Essays on Credit Booms and Rational Bubbles

by

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Abstract

Why are credit booms and bubbles harmful to the economy? A dominant view points to the risk of bust. Traditional theories of bank runs and recent theories of rational bubbles describe the costs of jumping to a bad equilibrium when the economy accumulates too much debt. In this work, I propose a theory of rational bubbles where the boom, not the ensuing bust, reduces the output by promoting a misallocation of factors.

In the model presented in Chapter 2, financial markets are imperfect and the rise of a bubble alleviates credit constraints and boosts capital accumulation. However, capital accumulation occurs in unproductive sectors and aggregate output is reduced. The result is driven by the fact that heterogeneous borrowers have an advantage with respect to issuing different types of debt contracts. In normal times, High-productive borrowers have higher collateral and thereby attract most of the funds. In bubbly times, borrowers can also issue "bubbly debt," a debt that is repaid with future debt. The possibility to keep a pyramid scheme and raise bubbly debt depends on the probability of surviving in the market. Therefore, a bubble misallocates resources towards borrowers with low *fundamental* risk, even if they invest in projects with lower productivity.

In Chapter 3, I propose an augmented version of the model with nominal rigidities. The goal is to explain the timing of expansions and recessions during "bubbly episodes." In this version of the model, the initial boom in output is caused by a positive demand effect; the long run reduction in TFP is driven by a misallocation process. In this chapter, I also analyze the optimal policy prescriptions. In particular, I stress the importance of the central bank monopoly on the issuing of bubble-like instruments.

Finally, Chapter 4 presents an investigation of American banks' balance sheets motivated by the theory of the previous chapters. I test models of credit bubbles versus models of liquidity transformation. I provide evidence that the recent expansion in liquid debt instruments can be interpreted by the emergence of a bubble on bank's liabilities.

Lay Summary

Big financial crises typically burst in the midst of a credit boom. In traditional macroeconomic theories, this is explained by the fragility of financial markets: too much debt increases the risk of freezing in the supply of credit. According to this sources, credit booms are not bad per se; the problem is the bust. In this dissertation, I provide support to the alternative view that credit booms are inherently harmful to the economy because they misallocate resources toward lower productive sectors. I propose a theory of credit bubbles in which the emergence of the bubble induces a worse allocation of factors. From a theoretical and empirical point of view, I show that bank debt can be interpreted as a credit bubble.

Preface

This dissertation is original, unpublished, independent work by the author, Pierluca Pannella.

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Infine, ringrazio mia madre e mio padre, che non hanno mai smesso di sostenermi nelle mie scelte.

This work is dedicated to Ana Luiza.

