Optimal Design of Water Distribution Networks under Uncertainty

by

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Abstract

Generally, the use of the water distribution network (WDN) modeling is divided into two main categories, WDN design, and WDN hydraulic analysis, which itself is employed as part of WDN design. The former generally identifies components of the network, considering the system cost and the ability of the network to satisfy consumer demands for water availability, pressure, and quality. The latter evaluates the distribution of nodal pressures and pipe flows under a specified network design and known or estimated water consumption levels. WDN analysis is often embedded within design algorithms, where, for every potential design considered, the hydraulic energy and continuity equations that govern the system are solved. If water demands and the physical characteristics of the network design are known with certainty, deterministic approaches for solving these equations may be used. If some information is uncertain, non-deterministic approaches are used for identifying the Probability Density Function or the fuzzy membership functions of the pressure and flow conditions at all locations in the network.

This dissertation is divided into three main parts, 1) the application and development of evolutionary algorithms for single- and multi-objective optimization of WDNs, 2) the introduction of new frameworks and performance surrogates as objectives in the optimization of WDNs, and 3) the advancement of an efficient gradient-based technique for fuzzy analysis of WDNs under uncertainty. First, efficient Evolutionary Algorithms (EAs) are compared and advanced for reducing the computational burden of single- and multi-objective design of WDNs, respectively. We investigate and identify the most appropriate operators and characteristics of EAs for optimization of realistic-sized networks. Based on the experiences and capabilities of EAs obtained, a new EA is introduced for single-objective optimization of WDNs which is faster and more reliable than other popular algorithms presented in the literature. Next, two different frameworks are introduced for implementing many objectives in the optimization of realistic-sized WDNs. Both approaches can distinguish appropriate design solutions with minimum cost and maximum hydraulic and mechanical reliability. Finally, a fuzzy method is introduced for analysis of WDNs under uncertainty. The proposed technique significantly reduces the CPU time of uncertainty analysis of large-scale networks.

Lay Summary

Water distribution projects that provide drinking water for municipalities are among the most expensive of installed infrastructure. These systems include many components and thousands of junctions and pipes, connected together for transmission of water. They are usually constructed in a loop-shape to minimize the risk of failing to supply water at all demand sites when one or many pipes have been removed for scheduled repair, or unexpectedly broken due to pipe decay or high pressures. There are many uncertain conditions in these systems, including consumption levels, pipe age and operability. Design and maintenance of these systems requires consideration of many details and plans to respond to abnormal circumstances. This dissertation develops techniques to improve the quality of the designs of these systems by minimizing cost and maximizing reliability. This work may be used to undertake appropriate simulation and optimization of designs before construction and implementation of these systems.

Preface

Chapter 2 is published in the Canadian Journal of Civil Engineering (CSCE); I have conducted all the testing, computational analysis and wrote most of the manuscript. Dr. Lence edited all spelling the manuscript and directed the arrangements and presenting the results. This paper is:

Moosavian N., Lence B., Testing evolutionary algorithms for optimization of water distribution networks, *Canadian Journal of Civil Engineering (CSCE)*, (2018). Canadian Journal of Civil Engineering, https://doi.org/10.1139/cjce-2018-0137

Chapter 3 is published in the ASCE Journal of Water Resources Planning and Management. I conducted the computational analysis and wrote 70% of the manuscript. Dr. Lence revised the manuscript, and directed the development of methods for analyzing the results and frameworks for presenting these. This paper is:

Moosavian N., Lence B.J., Nondominated sorting differential algorithms for multi-objective optimization of water distribution systems, *Journal of Water Resources Management and Planning (ASCE),* (2016). http://ascelibrary.org/doi/abs/10.1061/(ASCE)WR.1943-5452.0000741

Chapter 4 is based on a submitted journal paper; I have conducted the testing, computational analysis and wrote most of the manuscript. Dr. Lence edited the manuscript and directed the additions of conceptual proofs of the algorithm developed, as well as the testing of the algorithm, and directed the arrangements and presenting of the results.

Chapter 5 is published in the ASCE Journal of Water Resources Planning and Management. Dr. Lence, Hanieh Daliri and I suggested the initial idea related to the application of the game theory in the area of multi-objective optimization of WDNs. I developed the fuzzy programming idea and conducted the computational analysis, provided the results and wrote 50% of the manuscript. Dr. Lence wrote 50% of the document and directed the computational work, and Hanieh Daliri conducted literature review. This paper is:

Lence B.J., Moosavian N., Daliri H., A fuzzy programming approach for multi-objective optimization of water distribution systems, *Journal of Water Resources Management and Planning (ASCE),* (2017). http://ascelibrary.org/doi/abs/10.1061/(ASCE)WR.1943-5452.0000769

Chapter 6 is based on a submitted journal paper. I proposed the initial idea and conducted computational analysis and Dr. Lence developed the methodology and the results sections and revised the manuscript.

Chapter 7 has been published in the ASCE Journal of Hydraulics. I proposed the idea, conducted computational analysis and wrote the 80% of the paper. Dr. Lence revised the manuscript and directed the presentation of the results. This paper is:

Moosavian N., Lence B. J., Fuzzy analysis of water distribution networks, *Journal of Hydraulic Engineering, (ASCE),* (2018). DOI: 10.1061/(ASCE)HY.1943-7900.0001483

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List of Abbreviations

ABC = Artificial Bee Colony

ACO = Ant Colony Optimization

CMAES = Covariance Matrix Adaptation Evolutionary Strategy

DE = Differential Evolution

EA = Evolutionary Algorithm

FDE = Fast Differential Evolution

FORM = First Order Reliability Analysis

GA = Genetic Algorithm

GGA = Global Gradient Algorithm

HS = Harmony Search

INLP = Integer Nonlinear Programming

ILP = Integer Linear Programming

LP = Linear Programming

MCS = Monte-Carlo Simulation

MEIGO = MEtaheuristics for Systems Biology and BIoinformatics Global Optimization

MF = Membership Function

NLP = Nonlinear Programming

NSDE = Nondominated Sorting Differential Evolution

NSGA = Nondominated Sorting Genetic Algorithm

PDF = Probability Density Function

PSO = Particle Swarm Optimization

PSOGSA = Particle Swarm Optimization Gravitational Search Algorithm

SA = Simulated Annealing

SLC = Soccer League Competition

SQP = Sequential Quadratic Programming

WDN = Water Distribution Network

WOA = Whale Optimization Algorithm

List of Symbols

Related to the Chapter 2

D_k = diameter of pipe k;

$c_k(D_k)$ = cost of pipe k per unit length with diameter D_k;

L_k = length of pipe k;

H_j = pressure at node j;

hf_k = head loss due to friction in the pipe k;

nl = number of loops;

nn = number of nodes in the network;

np = number of pipes in the network;

ns = number of candidate diameters;

Q_k = flow of pipe k;

q_j = demand at node j;

R_k = resistance coefficient of the pipe k;

ΔH = difference between nodal pressures at both ends of a path.

Related to the Chapter 3

C = cost;

CH = Hazen-Williams coefficient;

CP = penalty;

CR = crossover rate;

D = diameter of pipe;

F = mutation scale factor;

L = length of pipe;

H = head pressure at node;

H_0 = elevation head of a reservoir;

H^{min} = minimum pressure head required;

hf = head loss;

I_r = resilience index;

nl = number of loops;

nn = number of nodes;

np = number of pipes;

no = number of reservoirs;

$NPOP$ = number of population;

P = pump;

Pr = probability of selection;

Q = flow in pipe;

q = nodal demand;

$Rank$ = rank of each solution in the population;

S = number of candidate diameters;

sn = random integer number;

X = solution vectors;

γ = specific weight of water;

λ = penalty multiplier.

Related to the Chapter 4

C = cost;

CH = Hazen-Williams coefficient;

CP = penalty;

CR = crossover rate;

D = diameter of pipe;

F = mutation scale factor;

L = length of pipe;

H = head pressure at node;

H_0 = elevation head of a reservoir;

H^{min} = minimum pressure head required;

hf = head loss;

nl = number of loops;

nn = number of nodes;

np = number of pipes;

no = number of reservoirs;

$NPOP$ = number of population;

Pr = probability of selection;

Q = flow in pipe;

q = nodal demand;

ns = number of candidate diameters;

X = solution vectors;

X_{best} = best solution vector;

γ = specific weight of water;

λ = penalty multiplier.

Related to the Chapter 5

C = cost;

CH = Hazen-Williams coefficient;

d_α = minimum distance to the ideal solution;

D = diameter of pipe;

H = pressure head at node;

H_0 = elevation head of a reservoir;

H^{min} = minimum pressure head required;

hf = head loss;

Ir = resilience index;

Is = minimum surplus head index;

Id = diameter uniformity index;

L = length of pipe;

nk = number of pipes connected to node k;

nn = number of nodes;

np = number of pipes;

no = number of reservoirs;

nl = number of loops;

NRI = network resilience index;

P = pump;

Q = flow in pipe;

q = nodal demand;

Z_{kk}^* = ideal solution for objective kk;

$Z_{kk}(x)$ = optimal solution for objective kk;

α = metric between one and infinity;

γ = specific weight of water;

μ = fuzzy membership function;

λ = auxiliary decision variable.

Related to the Chapter 6

C = cost;

CH = Hazen-Williams coefficient;

D = diameter of pipe;

H = pressure head at node;

H^{min} = minimum pressure head required;

H_0 = elevation head of a reservoir;

hf = head loss;

Ir = resilience index;

Iu = flow uniformity index;

L = length of pipe;

$nkin$ = number of pipes with flows that enter the node ;

$nkout$ = number of pipes with flows that exit the node;

nl = number of loops;

nn = number of nodes;

np = number of pipes;

no = number of reservoirs;

Q = flow in pipe;

q = nodal demand;

Q_{kin} = flow in pipe *kin* which is entering the node;

Q_{kout} = flow in pipe *kout* which is exiting the node.

Related to the Chapter 7

CC = co-content model;

E_k = energy loss in pipe k;

H_j = pressure at node j;

h_k = head loss in pipe k;

Q_k = flow of pipe k;

q_j = demand at node j;

R_k = resistance coefficient of the pipe k.

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I would like to express my sincere gratitude to my supervisor, Dr. Barbara Lence, for her motivation, thoughtful advice, and continuous support. Her guidance helped me in all the time of research and writing of this thesis. Dr. Lence, thank you very much for your trust on me in these three years.

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Dedication

To my wife, Hanieh

Chapter 1 : Introduction

1.1 Introduction

Next to air, water is the most indispensable element required for human survival. Municipalities classify water uses as domestic, public, commercial, and industrial. Domestic uses include water required for drinking, cooking, bathing, washing, heating, cooling, air-conditioning, sanitary purposes, private swimming pools, and watering lawns and gardens. Public uses include water needed for public places and buildings such as parks and fountains, public gardens and swimming pools, hospitals, universities and other educational institutions, prisons, public sanitary places, street and sewer flushing, and firefighting. Commercial uses include water for hotels and restaurants, office buildings, car washes, laundries, shopping centres, bus and railway terminals, and airports. Industrial uses are typically for manufacturing and processing, including heat exchange and cooling (Bhave and Gupta 2006). For all classifications, water transmission lines and pipe networks are employed to convey large amounts of water to demand and consumption sites. The optimal design of water distribution network (WDNs) is often considered a least-cost optimization problem with pipe sizes for commercially available pipes being the primary discrete decision variables. The total cost is a function of the pipe design diameters and material, and the diameters in turn affect the network pressures. The main requirement of the problem is to satisfy minimum desired pressures at all network nodes. For each design solution, estimation of the pressure level at all consumption nodes requires the simultaneous solution of nonlinear energy and linear flow equations for the entire network, which significantly complicates the design problem. During the design optimization process, considering uncertain parameters such as nodal consumptions, further increases the complexity of the problem.

The first underground pipe network for water distribution is located in Knosis on the Island of Crete, and is 2500 years old. The first public water system was constructed in 700 BC when sloped hillside tunnels, called Qanats, were built to transport water in Persia. In 312 BC, Romans began to construct aqueducts. The first public water supply system in the United States was installed in 1652 AD when the City of Boston incorporated its water works to provide water for fire-fighting and domestic use. The first municipal water pumping station in Canada was installed in Toronto by 1841. Since then many public water supply systems have been installed (Ormsbee 2006).

The US Environmental Protection Agency (EPA), in the fifth report on drinking water infrastructure (2013), stated that a total of $384.2 billion US is required in water infrastructure investments over the next 20 years, 64.4% of which, is needed for the transmission and distribution. Thus, it is essential that the water infrastructure functions properly to ensure the continuous supply of safe water with the required quality and pressures.

Generally, the use of WDN modelling is divided into two main categories, WDN design, and WDN hydraulic analysis, which itself is employed as part of WDN design. The WDN design generally selects elements of the system, such as pipe diameters and configurations, and pump and valve characteristics, considering the system cost and the capability of the network to satisfy consumer demands for water availability, pressure, and quality. WDN hydraulic analysis estimates the distribution of nodal pressures and pipe flow under a specified network design and known or probable water consumption levels.

This thesis investigates the application of single- and multi-objective optimization algorithms and fuzzy programming formulations for WDN design, and introduces a new algorithm for identifying, and a new surrogate for improving, design solutions. An efficient numerical technique for fuzzy analysis to accommodate uncertainty is also developed.

Figure 1.1 provides the modelling framework for the design and analysis of WDNs, including how and where uncertainty may be addressed. The optimization of WDN designs may be divided into methods that formulate the problem as having a single or multiple objectives. In the single-objective models, the problem has only one main goal (such as to minimize network cost) as the objective function. Other minor goals can be considered as constraints. In the multi-objective models, the problem has more than one main goal (such as minimize cost and maximize reliability indices), and evaluates all objectives as important. Examples of methods for undertaking WDN design are shown in the figure. Given the discrete nature of the decision variables (e.g., commercially available diameters of pipes), early applications of optimization to this problem used continuous and integer formulations of classical mathematical techniques. Given their computational power and ability to overcome the limitations of classical methods, heuristic approaches, such as Evolutionary Algorithms (EAs), have been increasingly applied, and are currently considered a standard techniques for solving this problem.

Under optimization models that minimize network cost, the best set of pipe size diameters is selected to assure that required pressure at all nodes in the network is satisfied. In the literature, this single-objective optimization problem is considered to be a nonlinear, non-convex, NP-hard problem, and is one of the most well-known benchmark problems for testing the performance of different heuristic algorithms. The earliest version of a Genetic Algorithm (GA) also was tested in a single-objective gas pipeline optimization problem by Goldberg (1983). The complexity of this problem is mainly related to the hydraulics of the network. For each design solution, hydraulic analysis is undertaken to calculate the amount and direction of flows in pipes and the pressures at all nodes. Since different solution designs have different pipe flow directions and values, finding the global optimum solution is difficult using classical optimization algorithms. Therefore, only powerful algorithms with specific exploration properties are capable of efficiently solving this problem. In Chapter 2, the investigation of convergence characteristics and efficiency of ten popular EAs is undertaken for the single-objective design of three pipe networks. Some of these algorithms are tested and analyzed for the first time, and when compared with other popular EAs, show promise for rapid convergence to the global optimum. Such efficient EAs may be suitable alternatives in the design of large-sized WDNs with thousands of pipes and nodes.

Generally, design optimization of WDNs faces more than one objective. For example, to identify robust network designs, reliability and resilience surrogates are estimated and integrated in constraints or objectives to improve the quality of the optimal design in the face of hydraulic uncertainty and pipe failures. When considered as objectives in multi-objective models, populations of solutions with different values of objective functions are evaluated and the Pareto optimal frontier for these objectives is investigated. There are many multi-objective EAs for determining the Pareto frontier effectively. In Chapter 3, the investigation of convergence characteristics and efficiency of four popular nondominated sorting EAs, including the nondominated sorting genetic (NSGA_II) and nondominated sorting differential evolution (NSDE) algorithms and variations of these, are undertaken for the multi-objective design of a wide range of networks. In the WDN design model, one objective is minimization of cost function and another objective is maximization of a reliability index. The NSDE algorithm with ranking based mutation is applied for optimization of the multi-objective WDN design model for the first time in this chapter, and results show that this algorithm can effectively find the optimal Pareto frontier and has the highest convergence rate. These results, and the simple structure of the differential evolution (DE) algorithm and coding process inspired the author to develop an improved DE algorithm for WDN optimization. Chapter 4 presents a modified DE algorithm which adds the most effective characteristics of other EAs and is highly effective for solving single-objective WDN design problems. The proposed algorithm is implemented for optimization of three popular benchmark problems and the Farhadgerd City WDN in Iran, and compared with other EAs for WDN design.

In recent years, the ability of networks to satisfy water demands under normal and abnormal operating conditions, including during pipe bursts and as components deteriorate, has been defined in terms of hydraulic and mechanical reliability of the system. Hydraulic reliability is a representative measure of the ability of the system to respond to gradual changes in the system characteristics, e.g., available pipe diameters and pipe roughness, and short- or long-term shifts in consumption, e.g., hourly variations in withdrawal rates or community-driven changes in required fire flows. Mechanical reliability is a measure of the ability of the system to adapt to mechanical failures, e.g., to maintain operating goals during pipe, valve, or pump failures.

For enhancing both mechanical and hydraulic reliability in WDN design, researchers propose several reliability surrogates and apply these individually and in combination to improve WDN design solutions. Chapter 5 proposes a fuzzy multi-objective programming model for combining these surrogates in the optimal design of WDNs. Unlike the nondominated sorting-based multi-objective models in Chapter 3, which provide populations of solutions on the Pareto optimal frontier, the fuzzy programming model converges to only one good compromise solution.

While effective at identifying solutions that exhibit hydraulic reliability, the above-mentioned surrogates may be inaccurate measures of network reliability in the face of mechanical failures. In Chapter 6, a flow uniformity index is proposed to address this limitation. The uniformity index may be considered as a surrogate for mechanical reliability, i.e., increasing it generally improves mechanical reliability of a system. For all pipes and nodes in the network, the scaled ratios of input (and output) pipe flows to the total input (and output) pipe flows are evaluated. The uniformity index is the minimum of these ratios among all pipes at each node, and among all nodes in the network. For each node, the scaling factor for each input (or output) pipe is equivalent to the number of input (output) pipes for that node. The use of scaling factors reduces the dependence on network configuration, resulting in an indicator that describes the degree of uniformity at each node in the system, the minimum of which is the least amount of uniformity at any node in the system. The index can be used to compare the uniformity of different designs of a single network configuration, or of different configurations. The maximization of the uniformity index leads to a network design with increasingly uniform flows throughout, where the burden of flow conveyance is shared more evenly among all input (or output) pipes, and where the surplus hydraulic power is gradually decreased from the supply reservoir(s) to the extremes of the network.

WDN hydraulic analysis is generally embedded within design algorithms, where, for every potential design considered, the hydraulic energy and continuity equations that govern the system must be solved. If input parameters such as water demands are known with certainty, deterministic approaches for solving these equations may be used. If this information is uncertain, non-deterministic approaches are used for approximating the measures of performance under uncertain conditions. In non-deterministic analyses, the aforementioned input parameters do not have certain values. Rather, Probability Theory may be used to identify Probability Density Functions (PDFs) for quantifiable parameter and Fuzzy Set Theory is used to identify Fuzzy Membership (FM) functions for linguistic parameters. There are several methods for non-deterministic analysis such as Monte-Carlo Simulation for incorporating both Probability and Fuzzy Set Theory, Reliability Analysis for incorporating Probability Theory, and Fuzzy Analysis techniques for incorporating Fuzzy set theory. The main advantage of the application of Fuzzy Set Theory is the simple construction of the FMs for variables that can only be described linguistically. This technique can identify the extreme values of unknown variables when uncertain input information ranges between pre-specified extremes, and when the PDFs of the information cannot be obtained. The FMs help the designer to estimate the worst conditions for providing the sufficient amount of water in the network facing nodal consumption variations, and provide useful insight toward making decisions for avoiding these conditions. There are two main approaches for conducting the Fuzzy Analysis in WDN design: 1) optimization methods and 2) gradient-based methods. The optimization methods are very time-consuming processes which need many hydraulic analyses for constructing the MFs. The initial applications of the gradient-based method improves the efficiency of Fuzzy Analyses for middle-sized networks. However, this method is not readily applicable for large-sized networks. Approximations of the gradients of the equations that govern WDN analysis, with respect to nodal demands and pipe resistance, are identified in Chapter 7 and harnessed to accelerate Fuzzy Analysis of system hydraulics.

1.2 Research Questions and Guide to the Reader

This thesis is a published paper-based document. The research centers on seeking answers to eight questions.

(i) How do evolutionary algorithms (EAs) perform when applied for single-objective design and optimization of WDNs?;

(ii) How do EAs perform when applied for multi-objective design and optimization of WDNs?;

(iii) Can one apply fuzzy programming to develop a framework for addressing more than two objectives in design and optimization of WDNs?;

(iv) How should several reliability surrogates for identifying appropriate design solutions of WDNs be integrated in the design and optimization of WDNs?;

(v) How can flow distribution in networks influence the design quality?:

(vi) How should fuzzy set theory for hydraulic analysis of large-scale WDNs under uncertainty be advanced?: and

(vii) How should advanced numerical techniques for fuzzy analysis of pipe networks be developed to reduce the computational burden?;

1.2.1 Published Work

Generally, this thesis investigates issues related to pipe network analysis and design. Four chapters of this research are published and two chapters have been submitted and are currently being peer-reviewed for publication. A brief explanation of the thesis contributions to the literature includes:

EAs Applied to Single-Objective WDN Design: to address some portions of Research Question (i), in Chapter 2, ten evolutionary algorithms are tested for optimal single-objective design of WDNs. Here, the objective function is the total cost of the network and the main constraint is the satisfaction of minimum required pressure head at all nodal demand sites. An early form of this work, testing only 6 algorithms, was presented at the Canadian Society of Civil Engineering Conference (Moosavian and Lence 2017), and the full Journal Paper has been published in the *Canadian Journal of Civil Engineering*, (Moosavian and Lence 2018).

EAs Applied to Multi-Objective WDN Design: to address Research Question (ii), in Chapter 3, four efficient evolutionary algorithms are tested for optimal multi-objective design of WDNs.

Here, the objectives are minimization of the total cost of the network and maximization of the resilience index proposed by (Todini 2000). This work demonstrates that by slightly modifying the Differential Evolution (DE) algorithm, the speed of convergence of the algorithm is improved for large-scale networks. The paper related to this chapter has been published as a Technical Paper in the *ASCE Journal of Water Resources Planning and Management*, 2016.

Fittest Individual Referenced Differential Evolution (FDE) algorithm for Single-Objective WDN Design: to address Research Question (i), based on the experiences obtained from Chapter 2 and 3, in Chapter 4, an efficient EA is proposed for minimization of the total cost of WDNs, which performs significantly better than other popular algorithms presented in the literature. The paper related to this chapter has been submitted as a Journal Paper to the *ASCE Journal of Computing in Civil Engineering*, July 2018.

Fuzzy Programming Approach for Multi-Objective WDN Design: to address Research Questions (iii) and (iv), in Chapter 5, a fuzzy programming approach based on the concept of Game Theory is proposed to address more than two objectives in the design of WDNS. These objectives are the total cost of the network and several indicators of hydraulic reliability (i.e., able to withstand hydraulic disturbances) presented in the literature. Unlike other multi-objective models (which create many optimal solutions on the Pareto optimal frontier), the fuzzy programming approach converges to only one preferred solution. This work also indicates that in addition to the resilience index, adding a new objective function, that focuses on increased uniformity in pipe diameters, will enhance the network mechanical reliability (i.e., able to withstand pipe failures). The paper related to this chapter has been published as a Technical Paper in the *ASCE Journal of Water Resources Planning and Management*, 2017.

Flow Uniformity Index for Multi-Objective WDN Design: to address Research Questions (iv) and (v), in Chapter 6, a new surrogate, along with the resilience index, is proposed and integrated in a triple-objective model which gives priority to solutions that exhibit increased flow uniformity. This work shows that for a similar cost and resilience index value, there are several design solutions with different flow distributions, and that designs with more uniform flow distribution have more mechanical reliability when facing a network pipe failure. The paper related to this chapter has been submitted as a Journal Paper in the *ASCE Journal of Water Resources Planning and Management*, June 2018.

Fuzzy Analysis of WDNs: to address Research Questions (vi) and (vii), in Chapter 7, a gradient-based fuzzy analysis technique for accounting for uncertainty in water distribution networks is developed which is computationally efficient in comparison with previous analytic methods. The paper related to this chapter has been published as a Technical Paper in the *ASCE Journal of Hydraulic Engineering*, March 2018.

In this thesis, seven pipe systems are employed for demonstrating the performance of the proposed analysis and optimization methods. These are the Two-loop, Hanoi, NewYork and EXNET benchmark networks, the water main networks for the Cities of Farhadgerd, Iran and Nagpur, India, and a set of large Hypothetical networks developed by the author. These networks and their use in the thesis are described in Section 1.5, in order of increasing network complexity and size.

1.2.2 Summary

In this chapter, a general framework for WDN modeling is presented and discussed. A review of previous work in the application of optimization algorithms for WDNs and different aspects of hydraulic analysis of WDNs are then presented for the reader. This literature review is gathered from sections of the published or submitted papers in Chapters 2-7 and is provided as a general overview for the novice reader who may not be familiar with the concepts of WDN analysis and design. Expert readers can skip this chapter and begin their reading with Chapter 2.

1.3 Literature Review

1.3.1 WDN Design

1.3.1.1 Single-Objective Minimum Cost Optimal Design

The single-objective optimal design of WDNs is often considered a least-cost optimization problem with pipe sizes for commercially available pipes being the primary discrete decision variables. The total cost is a function of pipe size diameters and these pipe diameters in turn affect the network pressures. The main constraint of the problem is the satisfaction of minimum required pressures at all network nodes. Early solution approaches formulate these problems as continuous optimization models and round off the optimum values of the design variables to the nearest commercial pipe sizes, However, in many cases, the rounding off of some variables changes the values of others, and the resultant solutions violate constraints or are suboptimal. Early applications of discrete optimization to this problem uses integer formulations of classical mathematical techniques such as Integer Linear Programming (ILP) and Nonlinear Programming (INLP). However, these approaches are computationally burdensome and may not converge to the global optimum solution.

Recently, EAs are becoming increasingly popular for their use in solving engineering decision problems and in practical applications, such as calibration, because they: (i) are based on rather simple concepts and are easy to implement; (ii) do not require gradient information; (iii) can consider and bypass local optima; and (iv) can be utilized to address a wide range of problems covering different disciplines. Generally, the main structure and search process of different EAs are similar, however, their operators may vary. Over the last three decades, many heuristic optimization techniques have been successfully used to identify water network designs, see, e.g., applications of Genetic Algorithms,GA (Murphy and Simpson 1992); Simulated Annealing, SA (Cunha and Sousa 2001); Harmony Search, HS (Geem 2006); Shuffled Frog Leaping, SFL (Eusuff and Lansey 2003); Ant Colony Optimization, ACO (Maier et al. 2003); Particle Swarm Optimization, PSO (Suribabu and Neelakantan 2006); Cross Entropy, CE (Perelman and Ostfeld 2007); Scatter Search, SS (Lin et al. 2007); Differential Evolution, DE (Vasan and Simonovic 2010); Self-Adaptive Differential Evolution, SADE (Zheng et al. 2012); Soccer League Competition, SLC (Moosavian and Roodsari 2014); and improved Genetic Algorithms (Bi et al. 2015).

Because these heuristic algorithms are originally developed to address specific engineering problems, there is no guarantee that the global optimum may be found or that the method will be efficient in the solution of specific problems, such as the design of WDNs. Wolpert and Macready (1997) cite the No-Free-Lunch theorem, assert that no one optimization algorithm may be suited for solving all kinds of optimization problems, and underscore the need for new algorithms that may address a wide range of problems or improve on efforts to reach global optima. In the area of water distribution systems, some comparisons of EA algorithms have been undertaken (Zheng et al. 2012; Moosavian and Roodsari 2014; Bi et al. 2015), though these studies focus on a select few methods. Moosavian and Lence (2017) present a rigorous comparison of six EAs in application to the optimum design of WDNs, and evaluate the algorithms in terms of the best solution obtained, the speed of convergence, and the numbers of function evaluations.

In applications of EAs, at each evaluation of the objective function, hydraulic analysis of the network must be conducted, which exacts a high computational burden, and is in fact the most computationally intensive portion of the optimization process. Moosavian and Lence (2017) show that the SLC algorithm has the best performance in terms of computational time and accuracy for single-objective optimization and design of WDNs, in comparison with other EAs.

1.3.1.2 Single-Objective Maximum Reliability Optimal Design

Given the output from probability-based WDN hydraulic analyses, i.e., the PDFs of pressure and flow, early methods for maximizing reliability of WDNs focus on stochastic optimization (Lansey et al., 1989). Recent work uses these output in linked-optimization-reliability analysis (Xu and Goulter, 1999; Tolson et al., 2004; Piralta and Ariaratnam, 2012; Torii and Lopez, 2012; Basupi and Kapelan, 2014).

Xu and Goulter (1999) present the application of First Order Reliability Method (FORM) for the reliability analysis of WDNs. This approach is capable of jointly identifying uncertainty in the nodal demands and pipe hydraulic capacities in addition to the influences of mechanical failure of system components. The main drawback of their work is the assumption of continuous pipe size diameter in the optimization model.

In order to address this drawback, Tolson et al. (2004) apply MCS and FORM for conducting the reliability analysis and GA for conducting the optimization of the network. They connect WADISO (an hydraulic solver), RELAN (a reliability analyzer), and a GA source code (as the optimizer) for conducting the reliability-based optimal design of a simple network. However, they only consider six predefined designs, and use RELAN to find failure probability of these designs initially and then GA to choose the best design among these solutions.

Piralta and Ariaratnam (2012) present a model for designing sustainable WDNs by minimizing life cycle costs and life cycle CO2 emissions, while ensuring reliability for the life time of the system. Torii and Lopez (2012) approximate the response of the system by an analytical solution (response surface) and then the reliability problem is solved using FORM. This approach is then extended to the analysis of systems that may have to work under several different configurations, which can be a consequence of repairing operations or some other unexpected event. Basupi and Kapelan (2014) present an optimization technique that can effectively identify adaptable solutions in the long-term planning and design of WDNs under uncertain future scenarios, such as those resulting due to climate change and urbanization.

1.3.1.3 Multi-Objective Minimum Cost/Maximum Reliability Optimal Design

Thus far, multi-objective design of WDNs is comprised of minimization of cost and maximization of reliability surrogates, as a means of improving hydraulic and mechanical reliability. Optimizing these reliability surrogates provides required water for users under all conditions, including during pipe bursts or as components deteriorate. Todini (2000) demonstrates the shortcomings of single-objective least-cost optimization approaches and explores the application of optimization for meeting the competing objectives of minimizing cost and maximizing reliability. Evaluation of the nondominated set of WDN solutions for these objectives, i.e., the Pareto optimal front, has been achieved using meta-heuristics such as ranked-based fitness assignment methods for Multi-Objective Genetic Algorithms, MOGAs (Fonseca and Fleming 1993), Strength Pareto EAs, SPEAs (Zitzler and Thiele 1999), and Nondominated Sorting Genetic Algorithms, NSGAs (Srinivas and Deb 1994), and more recently with hybrid meta-heuristics such as the multi-algorithm, genetically adaptive multi-objective, AMALGAM (Vrugt and Robinson 2007) method, which applies a simultaneous multi-method search of the fitness landscape and genetically adaptive offspring creation. For applications and comparisons of these, see, e.g., Farmani et al. (2005a) and Raad et al. (2009).

NSGA-II (Deb et al. 2002), an advanced nondominated sorting algorithm, is arguably the most popular multi-objective EA, and is increasingly used in WDN design (Farmani et al. 2006). It features implicit elitist selection based on the Pareto dominance rank, and a secondary selection method based on crowding distance, which significantly improves the performance of NSGA on difficult multi-objective problems. Having adopted the mechanisms of crossover and mutation in its GA, however, NSGA-II faces many of the challenges of GA, such as unstable and slow convergence, and difficulty in escaping from local optima (Storn and Price 1997; Iorio and Li 2004; Peng et al. 2009).

To overcome these limitations, Angira and Babu (2005) substitute the DE Algorithm for the GA in NSGA-II and develop the Nondominated Sorting Differential Evolution Algorithm (NSDE). A variation of NSDE, NSDE with ranking-based mutation (NSDE-RMO), uses the ranks of the population members to modify the mutation operator (Chen et al. 2014). Both NSDE and NSDE-

RMO exhibit improved stability, accelerated convergence, and increased diversity of solutions in applications to continuous multi-objective problems, and given the high performance of DE in single-objective WDN design, show potential for optimizing WDNs in terms of cost and reliability.

While similar surrogates have been proposed (Todini 2000; Geem 2015), the most widely used approach for estimating mechanical reliability is a cut-set analysis in which a given design is evaluated based on the number of single pipes that could be removed from operation without causing the network design to fail to meet specified nodal pressures. This approach exacts a high computational burden because every possible pipe removal scenario requires a re-evaluation of the hydraulic conditions, and thus it is generally undertaken post-optimization. Typically, in multi-objective design of WDNs, a large number of Pareto optimal solutions are generated, though generally only a small set of these are evaluated in depth for mechanical reliability (Farmani et al 2005b), and no standard approach for selecting which WDN solutions to investigate has been adopted. Several approaches for incorporating decision-making preferences to identify WDN solutions that achieve the best compromise among the objectives have been proposed (Vamvakeridou et al. 2006; Atiquzzaman et al. 2006), although these approaches are generally challenged as the number of solutions and objectives, and the complexity of the WDN, are increased.

Fuzzy programming models have been adapted as an alternative for evolutionary multi-objective optimization, for WDN designs comprised of fuzzy constraints (Goulter and Bouchart 1988; Xu and Goulter 1999; Bhave and Gupta 2006; Xu and Qin 2013) and objective functions that account for a single fuzzy benefit or weighted benefits (Vamvakeridou et al. 2006; Geem, 2015), and the degree of satisfaction of fuzzy constraints through penalty functions (Amirabdollahian et al. 2011).

1.3.2 WDN Hydraulic Analysis

WDN analysis is described here in terms of 1) deterministic analysis, 2) non-deterministic, henceforth referred to as uncertainty analysis, and 3) surrogate analysis. In deterministic analysis, all parameters and variables are assumed to have known values, and as a result of hydraulic simulation, a unique solution is obtained. In uncertainty analysis, all or some input parameters, such as nodal demands are assumed to be uncertain and hydraulic simulation of the network leads to a set of solutions with different likelihoods.

1.3.2.1 Deterministic Analysis

The flows and pressures in a WDN are governed by two sets of equations that describe the conservation of energy for each pipe and the conservation of mass for each node. Usually a steady-state network analysis problem is solved for a given set of boundary conditions, i.e., reservoir levels, nodal demands, pipe hydraulic resistance, pump characteristics, and minor losses. The related mathematical problem is partly nonlinear (i.e., the energy balance equations) and partly linear (i.e., the mass balance equations), provided that the demands are fixed *a priori*.

Generally, there are four kinds of formulations of the WDN model, the: 1) Q-formulation for which the unknown variables are pipe flows (Wood and Charles 1972); 2) H-formulation where the unknown variables are nodal pressures (Martin and Peters 1963); 3) ΔQ-formulation where the unknown variables are the flow corrections required to balance the energy equations in each pipe loop (Todini and Pilati 1988); and 4) Q-H-formulation where the unknown variables are both nodal pressures and pipe flows (Todini and Pilati 1988; Todini and Rossman 2013).

Arora (1976) suggested an approach based on the principle of conservation of energy. According to the principle, "Flow in the pipes of a hydraulic network adjusts so that to minimize the expenditure of the system energy." In this work, an expenditure of energy as a function of pipe flows and head-loss is defined. The minimization of this term for all pipes, along with the continuity equations as linear constraints, leads to the hydraulic analysis of the WDN. Then, Collins et al. (1978) propose two optimization models termed the content and co-content models, which are based on the Q-formulation and H-formulation, respectively. The Q-formulation or content model is similar to Arora's work and minimizes the expenditure of the energy in a constrained optimization model. The H-formulation or co-content model transforms all flow-based terms into pressure-based terms. This transformation makes an unconstrained model for which the continuity equations is satisfied automatically by the minimization of the model. Collins et al. (1978) use three classical piece-wise LP and NLP solvers for minimization of these models.

Cross (1936) presents a local linearization method for hydraulic analysis based on successive corrections of pipe flows in each loop (the ΔQ-equations). Cross's method is widely used for small-scale networks, due to its simplicity. The major drawback of this method is its difficulty in selecting the initial solution as the network size increases. Also, the speed of convergence of the approach decreases with increases in size, and in some cases divergence may occur.

Martin and Peters (1963) first used the Newton-Raphson method in the analysis of water supply networks. In their method, all equations are written in terms of nodal heads (the H-equations). Then the solution is obtained by correcting the nodal heads in successive iterations. Again, one of the disadvantages of this method is the lack of optimal convergence for large-scale networks. To eliminate this problem, some pipes of the network may be temporarily removed in the analysis process. Another disadvantage is high oscillations in achieving optimal convergence. To reduce the oscillations, the value of ΔH may be reduced by one-half, though this will increase the number of iterations.

Liu (1969) presents a simpler version of the Newton-Raphson method for solving the H-equations. In this method, the Jacobian matrix is decomposed into diagonal and non-diagonal matrices. Considering the importance of the diagonal matrix, the non-diagonal matrix is eliminated for simplification. Consequently, solving the remaining linear equations becomes very easy. However, if an inappropriate initial solution is chosen, this method may diverge.

The use of Linear Theory was first proposed by Wood and Charles (1972) for solving the Q-equations. In this method, for each iteration, the nonlinear equations of energy are linearized and the linearized equations are solved simultaneously. Convergence to a final solution is obtained by updating the flow rate in each iteration, and correcting the linearization approximations. One of the disadvantages of this method is that oscillations commonly occur in the convergence process. To reduce these oscillations, the flow rate is assumed to be equal to the average discharge rates of the previous and past previous iterations (Wood and Charles, 1973). Wood and Funk (1993) use an extended Taylor series to propose a new method for solving the Q-equations. This approach is exactly the same as the Newton-Raphson method for solving the Q-equations. Currently, this algorithm is used to analyze WDNs in the commercial software KYPIPE (Wood 1980).

Given the initial success of the Linear Theory-based approach, Isaac and Mills (1980) develop a version of this method based on the H-equations. The advantage of this method compared with that of Wood and Charles (1972) is its modelling simplicity and the symmetry of its coefficient matrix, in which the nodal heads are estimated without providing for mass conservation. This method becomes unstable for discharge rates that are close to zero. To avoid this problem, Wood and Charles (1972) suggest eliminating the pipes with flow rates close to zero.

Todini and Pilati (1988) apply the Global Gradient Algorithm (GGA) for hydraulic analysis of pipe networks for solving the Q-H-equations. In this method, the energy loss equations in the loops, and the continuity equations at the nodes, are defined as an objective function and constraints, respectively, for an optimization problem. Then, using the Lagrange multipliers, the constraints of the optimization problem are removed and the Lagrange equations are solved using the Newton-Raphson method. GGA exhibits excellent convergence characteristics and is used in the freely available EPANET2 software (Rossman, 2000).

Basha and Kassab (1996) present a method for solving the Q-H-equations based on the theory of disturbances. For most networks, this approach results in a nearly-final solution within three iterations.

Giustolisi et al (2012) present a matrix-based transformation method that improves the numerical and computational behavior of GGA, referred to as the Enhanced GGA. This algorithm allows for a simplified topological representation of the network whilst preserving hydraulic accuracy. It achieves superior computational performance when compared with GGA.

Recently, Moosavian and Jaefarzadeh (2014) develop a multistage linearization method in the hydraulic analysis of water distribution networks for solving the Q- and Q-H-equations. Moosavian (2017) advances this technique and shows that this method can accurately simulate WDNs and is significantly better than GGA for large-size networks.

1.3.2.2 Uncertainty Analysis

In real WDNs, many of the known parameters, such as nodal demands and pipe roughness coefficients, are uncertain and therefore calculation of crisp values for pressures and flows is not logical. Two approaches for addressing uncertainty of WDNs are proposed in the literature, these are based on: 1) probability theory and probability theory in support of reliability analysis, and 2) fuzzy set theory. Under the first approach, a PDF of the hydraulic input information is developed based on historical measurements, and unknown hydraulic output parameters are estimated using e.g., MCS. In analysis of large WDNs, undertaking MCS requires significant computational time because of the large numbers of hydraulic simulations required.

Probability theory may also be used to support tools for undertaking reliability analysis. A broad definition of reliability is the degree of accuracy of a measurement, calculation, or specification.

In engineering design, reliability is most often viewed as the probability that satisfactory performance will occur, given uncertainty regarding knowledge of the physical conditions that govern a problem, and its inherent variability. Reliability analysis estimates the failure probability of an element, system, or several systems working together, given the limit-state function that describes the boundary between domains of failure and success based on the difference between the PDF of the resistance of, and the load on, the element or system. Approaches for assessing uncertainty in WDN simulation focus on implementation of probabilistic analyses of the condition of network components (Damelin et al., 1972; Santinelli, 2016), the level of system demands (Mays and Cullinane, 1986; Giustolisi et al., 2005; Giustolisi et al., 2009), or both (Lansey et al., 1989; Xu and Goulter, 1999; Tolson et al., 2004; Kapelan et al., 2005), where the inherent variations in these parameters are characterized with estimates of their PDFs.

WDN uncertainty comprises not only random input variability, but also factors that are distinct from randomness, including incomplete or qualitative knowledge about the physical system, and imprecise data and observations. Imprecision in measurable information, such as a pressure level, may render the results of the probabilistic analysis of pressure, itself, as uncertain. Subjective information, such as the relationship between consumptive behavior and economic development, and semi-qualitative information, such as the degradation of pipe conditions over time, may need to be described based on engineering experience or judgement. Recognizing that such uncertainties limit the applicability of probability-based WDN analyses, and that any approach for assessing system dependability in the face of these subjective uncertainties should be logically consistent, parallel approaches are proposed that employ fuzzy variables (Zadeh, 1965) to describe subjective and imprecise information regarding WDN inputs for simulation (Revelli and Ridolfi, 2002; Bhave and Gupta, 2006; Spiliotis and Tsakiris, 2012) and calibration and optimization (Goulter and Bouchart, 1988; Xu and Goulter, 1999; Vamvakeridou-Lyroudia et al., 2005; Pitratla and Ariaratnam, 2012; Basupi and Kapelan, 2014). While fuzzy variables convey less information than statistical distributions, fuzzy set theory may be used to calculate Fuzzy Membership (FM) functions that describe these variables, and thereby translate imprecise or subjective information into mathematical language. Typical nodal pressures and demands, and pipe roughness coefficients, may be described with triangular or trapezoidal FM functions. These functions are then applied as input to models of the system behavior, e.g., hydraulic simulation, and used to determine the membership functions of system outputs, such as pressure heads at the nodes.

Fuzzy set theory was first developed for the analysis of pipe networks considering uncertain demands, and roughness coefficients, based on the Q-equations, by Revelli and Ridolfi (2002). Since both nodal demands and pipe flows are unknown, they suggest a simple single-objective optimization model to find the extreme values of pipe discharges. A sequential quadratic programming (SQP) technique is used to solve this optimization model. Xu (2003) demonstrate that this approach provides very reliable estimates of uncertainty in the analysis of WDNs. However, its application to real sized networks may be limited by its excessive computational requirements. For example, for a network with nn nodes and np pipes and for FM functions with nf levels of membership, solutions of $2 \times (nf - 1) \times (nn) + 1$ nonlinear optimization problems would be required in order to provide a full representation of the uncertainty associated with each nodal head and pipe flow. Xu (2003) also claims that for large networks, which may involve thousands of nodes and pipes, the method of Revelli and Ridolfi (2002) may be less computationally efficient than MCS.

In order to reduce the computation of burden, Gupta and Bhave (2007) propose a direct approach to accelerate the search technique and remove the optimization model of Revelli and Ridolfi (2002). In this methodology, a hydraulic analysis is conducted based on the most likely values of the pipe friction coefficients and nodal demands, and the gradients of the hydraulic response at each node, to changes in this input information, are estimated. Assuming that the gradients are constant over the entire variable space, i.e., responses are either monotonically decreasing or increasing, with changes in input information, the extreme responses are then obtained by substituting the upper or lower bound of the input information into the hydraulic simulator. In order to fully represent the uncertainty associated with each nodal head and pipe flow, for a network with nn nodes and np pipes and for FM functions with nf levels of membership, $(nf - 1) \times (3 \times nn + np) + 1$ hydraulic simulations would be required. Fu and Kapelan (2011) propose the use of fuzzy random variables to characterize uncertainties in WDN design. For each node in the WDN, they develop a fuzzy random variable that describes the nodal pressure head and define the magnitude of failure as the FM at which this fuzzy variable is less than the fuzzy required pressure head. For a given WDN design, they use MCS of nodal demands and pipe roughness coefficients to estimate the extreme values of nodal pressures at each level. Thus, analysis of each potential design is computationally intensive as it requires the same number of hydraulic simulations as Monte Carlo realizations. Spiliotis and Tsakiris (2012) apply a Newton-

Raphson method for fuzzy analysis of pipe networks facing only fuzzy demands. In order to provide all FM functions for nodal heads, for a network with nn nodes and np pipes and for FM functions with nf levels of membership, solutions of $(nf - 1) \times (2 \times nn) + 1$ nonlinear systems of H-equations would be required. Unlike the work of Revelli and Ridolfi (2002), this method assumes that the minimum and maximum value of head pressures occur when the fuzzy demands are at their extreme values, and does not consider uncertainty of pipe roughness coefficients in the networks. Sabzkouhi and Haghighi (2016) apply an EA (many-objective PSO), coupled with a network hydraulic simulation model, to find extreme values of nodal pressures and pipe velocities. Their approach needs 29,376 hydraulic analyses for a small network (three nodes and five pipes) and 590,040 for a network with 45 nodes and 65 pipes. Therefore, it can be concluded that the technique is also computationally inefficient for the fuzzy analysis of large WDNs.

1.3.2.3 Surrogate Analysis

As mentioned in section 1.3.1.3, a related concept, called resilience, can be applied to assess reliability implicitly in WDN analysis and design (Basupi and Kapelan, 2014). Resilience is defined as the capability of system to "bounce back" following some failure and therefore can be approximated as the capacity of a network to absorb disturbance while experiencing alterations so as to retain essentially the same function, structure, identity, and feedback information. Todini (2000) introduces a surrogate for reliability, the resilience index, and proposes a simple heuristic that emulates the reasoning of an engineer whose goal is to reduce cost while preserving an acceptable level of reliability of the system to handle possible failures. He defines the resilience index as the ratio of the total excess power in the network, i.e., the product of the consumption and excess pressure heads at all nodes, and the excess hydraulic power provided by the source reservoirs and pumps, after satisfying all demands. While it is not based on probabilistic failure considerations, increases in Todini's resilience index has been shown to lead to improved network reliability or robustness under cases of mechanical failure. While other surrogates for reliability have been examined, e.g., modifications of Todini's index as a function of the diameters of pipes selected (Prasad and Park 2004), maximum head deficiency (Farmani et al. 2005b), flow entropy (Prasad and Tanyimboh 2008), and a combination of these (Raad 2011), Todini's resilience index is the most widely used.

1.5 Example Networks

In this section, the WDNs used to demonstrate the techniques developed in Chapters 2-7 are introduced and relevant data such as pipe characteristics and nodal demands for each network are provided in detail.

1.5.1 Two-loop network

This network, shown in Figure 1.2, is a hypothetical benchmark which has seven nodes and eight pipes with two loops, and is fed by a reservoir with a 210-m fixed head (Alperovitz and Shamir 1977). The pipes are all 1,000 m long with a Hazen-Williams coefficient of 130. The minimum pressure limitation is 30 m above ground level for all nodes. There are 14 available commercial pipe diameters. Although the Two-loop network is small, a complete solution enumeration comprises $14^8 = 1.48 \times 10^9$ different network designs, thus making this illustrative example difficult to solve. Geem (2006) determines the minimum cost of this network to be $419,000 US. Relevant data regarding to this network are provided in Tables 1.1 to 1.3. This network has been used in the optimization results of Chapter 2-4.

1.5.2 Hanoi network

The Hanoi network, shown in Figure 1.3, consists of 31 nodes, 34 pipes with 3 loops, and is fed by gravity from a reservoir with a 100-m fixed head. The pipe lengths vary from 100 to 3500 m, with a Hazen-Williams coefficient of 130, and the minimum head limitation at all nodes is 30 m above ground level (Alperovits and Shamir 1977). There are six possible pipe diameters and 34 pipes in the system. Therefore, the total solution space is $6^{34} = 2.865 \times 10^{26}$. The global optimum solution has the total cost of $6.1 million US (Alperovits and Shamir 1977). Relevant data regarding to this network are provided in Tables 1.4 and 1.5. This network has been used to demonstrate the techniques developed in Chapters 2-4.

1.5.3 NewYork network

Schaake and Lai [25] first presented the New York City network, shown in Figure 1.4. It consists of 20 nodes, 21 pipes and 1 loop, and is fed by gravity from a reservoir with a 91.44-m fixed head. The objective of the problem is to add new pipes parallel to existing ones because the existing network cannot satisfy the pressure head requirements at certain key nodes (nodes 16–20). The Hazen–Williams constant C for all pipes is 100. For each duplicate tunnel there are 16 allowable decisions including 15 available diameters and the ‘do nothing'' option; therefore the search space of this optimization problem is $16^{21} = 1.93 \times 10^{25}$ possible designs. Relevant data regarding this network are provided in Tables 1.6 and 1.7. This network has been used to demonstrate the technique developed in Chapter 4.

1.5.4 Farhadgerd network

The Farhadgerd WDN serves a population of approximately 8200 in the town of Farhadgerd, Iran, a residential community near the regional capital, Mashhad. The network shown in Figure 1.5, comprises 68 pipes and 53 nodes, has one reservoir with a head of 510 m, and a minimum pressure requirement of 20 m at all nodes. There are nine possible pipe diameters, therefore the search space of this optimization problem is $9^{68} = 7.74 \times 10^{68}$. The recorded optimal cost for Farhadgerd is \$18 million US. Relevant data regarding to this network are provided in Tables 1.8, 1.9 and 1.10. This network was developed by the author and has been used to demonstrate the techniques developed in Chapters 2-6.

1.5.5 Nagpur network

The Nagpur network is introduced by Gupta and Bhave (2007), and is based on that for the Gittikhadan Zone of Nagpur, India. It has one source node, 179 pipes, 141 demand nodes, and 38 loops. A representation of the network is shown in Figure 1.6. The source node is a reservoir with a surface elevation of 335.84 m and supplies 755.26 L/s to the system. Relevant network data, such as nodal demands, pipe diameters, lengths, and connectivity, are provided in Gupta and Bhave (2007). This network has been used for demonstrating the performance of Fuzzy Analysis methods in Chapter 7.

1.5.6 EXNET network

The EXNET network, introduced by Farmani et al. (2003), has 1,892 nodes, 2,467 pipes, and two reservoirs. A representation of the network is shown in Figure 1.7. The range of normal nodal demands were between zero and 0.1255 cubic meter per second (cms); and the range of resistance factors for pipes were set to be between 0.0116 and 9.9952. Relevant data regarding to this network are provided in Farmani et al. (2003). This network has been used for demonstrating the performance of Fuzzy Analysis methods in Chapter 7.

1.5.7 Hypothetical networks

Hypothetical networks are randomly generated by the author to expand the range of network sizes. In all cases, for the specified numbers of nodes and pipes, whether a pipe connects any two nodes is randomly determined with equal likelihood, and the network is formed under the conditions that each pipe connects only two nodes and that each node is connected to at least two pipes. For every 20 nodes, a source reservoir with a constant fixed pressure head is randomly assigned to one of the nodes, and all other nodes of the network are assumed to be at sea level (e.g., Zecchin et al. 2012 and Moosavian 2017). The parameters of each reservoir and pipe are sampled independently from uniform distributions as follows: reservoir elevation = u (50, 100), nodal demands = u (0, 0.1), and pipe length = u (500, 650)], in which u (LB, UB) symbolizes a random variable uniformly distributed between LB and UB. The Hazen-Williams coefficient of all pipes is assigned a constant value of 130. For these networks, 20 candidate commercial pipe diameters are possible, ranging from 50 to 1,000 mm, in increments of 50 mm. The smallest of these networks has 100 nodes and 200 pipes, and the largest network has 400 nodes and 800 pipes. The number of combinatorial sets of pipe diameter options for these networks range from 20^{200} to 20^{800}. These networks are used for measuring the performance of multi-objective algorithms in Chapter 3.

1.6 Summary

In this chapter, the different aspects of WDN modelling are presented and recent research is reviewed. Generally, WDN modelling is divided into the two subcategories of WDN design and WDN analysis. Each category has many options for solution which directly or indirectly influence the design and analysis of WDNs. Researchers usually combine different features of WDN design with WDN analysis to develop new tools for evaluation or reliability-based optimization of WDNs.

