PARTICLE IMPINGEMENT ONTO A MOVING SUBSTRATE

by

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Abstract

Particle sprays are salient in processes such as erosion in turbomachinery and traction enhancement in the railroad industry. In this study, particles with a Stokes number at the nozzle much larger than unity were sprayed at an acute angle between a horizontal, flat, moving substrate, representing the rail, and a wheel. A device was created that contained a sprayer aimed between the wheel and moving substrate, with the rim of the wheel hovering above the flat substrate. The particles form a spray into the nip that can ricochet between both the wheel and substrate. The standard deviation of the angle that the particle trajectories make as they exit the nozzle can be used to describe the geometry of the spray. Similarly, the normal and tangential coefficients of restitution describe ricochet effects by quantifying the amount of energy dissipated through the particle-substrate interaction. Initial work in this study focuses on the spray and bounce properties to determine their correlation to the particle’s efficiency, which was defined as the percentage of particles leaving the nozzle that make it into the nip. Next, the effects of shape and size were determined using particles with similar compositions. Then, the effect of coating the particles and substrate speed was determined. For silica sand with diameters from 0.2-0.6 mm, the typical efficiency was 68% with a flat wheel and rail profile. The larger and rounder particles were found to deposit the best, with coated sand having an efficiency of 91%. It was discovered that the improvement in the efficiencies may be from reducing the spread of the particle spray from the nozzle from $8^\circ$ to $3^\circ$, which increased the likelihood that the particles make it between the wheel and belt. Lastly, decreasing the substrate speed below 18 km/h produced lower efficiencies due to the particle-particle interactions as they approach the wheel-substrate interface unless the mass flowrate of the particles was reduced.
Lay Summary

This thesis investigates particle sprays when trains use sanders to deliver sand between their wheels and the rail for traction. It was found that less material makes it between the wheel and rail when the sander sprays the particles in a wider fan. Also, increasing the size and decreasing the roundness of the particles caused lower efficiency. Finally, when the train is moving slower than 18 km/h, the amount of particles sprayed should be reduced to minimize particle waste. The results of this thesis can be used to help optimize the efficiency of locomotive sanders. An increase in sanding efficiency allows for more effective braking and better traction while carrying less sand on the train. This can lead to fewer delays and less waste.
Preface

The author of chapter 2 is Justin Roberts, who worked under the supervision of Dr. Sheldon Green. Representatives from L.B. Foster® determined the need for a testing apparatus and helped define its constraints. The device was then created by Justin Roberts under the close supervision of Dr. Green.

Chapter 3 contains specimens obtained from L.B. Foster®. Testing was conducted by Justin Roberts under the supervision of Dr. Sheldon Green.
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List of Symbols

\( \alpha \) Volume Fraction of the Particle-Laden Flow

\( A \) Area

\( \beta \) Angle Between the Sander Nozzle and Belt

\( C \) Chord Length

\( d \) Diameter

\( \delta \) Percent Difference

\( e \) Restitution of the Particle, \( \frac{v_f}{v_o} \)

\( \epsilon \) Percent Error

\( h \) Height

\( L \) Length

\( \dot{m} \) Mass Flow Rate

\( \mu \) Kinematic Viscosity

\( N \) Number of Samples

\( \eta \) Efficiency, \( \frac{m_{nip}}{m_{sander}} \)

\( \rho \) Density

\( R^2 \) Coefficient of Determination

\( \sigma \) Standard Deviation

\( \theta \) Angle of the Particle Trajectories

\( \bar{x} \) Median

\( \bar{x} \) Mean
\( \nu \) Velocity

\( V \) Volume

\( \dot{V} \) Volumetric Flowrate

\( v \) Kinematic Viscosity

\( W \) Width

\( Z \) Loading Ratio

Subscripts and Postscripts

1 Substrate 1

2 Substrate 2

\( \perp \) Normal

\( || \) Tangential

\( b \) A Bulk Particle Property

\( e \) Exit

\( f \) Fluid

\( fric \) Friction

\( i \) Initial

\( ins \) Insignificant

\( n \) Nozzle

\( N \) Normal

\( p \) Particle

\( w \) Weight
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>Two dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three Dimensional</td>
</tr>
<tr>
<td>A.R.</td>
<td>Aspect Ratio</td>
</tr>
<tr>
<td>CoR</td>
<td>Coefficient of Restitution (Defined as Normal Restitution)</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>RR</td>
<td>Restitution Ratio (Defined as Tangential Restitution)</td>
</tr>
<tr>
<td>PTV</td>
<td>Particle Tracking Velocimetry</td>
</tr>
<tr>
<td>Re</td>
<td>Reynold Number</td>
</tr>
<tr>
<td>St</td>
<td>Stokes Number</td>
</tr>
</tbody>
</table>
Acknowledgements

Thank you, Dr. Sheldon I. Green, for giving me this opportunity to learn about conducting research. Your guidance, patience, and teaching have helped immensely. You always made time to answer questions and give feedback, despite an incredibly overwhelming schedule. You are a role model to your students and my graduate experience would not have been nearly as fulfilling without your help. I would also like to thank Dr. Boris Stoeber for his input throughout the design phase.

Without the support and funding of L.B. Foster ® and NSERC, this research would not be possible. Likewise, without the help of George, Glenn, and Markus, construction of the device would not have been so quick. Thank you.

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To my mother and father.
Chapter 1: Introduction

Particle-laden flows impinging onto solid substrates are observed in applications such as pneumatic conveying of particles, fluidized beds, coatings, particle separators, and in the compressor blades of jet engines where erosion may be important [1]–[4]. This study focuses on jets with a high Stokes number at the nozzle and their interaction with moving substrates of moderate train speeds up to 30 km/h. Furthermore, this work is motivated by the locomotive industry to improve the efficiency of locomotive sanding that occurs during braking. Fundamentally, a locomotive sander consists of three parts: (i) a hopper to store the sand, (ii) a conveyance system that sends the sand through (iii) the nozzle before it is sprayed into the nip (Figure 1.1). This investigation focuses on the latter to improve the efficiency of the particles as they exit the nozzle and go into the nip. Here, the efficiency is defined as the percentage of sand that leaves the sander that passes into the wheel rail interface. Better braking on trains implies safer and faster transportation, with a reduction in delays caused by low traction on the rails. In addition to an increase in performance, there is a motivation to substitute sand with alternative materials since sand may create silica particulate that can lead to silicosis if inhaled by workers [5]–[7]. Improvements in the performance and safety of trains are of interest because the rail industry has shown promise as a more sustainable alternative to trucking, which is currently the most common method of shipping [8]. With online retailers and shipping expected to grow significantly, the rail industry may be expected to fulfill the quickly growing need to ship goods whilst still fulfill environmentally sustainable goals for the future [8].
Figure 1.1 A Schematic of a train sander highlighting its three fundamental parts: (i) a hopper or storage tank, (ii) a conveyance system, and (iii) a nozzle.

1.1 Sanding Systems in the Locomotive Industry

Sanding is used to improve the safety and braking performance of locomotives by controlling the wheel slip. When the wheel slip is large, and the train begins to slide, the sanders pour sand between the wheel and rail to restore traction. Recently, products to increase the sander’s effectiveness have been created with systems such as the SmartSander® by DeltaRail that adjusts the amount of sand being sprayed and different sanding materials made by MBM Industry & Rail Tech GmbH [7], [9]. The SmartSander® controls the rates of sand to reduce excess sanding at low speeds [9]. Tests conducted with the SmartSander® system showed that the delays were reduced by 30% and had $750,000 savings over the course of three years in comparison to trains without any sanders [9]. Train sanding companies have not just tried to increase efficiency but are also
making sanding more effective. Here, the lifespan of the material as it passes through the nip can be extended by utilizing alternative materials to sand that have high hardness. MBM Industry and RailTech® have started to use aluminum oxide and silicon carbide as alternatives to silica sand, claiming that 10% of the material yields the same amount of traction and with no detrimental effect on health resulting from its dust [7].

In addition to new products created by the locomotive industry, organizations and academia have started to investigate the major causes of inefficiencies in sanding systems. The Rail Safety and Standards Board in the United Kingdom took a survey from different manufacturer’s sanders in the field to determine the effects of the sanding hose length, diameter, and nozzle angle on sanding discharge rate [10]. Moreover, Lewis et al. investigated the efficiencies at train speeds of 0.18 km/h resulting from varying wind velocities and directions, changes in the nozzle position, and angles in wet and dry conditions [11]. The efficiency in this study is defined as the amount of particles that made it through the nip compared to those that were sprayed by the sander. Next, computer simulations with spherical particles also determined the most optimal position of the nozzle were investigated by Gorbunov et al. [12]. Finally, Gorbunov et al. also used experiments to determine the effects of different parameters on the friction coefficient between the wheel and rail during sanding conditions [13]. However, it is not clear if the decrease in traction is because of a change in the particle’s friction or sander efficiency.

1.2 Properties for Sanding on Locomotives

As mention in Section 1.1 above, there are various settings that the sanders operate under while the train is braking. First, their airflow rate ranges between 150-200 l/min and their mass flow of
sand out of the nozzle is recommended to be 2kg/min [14],[3],[23]. The particles that are commonly used in sanders range from 0.15mm to 1.6mm in diameter [14]. Similarly, the nozzles typically range from 12mm to 19mm in diameter, but have been found to be as high as 24mm [11], [14]. These values correlate to a particle volume fraction of 0.45% to 0.60%, and airspeeds of 12.4 m/s to 29.5 m/s at the nozzle [3],[18]. The most optimal nozzle orientation was found by Lewis et al. to be as close to the nip as possible and aimed directly into the nip [11]. However, it was also found that 3” spacing is kept from the wheel, and so the closest nozzle position was calculated to be 373.4mm in the horizontal axis from the wheel-rail interface, and 76.2mm above the nip, with the nozzle pointed at 11.5° into the nip [11], [16]. These properties are summarized below (Table 1.1).

In addition to the information on the sander settings, the wheel and rail properties were also found from literature. Here, freight train speeds were discovered to be as high as 120km/h, with passenger trains even faster depending on the class of the rail track and high speed defined as 200+ km/h [17]–[20]. The typical train wheel consists of a flange to stop derailment in turns and a tread that contacts the top of the railhead during straight sections (Figure 1.2). The tread was found to be flat with a slope of 1:40 and is 40mm wide [21]–[23]. Furthermore, the wheel diameter was determined to be ~1m with ±0.5mm maximum runout [3],[24]. Lastly, train wheels were found to be between

Table 1.1 Train Sander Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value (s)</th>
<th>Unit</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Flow Rate</td>
<td>150-200</td>
<td>l/min</td>
<td>[29]</td>
</tr>
<tr>
<td>Mass Flow Rate</td>
<td>2</td>
<td>kg/min</td>
<td>[10], [11], [30]</td>
</tr>
<tr>
<td>Particle Diameter</td>
<td>0.15-1.6</td>
<td>mm</td>
<td>[29]</td>
</tr>
<tr>
<td>Nozzle Width</td>
<td>12-24</td>
<td>mm</td>
<td>[11], [29]</td>
</tr>
</tbody>
</table>
229 Brinell hardness to 277 Brinell and could have up to 6.3 µm Ra roughness [25],[17]. Rails can also be flat and have a similar width, and hardness (235 to 370 Brinell Hardness) [22], [26]. The pressure where the wheel and the rail contact can reach a maximum of 2.7 GPa occur in the center of a ~1cm² contact patch [27]. The properties of the wheel, rail, and their contact are outlined below (Table 1.2).

