since the actual signal is larger than zero for less than half of the switching period. In order to overcome this last complication, a method has been developed to improve the accuracy of the waveforms produced by FHA for \( f_{sw} < f_{res} \). It consists on considering the on-time \( t_{on} \) during which the SR MOSFET is conducting to have a fixed value of \( t_{on} = \pi \sqrt{L_{r}/C_{r}} \). This produces a waveform that is closer in magnitude and shape to that presented by the circuit, as shown in Fig. 3.21.
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

Nevertheless, the waveform produced with the modified FHA is still symmetrical $0 < t < \pi \sqrt{L_r C_r}$, so it cannot replicate the effect that is observed in Fig. 3.20. Considering the case study shown in Fig. 3.22, the maximum error observed when only $t_{d(on)}$ is considered is reduced from 41.1% to 28.8% when employing the modified FHA versus conventional FHA, and from 56.4% to 29.3% when only $t_{d(off)}$ is accounted for.

Figure 3.22: a) $P_{loss}$ considering different values of $t_{d(on)}$ and $t_{d(off)} = 0$. b) $P_{loss}$ considering different values of $t_{d(off)}$ and $t_{d(on)} = 0$
3.3.3 Design and Control Considerations for SR Losses

As discussed previously, while FHA provides interesting insights regarding the operation of LLC resonant converters, it lacks the level of refinement which is necessary for the accurate determination of $P_{\text{loss}}$ when compared to RCE. Incorporating different values for $t_d$ may result in even further discrepancies between the calculated values using both methods. In addition, the inductance ratio $m$, one of the fundamental design considerations of this topology, is not accounted for in the FHA calculations for SR current determination. As a result, while RCE is able to determine $i_{\text{SR}}(t)$ with accuracy, considering the dependency of $m$ for $P_{\text{loss}}$ calculation, FHA only provides a single $P_{\text{loss}}$ estimation for different $m$, as shown in Fig. 3.23.

As can be observed in Fig. 3.23, $P_{\text{loss}}$ assessment using FHA always results in underestimated values, which can be detrimental for the design of an appropriate thermal management solution for the switches of the rectifier. When the converter operates below $f_{\text{res}}$, it is possible to see that larger values of $m$ such as $m = 6$ result in smaller levels of $P_{\text{loss}}$, which approximate to those estimated with FHA. When the converter operates above $f_{\text{res}}$, similar values of $P_{\text{loss}}$ are obtained using RCE considering the different $m$. Nevertheless, larger

![Figure 3.23: $P_{\text{loss}}$ estimation considering different $I_{\text{load}}$ and $m$](image-url)
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

values of $m$ still yield slightly lower $P_{\text{loss}}$ estimation. FHA, on the other hand, substantially underestimates $P_{\text{loss}}$ for operation under these conditions.

Even though lower values of $m$ result in larger $P_{\text{loss}}$ for a considered $f_{\text{sw}}$, $P_{\text{load}}$ differs from case to case, since the relationship between $V_{\text{out}}$ and $f_{\text{sw}}$ depends on $m$. As a result, a more sensible comparison can be performed in which the the ratio of power being dissipated in an SR MOSFET versus the total power being transferred to the load, namely $P_{\text{loss}}/P_{\text{load}}$, is analyzed. Because the determination of $P_{\text{loss}}$ using FHA does depend on $m$, different values of $P_{\text{loss}}/P_{\text{load}}$ can be obtained for various $m$ using both methods: RCE and modified FHA.

Figure 3.24 displays different levels of $P_{\text{loss}}/P_{\text{load}}$ considering various loading conditions and values of $m$, using as estimation both RCE and FHA. The first conclusion that can be drawn from the RCE calculations is that the relationship $P_{\text{loss}}/P_{\text{load}}$ increases as $I_{\text{load}}$ becomes higher. This implies that $P_{\text{loss}}$ increases more significantly than $P_{\text{load}}$ does. When looking at the results obtained for operation below $f_{\text{res}}$, lower values of $m$ result in a better performance, even though this condition results in larger $P_{\text{loss}}$, as observed in Fig. 3.23. When the converter operates above $f_{\text{res}}$, lower values of $m$ result in a better performance, which is in accordance with the behaviour observed in Fig. 3.23. The $P_{\text{loss}}/P_{\text{load}}$ behaviour

![Figure 3.24: $P_{\text{loss}}/P_{\text{load}}$ estimation considering different $I_{\text{load}}$ and $m$](image)

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3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

is of especial interest when a converter is designed to perform predominantly as a step-up or step-down topology, since SR losses can be reduced independently of the control strategy employed. Even though the same conclusion can be obtained with either FHA or RCE, the calculated ratio with FHA can be inaccurate, especially for operation under heavy loading conditions and with high values of $m$. Therefore, FHA is not the best approach to determine $P_{\text{loss}}$ accurately, especially for lower values of $m$, and it cannot determine $P_{\text{loss}}/P_{\text{load}}$ with precision for higher values of $m$.

