Hadron Production Measurements of the EMPATHIC Group

by

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Abstract

Long-baseline neutrino experiments produce neutrinos by colliding a beam onto a target. Hadrons produced in the target can be magnetically focused into a beam that decays in flight, yielding a collimated beam of neutrinos. Neutrino flux predictions are used to constrain the uncertainties of the detected neutrino flux and its uncertainty is dominated by the uncertainties in hadronic interactions. The reduction of this uncertainty would further the physics goals for these experiments. These uncertainties can be sourced to the production cross section uncertainty which has been set to a conservative value due to previous discrepancies in the estimated production cross section seen in the NA61/SHINE 2cm and 90cm carbon target data, used by the T2K collaboration. In addition, untuned secondary interactions within the horn and decay volume contribute to the uncertainty.

The EMPHATIC group has taken data using the Fermilab test beam on carbon, aluminum, and steel targets for hadrons ranging from 2GeV/c to 120GeV/c. EMPHATIC uses gas Cherenkov and lead glass calorimeters for particle identification, semiconductor tracking detectors for position and timing information, and emulsion situated on a moving table for precise position measurements. In addition, an aerogel Cherenkov detector was constructed at TRIUMF which was used for testing at the Fermilab Test Beam Facility. The silicon strip tracking detectors were instrumental in the data acquisition of the EMPHATIC experiment. The data acquired by the silicon strip detector for 30GeV/c protons impinging on a carbon target was fit with the Bellettini et al. model [1]. The parameters of the total cross section, the black body radius, and the nuclear transparency term were fit and agree within one standard deviation with previous measurements of Bellettini et al. The elastic cross section agrees within two standard deviations.
Lay Summary

To increase the precision and accuracy of neutrino oscillation measurements uncertainties on the number of neutrinos produced in the beam need to be reduced. The EMPHATIC collaboration performed measurements at Fermilab in hopes to reduce these uncertainties. A beam of protons with various energies was directed at a number of target materials. Sensitive tracking detectors measured the number of particles produced in these collisions, in order to determine how many particles would be generated that could produce neutrinos. An analysis of the data gave results consistent with previous measurements.
Preface

This thesis describes work I have done as part of the T2K and EMPHATIC collaboration. Chapters 1 and 2 which describe aspects of the T2K collaboration have been all properly referenced and no texts have been copied directly from other sources. I contributed directly to the data acquired by the EMPHATIC collaboration, described in Chapter 2. All my work presented in this thesis was done under the excellent guidance of Professor Akira Konaka and Professor Scott Oser.

The simulation of the aerogel Cherenkov detector in Chapter 3 was my original creation, based off of aerogel scattering information from the Belle experiment [2]. The subsequent design and construction of the aerogel detector was my own, with the indispensable help of Steve Chan, Chapman Lim, and Philip Lu. In addition, during the data acquisition in Fermilab the aerogel of Chiba University was used. Thomas Lindner was instrumental in setting up the MIDAS frontend necessary in acquiring the data. Direct help from Makoto Tabata was necessary in maintaining the detector during operation in Fermilab. The efficiency analysis code was designed by myself with help from the EMPHATIC collaboration.

The work shown in Chapter 4 was on the silicon detector. The Monicelli software was designed by Fermilab, Istituto Nazionale di Fisica Nucleare, Sezione di Milano Bicocca, and Universit degli Studi di Milano Bicocca. The analysis software EMA was designed by Matej Pavin. The efficiency code and testing were all created by myself with the guidance of EMPHATIC collaboration. I created the code used to fit the data with the Bellettini et al. [1] model and the $\chi^2$ test, with the assistance of Matej Pavin.
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List of Acronyms and Abbreviations

ADC  Analog to Digital Conversion.

CAPTAN  Compact And Programmable daTa Acquisition Node.

CCQE  Charged Current Quasi-Elastic.

CP  Charge Parity.

DCB  Data Conversion Board.

ECAL  Electromagnetic CALorimeter.

EMPHATIC  Emulsion-based Measurement of the Production of Hadrons At a Test beam In Chicagoland.

FGD  Fine-Grained Detector.

FTBF  Fermilab Test Beam Facility.

MINERvA  Main INjector ExpeRiment for \(\nu_{e}\)..

MINOS  Main Injector Neutrino Oscillation Search.

ND280  off-axis Near Detector located 280m from the target.

NOvA  NuMI Off-axis \(\nu_{e}\) Appearance.
NPCB  Node Processing and Control Board.

NuMI  Neutrinos at the Main Injector.

P0D  $\pi^0$ Detector.

PDB  Power Distribution Board.

PMNS  Pontecorvo-Maki-Nakagawa-Sakata.

PMT  Photomultiplier Tube.

RICH  Ring Imaging CHerenkov detector.

SK  Super-Kamiokande.

SMRD  Side Muon Range Detector.

T2K  Tokai to Kamiokande.

TPC  Time Projection Chamber.
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Thanks, specifically, to Akira for helping me experience what only a few graduate students get to see in their careers, the setup of an experiment from the ground up. It was hectic at times but it has been one of the most rewarding experiences, as a scientist, and I thank you for that.

Scott has always been a guiding force in my work. Whenever I was confused, he would help me prioritize how to solve my analysis problems. The time he took and his patience in assisting me is what makes him an excellent supervisor.

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Chapter 1

Introduction

1.1 Neutrino Physics

1.1.1 A History of Neutrinos

The search for neutrinos can be traced back to the discovery of radiation by Henry Becquerel [3] and subsequently Marie and Pierre Curie in 1886 [4]. This quickly led to the discovery of the different types of radiation by Rutherford in 1899 [5] and by 1904 [6] he found that there are three different processes of radioactive decay: alpha decay where an alpha particle (helium nucleus) is ejected, beta decay where an electron/positron is released, and gamma decay where a photon is emitted.

After the discovery of these decays beta radiation was shown to have a continuous energy spectrum which conflicted with the idea that only a beta particle was ejected from the nucleus. This discovery, in 1914 by Chadwick [7], Hahn [8], Meitner [9], and others, forced physicists to rethink what is truly occurring in beta decay. This led to Pauli positing a neutral particle lighter than the electron, which was subsequently named the neutrino [10].

In 1956 a neutrino detector was built by Reines and Cowan [11] near a nuclear power plant as a source for anti-neutrinos. The detector consisted of 400L of water.

\footnote{The particle was dubbed the neutrino after the neutron was found: due to the difference in the theorized size.}
and CdCl. The processes below show how the anti-neutrino was measured:

\[ \bar{\nu}_e + p \rightarrow n + e^+ , \]
\[ e^+ + e^- \rightarrow \gamma + \gamma , \]
\[ n +^{114}_2 \text{Cd} \rightarrow^{115}_2 \text{Cd} + \gamma . \]  

The concept was that the anti-neutrino would interact with protons in the liquid leading to a positron and a neutron being emitted. The positron would then annihilate with an electron thus generating two photons. Afterwards the neutron would thermalize and get captured by a cadmium nucleus which would then emit photons 15 microseconds afterwards. These two photon signatures would yield a definitive signal for the neutrino. After these measurements other experiments were then made to detect the muon \[12\] and tau \[13\] neutrino. The next momentous discovery would come from the measurement of solar neutrinos and the questions they would raise.

1.1.2 Neutrino Oscillation

The solar neutrino problem

Solar neutrinos are produced from nuclear fusion occurring in the sun. This fusion is the production of alpha particles from protons within the sun. The many interactions required to produce the alpha particles in the sun are known as the \( pp \) chain. Important to this discussion is the fact that multiple steps within the \( pp \) chain produce electron neutrinos. These reactions vary in neutrino abundance and energy. The reaction most important in neutrino detection comes from the decay of boron-8 \[14\]

\[ ^8\text{B} \rightarrow ^8\text{Be} + e^+ + \nu_e . \]  

Its importance is due to the fact that this reaction produces neutrinos with the required energy for detection.

With the theory of solar neutrinos posed, Raymond Davis in 1968 attempted to measure solar neutrinos in the Homestake mine in South Dakota \[15\]. His ap-
paratus consisted of a tank of $C_2Cl_4$. The underlying process is the capture of the neutrino by the chlorine nucleus and the subsequent production of an argon atom and an electron

$$\nu_e + ^{37}_\text{Cl} \rightarrow ^{39}_\text{Ar} + e. \quad (1.3)$$

The production rate of argon could then be compared with the theoretical expectation for the rate of neutrinos reacting within the chlorine tank [14]. This experiment showed that the argon production rate was a third of its expected value. Soon after, similar experiments verified the results [16–18]. Bruno Pontecorvo had suggested that electron neutrinos may transition to their antiparticles, however that has yet to be proven [19]. This was followed by Maki et al. in 1962 who instead suggested that neutrinos oscillate between flavours [20].

**The theory of neutrino oscillations**

Neutrinos are most easily observable through charged current weak interactions, where each neutrino’s respective lepton is involved ($e, \mu, \tau$)

$$e^- \rightarrow \nu_e + W^-, \quad \mu^- \rightarrow \nu_\mu + W^-, \quad \tau^- \rightarrow \nu_\tau + W^-.$$

Equation 1.4 shows that the neutrino flavours correspond to the weak (or flavour) eigenstates ($\nu_e, \nu_\mu, \nu_\tau$) with relation to the charged weak boson ($W$) that acts as a virtual particle. These flavour eigenstates are comprised of coherent linear combinations of the mass eigenstates ($\nu_1, \nu_2, \nu_3$) [21], described by

$$\begin{bmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{bmatrix} =
\begin{bmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{bmatrix}
\begin{bmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{bmatrix}. \quad (1.5)$$

The equation shows that the weak eigenstates and mass eigenstates are related by a unitary matrix known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. Thus, the wave equation of each neutrino can be written as a superposition of the
mass eigenstates \[ (14) \]

\[ |\nu_e\rangle = U_{e1}^* |\nu_1\rangle + U_{e2}^* |\nu_2\rangle + U_{e3}^* |\nu_3\rangle. \tag{1.6} \]

This wave equation can now be used to determine the probability of oscillation between the different neutrino species.

**Two neutrino oscillation**

To simplify the problem one can consider the oscillation of only two hypothetical neutrinos \( \nu_a \) to \( \nu_b \) where \( a \) and \( b \) are placeholders for the other neutrino flavours and are not equal. This means a reworking of Equation (1.5) to a \( 2 \times 2 \) matrix:

\[
\begin{bmatrix}
\nu_a \\
\nu_b
\end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} \\
U_{\mu 1} & U_{\mu 2} \end{bmatrix} \begin{bmatrix}
\nu_1 \\
\nu_2
\end{bmatrix}, \tag{1.7}
\]

\[
\begin{bmatrix}
\nu_a \\
\nu_b
\end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\
-\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix}
\nu_1 \\
\nu_2
\end{bmatrix}. \tag{1.8}
\]

Equation (1.8) shows that the mixing matrix for two neutrinos is analogous to a rotation along an orthogonal axis. Therefore, by using this matrix the wavefunction can be written as:

\[ |\nu_a\rangle = \cos(\theta) |\nu_1\rangle + \sin(\theta) |\nu_2\rangle, \tag{1.9} \]

where \( |\nu_1\rangle \) and \( |\nu_2\rangle \) represent the time-dependent plane wavefunctions with the time-varying phase \( \phi_i \) \((i = 1 \text{ or } 2) \) \([21]\]

\[ |\nu_i(t)\rangle = |\nu_i\rangle e^{-i\phi_i}. \tag{1.10} \]

In addition, the inverted form of Equation (1.8) can be used to expand Equation (1.9) in terms of \( \nu_a, \nu_b \), and the phases

\[ |\psi(t)\rangle = e^{-i\phi_1} \left[ (\cos^2(\theta) + e^{-i\Delta\phi_{12}} \sin^2(\theta)) |\nu_a\rangle - (1 - e^{-i\Delta\phi_{12}} \cos(\theta) \sin(\theta)) |\nu_b\rangle \right], \tag{1.11} \]
where the $\Delta \phi_{ij}$ terms represent the difference in the mass eigenstates phases.

This comes from the original form of $\phi_i$ which is represented as

$$\phi_i = E_i t - \vec{p}_i \cdot \vec{x}. \quad (1.12)$$

Here $E_i$ is the energy of the mass eigenstates, $\vec{p}_i$ is the momentum vector of the particle, $\vec{x}_i$ is the position of the particle, and $t$ is the time. When the ultrarelativistic limit is assumed ($p_i >> m_i$) the energy can be approximated as

$$E_i = \sqrt{p_i^2 + m_i^2} \approx p_i + \frac{m_i^2}{2p_i}. \quad (1.13)$$

Here $p_i$ is approximately the total energy ($E$) assumed to be the same for each mass eigenstate and $m_i$ is the neutrino mass eigenstates. If the time is approximated by the length ($L$) the final approximation of the phase is

$$\phi_i \approx \frac{m_i^2 L}{2E}. \quad (1.14)$$

The probability of oscillation for $\nu_a$ to become $\nu_b$ can be determined by applying the $\langle \nu_b \rangle$ state to the above equation and calculating the resulting magnitude

$$P_{a \to b} = |\langle \nu_b | \nu(t) \rangle|^2 = \sin^2(2\theta) \sin^2 \left(\frac{\Delta \phi_{12}}{2}\right), \quad (1.15)$$

where $\Delta \phi_{12}$ equates to (in natural units) [14]:

$$\Delta \phi_{12} \approx \frac{m_1^2 - m_2^2}{2E} L. \quad (1.16)$$

Thus, the probability of oscillation of $\nu_a$ to $\nu_b$ is:

$$P_{a \to b} = \sin^2(2\theta) \sin^2 \left(\frac{m_1^2 - m_2^2}{4E} L\right), \quad (1.17)$$
The PMNS matrix

Returning to the mixing matrix of the three neutrinos flavours, one can follow a similar process to determine the form of the PMNS matrix. The matrix can first be simplified by expanding it to three orthogonal matrices that rotate between the different neutrino mass eigenstates \[14\]:

\[
R = \begin{bmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{bmatrix}
\begin{bmatrix}
c_{13} & 0 & s_{13} e^{-i\delta_{CP}} \\
0 & 1 & 0 \\
-s_{13} e^{i\delta_{CP}} & 0 & c_{13}
\end{bmatrix}
\begin{bmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{bmatrix}
\tag{1.18}
\]

Here \( R \) represents the mixing matrix and \( \theta_{ij} \) is the mixing angle between two neutrino species, in addition \( c_{ij} \) and \( s_{ij} \) are equal to the sine and cosine of the variable \( \theta_{ij} \). This form also agrees with the fact that the mixing matrix must be unitary (i.e. \( R R^\dagger = 1 \)), which gives a constraint on the form of the matrix. In addition to three independent mixing angles, there is a fourth variable corresponding to the complex phase \( \delta_{CP} \) \[14\]. The phase, also known as the Charge Parity (CP) violating phase, introduces a complex phase into the mixing matrix that results in neutrinos and anti-neutrinos potentially oscillating differently \[21\].

1.2 The T2K Experiment

1.2.1 Overview of Experiment

Tokai to Kamiokande (T2K) is a Japanese long baseline neutrino experiment \[22\]. The experiment aims to explore the oscillation of muon neutrinos to electron neutrinos. Muon neutrinos are produced at the J-PARC Neutrino Beam Facility in Tokai by a 30GeV proton beam hitting a 91.4cm carbon target. The impinging protons can create pions and kaons. These hadrons are then focused using magnetic horns into the decay volume where they decay into muons:\[2\]

\[
\pi^+ \rightarrow \mu^+ + \nu_\mu, \\
\pi^- \rightarrow \mu^- + \bar{\nu}_\mu.
\] \tag{1.19}

\[2\]This is similar for \( K^+ \) and \( K^- \) decay.
It should be noted that the polarity of these magnets can be changed to select either positive or negative pions which produce muon neutrinos and anti-neutrinos respectively. Figure 1.1 shows that the off-axis angle can be used to tune the energy of the neutrino beam and thus vary the oscillation probability. The off axis angle occurs from pion decay (π → μ + νμ) where the neutrino is produced at an angle with respect to the direction of the parent pion. This angle is correlated with the energies of the pion and the neutrino. An angle of 2.5° was chosen to maximize the neutrino oscillation probability for the given travel length and energy. The neutrinos are then measured by the off-axis Near Detector located 280m from the target (ND280), situated 280m away from the target. The neutrinos then travel 295km to Super-Kamiokande (SK) to be measured again by the water Cherenkov detector. From the neutrino spectrum, measured by these detectors, the other oscillation parameters can be determined [22].

