DETERMINING THE CONTRIBUTIONS OF EXECUTIVE FUNCTIONING TO MATHEMATICAL SKILLS

by

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Abstract

The importance of executive functioning (EF) skills in general mathematical achievement has been well established. However, a deeper understanding about the degree to which distinct EF skills uniquely contribute to not just math as a whole, but to individual mathematical abilities is needed. The present study assessed the unique contributions of EF skills (working memory, inhibition, and shifting) to performance on math fluency, calculation, and problem solving tasks. A secondary goal of this study was to investigate whether performance based tests or parental ratings of EF better predict mathematical performance for each mathematical subskill. Participants were individually administered tests of EF and mathematics and parents completed an EF rating scale. A hierarchical multiple regression was conducted for each mathematic subskill, including lower level math skills in the first block and either performance-based or parent ratings of EF in the second block. Results show a strong relationship between verbal working memory and math fluency, calculation, and problem solving over and above lower level math skills. Math fluency had a significant relationship with calculation, but only calculation was related to problem solving. Performance based measures of EF were superior to parental ratings in terms of their relationship with calculation and problem solving. These findings have implications for our understanding of how EF skills may contribute to math achievement, which can inform early identification and provision of accommodations that will better support mathematical success and the remediation of math difficulties. Furthermore, the findings have implications for the validity of performance based and parental ratings of EF for identifying skills related to math. Lastly, the findings highlight the importance of lower level math skills in calculation and problem solving, which reiterates the importance of the mastery of prerequisite skills as well as areas of targeted intervention.

Lay Summary

Executive functioning (EF) is a set of cognitive skills that are essential for goal directed behaviour and goal attainment, especially in the school setting. It is important for researchers and educators to understand how this set of cognitive skills contributes to the development of mathematical achievement and most importantly mathematics difficulties. Elucidating how EF skills contribute to mathematics will inform mathematics instructional strategies, interventions, and accommodations that can be provided to students to support their success and remediation of difficulties.

Preface

This thesis is the original and unpublished work of the author, Marley Morton. The UBC Ethics Certificate number H17-00324 approved the work reported in the methodology and results sections of this thesis. The research design and analysis of research data was done independently by the author. Data collection and entry was completed by the author, Meagan Murphy, as well as with the assistance of Raveena Mahal and Sarah Gutri.

Table of Contents

List of Tables

List of Figures

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Introduction

Executive Functioning

Executive functioning (EF) is a set of cognitive skills that allow individuals to manage and direct their attention, thoughts, and actions to achieve goal directed behaviour (Best & Miller, 2010; Blair & Raver, 2012; Diamond & Lee, 2011). EF is comprised of subcomponents including working memory, response inhibition and cognitive shifting (Miyake et al., 2000; Senn, Espy, & Kaufmann, 2004). The ability to use EF to manage and direct behaviours towards goal attainment is critical for completing everyday tasks, and especially for more demanding ones such as academic tasks. Throughout the school day, students are expected to perform numerous cognitive demanding activities that require EF including organizing their ideas, thoughts and school materials, attending continuously to instruction, remembering instructions, following rules, planning future actions, ignoring external distractions, inhibiting inappropriate or ineffective behaviors, and shifting fluidly between tasks (Blair & Diamond, 2008; Jacobson, Williford, & Pianta 2011; Langberg et al., 2011). When students are successful in completing these tasks, they are exhibiting effective executive functions (Anderson, 2002; Blair, 2002; Blair & Razza, 2007; Diamond & Lee, 2011; Isquith, Gioia, & Espy, 2004).

Research has established that EF is critical for learning and school readiness (e.g., Bull, Johnston, & Roy, 1999; McLean & Hitch, 1999; Ozonoff & Jensen, 1999). Students with poor executive functioning demonstrate weak school performance (Gathercole & Pickering, 2000; Swanson & Beebe-Frankberger, 2004) and perform worse than peers on measures of reading and mathematics (Alloway, Banner, & Smith, 2010; Bull & Scerif, 2001; Clark, Pritchard, & Woodward, 2010; Espy et al., 2004; Gathercole, Pickering, Knight, & Stegmann, 2004). One longitudinal study found that parental ratings of EF, as measured by the Behavior Rating

Inventory of Executive Function (BRIEF), predicted students' academic grade point averages four years later, even after controlling for poverty and having an individual education plan (Samuels, Tournaki, Blackman, & Zilinski, 2016). In sum, EF appears to be strongly related to learning and academic achievement. Thus, questions remain about the relationship between EF and achievement, such as how EF skills contribute to the acquisition of achievement skills and application, and how academic achievement may even contribute to the development of EF. Even still, some researchers have proposed that EF may not be a predictor of achievement after controlling for other unobserved characteristics of children such as socio-economic status (SES) or a parent's level of education (Jacob & Parkinson, 2015), while others propose that EF is a mediator of SES and mathematic achievement rather than a direct contributor (Nesbitt, Baker-Ward, & Willoughby, 2013**)**.

Because such a strong relationship exists between EF and achievement, in theory, strengthening and promoting the development of EF behaviours and capacities should result in academic achievement gains. Unfortunately, the research that has been conducted in this area does not demonstrate the clinical utility of executive functioning training programs in producing transfer effects, meaning improvement in other areas of functioning such as academics and behaviour is rarely documented (Cortese et al., 2015; Melby-Lervag, 2012; Rapport, Orban, Kofler, & Friedman, 2013).

However, understanding the relationship between EF and academics is crucial for early identification of difficulties, which ultimately facilitates early intervention. In fact, research has shown the clinical utility of early identification of EF weaknesses for the purpose of predicting later mathematical difficulties (Gathercole & Pickering, 2001; Geary et al., 2007; Samuels et al, 2016). Therefore, understanding how EF contributes to learning, and identifying EF weaknesses early on before students experience frustration is crucial for providing early identification and remediation.

Mathematical Difficulties

Mathematics is a broad subject that covers measurement, properties, and relations of quantities as expressed in numbers or symbols. A significant portion of each school day and a student's education experience is devoted to mathematical education. In the early school years, the math curriculum typically includes, but is not limited to, number concepts, numeration, measurement, arithmetic, algorithmic computation, and problem solving. As students advance to higher grades, more complex subjects like trigonometry and calculus are covered.

Mathematics proficiency is not only crucial for school success, but also for daily living and later life outcomes. Mathematical difficulties can result in significant societal obstacles as minimal math competency is necessary for school success, daily living (e.g., managing one's own finances, determining sale prices) and employment (Geary, Hoard, Nugent, & Bailey, 2013; Light & DeFries, 1995). Increased time spent studying mathematics is positively correlated with longer education duration, higher qualification attainment, and SES (Duncan et al., 2007; Kutner et al., 2007). On the other hand, when one is poorly skilled in mathematics it can lower employability and wages (Rivera-Batiz, 1992). Unfortunately, it is estimated that 4-7% of school aged children experience some form of mathematics difficulty (Fuchs et al., 2005), and 5-8% of students show a significant deficiency in mathematical achievement (Geary, 2004; Proctor, Floyd, & Shaver, 2005).

Researchers and practitioners have some knowledge about the cognitive correlates of mathematical difficulties. This is especially true for research investigating math in the early school years (Gersten, Jordan, & Flojo, 2005) as well as longitudinal study designs (Chong &

Siegel, 2008; Guarino, Hamilton, Lockwood, & Rathburn, 2006). Although overall intelligence has been shown to be an important cognitive correlate of math, it only explains 9-25% of the variance in mathematical performance, as many children with mathematical learning difficulties have average intelligence (Resing, Ruijssenaars, & Bosma, 2002). Alternatively, math difficulties may be explained by deficits in specific cognitive skills such as retrieval from long-term memory, working memory, processing speed, and fluid reasoning, rather than overall intelligence (e.g., Bull & Johnston, 1997; Floyd, Evans, & McGrew, 2003; Geary, 1994; Passolunghi & Siegel, 2004). Bull and Scerif (2001) suggest that weaknesses in EF skills including inhibition and working memory contribute to mathematics learning disabilities. Yet it remains unknown exactly which cognitive skills, such as EF, underlie mathematical disability (Geary, 2005; Rourke & Conway, 1997) as well as whether the cognitive skills that contribute to different aspects of mathematical performance (e.g., calculation versus problem solving) are shared or distinct (Fuchs et al., 2006).

More research is required to elucidate the relationships between cognitive abilities and mathematical performance. Understanding how cognitive abilities contribute to mathematics difficulties is important for providing appropriate accommodations to students. For example, students with working memory deficits will benefit from accommodations that reduce their working memory load (e.g., formula sheet or calculator). Furthermore, identifying which cognitive abilities contribute to mathematical performance could assist in the early identification of children with mathematical difficulties. If certain cognitive abilities are shown to be consistently related to mathematical difficulties, then cognitive assessments may be useful for screening in young children, which may help to identify those at risk of mathematical difficulties (Gathercole & Pickering, 2001).

The Relationship between Executive Functioning and Mathematics

Executive functioning is theorized to contribute significantly to students' mathematical performance. Prior research has demonstrated that children with strong EF skills also exhibit well-developed mathematics skills, such as mastering basic number facts, utilizing procedural knowledge, and problem solving (Zentall & Ferkis, 1993). Studies have examined the role of several EF skills (e.g., shifting, attentional control, inhibition and working memory) and their associations with mathematical performance (Gold et al., 2013). These EF skills are interrelated, yet distinctly linked to mathematical skills (Tsujimoto, Kuwajima, & Sawaguchi, 2007; Verte et al., 2006). For example, inhibition is thought to be important for ignoring the following: automatic and incorrect answers, previously applied strategies, task-irrelevant information, impulsive responding, and urges to turn in assignments without checking one's work. Working memory likely plays a critical role in tasks such as counting, arithmetic, and problem solving, as counting requires one to keep numbers and sequences in one's mind, arithmetic requires the ability to store numbers and strategies in one's mind while reaching an answer, and during problem solving one must simultaneously hold incoming information from text. Finally, shifting is thought to be important during math problem solving, as one must be able to alternate between problem solving strategies and various steps when solving multistep problems.

While there is at least some evidence of a relationship between executive functioning and mathematics performance, this research has primarily focused on investigating how EF deficits, mainly in working memory, contribute to mathematical performance. In addition, these studies have typically recruited special populations that commonly demonstrate impairments in EF, such as children with mathematics disabilities, reading disabilities, Autism Spectrum Disorder, and/or Attention Deficit Hyperactivity Disorder. In other words, the full spectrum of possible executive functions, including average or strong EF skills, has not been explored in regards to its contribution to mathematics performance, nor has this research often included typically developing populations. In addition, most prior research has assessed only one or a few executive functions and how they contribute to a single mathematical skill (e.g., calculation), or a composite mathematics score, disregarding how distinct EF skills may uniquely contribute to separate and distinct mathematical tasks within a multivariate framework (Fuchs et al., 2006). Research that investigates cognitive abilities and general mathematical abilities, yielding an overall mathematics composite score as the outcome variable, fails to elucidate how specific cognitive skills are involved in distinct mathematical processes. Therefore, studying the various mathematics abilities rather than an overall mathematics composite is important to determine how EFs are related to specific mathematical skills, as EF associations should vary by math achievement area (Geary, Hoard, Nugent, & Byrd-Craven, 2005). Furthermore, when research assesses how a single EF skill (e.g. working memory) predicts mathematical performance, conclusions cannot be made about the unique contributions to math skill variance of that single EF skill if it is not considered concurrently with other EF skills. This study aims to extend the literature by concurrently assessing multiple and distinct EF skills in children of all EF ability levels to investigate how EF contributes to multiple and distinct mathematical processes.