Another limitation of employing FHA is that the determination of $t_{\text{on}}$ during which each MOSFET should conduct is fixed at $t_{\text{on}} = \pi \sqrt{L_r/C_r}$ for operation below $f_{\text{res}}$. However, this parameter can change significantly based on operating conditions, such as $I_{\text{load}}$ and $f_{\text{sw}}$, and on design parameters, such as $m$, for operation below $f_{\text{res}}$, as can be observed in Fig. 3.25. Because of that, RCE can be used to analyze the impact of different control techniques in $P_{\text{loss}}$, since it is able to predict with accuracy the value of $t_{\text{on}}$ as a function of different operating conditions and circuit parameters, as shown in Fig. 3.26. From the figure, it is possible to observe that the maximum error observed between calculated and measured $t_{\text{on}}$ is of 4%.

![Figure 3.25: $t_{\text{on}}$ as a function of $f_{\text{sw}}$ and $I_{\text{load}}$ for different values of $m$](image)

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</table>

Figure 3.25: $t_{\text{on}}$ as a function of $f_{\text{sw}}$ and $I_{\text{load}}$ for different values of $m$
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

Different control techniques can be employed to operate the SR, which will result in various combinations of $t_{d(on)}$ and $t_{d(off)}$ and in different levels of $P_{loss}$. In general, a simpler and low-cost control method, such as assuming a fixed on-time $t_{on}$, will result in more elevated levels of $P_{loss}$; employing a more sophisticated control technique with sensing elements may reduce $P_{loss}$, at the expense of a bulkier, heavier, and costlier converter.

From Fig. 3.25, it is possible to observe that higher values of $m$ result in smaller variations in $t_{on}$ with $f_{sw}$ and $I_{load}$, which is convenient for the implementation of simpler control techniques, such as constant $t_{on}$. In cases where $m$ is smaller, more complex control algorithms may be recommended so to avoid excessive $P_{loss}$, since an approach such as a constant fixed $t_{on}$ may result in excessively long $t_d$.

Fixed $t_{on}$

One of the simplest control algorithms for operating the SR of an LLC resonant converter consists of determining a constant and fixed $t_{on}$ for operation below $f_{res}$ which ensures no shoot through during any operating condition. Even though this approach can be easily implemented, it may result in increased $P_{loss}$.

In order to assess the impact of employing this control method, it is possible to calculate

![Figure 3.26: Experimental, simulated an calculated $t_{on}$ for different values of $f_{sw}$ and $I_{load}$](image)
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

$P_{\text{loss}}$ using the developed RCE. First, the smallest $t_{\text{on}}$ which occurs under heavy loading conditions at the lowest $f_{\text{sw}}$ is determined using TIA. In order to ensure no shoot through, a minimum $t_{\text{d(on)}}$ and $t_{\text{d(off)}}$ of 200 ns is considered. Finally, a $t_{\text{d(on)}}$ of 200 ns is considered for all operating conditions, with a variable $t_{\text{delay(\text{off})}}$ which depends on the constant $t_{\text{on}}$ calculated. The $P_{\text{loss}}$ obtained using this method is shown in Fig. 3.27, as well as $P_{\text{loss}}$ for $t_{\text{delay(on)}} = t_{\text{delay(\text{off})}} = 200$ ns for comparison, taken as a benchmark.

It is possible to observe from Fig. 3.27 that this simple control algorithm can be effective for high values of $m$ if $t_{\text{on}}$ is appropriately determined, which cannot be done using FHA. In cases where $m$ is small, such as $m = 2$, because $t_{\text{on}}$ varies considerably with operating conditions, as shown in Fig. 3.25, determining an effective fixed value for $t_{\text{on}}$ may not be possible since $P_{\text{loss}}$ are considerably high. In the considered scenario, using a fixed $t_{\text{on}}$ more than doubles the $P_{\text{loss}}$ when compared to the benchmark for $m = 2$, but only increases losses by up to 19% for $m = 6$. Thus, a fixed $t_{\text{on}}$ can be suitable for cases where $m$ is large.