Starting with Equation (1.18) the oscillation probability for muon neutrinos to turn into electron neutrinos can be found to be [22]

\[ P_{\nu_\mu \rightarrow \nu_e} \approx \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2\left(\frac{\Delta m_{32}^2}{2}\right). \] (1.20)

This agrees with the form of the probability determined in Section 1.1.2 with the slight addition that now there is a mixing term for the ν1 and ν3 mass eigenstates. This oscillation probability is the only one that contains θ_{13} as a leading order term thus making this oscillation a good candidate to probe the parameter. Furthermore, the second order terms (not shown) contain the δ_{cp} variable, thus this oscillation can probe that variable as well. This means that by using the mixing angle of θ_{23} ≈ 45° and mass splitting value of Δm_{32}^2 ≈ 2.4 × 10^{-3}eV^2, taken from ν_μ → ν_μ measurements [24], the other oscillation parameters can be derived.

### 1.2.2 The Near and Far Detector

The off-axis Near Detector located 280m from the target (ND280) is a multi-part detector used to measure the neutrino flux, energy, and electron neutrino contamination prior to oscillation. The detector is encased in a magnetic field housing a π^0 Detector (P0D), Side Muon Range Detector (SMRD), Electromagnetic CALorimeter (ECAL), and tracking components which consist of two Fine-Grained Detectors.
Figure 1.1: The figure above shows the shared energy dependence of the neutrino beam flux and the oscillation probabilities of $\nu_\mu \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_\mu$. Furthermore, it is shown that the off-axis angle determines which energy the neutrino flux is peaked at and thus can be used to tune the oscillation probability. ©, American Physics Society, by permission[23].
(FGDs) sandwiched between three Time Projection Chambers (TPCs) [24]. The 
P0D is a detector used to count the number of neutral pions to characterize back-
ground generated by $\pi^0$. The ECAL is used to identify electrons, positrons, and 
normal particles [22]. The SMRD measures outgoing muons at high angles to the 
beam direction. It also triggers on cosmic ray muons to protect ND280 from the 
influx of these particles. The tracker is used for particle identification, momentum 
measurement, charged detection, and position reconstruction of the generated par-
ticles. The first FGD is constructed solely of scintillator bars, while the second has 
interlaced water targets. These targets are used to determine neutrino interaction 
cross sections necessary in calculating the oscillation effects.

The SK detector is a water Cherenkov detector used as T2K’s far detector [25]. 
The detector is filled with 50 kilotons of pure water and uses approximately 13000 
Photomultiplier Tubes (PMTs) [24]. The detector is cylindrical in shape and has 
inner and outer detecting regions. The outer detector, consisting of 2000 PMTs, 
is used to veto particles coming from outside the tank. The inner detector is where 
the neutrino events from T2K are measured. The detector optimally measures the 

Charged Current Quasi-Elastic (CCQE) interactions where a neutrino ($\nu_l$) interacts 
with a neutron to produce a proton and a detectable lepton

$$\nu_l + n \rightarrow p + l.$$  

These leptons are detected through Cherenkov radiation. Due to the distinct differ-
ences of the rings of muon and electron events they can be accurately separated and 
counted for the oscillation parameter analysis. The muon ring has a sharp outline 
while the electron is “fuzzy” because electrons (due to their smaller masses) scatter 
more easily and induce electromagnetic showers which distort the ring [22]. 

The energy of the neutrino can be reconstructed ($E_{\nu}^{\text{rec}}$) from the lepton pro-
duced in the CCQE interaction

$$E_{\nu}^{\text{rec}} = \frac{m_p^2 - (m_n - E_b)^2 - m_l^2 + 2(m_n - E_b)E_l}{2(m_n - E_b - E_l + p_l \cos \theta_l)}.$$  

Here $\theta_l$, $E_l$, and $p_l$ are the angle, energy, and momentum of the lepton. The variable 
$E_b$ is the binding energy of the neutron. The variables of $m_p$, $m_n$, and $m_l$ are the
masses of the proton, neutron, and lepton. This energy is used directly in the estimation of the predicted events at SK [26].

1.2.3 T2K Analysis Overview

To determine the neutrino oscillation parameters the number of $\nu_\mu$ events measured at SK ($N_{SK}^{obs}$) must be compared with the number of predicted events at SK ($N_{SK}^{exp}$), thus determining the oscillation probability ($P_{\nu_\mu \rightarrow \nu_e}(E_\nu)$). The measured number of events is dependent on the neutrino energy ($E_\nu$), the predicted events vary as a function of $E_{rec}^{\nu}$ [23]

$$N_{SK}^{exp} = \int dE_\nu \Phi_{SK}(E_\nu) \sigma_{SK}(E_\nu) \varepsilon_{SK}(E_\nu) P_{\nu_\mu \rightarrow \nu_e}(E_\nu) M_{SK}. \quad (1.23)$$

The above equation contains the neutrino cross section $\sigma_{SK}(E_\nu)$, the neutrino flux $\Phi_{SK}(E_\nu)$ at SK, and $\varepsilon_{SK}(E_\nu)$ is the efficiency of detection, and $M_{SK}$ is the mass of the SK target. This equation should also account for the different flux types as well as different interaction modes. The systematic uncertainty introduced by $\sigma_{SK}(E_\nu)$ and $\Phi_{SK}(E_\nu)$ is reduced by measurements of $\nu_\mu$ events at the near detector ($N_{ND}^{obs}$) [24]. This is done by predicting the candidate $\nu_e$ using the neutrino flux and prediction models constrained by ND280. By performing a fit on the ND280 charge current quasielastic-like and nonquasielastic-like $\nu_\mu$ events that determines the values of the $\nu_\mu$ and $\nu_e$ tuned flux parameters are determined which are then used to predict the $\nu_e$ signal at SK [23].

To determine the oscillation parameters multiple models are used in the analysis. The far detector requires a model to determine its detection efficiency as well as its response to the reconstructed energy [24]. The near detector, because of the extrapolation to the far detector, requires three models. First, the ND280 detector model that determines $\varepsilon_{ND}$ the efficiency of the near detector. Second, the cross section model $\sigma_{ND}$ that is based on external cross section data. Third, the flux model that depends on hadron production and beam monitor data. These models all filter into the oscillation fit to determine the parameters. The focus of this thesis will be the neutrino flux and hadron production elements of the analysis.
1.3 Neutrino flux predictions

Neutrino flux predictions are necessary in determining neutrino oscillation parameters. They, along with the neutrino-nucleus interaction, account for the differences in the near and far detector. These differences include that the near detector is a multi-component detector and the far is a water Cherenkov detector. Another more subtle difference is that although both detectors measure the same source of neutrinos the far detector measures it as a point source while the near as a line source [24].

The process of predicting neutrino flux for the T2K experiment is by means of data driven simulations. Initially, FLUKA is used to simulate interactions within the target when the 30GeV proton beam first impinges upon it [27]. The resulting particles and their momenta are then saved and sent to a JNUBEAM simulation [24]. This GEANT3 simulation models most of beamline components that the particles interact with. This includes components of the baffle, target, horn, helium vessel, decay volume, beam dump, beam collimator, and muon monitor that are all simulated in JNUBEAM [27]. Thus, the particles generated by FLUKA are then tracked through these geometries and any resulting neutrino parents (pions, muons, and kaons) are saved and their interaction list recorded. Hadron interaction data is then used to re-weight the simulation.

1.3.1 Reweighting

Currently, re-weighting is only performed with the NA61/SHINE thin target data set measured in 2009 [27]. NA61/SHINE is a particle physics experiment at CERN which aims to measure hadron production of interactions which include protons, hadrons, and nuclei [28]. NA61/SHINE performed measurements of 31GeV/c protons on a thin target and a T2K replica target [27]. The experiment measured two values relevant for reweighting: production cross section and differential multiplicities [27]. Production cross section is the probability that a secondary particle is produced in the target as a function of particle energy and scattering angle. Differential multiplicities are the number of particles produced in each interaction.

The production cross section is reweighted to account for differences between the measurements and the Monte Carlo model [27]. The probability \( P(x, \sigma_{prod}) \)
that a hadron is produced by a particle with the cross section of $\sigma_{\text{prod}}$ after the particle moves a distance $x$ and interacts after an infinitesimally small distance $\Delta x$ is defined by

$$P(x, \sigma_{\text{prod}}) = \int_x^{x+\Delta x} \sigma_{\text{prod}} \rho e^{-x'} \sigma_{\text{prod}} \rho \, dx'$$

$$= \Delta x \sigma_{\text{prod}} \rho e^{-x} \sigma_{\text{prod}} \rho,$$

where $\rho$ is the target density (g/cm$^3$). Thus, to reweight for different production cross sections the ratio of the probabilities is used:

$$W = \frac{P'_{\sigma_{\text{prod}}}}{P_{\sigma_{\text{prod}}}}$$

$$= \frac{\sigma'_{\text{prod}}}{\sigma_{\text{prod}}} e^{-x p' (\sigma'_{\text{prod}} - \sigma_{\text{prod}})}.$$

Here $\sigma'_{\text{prod}}$ is the measured production cross section. NA61/SHINE defines $\sigma_{\text{prod}}$ as the subtraction of the quasielastic ($\sigma_{\text{qe}}$) from the inelastic cross section ($\sigma_{\text{ine}}$) [27]

$$\sigma_{\text{prod}} = \sigma_{\text{ine}} - \sigma_{\text{qe}}.$$

Other experiments only measure the inelastic cross section, therefore $\sigma_{\text{qe}}$ is estimated by an empirical equation. In the following subsection discrepancies between the different production cross sections are described.

Differential multiplicities ($\frac{d^2n}{dp d\theta}$) can be determined by the ratio of the double differential cross section ($\frac{d^2\sigma}{dp d\theta}$) and the $\sigma_{\text{prod}}$ for the incoming particle momentum ($p_{\text{in}}$) and target nucleus (A) [27]

$$\frac{dn}{dp d\theta}(p_{\text{in}}, A) = \frac{1}{\sigma_{\text{prod}}(p_{\text{in}}, A)} \frac{d\sigma}{dp d\theta}(p_{\text{in}}, A).$$

The subsequent weight is defined as the ratio of the measured and the simulated
differential multiplicities

\[ W = \frac{\left| \frac{dn}{dpd\theta}(p_{in}, A) \right|_{Data}}{\left| \frac{dn}{dpd\theta}(p_{in}, A) \right|_{Simu}}. \]  

(1.28)

The use of the weights is simplified when used for simulations covered by the NA61/SHINE data set (31GeV p+C). However, to extend the coverage of the data to other particles, momenta, and targets not taken by NA61/SHINE, other scaling and extrapolation techniques are used to include other data sets [27].

Hadrons produced in T2K are categorized as either secondary or tertiary. Secondary hadrons are produced by the original protons scattering on carbon [24]. Tertiary hadrons are all hadrons which are not created by the original protons interacting directly on the target. They occur from re-interacting secondary hadrons either within the target or the surrounding material such as the aluminum and iron in the magnetic horns [27]. These tertiary hadrons prove to be a significant contribution to the neutrino flux. An example from Abe et al. [27] is that tertiary pions (\(\pi^\pm\)) contributes approximately 50% of the \(\bar{\nu}_\mu\) contamination to the \(\nu_\mu\) flux.

The T2K replica target (90cm graphite target) reweighting procedure does not substantially differ from the thin target. The only difference is that rates of \(\pi^\pm\) exiting the target are reweighted as well. Any particles not covered by the replica target are still reweighted using the thin target procedure. The weights were binned based on out-going particle momenta, angle, and z position along the replica target. The target was separated into six bins along z for reweighting to occur.

1.3.2 Neutrino Flux Prediction Systematic Errors

The systematic error of the neutrino flux prediction can be separated into five major sources: hadronic interactions, proton beam profile and off-axis angle, horn current and field, horn and target alignment, and material modelling [27]. Figure 1.2 shows the contribution of each term. It can be seen that the hadronic interactions are the dominating contribution for the most recent error calculation, labelled 13av2 in the figure. The hadronic interaction uncertainty comprises many components. This includes the experimental uncertainties of the NA61/SHINE data, including the sys-
Figure 1.2: Neutrino flux prediction fractional error with respect to the far detector muon neutrino (top) and anti-neutrino flux (bottom) [29]. The curve denoted as 11bv3.2 is the previous uncertainty estimate and should be ignored. The 13av2 error represents the total most recent uncertainty of the neutrino flux prediction. This uncertainty is comprised of all the remaining coloured lines, including the hadronic interactions. Figure used with permission of the T2K collaboration.
tematic and statistical error [24]. Another component is the method used to scale the differential production yields to different particle momenta [27]. In addition, an extrapolation to different target materials also adds to the uncertainty. Another extrapolation is performed for regions in the momenta, particle, and target phase space not covered by experimental data. The errors due to extrapolation are either estimated by data or simulation. The difference is then taken to be the value of the uncertainty. These usually account for a small component of the hadronic uncertainty. Finally, discrepancies in the production cross section should be accounted for as well [27].

The decomposition of the hadronic interaction uncertainty can be seen in Figure 1.3. It should be noted that this data is for the SK νμ and ¯νμ flux prediction [29]. The dominant uncertainty comes from the production cross section (labeled the interaction length in pink) and secondary interactions (labelled as the multiplicities in red).

To scale for secondary interactions, data from experiments such as HARP [30], BNL-E802 [31], and Allaby et al. [32] are used. A parameterized fit is performed to this data to then scale to different target nuclei and particle momenta. In this process their uncertainties are propagated. The HARP experiment uses a TPC to measure the double differential pion production cross section for momenta between 0.5 and 0.8 GeV/c and angles between 0.025 and 0.25 radians, thus its data is specifically used to tune the pion interaction on carbon and aluminum targets [30]. Its systematic uncertainties of ∼ 10% are dominated by the pion proton absorption and tertiary particle interactions. These effects occur due to interactions within the outer TPC area, as well as re-interactions in the targets used.

The production cross section measurements come from multiple sources: Bellettini et al. [1], Carol et al. [33], Denisov et al. [34], and NA61/SHINE [27]. All of these experiments measure the production cross section of proton scattering off a carbon target. However, their lack of agreement can be seen in Figure 1.4, where each production cross section is plotted as a function of proton momentum [27]. It would be first pertinent to explain the confusion of the Denisov et al. measurements, where the reported measurements of production cross section might be

---

[3] This refers to the measurement of double differential pion production cross section only and not the integrated cross section measurements.
Figure 1.3: Decomposition of the hadron production component data for the T2K 13av2 uncertainty for runs 1-7. This corresponds to the far detector neutrino flux error for $\nu_\mu$ (top) and $\bar{\nu}_\mu$ (bottom) [29]. Figure used with permission of the T2K collaboration.
defined as only the inelastic cross section, without the subtraction of the quasielastic term [27]. The reason this is assumed to be the case is once the Denisov et al. measurement is subtracted by the quasielastic cross section it agrees with the Carol et al. measurement. Nevertheless, the NA61/SHINE measurement does not agree (at one sigma) with Denisov et al. and lies in between the two definitions of the cross section [27]. Therefore, due to these distinct discrepancies the uncertainty of the production cross section is currently set to be the value of the quasielastic cross section, approximately 30mb, thus it encompasses the entire region.

The purpose of this thesis is to describe measurements performed at the Fermilab Test Beam Facility by the EMPHATIC experiment to reduce the uncertainties introduced by the hadronic cross section and multiplicities.
Chapter 2

The EMPHATIC Experiment

The following chapter describes the work done by the Emulsion-based Measurement of the Production of Hadrons At a Test beam In Chicagoland (EMPHATIC) group. EMPHATIC represents a collaboration of Fermilab, Chiba University, Kavli IPMU (Institute for the Physics and Mathematics of the Universe), KEK (the High Energy Accelerator Research Organization), Kobe University, Nagoya University, and TRIUMF. The focus of this section will be on the physics goals and plans of the group. This includes sections on the NA61/SHINE neutrino flux measurements and the coverage of the data used for reweighting. It also includes a section on the detector components and experimental setup. Finally, it will end with a summary of the measurements taken in the Fermilab Test Beam Facility (FTBF) in January 2018.