Table 1.1 Candidate pipe size diameters and corresponding costs for Two-loop network

Diameter (mm)	Cost ($/meter length)
25.4	2
50.8	5
76.2	8
101.6	11
152.4	16
203.2	23
254	32
304.8	50
355.6	60
406.4	90
457.2	130
508	170
558.8	300
609.6	550

Table 1.2 Nodal information for Two-loop network

Node	Elevation (m)	Demand (m^3/h)
R-1	210	-
J-2	150	100
J-3	160	100
J-4	155	120
J-5	150	270
J-6	165	330
J-7	160	200

Table 1.3 Pipe information for Two-loop network

Pipe	L (m)	Hazen-Williams factor, C
P-1	1000	130
P-2	1000	130
P-3	1000	130
P-4	1000	130
P-5	1000	130
P-6	1000	130
P-7	1000	130
P-8	1000	130

Table 1.4 Candidate pipe size diameter and corresponding costs for Hanoi network

Diameter (mm)	Cost ($/meter length)
304.8	45.726
406.4	70.40
508.0	98.378
609.6	129.333
762.0	180.748
1016	278.28

Table 1.5 Hydraulic data for Hanoi network

Node	Demand(m^3/h)	Node	Demand(m^3/h)	Pipe	L (m)	Pipe	L (m)
1	-19,940	17	865	1	100	18	800
2	890	18	1345	2	1350	19	400
3	850	19	60	3	900	20	2200
4	130	20	1275	4	1150	21	1500
5	725	21	930	5	1450	22	500
6	1005	22	485	6	450	23	2650
7	1350	23	1045	7	850	24	1230
8	550	24	820	8	850	25	1300
9	525	25	170	9	800	26	850
10	525	26	900	10	950	27	300
11	500	27	370	11	1200	28	750
12	560	28	290	12	3500	29	1500
13	940	29	360	13	800	30	2000
14	615	30	360	14	500	31	1600
15	280	31	105	15	550	32	150
16	310	32	805	16	2730	33	860
				17	1750	34	950

Table 1.6 Candidate pipe size diameter and corresponding costs for New York network

Diameter (in)	Cost ($/ft length)
0	0
36	93.5
48	134
60	176
72	221
84	267
96	316
108	365
120	417
132	469
144	522
156	577
168	632
180	689
192	746
204	804

Table 1.7 Hydraulic data for New York tunnel network

Node	Demand	Minimum head (ft)	Pipe	L(ft)	Existing diameter (ft)
1	-22018	300	1	11,600	180
2	92.4	255	2	19,800	180
3	92.4	255	3	7,300	180
4	88.2	255	4	8,300	180
5	88.2	255	5	8,600	180
6	88.2	255	6	19,100	180
7	88.2	255	7	9,600	132
8	88.2	255	8	12,500	132
9	170	255	9	9,600	180
10	1	255	10	11,200	204
11	170	255	11	14,500	204
12	117.1	255	12	12,200	204
13	117.1	255	13	24,100	204
14	92.4	255	14	21,100	204
15	92.4	255	15	15,500	204
16	170	260	16	26,400	72
17	57.5	272.8	17	31,200	72
18	117.1	255	18	24,000	60
19	117.1	255	19	14,400	60
20	170	255	20	38,400	60
			21	26,400	72

Table 1.8 Candidate pipe size diameter and corresponding costs for Farhadgerd network

D (mm)	Cost ($/m)
63.8	638
79.2	792
96.8	968
150	1500
200	2000
250	2500
300	3000
350	3500
400	4000

Table 1.9 Nodal information for Farhadgerd network

Node	Elv (m)	Demand (L/s)	Node	Elv (m)	Demand (L/s)
R-1	510	0	J-28	454.5	0.9242
J-1	479	2.1037	J-29	458	1.0336
J-2	468.7	3.3318	J-30	455.8	1.2403
J-3	466.5	1.4227	J-31	437	1.4462
J-4	467.1	0.5594	J-32	434.6	0.9990
J-5	469	0.1581	J-33	430	1.3035
J-6	467.3	1.2282	J-34	439.4	0.3996
J-7	469.2	2.4685	J-35	433.3	16.3083
J-8	460.5	0.3891	J-36	437	1.1037
J-9	461.3	4.5357	J-37	455.6	0.5594
J-10	465.4	0.9850	J-38	468.5	10.3238
J-11	476.3	0.4621	J-39	472.7	0.9850
J-12	477	0.6931	J-40	478.7	0.9485
J-14	472.5	0.1581	J-41	486	0.9971
J-15	478.7	0.6688	J-42	477.5	0.3770
J-16	478.5	0.5594	J-43	473.3	0.7053
J-17	475.5	4.7546	J-44	471.4	2.1037
J-18	475	0.2067	J-49	431.2	0.3140
J-19	480	0.3162	J-50	431.8	0.1618
J-20	472	1.3133	J-51	462.8	3.5872
J-21	456.2	0.6931	J-52	440	0.0000
J-22	458.3	1.2890	J-53	454.5	0.1000
J-23	446.3	0.8147	J-13	455.8	0.0000
J-24	440	0.5715	J-45	455.8	0.0000
J-25	445.4	1.5200	J-47	440	0.0000
J-26	438	2.7688	J-48	440	0.0000
J-27	441	2.0076	J-46	455.8	0.0000

Table 1.10 Pipe characteristics for Farhadgerd network

Pipe	J-in	J-out	L (m)	C	Pipe	J-in	J-out	L (m)	C
P-1	R-1	J-1	885.3851	120	P-41	J-35	J-36	437.3994	130
P-2	J-1	J-2	328.5421	120	P-42	J-36	J-26	359.4174	130
P-3	J-2	J-3	86.0021	120	P-43	J-30	J-37	141.5928	130
P-4	J-3	J-4	195.1899	120	P-44	J-37	J-38	667.3086	130
P-5	J-4	J-5	30.9500	120	P-45	J-38	J-39	242.5379	120
P-6	J-5	J-6	68.4875	130	P-46	J-39	J-40	229.1806	130
P-7	J-6	J-7	563.0813	130	P-48	J-41	J-42	203.4631	130
P-8	J-7	J-8	417.1908	130	P-49	J-42	J-43	100.3686	130
P-9	J-8	J-9	416.8621	120	P-50	J-43	J-29	503.9709	120
P-10	J-9	J-4	294.4953	120	P-51	J-38	J-44	385.2943	130
P-11	J-7	J-10	367.3761	130	P-57	J-32	J-49	190.6523	130
P-12	J-10	J-11	372.3605	130	P-58	J-49	J-50	208.6375	130
P-13	J-11	J-12	147.7453	130	P-59	J-50	J-33	227.8588	130
P-16	J-14	J-15	213.1492	130	P-60	J-34	J-31	278.1212	120
P-17	J-15	J-16	233.7499	130	P-61	J-7	J-14	322.7427	130
P-18	J-14	J-17	315.1370	130	P-62	J-12	J-16	193.8307	130
P-19	J-17	J-18	247.3020	130	P-63	J-27	J-35	354.9940	130
P-20	J-18	J-19	155.8379	130	P-64	J-2	J-51	342.1014	130
P-21	J-19	J-20	138.7134	130	P-56	J-51	J-28	299.1446	130
P-22	J-20	J-5	302.7770	130	P-55	J-1	J-43	135.5139	120
P-23	J-9	J-21	202.5051	130	P-54	J-43	J-39	444.0197	120
P-25	J-22	J-3	421.6994	130	P-53	J-45	J-31	505.6546	120
P-26	J-21	J-23	451.5464	130	P-52	J-47	J-26	247.9111	130
P-27	J-23	J-24	94.9841	130.013	P-47	J-53	J-27	445.0974	130
P-28	J-24	J-25	138.8879	130	P-38	J-52	J-53	504.6454	130
P-29	J-25	J-22	381.8934	130.013	P-36	J-53	J-13	468.0775	130
P-33	J-28	J-25	388.1121	130	P-32	J-48	J-52	16.5572	130
P-34	J-28	J-29	348.5208	130	P-31	J-46	J-13	16.3047	120
P-35	J-29	J-30	125.7833	120	P-30	J-45	J-13	52.5861	120
P-37	J-31	J-32	333.0668	130	P-24	J-46	J-30	14.3932	120
P-39	J-33	J-34	286.3284	130	P-15	J-47	J-52	74.3145	130
P-40	J-34	J-35	436.9019	130	P-14	J-48	J-24	13.5482	130

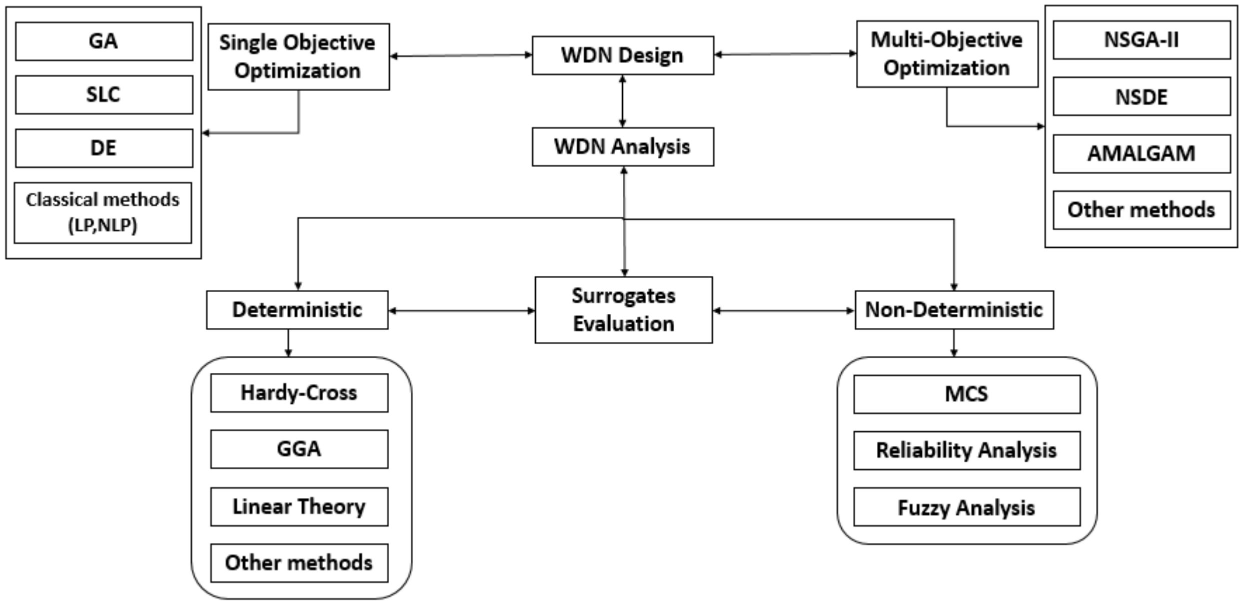


Figure 1.1 WDN modeling framework

GA = Genetic Algorithm;
DE = Differential Evolution;
SLC = Soccer League Competition;
GGA = Global Gradient Algorithm;
MCS = Monte-Carlo Simulation;
NSGA-II = Nondominated Sorting Genetic Algorithm;
NSDE = Nondominated Sorting Differential Evolution;
LP = Linear Programming;
NLP = Nonlinear Programming;

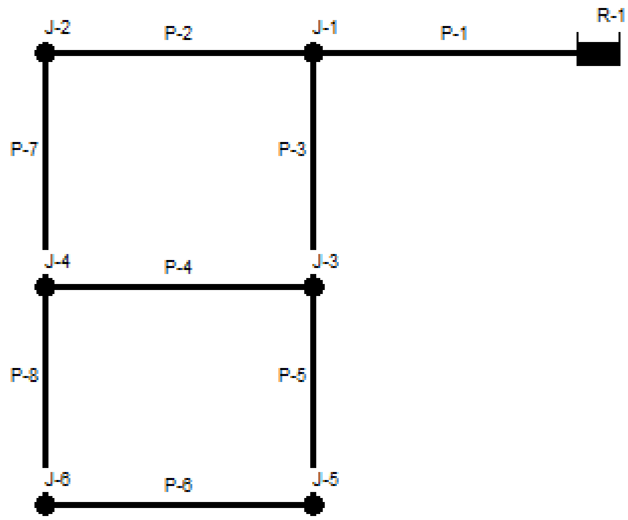


Figure 1.2 Two-loop network

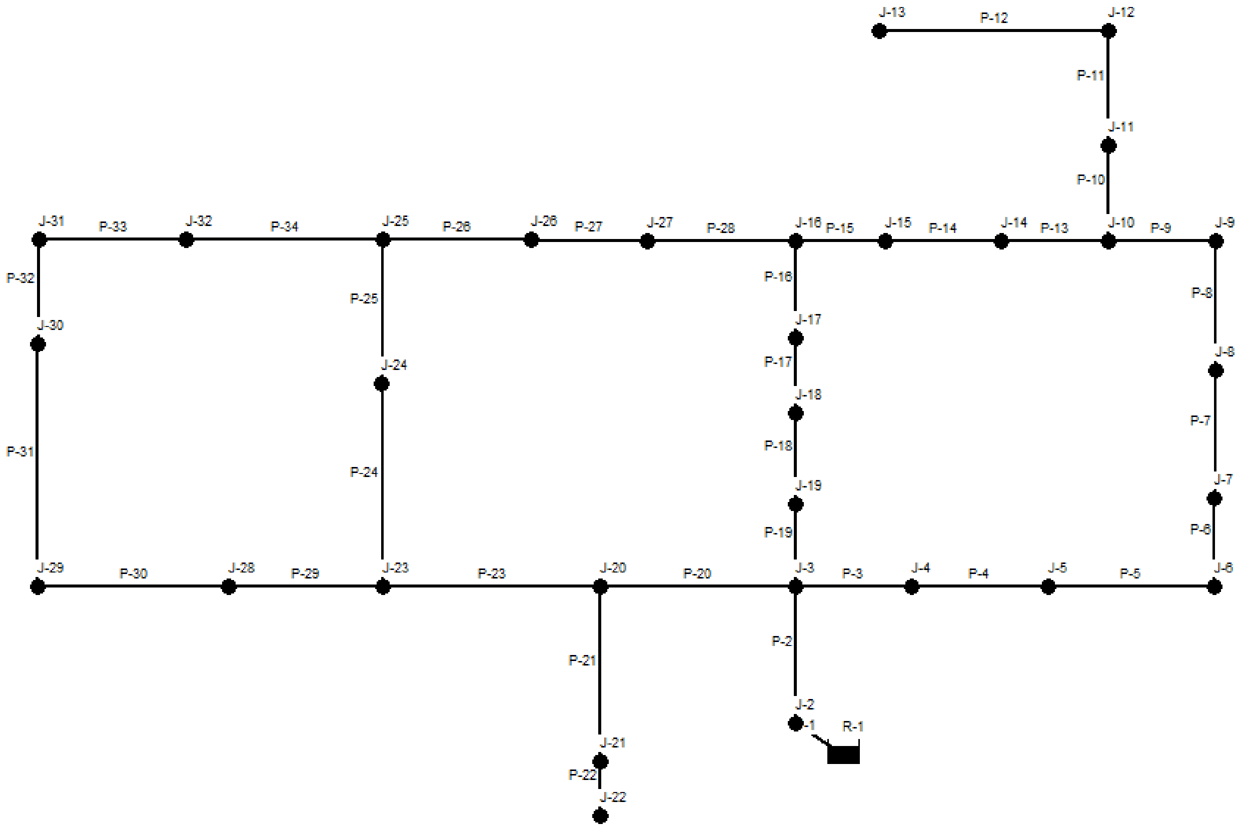


Figure 1.3 Hanoi network

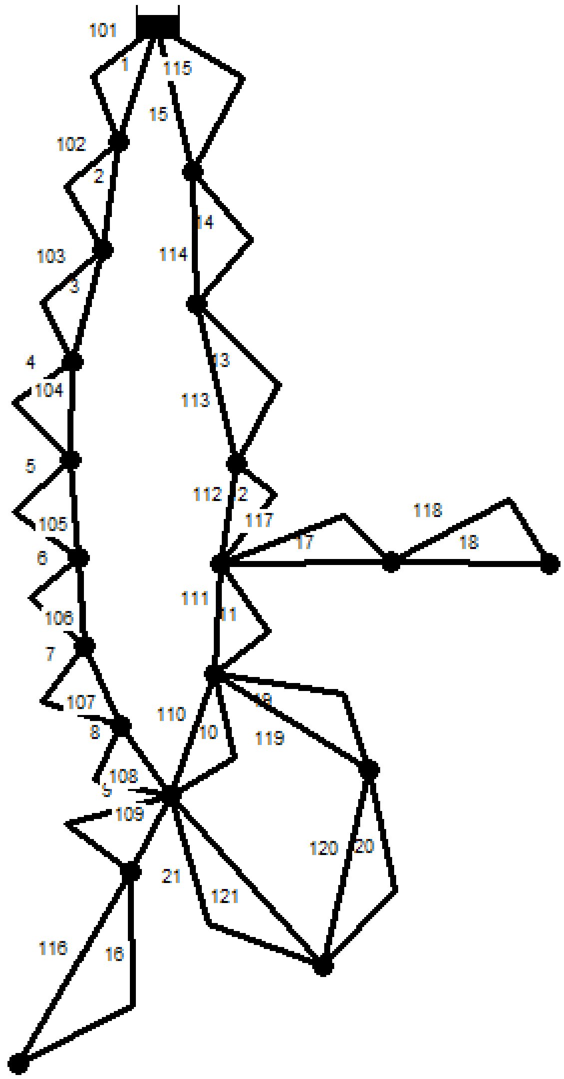


Figure 1.4 New York tunnel network

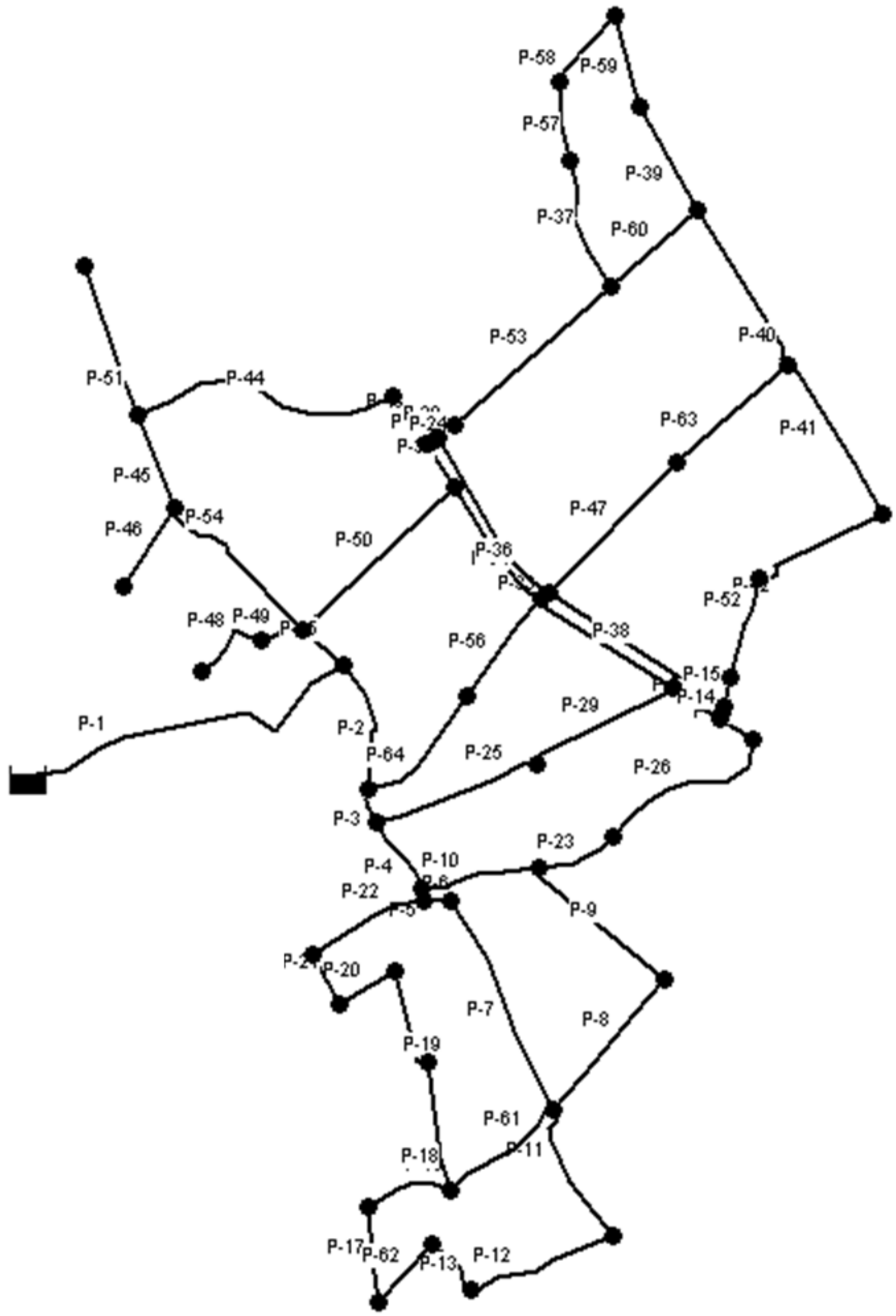


Figure 1.5 Farhadgerd network

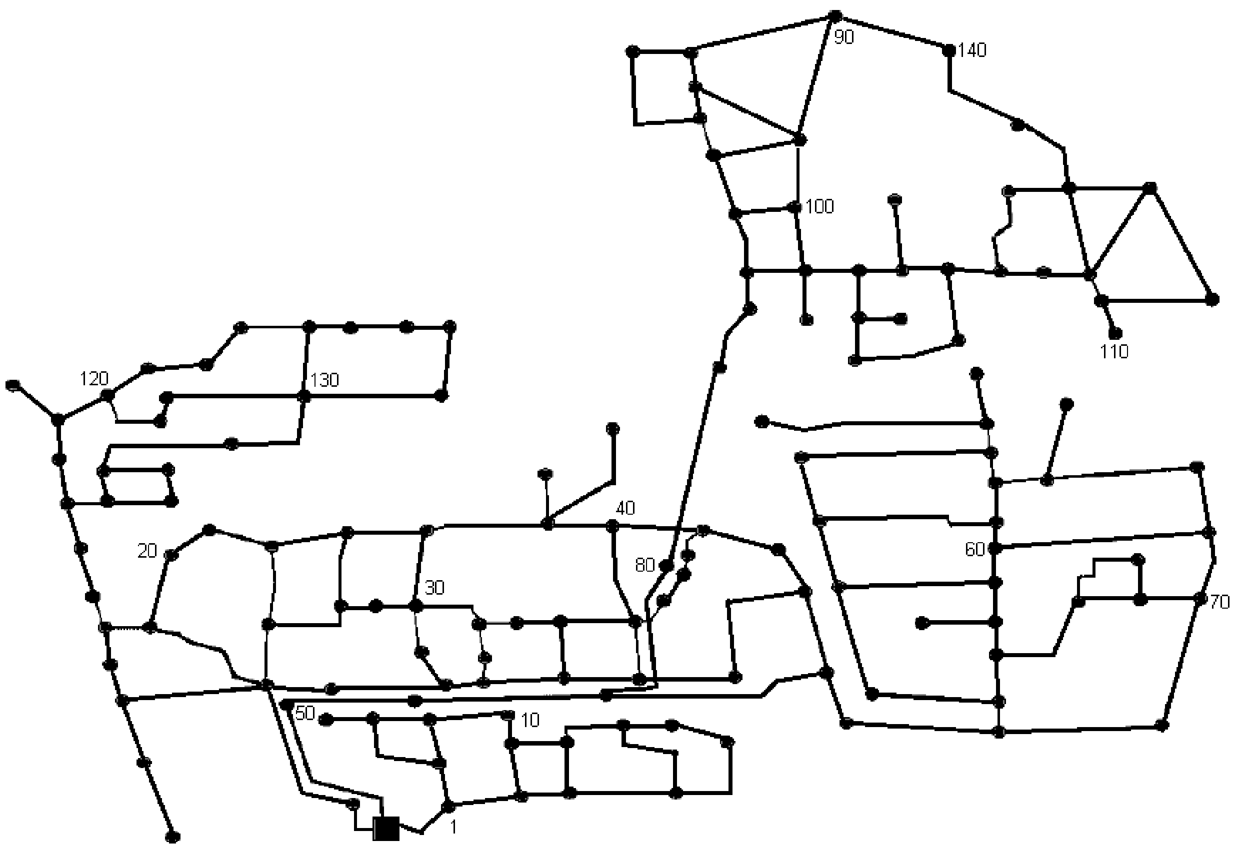


Figure 1.6 Nagpur network

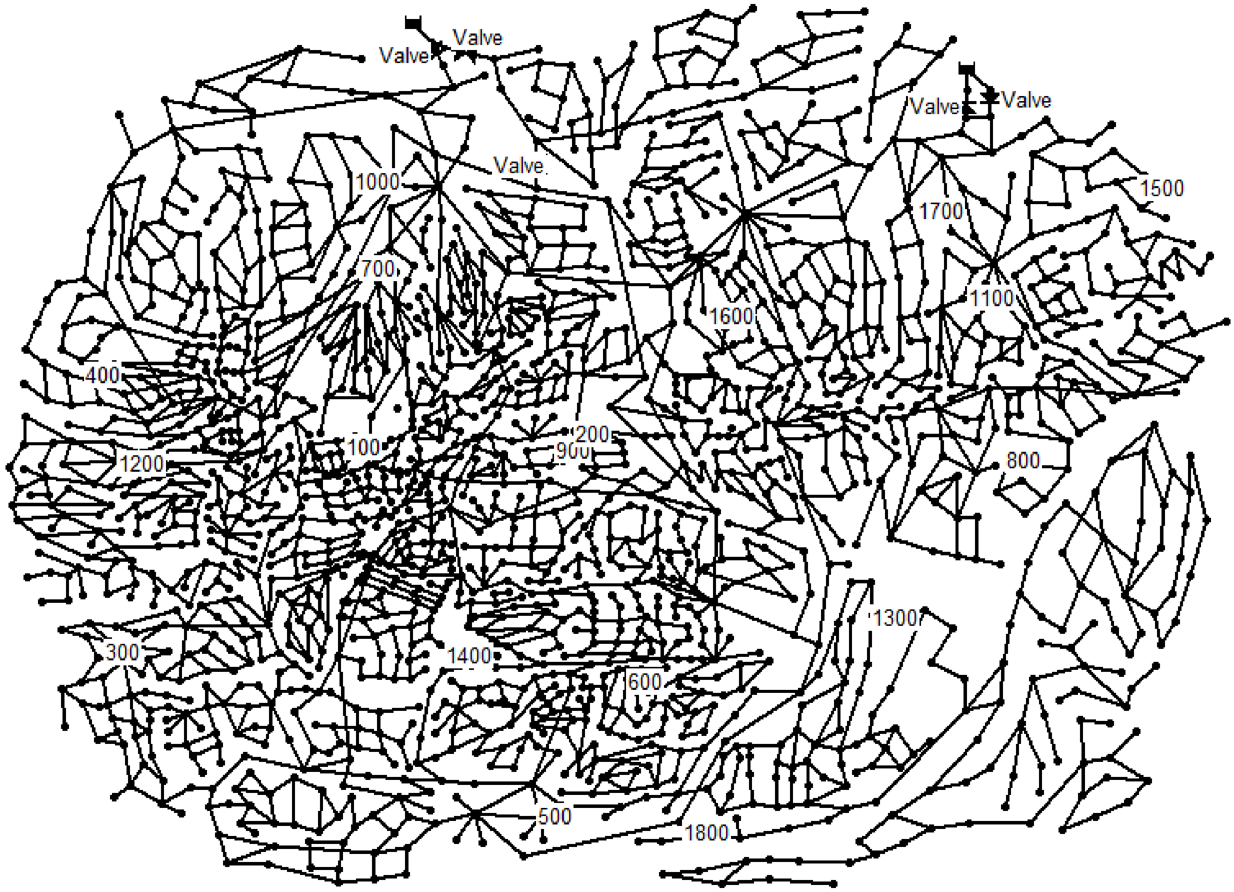


Figure 1.7 EXNET network

Chapter 2 : Testing Evolutionary Algorithms for Optimization of Water Distribution Networks

2.1 Preface

Based on the WDN framework presented in Figure 1.1, WDN design may be divided into single- and multi-objective optimization. Under either approach, WDN design may be classified as a large combinatorial discrete nonlinear optimization problems. The main concerns associated with optimization of WDNs are related to the nonlinearity of the discharge-head loss relationships for pipes and the discrete nature of pipe sizes. Faced with these problem characteristics, classical optimization methods, such as Linear (LP) and Nonlinear Programming (NLP), cannot find the global optimum solutions to WDN design problems. Over the last three decades, many heuristic optimization algorithms have been successfully used to identify WDN designs. In this chapter, the investigation of convergence characteristics and efficiency of ten EAs is undertaken for the single-objective design of three networks. The algorithms include the: Genetic Algorithm (GA); Ant Colony Optimization (ACO); Harmony Search (HS); Differential Evolution (DE); Particle Swarm Optimization Gravitational Search Algorithm (PSOGSA); Artificial Bee Colony (ABC); Soccer League Competition (SLC); Covariance Matrix Adaptation Evolution Strategy (CMAES); MEtaheuristics for Systems Biology and BIoinformatics Global Optimization (MEIGO); and Whale Optimization (WOA) meta-heuristic algorithms. All of these algorithms search for the global optimum with populations of solutions, rather than by improving a single solution, as in Newton-based search methods. They start with an initial population and use different operators to improve the population's performance over repeated iterations. The algorithms are compared in terms of the mean, median, standard deviation, minimum, and maximum of all executions for each network, and in terms of the results of Friedman tests. Small standard deviation values indicate that an algorithm provides an attractive and consistent performance for different initial random populations. Results show that the CMAES and SLC algorithms consistently converge to the global optimum for all networks, DE and ABC algorithms along with SLC are suitable for small networks, and the WOA algorithm is not well-suited for optimization of WDNs.

2.2 Abstract

Water distribution networks (WDNs) are one of the most important elements of urban infrastructure and require large investment for construction. Design of WDNs is classified as a

large combinatorial discrete nonlinear optimization problem. The main concerns associated with the optimization of such networks are the nonlinearity of the discharge-head loss relationships for pipes and the discrete nature of pipe sizes. Due to these issues, this problem is widely considered to be a benchmark problem for testing and evaluating the performance of nonlinear and heuristic optimization algorithms. This paper compares different techniques, all based on evolutionary algorithms (EAs), which yield optimal solutions for least-cost design of WDNs. All of these algorithms search for the global optimum starting from populations of solutions, rather than from a single solution, as in Newton-based search methods. They use different operators to improve the performance of many solutions over repeated iterations. Ten EAs, four of them for the first time, are applied to the design of three networks their performance in terms of the least cost, under different stopping criteria, are evaluated. Statistical information for 20 executions of the ten algorithms are summarized, and Friedman tests are conducted. Results show that, for the Two-Loop benchmark network, the particle swarm optimization gravitational search (PSOGSA) and biology and bioinformatics global optimization (MEIGO) algorithms efficiently converge to the global optimum, but perform poorly for large networks. In contrast, given a sufficient number of function evaluations, the covariance matrix adaptation evolution strategy (CMAES) and soccer league competition (SLC) algorithm consistently converge to the global optimum, for large networks.

Key words: Evolutionary algorithms, genetic algorithm, water distribution networks, optimization.

2.3 Introduction

Evolutionary algorithms (EAs) are becoming increasingly popular for solving engineering decision problems and in practical applications such as calibration, because they: (i) are based on rather simple concepts and are easy to implement; (ii) do not require gradient information; (iii) can consider and bypass local optima; (iv) do not require continuous objective functions; and (v) can be utilized in a wide range of problems covering different disciplines. Generally, the main structure and searching process of different EAs are similar, however, their operators may vary. Being a nonlinear, nonconvex, nonpolynomial hard problems, the least-cost water distribution network (WDN) design problem, in its simplest form, is an attractive benchmark for evaluating innovations in the development of these algorithms. Over the years, the performance of various heuristic optimization techniques have been evaluated with this problem, and its variations, see, e.g., investigations of the efficiencies of genetic algorithms (Murphy and Simpson 1992); simulated annealing (Cunha and Sousa 2001); harmony search (Geem 2006); shuffled frog leaping (Eusuff and Lansey 2003); ant colony optimization (Maier et al. 2003); particle swarm optimization (Suribabu and Neelakantan 2006); cross entropy (Perelman and Ostfeld 2007); scatter search (Lin et al. 2007); differential evolution (Vasan and Simonovic 2010); self-adaptive differential evolution (Zheng et al. 2013); soccer league competition (Moosavian and Roodsari 2014); and improved genetic algorithms (Bi et al. 2015). Variations of some of these algorithms, e.g., those that include nondominated sorting, such as the nondominated sorting genetic algorithm (NSGA), and nondominated sorting differential evolution (NSDE), have been tested in their application to multi-objective WDN design problems such as those that optimize cost as well as the network's reliability in the face of hydraulic and mechanical failure (see, e.g. Moosavian and Lence 2016). While these problems address the practical need for networks that are robust to uncertainty, by far, the most widely recognized benchmark is the single objective least-cost WDN design problem.

Though these heuristic algorithms show success for specific WDN design problems, there is no guarantee that the global optimum, or the complete Pareto optimal frontier, may be found. Furthermore, in the area of WDN design, only a few comparisons of EA algorithms have been undertaken (Zheng et al. 2013; Moosavian and Roodsari 2014; Bi et al. 2015), and these studies focus on a select few methods. This paper presents a rigorous comparison of ten EAs in application to the single objective least-cost design of WDNs, and evaluates the algorithms in terms of Freidman test results, the best solution obtained, and the distribution of the solutions.

The cost of a WDN design is a function of pipe size diameters and these pipe diameters in turn affect the network pressures. The main constraint of the problem is to satisfy the minimum required pressure at all network nodes. In applications of EAs, at each evaluation of the objective function, hydraulic analysis of the network must be conducted, which exacts a high computational burden, and is in fact the most computationally intensive portion of the optimization process. Therefore, the optimal solution, for a fixed number of function evaluations, is considered a suitable comparator for EAs. In this paper, ten meta-heuristic algorithms are evaluated in terms of the least cost of the WDN design, for three example networks, and under different stopping criteria based on the allowable number of function evaluations. To guard against selection of local optima, for each stopping criterion and each pipe network, these algorithms are implemented 20 times. The algorithms include current versions of: genetic (GA); ant colony optimization (ACO); harmony search (HS); differential evolution (DE); particle swarm optimization gravitational search (PSOGSA); artificial bee colony (ABC); soccer league competition (SLC); covariance matrix adaptation evolution strategy (CMAES); biology and bioinformatics global optimization (MEIGO); and whale optimization (WOA) meta-heuristic algorithms. Several of these algorithms, i.e., GA, ACO, HS, DE, ABC, and SLC have either been commonly applied or are beginning to be applied in the analysis of WDNs. Others have shown high performance when applied to other complex, nonpolynomial hard optimization problems. In the following sections, the general WDN optimization model is introduced, the algorithms are briefly described, their application to three example networks are assessed, and conclusions are drawn from this comparison.

2.4 Problem formulation

A water distribution system is a collection of many components such as pipes, reservoirs, pumps and valves which are connected in order to provide water to consumers. The optimal design of such a system can be defined as the best combination of component sizes and component settings (e.g., pipe size diameters, pump types, pump locations and maximum power, and reservoir storage volumes) that exacts the minimum cost for a given network layout, such that hydraulic laws governing continuity of flow and energy are maintained and demands for water quantities and pressures at the consumer nodes are satisfied. In its simplest form, WDN design is formulated as a least-cost optimization problem with the selection of pipe sizes as the decision variables, while pipe layout and node connectivity, nodal demands, and minimum pressure head requirements, are imposed. The optimization problem is stated mathematically as:

$$Min\ C = \sum_{k=1}^{np} c_k\left(D_k\right) \times L_k \tag{2.1}$$

where $c_k\left(D_k\right) \times L_k$ = cost of pipe k per unit length with diameter D_k; L_k = length of pipe k; and np = number of pipes in the network. This objective function is minimized under the following constraints:

Flow continuity at nodes

For each node, flow continuity must be satisfied;

$$\sum Q_{in} - \sum Q_{out} = q_j, \qquad \forall\ k \in nn \tag{2.2}$$

where q_j = water demand at node j (meters^3/second); nn = number of nodes; and Q_{in} and Q_{out} = flow into and out of node j (meters^3/second), respectively.

Energy conservation in loops

The total head loss around a closed pipe loop should be equal to zero, or the head loss along a loop between two fixed head reservoirs should be equal to the difference in water level of the reservoirs:

$$\sum_{k \in loop\ l} hf_k = \Delta H, \qquad \forall\ l \in nl \tag{2.3}$$

where ΔH = difference between nodal pressures at both ends of a path (meters), and $\Delta H = 0$, if the path is closed; nl = number of loops; and hf_k = head loss due to friction in pipe k (meters) which is obtained from following equation:

$$hf_k = H_i - H_j = R_k Q_k^n \tag{2.4}$$

where H_i and H_j = nodal heads at the start, and at the end node of the pipe (meters), respectively; R_k = resistance coefficient of the kth pipe; Q_k = flow rate in the kth pipe (second/ meters^2); and n = constant depending on the head loss equation, and is 1.852 for the most common expression for head loss, the Hazen-Williams head loss formulation.

Minimum pressure at nodes

For each junction node in the network, the pressure head should be greater than the prescribed minimum pressure head:

$$H_j \geq H_j^{min}, \forall\ j \in nn \tag{2.5}$$

where H_j = pressure head at node j (meters); nn = number of nodes; and H_j^{min} = minimum required pressure head (meters).

In the comparison of EA performance undertaken in this paper, the Global Gradient Algorithm, GGA (Todini and Pilati 1988), in a MATLAB environment, is applied to conduct the hydraulic analysis of the networks. GGA satisfies the continuity and energy conservation equations (Eq 2.2-2.4), while calculating the pressure head, H_j, at each junction node and the discharge, Q_k, in each pipe.

Pipe size availability

The diameter of the pipes should be selected from a set of commercially available sizes, and are thus discrete:

$$D_k = \{D(1), D(2), \dots., D(ns)\}, \quad \forall\, k \in np \tag{2.6}$$

where ns = number of candidate diameters.

2.5 Evolutionary Algorithms (EAs)

Generally, EAs are implemented in five main steps, including: (i) creation of a random initial population; (ii) evaluation of the objective function or fitness; (iii) selection of two or more solution vectors for the evolution process; (iv) implementation of an evolution strategy with different operators; and (v) selection of good solutions, and replacement of these in the population. Differences in EAs most often include variations in how they evolve new solution vectors, how they select good solutions from the population, and how they replace solutions in the updated population. With the exception of the ACO algorithm, which is designed for discrete problems, these algorithms are designed to solve continuous problems. Problems with discrete variables may be solved by obtaining solutions based on continuous variables, rounded to the nearest allowable integer value. In this section, a summary of the different EAs compared in this paper is presented. Links to the source code of the algorithms are provided below. Unless otherwise noted, all algorithms are Matlab File Exchange Codes obtained from Mathworks.com.

2.5.1 Genetic Algorithm (GA)

Implementation of the GA (Holland 1972) begins with identification of a random set, or population, of solutions (called chromosomes). The objective function value, or fitness, of each solution is determined and compared to determine those solutions, or population members, that are allowed to evolve. Typically, the selection process employs tournament selection, which involves a series of competitions, or "tournaments," among pairs of solutions chosen at random from the population. The winner of each tournament is the solution with the best fitness. These winners are selected to undergo evolution with crossover, mutation, and selection operations, probabilistic-based mechanisms through which new solutions are constructed. In crossover, portions of the set of variable values in two solutions are exchanged, and in mutation, randomly selected variable values for one solution are changed. Under the selection operation, the new solutions are evaluated and may be used to further evolve the population, if they provide better fitness (i.e., improved objective function value), relative to other population members. The process is continued for a large number of generations or until no further improvement in the best objective function is obtained.

https://www.mathworks.com/help/gads/examples/coding-and-minimizing-a-fitness-function-using-the-genetic-algorithm.html

(2017 Version)

2.5.2 Ant Colony Optimization (ACO)

ACO is a probabilistic approach that mimics the normal behavior of ants that tend to find the shortest route while searching for food. Initially, agents that are akin to ants move randomly toward food, and their pathway is considered one solution. In moving along a pathway, they indicate the shortest path to follow (e.g., the best objective function value) probabilistically, in a manner that is akin to an ant depositing a pheromone trail.

For example, in WDN design, an ant's path is the set of pipe diameters selected for each pipe in the network. In the first iteration, each ant randomly selects a set of pipe diameters as a path or solution vector. The objective function (WDN cost) is evaluated for each solution vector and a pheromone value is assigned to that vector based on the value of the objective function. This process is repeated for all ants in the population. In subsequent iterations, the probability that an ant will select a given solution vector is a function of the total pheromone concentration assigned to that vector thus far, where paths with higher pheromone values are selected to be followed. At each iteration the pheromone concentration of each solution vector is updated, and this process continues until a termination criterion is satisfied. Throughout this process, the shortest path will continuously be reinforced by an increasing amount of pheromones, and the pheromone trails along less preferred paths will decrease in a manner akin to the evaporation process (for an application of ACO to WDNs, see Maier et al. 2003).

https://www.mathworks.com/matlabcentral/fileexchange/52859-ant-colony-optimization-aco

(2015 Version)

2.5.3 Harmony Search (HS)

The HS algorithm (Geem, 2006) is conceptually based on the musical process of searching for a 'perfect state' of harmony, such as in jazz improvisation. Jazz improvisation seeks a best state (fantastic harmony) determined by aesthetic estimation, and analogously, HS seeks a best state (global optimum) determined by evaluating the objective function. In music, the aesthetic estimation is evaluated by the set of pitches played by each instrument, and in HS the objective function evaluation is performed for the set of values assigned to each decision variable. In music, the harmony quality is enhanced with practice, and in HS the solution quality is enhanced over many iterations.

Initially, the population comprises many randomly generated solution vectors, each possessing an objective function value, or harmony quality. In subsequent iterations, each new harmony, or solution vector, is generated based on three rules: memory consideration, pitch adjustment, and random selection. In memory consideration, the value of the first decision variable for the new vector is chosen from any value within its specified memory range, i.e., the range of variable values previously selected. Values of the other decision variables are chosen in the same manner. In pitch adjustment, a randomly selected variable is changed slightly, relative to its current value. And in random selection, a randomly selected variable is changed to any value, regardless of its memory consideration or current value. If the harmony of any new vector is better than the existing worst harmony in memory, the new vector is included in memory and the worst harmony is excluded. This procedure is repeated until satisfaction of a termination criterion is obtained and thus a fantastic harmony is found. This algorithm was obtained, with permission from the author, at:

https://sites.google.com/a/hydroteq.com/www/

(2006 Version)

2.5.4 Differential Evolution (DE)

DE (Price et al. 2005) is an improved version of the GA. Similar to the GA, there are three important operators involved in DE, including the mutation, crossover, and selection operators. The main difference between GA and DE is that GA relies on its crossover operator to exchange information among solutions, while DE primarily relies on its mutation operator to form new solution vectors.

Initially, the population comprises many randomly generated solution vectors, each possessing an objective function value. In the mutation process, the genetic material, or decision variable values, from three or more parent solutions are exchanged to produce one new solution. The crossover operator determines whether and where this new vector is allowed to breed with a vector from a previous generation, or iteration. Subsequently, the selection process compares the bred and existing vector, and allows the best among these to survive to the next generation. By considering components of previous generations in the construction of new vectors, the crossover operator efficiently shuffles information about successful solutions, enabling the search for an optimum to focus on the most promising area of the solution space. Over multiple iterations, DE refines the search by adapting the mutation increments (i.e., the step size of the allowable variable changes), based on the stage of the evolutionary process. At the beginning of the evolution process, the mutation operator favors exploration, i.e., larger incremental changes of variable values, and as evolution progresses, it favors exploitation, i.e., smaller incremental changes of variable values. This process continues until satisfaction of a termination criterion is achieved.

https://www.mathworks.com/matlabcentral/fileexchange/18593-differential-evolution

(2017 Version)

2.5.5 Particle Swarm Optimization and Gravitational Search Algorithm (PSOGSA)

PSO (Kennedy and Eberhart 1995) originated as a simulation of a simplified social system, such as birds flocking or fish schooling. Similar to GA, PSO is also population-based and searches for optimal solutions by updating generations. However, unlike GA, PSO possesses no evolution operators. Instead, PSO relies on the exchange of information between individuals, or particles, of the population, or swarm. In order to select fitter particles for further populations, each particle adjusts its trajectory as a function of its current position (or solution), its previous best position (solution) over all previous generations, and the current best position attained among all other population members in the most previous generation. PSO presents the advantage of being conceptually simple, however, its main disadvantage is the risk of a premature search convergence due to the lack of diversity resulting from its simple operator.

PSOGSA (Mirjalili and Hashim 2010), a modification of PSO, employs the gravitational search algorithm (GSA) for exploring the fitness landscape (Rashedi et al. 2009), which is inspired by the laws of motion, where particles are attracted proportional to the product of their masses and inversely proportional to the square of the distance between them. In GSA, for each solution in a generation of solutions, a directional move is evaluated as a function of two masses, and the distance between them. The masses are the objective function value of the solution divided by the sum of the objective function values for all solutions, and the difference between the objective function value and the worst objective function value in the given generation, divided by the range of objective function values for that generation. The distance between the masses is the difference between the solution vector, and the previous solution vector for this population member. PSOGSA uses PSO to exploit good solutions and GSA to explore a wider array of solutions.

https://www.mathworks.com/matlabcentral/fileexchange/35939-hybrid-particle-swarm-optimization-and-gravitational-search-algorithm-psogsa

(2018 Version)

2.5.6 Artificial Bee Colony (ABC)

In the ABC algorithm (Karaboga 2005) the position of a food source represents a possible solution to the optimization problem and the nectar amount of a food source corresponds to the quality (fitness) of the associated solution. Search processes are based on three kinds of bees: employed bees; onlooker bees; and scout bees. Initially, the number of the employed and onlooker bees is equal to the number of solutions considered in the population, and there are no scout bees. Each cycle of the search procedure consists of the following actions: employed bees are sent onto their food sources (solutions) and the nectar amounts of the sources (performance) are evaluated; employed bees share the nectar information of food sources with the onlooker bees, and based on this information, the onlooker bees produce a new random food source (solution) and evaluate the nectar amount of these food sources. This process continues iteratively, and if a solution represented by a food source is not improved by a pre-determined number of iterations, then the food source is abandoned by the onlooker and employed bees for that source, and the employed bee is converted to a scout bee, and is sent randomly onto possible new food sources. In this case, a new solution vector is randomly created and replaces the abandoned solution.

https://www.mathworks.com/matlabcentral/fileexchange/52966-artificial-bee-colony-abc-in-matlab

(2015 Version)

2.5.7 Soccer League Competition (SLC) Algorithm

The SLC algorithm (Moosavian and Roodsari 2014) is inspired from professional soccer leagues. It involves different teams, or collections of solutions, where each solution is a team member, and a number of operators act on the team members to perform an effective search for the near optimal solution. SLC mimics matches between teams and determines the winners based on their relative power, or the objective function value, where the winner of a match has a higher probability of increasing its power for future matches. After each match, all players (decision vectors) on the winning team undergo operations that change their values, and thus their power (i.e., strengthening or weakening each player), producing modified team members. Finally, the original strength of each player on this winning team is compared with its modified strength and the player, i.e., decision vector, with the greater strength, is allowed to play on the team in the next match.

An iteration, or round, is defined as a set of matches that allows each team, or modified team, to play with each other team in the league. Thereafter, the next round is played with these new teams. The user specifies the stopping criteria to be either based on a limit of the number of rounds undertaken, or the number of function evaluations made.

https://www.mathworks.com/matlabcentral/fileexchange/56480-soccer-league-competition-slc-algorithm-for-discrete-problems

(2016 Version)

2.5.8 Covariance Matrix Adaptation Evolution Strategy (CMAES)

CMAES generates new populations of solutions by sampling from a probability distribution that is constructed during the optimization process. It is derived from the concept of self-adaptation in evolution strategies, which adapts the covariance matrix of a multivariate normal search distribution. An initial population of solutions is generated, a high-performing fraction of this population is selected, and future populations are evolved based on the adaptation of the: (i) mean vector of the selected solutions; (ii) covariance matrices of the selected solutions and the total population of solutions; and (iii) an evolving global step size. The evolution path of the high-performing solutions is thus based on the vector difference between the best individuals in the current and previous generations, which accelerates the convergence of the algorithm. As the algorithm progresses, the global step size changes dynamically as a function of the distribution of the high-performing solutions, where larger increments favor exploration and smaller increments favor exploitation. While the algorithm outperforms other similar classes of learning algorithms on benchmark multimodal objective functions (Hansen, 2004), there is no available information about the performance of CMAES in discrete optimization problems, such as in WDN design.

This algorithm was one of five winners of the 2013 Congress on Evolutionary Computation (CEC) Competition, see:

http://www.ntu.edu.sg/home/EPNSugan/index_files/CEC2013/CEC2013.htm

https://www.mathworks.com/matlabcentral/fileexchange/52898-cma-es-in-matlab

(2015 Version)

2.5.9 MEtaheuristics for Systems Biology and BIoinformatics Global Optimization (MEIGO)

MEIGO is a black-box optimizer which incorporates the metaheuristic search algorithms of an Enhanced Scatter Search (ESS) and a Variable Neighbourhood Search (VNS). A diverse initial population of solutions is generated, VNS and ESS are applied to transform each solution into enhanced solutions, a subset of the enhanced solutions is further combined, and the best solutions among these are retained for iterative applications of the search algorithms. This process is repeated until a termination criterion is satisfied.

An ESS applies the same operator used in DE to explore the current population, or reference Set (RefSet). The ESS operator enhances diversity and prevents the search from premature stagnation by creating dissimilar solutions and retaining these in a revised RefSet. A VNS is a trajectory-based metaheuristic that performs a local search for each solution in the RefSet to move it toward the incumbent high-performing solutions, thus exploiting different local optima, among which the global optimum is expected to be found. One of the key operators of the algorithm is the strategy followed to change the degree of movement toward a given high-performing solution, which perturbs the decision variables using a dynamically adapting distance criterion. The typical strategy consists of visiting the neighbourhood close to the current solution (i.e., small perturbations of the solution), until no further improvement is achieved.

MEIGO has been used to solve problems with real-valued and mixed-integer decision variables, and problems with integer and binary decision variables. It is viewed as efficient because it allows the user to apply parallel computation efforts (details of the algorithm are provided in Egea et al., 2014).

This algorithm was obtained, with permission from the author, at:

http://nautilus.iim.csic.es/~gingproc/

(2016 Version)

2.5.10 Whale Optimization Algorithm (WOA)

WOA mimics the social behavior of humpback whales and has recently been applied for optimization of engineering problems (Mirjalili and Lewis, 2016). The WOA algorithm starts with a set of random solution vectors. At each iteration, the positions of solution vectors are updated with respect to either a randomly chosen solution vector or the best solution obtained thus far. The WOA employs three operators to simulate the search for prey, encircling prey and bubble-net foraging behavior of humpback whales. In the prey operation, new solutions in the current iteration are created by selecting a random solution from the previous iteration and subtracting the difference between a randomly changed version of it and a given solution vector from the previous iteration. The encircling prey operator generates a new solution in the current iteration by selecting the best solution from the previous iteration and subtracting the difference between a randomly changed version of it and a given solution vector from the previous iteration. The bubble-net foraging operator generates a new solution vector in the current iteration by selecting the best solution from the previous iteration and adding a random logarithmic spiral of the difference between it and a given solution vector from the previous iteration.

https://www.mathworks.com/matlabcentral/fileexchange/55667-the-whale-optimization-algorithm

(2017 Version)

2.6 Applications to WDN design

The performance of the ten above-mentioned EAs is tested for three water distribution networks, the Two-Loop and Hanoi benchmark networks, and the WDN for the City of Farhadgerd, Iran. For consistent comparison, three stopping critera are defined as: (i) the maximum number of function evaluations is set to 1000. Given this criteria, the initial speed of convergence and performance of the algorithms may be evaluated.; (ii) the maximum number of function evaluations is set to 10000. Here, the performance of the algorithms is evaluated for a more mature evolution, compared with (i).; and (iii) the maximum number of function evaluations is set to 40000, for which the evolution of the algorithms is considered to be fully mature. For each of these criteria, all algorithms are executed 20 times, i.e., 20 sets of initial populations of solutions are generated based on 20 different random number sequences.

Statistical analysis of results for all algorithms are performed and the mean, median, standard deviation, minimum, and maximum for the 20 executions of the algorithms are calculated. The mean shows the overall performance of the algorithm for the different runs. The standard deviation indicates the variation of the final solution with respect to mean value. Small standard deviation values show that an algorithm has a similar behavior regarding its search process for different initial random populations. All of the computations are implemented in the MATLAB programming language environment with an Intel(R) Core(TM) 2Duo CPU P8700 @ 2.53GHz and 4.00 GB RAM.

2.6.1 Two-loop benchmark network

This network, shown in Figure 2.1, is a hypothetical benchmark which has seven nodes and eight pipes with two loops, and is fed by a reservoir with a 210 m fixed head (Alperovits and Shamir 1977). The pipes are all 1,000 m long with a Hazen-Williams coefficient of 130. The minimum pressure limitation is 30 m above ground level for all nodes. There are 14 available commercial pipe diameters. The degree of candidate diameters is an indicator of the size and complexity of the network problem, and is defined as the number of candidate diameters divided by the number of pipes. For this case, the degree of candidate diameters is equivalent to 14 / 8 =1.75. Geem (2006) determines the minimum cost of this network to be $419,000 US.

The performance of the algorithms, in terms of the minimum total cost obtained, under the three stopping criteria, is shown in the Tables 2.1, 2.2, and 2.3, for the 1000-, 100000, and 40000-

function evaluation stopping criteria, respectively. The results show that, for the first stopping criterion, the PSOGSA exhibits the best mean cost of $433,000 US and the MEIGO algorithm second best mean of $436,000 US, while the MEIGO algorithm exhibits the best minimum of $419,000 US and PSOGSA the second best minimum of $420,000 US. The PSOGSA also has the least standard deviation of 1467 which indicates that it exhibits the best overall performance among the remaining algorithms. The GA and HS algorithms result in the third and fourth lowest mean cost, respectively, and the ACO and WOA algorithms exhibit the worst performance among all algorithms.

As the stopping criterion is relaxed to 10000 function evaluations, the performance of the ABC, SLC, HS, CMAES and DE algorithms improves significantly, and each of these algorithms achieve the global optimal minimum cost. Here, the ABC, DE, and SLC algorithms have the least standard deviation, in this order. These small standard deviation values show that the algorithms perform the search process consistently, for the breadth of different initial random populations. In contrast, the PSOGSA algorithm also achieves the global optimum, but its average performance over 20 executions is relatively poor. Similar results are found with stopping criteria is relaxed to 40000 function evaluations.

2.6.2 Hanoi benchmark network

The Hanoi network, shown in Figure 2.2, consists of 32 nodes, 34 pipes with 3 loops, and is fed by gravity from a reservoir with a 100 m fixed head. All pipes have a Hazen-Williams coefficient of 130 and the minimum head limitation at all nodes is 30 m above ground level (Alperovits and Shamir 1977). There are six possible pipe diameters and 34 pipes in the system, thus the degree of candidate diameter is equivalent to 6 / 34 =0.1765. The global optimum solution has the total cost of $6.1 million US (Alperovits and Shamir 1977).

Optimization results for this network are provided in Tables 2.4, 2.5, and 2.6 for 1,000-, 10,000-, and 40,000-function evaluation stopping criteria, respectively. For the first criterion, the SLC algorithm exhibits the highest performance with the minimum and mean least cost equal to $7.0 and $8.3 million, respectively. It also has the smallest standard deviation. The MEIGO and WOA algorithms obtain equal minimum costs of $7.9 million US, and mean costs of $8.5 and $8.7 million US, respectively. For the 10000-function evaluation criterion, the SLC also exhibits the best performance, and has a minimum and mean least cost of $6.1 and $6.3 million US, respectively. The CMAES algorithm has the smallest standard deviation, but does not find the lowest cost overall. The CMAES, GA and MEIGO algorithms are the second, third and forth best algorithms, respectively. A similar trend is exhibited for the 40000-function evaluation criterion, where the SLC and CMAES algorithms exhibit the highest performance, in this order, and WOA, PSOGSA and ACO exhibit the worst performance. This indicates that the convergence properties of the PSOGSA algorithm do not improve when the number of iterations increases. A comparison of the Hanoi and Two-Loop benchmark networks indicates that as the degree of candidate diameter decreases, the convergence potential of the SLC and CMAES algorithm improves significantly.