Finally, research investigating math and EF skills has been inconsistent in the methods used to assess executive functioning, as demonstrated by some studies having utilized direct, performance-based EF measures while others have used indirect measures of parental ratings (e.g., BRIEF rating scales). This is important to note because few significant associations have been reported between parental ratings and performance-based measures of executive functioning (e.g., Anderson, Anderson, Northam, Jacobs, & Mikiewicz, 2002; Bodnar, Prahme, Cutting, Denckla, & Mahone, 2007; Vriezen & Pigott, 2002). The lack of associations between these direct and indirect measures suggests that the two measures are not measuring the same EF construct (Toplak, West, & Stanovich, 2013). Indeed, performance-based measures of EF are direct measures of cognitive EF, and some have argued that they lack ecological validity due to their un-naturalistic administration procedures, and consequently, inferences cannot be made between tests of EF and an examinee's behaviour in the classroom or other real-world settings (Isquith, Roth, & Gioia, 2013; Sbordone, 1996). On the other hand, parental ratings of EF are indirect measures of behavioural EF and are inherently challenged as they depend on parents as raters, who may be biased. While parental ratings have been argued to capture real-world behaviours, they may not assess underlying cognitive processes (Gioia & Isquith, 2004; Silver, 2000). Despite their differences, it is important to note that studies have found that each of these measures have demonstrated at least some significant associations with math achievement (Cirino, Morris, & Morris, 2002; Gerst, Cirino, Fletcher, & Yoshida, 2015)**.** It is unclear at this point whether parental ratings or performance-based measures of EF better predict math performance.

Purpose of the Present Study

The purpose of the current study was to investigate the relationship between executive functioning and three areas of mathematics achievement: math fluency, calculation, and problem solving. This study significantly contributes to the existing body of literature in several ways. First, this study recruited a sample with a broad range of EF ability, including those exhibiting EF normative weaknesses, those with age-appropriate EF, and those who demonstrate EF strengths relative to the normative population for their age. This allowed for a fuller understanding of how EF is associated with math abilities. This study also explored the individual relationships of EF skills to various yet distinct mathematical abilities, by including measures of three EF skills and three math achievement areas. Doing so allowed for more precise examinations of how each EF skill is related to each area of mathematics achievement.

Research Question #1: How are performance-based measures and parental ratings of executive functioning skills (i.e., working memory, inhibition and shifting) related to math fluency?

Research Question #2: How are performance-based measures and parental ratings of executive functioning skills related to calculation?

Research Question #3: How are performance-based measures and parental ratings of executive functioning skills related to math problem solving?

A second aim of this study was to compare the relationships of performance-based tests of EF and parental ratings of EF with mathematical performance. As discussed earlier, it is important to compare these two methods as one is an indirect measure of behavioural EF, and the other is a direct measure of cognitive EF, and it remains unclear whether cognitive or behavioural EF relates more to mathematical achievement. Thus, this study investigated which measure (i.e., performance based or parental ratings of EF) better predicts mathematical performance.

Research Question #4: Do performance-based measures of EF or parental ratings of EF better correlate with math fluency, calculation, and problem solving?

Significance of the Present Study. This study explored how well EF skills correlate with performance on three distinct areas of mathematical achievement. In doing so, this study allowed for the examination of the unique relationships of working memory, inhibition, and shifting with math fluency, calculation, and problem solving. The inclusion of all three of these EF skills and areas of math achievement allows this study to elucidate the relationship between EF and children’s mathematical abilities beyond previous literature.

Numerous experts argue that it is critical that school professionals have access to instruments that accurately detect early mathematical difficulties (Dowker, 2005; Fuchs et al., 2007). Math difficulties and disabilities are typically identified late into the education years, at which point children’s mathematical difficulties have increased and caused these children to experience frustration (Desoete, Roeyers, & De Clerq, 2004). Some prior research has demonstrated the promising utility of EF assessments for identifying mathematical difficulties (Gathercole & Pickering, 2001). For example, Toll, Van der Ven, Kroesbergen and Van Luit (2011) used a longitudinal design to predict learning disabilities using measures of EF. The authors found that one year later, working memory tasks predicted math learning disabilities over and above earlier mathematical abilities. Other studies have also had success in predicting learning disabilities using measures of EF, or discriminating between high and low achievers using EF measures (Geary et al., 2007; Swanson & Sachse-Lee, 2001). If early EF assessments did lead to the development of early identification for children at risk for math difficulties, this would consequently enable earlier treatment and access to accommodations for the identified children, potentially preventing the worsening of the gaps in their math achievement and their growing frustration. In addition, treatment of the EF deficits that may impact achievement could also prove important in the early years when EF skills are malleable and training may be more effective (reviewed in Wass, Scerif, & Johnson, 2012).

The understanding provided by this study regarding how each EF skill uniquely relates to each math skill is a prerequisite for appropriately selecting valid tests. It is essential that school professionals have access to valid tests that allow for the identification of EF strengths and weaknesses that contribute to math achievement. Identifying EF skills that may cause either limitations in acquiring math skills, or promotion of math achievement, is an important step towards identifying a student's learning profile, and the type of learning support they require.

Furthermore, understanding whether performance based or parental ratings of EF better correlate with mathematics performance allows for more valid assessment of the EF skills that contribute to mathematics achievement. Parental ratings and performance based measures of EF are common assessment tools used by school professionals. Often, the results are used to make inferences about the relationship between a student's EF and their academic achievement. By determining which measurement method seems to better relate with math achievement, the results inform the validity of using such measures for identifying the EF skills that contribute to mathematical skills.

Review of the Relevant Literature

Overview of Executive Functioning

The concept of executive functioning (EF) has been discussed for over 160 years, dating back to the 1840s when scientists were beginning to explore the functions of the frontal lobes (e.g., Harlow, 1848; 1868). One of the first to use the term "executive" was Pribram in 1973 when he described the prefrontal lobes as the "executive brain" (Barkley, 2012, p.1), consequently spurring the development of over thirty constructs under the umbrella term EF. Executive function continues to be a widely studied topic in contemporary psychological research, yet no universally accepted definition or operational definition has emerged (Jurado & Rosselli, 2007). One of the most widely cited definitions comes from Welsh and Pennington (1988), who defined EF as "the ability to maintain an appropriate problem solving set for attainment of a future goal" (pp. 201-202). Similarly, Lezak (1995) defines EF as "a collection of interrelated cognitive and behavioural skills that are responsible for purposeful, goal-directed activity" (p. 42). Both definitions highlight that EF plays a critical role in the efficiency of goal-directed behaviour, which is a common feature of EF definitions (e.g. Barkley, 2012; Best, Miller, & Jones, 2009; Dawson & Guare, 2010; Funahashi, 2001; Hughes, Graham, & Grayson, 2004; Lezak, 1995; Luria, 1966; Malloy, Cohen, & Jenkins, 1998; Oosterlaan, Scheres, & Sergeant, 2005; Pennington & Ozonoff, 1996; Strauss, Sherman, & Spreen, 2006; Stuss & Benson, 1984; Welsh & Pennington, 1988).

While lacking an agreed upon definition, EF is regarded as distinct from other cognitive domains such as sensation, perception, language and long-term memory (Pennington & Ozonoff, 1996). There is disagreement, however, among experts as to whether EF is a unitary construct or comprised of sets of independent cognitive processes and abilities (e.g., Barkley, Edwards, Laneri, Fletcher, & Metevia, 2001; Brocki & Bohlin, 2004; Isquith, Gioia, & Espy, 2004; Miyake et al., 2000). Many authors have defined EF as a unitary construct (e.g. Goldstein, Naglieri, Princiotta, & Otero, 2014), while others include multiple components in their definition (e.g. Baddeley, 1996; Diamond, 2013; Godefroy, Cabaret, Petit-Chenal, Pruvo, & Rousseaux, 1999; Lehto, Juujärvi, Kooistra, & Pulkkinen, 2003; Miyake et al. 2000; Shallice, 1988).

A review of all assumed components of EF results in an extensive list, with fifteen components appearing in contemporary research (Best et al., 2009). Typical lists of EF include, but are not limited to, set-shifting and set maintenance, interference control, inhibition, integration across space and time, planning, and working memory (Pennington & Ozonoff, 1996), with many including additional processes such as organization, vigilance, and visual-spatial orienting (Castellanos & Tannock, 2002; Huang-Pollock & Nigg, 2003; Willcutt, Doyle, Nigg, Faraone, & Pennington, 2005). According to Anderson (2002), "Processes associated with EF are numerous, but the principal elements include anticipation, goal selection, planning, initiation of activity, self-regulation, mental flexibility, deployment of attention, and utilization of feedback." (p. 71). While the literature reports numerous EF processes, the four most commonly reported components are inhibition, working memory, shifting, and planning (Best et al., 2009).

Despite the complexity and lack of consensus surrounding the construct of EF, research has demonstrated evidence supporting its validity. For example, studies have shown that frontal lesions in both animals and human patients commonly produce EF deficits such as attention and inhibition difficulties (e.g. Fuster, 1989; Levin et al., 1991; Stuss & Benson, 1986), while leaving intelligence unaffected (e.g., Stuss & Benson, 1984), thereby demonstrating ecological validity. Additionally, factor analyses of EF batteries provide construct validity (Mariani & Barkley, 1997; Miyake et al., 2000; Pennington, 1997; Willcutt et al., 2001). In their seminal confirmatory factor analysis, Miyake et al. (2000) found three correlated yet independent variables contributing to performance on complex EF tasks: inhibition of prepotent responses (inhibition), updating and monitoring of working memory contents (working memory), and shifting between mental sets or tasks (shifting). Support for this finding also comes from Lehto et al. (2003), who confirmed that Miyake's three-factor model of EF was the best fit for their data when using exploratory and confirmatory factor analyses with their sample of children ages 8 to 13. Miyake's three-factor model of EF (inhibition, working memory and shifting) provides a simple yet sound model of EF that will be used in the current study.

Executive Functioning Processes

Working Memory. Working memory is one of the more popular executive functions in recent literature. Working memory is the ability to constantly monitor and code incoming information and its relevance for the task, and to update irrelevant information with newer, more relevant information (Morris & Jones, 1990). Working memory is not merely storing and coding incoming information, but also the manipulation of this information over a short period of time, thus related to but distinguishable from short term memory. In general terms, working memory can be considered as the ability to remember and update information while simultaneously engaging in other cognitively demanding tasks (Gathercole, Alloway, Willis, & Adams, 2006).

One of the most influential models of working memory has been proposed by Baddeley and Hitch's three component model of working memory (Baddeley & Hitch, 1974). Their model is comprised of a control system termed the central executive, which is supported by two secondary systems, one based on language and the other based on visual and spatial representations, which are the phonological loop and the visual-spatial sketchpad, respectively. Morris and Jones (1990) have demonstrated that the central executive is responsible for the process of updating information in working memory. On the other hand, the phonological loop stores verbal memory information before fading after a short period of time, sometimes lasting only seconds. The phonological loop works as an auditory rehearsal process that aids in retaining information that can be thought of as subvocal speech (Baddeley, 2003). The visual-spatial sketchpad works similarly to the phonological loop by temporarily storing and rehearsing visual or spatial information before disappearing. These visual or spatial representations can feature colours, locations, or shapes. Although the two subcomponents of verbal and visual-spatial working memory are separate, research has established that they are utilized simultaneously (Brandimonte & Gerbino, 1993).