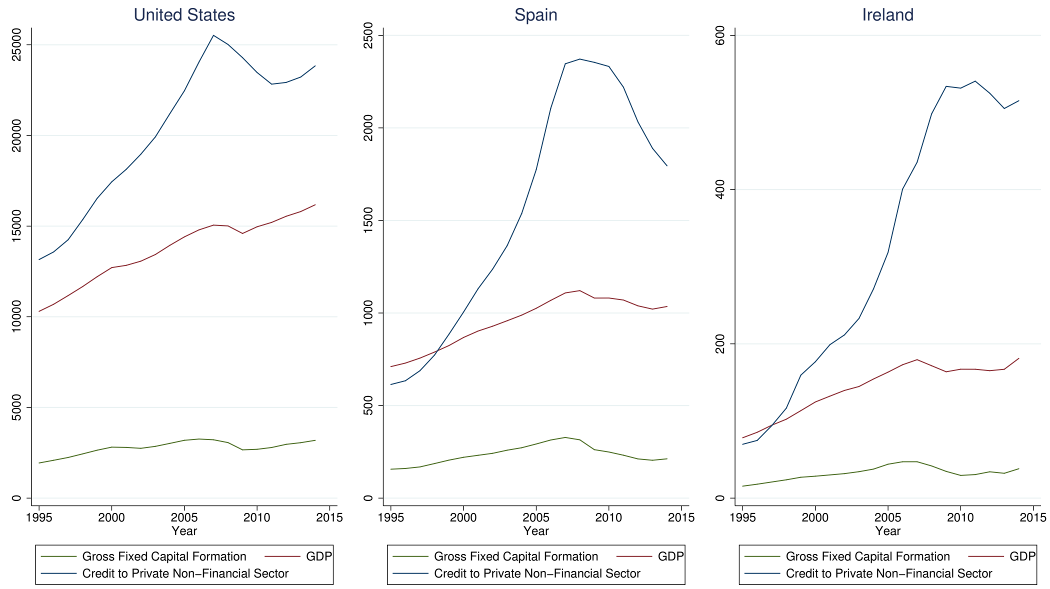
Chapter 1

Introduction

In recent times, the macroeconomy of Western countries has been characterized by unprecedented fluctuations in aggregate credit. These credit cycles have been correlated with fluctuations in overall output and capital. Nonetheless, the housing sector was at the core of these expansions and contractions: it is widely documented how credit and property prices tended to strictly co-move. Observers from many different fields have often associated these boom-and-bust cycles to the appearance of a bubble. Even though the concept of financial bubble often arises in the public debate, most macroeconomists have been reluctant to introduce bubbles in their formal models. One exception is represented by recent developments in the literature on rational bubbles. This work wants to add to this new literature, by focusing, in particular, on the role of financial bubbles in the allocation of funding.

In Figure 1.1 I report the dynamics of the total credit to the private non-financial sector, output and fixed capital formation in the United States, Spain, and Ireland, between 1995 and 2015. The three countries have famously experienced boom-and-bust cycles in the credit and housing markets in the beginning of the century and were at the origins of the 2008 Great Recession. The images confirm a well-known fact: credit is correlated with economic fundamentals. Another fact is documented in Figure 1.2: the fast rise and the sudden contraction in aggregate credit are similarly replicated by the dynamics of property prices. Finally, the graphs in Figure 1.3 show the dynamics of credit and Total Factor Productivity in the three countries. These last graphs are particularly interesting because they reveal an aspect that is not accounted by traditional models of business and credit cycles: a higher amount of credit in the economy can be associated with a reduction in the aggregate productivity. In the United

Figure 1.1: Credit and Growth

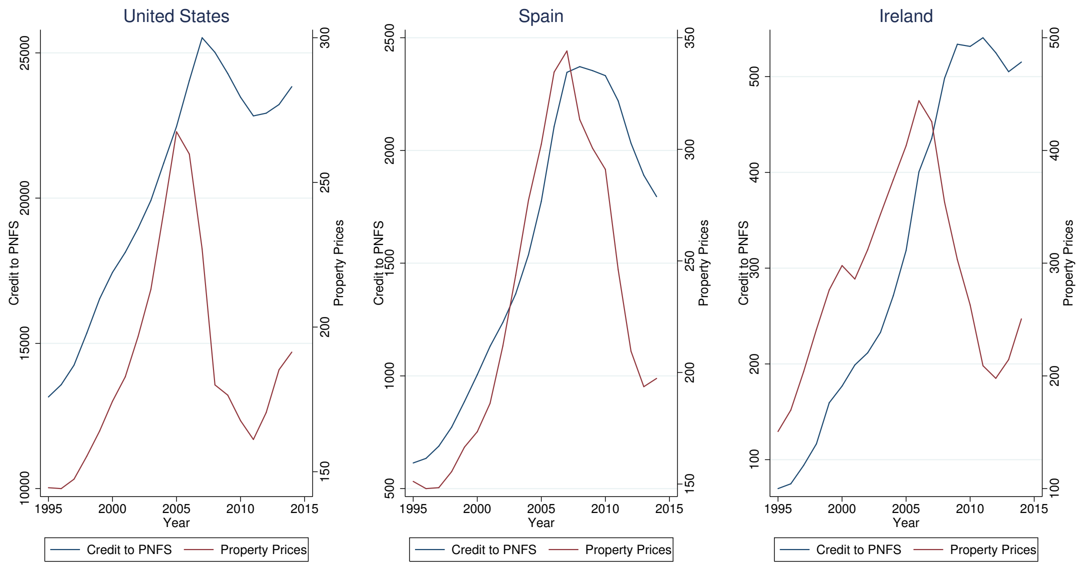


Notes: Data on GDP and fixed capital formation are from OECD.Stat. Data on aggregate credit are from the "Total credit to the non-financial sector" database by the Bank for International Settlements. The unit of measure is billions of national currency (US dollar for the United States, Euro for Spain and Ireland). All quantities are deflated by the CPI with 2010 as base year.