2.6.3 Farhadgerd network

The Farhadgerd WDN serves a population of approximately 8200 in the City of Farhadgerd, Iran, a residential community near the regional capital, Mashhad. The network, shown in Figure 2.3, comprises 68 pipes and 53 nodes, has one reservoir with a head of 510 m, and a minimum pressure requirement of 20 m at all nodes. There are nine possible pipe diameters and thus the degree of candidate diameter for this network is equivalent to 9 / 68 =0.13. Relevant data for this network is provided in Appendix B.

Optimization results for the network are provided in Tables 2.7, 2.8, and 2.9 for the 1000-, 10000- , and 40000-function evaluation stopping criteria, respectively. In this paper, the recorded optimal cost for Farhadgerd is $1.78E7 US.

For the first first criterion, the PSOGSA exhibits the best mean cost of $3.03E07 US and the MEIGO algorithm second best mean of $3.2 E07 US, while the MEIGO algorithm exhibits the best minimum of $2.2E07 US and PSOGSA the second best minimum of $2.77E07 US. The ACO, DE and ABC algorithms exhibit the worst performance with respect to the other algorithms, under this criterion. For the 10000-function evaluation criterion, the CMAES algorithm again out-performs all other algorithms, with a minimum and mean least cost of $1.8E07 and $1.9E07 US, respectively. It also exhibits the smallest standard deviation. The SLC algorithm performs second-best, under this criterion. The results of CMAES and SLC are similar for the cases with a stopping criterion of 40000 function evauations. The minimum and maximum least cost solutions obtained by both CMAES and SLC algorithm are $1.78E07 and $1.8E07 US, respectively, which are less than all solutions obtained by other algorithms.

2.7 The Friedman test

The Friedman test (Friedman(1937; Friedman, 1940), a nonparametric analysis of variance, implemented using IBM-SPSS Statistics 23 software, is used for comparing the performance of the algorithms. The test is a multiple comparisons test that may be used to detect significant differences between the responses of two or more subjects to various treatments. For the results presented herein, the response is the mean least cost values, the subjects are the example networks, and the treatments are the applications of the various algorithm. Three Friedman tests were conducted, one for each of the stopping criteria. The test determines the ranks of the responses, or mean least cost solutions, for each of the three networks, resulting under each of the algorithms. For each algorithm, the mean of these ranks is evaluated. The Friedman test statistic, based the number of subjects, the number of is treatments, and mean ranks of the responses for each treatment, is tested against a Chi-Square critical value, and indicates whether the various treatments, or algorithms, are significantly different.

Results of the Friedman test are provided in Figures 2.4a, 2.4b, and 2.4c, for the stopping criteria based on 1000, 10000, and 40000 function evaluations, respectively. In these figures, the mean rank of the algorithms is the mean of the ranks of the resulting mean least cost solutions over each of the three networks, where 10 is the maximum possible mean rank, N is equal to 3 subjects, Chi-Square is the Chi-Square critical value for N=3, 10-1 degrees of freedom (df), the given mean ranks and a significance level of 0.05. Asymp. Sig., the asymptotic significance, is the p-value for the test and indicates similarity of algorithms if this value is greater than 0.05.

The Friedman test results show that for a stopping criteria of 1000 function evaluations, PSOGSA, MEIGO and GA have the lowest mean ranks, in increasing order, and as a result, higher performance. For larger stopping criteria, SLC, CMAES and DE have the lowest mean ranks, in increasing order, and thus better performance. Furthermore, rejection of the null hypothesis for all three Friedman tests (i.e., Asymp. Sig. or p-value < 0.05), in all cases, indicates that the performance of the algorithms differs significantly.

2.8 Conclusion

In this paper, ten EAs are evaluated for finding the optimal least-cost design of three water distribution networks, under three pre-defined stopping criteria: 1000, 10000, and 40000 function evaluations. The performance of the algorithms is evaluated based on statistical analysis of 20 different executions of the algorithms and Friedman tests of the mean of the resulting solutions. The algorithms are compared in terms of the mean, median, standard deviation, minimum, and maximum of all executions for each network, as well as the mean rank in the Friedman test. Small mean and standard deviation values indicate that an algorithm provides an attractive and consistent performance for different initial random populations. For the Two-loop network nearly all ten algorithms are able to identify the global optimum solution, though the small standard deviations of the ABC, DE, and SLC algorithms indicate high consistency. For the Hanoi network the SLC exhibits the best performance, and has the lowest minimum and mean cost. The CMAES, GA and MEIGO algorithms are the second, third and fourth best algorithms, respectively, in terms of mean cost. For the Farhadgerd network, the CMAES algorithm provides the lowest mean and standard deviation, and the CMAES and SLC algorithms provide the lowest minimum cost among all ten algorithms. Friedman test results show that the performance of the algorithms differ significantly, and that at early stages of the solution process, PSOGSA, MEIGO and GA perform well, but that as the algorithms proceed to undertake more function evaluations, the SLC, CMAES and DE out-perform the others.

Due to the requirement to satisfy the minimum required pressure throughout the network, a WDN design problem may have various local optima which may be spread over the feasible region. The minimum required pressure is a nonlinear function of the pipe size diameters and their configuration. For a given design, if any one pipe diameter were to be changed, a corresponding change in the direction of flow in the pipe may occur and the distribution of the pressures throughout the entire system may also change in response. Therefore, when the optimization algorithm is close to the global optimum, there is no guarantee of finding it. This hydraulic sensitivity makes the optimization of the network model difficult to solve.

To overcome this problem and effectively escape from local optima, EAs should be designed to provide diverse solutions. The solution approach should not concentrate solely on the evolution of the best set of solutions, but on a wide range of solutions, making it independent of the initial population, which has typically been used to determine the direction of search. The CMAES and SLC algorithms exhibit these properties and can successfully avoid being trapped at local optimum solutions. CMAES creates new generations of solutions based on the entire distribution of a wide range of solutions, thus enhancing the exploration process, and adapts the global step size dynamically to efficiently conduct the exploitation process. By applying a combination of coarse (exploration)- and fine (exploitation)-scale search processes, the SLC algorithm is less dependent on a single initial population than most EAs. The coarse-scale search process of the SLC is conducted among several different populations (i.e., teams) that are distributed across the feasible region, while the fine-scale search process is undertaken within each of the populations (i.e., among players of the teams). Another advantage of the SLC is that it applies different evolutionary operators for different teams and different players based on their performance (i.e., power).

The CMAES and SLC algorithms show promise for applications to more complex WDNs, and other nonpolynomial hard problems. Because it uses the distribution of the population at a given generation to provide a wide and deep exploration of the feasible region, the CMAES may be integrated with other algorithms, such as the SLC, to enhance their exploration and exploitation processes. Furthermore, in the CMAES algorithm the step size of the search may shrink to the point of restricting exploration, and to guard against such cases, the mutation operators of DE or ABC may be adapted and integrated in CMAES. While the high performing algorithms evaluated in this paper were analyzed for the single objective least-cost WDN design problem, consideration of other objectives, such as network reliability and robustness, requires the advancement of multi-objective versions of the algorithms for evaluating the Pareto optimal frontier for the collection of objectives. One approach for doing so is to integrate nondominated sorting with existing algorithms. Given the performance of the CMAES and SLC algorithms for addressing the presented network problems, future work in the advancement of these algorithms should develop their multi-objective versions, and evaluate these against the performance of other widely used algorithms, such as NSGA, NSGA-II and NSDE.

Table 2.1 Least cost solutions for the Two-loop network: 1000 function evaluations

	GA	ACO	HS	DE	PSOGSA	ABC	CMAES	SLC	MEIGO	WOA
Mean	4.57E05	5.84E05	4.73E05	5.38E05	4.33E05	4.86E05	5.23E05	4.98E05	4.36E05	5.49E05
Median	4.54E05	5.73E05	4.72E05	5.34E05	4.30E05	4.79E05	5.21E05	4.90E05	4.39E05	5.28E05
Std	1.77E04	6.09E04	1.76E04	3.19E04	14.67E3	3.10E04	2.32E04	3.42E04	1.11E04	6.78E04
Minimum	4.29E05	4.61E05	4.41E05	4.82E05	4.20E05	4.37E05	4.82E05	4.42E05	4.19E05	4.43E05
Maximum	5.02E05	7.19E05	5.02E05	6.00E05	4.78E05	5.58E05	5.59E05	5.81E05	4.60E05	7.23E05

Table 2.2 Least cost solutions for the Two-loop network: 10000 function evaluations

	GA	ACO	HS	DE	PSOGSA	ABC	CMAES	SLC	MEIGO	WOA
Mean	4.47E05	4.54E05	4.21E05	4.21E05	4.30E05	4.20E05	4.21E05	4.20E05	4.25E05	5.03E05
Median	4.48.E05	4.55.E05	4.20.E05	4.20.E05	4.29.E05	4.20.E05	4.20.E05	4.20.E05	4.20.E05	4.98.E05
Std	1.77.E04	8.50.E03	3.62.E03	1.11.E03	9.53.E03	4.53.E02	6.98.E03	1.28.E03	8.91.E03	3.91.E04
Minimum	4.20.E05	4.34.E05	4.19.E05	4.19.E05	4.19.E05	4.19.E05	4.19.E05	4.19.E05	4.19.E05	4.43.E05
Maximum	4.95.E05	4.69.E05	4.32.E05	4.22.E05	4.46.E05	4.20.E05	4.27.E05	4.24.E05	4.43.E05	5.81.E05

Table 2.3 Least cost solutions for the Two-loop network: 40000 function evaluation

	GA	ACO	HS	DE	PSOGSA	ABC	CMAES	SLC	MEIGO	WOA
Mean	4.31E05	4.37E05	4.20E05	4.19E05	4.24E05	4.19E05	4.22E05	4.19E05	4.22E05	4.67.E05
Median	4.27E05	4.38E05	4.19E05	4.19E05	4.20E05	4.19E05	4.20E05	4.19E05	4.20E05	4.53.E05
Std	1.06E04	6.13E03	1.90E03	3.48E02	8.81E03	1.87E02	2.29E03	5.10E02	6.93E03	4.85.E04
Minimum	4.20E05	4.23E05	4.19E05	4.19E05	4.19E05	4.19E05	4.19E05	4.19E05	4.19E05	4.22.E05
Maximum	4.51E05	4.46E05	4.27E05	4.20E05	4.45E05	4.20E05	4.43E05	4.20E05	4.43E05	6.48.E05

Table 2.4 Least cost solutions for the Hanoi network: 1000 function evaluations

	GA	ACO	HS	DE	PSOGSA	ABC	CMAES	SLC	MEIGO	WOA
Mean	1.7E07	2.2E08	4.5E07	8.2E07	1.34E07	3.2E08	6.5E07	8.3E06	8.5E06	8.7E06
Median	1.4E07	2.2E08	4.2E07	7.9E07	9.88E06	3.5E08	5.6E07	7.3E06	8.4E06	8.6E06
Std	8.6E06	5.9E07	1.6E07	4.3E07	7.35E06	1.4E08	4.4E07	3.3E06	4.0E05	5.0E05
Minimum	8.4E06	1.3E08	2.7E07	2.9E07	7.35E06	1.1E08	9.7E06	7.0E06	7.9E06	7.9E06
Maximum	3.5E07	3.5E08	8.3E07	2.1E08	3.01E07	5.4E08	2.2E08	2.0E07	9.3E06	9.7E06

Table 2.5 Least cost solutions for the Hanoi network: 10000 function evaluations

	GA	ACO	HS	DE	PSOGSA	ABC	CMAES	SLC	MEIGO	WOA
Mean	6.6E06	8.3E06	7.1E06	6.7E06	7.74E06	7.0E06	6.4E06	6.3E06	6.6E06	8.4E06
Median	6.6E06	7.9E06	7.1E06	6.7E06	7.49E06	7.0E06	6.4E06	6.3E06	6.6E06	8.5E06
Std	1.4E05	1.5E06	1.1E05	7.7E04	8.49E05	1.9E05	3.6E04	1.5E05	1.7E05	3.5E05
Minimum	6.4E06	7.3E06	6.7E06	6.5E06	7.08E06	6.6E06	6.3E06	6.1E06	6.4E06	7.7E06
Maximum	6.9E06	1.4E07	7.2E06	6.8E06	1.1E07	7.4E06	6.4E06	6.6E06	7.0E06	9.3E06

Table 2.6 Least cost solutions for the Hanoi network: 40000 function evaluations

	GA	ACO	HS	DE	PSOGSA	ABC	CMAES	SLC	MEIGO	WOA
Mean	6.5E06	7.0E06	6.7E06	6.4E06	6.72E06	6.7E06	6.2E06	6.2E06	6.4E06	8.0E06
Median	6.5E06	7.0E06	6.7E06	6.4E06	6.70E06	6.7E06	6.2E06	6.3E06	6.4E06	7.9E06
Standard Deviation	1.8E05	7.8E04	7.8E04	2.8E04	1.65E05	8.6E04	1.3E04	1.4E05	1.2E05	3.8E05
Minimum	6.1E06	6.9E06	6.5E06	6.3E06	6.52E06	6.5E06	6.2E06	6.1E06	6.3E06	7.3E06
Maximum	6.9E06	7.2E06	6.8E06	6.4E06	7.10E06	6.8E06	6.3E06	6.6E06	6.7E06	8.7E06

Table 2.7 Least cost solutions for the Farhadgerd network: 1,000 function evaluations

	GA	ACO	HS	DE	PSOGSA	ABC	CMAES	SLC	MEIGO	WOA
Mean	3.0E07	3.5E07	3.2E07	3.5E07	3.03E07	3.5E07	3.1E08	3.2E07	3.2E07	3.5E07
Median	2.9E07	3.5E07	3.2E07	3.5E07	3.03E07	3.5E07	5.2E07	3.24E07	3.4E07	3.5E07
Standard Deviation	1.1E06	9.1E05	1.4E06	9.6E05	2.11E06	1.6E06	6.7E08	1.67E06	4.7E06	3.4E06
Minimum	2.8E07	3.4E07	2.9E07	3.3E07	2.77E07	3.2E07	4.7E07	2.87E07	2.2E07	2.9E07
Maximum	3.2E07	3.7E07	3.4E07	3.6E07	3.58E07	3.8E07	2.5.E+9	3.43E07	3.6E07	4.1E07

Table 2.8 Least cost solutions for the Farhadgerd network: 10,000 function evaluations

	GA	ACO	HS	DE	PSOGSA	ABC	CMAES	SLC	MEIGO	WOA
Mean	2.3E07	3.1E07	2.4E07	2.3E07	2.65E07	2.5E07	1.9E07	2.09E07	2.3E07	3.5E07
Median	2.3E07	3.1E07	2.4E07	2.3E07	2.63E07	2.5E07	1.9E07	2.09E07	2.3E07	3.5E07
Standard Deviation	1.0E06	8.5E05	4.7E05	4.4E05	1.29E06	1.1E06	2.1E05	5.65E05	7.9E05	2.9E06
Minimum	2.1E07	2.8E07	2.3E07	2.2E07	2.44E07	2.3E07	1.8E07	2.01E07	2.1E07	2.9E07
Maximum	2.6E07	3.3E07	2.5E07	2.4E07	2.94E07	2.7E07	1.9E07	2.23E07	2.4E07	4.0E07

Table 2.9 Least cost solutions for the Farhadgerd network: 40,000 function evaluations

	GA	ACO	HS	DE	PSOGSA	ABC	CMAES	SLC	MEIGO	WOA
Mean	2.1E07	2.8E07	2.2E07	2.0E07	2.28E07	2.3E07	1.8E07	1.8E07	2.0E07	3.3E07
Median	2.1E07	2.8E07	2.2E07	2.0E07	2.22E07	2.3E07	1.8E07	1.8E07	2.0E07	3.3E07
Standard Deviation	8.9E05	9.2E05	3.3E05	2.7E05	1.8E06	8.3E05	9.2E04	3.0E05	3.4E05	3.2E06
Minimum	1.9E07	2.6E07	2.1E07	2.0E07	2.05E07	2.1E07	1.78E07	1.78E07	1.9E07	2.8E07
Maximum	2.3E07	2.9E07	2.3E07	2.1E07	2.70E07	2.4E07	1.8E07	1.87E07	2.0E07	3.9E07

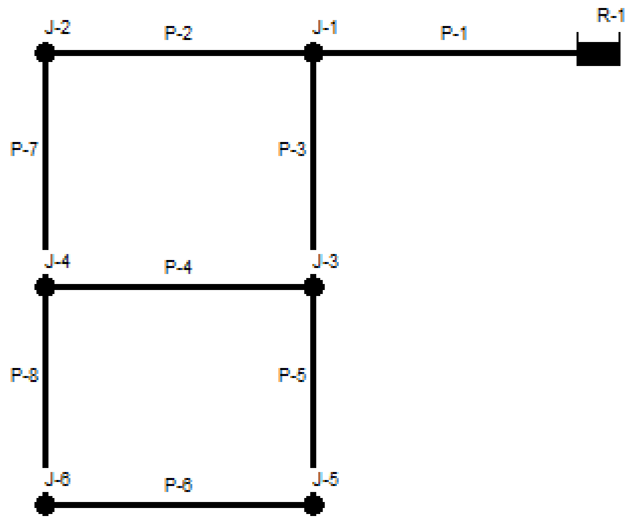


Figure 2.1 Two-loop network

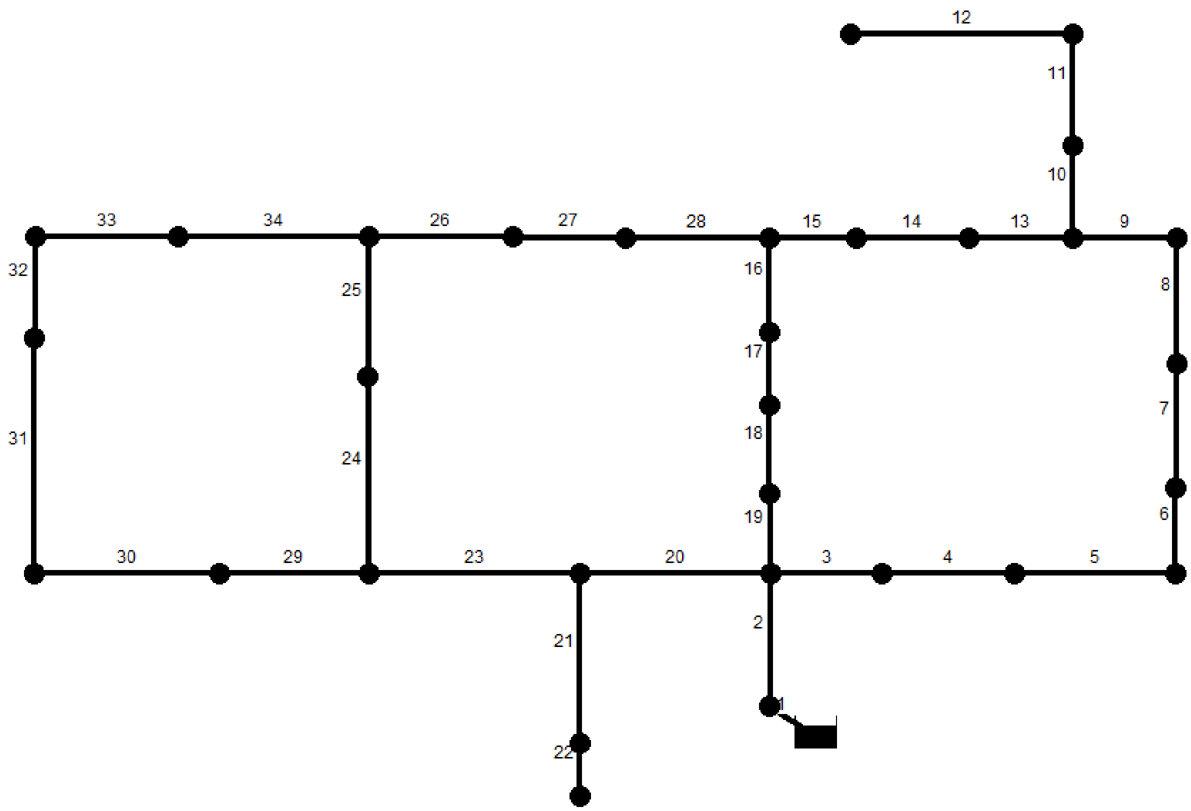


Figure 2.2 Hanoi network

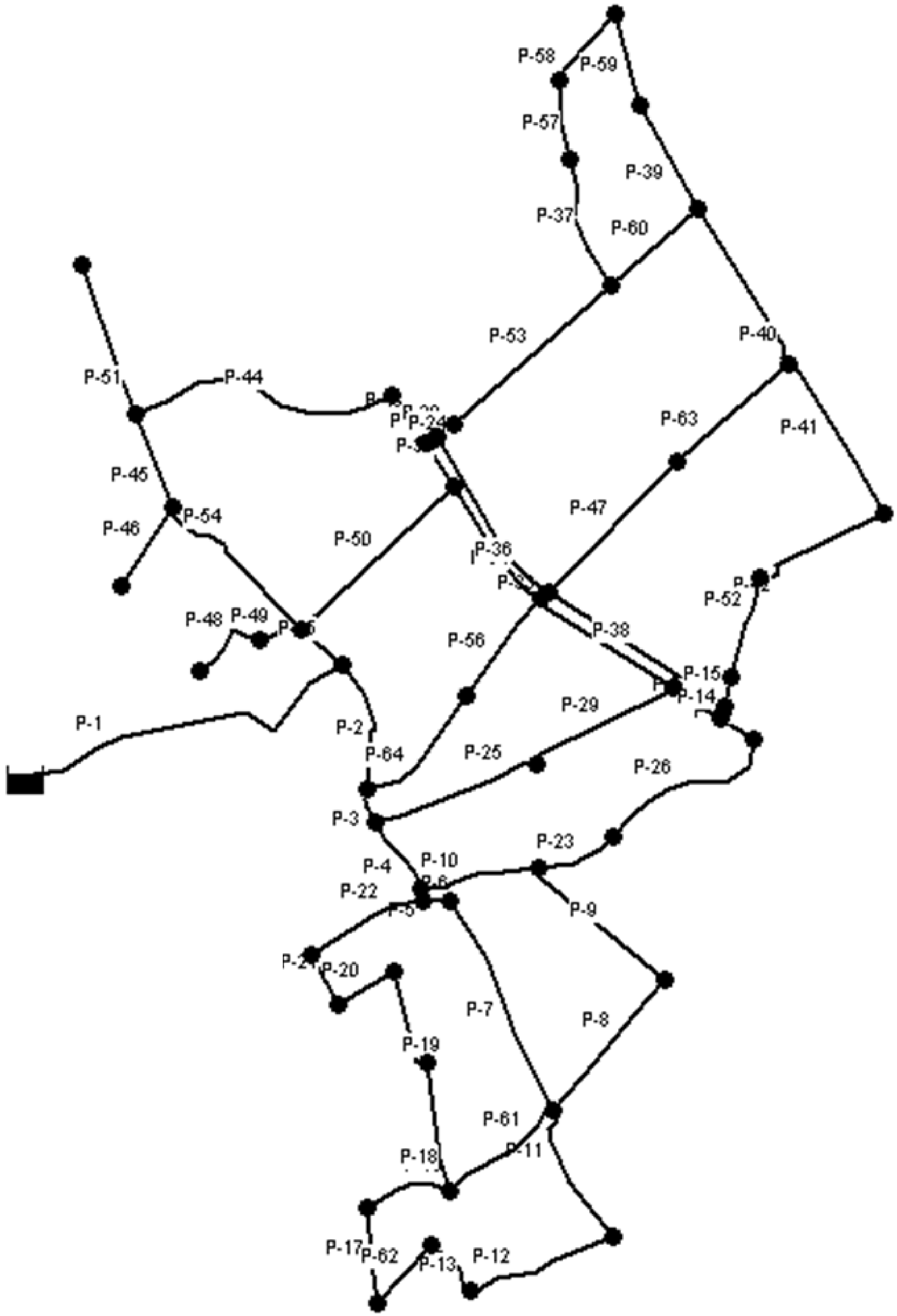


Figure 2.3 Farhadgerd network

Ranks

	Mean Rank
GA	3.00
WOA	6.50
SLC	3.67
CMAES	8.00
ABC	7.50
PSOGSA	2.33
MEIGO	2.67
DE	7.83
HS	4.67
ACO	8.83

Test Statistics[a]

N	3
Chi-Square	18.917
df	9
Asymp. Sig.	.026

a. Friedman Test

a) 1,000-Function Evaluation

Ranks

	Mean Rank
GA	5.17
WOA	10.00
SLC	1.50
CMAES	2.33
ABC	4.83
PSOGSA	7.67
MEIGO	4.50
DE	4.33
HS	5.67
ACO	9.00

Test Statistics[a]

N	3
Chi-Square	22.101
df	9
Asymp. Sig.	.009

a. Friedman Test

b)10,000-Function Evaluation

Ranks

	Mean Rank
GA	6.00
WOA	10.00
SLC	1.67
CMAES	2.83
ABC	5.50
PSOGSA	7.33
MEIGO	4.17
DE	3.00
HS	5.50
ACO	9.00

Test Statistics[a]

N	3
Chi-Square	22.027
df	9
Asymp. Sig.	.009

a. Friedman Test

c)40,000-Function Evaluation

Figure 2.4 Results of Friedman test for: a) 1,000, b) 10,000, and c) 40,000 function evaluations of three networks

Chapter 3 : Nondominated Sorting Differential Evolution Algorithms for Multi-Objective Optimization of Water Distribution Systems

3.1 Preface

Figure 1.1 has listed some population-based EAs for multi-objective design of WDNs such as NSGA and NSDE. Similar to single-objective design of WDNs in Chapter 2, multi-objective design of reliable looped WDNs is challenging because these problems comprise nonlinear relationships between discharge and energy in the pipes and junctions, and nonconvex objective functions that relate cost and reliability to discrete choices of pipe diameters. The second objective in these problems is reliability of service, often measured with surrogates such as the resilience index of Todini (2000). By examining the entire population and evaluating those members that dominate in terms of two objectives, nondominated sorting algorithms efficiently identify the Pareto optimal front in multi-objective problems, and one such algorithm, the improved NSGA-II, has been applied successfully to evaluate the cost-reliability tradeoff for WDNs. The NSDE algorithm, which takes advantage of the mutation operations in DE and nondominated sorting, and its variation NSDE-RMO, have been demonstrated as efficient for solving multi-objective problems. In this chapter, NSDE and NSDE-RMO are applied to discrete WDN optimization for the first time, and their high performance is demonstrated and compared with NSGA-II and the AMALGAM method, a widely applied hybrid metaheuristic multi-objective algorithm. For several networks, NSDE and its variation perform similarly to or better than NSGA-II and AMALGAM.

3.2 Abstract

Optimal design of reliable looped water distribution systems (WDSs) is challenging because these problems comprise non-linear relationships between discharge and energy in the pipes and junctions, and non-convex objective functions that relate cost and reliability to discrete choices of pipe diameters. Efficient algorithms for identifying optimal WDS designs must also be practical for realistic systems. Differential Evolution (DE) Algorithms, which harness the mutation operator to identify populations containing very different alternatives, have been shown to be more efficient than other heuristics for solving single objective versions of these problems, e.g., minimizing the cost of WDSs. By examining the entire population and evaluating those members that dominate in terms of two or more objectives, nondominated sorting algorithms efficiently identify the Pareto optimal front in multi-objective problems, and one such algorithm, the Improved Nondominated

Sorting Genetic Algorithm, NSGA-II, has successfully been applied to evaluate the cost-reliability trade-off for WDSs. The Nondominated Sorting Differential Evolution (NSDE) Algorithm, which takes advantage of the mutation operations in DE and nondominated sorting, and its variation, NSDE with ranking-based mutation (NSDE-RMO), have been demonstrated as efficient for solving multi-objective problems. In this paper we apply NSDE and NSDE-RMO to discrete WDS optimization for the first time, and demonstrate their high performance compared with NSGA-II and the multi-algorithm, genetically adaptive multi-objective, AMALGAM method, a widely applied hybrid meta-heuristic multi-objective algorithm. For three benchmark networks, NSDE and its variation perform similarly to or better than NSGA-II and AMALGAM except at high cost levels. For hypothetical randomly generated networks ranging from 100 to 400 nodes, and 100 to 800 pipes, the Pareto optimal front of the NSDE Algorithms dominate all other algorithms, exhibiting more, and more-varied, Pareto optimal solutions, and converging sooner.

3.3 Introduction

The optimal design of water distribution systems (WDSs) is often considered a least-cost optimization problem with pipe sizes being the primary discrete decision variables. Early solution approaches formulate these problems as continuous optimization models and round off the optimum values of the design variables to the nearest commercial pipe sizes. However, in many cases, the rounding off of some variables changes the values of others, and the resultant solutions violate constraints or are suboptimal. Given their efficiency in globally searching the solution space, evolutionary search methods such as Genetic Algorithms (GAs) (Dandy et al. 1996; Savic and Walters 1997), Ant Colony Optimization (Maier et al. 2003), Shuffled Frog Leaping (Eusuff and Lansey 2003), Particle Swarm Optimization (Suribabu and Neelakantan 2006), Harmony Search (Geem 2006), Dynamically Dimensioned Search Algorithms (Tolson et al. 2009), Honey Bee Mating Optimization (Mohan and Babu 2010), Differential Evolution (DE) Algorithms (Suribabu 2010; Zheng et al. 2012; Dong et al. 2012), and Soccer League Competition (Moosavian and Roodsari 2014), have been applied for solving these and other discrete optimization problems.

The Differential Evolution (DE) Algorithm (Storn and Price 1997), which is similar to, but more efficient than GA, emphasizes mutation with a mutation operator that sources its genetic information for trial vectors from three or more random solution vectors, combines trial vectors to seek better solutions, and thereafter focuses on the most promising solutions. It converges to stable solutions rapidly, is readily adaptable to a wide array of non-linear functions, has been proven to outperform most other evolutionary search methods for continuous problems (Rozenberg et al. 2005), and is earning the reputation as the first choice in discrete WDS least-cost optimization (Suribabu 2010; Zheng et al. 2012; Dong et al. 2012).

The reliability of a WDS to provide required water for users under all conditions, including during pipe bursts or as components deteriorate, is increasingly identified as a key consideration and competing objective in WDS design. Todini (2000) demonstrates the shortcomings of single-objective least-cost optimization approaches and explores the application of multi-objective optimization for minimizing cost and maximizing reliability. He introduces a surrogate for reliability, the resilience index, and proposes a simple heuristic that emulates the reasoning of an engineer whose goal is to reduce cost while preserving an acceptable level of reliability of the system to handle possible failures. While other surrogates for reliability have been examined, e.g., modifications of Todini's index as a function of the diameters of pipes selected (Prasad and Park 2004), maximum head deficiency (Farmani et al. 2005b), flow entropy (Prasad and Tanyimboh 2008), and a combination of these (Raad 2011), Todini's resilience index is the most widely used, and in all cases, reliability and cost are competing objectives, requiring multi-objective analysis. Evaluation of the nondominated set of WDS solutions, the Pareto optimal front, has been achieved using meta-heuristics such as ranked-based fitness assignment methods for Multi-Objective Genetic Algorithms, MOGAs (Fonseca and Fleming 1993), Strength Pareto Evolutionary Algorithms, SPEAs (Zitzler and Thiele 1999), and Nondominated Sorting Genetic Algorithms, NSGAs (Srinivas and Deb 1994), and more recently with hybrid meta-heuristics such as the multi-algorithm, genetically adaptive multi-objective, AMALGAM (Vrugt and Robinson 2007) method, which applies simultaneous multi-method search of the fitness landscape and genetically adaptive offspring creation. For applications and comparisons of these, see, e.g., Farmani et al. (2005a) and Raad et al. (2009).

NSGA-II (Deb et al. 2002), an advanced nondominated sorting algorithm, is arguably the most popular multi-objective evolutionary algorithm, and is increasingly used in WDS design (Farmani et al. 2006). It features implicit elitist selection based on the Pareto dominance rank, and a secondary selection method based on crowding distance, which significantly improves the performance of NSGA on difficult multi-objective problems. Having adopted the mechanisms of crossover and mutation in its GA, however, NSGA-II faces many of the challenges of GA, such as unstable and slow convergence, and difficulty in escaping from local optima (Storn and Price 1997; Iorio and Li 2004; Peng et al. 2009).

To overcome these limitations, Angira and Babu (2005) substitute the DE Algorithm for the GA in NSGA-II and develop the Nondominated Sorting Differential Evolution Algorithm (NSDE). A variation of NSDE, NSDE with ranking-based mutation (NSDE-RMO), uses the ranks of the population members to modify the mutation operator (Chen et al. 2014). Both NSDE and NSDE-RMO exhibit improved stability, accelerated convergence, and increased diversity of solutions in applications to continuous multi-objective problems, and given the high performance of DE in single objective WDS design, show potential for optimizing WDSs in terms of cost and reliability. This paper explores this potential with a comparative analysis of these algorithms.

In the following section, we review the two objective functions of cost and reliability, as well as the hydraulic and design limitations of WDSs, and develop the general WDS design problem. In Section 3, the NSDE and NSDE-RMO Algorithms are described in the context of WDS design. Next, the performance of the NSDE Algorithms for designing multi-objective WDSs is compared with that of NSGA-II and the AMALGAM method in applications to the Two-loop and Hanoi (Alperovits and Shamir 1977), and the Farhadgerd (Moosavian and Jaefarzadeh 2014) benchmark networks, and in numerical examples of hypothetical randomly generated networks. Conclusions are summarized in Section 5.

3.4 Cost and Reliability in Water Distribution Systems (WDSs)

The optimal design of a WDS is defined as the best combination of component sizes and settings, e.g., pipe size diameters, pump types, pump locations and maximum power, and reservoir storage volumes, that satisfies the network objectives, such that hydraulic laws of continuity of flow and energy are maintained, and flow and pressure requirements at the consumer nodes, or other constraints reflecting network-specific considerations, are met. In this paper, the design of a new WDS is formulated to optimize cost and reliability objectives with the selection of pipe sizes as the decision variables, while pipe layout and its connectivity, nodal demands, and minimum head requirements are imposed. The optimization simultaneously minimizes cost (Eq. 3.1) and maximizes reliability (Eq. 3.2):

$$Min\ Cost = \sum_{i=1}^{np} C_i(D_i) \times L_i \tag{3.1}$$

$$Max\ I_r = \frac{\sum_{k=1}^{nn} q_k\left(H_k - H_k^{min}\right)}{\left(\sum_{k=1}^{no} Q_k H_{ok} + \sum_{i=1}^{np}\left(\frac{P_i}{\gamma}\right) - \sum_{j=1}^{nn} q_j H_j^{min}\right)} \tag{3.2}$$

where L_i = length of each pipe (meters); $C_i\ (D_i)$ = cost of a pipe of a given diameter ($/meter); D_i = diameter of the selected pipe (meters); *np* = number of pipes; *nn* = number of nodes; *no* = number of reservoirs; q_k = flow demand at node *k* (meters^3/second); H_k = available pressure head at node *k* (meters); $H_k^{\min}$ = minimum pressure head required at all nodes (meters); Q_k = supply at a reservoir located at node *k* (meters^3/second); H_{0k} = elevation head of a reservoir located at node *k* (meters); P_i = power of a pump located in pipe *i* (watts); and γ = specific weight of water (Newtons/ meters^3).

Eq. (3.2) is the expression for Todini's resilience index (Todini 2000) and ranges from 0 to 1, where a higher value represents a higher reliability. This multi-objective problem is solved subject to the following constraints:

Continuity at nodes

For each node, continuity should be satisfied,

$$\sum Q_{in} - \sum Q_{out} = q_k, \qquad \forall\ k \in nn \tag{3.3}$$

where Q_{in} and Q_{out} = flow into and out of node *k* (meters^3/second), respectively.

Energy conservation in loops

The total head loss around a closed loop should be equal to zero, or the head loss along a loop between the two fixed head reservoirs should be equal to the difference in water level of the reservoirs:

$$\sum_{i\ \in\ loop\ l} hf_i = \Delta H, \qquad \forall\ l \in nl \tag{3.4}$$

where hf_i = head loss due to friction in the pipe *i* (meters). We use the Hazen-Williams formula to define the pressure head loss in each pipe of the network (Williams and Hazen 1914).

Minimum pressure at nodes

For each node, the pressure head should be greater than the prescribed minimum pressure head.

$$H_k \geq H_k^{min}, \ \forall k \epsilon nn \tag{3.5}$$

Pipe size availability

The diameter of the pipes should be available from a commercial size set:

$$D_i = \{D(1), D(2), \ldots, D(S)\}, \quad \forall\, i \,\epsilon\, np \tag{3.6}$$

where S = the number of candidate diameters.

3.5 Differential Evolution Algorithms for Multi-Objective Design of WDSs

Optimal WDS design is a combinatorial Non-deterministic Polynomial-hard (NP-hard) problem, requiring hydraulic simulation to evaluate Eqs. (3.3)-(3.4) for every candidate solution. As the size of the WDS increases, the combinatorial set of pipe diameter options is on the order of np^S, and the computational burden of the hydraulic simulations increases exponentially. Because they generate an array of solutions distributed throughout the feasible region at each iteration of the algorithm, thus reducing the potential for getting trapped at local optima, evolutionary search algorithms are attractive for addressing NP-hard problems. Among these, DE algorithms obtain a wider distribution of solutions within the feasible region, and for NP-hard problems have been shown to accelerate the optimization process. The dominance of DE over other evolutionary and search algorithms has been demonstrated in the least-cost design of WDS benchmark networks (Suribabu 2010; Zheng et al. 2012; Dong et al. 2012). DE Algorithms are even more beneficial in the face of multiple objectives, where evaluation of the Pareto optimal front is enhanced by the breadth of the arrays of candidate solutions. NSDE has been successfully applied to many nonlinear and NP-hard multi-objective problems (see, e.g., Angira and Babu 2005; Peng et al. 2009) and a new variation, NSDE-RMO (Chen et al. 2014), which has been applied to continuous nonlinear optimization test problems, shows promise for discrete cases.

3.5.1 Differential Evolution (DE)

As in GA, the set of decisions, e.g., the candidate design diameters for all pipes in the network, of the DE Algorithm, are represented as a chromosome, and each decision, or diameter of a given pipe, is represented as a gene in the chromosome. Candidate diameters are limited to commercially available pipe sizes and the iterative global search for the best set of chromosomes begins with an initial population of chromosomes, or combinatorial sets of pipe diameters, *NPOP* in size, that is randomly generated over the range of commercially available sizes. At each iteration, or generation, *NPOP* chromosomes are defined and allowed to reproduce as a function of their performance, and via crossover of genetic material and mutation. Because both GA and DE are unconstrained search algorithms, both use penalty functions to track and account for violations of problem constraints. In DE, a chromosome is designated as a set of real numbers, which are equivalent to the commercial pipe size selected for the corresponding pipe, and the mutation operator plays the dominant role in reproduction. At each generation, each chromosome is defined as X_j, where $j = 1, \ldots NPOP$. Each X_j experiences mutation, and mutation is achieved by acquiring genetic material from three or more randomly selected parent solutions in the current generation, forming a new chromosome X_j^{new}. Thereafter, each X_j has the potential to crossover with X_j^{new}. Storn and Price (1997) provide several formulations for X_j^{new}, where the formulation most commonly used in WDS design (see, e.g., Suribabu 2010; Zheng et al. 2012; Dong et al 2012) is:

$$X_j^{new} = X_C + F(X_A - X_B) \tag{3.7}$$

where X_A, X_B, and X_C = random solution vectors; and F = mutation scale factor that is designated to be between 0 and 1.

For a given chromosome, X_j, a gene is designated as X_{ji}, for all i= 1, …*np*, and correspondingly for every new chromosome, X_j^{new}, the genes are designated X_{ji}^{new}, for all i= 1, 2,…*np*. For every member of the population, the crossover operation governs whether and where the mutant vector X_j^{new} is allowed to breed with X_j, and is defined as follows:

$$X_j^{new} = \begin{cases} X_{ji}^{new} & if\ rand < CR \quad or \quad i = sn, \quad i = 1,2, \ldots np \\ X_{ji} & otherwise \end{cases} \tag{3.8}$$

where *rand* generates a random value between 0 and 1; *CR* = crossover rate, designated to be between 0 and 1; and *sn* = random integer value between 1 and *np* that corresponds to a location in the string of genes. This ensures that the child of X_j inherits at least one decision from X_j^{new}. Finally, selection of the fittest chromosomes in each generation is based on a comparison of the objective function values of X_j and X_j^{new} after mutation and crossover occur.

The three steps of DE, mutation, crossover, and selection, are performed sequentially and are repeated at every generation throughout the optimization cycle. The algorithm stops when a stopping criterion is met, e.g., when there is no further improvement of the objective function, or when the allowable number of function evaluations is reached.

3.5.2 Nondominated Sorting Differential Evolution (NSDE)

In NSDE, the accelerated mutation and crossover process of DE is employed, but like NSGA-II, its goal is to identify the non-inferior set of solutions to a multi-objective problem. Here the selection process is significantly different than that of DE. Once X_j and X_j^{new} are determined, rather than comparing them with each other in terms of a single objective, they are compared with the entire 2**NPOP* set of solutions for the given generation, in terms of the multiple objectives. Sorting of the 2**NPOP* solutions is undertaken firstly by fast nondominated sorting, i.e., solutions sorted based on their nondominance rank, and secondly by crowding distance, i.e., solutions within a non-dominance rank sorted in descending order of their normalized crowding distance. After sorting is achieved, the top ranked *NPOP* solutions are then allowed to survive to the next generation. An advantage of this approach is that, since crowding distance is incorporated in the sorting procedure, selected solutions are more likely to be distributed along the Pareto optimal front, see Srinivas and Deb (1994) and Deb et al. (2002) for additional details and discussion of the advantages of the nondominated sorting procedure.

3.5.3 Nondominated Sorting Differential Evolution with Ranking-based Mutation (NSDE-RMO)

As in the NSDE approach, NSDE-RMO ranks population members according to their objective performance and crowding distance, but uses these ranks to modify the mutation operator, as well as to sort in the selection process. The probability of each chromosome to mutate is defined as follows:

$$Pr_j = \frac{NPOP - Rank_j}{NPOP} \tag{3.9}$$

where Pr_j = probability of selection for the jth population member; and $Rank_j$ = rank of the solution for member j in the population (Chen et al. 2014).

By assigning higher ranked solutions with a higher probability of mutation, NSDE-RMO enhances the transfer of genetic information that is beneficial for producing high performance solutions and accelerates convergence to the Pareto optimal front.

3.6 Multi-objective Design of WDSs

For each combinatorial set of pipe diameters for the network, i.e., each chromosome, hydraulic simulation is applied to evaluate Eqs. (3.3)-(3.4) to obtain pressure heads, H_k, for all nodes of the network. Since Eq. (3.5) must be satisfied, and evolutionary algorithms do not accommodate such constraints, violations of Eq. (3.5) are tracked, assigned a penalty, and added to the cost objective function in Eq. (3.1), The penalty is defined as:

$$CP_k = \lambda \left| min\left(\frac{H_k}{H_k^{min}} - 1, 0\right)\right| \tag{3.11}$$

where CP_k = penalty associated with node k; and λ = penalty multiplier.

H_k are also used to calculate the reliability, Eq. (3.2). To assure solutions that satisfy Eq. (3.5), the magnitude of λ should be sufficiently large. The hydraulic simulation is undertaken with a Global Gradient Algorithm (GGA) (Todini 2000) implemented in MATLAB.

The performance of the NSDE Algorithms is compared with that of NSGA-II and AMALGAM, for three benchmark networks, the Two-loop, the Hanoi, and Farhadgerd benchmark networks and for three hypothetical networks chosen to represent a range of larger WDSs. In most cases, the minimum pressure limitation is 30 m above ground level, with the exception of the Farhadgerd network, which must meet the local standard of 20 m. In all cases, the population, *NPOP*, is 100. For each of the algorithms, parameter values are selected based on those recommended from the literature. For NSGA-II, these values are 90% probability of crossover and 3% probability of mutation based on Farmani et al. (2005a), and for NSDE and NSDE-RMO, 30% probability of crossover and mutation scale factor of 0.5 (Chen et al. 2014). AMALGAM self-adapts, given a range for each parameter value, and these are provided in Vrugt (2015). Calibration of the parameters for multi-objective network problems is not trivial and is beyond the scope of this paper. For a discussion of calibration of evolutionary algorithms for single objective network problems, see Marchi et al. (2014).

3.7 Benchmark and Experimental Networks

The Two-loop network has seven nodes, eight pipes, and two loops, and is fed by gravity from a reservoir with a 210-m fixed head. The pipes are all 1,000 m long with a Hazen-Williams coefficient of 130, and there are 14 candidate commercial pipe diameters available. Although the

Two-loop network is small, a complete enumeration comprises $14^8 = 1.48 \times 10^9$ different network designs, thus making this illustrative example difficult to solve. The Hanoi network is a more complex compilation of loops and branches, has 32 nodes, 34 pipes and three loops, and is fed by gravity from a reservoir with a 100-m fixed head. The pipe lengths vary from 100 to 3500 m, with a Hazen-Williams coefficient of 130, and six candidate commercial pipe diameters. The Farhadgerd, Iran, network is a detailed representation of the water mains for a community of 8200 people. It has 68 pipes, 53 nodes, and 12 loops, and is fed by gravity from a reservoir with a 510-m fixed head. The pipe lengths vary from 5 to 885 m, with a Hazen-Williams coefficient of 130, and nine candidate commercial pipe diameters. For detailed hydraulic and cost information for these benchmark networks see Alperovits and Shamir (1977) and Moosavian and Jaefarzadeh (2014), for the Two-loop and Hanoi, and Farhadgerd benchmark networks, respectively.

Three additional hypothetical networks are randomly generated to expand the range of network sizes. In all cases, for the specified numbers of nodes and pipes, whether a pipe connects any two nodes is randomly determined with equal likelihood, and the network is formed under the condition that each pipe connects only two nodes, and each node is connected to at least two pipes. For every 20 nodes, a source reservoir with a constant fixed pressure head is randomly assigned to one of the nodes, and all other nodes of the network are assumed to be at sea level (see, e.g., Zecchin et al. 2012). The parameters of each reservoir and pipe are independently sampled from uniform distributions as follows: reservoir elevation = u [50, 100], nodal demands = u [0, 0.1], and pipe length = u [500, 650], where u [LB, UB] symbolizes a random variable uniformly distributed between LB and UB. The Hazen-Williams coefficient of all pipes is assigned a constant value of 130 and as for the benchmark networks, the direction of flow is determined with the hydraulic simulator. For these networks, 20 candidate commercial pipe diameters are possible, ranging from 50 to 1000 mm, in increments of 50 mm. The smallest of these networks has 100 nodes and 200 pipes and the largest network has 400 nodes and 800 pipes. The number of combinatorial sets of pipe diameter options for these networks range from 20^{200} to 20^{800}, complicating the identification of the multi-dimensional fitness landscape, and increasing the difficulty in identifying Pareto optimal decisions. For all networks, the value of λ, the penalty weight in Eq. (3.11), is selected to be the maximum cost for the given network, i.e., the sum of the costs, assuming that for all pipes, the maximum diameter is chosen.

3.8 Performance of Multi-objective Optimization Algorithms

In order to evaluate the number of required generations to approximate the Pareto optimal front, for each benchmark and hypothetical network, all nondominated sorting algorithms are executed 20 times. Each algorithm is stopped when there is no further change in the cost-reliability Pareto optimal front, expressed by the minimum, median and maximum solutions, for 50 generations. Table 3.1 provides the average values of the costs, resilience indices, and generations for the 20 algorithm executions. In all cases, the pressure constraints, see, e.g., Eq. (3.5), are satisfied, and thus there are no infeasible solutions. Being a black-box, hybrid meta-heuristic, the AMALGAM method uses the number of generations as the stopping criteria, to be pre-specified by the user. Thus our IGD-based stopping criteria could not be applied for this method. For the benchmark networks, the single objective least-cost solutions identified in previous studies are also provided in the table. For all executions of the algorithms, the minimum cost calculated for the multi-objective WDS design is greater than the single objective least-cost solution. This shows that no algorithm is able to explore the entire Pareto front using the specified algorithm parameters. However, as shown in Table 3.1, NSDE and NSDE-RMO have explored this part of the Pareto front better than NSGA-II and, with the exception of the Hanoi case study, found cheaper solutions.

As one measure of the accuracy of the approximated set of nondominated solutions, or the speed of convergence of an algorithm to a near optimal solution, the Inverted Generational Distance (IGD) determines the average distance of the uniformly distributed Pareto optimal solutions obtained after a very large number of generations to the approximated nondominated surface. Small values of IGD are preferred and the rate at which small values are achieved indicates the rate of convergence. For a detailed development of the IGD measure and a discussion of its shortcomings see Zitzler et al. (2003) and Maier et al. (2014).

Based on the findings in Table 3.1, 20 additional algorithm executions were undertaken, for 500, 1000, and 2000 generations, for the Two-loop, Hanoi and Farhadgerd, and hypothetical networks, respectively. Figures 3.1-3.3 show one Pareto optimal front for the benchmark networks. Figures 3.4-3.6 are similar figures for the hypothetical networks. These are the Pareto optimal fronts with the lowest final IGD, among the 20 executions of the algorithms. For the Two-loop network, the cost-reliability trade-off is nearly the same for all algorithms, where the cost ranges from \$420,000 to $4.4 million for all four algorithms, and the resilience index from 0.029 to 0.113, for the NSDE Algorithm, and approximately 0.043 to 0.113 for all other algorithms.

For the Hanoi network, the performance of the nondominated sorting algorithms is also similar, although they are unable to find solutions at the extreme upper range of the frontier. Here the performance of NSGA-II and AMALGAM are high, though AMALGAM generates more of the upper range of the frontier and NSGA-II generates more of the lower range. This reinforces similar findings in Wang et al (2014), where, for the Hanoi network, NSGA-II shows better convergence than AMALGAM for the majority of the Pareto optimal front, but only AMALGAM generates solutions at high reliability levels. As Vrugt and Robinson (2007) note, the Strength Pareto sorting approach in AMALGAM ensures that unique individuals survive to the next generation, thus enhancing its performance at the extreme ends of the frontier, and this characteristic is demonstrated for this network.

For the Farhadgerd network, NSGA-II generates a narrow portion of Pareto optimal front, with costs ranging from $1.87 to $4.65 million and resilience index from 0.56 to 0.66. Here AMALGAM generates solutions at the upper range of the frontier, and NSDE and NSDE-RMO generates those at the lower range of the frontier. The mutation operator in the NSDE Algorithms enhances their ability to rapidly identify points on the Pareto optimal front, and the ranked-based sorting mutation operator in NSDE-RMO enhances its ability to expand the frontier as more generations pass.

As shown in Figures 3.4-3.6, the performance of nondominated sorting algorithms is comparable for the smallest hypothetical network, with NSDE and NSDE-RMO identifying a wider range of non-inferior points. In general, as the network size increases, the NSDE-RMO solutions dominate those of all other algorithms, and the NSDE Algorithm performs well, though not as well as the NSDE-RMO. The performance of the NSGA-II Algorithm and AMALGAM method drops significantly as the network size increases.

The results for the WDS with 400 nodes and 800 pipes, WDS400x800, indicate that, while the NSDE and NSDE-RMO Algorithms appear to be trending toward dominance, the solutions have not converged to the Pareto Optimal Front for any of the algorithms, suggesting that the population number or the number of generations is insufficient for all algorithms. However, NSDE-RMO does provide the solution with the minimum cost ($116 million) and the highest reliability with a resilience index of 0.615.

Figures 3.7-3.12 provide the IGD values for the benchmark and hypothetical networks, where the final Pareto optimal fronts in Figures 3.1-3.6 are considered as the reference ideal trade-off. For the Two-loop and Hanoi networks, the ALMALGAM method converges faster than other algorithms, but in general, as the network size increases its convergence slows relative to that of all other algorithms. With the exception of WDS400X800, which has not yet converged, the performance of NSDE and NSDE-RMO are similar for most network sizes, and are greater than for other algorithms, though NSDE-RMO performs slightly better overall.

3.9 Conclusion

NSDE and NSDE-RMO are efficient for evaluating trade-offs between cost and reliability in WDS design. Moreover, they are particularly effective for evaluating solutions in the highest cost-benefit portion of the solution space (i.e., high-resilience, low-cost solutions), which may be of high interest to practitioners. Discrete large NP-hard problems like WDS design require the consideration of many different solutions and the mutation advantage of DE is particularly effective at generating diverse sets of solutions. The mutation operator of DE, compared with that of GA and other evolutionary algorithms, provides candidate solutions with very different characteristics. This feature enhances its ability to escape local optima and, combined with the selection function of nondominated sorting, is the main reason that NSDE is attractive for analyzing multi-objective discrete large problems.

In a comparison analysis of benchmark and hypothetical networks, NSDE and NSDE-RMO generally perform as well or better than NSGA-II and AMALGAM except for the Hanoi and Farhadgerd networks where, at the extreme end of the Pareto optimal front, AMALGAM performs well. NSDE-RMO is typically more effective than NSDE. Its ability to rapidly identify the Pareto optimal front, due to its ranked-based selection mechanism, makes it an attractive algorithm overall.

The WDS analysis results show promise for the use of NSDE and NSDE-RMO in the multi-objective analysis of large and more realistic networks. The use of these algorithms may provide opportunities for expanding the objectives to be considered in WDS design, and for including additional definitions of reliability in the objective set.

The high performance of the NSDE Algorithms is further evidence of the advantages of nondominated sorting, and of the clear potential of ranked-based sorting, in multi-objective evolutionary optimization. Advances in multi-objective optimization and WDS design may be directed at harnessing this potential, for example, by modifying AMALGAM to include ranked-based sorting as part of its sorting procedure. Moreover, the efficiency of NSDE may be further improved by incorporating other sorting procedures, such as the Strength Pareto sorting approach, which shows potential for defining the extreme end of the Pareto frontier.