Various working memory measures are utilized in research; however, working memory is most often assessed using backwards digit or word span tests whereby an individual is verbally presented with a list of words or numbers and is then asked to repeat the list in backwards order.

Another common measure of working memory is the Corsi Block test (Lezak, 1983). During this task, the examinee watches the examiner touch a series of blocks, and after is asked to touch the blocks in the same order. Computerized versions of these tasks are included in the Automated Working Memory Assessment (AWMA) battery (Alloway, 2007). Other common measures of working memory include the Self-Ordered Pointing task (Petrides, Alivisatos, Meyer, & Evans, 1993), counting span or reading span tasks (Barrouillet, Gavens, Vergauwe, Gaillard, & Camos, 2009; Case, 1995; Conway et al., 2005; Daneman & Carpenter, 1980), and N-back tasks (Owen, McMillan, Laird, & Bullmore, 2005; Verhaeghen & Basak, 2005).

Inhibition. Inhibition is a complex executive function as numerous authors have argued for several similar yet distinct inhibitory processes (Friedman & Miyake, 2004). These include response or motor inhibition, cognitive inhibition, interference control, motivational inhibition, and automatic inhibitors of attention. However, for the purpose of the current study, inhibition refers to the ability to deliberately suppress a dominant, automatic prepotent response in favour of a more appropriate response, or no response at all. One of the most common tasks used to measure inhibition is the Stroop Color-Word test (Stroop, 1935), in which the examinee is asked to name the ink colour of coloured words printed in an incongruent ink colour (e.g., the word ‘red’ printed in green ink). For most examinees, it is difficult to inhibit the impulse to read the word instead of naming the colour of the ink. Another common inhibition measure is the Flanker task (Eriksen & Eriksen 1974). During the Flanker task, the examinee is required to attend to the central presented stimulus and to ignore the surrounding flanking stimuli. The task becomes increasingly difficult when the surrounding flanker stimuli become opposite and incompatible to the central stimulus and examinees need to exercise top-down control (Eriksen & Eriksen 1974) to inhibit their automatic response. The Flanker task has been shown to be sensitive to developmental trajectories throughout late childhood and adolescence (Luna, 2009; Luna, Garver, Urban, Lazar, & Sweeney, 2004). Other examples of inhibition include the Simon task (Hommel, 2011), antisaccade tasks (Luna 2009, Munoz, & Everling, 2004), delay-of-gratification tasks (Kochanska, Coy, & Murray, 2001; Sethi, Michel, Aber, Shoda, & Rodriguez, 2000), go/no-go tasks (Cragg & Nation, 2008), and stop-signal tasks (Verbruggen & Logan, 2008).

Shifting. Shifting is the ability to switch between mental sets, rules, tasks, or strategies, for example alternating between sorting objects according to color or shape. To be able to change our mental sets and strategies, one must inhibit (or deactivate) the previous mental set or strategy and activate a different perspective (Diamond, 2013). When students can do this, they are successfully shifting, such as generating alternative problem solving strategies that have not been considered previously, like when solving a riddle. One of the oldest shifting measures is the Wisconsin Card Sorting Task (Milner, 1964; Stuss et al., 2000). In this task the examinee is asked to sort cards by colour, shape, or number. The examinee then needs to deduce the correct sorting rule based on feedback and to flexibly switch sorting rules. A similar, but more simple task, the Dimensional Change Card Sort Test (DCCS), also requires examinees to sort cards based on two rules, but only one switch occurs during the entire test. Once the rules shift, examinees have difficulty overcoming the tendency to continue to focus attention on the previously learned rule (Kirkham, Cruess, & Diamond, 2003; Kloo & Perner, 2005).

Executive Functioning, Academic Skills and Achievement

The literature on EF and academic achievement has consistently shown medium to large relationships between these variables in large general education samples (Best, Miller, & Naglieri, 2011; Bull & Scerif, 2001; Peng, Namkung, Barnes. & Sung, 2016), as EF is critical for acquiring and using academic skills. EF skills are important for school functioning as impairments can result in difficulties with attention and planning, developing, implementing and switching between strategies, failure to use feedback, and inflexibility of thinking (Anderson et al., 2001). All of these skills are critical for school success. Research has also established that EF is critical for learning (e.g., Bull, Johnston, & Roy, 1999; McLean & Hitch, 1999; Ozonoff & Jensen, 1999). Considerable evidence links EF and mathematical performance (e.g., Bull & Scerif, 2001; Cirino, et al., 2002; Espy et al., 2004; Gathercole & Pickering, 2000; Mazzocco & Kover, 2007; Passolunghi & Siegel, 2004). This research has shown that children with greater EF skills demonstrate greater mathematics skills such as mastering basic number facts, utilizing procedural knowledge, and problem solving (Zentall & Ferkis, 1993). Several EF processes such as shifting, sustained attention, inhibition and working memory are most often examined and associated with mathematical performance (Gold et al., 2013). Each of these EF processes are interrelated yet distinctly linked to mathematical skills (Tsujimoto, Kuwajima, & Sawaguchi, 2007; Verté, Geurts, Roeyers, Oosterlaan, & Sergeant, 2006).

While the literature consistently reports a relationship between EF and mathematics, the degree to which each EF skill is related to mathematics as a whole, and particularly how they uniquely relate to various mathematical skill areas remains unknown. Most of the literature focuses on working memory as the key EF skill that contributes to math (van der Ven, Kroesbergen, Boom, & Leseman, 2012), while other studies focus on inhibition (Blair & Razza, 2007; Jenks, van Lieshout, & de Moor, 2012). Even still, some researchers have failed to find a contribution of working memory or inhibition to math, and instead found that mathematical difficulties resulted from difficulties with switching and evaluating new strategies for dealing with a particular task (Bull & Scerif, 2001; Lan, Legare, Ponitz, Li, & Morrison, 2011). In sum, numerous studies have established significant relationships between EF and math, and some have failed to find them. The complexity of the interactions among EF variables, as well as with other cognitive processes is unknown. As such, much remains to be discovered about the complex relationship between EF and mathematics, such as how EF skills are used to acquire mathematic skills and meet math demands, how math achievement contributes to the development of EF, and for which math skills EF matters most (Blair, Knipe, & Gamson, 2008).

Working Memory and Mathematics. Numerous studies have highlighted the importance of working memory for mathematical outcomes (e.g., Alloway & Passolunghi, 2011; Bull, Espy, & Wiebe, 2008; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Li & Geary, 2013; Rogers, Hwang, Toplak, Weiss, & Tannock, 2011), with some studies indicating working memory is a unique predictor of mathematical performance (Bull et al., 2008; Holmes & Adams, 2006; LeFevre et al., 2010; Swanson & Jerman, 2006). Working memory plays an important role in mathematical performance as it supports the encoding, retention, and manipulation of the verbal and visual codes necessary for tasks such as counting, arithmetic, and problem solving. While counting, one must keep the numbers in working memory or errors will result. For accurate arithmetic and calculation, working memory is especially important for holding incomplete calculation products during multi-step problems, as well as remembering procedures. Without the ability to acquire and temporarily store numerical information in working memory, basic arithmetic and more complex operations will be affected (Ackerman, Anhalt, Holcomb, & Dykman, 1986; Ackerman, Anhalt, & Dykman, 1986; DeStefano & LeFevre, 2004; Swanson & Sachse-Lee, 2001). Working memory has also been implicated in math fluency, or the ability to quickly solve simple calculation problems automatically (Geary, Brown, & Samaranayake, 1991; Hitch & McAuley, 1991; Siegel & Linder, 1984; Webster, 1979; Wilson & Swanson, 2001). To acquire math fluency skills, working memory is necessary to associate problems with the correct answers so that this association can be stored and later retrieved from long-term memory (Geary, 1993). During mathematical word problem solving, one must simultaneously hold incoming information from the text while continuing to understand and make sense of the text one is reading. Many studies have linked working memory and its importance to solving arithmetic word problems (LeBlanc & Weber-Russell, 1996; Passolunghi & Siegel, 2004; Swanson & Sachse-Lee, 2001) and multi-step multiplication word problems (Agostino, Johnson, & Pascual-Leone, 2010). It has been suggested that working memory is more important than arithmetic skills, and reading skills for math problem solving (Swanson & Beebe-Frankenberger, 2004; Fuchs et al., 2006).

Studies have shown that children with lower mathematical ability have difficulty maintaining information in working memory (Bull & Scerif, 2001). Swanson and Beebe-Frankenberger (2004) assessed math problem solving and working memory abilities in first, second, and third graders with average and low mathematical performance. The results showed that working memory was uniquely related to problem solving ability over and above inhibition and contributed 30% of the variance in problem solving accuracy. Passolunghi and Siegel (2001) classified a sample of children as “poor” or “good” arithmetic problem solvers, finding that “poor” problem solvers had greater difficulty simultaneously using inhibition (ignoring irrelevant information) and working memory than children classified as “good” problem solvers. Passolunghi and Siegel (2004) later reassessed the same sample and confirmed their earlier findings by demonstrating that children with difficulties in mathematics had consistent difficulties across working memory tasks, with specific weaknesses on auditory working memory tasks (i.e., digit span backward). The authors concluded that children with mathematics learning disabilities are less efficient in utilizing their storage capacities. Although these results show a strong relationship between working memory and word problem solving, many studies have failed to find robust relationships. Swanson, Cooney, and Brock (1993) found only a weak relation between working memory and problem solution accuracy among 100 typically developing children, which was no longer significant once reading comprehension was considered.

Few longitudinal studies have been conducted to investigate working memory and mathematics, but the limited studies thus far show that working memory abilities are related to later mathematical skills (Welsh, Nix, Blair, Bierman, & Nelson, 2010; Ven, Kroesbergen, Boom, & Leseman, 2013; De Smedt et al., 2009). Specifically, Ven et al. (2013) investigated low and typically achieving students and found that working memory was a significant predictor of mathematic abilities 18 months later. As it remains, little is known about how working memory affects mathematical processes, especially for long-term outcomes.

Moreover, additional work is needed to investigate a larger pool of cognitive processes simultaneously to determine the unique contributions of working memory. When working memory is considered along with other cognitive processes, we may see a reduction of the robustness of its contribution. For example, Swanson and Beebe-Frankenberger (2004) found that when working memory was considered with other cognitive processes such as processing speed, working memory only accounted for 1% of the variance in math calculation. This study will extend this literature by investigating the unique contribution of working memory to math skill development while also considering the effects of inhibition and shifting.

Some studies have proposed theories as to how working memory contributes to math. For example, working memory may contribute to procedural knowledge by holding temporary answers in the mind while performing other parts of a calculation, and for holding factual knowledge online. Working memory is likely to play a role in acquiring new facts as both sum and answer need to be held in mind together to strengthen the relationship between them (Cragg & Gilmore, 2014). Yet there is an absence of theories about how working memory interacts with other cognitive processes during math fluency, calculation, and problem solving. Therefore, as the complexity of EF interactions is unknown, this study will contribute to understanding the role of working memory in mathematical tasks and aid in the development of stronger theoretical models that explain how working memory may contribute to distinct mathematical skills including fact fluency, calculation and problem solving.