States, and Ireland the growth in TFP stopped respectively two and four years before the aggregate credit reached its peak. In Spain, the TFP growth was negative for the entire period of boom.

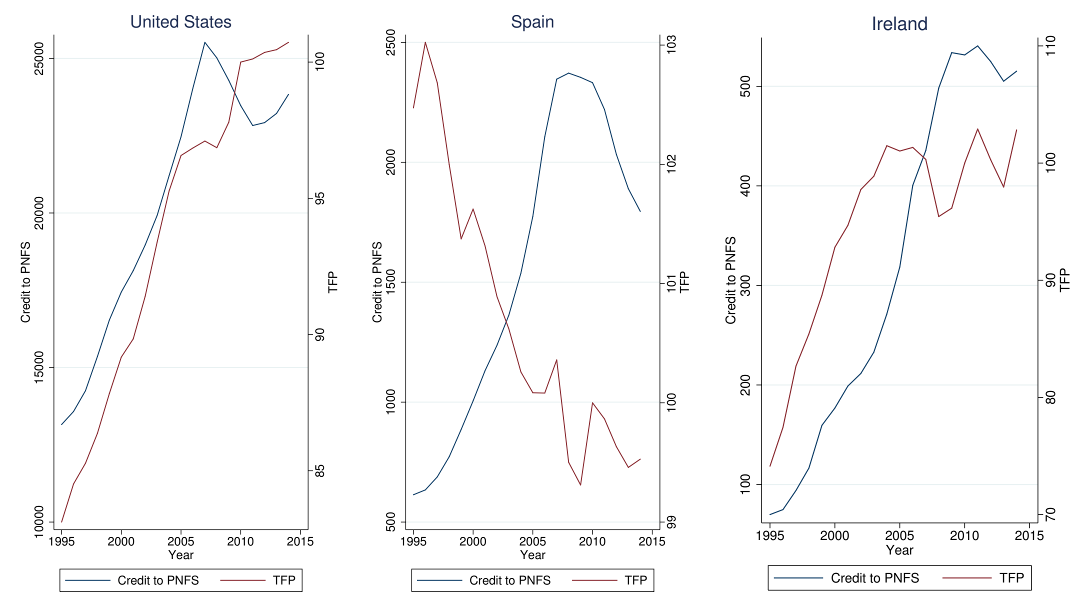
In Chapter 2 I will start by showing that the negative relation between credit and productivity can be explained by a causal effect from credit to productivity through a worsening in the allocation of factors. Specifically, I reveal that those Western countries that experienced a larger credit boom allocated this funding toward less productive industries. Many observers would interpret this fact as a proof that credit booms were associated with bubbles. Interestingly, the most prominent theory of bubbles, the rational bubble one, typically produces opposite predictions. Indeed, in both the original theory by Tirole (1985) and the recent papers by Kocherlakota (2009), Martin and Ventura (2012, 2016), and Miao and Wang (2012), bubbles play an efficient role in the economy as they improve the intertemporal or intratemporal allocation of funding. In the same chapter, then, I propose a theory of rational credit bubbles with misallocation of factors. I describe the necessary condition to have bubbles misallocating capital in the economy and simulate the dynamics for the rise and burst of a bubble in a simple model.

Figure 1.2: Credit and Housing



Notes: Data on property prices are from the long series of the BIS "Residential Property Price" database. The quantities are the original indices adjusted for inflation.

Figure 1.3: Credit and TFP



Notes: Data on TFP are the Multi-factor Productivity series from the OECD.Stat. The variable is built as a residual from GDP growth and re-expressed as an index.

In the framework presented in Chapter 2 the emergence of a bubble produces a negative effect on the aggregate productivity and output by promoting a misallocation of factors toward lower productivity sectors. While this outcome replicates the negative relation between credit and productivity, it also implies a counterfactual dynamics for the output. If the emergence of a bubble is not the source for a better allocation of factors and a higher GDP, as suggested by the recent literature, then there must be an alternative channel explaining the increase in GDP during "bubbly episodes". In Chapter 3 I extend my model by adding nominal rigidities and shocks. I will show that a nominal increase in the value of credit assets, can trigger a demand effect that ultimately boosts the entire economy. When this demand effect is eventually absorbed, real values may not return to their original levels by inducing the rise of a bubble scheme and a misallocation of factors. In the same chapter, I also analyze the optimal policy of a social planner. I show that the emergence of a bubble can be prevented by setting a cap on debt creation.