2.1 Introduction

2.1.1 Motivation

The objective of the EMPHATIC experiment is to measure the interaction cross section and hadron multiplicities of particles impinging on various target nuclei. This is to reduce the uncertainties introduced in Section 1.3.2 from the production cross section and the secondary interactions. To perform this measurement it is necessary to have accurate position measurements to reconstruct the scattering an-
gle. Furthermore, the reduction of any surrounding material is favourable to lower the systematic uncertainty caused by out-of-target interactions. The EMPHATIC collaboration has chosen to solve both of these problems by using emulsion film for tracking. This is because of its small interaction length and high position resolution.

The purpose of the following subsections are to explain other issues EMPHATIC aims to address. First, to show how the experiment can be used to help bolster the NA61/SHINE experiment by understanding the discrepancy between their thin and replica target data. Second, to explain the EMPHATIC detector coverage and how that can aid in increasing the coverage of T2K and other experiments.

**NA61/SHINE Thin and Replica Target**

In Section 1.3 the neutrino flux prediction depends on model predictions that are constrained by external production data mainly from the NA61/SHINE experiment. The measurements of the experiment are split into two: the thin (2cm) [35] and replica (90cm) target data sets [36]. Currently, the flux prediction is only based on the thin target measurements performed in 2009 [29]. The thin target data set directly constraints $\sim 60\%$ of the neutrino flux, while the replica target has the potential to directly constrain up to $\sim 90\%$. However, the replica target currently has not been implemented into the T2K analysis.

The procedure for reweighting the simulated data with the thin and replica target is described in Section 1.3.1. To compare the predicted neutrino flux when reweighted by the thin target procedure with the replica target results, consistency in the settings, code, and the beam profile are needed. The study was performed first for the reweighted multiplicities and then for both the multiplicities and production cross section. The production cross section measured by NA61/SHINE for the thin target measurement is [35]

$$\sigma_{\text{prod}} = 230.7 \pm 2.8(\text{stat}) \pm 1.2(\text{det}) ^{+6.3}_{-3.3}(\text{mod}) \text{ mb.} \quad (2.1)$$

The uncertainty is split into three components: the statistical uncertainty ($\text{stat}$), the detector-dependent ($\text{det}$), and the model-dependent ($\text{mod}$). When a comparison was made between the thin and replica target it was seen that the replica target
\( \sigma_{\text{prod}} \) fit preferred a value closer to 200mb \([29]\).

Currently, this difference is still accounted for when the magnitude of the production cross section uncertainty is set to the magnitude of the quasielastic cross section

\[
\sigma_{\text{prod}} \pm \sigma_{\text{qe}}.
\] (2.2)

It may suggest that there is an inconsistency with the NA61/SHINE cross section measurement. In addition, it may show that there is a problem with the replica target fit. Finally, it could indicate that the thin target reweighting procedure requires corrections. The EMPHATIC experiment can aid in solving this discrepancy by performing another measurement of \( \sigma_{\text{prod}} \) for 30GeV/c protons on carbon. This can be done by providing reliable tracking and high efficiency measurements that can be used with or without production models.

**Coverage**

The hadronic interactions that contribute to the flux should be considered when determining the desired coverage of the detector. Figure 2.1 shows the expected muon neutrino and antineutrino fluxes at SK \([29]\). This shows the different contribution of each parent particle (\( \pi, K, K^0, \mu, \) and \( p \)).

The contribution of the muon neutrino flux can be separated into hadronic interactions inside the target and interactions outside the target. On average, for every one neutrino produced 1.35 hadronic interactions take place within the target. Approximately 75\% of these are primary interactions where pions and kaons are produced by the direct collision of the 30GeV/c protons on the graphite target. The other \( \sim 25\% \) originate from secondary interactions within the target, where the produced pions and kaons re-interact within the target. Out-of-target secondary interactions occur at a rate of 0.12 per produced neutrino. These occur predominantly from proton and pion interactions within the horn or the decay volume. The antineutrino muon flux sees a higher contribution of out-of-target interactions ranging from 0.43 to 0.5 per produced neutrino \([29]\).

Tables which describe the interaction types that contribute to the SK flux are included in Friend et al. \([29]\). From these tables a few important interactions
Figure 2.1: The muon neutrino and antineutrino flux expected at SK broken down into parent hadrons [29]. Figure used with permission of the T2K collaboration.
Table 2.1: Proton interaction data used for out-of-target secondary interaction target nuclei scaling. Table used with permission of the T2K collaboration. [29]

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Target Nuclei</th>
<th>$p_{in}$ (GeV/c)</th>
<th>$p_{out}$ (GeV/c)</th>
<th>$\theta_{out}$ (rad.)</th>
<th>Produced Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eitchen et al.</td>
<td>Be, Al, Cu</td>
<td>24.0</td>
<td>4.0-18.0</td>
<td>0.017-0.127</td>
<td>$\pi^\pm, K^\pm, p$</td>
</tr>
<tr>
<td>Allaby et al.</td>
<td>Be, Al, Cu</td>
<td>19.2</td>
<td>6.0-16.0</td>
<td>0.013-0.07</td>
<td>$\pi^\pm, K^\pm, p$</td>
</tr>
<tr>
<td>BNL-E802</td>
<td>Be, Al, Cu</td>
<td>14.6</td>
<td>0.5-4.5</td>
<td>0.1-0.9</td>
<td>$\pi^\pm, K^\pm, p$</td>
</tr>
<tr>
<td>HARP [30]</td>
<td>Be, C, Al, Cu</td>
<td>12.0</td>
<td>0.5-8.0</td>
<td>0.025-0.25</td>
<td>$\pi^\pm$</td>
</tr>
</tbody>
</table>

can be identified. First, the production of secondary hadrons ($p, n, K,$ and $\pi$) from primary interactions should be constrained. This is already constrained using the NA61/SHINE data set. Second, the production of protons and neutrons from secondary interactions must be measured. Third, we should measure the pions produced by secondary proton interaction inside and outside of the target. Finally, we must account for pions produced by secondary pion interactions inside and outside the target.

As described in Section 1.3.2, the process of scaling for secondary interactions requires external data sets. These data sets are shown in Table 2.1 for protons of various momenta. The table shows the target nuclei, beam momentum ($p_{in}$), out-going particle momenta ($p_{out}$), out-going particle angles ($\theta_{out}$), and produced particles. This data is used to cover the particle production phase space relevant to the T2K experiment.

Figure 2.2 categorizes the unconstrained interactions that contribute to the neutrino flux, coming from Monte Carlo simulations. The main contributions come from the pion scattering. The unconstrained contribution of the neutrino flux is dominated by $\pi^+ + X \rightarrow \pi^+ + ...$ which occur in the target, horn, and decay volume. In addition, the unconstrained contribution to the anti-neutrino flux is dominated by the $\pi^-$ rescattering of a similar nature in the target, horn, and decay volume.
2.2 Experimental Setup and Phases of Data Taking

The phases of data taking are currently split up into two. The first was performed January 2018 in FTBF. This was used to measure the interaction cross sections for multiple particles, target nuclei, and momenta. Furthermore, efficiency studies were performed on the aerogel Cherenkov detectors for the second phase. The second phase consists of an upgrade to the detector setup where a Ring Imaging CHerenkov detector (RICH) and spectrometer will be added to the experimental setup to measure the hadron production downstream of the target.

Figure 2.3 shows the first phase of the EMPHATIC experimental setup. The FTBF beam ranges from 2GeV to 120GeV with proton and negative pion beams. However, other particles may contaminate the beam. The setup contained two gas Cherenkov detectors used for particle identification. When used together these detectors can optimally identify protons, pions, and kaons above the momenta of 15, 2.4, and 8.0GeV/c levels respectively. Minimum bias triggers were taken based on the coincidence of the two scintillation counters, with an electron veto on the first gas Cherenkov counter applied for some data sets. The tracking system, which surrounded the target, included silicon strip detectors, pixel detectors, and occasionally emulsion. There were four silicon strip detectors upstream of the target and three detectors downstream of the target. The pixel detector, which had smaller dimensions and thus lower statistics, was also upstream of the target and contained eight detector planes. The semiconductor detectors will be described in Chapter 4. In some measurements, the emulsion was sealed upstream and downstream of a 1cm graphite target. The targets and emulsion were placed on a moving table to allow for quick target switching during measurements. This was also necessary to avoid over-exposure of a single region of emulsion. Downstream, in room MT6.1-B of the FTBF, the aerogel Cherenkov threshold detectors were installed for testing, described in Chapter 3. Finally, the setup ends with a lead glass calorimeter used to detect the muon and electron content of the beam.

The two gas Cherenkov detectors, provided by FTBF, are used to identify particles for selection in the analysis. The first detector is a threshold detector which is
Figure 2.2: The plot is of the unconstrained hadronic interactions of neutrino parent particles that contribute to the $\nu_\mu$ (top) and the $\bar{\nu}_\mu$ (bottom). The material numbering follows the T2K beam Monte Carlo numbering scheme. Some different numbers might represent the same material however in different detector components. For example, the aluminum is numbered 70, 71, and 72 because it is found in the three horns. Materials of note are graphite (-1,14), iron (21), and aluminum (70, 71, and 72). Figure used with permission of the T2K collaboration.
Figure 2.3: The image above is a diagram of the EMPHATIC detector setup. MT6.1 A and B represent the room numbers which house the detector components. CH stands for cherenkov, SSD stands for the silicon strip detectors, and AC stands for aerogel counter. The diagram was designed by Tetsuro Sekiguchi for the EMPHATIC collaboration. Figure used and modified with permission of the EMPHATIC collaboration.

used to identify pions, electrons, and muons when set at a pressure of 5.99 psia for a beam of 30GeV/c. This can be used as a veto to reduce the load on the electronics when measuring protons. The theory of Cherenkov radiation and Cherenkov threshold detectors are also explained in Chapter 3. The second detector is a differential Cherenkov detector which uses a curved mirror to add a secondary constraint on the particle’s Cherenkov angle. This requires a particle to be within the Cherenkov threshold and the critical angle needed to reflect on the mirror and be measured by the inner PMT. Most of the photons missed by the inner PMT are reflected into an outer PMT. The differential Cherenkov counter is set to select (for a 30GeV proton measurement) protons, however there is a kaon contamination in the inner PMT. This is done by setting the pressure of the gas to be 13.48psia. The gas used in both Cherenkov detectors is nitrogen. The refractive index \((n - 1)\) can be estimated, for an ideal gas, by the equation

\[
(n - 1) = 0.0002984 \times \frac{P}{P_0} \times \frac{T_0}{T}.
\]

Here the value of the scaling parameter of 0.0002984 comes from the refractive in-
dex measured at the temperature ($T_0$) of 0°C and pressure ($p_0$) of 1atm. Taking the temperature to be 20°C the refractive index of the gas is 1.000113 and 1.000255.

The emulsion, produced by Nagoya University, was used for increased position accuracy for the scattering measurements. Emulsion acts as a photograph which captures the particles passing through it. This is done by using silver halide crystals (AgBr and AgCl) immersed in a gelatin substrate [38]. When a charged particle passes though the substrate it leaves an image within the emulsion. The silver halide is then chemically reduced in the emulsion development process. This process also converts silver halide which was ionized by a particle into metallic silver. All the remaining silver halide is removed leaving the silver as a record of the particle’s trajectory. The emulsion is then scanned and analyzed. Each emulsion brick was designed to have two emulsion sheets (70µm thick) on either side of a rohacell base (180µm thick). These sheets of emulsion can record up to $10^4$ particles/cm$^2$. Currently, the emulsion analysis is still underway.

The lead glass calorimeter was used for detection of muon and electron contamination in the beam. The detector is an example of a homogeneous calorimeter which means its total volume is used to measure the deposited energy of the particle. Lead glass calorimeters use Cherenkov radiation to detect the energy deposited by electrons and muons.

After all the data were collected, data from different detectors were merged into events. The majority of the detectors were measured by a CAMAC digitizer using the MIDAS data acquisition software. The silicon strip and pixel telescope detectors were digitized separately. This means that afterwards both sets of data were merged for analysis based off of trigger number and timestamp.

### 2.3 Measurements

During a data-taking period in January 2018 the EMPHATIC group used silicon detector data, emulsion data, and aerogel data. The silicon strip data is used in the analysis described in Chapter 4. The emulsion data will be used in a future study. In addition, the aerogel data was used to measure the efficiency and response of the aerogel detectors, described in Chapter 3.

Table 2.2 summarizes the data taken without the emulsion. The table separates...
Table 2.2: Measurements of EMPHATIC during the January 2018 data-taking period. These are measurements without emulsion. Negative momenta represent a π⁻ beam, the remaining are proton beams.

<table>
<thead>
<tr>
<th>Momentum (GeV/c)</th>
<th>Graphite (Events)</th>
<th>Aluminum (Events)</th>
<th>Iron (Events)</th>
<th>Empty (Events)</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>1.63 · 10⁶</td>
<td>0</td>
<td>0</td>
<td>1.21 · 10⁶</td>
</tr>
<tr>
<td>30</td>
<td>3.42 · 10⁶</td>
<td>9.76 · 10⁵</td>
<td>1.01 · 10⁶</td>
<td>2.56 · 10⁶</td>
</tr>
<tr>
<td>-30</td>
<td>3.13 · 10⁵</td>
<td>3.08 · 10⁵</td>
<td>1.28 · 10⁵</td>
<td>3.12 · 10⁵</td>
</tr>
<tr>
<td>20</td>
<td>1.76 · 10⁶</td>
<td>3.08 · 10⁵</td>
<td>1.72 · 10⁶</td>
<td>1.61 · 10⁶</td>
</tr>
<tr>
<td>10</td>
<td>1.18 · 10⁶</td>
<td>1.76 · 10⁶</td>
<td>9.67 · 10⁵</td>
<td>1.17 · 10⁶</td>
</tr>
<tr>
<td>2</td>
<td>1.05 · 10⁵</td>
<td>1.1 · 10⁶</td>
<td>1.83 · 10⁵</td>
<td>1.08 · 10⁵</td>
</tr>
</tbody>
</table>

The events based on particle momenta and target nuclei. The graphite target momenta with the highest number of events are 20GeV/c, 30GeV/c, and 120GeV/c. The 20GeV/c data was taken to compare with Bellettini et al. [1]. Similarly, the 30GeV/c data was taken to compare to NA61/SHINE production cross section measurements. The 120GeV/c momentum was used to measure the production cross section of the Neutrinos at the Main Injector (NuMI) beamline used by the NuMI Off-axis ν_e Appearance (NOvA), Main Injector Neutrino Oscillation Search (MINOS), and Main Injector ExpeRiment for ν-A, (MINERνA) experiment [39]. The corresponding empty target data is to account for the particle interactions within the excess material of the beamline. Furthermore, the aluminum and iron targets are used to constrain the cross section of secondary out-of target interactions at NuMI and T2K beam-lines.

All emulsion events are summarized by Table 2.3. The table separates the data into each individual brick. Brick “M” is labelled refurbished because it recycled emulsion from a previous brick that had a broken seal, which means that the planes were misaligned. The refurbishment process involves creating a new brick with new and old emulsion, thus the previously recorded tracks act as a background.

The following chapters will discuss some aspects of the work contributed to the EMPHATIC collaboration. The next chapter will be on the aerogel Cherenkov detector designed and constructed in TRIUMF, which was then brought during data acquisition. In addition, it will go over efficiency tests performed on the Cherenkov
Table 2.3: Measurements of EMPHATIC during the January 2018 data taking period. These are measurements with emulsion.