Inhibition and Mathematics. Inhibition is important for general learning (St Clair-Thompson & Gathercole, 2006) and appears to contribute specifically to the acquisition of math skills, as children with lower mathematical ability tend to also show a deficiency in inhibition (Bull et al., 1999; Bull & Scerif, 2001). In fact, inhibition has been proposed as an even bigger contributor than working memory and shifting in early math skill development (Espy et al., 2004). It has been theorized that inhibition allows children to suppress their initial responses and immature strategies to math questions in order to wait for their processing of more correct and use of mature responses. Without inhibition, students may answer questions incorrectly and without adequate contemplation (Raggi & Chronis, 2006; Zentall, 1993). Indeed, it has been shown that children with lower mathematical ability have difficulty inhibiting prepotent responses and tend to rely on previously learned strategies that are less efficient than new strategies (Bull & Scerif, 2001). Moreover, a lack of inhibition may result in inadequately checking one's work, resulting in more computational errors (Engelhardt, 1978). It has also been theorized that a lack of inhibition interferes with students' ability to block out irrelevant information, which likely interrupts working memory processes and may impede the automaticity of general math skills, providing evidence of a potential interaction between EFs that supports math development (Salthouse & Meiz, 1995). For example, for students to successfully solve problems they must be able to ignore task-irrelevant information during problem solving that might act as a distractor. Passolunghi, Cornoldi, and De Liberto (1999) demonstrated the importance of inhibition when solving math word problems by comparing a group of children with good problem solving skills to a group with poor problem solving skills. They found that the poor problem solvers remembered more non-target and irrelevant information and less relevant information from the math word problem than the good problem solvers.

However, other studies have failed to find a relationship between inhibition and math performance. For example, in Toll, Van der Ven, Kroesbergen and Van Luit's (2011) longitudinal study examining low achieving and typically achieving children, inhibition did not play a crucial role in mathematical abilities. However, the absence of inhibition as a contributor to mathematical abilities may be explained by the study's young sample in that the mathematical task demands for this age group are so simple (i.e., single-step processes that do not include distractors) that they do not require a great deal of inhibition. Moreover, Van der Sluis, De Jong and Van der Leij (2004, 2007) also failed to find a direct relationship between inhibition and mathematics; instead, they found that children with mathematics difficulties struggled the most on more complex tasks that assessed inhibition and shifting simultaneously.

In sum, there is mixed evidence of a direct relationship between inhibition and mathematics. It is important for research to continue to investigate this relationship and the degree, if any, to which inhibition uniquely contributes to math. This study will aim to determine the unique relationship of inhibition with various mathematical skills, building on previous research that has commonly used only a general mathematics composite score. Research in this area is essential to advance our understanding, as currently the mechanism by which inhibition may contribute to mathematics is unconfirmed.

Shifting and Mathematics. Shifting is believed to be important in mathematics as one must be able to alternate between problem solving strategies and solutions in multistep problems. Studies have found that children with mathematics difficulties show poorer shifting abilities (Bull et al., 1999; Bull & Scerif, 2001; McLean & Hitch, 1999). Bull and Scerif (2001) found that children with lower mathematical ability had difficulties in their abilities to shift from a previously learned strategy to a new one. The authors propose that these difficulties stem from a lack of inhibition and poor working memory, which affects the ability to switch and evaluate new strategies when completing a task (Andersson, 2008; Van der Sluis et al., 2007). However, other studies have been unsuccessful in connecting shifting abilities to mathematics (Blair & Razza, 2007; Espy et al., 2004; Toll et al., 2011; Van der Sluis et al., 2004). Some of these studies included young samples, in which all participants were below grade 2, which may explain their lack of findings (Blair & Razza 2007; Espy et al., 2004; Toll et al., 2011), as early mathematics are relatively simple and do not require shifting between strategies and alternate solutions. Although Van der Sluis et al. (2004) included older children in their sample (grades 4 and 5), their lack of findings can be explained when considering that the mathematics measure they employed was The Arithmetic Tempo Test (De Vos, 1992), which is a timed test of math fluency. This measure uses separate tests for addition, subtraction and multiplication, minimizing the use of shifting.

Although the literature demonstrates that shifting is related to mathematics, with meta-analyses demonstrating the strongest evidence (Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013; Yeniad, Malda, Mesman, van Ijzendoorn, & Pieper, 2013), overall, the findings are not robust. When researchers consider the effects of participants' characteristics, the relationship becomes nonsignificant (Jenks et al., 2012). Additionally, shifting is not consistently related to all skill areas of math (Andersson, 2010). Furthermore, not all studies control for the effects of other EF skills (e.g., Yeniad et al., 2013), therefore calling into question the unique contribution of shifting to math. More research is required to determine if shifting is related to mathematics in general, and whether it uniquely relates to math fluency, calculation and problem solving; this study aims to further this knowledge.

Summary and Rationale for Study

Compelling evidence acknowledges the importance of EF for learning, academic achievement, long term outcomes (Alloway, 2009; Samuels et al., 2016), and in particular, the role of EF for mathematical performance and skill development (LeFevre et al., 2010). Much of the research has reported strong relationships between EF processes such as working memory, inhibition, shifting and mathematical performance (Alloway & Passolunghi, 2011; Andersson, 2008; Bull & Scerif, 2001), while other studies have failed to find strong relationships, or any relationship at all (Blair & Razza, 2007; Swanson et al, 2013; Toll et al, 2011). More research is required to determine the degree to which EF is related to mathematics. Furthermore, much remains to be uncovered regarding how EF skills interact with one another, how these interactions impact math, and how individual EF skills uniquely relate to different mathematical skills, and not just a general math composite score. Therefore, this study will explore how working memory, inhibition, and shifting, when studied concurrently, uniquely relate to different mathematical tasks including math fluency, calculation, and problem solving.

Some studies have shown promising findings that may allow for the identification of early mathematics difficulties (e.g., Gathercole & Pickering, 2001), yet instruments that can identify EF strengths and weaknesses that can accurately identify, and even predict later math difficulties are greatly needed. Therefore, this study will aim to examine the utility of using both direct (i.e., performance based tests) and indirect (i.e. parental ratings) measures of EF, and will seek to determine which method better relates to mathematical performance.

Method

Participants

Eighty four participants were recruited from three elementary schools in the Catholic Independent Schools Vancouver Archdiocese (CISVA) school district. However, due to student absences and incomplete measures, complete data are only available for 77 participants. Missing data was handled using a list-wise deletion method. The sample was composed of 55% males and 45% females spanning across grades three (21%, n = 17), four (46%, n = 38), five (17%, n = 14), and six (17%, n = 14). Parental reports of ethnicity indicated that the sample was primarily composed of children of East Asian (52%, n = 40) and Southeast Asian (30%, n = 23) origins. Inclusion criteria included proficiency in English and overall cognitive abilities within two standard deviations of the normative mean (standard score ranging from 70 to 130). Recruitment packages were sent to students' homes by participating classroom teachers. Approval was obtained through the UBC Research Ethics Board and the CISVA school district. Recruitment packages included a written description of the study, an informed consent form, and a demographic questionnaire to be completed if they consented to participate in the study. Parental permission was required for student participation in the study, and was indicated by a completed and returned informed consent form. BRIEF rating scales were sent home to be completed by participants' parents after consent was obtained. Before the administration of measures, participant assent was obtained.

Procedures

All testing took place at participating schools in a quiet testing environment. Child participants were individually administered all measures according to the standardized procedures of each instrument. Counterbalancing was used when administering measures to minimize order effects. Half the participants were administered the EF measures first, and then the math measures (procedure 1). The other half of participants were administered the math measures first, and then EF measures (procedure 2). Procedures were randomly assigned to participants.

Participants were screened for intelligence before the administration of EF and mathematics measures to ensure that the inclusion criteria were met. All participants were offered a break half-way through the testing session. Participants were given a toy as compensation for their participation regardless of whether they completed the entire testing session in full. Parents were instructed to complete the demographic survey and BRIEF rating scale and to return the completed forms to the participating classroom teacher. The researchers then collected completed forms from the classroom teachers. All forms were sealed in envelopes to ensure confidentiality.

Measures

Executive Functioning.

NIH Toolbox Dimensional Change Card Sort (DCCS). The DCCS served as the performance-based measure of shifting. The DCCS was designed by Zelazo and colleagues (Frye, Zelazo, & Palfai, 1995; Zelazo, 2006) based on Luria's influential work on rule use. During the DCCS task, two target stimuli are presented that differ along two dimensions (e.g., shape and color). The participants are then asked to categorize the target stimuli first according to one dimension (e.g., colour). After a significant amount of trials, participants are asked to sort the target stimuli according to the other dimension (e.g., shape). In the third trial, participants need to match the target stimuli to one of the two dimensions (e.g., colour or shape), thereby requiring quick shifting between the dimension rules. By age 5, most children can switch flexibly between dimensions (e.g., Dick, Overton, & Kovacs, 2005; Kirkham et al., 2003; Zelazo et al., 2003). The DCCS and adapted versions of the task demonstrate excellent test-retest reliability (Beck, Schaefer, Pang, & Carlson, 2011) and good construct validity in children and adults (Zelazo et al., 2013; Zelazo et al., 2014). A standardized shifting score was produced based on a combination of accuracy and reaction time.

NIH Toolbox Flanker Inhibitory Control and Attention Test. The Flanker task is a common measure of inhibitory control and attention that was adapted from a version of the Eriksen flanker task (Eriksen & Eriksen, 1974). The flanker task requires participants to attend to the target stimulus (e.g., the center stimulus) while inhibiting attention to the distractor stimuli (e.g., surrounding, or flanking stimuli/flankers). Participants are required to indicate the direction of the target stimulus; this varies in that it can be pointing in the same direction as the "flanker" stimuli (i.e., congruent) or in the opposite direction as the flankers (i.e., incongruent). When the target stimulus is incongruent with the flanker stimuli, subjects often respond more slowly because they must exercise top-down control to inhibit their automatic response, which would be to select the direction in which the flanker stimuli are pointing (Eriksen & Eriksen, 1974). Flanker Inhibitory Control and Attention Test demonstrates excellent test-retest reliability and good construct validity in children and adults (Zelazo et al., 2013; Zelazo et al., 2014). A single inhibition score was assigned to each participant based on a combination of accuracy and reaction time.

Automated Working Memory Assessment: Short Form (AWMA-S). The AWMA is a computer-based assessment that provides measures of verbal and visual-spatial short-term and working memory. The purpose of the AWMA is to identify individuals ages 4-22 with working memory problems. The AWMA has a normative sample of 1269 individuals and has high internal consistency, excellent test-retest reliability and good construct validity (Alloway, 2007). This study used the AWMA-S which is the short version of the AWMA and consists of four subtests (Digit Recall, Listening Recall, Dot Matrix, and Spatial Recall). Scores from the Listening Recall and Spatial Recall were used as verbal and visual-spatial working memory scores, respectively. Each participant had two separate standard scores, one for visual-spatial and one for verbal working memory. During the Listening Recall task, the examinee listens to a series of short sentences and has to decide whether each sentence is true or false (e.g., "Lions have four legs."). After they respond, the examinee is asked to recall the last word of each sentence in the exact order they were presented (e.g. "legs"). During the Spatial Recall task, two identical shapes are presented, one with a red dot on top. During the learning portion of this task, the examinee is shown that the shape with the red dot on top can be the same, or identical, to the shape without the dot, even when it is rotated. But, the shape will be different if it is rotated *and* flipped. During learning trials, the examinee practices determining whether the shape with the red dot is the same or the opposite to the one without the dot for each set of shapes. During the real task, the examinee determines if the shape with the red dot is the same or opposite, then, the stimuli disappear and the examinee is asked to point to the location where the dot on the rotating shape had been pointing for each display, in sequence.