![Figure 1.2 A typical wheel and rail profile with the railhead and sections of the wheel profile outlined.](image)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value (s)</th>
<th>Unit</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Steel</td>
<td>R7</td>
<td>-</td>
<td>[26], [28]</td>
</tr>
<tr>
<td>Roughness</td>
<td>&lt;6.3</td>
<td>Ra µm</td>
<td>[25]</td>
</tr>
<tr>
<td>Hardness</td>
<td>229-277</td>
<td>HB</td>
<td>[24]</td>
</tr>
<tr>
<td>Width</td>
<td>40-60</td>
<td>mm</td>
<td>[22], [23],[26]</td>
</tr>
<tr>
<td>Taper</td>
<td>1:40</td>
<td>-</td>
<td>[21]</td>
</tr>
<tr>
<td>Contact Patch</td>
<td>~1</td>
<td>cm²</td>
<td>[27]</td>
</tr>
<tr>
<td>Contact Pressure</td>
<td>2.7</td>
<td>GPa</td>
<td>[20]</td>
</tr>
</tbody>
</table>

### 1.3 Particle-Laden Jets

A particle-laden jet is formed when the particles are sprayed out of the sander’s nozzle, and can be characterized by their Stokes number, the jet’s Reynolds number, and volume fraction or
These parameters describe if particle collision is common, if the particles follow the flow, and if the fluid flow field is turbulent, respectively. The Stokes number uses the time scale of the particle velocity over that of the fluid relaxation time or how quickly the particle can change for a given change in the flow. Therefore, the time it takes for the particle to respond to a flow is long with high Stokes numbers, and the particle does not follow the fluid streamlines. For flow going through a nozzle, the Stokes number is defined as:

$$St = \frac{\rho_p d_p^2 v_n}{\mu_f d_n}$$  \hspace{1cm} (1.1)

Here, $\rho_p$ is the particle density, $d_p$ is the particle diameter, $v_n$ is the air speed at the nozzle, $\mu_f$ is kinematic velocity of the fluid, and $d_n$ is the nozzle diameter. This characterizes how well entrained the particles are in the air jet, and is a common way to describe a particle-laden jet [29]. Similarly, the Reynolds number is used to measure if viscous effects are dominant in the flow. For high Reynolds numbers, the fluid flow from the jet is dominated by the inertial effects and is therefore turbulent. The Reynolds number is given by:

$$Re = \frac{v_n d_n}{\nu_f}$$  \hspace{1cm} (1.2)

Here, $\nu_f$ is the dynamic viscosity of the fluid. The volume fraction and loading can help determine if collisions are common in the flow, or if the particles are isolated. The volume fraction is defined as [3]:

$$\alpha_p = \lim_{\Delta V \to \Delta V_{ins}} \frac{\Delta V_p}{\Delta V}$$  \hspace{1cm} (1.3)

In Equation 1.3, $\Delta V_p$ is the volume of the particles for a given control volume, $\Delta V$ is the volume of that control volume, and $\Delta V_{ins}$ is the critical volume size that in which the properties can be
treated as continuous, and not change dramatically from one point to another [3]. The loading fraction is given as [3]:

\[ Z = \frac{\dot{m}_p}{\dot{m}_f} \]  

Which is the ratio between the mass flowrate of the particles \( \dot{m}_p \), and the mass flowrate of the fluid, \( \dot{m}_f \). Finally, the particle spacing for three dimensions can be calculated from redefining volume fraction, \( \alpha_p \), as the space that a spherical particle occupies versus the spacing between each particle [3]. The 2D particle spacing can be defined analogously and, on a plane, as the ratio of the area that the particle occupies to the area between particles. Solving for \( \frac{l}{d_p} \) yields:

\[ \frac{l}{d_p} = \sqrt{\frac{\pi}{4 \times \alpha_p}} \]  

Here, \( l \) is the spacing between particles, \( d_p \) is the particle diameter, and \( \alpha_p \) is the volume fraction from Equation 1.3. When \( \frac{l}{D} \approx 10 \), then the flow is treated as dilute and the particles can be treated as isolated from one another [3].

In addition to their characterization, particle-laden jets have been observed in depth for downward spraying flows in ambient air. It was found that three regions exist for different particle diameters and particle coupling in the turbulent jet [30], [31]. Authors Levy and Lockwood showed that the silica sand particle velocities at 20 diameters downstream of the nozzle had between 90-100% the speed of their exit velocity at the nozzle in a downward facing jet [31]. In this study, the larger particles (850-1200µm) did not follow the flow, while the medium particles (700-300µm) were slightly entrained in the flow but did not follow the fluctuations, and the smaller particles (180-
250 µm) where fully entrained. The medium sized particles (700-300µm) had the largest distribution downstream. The velocity profiles of polystyrene beads have also been studied by for sizes of 210µm, 460µm, and 780µm in loading ratios from 0-3 [30]. The velocity profiles at 20 to 60 jet diameters downstream show that larger particles sustain velocities between 80-100% of their initial velocity out of the nozzle. Lastly, both studies show that in the center of the jet, the particles have a higher speed and maintain that speed more than the outer radius.

1.4 Particle-Substrate and Particle-Particle Interactions

As the particles approach the nip, they begin to contact both the wheel and rail. Here, how they ricochet may cause the particle trajectory to diverge from a path line that would have been aimed towards the nip. In addition, particle-particle interactions occur when the particle laden flow becomes denser. In both cases, surface forces may occur to help dampen the bounce of the particles. The term adhesion is used to describe these forces with particle-substrate interactions, and cohesive for particle-particle interactions [3]. However, the same forces occur regardless if the particle is interacting with a substrate or another particle: Van Der Waals, Electrostatic, interstitial fluid forces (i.e. liquid bridges) can help dampen the bounce [3],[8].
The models that describe impact include these forces as well as consider how elastic the collision is. With this thesis, the hard sphere model is considered since it is easy to characterize in experiments. In the hard sphere model, the particle’s interaction with the substrate is summarized by the initial and post-bounce states of the particles to determine impaction of the particles (Figure 1.3 a). The coefficient of restitution, $e$, is used as a metric for how well the particle bounces [3]:

$$e = \frac{|v_{e1}|}{|v_{i1}|}$$  (1.6)

When $e \rightarrow 1$, little energy is lost during the bounce. A more specific definition can be considered when deconstructing Equation 1.6 into normal and tangential components. Even more, the relative velocities of each of the particles/particle-substrate may be considered to yield:

$$e_{i} = \frac{v_{i1} - v_{i2}}{v_{i1} - v_{i2}}$$  (1.7)
\[ e_\perp = \frac{v_{\perp e1} - v_{\perp e2}}{v_{\perp i1} - v_{\perp i2}} \]  

(1.8)

This can help to decouple the effects of the particle sliding to the restitution in the parallel direction from the particle’s ability to bounce in the perpendicular direction. In this work, the Restitution Ratio (RR) is defined as the restitution in the tangential direction \( (e_{|\parallel|}, \text{ Equation } 1.7) \) and the Coefficient of Restitution (CoR) is the restitution in the normal direction \( (e_{\perp}, \text{ Equation } 1.8) \).

Irregularities in the wall and particle shape can cause large variations in the normal and tangential restitution of the particle [1], [32]. Thus, particle bouncing with irregular particle shapes and wall roughness have been investigated to help account for such variations [3], [4], [32]. For rough walls, it is assumed that the angle of contact between the particle and wall has a Gaussian distribution [32]. Similarly, for irregular particles, the variance can be explained if the angle from the point of contact to the weight vector from the center of mass is considered [3], [4]. For a sphere, these two vectors align to be the same [3], [4]. However, they differ for very irregular particles, which causes a variation in the post-bounce trajectories and rotation of the particle (Figure 1.3 b). Thus, an induced particle spin or tumbling along the boundary may occur [3], [4]. Finally, particle jet impingement on steel coupons has been studied for glass beads and road dust was studied at various impingement angles [1]. It was found that varying impaction angles of glass beads from 56° to 83° resulted in a constant CoR between 0.79 and 0.83, respectively [1].

Due to the non-uniform nature of particles entrained in a flow before and after ricocheting, a Lagrangian approach is ideal to determine their bounce properties where single particles are tracked from before they bounce to after the bounce. Particle tracking velocimetry (PTV) follows
the particle frame by frame recorded through videos and can determine the velocity of the particle. To find the bouncing properties, the trajectory line from the PTV can be analyzed, regression fitted, and an impaction point can then determine where a post-bounce and pre-bounce line intercepts [32]. However, since it is often difficult to distinguish the particle during bouncing with PTV, simply averaging of the incoming and exiting particle velocities has been achieved [1].

1.5 Research Objective and Scope

This thesis focuses on two questions: (i) What is the sanding efficiency at train speeds much higher than those conducted by the previous studies at 0.18 km/h? and (ii) How can changes in the addition of a coating or particle properties and sanding system improve the deposition efficiency? As mentioned earlier, the deposition efficiency is defined as the mass fraction of particles ejected by the sander that make it into the nip.

The initial section of this work analyzes the particle trajectories. It determines the nozzle and bounce characteristics of the particles and their effect on efficiency. The subsequent chapters focus on the deposition of sand with different properties such as shape and size. Next, the addition of a propriety coating on sample particles was investigated. Finally, Section 3.5 explores the deposition of silica sand as a function of the train speed (in the range 0-30 km/h). The thesis closes with a summary of key findings and recommendations for further investigations to optimize deposition.
Chapter 2: Setup and Methodology

2.1 Experimental Setup

Previous experiments for studying train sanders occurred at speeds much less than the operational speed of trains (as high as 120 km/h for freight trains) and literature of particle impingement with fast moving surfaces is lacking. Also, experimental results are already needed as a basis for simulations because of the complex behavior of the particles leaving the sander and their interaction with the wheel and rail. Consequently, a device had to be created to simulate a train sander to determine how to better optimize sanding. It would do this by comparing the amount of particles that pass through the nip from what leaves the sander to measure the efficiency. Visual access to the particle impingement was also desired to be able to determine how the losses occur and study this phenomenon. The wheel and rail were scaled down to further simplify the manufacturing of the device. Furthermore, the full-scale sander was replaced with a commercial air etcher (hand held sand-blasters commonly used to etch glass).

Below is a schematic and pictures outlining the resulting device (Figure 2.1). The following figure displays the geometry and adjustable parameters of air etcher in the testing apparatus (Figure 2.2). Finally, the subsequent paragraphs outline the design details for each subsystem.
Figure 2.1 Testing apparatus (a) schematic (b) airflow and belt controls, (c) sander, wheel, collector and moving belt to model the rail.
Figure 2.2 Settings of the device that were held constant. The volume fraction, $\alpha_p$, was found with silica sand, and the setting was held constant across different materials.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_p$</td>
<td>0.46</td>
<td>%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>11.5</td>
<td>°</td>
</tr>
<tr>
<td>$d_{\text{wheel}}$</td>
<td>502</td>
<td>mm</td>
</tr>
<tr>
<td>$v_f$</td>
<td>12.8</td>
<td>m/s</td>
</tr>
<tr>
<td>$h$</td>
<td>207.5</td>
<td>mm</td>
</tr>
<tr>
<td>$L$</td>
<td>42</td>
<td>mm</td>
</tr>
</tbody>
</table>

2.1.1 Modeling Locomotive Sanding Experimentally

Since the tread of the train wheel and track in the vicinity of the nip is approximately flat ($<1^\circ$ slope), the geometry of the locomotive wheel and track were simplified to that of a wheel without camber and a flat belt. In addition to simplifications made to the profiles, the device was scaled to approximately 50% of full scale, since the particle diameter was much smaller than the diameter of the wheel. (Approximately, because the variation is because train wheels in North America do not have a consistent diameter, but rather have diameters between 28” and 42” (0.7m to 1.07m) depending on the type of train)[33]–[35]. As the particles are much smaller than the wheel, by the time they are simultaneously in contact with both the wheel and rail, the slope tangent to the wheel at the contact point is very small ($<6^\circ$). Owing to the small tangent angle, the friction of sand is large enough to ensure that it gets pulled by the wheel and rail into the nip and does not squeeze back out of the nip. The effects from the particles being crushed were neglected as well, though they were found to crush at forces much less than would be applied by a train. Because of this, the
wheel was set about 1.5mm above the rail to allow for less wear, power consumption, and hazard from crushing silica sand to the user. For the wheel, a solid cast iron bandsaw fly-wheel was used that had a diameter of 19” and a width of 1.33”. This was rotated by a speed control motor or an electric drill powered by a variac with an encoder. The encoder gave 200 pulses per revolution which was then measured over a set amount of time and converted to a surface speed in LabVIEW™. The user then adjusted the voltage to the desired speed as needed. The belt consisted of a 0.035” bandsaw blank blade welded to the correct length and was propelled by a Burr King® Belt Grinder that could reach a maximum speed of 150 km/h [36]. The grinder was controlled by a motor speed controller to match the surface speed of the moving belt. An encoder was used to verify the speed of the belt. Tests showed that the CoR of the belt were comparable to that of a section of railroad rail (profile shown in Figure 2.3), and that for belts thicker than 0.010”, the thickness of the belt had no significant impact on how the silica sand bounced (Figure 2.4, Figure 2.5). Additionally, it was found that varying the material hardness of the belt did not significantly change normal impaction (Figure 2.6). The roughness of the wheel and rail were found to not be important since the particle shapes already varied greatly and caused a large variance in the bouncing properties.