Table 3.1 Average of twenty executions of the algorithms

Network Type (diameter options) Least cost		Average Min Cost($)	Average Median Cost ($)	Average Max Cost($)	Average Min Resilience Index	Average Median Resilience Index	Average Max Resilience Index	Average Stopping Generation
Two-Loop Benchmark (14) $4.19E+5[1]	NSGA-II	4.54E+05	1.21E+06	4.28E+06	4.99E-02	1.05E-01	1.12E-01	224
	NSDE	4.29E+05	1.12E+06	4.27E+06	4.24E-02	1.03E-01	1.12E-01	216
	NSDE-RMO	4.28E+05	1.11E+06	4.29E+06	4.33E-02	1.03E-01	1.12E-01	179
Hanoi Benchmark (6) $6.08E+6[2]	NSGA-II	6.38E+06	7.37E+06	9.77E+06	2.51E-01	3.25E-01	3.53E-01	520
	NSDE	6.45E+06	7.27E+06	9.08E+06	2.60E-01	3.22E-01	3.48E-01	405
	NSDE-RMO	6.42E+06	7.25E+06	9.14E+06	2.54E-01	3.21E-01	3.48E-01	371
Farhadgerd Benchmark (9) $1.78E+7	NSGA-II	1.94E+07	2.48E+07	4.29E+07	5.60E-01	6.47E-01	6.56E-01	835
	NSDE	1.91E+07	2.59E+07	4.35E+07	4.93E-01	6.49E-01	6.56E-01	503
	NSDE-RMO	1.90E+07	2.56E+07	4.44E+07	4.95E-01	6.49E-01	6.56E-01	582
WDS100x200 hypothetical network (20)	NSGA-II	2.34E+07	3.02E+07	3.93E+07	8.39E-01	8.77E-01	8.78E-01	617
	NSDE	1.95E+07	2.80E+07	5.04E+07	8.24E-01	8.77E-01	8.78E-01	1018
	NSDE-RMO	2.00E+07	2.68E+07	4.17E+07	8.06E-01	8.75E-01	8.78E-01	726
WDS200x400 hypothetical network (20)	NSGA-II	5.88E+07	6.62E+07	8.15E+07	5.73E-01	6.21E-01	6.28E-01	1118
	NSDE	5.54E+07	6.57E+07	8.55E+07	5.89E-01	6.24E-01	6.32E-01	1145
	NSDE-RMO	5.24E+07	6.29E+07	9.01E+07	5.65E-01	6.22E-01	6.32E-01	1503
WDS400x800 hypothetical network (20)	NSGA-II	1.40E+08	1.52E+08	1.73E+08	5.20E-01	6.06E-01	6.11E-01	1352
	NSDE	1.30E+08	1.44E+08	1.70E+08	5.87E-01	6.07E-01	6.14E-01	1287
	NSDE-RMO	1.16E+08	1.30E+08	1.57E+08	5.73E-01	6.07E-01	6.14E-01	1697

[1] From Geem (2006)

[2] From Geem (2006)

[3] From Lence et al. (2017)

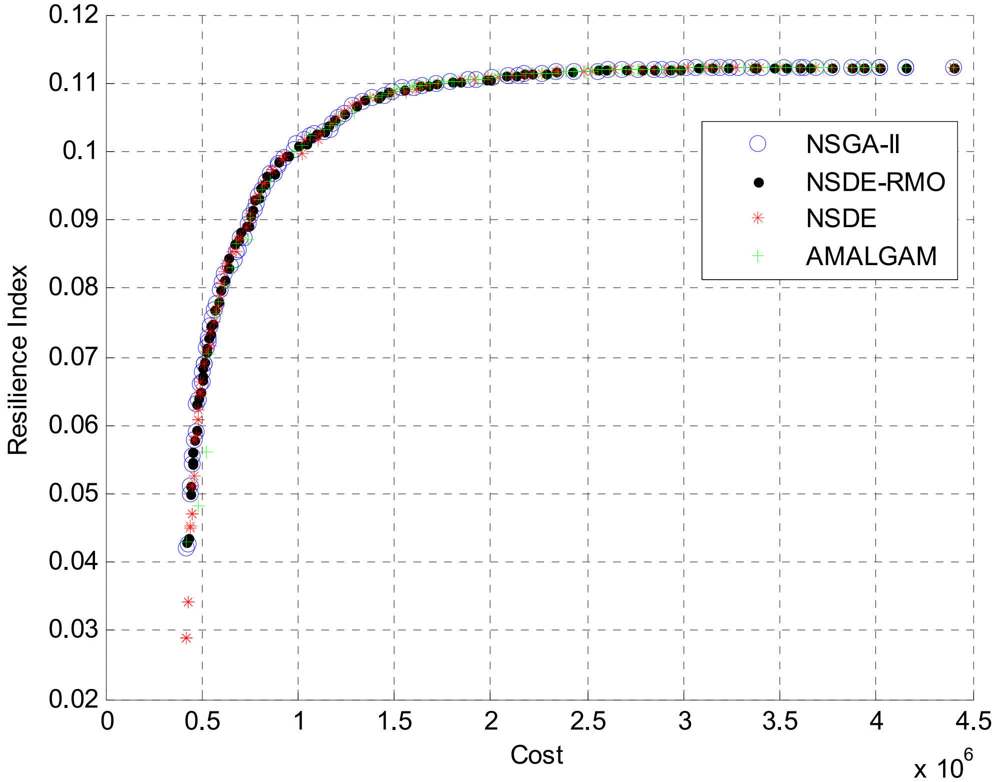


Figure 3.1 Pareto optimal front of Two-loop network

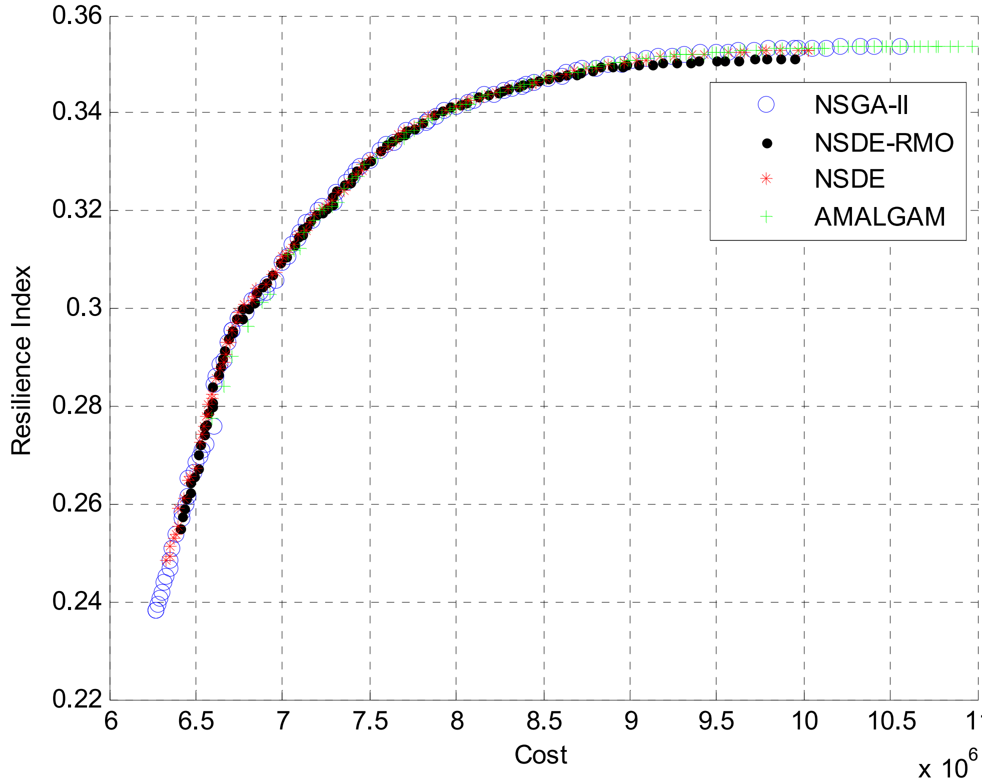


Figure 3.2 Pareto optimal front of Hanoi network

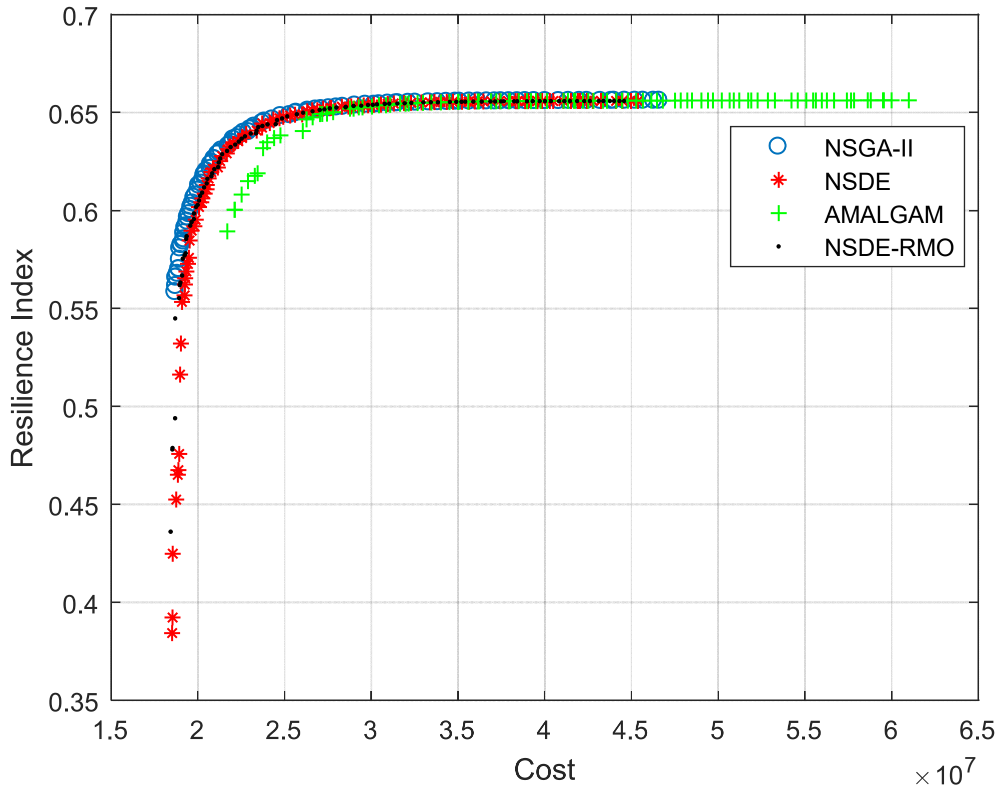


Figure 3.3 Pareto optimal front of Farhadgerd network

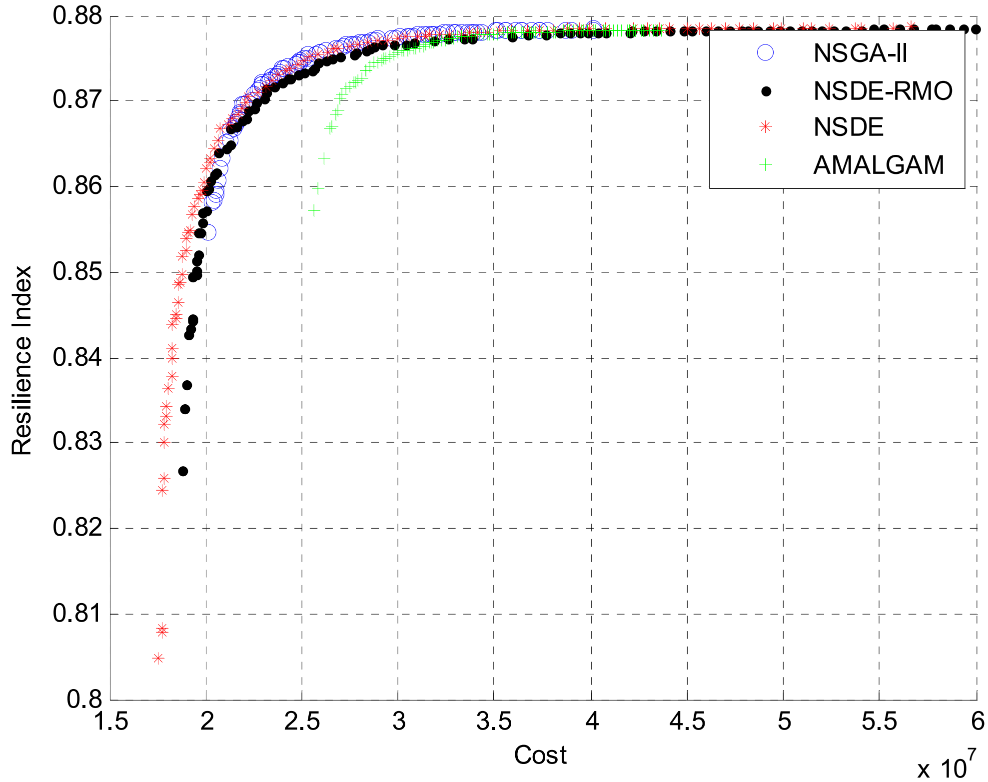


Figure 3.4 Pareto optimal front for WDN 100x200, 100 nodes and 200 pipes

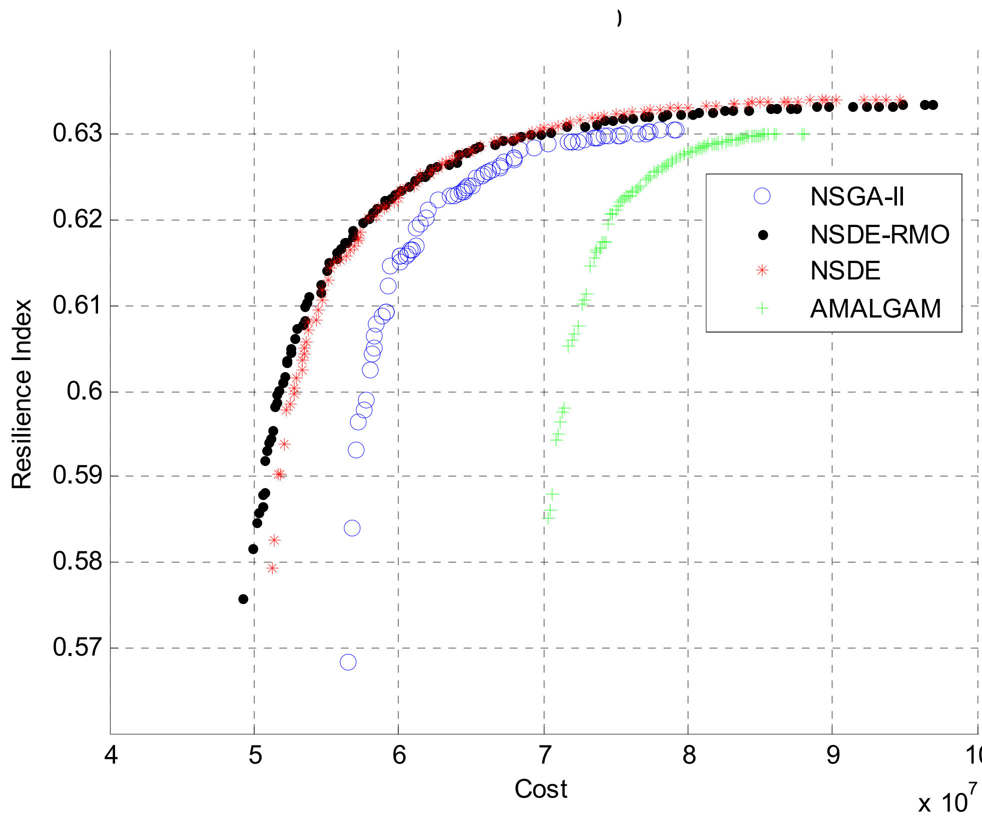


Figure 3.5 Pareto optimal front for WDN 200x400, 200 nodes and 400 pipes

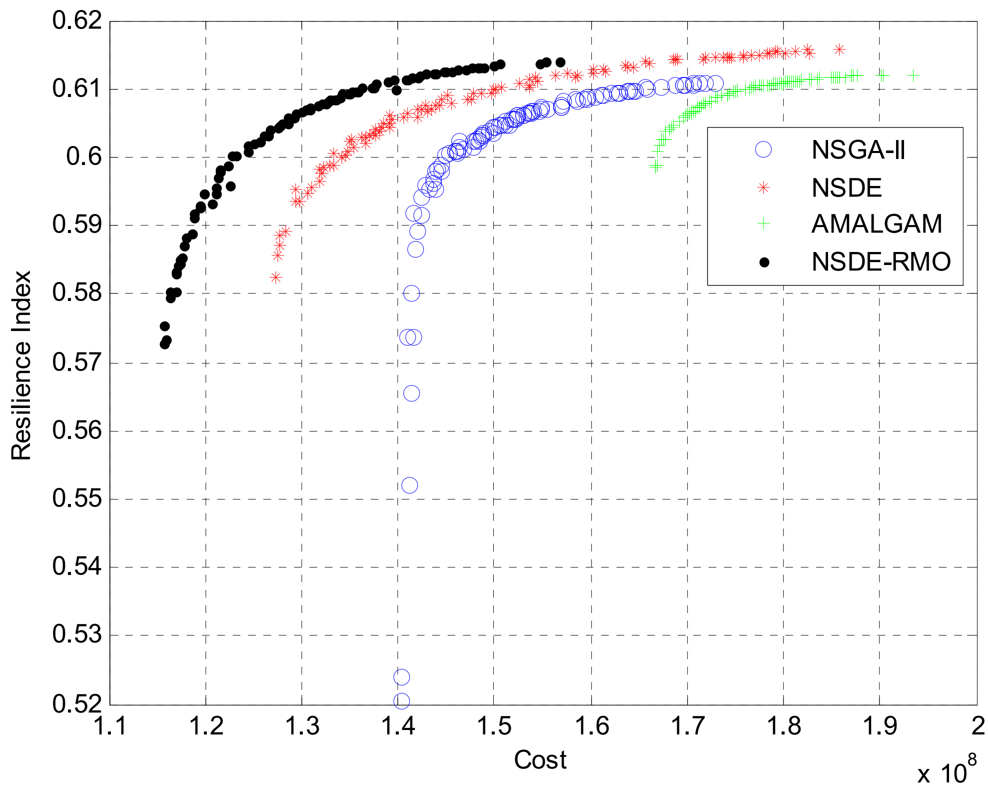


Figure 3.6 Pareto optimal front for WDN 400x800, 400 nodes and 800 pipes

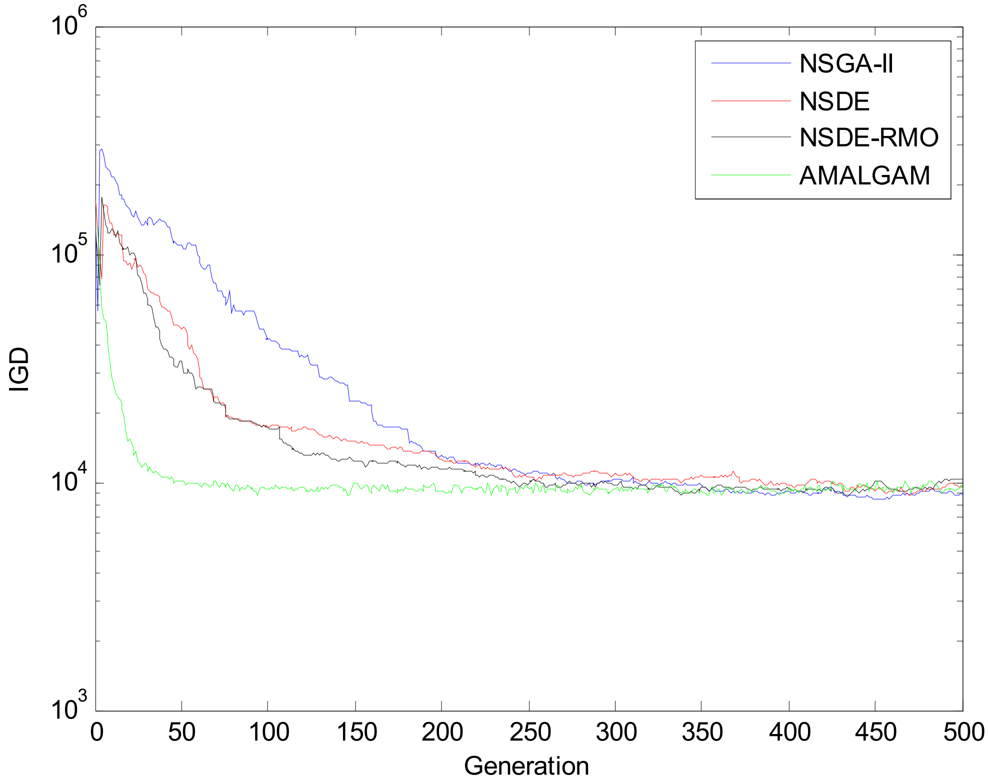


Figure 3.7 IGD at each generation for Two-loop network

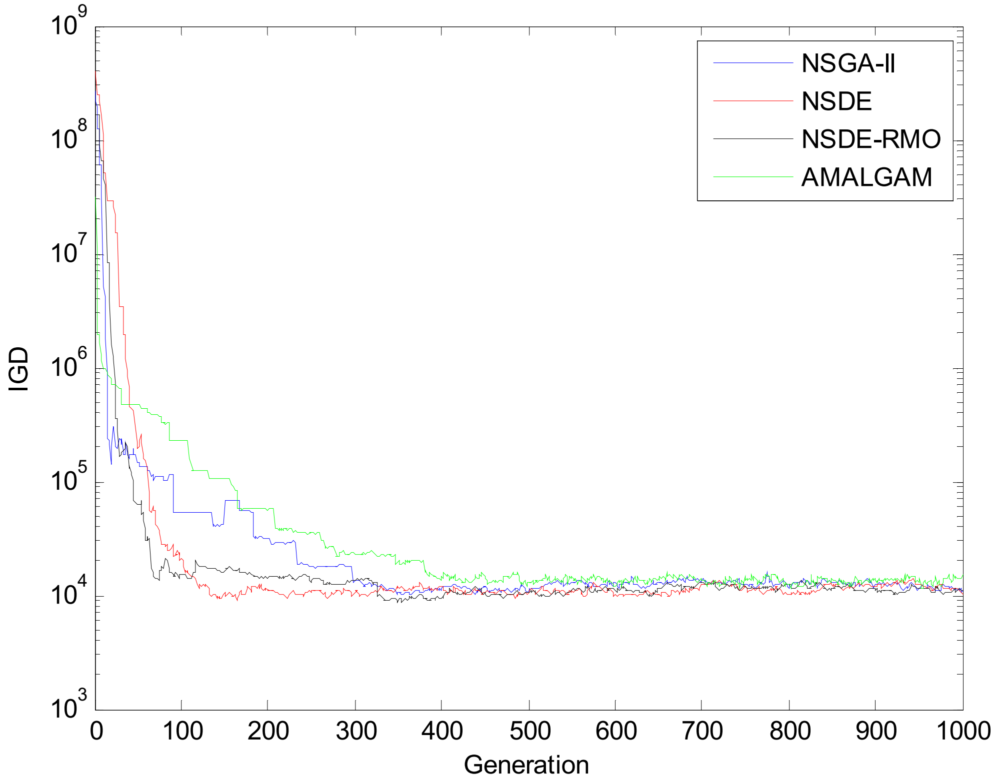


Figure 3.8 IGD at each generation for Hanoi network

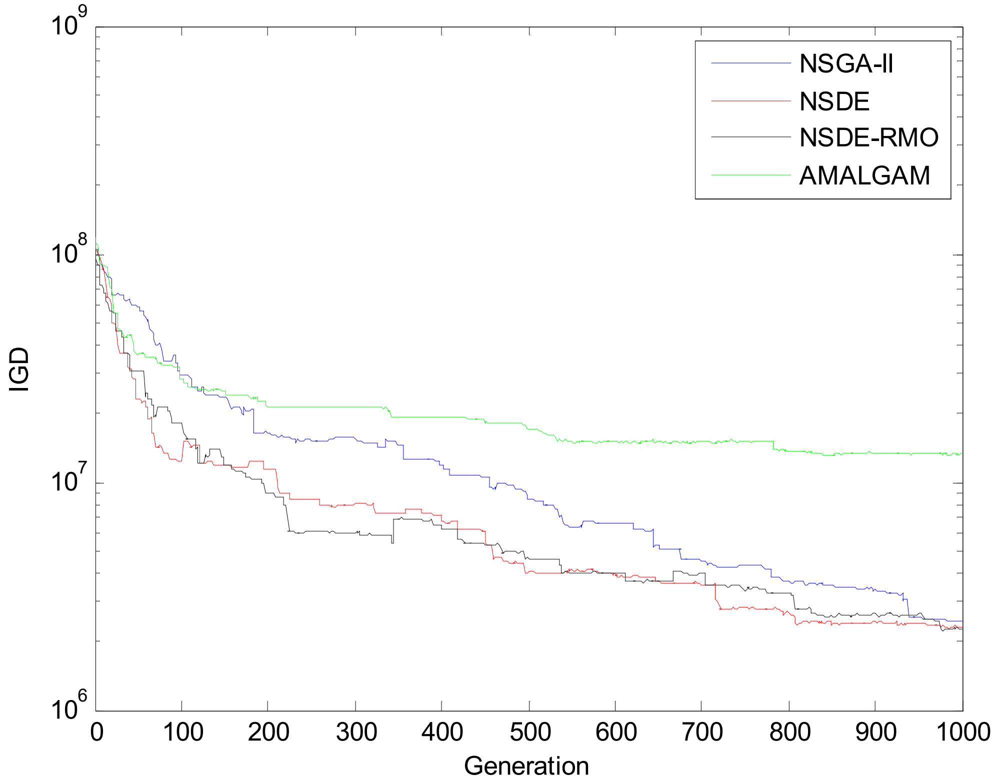


Figure 3.9 IGD at each generation for Farhadgerd network

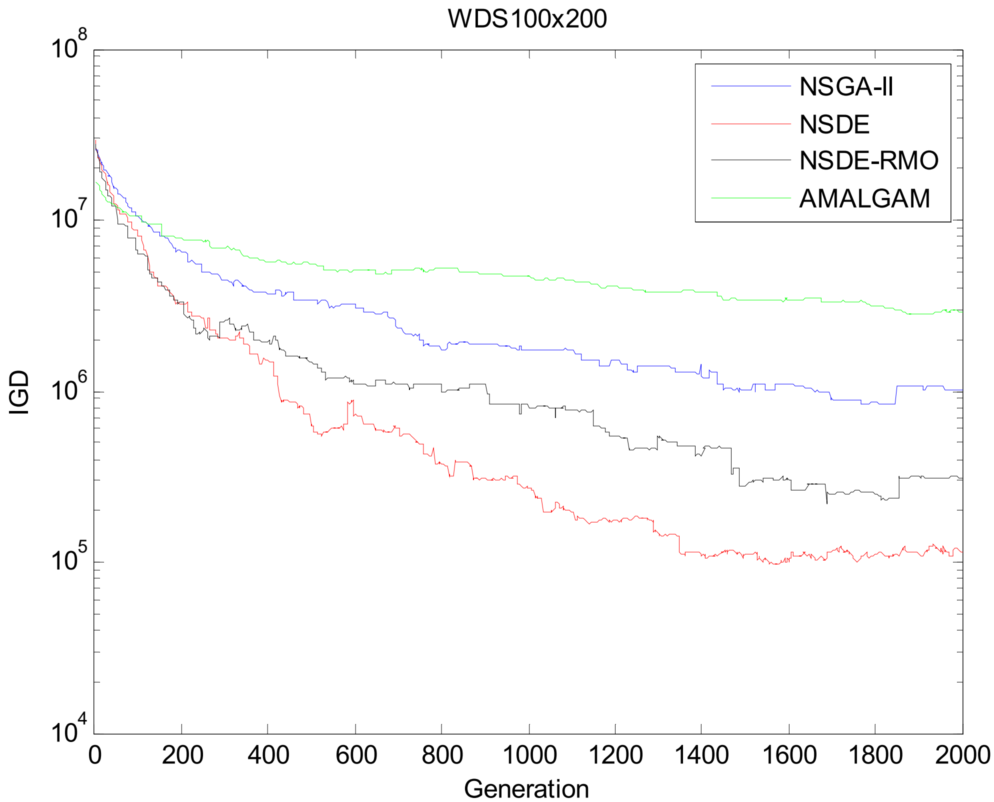


Figure 3.10 IGD at each generation for WDN100x200

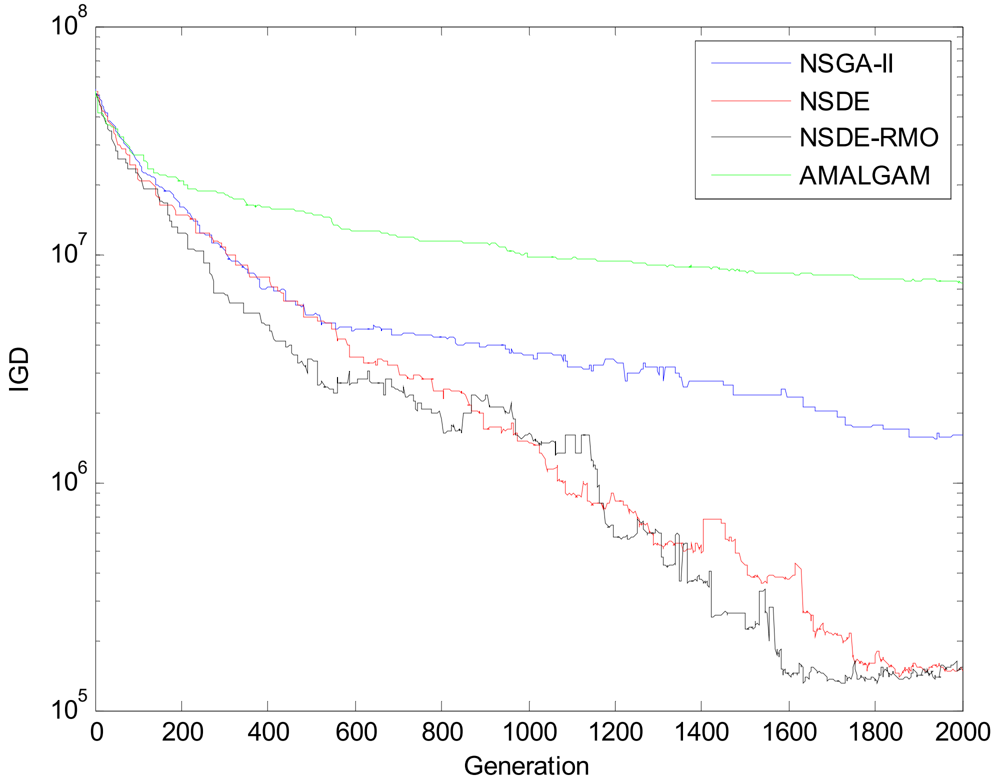


Figure 3.11 IGD at each generation for WDN200x400

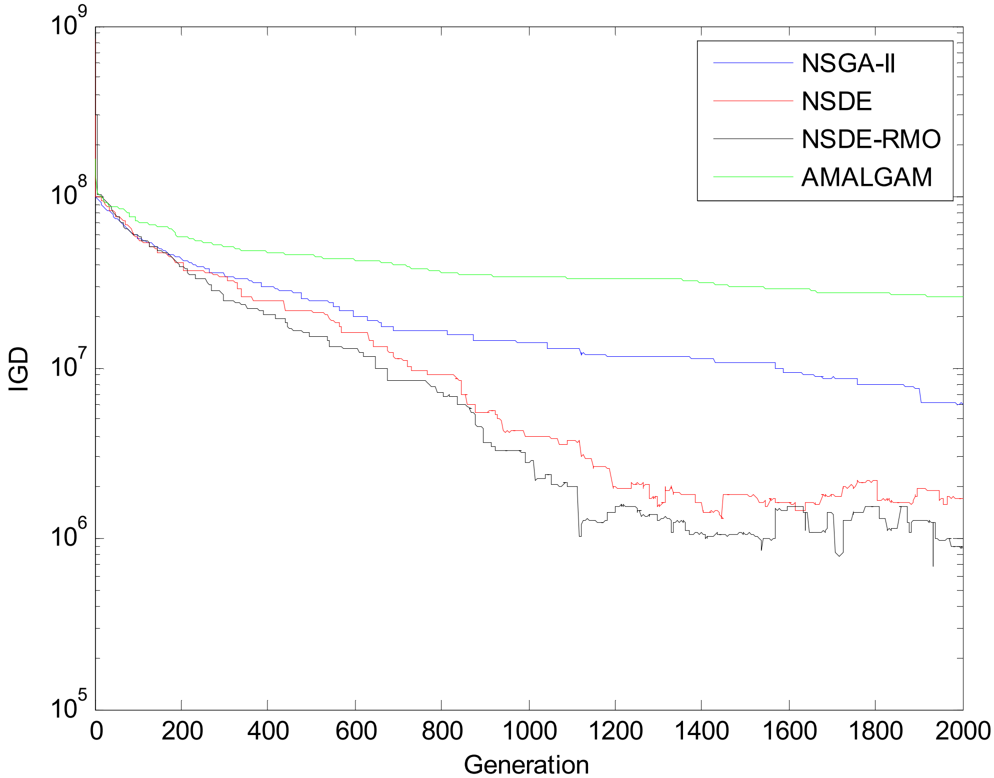


Figure 3.12 IGD at each generation for WDN400x800

Chapter 4 : Fittest Individual Referenced Differential Evolution Algorithm for Optimization of Water Distribution Networks

4.1 Preface

In Chapter 2, ten popular EAs are tested and compared for single-objective design of three networks. While these heuristic algorithms show success in addressing specific WDN design problems, there is no guarantee that the global optimum may be found or that any one method will be efficient. Wolpert and Macready (1997) cite the No-Free-Lunch (NFL) theorem, assert that no one optimization algorithm may be suited for solving all kinds of optimization problems, and underscore the need for new algorithms that may improve on efforts to reach global optima. Furthermore, results and analysis of EAs in Chapter 2 show that some characteristics and properties of the CMAES, SLC, ABC and DE have significant impact on the convergence properties of these algorithms. Among all other algorithms, DE has the simplest framework and computing code structure which makes it efficient for solving a wide range of optimization problems. In this chapter, by maintaining the simple framework of DE and adding some worthy characteristics of SLC and ABC, a new efficient algorithm, called the Fittest Individual Referenced Differential Evolution (FDE), is proposed which significantly increases the speed of convergence in the optimization of WDN design. The fundamental concept of FDE is similar to DE, however, a counter for each solution vector is defined and used to locate local optimum traps and restart the algorithm, thus avoiding stagnation positions. This capability is beneficial in network design, because the problem has various local optima which are sometimes far from eachother and make finding global optima difficult. Results of optimization for three benchmark networks, the Two-loop, Hanoi, and NewYork WDN, show that the number of function evaluation under FDE required to reach the global optimum is fewer than DE. For example for Two-loop network, the minimum number of function evaluations to find first occurrence of the best solution for FDE is 197 while for DE is 1,320.

4.2 Abstract

Water distribution networks (WDNs) are one of the most essential components in the urban infrastructure system and require large investment for construction. Design of such networks is classified as a large combinatorial discrete nonlinear non-deterministic polynomial-hard (NP-hard) optimization problem. The main concerns associated with optimization of WDNs are related to the nonlinearity of the discharge-head loss relationships for pipes and the discrete nature of pipe sizes. This paper proposes a new evolutionary algorithm, called Fittest Individual Referenced Differential Evolution (FDE), which is significantly more efficient and reliable than other algorithms for optimization of WDN DESIGN problems. The fundamental structure of FDE is similar to conventional Differential Evolution (DE), though the functions of the mutation and crossover operators are to exploit good solutions, and explore very different solutions, respectively, and counting and re-starting mechanisms are introduced to identify local optima and avoid stagnation. These features are beneficial in WDN design because such problems may have an array of local optima, which are, at times, far from one another, making finding the global optima difficult. The mutation and crossover operators are developed to accelerate convergence and, while the optimal mutation and crossover parameters may vary among networks, the efficiency of the algorithm reduces the need for adaptive parameters or population size. A sensitivity analysis is conducted for algorithm parameters and counting and re-starting limits, based on eight evolutionary algorithm (EA) benchmark test problems, and the efficiency of the FDE for solving two- and ten-dimensional versions of these problems is demonstrated, using the most effective values of the parameters and limits. FDE-based optimal designs of three benchmark WDNs, the Two-loop, Hanoi, and New York Networks, show that a minimum number of function evaluations (or hydraulic simulations) are required to reach the global optimal, less than 300 in all cases, compared with the more than 1300 function evaluations required for DE. Furthermore, an application of FDE to a large-sized network, that for the City of Farhadgerd, Iran, shows that FDE is significantly more effective than other EAs in terms of its speed of convergence and reliability.

Key words: Evolutionary algorithms, differential evolution algorithm, water distribution networks, optimization.

4.3 Introduction

Approaches for determining optimal design of water distribution networks (WDNs) and their appurtenances have been developed based on linear programming, LP (Schaake and Lai (1969); Alperovitz and Shamir (1977); Quindry et al. (1982); Kessler and Shamir (1989)), and nonlinear programming, NLP (Lansey and Mays (1989); Duan et al. (1990)) formulations that maximize design objectives while satisfying the hydraulic rules that govern such systems, and consumer demands for water quantity, quality and pressure. In an illustration based on the Two-loop benchmark WDN, Bhave and Sonak (1992) demonstrate that LP is less efficient in terms of computational effort, compared with a heuristic method which initially identifies good branching configurations for obtaining the global optimum. Cunha and Sousa (1999) posit that NLP approaches are sophisticated in terms of their use of the fundamental hydraulic equations to recast the form of the optimization problem and to yield gradient and Hessian information, however, their inability to restrict the search space to discrete pipe sizes is a significant practical limitation.

In order to identify designs for large networks, and more readily accommodate the discrete and nonlinear nature of the problem, this paper presents a new evolutionary optimization algorithm (EA), the Fittest Individual Referenced Differential Evolution (FDE) Algorithm, which significantly increases the efficiency and consistency of the conventional Differential Evolution (DE) Algorithm (Storn and Price 1997). DE, like other EAs, is becoming increasingly more popular for optimization of WDNs and in practical applications, because it: (i) is based on simple concepts and is easy to implement; (ii) can identify and avoid local optima; and (iii) does not require a continuous or continuously differentiable objective function. It employs operators, similar to mutation, crossover (or mating), and selection, to explore the fitness surface, or range of solutions, and exploit attractive solutions, i.e., mutation seeks to assess all possible good solutions, and crossover seeks to concentrate these solutions. Because they generate an array of distributed solutions at each iteration of the algorithm, thus reducing the potential for getting trapped at local optima, EAs are attractive for addressing NP-hard problems. Among these, DE algorithms have been shown to obtain a wider distribution of solutions within the feasible region, and accelerate the optimization process (Storn and Price 1997; Noman and Iba, 2005; Yang et al., 2007).

Many EAs are successfully used to identify WDN designs, see, e.g., applications of Genetic Algorithms, GAs (Brooke et al. 1988; Lansey and Mays 1989; Duan et al. 1990); Simulated Annealing, SA (Cunha and Sousa 1999); Harmony Search, HS (Geem 2006); Shuffled Frog Leaping (Eusuff and Lansey 2003); Ant Colony Optimization, ACO (Maier et al. 2003); Max-min Ant System (Zecchin et al. 2006); Particle Swarm Optimization, PSO (Suribabu and Neelakantan 2006a and Suribabu and Neelakantan 2006b); Cross Entropy (Perelman and Ostfeld 2007); Scatter Search (Lin et al. 2007); DE (Vasan and Simonovic 2010); Self-adaptive Differential Evolution, SaDE (Zheng et al. 2013); Soccer League Competition, SLC (Moosavian and Roodsari 2014); and Improved Genetic Algorithms (Bi et al. 2015). Recently, combinations of classical optimization methods and EAs are suggested for WDN design, e.g., Cisty (2010) applies a combination of a simple GA and LP; Haghighi et al. (2011) apply a combination of GA and integer linear programming (ILP); Zheng et al. (2011) apply a combination of NLP and DE; and Zheng et al. (2014) apply a combination of binary linear programming (BLP) and DE. All of these algorithm combinations eventually converge to an appropriate design, although their computational costs are often significantly greater than those of the aforementioned stand-alone EAs. To accelerate convergence, hybrid algorithms are developed to improve the efficiency of the search process, including: Keedwell and Khu (2005) who use a hybrid method in which a cellular automata approach provides a good initial population for a GA; Tolson et al. (2009) who propose a hybrid approach which combines two local search heuristics with a discrete search strategy adapted from a continuous dynamically dimensioned search algorithm; Sedki and Ouazar (2012) who apply a combination of DE and PSO algorithms; Babu and Vijayalakshmiz (2013) who apply a combination of PSO and GA; Geem (2013) who applies a combination of PSO and HS algorithms; Sheikholeslami et al. (2016) who apply a combination of Cuckoo search and HS algorithms; and Reca et al. (2017) who apply a combination of a search space reduction algorithm and GA.

The dominance of DE over other algorithms has been demonstrated in the least-cost design of WDS benchmark networks (Suribabu 2010; Zheng et al. 2011; Dong et al. 2012). DE Algorithms are also beneficial in the face of multiple objectives, where evaluation of the Pareto optimal front is enhanced by the breadth of the arrays of candidate solutions. Nondominated Search Differential Evolution (NSDE) has been successfully applied to many nonlinear and NP-hard multi-objective problems (see, e.g., Angira and Babu 2005; Peng et al. 2009) and a new variation, Nondominated Search Differential Evolution with Ranking-based Mutation Operators, NSDE-RMO (Chen et al. 2014), which has been applied to continuous nonlinear optimization test problems and multi-objective WDN design (Moosavian and Lence 2016), shows promise for discrete cases.

Given the efficiency of unaided DE, or DE in combination with other algorithms, for solving an array of discrete and continuous optimization problems, improvements in the algorithm abound. These can be classified into: (i) tuning the control parameters, i.e. tuning the mutation and crossover parameters, (Zheng et al. 2014; Gamperle et al. 2002; Omran et al. 2005; Brest et al. 2006; Das et al. 2005) or population size (Teo 2006); (ii) advancing the selection operator, such as by introducing ranking-based selection (Gong and Cai 2013; Chen et al. 2014) and statistically-based selection (Mininno et al. 2011); and (iii) adding new schemes for advancing the operators, such as by introducing enhanced mutation formulae (Zhao and Liu 2014; Kumar 2017; Cail et al.

2017; Maucec et al. 2018), employing a neighborhood search (Yang et al. 2007; Das et al. 2009), and improving the crossover process (Liang et al. 2014), and (see, e.g., Das et al. (2009) for an in-depth review of these and other advances in DE). In one example of the latter, the Fittest Individual Refinement, FIR-modified DE (Noman and Iba 2005), the crossover operator compares each mutated individual with the individual with the highest fitness, and uses the Simplex Crossover approach for multiparent recombination of individuals. Thus, in FIR, crossover allows for both accelerated exploitation of the current best solution, along with exploration of the range of solutions.

This paper develops and analyzes applications of an improved DE, FDE, which modifies mutation and crossover operators to accelerate convergence, and employs a counting mechanism to find local optima traps. Each individual member of the population of solutions undergoes mutation with the current best solution, and crossover with other randomly selected members of the population. Thus, in contrast to conventional DE, the mutation operator encourages exploitation of good solutions, and the crossover encourages exploration of the range of solutions. Local optima are identified by counting the number of generations in which a given individual solution survives and, upon reaching a counting limit, eliminating this individual solution from the population, thereby shrinking the population size. As further local optima are identified, the population size reduces further, and this process continues until a re-starting limit on the minimum population size is reached. Thereafter, the algorithm restarts the entire optimization search, continuing to explore the range of solutions. The FDE Algorithm is efficient for optimization of WDNs, because this problem has many local optimum solutions, while other EAs cannot always escape from these locations (Liong and Atiquzzaman 2004; Reca and Martinez 2006; Zecchin et al. 2006). The performance of FDE is demonstrated for eight EA benchmark test problems, for three benchmarks WDNs, the Two-loop, Hanoi, and New York networks, and for the WDN for the City of Farhadgerd, Iran. The results of the application of FDE to eight EA benchmark test problems are compared with those of DE, and the results of the application of FDE to the benchmark networks are compared with results from a large array of EA applications reported in the literature for these problems. For the Farhadgerd WDN, FDE and nine popular EA Algorithms and metaheuristics are implemented in this work, and all ten of these algorithms are compared in terms of their efficiency, i.e., the rate at which the global optimum is found, and reliability, i.e., its consistency in finding the global optimum. The algorithms assessed for the Farhadgerd WDN are

GA, SA, HS, DE, PSO, ABC, SLC, the covariance matrix adaptation evolution strategy (CMAES) (Hansen and Kern 2004), and biology and bioinformatics global optimization (MEIGO) (Egea et al., 2010; Egea et al., 2014). Results show that the proposed FDE is superior in terms of the number function evaluations required to reach the global optimum, and consistently does so, over a large number of applications of the algorithm.

In the following sections, the DE and the proposed FDE algorithms are presented in detail. Next, a sensitivity analysis regarding the parameters and limits employed in FDE, and a comparison of the perfomance of DE and FDE, is conducted for eight benchmark test problems for EAs. Thereafter, the general optimization model formulation for least-cost design of WDNs is briefly described. Application of FDE to three benchmark and the Farhadgerd WDNs is then demonstrated and performance of FDE for these networks is assessed, compared with that for an array of EAs reported in the literature, and nine efficient EAs or metaheuristics, respectively. Finally, a discussion of the advantages of FDE, and recommendations for future research are provided.

4.4 Differential Evolution (DE) and Fittest Individual Referenced Differential Evolution (FDE) Algorithms

Table 4.1 outlines the different steps of the DE and FDE optimization algorithms and describes their evolution strategies in detail. In both algorithms, and similarly for all EAs, **Step 1** is the initialization of the parameters, i.e.: the mutation, *F*, and crossover, *CR*, rates; the coding of the strings that define the possible decision variable values, for the vectors of decisions that represent the members of a population; and the number of individuals that comprise the population (*NPOP*). In addition to this information, FDE also assigns a counter limit, *Counterlimit*, and a re-start limit, *Re-startlimit*.

Step 2 is the generation of the initial population of decision variable values, for vectors of decisions, or individual members of the population, and calculation of the corresponding objective function values for, or the fitness value of, these vectors, i.e., , the generation of random vectors of decision variable values for an array of solutions, X_j, where $j = 1,\ldots, NPOP$, that represent the first generation of a population. In this step, DE assigns a solution vector and the corresponding objective value for each population member. In addition to these vectors, FDE also assigns a new variable, called the counter, $Counter_j$, and sets its value to zero.

In **Step 3**, both algorithms conduct two different mutation operations. In DE, the mutation operator selects three random solution vectors for exploration purposes, X_A, X_B, and X_C. In so doing, it provides a random search direction for each member of the population. In FDE, X_A, X_B, and X_C are identified, but the mutation operator is based on a random movement by selection of two random solution vectors, X_A, and X_B, around the member of the current population with the highest fitness, or best objective function value, X_{best}, and thus provides dissimilar random search directions for the best solution thus far. The mutation operation in FDE is designed for exploitation purposes and by improving the best solution vector at each stage, it accelerates the convergence rate of the algorithm. Figure 4.1 shows a conceptualization of the different nature of the mutation operators in DE and FDE.

In **Step 4**, both algorithms conduct crossover operations. In DE, the crossover operation is conducted for each X_j, to exploit the search direction which is created through mutation. FDE introduces a shuffled crossover operator to explore the range of solutions, the fitness landscape, by randomly moving the modified best solution vector created through mutation. The FDE crossover operator conducts its process with the random solution vector, X_C, instead of X_j. The shuffling process provides more extensive exploration of the search space in different directions, thereby reducing the chance of the search getting trapped in local optima. Therefore, by application of both the mutation and crossover mechanisms, new positions of the population members for both algorithms may be widely distributed.

In **Step 5**, both algorithms update the population based on the new solution vector created by the mutation and crossover operators. In this step, the objective function value corresponding to the new solution vector is evaluated, and this value is compared with the current solution vector, X_j. If the objective value of the new solution vector is better than that of X_j, then it enters the population in place of X_j. Otherwise, X_j remains in the population. A similar comparison is conducted with the best solution vector (X_{best}) to update this member after each stage.

In this step, FDE also updates the counter value of the selected member, $Counter_j$, i.e., if the new solution vector is better than that of X_j, the counter is kept at its current value, i.e., zero in the first generation, otherwise the corresponding counter is increased by one unit, $Counter_j = Counter_j + 1$. To show the difference in performance of mutation and crossover operators in the DE and FDE Aalgorithms, a two-dimensional Sphere Function EA benchmark test problem is employed and the first two iterations of both algorithms are executed. The EA benchmark test problem is minimization of $f(x,y) = x^2 + y^2$, with the global optimum solution of $(0,0)$. Assume that the initial population for both algorithms is randomly generated as in Table 4.2.

The distribution of the initial population in the X-Y plane is shown in Figure 4.2. The first iteration of both algorithms, including mutation, crossover and selection, are shown in Table 4.3. After the first iteration, the best solution obtained by DE is its new point D (2,1) with an objective function value equal to 5, and an average objective function value for all five points of 14. The best solution vector for FDE is its new point B (-1, 0.5) with an objective function value equal to 1.25, and an average objective function value for all five points of 13.15. Furthermore, the number of function evaluations required for DE to obtain point D is five, and is already 25% higher than that required for FDE to reach point B, which is 4. Therefore, FDE has slightly better performance even in the first iteration and can accelerate the convergence process with a lower computational burden.

A similar progression may be observed in the second iteration, as reported in Table 4.4. For the DE-based solution, the average value of the objective function for all points remains at 14 requiring ten function evaluations, while for the FDE-based solution, the average value of the objective function is reduced to 8.86 requiring eight function evaluations.

In **Step 6**, the DE algorithm checks the termination criteria, e.g., some maximum number of generations or function evaluations, or some lower limit on the change in global optimum fitness value, and if the criteria are not satisfied, **Steps 3-5** are repeated. The FDE algorithm first checks the counter value, $Counter_j$, of all j = 1, *NPOP* population members. The counter value of each member shows how many times this member could not be improved during the evolution process. If the counter value of a member is greater than the maximum counter value, *Counterlimit*, it indicates that stagnation at a local optima has occurred, and that the mutation and crossover operators alone cannot escape from this local optimum position. Therefore, more evolutions would not lead to improvement in this vicinity and will simply increase the computational burden ungainly. In this case, the algorithm eliminates this unprofitable solution from the population and shrinks the population size accordingly. Reducing the size of the population, reduces the number of function evaluations in subsequent generations, yet provides for the opportunity for other members to evolve and progress.

If the shrinking process reduces the population size to less than the *Re-startlimit*, the remainder of the population is likely to experience difficulty in escaping from the current local or global optima. Thus, there may be no benefit in further evolving this small-sized population. In this case, FDE deletes the entire population and re-starts a new population, i.e., the algorithm generates a new random set of initial population members, and **Steps 2-5** are undertaken. This process provides a renewed chance for the algorithm to find other improved solutions of the problem.

Finally, in **Step 7**, the FDE algorithm checks the termination criteria and if it is not satisfied, **Steps 3-6** are repeated. The FDE code for minimization of the Sphere Function in a Python environment is provided in Appendix A.

4.5 Sensitivity Analysis and Performance of FDE for EA Benchmark Test Problems

In order to assess the general performance of FDE, relative to DE, eight EA benchmark test problems are employed, see Table 4.5 for a summary of these, including their sources. The variables of these problems are x_i, where $i = 1, \ldots n$, and n is the number of variables. In some cases, a number of constants are employed, denoted by *a, b,* and *c.* Increases in the value of n, increases the dimensionality of the problem, and thus the complexity of its solution. The mathematical formulations and two-dimensional surfaces of these problems are provided in Table 4.5 and Figure 4.3, respectively. EA benchmark test problems 1, 2, 5 and 8 have only one optimum point, and the other EA benchmark problems have many local and global optimum points. Also, EA benchmark test problems 1, 5 and 8 are convex while all other problems are non-convex.

In order to identify the sensitivity of FDE to the mutation, *F*, and crossover, *CR*, rates and the values of *Counterlimit* and *Re-startlimit*, and to select attractive values of these parameters and limits, a sensitivity analysis of the global optimum solution for the eight EA benchmark test problems, based on their ten-variable formulations, is undertaken. For these problems, the population number, *NPOP*, and the number of generations, are assumed to be 100 and 1000, respectively. Table 4.6 summarizes the sensitivity analysis. In the first section of the table, *CR* is varied from 0.1 to 0.9, while *F*, *Counterlimit*, and *Re-startlimit* are set to be 0.5, *NPOP*/3, and 0.*1NPOP,* respectively. The results of the eight EA benchmark test problems show that the most suitable value for *CR* is between 0.3 and 0.5, which is similar to that identified by Storn and Price (1997) for DE algorithm.

In the second section of the table, *F* is varied, keeping *CR*, *Counterlimit*, and *Re-startlimit* constant at 0.4, *NPOP*/3, and 0.*1NPOP,* respectively. The value of *CR* is either a random number that lies in the ranges (0-1), (0.2-0.8), or (0.4-0.6), or a constant value equal to 0.5. Results of this analysis show that the best value for *CR* is 0.5, which is also consistent with that identified by Storn and Price (1997) for the DE algorithm.

The third section of the table shows the sensitivity of the optimal solution to the variations of *Counterlimit*, where *Counterlimit* is either *NPOP*/2, *NPOP*/3, or *NPOP*/4. In this case, *F*, *CR*, and *Re-startlimit* are set be 0.5, 0.4, and 0.*1NPOP,* respectively. The best value for the counter limit is *NPOP*/3, or approximately 33 for *NPOP* equal to 100. Karaboga (2005) proposes a similar counter limit of 30 as the optimal limit, for the ABC algorithm, with a population of 100.

Finally, by assuming an *F*, *CR*, and *Counterlimit* of 0.5, 0.4, and *NPOP*/3*,* respectively, the sensitivity of FDE to the *Re-startlimit* value is analysed as described in the fourth section of the table. In this case, the optimal value is 0.1*NPOP* or 10% of the original number of individuals in the population.

To demonstrate the performance of the DE and FDE algorithms for optimization of the EA benchmark test problems, *F* and *CR* are set equal to 0.5 and 0.4, for both the DE and FDE Algorithms, the former based on values recommended in the literature for DE (see, e.g., Das et al. 2009), and the latter, the sensitivity analysis for FDE. In all DE and FDE applications, *NPOP* is equal to 100. In all FDE applications, the *Counterlimit* and *Re-startlimit* values are set at *NPOP*/3 and 0.1*NPOP*, respectively. For two-dimensional formulations of the EA benchmark test problems, the convergence properties of DE and FDE for all problems are shown in Figure 4.4. Each figure also provides the number of function evaluations required to reach the DE and FDE solutions in the top portion of the figure. For all problems, the slope of convergence of FDE is steeper than that of the DE algorithm. Also, the number of function evaluations of FDE in all problems is significantly fewer than those of DE. This reduction in function evaluations is due to the fact that FDE has a faster convergence rate, and that the shrinkage process in FDE reduces the population size and eliminates extra function evaluations. In benchmark test problem 3, FDE initially stagnates at a local optimum point, but after significant shrinkage triggers the re-start process, it can successfully find the global optimum point, again, at an accelerated rate.