Behavior Rating Inventory of Executive Function (BRIEF). The BRIEF is an 86-item teacher and parent questionnaire that provides eight clinical scales (Inhibit, Shift, Emotional Control, Initiate, Working Memory, Plan/Organize, Organization of Materials, and Monitor) that form two broader Indexes (Behavioral Regulation and Metacognition) and an overall score, the Global Executive Composite. The BRIEF was standardized using a normative sample of 1419 parents and 720 teachers. The BRIEF shows high internal consistency and test-retest reliability

(Gioia, Isquith, Guy, & Kenworth, 2000). One parent of each participant completed the BRIEF parent form. The inhibit, shift, and working memory clinical scales were used for this study. T-scores from these three clinical scales were used; of note, higher T-scores indicate poorer working memory, inhibition, and shifting skills.

Mathematics.

KeyMath 3 Diagnostic Assessment: Canadian Edition (KeyMath 3). The KeyMath 3 is a comprehensive mathematics assessment that measures three broad areas: basic concepts, operations, and applications. The KeyMath 3 was standardized using a large sample of 4000 individuals with updated Canadian norms. The KeyMath 3 has good internal consistency, test–retest, and alternate-form reliabilities, and good construct validity. The following tests were administered to participants: Addition & Subtraction (Written Computation), Multiplication & Division (Written Computation), and Applied Problem Solving. Scores from the two-written computation subtests (Addition & Subtraction and Multiplication & Division) were added together to produce a sum of scaled scores for calculation. Results from the Applied Problem Solving subtest yielded a scaled score for problem solving.

Math Fluency. A paper and pencil math fact fluency probe was administered to measure participants' ability to quickly and accurately retrieve math facts from memory (i.e., math fluency). Each participant had 1 minute to answer as many simple questions as possible. The probe contained single digit addition, subtraction, multiplication and division questions. The measure contained 14 addition questions, 20 subtraction questions, 11 multiplication questions, and 13 division questions. Numerals ranging from 1 to 10 were used. A raw score was used for the data analysis.

Intelligence.

Kaufman Brief Intelligence Test—Second Edition (KBIT-2). The KBIT-2 is a brief, individually administered measure of verbal and nonverbal intelligence in children and adults. The KBIT-2 consists of 3 subtests and yields Verbal, Nonverbal, and a Composite IQ standard score. The test is designed for the purpose of screening intellectual abilities. The KBIT-2 has a large normative sample (N=2120), and has high internal consistency, split-half reliability, test-retest reliability, high reliability, and concurrent validity. The Composite IQ was used to determine if the participants met the inclusion criteria of having overall cognitive abilities within two standard deviations of the normative mean (IQ between 70 and 130).

Demographics.

Demographic Survey. The demographic survey included questions regarding the participant's gender, age, ethnicity, socio-economic status (as measured by parent's highest education level), and any previous diagnoses of Specific Learning Disorder (SLD), Attention Deficit Hyperactivity Disorder (ADHD), Autism Spectrum Disorder (ASD), Developmental Coordination Disorder (DCD), Specific Language Impairment, (SLI) or any other relevant condition. Also, the survey requested information about participants receiving any academic interventions.

Analysis

Hierarchical multiple regression is the practice of building successive linear regression models by adding more predictors at each step and was used to address research questions one to three of this study. By performing a hierarchical multiple regression, this study was able to evaluate models that controlled for lower level mathematical skills to determine the contribution of EF skills (working memory, inhibition, and shifting) to the models, over and above lower level math skills. Since math skills develop hierarchically (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004), it is important to account for the contribution of lower mathematical skills to high level math skills; this is also recommended by accomplished researchers in the field (e.g. Fuchs et al., 2006). All statistical analyses were run using SPSS.

Three hierarchical multiple regressions were run using performance-based measures of EF as the independent variables. Each regression used a different mathematical skill as the dependent variable (math fluency, calculation, and problem solving). Three additional hierarchical multiple regression using parent ratings of EF were conducted, one for each mathematical skill as the dependent variable. The two regression models predicting math fluency included age as a predictor, as the math fluency measure was not normed and therefore did not inherently control for age. Therefore, the two regression models predicting math fluency included age as an independent variable in the first sequential block. In the regression models predicting calculation skills, math fluency was included as the independent variable in the first sequential block. Finally, in the regression model predicting problem solving, math fluency and calculation were included as independent variables in the first sequential block.

The models in Figures 1 and 2 depict the hierarchical regression that was conducted, which is based on Fuchs et al. (2006) work investigating the cognitive correlates of arithmetic, calculation and word problem solving in grade three children. For the regression using performance-based measures of EF as predictors, the regression model included age, verbal and visual-spatial working memory, inhibition, and shifting as predictors of math fluency. For the regression using parenting ratings of EF as predictors, the regression model included age, working memory, inhibition and shifting as predictors of math fluency. It was hypothesized that working memory, inhibition, and shifting will predict math fluency above and beyond age.

Figure 1. Math Fluency Regression; Sequential Block 1

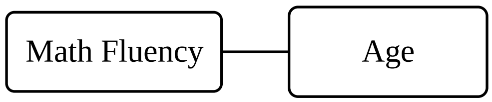
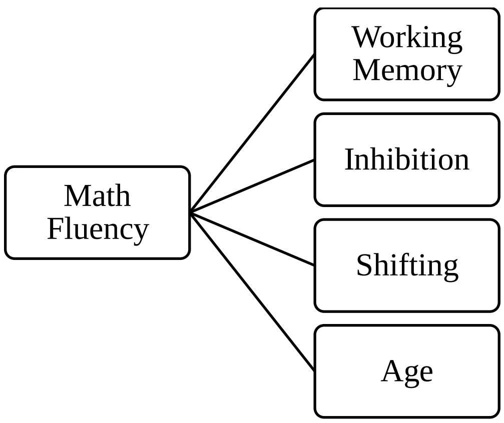


Figure 2. Math Fluency Regression; Sequential Block 2



The model in figures 3 and 4 depict the hierarchical regression model for calculation, which included math fluency, verbal and visual-spatial working memory, inhibition, and shifting as predictors of calculation. For the regression using parenting ratings of EF, the regression model included age, verbal and visual-spatial working memory, inhibition, and shifting as predictors of calculation. Age was not included as a predictor because age was controlled for inherently in the norm-based measure of calculation. It was expected that working memory, inhibition, and shifting would predict calculation performance above and beyond math fluency. Furthermore, it was expected that math fluency would contribute a significant amount of variance to the model.

Figure 3. Calculation Regression; Sequential Block 1

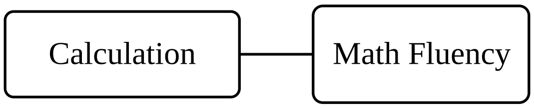
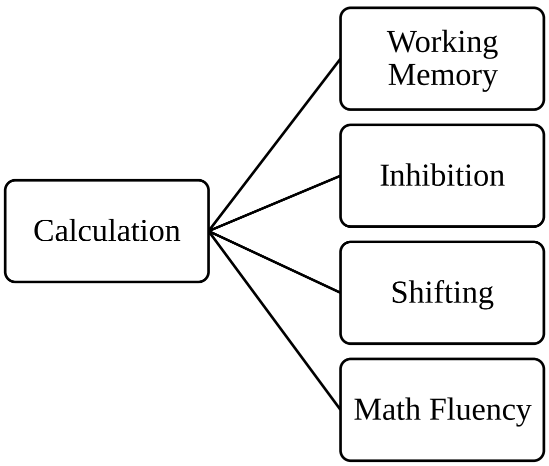


Figure 4. Calculation Regression; Sequential Block 2



The models in figures 5 and 6 represent the hierarchical regression that was conducted for problem solving as the dependent variable, which included math fluency, calculation, working memory, inhibition, and shifting as predictors. It was hypothesized that working memory, inhibition, and shifting would predict problem solving above and beyond calculation and math fluency performance, and that math fluency and calculation would add significant variance to the model.

Figure 5. Problem Solving Regression; Sequential Block 1

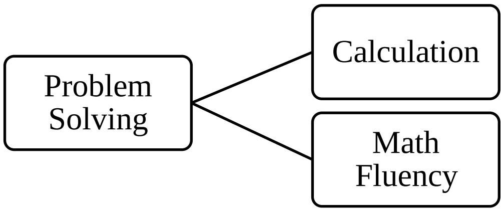
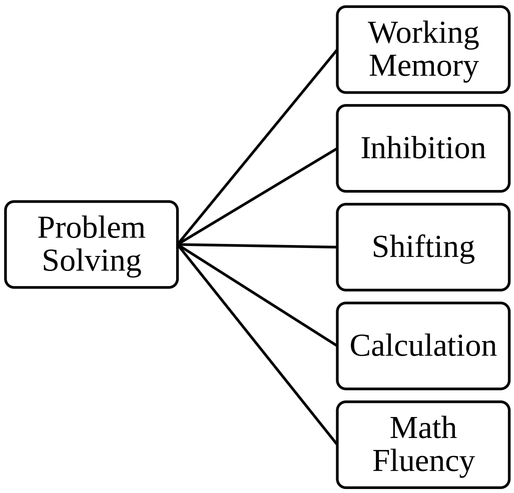


Figure 6. Problem Solving Regression; Sequential Block 2



To address research question four, the amount of R^2 in the models using performance based measures versus parental ratings of EF was compared. A comparison was made for each mathematical skill; the better model was determined by having a larger proportion of variance, as indicated by a higher R^2.

Results

Before addressing the research questions, a one-way between subjects analysis of variances (ANOVA) was conducted to compare the effect of SES on the three dependent variables: math fluency, calculation, and problem solving. SES was measured using parental education levels. The levels of SES included high school, college, university, and post-grad education. There was no significant effect of SES on math fluency, calculation, nor problem solving. Refer to Table 1 for a summary of statistics.

Table 1

One-Way ANOVA of Parent Education Levels by Math Skills

	Sum of Squares	df	Mean Square	*F*
Math Fluency				
Between Groups	55.25	3	18.42	.23
Within Groups	4862.50	61	79.71	
Total	4917.75	64		
Calculation				
Between Groups	8.30	3	2.77	1.37
Within Groups	1252.32	62	20.20	
Total	1260.62	65		
Problem Solving				
Between Groups	39.28	3	13.09	1.62
Within Groups	492.97	61	8.08	
Total	532.25	64		

Note. *$p<.05$. **$p<.01$.

Additionally, a one-way between subject ANOVA was conducted to compare the effects of SES on both performance based measures and parental measures of EF. Refer to Table 2 for a summary of statistics. There was no significant effect of SES on performance-based measures of verbal WM, inhibition, or shifting. However, there was a significant effect of SES on visual-spatial WM F (3, 62) = 3.31, p <.05. Post hoc comparisons using the Least Significant Difference (LSD) indicated that the mean score for parents who obtained a university degree (M = 115.37, SD = 14.7) was significantly different from those who completed high school (M = 101.63, SD = 11.63) and college (M = 104.8, SD = 12.26). However, the post-grad group (M = 111.13, SD = 15.82) did not significantly differ from the other education levels. Of note, all of these mean scores are within the broad average range. There was no significant effect of SES on parental ratings of WM, shifting, and inhibition.