Figure 2.3 A cross sectional view of the sample rail used in the restitution test to benchmark the device.
Figure 2.4 Comparison of the bounce properties between the rail and belt.

Figure 2.5 Restitution for various belt thickness. An average line is displayed for reference.

Figure 2.6 CoR and RR for different surface roughness.
Figure 2.7 The loading of sand in the nozzle at each turn on the air etcher nozzle.

The sander was modeled by a Paasche air etcher LAC#3, which gave a fairly homogeneous mixture of particles. A funnel served as the hopper and tubing connected the funnel to the where the particles were entrained into the fluid jet. The LAC #3 uses an air nozzle in a chamber to create a vacuum pressure to help draw the particles out of the hopper and through the nozzle [37]. The size of the chamber could be expanded to increase the flow rate by screwing the nozzle out. However, it was found that at angles close to horizontal the mass flowrate of the sand decreased so the angle of the etcher was increased, and a tube was added to the end of the etcher that reduced the angle of the jet from 25° to 11.5±1°, which is consistent with industrial sanders. The resulting mass flowrate from the etcher yielded a volume fraction that matched that of an actual sander (Figure 2.7). However, the mass flowrate of the sander had to be reduced from the order of 2 kg/min for a full-scale sander to ~0.5 kg/min for the air etcher in the lab to maintain the same volume fraction as a locomotive sander. Still, visual inspection of the flow saw no change to the particle-laden flow from the addition of a bend in the tube: the mass of the sand was evenly spread across the jet (Figure 2.8). In addition, the jet expansion was visually estimated to be 10°-15° with silica sand. Based on this observation, the etcher was moved back 10mm from the nip to mimic a scaled down
replica of a 12mm nozzle in the field. The airflow rate was set to 11.5 scfh, which correlates to an airspeed of ~12.5 m/s with a Reynolds number in the edge of the turbulent regime. This gave Stokes Numbers on the order of 500-32000 at the nozzle. To ensure that the air was clean, a moisture trap and filter was added to the lab air supply. To adjust the flowrate, an inline relieving regulator was used in conjunction with a needle valve. The regulator would lower the pressure into the needle valve and ensure a steady pressure. The needle valve would then be used to fine-tune the pressure. Lastly, the diameter of particles was restricted between approximately 0.12mm to 1.65mm. The larger diameter particles of 1.65mm started to clog the nozzle whereas the small diameter glass beads of 0.12mm yielded inconsistent results from the collector. The error of the sensors used in the device are given with their operational range below (Table 2.1).

### Table 2.1 The list of sensor and actuators in the experimental apparatus with their resulting error.

<table>
<thead>
<tr>
<th>Property</th>
<th>Actuator/Sensor Model</th>
<th>Range Tested</th>
<th>( \varepsilon )</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel Speed</td>
<td>Dewalt DWD201G/ Yumo E6A2-CS3E</td>
<td>0-24</td>
<td>( \pm 1 ) (at 18 km/h)</td>
<td>km/h</td>
</tr>
<tr>
<td></td>
<td>Burr King 960-272/ Yumo E6A2-CS3E</td>
<td>0-24</td>
<td>( \pm 0.5 ) (at 18 km/h)</td>
<td>km/h</td>
</tr>
<tr>
<td>Belt Speed</td>
<td>N/A</td>
<td>0-20</td>
<td>( \pm 0.5 )</td>
<td>scfh</td>
</tr>
<tr>
<td>Airflow Rate</td>
<td>Paasche LAC #3 Setting</td>
<td>0-114 (for silica sand)</td>
<td>( \pm 3.3 )</td>
<td>grams/min</td>
</tr>
<tr>
<td>Mass Flow Rate</td>
<td>Mettler PM 4000</td>
<td>0-1000</td>
<td>( \pm 0.01 )</td>
<td>grams</td>
</tr>
<tr>
<td>Sample Mass</td>
<td>Phantom V12</td>
<td>8000-27000 fps; 1024x512 (162.5mm x 81.3mm) to 512x384 (26.5 mm x 19.9 mm) and 10 mm depth of focus</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
2.1.2 Deposition Measurements

Two methods were used to study efficiency: the strict definition that compares the amount of sand going into the etcher to what passes through the nip and a closer examination on how the particles behave from the nozzle to the nip. It was suspected that certain particle properties lead to different behavior from the nozzle to the nip and resulted in different efficiencies. Particle trajectories and the interactions of the particles were visualized using high speed imaging and quantified with Particle Tracking Velocimetry (PTV). The nozzle and particle-interface interactions could then be correlated to the deposition values. For example, the trajectories of the particles could be investigated to determine if the trajectories from the nozzle correlated with the particle properties and deposition. The efficiency of a sander is defined as the mass fraction of the particles that make it into the nip compared to the amount of particles that were ejected by the sander. Thus, the loss in the system is the mass fraction that left in the region between the nip and the sander (Figure 2.9). The mass collected by the etcher was negligible, and the amount of particles could be weighed before being poured into the device. The particles passing through the nip were guided by a fender with two prongs that went around the side of the wheel and were collected in a bin. The bin was
placed behind the wheel and sat on top of a scale that measured the weight in real time to help determine when the test would end (Figure 2.1). Dust covers were built over the scale for more accurate results. Equation 2.1 below shows the efficiency calculation from the simplifications and methodology outlined above.

\[
\eta (\%) = \frac{m_{p,\text{out}}}{m_{p,\text{in}}} \times 100
\]  

(2.1)

Here, \( m_{p,\text{out}} \) is the mass of the particles that passed through the nip and \( m_{p,\text{in}} \) is the mass of the particles that pass through the sander during a test.

In addition to determining the efficiency, the CoR and RR values were calculated to study if their bouncing properties had a large effect on efficiency. It was expected that with more ricochet (higher CoR) there would be a higher chance of the particles bouncing out of the pathway into the nip. To determine CoR and RR values, highspeed imaging (with a Phantom v12 camera, see Table 2.1 for details) and a particle tracking velocimetry (PTV) application called TrackMate in Fiji was used to find the trajectories of particles as they entered the nip [38]–[40]. The bounce properties
were calculated using data from the trajectories. In Trackmate, a Lagrangian of a Gaussian method was implemented to determine the particle positions during each frame [41]. A Kalman filter, Linear Assignment Problem, or nearest neighbor search was implemented to detect the particle motion between time frames depending on which one yielded the best visual result with the least amount of missed calculated tracks [41]. The results from the PTV analysis were post-processed to determine when the particles contacted the interface and then calculate the tangential and normal restitution values (Appendix B ). This was achieved by first finding the edge of the substrate, then finding a change in the relative normal velocity of the particles to the substrate. Incorrect bounces were mitigated through applying filters to each of the “bounces” found. The primary filters looked

![Figure 2.10 Typical bounces on the (a) belt and (b) wheel.](image)

for the bounces that were near the substrates, changed direction after impacting substrate, and had enough time frames before or after the bounce in their trajectory to reduce error in calculating their velocities. In other words, the bounces looked like a “V” when the particle’s trajectory was plotted. The bottom of the “V” should be near the substrate and “V’s” should be smooth with long enough trajectories before and after the bounce to be considered (anything similar to a “W” would indicate a double bounce and be neglected). Figure 2.10 shows typical bounces that would be accepted by the code. To verify the results, the trajectories that were included in the calculations were plotted so that the user could visually check them and see any miscalculations in the PTV algorithm. All
of the filters mentioned above were adjustable so that the algorithm could be implemented in different cases. The parameters become more important the denser the flow since the PTV was more likely to make erroneous links between two different particles in the flow. Finally, it should be noted that the PTV software could not perform as well with a denser flow of particles at smaller diameters (~<0.2mm). For improving the PTV and ricochet calculations, a lens that has a higher zoom may be used in conjunction with a laser sheet to make the smaller particles more distinguishable.

Figure 2.11 Benchmark results of the CoR and RR code. Here, the program was compared to values calculated by tracing the particle trajectories by hand in the PCC ® software; (a) the values for CoR and RR are displayed; (b) the difference between the bounces that were not found automatically by the code and those that could be seen while analyzing the raw videos.
2.2 Particle Samples

The particles were measured with 1x magnification on a backlit Nikon Eclipse TE 2000-U microscope. The image was then post-processed in Fiji via automatic thresholding and particle analysis [38]. The output was compared to the original image to determine if there were any particles that were not found by the particle analysis. Though the sander efficiencies were measured using the mass of the particles, the particle distributions were determined based on their projected area diameter. To find the diameter of each particle, an area diameter was used that used the projection of each particle onto the plane that the microscope was viewing [42]:

\[
d_{\text{area}} = \sqrt{\frac{4 \times A}{\pi}}
\]  

(2.2)

The roundness and aspect ratio of each particle was used to describe the shape and are defined below [42]:

\[
\text{Roundness} = \frac{4A}{\pi C_{\text{major}}^2}
\]  

(2.3)

\[
A.\ R. = \frac{C_{\text{major}}}{C_{\text{minor}}}
\]  

(2.4)

The roundness of a particle compares the area of the particle to that of a circle that encompasses the particle. With irregular particles, it was assumed that with many particle projections onto a plane, the area diameter, roundness, and aspect ratio would give reasonable approximations for the 3D shape and size properties of the particle. Lastly, it should be noted that each particle material contained a sample that had a black paint-like coating with the underlying material the same as its uncoated counterpart. The following outlines each of the respective particles tested, with the
important details summarized (Table 2.2- Table 2.4). For images and size distributions of the particles, please see Appendix C.

Table 2.2 Size, shape, and appearance of the aluminum oxide samples.

<table>
<thead>
<tr>
<th>Sample Name</th>
<th>ID</th>
<th>$\bar{x}_{\text{guassion}}$(mm)</th>
<th>$\bar{x}$(mm)</th>
<th>Roundness</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum Oxide #46</td>
<td>A1</td>
<td>0.25</td>
<td>0.24</td>
<td>0.73</td>
<td>Brown</td>
</tr>
<tr>
<td>Aluminum Oxide #40-60</td>
<td>A2</td>
<td>0.30</td>
<td>0.31</td>
<td>0.64</td>
<td>Black</td>
</tr>
<tr>
<td>Aluminum Oxide #24</td>
<td>A3</td>
<td>0.91</td>
<td>0.90</td>
<td>0.70</td>
<td>Brown</td>
</tr>
<tr>
<td>Aluminum Oxide #24 (Coated)</td>
<td>C1</td>
<td>0.89</td>
<td>0.92</td>
<td>0.70</td>
<td>Black</td>
</tr>
</tbody>
</table>

Table 2.3 Size, shape, and appearance of the bauxite beads samples.