The theory of bubble I present in this work is suited to explain fluctuations in the value of debt contracts. While recent papers have usually applied the theory of rational bubbles to stock and housing prices, in my model, a bubble is instead a money-like asset, as in the original interpretation by Samuelson (1958). In Chapter 4, then, I test my theory of bubble on banks' balance sheet data. Specifically, I investigate if the mismatch between liabilities and assets is justified by the process of liquidity transformation, as described in the theories by Diamond and Dybvig (1983) or Dang, Gorton, Hölmstrom, and Ordoñez (2016), or if instead it is associated with the issuing of bubbly debt. My analysis provides support to the second hypothesis in the years after 2000.

All the three main chapters composing this work introduce novel elements in the literature. The model in Chapter 2 provides a novel formal explanation for the relation between bubbles and misallocation. The addition of nominal rigidities in Chapter 3, allows for an original interpretation of the events associated with a credit boom-and-bust cycle. Finally, the exercise proposed in Chapter 4 provides a new perspective to interpret the liquid debt instruments appearing on the liability side of banks' balance sheets.

Chapter 2

Credit Bubbles and Misallocation

2.1 Introduction

In recent decades modern economies have experienced large fluctuations in aggregate credit. Periods of high growth have typically been followed by periods of decline or sudden busts. What drives these cycles is a current subject of research and no consensus has yet been reached. A growing literature links these periods of extraordinary credit growth to the emergence of a bubble. In particular, recent papers on rational bubbles point to the role of asset bubbles in easing the transfer of funds when credit is constrained. According to these sources, bubbles boost the productive efficiency of the economy by improving the allocation of financing - the burst of the bubble initiates a recession. However, there exists an alternative view proposing that credit booms and bubbles actually induce a direct misallocation of resources in the economy.

This work contributes to the debate in two ways. First, it provides evidence that favors the misallocation view by analyzing the between-industry allocation of factors across Western countries in the years prior to the 2008 financial crisis. Second, it builds on recent theories put forward in the literature on rational bubbles to support this alternative hypothesis. I propose that it is the emergence of a bubble that reduces the output by promoting a misallocation of resources.

The original theory of rational bubbles was introduced by Tirole (1985). In Tirole's framework, a bubble, defined as an asset with a zero market fundamental, can appear when the economy is dynamically inefficient; i.e., when the marginal return on capital is smaller than the growth rate of the economy. Bubbles, then, enhance the inter-temporal allocation of resources and reduce the stock of capital. However, dynamic inefficiency was considered empirically irrelevant by most economists at the time.[1] In addition, real bubbly episodes are typically characterized by a boom in capital accumulation, a phenomenon that is counterfactual to the capital crowding-out predicted by the model. Recent papers relax the condition for the existence of rational bubbles and relate the arrival and burst of a bubble to credit dynamics. In fact, market returns can be lower than the growth rate even if the economy is dynamically efficient once we allow for imperfections in financial markets.[2] According to Kocherlakota (2009), Martin and Ventura (2012, 2016), and Miao and Wang (2012) a bubble improves the intra-temporal allocation of funds, from unproductive agents to credit-constrained productive ones.[3] Intuitively, a bubble in the asset market raises the value of collateral, relaxes the borrowing constraint, and therefore increases the amount of credit in the economy. In these models the positive reallocation of investment supports a crowding-in of capital.

These recent papers on rational bubbles can replicate aggregate macroeconomic facts, such as the rise in investment rate during a credit boom and the start of a recession at the bust. Nonetheless, I question the reallocation channel which drives their result. My theory suggests that a bubble still alleviates credit constraints and raises the stock of capital. However, this is in favor of low productivity sectors.[4]

In Section 2.2, I provide the evidence that motivates my model. I investigate the relationship between credit growth and factor allocation in the years preceding the 2008 financial crisis. Specifically, I compare the change in between-industry allocation for a sample of Western countries that experienced a differential growth in credit. The result is that larger credit booms favored the expansion of industries with low Total Factor Productivity growth. In particular, companies from less productive industries relatively increased their leverage in the countries with a higher credit growth. In the following sections I place these facts inside the rational bubble framework.[5]

[1] See Abel, Mankiw, Summers and Zeckhauser (1989) and Geerolf (2013) for an empirical investigation on dynamic inefficiency.