Table 2

One-Way ANOVA of Parent Education Levels by EF Variables

	Sum of Squares	df	Mean Square	*F*
Verbal WM				
Between Groups	1965.80	3	655.27	2.63
Within Groups	15475.65	62	249.61	
Total	17441.46	65		
Visual-Spatial WM				
Between Groups	1945.71	3	648.57	3.31*
Within Groups	12139.32	62	195.80	
Total	14085.03	65		
Inhibition				
Between Groups	112.54	3	37.51	.18
Within Groups	12266.07	59	207.90	
Total	12378.60	62		
Shifting				
Between Groups	282.39	3	94.13	.34
Within Groups	16164.21	59	273.97	
Total	16446.60	62		

BRIEF WM				
Between Groups	15.5	3	5.17	.06
Within Groups	3781.48	40	94.54	
Total	3796.98	43		
BRIEF Inhibition				
Between Groups	248.39	3	82.8	.86
Within Groups	3834.79	40	95.87	
Total	4083.18	43		
BRIEF Shifting				
Between Groups	30.24	3	10.1	.10
Within Groups	3984.56	40	99.61	
Total	4014.8	43		

Note. **p* < .05. ***p* < .01.

Furthermore, a correlational analysis was conducted; intercorrelations among variables are presented in Table 3, and central tendency is presented in Table 4. The correlational analysis revealed that SES was not significantly correlated with any of the dependent variables, therefore it was determined that SES would not be included as a predictor in the multiple regression models. Additionally, the analysis revealed that verbal working memory and visual-spatial working memory (as assessed by performance-based measures) were significantly correlated with all math skills. Inhibition was significantly correlated with problem solving only. Shifting was not significantly correlated with any of the math measures. Parental ratings of working memory were significantly correlated with calculation and problem solving. Parental ratings of shifting were significantly correlated with math fluency and calculation.

Several of the EF measures were also significantly correlated with one another. Verbal working memory, visual-spatial working memory, inhibition, and shifting were all significantly correlated with one another. Parental ratings of working memory were significantly correlated with all performance based EF measures. Parental ratings of shifting were significantly correlated with performance based EF measures of verbal working memory and inhibition. All of the parental ratings of EF, including working memory, inhibition, and shifting, were significantly correlated with one another.

Table 3

Intercorrelations for Predictor and Outcome Variables

Variables	1	2	3	4	5	6	7	8	9	10
1. SES	-									
2. Math Fluency	.08	-								
3. Calculation	-.01	.52*	-							
4. Problem Solving	.19	.22*	.55**	-						
5. Verbal WM	.33**	.33**	.42**	.58**	-					
6. Visual-Spatial WM	.31*	.23*	.21*	.44**	.55**	-				
7. Inhibition	.01	.08	.13	.34**	.32**	.27**	-			
8. Shifting	-.03	.18	.16	.10	.20*	.28**	.33**	-		
9. BRIEF WM	-.10	-.15	-.29*	-.37**	-.36**	-.33**	-.27*	-.30*	-	
10. BRIEF Inhibition	-.08	-.02	-.10	-.05	.07	-.13	-.19	-.08	.45**	-
11. BRIEF Shifting	-.01	-.26*	-.25*	-.25*	-.32*	-.23	-.42**	-.18	.66**	.55**

Note. *$p < .05$. **$p < .01$. Higher BRIEF scores indicate more EF problems.

Table 4

Means, Sample Size, and Standard Deviations for Predictor and Outcome Variables

Variables	*N*	*M*	*SD*
1. SES	67	2.67	.86
2. Math Fluency	81	15.58	8.78

3. Calculation	82	20.93	4.53
4. Problem Solving	79	11.44	2.8
5. Verbal WM	82	103.01	15.36
6. Visual-Spatial WM	82	112.32	14.76
7. Inhibition	79	100.68	13.6
8. Shifting	79	104.43	16.31
9. BRIEF WM	53	51.96	9.03
10. BRIEF Inhibition	53	49.83	9.3
11. BRIEF Shifting	53	49.72	9.5

Math Fluency

Prior to addressing the first research question, the relevant assumptions of the multiple regression analysis were tested. The assumption of linearity was met and examined using scatterplots, and multicollinearity was met and examined using VIF scores. Independency of the residuals was met and examined using the Durbin-Watson statistic, for which all values were close to 2. Homoscedasticity and normality were checked by evaluating plots, Q-Q plots, and Kolmogorov-Smirnov and Shapiro-Wilk tests. Both assumptions were met. No outliers or influential data points were found, as all Cooks D and leverage values were less than 1.

To investigate the first research question, whether performance based or parental measures of EF better predict math fluency, two separate hierarchical multiple regression analyses were conducted. The first regression included age in the first sequential block. Performance based EF measures of verbal WM, visual-spatial WM, inhibition and shifting were added into the second sequential block. Regression statistics are reported in Table 5. The results of the hierarchical regression revealed that the model for block one was significant, F (2,58) = 11.30, p<.001, accounting for 28% of the variance in math fluency. Adding verbal and visual-spatial WM, inhibition, and shifting explained an additional 12% of variance, and this change in R^2 was significant, F (4,71) = 3.65, p < .01. When all independent variables were included in the model, they accounted for 41% of the variance in math fluency skills, *F* (5,71) = 9.69, $p < .001$. Only age (β = .54, $p < .001$) and verbal WM (β = .30, $p < .01$) were significant predictors in the model.

Table 5

Hierarchical Regression Analysis for Performance Based EF Variables Predicting Math Fluency

Variable	β	*R*	R^2	ΔR^2
Block 1		.53	.28	.28
Age	.53			
Block 2		.64	.41	.12
Age	.54***			
Verbal WM	.30*			
Visual-Spatial WM	.07			
Inhibition	.07			
Shifting	.06			

Note. *N* = 77, **p* < .05, ***p* < .01, ****p* < .001

The second hierarchical multiple regression was conducted and included age in the first sequential block, and additional variables of BRIEF measures of WM, inhibition and shifting in the second block. Regression statistics are reported in Table 6. The results of the hierarchical regression revealed that the model for block one was significant, *F* (1,50) = 15.22, $p < .001$, accounting for 23% of the variance. Adding WM, inhibition and shifting explained an additional 12% of variance to the model, and this change in R^2 was significant, *F* (3,47) = 3.03, $p < .05$. When all independent variables were included in the model, they accounted for 30% of the variance in math fluency skills, *F* (4,47) = 6.54, $p < .001$. Only age (β = .52, $p < .001$) and shifting (β = -.41, $p < .05$) were significant predictors in the model.

Table 6

Hierarchical Regression Analysis for BRIEF EF Variables Predicting Math Fluency

Variable	β	*R*	R^2	ΔR^2
Block 1		.48	.23	.23
Age	.48**			
Block 2		.60	.36	.12
Age	.52***			
WM	.00			
Inhibition	.11			
Shifting	-.41*			

Note. *N* = 52, **p* < .05, ***p* < .001

Calculation

Prior to addressing the second research question, the relevant assumptions of the multiple regression analysis were tested. The assumptions of linearity, multicollinearity, independency, homoscedasticity and normality were met. No outliers or influential data points were found, as all Cooks D values were less than 1. All missing data was deleted using a list-wise deletion method.

To investigate the second research question, whether performance based or parental measures of EF better predict calculation skills, two separate hierarchical multiple regression analyses were conducted. The first regression included math fluency in the first sequential block. Performance based EF measures of verbal WM, visual-spatial WM, inhibition and shifting were added into the second sequential block. Regression statistics are reported in Table 7. The results of the hierarchical regression revealed that the model including only math fluency was significant, F (1,75) = 31.91, p<.001, accounting for 29% of the variance in calculation. Adding verbal and visual-spatial WM, inhibition, and shifting explained an additional 6% of variance to the model, however the change in R^2 was not significant, F (4,71) = 1.40, *ns*. Thus, performance based measures of EF do not have a significant relationship with calculation skills over and above math fluency. When all independent variables were included in the model, they accounted for 36% of the variance in calculation skills, F (5,71) = 8.11, $p < .001$. Only math fluency (β = .46, $p < .001$), and verbal WM (β = .28, $p < .05$) were significant predictors.

Table 7

Hierarchical Regression Analysis for Performance Based EF Variables Predicting Calculation

Variable	β	R	R^2	ΔR^2
Block 1		.55	.30	.30
Math Fluency	.55***			
Block 2		.60	.36	.06
Math Fluency	.46***			
Verbal WM	.28*			
Visual-Spatial WM	.00			
Inhibition	-.02			
Shifting	-.01			

Note. N = 77, *$p < .05$, **$p < .001$

The second hierarchical multiple regression included math fluency in the first sequential block and parental ratings of WM, inhibition and shifting were added into the second block. Regression statistics are reported in Table 8. The results of the hierarchical regression revealed that the model including only math fluency was significant, F (2,50) = 32.96, $p < .001$, accounting for 40% of the variance. Adding WM, inhibition and shifting explained an additional 4% of variance to the model, and this change in R^2 was not significant, F (3,47) = 3.51, *ns*. Thus, BRIEF measures of EF do not have a significant relationship with calculation skills over and above math fluency skills. When all independent variables were included in the model, they accounted for 44% of the variance in calculation skills, F (4,47) = 9.14, $p < .001$. Only math fluency (β = .63, $p < .001$) was a significant predictor in the model.

Table 8

Hierarchical Regression Analysis for BRIEF EF Variables Predicting Calculation

Variable	β	*R*	R^2	ΔR^2
Block 1		.63	.40	.40
Math Fluency	.63***			
Block 2		.66	.44	.04
Math Fluency	.63***			
WM	-.08			
Inhibition	.11			
Shifting	-.22			

Note. $N = 52$, $*p < .05$, $**p < .01$

Problem solving

Prior to addressing the third research question, the relevant assumptions of the multiple regression analysis were tested. The assumptions of linearity, multicollinearity, independency, and homoscedasticity were met. The Kolmogorov-Smirnov test was significant, therefore violating the assumption of normality. However, the Q-Q plot appeared normal, and all Cooks D and leverage values were less than 1. Therefore, no significant outliers were found. All missing data was deleted using a list-wise deletion method.

To investigate the third research question, whether performance based or parental measures of EF better predict problem solving skills, two separate hierarchical multiple regression analyses were conducted. The first regression included math fluency and calculation skills in the first sequential block. Performance based EF measures of verbal WM, visual-spatial WM, inhibition and shifting were added into the second sequential block. Regression statistics are reported in Table 9. The results of the hierarchical regression revealed that the model including only math fluency and calculation was significant, F (2,71) = 14.66, $p < .001$, accounting for 29% of the variance. Adding verbal and visual-spatial WM, inhibition, and shifting explained an additional 21% of variance to the model and the change in R^2 was significant, F (4,67) = 7.27, $p < .001$. When all independent variables were included in the model, the model accounted for 51% of the variance in problem solving, F (6,67) = 11.46, $p < .001$. Only calculation skills (β = .44, $p < .001$) and verbal WM (β = .29, $p < .05$) were significant predictors.