<table>
<thead>
<tr>
<th>Emulsion Brick</th>
<th>Momentum (GeV/c)</th>
<th>Trigger Rate</th>
<th>Total Exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30</td>
<td>4,200</td>
<td>2.95 \cdot 10^5</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
<td>4,200</td>
<td>3.74 \cdot 10^5</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>4,200</td>
<td>2.59 \cdot 10^5</td>
</tr>
<tr>
<td>D</td>
<td>30</td>
<td>4,200</td>
<td>3.30 \cdot 10^5</td>
</tr>
<tr>
<td>E</td>
<td>30</td>
<td>4,200</td>
<td>2.90 \cdot 10^5</td>
</tr>
<tr>
<td>F</td>
<td>30</td>
<td>4,200</td>
<td>2.94 \cdot 10^5</td>
</tr>
<tr>
<td>G</td>
<td>30</td>
<td>4,500</td>
<td>3.27 \cdot 10^5</td>
</tr>
<tr>
<td>H</td>
<td>30</td>
<td>5,000</td>
<td>3.51 \cdot 10^5</td>
</tr>
<tr>
<td>I</td>
<td>30</td>
<td>5,000</td>
<td>3.53 \cdot 10^5</td>
</tr>
<tr>
<td>J</td>
<td>30</td>
<td>5,000</td>
<td>3.15 \cdot 10^5</td>
</tr>
<tr>
<td>K</td>
<td>120</td>
<td>5,000</td>
<td>3.88 \cdot 10^4</td>
</tr>
<tr>
<td>L</td>
<td>120</td>
<td>7,500</td>
<td>5.17 \cdot 10^5</td>
</tr>
<tr>
<td>M (refurbished)</td>
<td>30</td>
<td>2,500</td>
<td>6.58 \cdot 10^4</td>
</tr>
</tbody>
</table>

detector. Chapter 4 describes the performance of the tracking detectors used by the EMPHATIC group. The chapter ends with a preliminary analysis of 30GeV/c proton on carbon data taken by the silicon strip detectors.
Chapter 3

Aerogel Cherenkov Detector

3.1 Introduction

3.1.1 Cherenkov Radiation

Cherenkov radiation occurs when a charged particle moves faster than the speed of light in the same dielectric medium. This is analogous to a sonic boom however with light. This occurs because the particle polarizes the atoms in the direction of its motion causing a non-vanishing electric dipole which then emits radiation [38]. If the particle is slower than the speed of light in the medium then the electric dipoles are symmetric and do not emit radiation. This gives a threshold for Cherenkov radiation:

\[ v > \frac{c}{n}, \]

(3.1)

where \( v \) is the speed of the particle in the medium, \( n \) is the refractive index of the medium, and \( c \) is the speed of light. Rearranging Equation (3.1) and defining \( \beta \) as \( v/c \) yields:

\[ \beta > \frac{1}{n}. \]

(3.2)

Cherenkov radiation is emitted in a conical shape along the direction of the...
particle. The angle of the apex of the cone, known as the Cherenkov angle ($\theta_C$), is determined by dividing the distance travelled by the photon over the distance travelled by the particle resulting in the following relation [38]:

$$\cos(\theta_C) = \frac{1}{n\beta}.$$  \hspace{1cm} (3.3)

Due to the fact that $\beta$ cannot be greater than one an upper limit can be placed on the value of $\theta_C$

$$\theta_C^{\text{max}} = \arccos\left(\frac{1}{n}\right).$$  \hspace{1cm} (3.4)

In addition, the number of Cherenkov photons produced per unit path length depends on the electrical charge of the particle ($z$), the fine structure constant ($\alpha$), the range of wavelengths emitted by the photons ($\lambda_1$ to $\lambda_2$), $\beta$, and the refractive index of the medium [38]:

$$\frac{dN}{dx} = 2\pi\alpha^2 \int_{\lambda_1}^{\lambda_2} \left(1 - \frac{1}{n(\lambda)^2\beta^2}\right) \frac{d\lambda}{\lambda^2}. $$  \hspace{1cm} (3.5)

The refractive index is wavelength-dependent in the above equation due to dispersion in the material. If dispersion is assumed to be negligible, thus removing the $\lambda$ dependence, then the integral can be easily solved. In addition, the theta dependence can be included by substitution using Equation (3.4), resulting in

$$\frac{dN}{dx} = 2\pi\alpha^2 \sin^2(\theta_C) \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right). $$  \hspace{1cm} (3.6)

### 3.1.2 Cherenkov Threshold Detector

The design of a Cherenkov threshold detector is quite simple. A threshold detector is essentially two materials of different refractive indices ($n_1$ and $n_2$) and a method to detect their light separately. By comparing the number of photons emitted by $n_1$ and $n_2$ the particle can be identified.

For example, a proton whose beta value is lower than the threshold of both

---

1 This range is set to the quantum efficiency of the PMTs, meaning it ranges from 300-600nm.
refractive indices would emit no light. A kaon whose beta value satisfies the condition for \( n_1 \) but not \( n_2 \) would only emit half the maximum detectable light. A pion who met both Cherenkov thresholds would emit the maximum light. These distinctions allow for the particles to be identified.

The properties of Cherenkov radiation make it an excellent candidate for particle identification. First and foremost its ability to discriminate based on particle velocity, due to the threshold set in Equation (3.2), means that particles with high enough energies can be vetoed from samples, thus reducing background [40]. Moreover, the time it takes for the radiation to occur is on the order of picoseconds, allowing for fast particle identification, and is only limited by the speed of the PMTs. Also the energy loss of gas and aerogel Cherenkov radiators are small compared to that of other light emitting detectors such as scintillators or calorimeters, which in some cases stop the beam. This means that a Cherenkov detector can be placed in the line of the beam and be minimally destructive. The biggest drawback for the use of Cherenkov radiation is that the light yield is low and thus limits the sensitivity of the detector. Cherenkov radiation is lower by two orders of magnitude when compared to the light yield of a plastic scintillator.

**Photomultiplier Tube:**

The Cherenkov detectors used by the EMPHATIC collaboration detect light by using PMTs. The PMT uses a photocathode which converts the incident photons into electrons, using the photoelectric effect. The electrons are then accelerated by an electric field to a chain of dynodes [38]. These dynodes multiply the number of electrons emitted, which causes an avalanche effect that increases the number of electrons by many orders of magnitude. This amplification process is categorized as the gain of the PMT which is usually of the order of \( 10^5 \) to \( 10^7 \) electrons [40]. Finally, the electrons are then collected by an anode and converted into a signal for processing.

Another important factor in determining the quality of a PMT is the quantum efficiency. This efficiency quantifies the probability that an incident photon will produce one free electron by the photoelectric effect. Therefore, it can be thought of as a conversion efficiency. The value of quantum efficiency is dependent on the
wavelength and can vary between 300-600nm and ranges from 15% – 30% [40]. For shorter wavelength there is a limit due to the transparency of the PMT’s glass.

3.1.3 Aerogel

Materials that are transparent can be used for Cherenkov detectors. Gas Cherenkov detectors are most commonly used in beam lines because their refractive indices vary as a function of pressure. However, their indices are usually lower than 1.002 which limit their usability for lower β value particles [40]. Liquid radiators usually have indices greater than 1.33, which is approximately the index of water. Aerogel however, has the ability to cover the region in between these two limits.

Aerogels are 2:1 mixtures of SiO$_2$ and water. This mixture forms micro-bubbles of air within the structure. These micro-bubbles within the silica are small enough when compared to the wavelength of light, and thus make for an ‘average’ refractive index seen by the particle [38]. The density ($\rho$) of aerogels can be tuned by increasing or decreasing the amount of micro-bubbles. This means the refractive index ($n$) can be tuned as well:

$$n = 1 + 0.21 \left( \frac{\rho}{1 \text{ g/cm}^3} \right). \quad (3.7)$$

Currently, aerogel refractive indices can range from 1.004 to 1.15 and thus bridge the majority of the gap.

3.2 Design and Testing

The following section describes work I did at TRIUMF to design and build a Cherenkov threshold detector for use with EMPHATIC. The first section talks about a toy simulation used to simulate the Cherenkov threshold detector as well as the light propagation properties of the aerogel. The second section explains the 3D printed detector enclosure designed in Autocad as well as the initial characteristics of the threshold detector. To give context to the following sections it should be noted that the initial limitations were that the design only used two PMTs, that the aerogel dimensions are $11 \times 11 \times 1 cm$, and the detector was initially designed to use only four pieces of aerogel (with 2 different refractive indices). Another im-
Important design constraint is that the PMTs cannot be in line with the beam to avoid any damage to the PMTs or the purity of the beam.

3.2.1 Toy Simulation

A simulation was designed to test the light propagation capabilities of the aerogel Cherenkov threshold detector depicted in Figure 3.1. The purpose of the simulation was threefold:

1. To analyze the efficiency of the reflective material and the geometry of the detector.
2. To measure the number of photons scattered or absorbed by the aerogel.
3. To determine an optimum position for the PMTs.

This analysis was achieved by a simple simulation of photon scattering within the detector. The detector geometry was modelled as a rectangular prism with the dimensions of $11 \times 11 \times 15\text{cm}$, with the additional 4cm of height allowing enough room for the pieces of aerogel. The PMTs’ faces are simulated by circles with 3 inch radii on either side of the threshold detector. Figure 3.1 shows the detector geometry used in the simulation, with direction of the beam towards the positive $z$ direction.

The simulation follows the photon from its generation in the aerogel to the point it enters a PMT or is absorbed by the reflective material or the aerogel. Initially, the photons are simulated to be produced at positions centred at 0cm in the $x$ and $y$ directions and a uniform distribution of $z$ positions with mean of -1cm or -3cm. The angle of the photons are determined by Equation (3.4). Also, the $\phi$ angle of the photons is randomly thrown between 0 and $2\pi$. The detector walls are simulated to be covered in Tyvek with a diffuse reflectivity of approximately 99% [41]. Thus, if the photon hits any of the walls its $\theta$ and $\phi$ angles are randomized in a direction away from the wall, or the photon is absorbed. Each reflection is counted for every photon and a histogram is produced of the mean number of reflections.

The photon scattering and absorption in the aerogel are also simulated. This is done by determining the transmission probability ($P_T$) of the photon through the
• For the first phase of the detector a Cherenkov threshold detector will be used to determine the purity of the beam.

Currently we are in the process to optimize and design the detector.

The direction of the beam

**Figure 3.1:** The image above shows the Cherenkov threshold detector design for the simulation. The direction of the beam is added to give context for future plots. The aerogel is shown in orange to delineate the extra space required to place them.

Aerogel which depends on the photon’s step size ($x$) and the transmission length ($\Lambda_T$):

$$P_T = e^{-\frac{x}{\Lambda_T}}.$$  \hspace{1cm} (3.8)

If the photon is transmitted for that step then it does not interact, otherwise the photon will either be scattered or absorbed. The scattering ($\Lambda_S$) and absorption lengths ($\Lambda_A$) are related to $\Lambda_T$ by

$$\frac{1}{\Lambda_T} = \frac{1}{\Lambda_S} + \frac{1}{\Lambda_A}.$$  \hspace{1cm} (3.9)
Given that the photon interacted, the probability of scattering and absorption are described by the following equations

\[
P_S = \frac{\Lambda_S}{\Lambda_T} = \frac{\Lambda_A}{\Lambda_S + \Lambda_A},
\]

\[
P_A = \frac{\Lambda_S}{\Lambda_S + \Lambda_A},
\]

(3.10)

To determine all transmission, scattering, and absorption probabilities values of \(\Lambda_A\) and \(\Lambda_S\) are taken from [2] to be 5.4m and 2.5cm respectively\(^2\). This returns a transmission probability of 96% for 0.1cm of aerogel. The other 4% is split between scattering (99.5%) and absorption (0.5%). These values agree with the characteristics of aerogel described in Section 3.1.3, specifically its high transparency. If the photon is scattered randomly generated \(\theta\) and \(\phi\) angles are produced, and absorbed photons are lost.

The simulation was run for 10000 photons. If the photon enters the PMT then it is added as an entry for the histogram which records the number of reflections in the dark box. Thus, the ratio of the entries of the histogram to the number of initial photons is the efficiency of the photon propagation within the counter. Figure 3.2 shows an event display of the toy simulation, a single photon being emitted from the aerogel. It is then transmitted through the aerogel, and then it reflects off the furthermost wall. Afterwards, it enters the PMT. Figure 3.3 show the histogram for the number of reflections on the Tyvek and a histogram of the scattering within the aerogel. Thus, the efficiency of the detector is 87.5% and the most probable number of reflections are two and three, although there is a long tail to higher numbers of reflections. This means that the photon usually scatters more than just off the furthermost wall. The histogram describing the amount of scattering in the aerogel shows that the majority of photons pass through the aerogel without scattering. In addition, the position of the PMTs was determined to not have a major effect, because the diffuse reflectivity of the detector removes the \(\theta\) angular constraint given by the Cherenkov angle.

\(^2\)These were taken for a piece of aerogel of refractive index of 1.01 and at a wavelength of 400nm.
Figure 3.2: The above figure is an event display of the simulation. The red rectangular boxes represent the aerogel. The green dashed line represents the path of the photon. The indigo circles are the PMTs. The figure on the top and bottom are of the same event viewed at different angles.
Figure 3.3: The top figure shows the distribution of the photon reflections within the aerogel Cherenkov detector. The number of events is dictated by the photons that enter the PMT. The bottom histogram is of the photons scattered within the aerogel.
3.2.2 Designing the Detector Enclosure

To design the threshold detector many considerations were made:

- The enclosure needed to be light tight to avoid any damage to the PMTs.
- The aerogel should be easily removable to facilitate any quick exchange between beam time, and not be damaged in the process. This allows for easy testing of multiple pieces of aerogel.
- The enclosure should minimize any material along the beam path to reduce the probability of scatter.
- The enclosure’s size should be minimized to save the available detector space.

It was decided early on, for convenience and to reduce cost, that the design would be 3D-printed at TRIUMF, using PLA as the printing material. The design was developed using Autocad 3D design software. The main components of the design were the aerogel holder, the main enclosure with PMT supports, the lid, and the PMT cover and cap. The concept of the enclosure is to have a removable lid that can attach to the aerogel holders to allow for quick removal of the aerogel. The interior of the detector was lined with UV-stabilized Tyvek provided by the TRIUMF facility.

Each aerogel holder was designed for two pieces of aerogel of $11.5 \times 11.5\text{cm}^2$ area. This is to allow for some size variation. The holder consists of two identical frames that encase the pieces of aerogel. In addition, there is a $1.125\text{cm}$ structure along the interior frame to allow for the aerogel to have a base to lay on. Holes were placed on the corners of the interior frame to avoid any issues with precision of 3D printed corners. The thickness of the outer wall of the holder is $1.325\text{cm}$ to fit the screw used to connect the two aerogel holders with the lid. The benefit of using a screw to attach multiple aerogel holders is that it allows for more flexibility in the number of holders used. Figure 3.4 shows both the aerogel holder design in Autocad and the final printed design.

The main enclosure, shown in Figure 3.5, is a hollow rectangular prism with a side removed to allow for the lid. Two sides of the enclosure are used to slot in the
Updates to Cherenkov threshold detector

- Aerogel Holder has been extended to properly fit a 11x11cm piece of aerogel
- The screw placement has been maintained as well
- Update not shown in picture: Corners of the holder are flattened to not impede fitting
- One side of the aerogel holder can fit a 11.5cm piece of aerogel

Figure 3.4: The aerogel holder design (top) and the completed print (bottom). The design consists of three squares: an outer, middle, inner square. These can be used to define the different borders. The border used to hold the aerogel is between the middle and inner square. The border that attaches to the lid is between the outer and middle square.
three inch PMTs. This is done by attaching PMT support structures designed by Philip Lu to either side. In addition, a $9 \times 9\text{cm}$ hole was cut out of the bottom side (or side opposite of the lid) to allow the beam to pass through and not interact with the PLA. This hole was covered by two layers of Tedlar® to make it light tight.

The lid was printed out separately however its design was heavily dependent on the enclosure and the aerogel holder. The only notable design choice is that the side walls were added to the lid so that they would slot over the main enclosure ensuring a light tight seal when taped.