<table>
<thead>
<tr>
<th>Sample Name</th>
<th>ID</th>
<th>$\bar{x}_{\text{guassion}}$(mm)</th>
<th>$\bar{x}$(mm)</th>
<th>Roundness</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bauxite Beads #46</td>
<td>B1</td>
<td>0.41</td>
<td>0.41</td>
<td>0.80</td>
<td>Black</td>
</tr>
<tr>
<td>Bauxite Beads</td>
<td>B2</td>
<td>0.88</td>
<td>0.87</td>
<td>0.92</td>
<td>Black</td>
</tr>
<tr>
<td>Bauxite Beads (Coated)</td>
<td>C2</td>
<td>0.91</td>
<td>0.91</td>
<td>0.92</td>
<td>Black</td>
</tr>
</tbody>
</table>

Table 2.4 Size, shape, and appearance of the silicon carbide samples.

<table>
<thead>
<tr>
<th>Sample Name</th>
<th>ID</th>
<th>$\bar{x}_{\text{guassion}}$(mm)</th>
<th>$\bar{x}$(mm)</th>
<th>Roundness</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon Carbide #24</td>
<td>SC</td>
<td>0.92</td>
<td>0.94</td>
<td>0.69</td>
<td>Black</td>
</tr>
<tr>
<td>Silicon Carbide #24 (Coated)</td>
<td>C3</td>
<td>1.23</td>
<td>1.22</td>
<td>0.72</td>
<td>Black</td>
</tr>
</tbody>
</table>

Table 2.5 Size, shape, and appearance of the silica sand samples.

<table>
<thead>
<tr>
<th>Sample Name</th>
<th>ID</th>
<th>$\bar{x}_{\text{guassion}}$(mm)</th>
<th>$\bar{x}$(mm)</th>
<th>Roundness</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-35</td>
<td>S1</td>
<td>0.50</td>
<td>0.52</td>
<td>0.74</td>
<td>Cream</td>
</tr>
<tr>
<td>F-50</td>
<td>S2</td>
<td>0.32</td>
<td>0.32</td>
<td>0.74</td>
<td>Cream</td>
</tr>
<tr>
<td>F-75</td>
<td>S3</td>
<td>0.18</td>
<td>0.17</td>
<td>0.74</td>
<td>Cream</td>
</tr>
<tr>
<td>HI-50</td>
<td>S4</td>
<td>0.26</td>
<td>0.24</td>
<td>0.70</td>
<td>Yellow</td>
</tr>
<tr>
<td>0.75mm Sand</td>
<td>S5</td>
<td>1.05</td>
<td>1.04</td>
<td>0.79</td>
<td>Yellow-Cream</td>
</tr>
<tr>
<td>0.75mm Silica Sand (Coated)</td>
<td>C4</td>
<td>1.15</td>
<td>1.15</td>
<td>0.82</td>
<td>Black</td>
</tr>
<tr>
<td>Coated Sand</td>
<td>C5</td>
<td>0.51</td>
<td>0.49</td>
<td>0.77</td>
<td>Black</td>
</tr>
<tr>
<td>Derbyshire Sand</td>
<td>S6</td>
<td>0.44</td>
<td>0.43</td>
<td>0.78</td>
<td>Grey &amp; Yellow</td>
</tr>
<tr>
<td>B.C. Rail Sand</td>
<td>S7</td>
<td>0.61</td>
<td>0.54</td>
<td>0.70</td>
<td>Grey &amp; Yellow</td>
</tr>
</tbody>
</table>
2.3 Methodology

All tests were run at a substrate speed of 18 km/h, and an air flow rate of 11.5 scfh, unless stated otherwise. The nozzle adjustment was held constant between all tests. This meant that the mass flow rate from the etcher varied as the type of particle was varied, but this behavior would also occur in an actual sander. To run a test, the airflow rate was set first so that the jet was fully developed. The wheel and belt were allowed to reach steady state at the desired speed. The scale holding the bucket that collected the particles was then tared to zero. The particles would be poured into the hopper with test sample sizes ranging from about 50 grams to 100 grams. An average of 4 trials were run at every test condition. Post-processing of the results showed no effect of the sample size on the deposition results. As soon as the particles stopped flowing out of the nozzle, and the mass in the collector did not change, then the system would be turned off starting with the substrates and then the air to the etcher. The bucket that collected the particles would then be weighed on a separate, higher accuracy scale (Mettler PM4000). This ensured that the particles that landed on the scale outside of the collector did not bias the measurements. Additionally, the device was vacuumed between different tests and the bin that collected the particles was cleaned. The nozzle was then realigned after cleaning, and the collector could be adjusted if needed. Lastly,

### Table 2.6 The size, shape, and appearance of the glass samples.

<table>
<thead>
<tr>
<th>Sample Name</th>
<th>ID</th>
<th>$\bar{x}_{\text{guassion}}$ (mm)</th>
<th>$\bar{x}$ (mm)</th>
<th>Roundness</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass Beads # 3</td>
<td>G1</td>
<td>0.60</td>
<td>0.59</td>
<td>0.92</td>
<td>Clear</td>
</tr>
<tr>
<td>Glass Beads #5</td>
<td>G2</td>
<td>0.33</td>
<td>0.33</td>
<td>0.92</td>
<td>Clear</td>
</tr>
<tr>
<td>Glass Beads #8</td>
<td>G3</td>
<td>0.125</td>
<td>0.129</td>
<td>0.91</td>
<td>Clear</td>
</tr>
<tr>
<td>Crushed Glass</td>
<td>G4</td>
<td>0.41</td>
<td>0.41</td>
<td>0.65</td>
<td>Clear &amp; Tinted</td>
</tr>
</tbody>
</table>
the belt was inspected for any cracks before and after each test. Additional details on the operation of the testing apparatus are included in Appendix A.

2.4 Repeatability

A small number of tests were run where the height of the wheel above the belt was changed. Those tests revealed no significant difference in the deposition at 18km/h. Additionally, the device was run with the same parameters on multiple trials held on seven different days to ensure that the tests were repeatable and benchmark the device before running different materials. The average deposition for the sample HI-50 (S4) sand was found to be 69.3 ±1.3% (Figure 2.12). The low standard deviation in the results is indicative of the high degree of repeatability of the tests.

![Graph showing efficiency of HI-50 (S4) for multiple days](image)

**Figure 2.12** The efficiency of HI-50 (S4) with the same settings across multiple days.
Chapter 3: Results

3.1 General Observations of the Particle Trajectories

A general view of the particle trajectories reveals that the particles travel about half way from the nozzle to the nip before they start interacting with the wheel or rail (Figure 3.1). After this initial contact, they begin to interact with both substrates more frequently as they are funneled between both the wheel and the rail into the nip. The flow of particles from the nozzle into the nip can then be divided into two regions: (i) the particle-laden jet geometry which is characterized by the nozzle parameters and aerodynamics of the system until (ii) the particles begin to ricochet off the wheel and rail with changes to their trajectories being influenced from their bouncing properties. These regions are investigated in the subsequent paragraphs.
Figure 3.1 The particle trajectory as they travel to the nip; the two different regions are shown in both (a) the schematic above, and an image of the trajectories are shown below in (b).
For 16 different particle samples with varying shapes and compositions, it was found that the spread of the particle-laden jet as it leaves the nozzle was the only parameter that was significantly correlated to deposition (Figure 3.2). The spread of the particles is defined here by the standard deviation of the particle trajectory as it is exited from the nozzle:

\[
\sigma = \sqrt{\frac{\sum_{j=1}^{N} (\theta_j - \bar{\theta})^2}{(N - 1)}}
\]  

(3.1)

In this test, the nozzle was studied independently from the belt and rail. The particles were then analyzed as they left the nozzle at one diameter and 15 diameters away from the exit of the nozzle. The speed and angle of the particle trajectory were analyzed over 2.6 ms. This allowed for gravitational effects on the trajectories to be small and therefore neglected. However, to ensure that the spread of the particle-laden jet was independent of gravity, the spread was also analyzed from the top (Figure 3.2). Comparing Figure 3.2 (a) and Figure 3.2 (b), it is apparent that the lateral and vertical spread for the particle-laden jet is similar.
Figure 3.2 Efficiencies of deposition vs. the spread of the particle-laden jet in the (a) vertical (looking from the side) noting the various particle diameters and (b) lateral (looking from the top direction). The spread of the jet was calculated by taking the standard deviation of the trajectory angles as the particles exit the jet. A linear trendline was calculated by the least square method to show correlation. The labels in the figure refer to the particle IDs provided in Tables 2.3-2.7.
In both the vertical and lateral results of the particle laden-jet, the particle’s efficiency is dependent on the spread of the jet. This high correlation may be expected since the primary method for losing particles is their loss out of the sides of the wheel and rail geometry. More particles would be lost if, owing to the spread of their trajectories, more were directed out of the sides of the rail. It should be noticed that the particles did not have a perfect correlation to the spread of the jet. It will be shown in Section 3.2 that part of this may be from some of the larger particles being less entrained and falling from the air-jet earlier, as in the case of silica sand. Additional tests were done to make sure that the particles with the smaller spread were not caused by having a more dilute flow. The mass flow rate of eight different samples was tested by measuring the amount of time it took to dispense 100 grams of the sample particles. It was found that there was a negligible correlation between the mass flow rate or loading of the particles and

Figure 3.3 Efficiency vs. the loading and mass flow rate of different particles.
their efficiencies (Figure 3.3, Table 3.1). The correlation values of the mass flow rate and the other nozzle properties are given in Table 3.1.

In the regime where the particles interacted with the wheel and rail, their bounce properties were characterized by the coefficient of restitution and restitution ratio to determine how well the particles were deposited. The particles that had high deposition would have larger sanding efficiencies since the surfaces of the wheel and rail both move through the nip. In this test, the high-speed imaging magnified a region in front of the nip to observe the particle bounces. Section 2.1.2 describes the complete test set-up in more detail. The variation in the coefficient of restitution was found to be large since the normal velocity of the particles is small during impacts at angles near horizontal (Figure 3.4). In addition, irregularities in the particle’s shape can also increase the variation. Therefore, no strong correlation could be found between the bounce properties and the sanding efficiencies because of these high variations and low coefficients of determination (Figure 3.4). To reinforce this conclusion, the restitution values were found for various particle with a normal impact at a substrate speed of 0 km/h (Figure 3.5). This allowed for a more controlled analysis of each particle’s bouncing properties since the particles could be analyzed with little spin. It was found that the efficiency still had no dependence on restitution. Likewise, no dependence was found in the small range of restitution ratios from the particles tested. However, it should be noted that if both the coefficient of restitution and the restitution ratio reached values near zero, then the efficiency would theoretically increase since the particles would analogous to something like spray paint and stick to the surface. Lastly, it should also be noticed that the particle velocities from the nozzle ranged from 4.2 km/h to 15.4 km/h, and the substrates were held at a constant
speed of 18 km/h. Still, as shown in Section 3.5, because the deposition efficiency does not change at speeds above 18 km/h, then the bounce properties remain unimportant for higher speeds.

Table 3.1 Correlation values between different nozzle properties and efficiency.

<table>
<thead>
<tr>
<th>Property</th>
<th>Coefficient of Correlation to Efficiency ($R^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity at the Nozzle</td>
<td>0.09</td>
</tr>
<tr>
<td>Deviation of Trajectory Angle at the Nozzle (Vertical)</td>
<td>0.88</td>
</tr>
<tr>
<td>Deviation of Trajectory Angle at the Nozzle (Lateral)</td>
<td>0.91</td>
</tr>
<tr>
<td>Variance from Centerline of the Jet 15 Diameters</td>
<td>0.06</td>
</tr>
<tr>
<td>Mass Flow Rate</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

Figure 3.4 The Efficiency vs. (a) Coefficient of Restitution and (b) Restitution Ratio. The CoR and RR were taken in-situ during a test at substrates speeds of 18 km/h.
Figure 3.5 The efficiency vs. restitution. The restitution was taken by dropping particles vertically onto a stationary belt. Since the particles were dropped vertically onto the belt, the restitution value is approximately the same as their CoR.

To summarize the general observations made, the spread of the jet may have predicted the deposition whereas the bouncing characteristics were unrelated. Nonetheless, though there is a good correlation between the spread of the jet and deposition, the cause of this behavior can only be speculated. The spread of the nozzle may be from its internal geometry, particle-particle and particle-wall interactions, and entrainment into the airflow.