[2] Woodford (1990) had already shown that financial frictions could relax the conditions for rational bubbles.

[3] Kocherlakota (2009) and Miao and Wang (2012) present models with infinite lived agents facing productivity shocks. Martin and Ventura (2012, 2016) rely on an Over-Lapping Generations model with generations of productive and unproductive agents.

[4] To my knowledge, factor misallocation in a rational bubble environment has only been discussed in Miao and Wang (2014). According to them a bubble can arise in a specific sector. However, a sector-specific bubble does not produce any direct misallocation. In keeping with the rest of the literature, the bubble still increases the productive efficiency of the sector. The overall productivity of the economy is negatively affected because the specific sector produces a negative externality on the rest of the economy.

[5] There are alternative theories that link credit booms and misallocation. For example Cecchetti and Kharroubi (2015) show that an expansion of the financial sector misallocates high-skilled workers from more pro-

The theoretical contribution is presented in two steps, described in Sections 2.3 and 2.4. First, in a stylized model I derive the necessary conditions for bubbles inducing a misallocation of factors. Second, in a richer model I introduce a motivation for bubbles appearing and boosting capital accumulation in low productivity sectors.

My setup is based on the classical Over-Lapping Generations framework. In the model there are two types of agents: workers and investors. Workers earn their wage when young but have no technology to store their income for consumption when old. Investors, on the other hand, can invest today in order to obtain working capital tomorrow. A borrowing constraint limits the credit between workers and investors. However, the latter can potentially expand the funds they raise by issuing bubbly debt, a debt that will not be repaid with future income but with the purchase of this debt by a new generation of workers. It is worth noting that the emergence of bubbly debt is subject to workers' beliefs regarding future repayment.

A main feature in the model is heterogeneity in investor productivity. In Section 2.3, agents' beliefs will not only determine the rise of bubbly debt but also the identity of the issuers. Notably, the ability to issue bubbly debt does not depend on the productivity of an investor, since he will not be responsible for repayment. If workers buy bubbly debt issued by low productive investors, the outcome is a misallocation of resources away from more productive investors.

The mechanism described in Section 2.3 illustrates how a credit bubble can drag the economy into an inefficient allocation of factors. There are, however, two drawbacks to this model. First, it does not explain how the borrowers issuing bubbly debt are selected. Second, it predicts a reduction in aggregate capital when factors are misallocated. This prediction is counterfactual to the large accumulation of capital that preceded the 2008 financial crisis.

In Section 2.4, I address both issues by making a substantial addition to my model. I assume that the possibility of sustaining a bubbly scheme is subject to the survival of the issuer on the market: when a singular investor leaves the market, his bubbly debt must burst. In this section, bubbly debt is effectively repaid with future debt until such a time that a borrowing investor dies or fails. In the real world, long-lived investors may be intermediaries that finance traditional sectors such as housing and real estate, activities with typically low productivities that, nonetheless, have low *fundamental* risk. Assuming that low productive investors also

ductive sectors. Alternatively, Gopinath, Kalemli-Ozcan, Karabarbounis and Villegas-Sanchez (2015) describe an environment in which larger firms have an advantage in accessing credit. However, neither paper takes into account the boom-and-bust nature of credit cycles.

face a lower risk of leaving the market, they have a higher chance of issuing bubbly debt.[6] In addition, their longer life expectancy allows them to accumulate more capital over time. This implies that a bubble can boost aggregate capital even if resources are misallocated and the economy is contracting.

A crucial aspect of both versions of my model is the possibility of initiating a new bubbly scheme by issuing bubbly debt. This possibility is also included in the framework set out by Martin and Ventura (2012) where the agent who issues a bubbly asset effectively earns a rent. The authors identify two types of bubbly episodes: in contractionary episodes capital is crowded-out as in Tirole's framework; in expansionary episodes capital is crowded-in.[7] My model proposes a third type of bubbly episodes: capital is crowded-in while output is reduced.