As these problems become more complex, i.e., the dimensionality is increased to *n* equal ten, the efficiency of FDE improves, relative to DE. Figure 4.5 shows the solution results of the algorithms, where the slope of convergence of FDE is steeper than DE, and again, the number of function evaluations required for FDE is significantly fewer than DE.

4.6 Water Distribution Network (WDN) Design Problem Formulation

A WDN is a group of several components such as pipes, reservoirs, pumps and valves which are linked in order to provide water to users. The optimal design of such a system may be defined as the best combination of component sizes such as pipe size diameters and reservoir storage volumes, and component settings such as pump types, pump locations and maximum power, that exacts the minimum cost for a given network layout, and satisfies hydraulic laws governing continuity of flow and energy and demands for water quantities and pressures at the user nodes. In this work, WDN design is formulated as a least-cost optimization model with the selection of pipe size diameters as the decision variables, while pipe layout and its connectivity, nodal demands, and minimum pressure head requirements, are imposed. The optimization model is stated mathematically as:

$$Min\ C = \sum_{k=1}^{np} c_k\left(D_k\right) \times L_k \tag{4.1}$$

where $c_k(D_k) \times L_k$ = cost of pipe *k* with length L_k and diameter D_k; and *np* = number of pipes in the network. This objective function is minimized under the following constraints:

<u>*Flow continuity at nodes*</u>

For each node, continuity of flow must be satisfied;

$$\sum Q_{in} - \sum Q_{out} = q_j, \qquad \forall k \epsilon nn \tag{4.2}$$

where q_j = water consumption at node *j* (meters^3/second); *nn* = number of nodes; and Q_{in} and Q_{out} = flow into and out of node *j* (meters^3/second), respectively.

<u>*Energy conservation in loops*</u>

The total head loss around a closed pipe loop should be equal to zero, or the head loss along a loop between two fixed head reservoirs should be equal to the difference in water level of the reservoirs:

$$\sum_{k \in loop\, l} hf_k = \Delta H, \qquad \forall l \epsilon nl \tag{4.3}$$

where ΔH = difference between nodal pressures at both ends of a path (meters), and $\Delta H = 0$, if the path is closed; nl = number of loops; and hf_k = head loss due to friction in the pipe k (meters) which is obtained from following equation:

$$hf_k = H_i - H_j = R_k Q_k^n \tag{4.4}$$

where H_i and H_j = the nodal heads at the start, and at the end node of the pipe (meters), respectively; R_k = the resistance coefficient of the kth pipe with flow rate Q_k (second/ meters^2); and n = a constant depending on the head loss equation, and is 1.852 for the most common expression for head loss, the Hazen-Williams head loss formulation.

Minimum pressure at nodes

For each junction node in the network, the pressure head should be greater than the prescribed minimum pressure head:

$$H_j \geq H_j^{min}, \forall j \in nn \tag{4.5}$$

where H_j = pressure head at node j (meters); nn = number of nodes; and H_j^{min} = minimum required pressure head (meters).

Pipe size availability

The diameter of the pipes should be selected from a set of commercially available sizes, and are thus discrete:

$$D_k = \{D(1), D(2), \ldots., D(ns)\}, \quad \forall\, k \,\epsilon\, np \tag{4.6}$$

where ns = number of candidate diameters.

4.7 Differential Evolution Algorithms for Design of WDNs

For each combinatorial set of pipe diameters for the network, hydraulic simulation is applied to evaluate Eqs. (4.3)-(4.4) to obtain pressure heads, H_k, for all nodes of the network. Since Eq. (4.5) must be satisfied, and all EAs do not accommodate such constraints, violations of Eq. (4.5) are tracked, assigned a penalty, and added to the cost objective function in Eq. (4.1), where the penalty is defined as:

$$CP_k = \lambda \left| min\left(\frac{H_k}{H_k^{min}} - 1, 0\right) \right| \tag{4.7}$$

where CP_k = penalty associated with node k; and λ = penalty multiplier. To assure solutions that satisfy Eq. (5), the magnitude of λ should be sufficiently large. In this work, the value of λ is set equal to 10*max (C) and the Global Gradient Algorithm, GGA (Todini and Pilati 1988), is applied to conduct the hydraulic analysis of network. GGA satisfies the continuity and energy conservation equations (Eq 2-4), while calculating the pressure head H_j at each node and the flow Q_k in each pipe.

4.7.1 Application of FDE for Benchmark Networks

Applications of FDE to three benchmark networks, the Two-loop, Hanoi, and New York WDNs, are compared with results of EA algorithms widely-reported in the literature. A summary of the configuration, and all relevant optimization model information, for the networks are provided in Table 4.7. For detailed hydraulic, nodal demand and cost information for the three benchmark networks see (Schaake and Lai 1969; Alperovitz and Shamir 1977). Tables 4.8-4.10 provide the results for four applications of the FDE, corresponding to executions with stopping criteria of 1,000, 5,000, 10,000, and 100,000 function evaluations, for the Two-loop, Hanoi, and New York WDN, respectively. For each of these criteria, the algorithm is executed 100 times, *NPOP* is 100, *F* is 0.5, *CR* is 0.4, *Counterlimit* is *NPOP*/3, and *Re-startlimit* is 0.1*NPOP*. These results are compared with those identified in an in-depth review of the literature on applications of EAs to these networks. Columns 1-8 of the tables provide the EA or metaheuristic algorithm applied and the source of the data reported in the tables, the number of executions of the algorithm investigated, the optimal value of the least-cost solution for this WDN, the percent of the trial executions for which the global optima is obtained, the average least-cost over all trial executions, the average number of function evaluations required to find the first occurrence of the optimal solution, the minimum number of function evaluations required to find the first occurrence of the optimal solution, and the maximum allowable function evaluations, respectively. If a number is not provided in the table, either the source literature does not measure this number, or the number cannot be estimated based on the information provided in the source. For the algorithm results for

Two-loop network summarized in Table 4.8, the references for HS and PSHS Algorithms provide only the median number of function evaluations required to reach the first occurrence of the global minimum.

The global minimum cost found by all algorithms for the Two-loop network is $419,000 US. The minimum number of function evaluations required for DE and PSO-DE to converge to the global optimum is 1,320 and 3,080, respectively. Among the different algorithms SLC, PSHS and FDE have the best performance in terms of the number of function evaluations required to reach the global optimum solution, i.e., 968, 204, and 197, respectively, indicating that FDE has the best overall speed of convergence. Of the 100 executions of FDE, 45%, 86%, 88%, and 99% of these executions converge to the global optimum solution, when the number of function evaluations is less than or equal to 1,000, 5,000, 10,000, and 100,000, respectively. These percentages may be thought of as the reliability of the solution approach, and, though not based on the same number of algorithm executions, may nominally be compared with analyses of other EAs, that use the same stopping criteria. For example, the application of DE for the Two-loop WDN, with a stopping criteria of 10,000 function evaluations, while executed only 30 times, has an average number of function evaluations of 4,750 to find the first occurrence of the optimal solution, compared with 1,300 for FDE for the same stopping criteria, and a percent of trials with the global optima of 40%, compared with 88% for FDE.

The global minimum cost found by all algorithms for the Hanoi network is $6,081,087 US. Results of Table 4.9 show that SCE, GA, B-GA, MMAS, GAtrad, GAmod, and PSO-GA do not converge to the global optimum solution, while other algorithms do. NLP-DE1, NLP-DE2, SADE, and BLP-DE, have high reliability, or consistency of performance, with 97%, 98%, 84%, and 98%, respectively, of the trials locating the optimal solution. The average evaluations to find first occurrence of the best solution for one application of DE, SADE, BLP-DE, NLP-DE1 and NLP-DE2 are more than 30,000, indicating a time consuming process, particularly for BLP-DE, NLP-DE1 and NLP-DE2, which need extra calculations to undertake the binary linear and nonlinear programming aspect of the algorithm. SLC is the most reliable algorithm, requiring an average 71,789 function evaluations to reach the global optimum. FDE converges to the global optimum solution with 63%, 90%, 95%, and 97% reliability, when the number of function evaluations is less than or equal to 1,000, 5,000, 10,000, and 100,000, respectively. For these cases, the average number of evaluations to find first occurrence of the best solution is 528, 822, 1146, and 2,065 function evaluations, respectively, which is significantly less than those for other algorithms. In contrast, for DE, the smallest average number of function evaluations required to reach the optimal solution is 6,244. For FDE a minimum of 241 function evaluations are required to reach the global optimum, affording it the best performance among all EAs reported in the literature thus far.

The global minimum cost found by different EAs for the New York network is US $38.64 million. Results of Table 4.10 show that GA, PSO, ACO, MA, and SFLA do not converge to the global optimum solution, while other algorithms do. Two research papers present the results of DE and one shows that the average number of function evaluations to find the first occurrence of the global optimum is 5,494 and the percent of these trials reaching the global optimum is 71%. BLP-DE, SLC, and FDE, with a 100,000 function evaluation stopping criterion, have the best reliability, with 100% of the trials converging to the optimum solution. The average number of function evaluations to find the first occurrence of the optimal solution for BLP-DE, SLC, and FDE are 3,486, 15,764, and 4,193, respectively. In addition to its high performance under the least conservative stopping criterion, FDE can converge to the global optimum solution with 47%, 84%, and 89%, reliability when the number of function evaluations is less than 1,000, 5,000, and 10,000, respectively. For these cases, the average number of function evaluations to find the first occurrence of the global optimum is 472, 1,454, and 1,685 function evaluations, respectively, which is significantly fewer than those for other algorithms. For the DE algorithm, the minimum number of function evaluations required to reach the optimal solution is 3,220 (Suribabu 2010) compared with 238 for FDE, which demonstrates the best overall performance among all EAs reported in the literature thus far.

4.7.2 Application of FDE for Farhadgerd WDN

The Farhadgerd, Iran, WDN, shown in Figure 4.6, is a detailed representation of the water mains for this community of 8200 people. It has 68 pipes, 53 nodes, and 12 loops, and is fed by gravity from a reservoir with a 510-m fixed head. The pipe lengths vary from 5 to 885 m, with a Hazen-Williams coefficient of 130, and nine candidate commercial pipe diameters, i.e., a total enumeration of 9^{68}=7.735 $*10^{64}$ different network designs. Relevant data related to Farhadgerd network are given in Appendix B.

As there is little information about the least-cost and optimization results for the Farhadgerd Network in the literature, here, the performance of FDE is evaluated relative to the performance of nine popular EAs or metaheuristics. The algorithms include the: GA, SA, HS, DE, PSO, ABC, SLC, CMAES, and MEIGO. For consistent comparison, the three stopping criteria are limitations on the maximum number of function evaluations of 1000, 10,000 and 40,000, which may be thought of as evaluating the initial speed of convergence, and the performance of the algorithms at more mature and fully mature evolutions, respectively. For each of these criteria, all algorithms are executed 20 times, and the size of population is 100, i.e., $NPOP=100$. For FDE, F is 0.5, CR is 0.4, *Counterlimit* is *NPOP*/3, and *Re-startlimit* is 0.1*NPOP*, while for other algorithms, the parameter values are as listed in Table 4.11.

The results of the application of all ten algorithms are presented in in Tables 4.12-4.14 for the 1000-, 10000-, and 40000-function evaluation stopping criteria, respectively. Statistical analysis of results for all algorithms are performed and the average, standard deviation, minimum, and maximum solutions for different executions of the algorithms are provided in the tables. The average least-cost indicates the overall performance and the standard deviation indicates consistency of the search process for the 20 different initial random populations. Based on an application of FDE using 100,000 function evaluations, and 100 executions of the algorithm, the global optimum cost for the Farhadgerd WDN is US $17.782 million.

For the 1,000-function evaluation criterion, FDE surpasses the other algorithms with a minimum and average least-cost of US $18.950 and US $21.800 million, respectively. The CMAES and ABC algorithms exhibit the worst performance in comparison with the other algorithms, under this criterion. The average least-cost of the PSO algorithm indicates that it performs second-best, with a value of US $28.920 million. GA has the least standard deviation. However the average and minimum least-costs are far from those of FDE.

For the 10,000-function evaluation criterion, the FDE algorithm again out-performs all other algorithms, with a minimum, maximum, and average least cost of US $17.920, US $19.570 and US $18.540 million, respectively. CMAES exhibits the smallest standard deviation and performs second-best, under this criterion. For the 40,000-function evaluation criterion, the FDE algorithm out-performs all other algorithms, with a minimum and average least-cost of US $17.780 million, and US $17.920 million, respectively. CMAES exhibits the smallest standard deviation and performs second-best, under this criterion. For the 40000-function evaluation criterion, the FDE algorithm out-performs all other algorithms, with a minimum and average least-cost of $1.778E07, and \$1.792E07 US, respectively. CMAES is the second best algorithm with least standard deviation. This paper marks the first time that CMAES is applied to determine the optimal least-cost of a WDN model and it performs very well, in terms of the standard deviation of the solutions found. While only FDE achieves the global optimum solution for this WDN, SLC, CMEAS, MEIGO and DE are among the higher performing algorithms, and are further investigated, along with FDE, for solution with a 100,000 function evaluation criterion. These results are provided in Table 4.15. Here, even for a large number of function evaluations, only DE and FDE succeed in achieving the optimum solution, and the reliability of the DE algorithm is only 30%, compared with 74% for FDE.

4.8 Discussion and Conclusion

Generally, the WDN optimization problem has an array of local optima which are, at times, very different from each other. This characteristic is related to the need to satisfy minimum required pressures at junctions throughout the network, and the required pressure is a nonlinear function of the pipe size diameters and their configurations. By changing one pipe diameter, the flow direction in the pipes, at least in the vicinity of the pipe, may change, and the distribution of the pressures throughout the system may also change. This property makes the optimization of a network model difficult to solve. Therefore, when the optimization algorithm is close to the global optimum, there no guarantee that the optimum has been located, because the path for reaching the global optimum may not necessarily be in the path taken near the best local optimum. EAs provide more diverse solutions to escape from local optimum solutions effectively. They also exhibit independence from the initial population of solutions, which, under classical optimization, determines the direction of search. Finally, they do not concentrate solely on the evolution of the best solution (or solutions) of the population.

Results of optimization for three benchmark problems, the: Two-loop, Hanoi, and New York WDN show that the number of function evaluations required to reach the global optimum using FDE is 197, 241, and 238, respectively, which are much less than those required by all other EAs reported in the literature. The performance of FDE is also demonstrated with 20 optimization executions for a realistic-sized pipe network. The small average and standard deviation values for the number of function evaluations required to reach the global optimum under FDE indicate that the algorithm has a high and consistent performance relative to other widely-applied EAs. This is due to its advanced operators for exploration and exploitation.

In future work, FDE may be advanced for solving multiobjective WDN design problems and for optimization of an array of water supply problems such as the management of reservoir operations and leakage detection. It may also be applied to calibrate hydraulic simulation models of such systems, and adapted for use in to improve the efficiency of other network and N-P hard engineering optimization problems.

Table 4.1 DE and FDE Algorithms for solving optimization problems

DE	FDE
1) Initialize mutation, *F*, and crossover, *CR*, rates; coding of strings for decision variable values; and population number (*NPOP*).	1) Initialize mutation, *F*, and crossover, *CR*, rates; coding of strings for decision variable values; population number (*NPOP*); counter limit, *Counterlimit*; and re-start limit, *Re-startlimit*.
2) Initialize population: for *NPOP* number of individual solutions, generate decision variable values for vectors of decisions, X_j, where j = 1,.., *NPOP*, and calculate corresponding objective function values for these. Find the solution vector with highest fitness, i.e., X_{best}.	2) Initialize population: for *NPOP* number of individual solutions, generate decision variable values for vectors of decisions, X_j, where j = 1,..., *NPOP*, and calculate corresponding objective function values for these. Find solution vector with the highest fitness, i.e., X_{best}. Set counter, $Counter_j$, equal to zero for all X_j.
3) For each solution vector (X_j), mutation operator is conducted as follows: Mutation: Select three random solution vectors, X_A, X_B, and X_C. Calculate new solution vector by: $X_j^{new} = X_C + F(X_A - X_B)$	3) For each solution vector (X_j), mutation operator is conducted as follows: Mutation: Select three random solution vectors, X_A, X_B, and X_C. By considering best solution vector thus far, calculate new solution vector by: $X_j^{new} = X_A + F(X_{best} - X_B)$
4) For each solution vector (X_j), crossover operator is conducted as follows: Crossover: for each element, i, of X_j, randomly select a value from X_{ji}^{new} or X_{ji}: $X_j^{new} = \begin{cases} X_{ji}^{new} & if\ rand < CR \\ X_{ji} & otherwise \end{cases}$	4) For each solution vector (X_j), crossover operator is conducted as follows: Crossover: using X_C random solution vector identified in **Step 3,** for each element, i, of the X_j, randomly select a value from X_{ji}^{new} or X_{Ci}: $X_j^{new} = \begin{cases} X_{ji}^{new} & if\ rand < CR \\ X_{Ci} & otherwise \end{cases}$
5) If new solution vector created by mutation and crossover operators is better than X_j, replace X_j, otherwise X_j remains in population. Also, if new solution vector created by the mutation and crossover operators is better than X_{best}, it is considered to be the current X_{best}, otherwise X_{best} does not change.	5) If new solution vector created by mutation and crossover operators is better than X_j, replace X_j, and $Counter_j$ remains at zero, otherwise X_j remains in population and $Counter_j = Counter_j + 1$. Also, if new solution vector created by the mutation and crossover operators is better than X_{best}, it is considered to be the current X_{best}, otherwise X_{best} does not change
6) If the convergence criteria has been met, terminate algorithm, otherwise go to the **Step 3**.	6) For each solution vector, if $Counter_j$ is greater than a $Counterlimit$, eliminate that solution vector and shrink the population size. If population size is less than *Re-startlimit*, restart population, i.e., create new random initial population, and go to **Step 2**.
	7) If the convergence criteria has been met, terminate algorithm, otherwise go to the **Step 3**.

Table 4.2 Initial population for $f(x,y)=x^2+y^2$ EA benchmark test problem

Solution vector	X	Y	Objective function
A	-1	4	17
B	-5	0	25
C	-3	-4	25
D	2	1	5
E	5	-3	34

Table 4.3 First iteration of DE and FDE for optimization of $f(x,y)= x^2+y^2$ EA benchmark test problem

DE — Mutation operation

member	Xrand	Mutation	Xnew	Ynew
A	B,D, E	B+0.5(D-E)	-6.5	2
B	C,A,D	C+0.5(A-D)	-4.5	-2.5
C	A,B,E	A+0.5(B-E)	-6	5.5
D	E,C,A	E+0.5(C-A)	4	1
E	D,B,C	D+0.5(B-C)	1	3

FDE — Mutation operation

member	Xbest	Xrand	Mutation	Xnew	Ynew
A	D	B,C,E	B+0.5(D-C)	-2.5	2.5
B	D	E,C,A	E+0.5(D-B)	7.5	-0.5
C	D	A,B,C	A+0.5(D-B)	2.5	4.5
D	D	D,B,D	-	-	-
E	D	C,E,B	C+0.5(D-E)	-4.5	-2

DE — Crossover Operation

Crossover	Xnew	Ynew
A,(Xnew,Ynew)	-1	2
B,(Xnew,Ynew)	-5	-2.5
C,(Xnew,Ynew)	-6	-4
D,(Xnew,Ynew)	2	1
E,(Xnew,Ynew)	1	-3

FDE — Crossover Operation

Crossover	Xnew	Ynew
E,(Xnew,Ynew)	5	2.5
A,(Xnew,Ynew)	-1	-0.5
C,(Xnew,Ynew)	2.5	-4
D,(Xnew,Ynew)	-	-
B,(Xnew,Ynew)	-4.5	0

DE — Selection Operation

Selection	Xnew	Ynew	Obj
A,(Xnew,Ynew)	-1	2	**5**
B,(Xnew,Ynew)	-5	0	25
C,(Xnew,Ynew)	-3	-4	25
D,(Xnew,Ynew)	2	1	**5**
E,(Xnew,Ynew)	1	-3	10

FDE — Selection Operation

Selection	Xnew	Ynew	Obj
A,(Xnew,Ynew)	-1	4	17
B,(Xnew,Ynew)	-1	-0.5	1.25
C,(Xnew,Ynew)	2.5	-4	22.25
D,(Xnew,Ynew)	2	1	**5**
E,(Xnew,Ynew)	-4.5	0	20.25

Table 4.4 Second iteration of DE and FDE for optimization of the benchmark problem

DE

Mutation Operation

member	Xrand	Mutation	Xnew	Ynew
A	B, D, E	B+0.5(D-E)	-4.5	2
B	C,A,D	C+0.5(A-D)	-4.5	-3.5
C	A,B,E	A+0.5(B-E)	-4	3.5
D	E,C,A	E+0.5(C-A)	4	2
E	D,B,C	D+0.5(B-C)	1	3

Crossover Operation

Crossover	Xnew	Ynew
A,(Xnew,Ynew)	-4.5	2
B,(Xnew,Ynew)	-5	-3.5
C,(Xnew,Ynew)	-4	-4
D,(Xnew,Ynew)	2	2
E,(Xnew,Ynew)	1	-3

Selection Operation

Selection	Xnew	Ynew	Obj
A,(Xnew,Ynew)	-1	2	**5**
B,(Xnew,Ynew)	-5	0	25
C,(Xnew,Ynew)	-3	-4	25
D,(Xnew,Ynew)	2	1	**5**
E,(Xnew,Ynew)	1	-3	10

FDE

Mutation Operation

member	Xbest	Xrand	Mutation	Xnew	Ynew
A	D	B,C,E	B+0.5(D-C)	-1.25	2
B	D	E,C,A	E+0.5(D-B)	-4.75	2.5
C	D	A,B,C	A+0.5(D-B)	0.5	4.75
D	D	D,B,D	-	-	-
E	D	C,E,B	C+0.5(D-E)	5.75	-3.5

Crossover Operation

Crossover	Xnew	Ynew
E,(Xnew,Ynew)	-1.25	0
A,(Xnew,Ynew)	-1	2.5
C,(Xnew,Ynew)	0.5	-4
D,(Xnew,Ynew)	-	-
B,(Xnew,Ynew)	5.75	-0.5

Selection Operation

Selection	Xnew	Ynew	Obj
A,(Xnew,Ynew)	-1.25	0	1.56
B,(Xnew,Ynew)	-1	-0.5	**1.25**
C,(Xnew,Ynew)	0.5	-4	16.25
D,(Xnew,Ynew)	2	1	5
E,(Xnew,Ynew)	-4.5	0	20.25

Table 4.5 EA benchmark test problems

Problem No.	Functional Form	Variable Limits and Constant Definitions	Initial Conditions
1	$f(x)=\sum_{i=1}^{n} x_i^2$	$-100 \le x_i \le 100, i=1,\dots,n$	$f(x)=0$ $x_i=0, i=1,\dots,n$
2	$f(x)=-a.\exp\left(-b.\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right)-\exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(cx_i)\right)+a+\exp(1)$	$-32.768 \le x_i \le 32.768, i=1,\dots,n$ $a=20,\ b=0.2, c=2\pi$	$f(x)=0$ $x_i=0, i=1,\dots,n$
3	$f(x)=\frac{1}{4000}\sum_{i=1}^{n} x_i^2-\prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	$-600 \le x_i \le 600, i=1,\dots,n$	$f(x)=0$ $x_i=0, i=1,\dots,n$
4	$f(x)=\sum_{i=1}^{n-1}\left[100(x_{i+1}-x_i^2)^2+(1-x_i)^2\right]$	$-2.048 \le x_i \le 2.048, i=1,\dots,n$	$f(x)=0$ $x_i=1, i=1,\dots,n$
5	$f(x)=-\sum \sin(x_i)\left[\sin\left(\frac{ix_i^2}{\pi}\right)\right]^{2m}$	$0 \le x_i \le \pi, i=1,\dots,n$	$f(x)=-1.8013,$ $x_1=2.2029, x_2=1.5708$
6	$f(x)=10n+\sum_{i=1}^{n}\left[x_i^2-10\cos(2\pi x_i)\right]$	$-5.12 \le x_i \le 5.12, i=1,\dots,n$	$f(x)=0$ $x_i=0, i=1,\dots,n$
7	$f(x)=\sum_{i=1}^{n}\left[-x_i \sin\left(\sqrt{\lvert x_i\rvert}\right)\right]$	$-500 \le x_i \le 500, i=1,\dots,n$	$f(x)=-418.9829n$ $x_i=420.9687, i=1,\dots,n$
8	$f(x)=\sum_{i=1}^{n}\lvert x_i\rvert^{i+1}$	$-1 \le x_i \le 1, i=1,\dots,n$	$f(x)=0$ $x_i=0, i=1,\dots,n$

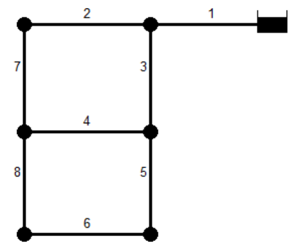
1- De Jong’s Function or Sphere Function (Dixon and Szego, 1978)
2- Ackley’s Function (Ackley, 1987)
3- Griewangk’s Function (Griewank, 1981)
4- Rosenbrock’s Valley (Rosenbrock, 1960)
5- Michalewicz’s Function (Molga and Smutnicki, 2013)
6- Rastrigin’s Function (Rastrigin, 1974)
7- Schwefel’s Function (Schwefel, 1981)
8- 8-Sum of Power Functions (Molga and Smutnicki, 2013)

Table 4.6 Sensitivity analysis based on EA benchmark test problem

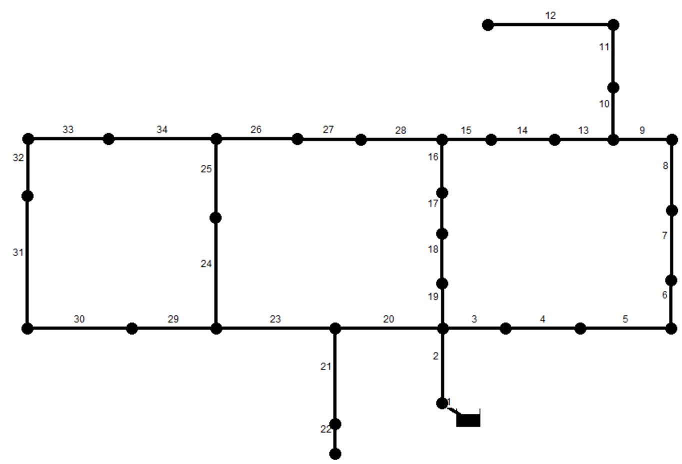
				EA Benchmark Test Problem, Based on 10-Variable Formulations							
Re-start Limit	Counter Limit	Mutation Rate	Crossover Rate	1	2	3	4	5	6	7	8
0.1*NPOP*	*NPOP*/3	0.5	0.1	39.06	0.44	0.4218	14.29316	-9.12475	7.685384	-3845.80	4.62E-06
0.1*NPOP*	*NPOP*/3	0.5	0.2	4.15E-34	4.44E-15	0.0378	4.576513	-9.57815	1.83425	-4110.40	3.69E-127
0.1*NPOP*	*NPOP*/3	0.5	0.3	2.13E-95	4.44E-15	0.0185	4.257142	-9.598	0.1292	-4177.80	2.67E-184
0.1*NPOP*	*NPOP*/3	0.5	0.4	5.33E-92	4.44E-15	0.0146	1.5045	-9.6227	0.0995	-4178.00	4.23E-179
0.1*NPOP*	*NPOP*/3	0.5	0.5	1.35E-87	4.44E-15	0.0241	0.189339	-9.65524	0.9322	-4186.60	4.74E-166
0.1*NPOP*	*NPOP*/3	0.5	0.6	8.86E-82	4.44E-15	0.144	1.13E-05	-9.65524	7.396701	-4154.30	9.07E-157
0.1*NPOP*	*NPOP*/3	0.5	0.7	1.56E-75	4.44E-15	0.2178	6.94E-16	-9.57728	11.33187	-4142.40	1.22E-146
0.1*NPOP*	*NPOP*/3	0.5	0.8	4.94E-70	4.44E-15	0.352	6.51E-25	-9.5765	20.45067	-4057.40	3.67E-141
0.1*NPOP*	*NPOP*/3	0.5	0.9	6.54E-67	4.44E-15	0.3204	0	-8.53538	22.86157	-4035.20	6.37E-140
0.1*NPOP*	*NPOP*/3	0-1	0.4	7.0025	1.699528	0.2226	6.476	-9.6077	0.7629	-4177.50	1.18E-09
0.1*NPOP*	*NPOP*/3	0.2-0.8	0.4	3.46E-47	4.44E-15	0.0174	5.0722	-9.6473	0.9106	-4188.50	4.57E-19
0.1*NPOP*	*NPOP*/3	0.4-0.6	0.4	1.49E-89	4.44E-15	0.0104	2.4312	-9.6447	0.1707	-4189.80	2.37E-173
0.1*NPOP*	*NPOP*/3	0.5	0.4	5.33E-92	4.44E-15	0.0111	1.5045	-9.6299	0.0995	-4189.80	4.23E-179
0.1*NPOP*	*NPOP*/2	0.5	0.4	1.85E-91	4.44E-15	0.0025	2.362	-9.6278	0	-4178.00	6.76E-177
0.1*NPOP*	*NPOP*/3	0.5	0.4	5.33E-92	4.44E-15	0.0111	1.5045	-9.6299	0.0995	-4189.80	4.23E-179
0.1*NPOP*	*NPOP*/4	0.5	0.4	1.69E-91	4.44E-15	0.0198	1.3348	-9.6376	0.0146	-4188.90	1.17E-175
0.1*NPOP*	*NPOP*/3	0.5	0.4	5.33E-92	4.44E-15	0.0111	1.1345	-9.6299	0	-4189.80	4.23E-179
0.2*NPOP*	*NPOP*/3	0.5	0.4	2.12E-92	4.44E-15	0.0073	1.3077	-9.6518	0	-4189.80	7.97E-178
0.3*NPOP*	*NPOP*/3	0.5	0.4	1.71E-91	4.44E-15	0.0109	1.4569	-9.6467	0	-4189.80	2.73E-179

Table 4.7 Relevant optimization model information for three benchmark networks

Two-loop network
Global Optimum = 0.419E+6(US $)
Number of Pipes = 8
Number of Nodes = 7
Number of Candidate Diameters = 14
Enumeration = 1.48×10^{9}
Reservoir Level = 210 m
Pipe Lengths = 1,000 m
Hazen-Williams Coefficients = 130
Gravity Fed



Hanoi network
Global Optimum =6.081E+6(US $)
Number of Pipes = 34
Number of Nodes = 32
Number of Candidate Diameters = 6
Enumeration = 2.865×10^{26}
Reservoir Level = 100 m
Pipe Lengths = 100 to 3,500 m
Hazen-Williams Coefficients = 130
Gravity Fed



NewYork network
Global Optimum = 38.64E+6(US $)
Number of Pipes = 21
Number of Nodes = 20
Number of Candidate Diameters = 16
Enumeration = 1.93×10^{25}
Reservoir Level = 91.44 m
Pipe Lengths = 2,225 to 11,705 m
Hazen-Williams Coefficients = 100
Gravity Fed

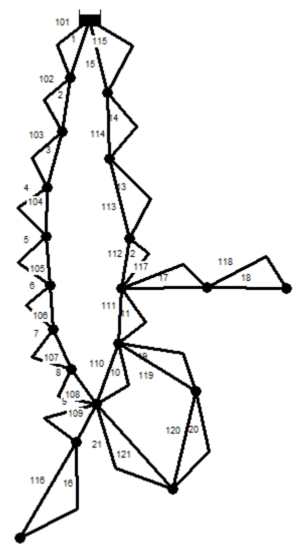


Table 4.8 Optimization results obtained based on different EAs and meta-heuristic algorithms for Two-loop network

Algorithm	Number of Executions	Best Cost ($M)	Percent of trials with the global best found	Average Cost ($M)	Average evaluations to find first occurrence of the best solution	Minimum evaluations to find first occurrence of the best solution	Maximum allowable evaluations
SA(1)	50	0.419	-	-	25,000	-	100,000
SFLANET(2)	5	0.419	-	-	11,323	11,155	17,000
SCE(3)	-	0.419	-	-	1,345	1,091	-
PSO(4,8)	10	0.419	-	-	3,120	760(4)	6,000
SS(5)	50	0.419	100%	0.419	5,300	3,215	4,000
HS(6)	27	0.419	-	-	1,958*	1,121	5,000
PSHS(6)	27	0.419	-	-	346*	204	5,000
DE(7)	30	0.419	40%	-	4,750	1,320	10,000
PSO-DE(8)	10	0.419	100%	0.419	3,080	-	6,000
PSO-GA(9)	-	0.419	-	-	-	1,300	5,000
SLC(10)	50	0.419	40%	-	2,051	968	5,000
GA(11)	50	0.419	4%	0.437,880	-	6,060	50,000
PSO(11)	50	0.419	24%	0.430,100	-	1,650	50,000
ACO(11)	50	0.419	14%	0.434,450	-	2,650	50,000
MA(11)	50	0.419	32%	0.425,440	-	11,402	50,000
SFLA(11)	50	0.419	22%	0.425,771	-	6,921	50,000
FDE	100	0.419	45%	0.419,550	600	197	1,000
FDE	100	0.419	86%	0.419,140	1,116	197	5,000
FDE	100	0.419	88%	0.419,120	1,300	197	10,000
FDE	100	0.419	99%	0.419,010	4,172	197	100,000

(1) Cunha and Sousa (1999)
(2) Eusuff and Lansey (2003)
(3) Liong and Atiquzzaman (2004)
(4) Suribabu and Neelakantan (2006)
(5) Min-Der et al. (2007)
(6) Geem (2013)
(7) Suribabu (2010)
(8) Sedki (2012)
(9) Babu and Vijayalakshm (2013)
(10) Moosavian and Roodsari (2014)
(11) El-Ghandour and Elbeltagi (2017)
*median

Table 4.9 Optimization results obtained based on different EAs and meta-heuristic algorithms for Hanoi network

Algorithm	Number of Executions	Best Cost ($M)	Percent of trials with the global best found	Average Cost ($M)	Average evaluations to find first occurrence of the best solution	Minimum evaluations to find first occurrence of the best solution	Maximum allowable evaluations
SCE(1)	10	6.220	-	-	25,402	-	-
GA(2)	10	6.208	-	6.296	-	-	30,000
B-GA(2)	10	6.182	-	6.219	-	-	30,000
MMAS(3)	20	6.134	-	6.394	85,571	35,433	100,000
HD-DDS(4)	50	6.081	8%	6.252	100,000	-	100,000
SS(5)	100	6.081	64%	-	43,149	-	-
HS(6)	81	6.081	-	6.319	-	27,721	50,000
PSHS(6)	81	6.081	-	6,340	-	17,980	50,000
GAtrad(7)	30	6.122	-	6.278	-	-	100,000
GAmod(7)	30	6.221	-	6.347	-	-	100,000
PSO(7)	30	6.081	-	6.265	-	-	100,000
DE(7)	30	6.162	-	6.227	-	-	100,000
GHEST(8)	60	6.081	38%	6.175	50,134	16,600	-
DE(9)	300	6.081	20%	-	6,244	-	10,000
DE(9)	300	6.081	80%	-	48,724	-	100,000
NLP-DE1(10)	100	6.081	97%	6,082	34,609	-	80,000
NLP-DE2(10)	100	6.081	98%	6,081	42,782	-	80,000
SADE(11)	50	6.081	84%	6.09	60,532	-	74,876
PSO-GA(12)	-	6.117	-	-	15,200	-	2000
BLP-DE(13)	100	6.081	98%	6,085	33,148	-	40,000
SLC(14)	20	6.081	80%	6.110	29,108	-	100,000
SLC(14)	50	6.081	100%	6.081	71,789	-	100,000
MBA(15)	10	6.081	-	6.275	-	22,450	-
IMBA(15)	10	6.081	-	6.187	-	16,400	-
CS(16)	200	6.081	-	6.224	-	52,890	>54,000
CSHS(16)	200	6.081	-	6.160	-	31,800	>54,000
FDE	100	6.081	63%	6.581	528	241	1,000
FDE	100	6.081	90%	6.12	822	241	5,000
FDE	100	6.081	95%	6.093	1,146	241	10,000
FDE	100	6.081	97%	6.088	2,065	241	100,000

(1) Liong and Atiquzzaman (2004)
(2) Reca and Martinez (2006)
(3) Zecchin et al. (2006)
(4) Tolson et al. (2009)
(5) Min-Der et al. (2007)
(6) Geem (2013)
(7) Marchi et al. (2014)
(8) Bolognesi et al. (2010)
(9) Suribabu (2010)
(10) Zheng et al. (2011)
(11) Zheng et al. (2013)
(12) Babu and Vijayalakshm (2013)
(13) Zheng et al. (2014)
(14) Moosavian and Roodsari (2014)
(15) Sadollah et al. (2014)
(16) Sheikholeslami et al. (2016)

Table 4.10 Optimization results obtained based on different EAs and meta-heuristic algorithms for New York Tunnels network

Algorithm	Number of Executions	Best Cost ($M)	Percent of trials with the global best found	Average Cost ($M)	Average evaluations to find first occurrence of the best solution	Minimum evaluations to find first occurrence of the best solution	Maximum allowable evaluations
MMAS(1)	20	38.640	60%	38.84	30,711	22,635	50,000
SS(2)	100	38.640	65%	-	57,583	-	-
HD-DDS(3)	50	38.640	86%	-	47,000	-	50,000
HS(4)	81	38.640	-	-	5,991	3,373	10,000
PSHS(4)	81	38.640	-	-	5,923	4,475	10,000
GAtrad(5)	30	38.640	-	40.99	-	-	100,000
GAmod(5)	30	38.640	-	39.013	-	-	100,000
PSO(5)	30	38.640	-	39.845	-	-	100,000
DE(5)	30	38.640	-	38.667	-	-	100,000
GHEST(6)	60	38.640	92%	-	11,464	2,100	-
DE(7)	300	38.640	71%	-	5,494	3,220	10,000
NLP-DE1(8)	100	38.640	99%	38.64	8,277	-	20,000
NLP-DE2(8)	100	38.640	99%	38.64	10,631	-	20,000
SADE(9)	50	38.640	92%	38.64	6,598	-	9,227
BLP-DE(10)	100	38.640	100%	38.64	3,486	-	7,500
SLC(11)	20	38.640	80%	38.81	7,821	-	100,000
SLC(11)	50	38.640	100%	38.64	15,764	-	100,000
GA(12)	50	38.796	8%	40.96	23,070	-	30,000
PSO(12)	50	38.796	28%	40.82	6,100	-	100,000
ACO(12)	50	38.796	16%	39.24	55,950	-	150,000
MA(12)	50	38.796	30%	39.23	43,482	-	15,000
SFLA(12)	50	38.796	14%	39.51	7,963	-	10,000
FDE	100	38.640	47%	52.87	472	238	1,000
FDE	100	38.640	84%	38.92	1,454	238	5,000
FDE	100	38.640	89%	38.69	1,685	238	10,000
FDE	100	38.640	100%	38.64	4,193	238	100,000

(1) Zecchin et al. (2006)
(2) Min-Der et al. (2007)
(3) Tolson et al. (2009)
(4) Geem (2013)
(5) Marchi et al. (2014)
(6) Bolognesi et al. (2010)
(7) Suribabu (2010)
(8) Zheng et al. (2011)
(9) Zheng et al. (2013)
(10) Zheng et al. (2014)
(11) Moosavian and Roodsari (2014)
(12) El-Ghandour and Elbeltagi (2017)

Table 4.11 parameter values of ten EAs

Algorithm	nPop	Parameters	
GA[(1)]	100	Mutation= 0.01	Crossover= 0.8
SA[(2)]	100	Initial Temperature =100	Reannealing Interval =100
HS[(3)]	100	HMCR =0.8	PAR = 0.4
DE[(4)]	100	Mutation= 0.5	Crossover= 0.3
PSO[(5)]	100	Inertia Weight = 0.99	Evolution Parameters = 2
ABC[(6)]	100	Food Number = 50	Limit Trials = 100
SLC[(7)]	100	Imitation = U(0.2,0.8)	Provocation = 1 and 0.5
MEIGO[(8)]	100	Black Box Optimizer with self adaptive parameters	
CMAES[(9)]	100	Sigma = 0.5	Mu = 50

(1) Matlab
(2) Matlab
(3) Geem (2006)
(4) Suribabu 2010
(5) Suribabu and Neelakantan (2006)
(6) Karaboga (2005)
(7) Moosavian and Roodsari (2014)
(8) Egea et al., (2014)
(9) Hansen and Kern (2004)

Table 4.12 Least cost solutions for the Farhadgerd network: 1000 function evaluations

1,000 evaluations	Min($M)	Max($M)	Mean($M)	Standard deviation (M)
GA	27.730	32.180	29.500	**1.115**
SA	29.190	36.670	32.700	2.269
HS	29.210	34.220	31.740	1.374
DE	33.100	36.170	34.750	0.931
PSO	26.170	**30.850**	28.920	1.410
ABC	32.400	38.110	34.920	1.596
SLC	28.710	34.260	32.030	1.671
MEIGO	21.930	36.280	31.840	4.595
CMAES	45.690	818.800	171.700	225.800
FDE	**18.950**	30.920	**21.800**	3.374
Best	FDE	PSO	FDE	GA

Table 4.13 Least cost solutions for the Farhadgerd network: 10,000 function evaluations

10,000 evaluations	Min($M)	Max($M)	Mean($M)	Standard deviation (M)
GA	21.300	26.100	22.880	0.994
SA	24.050	34.440	27.940	2.454
HS	22.800	24.600	23.850	0.455
DE	22.100	23.940	23.070	0.427
PSO	22.280	29.220	25.040	1.695
ABC	23.430	27.380	25.330	1.084
SLC	20.070	22.290	20.940	0.565
MEIGO	21.080	24.370	22.740	0.769
CMAES	18.490	19.710	19.060	**0.273**
FDE	**17.920**	**19.570**	**18.540**	0.540
Best	FDE	FDE	FDE	CMAES

Table 4.14 Least cost solutions for the Farhadgerd network: 40,000 function evaluations

40,000 evaluations	Min($M)	Max($M)	Mean($M)	Standard deviation (M)
GA	19.460	23.020	21.080	0.8677
SA	22.750	27.280	24.950	1.327
HS	21.380	22.750	22.240	0.324
DE	19.560	20.610	20.060	0.262
PSO	20.210	26.530	24.070	1.380
ABC	20.830	23.980	22.640	0.805
SLC	17.790	18.730	18.290	0.301
MEIGO	19.090	20.300	19.700	0.335
CMAES	17.840	**18.180**	18.040	**0.0910**
FDE	**17.780**	18.420	**17.920**	0.209
Best	FDE	CMAES	FDE	CMAES

Table 4.15 Optimization results obtained based on different EAs and meta-heuristic algorithms for Farhadgerd network

Algorithm	Number of Executions	Best Cost ($M)	Percent of trials with the global best found	Average Cost ($M)	Average evaluations to find first occurrence of the best solution	Minimum evaluations to find first occurrence of the best solution	Maximum allowable evaluations
SLC	100	17.79	-	18.27	-	-	100,000
CMAES	100	17.84	-	18.01	-	-	100,000
MEIGO	100	17.98	-	18.92	-	-	100,000
DE	100	17.78	30%	17.82	88,590	67,800	100,000
FDE	100	17.78	74%	17.80	24,294	638	100,000

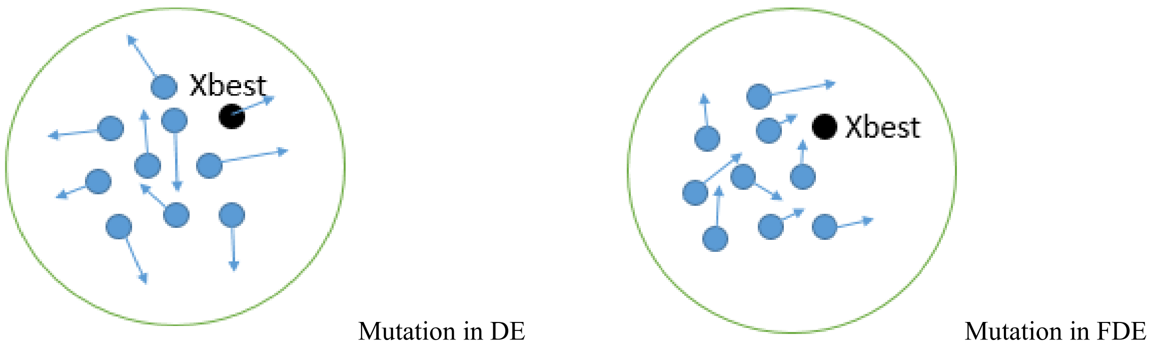


Figure 4.1 Conceptualization of mutation operation for DE and FDE

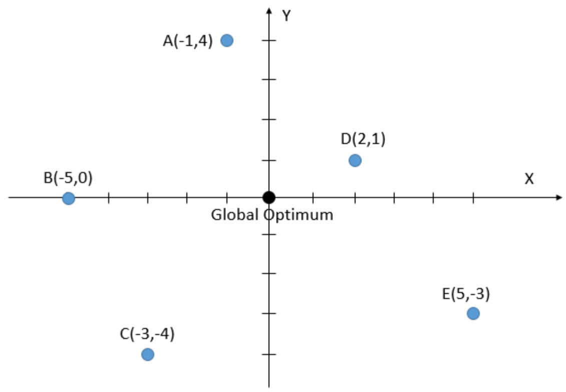
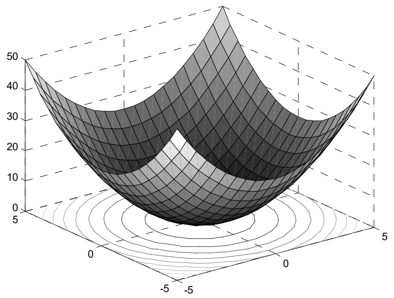
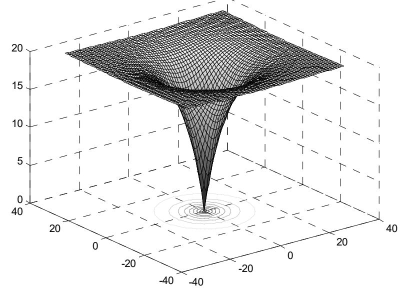


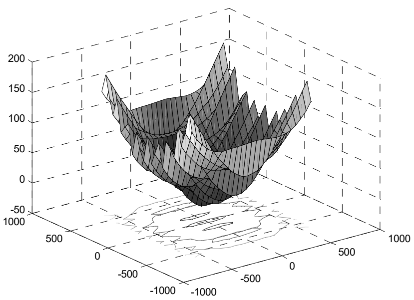
Figure 4.2 Initial solution vectors for $f(x,y)=x^2+y^2$ EA benchmark test problem



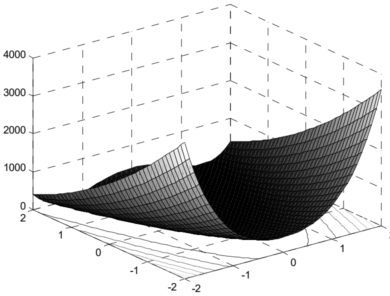
1- De Jong's Function (Sphere Function)



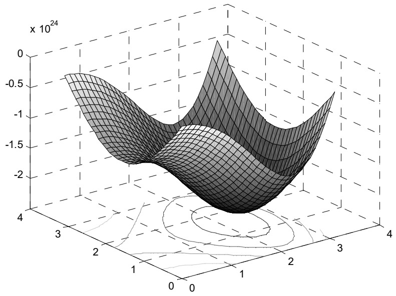
2-Ackley's Function



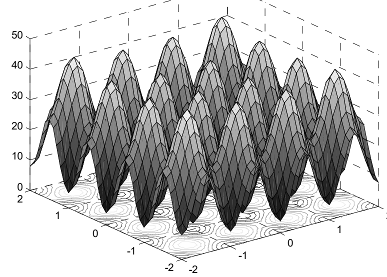
3- Griewangk's Function



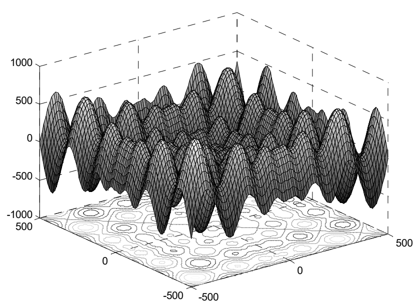
4- Rosenbrock's Valley



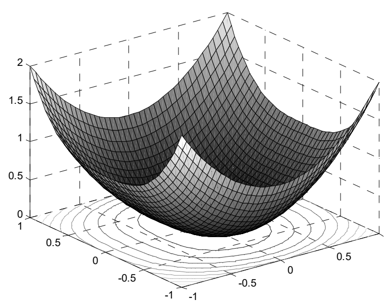
5- Michalewicz's Function



6-Rastrigin's Function



7- Schwefel's Function



8-Sum of Power Functions

Figure 4.3 EA benchmark test problems

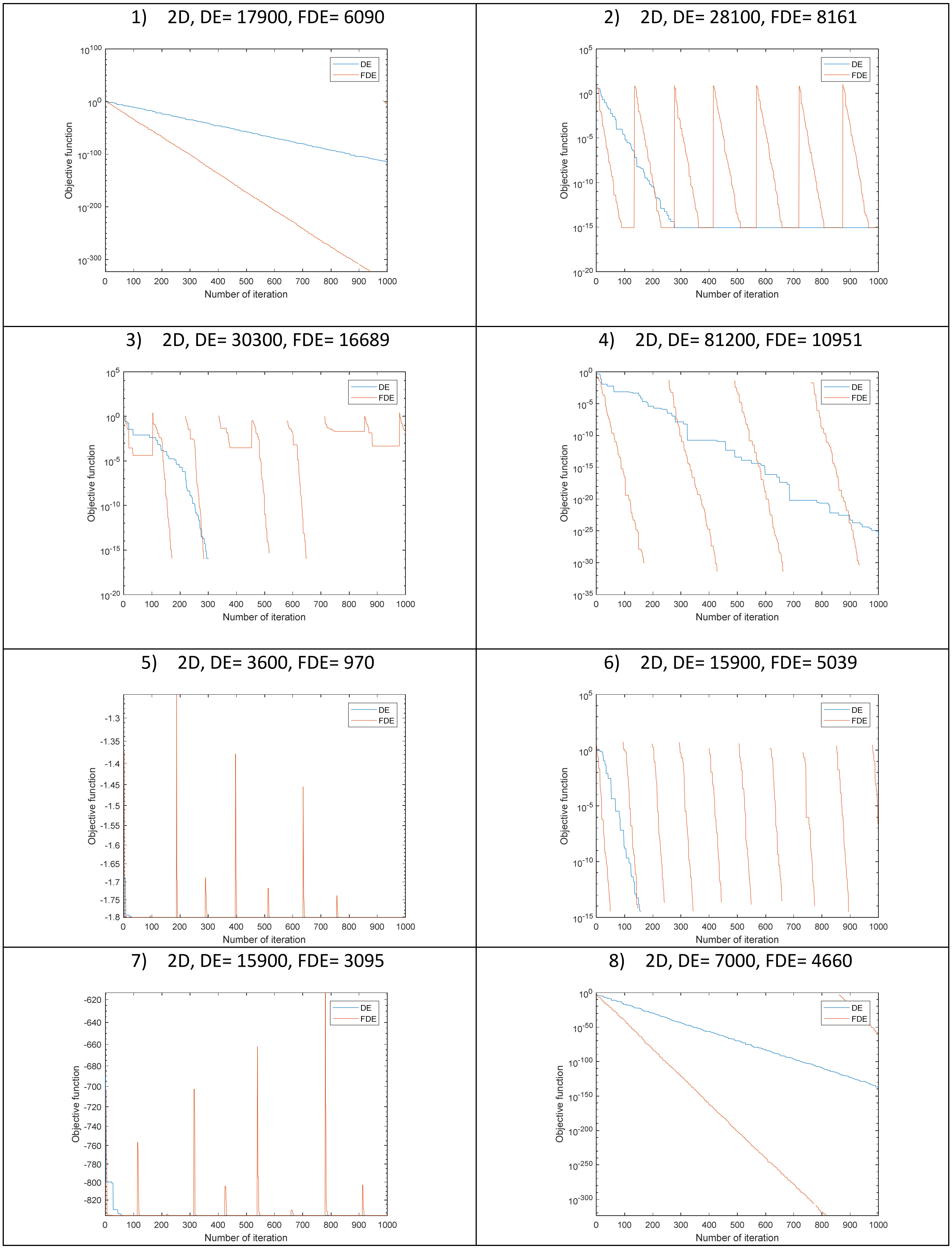


Figure 4.4 Convergence properties of DE and FDE for EA benchmark test problems, based on two-variable formulations

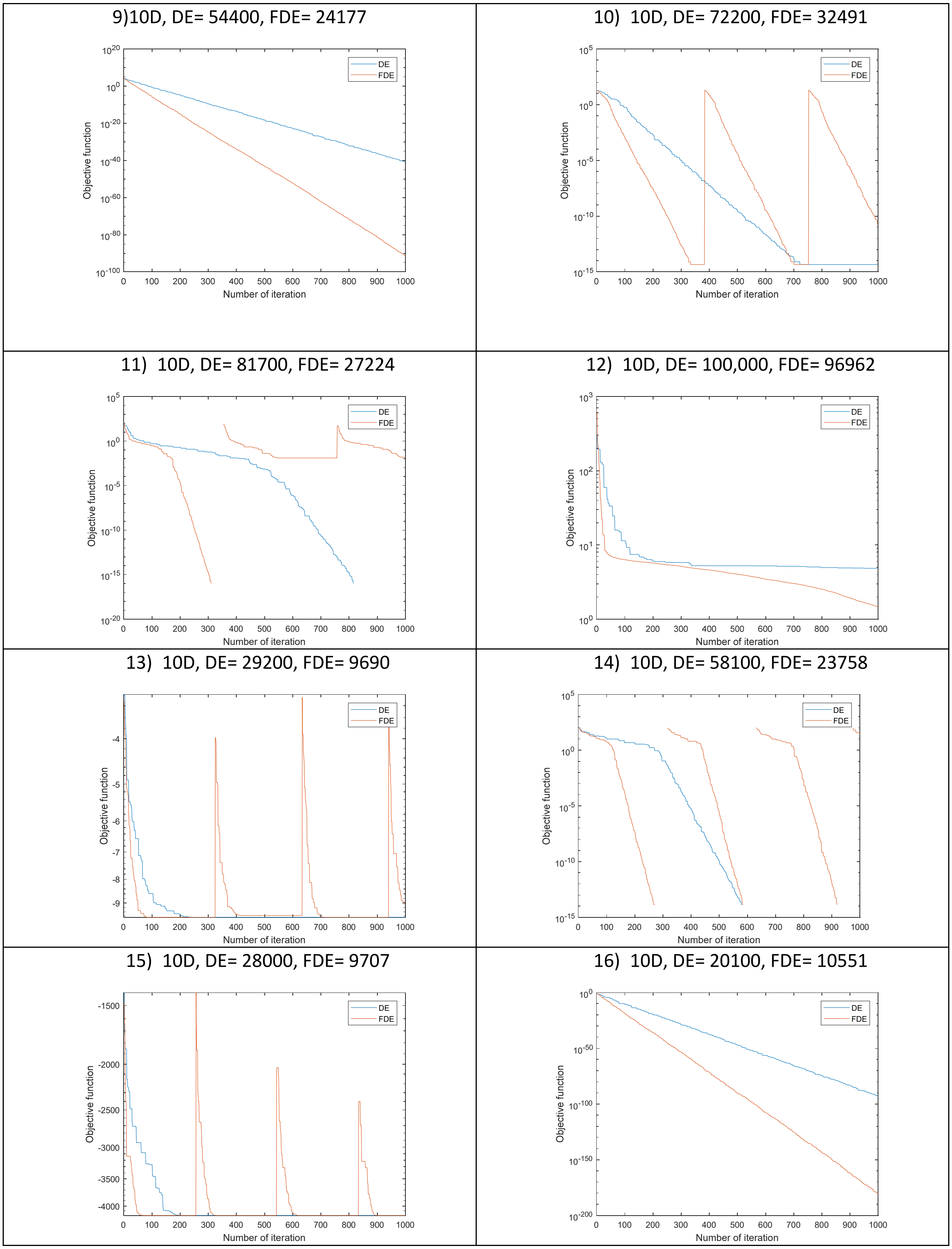


Figure 4.5 Convergence properties of DE and FDE for EA benchmark test problems, based on ten-variable formulations

Chapter 5 : A Fuzzy Programming Approach for Multi-Objective Optimization of Water Distribution Systems

5.1 Preface

As mentioned in Chapters 1 and 3, the resilience index can be applied to implicitly assess reliability of WDN design. It is defined as the capability of system to "bounce back" following some failure and can be approximated as the capacity of a network to absorb disturbance while experiencing alterations, so as to retain essentially the same function, structure, identity, and feedback information. In Chapter 3, a nondominated sorting population-based multi-objective design framework, based on the minimization of cost and maximization of the resilience index, is presented. Maximization of the resilience index increases the hydraulic reliability of the system. For enhancing both mechanical and hydraulic reliability in WDN design, researchers propose several additional reliability surrogates. These include: (1) the sum over all nodes of the relative surplus of energy at each node; (2) the minimum surplus head at a critical node; (3) the sum over all nodes of the relative surplus of energy at each node modified by the degree of pipe diameter uniformity of its associated loop; and (4) the minimum pipe diameter uniformity of the associated loops over all nodes. For employing these surrogates, this chapter proposes a fuzzy multi-objective programming model for meeting more than three competing objectives in the optimal design of WDNs. FM functions for minimizing the pipe network cost and maximizing the reliability surrogates are defined, and the model maximizes the degree of satisfaction of these membership functions. Based on post-optimization cut-set analyses, the proposed model that minimizes cost and maximizes the resilience index and the minimum pipe diameter uniformity of the associated loops over all nodes, exhibits the highest level of mechanical reliability. Unlike the nondominated multi-objective models discussed in Chapter 3 which provide populations of solutions on the Pareto optimal frontier, the fuzzy programming model converges to only one good compromise solution.