Table 9

Hierarchical Regression Analysis for Performance Based EF Variables Predicting Problem Solving

Variable	β	R	R^2	ΔR^2
Block 1		.54	.29	.29
Math Fluency	-.07			
Calculation	.57***			
Block 2		.71	.51	.21
Math Fluency	-.15			
Calculation	.44***			
Verbal WM	.29*			
Visual-Spatial WM	.20			
Inhibition	.16			
Shifting	-.05			

Note. N = 74, *p < .05, **p < .001

The second hierarchical multiple regression included math fluency and calculation skills in the first sequential block and parental ratings of WM, inhibition, and shifting were added into the second block. Regression statistics are reported in Table 10. The results of the hierarchical regression revealed that the model including math fluency and calculation was significant, F (2,46) = 15.14, $p < .001$, accounting for 40% of the variance. Adding WM, inhibition and shifting explained an additional 5% of variance to the model, and this change in R^2 was not significant, F (3,43) = .1.30, *ns*. Thus, BRIEF measures of EF do not predict problem solving skills over and above lower level math skills. When all independent variables were included in the model, they accounted for 45% of the variance in problem solving, F (5,43) = 6.95, $p < .001$. Only calculation skills (β = .63, $p < .01$) was a significant predictor in the model.

Table 10

Hierarchical Regression Analysis for BRIEF EF Variables Predicting Problem Solving

Variable	β	R	R^2	ΔR^2
Block 1		.63	.40	.40
Math Fluency	-.10			
Calculation	.69***			
Block 2		.67	.45	.05
Math Fluency	-.05			
Calculation	.63***			
WM	-.31			
Inhibition	.11			
Shifting	.14			

Note. N = 49, *$p < .05$, **$p < .01$

To address research question four, whether performance-based measures of EF or parental ratings of EF better predict math fluency, calculation and problem solving skills, the R^2, or variance accounted for the performance based measures versus parental ratings of EF were compared. For math fluency skills, both performance based measures of EF and parental ratings of EF accounted for 12% of variance when added to the model. For calculation, performance based measures of EF better predicted calculation skills as this model accounted for 6% of variance over and above lower level math skills (i.e., math fluency), while parental measures of EF only accounted for an additional 4%. However, while performance based measures accounted for more variance, these measures only account for 2% more than parental measures, which is a small amount of variance. For problem solving, performance based measures of EF better predicted problem solving skills as this model accounted for 21% of variance over and above lower level math skills (i.e., math fluency and calculation), while parental measures of EF only accounted for an additional 5%.

Discussion

The first objective of this study was to determine if working memory, inhibition, and shifting are related to mathematics achievement across the individual math skills of math fluency, calculation, and problem solving. The second objective was to determine whether performance based tests or parental ratings of EF were more strongly related to these three math skills. When considering the results from this study, it is critical to consider the sample. Most previous studies in this literature have used samples of special populations that demonstrate EF deficits, including those with specific learning disorders in math or ADHD. The current study aimed to explore the relationship between EF and math using a typically developing sample. As such, it is difficult to compare the results of this study to those that exclusively sampled special populations. In fact, in a recent meta-analysis the authors concluded that the relation between WM and mathematics was significantly influenced by the types of mathematics skills assessed and sample type (Peng et al., 2016). In other words, there may be a good reason not to necessarily expect this study to produce the same results as those previously conducted on this topic.

Regarding math fluency, this study hypothesized that EF skills (working memory, inhibition, and shifting) would predict performance, and that each skill would contribute unique variance to the model. In theory, working memory is important during math fluency tasks for the retrieval of number facts from long-term memory, and the verbal rehearsal of these facts, while shifting is important as one is required to shift between the use of different solutions and calculation signs (i.e., addition, subtraction, multiplication, and division). For math fluency skill development, Geary (1993) argues that working memory is necessary to associate problems and their corresponding correct answers so that the association can be stored and later retrieved from long-term memory. Inhibition is also critical during math fluency tasks, in order to suppress previously used inefficient strategies and calculations, while ignoring external distractors that may exist in the environment.

The results of this study suggest that EF skills are correlated with math fluency over and above age. Specifically, verbal working memory was a significant predictor in this model. Although this finding is contrary to the numerous studies that have established the important role of visual-spatial working memory in mathematics ability in general (Jarvis & Gathercole, 2003; Maybery & Do, 2003), the results are best understood when considering the age range of the sample, and the importance of verbal working memory during math fluency. Many researchers have argued that younger children rely more on visual representations (i.e., finger counting, number lines) (e.g., De Smedt et al., 2009) but as children grow older, learning becomes more dependent on verbal rehearsal and the phonological loop (De Smedt et al., 2009; Holmes & Adams, 2006). When children learn to memorize basic math facts (e.g., 2 + 2, 3 X 4), children transform math numbers and symbols into verbal code (Geary et al., 1996) which they memorize and later retrieve during math fluency recall. They use this strategy rather than computing an answer. This study was composed of students in grades 3 to 6 who are old enough to be less reliant on visual-spatial strategies, and instead may have developed verbal codes for retrieving math facts fluently. Therefore, the participants in this sample were likely retrieving and holding verbal codes in their verbal working memory while completing this task.

Since the math fluency task was designed to access participants' previously stored basic math fact knowledge and not a higher-level skill, it would not necessarily be expected that it would relate strongly with all EF components. This is consistent with Bull & Scerif's (2001) claim that executive functioning is less important once a skill becomes more automatic and individuals have well-practiced strategies. Math fluency tasks are a common practice in today's classroom, and it is likely that the participants' familiarity with this task required limited executive functioning demands such as shifting and inhibition. This is further confirmed by the sample's performance on the math fluency task, which suggests that most of the participants were able to fluently complete a reasonable amount of questions within the time limit.

When examining calculation, the performance based measures of EF did not significantly correlate with calculation skills over and above math fluency, though verbal working memory was a significant predictor in the final model. It is likely that verbal working memory contributes to calculation performance as children recall procedural strategies from long term memory, and then keep this information online in memory using while performing other mental operations. For example, when a student is presented with a multi-step computation problem (108+57), they must first recall the procedural steps for solving the problem. Once this procedure is recalled, they must temporarily store this information while they perform other mental operations such as adding the ones, tens and hundreds columns, and/or counting. Furthermore, intermediate values must be temporarily stored when carrying values across columns. Many studies have established relationships between the phonological loop (often referred to as verbal working memory) and mathematical calculations including addition and subtraction (Siegler & Jenkins, 1989), subtraction (Seyler, Kirk, & Ashcraft, 2003), and multiplication (Seitz & Schumann-Hengsteler, 2000), however, the exact role of verbal working memory remains unclear.

While few EF predictors significantly correlated with calculation, math fluency provided a strong relationship. This finding is in line with Fuchs et al. (2006) correlational study that simple arithmetic was a significant path to multi-step computation and one of the few predictors of calculation. Indeed, one must be able to quickly recall basic math facts during more complex multi-step calculations, and if one has poor math fluency skills, this can lead to difficulties executing calculation procedures (Geary, 1993). If one is allocating their working memory resources to count and/or compute simple calculations rather than quickly retrieving facts, they may not be able to keep procedures and next steps online and/or store intermediate values, which consequently interferes with efficient and accurate computations.

In regards to math problem solving, the addition of EF performance based tests explained a significant portion of the variance over and above lower level math skills (i.e., math fluency and calculation). Of the two lower level math skills, only calculation was a significant predictor of problem solving. It is probable that calculation was a predictor of problem solving as the current study's measure of problem solving did require examinees to perform multi-step calculations. The following example illustrates the type of questions included in the KeyMath 3 problem solving subtest: Train riders can buy a single ride for 4 dollars, or a monthly pass for 60 dollars. Molly rode the train 20 times last month. How much did she save buying a monthly pass? To solve this question, one must complete a series of steps. First, one must identify the problem and determine what the question is asking. Next, one must develop a strategy for solving the problem (e.g., algebraic equation, trial and error, etc.). Next, one must solve the computation using information from the problem. Then, one must assess the solution to determine if it answers the problem. Finally, one must interpret the solution and convert the number into a verbal response. This item in particular required solving a multi-step computation. First, the examinee must calculate how much it costs to ride the train 20 times. Then, one must use subtraction to determine the difference between the cost of riding the train 20 times and the cost of the monthly train pass. Therefore, the finding that calculation significantly predicts problem solving contradicts Fuchs and colleagues' null finding, but is consistent with their argument that lower level skills are a prerequisite for higher level mathematics skills.

Of the EF skills, again, only verbal working memory was a significant predictor, yet all of the EF skills together contributed a large amount of variance in the final model. Many studies have established the importance of the phonological loop in problem solving (Gathercole & Pickering, 2000; Gathercole et al., 2004; Geary, Brown, & Samaranayake, 1991; Swanson & SachseLee, 2001). Kintsch & Greeno (1985) and Mayer (1992) propose a theoretical framework for understanding the role of working memory in arithmetic word problems. They theorize that when a child is presented with a problem, new problem solving sets are formed online as the story, or problem, is processed. Once the strategy schema is completed, previous sets that had been active in the memory buffer are discarded. All of the incoming information, as well as the schema development must be kept in memory and tracked.

Interestingly, visual-spatial working memory was not a significant predictor of problem solving, despite the literature that establishes the relationship between the two. The lack of visual-spatial working memory findings can be understood when one considers the age of the participants and the problem solving measure. First, participants were enrolled in grades 3 to 6. At this point in their education, the curriculum demands are significantly higher than grades 1 and 2 as students are expected to solve multi-step multiplication and division calculations, and complex word problems. Research has shown that visual-spatial working memory, sometimes referred to as the visual-spatial sketchpad, is more important and relied on in earlier grades (Holmes & Adams, 2006). Typically, before the age of 7 and before the onset of spontaneous verbal rehearsal, children rely on visual-spatial representations to keep information on line in memory (McKenzie, Bull, & Gray, 2003). A second reason why we believe that visual-spatial working memory was not a significant predictor of problem solving has to do with the problem solving measures used in the current study. The Applied Problem Solving subtest of the KeyMath 3 provides examinees with a visual representation for each problem; these representations included graphs, charts, figures, labeled shapes, as well as numbers needed to make calculations. Thus, the visual-spatial demands of this task were minimized as examinees did not need to keep a visual-spatial representation online in their sketchpad.

Inhibition was correlated with problem solving, but surprisingly was not a significant predictor. Inhibition has been implicated in problem solving by acting as a buffer from preventing unnecessary information from entering working memory. When this information is cognitively inhibited, there is more working memory capacity to consider alternative interpretations or solutions. For example, Passolunghi, Cornoldi, and DeLiberto (1999) found that poor problem solvers remembered less relevant but more irrelevant information in mathematics problems.

Overall, the lack of findings for inhibition and shifting across math skills was not expected as numerous studies have found a relation between these variables. Although strong relationships have been established, the literature remains unclear as to what extent inhibition contributes to problem solving, as relationships are not always evident. Further, the literature is also unclear as to *how* inhibition and shifting play a role in math. For example, Andersson's (2008) and Swanson's and Frankenberger's (2004) study failed to demonstrate that inhibition or shifting contributed to math performance. The authors postulate that inhibition and shifting are central processes that are critical during all working memory tasks and contribute to working memory, which in turn affects math performance. In relation to this study, it may be that the inhibition and shifting skills that were required during the verbal working memory task used in this study, and their contributed variance was accounted for in the verbal working memory score, thus eliminating the correlations between inhibition, shifting, and math skills. The verbal working memory task required participants to listen to, and shift quickly between a recall and fact retrieval task, and then to recall a list of words, which required inhibition of interference during the fact retrieval task. A second reason why inhibition and shifting did not contribute a unique amount of variance may be due to the math task demands. It may be that the math fluency, calculation, and problem solving tasks were not cognitively demanding enough, and thus did not require the use of inhibition and shifting skills. In fact, the ability to shift flexibly and inhibit certain procedures or information may be more important for more complex mathematical problem solving required in later grades (Bull et al., 1999; Bull & Scerif, 2001; van der Sluis et al., 2004).