Also a PMT cover and cap were designed to protect the PMT from any outside light. The PMT cover was simply a cylinder with a radial wall that slots into a groove in the exterior side of the main enclosure. The caps were designs as two pieces that would be taped together with holes for the wiring to be slotted through. They were also designed to slot into each other and eliminate as much light leak as possible. Finally, small cylinders were added around the wire holes with the idea that they would tighten around the BNC and high voltage cables and prevent light from entering, as shown in Figure 3.6.

### 3.3 Results from Aerogel Studies

Figure 3.6 shows the final design of the detector, as it was used in the Fermilab test beam facility. Due to issues with a faulty PMT only one was used. This meant that a PMT cover was removed and the hole was covered with diffuse reflective material. To correct for the effect that a single PMT would have on the detection efficiency more aerogel was placed inside the detector. The refractive indices and the thicknesses of the pieces of aerogel are shown in Table 3.1. Some design improvements are suggested in the conclusion. All data taken here is with an operational voltage of 1310V.

#### 3.3.1 Efficiency of the Aerogel Cherenkov Detector

The efficiency of the aerogel detector was determined with a 120GeV proton beam, without a target. Due to the chosen momentum the beam is purely protons of $\beta \approx 1$. The efficiency of light detection was determined by performing a fit of the signal and pedestal, described by [42]. The concept of the fit is to account for the
Further extensions were made to all other parts of the detector.

Figure 3.5: The image shows the design (top) and the printed (bottom) Cherenkov threshold detector. The PMT supports on either side are shown. They are extruded outward to help stabilize the PMT and to aid in the attachment of the PMT covers.
Table 3.1: Thickness and refractive index of aerogel pieces.

<table>
<thead>
<tr>
<th>Refractive Index</th>
<th>Thickness (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0252</td>
<td>1.9</td>
</tr>
<tr>
<td>1.0254</td>
<td>1.8</td>
</tr>
<tr>
<td>1.0257</td>
<td>1.1</td>
</tr>
<tr>
<td>1.0257</td>
<td>1.1</td>
</tr>
<tr>
<td><strong>Average Refractive Index</strong></td>
<td><strong>1.0255</strong></td>
</tr>
<tr>
<td><strong>Total Thickness</strong></td>
<td><strong>5.9</strong></td>
</tr>
</tbody>
</table>

Figure 3.6: Picture of the final detector with the PMT cover and the cap.

The photoelectric effect and to model the effect of the amplification. The photoelectric effect is a random process and as such is fit with the Poisson distribution

\[
P(\mu, k) = \frac{\mu^k e^{-\mu}}{k!}. \tag{3.11}
\]

Here \(P(\mu, k)\) is the probability that \(k\) photoelectrons are detected and \(\mu\) as the mean number of photoelectrons. The detection efficiency can be determined by \(\mu\)
using the equation

\[ P_{\text{det}} = 1 - e^{-\mu}. \quad (3.12) \]

The above equation can be read as one minus the probability of not detecting a photon. The mean number of photoelectrons in this context accounts for the quantum efficiency and the collection efficiency of the PMT.

The amplification process can be modelled by a Gaussian distribution if the first dynode electron emission is greater than that of the other dynodes [42]. Another assumption made is that the amplification processes for photoelectrons remain independent of one another. Therefore, the Gaussian distribution of the amplification of \( k \) photoelectrons is used to fit \( Q_1 \) and \( \sigma_1 \), which are the mean and standard deviation for the measured charge for one photoelectron, with respect to the ADC value \( x \):

\[ G(x) = \frac{1}{\sigma_1 \sqrt{2\pi k}} \exp \left( -\frac{(x - kQ_1)^2}{2k\sigma_1^2} \right). \quad (3.13) \]

Equation (3.12) and Equation (3.14) are convolved to properly model the ADC spectrum:

\[ S(x) = \sum_{k=1}^{\infty} \frac{\mu^k e^{-\mu}}{k! \sigma_1 \sqrt{2\pi k}} \exp \left( -\frac{(x - kQ_1)^2}{2k\sigma_1^2} \right). \quad (3.14) \]

The pedestal must be fit to subtract its mean (\( \mu_{\text{ped}} \)) charge from \( x \) to properly determine the number of photoelectrons. This can be done by fitting the pedestal with a Gaussian distribution. The new fit with the added pedestal contribution is

\[ S(x) = \frac{1}{\sigma_{\text{ped}} \sqrt{2\pi}} \exp \left( -\frac{(x - \mu_{\text{ped}})^2}{2\sigma_{\text{ped}}^2} \right) + \sum_{k=1}^{\infty} \frac{\mu^k e^{-\mu}}{k! \sigma_1 \sqrt{2\pi k}} \exp \left( -\frac{(x - \mu_{\text{ped}} - kQ_1)^2}{2k\sigma_1^2} \right). \quad (3.15) \]

The results of the fit for the 120GeV are shown in Figure 3.7 where \( \mu = 9.1 \pm 0.3 \). This equates to an efficiency of \( \sim 99.999% \), suggesting the detector is very efficient.
Figure 3.7: The plot shows the fit of the ADC spectrum pedestal and signal. The signal is fit with a Poisson and Gaussian convolved fit. The ADC values within the signal region are subtracted by the mean of the pedestal. This results in the mean number of photons being directly fit. It should be noted that the first bin represents overflow and is ignored.

The efficiency can also be determined by applying quality cuts to the data that ensure a 120GeV proton track that passes through the aerogel detector and deposits a hit in the downstream lead-glass calorimeter. Figure 3.8 shows the quality cuts applied to the data. The first set of cuts ensures that the $\chi^2$ of the upstream and downstream tracks is less than four. The other set of cuts allow for a single track in both downstream and upstream. This is followed by a cut to make the tracks pass through all silicon strip planes (upstream of the aerogel Cherenkov detector). Finally, the lead-glass calorimeter cut allows for ADC values greater than 200 which is well within the signal region of the lead-glass calorimeter. This means that every event within the aerogel ADC spectrum should be a proton that deposits a signal distribution of $\mu = 9.1$. Thus, any pedestal contribution can be deemed as an inefficiency of the detector:

$$Eff \ = \ \frac{N_{tot} - N_{ped}}{N_{tot}}.$$  \hspace{1cm} (3.16)
Figure 3.8: The graph depicts the number of remaining events as a function of the applied quality cuts.

The equation above shows that the efficiency can be determined by the ratio of the number of non-pedestal events \(N_{\text{tot}} - N_{\text{ped}}\) divided by the total number of events \(N_{\text{tot}}\). Figure 3.9 shows the ADC spectrum of the aerogel Cherenkov detector after the quality cuts have been applied. It can be seen that the number of events in the pedestal are not measurable and therefore indicates that the detection efficiency is very high.
**Figure 3.9:** The distribution of the aerogel ADC after the quality cuts were applied. The pedestal can not be seen. It should be noted that the first bin represents overflow and is ignored.
Chapter 4

Silicon Telescope Data
Acquisition and Analysis

The purpose of this chapter is to describe work performed with the EMPHATIC silicon telescope. This includes an introduction to the solid state detectors used in this experiment. Afterwards the data acquisition of the silicon telescope is discussed. In addition, a brief overview of the Monicelli alignment procedure is given. This then becomes a discussion of the efficiency of the silicon strip. The pixel detector and its inefficiency will be explained in Appendix A. Finally, the chapter is concluded with a preliminary analysis of the data acquired by the silicon strip detectors.

4.1 Solid State Detectors

Solid state detectors operate based on the principle of the electronic band model. The band gap model describes the energy levels of an electron, and subsequently holes, within a solid state detector [38]. Holes are described as the absence of electrons within an energy level. The discussion begins with the fact that an isolated electron occupies discrete energy levels. The Pauli exclusion principle says that two or more identical fermions of the same spin state cannot occupy the same energy level [43]. Thus, electrons in a diatomic molecule are split into different energy levels. In a crystal lattice the energy levels of the electrons form into en-
ergy bands. Band gaps are formed when energy levels are not covered by any band. This band structure forms the basis of semiconductor design.

Semiconductors utilize the relationship of the conduction and valence band to control electron migration \[38\]. These bands (and their band gap) are closest to the Fermi level which is the theoretical energy level that has 50% occupancy of electrons at thermal equilibrium. The valence band, below the energy gap, is where electrons can become excited and move into the conduction band. The conduction band, above the energy gap, is partially filled with electrons and is where electrons can move freely within the semiconductor \[43\]. When an electron moves from the valence band to conduction band, it leaves a hole within the valence band.

Semiconductors can be doped to have either an abundance of holes or electrons. A semiconductor which has an excess of electrons is labelled “n” type for the negative charge of the electrons. The semiconductor with an excess of holes is known as a “p” type \[38\]. The interface between the p-type and n-type semiconductor is known as a PN junction. This junction allows for electrons in the n-type semiconductor to diffuse into the p-type and vice versa. Consequently, this causes what is known as a depletion region to be formed along the PN junction, where an electric field is formed by the holes in the n-type and the electrons in the p-type. Then this field pushes electrons (in the n-type) and holes (in the p-type) away from the junction. Thus, the depletion region has a lower concentration of charge carriers.

The depletion region is dictated by the specific resistivity and the charge carrier concentrations. The specific resistivity of the material (\(\rho\)) is dependent on the concentration of the charge carriers (\(n\) for electrons and \(p\) for holes) and the electron (\(\mu_e\)) and hole (\(\mu_p\)) mobility \[38\]:

\[
\rho = \frac{1}{e(n\mu_e + p\mu_e)}.
\]  

The charge carrier concentrations are equal for a pure semiconductor and are dictated by the following equation

\[
n = p \approx 5 \times 10^{15} (T)^{3/2} e^{-E_g/(2kT)} \text{ cm}^{-3},
\]  

48
where $T$ is the temperature and $E_g$ is the band gap energy. The width of the depletion region ($d$) is

$$d = \sqrt{2\varepsilon(U + U_c)\mu\rho_d}. \quad (4.3)$$

The width becomes dependent on the main carrier mobility ($\mu$) and the specific resistivity ($\rho_d$) of the low-doped semiconductor. The voltages, $U$ and $U_c$, are the reverse-bias voltage and the contact voltage. Also, $\varepsilon$ is the dielectric constant. The reverse-bias voltage is the positive voltage applied to the n-type region [38]. This means that when the system is in reverse-bias the reverse-bias voltage increases and thus the depletion region increases. When electron-hole pairs are formed in the depletion region they then migrate to electrodes to be collected, creating an electrical signal.

Another consideration in the design of a semiconductor is to mitigate the effects of drift and diffusion. Diffusion is the random movement of the electrons and holes from their point of origin due to thermal energy [40]. This diffusion broadens the detected charge distribution by

$$\sigma = \sqrt{\frac{2kT\chi}{eE}}. \quad (4.4)$$

Here $T$ is the temperature, $E$ is the electric field, and $\chi$ is the drift distance. Diffusion also has a direct effect on the position resolution. The electron-hole pairs also drift due to the effects of the electric field. The drift velocity ($v$) is related to the electric field by the mobility of the charge carriers which acts as a proportionality constant

$$v_{h/e} = \frac{\mu_{h/e}E}{e}. \quad (4.5)$$

As the electric field increases the drift velocity begins to saturate and loses its $E$ dependency. The velocity saturates on the order of $10^7$ cm/s. This equates to a response time of 10ns for common semiconductor detectors.

Semiconductor detectors are used to measure the energy deposited by a photon or a charged particle. The charge collected by the detector, from the electron-hole
pairs in the depletion region, is proportional to the energy deposited by the particle. Semiconductor detectors are also well suited as tracking and vertex detectors due to their high granularity and precision [38]. This comes from the fact that they can be segmented into strips or pads which can accurately measure the position.

4.2 Semiconductor Tracking Detectors

As said earlier, semiconductor detectors can be segmented to obtain accurate position information. These segments can be either strips, pads, or pixels. For this section the discussion will focus on strips and pixels detectors. However, these two detectors share many fundamental design characteristics. When multiple segmented detectors are used in conjunction, a particle can be tracked through its reconstructed positions [38]. Semiconductor tracking detectors still adhere to the processes that dictate all semiconductor detectors. Thus, when a charged particle traverses a strip or pixel a charge proportional to the energy deposited by the particle is still detected.

A defining characteristic of the semiconductor tracking detectors is the pitch. The pitch is the distance between the strips or pixels. Thus, for high resolution measurements the pitch would ideally be as small as possible [44]. However, this is not feasible due to design and construction limitations. The pitch used in this experiment is approximately 60 µm and has a special importance on the determination of the position measurement and the resolution of the detector.

The position measurement of a pixel or a strip detector is determined based on the number of strips detected. If the drift distance is not far enough only a single strip (or pixel) is detected and as such the position would be the strip center $x$. This results in a resolution ($\sigma_x$) of [38]

$$\sigma_x = \frac{p}{\sqrt{12}}.$$  \hspace{1cm} (4.6)

Here the resolution is directly proportional to the pitch ($p$), when a uniform density of tracks is assumed. If diffusion is sufficient at the point of charge collection then two or more strips collect charge. Thus, a more accurate position measurement can
be calculated using the so-called center-of-gravity method:

\[ x = \frac{x_1 c_1 + x_2 c_2}{c_1 + c_2}. \]  

(4.7)

The equation above shows that the charges of strip 1 and 2 \((c_1 \text{ and } c_2)\) are used to find a weighted mean of the strip positions \((x_1 \text{ and } x_2)\). The position resolution for this method is directly related to the pitch and the signal-to-noise ratio (SNR) [45]:

\[ \sigma_x \propto \frac{P}{SNR}. \]  

(4.8)

The signal is the mode of the charge distribution of a minimum ionizing particle of a cluster of strips or pixels. The noise is defined as the noise of each strip or pixel.

Silicon strip detectors require two detectors in different orientations to measure both \(x\) and \(y\) coordinates. This can be done by having two separate detectors operate independently or by installing the second orientation directly on the substrate, creating a double-sided silicon strip detector [40]. However, this method may introduce additional noise in the measurements.

The pixel and strip detectors used in the EMPHATIC experiment were provided by the Fermilab Test Beam Facility (FTBF). Four silicon strip detectors upstream of the target and three downstream of the target return both \(x\) and \(y\) measurements of the beam profile. The size of the silicon strip detectors is approximately \(3.84 \times 3.84\text{cm}\). More specifications on the silicon strip and pixel detector are given in Appendix B. The pixel detector is situated upstream of the target and contains eight pixel planes of different sizes and orientations. The operators of these detectors were Ryan Rivera and Lorenzo Uplegger of the FTBF.

### 4.3 Data Acquisition and Monicelli

Data acquisition of the pixel telescope and the silicon strip detectors is performed through the system of readout nodes known as the Compact And Programmable daTa Acquisition Node (CAPTAN) [46]. A CAPTAN node can contain a Node Processing and Control Board (NPCB), a Data Conversion Board (DCB), and a Power Distribution Board (PDB). The NPCB dictates the processes of the other boards and transfers the data through a Gigabit Ethernet connection. The DCB is
used to read the data from the readout chips and convert them into an ADC signal [46]. The PDB is used to power each node. Thus, when a beam trigger is received a signal is generated by a semiconductor detector which is then received by the node and converted to an ADC signal. The ADC signal then sends the digitized values to a field programmable gate array (FPGA) which passes the data to a PC for further processing. A separate card obtains an external trigger that is sent to the other CAPTAN nodes.

The geometry of the detectors must be correctly determined to properly obtain the particles’ scattering information. This ensures that the position of the detectors are accurate with respect to the alignment of the detecting planes. The alignment process can be summarized as the accurate preparation of the geometry file, which rotates the local geometry of the planes to the global geometry of the laboratory. The local geometry is defined by a coordinate system of each detecting plane where the centre of the plane is set to the origin. The global geometry is determined by all the planes. The important distinction is that in this geometry the z positions of the planes, which are along the direction of the beam shown in Figure 4.1, are set relative to the centre of the pixel detector. Therefore, upstream silicon strips will have negative z values and downstream silicon strips will have positive. This is an important step because the alignment software uses the z positions of planes as a user input, therefore they were measured properly by the FTBF staff with an accuracy of 1mm.