5.2 Abstract

This paper proposes a fuzzy multi-objective programming model for meeting competing objectives in the optimal design of water distribution systems (WDSs). Fuzzy membership functions for minimizing the pipe network cost and maximizing a number of reliability surrogates are defined, and the model maximizes the degree of satisfaction of these membership functions. Reliability

surrogates investigated include: i) the sum over all nodes of the relative surplus of energy at each node; ii) the minimum surplus head at a critical node; iii) the sum over all nodes of the relative surplus of energy at each node modified by the degree of uniformity of its associated loop; and iv) the minimum uniformity of the associated loops over all nodes. The model is applied to various combinations of objective functions, to identify the design pipe diameters of the water-main network of Farhadgerd, Iran. Optimal solutions of the various model formulations show that for this WDS the third surrogate (iii) may be a reasonable substitute for the first surrogate (i), and that while the inclusion of the last surrogate (iv) may be beneficial, the inclusion of the second surrogate (ii) may not. Based on post-optimization cut-set analyses, the model that minimizes cost and maximizes surrogates (iii) and (iv) exhibits the highest level of mechanical reliability, while the model that minimizes cost and maximizes the second surrogate (ii) exhibits the lowest level of mechanical reliability.

5.3 Introduction

In optimal design of water distribution systems (WDSs), network components are selected considering the system cost of construction and operation and the ability of the network to satisfy consumer demands for water availability, pressure, and quality, under normal and abnormal operating conditions, including during pipe bursts and as components deteriorate. Hydraulic reliability is a representative measure of the ability of the system to respond to gradual changes in the system characteristics, e.g., available pipe diameters and pipe roughness, and short or long-term shifts in consumption, e.g., hourly variations in withdrawal rates or community-driven changes in required fire flows (Shamir and Howard 1981). Mechanical reliability is a measure of the ability of the system to adapt to mechanical failures, e.g., to maintain operating goals during pipe, valve, or pump failures (Mays 1989). Approaches for accounting for reliability include stochastic analysis of uncertainty regarding the condition of network components (Damelin et al. 1972) and the level of system demands (Mays and Cullinane 1986), and the estimation of deterministic surrogate measures of reliability (Todini 2000). The latter approach has gained wide recognition in recent years due to the ease with which the surrogates may be incorporated as constraints or objectives in optimization models and to advances in Computational capabilities for solving these models (Babayan et al. 2005; Giustolisi et al. 2009; Schwartz et al. 2016).

Surrogates of hydraulic reliability include the: relative surplus of pressure head over all system nodes and the minimum surplus head at a critical node, as introduced by Todini (2000), modifications of the former approach as a function of the degree of uniformity of the network loops (Prasad and Park 2004), minimum flow entropy (Ang and Jowitt 2003; Atkinson et al. 2014), or variations and combinations of these (Saleh and Tanyimboh 2014; Jayaram and Shrinivasan 2008; Ostfeld et al. 2014; Creaco et al 2016). While similar surrogates have been proposed (Todini 2000; Geem 2015), the most widely used approach for estimating mechanical reliability is a cut-set analysis in which a given design is evaluated based on the number of single pipes that could be removed from operation without causing the network design to fail to meet specified nodal pressures. This approach exacts a high computational burden because every possible pipe removal scenario requires a re-evaluation of the hydraulic conditions, and thus it is generally undertaken post-optimization.

Recent progress in WDS optimization successfully applies multi-objective evolutionary algorithms for evaluating the cost-reliability Pareto Optimal Frontier (Prasad et al. 2003; Farmani et al. 2005a; Farmani et al. 2006; Raad et al. 2009; Baños et al. 2010; Greco et al. 2012). The most attractive of these algorithms use nondominated sorting in the selection process, e.g., the modified nondominated sorting genetic algorithm, NSGA-II (see Farmani et al. 2006), the nondominated sorting differential evolution algorithm, NSDE, and the nondominated sorting differential evolution algorithm with ranking-based mutation operator, NSDE-RMO (for the latter two, see Moosavian and Lence 2016), to identify well-distributed solutions along the entire cost-reliability domain. Typically, a large number of Pareto optimal solutions are generated, though generally only a small set of these are evaluated in depth for mechanical reliability (Farmani et al 2005a), and no standard approach for selecting which WDS solutions to investigate has been adopted. Several approaches for incorporating decision-making preferences to identify WDS solutions that achieve the best compromise among the objectives have been proposed (Vamvakeridou et al. 2006; Atiquzzaman et al. 2006), although these approaches are generally challenged as the number of solutions and objectives, and the complexity of the WDS, are increased.

Considered an appropriate framework for representing imprecise information and linguistic ambiguity, fuzzy set theory (Zadeh 1965) has been applied in the design and operation of WDSs to describe uncertain conditions such as nodal demands and pipe roughness information, and system constraints such as desirable pressure head and velocity levels. Fuzzy programming models for WDS optimization comprised of fuzzy constraints (Goulter and Bouchart 1988; Xu and Goulter 1999; Bhave and Gupta 2007; Xu and Qin 2013) and objective functions that account for: a single fuzzy benefit or weighted benefits (Vamvakeridou et al. 2006; Geem, 2015), the degree of satisfaction of fuzzy constraints through penalty functions (Amirabdollahian et al. 2011), and the probability of meeting fuzzy constraints (Fu and Kapelan 2011), have been investigated. The findings of these studies indicate that, by accounting for uncertainty in the system limitations and goals, fuzzy programming models may lead to improved WDS designs that are robust to changes in system conditions.

We present a mixed-integer fuzzy programming model for identifying the WDS design solutions that minimize cost while maximizing one or more surrogates for hydraulic reliability, and implement this model with the Soccer League Competition Algorithm, SLC (Moosavian and Roodsari 2014a), an evolutionary algorithm for which the decision variables may take any form. This approach defines fuzzy membership functions for each of the objectives and maximizes the degree of satisfaction of all membership functions. Attractive solutions are identified directly, without the need to generate the entire Pareto Optimal Frontier. By maximizing the satisfaction of the various fuzzy objectives, it is possible to substantially increase the probability of finding an attractive compromise among them. Depending on the composition of the objective set, a number of compromise solutions on the Pareto Optimal Frontier may be identified, and post-optimization analysis of mechanical reliability for these points may be used as comparators, or in conjunction with other information, to evaluate the potential performance of a given WDS design.

The following section presents the general model for maximizing membership of fuzzy objectives for cost and reliability. This model may be extended for any number of objectives, and thus may include a number of surrogates for reliability. Next, the objectives for designing WDSs applied in this paper, and the mixed-integer WDS fuzzy optimization model are described. In Section 4, the SLC is briefly described in the context of the WDS optimization problem. Finally, the application of the fuzzy programming model for finding compromise solutions for design of the water-main network for the town of Farhadgerd, Iran is provided, post-optimization cut-set analyses are conducted for these solutions, and ideal solutions for these multi-objective problems are computed and compared with the fuzzy programming results.

5.4 Fuzzy Programming for Multi-objective Optimization of Cost and Reliability

Most multi-objective optimization approaches fall into three categories, approaches that: i) identify the nondominated trade-offs between objectives, or the Pareto Optimal Frontiers; and identify the solution which best meets all objectives, or an attractive compromise solution on the Pareto Optimal Frontier, either with (ii); or without (iii) considering the preferences of the decision makers, or the welfare of the users (Cohen 1978). Our fuzzy programming model for multi-objective WDS problems is an example of the latter category. Here, the best compromise WDS design for the competing objectives of cost and reliability maximizes the satisfaction of fuzzified values of these objectives, expressed as linear fuzzy membership functions. By expressing the objectives in a fuzzified manner, it is possible to further explore, or stretch, the Pareto Optimal Frontier, so as to increase the potential for finding the most attractive compromise solution. While different fuzzy membership functions exist, linear functions in which the maximum membership represents the best possible value for a given objective, and the minimum membership the worst, are used here. If the resulting cost and reliability for a given set of decisions, x, (i.e., the diameter of pipe selected for each pipe of the network) are defined as $Cost(x)$ and $Reliability(x)$, and the minimum and maximum possible values of cost and reliability are defined as $Cost_{min}$, and $Cost_{max}$, and $Reliability_{min}$, and $Reliability_{max}$, respectively, the linear membership function for the cost objective is a normalized cost function, for costs that range from $Cost_{min}$ to $Cost_{max}$, and is defined as a function of discrete variables as follows:

$$\mu_1\left(Cost\left(x\right)\right)=\begin{cases} 1 & if\ \ Cost\left(x\right)\leq Cost_{\min} \\ \dfrac{Cost_{\max}-Cost\left(x\right)}{Cost_{\max}-Cost_{\min}} & ,\ if\ Cost_{\min}\leq Cost\left(x\right)\leq Cost_{\max} \\ 0 & if\ Cost\left(x\right)\geq Cost_{\max} \end{cases} \tag{5.1}$$

And similarly, the membership function of the reliability objective is a normalized reliability function, for reliability levels that range from $Reliability_{min}$ to $Reliability_{max}$, and is defined as a function of discrete variables as follows:

$$\mu_2\left(Reliability\left(x\right)\right)=\begin{cases} 0 & if\ Reliability\left(x\right)\leq Reliability_{\min} \\ \dfrac{Reliability\left(x\right)-Reliability_{\min}}{Reliability_{\max}-Reliability_{\min}} & ,if\ Reliability_{\min}\leq Reliability\left(x\right)\leq Reliability_{\max} \\ 1 & if\ Reliability\left(x\right)\geq Reliability_{\max} \end{cases} \tag{5.2}$$

Using Zimmermann's minimum operator (Zimmermann 1991), the general multi-objective model for optimizing cost and reliability, as a function of discrete variables, is as follows:

$$Min \;\; \mu_1\big(Cost(x)\big), \;\; and \;\; Max \;\; \mu_2\big(Reliability(x)\big) \tag{5.3}$$

Subject to:

$$g_j(x) \leq 0 \;\; for \;\; j = 1,2,\ldots,m \tag{5.4}$$

$$x_t \geq 0 \;\; for \;\; t = 1,2,\ldots,T \tag{5.5}$$

where $g_j(x)$= *j*th constraint function and x_t= *t*th decision variable.

According to Zimmermann (1991), this model may be solved by introducing an auxiliary continuous variable, λ, known as the overall satisfaction, which ranges from 0 to 1, and by transforming Eq. 5.3-5.5 into the equivalent single objective mixed-integer optimization problem of maximizing λ. The transformed problem finds the intersection of the fuzzy member functions of the objectives (Eq. 5.1-5.2) at a point that maximizes the level of satisfaction of the fuzzy membership function for each of the objectives. It is:

$$Max \, \lambda \tag{5.6}$$

Subject to:

$$\mu_1(Cost(x)) \geq \lambda \tag{5.7}$$

$$\mu_2(Reliability(x)) \geq \lambda \tag{5.8}$$

$$0 \leq \lambda \leq 1 \tag{5.9}$$

and Eqs (5.4) and (5.5) above.

5.5 Multi-objective Optimization of Water Distribution Systems (WDSs)

For purposes of this paper, the WDS decisions to be made are limited to the selection of pipe sizes, while pipe layout and connectivity, nodal demands, and minimum head requirements are imposed. The objective of minimizing the cost of a WDS design may be expressed as:

$$Min\ Cost = \sum_{i=1}^{np} Cost_i(D_i) \times L_i \tag{5.10}$$

where L_i = length of each pipe (meters); $Cost_i\ (D_i)$ = cost of a pipe of a given diameter (\$/meter); D_i = diameter of the selected pipe (meters); and np = number of pipes.

Among the possible surrogates proposed for estimating the reliability of pipe networks, the most widely used surrogate is the resilience index (*Ir*) introduced by Todini (2000), which characterizes the intrinsic capability of the system to overcome failures while still satisfying demands and pressures in nodes. The objective of maximizing the reliability of a WDS design is expressed as:

$$Max\ \ Ir = \frac{\sum_{k=1}^{nn} q_k\left(H_k - H_k^{min}\right)}{\left(\sum_{k=1}^{no} Q_k H_{0k} + \sum_{i=1}^{np} \frac{P_i}{\gamma} - \sum_{k=1}^{nn} q_k H_k^{min}\right)} \tag{5.11}$$

where nn = number of nodes; no = number of reservoirs; q_k = flow demand at node k (meters^3/second); H_k = available pressure head at node k (meters); $H_k^{\min}$ = minimum pressure head required at all nodes (meters); Q_k = supply at a reservoir located at node k (meters^3/second); H_{0k} = elevation head of a reservoir located at nPode k (meters); and P_i = power of a pump located in pipe i (watts); γ = specific weight of water (Newtons/ meters^3). Being the ratio of the energy available in the network to that required, *Ir* ranges from 0 to 1.

Another important indicator introduced by Todini is the minimum surplus head (*Is*), which is defined as the smallest nodal pressure head difference between the minimum required and observed pressure. Farmani et al. (2005a) found that increasing the value of this index can improve mechanical reliability. The objective function based on this measure is formulated as:

$$Max\ \ Is = MaxMin\left(H_k - H_k^{min}\right),\ \ k = 1, \dots, nn \tag{5.12}$$

Prasad and Park (2004) introduced the network resilience index (*NRI*), which incorporates the effects of both surplus power and reliable loops. Reliable loops can be ensured if the pipes connected to a node do not vary greatly in diameter. If the uniformity of pipes connected to a node k is defined as C_k, the objective function for the *NRI*, is formulated as:

$$Max\ \ NRI = \frac{\sum_{k=1}^{nn} C_k q_k \left(H_k - H_k^{min}\right)}{\left(\sum_{k=1}^{no} Q_k H_{0k} + \sum_{i=1}^{np} \frac{P_i}{\gamma} - \sum_{k=1}^{nn} q_k H_k^{min}\right)} \tag{5.13}$$

where

$$C_k = \frac{\sum_{i=1}^{nk} D_{ik}}{nk * \max(D_{ik})} \tag{5.14}$$

where nk = number of pipes connected to node k, and D_{ik} = diameter of each of the nk pipes connected to node k.

As a fourth surrogate, we propose a version of *NRI*, which seeks to maximize the least uniformity of pipes over all nodes as a means of potentially improving mechanical reliability. That is, the more uniform the loops, the more likely they are to be able to reroute flow during unforeseen pipe closures. The objective function for this surrogate is:

$$Max\ Id = MaxMis(C_k) = MaxMin\left(\frac{\sum_{i=1}^{nk} D_{ik}}{nk * \max(D_{ik})}\right) \tag{5.15}$$

Since the design pipe sizes must be selected from those commercially available, the fuzzy transformed optimization model of the WDS design problem is a mixed-integer problem:

$$Max\ \ \lambda \tag{5.16}$$

Subject to:

$$\sum Q_{in} - \sum Q_{out} = q_k, \quad \forall\ k \in nn \tag{5.17}$$

$$\sum_{i\ \in\ loop_l} hf_i = \Delta H, \quad \forall\ l \in nl \tag{5.18}$$

$$H_k \geq H_k^{min}, \quad \forall\ k \in nn \tag{5.19}$$

$$D_i = \{D(1), D(2), \ldots, D(S)\}, \quad \forall\ i \in np \tag{5.20}$$

$$\mu_1(Cost(x)) \geq \lambda \tag{5.21}$$

$$\mu_2(Ir(x)) \geq \lambda \tag{5.22}$$

$$\mu_3(Is(x)) \geq \lambda \tag{5.23}$$

$$\mu_4(NRI(x)) \geq \lambda \tag{5.24}$$

$$\mu_5(Id(x)) \geq \lambda \tag{5.25}$$

$$0 \leq \lambda \leq 1 \tag{5.26}$$

where Q_{in} and Q_{out} = flow into and out of node k (meters^3/second), respectively; hf_i = head loss due to friction in the pipe i (meters) computed by the Hazen-Williams or Darcy-Weisbach formulae; nl = loop set; $\Delta H = 0$, if the path is closed, and ΔH = difference between nodal heads at both reservoirs (meters), if the loop is between two fixed head reservoirs; and S = the number of candidate diameters commercially available.

Eq. (5.17) represents the continuity constraint for each node in the network. Eq. (5.18) represents the conservation of energy constraint for each loop in the network, that is, the total head loss around a closed loop should be equal to zero, or the head loss along a loop between the two fixed head reservoirs should be equal to the difference in water level of the reservoirs. Eq. (5.19) constrains the pressure head at each node to be greater than the prescribed minimum pressure head. Eq. (5.20) requires that the diameter of the pipes selected be commercially available, and Eqs (5.21)-(5.26) are analogous to Eqs (5.7)-(5.9) for the case where all four surrogates for hydraulic reliability are considered simultaneously.

The Hazen-Williams formula which defines the pressure head loss equation for pipe i connecting nodes n and m is:

$$hf_i = H_n - H_m = 10.667\left(L_i / \left(CH_i^{1.852} D_i^{4.871}\right)\right) Q_i \left|Q_i\right|^{0.852} \tag{5.27}$$

where Q_i = the flow in the pipe i (meters^3/second); and CH_i = Hazen-Williams coefficient for pipe i. Thus Eq. (5.16)-(5.27) generate a unique compromise Pareto-optimal solution for the multi-objective WDS problem and this problem may be solved with any powerful mixed-integer programming algorithm.

5.6 Soccer League Competition (SLC) Algorithm

Given the non-linearity and non-convexity of the multi-objective WDS design problem and the implicit constraints requiring hydraulic simulation to satisfy the WDS continuity and energy equations (Eq. 5.17 and 5.18), we apply the SLC evolutionary algorithm with mixed-integer solution vectors. The SLC algorithm has successfully achieved high performance in optimization of discrete problems, such as WDS design (Moosavian and Roodsari 2014b), knapsack problems (Moosavian 2015) and the set-covering problem (Jaramillo et al. 2016), though it can also accommodate continuous and mixed-integer variables.

The SLC algorithm is inspired from professional soccer leagues. It involves different teams, or collections of solutions, where each solution is a team member, and a number of operators that act on the team members to perform an effective search for finding the near optimal solution. For the network optimization problem each of the team members, or solutions, may comprise the set of design pipe diameters for the system.

The key organizing structure of the algorithm is the soccer teams, and each team includes fixed players (*FPs*), or fixed sets of pipe diameter solution vectors, and substitute players (*Ss*), or substitute sets. The number of fixed players, *nFP*, and the number of substitute players, *nS*, is equivalent for all teams. The power of each team member, is calculated based on its objective function value. For minimization problems, e.g., that minimize the design cost of a network, the power of player *j* on team *i*, *PP(i,j)* is the inverse of its objective function value. The players are rank ordered as a function of their power, and the *nFP* players with the highest power are considered to be the fixed players, while the *nS* players with the lowest power are considered to be substitutes. The power of a team is the average power of the fixed players of the team, i.e.:

$$TP(i) = (1/nFP) \sum_{j=1}^{nFP} PP(i,j) \tag{5.28}$$

where *TP(i)* = the power of team *i*.

The algorithm mimics matches between teams and determines the winners based on their relative power, and the winner of a match has a higher probability of increasing its power for future matches. After each match, all players (decision vectors) on the winning team undergo operations that change their decisions, and thus their power (i.e., strengthen or weaken each player), producing modified team members. Finally, the original strength of each player on this winning team is compared with its modified strength and the player, i.e., decision vector, with the greater strength, is allowed to play on the team in the next match.

A round is defined as a set of matches that allows each team, or modified team, to play with each other team in the league. If *NT* is the number of teams in the league, at each round of the tournament the number of matches is:

$$\frac{NT\ (NT-1)}{2} \tag{5.29}$$

After finishing of the total matches in a given round, all players in the league are re-sorted in descending order of their power and are assigned to teams. The *nFP* highest ranked players are assigned to team one, the next *nFP* highest ranked players are assigned to team two, and so on until all $NT \times nFP$ fixed players are assigned to a team. And finally, the next *nS* highest ranked players are assigned to team one, and the next *nS* highest ranked are assigned to team two, and so on until all $NT \times nS$ substitute players are assigned to a team.

Thereafter, the next round is played with these new teams. The user specifies the stopping criteria to be either based on a limit of the number of rounds undertaken, or the number of function evaluations made (in this paper, the limit on the number of function evaluations is set to be 10^6). Details of the algorithm are described in Moosavian and Roodsari (2014a and 2014b).

5.7 Fuzzy Multi-objective Optimization of the Farhadgard WDS

Hydraulic simulation to satisfy the energy and continuity equations for each SLC solution vector is undertaken with the Global Gradient Algorithm (GGA) (Todini 2000), and the linked simulation-optimization process is implemented in MATLAB. Fuzzy multi-objective optimal solutions to the cost-reliability WDS design problem are examined for a variety of surrogates for reliability, for a water supply network serving a population of approximately 8200 in the town of Farhadgard, Iran, a residential community near the city of Mashhad. Post-optimization cut-set analyses are also undertaken as a means of further evaluating the mechanical reliability of these design solutions. The Farhadgard network (Figure 5.1) comprises 68 pipes, 53 nodes, and two gate valves. The network has one reservoir with a head of 510 m and the minimum pressure required in the network is 20 m. Other information regarding the hydraulic simulation of this network is provided in Moosavian and Jaefarzadeh (2014).

5.7.1 Hydraulic Reliability

Table 5.1 presents the objective function values for cost and reliability for the *minimum cost*, *maximum cost, and* fuzzy multi-objective optimal models with a variety of objectives. The *minimum cost* model for this network, considers cost as the sole objective function (i.e., Eqs (5.10), and (5.17)-(5.20) and represents the solution with the least reliability. The *maximum cost* solution, for which all pipe diameters are set to their largest possible values, represents the solution with the overall maximum reliability. These solutions provide the range of possible values for cost and each of the reliability surrogates. When two objectives are considered, i.e., cost and any one of the reliability surrogates, the relative value of reliability surrogate increases significantly with only moderate increases in the relative design cost. The fuzzy *Cost*+*Ir* and *Cost*+*NRI* models provide nearly the same results for the *Ir* surrogate (0.5940 and 0.5900) and the *NRI* surrogate (1.907 and 2.01), though the *Cost*+*NRI* model results in a somewhat higher cost. This indicates that the *NRI* may be a reasonable substitute for the *Ir* surrogate. The optimal pipe diameters for these two models for pipe numbers 1-68 are similar, as shown in Figure 5.2. The fuzzy *Cost*+*Ir*, *Cost*+*NRI*, and *Cost*+*Is* models all result in significantly improved values for the *Is* surrogate (1.907, 2.010, and 2.850, relative to its range of from 0.015 to 2.99), indicating that this measure may be well represented by the inclusion of *Ir* or *NRI* surrogates. In contrast, the results of all of the fuzzy two-objective models indicate that the only way to assure that the *Id* surrogate increases from its lowest value of 0.0638 is to account for it explicitly in the objective function set. The *Id* surrogate describes the uniformity of the diameters at a given node, or entering a given junction, for all nodes/junctions in the network. Unlike *Ir*, *NRI*, or *Is*, it does not include energy considerations between nodes or discharge requirements at nodes. The increase in the value of the *Id* surrogate to 0.279 in the fuzzy *Cost*+*Id* model solution is accompanied by corresponding decreases in all reliability surrogates and an increase in the cost. The significant difference in pipe diameters induced by including the *Id* surrogate is illustrated in Figure 5.3, which provides optimal pipe diameters for the *Cost*+*Ir* and *Cost*+*Id* models.

Thus, solutions for models that contain only select sets of objectives, the fuzzy *Cost*+*Ir*+*Is*, *Cost*+*NRI*+*Is*, *Cost*+*Ir*+*Id*, *Cost*+*NRI*+*Id*, *Cost*+*Ir*+*Id*+*Is*, and *Cost*+*NRI*+*Id*+*Is* models, are examined further. The solution of the fuzzy *Cost*+*Ir*+*Is* model shows that, relative to the fuzzy *Cost*+*Ir* model, the addition of the *Is* surrogate results in an improved value of *Is* (from 1.9070 to 2.6233), with an accompanying increase in cost and very little change in the other reliability surrogates. As expected, given the similarities of the fuzzy *Cost*+*Ir* and *Cost*+*NRI* models, similar results are observed for the fuzzy *Cost*+*NRI* and *Cost*+*NRI*+*Is* models where the *Is* surrogate is improved from 2.010 to 2.373, and very little change occurs in all other reliability surrogates. As was observed for the two-objective fuzzy models, the value of the *Id* surrogate may only be improved by including this objective explicitly in the fuzzy model. For the fuzzy *Cost*+*Ir*+*Id* and *Cost*+*NRI*+*Id* models the increase in the value of the *Id* surrogate is accompanied by corresponding decreases in all reliability surrogates and an increase in the cost. Again, the addition of the *Is* surrogate as an objective in the fuzzy *Cost*+*Ir*+*Id*+*Is* and *Cost*+*NRI*+*Id*+*Is* models serves only to improve the *Is* surrogate (from 1.244 to 1.978, and from 0.740 to 1.791, respectively), with an accompanying increase in cost, and very little change in the other reliability surrogates.

5.7.2 Mechanical Reliability

Table 5.2 provides the results of the post-optimization cut-set analyses for these models, where columns 2-4 provide the number of individual pipes that may be closed while maintaining pressure heads of 0, 10 and 20 m throughout the network, and columns 5-7 provide the percentage of the 68 pipes in the network that the cut-set represents. The cut-set analysis of the *maximum cost* (maximum diameter) solution has the highest level of mechanical reliability, where 64 of the 68 pipes may be damaged without shortfalls in pressure requirements. Pipe number 1 connected to the reservoir source and is crucial for operation of the network. Three additional dendritic pipes (pipe numbers 46, 48 and 51), are critical to the mechanical reliability of the network, and should any one of these pipes fails, the network will fail to meet the pressure requirements at the extreme junction for the given pipe. As expected, the *minimum cost* solution has the lowest level of mechanical reliability, where only 35, 24, and 4 pipes may be removed while still meeting pressure head targets of 0, 10, and 20 m, respectively. Among the two-objective models, the fuzzy *Cost*+*NRI* model results in the highest mechanical reliability (with cut-sets comprised of 51, 47, and 40 pipes for pressure head targets of 0, 10, and 20 m, respectively) and the addition of the *Is* surrogate to this model results in a decrease in the mechanical reliability (with cut-sets comprised of 45, 45, and 39 pipes for pressure head targets of 0, 10, and 20 m, respectively). The fuzzy *Cost*+*NRI*+*Id* model has the highest mechanical reliability among all fuzzy multi-objective models (with cut-sets comprised of 54, 51, and 47 pipes, for pressure head targets of 0, 10, and 20 m, respectively), though the fuzzy *Cost*+*Ir*+*Id* model has nearly the same mechanical reliability, with cut-sets comprised of 53, 52, and 43 pipes, for pressure head targets of 0, 10, and 20 m, respectively. Maximization of the *NRI* or *Id* surrogate favors solutions in which the diameters of pipes connected to any given junction are as similar as possible, thus the satisfaction of demands at the junctions is less likely to be dependent on any one pipe. Maximization of the *Ir* or *NRI* surrogate maximizes the total energy available to the network. These two features explain the high mechanical reliability of the fuzzy *Cost*+*NRI*, *Cost*+*Ir*+*Id*, and *Cost*+*NRI*+*Id* models. High uniformity among pipes at a junction increases the uniformity of the pipes in the entire network, thus reducing stress on any one pipe, without sacrificing the satisfaction of discharge and pressure requirements. The slight difference in the mechanical reliability between the *Cost*+*Ir*+*Id* and *Cost*+*NRI*+*Id* models may be attributed to the difference in the optimal pipe diameters selected, see Figure 5.4.

The sensitivity of the mechanical reliability to increasing demands is analyzed by performing cut-set analyses assuming a 10% increase in demands at all nodes (Table 5.3). Again, the *maximum cost* and *minimum cost* solutions represent the highest and lowest mechanical reliability, though the *minimum cost* solution is the least robust of all solutions to changes in demand. Among the two-objective models, the fuzzy *Cost*+*Ir* and *Cost*+*NRI* models are the most robust to changes in demands, and the fuzzy *Cost*+*Is* and *Cost*+*Id* models are the least robust. Since all of the three- and four- objective fuzzy models include either *Ir* or *NRI*, they are all robust to changes in demand. The fuzzy *Cost*+*Ir*+*Id* and *Cost*+*NRI*+*Id* models again have the highest mechanical reliability among all fuzzy multi-objective models (with cut-sets comprised of 52, 49, and 37 pipes, and 54, 49, and 36 pipes, respectively, for pressure head targets of 0, 10, and 20 m, respectively).

5.7.3 Comparison with Compromise Set Ideal Solution

As a comparison with the fuzzy programming-based compromise solutions, we evaluate the compromise solution of the WDS multi-objective problem using the compromise set solution approach as defined by Yu (1973) and Zeleny (1974a; 1974b). Here, the constraints to the problem are defined as in Eqs (5.17)-(5.20), and the objective function minimizes the distance from the ideal solution (minimum cost or maximum reliability) for each of the objectives of the multi-objective model according to the following relationship:

$$Min\ d_\alpha = \left\{ \sum_{kk=1}^{KK} \left| Z_{kk}^* - Z_{kk}(x) \right|^\alpha \right\}^{\frac{1}{\alpha}} \qquad (5.30)$$

where $d\alpha$ = minimum distance to the ideal solution; Z_{kk}^* = ideal solution for objective *kk*; $Z_{kk}(x)$ = optimal solution for objective *kk*; *KK* = number of objectives; and α = metric between one and infinity.

When α is one, Equation 28 gives the absolute deviations from the ideal, and when α is infinity, only the largest deviation is considered. Yu (1973) shows that, in general, the best-compromise solutions define a subset of the non-inferior set, for which α ranges from one to infinity.

For purposes of this comparison, we evaluate the compromise solution for a midpoint between these values, at α equal to two. Tables 5.4 and 5.5 provide the objective function values, hydraulic reliability, and cut-set analyses (mechanical reliability) for this model. In general, the model solutions shown in Table 5.4 are different and non-inferior to analogous solutions for the fuzzy programming models (Table 5.1). However, except for the fuzzy *Cost+Is* model, the fuzzy programming models demonstrate a significantly higher mechanical reliability than models based on a compromise objective function with α equal to two. For more examples of the use of the compromise set solution approach, i.e., Eq. 30, see Cohen (1978).

5.8 Conclusions

The fuzzy multi-objective WDS model presented in this paper provides a framework for investigating an assortment of design objectives and for identifying attractive options among these. An advantage of this modeling framework is that it may be used to identify compromise solutions that may be investigated in greater detail, e.g., with cut-set analyses. The benefits of this feature increase as the number of objectives increase, i.e., when the dimensions of the objective space increase.

The model is applied for evaluating combinations of objectives for minimizing cost and maximizing reliability, and demonstrated for designing the water-main network of Farhadgerd, Iran. Among several surrogates for reliability, the *NRI* and the proposed *Id* surrogates favor solutions in which the diameters of pipes connected to any given node are as similar as possible, and thus are robust to pipe failures. The *Ir* and *NRI* surrogates maximize the total energy available to the network, and are shown to be reasonable substitutes for each other. Post-optimization cut-set analyses indicate that the most effective objective combinations for assuring mechanical reliability are the fuzzy *Cost+NRI+Id* and *Cost+Ir+Id* models. Both models are also robust to realistic (+10%) changes in demand. In contrast, the *Is* surrogate may be well represented by the

Ir or *NRI* surrogates, and is likely redundant. Furthermore, all cut-set analyses indicate that sole consideration of the *Is* surrogate, along with cost, result in low mechanical reliability.

As more objectives are identified, e.g., new expressions for reliability, they may be easily added to the model with little additional programming and computational burden. The modeling framework is flexible, even in cases where additional simulations may be required, e.g., in considering water quality in the network as a system objective. A natural extension of the fuzzy multi-objective model is to fuzzify the constraints (e.g., Equations 17, 18 and 27) to account for uncertainties in nodal demands and pipe roughness information.

The advantage of the *NRI* surrogate is that it enhances mechanical reliability by incorporating the effects of both surplus energy and reliable loops. The results of this paper show that inclusion of the *Id* surrogate, which optimizes the uniformity of the loops, further improves mechanical reliability. Future work will investigate the benefit of applying these surrogates in the optimization of larger and more complex WDSs.

Table 5.1 Fuzzy Multi-objective WDN Solutions for Town of Farhadgerd

Objective Functions Included	Objective Function Values				
	Cost ($)	*Ir*	*Is*	*NRI*	*Id*
Minimum Cost	1.78E+07	0.017	0.015	0.015	0.064
Maximum Cost	7.32E+07	0.066	2.990	0.066	0.400
Cost+Ir	2.15E+07	0.059	1.907	0.044	0.064
Cost+Is	1.92E+07	0.022	2.850	0.019	0.064
Cost+NRI	2.35E+07	0.059	2.010	0.055	0.064
Cost+Id	2.77E+07	0.031	0.352	0.025	0.279
Cost+Ir+Is	2.23E+07	0.058	2.623	0.043	0.064
Cost+NRI+Is	2.42E+07	0.059	2.373	0.054	0.064
Cost+Ir+Id	2.78E+07	0.049	1.244	0.033	0.279
Cost+NRI+Id	2.89E+07	0.054	0.740	0.046	0.264
Cost+Ir+Id+Is	2.87E+07	0.047	1.978	0.035	0.264
Cost+NRI+Id+Is	2.90E+07	0.054	1.791	0.045	0.264

Table 5.2 Cut-set Analyses of Fuzzy Multi-objective WDN Solutions for Town of Farhadgerd

Objective Functions Included	min(H) > 0 m	min(H) > 10 m	min(H) > 20 m	% >0 m	% >10 m	% >20 m
Minimum Cost	35	24	4	51.5	35.3	5.9
Maximum Cost	64	64	64	94.1	94.1	94.1
Cost+Ir	42	41	35	61.8	60.3	51.5
Cost+Is	34	29	12	50.0	42.7	17.7
Cost+NRI	51	47	40	75.0	69.1	58.8
Cost+Id	41	36	20	75.0	52.9	29.4
Cost+Ir+Is	40	39	34	58.8	57.4	50.0
Cost+NRI+Is	45	45	39	66.2	66.2	57.4
Cost+Ir+Id	53	52	43	77.9	76.5	63.2
Cost+NRI+Id	54	51	47	79.4	75.0	75.0
Cost+Ir+Id+Is	48	46	39	70.6	67.7	57.4
Cost+NRI+Id+Is	50	49	42	73.5	72.1	84

Table 5.3 Cut-set Analyses of Fuzzy Multi-objective WDN Solutions with 10% Increase in Demands

	min(H) > 0 m	min(H) > 10 m	min(H) > 20 m	% >0 m	% >10 m	% >20 m
Minimum Cost	28	0	0	41.2	0	0
Maximum Cost	63	63	63	92.6	92.7	92.7
Cost+Ir	42	40	31	61.8	58.8	45.6
Cost+Is	29	13	0	42.7	19.1	0
Cost+NRI	48	43	38	70.6	63.2	55.9
Cost+Id	40	31	1	58.8	45.6	1.5
Cost+Ir+Is	40	37	32	58.8	54.4	47.1
Cost+NRI+Is	44	41	37	64.7	60.3	54.4
Cost+Ir+Id	52	49	37	76.5	72.1	54.4
Cost+NRI+Id	54	49	36	79.4	72.1	52.9
Cost+Ir+Id+Is	48	42	35	70.6	61.8	51.5
Cost+NRI+Id+Is	50	47	34	73.5	69.1	50

Table 5.4 Compromise WDN Solution, with α = 2, for Town of Farhadgerd

Objective Functions Included	Objective Function Values				
	Cost ($)	*Ir*	*Is*	*NRI*	*Id*
Cost+Ir	1.82E+07	0.035	0.088	0.025	0.064
Cost+Is	1.97E+07	0.025	2.979	0.020	0.064
Cost+NRI	1.83E+07	0.031	0.066	0.026	0.064
Cost+Id	2.06E+07	0.017	0.018	0.014	0.176
Cost+Ir+Is	1.93E+07	0.025	2.978	0.020	0.064
Cost+NRI+Is	1.91E+07	0.024	2.976	0.020	0.064
Cost+Ir+Id	2.05E+07	0.022	0.071	0.019	0.176
Cost+NRI+Id	2.04E+07	0.020	0.318	0.017	0.176
Cost+Ir+Id+Is	2.00E+07	0.023	2.971	0.020	0.143
Cost+NRI+Id+Is	2.10E+07	0.027	2.990	0.023	0.158

Table 5.5 Cut-set Analysis of Compromise WDN Solution, with α = 2, for Town of Farhadgerd

Objective Functions Included	min(H) > 0 m	min(H) > 10 m	min(H) > 20 m	% >0 m	% >10 m	% >20 m
Cost+Ir	42	36	25	61.8	52.9	36.8
Cost+Is	35	29	15	51.5	42.7	22.2
Cost+NRI	40	36	16	58.8	52.9	23.5
Cost+Id	34	21	0	50.0	30.9	0.0
Cost+Ir+Is	38	31	21	55.9	45.6	30.9
Cost+NRI+Is	39	30	17	57.4	44.1	25.0
Cost+Ir+Id	35	28	3	51.5	41.2	4.4
Cost+NRI+Id	35	26	4	51.5	38.2	5.9
Cost+Ir+Id+Is	35	31	15	51.5	45.6	22.1
Cost+NRI+Id+Is	34	29	13	50	42.65	19.12

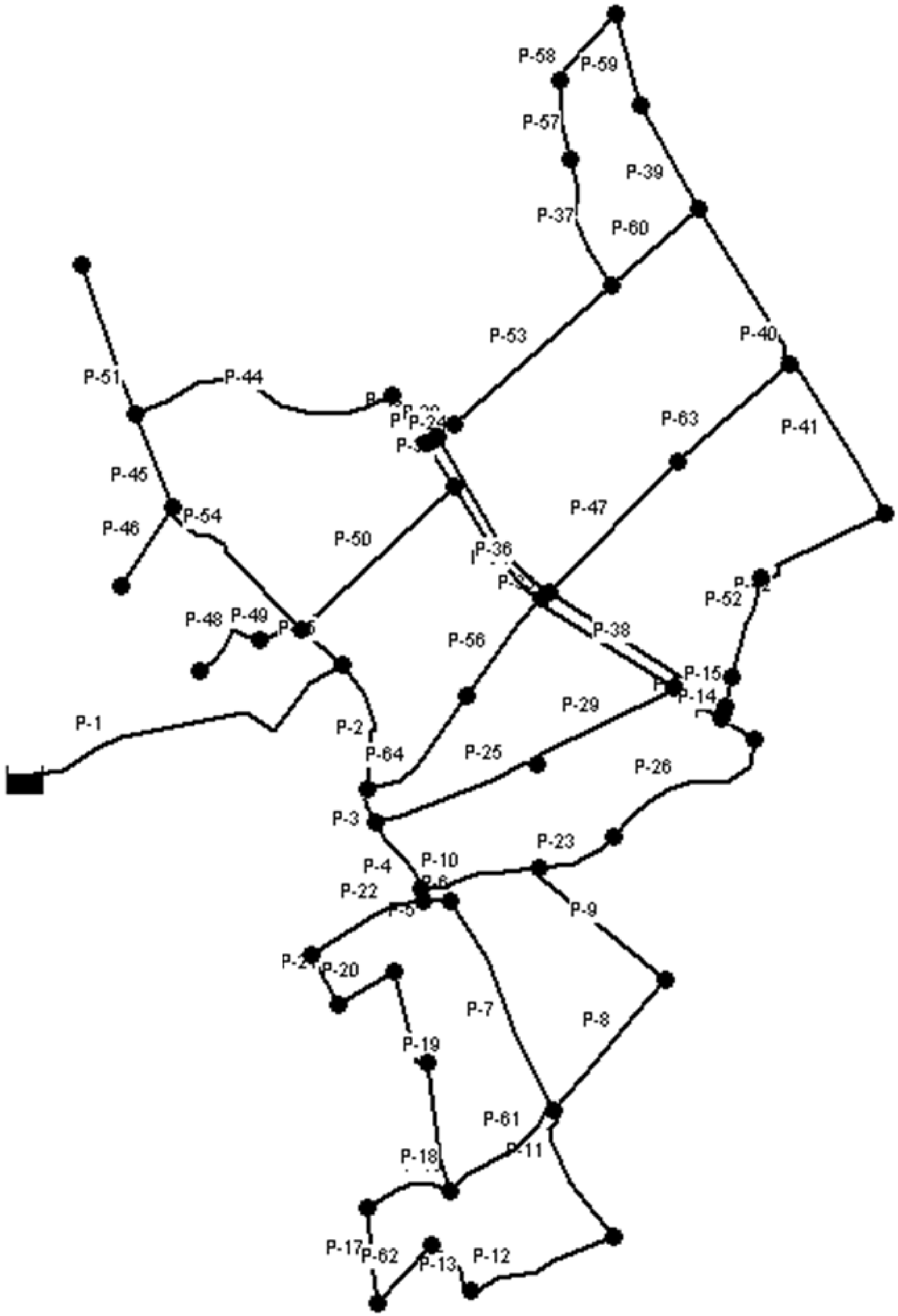


Figure 5.1 Water-main network of Farhadgerd, Iran

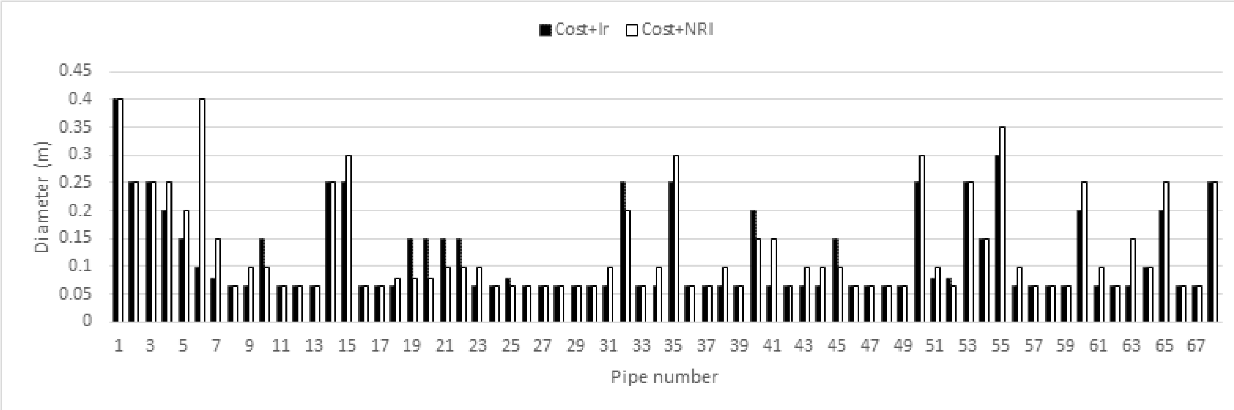


Figure 5.2 Design diameter for the fuzzy *Cost+Ir* and *Cost+NRI* models

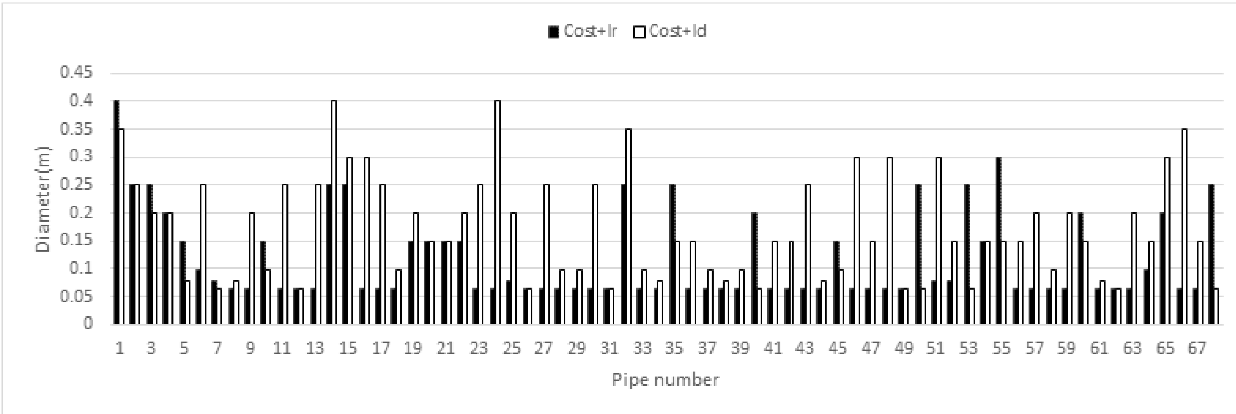


Figure 5.3 Design diameter for the fuzzy *Cost+Ir* and *Cost+Id* models

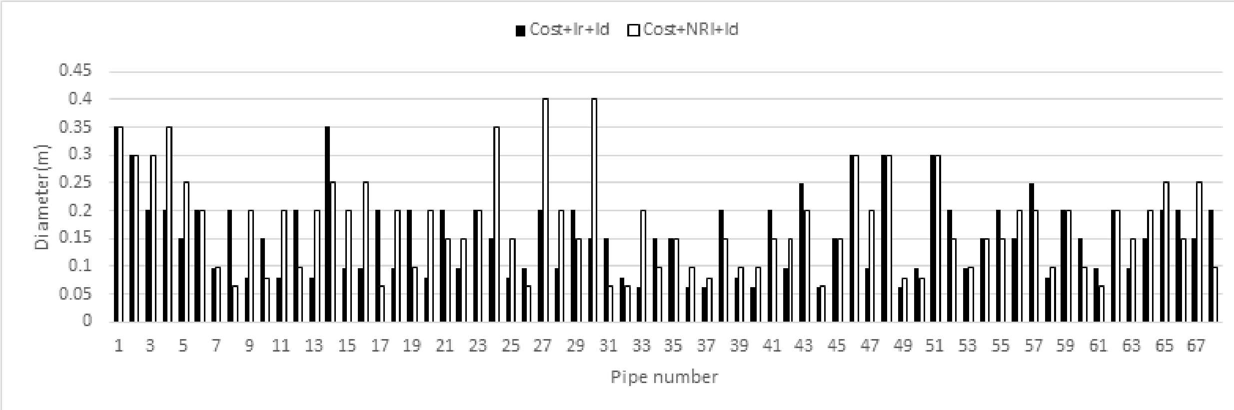


Figure 5.4 Design diameter for the fuzzy *Cost+Ir+Id* and *Cost+NRI+Id* models

Chapter 6 : Flow Uniformity Index for Reliable-Based Optimal Design of Water Distribution Networks

6.1 Preface

Many researchers have analyzed and tested different reliability surrogates for optimal design of WDNs. Chapter 5 shows that the incorporation of several combinations of reliability surrogates can improve the mechanical reliability of a WDN design. However, these surrogates may be inaccurate measures of network reliability in the face of mechanical failures. In this chapter, a flow uniformity index is proposed to address this limitation. The definition of the index is simple, and the maximization of it is expected to lead to network designs with a balanced, increasingly uniform flow, at each node, where the burden of carrying inflows (or outflows) at each node is shared evenly among all input (or output) pipes. The incorporation of the uniformity index, along with the goals of maximizing the resilience index and minimizing design costs, in a nondominated population-based triple-objective optimization modelling framework, is likely to result in WDN design solutions with increased mechanical reliability, relative to designs based solely on the resilience index and cost. Results show that by including uniformity as a design objective, solutions that may be similar in terms of the resilience index and cost may have significantly different levels of uniformity, and thus allow for additional design considerations, and have a potential for improved solutions. Cut-set analyses of these WDN designs indicate improved mechanical reliability, and cost, with only some degree of sacrifice in terms of hydraulic reliability.

6.2 Abstract

The reliability of water distribution network (WDN) designs for satisfying water demands under normal and abnormal operating conditions, including during pipe bursts and as systems age, is typically evaluated with surrogate measures of system resistance to potential hydraulic and mechanical stressors. The resilience index and its modifications have been applied to assess the reliability of a WDN in the face of hydraulic variances, including alterations in network friction characteristics, inputs and demands. A limitation of these indices is that they may only be used as an indirect measure of network reliability in the face of mechanical failures. A uniformity index is proposed to address this limitation. While the definition of the index is simple, the maximization of it is expected to lead to network designs with an increasingly uniform flow, at each node, where the burden of carrying inflows (or outflows) at each node is shared evenly among all input (or

output) pipes. Using a single-loop network example, this paper demonstrates that increasing this index leads to increased mechanical reliability. The incorporation of the uniformity index, along with the goals of maximizing the resilience index and minimizing design costs, in a multi-objective optimization modelling framework, is likely to result in WDN designs with increased mechanical reliability, relative to those based solely on resilience and cost. The efficacy of the multi-objective approach for improving overall reliability, in response to mechanical and hydraulic disturbances, is demonstrated for the WDN municipality of Farhadgerd, Iran.

6.3 Introduction

The reliability of networks for satisfying water demands under a wide range of operating conditions has recently been evaluated with surrogate measures of the system's ability to withstand potential hydraulic stressors. Hydraulic reliability indicates the ability of the system to absorb hydraulic disturbances, in response to degradation, e.g., reduction of available pipe diameters and pipe roughness, and variations in consumption, e.g., hourly fluctuations in withdrawal rates. Mechanical reliability indicates the ability of the system to adapt to mechanical failures, i.e., to maintain operating goals during pipe, valve, or pump failures. In this paper, a new surrogate for estimating mechanical reliability of WDNs, the uniformity index, I_u, is proposed and its applicability, when integrated in multi-objective WDN design optimization, is demonstrated.

As a means of addressing the risks of WDN designs, Todini (2000) introduces the resilience index, I_r, which describes the capability of a system to return to normal operations following hydraulic disturbances. The index is defined as the ratio of the total surplus power in the network and the surplus hydraulic power provided by the source reservoirs and pumps, after satisfying all demands. The former is the sum of the products of the consumption and excess pressure heads at all nodes, and the latter is the difference between the sum of the power provided by all reservoirs and pumps and the sum of the power demanded at all nodes. While it is not based on probabilistic failure considerations, increases in I_r have been shown to improve hydraulic reliability, and may indirectly provide for robustness under cases of mechanical failure. While other surrogates for reliability have been proposed, e.g., modifications of I_r (Prasad and Park 2004; Jayaram and Srinivasan 2008; Reca et al. 2008), measures based on pressure head deficiencies and system entropy (Farmani et al. 2005; Prasad and Tanyimboh 2008; Liu et al. 2014), and combinations of these (Raad 2011; Banos et al. 2011; Liu et al. 2014; Jeong et al. 2017), I_r is most commonly used.