![Figure 3.27: Comparison between $P_{\text{loss}}$ with constant $t_{\text{on}}$ and $t_{\text{d(on)}} = t_{\text{d(\text{off})}} = 200$ ns, taken as benchmark](image)
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

**Variable \( t_{on} \) using sensing elements**

Another technique that is commonly used for operating the SR is to determine a variable \( t_{on} \) that is adjusted by sensing either \( I_D \) or \( V_{DS} \) across the SR MOSFET. This control algorithm turns the MOSFET on by comparing the measured signal with a threshold level. When the MOSFET is turned off, the control algorithm determines if the device was turned off prematurely or belatedly based on the sensing signals at the moment of turn-off, and \( t_{on} \) is updated accordingly at a frequency lower than \( f_{sw} \). As a result, \( t_d(\text{on}) \) is approximately constant while \( t_d(\text{off}) \) varies over time, “sweeping” between larger and smaller values. While this control technique is more advanced since it relies on measurements from a sensing element, thus being able to adapt to circuit changes, it still can yield poor \( P_{loss} \) levels, especially due to \( t_d(\text{off}) \).

Supposing that \( t_{delay(\text{on})} \) is fixed at 200 ns and \( t_d(\text{off}) \) varies between 100 and 400 ns, which would be a result of the “sweeping” technique, \( P_{loss} \) can be calculated using RCE, and then compared to the benchmark with \( P_{loss} \) for \( t_{delay(\text{on})} = t_{delay(\text{off})} = 200\text{ns} \). In order to obtain the \( P_{loss} \) using the “sweeping” control algorithm, the average value of \( P_{loss} \) can be obtained from different calculations using the various values of \( t_d(\text{off}) \) that the converter assumes during operation. Figure 3.28 shows the \( P_{loss} \) obtained using this technique versus the benchmarked value for different operating conditions. It is possible to observe that this control technique results in a \( P_{loss} \) that can be up to 18% higher than that of the benchmark, which is considerably smaller than that observed for the constant \( t_{on} \) control technique, but still not optimal considering that the benchmark \( P_{loss} \) presented already includes losses originated from a \( t_d \) of 200 ns for turn-on and turn-off.

If the \( t_d(\text{off}) \) varied between 100 and 300 ns instead, the obtained \( P_{loss} \) would be similar to that obtained for \( t_d(\text{off}) = 200\text{ns} \), which is taken as benchmark. This shows that an increase of 100 ns in the tolerance for \( t_d(\text{off}) \) from 300 ns to 400 ns results in a substantial increase
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

Figure 3.28: Comparison between $P_{\text{loss}}$ with variable $t_{\text{on}}$ using the “sweeping” technique and $t_{d(\text{on})} = t_{d(\text{off})} = 200$ns, taken as benchmark

of $P_{\text{loss}}$. Therefore, in order for this control algorithm to produce sensible results, the upper limit of $t_{d(\text{off})}$ must be kept as reduced as possible, since $P_{\text{loss}}$ can increase quickly with changes in $t_{d(\text{off})}$.

Variable $t_{\text{on}}$ using a sensing-less approach

In order to implement a variable $t_{\text{on}}$ using a sensing-less approach, it is possible to employ the TIA equations developed to determine the behaviour of $t_{\text{on}}$ as different circuit parameters are changed. Considering the circuit parameters from Table 3.3, it is possible to obtain different values of $t_{\text{on}}$ as certain circuit design properties change. Fig. 3.29 shows the behaviour of $t_{\text{on}}$ as $f_{\text{sw}}$ changes, where Fig. 3.29(a) depicts the behaviour when $L_m$ is modified and $L_r$ and $C_r$ are kept constant; Fig. 3.29(b) considers a fixed value of $L_m$ and $f_{\text{res}}$, with $L_r$ and $C_r$ being adjusted accordingly; and Fig. 3.29(c) considers a constant $L_m$ and $C_r$, with different values of $L_r$ and thus $f_{\text{res}}$. It is possible to observe from the considered cases that $t_{\text{on}}$ can vary substantially with changes in loading condition and $f_{\text{sw}}$, and this parameter can present different behaviours as circuit design properties change.