Before an alignment can be performed some beam conditions must be met. The beam should be centred along all silicon strip and pixel detectors to ensure all planes can be adjusted. In addition, the beam should be narrow to assist in the centring. These conditions are achieved by a 120 GeV/c proton beam. No target should be placed in the beam because any scattering may yield an incorrect alignment. With these conditions met, the raw data can now be analyzed by Monicelli.

Monicelli is the alignment software used by the FTBF data tracking group [46]. The software was designed to provide the user the ability to accurately adjust the variables of the geometry file to minimize the fitting error. This is done by first generating a default geometry file with measured values of the z positions, x positions, y positions, and angles for each detector. Events, which are measurements that share the same timestamp, are first extracted from raw data files to be used by
Monicelli. These events are then converted to the global geometry using the default geometry file. These “global events” are then formed into clusters for each plane. Currently, silicon strip clusters are defined as the weighted position between two or more strips. The pixel clusters are defined as the weighted position of one, two, or four pixels. The other options will be tested in the future.

Once the clusters are formed then the tracks can be fitted to determine their slope, residuals, and $\chi^2$. The tracks are determined by using the slope of the measured x and y positions with respect to the user defined z values. There are two track-finding methods: “first and last” and “road search”. As the name suggests, the first and last method is when a track is found by fitting a line between the first and last planes. Once the line is fit a user adjustable window, in x and y, is set to allow the points from the other planes to contribute. When this window is used with a cut on the minimum number of points per fit, an effective threshold is placed on tracks with high residuals. The road search algorithm begins similarly to the first and last method where the initial and final planes are fit. However, the points are then tested for goodness of fit along the trajectory of the beam (increasing z).

In addition to the track-finding methods there are two fitting methods: simple and Kalman. The simple fit applies a linear regression with respect to the x and y values of the track. The Kalman method applies a Kalman filter which aims at reducing the noise caused by multiple scattering. Currently, the EMPHATIC collaboration has chosen to use the combinations of the first and last method and the simple fit to obtain fitted parameters for all the tracks.

With the tracks fitted, x and y slope distributions can be plotted. Since the tracks are narrow and centred for all detectors the expected means of the slope distributions are zero. Thus, these distributions are fitted with a Gaussian to determine their means and the geometry file is then adjusted so that their means are zero. Similar distributions can be obtained for the residuals of each detector and these are also fitted and centred at zero. Finally, a “fine alignment” was performed by iterating through each track and by adjusting each variable of the geometry file (except for the z positions and specific angles corresponding to the x and y planes). This should have resulted in a properly aligned geometry file used for track analysis.

It should be noted that to achieve the correct conditions for the analysis the
planes of the tracks were separated upstream and downstream of the target. Figure 4.1 shows the breakdown of the different silicon strip and pixel planes (also referred to as plaques). This figure should be used in conjunction with Appendix B for a detailed description of the plane positions, sizes, angles, and thicknesses. It can be seen from the figure that the silicon strip planes are treated individually by the data acquisition code, that is, each plane regardless of what direction it measures returns a coordinate with an x and y value. This means that a duplication is occurring between the x strips and y strips\(^1\). It is currently believed that this arouse as an artifact of rotating between the local geometry of the strips to the global geometry. This means that, ideally, an upstream track would contain 16 upstream points (4 × 2 silicon strips and 8 pixel detectors). After the alignment procedure is completed the geometry file can be used by Monicelli Express. This

\(^1\)Currently, the effects of this duplication are being looked into, although the assumption is that its affect are minimal.
code works similarly to Monicelli, however it is more streamlined for processing after an alignment is performed. It creates tracks with the input of the data and the geometry file. The Monicelli Express code is wrapped in a larger script known as the “Hyperscript”. This code is used to manage the conversion of the raw data to ROOT files.

Once analyzed by the Hyperscript the runs are in the form of a ROOT file and geometry file. This is not directly readable by ROOT scripts therefore they must be extracted using the C++ code “T1396Extraction”. This code reads the events and converts them into a TTree saved into a ROOT file. The TTree stores track parameters, cluster positions, and raw data. After the events are extracted the upstream, downstream, and MIDAS data are merged into one TTree for future analysis. This is then analyzed using EMA (EMPHATIC Analysis code). This code, designed by Matej Pavin, allows for cuts to be performed on the data to extract the desired particles and tracking efficiency.

4.4 Silicon Telescope Detector Efficiency Studies

The efficiency was determined by a ratio of true hits ($N_{\text{true}}$) over expected hits ($N_{\text{exp}}$). The expected hits were calculated by an iterative process where, for a single track, a single plane was chosen and removed (i.e the plane’s measured x and y coordinate including the copied values from the other planes) from the fitted points of the track. The track was then refit to determine new slopes and intercepts in x and y. These slopes and intercepts were used to interpolate the tracks back to the removed plane. If the interpolated point was within the geometry of the detector then it was considered to be an expected hit, and thus the value of $N_{\text{exp}}$ increased by one. After this the value $N_{\text{true}}$ increased if the plane was seen to have a hit for that track. Therefore, the efficiency and its uncertainty would be

$$Eff = \frac{N_{\text{true}}}{N_{\text{exp}}} \pm \sqrt{\frac{N_{\text{exp}} - N_{\text{true}}}{N_{\text{exp}}}}.$$ \hspace{1cm} (4.9)

To test the efficiency of the silicon strip detectors a tighter constraint is placed on track quality. The tracks must satisfy the following conditions:

- The track must be the only track for that event, meaning that events with two
or more tracks are not considered.

- The track must have a reduced $\chi^2$ that is less than four.
- The track must have an associated downstream track.
- The track must pass through all other planes than the selected plane.

The first condition was used to ensure that the track is not caused by back scatter or electronic noise. Then the second condition was applied to the $\chi^2$ to reduce the probability of scattering within the components of the tracker\(^2\). Finally, the last condition is used to determine if a hit was registered in the downstream planes. The final constraint is that the tracks must pass through all other strip planes other than the tested plane. So if plane 3 (which corresponds to the second-most upstream x detecting silicon strip) is selected all other 13 strips (7 upstream and 6 downstream) must have points. In addition, a constraint has been placed on the lead glass calorimeter that it must register the track within its signal region (between 100 and 400 ADC counts for all beam conditions). This condition ensures that the downstream-most plane should have detected a hit. A lead-glass calorimeter ADC spectrum is provided in Figure 4.2.

An important note should be on the geometric acceptance of each plane. Currently, no in-depth tests were performed on the uniformity of the efficiency as a function of strip position. Therefore, to account for this lack of understanding the geometric acceptance was taken to be a narrow region in x and y that encompasses both the silicon strip and pixel detectors. This was set to be between 1.25cm and 2.25cm in x and between 1.85cm and 2.60cm in y. The effect of this cut can be seen in Figure 4.3. These plots show the hit distribution along x and y for each plane, separated based on their z value. The lines show the accepted region where hits are considered.

These test were performed for a smaller set of data acquisition runs to sample the efficiency of the detector. The runs range from before the installation of the moving table to after the installation of the moving table. This is an important point because this represent two different geometry files being used during the

\(^2\)These components include the detecting planes as well as the outer material of the pixel detector.
Figure 4.2: Lead-glass calorimeter ADC spectrum. The first peak represents the pedestal and the second represents the signal, ranging from 100 to 400 ADC.

analysis. Thus, by looking at inconsistencies in the efficiency before and after, the possibility of a moving table systematic efficiency loss can be studied.

A sample efficiency plot is shown in Figure 4.4. This plot is of 30 GeV/c protons with an empty target. Only the silicon strip detecting planes’ are plotted. The plot shows that all planes operate at acceptable efficiencies ranging from 98.8% to 100%. The only noteworthy aspect of this plot is that planes 1, 8, 17, and 22 each have efficiencies of 100%. This is also prevalent in most of the tested runs. The high value of the efficiency arises from the “first and last” track finding method. This method mandates that a track must pass through the first available cluster and last available cluster. Therefore, for upstream tracks planes 1 and 8 usually include both a true and expected hit. The same logic applies to planes 17 and 22 downstream of the target. An estimate of these planes efficiencies is given at the end of the section to see if a global efficiency loss is applied. This process was
Figure 4.3: The figures above show the distribution of hits in x and y as a function of z. The reason that discrete z values are shown is because the z value of each detecting plane is fixed by the geometry file. The value of x and y can be used to show the general shape of the silicon strip detectors (3.85cm in x and y) and the pixel detector planes which vary in size (approximately 1.5cm to 2.5cm). The first four lines represent the upstream silicon strip detector. The next eight are the pixel detector. The final three lines are the downstream silicon strip detectors.
Figure 4.4: The efficiency of the silicon strip detectors for a 30GeV/c proton beam. The efficiency is plotted as a function of plane number.

repeated for multiple runs before and after the installation of the moving table and for multiple beam momenta. The results are shown in Figure 4.5.

The summary plot of the efficiency tests on the silicon strip detectors shows a consistency between multiple runs. Each run contains 14 points for the different silicon strip planes. This plot depicts the fact that the high efficiency seen in planes 1, 8, 17, and 22 are consistent between multiple runs.

To estimate the efficiency of these planes a simple analysis was performed by using the total efficiency (\(\varepsilon_{\text{tot}}\)) and dividing it by the efficiency of all the other planes. The value of \(\varepsilon_{\text{tot}}\) can be determined by

\[
\varepsilon_{\text{tot}} = \frac{\text{Number of Events when all 14 planes have a hit observed}}{\text{Total Number of Events}}.
\]

Here the total number of events are without the constraint applied on the planes.
Figure 4.5: The above plot summarizes the silicon strip efficiency test. For a given run, each point represents the efficiency of the plane.

This \( \varepsilon_{\text{tot}} \) represents the product of all the planes' efficiencies (\( \varepsilon_n \)):\[
\varepsilon_{\text{tot}} = \prod_{n=0}^{14} \varepsilon_n. \tag{4.11}
\]

The product of the four overestimated planes (1, 8, 17, 22) will be represented by\[
\varepsilon_x = \frac{\varepsilon_{\text{tot}}}{\prod \varepsilon_n \neq 1, 8, 17, 22}. \tag{4.12}
\]

Here \( \varepsilon_x \) represents an estimate of the 4 plane efficiency as a function of the total efficiency. The following equation was used to estimate the contribution of an individual plane (\( \varepsilon_0 \))\[
\varepsilon_0 = \varepsilon_x^{1/4}. \tag{4.13}
\]

This should only be considered as an estimate because any efficiency loss can not
Figure 4.6: This is a plot of the estimated efficiency of an individual plane ($\varepsilon_0$) for the planes 1, 8, 17, 22.

be attributed to an individual plane, however it will indicate whether these planes are within the expected 98% to 100% efficiency region. The results of this efficiency estimate is shown in Figure 4.6, which is a plot of $\varepsilon_0$ as a function of the global run number.

### 4.5 30GeV/c Proton on Carbon Analysis

The following preliminary analysis will attempt to mimic the procedure performed by Bellettini et al. [1]. The purpose of the experiment was also to measure the differential cross section of 20GeV/c protons scattering off of multiple target nuclei. After the target the scattered particles would be measured by scintillators and analyzing magnets to determine their scattering angle. For the measurements performed on carbon the minimum angular acceptance of the detector was estimated to be 1.5mrad. Therefore, particles with angle below 1.5mrad would not be measured. The analysis will be on a 30GeV/c proton on carbon data set, prepared by
Figure 4.7: The graph depicts the number of remaining events as a function of the applied quality cuts used in the 30GeV/c proton on carbon analysis.

Matej Pavin. The data is a combination of multiple 30 GeV/c proton on carbon data sets. Issues with the data set used will be explained in the conclusion. It should also be stated that the data was prepared by the EMA code designed and programmed by Matej Pavin. All the cuts used on the data are shown in Figure 4.7. The noteworthy cuts are:

- Gas Cherenkov cuts
- Reduced $\chi^2$ cuts on both upstream and downstream tracks so that the goodness-of-fit is no greater than four
- Cuts on the difference between the extrapolated position $(x, y)$ of the upstream and downstream tracks at each $z$ position of the target

A full description of each cut is shown in Appendix C.

The gas Cherenkov cuts were used to select the protons, with a 3% kaon contamination, as calculated by Matej Pavin. The $\chi^2$ cuts ensure the quality of the fit. The cuts on the difference in $x$ and $y$ were used as another quality check to ensure they are distributed with a mean of zero. Through these cuts 30GeV/c proton on carbon scattering events can be analyzed.
Each event contains upstream and downstream tracks with x and y slopes. The slopes for the upstream track can be written as a vector \((S^u_x, S^u_y, S^u_z)\) where \(S^u_x\) and \(S^u_y\) are fitted by Monicelli and \(S^u_z\) is taken to be one. The same can be done to the downstream track \((S^d_x, S^d_y, S^d_z)\). Therefore, the scattering angle \((\theta)\) can be determined by the equation:

\[
\begin{align*}
S^u &= (S^u_x, S^u_y, S^u_z), \\
S^d &= (S^d_x, S^d_y, S^d_z), \\
\theta &= \arccos \left( \frac{S^u \cdot S^d}{|S^u||S^d|} \right).
\end{align*}
\] (4.14)

The angle is then used to find the squared four momentum transfer variable \((t)\). The \(t\) variable also known as the time-channel Mandelstrom variable is proportional to the mass of the target nucleus and the total kinetic energy. This means that \(t\) represents the scale that is being probed by the incoming particle. At the relativistic limit and after applying the small angle approximation the calculation of \(t\) can be simplified so that it only depends on the scattering angle and the input momentum \((p)\):

\[ t \approx \theta^2 p^2. \] (4.15)

A distribution of \(t\) values is then obtained to describe the scattering processes going on.

The \(t\) distribution contains three regions describing Coulomb scatter, elastic scatter, and quasielastic scatter. Coulomb scattering is when a charged particle scatters off the Coulomb potential of the nucleus. Single Coulomb scattering is where the target is thin, thus lowering the probability of multiple interactions. Multiple Coulomb scattering is where the thickness of the target is substantial enough to model the angular distribution as Gaussian. The particles that go through 2cm targets experience multiple scattering. The standard deviation of the angular distribution projected onto a plane \((\theta_0)\) can be defined as [47]:

\[
\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p z \sqrt{x/X_0}} \left[ 1 + 0.038 \ln x/X_0 \right].
\] (4.16)
Here $x$ represents the thickness of the material, $X_0$ the material’s radiation length, $z$ the charge of the particle, $\beta$ the speed of the particle, and $p$ the momentum of the particle. Thus, for a 30GeV/c proton travelling through 2cm of graphite a deflection angle of 0.135mrad is expected. Elastic scatter, also known as coherent nuclear scatter, describes the process of the particle scattering off the nucleus where the total kinetic energy of the system is conserved. Scattering where the total kinetic energy is not conserved is known as inelastic scattering. Quasielastic scattering, which in this context can also be referred to as incoherent proton-nucleon scattering, is a subset of inelastic scattering where the particle has scattered directly off of the nucleons. Bellettini et al. uses a simplified model to determine how these regions vary with respect to $\omega$ the solid angle:

$$\omega = 2\pi(1 - \cos \theta)$$

$$\approx \pi \theta^2$$

$$\approx \pi \frac{t}{p^2}.$$ \hspace{1cm} (4.17)

Here $\omega$ has the units of radians and $t$ has the units of $GeV^2/c^2$. The units of $t$ will remain the same for all the proceeding equations.