The second goal of the current study was to determine whether performance-based measures of EF or parental ratings of EF exhibited stronger relationships with math fluency, calculation, and problem solving. In theory, both models should account for a similar amount of variance and should have the same significant predictors, as the models include similar EF variables. In this study, the parental ratings of EF were generally less related to math skills than the performance based tasks, with one exception. For math fluency skills, both performance based measures of EF and parental ratings of EF accounted for the same amount of variance (12%). In this case, no method of measurement was superior to the other. This was also the only instance in which a parental rating of EF was a significant predictor; shifting was a significant predictor of math fluency in this final model. The biggest discrepancy between the variance explained between performance based and parent rating measurements of EF occurred when predicting problem solving; in this case, performance based tests accounted for 16% more of the variance than parental ratings of EF.

Parental ratings and performance based measures of EF purport to measure the same construct (i.e., EF). Yet, as evidenced by the inconsistencies between the performance based and parental ratings of EF as predictors of math skill, and the lack of correlations between these variables, it is unlikely that the same EF constructs are being measured by these two methods. Parental ratings of EF as measured by the BRIEF are ‘real world’ behaviour observations within the home. As such, it is unlikely that these ratings were able to validly assess the participants’ behavioural EF that occurs in the classroom, particularly during performance of academic tasks. The use of teacher ratings rather than parental ratings may have provided a more valid assessment of the participants’ behavioural EF that contributes to academic achievement. For example, Samuels and colleagues’ (2016) longitudinal study found that *teacher* BRIEF scores predicted math grade point average consistently over the course of 4 years. Additionally, Waber, Gerber, Turcios, Wagner and Forbes (2006) found that *teacher* ratings on the BRIEF were significant predictors of performance on academic testing.

Lastly, the lack of relationship between these two measures may also be due to the types of scores provided by the BRIEF, as well as the sample composition. The BRIEF was developed to identify problematic EF for the purposes of accommodation or intervention. As such, it is not intended to identify EF strengths, only a level of EF that is more problematic than the norm for a child’s age. As this sample was composed of typically developing children, a problem-based parent rating may not have been able to adequately capture the EF of this sample. This may also explain the weaker correlations between this measure and the sample’s age appropriate math development.

General Conclusion

In conclusion, in this study, verbal working memory was consistently related to all math skills, above and beyond age or any lower level skill. Lower level math skills were typically correlated with higher order math, though only calculation was important for predicting problem solving. These findings are similar to those found in a recent meta-analysis summarizing the research of the relationship between working memory and mathematics (Peng et al., 2016). In this work, the authors found that math problem solving had the strongest relations with working memory. This is likely due to problem solving being the highest order math skill, meaning it requires the most application of EF skills. Furthermore, the authors found that the relation between working memory and mathematics was stronger among individuals with math disabilities than those with typical math skill development. This is a fairly common finding in the literature as well (e.g., Swanson & Jerman, 2006). Because the sample used in this study was composed of typically developing children, the association between visual-spatial working memory and math may not have been strong enough to detect. This relationship may have otherwise been detected in a sample of individuals with math disabilities.

The literature has consistently established a robust relationship between working memory and math (Peng et al., 2016). However the exact role of working memory in math, and the interaction with inhibition and shifting remains unclear. What is clear, is that the sample type studied and how EF and math skills are assessed influence findings across studies. Recruiting a typically developing sample or one composed of individuals with math disabilities may lead to variability in the relationships observed between EF and math. The way researchers define and measure EF constructs also influences results; many EF constructs have varying and inconsistent definitions and numerous tests are assumed to measure the same construct. For example, one common measure of verbal working memory is the digit span task. This task requires examinees to remember a string of digits and to repeat them backwards. The verbal working memory task used in this study used language stimuli rather than digits. Yet both tasks theoretically measure the same construct. Raghubar, Barnes and Hecht (2010) argue that the digit span measure may overestimate the role of EF skills in mathematics compared to non-numerical tasks such as the one used in this task. This is because of the digit span task uses numeric stimuli which are domain specific to math, where the verbal working memory task used in this study does not.

Performance based measures of EF were superior to parental ratings for predicting calculation, and especially for problem solving. Additionally, little to no correlations were found between performance based measures and parental ratings, suggesting that these measurement tools assess different EF constructs. These results highlight the utility of using performance based measures of EF instead of parental ratings to identify EF strengths and weaknesses to identify math learning profiles, inform intervention and accommodations, and instructional practices.

Implications for Practice

Although findings from this study do not have direct implications for intervention or instruction, the findings do speculate about the importance of verbal working memory in the classroom. Educators may want to consider the importance of verbal working memory for math and consider this relationship in delivering instruction and when targeting strategies for students who struggle. For example, if a teacher is aware of a student’s difficulties with verbal working memory, the teacher can provide individualized instruction and accommodations to this student such as direct instruction, scaffolding, memory aids and strategies, formula sheet, and/or a calculator. They will also know that this student will likely struggle with the verbal working memory demands of most areas of math and be able to prevent them from experiencing considerable frustration by reducing the working memory demands of tasks. Additionally, it is critical that teachers understand the importance of verbal working memory during the completion of math and promote the use of verbal rehearsal strategies that are essential during math fluency, calculation, and problem solving.

Second, these findings have implications for the way in which professionals choose assessment tools to understand students' learning profiles and to tailor instruction and supports. For example, the findings from this study suggest that parental ratings of EF are not strongly related to math fluency, calculation or problem solving. Parental ratings are a common assessment tool and often connections are made between a child's EF deficit as identified by parental ratings and their math difficulties. Yet, findings from this study suggest that parental ratings are not strong correlates and predictors of math performance. However performance based tests are more strongly associated with math performance. It is important that professionals choose valid assessment tools that can make inferences about a student's education. As such, it is important that professionals understand that performance based measures are more effective than parent ratings at identifying EF strengths and weaknesses that are related to math. Rourke and Conway (1997) iterate the utility of early psychological assessments that reveal patterns of strengths and weaknesses that may be predictive of later academic performance. By identifying these strengths and weaknesses, students who struggle in math can be identified early, which can lead to earlier provision of services and accommodations, and ultimately reduced frustration.

Lastly, given that lower level math skills contribute to the development and achievement of higher order math skills, it is essential that students master prerequisite skills before learning the next skill in the hierarchy. The results from this study will help teachers understand the importance of mastering lower level skills, as well as the potential areas of intervention for remediating difficulties. For example, if a student is struggling with word problems, and the teacher is able to identify weak calculation skills, then calculation may be an appropriate area to target to ultimately improve achievement on both calculation and word problem tasks.

Limitations

There are several limitations to acknowledge in this study. To begin with, EF lacks a consistent conceptual definition, as does many of its components (working memory, inhibition, and shifting). Furthermore, many studies use a wide variety of measures to assess the same construct, making comparison of findings across studies difficult. Fuchs et al. (2006) argue that many studies do not include enough predictors in their model when looking at math skills. This study did not include many EF skills that may be important for mathematical achievement including planning, organization, self-monitoring, and attention. Also, in their meta-analysis Peng et al., (2016) highlight the influence of EF assessments. While this study used common measures of EF with good psychometrics, the findings may still depend on instrumentation.

Additionally, it is important to consider the possible predictive validity of performance based tests of EF; they are administered individually in a quiet environment. Arguably, in such a setting EF demands are minimized. It would be interesting to assess participants EF abilities within a typical busy classroom, the environment in which students are expected to operate daily.

Due to the nature of the research design, the results from the current study do not imply causation. Verbal working memory was shown to be well correlated with math fluency, calculation and problem solving, but true prediction and directionality of the developmental relationship cannot be established without a longitudinal design. For instance, it remains unclear as to what extent math skills may predict EF skills.

SES was not included as a predictor in this study as it was not correlated with math skills, nor did group differences exist on measures of math. However, SES is an important variable to consider when examining the relationship between cognitive skills and academic achievement. The lack of variability in the obtained SES indicator could be due to a couple of reasons. First, it could be that the way in which parental education level was measured was not a valid indicator of the socioeconomic status of this sample. Only one parent of each child participant indicated their education level, and therefore we may not have obtained the highest level of education within the home. Second, all participants were recruited from a private school where parents pay student tuition. Therefore, it could be that participants were generally from middle income to higher income homes.

Another limitation of this study concerns the mathematics measures used in this study. Given that the sample performed well within the average range on both norm-referenced measures, it is likely that these tests were likely not too difficult for them. As such, the tasks were not overly demanding, and likely did not require the application of EF skills. Furthermore, only three math skills were assessed (fluency, calculation, and problem solving), which is only a small subset of mathematics skills that are taught at school. A more comprehensive study may be interested in examining the relationship of EF and a broad range of math skills including the ones in this study as well as number sense, algebra, measurement, geometry, etc.

Another limitation to this study concerns the composition of the sample. First, the sample consisted of a larger proportion of East and Southeast Asian participants (82%) than what is representative within Vancouver (31%; Statistics Canada, 2016). The sample consisted of 2.6% European participants, which is much lower than what is representative in Vancouver (29%). As such, readers are cautioned when considering the findings and should not make generalizations beyond these ethnicities or the grades of the participants (grades 3-6). An additional limitation concerning the sample pertains to the size. Stevens (1996) recommends 15 cases per predictor. This recommendation was not met for any of the analyses using parental ratings of EF. All parents were sent home BRIEF forms to complete, however only approximately 50% were returned. For performance-based measures, this recommendation was met for the math fluency analysis, but not the calculation and problem solving analyses. A bigger sample size may have resulted in a wider range of scores on EF and math measures, and may have resulted in more power to detect significant results.

Future Research

Future studies may consider assessing a larger number of predictors of math. The independent variables in this study were able to predict a significant amount of variance, yet there was still a lot of variance left to be explained. Fuchs and colleagues (2006) highlight the importance of including phonological processing, long-term memory, processing speed, nonverbal intelligence, and reading abilities have been shown to correlate with math. Additionally, future studies may also want to assess more demanding math tasks and additional areas of math achievement, or even older grades where math is more complex and requires more EF. Furthermore, more investigation of the utility of using parental ratings to inform assessments, educational practices, and their incremental validity over performance based tests is needed.

Lastly, it may also be of importance to not only assess math achievement on standardized measures, but to also consider curriculum mastery as this is the most ecologically valid assessment of a student's achievement. In fact, relationships between EF and curriculum attainment in math have been established (St Clair-Thompson & Gathercole, 2006). Using curriculum attainment over standardized math measures may be advantageous as it would reduce the contrived testing environment that minimizes the use of EF. Curriculum mastery as measured by report cards or grade point average may demonstrate stronger associations with executive functioning as classroom achievement requires higher-order EF skills such as task initiation, self-monitoring, and attention.

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