Besides the rational bubble literature, this work is related to the wider literature on credit cycles and financial crisis. Empirical works by Borio and Drehmann (2009), Reinhart and Rogoff (2011), and Schularick and Taylor (2012) recognize that credit growth is a main predictor for financial crises. More recently, additional papers have addressed the effect of credit booms on factor allocation. Gopinath, Kalemli-Ozcan, Karabarbounis and Villegas-Sanchez (2015) illustrate how the allocation of capital in Spain deteriorated during the period of rapid inflows following the introduction of the euro in 1999; alternatively Borio, Kharroubi, Upper and Zampolli (2016) present a decomposition of labor productivity across Western economies and claim that credit booms provoke a misallocation of the labor force.[8] My theory is also related to the over-accumulation view of crises.[9] Note that, in the model described here, a recession does not originate from an over-accumulation of capital, but rather from an accumulation in the wrong sector.

Finally, the work is linked to the empirical and theoretical research on liquid debt. Indeed, our bubbly debt can be naturally interpreted as a short-term or liquid bank note. Growth in aggregate credit is associated with a near-symmetric increase in bank debt. For example, Krishnamurthy and Vissing-Jorgensen (2015) describe the relation between loans and liquid debt on the two sides of the balance sheets for the US financial sector. From a theoretical perspective, our bubbly debt has similarities to the information-insensitive bank debt described

[6] A low fundamental risk, clearly, does not imply an overall low risk. Interestingly, the framework predicts a negative relation between fundamental and non-fundamental risk.

[7] The crowd-in and crowd-out effects of bubbles is explored also in Hirano and Yanagawa (2016) in a model with infinite-lived agents. The authors analyze how the degree of financial imperfections influences the effect of bubbles on economy's growth.

[8] The first paper focus on within-industry misallocation, while the second one looks at between-industry misallocation.

[9] Friedrich Hayek was the most notable proponent of this view on recessions.

by Dang, Gorton, Hölmstrom, and Ordoñez (2016) where repayment does not depend on the borrower's productivity. However, in the model set out here there is no liquidity mismatch between the assets and the liabilities of a borrower.

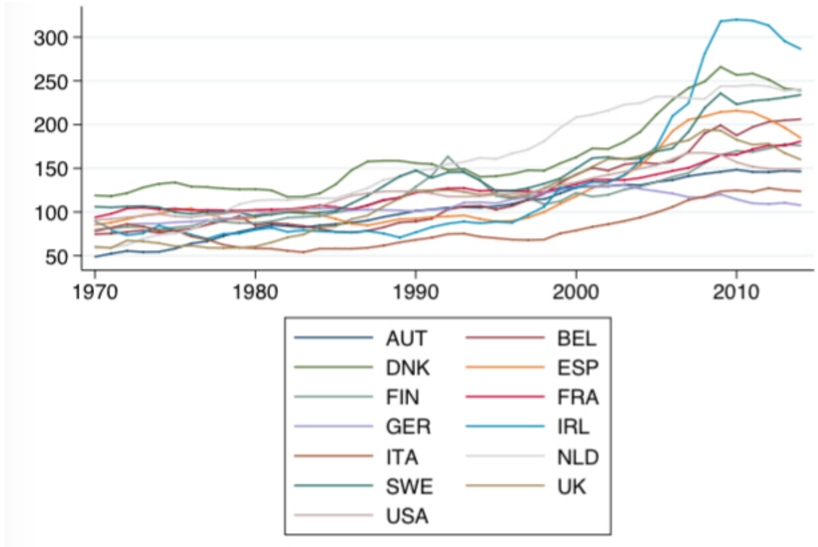
The remainder of the chapter is organized as follows: Section 2.2 presents the empirical results that inform the theory. In Section 2.3, I describe the stylized version of the model in which workers' beliefs determine who can issue bubbly debt. In Section 2.4, I add a risk component to the activity of investors which influences their survival on the market. Here low risk investors have an advantage in the issuing of bubbly debt. Section 4 also describes the dynamics of the model. Section 2.5 concludes.

2.2 Credit Booms and Between-Industry Misallocation

I motivate my theory on the basis of evidence on the allocation of factors across industries in the US and western Europe prior to the 2008 financial crisis. In Figure 2.1, I show the path of total credit to the Private Non-Financial Sector (PNFS) normalized by GDP. As we can see, from the late 1990s to 2008, the majority of sample countries experienced an unprecedented credit rise. For some countries, this boom was particularly dramatic: in Ireland the credit ratio rose from 100% at the end of the 1990s to over 300% at the peak of the cycle. In my empirical analysis I will exploit variation across countries to assess the impact of a credit boom on the allocation of factors.