3.2 Effect of Particle Size on Deposition

Four sizes of silica sand were considered to more precisely determine the effect of particle size on deposition. The composition was held the same, but only the sand with similar gaussian-like distributions (though their average diameter was changed), color, and roundness were considered since there were many samples of silica sands. Thus, the F-Series sand (S1-S3) and the 0.75mm diameter sand (S5) were used. For their properties, please refer to Section 2.2 and Appendix C.
It was seen that the larger particles became less entrained, which helped them fall out of the jet and contact the belt earlier (Figure 3.6). In comparison, the smaller particles were more entrained and had a trajectory went more directly into the nip (Figure 3.6). Thus, the larger sand had a less efficient deposition efficiency since its path into the nip was less direct. This correlated to a decrease in efficiency with an increase in the particle diameter (Figure 3.7). The trend line shows a log-linear least square curve fit [43]. It should be mentioned that these findings differ from the previous and subsequent sections in that the primary cause of inefficiencies do not stem directly

![Figure 3.6 Trajectories of Silica Sand with a diameter of (a) 0.32mm (S2) and (b) 1.05mm (S5).](image)

![Figure 3.7 Deposition vs. the area diameter of silica sand.](image)
from a change in the spread of the particle-laden jet. Furthermore, the change in efficiency from varying the spread of the jet was ~30% in Section 3.1, compared to only ~7% here.

The simulations done in conjunction with this work also show a decrease in efficiency for larger particles because the particles become less entrained [44]. However, the decrease in efficiencies found here is less drastic than observed in the simulations. This difference may stem from the fact that the simulations use only one particle diameter and therefore do not capture the full particle distribution used in the testing apparatus. Similarly, the collection device has small leaks due to wear and vibrations in the belts, which will lead to lower measured efficiencies, particularly for smaller particles. Lastly, it should be noted that this test held the nozzle position the same across all tests to simulate the effect of using different particles with the same sanding set-up. Changing the nozzle angle may be one solution to mitigate the losses that occur from larger particles settling.

### 3.3 Effect of Particle Shape on Deposition

Tests were performed comparing crushed glass with glass beads to isolate the effect of shape on deposition. More specifically, the aim was to determine the differences in deposition between an irregular particle and a similarly sized round particle that had the same composition. The roundness of the particle was used to characterize its shape (Equation 2.2). Figure 3.8 shows that the crushed glass has a wider spread than glass beads as they exit the nozzle. As a result, the crushed glass had a lower efficiency (67%) relative to the nearly round glass beads (77%) (Figure 3.9). This suggests that the properties of the nozzle may be the cause for such a difference. However, due to the complex dynamics which occur within the nozzle, the exact cause of why the spread is larger is out of the scope of this work and will be reserved for later studies.
Figure 3.8 Trajectories of (a) glass beads (G2) and (b) the irregular crushed glass (G4).

Figure 3.9 Efficiencies of crushed glass and glass beads
3.4 Comparison of Coated Particles to Uncoated Particles

Another difference was found when testing coated particles, whose coating was observed to cause a narrower spread in the particle-laden jet at the nozzle than their uncoated counterparts (Figure 3.10). This led to an average increase of $18.4\pm7.9\%$ in deposition (Figure 3.11). The rounder particles had the best efficiencies, with large silica sand (S4) displaying the largest increase of 30%. To ensure that the increased diameter from coating the particle did not cause the increase in efficiency, coated sand with an average diameter of 0.51mm was tested. It was found to have an efficiency of 77%, which is 4% above the most efficient sand (S7) and 14% higher than the comparable sized F-35 sand (S1).

![Figure 3.10 Trajectories of (a) coated silica sand (C4) and (b) uncoated silica sand (S5).](image)
Figure 3.11 Efficiency sample particles with and without coatings.

3.5 The Effect of Substrate Speed on Deposition

The device was ran at speeds of 0.1 km/h, 0.18 km/h, 1 km/h and 2 km/h, 12 km/h, 18 km/h, 24 km/h, and 30 km/h. For speeds under 12 km/h, the wheel was lowered so it was in contact with the belt, but the belt was still unsupported beneath the nip. Thus, the sand was not crushed since the

Figure 3.12 The maximum amount of mass that can pass through the nip. For low speed, this amount may be less than what the sander supplies to the nip and a buildup occurs.
belt warped beneath the wheel. The speed was measured from the wheel, which was moved by rotating the belt. Since the belt was in firm contact with the wheel, no slip occurred during these tests. HI-50 was used in all tests and had a mass flowrate of ~58 g/min (which scaled to a full-size sander correlated to ~1 kg/min). For the particle properties of HI-50, please refer to Section 2.2.

In the most ideal case for deposition through the nip, it was assumed that the particles align in a perfect packing structure as they pass between the wheel and rail. This may occur at low speeds where the sander can supply all of the particles that form this structure to the nip. If any more sand is sprayed into the nip, then it builds up and eventually falls off the sides of the substrate (Figure 3.12, Figure 3.13). Thus, the maximum potential mass flowrate of the particles going through the nip is the bulk density of the sand multiplied by the volume of the single-layered particles passing underneath the nip (Equations 3.2). The maximum efficiency at low speeds can then be given by the ratio of the mass flow rate drawn through the nip over the mass flowrate sprayed by the sander (Equation 3.3).

\[
\dot{m}_{\text{maximum}} = \rho_b \times W \times \bar{d}_p \times v \quad (3.2)
\]

\[
\eta_{\text{max}} (%) = \frac{\dot{m}_{\text{maximum}}}{\dot{m}_p} \times 100 = \frac{\rho_b \times W \times \bar{d}_p \times v}{\dot{m}_p} \times 100 \quad (3.3)
\]
Here, $\rho_b$ is the bulk density of the sand, $W$ is the width of the nip, $\bar{d}_p$ is the mean particle diameter, $\nu$ is the train speed and $\dot{m}_p$ is the mass flowrate of sand out of the sander. The experimental results are consistent with Equation 3.3 at low substrate speeds, but they diverge from this when the speed increases past ~0.1 km/h (Figure 3.14). This is because the sand stops accumulating in front of the nip as the speed increases, and the losses from particle to particle interactions become dominant. To ensure that the decrease in efficiency was from the denser flow of particles in front of the nip, the mass flowrate of the etcher was limited from about ~58 g/min to ~40 g/min at 1 km/h. This resulted in an improvement from 32% to 48% in efficiency. Furthermore, the particle spacing can describe how tightly packed the particles are going through the nip and indicate how common particle to particle interactions are at these larger speeds. The 2D particle spacing can be applied to the sand entering into the nip by using Equation 1.5 and redefining the volume fraction as the volume of sand supplied the etcher to the volume being drawn through the nip (Equation 3.4).

Figure 3.14 Sanding efficiencies at speeds up to 2 km/h.
\[ \frac{l}{d_p} = \sqrt{\frac{\pi \rho_p W d_p v}{4 \dot{m}_p}} \] (3.4)

Here, \( \rho_p \) is the particle density, and \( l \) is the spacing between particles. The rest of the variables were outlined earlier, while describing Equation 3.3. When the particle spacing surpasses \( \sim 10 \) particle diameters then the particles become more isolated from each other as they pass through the nip and interactions are negligible [3]. However, the efficiency approaches an asymptotic limit when the spacing between particles is larger than approximately eight diameters (Figure 3.15). This suggests that the primary losses start to occur from the particle-laden jet geometry even when the particles are not fully isolated from one another as in a dilute flow. The critical substrate speed that this occurs at is between 14 to 18 km/h. Above this speed, the efficiency approaches its asymptotic limit that occurs from the particle laden jet inefficiencies previously outlined in Section 3.1.

Comparing the efficiency at 0.18 km/h with literature, an improvement of 16% was noticed over studies that used an actual locomotive sander, wheel, and rail [11]. This means that the device created may overestimate what occurs in the field. However, a direct comparison cannot be made since efficiency values given by Lewis et al. were conducted with a wet rail [11]. Although this difference may be large, the effects of having a smaller nozzle (12mm vs 19mm), the flat geometry of the wheel, different mass flowrate (the one in this device simulated 1kg/min instead of 2 kg/min), and different sized particles (2mm-1mm vs. 0.21mm used here) may be the root of the difference [11].
Summary of Deposition Results for Selected Particles and Recommendations

A total of 23 different particles were tested comprising of five different materials. The materials were various sands, aluminum oxide, silicon carbide, glass beads, and bauxite beads. All the materials besides the glass beads contained at least one sample that was tested with a coating added to it. The following are the main discoveries from these tests:

- A reduction in the spread of the particle-laden jet is related to higher efficiencies. Typically, the larger round particles had the least spread from the nozzle whereas the larger irregular particles had the greatest.
- For silica sand, smaller particles had better entrainment and did not fall out of the air-jet which may cause slightly higher efficiencies. Furthermore, the typical efficiency was found to be ~68% for sand with diameters from 0.2-0.6 mm. However, the nozzle orientation was fixed across all tests.

Figure 3.15 Efficiency and particle spacing on the belt vs. speed.
- For lower speed sanding, limiting the flow rate to reduce particle to particle interactions at the nip is recommended. Keeping the mass flow rate the same may cause excess build up and losses.
Chapter 4: Conclusions and Future Work

4.1 Conclusion

The main achievements of this investigation are as follows:

- **Experimental Apparatus and Methodology:** A device was created that adequately modeled sanding on locomotives to study the effects from changing various particle properties. Furthermore, by using TrackMate in Image J, PTV in conjunction with a post-processing code could be done to determine the jet and bounce properties of the particles. The device may overpredict values that can occur in the field due to some simplifications made, but the trends displayed in the results can be applied to train sanders to improve their efficiency.

- **Nozzle Properties:** The spread of the trajectories as particles exit the nozzle is correlated to deposition. Changes to the spread of the jet was found to be the primary cause for increasing efficiency for all properties investigated in this work, except while analyzing the different sized sand. Similarly, it was determined that there was no correlation between the bouncing properties of the particles and deposition at 18 km/h.

- **Silica Sand Size:** Increasing particle size of silica sand was correlated with decreasing deposition. Analyzing the nozzle properties showed that jet entrainment and how much the particles fall out of that jet may have been the cause of this behaviour.

- **Particle Shapes:** It was observed that the spread of the crushed glass shards as they exit the nozzle was larger than the spread of the glass beads. Accordingly, the irregular crushed glass had a lower deposition efficiency than round glass beads of a similar size.

- **Coating the Particles:** Adding a proprietary coating has shown to consistently increase deposition due to a decrease in the spread of the particle trajectory as they leave the nozzle.
4.2 Limitations and Future Work

The major limitations and recommendations of this work are:

- The testing apparatus simplifies what occurs during actual sanding so specific parameters can be isolated. However, the results in this thesis may overpredict that in the field. Most notably, modeling the wheel and rail as flat can lead to a large increase in deposition. Likewise, crosswinds were not present in this study and may also reduce deposition in the field. The effects of these can be studied in future work.

- This study neglected the aerodynamics of the train underneath the carriage. Since the geometry of the train undercarriage can result in a complex flow pattern around the sander, another study would be required to realistically simulate the flow in the experiments. Also, when the train speed becomes larger, the aerodynamics of the jet may vary from the oncoming flow. Author Lewis et al. found an increase in deposition when an oncoming airflow was added to their tests [11].

- A more in-depth analysis of the particle-laden jet could be done. As mentioned in Section 3.1, the cause of the different spreads as the particle leaves the nozzle should be studied. One possible study could investigate how well the particles are entrained in the jet as they leave the tube. For the larger particles, we saw a very large efficiency if they were round, or a very small efficiency if they were irregular. This may be because the larger particles become less entrained into the jet and thus roll out of the nozzle. If the particles are irregular, then they tumble as they roll out of the nozzle. This would lead to a large spread
for irregular particles and funnel the round particles into a stream like that observed. However, this is just speculation that could be confirmed with additional study.