The model used in Bellettini et al. [1] involves splitting the distribution into two regions. The first region is defined as $t < 0.03GeV^2/c^2$, which is where the Coulomb and elastic scattering dominate. The second region, defined as $t > 0.08GeV^2/c^2$, is where quasielastic scattering dominates. The first region is described by the following equation$^3$

$$\frac{d\sigma}{d\omega} = C^2 + I^2 + R^2 + 2CR.$$ \hspace{1cm} (4.18)

Here $C$ is the Coulomb contribution, $I$ is the imaginary component of the elastic scatter, and $R$ is the real component of the elastic scatter. The $R$ component is treated as negligible in this analysis due to the assumption that its magnitude is

---

$^3$The equation will be converted into $\frac{d\sigma}{dt}$ in the following lines.
significantly less than that of \( I \) and \( C \), therefore

\[
\frac{d\sigma}{d\omega} \approx C^2 + I^2. \quad (4.19)
\]

Here \( C \) is described by the Rutherford scattering equation and the Coulomb form factor \( (f_c) \):

\[
C = 2 \left( \frac{Zr_emc}{t} \right) \times f_c = 2 \left( \frac{Zr_emc}{t} \right) \times \exp\left\{ -\frac{a^2}{6\hbar^2} |t| \right\}, \quad (4.20)
\]

where the Rutherford scattering component contains: \( Z \) the atomic number, \( r_e \) the electron radius, \( p \) the momentum of the impinging particle, \( m \) the mass of the electron, and \( c \) the speed of light. The form factor depends on Planck’s constant \( (\hbar) \) and the RMS radius of the target \( (a) \), a parameter given in Herman and Hofstadter [48] as \( 2.4 \pm 0.1 \) fm. The imaginary component of elastic scattering is described by

\[
I = \left( \frac{p\sigma_{tot}}{4\pi\hbar} \right) \times f_n = \left( \frac{p\sigma_{tot}}{4\pi\hbar} \right) \times \exp\left\{ -\frac{B|t|}{2} \right\}. \quad (4.21)
\]

Here \( f_n \) is the nuclear form factor which depends on the elastic slope parameter \( B \). In addition, the coefficient is dependent on \( p \) the momentum and \( \sigma_{tot} \) the total cross section. Therefore, Equation (4.21) shows that, when plotted on log scale, \( B \) can be determined by the slope and \( \sigma_{tot} \) can be determined by the intercept. Moreover, Equation (4.21) can be integrated out to determine \( \sigma_{ela} \). The \( B \) parameter can determine the radius \( (R) \), measured in femtometres, used in spherical black body diffraction [1]:

\[
R = \frac{\sqrt{B}}{4\hbar^2}. \quad (4.22)
\]
Quasielastic scattering is modelled by an equation similar to Equation (4.21):

\[
\frac{d\sigma}{d\omega} = \frac{N(A)(p\sigma_{tot}(pN))^2}{4\pi\hbar} \exp\{\sqrt{-D}|t|}\]

\[\approx N(A) 11.0 \exp\{-10.0|t|\} \text{ b/sr.}\]  \(4.23\)

Here \(\sigma_{tot}(pN)\) is the total scattering cross section of protons off the nucleons, \(D\) is the quasielastic slope term, and \(N(A)\) represents the number of free nucleons that can produce inelastic scattering similar to that of the target nuclei of atomic mass \(A\). In addition, it should be noted that \(t\) is of the units of \(GeV^2/c^2\). The second form of the equation uses previously measured values of \(\sigma_{tot}(pN)\) and \(D\) from proton-proton scattering experiments, described in a separate paper by Bellettini et al. [49]. This assumes that proton-proton scattering \((\sigma_{tot}(pp))\) values are comparable to \(\sigma_{tot}(pN)\). These measurements were made for 19.3\(GeV/c\) proton on proton scattering however a simple conversion, explained in the next paragraph, can make it applicable to 30\(GeV/c\).

To make the model described above be applicable to our data a conversion must be made, using Equation (4.17), from \(\frac{d\sigma}{d\omega}\) to \(\frac{d\sigma}{dt}\) which can be done by the following

\[
\frac{d\sigma}{dt} = \frac{d\sigma}{d\omega}\frac{d\omega}{dt}
\approx \frac{d\sigma}{d\omega}\frac{\pi}{p^2}.
\]  \(4.24\)

Applying this factor to Equation 4.19 and subbing in 4.20 and 4.21 yields

\[
\frac{d\sigma}{dt} = \frac{\pi}{p^2} (C^2 + I^2)
= \pi \left( \frac{2Zr_emc}{t} \right)^2 \exp\left\{ -\frac{a^2}{3\hbar^2}|t| \right\} \exp\{1\} \times \exp\{-B|t|\}.
\]  \(4.25\)

The same transformation can be applied to the simplified form of Equation (4.23) however now the value of momentum will be taken to be 19.3\(GeV/c\) to make it momentum independent. This equation still applies for higher momentum because \(\sigma_{tot}(pN)\) does not vary significantly between 19.3\(GeV/c\) (\(\sigma_{tot}(pp) = 38.9 \pm 0.3\text{mb}\)
and 26.4GeV/c ($\sigma_{tot}(pp) = 38.8 \pm 0.3 mb$) [49]. The equation now becomes

$$\frac{d\sigma}{dt} \approx N(A)(92.77) \exp\{-10.2|t|\}. \quad (4.26)$$

Equations 4.25 and 4.26 are of the units $mb/(GeV^2 c^{-2})$. Therefore by applying these two equations the desired parameters can be extracted. The analysis performed in this section is centred around the derivation of four variables: the total cross section ($\sigma_{tot}$), the radius of the spherical black body (R), the elastic cross section ($\sigma_{ela}$), and $N(A)$.

Difficulty occurred while selecting the lower limit of the joint Coulomb elastic fit, when attempting to fit the model to the 30GeV/c proton on carbon data. This resulted in a secondary analysis on the validity of the model at low t. It is valid to not include the first bin of the histogram into the fit because at $t \approx 0.0 GeV^2 c^{-2}$ no scattering occurs and the Coulomb scatter model diverges. However, when a fit was attempted at the next bin the $\chi^2/DoF$ value was too high for a valid fit. This can be due to multiple reasons:

- Due to the real part of the elastic scatter being neglected the cross term between the real component and the Coulomb scatter is not considered.
- The Coulomb region also contains an imaginary phase that also needs to be considered for the cross term [50].

The reason that these effects are not seen in Bellettini et al. is that the resolution of the $t$ distribution is not fine enough to discriminate between these effects, as shown in Figure 4.8. The black points represent the true histogram values of the $t$ distribution and the white points represents the points corrected for the Coulomb contribution. The point labelled “O.T” is the calculated $\sigma_{tot}$ that comes from the model. Thus, from Figure 4.8 it can be seen that the carbon histogram only has two points, at $t < 0.01 GeV^2 c^{-2}$, that contribute to the Coulomb region. Thus, to attempt to see where this model is valid the $\chi^2/DoF$ of the fit was plotted as function of the lower boundary ranging from $0.0 GeV^2 c^{-2} \leq t < 0.03 GeV^2 c^{-2}$.

Figure 4.9 shows the variation of the $\chi^2/DoF$ as a function of the lower boundary. It can be seen from the plot that the plateau region was reached around $0.003 GeV^2 c^{-2}$, which will be chosen as the lower boundary. In addition, when
scattering was less than $10^{-2}$ of that due to single scattering. At larger angles the relative importance of plural scatterings decreased rapidly and became smaller than $10^{-1}$ at $\theta > 5 \text{ mrad}$. Multiple and plural scatterings were evaluated with the Molière theory, using the formulae given by Bethe and Ashkin. The data presented below are for $\text{Li}_6$ and $\text{Li}_7$, $P_0 = 19.3 \text{ GeV/c}$, $\text{Be}^9$, $P_0 = 19.3 \text{ GeV/c}$, $\text{C}^{12}$, $P_0 = 21.5 \text{ GeV/c}$, and $\text{Al}^{27}$, $P_0 = 19.3 \text{ GeV/c}$.

Figure 4.8: The above distribution is the differential cross section as a function of $t$ taken by Bellettini et al. The plot is of proton scattering off multiple target nuclei (Li, Be, C, and Al). The first fit ($t < 0.03 \text{GeV}^2 \text{c}^{-2}$) is of the elastic scatter region. The secondary fit is of the quasielastic region (approximately $t > 0.08 \text{GeV}^2 \text{c}^{-2}$). ©, Elsevier, by permission[1].
the lower boundary increases past the $0.02\text{GeV}^2c^{-2}$ so does the $\chi^2$ however this is due to the number of point being too low to maintain a stable fit. Therefore, the range chosen for this analysis is from $0.003\text{GeV}^2c^{-2}$ to $0.03\text{GeV}^2c^{-2}$.

With the region chosen the fit can be performed on the $t$ distribution. The fit of the $30\text{GeV}/c$ proton on carbon target is shown in Figure 4.10. The first plot is the fit of the $t$ distribution with the models described in Equations 4.25 and 4.26. The second plot is a distribution with the Coulomb component subtracted and the models are the same however in this scenario $C = 0$. The results of the fit are summarized in Table 4.1. The EMPHATIC results are from the second plot because that most closely follows the Bellettini et al. method. The uncertainty of $N(A)$ was not explicitly given in Bellettini et al. however in the paper two methods are described in deriving $N(A)$. The first is by applying the model to the quasielastic region. The second is by determining $N(A)$ through the following equation

$$N(A) = A \left(1 - \frac{\sigma_{abs}}{\pi R^2}\right),$$

(4.27)
Table 4.1: Results of Bellettini et al. [1] compared with EMPHATIC (30GeV/c proton on Carbon) data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bellettini et al. measurement (p+C 21.5GeV/c)</th>
<th>EMPHATIC measurement (p+C 30GeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cross section (mb)</td>
<td>335±5</td>
<td>334±2</td>
</tr>
<tr>
<td>R (fm)</td>
<td>3.2±0.1</td>
<td>3.18±0.08</td>
</tr>
<tr>
<td>Elastic Cross Section (mb)</td>
<td>81±4</td>
<td>88±1</td>
</tr>
<tr>
<td>$N(A)$</td>
<td>3.4±0.4</td>
<td>3.9±0.2</td>
</tr>
</tbody>
</table>

where $\sigma_{abs}$ is the absorption cross section. The difference between these two values is taken as an uncertainty to the measurement of $N(A)$. With this applied it is shown that $\sigma_{tot}$, $R$, and $N(A)$ agree within one standard deviation with Bellettini et al. The elastic cross section agrees within two sigma. A comparison between these parameters at different momenta (20GeV/c and 30GeV/c) is valid because these parameters are not strongly dependent on the momentum, as shown in the $\sigma_{prod}$ shown in Figure 1.4. In addition, the value of the quasielastic cross section was measured to be $35\pm2mb$ and the value of the inelastic cross section was determined to be $247\pm2mb$. 

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Figure 4.10: The above plots are the analysis performed on 30GeV/c proton on carbon. The analysis follows the Bellettini et al. model where the Coulomb and elastic region is fit separately from the quasielastic region. The second plot is the same as the first plot however with the Coulomb component subtracted and only the elastic is fitted within the region.
Chapter 5

Conclusion

The purpose of the EMPHATIC experiment is to reduce the hadronic interaction uncertainties that dominate the neutrino flux predictions. This is done by minimizing the uncertainty of the production cross section and the out-of-target secondary interactions. Another goal of the experiment is to understand the discrepancy between the NA61/SHINE thin and replica target data sets. This can be done by taking additional measurements of proton and pion scattering off of carbon, aluminum, and iron targets at multiple momenta. In January 2018 the EMPHATIC group took its first measurements at the Fermilab Test Beam Facility (FTBF). The work shown in this thesis represents aerogel detector design, efficiency tests, and preliminary analysis of this data.

The aerogel detector design began as a simulation of the light propagation properties of the detector. A simulation suggested that the initial design operated efficiently and the decision was made to produce the detector. The design and the construction of the detector was in Autocad. Subsequently, the detector was 3D printed in the detector facility in TRIUMF. When tested at FTBF, one of the PMTs did not operate. Therefore, it was chosen to reformat the detector to use a single PMT. Afterwards, the efficiency of the detector was tested at a proton beam momentum of 120GeV/c ($\beta \approx 1$). The efficiency of the detector was proven to be 99.999%. This suggests that only one PMT is sufficient to operate the detector. Another important finding for the second phase of the EMPHATIC data taking is the number of photons detected by the detector. The number of photons estimated
was 9.1 for a thickness of 5.9cm and an average refractive index of 1.0255. When compared to the number of photons for the Belle 2 experiment aerogel Ring Imaging CHERENKOV detector (RICH) of 11.4 for a thickness of 4cm and a refractive index of 1.05 suggests that the number of photons detected are sufficient for particle identification by a ring imaging Cherenkov detector [51]. It also suggests that the majority of the design decisions help yield an optimal result. However, some changes would be suggested to help with ease of use.

These changes would involve some redesign and some additional components. The main changes are for the PMT covers and their lids. These could be changed to add a mechanism for simple removal and insertion such as a clip on the sides of the detector enclosure. Another suggestion, from Matej Pavin, is that the cover could be screwed onto the detector enclosure which would give both an easy attachment and additional light-tightness. The lids can be improved by adding BNC and high voltage cables barrels eliminating the need to remove of the lid when the enclosure is moved. Another thing to add would be ledges on the sides of the enclosure that would help the user place the lid accurately. The lid could have been made slightly wider to facilitate an easier placement, however this is not necessary. The aerogel holders should be reworked to be even thinner and lighter. During operation, they seem to be too bulky for the aerogel. An improvement would be to have a similar, but thinner, upper and lower aerogel holders and thin walls of varying height that can be slotted in.

The efficiency test of the aerogel detector should be extended to other detectors (the aerogel detectors of the Chiba group) as well as lower momentum. This will help probe the efficiency as a function of momentum for the different aerogel giving a better understanding of the requirements needed for future measurements performed by EMPHATIC.

The semiconductor test have been split into two, the pixel and silicon strip detector. The pixel detector has been found to have a lower than expected efficiency and has been chosen to be removed from the current analysis. More is explained in Appendix A. The efficiency of the silicon detector was found to be greater than 98%. Meaning that these detectors work optimally and do not cause major issues. In addition, the detecting planes 1, 8, 17, and 22 were found to have an inflated efficiency of 100% arising from the “first and last” track finding method. This
method requires that any track pass through the first and last cluster, which is likely
to be those four planes. To account for this method an estimated efficiency for
these planes were calculated to be around 98%. In future data taking the silicon
detectors will only be used. This allows a slight more reliable detector set up as
well as the ability to bring the silicon detector closer to the target. As can be seen in
Figure 4.1 the pixel detector is closer to the target which means that approximately
160cm needs to be extrapolated to determine the position of the particle at the
target. This large lever arm will increase systematics if there are slight variations
with the track parameters (slope and intercepts). In addition, the silicon detector
has a larger detecting area, when compared to the pixel detector meaning a wider
beam can be used.

The analysis of the silicon strip detector was done following the method ex-
plained by Bellettini et al. [1]. The use of the model yielded results that agree
between Bellettini et al. and the EMPHATIC for the variables \( \sigma_{\text{tot}}, \sigma_{\text{ela}}, R, \) and
\( N(A) \). Although these measurements were performed at different momentum it is
not expected that these parameters are strongly momentum dependent. However,
it was seen that there were problems with the model at lower \( t \) values. This is due
to the simplified form of the Coulomb component. In addition, this model does not
consider the real component of the elastic scattering. Another model being tested is
from the ATLAS collaboration used in high energy proton-proton scattering [50]:

\[
\frac{d\sigma}{dt} = \frac{1}{16\pi} \left[ -\frac{8\pi\alpha Zhc}{t} \exp \left\{ -\frac{a^2 t}{4} + i\phi(t) \right\} \right] \\
+ \left( \rho + i \right) \frac{\sigma_{\text{tot}} \bar{h} c}{\pi} \times \exp \left\{ \frac{-Bt}{2} \right\}^2
\]

(5.1)

This equation shares many of the same variables as Equation (4.25). Here \( \rho \) is the
ratio of real to imaginary component of the elastic cross section\(^1\) and \( \phi(t) \) is the
imaginary Coulomb phase. This model can also include a quasielastic model to

\(^1\)This value was set to zero in the Bellettini et al. method.
properly measure the entire region:

\[
\frac{d\sigma}{dt} = \frac{1}{16\pi} \left| -\frac{8\pi\alpha Z hc}{t} \exp\left\{ -\frac{a^2 t}{4} + i\phi(t) \right\} \right|
\]

\[
+ \frac{(\rho + i)\sigma_{tot}}{hc} \times \exp\left( -\frac{Bt}{2} \right)
\]

\[
+ \frac{N(A)(\rho(pN) + i)\sigma_{tot}(pN)}{hc} \times \exp\left( -\frac{Ct}{2} \right)^2
\]

(5.2)

where \(\rho(pN)\) is the ratio of real to imaginary for the quasielastic. The equation represents Equation (5.1) with an additional term for the quasielastic component. This would eliminate the need for separating the data into regions as well, meaning the entire \(t\) distribution can be utilized. In addition to the Coulomb region the quasielastic region was fit in a very simplified model, shown in Equation (4.26). The issue with this model is that the quasielastic slope \((C)\) was fixed to previously measured values in Bellettini et al. for proton-proton scattering. It can clearly be seen in Figure 4.8 that for some quasielastic regions this slope is not optimal. It can be shown in the carbon target that there is a clear discrepancy between the fit and the point at \(t \approx 0.07 GeV^2 c^{-2}\). For the EMPHATIC data at 30GeV/c proton on carbon scattering if the \(D\) parameter, in Equation (4.23), is allowed to vary it prefers a value of \(14 GeV^{-2} c^2\) as apposed to the \(10 GeV^{-2} c^2\). In addition, to these model dependent issues there are also systematic uncertainties that require consideration.