However, when the x-axis of the plots from Fig. 3.29 are normalized with respect to $f_{\text{res}}$,
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

Figure 3.29: Determination of $t_{on}$ considering different $I_{load}$, $f_{sw}$ and $m$, for a) constant $L_r$ and $C_r$, b) constant $L_m$ and $f_{res}$ and c) constant $L_m$ and $C_r$ after normalization with respect to $t_{sw} = \frac{1}{f_{sw}}$. It is possible to observe that all considered scenarios behave similarly, as can be observed in Fig. 3.30. Additionally, in all considered cases the relationship between $t_{on}/t_{sw}$ and $f_{sw}/f_{res}$ is close to linear, and the smallest value of $t_{on}/t_{sw}$ is obtained at the lowest $f_{sw}/f_{res}$ and highest $I_{load}$. Therefore, it is possible to suppose a linear relationship between $f_{sw}/f_{res} = 1$ and $t_{on}/t_{sw} = 0.5$ with

Figure 3.30: Determination of $t_{on}$ considering different $I_{load}$, $f_{sw}$ and $m$, for a) constant $L_r$ and $C_r$, b) constant $L_m$ and $f_{res}$ and c) constant $L_m$ and $C_r$ after normalization.
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

the lowest \( f_{sw}/f_{res} \) that will be considered for the circuit and \( t_{on}/t_{sw} \) for the highest \( I_{load} \) and smallest \( f_{sw} \), where \( t_{on} = t_1 \) from (3.63), (3.66) and (3.67). The obtained relationship between \( t_{on} \) and \( f_{sw} \) is:

\[
t_{on} = \frac{t_{on(low)} f_{sw(low)} f_{sw} - f_{sw(low)}}{f_{res} - f_{sw(low)}} + \frac{t_{on(low)} f_{sw(low)}}{f_{sw}}
\]

where \( f_{sw(low)} \) represents the lowest \( f_{sw} \) being considered and and \( t_{on(low)} \) is the value of \( t_{on} \) calculated for \( f_{sw(low)} \) and the highest \( I_{load} \). Considering the scenarios presented with different combinations of circuit parameters, simulation results indicate that it is possible to obtain an improvement of at least 80% in \( P_{loss} \) when using the linear \( t_{on} \) relationship versus using diode rectification, and up to 50% of improvement when comparing the sensing-less linear \( t_{on} \) relationship versus a constant \( t_{on} \) approach.

3.3.4 Experimental Validation of Power Loss Assessment

While the validation of the RCE waveforms has already been observed in Fig. 3.19, it is also necessary to compare expected values of \( P_{loss} \) with measured ones for operation of the SR MOSFETs. In order to explore the impact of different \( f_{sw} \), \( I_{load} \) and combinations of \( t_{d(on)} \) and \( t_{d(off)} \), the circuit with parameters listed in Table 3.3 was operated under different conditions. In order to investigate the impact of different \( t_{d(on)} \) and \( t_{d(off)} \), the \( t_{on} \) was controlled manually, so no specific control algorithm was employed.

Because the measurement of \( P_{loss} \) directly using electrical parameters is not recommended, as discussed previously, a different approach was employed in this case: using a high-precision power analyzer, the difference of \( P_{loss} \) observed when the SR was turned on or off manually was studied. This \( \Delta P_{loss} \) can be obtained by comparing \( P_{in} \) with \( P_{load} \) when the SR is turned off and the body diodes of the MOSFETs conduct with \( P_{in} \) and \( P_{load} \) when the SR is turned on and \( t_{d(on)} \) and \( t_{d(off)} \) are determined manually. From Fig. 3.31, is possible to observe
3.3. Power Loss Estimation of LLC MOSFETs in the Rectifying Stage

Figure 3.31: Measured Power Losses in LLC rectifying MOSFETs under different $f_{sw}$ and $I_{load}$
that FHA overestimates $i_{SR}(t)$ during the first half of the waveform, while providing an underestimation of the current value for the second half of the waveform. As a result, FHA will overestimate $P_{\text{loss}}$ if $t_{d(\text{on})}$ is large, and underestimate it when $t_{d(\text{off})}$ is elevated. This situation can be observed from the error bars presented in Fig. 3.31: in Scenario 2, a large $t_{d(\text{off})}$ causes $\Delta P_{\text{loss}}$ to be high using FHA, and in Scenario 3, a large $t_{d(\text{on})}$ causes $\Delta P_{\text{loss}}$ to be too low when FHA is employed. In addition, an increased discrepancy is observed as the $t_d$ is increased, which can be seen when comparing Scenarios 1 and 2. The RCE using TIA, on the other hand, provide very good estimation of losses independently of the combination of $t_{d(\text{on})}$ and $t_{d(\text{off})}$.