Despite being widely applied in multi-objective WDN design (Prasad and Park 2003; Farmani et al. 2005; Reca et al. 2008; Jayaram and Srinivasan 2008; Raad et al. 2010; Di Nardo et al. 2010; Banos et al. 2011; Greco et al. 2012; Creaco et al. 2014; Atkinson et al. 2014; Creaco et al. 2016), the definition of I_r presents several challenges, particularly for providing a consistent measure of mechanical reliability. While it ranges from 0, i.e., completely unreliable, to 1, i.e., completely reliable, there is no pre-determined scale for the degree of reliability. Its value is a function of reservoir heads, pipe size diameters, and nodal demands, which are specific for each network configuration. Thus, it is useful in comparing alternative designs of the same network configuration, but may not be useful in comparing the relative reliability of different configurations.

Even when comparing alternative designs for a given network configuration, designs with similar I_r values may perform very differently under mechanical failures (Farmani et al. 2005; Banos et al. 2011; Raad 2011; Jeong et al. 2017). Given the definition of the total surplus power in the network, nodes with a high degree of surplus power (i.e., high excess pressure heads), such as those close to reservoirs, are not distinguished from those with a low degree of surplus power, such as those at the extremes of the network. Thus, network designs with very different pipe size and flow distributions, i.e., very different power gradients and net changes in pressure heads, and therefore very different vulnerabilities to mechanical failures, may have similar I_r values.

Prasad and Park (2004) introduce the network resilience index, a modification of I_r, in which the surplus power at a node is scaled by the degree of uniformity of the pipe sizes entering and exiting that node, where high degrees of pipe size uniformity are ideal. However, in cases where demands vary across the network, application of the network resilience index may lead to a non-uniform distribution of flows, and in some cases, over- or under-design of pipe size diameters. Jayaram and Srinivasan (2008) show that for WDNs with multiple reservoirs, the definition of surplus power in the denominator of I_r does not distinguish between reservoirs with high water levels and local reservoirs with lower water levels. Thus, they propose a modified resilience index for which the term representing the sum of the power provided by reservoirs is removed. Banos et al. (2011) compare I_r and its modifications in the face of demand uncertainty, show that no one index outperforms another, and call for the development of an index that considers the network configuration in its definition. Moreover, because all of the modified resilience indices employ the same definition of surplus power, none of them guarantee high network performance under pipe failure events.

The I_u index, proposed in this paper, addresses these limitations, and may be considered a surrogate for mechanical reliability, i.e., increasing I_u generally improves mechanical reliability of a system. For all pipes and nodes in the network, the scaled ratios of input (and output) pipe flows to the total input (and output) pipe flows are evaluated. The I_u index is the minimum of these ratios among all pipes at each node, and among all nodes in the network. For each node, the scaling factor for each input (or output) pipe is equivalent to the number of input (output) pipes for that node. The use of scaling factors reduces the dependence on network configuration, resulting in an indicator that describes the degree of uniformity at each node in the system, the minimum of which is the least amount of uniformity at any node in the system. The index can be used to compare the uniformity of different designs of a single network configuration, or of different configurations. The maximization of I_u leads to a network design with increasingly uniform flows throughout, where the burden of flow conveyance is shared more evenly among all input (or output) pipes, and where the surplus power is gradually decreased from the supply reservoir(s) to the extremes of the network. When considered in tandem with other measures of system performance, such as I_r, which indicates the total power in the network, and the total network cost, it may be used to enhance WDN designs to improve mechanical reliability.

6.4 The Uniformity Index

The proposed index, I_u, is defined as follows:

$$I_u = min \begin{cases} min\left\{\left(\dfrac{nkin \times Q_k}{\sum_{kin=1}^{nkin} Q_{kin}}\right)_j, \text{for all } k \in nkin\right\}, & (6.1a) \\ & \text{for all } j \in nn \\ min\left\{\left(\dfrac{nkout \times Q_k}{\sum_{kout=1}^{nkout} Q_{kout}}\right)_j, \text{for all } k \in nkout\right\}, & (6.1b) \end{cases}$$

where nn = number of nodes; Q_k = flow in pipe k (meters^3/second); $nkin$ = number of pipes with flows that enter node j; Q_{kin} = flow in pipe kin which is entering node j (meters^3/second); $nkout$ = number of pipes with flows that exit node j; and Q_{kout} = flow in pipe $kout$ which is exiting node j (meters^3/second).

The value of I_u ranges from 0, i.e., no uniformity, to 1, i.e., complete uniformity. The values for *nkin* and *nkout* vary with the network configuration, and for a given configuration, with the selected design, i.e., pipe diameters and resulting flow and flow directions. They may be viewed as scaling factors that allow for consistent comparisons among different configurations and designs. For example, consider a node connected to five pipes, two of which convey flow into the node, and three of which convey flow out of the node. Assume that the incoming flow is evenly distributed among the two entering pipes and that the outgoing flow is also evenly distributed among the three outgoing pipes. If the ratios in Eq. (6.1) were not scaled, Eq. (6.1a) would be equal to 1/2 and Eq. (6.1b) would be equal to 1/3, and the lowest of these values would incorrectly indicate a low degree of uniformity for this node. This would then be compared with corresponding values at other nodes. If instead, values of *nkin* = 2 and *nkout* = 3 were assigned for this node, Eq. (6.1a) would be equal to one, as would Eq. (6.1b), and these values would correctly indicate a high degree of uniformity for this node, thus providing an accurate comparator. Given the proposed scaling convention, the minimum value among all nodes, of Eqs. (6.1a) and (6.1b), represents the largest imbalance of flow at any one point in a network, and provides a comparator for evaluating different network designs and configurations. Because, by definition, I_u incorporates the topology of the network in the scaling factors, it may also be used to compare the mechanical reliability of WDNs at different locations, i.e., for different municipalities.

Maximizing I_u leads to improved uniformity of flows at as many locations in the network as possible, and thus provides for a gradual decrease in power from the source reservoir(s) to the extremes of the network. This feature may be expected to result in a higher potential to recover from pipe failure events, thus improving mechanical reliability. Consider two network designs for distributing **q** units of flow in a simple gravity-fed single-loop network, shown in Figure 6.1. The reservoir elevation and pressure head at the outlet are equal for both designs, and are referred to as H_0 and H_1, respectively. The design diameters and resulting head losses for the most extreme pipe failure events are provided in Table 6.1. For the network with uniform flows, WDN(a), shown in Figure 6.1a, the flows, and thus the diameters, D_1 each, as determined in Table 6.1, are uniformly distributed. For the network with non-uniform flows WDN(b), shown in Figure 6.1b, the flows are apportioned at a ratio of 9 to 1, resulting in diameters of D_2 and D_3, for the north and south branches of the network, respectively. As may be expected, due to the flow allocation, $D_3 < D_1 < D_2$. Should any one branch of WDN(a) experience pipe failure, in the worst case, the flow in the remaining branch would double, accompanied by an increased head loss of 4 (H_0 - H_1). In contrast, should the north branch of WDN(b) experience pipe failure, the flow in the remaining branch would increase to 10 times its design value, accompanied by an increased head loss of 100 (H_0 - H_1), more than 20 times that of WDN(a). Therefore, in the worst case, WDN(a) will provide a power at the network outflow that is greater than or equal to that for WDN(b), and will thus have a greater capacity to maintain sufficient power during extreme events. Of course, for the less severe case in which the south branch of both networks fail, WDN(b) experiences an increase head loss of 1.2346 (H_0 - H_1), on the order of three times smaller than the similar failure event in WDN(a), but this disadvantage is greatly offset by the advantage that the uniform flow design maintains under severe failure events.

The I_u index is directly dependent on the nodal demands and topology of the network (i.e., the continuity equations). It is an indicator of flow uniformity, and thus reliability in the face of flow disturbances, but not an indicator of the system's ability to absorb a change in energy, for example, in response to an increase in demand at a given node. The I_r index, on the other hand, indicates the total power of the system but does not necessarily satisfy the uniformity of flows, because it is not based on the continuity equations. For different values of the resilience index, there may be many solutions with different levels of flow uniformity, and for different values of the uniformity index, there may be many solutions with different levels of total system power. Therefore, it is recommended that these indicators be applied in tandem, when evaluating alternative network designs.

6.5 Multi-objective Optimization Model

The multi-objective optimization model may be used to consider cost and reliability, in response to both hydraulic and mechanical disturbances. It may be applied to find the set of Pareto-optimal design solutions, i.e., sets of design pipe diameters, that optimize two or more of the following objectives.

6.5.1 Minimize the total system cost

$$Min\ Cost = \sum_{s=1}^{np} Cost_s(D_s) \times L_s \tag{6.2}$$

where L_s = length of pipe *s* (meters); $Cost_s(D_s)$ = cost of a pipe of a given diameter ($/meter); and D_s = diameter of pipe *s* (meters); and np = number of pipes in the network,

6.5.2 Maximize resilience, I_r

$$I_r = \frac{\sum_{j=1}^{nn} q_j\left(H_j - H_j^{min}\right)}{\sum_{i=1}^{no} Q_i H_{oi} - \sum_{j=1}^{nn} q_j H_j^{min}} \tag{6.3}$$

where *no* = number of reservoirs; q_j = flow demand at node *j* (meters^3/second); H_j = available pressure head at node *j* (meters); H_j^{min} = minimum pressure head required at all nodes (meters); Q_i = discharge from a reservoir located at node *i* (meters^3/second); and H_{oi} = elevation head of a reservoir located at node *i* (meters). The numerator in this equation is considered to be the total surplus power in the network, and the denominator is considered to be the surplus hydraulic power provided by the source reservoirs and pumps, after satisfying all demands. In this paper, the term representing the demands is slightly different from that reported in Todini (2000). Todini includes the elevation of the node in the value of H_j^{min} , while in this paper, H_j^{min} is only the pressure target.

6.5.3 Maximize uniformity, I_u , or Eq. (6.1)

These objective functions may be optimized under the following constraints:

Flow continuity at nodes:

For each node, flow continuity must be satisfied,

$$\sum Q_{in} - \sum Q_{out} = q_j, \qquad \forall j \epsilon nn \tag{6.4}$$

Energy equation in pipes:

For each pipe, energy conservation must be satisfied,

$$hf_s = H_i - H_j = R_s Q_s^n \tag{6.5}$$

where hf_s = head loss due to friction in the pipe *s* (meters) computed by the Hazen-Williams or Darcy-Weisbach Equations; H_i and H_j = the nodal heads at the start node and the end node of pipe *s* (meters); R_s = the characteristic pipe resistance coefficient which depends on the definition of pipe roughness selected, e.g., for head loss based on the Hazen–Williams Equation, it is a function of the friction factor (C), length (L), and diameter (D) and n is the exponent of the head loss equation and is specified for a given pipe type.

Pressure requirements at node:

For each junction node in the network, the pressure head should be greater than the prescribed minimum pressure head:

$$H_j \geq H_j^{min}, \forall\, j \in nn \tag{6.6}$$

where H_j^{min}= the minimum required pressure head (meters) at node j.

Pipe size availability:

The diameter of the pipes should be available from a commercial size set:

$$D_k = \{D(1), D(2), \ldots., D(ns)\}, \quad \forall\, k \in np \tag{6.7}$$

where ns = the number of candidate diameters.

6.6 Multi-objective Optimization Algorithm

The Pareto-optimal frontiers are determined with the Ranking Based Nondominated Sorting Differential Evolution (NSDE-RMO) Algorithm (Chen et al., 2014). The NSDE-RMO uses the accelerated mutation and crossover process of the Nondominated Sorting Differential Evolution (NSDE) Algorithm (Angira and Babu, 2005), ranks population members according to their objective performance and crowding distance, and uses these ranks to modify the mutation operator, as well as to sort in the selection process. Chen et al. (2014) provide details of the NSDE-RMO Algorithm and Moosavian and Lence (2016) apply it successfully for the multi-objective optimal design of WDNs.

For each combinatorial set of pipe diameters for the network, i.e., each potential solution, hydraulic simulation is applied to evaluate Eqs. (6.4) and (6.5) to obtain pressure heads, H_j, for all nodes and pipe flows, Q_k, and directions, in all pipes of the network. The resultant flows are then used to estimate I_u in Eq. (6.1), and the pressure heads to estimate I_r in Eq. (6.3). Since Eq. (6.6) must be satisfied, and, like all evolutionary algorithms, NSDE-RMO does not accommodate such constraints, violations of Eq. (6.6) are tracked, assigned a penalty, and added to the cost function, Eq. (6.2). The penalty is defined as:

$$CP_j = \lambda \left| min\left(\frac{H_j}{H_j^{min}} - 1, 0\right) \right| \tag{6.8}$$

where CP_j = penalty associated with node j; and λ = penalty multiplier. To determine solutions that satisfy Eq. (6.6), the magnitude of λ should be sufficiently large. For the example network examined herein, the penalty multiplier is based on the maximum possible cost for the given network, i.e., the sum of the costs associated with the maximum available diameter for all pipes.

6.7 Multi-objective Optimal WDN Designs

Application of the multi-objective model is demonstrated for the Farhadgerd WDN. The NSDE-RMO-based Pareto-optimal frontiers are generated for two model variations: one optimizing cost, I_r and I_u, as described in Eq. (1-8), and one optimizing cost and I_r as described in Eq. (2-8). These are referred to as the minimize-cost-maximize-I_r-maximize-I_u, and the minimize-cost-maximize-I_r model, respectively. The NSDE-RMO Algorithm is implemented using a population size of 1,000 and is terminated after 10,000 function evaluations. For this network, the algorithm converges to a consistent Pareto-optimal frontier and no improvement is observed beyond 10,000 function evaluations. The mutation and crossover operators selected for the NSDE-RMO optimization are 0.5 and 0.3, respectively, as suggested by Chen et al. (2014), and to be consistent with the approach and results of Moosavian and Lence (2016), against which the multi-objective models are compared.

The mechanical reliability of the NSDE-RMO-generated Pareto-optimal WDN designs is investigated using cut-set analyses. The cut-set analyses for meeting the 20-meter and 0-meter pressure targets are conducted. Under a cut-set analysis, for each Pareto-optimal WDN design, potential pipe bursts are simulated by removing a given pipe, conducting an hydraulic analysis for the remaining WDN, recording the minimum pressures and other information for the system, and returning the pipe to its in service position. This process is repeated for each pipe in the network, and the set of Pareto-optimal designs that meet the cut-set pressure target, for a given number of pipe removals (i.e., cut-set level), are summarized for comparison. That is, for a cut-set of one pipe, the Pareto-optimal designs that meet the pressure target when any one pipe is removed are grouped and summarized. For a cut-set of two pipes, the Pareto-optimal designs that meet the pressure target when any two pipes are removed, are grouped and summarized, and so on. The summary includes information that characterizes the cost, resilience, uniformity, number of designs, and hydraulic conditions, of the cut-set. Hydraulic reliability in the face of demand uncertainty is the focus of several studies (Prasad and Park 2003; Farmani et al. 2005; Reca et al. 2008; Jayaram and Srinivasan 2008; Raad et al. 2010; Creaco et al. 2016). Thus, in this paper, hydraulic reliability is examined cursorily, by determining the minimum pressure that results under a 10% increase in nodal demands throughout the network, for example low-cost-high-I_u designs.

6.7.1 Farhadgerd Network

The Farhadgerd WDN serves a population of approximately 8200 in the town of Farhadgerd, Iran (Lence et al. 2017). The network comprises 68 pipes, some in parallel, and 53 nodes, has one reservoir with a head of 510 m, and a minimum pressure requirement of 20 m at all nodes. There are nine possible pipe diameters. A schematic view of the network is provided in Figure B1 and all relevant data for this network are provided in Tables B1-B3.

The three-dimensional Pareto-optimal frontier under the minimize-cost-maximize-I_r-maximize-I_u model, shown in Figure 6.2, is a relatively smooth curved surface which is distributed broadly from lower to higher uniformity levels. The range of cost is between 17.8 and 72.1 million dollars US, and the WDN designs for this range have corresponding I_r and I_u values of 0.018 and 0.045, and 0.066 and 0.0250, respectively. The range of I_r values is between 0.018 and 0.066, and thus the aforementioned WDN designs represent the extreme points for this indicator. Here, improvement in I_r requires significant increases in cost accompanied by a significant reduction in I_u. The range of I_u values is between 0.001 and 0.330, and the WDN designs for this range have corresponding cost and I_r values of 46.6 million dollars US and 0.066, and 36 million dollars US and 0.061, respectively. Thus, it is possible to achieve significant improvement in I_u and reduce costs, and sacrifice little in terms of I_r, though at both extremes I_r is low. These data demonstrate that there is significant trade-off between I_r and cost, where the correlation coefficient for I_r and cost is 0.708. On the other hand, there is only mild correlation between I_u and cost (with a correlation coefficient is -0.231), and nearly no correlation between I_u and I_r (with a correlation coefficient is -0.099). These relationships indicate that the inclusion of uniformity as a design objective allows for the exploration of other aspects of the decision-space.

Figure 6.3a is a two-dimensional conflation of this surface, in terms of cost and I_r. As shown in the figure, there are only a few Pareto-optimal designs that do not lie near the edge of the cost-I_r trade-off. Thus, at every cost-I_r level, a range of solutions may exist, each having different I_u

values. Figure 6.3b represents the two-dimensional Pareto-optimal frontier for the minimize-cost-maximize-I_r model. Because the network and design options are limited, the WDN designs representing the extremes of cost are the same as under the minimize-cost-maximize-I_r-maximize-I_u model, though the range of I_r and I_u values, and their corresponding costs differ. The resulting WDN designs in Figure 6.3b are identical to the cost-I_r values for the optimal designs obtained for a similar model using NSDE-RMO, reported in Lence et al. (2017). Here, owing to the network complexity, the range of design costs is still between 17.8 and 72.1 million dollars US, but the corresponding I_r and I_u values at these extremes are 0.018 and 9.8E-06, and 0.066 and 0.096, respectively, each with much lower uniformity levels. A significant benefit of explicitly accounting for uniformity in a multi-objective modelling framework is that highly non-uniform designs may be avoided. Figure 6.3c is a superposition of Figure 6.3a on Figure 6.3b, and shows the slight sacrifice required, in terms of maximizing I_r, in order to also consider uniformity.

To explore the degree of mechanical reliability achieved by incorporating uniformity of flow in design decisions, cut-set analyses for meeting the 20-meter and 0-meter pressure targets are conducted on the Pareto-optimal design solutions resulting from both multi-objective models. Tables 6.2 and 6.3 summarize the cut-set results for the minimize-cost-maximize-I_r model, and Tables 6.4 and 6.5 summarize these data for the minimize-cost-maximize-I_r-maximize-I_u model. The first column of these tables is the cut-set accommodated; these range from varies from 0-19 potential pipes to 60-64 potential pipes removed, although the analyses for the 0-meter pressure target did not result in cut-sets less than 30-39 pipes. Columns 2 to 8 of these tables provide the minimum, maximum and mean design cost, mean I_r, mean I_u, number of solutions included, and mean fraction of the nodes where nodal pressure targets are violated under these designs, for all cut-sets greater than the given cut-set, respectively.

Pipe P-1, the source pipe for the network, and pipes P-46, P-48, P-49 and P-51, which are the sole feeder pipes for five extreme nodes, nodes J-1, J-40, J-41, J-42, and J-44, can never be removed. For both models, the minimum, maximum, and mean cost of the solutions in each cut-set range increase, or in two cases remain the same, as the required minimum pressure increases. The number of solutions in the cut-sets respond to the pressure targets less systematically, though the number of solutions in the largest cut-set range, 60-64 pipes removed, decreases as expected, as the pressure targets are increased. Based on the mean fraction of nodal pressure targets violated, the hydraulic vulnerability of the poor, or small cut-set, designs increases as the pressure targets are increased, though the highest performing solutions under the minimize-cost-maximize-I_r model all ultimately result in approximately 1.3 of the 53 nodes on average (i.e., a fraction of 0.024) being vulnerable, though the vulnerable node(s) vary with the pipe(s) removed. Under the minimize-cost-maximize-I_r-maximize-I_u model, all pressure targets ultimately result in approximately 0.84 of the 53 nodes on average (i.e., a fraction of 0.016) being vulnerable, dependent on the pipe(s) removed.

The introduction of an objective that improves uniformity, results in solutions that have increased mean I_u values in all cut-sets, reduced minimum and mean costs, and only marginally decreased, and in some cases, unchanged, mean I_u values, and decreased mean fractions of nodal pressure targets violated for all larger cut-set levels. Table 6.6 provides a comparison of the three objectives, for the least cost solutions of the 60-64-pipe cut-set range, determined for both optimization models under both nodal pressure targets. The actual cut-set achieved is also provided in parentheses. In all cases, the least cost decreases and I_r decreases slightly or remains the same, with decreasing pressure target values, while, in all cases, the least cost decreases, I_r decreases and I_u increases, with the addition of the minimize I_u objective, though the decrease in I_r is only marginal. Figure 6.4 is a schematic representation of these WDN designs, and shows the general decrease in the pipe sizes of the WDN designs under both the 0-meter and 20-meter pressure targets, with the addition of the minimize I_u objective. The pipe sizes of the WDN designs under the minimize-cost-maximize-I_r model tend to increase, as pressure target values are increased, while the pipe sizes of the WDN designs under the minimize-cost-maximize-I_r-maximize-I_u model increase and decrease strategically in order to accommodate increasing pressure targets, while keeping design costs low.

For these four WDN designs, the minimum pressure that results when the nodal pressures throughout the network are increased 10% uniformly, are shown in Table 6.7. The minimum pressure throughout the system, occurring consistently at node J-41, under both of the optimization models, changes very little, with the level of the pressure target. Under both pressure targets, the addition of the minimize I_u objective, results in a very slight decrease in minimum pressure, though never below the pressure target. Thus, a significantly lower cost, and increased mechanical reliability is accompanied by only a small sacrifice in the degree of hydraulic robustness, and reliability, achieved.

6.8 Conclusion

This paper introduces a new surrogate, the I_u index, which describes the weighted allocation of flows among incoming pipes, and among outgoing pipes, in a network. Maximizing the minimum of I_u identifies the critical node in the network, which has the least degree of uniformity of flow. In contrast with the widely applied I_r index (Todini, 2000), which is based on the average power of all network nodes and is a measure of hydraulice reliability, the I_u index is based on the worst overall uninformity, and may be viewed as a measure of mechanical reliability because it indicates the system's ability to absorb disturbances in network links. Both indices range from zero to one, but the scale of the I_u index reflects the actual level of uniformity at the critical node, takes into consideration network topology, and may be used to compare networks with different configurations. The scale of the I_r index, on the other hand, is different for different networks. The I_u index improves on the network resilience index, a variation of I_r, that induces uniform pipe sizes for pipes entering and exiting nodes, but does not account for withdrawals at the nodes (Prasad and Park, 2004), and thus leads to an inefficient selection of pipe size diameters. Because the definition of I_u decouples incoming and outgoing pipes, and allocates incoming flows before withdrawal, and outgoing flows after withdawel, the WDN design tends toward efficient selection of design flows and pipe diameters.

Moreover, the use of I_u , in conjunction with I_r and design costs, as a goal in multi-objective optimal design of WDNs, may enhance the applicability of both indices, as demonstrated in this paper for the Farhadgerd network. For this network, under all WDN designs that consider these three objectives, there is a distinct trade-off between I_r and cost, only mild correlation between I_u and cost, and nearly no correlation between I_u and I_r. These results show that by including uniformity as a design objective, solutions that may be similar in terms of I_r and cost my have significantly different levels of uniformity, and thus allow for additional design considerations, and have a potential for improved solutions. Cut-set analyses of these WDN designs indicate improved mechanical reliability, and cost, with only some degree of sacrifice in terms of hydraulic reliability.

While in practice, the addition of uniformity considerations may increase the complexity of the multi-objective optimization process, the computational effort was kept constant (i.e., 10,000 function evaluations) under the minimize-cost-maximize-I_r-maximize-I_u and minimize-cost-maximize-I_r models applied herein, to show the benefit of the former model under identical computational conditions. Future exploration of the increased computational burden exacted when the former model is applied to increasingly more complex networks, should be undertaken. The objective of maximizing the minimum I_u over all nodes in the network does not explicitly require high I_u values at all nodes, and thus objectives that attempt to improve on this approach, such as those that maximize the average I_u , or some other goal that describes total system uniformity, may need to be developed and investigated. While only one resilience indicator was incorporated in the multi-objective framework presented here, a framework that includes other indicators, such as the network resilience index, or other approaches for risk-based network design, such as approaches that use fuzzy programming (Lence et al. 2017) and FORM-based reliability analyses (Tolson et al. 2004), may also benefit from the inclusion of uniformity considerations. The advantages of doing so, as well as any disadvantages, e.g., increased computational burden, for these and other networks, should be identified. Finally, future work should be undertaken to identify the merits of using the I_u index in optimal operations planning and in pipe, pump, and valve replacement strategies. Here, the index may provide additional information about replacement choices that improve mechanical reliability.

Table 6.1 Design diameters and pipe failure head losses for single loop network

Design Diameter For Uniform Flow Distribution, WDN(a)	Design Diameter For Non-uniform Flow Distribution, WDN(b)
$H_0 - H_1 = \Delta H = \frac{8fL(0.5q)^2}{g\pi^2 D_1^5}$ $D_1 = \sqrt[5]{\frac{8fL(0.5q)^2}{g\pi^2 \Delta H}}$ $D_1 = 0.7579\left(\sqrt[5]{\frac{8fLq^2}{g\pi^2 \Delta H}}\right)$	$H_0 - H_1 = \Delta H = \frac{8fL(0.9q)^2}{g\pi^2 D_2^5}$ $D_2 = \sqrt[5]{\frac{8fL(0.9q)^2}{g\pi^2 \Delta H}}$ $D_2 = 0.9587\left(\sqrt[5]{\frac{8fLq^2}{g\pi^2 \Delta H}}\right)$ $H_0 - H_1 = \Delta H = \frac{8fL(0.1q)^2}{g\pi^2 D_3^5}$ $D_3 = \sqrt[5]{\frac{8fL(0.1q)^2}{g\pi^2 \Delta H}}$ $D_3 = 0.3981\left(\sqrt[5]{\frac{8fLq^2}{g\pi^2 \Delta H}}\right)$

Pipe Failure Head Loss

Failure of Either Branch	Failure of North Branch (most extreme pipe failure)
$H_0 - H_{1new} = \frac{8fLq^2}{g\pi^2 D_1^5}$ $H_0 - H_{1new} = \frac{8fLq^2}{g\pi^2\left(\frac{8fL(0.5q)^2}{g\pi^2 \Delta H}\right)} = \frac{\Delta H}{0.25}$ $H_{1new} = H_0 - 4\Delta H = H_0 - 4(H_0 - H_1)$	$H_0 - H_{1new} = \frac{8fLq^2}{g\pi^2 D_3^5}$ $H_0 - H_{1new} = \frac{8fLq^2}{g\pi^2\left(\frac{8fL(0.1q)^2}{g\pi^2 \Delta H}\right)} = \frac{\Delta H}{0.01}$ $H_{1new} = H_0 - 100\Delta H = H_0 - 100(H_0 - H_1)$ Failure of South Branch $H_0 - H_{1new} = \frac{8fLq^2}{g\pi^2 D_2^5}$ $H_0 - H_{1new} = \frac{8fLq^2}{g\pi^2\left(\frac{8fL(0.9q)^2}{g\pi^2 \Delta H}\right)} = \frac{\Delta H}{0.81}$ $H_{1new} = H_0 - 1.235\Delta H = H_0 - 1.235(H_0 - H_1)$

Table 6.2 Cut-set analysis for Farhadgerd network with zero-meter pressure target, minimize-cost-maximize-I_r model

Cut-Set	Min Cost, $	Max Cost,$	Mean Cost, $	Mean I_r	Mean I_u	Number of Solutions	Mean Fraction of Nodes Violated
30-39	1.78E+07	1.80E+07	1.79E+07	0.021	0.078	3	0.116
40-49	1.82E+07	3.43E+07	2.42E+07	0.060	0.030	554	0.081
50-59	2.93E+07	5.16E+07	4.02E+07	0.066	0.010	443	0.046
60-64	4.49E+07	5.16E+07	4.80E+07	0.066	0.008	89	0.024

Table 6.3 Cut-set analysis for Farhadgerd network with 20-meter pressure target, minimize-cost-maximize- I_r model

Cut-Set	Min Cost, $	Max Cost,$	Mean Cost, $	Mean I_r	Mean I_u	Number of Solutions	Mean Fraction of Nodes Violated
0-19	1.78E+07	2.02E+07	1.86E+07	0.034	0.067	5	0.165
20-29	1.82E+07	2.05E+07	1.92E+07	0.045	0.036	21	0.132
30-39	1.87E+07	2.55E+07	2.13E+07	0.057	0.036	264	0.109
40-49	2.30E+07	4.64E+07	3.28E+07	0.065	0.016	564	0.072
50-59	3.51E+07	5.16E+07	4.55E+07	0.066	0.008	136	0.037
60-64	5.00E+07	7.21E+07	5.64E+07	0.066	0.010	10	0.024

Table 6.4 Cut-set analysis for Farhadgerd network with zero-meter pressure target, minimize-cost-maximize- I_r -maximize- I_u model

Cut-Set	Min Cost, $	Max Cost,$	Mean Cost, $	Mean I_r	Mean I_u	Number of Solutions	Mean Fraction of Nodes Violated
30-39	1.78E+07	2.48E+07	1.93E+07	0.028	0.190	97	0.102
40-49	1.80E+07	3.07E+07	2.09E+07	0.045	0.152	319	0.079
50-59	2.17E+07	4.74E+07	2.87E+07	0.061	0.181	168	0.038
60-64	2.97E+07	7.21E+07	5.09E+07	0.065	0.145	416	0.016

Table 6.5 Cut-set analysis for Farhadgerd network with 20-meter pressure target, minimize-cost-maximize- I_r -maximize- I_u model

Cut-Set	Min Cost, $	Max Cost,$	Mean Cost, $	Mean I_r	Mean I_u	Number of Solutions	Mean Fraction of Nodes Violated
0-19	1.78E+07	2.27E+07	1.95E+07	0.034	0.159	111	0.159
20-29	1.78E+07	2.67E+07	2.01E+07	0.040	0.163	149	0.122
30-39	1.81E+07	3.61E+07	2.19E+07	0.048	0.161	162	0.098
40-49	2.18E+07	4.52E+07	2.75E+07	0.060	0.189	115	0.060
50-59	2.32E+07	5.71E+07	3.61E+07	0.064	0.158	148	0.029
60-64	3.00E+07	7.21E+07	5.49E+07	0.065	0.140	315	0.016

Table 6.6 Least cost designs for cut-set analysis accommodating 60-64 pipe removals for Farhadgerd network

	Optimization Models			
	Min Cost Max I_r 0 m Target	Min Cost Max I_r, I_u 0 m Target	Min Cost Max I_r 20 m Target	Min Cost Max I_r, I_u 20 m Target
Cut-set	60-64	60-64	60-64	60-64
Cost	4.49E+07	2.97E+07	5.00E+07	3.00E+07
I_r	0.066	0.064	0.066	0.064
I_u	0.004	0.109	0.009	0.123
Fraction of Nodes Violated	0.024	0.021	0.024	0.019

Table 6.7 Minimum pressure and location for a 10% increase in nodal demands for Farhadgerd network, least cost designs

	Optimization Models			
	Min Cost Max I_r 0 m Target	Min Cost Max I_r, I_u 0 m Target	Min Cost Max I_r 20 m Target	Min Cost Max I_r, I_u 20 m Target
Cost	4.49E+07	2.97E+07	5.00E+07	3.00E+07
Min Pressure	22.6	22.1	22.6	22.1
Node Number	J-41*	J-41	J-41	J-41

* J-41 is located at downstream of pipe P-48 (a branch pipe close to the reservoir)

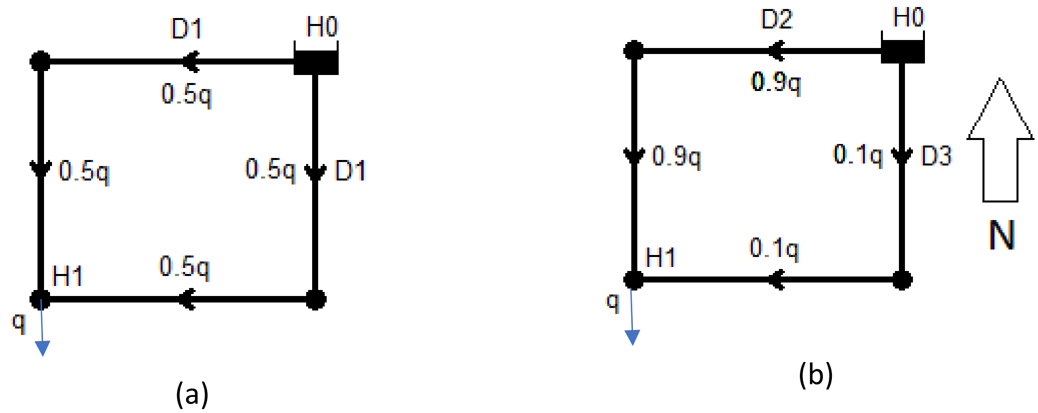


Figure 6.1 Uniform flow single loop network, WDN(a), (b) Non-uniform flow single loop network, WDN(b)

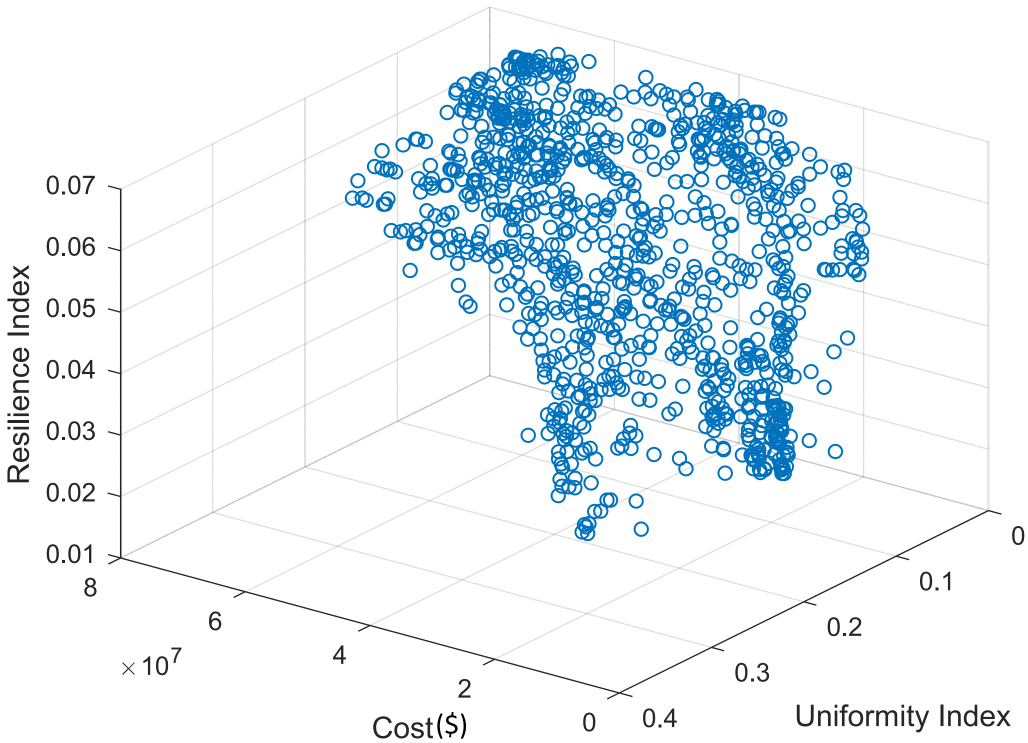
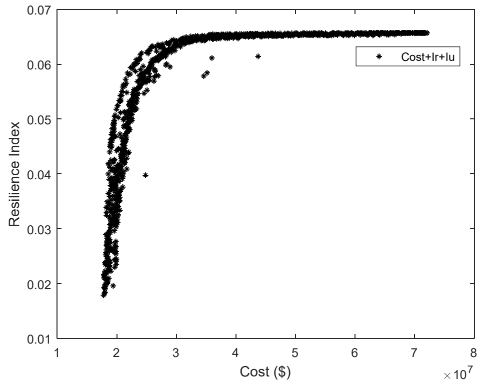
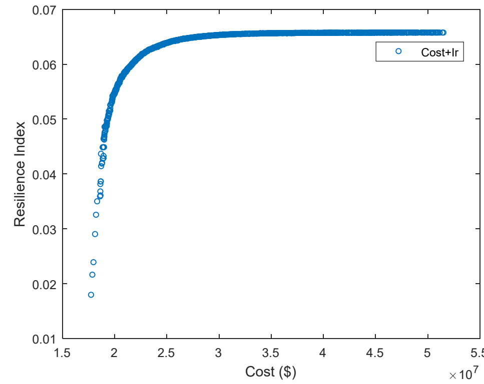


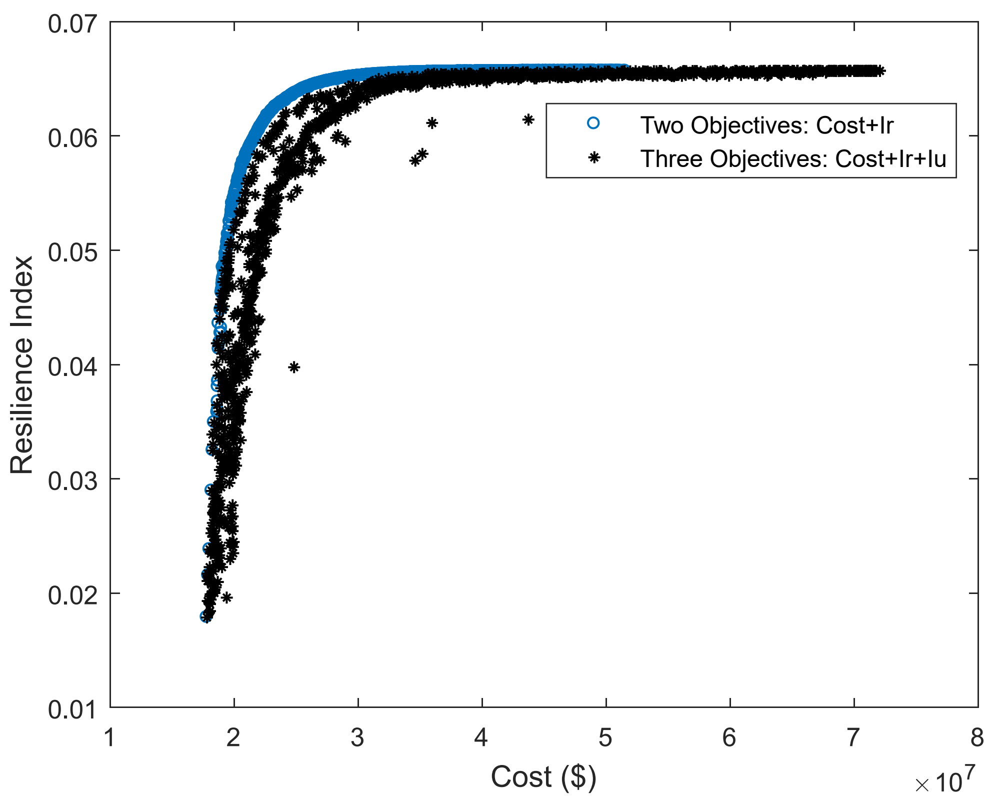
Figure 6.2 Pareto-optimal frontier for Farhadgerd network



(a) minimize-cost-maximize-I_r-maximize-I_u model

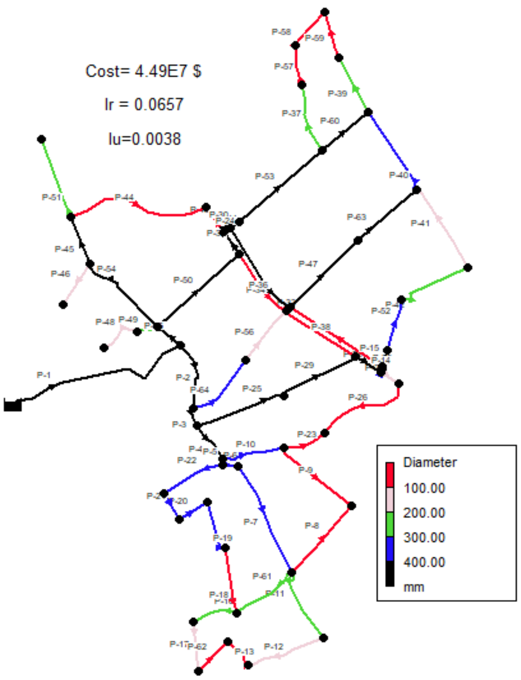


(b) minimize-cost-maximize-I_r

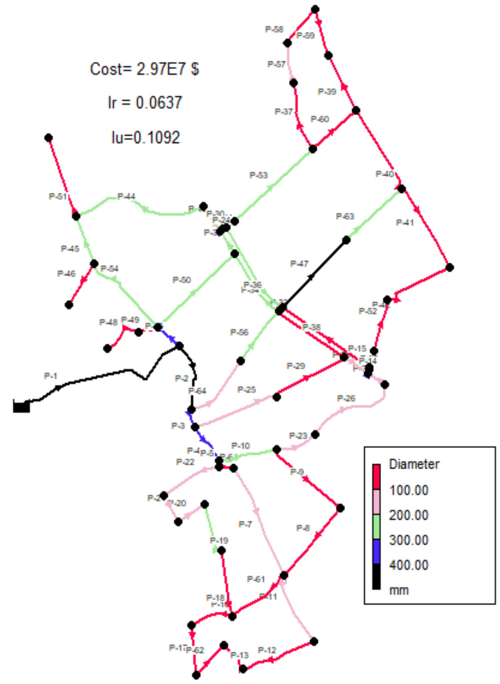


(c) both models

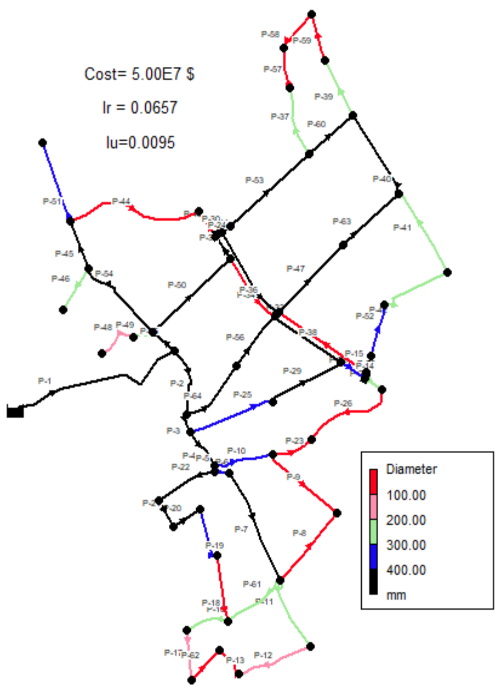
Figure 6.3 Pareto-optimal frontier for Farhadgerd network considering cost and I_r, based on (a) minimize-cost-maximize- I_r -maximize- I_u model, (b) minimize-cost-maximize- I_r , (c) both models



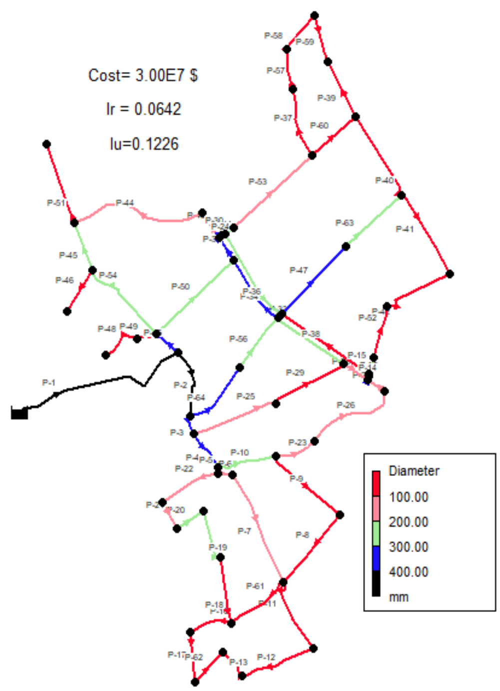
(a) pressure target zero-meter, minimize-cost-maximize-I_r



(b) pressure target zero-meter, minimize-cost-maximize-I_r-maximize-I_u



(c) pressure target 20-meter, minimize-cost-maximize-I_r



(d) pressure target 20-meter, minimize-cost-maximize-I_r-maximize-I_u

Figure 6.4 Diameter distribution for Farhadgerd network, least cost solutions for the pipe cut-set with (a) Zero-meter pressure target, minimize-*cost*-maximize-I_r model, (b) Zero-meter pressure target, minimize-*cost*-maximize-I_r-maximize-I_u model, (c) 20-meter pressure target, minimize-*cost*-maximize-I_r model, (d) 20-meter pressure target, minimize-*cost*-maximize-I_r-maximize-I_u model

Chapter 7 : Approximation of Fuzzy Membership Functions in Water Distribution Network Analysis

7.1 Preface

As shown in Figure 1.1, WDN analysis can be deterministic or non-deterministic. In deterministic analysis, input parameters such as nodal demands, pipe characteristics and reservoir water levels are assumed to be constant and certain and unknown variables such as pipe flows and nodal pressures are evaluated. In response to those non-deterministic analysis, the aforementioned input parameters do not have a certain value. In this case, Probability Theory identifies PDFs for quantitative parameters and Fuzzy Set Theory identifies FM functions for linguistic parameters. The unknown variables also may be reported as PDFs or FMs. There are several methods for non-deterministic analysis such as MCS for incorporating both Probability and Fuzzy Set Theory, FORM for incorporating Probability Theory, and Fuzzy Analysis techniques for incorporating Fuzzy set theory. The main advantage of the application of Fuzzy Set Theory is the simple construction of the FMs for uncertain linguistic variables. This technique can identify the extreme values of unknown variables when uncertain input information ranges between pre-specified extremes, and when the PDFs of the information cannot be obtained. There are two main approaches for conducting the Fuzzy Analysis in WDN design: 1) $\alpha -$level optimization and 2) gradient-based methods. The $\alpha -$level optimization technique is a very time-consuming process which needs many hydraulic analyses for constructing MFs. The gradient-based method improves the efficiency of the Fuzzy Analysis for middle-sized networks. However, this method is also not readily applicable for large-sized networks. Approximations of the gradients of the equations that govern WDN analysis, with respect to nodal demands and pipe resistance, are identified herein and harnessed to accelerate Fuzzy Analysis of system hydraulics. The resulting WDN nodal pressures are inversely proportional to nodal demands, and depending on flow directions, proportional to pipe resistance.

7.2 Abstract

Design and analysis of water distribution networks (WDNs) is laden with uncertainty, both, aleatory, i.e., natural randomness, such as variations in reservoir elevation heads, and epistemic, i.e., incomplete knowledge, imprecise data, and linguistic ambiguity such as that associated with characterization of pipe resistance, nodal demands, and hydraulic responses. To accommodate

aleatory uncertainty, stochastic analysis is applied to represent the input uncertainties and to estimate resulting uncertainty in nodal pressures and pipe flows. In the analysis of WDNs facing epistemic uncertainty, in particular, fuzzy set theory has widely been suggested as an alternative to stochastic analysis. This technique can identify the extreme values of unknown variables when uncertain input information ranges between pre-specified extremes, and when the probability distribution of the information cannot be obtained. Current approaches for conducting fuzzy analysis of WDNs to support operations and design are computationally demanding, and thus limited in their applicability to large networks. Approximations of the gradients of equations that govern WDN analysis, with respect to nodal demands and pipe resistance, are identified herein and harnessed to accelerate fuzzy analysis of system hydraulics. The resulting WDN nodal pressures are inversely proportional to nodal demands, and depending on flow directions, proportional to pipe resistance. Results of fuzzy analyses for two realistically-sized WDNs show that the proposed method performs with an acceptable level of accuracy and greatly reduces computational time, relative to existing fuzzy analysis approaches.

7.3 Introduction

In practice, many model parameters for hydraulic analysis of water distribution networks (WDNs), such as nodal demands and pipe roughness coefficients, are uncertain and therefore estimation of crisp values for pressures and flows may not be realistic. If a parameter, such as water demand, may be measured under differing system conditions, a probability density function of the parameter may be estimated, and the resulting distribution of the simulation model output may be obtained. When parameters cannot be measured directly, as is the case of pipe roughness coefficients, such stochastic methods are not applicable. Revelli and Ridolfi (2002) introduce an analysis of pipe networks that considers demands and pipe roughness coefficients as fuzzy parameters and evaluates fuzzy pipe flows and nodal pressures resulting from these parameters. A fuzzy parameter is represented by a fuzzy set, which defines the range of values for the parameter, i.e., its support, its most-likely value, i.e., its kernel, and the degree of membership, over the closed interval [0,1], at which the parameter is believed to range between any set of values. Rather than apply hydraulic simulation directly, for each node, and a given degree of membership, referred to as α, Revelli and Ridolfi (2002) employ an $\alpha-$level optimization approach and sequential quadratic programming (SQP) to approximate extreme values of the flow, and estimate extreme values of the pressure based on these flows. For each node, the α −level optimization is repeated for a range of α levels from 0 to 1. Haghighi and Keramat (2012) employ this approach with simulated annealing for estimating flows and pressures over time in transient hydraulic analysis. Sivakumar et al. 2016 analyze four water distribution networks and estimate the membership function of pipe discharges and hydraulic heads by assuming pipe roughness as uncertain. All approaches employ triangular membership functions and are applied to small networks.

Xu (2002) compares the fuzzy hydraulic model of Xu and Goulter (1999), which linearizes the headloss equations, with the fuzzy hydraulic analysis of Revelli and Ridolfi (2002), and demonstrates that, while the latter provides reliable estimates of uncertainty, its practical application to larger networks is limited by its excessive computational requirements. Xu shows that for a network with *nn* nodes and *np* pipes and for which *nf* degrees of membership are considered, the approach of Revelli and Ridolfi (2002) would require solution of $2 \times (nf - 1) \times (nn) + 1$ nonlinear optimization models to characterize the uncertainty associated with nodal pressure and pipe flow. He also asserts that for large networks, which may involve thousands of nodes and pipes, the method of Revelli and Ridolfi (2002) may be less computationally efficient than Monte Carlo Simulation. Furthermore, Gould et al. (2005) show that the SQP algorithm used by Revelli and Ridolfi (2002) may only be efficient for small- and certain classes of large-scale convex nonlinear problems.

In order to reduce the computational burden of fuzzy hydraulic analysis, Gupta and Bhave (2007) propose a direct approach for estimating the extreme values of nodal pressures and pipe flows which does not require iterative α −level optimization. Here, a hydraulic analysis is conducted based on the most likely values of the pipe friction coefficients and nodal demands, and the gradients of the hydraulic response at each node, to small changes in this input information, are estimated with trial analyses. Assuming that the gradients of the hydraulic responses are constant over the entire variable space, the extreme responses are then obtained by substituting the α −level upper or lower bound of the input information in the hydraulic simulator. This approach requires $(nf - 1) \times (3 \times nn + np) + 1$ hydraulic analyses, and is computationally demanding for large networks. Spiliotis and Tsakiris (2012) apply the Newton-Raphson Method to estimate the derivative of the coupled energy plus continuity equation, with respect to pressure and estimate the fuzzy nodal pressures resulting from fuzzy demands and deterministic pipe roughness coefficients. This approach requires solution of $(nf-1)\times(2\times nn)+1$ nonlinear systems of coupled energy plus continuity equations. Sabzkouhi and Haghighi (2016) apply an evolutionary algorithm (particle swarm optimization) to solve an α −level optimization model for all nodal pressures and pipe flows, simultaneously. In this case, the fuzzy parameters are demands, pipe roughness coefficients, and reservoir water levels. For a small network (three nodes, five pipes, and two loops) this approach requires 29,376 hydraulic simulations and for a network with 45 nodes, 65 pipes, and twenty loops, it requires 590,040 hydraulic simulations. Thus, this approach is also computationally burdensome for the fuzzy analysis of large WDNs.

Fu and Kapelan (2011) argue that much of municipal design relies on imprecise and ambiguous information and propose the use of fuzzy random variables to characterize such uncertainties in WDN design. For each node in the WDN, they develop a fuzzy random variable that describes the nodal pressures, and define the magnitude of failure of a given design as the fuzzy membership at which this fuzzy variable is less than the fuzzy required pressures. For a given WDN design, they use Monte Carlo sampling of the fuzzy nodal demands and pipe roughness coefficients to estimate the extreme values of nodal pressures at each α level. Thus, analysis of any WDN design is computationally intensive as it requires the same number of hydraulic simulations as Monte Carlo realizations. Dongre and Gupta (2017) formulate an optimization model for the least cost design of WDNs and link the fuzzy hydraulic analysis of Gupta and Bhave (2007) to a genetic algorithm to determine the optimal design. They apply the model to two benchmark networks and demonstrate its effectiveness for improving design.

The approach developed in this paper improves fuzzy hydraulic analysis by reducing the computational burden of finding the fuzzy membership functions of nodal pressures. First, the analytical solution of the Co-Content Model (Bhave and Gupta 2006) is evaluated and the solution is employed to approximate relationships between fuzzy nodal demands and/or pipe roughness coefficients and fuzzy nodal pressures. The kernel of each fuzzy nodal demand and pipe roughness coefficient is used to find the kernel of the fuzzy pressures at all nodes, using a hydraulic solver. Given the flow directions for this solution, at each α level, input values of fuzzy nodal demands and pipe roughness coefficients are selected that will result in the extreme values of the fuzzy nodal pressures as output to the hydraulic solver. In total, this approach requires $(nf-1)\times$

$(2 \times nn) + 1$ hydraulic analyses. In the following sections, the Co-Content Model, proposed fuzzy analysis approach, and application of the approach to two realistically-sized WDNs is presented. Results of these examples show that the proposed approach provides a close approximation of the fuzzy uncertainty of nodal pressures and is computationally efficient.

7.4 Co-Content Model for WDN Analysis

In the Co-Content Model (Collins et al. 1978) the nodal pressures are considered unknown variables and the coupled relationships for continuity at nodes and conservation of energy along pipes are described with the following expressions.

Consider the energy loss due to friction in pipe k, E_k, which is given by:

$$E_k = Q_k h_k \tag{7.1}$$

where Q_k is the flow at pipe k and h_k is headloss in pipe k. For pipe k, which begins at node i and ends at node j, h_k is defined as:

$$h_k = H_i - H_j = R_k Q_k^n \tag{7.2}$$

Where H_i is the pressure at node i, R_k is the characteristic pipe resistance coefficient which depends on pipe roughness (e.g., Collins et al. (1978) defined it as a function of the Darcy-Wiesbach friction factor (f) for pipe losses), length (L), and diameter (D), and n is the exponent of the headloss equation and is specified for a given pipe type.