- Studying an actual train sander is recommended to understand their operation and more accurately model those on actual trains. In these tests, the Reynolds number of the air jet at the nozzle and mass flowrate of particles is smaller than an actual train sander since the diameter of the sanding nozzle was reduced.

- If smaller particles are used, then a more robust collector is recommended. Wear on the collection fenders introduced leaks. Furthermore, surface variations in the wheel and rail make it difficult to keep the collector perfectly flat on the belt and against the wheel, which likely resulted in increased losses of finer particles.
Bibliography


Appendices

Appendix A  Standard Operating Procedures

Before operating the device, all of the relevant manuals should be reviewed so that the user can appropriately respond to any abnormal activities that may occur (references [36], [37], but may include more). There are many pinch points in the device so the door should always be shut while running tests. In addition, safety guards should not be removed. This standard operational procedure outlines the average procedure for running tests as well as some general maintenance protocols of the device.

Normal Operation:

1. Pre-run Inspection: ensure the following are in good working order:
   a. The air hose to the device is not damaged and is within its operational time frame.
   b. The belt to the device is not cracked at the weld.
   c. The belt can be freely moved.
   d. The wheel is free moving.
   e. All connections to the etcher are together.
   f. All of the components appear in good working order (no damage, loose parts/ bolts, or wear is observed)

2. Startup:
   a. Turn on the air to the device at the main air source by the air cannon and bleed the air to ensure that no moisture is going into the device.
   b. Turn on the electricity to the device and run the following programs on the lab computer:
      i. RS weight.
      ii. Wheel Speed
      iii. If using the high-speed camera, then PCC® software should be on.
   c. Plug in the encoder to the power strip.
3. Sample preparation:
   a. Screen the samples so that the particles are under 1.25mm in diameter.
   b. Weigh about 100 grams of the particles into a cup

4. Tests:
   a. Start the air through the nozzle by turning the gate valve to the on position.
   b. Adjust the pressure and needle valve to achieve a flow of 11.5 scfh given on on the flowmeter.
   c. Turn on the variac and increase the voltage until the desired speed is achieved, which is verified by the Wheel Speed.vi program. This speed should be monitored throughout the test with the voltage adjusted as needed.
   d. Turn on the belt grinder by flipping the switch shown on the front of the belt grinder.
   e. Once everything is at steady state, pour the particles into the funnel.
   f. Monitor the mass being collected into the bucket until steady state is achieved, then get a visual confirmation that no particles are leaving the nozzle.
   g. Shut the components down in the reverse of the startup order (i.e. turn off the belt grinder, then the motor to the wheel, then the air.)
   h. Once everything is stopped, then take off the front of the chamber and weigh the particles in the bin that is at the end of the collection fender. Record this weight and divide it by the initial weight to find the efficiency.

5. Post-Test/ Between Tests Servicing:
   a. Collect the particles that have gathered outside of the collection bin with a vacuum.
   b. Ensure that there are no particles still in the nozzle by unscrewing it and vacuuming the particles inside the chamber where they become entrained in the air-jet.

Maintenance:
Replacing the belt:
   1. Loosen the belt alignment adjustment by rotating screwed into the right side of the belt grinder. Then loosen the lever above that handle.
2. Work the belt off by rolling the wheels by hand and pulling the belt towards you, ensuring that it does not catch on the side of the collection fender.

3. Put two layers of duct tape about 7mm thick on the user side on the inside of the belt.

4. Install the new belt in a similar way that the old one was removed.

Removing the wheel:

1. In this device, the wheel can also be easily removed for servicing. To remove the wheel, first remove the motor assembly.

2. Remove the axle by sliding it out of the device in the direction of the motor.

3. The wheel should slide off of the axle during step 2.

Removing and adjusting the collection fender:

1. Remove the screws shown in the order below above the collector, or to adjust, loosen them and turn them as needed. Remove the clip that tensions the prongs if removing the collector.

2. To reassemble, tighten the screws in step 1 in the reverse order. This time, be careful to observe how well the prongs site next to the belt. The belt tension may affect the fit of the collection fender.
Appendix B  Post Processing Code

Nozzle Calculation Code:

```matlab
%Particle-Laden Jet Analyzer.
%This code was created by: Justin Roberts
%If this code is re-used or referenced, please cite this thesis.

clear all
clc
%Reads in the data from Trackmate as a .xlsx file, organizes the data.
trackmate_filename='C:\Users\justi\Desktop\Nozzle Data\Coated Al Ox new.xlsx'
G=xlsread(trackmate_filename);
x_pos=G(:,4);
y_pos=G(:,5);
time=G(:,7);
track_id=G(:,2);
spot_id=G(:,1);
Dia=G(:,18);

%User adjustments for the code. The time for each frame comes from the PPC video.
limit_nozzle=1; %The range of how many diameters away from the nozzle (more precisely, the beginning of the PTV) should the nozzle calculations be.
Ft=37.04*10^-6 %seconds per frame calculating velocity
min_o=1 %allowable distance in mm from downstream plane for particles to be considered in the calculation
dis=15 %nozzle diameters away from the nozzle for the downstream calculations
d_noz=3; %nozzle diameter in mm.
Ang_nozz=12 %angle of nozzle in degrees, this is just for reference and is replaced by the average angle of the trajectories for better precision.
% The following organizes the data with increasing track number,
for i1=0:1:max(track_id)
    ind=find(track_id==i1);
    X{i1+1}=ind;
    for j1=1:1:length(X{i1+1})
        T{i1+1}(j1,:)=G(X{i1+1}(j1),:);
    end
end

%%%Velocity out of the jet analyzer
figure
hold on
for i=1:1:length(T)
    plot(T{i}{:,4},T{i}{:,5});
end
hold off
k=1;
%The following finds the trajectory with the location that is farthest right, which is where the nozzle location is. This can be verified with %the graphs of the trajectories.
[X_nozzle, max_ind]=max(x_pos);
```
Y_nozzle=y_pos(max_ind);

%Nozzle Speed and Spread Calculator. Finds the start of each track and
%compares them to the location found above. They get rejected if the start
%point is outside of the 1 dia. threshold.
for j=1:length(T)
    x_i=T{j}(:,4);
    y_i=T{j}(:,5);
    diff_x=X_nozzle-x_i(1);
    diff_y=Y_nozzle-y_i(1);
    L=length(x_i);
    if diff_x<limit_nozzle && L>80 % A filter for the 1 dia. threshold and to
        ensure that the length of the trajectory is large enough for calculations.
        v_x_nozzle=(T{j}(79,4)-T{j}(9,4))/(Ft*70*1000);
        v_y_nozzle=(T{j}(79,5)-T{j}(9,5))/(Ft*70*1000);
        V=sqrt((v_x_nozzle)^2+(v_y_nozzle)^2);
        V_nozzle(k,1)=k;
        V_nozzle(k,2)=T{j}(3,2);
        V_nozzle(k,3)=T{j}(3,4);
        V_nozzle(k,4)=T{j}(3,5);
        V_nozzle(k,5)=V;
        V_nozzle(k,6)=atand(v_y_nozzle/v_x_nozzle);
        k=k+1;
    end
end

%The following is the calculations for the downstream particle properties.
angle_nozz=abs(mean(V_nozzle(:,6))); %uses average angle to get centerline jet
position
%x_dis=dis*d_noz*cosd(angle_nozz); %x length of 15d
%y_dis=dis*d_noz*sind(angle_nozz); %correlating distance of 15d
[X_dis_o, max_ind]=max(x_pos); %finding the relative distance from the nozzle
Y_dis_o=mean(V_nozzle(:,4)) %Finds the horizontal location of the particles
from the nozzle calculations shown above.
%y_pos(max_ind);
Y_dis=Y_dis_o+y_dis; %relative y position from the nozzle
for j_1=1:length(T)
    x_i=T{j_1}(:,4);
    y_i=T{j_1}(:,5);
    compare_x=abs(T{j_1}(:,4)-x_dist);
    %[min_y, in_y]=min(compare_y);
    [min_x, in_x]=min(compare_x);
    log{j_1}=compare_x;
    L_downstream=length(x_i);
    pass=L_downstream-in_x;
end
if min_o>min_x && pass>20 && in_x>20 %Finds the particles that pass within the preset threshold of the downstream particle plane, and ensures that the trajectory does not start less than 20 frames before or after the plane.

in_x3=in_x-3;

v_check=(T{j_1}(in_x3,4)-T{j_1}(in_x,4))/(Ft*15*1000);
if v_check>0

in_x2=in_x-15;
v_x_downstream=(T{j_1}(in_x2,4)-T{j_1}(in_x,4))/(Ft*15*1000);
v_y_downstream=(T{j_1}(in_x2,5)-T{j_1}(in_x,5))/(Ft*15*1000);
V_d=sqrt((v_x_downstream)^2+(v_y_downstream)^2);
V_downstream(f,1)=f;
V_downstream(f,2)=in_x;
V_downstream(f,3)=min_x;
V_downstream(f,4)=T{j_1}(1,2);
V_downstream(f,5)=T{j_1}(in_x,4);
V_downstream(f,6)=T{j_1}(in_x,5);
V_downstream(f,7)=V_d;
V_downstream(f,8)=atan2(v_y_downstream,v_x_downstream);
V_downstream(f,9)=-T{j_1}(in_x,5)+Y_dis;
V_downstream(f,10)=T{j_1}(in_x,18);
f=f+1;
end
end
end

%The following plots can be used by the user to verify the data and see if any anomalies occur.
figure
scatter(V_downstream(:,1),V_downstream(:,9)) %Plots the distance from the centerline of the jet to the particles horizontal location downstream (i.e. how far the particles drop from the original jet).
title('distance from centerline of jet downstream')
figure
scatter(V_downstream(:,1),V_downstream(:,8))
title('angle distribution at downstream')
figure
scatter(V_nozzle(:,1),V_nozzle(:,5))
hold on
scatter(V_downstream(:,1),V_nozzle(:,5));
title('jet exit and downstream velocities') %Compares nozzle velocities to jet velocities downstream. These should be about the same.
legend('Nozzle','Downstream')
hold off
figure
scatter(V_downstream(:,8),V_downstream(:,9))

%The following outputs table "sTable" which summarizes the data
Output=zeros(2,5);
Output(1,1)=mean(V_nozzle(:,5));
Output(1,2)=mean(V_nozzle(:,6));
Output(1,3)=mean(V_downstream(:,7));
Output(1,4)=-(mean(V_nozzle(:,6))-mean(V_downstream(:,8)));
Output(1,5)=mean(V_downstream(:,9));
Output(2,1)=std(V_nozzle(:,5));
Output(2,2)=std(V_nozzle(:,6));
Output(2,3)=std(V_downstream(:,7));
Output(2,4)=std(V_downstream(:,8));
Output(2,5)=std(V_downstream(:,9));
colNames = 
{'V_n', 'Alpha_n', 'V_downstream', 'Alpha_Downstream_vs_Upstream', 'DeltaY_downstream'};
rowNames = {'Average', 'StD'};
sTable = array2table(Output,'RowNames',rowNames,'VariableNames',colNames);

CoR and RR Calculator Code:

%Master code for CoR and RR Code
This code was created by: Justin Roberts
%If any part of this code and or functions was re-used or referenced, please cite this thesis.
clc
clear variables
clear all
lim=70;%Controls permitted minimum length of track in frames. must be larger than 4.
MM_p=19.2696;%pix per mm taken from ImageJ
F_T=37.04*10^-6; %milliseconds per frame tacked from Cine Viewer
V_t=18; %Velocity of the belt in km/hr
V_w=18; %Velcoity of the wheel in km/hr

%Filter values for the belt
P_t_conn=45;%distance from the belt that the bounce has to occur in pix
T_diff_pos=20;%dictates where the trajectory starts
T_traj_lim=40;%dictates number of frames before and after the bounce
T_v_c=90;% This value is used for the time step of before and after calculations to get the speeds

%Filter values for the wheel
P_w_conn=90;
Wheel_max_diff=30;
W_traj_lim=15;
W_v_c=30;

% Data inputs (One for determining the geometry and the other is the PTV results)
image_filename='C:\Users\justi\Desktop\Bounce Analysis\Coated Bauxite Beads 2.jpg';
trackmate_filename='C:\Users\justi\Desktop\Bounce Analysis\Uncoated Bauxite Beads 3.xlsx';

%Edge finders
[slope_1,slope_2,P1,P2,P3,I,f,c] = GeometryFinalDevice_Wheel_Finder(image_filename);
[slope_1_T, slope_2_T, P1_T, P2_T, I_T, f1, c1] = 
GeometryFinalDevice_Track_Finder(image_filename);

%Bounce Finding Algorithms
[T, Iter, Keep, Log_belt_con, Track] = Track_Bounce_Finder(
    trackmate_filename, P_t_conn,
    F_T, lim, MM_p, P1_T, P2_T, T_traj_lim, T_diff_pos, V_t, T_v_c);

[Log_wheel_con ] = Wheel_Bounce_1(
    trackmate_filename, MM_p, P1, P2, P3, lim, F_T, P_w_conn, Wheel_max_diff, W_traj_lim, I,
    V_w, W_v_c);

function [slope_1, slope_2, P1, P2, P3, I, a, f, c ] = 
GeometryFinalDevice_Wheel_Finder(image_filename)

% Geometry and Speed Initializing Code.
% Justin Roberts, UBC, 2017
% Purpose is to collect the wheel and belt speed and geometry. It
should
% output data which is used as part of a larger code for developing the
% coefficient of restitution.