Moreover, it is possible to observe that the measured $\Delta P_{\text{loss}}$ from Scenario 1 is comparable to that of Scenario 3. This reflects the fact that $t_{d(\text{on})}$ is not as impactful in $P_{\text{loss}}$ as $t_{d(\text{off})}$ is. In addition, a small change in $t_{d(\text{off})}$ from Scenario 1 to Scenario 2 significantly reduces $\Delta P_{\text{loss}}$. This leads to the conclusion that a good control algorithm for SR is required in order to reduce $t_{d(\text{off})}$ and achieve higher efficiencies for operation of the converter.

Considering all measurements performed, the maximum error observed between measured $\Delta P_{\text{loss}}$ and the calculation using RCE was of 5%, while FHA resulted in errors as large as 37%. The average error was reduced from 12% to 2% by using RCE instead of FHA for $P_{\text{loss}}$ estimation. As observed previously, the inaccuracy of FHA to determine $P_{\text{loss}}$ becomes critical for operation at high $f_{\text{sw}}$. Considering the measurements performed for operation at $f_{\text{sw}} = 250$kHz, the average error presented by FHA was of 18%, while RCE presented an average error of 3%. This is in accordance with the findings from the waveforms presented in Fig. 3.19 and in Fig. 3.31, since FHA cannot trace the expected waveforms as well as RCE for this operating condition.
Chapter 4

Conclusions

4.1 Summary

This thesis introduced and analyzed a method for developing a detailed characterization in MOSFET power loss ($P_{\text{loss}}$) key parameters, such as the on-state resistance ($R_{DS(on)}$), turn-off energy ($E_{off}$) and diode forward voltage drop ($V_{\text{diode}}$). This detailed characterization allows for the determination of the behaviour of these parameters in a more detailed fashion which goes beyond the information provided by datasheets. The impact of different operating conditions and parameters such as gate-source voltage ($V_{GS}$), drain-source voltage ($V_{DS}$), junction temperature ($T_j$) and drain current ($I_D$) are considered in the analysis, which allows for the determination of different loss mechanisms such as conduction ($P_{\text{cond}}$), switching ($P_{\text{sw}}$) and body diode losses ($P_{\text{diode}}$) with increased accuracy.

In addition, time-domain equations for the LLC converter are developed based on operating conditions and design parameters, which lead to an improved approximation of the power loss behaviour of the topology when compared with alternatives such as the well-established yet oversimplified First-Harmonic Approximation (FHA). The developed equations are especially useful for the calculation of $P_{\text{loss}}$ in both inverting and rectifying MOSFETs of this topology, since they provide steady-state information about the operation of the converter. This comes as an alternative to the simulation of losses using specialized software, which requires substantial computational resources. The $P_{\text{loss}}$ in both inverting and rectifying stages is determined using the developed equations based on the Time Interval Analysis (TIA),
and an analysis on the impact of different design parameters and operating conditions is performed. In addition, the limitations of FHA are presented and compared with the results obtained using TIA.

The developed models and equations are verified versus simulation and experimental results, and provide considerable improvement when compared with the traditional approach using FHA. In the inverting stage, the observed error in loss determination is reduced from an average of 19%, using FHA, to 2.8% using the proposed method. When it comes to the rectification portion of the circuit, the reduction in error observed is from 12% to 2%.

The contribution to the scientific community is proven by the presentation and subsequent publication of a conference paper at the IEEE Applied Power Electronics Conference and Exposition (APEC) 2018 [1], and by the submission of two papers to two IEEE Transactions journals, which are currently under review by the editorial board of the journals.

4.2 Future Work

The main advantage of the developed detailed characterization is that it is performed at the device level, which allows for the accurate determination of $P_{\text{loss}}$ in any topology. Other popular circuits used in power electronics such as the power factor correction topology (PFC) would definitely benefit from the accurate determination of different loss mechanisms in a similar fashion to that performed and presented in Chapter 2.

While the developed TIA equations are exclusively related to the LLC resonant converter, a similar approach which consists on dividing switching cycles into time intervals could be developed for other topologies in power electronics. This would facilitate the analysis on the impact of different design parameters and operating conditions, contributing to developing better design practices for various topologies.

Furthermore, an in-depth analysis of the viability of the variable on-time ($t_{\text{on}}$) control
4.2. Future Work

An approach using a sensing-less mechanism briefly introduced in Chapter 3 would be required for the full validation of said approach.
Bibliography


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