The unknown pipe flows may be expressed in terms of the nodal pressure and the known pipe resistance as:

$$Q_k = \frac{\left(H_i - H_j\right)^{\frac{1}{n}}}{R_k^{\frac{1}{n}}} \tag{7.3}$$

Therefore, energy loss as a function of H and R is expressed as:

$$E_k = \frac{\left(H_i - H_j\right)^{\frac{1}{n}+1}}{R_k^{\frac{1}{n}}} \tag{7.4}$$

where R_k is the headloss coefficient.

Bhave and Gupta (2006) reformulate the Co-Content Model as an unconstrained optimization that minimizes total energy loss:

$$Minimize\ CC = \sum_{k=1}^{np} \frac{\left(H_i - H_j\right)^{\frac{1}{n}+1}}{R_k^{\frac{1}{n}}} - \left(\frac{1}{n}+1\right)\sum_{i=1}^{nn} q_i H_i \tag{7.5}$$

The first term of the objective function represents the energy loss within the pipes, and the second term represents the energy loss due to withdrawals, where q_i is the nodal flow demand. Flow continuity at each node and energy conservation in each pipe are integral to the first term, and all other energy losses are expressed in the second, thus the optimal solution of this objective satisfies both nodal flow continuity and energy conservation relationships.

The matrix form of Co-Content Model is defined as:

$$Minimize\ CC = \sum_{k=1}^{np} \frac{1}{R_k{}^{\frac{1}{n}}} \sum_{i=1}^{nn} \left(A_{12}(k,i)H_i\right)^{\left(\frac{1}{n}+1\right)} - \left(\frac{1}{n}+1\right)\sum_{i=1}^{nn} q_i H_i \tag{7.6}$$

where $\mathbf{A}_{12}$ is an $np \times nn$ incidence matrix that describes the topological relationships of the network flows between nodes. Here, if a water source, or reservoir is located at a given node *i*, the known elevation head of the source is used instead of H_i. The elements of matrix $\mathbf{A}_{12}$ are defined as:

$$A_{12}(k,i) = \begin{cases} -1 & if\ the\ flow\ of\ pipe\ k\ exits\ node\ i \\ 0 & if\ pipe\ k\ is\ not\ connected\ to\ node\ i \\ +1 & if\ the\ flow\ of\ pipe\ k\ enters\ node\ i \end{cases} \tag{7.7}$$

7.5 Relationships for Demands, Pipe Resistance and Nodal Pressures

If the nodal demands and pipe resistance are known, the unknown H_i may be estimated by evaluating the derivative of *CC* with respect to H_i and setting this equation equal to zero.

$$\frac{\partial CC}{\partial H_i} = \left(\frac{1}{n}+1\right)\left\{\sum_{k=1}^{np} \frac{1}{R_k{}^{\frac{1}{n}}} \left(A_{12}(k,i)H_i\right)^{\left(\frac{1}{n}\right)} - q_i\right\} = 0,\ \ \forall i = 1, \dots, nn \tag{7.8}$$

By rearranging this equation an expression for the values of q_i at optimality may also be determined:

$$q_i = \sum_{k=1}^{np} \frac{1}{R_k^{\frac{1}{n}}} (A_{12}(k,i)H_i)^{\left(\frac{1}{n}\right)}, \quad \forall i = 1, \dots, nn \tag{7.9}$$

The relationship between nodal pressures and nodal demands may then be approximated, in order to conduct a fuzzy analysis of pressure as a function of fuzzy demands, by evaluating the derivative of the expression for q_i (Eq. 7.9) with respect to H_i:

$$\frac{\partial q_i}{\partial H_i} = -\frac{1}{n} \left\{ \sum_{k=1}^{np} \frac{1}{R_k^{\frac{1}{n}}} (A_{12}(k,i)H_i)^{\left(\frac{1}{n}\right)-1} \right\}, \quad \forall i = 1, \dots, nn \tag{7.10}$$

For a WDN operating under positive pressures, $\partial q_i / \partial H_i$ is always less than zero, and any variation of H_i is inversely proportional to any variation of q_i. That is, as nodal demands increase, required flows and corresponding friction losses increase, and resulting pressure heads decrease. Thus, the maximum value of H_i occurs at the minimum value of q_i and the minimum value of H_i occurs at the maximum value of q_i.

To find an approximation of the dependency of nodal pressures on pipe resistance, it is assumed that variations of the pressure at a given node are only dependent on the resistance factors of pipes which are connected to that node. Thus, Eq. (7.2) may be used to evaluate the derivative of the expression for headloss with respect to H_i and H_j.

$$\frac{\partial R_k}{\partial H_i} > 0 \tag{7.11}$$

$$\frac{\partial R_k}{\partial H_j} < 0 \tag{7.12}$$

For a given pipe *k*, which lies between nodes *i* and *j,* an approximation of the variation of R_k with respect to changes in upstream pressure, H_i, is positive and downstream pressure, H_j, is negative. Thus, for a given node, to find the maximum value of nodal pressure, resistance factors for all pipes with flow directions entering the node should be set to their maximum values, and resistance factors for all pipes with flow directions exiting the node should be set to their minimum values.

7.6 Fuzzy Analysis of WDNs

Given relationships between nodal demands, pipe resistance, and nodal pressures, the fuzzy analysis of a WDN is undertaken as follows. First, the hydraulic simulation of the pipe network is conducted using normal values (i.e., kernels of the fuzzy membership functions) of the nodal demands and pipe resistance, and resulting pipe flows and directions are evaluated. Next, to find the extreme values of pressures for each node, for a range of pre-defined α levels, hydraulic simulation of the network is undertaken using the appropriate $\alpha-$level extreme values of the fuzzy nodal demands and pipe resistance as input. To calculate the maximum value of pressure for a given node *i* and α level, $\alpha*$, all nodal demands should be assigned their minimum input values at the $\alpha*$ level:

$$\mathbf{q}_i = \left(q_j^{min}\right)_{\alpha=\alpha^*}, \quad \forall j = 1, \dots, nn \tag{7.13}$$

and all pipe resistance values for pipes with flow entering the node should be assigned their minimum values while all pipe resistance values for pipes with flow exiting the node should be assigned their maximum values:

$$\mathbf{R}_i = \bar{A}_{11}(i,k)_{\alpha=\alpha^*}, \quad \forall k = 1, \dots, np \tag{7.14}$$

where $\mathbf{R}_i$ is the vector of resistance values for all pipes in the network, and $\bar{A}_{11}(i,k)$ is defined as:

$$\bar{A}_{11}(i,k) = \begin{cases} \left(R_k^{min}\right)_{\alpha=\alpha^*} & if\ (Q_k)A_{12}(i,k) \geq 0 \\ \left(R_k^{max}\right)_{\alpha=\alpha^*} & if\ (Q_k)A_{12}(i,k) < 0 \end{cases} \tag{7.15}$$

where Q_k is obtained from hydraulic analysis of the network under normal conditions.

For calculation of the minimum value of the pressure for node *i* and α level $\alpha*$, all nodal demands should be assigned their maximum input values at the $\alpha*$ level:

$$\mathbf{q}_i = \left(q_j^{max}\right)_{\alpha=\alpha^*}, \quad \forall j = 1, \dots, nn \tag{7.16}$$

and all pipe resistance values for pipes with flow entering the node should be assigned their maximum values while all pipe resistance values for pipes with flow exiting the node should be assigned their minimum values:

$$\mathbf{R}_i = \bar{A}_{22}(i,k)_{\alpha=\alpha^*}, \quad \forall k = 1, \dots, np \tag{7.17}$$

Where the matrix $\bar{A}_{22}(j,k)$ is defined as:

$$\bar{A}_{22}(i,k)_{\alpha=\alpha^*} = \begin{cases} (R_k^{max})_{\alpha=\alpha^*} & if\ (Q_k)A_{12}(i,k) \geq 0 \\ (R_k^{min})_{\alpha=\alpha^*} & if\ (Q_k)A_{12}(i,k) < 0 \end{cases} \tag{7.18}$$

To obtain the complete membership functions of nodal pressures, this process is repeated for all desired α levels. While this analysis requires $(nf - 1) \times (2 \times nn) + 1$ simulations, if only nodal demands are considered to be fuzzy, only $(2nf - 1)$ simulations are needed.

The performance of the method is evaluated herein for two example networks. In the first, a medium-sized network, both demands at nodes and resistance factors in pipes are assumed to be fuzzy and the method is compared with those of Gupta and Bhave (2007) and Revelli and Ridolfi (2002), and in the second, a large-sized network, nodal demands are fuzzy and the method is compared with that of Gupta and Bhave (2007) only, due to computational limitations. The hydraulic simulations are conducted with the Global Gradient Algorithm (Todini and Pilati 1988), and are executed in MATLAB with an Intel(R) Core(TM) 2Duo CPU P8700 @ 2.53 GHz and 4.00 GB RAM. Given the difference between the hydraulic analytic approach and the optimization model of Revelli and Ridolfi, computational time is considered as a criteria for comparison of the methods.

7.7 Examples

7.7.1 Nagpur Medium-Sized Network

Introduced by (Gupta and Bhave 2007), this network is based on that for the Gittikhadan Zone of Nagpur, India. It has one source node, 179 pipes, 141 demand nodes, and 38 loops. A representation of the network is shown in Figure 7.1. The source node is a reservoir with a surface elevation of 335.84 m and supplies 755.26 l/s to the system. It is assumed that extreme values for nodal demands are $q_{min} = q - \sigma q$ and $q_{max} = q + \sigma q$, and for the Darcy-Weisbach coefficients of pipe roughness are $f_{min} = f - \sigma f$, and $f_{max} = f + \sigma f$, where q is the normal nodal demand, f is the normal Darcy-Weisbach pipe roughness coefficient, and σ is the variability which is set to 0.1. These assumptions and relevant network data, such as normal nodal demands, pipe diameters, lengths, and connectivity are provided in Gupta and Bhave (2007).

Example resulting pressures for fifteen key nodes, using the proposed approach and those developed by Revelli and Ridolfi (2002) and Gupta and Bhave (2007), are shown in Figure 7.2. The nodal pressures based on the proposed method are nearly identical to those based on that of

Gupta and Bhave (2007). At the supports, the maximum pressure variation over all nodes for these two approaches is almost three meters, but the maximum deviation between them is 6.57 cm. For all nodes, the SQP algorithm in Revelli and Ridolfi (2002) does not converge to the global optimum, and thus a suboptimal, narrower range, of nodal pressures are obtained. As summarized in Table 7.1, this approach requires the solution of 1129 optimization models, taking approximately 245 seconds per model to solve, and 76 hours in total to execute. The method of Gupta and Bhave (2007) requires 2409 hydraulic simulations and 48.24 seconds in total, and the proposed method requires 1129 hydraulic simulations and 22.64 seconds in total.

7.7.2 Hypothetical Large-Sized Network

This hypothetical network, introduced by Farmani et al. (2003), has 1892 nodes, 2467 pipes, and two reservoirs. A representation of the network is shown in Figure 7.3. The range of normal nodal demands are between zero and 0.1255 cms; and the range of resistance factors for pipes are set to be between 0.0116 and 9.9952. Two cases are analyzed, where the extreme values for nodal demands are $q_{min} = q - \sigma q$ and $q_{max} = q + \sigma q$, and the variability (σ) is 0.1 for the first case , and 0.2 for the second. For both cases, the two methods converge to identical membership functions for all nodes, and typical results are shown for nodes fourteen key nodes in Figure 7.4. For the case with 10% variability, the maximum pressure variation for both methods at the support is approximately 5% (i.e., nearly three meters), while for the case with 20% variability, it is approximately 10% (i.e., nearly six meters). The method of Gupta and Bhave requires 22,705 hydraulic simulations and 31.5 hours in total. The proposed method requires 9 hydraulic simulations and 45 seconds in total. Thus, for this large-sized network it may be readily used to characterize the uncertainty in nodal pressures due to nodal demands, and, unlike other techniques, is highly efficient, making it attractive for use in analysis of operations and design. While analysis of the effect of uncertainty due to both demands and resistance would require on the order of 2000 hydraulic simulations per membership level, this would take on the order of 10,000 seconds only, and thus considerably less effort than all existing methods.

7.8 Conclusion

While adoption of fuzzy set theory in hydraulic analysis has been gradual, its recent success in advancing reliability-based design of all engineering systems portends its attractiveness for simulating and optimizing WDNs. In the face of uncertainty, while probability theory is applicable when the system inputs have unique distributions, fuzzy set theory accommodates imprecise, incomplete, and ambiguous information in the design process, with an infinite number of possible membership functions. Existing techniques for fuzzy pipe network analysis and design, such as $\alpha -$level optimization, search methods for estimating the gradient of the hydraulic response, and Monte Carlo Simulation, are limited in application to small networks due to their high computational burden.

The fuzzy analytical approach introduced in this paper accelerates network analysis by approximating the gradients of the reformulated Co-Content Model. Application of this method to two example networks, which range in size and complexity, demonstrates its efficiency, accuracy, and applicability. When only the nodal demands are fuzzy, this analysis requires only two network simulations to find extreme pressures of all nodes. When both demands and pipe resistance are fuzzy, two matrices are introduced based on the topological information of the network under normal operating conditions, and these are used to find the extremes of the pressure boundaries. Unlike other methods, where the computational effort, or the number of computationally demanding optimization models to be solved, increases greatly with the size of the network, the number of simulations of the approach presented herein increases only by two for each additional node. This method results in comparable, and in some cases improved, accuracy, and exerts a reduced computational burden in comparison with other methods. Its efficiency for large networks allows for a broader analysis a range of possible scenarios for system conditions, stressors, and hazards that plague these networks, many of which are unpredictable. Like all methods for hydraulic analysis, it can also be used to assess the sensitivity of network operations to short- and long-term changes in demands and proposed operational and structural improvements, and as a means of measuring performance in optimal design of new systems.

Table 7.1 The comparison of different methods for Fuzzy analysis of the networks

Network	Method	CPU Time (s)	Hydraulic Analysis (EPANET Run)	Optimization of Nonlinear Model
Nagpur Medium-Sized Network	Revelli and Ridolfi	276605	-	1129
	Bhave and Gupta	48.24	2409	-
	Proposed Method	22.64	1129	-
Hypothetical Large-Sized Network	Revelli and Ridolfi	N.A.	N.A.	N.A.
	Bhave and Gupta	113400	22705	-
	Proposed Method	45	9	-

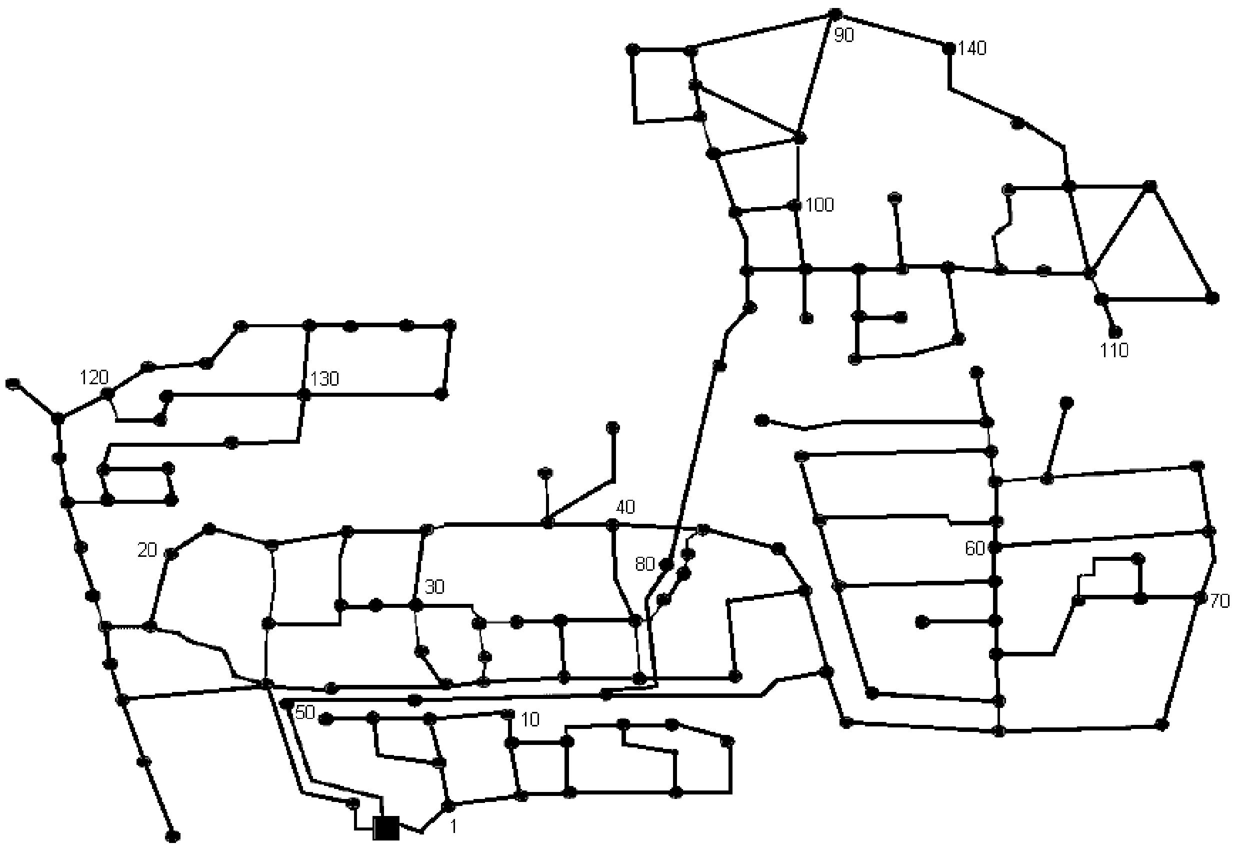
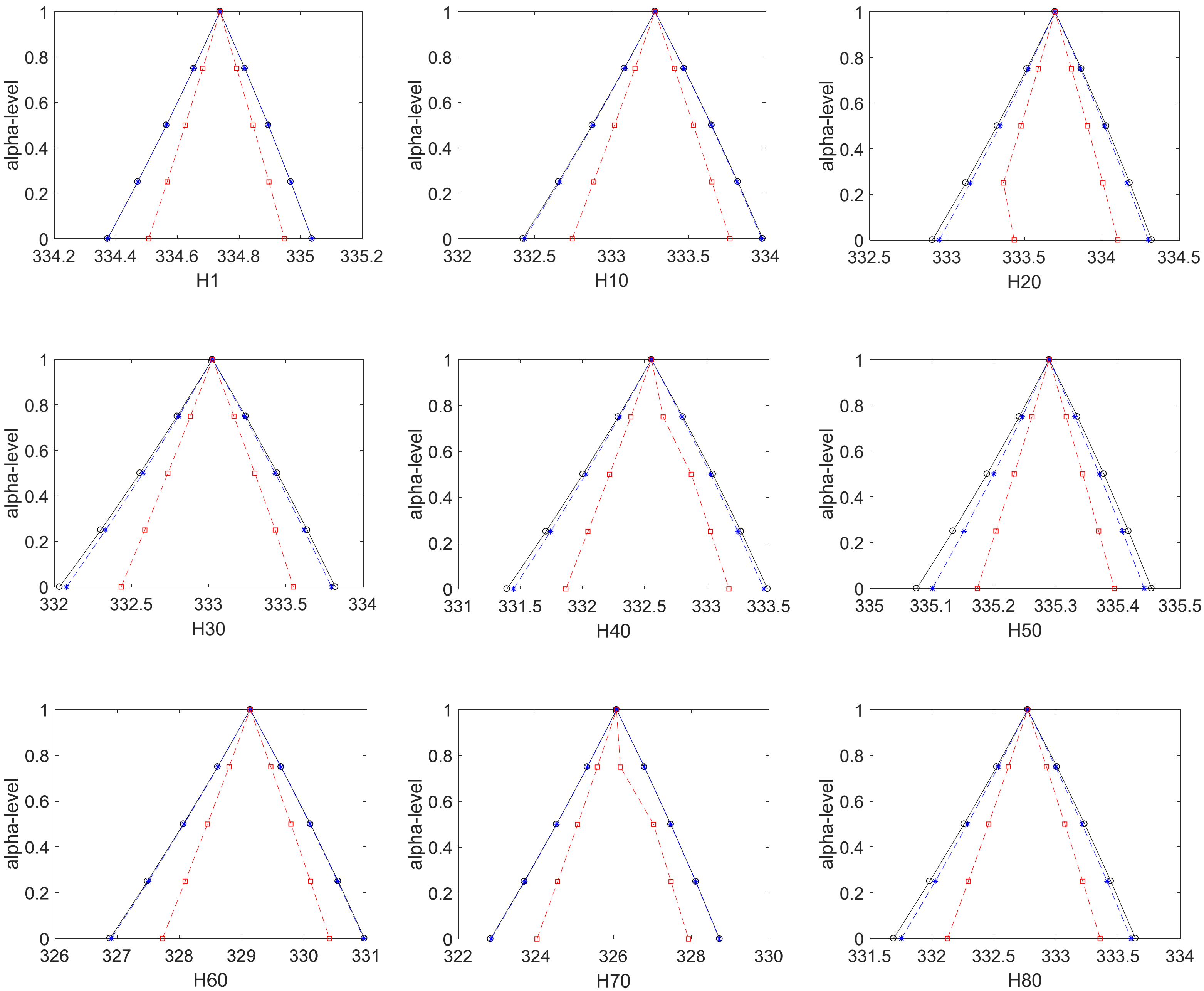


Figure 7.1 Nagpur network


alpha-level
0
0.2
0.4
0.6
0.8
1
334.2 334.4 334.6 334.8 335 335.2
H1
332 332.5 333 333.5 334
H10
332.5 333 333.5 334 334.5
H20
332 332.5 333 333.5 334
H30
331 331.5 332 332.5 333 333.5
H40
335 335.1 335.2 335.3 335.4 335.5
H50
326 327 328 329 330 331
H60
322 324 326 328 330
H70
331.5 332 332.5 333 333.5 334
H80

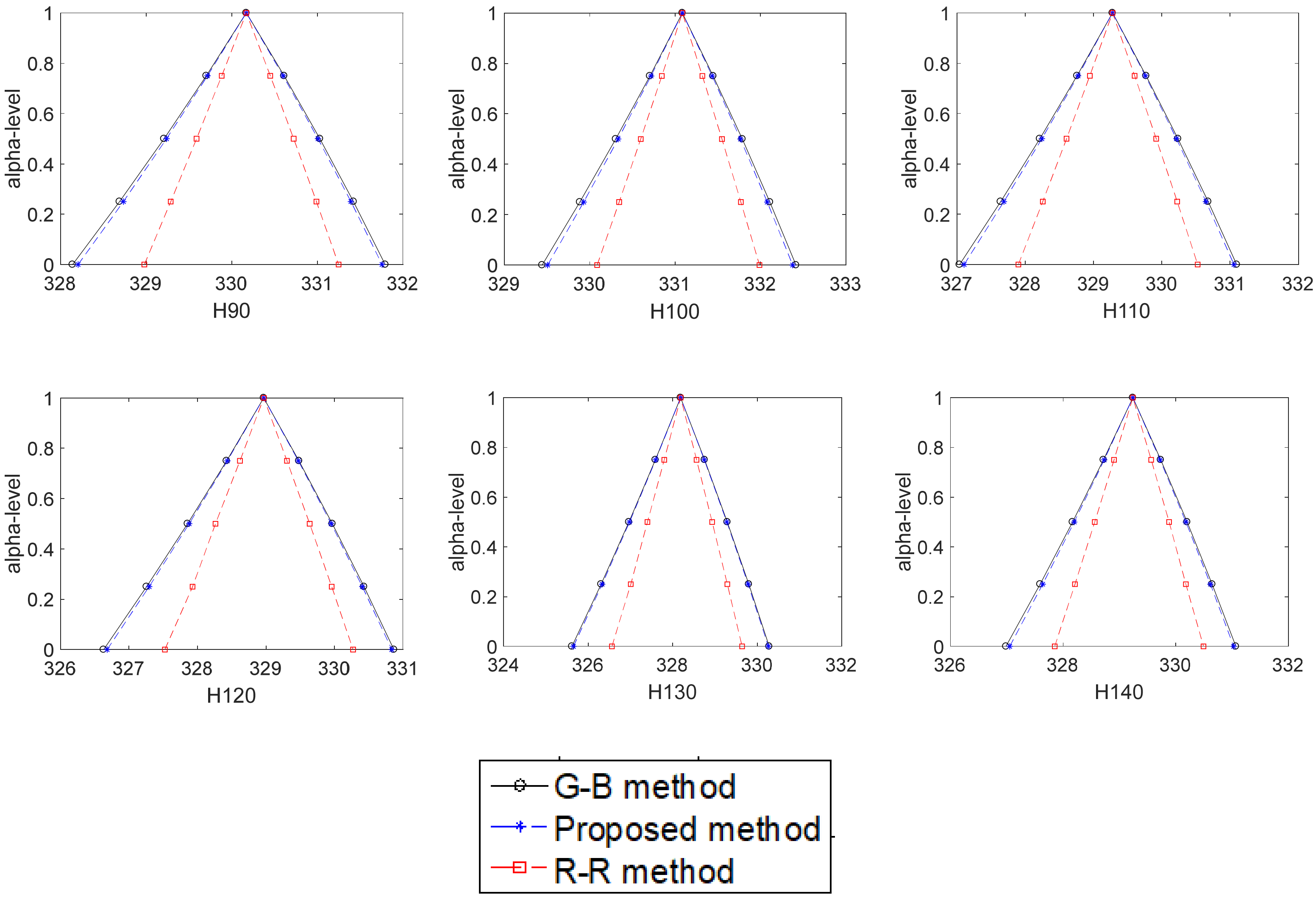


Figure 7.2 Fuzzy membership functions for H1, H10, H20, H30, H40, H50, H60, H70, H80, H90, H100, H110, H120, H130, and H140 obtained by the three fuzzy hydraulic analysis approaches

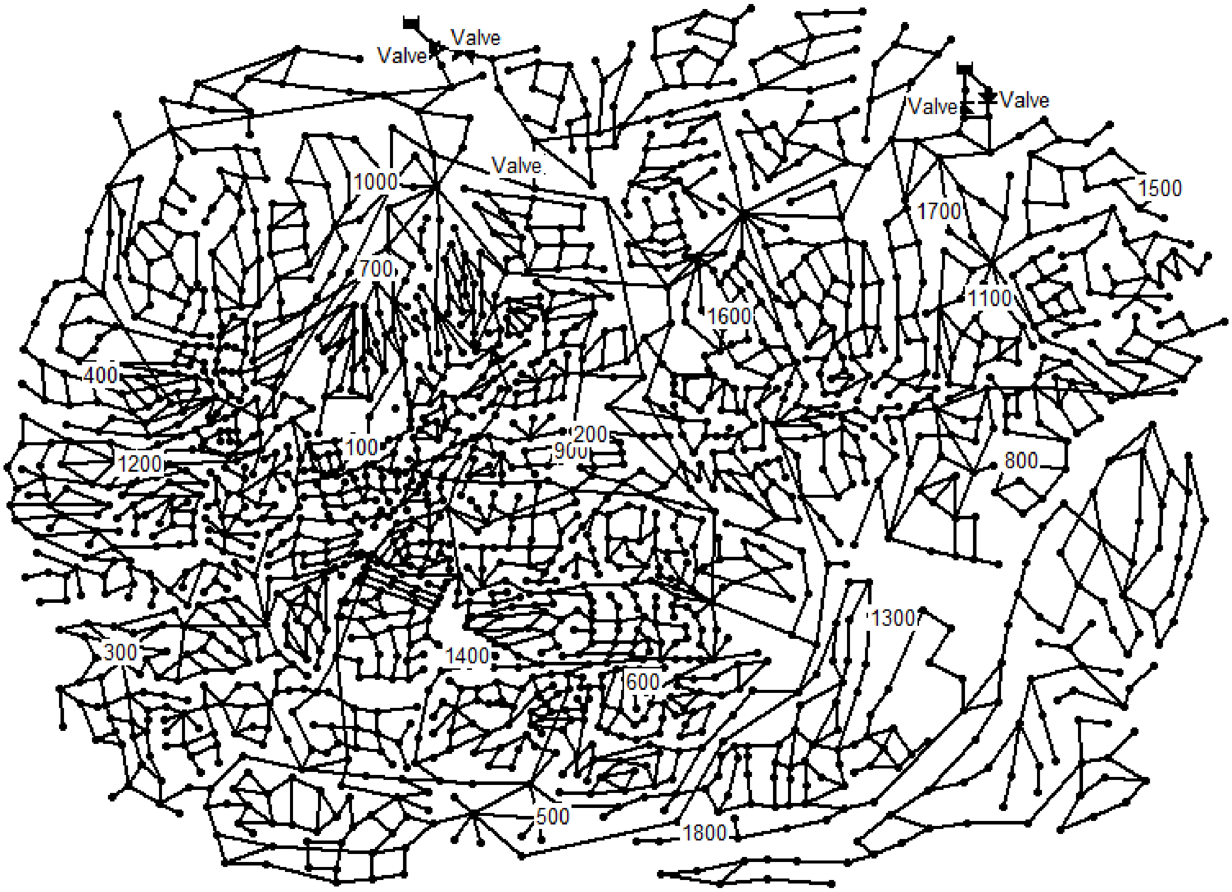
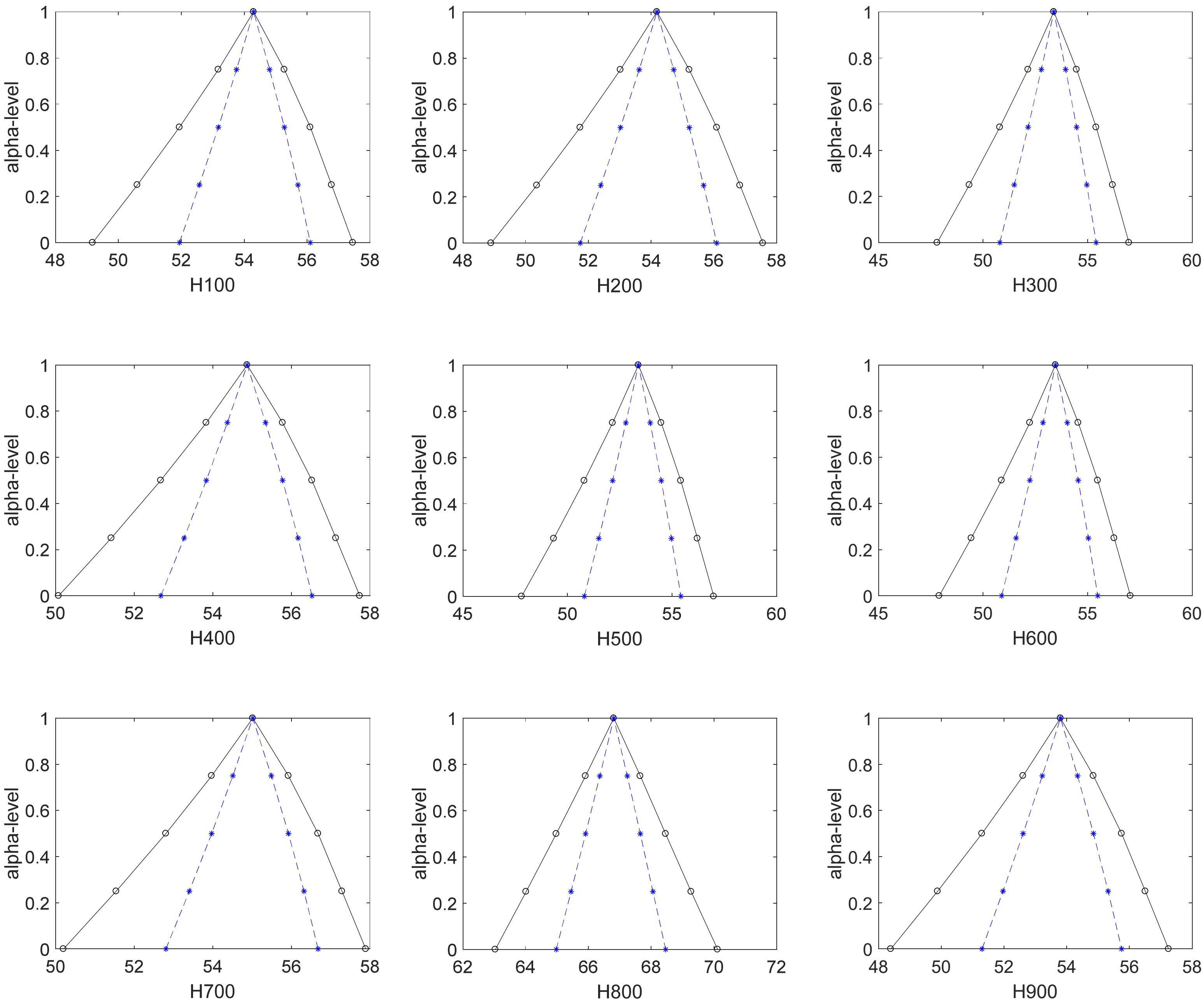


Figure 7.3 EXNET network


alpha-level
H100
H200
H300
H400
H500
H600
H700
H800
H900

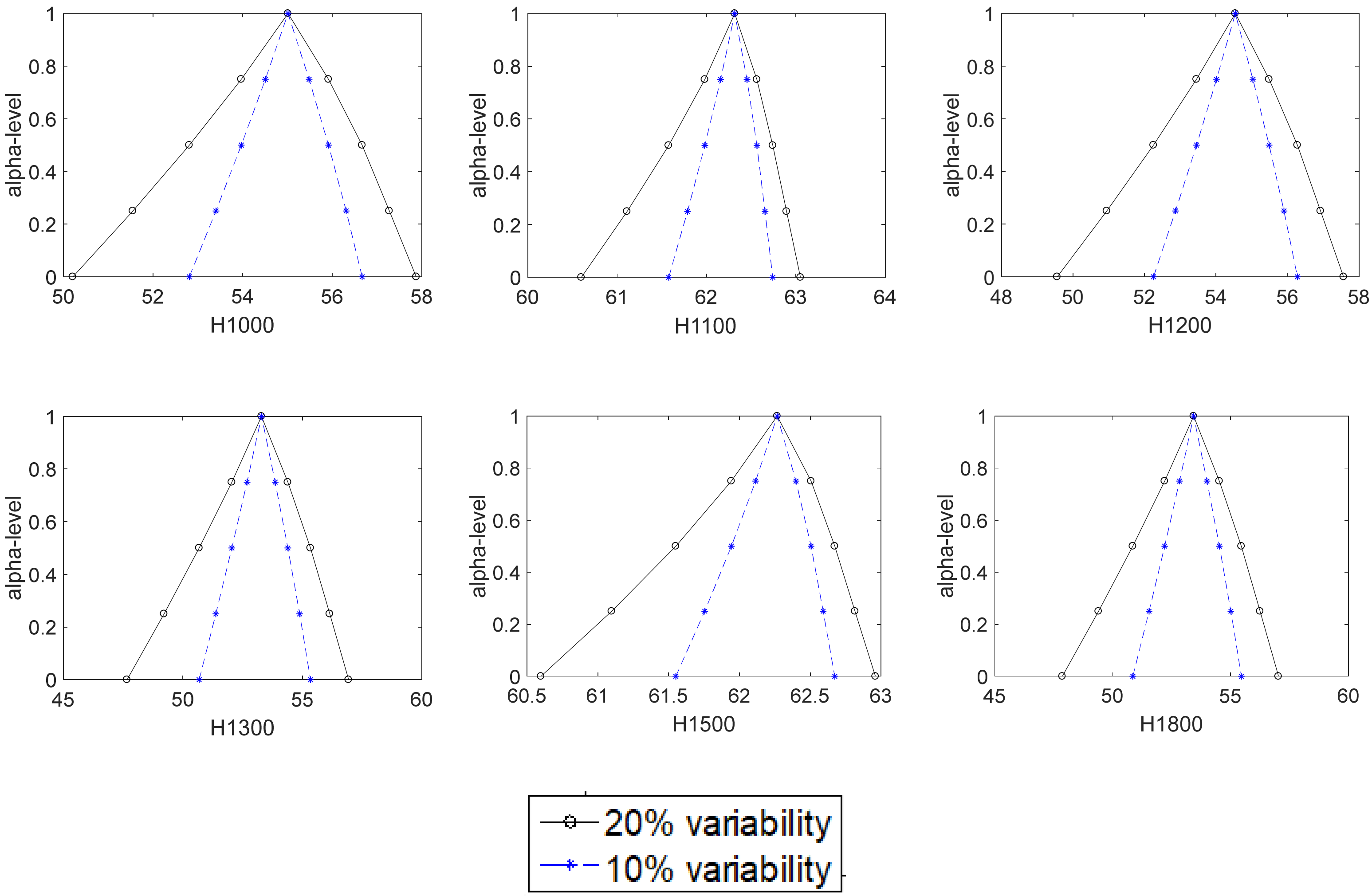


Figure 7.4 Fuzzy membership functions for H100, H200, H300, H400, H500, H600, H700, H800, H900, H1000, H1100, H1200, H1300, H1500, and H1800, with 10% and 20% variability of nodal demands, obtained by Gupta and Bhave, and the proposed method

Chapter 8 : Conclusions

8.1 Summary of thesis objectives and methodologies

This thesis is organized based on the framework presented in Chapter 1, Figure 1.1. In this framework, WDN modeling is classified into two efforts, WDN design and WDN analysis. WDN design is classified as single- and multi-objective optimization models. WDN analysis is also classified as deterministic or non-deterministic analysis. This thesis develops methods, algorithms, and frameworks for enhancing these efforts. One of the main purposes of this thesis is to propose efficient numerical methods for accelerating the design and/or analysis process for WDNs. Another goal is to investigate different surrogates and propose methodologies for improvement of the quality of WDN design. Research is conducted in three phases: 1) Chapters 2-4, investigate, test, and develop different EAs for reducing the computational burden of both single- and multi-objective WDN design; 2) Chapters 5 and 6, propose new frameworks and surrogates for improvement of the design quality in the face of mechanical failures; and 3) Chapters 7, develops numerical methods for acceleration of the Fuzzy analysis of WDNs. The research for each phase began with an extensive literature review to select and synthesize key features of previous works, evaluation approaches, and levels of detail. Then, fundamental equations of WDN modeling are described, briefly. Next, the proposed methodologies are introduced and presented in detail. Finally, several networks are used to demonstrate the efficiency of the proposed methodology in comparison with popular works in the literature.

8.2 Research conducted

With reference to the research objectives and questions presented in Chapters 2-7, a summary of the research conducted in this thesis is as follows:

8.2.1 Development of EAs for single- and multi-objective optimization of WDNs

Generally, the WDN optimization problem has an array of local optima which are, at times, very different from each other. This characteristic is related to the need to satisfy minimum required pressures at junctions throughout the network, and the required pressure is a nonlinear function of the pipe size diameters and their configurations. By changing one pipe diameter, the flow direction in the pipes, at least in the vicinity of the pipe, may change, and the distribution of the pressures throughout the system may also change. This property makes the optimization of a network model difficult to solve. Therefore, when the optimization algorithm is close to the global optimum, there no guarantee that the optimum has been located, because the path for reaching the global optimum may not necessarily be in the path taken near the best local optimum. EAs provide more diverse solutions to escape from local optimum solutions effectively. They also exhibit independence from the initial population of solutions, which, under classical optimization, determines the direction of search. Finally, they do not concentrate solely on the evolution of the best solution (or solutions) of the population.

The No-Free-Lunch (NFL) theorem asserts that no one optimization algorithm may be suited for solving all kinds of optimization problems, and underscores the need for new algorithms that may improve on efforts to reach global optima. Based on the NFL theorem and complexity of the pipe network optimization problem, finding the best procedure for solving this problem is an open question. Chapters 2-4, compare many popular EAs for single- and multi-objective optimization of pipe networks and tests their performance using statistical analysis. By evaluating and considering all benefits and demerits of different techniques, two modified versions of the DE algorithm, FDE for single-objective and NSDE-RMO for multi-objective design, are proposed for WDN design and are fast, reliable and have the least computational burden in comparison with other optimization methods. The FDE algorithm is developed for both continuous- and discrete-domain optimization problems and similar to the DE algorithm, it can solve nonlinear nonconvex NP-hard problems. The high performance of its operators in comparison with other EAs in WDN design and benchmark problems makes it an appropriate optimizer for other engineering optimization problems.

8.2.2 Development of a new framework and introduction of new surrogates for improving the quality of WDN design

In Chapter 5, a fuzzy multi-objective programming model for meeting competing objectives in the optimal design of WDNs is proposed. Fuzzy membership functions for minimizing the pipe network cost and maximizing a number of reliability surrogates are defined, and the model maximizes the degree of satisfaction of these membership functions. Reliability surrogates investigated include: i) the sum over all nodes of the relative surplus of energy at each node; ii) the minimum surplus head at a critical node; iii) the sum over all nodes of the relative surplus of energy at each node modified by the degree of pipe diameter uniformity of its associated loop; and

iv) the minimum pipe diameter uniformity of the associated loops over all nodes. An advantage of this modeling framework is that it may be used to identify best compromise solutions that may be investigated in greater detail. The benefits of this feature increase as the number of objectives increase. Results demonstrate that the resilience index and its modifications, may be used as an indirect measure of hydraulic reliability while they have some limitations in the face of mechanical failures. In Chapter 6, a flow uniformity index is proposed to address these limitations and may be considered a surrogate for mechanical reliability. The maximization of the uniformity index is expected to lead to network designs with a balanced, increasingly uniform flow, at each node, where the burden of carrying inflows (or outflows) at each node is shared evenly among all incoming (or outgoing) pipes. The incorporation of the uniformity index, along with the goals of maximizing the resilience index and minimizing design costs, in a multi-objective optimization modelling framework results in WDN design solutions with increased mechanical reliability.

8.2.3 Advancement of numerical methods for Fuzzy Analysis of WDNs

Fuzzy Analysis of WDNs is a time consuming process which needs a large number of hydraulic simulations to provide solutions. Chapter 7 shows how Fuzzy Analysis can effectively be applied for evaluation of networks under uncertainty and also indicates that current methods, such as $\alpha-$level optimization and MCS, are not efficient for generation of MFs for uncertain variables. In this dissertation, a gradient-based method is proposed to approximate these MFs, which requires the least amount of WDN simulations.

8.3 Suggestions for future work

Suggestions for future work are classified into four phases: 1) Application of efficient optimization algorithms for multi-objective WDN design; 2) Applications of new surrogates in the reliability-based optimization of WDNs; and 3) Development of a Fuzzy-Reliability approach for WDN design under uncertainty.

8.3.1 Application of efficient optimization algorithms for multi-objective WDN design

As presented in the Chapters 2-4, modifications of classical DE for single-objective design, i.e., FDE, and for multi-objective design, i.e., NSDE-RMO, can significantly reduce the computational burden of WDN optimization. Chapter 4 also demonstrates that FDE is an efficient algorithm for finding a global optimum solution in the cost minimization of several networks. Therefore, development of FDE for multi-objective models, considering cost and reliability surrogates is necessary future research. For example, the accelerated mutation and crossover process of FDE could be employed to identify the non-inferior set of solutions to a multi-objective problem. It may be adapted to rank population members according to their objective function performance crowding distance, and to use these ranks to modify the mutation operator and to sort in the selection process.

Furthermore, in Chapter 5, the optimization process inside the Fuzzy programming model which is currently conducted by the SLC algorithm, could be adapted to employ FDE. This application of FDE may quickly converge to the global optimum and may indeed require fewer hydraulic simulations with a reduced computational burden. A comparison among different multi-objective EAs and the Fuzzy programming model also can be conducted.

8.3.2 Applications of new surrogates in reliability-based optimization of WDNs

Chapter 6 demonstrates that the uniformity index can improve the reliability of the network in the face of mechanical failures. In this chapter, a multi-objective optimization model is employed for minimizing cost and maximizing resilience and uniformity indices. In future work, these objectives can be optimized in the Fuzzy programming model which converges to only one best compromise design. Comparison of optimal solutions in population-based nondominated sorting and Fuzzy programming models may reveal new characteristics of the uniformity index. Furthermore, other variations of the uniformity index, such as the average of flow uniformity for all nodes, may provide insightful information regarding design solutions. WDN optimization process can also be conducted under uncertainty. In this case, instead of using a deterministic constraints for minimum required pressures, failure probability can be approximated and implemented as an uncertain constraint and may improve the design quality in real networks.

8.3.3 Development of Fuzzy-reliability concept for WDN design under uncertainty

Chapter 7 shows that Fuzzy Analysis is an appropriate application when Probability Theory and Reliability analysis are not possible. However, the modification of Fuzzy analysis for WDN design, i.e., for assessing Fuzzy failure rates for each design solution, is an open question. Here, the main idea is to evaluate design solutions for networks under uncertainty to provide the degree of reliability, or failure rate, when input parameters are fuzzy. Fuzzy indicators can be introduced to describe the possible performance of design solutions. They may provide relative information about the potential for, degree of, and ability to cope with failure, and may be incorporated in multi-objective analysis to help decision makers select the best design(s), based on the available budget and performance-based priorities. Fuzzy analysis of uncertainty may be instrumental in achieving progress in several areas, most notably in the satisfaction of water quality targets throughout the network, in the adaptation of an existing network to population growth and shifting demands among user groups, and in the linked operation and design problems that plague water, and in particular gas, supply networks. The generality of this approach may indicate promise for its application in other engineering problems as well, such as in operation of networks of multi-objective reservoirs, planning of transportation networks, and design of structures.

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Appendices

Appendix A. The FDE code for minimization of the Sphere Function in Python

```
import numpy
Maxit=1000 # number of iteration
nPop=100 # number of population
NumVariable=10 # number of decision variable
Varmin=-100*numpy.ones(NumVariable)
Varmax=100*numpy.ones(NumVariable)
def Costfunction(x):
   a0= numpy.array(x)
   y0 = sum(a0*a0) # Sphere function
   return y0
class BestSol:
   def __init__(self,Cost,Position,Trial):
      BestSol.Cost=Cost
      BestSol.Position=Position
      BestSol.Trial=Trial
Beta=0.5 # Mutation rate
Pcr=0.4 # Crossover rate
nEval=0
BestSol.Cost=10e10
FCost=10e10
a=numpy.zeros((nPop,NumVariable))
f=numpy.zeros(nPop)
trial=numpy.zeros(nPop) # Counter
inum=numpy.zeros(nPop)
# Create inital population
for i in range (0,nPop):
   a[i,:] = numpy.random.uniform(Varmin,Varmax,NumVariable)
   f[i]=Costfunction(a[i,:])
   trial[i]=0
   nEval=nEval+1
   if f[i]<BestSol.Cost:
      BestSol.Position=a[i,:]
      BestSol.Cost=f[i]
      BestSol.Trial=trial[i]
nPop1=nPop
# Start FDE
for it in range (0,Maxit):
   k=0
   for i in range (0,nPop):
      if trial[i] > 30:
         inum[k]=i
         k=k+1
   if k>0 and nPop>0.1*nPop1:
```

```
        for ii in range (0,k-1):
            tt=int(inum[ii])
            a= numpy.delete(a, (tt), axis=0)
            f=numpy.delete(f, (tt), axis=0)
            trial=numpy.delete(trial, (tt), axis=0)
            nPop=nPop-1
            if nPop<=0.1*nPop1:
                break
    inum=numpy.zeros(nPop)
    if nPop<=int(0.1*nPop1): # Restart
        nPop1=int(1.1*nPop1)
        nPop=nPop1
        a=numpy.zeros((nPop,NumVariable))
        f=numpy.zeros(nPop)
        trial=numpy.zeros(nPop)
        BestSol.Cost=10e10
        for i in range (0,nPop1):
            a[i,:] = numpy.random.uniform(Varmin,Varmax,NumVariable)
            f[i]=Costfunction(a[i,:])
            trial[i]=0
            nEval=nEval+1
            if f[i]<BestSol.Cost:
                BestSol.Position=a[i,:]
                BestSol.Cost=f[i]
                BestSol.Trial=trial[i]
    for i in range (0,nPop): # Mutation and Crossover
        A = numpy.random.permutation(range(nPop))
        A=numpy.delete(A, (i), axis=0)
        A1=A[1]
        A2=A[2]
        A3=A[3]
        x=a[A1,:]
        TTT=sum(abs(BestSol.Position-a[A1,:]))>0 and sum(abs(BestSol.Position-a[A2,:]))>0 and
sum(abs(BestSol.Position-a[A3,:]))>0
        if TTT == True:
            Y=a[A1,:]+Beta*(BestSol.Position-a[A2,:])
            Y=numpy.minimum(Y,Varmax)
            Y=numpy.maximum(Y,Varmin)
            Z=numpy.zeros(NumVariable)
            for j in range (0,NumVariable):
                t=numpy.random.uniform(0,1,[1])
                if t<=Pcr:
                    Z[j]=Y[j]
                else:
                    Z[j]=x[j]
            NewSol=Z
            NewCost=Costfunction(NewSol)
            nEval=nEval+1
            if NewCost<f[i]: # Selection
                a[i,:]=NewSol
                f[i]=NewCost
                trial[i]=0
                if f[i]<BestSol.Cost:
                    BestSol.Position=a[i,:]
                    BestSol.Cost=f[i]
                    BestSol.Trial=trial[i]
```

```
        else:
          BestSol.Trial=BestSol.Trial+1
      else:
        trial[i]=trial[i]+1
      if BestSol.Cost<FCost:
        FCost=BestSol.Cost
  print('iteration',it,'Cost Function',FCost,)
```

Appendix B Pipe and nodal information for Farhadgerd network

Table B.1 Candidate pipe size diameters and corresponding costs ($/meter length) for Farhadgerd network

Diameter (mm)	Cost ($/meter length)
63.8	638
79.2	792
96.8	968
150	1500
200	2000
250	2500
300	3000
350	3500
400	4000

Table B.2 Nodal information for Farhadgerd network

Node	Elevation (m)	Demand (L/s)	Node	Elevation (m)	Demand (L/s)
R-1	510	-	J-28	454.5	0.9242
J-1	479	2.1037	J-29	458	1.0336
J-2	468.7	3.3318	J-30	455.8	1.2403
J-3	466.5	1.4227	J-31	437	1.4462
J-4	467.1	0.5594	J-32	434.6	0.9990
J-5	469	0.1581	J-33	430	1.3035
J-6	467.3	1.2282	J-34	439.4	0.3996
J-7	469.2	2.4685	J-35	433.3	16.3083
J-8	460.5	0.3891	J-36	437	1.1037
J-9	461.3	4.5357	J-37	455.6	0.5594
J-10	465.4	0.9850	J-38	468.5	10.3238
J-11	476.3	0.4621	J-39	472.7	0.9850
J-12	477	0.6931	J-40	478.7	0.9485
J-14	472.5	0.1581	J-41	486	0.9971
J-15	478.7	0.6688	J-42	477.5	0.3770
J-16	478.5	0.5594	J-43	473.3	0.7053
J-17	475.5	4.7546	J-44	471.4	2.1037
J-18	475	0.2067	J-49	431.2	0.3140
J-19	480	0.3162	J-50	431.8	0.1618
J-20	472	1.3133	J-51	462.8	3.5872
J-21	456.2	0.6931	J-52	440	0.0000
J-22	458.3	1.2890	J-53	454.5	0.1000
J-23	446.3	0.8147	J-13	455.8	0.0000
J-24	440	0.5715	J-45	455.8	0.0000
J-25	445.4	1.5200	J-47	440	0.0000
J-26	438	2.7688	J-48	440	0.0000
J-27	441	2.0076	J-46	455.8	0.0000

Table B.3 Pipes characteristics for Farhadgerd network

Node	J-in	J-out	L (m)	C	Node	J-in	J-out	L (m)	Hazen-Williams factor, C
P-1	R-1	J-1	885.3851	120	P-41	J-35	J-36	437.3994	130
P-2	J-1	J-2	328.5421	120	P-42	J-36	J-26	359.4174	130
P-3	J-2	J-3	86.0021	120	P-43	J-30	J-37	141.5928	130
P-4	J-3	J-4	195.1899	120	P-44	J-37	J-38	667.3086	130
P-5	J-4	J-5	30.9500	120	P-45	J-38	J-39	242.5379	120
P-6	J-5	J-6	68.4875	130	P-46	J-39	J-40	229.1806	130
P-7	J-6	J-7	563.0813	130	P-48	J-41	J-42	203.4631	130
P-8	J-7	J-8	417.1908	130	P-49	J-42	J-43	100.3686	130
P-9	J-8	J-9	416.8621	120	P-50	J-43	J-29	503.9709	120
P-10	J-9	J-4	294.4953	120	P-51	J-38	J-44	385.2943	130
P-11	J-7	J-10	367.3761	130	P-57	J-32	J-49	190.6523	130
P-12	J-10	J-11	372.3605	130	P-58	J-49	J-50	208.6375	130
P-13	J-11	J-12	147.7453	130	P-59	J-50	J-33	227.8588	130
P-16	J-14	J-15	213.1492	130	P-60	J-34	J-31	278.1212	120
P-17	J-15	J-16	233.7499	130	P-61	J-7	J-14	322.7427	130
P-18	J-14	J-17	315.1370	130	P-62	J-12	J-16	193.8307	130
P-19	J-17	J-18	247.3020	130	P-63	J-27	J-35	354.9940	130
P-20	J-18	J-19	155.8379	130	P-64	J-2	J-51	342.1014	130
P-21	J-19	J-20	138.7134	130	P-56	J-51	J-28	299.1446	130
P-22	J-20	J-5	302.7770	130	P-55	J-1	J-43	135.5139	120
P-23	J-9	J-21	202.5051	130	P-54	J-43	J-39	444.0197	120
P-25	J-22	J-3	421.6994	130	P-53	J-45	J-31	505.6546	120
P-26	J-21	J-23	451.5464	130	P-52	J-47	J-26	247.9111	130
P-27	J-23	J-24	94.9841	130.013	P-47	J-53	J-27	445.0974	130
P-28	J-24	J-25	138.8879	130	P-38	J-52	J-53	504.6454	130
P-29	J-25	J-22	381.8934	130.013	P-36	J-53	J-13	468.0775	130
P-33	J-28	J-25	388.1121	130	P-32	J-48	J-52	16.5572	130
P-34	J-28	J-29	348.5208	130	P-31	J-46	J-13	16.3047	120
P-35	J-29	J-30	125.7833	120	P-30	J-45	J-13	52.5861	120
P-37	J-31	J-32	333.0668	130	P-24	J-46	J-30	14.3932	120
P-39	J-33	J-34	286.3284	130	P-15	J-47	J-52	74.3145	130
P-40	J-34	J-35	436.9019	130	P-14	J-48	J-24	13.5482	130
P-65	J-46	J-30	14.3932	120	P-66	J-46	J-13	16.3047	120
P-67	J-48	J-24	13.5482	130	P-68	J-48	J-52	16.5572	130