%C:\Users\Justin\Desktop\0.77 s.jpg
buff =10;
I = imread(image_filename);

I = im2bw(I);

I2 = bwareaopen(I, 2000);

[a,b]=size(I);
a_2=.75*a;
I2(a_2:a,:) = 1;

I=edge(I2);
I(:,b-.25*b:b)=0;
I(:,1:2)=0;
I_in=I(:,3);
[~,r_in]=max(I_in);
r_cut=r_in+buff;
I(r_cut,:)=0;
[r_0, c_0] = find(I);

z = fit(c_0, r_0, 'poly1');
formula(z);
Z1 = z.p1;
Z2 = z.p2;

for i = 1:1:b
    cut = Z1*i + Z2 + buff;
    cut = round(cut, 0);
    if cut == 0
        break
    else
        I(cut,:)=0;
    end
end

figure
imshow(I);
hold on;
[r, c] = find(I);

f = fit(c, r, 'poly2');
plot(f(min(c):max(c)), 'blue', 'LineWidth', 3)

formula(f);
P1 = f.p1;
P2 = f.p2;
P3 = f.p3;

slope_1 = P1;
slope_2 = P2;
hold off
end

function [ slope_1_T, slope_2_T, P1_T, P2_T, I_T, f1, c1 ] = GeometryFinalDevice_Track_Finder(image_filename)
% Geometry and Speed Initializing Code.
% Justin Roberts, UBC, 2017
%Purpose is to collect the wheel and belt speed and geometry. It should
%output data which is used as part of a larger code for developing the
%coefficient of restitution.
I = imread(image_filename);
figure;
imshow(I);
hold on;
I2 = bwareaopen(I, 300);
[a, b] = size(I);
a_2 = .45 * a;
b_2 = .1 * b;
I2(1:a_2,:) = 1;
I2(:,1:b_2) = 1;
I2(:,b-b_2:b) = 1;
I_in = I2(:,.5*b);
[c_in, r_in] = min(I_in);
r_cut = r_in + 40;
r_cut_2 = r_in - 10;
I2(r_cut:a,:) = 1;
I2(1:r_cut_2,:) = 1;
I2 = edge(I2);
r_cut = r_in + 50;
I2(r_cut:a,:) = 0;
[r, c1] = find(I2);
f1 = fit(c1, r, 'poly1');
plot(f1(min(c1):max(c1)), 'red', 'LineWidth', 5);
formula(f1);
P1_T = -f1.p1;
P2_T = f1.p2;
slope_1_T = P1_T;
slope_2_T = P2_T;
I_T = I2;
end
function [T,Iter,Keep, Log_belt_con,Track, logger] = Track_Bounce_Finder(trackmate_filename,P_t_conn,F_T,lim,MM_p,P1_T,P2_T,T_traj_lim,T_diff_pos,V_t,T_v_c)
%algorithm that finds the bounces on the belt

G=xlsread(trackmate_filename);
x_pos=G(:,4);
y_pos=G(:,5);
time=G(:,8);
track_id=G(:,2);
spot_id=G(:,1);
Dia=G(:,18);

%organizes the data with increasing track number,
for il=0:1:max(track_id)
    ind=find(track_id==il);
    X{il+1}=ind;
    for j1=1:length(X{il+1})
        T{il+1}(j1,:)=G(X{il+1}(j1,:),:);
    end
end

% Filter for tracks
figure
for m=1:length(T)
    d=size(T{m});
    if d(1)<=lim
        T{m}=[];
    else
        [col3,~]=size(T{m});
        col4=col3/2;
        col4=round(col4,0);
        pos_y_i=T{m}(1,5);
        pos_y_r=T{m}(col4,5);
        pos_y_f=T{m}(col3,5);
        dif_pos_y=pos_y_f-pos_y_i;
        pos_x_i=T{m}(1,4);
        pos_x_r=T{m}(col4,4);
        pos_x_f=T{m}(col3,4);
        dif_pos_x=pos_x_f-pos_x_i;
        dif_pos_y_i= abs((pos_y_i*MM_p)-P2_T);
        dif_pos_y_r=abs((pos_y_r*MM_p)-P2_T);
        dif_pos_y_f= abs((pos_y_f*MM_p)-P2_T);

        % slope_part=abs(dif_pos_y/dif_pos_x);
        % belt_to_par_slope=abs(P1_T/slope_part); belt_to_par_slope>.7 && belt_to_par_slope< 1.3
if  dif_pos_y_l<T_diff_pos && dif_pos_y_r<T_diff_pos &&
  dif_pos_y_f<T_diff_pos  %Y position of the belt has to be
  outside of 40 pix to the belt  
  T(m)=[];
  elseif  T{m}(1,4)==T(m)(10,4)  
  T(m)=[];
  end
  end
k=1;
for i=1:1:length(T)

  [s,~]=size(T{i});
  sl=s-3;
  if  s==0
    continue
  end
  for j=4:1:sl
    j_o=j-1;
    j_e=j+1;
    %     j_o2=j-3;
    %     j_e2=j+3;
    %     j_o3=j-3;
    %     j_e3=j+5;

    y_diff_i=T{i}(j,5)-T{i}(j_o,5);
    y_diff_e=T{i}(j_e,5)-T{i}(j,5);
    % y_diff_i2=T{i}(j,7)-T{i}(j_o2,7);
    % y_diff_e2=T{i}(j_e2,7)-T{i}(j,7);
    %
    % y_diff_i3=T{i}(j,5)-T{i}(j_o3,5);
    % y_diff_e3=T{i}(j_e3,7)-T{i}(j,7);

    y_diff_iave=T{i}(j,5)-T{i}(1,5);
    y_diff_eave=T{i}(s,5)-T{i}(j,5);

    y_change=y_diff_i*y_diff_e;
    % y_change2=y_diff_i2*y_diff_e2*1;
    % y_change3=y_diff_i3*y_diff_e3*1;
    y_changeave=y_diff_iave*y_diff_eave;

    [Max,Indic]=max(T{i}(:,5));
    dif_max=abs(Max-T{i}(j,5));
    dif_ind=abs(j-Indic);

    diff_belt_part=abs((P1_T*(T{i}(j,4)*MM_p)+P2_T)-(T{i}(j,5)*MM_p));

    if  y_change<0 && diff_belt_part<P_t_conn && y_diff_i>0
        && y_changeave<0 && y_diff_i3>0 && y_diff_iave>0
        && y_change2>0 && y_change3>0 && dif_ind<2
    Has to be change direcetion in Y axis, be in belt region, have a max value there
        Iter{k}=j;
        Keep{k}=T{i};

    k;

end
% This algorithm logs tracks that have multiple bounces.

for i5 = 2:1:length(Keep)
    [s4,~] = size(Keep{i5});
    [s5,~] = size(Keep{i5-1});
    if s4 == 0
        continue
    elseif s5 == 0
        continue
    end
    track_num_i = Keep{i5-1}(1,2);
    track_num = Keep{i5}(1,2);
    if track_num == track_num_i
        logger(q3) = track_num;
        q3 = q3 + 1;
    end
end

for i6 = 1:1:length(Keep)
    if isempty(Keep{i6}) == 1
        continue
    end
    Track(i6) = Keep{i6}(1,2);
end

if ~exist('logger','var')
    % If the keep variable doesn't exist, outputs a blank matrix to avoid error.
    logger = [];
end

for i7 = 1:1:length(logger)
    % All of this is meant to delete multiple bounces should they happen.
    numb = logger(i7);
    [~,col] = find(Track == numb);
    for i8 = 1:1:length(col)
        coll = col(i8);
        Keep(coll) = [];
        Iter(coll) = [];
    end
end
if ~exist('Keep','var') % If the keep variable doesn't exists, outputs a blank matrix to avoid error.
    Keep=[];
    Iter=[];
else

    for i3=1:1:length(Keep)
        if isempty(Keep{i3})==1
            continue
        else
            [comp,~]=size(Keep{i3}); %excludes tracks that have a minimum near the end of the track.
            bounce_frame=Iter{i3};
            compare= abs(bounce_frame-comp);
            if compare<T_traj_lim %This should be pulled out.
                Keep{i3}=[];
                Iter{i3}=[];
            elseif bounce_frame<T_traj_lim
                Keep{i3}=[];
                Iter{i3}=[];
            end
        end
    end
end
n1=1;
figure
hold on
for i4=1:1:length(Keep)
    [s3,~]=size(Keep{i4});
    if s3==0
        continue
    else
        Bounce=Iter{i4}; %Calculates and Logs CoR and RR.
        Bounce_e=Iter{i4}+T_v_c+10;
        Bounce_i=Iter{i4}-T_v_c-10;
        Bounce_o=Iter{i4}-10;
        Bounce_p=Iter{i4}+10;
        if Bounce_i<0
            Bounce_i=5
        end
        if Bounce_e>s3
            Bounce_e=s3-5;
        end
        delta_t_i=Bounce_o-Bounce_i
delta_t_p=Bounce_e-Bounce_p
        y_diff_i=Keep{i4}(Bounce_o,5)-Keep{i4}(Bounce_i,5);
        y_diff_e=Keep{i4}(Bounce_e,5)-Keep{i4}(Bounce_p,5);
        x_diff_i=Keep{i4}(Bounce_o,4)-Keep{i4}(Bounce_i,4);
        x_diff_e=Keep{i4}(Bounce_e,4)-Keep{i4}(Bounce_p,4);
vel_i_y = \frac{(y\_diff\_i)}{(F\_T \* \delta_t\_i \* 1000)};
vel_e_y = \frac{(y\_diff\_e)}{(F\_T \* \delta_t\_p \* 1000)};
vel_i_x = \frac{(x\_diff\_i)}{(F\_T \* \delta_t\_i \* 1000)};
vel_e_x = \frac{(x\_diff\_e)}{(F\_T \* \delta_t\_p \* 1000)};
vel_i = \sqrt{vel_i_x^2 + vel_i_y^2};
vel_e = \sqrt{vel_e_x^2 + vel_e_y^2};