The systematic uncertainties may prove to be significant and require consideration. Monicelli and the alignment may add to the systematic errors of the analysis. Studies must be performed to understand the effect of the geometry file on the \(t\) distribution. Some of these may include measuring the change of the track \(\chi^2\) as a function of the variables of the geometry file. Other considerations include the different track fitting and finding methods. Ideally, the road search method should be used along with the Kalman fitter to account for multiple scattering. More fundamentally, the clusterization algorithm used for position measurements on the strips should be changed to use the charge distribution data more properly with a correct signal-to-noise ratio for the standard deviation. Currently, the standard deviation is only proportional to the pitch which limits the resolution and returns an inaccurate \(\chi^2\) for the tracks. Finally, the \(t\) distribution should be plotted as a function of the
different cuts used for data quality. This would aid in finding any dependencies and give more insight on the effects of these cuts on the final distribution.

The total cross section obtained in the Bellettini et al. model fit can be compared with other measurements. The cross sections taken in this comparison include, Bellettini et al. [1], NA61/SHINE [36], and the cross sections measured by the EMPHATIC. A separate analysis of the total cross section was performed by Matej Pavin of the EMPHATIC group data. This analysis was done by counting the number of particles that do not interact hadronically for 30GeV/c protons on carbon with the target in (\(N_{\text{in}}\)) and the target out (\(N_{\text{out}}\)). Multiple scattering for the target out data set was applied to properly compare the two. Then the probability of survival through the target (\(P_t\)) can be considered:

\[
P_t = \frac{N_{\text{out}}}{N_{\text{in}}} = \exp\left\{-n\sigma_{\text{tot}}d\right\}
\]  

(5.3)

where \(n\) is the density and \(d\) is the thickness of the target. The result of this comparison is shown in Figure 5.1. It is shown that all of these values agree within two standard deviations. In addition, both EMPHATIC measurements of \(\sigma_{\text{tot}}\) agree with value recorded by the NA61/SHINE collaboration. This gives increased support to the argument that the thin target data and model are correct. However, the fact that the Bellettini et al. model fit (the red point) and the probability of the survival analysis (the orange point) do not agree at one sigma should be investigated. Especially, considering that these two analysis are on the same data set.

Another comparison can be performed, shown in Figure 5.2, on the production cross section based on the Bellettini et al. model, with other data sets used in a comparison performed by the T2K experiment. The data sets used in the comparison were Denisov et al. [34], Bellettini et al. [1], and NA61/SHINE [35]. The Denisov et al. is also subtracted by the quasielastic cross section estimated in Abe et al. as 33.1mb and plotted as “Denisov et al. QE Subtracted” [27]. This was also performed on the Bellettini et al. calculated value of \(\sigma_{\text{qe}}\). It can be seen from the comparison that the EMPHATIC \(\sigma_{\text{prod}}\) agrees (the red point) within one standard deviation with the Bellettini et al. measurement (the green point) and the Denisov et al. \(\sigma_{\text{qe}}\) subtracted point (the orange point). However, when compared with the
NA61/SHINE value the cross sections agree within two standard deviations with the EMPHATIC value, although the values of $\sigma_{el}$ agreed at one sigma. This is due to both the $\sigma_{el}$ and $\sigma_{q}$ components being greater for the EMPHATIC collaboration than for NA61/SHINE which pulls the value of $\sigma_{prod}$ down because:

$$\sigma_{prod} = \sigma_{ine} - \sigma_{q}.$$  \hspace{1cm} (5.4)

However, it should be repeated that the full systematic uncertainty profile of the data point has yet to be considered.
Figure 5.2: The plot above shows a comparison between the production cross sections of multiple experiments. This includes the production cross section analysis following the Bellettini et al. model on the EMPHATIC data.
Bibliography


[34] SP Denisov et al. “Absorption cross sections for pions, kaons, protons and antiprotons on complex nuclei in the 6 to 60 GeV/c momentum range”. *Nuclear Physics B* 61 (1973), pp. 62–76.


Appendix A

Pixel Detector Efficiency

Pixel and pad detectors are formed when semiconductor detectors are set into a checkerboard pattern. The distinction between the pad and pixel detector is that the pad detector has dimensions of the order of 1mm and the pixel is less than that. Each pixel must be connected to each readout channel individually. These connections have been shown to reduce the low electronic noise due to a lower capacitance and leakage current [40]. The main difficulty of the pixel detector is the complexity of the connection between the detecting material and the readout chip. This has been mitigated, however, by the process of bump bonding where the readout chip is constructed with the same pitch as the detecting material.

Determining the pixel detector’s efficiency began as a general analysis of the efficiency of all tracking detectors upstream of the target. This was performed by taking all upstream tracks that satisfy the following conditions:

- The upstream track must be the only track for that event, meaning that events with two or more tracks are not considered.
- The upstream track must have a reduced $\chi^2$ that is less than four.
- The upstream track must have an associated downstream track.

The efficiency was determined based on the ratio of the number of true hits over the number of expected hits, as explained in Section 4.4. The geometric acceptance of this test is also shown in Figure 4.3. Figure A.1 shows a sample efficiency plot
Figure A.1: This plot shows the upstream efficiency of a 30GeV/c empty target run. The first eight points represent the silicon strip detector and the next eight planes are the pixel detector. There is an obvious drop in efficiency.

for the upstream silicon strip detectors for a 30GeV/c empty target run. The plot shows the efficiency as a function of plane number. The planes include separately both x and y silicon strip planes. This means there are eight upstream silicon strip planes (1-8) and eight pixel planes (9-16) totalling sixteen. From the plot it can be seen that silicon strip planes are observing an efficiency greater than 90% which is acceptable. The pixel detector, however, has an efficiency of approximately 50% for this run, which becomes the main concern for this section.

As said earlier, this process was repeated for multiple runs and the strip efficiency remains stable at 90% while the pixel ranges between 50% – 80%. After a discussion with the silicon telescope experts at the FTBF the theory for this drop in efficiency is that the trigger between the pixel and the silicon strip data acquisition became out of sync which resulted in the pixel dropping events. This syncing issue caused a global efficiency loss seen in the pixel detector.
Figure A.2 summarizes the tests on the pixel detector’s efficiency for different runs. The plot is of the average pixel efficiency as a function of global run number. The global run numbers are used to determine whether the run was performed before (negative value) or after (positive value) the moving table was installed. The runs include 30GeV/c and 120GeV/c empty target runs. Target-in runs were also used because the downstream efficiency was not considered in this scope. From the average pixel efficiency it can be seen that the highest efficiency obtained is 70%. The error bars on these values come from the standard deviation of the pixel detectors. The large error bars show runs where multiple pixel detectors ran with efficiencies of 10% or lower suggesting a critical data acquisition failure. After the global efficiency loss of the pixel detector was found, it was decided that to ease future data analysis the pixel detector measurements would be removed from the current data analysis. Thus, in the following section only analysis based on the silicon strip detector will be discussed.
Figure A.2: The above plot summarizes the efficiency tests performed on the pixel detector. The global run number is a number that is used to label each run separately from the internal MIDAS and CAPTAN run numbers to maintain consistency. The convention is that the first run after the moving table was installed is considered run zero. The runs before the installation are negative.
Appendix B

Semiconductor Detector Specifications

The following section describes the specifications of the semiconductor detectors. Table B.1 summarizes the Z position, XY dimensions, and rotational angle. The plane number goes from left to right of Figure 4.1. The second column describes the coordinates acquired by the semiconductor detecting plane. The Z position of each plane is relative to the center of the pixel detector and are previously recorded. The X and Y length make up the area of the detecting plane. The last two columns are the approximate angle and its axis of rotation giving the tilt of the detecting plane.

Tables B.2 and B.3 go over the pixel and silicon strip detector specifications. These specifications include the dimensions of a pixel detector, the number of pixels per readout chip, and the dimension of the readout chip cell. The last two rows describe the two possible pixel plane layouts. The first is a horizontal detector and the second is a vertical detector. The silicon strip specifications are the strip length, pitch, and number of strips per plane.

Tables B.4 and B.5 go over the radiation length and thickness of a single detecting plane. These values were given by Lorenzo Uppleger of the FTBF. The material FR4 represents a glass-reinforced epoxy laminate used on the silicon detectors. All these tables have been used in simulations currently being run by the EMPHATIC experiment.
Table B.1: Semiconductor planes position, dimensions, and angles.

<table>
<thead>
<tr>
<th>Plane Number</th>
<th>Detected Coordinate</th>
<th>Z Position (cm)</th>
<th>X Length (cm)</th>
<th>Y Length (cm)</th>
<th>Approximate Angle (°)</th>
<th>Axis of Rotation</th>
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</thead>
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<td>(x_1)</td>
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<td>3.85</td>
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<td>(y_1)</td>
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<td>Y</td>
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<tr>
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<td>(x_{p4}, y_{p4})</td>
<td>-24.9</td>
<td>1.62</td>
<td>2.43</td>
<td>25</td>
<td>Y</td>
</tr>
<tr>
<td>13</td>
<td>(x_{p5}, y_{p5})</td>
<td>25.9</td>
<td>1.62</td>
<td>2.43</td>
<td>25</td>
<td>Y</td>
</tr>
<tr>
<td>14</td>
<td>(x_{p6}, y_{p6})</td>
<td>29.7</td>
<td>1.62</td>
<td>2.43</td>
<td>25</td>
<td>Y</td>
</tr>
<tr>
<td>15</td>
<td>(x_{p7}, y_{p7})</td>
<td>42.7</td>
<td>3.24</td>
<td>1.62</td>
<td>-25</td>
<td>X</td>
</tr>
<tr>
<td>16</td>
<td>(x_{p8}, y_{p8})</td>
<td>46.5</td>
<td>3.24</td>
<td>1.62</td>
<td>-25</td>
<td>X</td>
</tr>
<tr>
<td>17</td>
<td>(x_5)</td>
<td>90.2</td>
<td>3.85</td>
<td>3.85</td>
<td>-15</td>
<td>Y</td>
</tr>
<tr>
<td>18</td>
<td>(y_5)</td>
<td>90.8</td>
<td>3.85</td>
<td>3.85</td>
<td>-15</td>
<td>Y</td>
</tr>
<tr>
<td>19</td>
<td>(x_6)</td>
<td>102.2</td>
<td>3.85</td>
<td>3.85</td>
<td>-15</td>
<td>Y</td>
</tr>
<tr>
<td>20</td>
<td>(y_6)</td>
<td>102.8</td>
<td>3.85</td>
<td>3.85</td>
<td>-15</td>
<td>Y</td>
</tr>
<tr>
<td>21</td>
<td>(x_7)</td>
<td>114.2</td>
<td>3.85</td>
<td>3.85</td>
<td>-15</td>
<td>Y</td>
</tr>
<tr>
<td>22</td>
<td>(y_7)</td>
<td>114.8</td>
<td>3.85</td>
<td>3.85</td>
<td>-15</td>
<td>Y</td>
</tr>
</tbody>
</table>

Table B.2: Specifications of the pixel detector area.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pixel Dimension</td>
<td>(100 \times 150 \times 285 \mu m^3)</td>
</tr>
<tr>
<td>Number of Pixels per ReadOutChips</td>
<td>(52 \times 80)</td>
</tr>
<tr>
<td>Readout Chip Module size</td>
<td>(0.81 \times 0.81 cm^2)</td>
</tr>
<tr>
<td>Module Configuration 1</td>
<td>(4 \times 2)</td>
</tr>
<tr>
<td>Module Configuration 2</td>
<td>(2 \times 3)</td>
</tr>
</tbody>
</table>
Table B.3: Specifications of the strip detector.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strip Length</td>
<td>3.85cm</td>
</tr>
<tr>
<td>Pitch</td>
<td>0.006cm</td>
</tr>
<tr>
<td>Number of Strips</td>
<td>640</td>
</tr>
</tbody>
</table>

Table B.4: Thickness and refractive indices of the aerogel detectors. The average refractive index and the total thickness is provided below.

<table>
<thead>
<tr>
<th>Material</th>
<th>Radiation Length (mm)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td>93.7</td>
<td>0.6</td>
</tr>
<tr>
<td>FR4 (plastic)</td>
<td>159.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Carbon Fiber</td>
<td>237</td>
<td>1</td>
</tr>
<tr>
<td>Total Thickness</td>
<td></td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table B.5: Thickness and materials of the pixel detectors.

<table>
<thead>
<tr>
<th>Material</th>
<th>Radiation Length (mm)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td>93.7</td>
<td>0.3</td>
</tr>
<tr>
<td>Carbon Fiber</td>
<td>237</td>
<td>0.5</td>
</tr>
<tr>
<td>Total Thickness</td>
<td></td>
<td>0.8</td>
</tr>
</tbody>
</table>
Appendix C

Data Quality Cuts

This appendix describes the calculation of the data quality cuts applied on the data set shown in Figures 3.8 and 4.7. Each cut in Figure 4.7 will be explained.

The “DownCherenkovOut” cut is performed on the outer PMT of the downstream gas Cherenkov to separate the electron and pion contribution from the data set. This contribution is shown in Figure C.1.

The “NUpTracks” and “NDownTrack” cuts are used to select the number of tracks presented in the event. Currently this cut is set as 1 upstream and 1 downstream track.

“ClustersUp” data quality cut is for the number of planes required for each upstream track. Currently this is set at 8 for the upstream silicon strip planes.

“Xup” and “Yup” cuts are used to account for any efficiency loss along the edges of the silicon strip detector. These cuts are geometrical cuts at the z position of 68cm (where the target is set at 69.6cm) along the x and y directions. Both directions allow tracks with an extrapolated between 0.5cm and 3.3cm, as shown in Figure C.2.

The “Chi2Up” and “Chi2Down” are quality cuts on the $\chi^2$ values of the upstream and downstream tracks. This is to ensure that the tracks are within the expected $\chi^2$ range. Recently, it has been found that the $\chi^2$ has not been calculated correctly so more work is necessary to understand the effect of these cuts. This is shown in Figure C.3.

Finally, the “DiffX” and “DiffY” cuts, shown in Figure C.4, are used to account
Figure C.1: This figure is of a downstream gas Cherenkov ADC distribution without the quality cuts applied. This shows the electron and pion contribution at $ADC > 50$ and to account for that a cut of $27 < ADC < 45$ is applied. The lower tail of the proton distribution is also cut.

for inconsistencies between the upstream and downstream tracks. This is done by extrapolating the hits to the target position (69.6cm) and calculating the difference between the $x$ and $y$ positions. Currently, these cuts are set at the $2\sigma$ width of the distribution (before the cuts are applied).
Figure C.2: This is a 2D distribution of the X and Y position, extrapolated at the 68cm position upstream of the target. All events outside of the red rectangular region (ranging from 0.5 to 3.3cm in x and y) have been excluded.
Figure C.3: The above plots are the $\chi^2$ distributions of the upstream and downstream fitted tracks. The cut of $\chi^2 < 4$ was applied to all the data to cut the tail of the distribution. The red line represents the cut placed on the data.
Figure C.4: The above plots are on the extrapolated differences of $x$ and $y$ values at the position of 69.6cm used to account for any inconsistencies between the upstream and downstream tracks. The interior of the two red lines (per plot) represents the data included in the fit.