Log\_belt\_con(n1,1) = n1;
Log\_belt\_con(n1,2) = Keep\{i4\}(1,2);  % Track ID
Log\_belt\_con(n1,3) = Keep\{i4\}(Bounce,3);  
Log\_belt\_con(n1,4) = Keep\{i4\}(Bounce,8);  % Frame
Log\_belt\_con(n1,5) = -vel_e_y/vel_i_y;
Log\_belt\_con(n1,6) = ((-vel_e_x)-(V_t*0.2778))/((-vel_i_x)-(V_t*0.2778));
Log\_belt\_con(n1,7) = vel_e/vel_i;
Log\_belt\_con(n1,8) = Keep\{i4\}(Bounce,1);  % spot ID
Log\_belt\_con(n1,9) = Keep\{i4\}(Bounce,4);
Log\_belt\_con(n1,10) = Keep\{i4\}(Bounce,5);
Log\_belt\_con(n1,11) = delta\_t\_i;
n1 = n1 + 1;

plot(Keep\{i4\}(:,4),Keep\{i4\}(:,5))

end
end
hold off
if ~exist('Log\_belt\_con','var')  % If the keep variable doesn't
exists, outputs a blank matrix to avoid error.
    Log\_belt\_con = [];
end
end

function [ Log\_wheel\_con,Keep\_w, logger\_w ] = Wheel\_Bounce\_1(
trackmate\_filename,MM\_p,P1,P2,P3,lim,F\_T,P\_w\_conn,W\_max\_diff,W\_traj\_lim,I,
V\_w, W\_v\_c)

%UNTITLED Summary of this function goes here
% Detailed explanation goes here

G = xlsread(trackmate\_filename);
x\_pos = G(:,4);
y\_pos = G(:,5);
%x\_vel = G(:,6);
%y\_vel = G(:,6);
time = G(:,8);
track\_id = G(:,2);
spot\_id = G(:,1);
Dia = G(:,18);

%organizes the data with increasing track number,
for il = 0:1:max(track\_id)
ind=find(track_id==i1);
X{i1+1}=ind;
for j1=1:1:length(X{i1+1})
    T{i1+1}(j1,:)=G(X{i1+1}(j1,:),:);
end
end

%% Filter for tracks
for m=1:1:length(T);
    d=size(T{m});
    if d(1)<=lim;
        T{m}=[];
    end
end
k=1;
for i=1:1:length(T)
    [s,~]=size(T{i});
    s1=s-9;
    if s==0
        continue
    end
    for j1=1:1:s
        diff_wheel_pos{i}(j1)=abs(P1*(((T{i}(j1,4)*MM_p)^2)+P2*(T{i}(j1,4)*MM_p)+P3)-
                        ((T{i}(j1,5)*MM_p)));
    end
    [MIN,Indic]=min(diff_wheel_pos{i});
    [~,siz]=size(diff_wheel_pos{i});
    compar=siz-Indic;
    Mean=mean(diff_wheel_pos{i});
    comparer=abs(diff_wheel_pos{i}-Mean);
    [Max_com,~]=max(comparer);
end

%Start of track finding algorithm.
for j2=3:1:s-3
    j_o=j2-1;
    j_e=j2+1;
    % j_o2=j2-3;
    % j_e2=j2+3;
    % j_o3=j2-5;
    % j_e3=j2+5;
    y_diff_i=diff_wheel_pos{i}(j_o)-diff_wheel_pos{i}(j2);
    y_diff_e=diff_wheel_pos{i}(j_e)-diff_wheel_pos{i}(j2);
    % y_diff_i2=diff_wheel_pos{i}(j_o2)-diff_wheel_pos{i}(j2);
    % y_diff_e2=diff_wheel_pos{i}(j_e2)-diff_wheel_pos{i}(j2);
\% \mbox{y\_diff\_i3} = \mbox{diff\_wheel\_pos\{i\}(j2)} - \mbox{diff\_wheel\_pos\{i\}(j\_o3)};
\% \mbox{y\_diff\_e3} = \mbox{diff\_wheel\_pos\{i\}(j\_e3)} - \mbox{diff\_wheel\_pos\{i\}(j2)};

\mbox{y\_diff\_iave} = \mbox{diff\_wheel\_pos\{i\}(j2)} - \mbox{diff\_wheel\_pos\{i\}(1)};
\mbox{y\_diff\_eave} = \mbox{diff\_wheel\_pos\{i\}(s)} - \mbox{diff\_wheel\_pos\{i\}(j2)};

\mbox{y\_change} = -y\_\mbox{diff\_i} \times \mbox{y\_diff\_e};
\% \mbox{y\_change2} = -y\_\mbox{diff\_i2} \times y\_\mbox{diff\_e2};
\% \mbox{y\_change3} = -y\_\mbox{diff\_i3} \times y\_\mbox{diff\_e3};
\mbox{y\_change\_ave} = \mbox{y\_diff\_iave} \times \mbox{y\_diff\_eave};

\begin{array}{l}
\{\text{Max,Indic}\} = \mbox{max}\{(T\{i\}\{(:,5))}\};
\text{dif\_max} = \mbox{abs}(\text{Max} - T\{i\}(j2,5)));
\text{dif\_ind} = \mbox{abs}(j2 - \text{Indic});
\end{array}

\mbox{if} \ y\_\mbox{change} < 0 \ && \ \mbox{diff\_wheel\_pos\{i\}(j2)} < \ P\_w\_\text{conn} \ && \ \text{Max}\_\text{com} > \text{Wheel}\_\text{max}\_\text{diff} \ && \ y\_\mbox{change2} < 0 \ && \ y\_\mbox{change3} < 0 \ && \ \text{MIN} < \ P\_w\_\text{conn} \ && \ \text{Indic} > 2 \ && \ y\_\mbox{change\_ave} > 0 \ && \ \text{compar} > 2

\begin{array}{l}
\text{Iter\{k\}} = \ j2; \ \%\text{Indic};
\text{Keep\_w\{k\}} = \mbox{diff\_wheel\_pos\{i\}};
\text{Keep\{k\}} = T\{i\};
\% \ k
\% \ T\{i\}(1,4)
\text{k} = \text{k} + 1;
\end{array}

\end{array}

\mbox{end}
\mbox{end}
\mbox{end}

\mbox{n1} = 1;
\mbox{q3} = 1;
\mbox{for} \ i6 = 1:1:length(\text{Keep\_w})
\begin{array}{l}
\text{if} \ \mbox{isempty}(\text{Keep\_w\{i6\}}) == 1
\text{continue}
\text{end}
\text{Track\{i6\}} = \text{Keep\{i6\}(1,2)};
\end{array}
\text{end}
\mbox{for} \ i5 = 2:1:length(\text{Keep}) \ % \text{this algorithm logs tracks that have multiple bounces.}
\begin{array}{l}
\{s4,\text{~}\} = \mbox{size}(\text{Keep\_w\{i5\}});
\{s5,\text{~}\} = \mbox{size}(\text{Keep\_w\{i5-1\}});
\text{if} \ s4 = 0
\text{continue}
\text{elseif} \ s5 = 0
\text{continue}
\text{end}
\text{track\_num\_i} = \text{Keep\{i5-1\}(1,2)};
\text{track\_num} = \text{Keep\{i5\}(1,2)};
\end{array}

if track_num==track_num_i
    logger_w(q3)=track_num;
    q3=q3+1;
end
end
for i6=1:1:length(Keep_w)
    if isempty(Keep_w{i6})==1
        continue
    end
    Track(i6)=Keep{i6}(1,2);
end
if ~exist('logger_w','var') % If the keep variable doesn't exists, outputs a blank matrix to avoid error.
    logger_w=[];
end
for i7=1:1:length(logger_w) %All of this is meant to delete multiple bounces should they happen.
    numb=logger_w(i7);
    [~,col]=find(Track==numb);
    for i8=1:1:length(col)
        col1=col(i8);
        Keep_w{col1}=[];
    end
end
if ~exist('Keep','var')
    Keep_w=[];
    Iter=[];
else
    for i3=1:1:length(Keep_w)
        if isempty(Iter{i3})==1
            continue
        else
            [~,comp]=size(Keep_w{i3});
            bounce_frame=Iter{i3};
            compare= abs(bounce_frame-comp);
            if compare<W_traj_lim && bounce_frame>W_traj_lim
                Keep_w{i3}=[];
                Iter{i3}=[];
            elseif Iter{i3}<W_traj_lim
                Keep_w{i3}=[];
                Iter{i3}=[];
            end
        end
    end
end
figure
hold on
for i4=1:length(Keep)
    [s3,~]=size(Keep_w{i4});
    if isempty(Keep_w{i4})==1
        continue
    end

    Bounce=Iter{i4};
    Bounce_o=Iter{i4}-5;
    Bounce_p=Iter{i4}+5;
    Bounce_e=Iter{i4}+W_v_c+5;
    Bounce_i=Iter{i4}-W_v_c-5;

    y_diff_i=Keep{i4}(Bounce_o,5)-Keep{i4}(Bounce_i,5);
    y_diff_e=Keep{i4}(Bounce_e,5)-Keep{i4}(Bounce_p,5);

    bounce_pos_pix=(MM_p*Keep{i4}(Bounce,4));
    slope=atand(2*P1*bounce_pos_pix+P2);

    x_diff_i=Keep{i4}(Bounce_o,4)-Keep{i4}(Bounce_i,4);
    x_diff_e=Keep{i4}(Bounce_e,4)-Keep{i4}(Bounce_p,4);

    vel_i_y=y_diff_i/(F_T*W_v_c*1000);
    vel_e_y=y_diff_e/(F_T*W_v_c*1000);

    vel_i_x=x_diff_i/(F_T*W_v_c*1000);
    vel_e_x=x_diff_e/(F_T*W_v_c*1000);

    vel_i=sqrt(vel_i_x.^2+vel_i_y.^2);
    vel_e=sqrt(vel_e_x.^2+vel_e_y.^2);

    slope_part=atand(vel_i_y/vel_i_x);
    slope_part_e=atand(vel_e_y/vel_e_x);

    V_n_i= sqrt(vel_i_x.^2+vel_i_y.^2).*sind((slope_part-slope))
    V_t_i=sqrt(vel_i_x.^2+vel_i_y.^2).*cosd((slope_part-slope))
    V_n_e=sqrt(vel_e_x.^2+vel_e_y.^2).*sind(-slope_part_e+slope)
    V_t_e=sqrt(vel_e_x.^2+vel_e_y.^2).*cosd(-slope_part_e+slope)

    CoR_w=(V_n_e)/(V_n_i);
    RR_w=(V_t_e-(V_w*0.2778))/(V_t_i-(V_w*0.2778));

    Log_wheel_con(n1,1)=n1;
    Log_wheel_con(n1,2)=Keep{i4}(1,2); %Track ID
    Log_wheel_con(n1,3)=Keep{i4}(Bounce,1); %Frame
    Log_wheel_con(n1,4)=Iter{i4};
    Log_wheel_con(n1,5)=CoR_w;
    Log_wheel_con(n1,6)=RR_w;
    Log_wheel_con(n1,7)=vel_e/vel_i;
    Log_wheel_con(n1,8)=Keep{i4}(Bounce,4);
    n1=n1+1;
plot(Keep{i4}(:,4),-Keep{i4}(:,5))
end
hold off
end

% [Max,~]=max(diff_wheel_pos{i});
  In_for=Indic+3;
  In_back=Indic-3;
  [col3,~]=size(diff_wheel_pos{i});
  col4=col3/2;
  col4=round(col4,0);
  pos_y_i=diff_wheel_pos{i}(1,7);
  pos_y_r=diff_wheel_pos{i}(col4,7);
  pos_y_f=diff_wheel_pos{i}(col3,7);
  pos_y_for=diff_wheel_pos{i}(In_for,7);
  pos_y_back=diff_wheel_pos{i}(In_back,7);
  diff_pos_y_i=(pos_y_i-MIN)
  diff_pos_y_r=(pos_y_r-MIN)
  diff_pos_y_f=(pos_y_f-MIN)
  diff_pos_y_for=(pos_y_for-MIN)
Appendix C  Particle Size Distributions and Appearances.

Note: The percent distributions are the area diameter of the particles (mm) are given with the particle’s ID. The red line in the images is a 5mm scale.
Appendix D  Nozzle Trajectories

The raw images of the particle-laden jet and the trajectories calculated from the PTV are given with the particle’s ID. Since only a single video can be considered, the trajectories may look sparse for small loadings.
Raw

Trajectories

S1  S2  S5  C4  S7