In recent years, a new literature focusing on factor misallocation has emerged. A crucial question is which measure should be considered to identify misallocation. Restuccia and Rogerson (2008), and Hsieh and Klenow (2009) assess the within-industry misallocation by measuring the dispersion of marginal products. Alternatively, Bartelsman, Haltiwanger and Scarpetta (2013) adopt a measure based on the covariance between size and productivity, where a weaker link denotes a worse allocation of factors. While the approach used here is similar to the latter, the analysis follows a separate line of inquiry in at least two ways. First, I rely primarily on industry-level data to detect between-industry rather than within-industry misallocation. Looking at the reallocation of factors between different industries is more appropriate to motivate my theory; it is also better suited to support the causal claims made by the empirical model set out here. Studies that measure misallocation typically deal with firm-level data and avoid between-industry considerations for comparability issues. However, the goal here is not to obtain an absolute measure of misallocation by doing an accounting of aggregate productivity, but rather to compare the allocation pathway across countries exhibiting differ-

Figure 2.1: Total Credit to the Private Non-Financial Sector (% of GDP)



Notes: Data are from the "Total credit to the non-financial sector" database by the Bank for International Settlements.

ent credit growth. The problems related to the lack of comparability of different industries are attenuated by the second point of departure from the literature: this analysis is based on growth rates rather than levels. Then, instead of looking at the correlation between size and productivity, I examine the correlation between input/output growth and productivity growth across industries and countries with a different credit growth prior to 2008.[10] Specifically, the model I will estimate is:

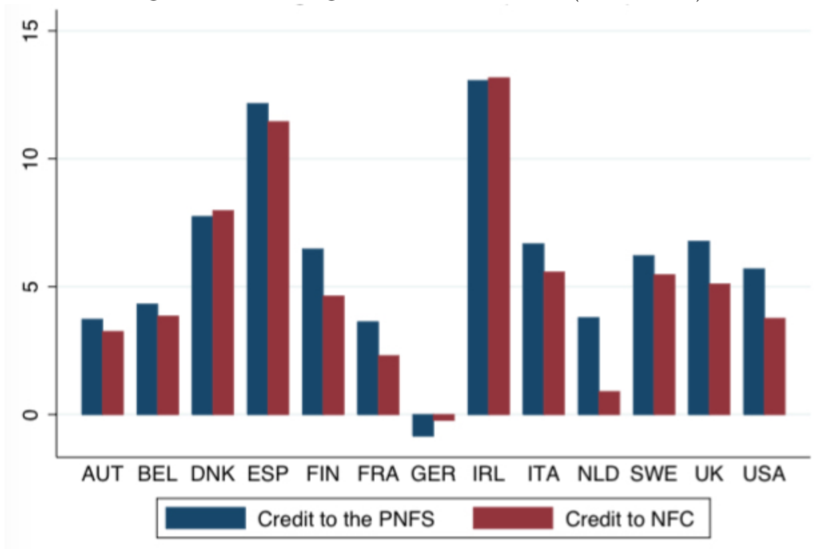
$$Y\_growth_{k,j} = \alpha_k \left(industry_k\right) + \beta_j \left(country_j\right)$$

$$+\delta \left(TFP\_growth_{k,j}\right) + \gamma \left(TFP\_growth_{k,j} \times credit\_growth_j\right) + controls_{k,j} + \varepsilon_{k,j}.$$

The dependent variables will include measures of growth in value added, capital, and labor for industry k in country j. $industry_k$ and $country_j$ are dummy variables respectively for industries and countries. The measure of productivity I will use is the Total Factor Productivity of each industry k in country j, $TFP\_growth_{k,j}$. Finally, $credit\_growth_j$ is the growth in aggregate credit in country j . While δ tells us about the overall correlation between productivity growth and input/output growth, γ tells us how this relation changes with credit

[10] Borio, Kharroubi, Upper and Zampolli (2016) provide the closest comparison to our study. The authors also look at the variation of between-industry allocation in relation to credit growth. However, they only focus on labor productivity and follow the same decomposition used by Bartelsman, Haltiwanger and Scarpetta (2013), originally introduced by Olley and Pakes (1996).

Figure 2.2: Average Growth in Credit (2001-2007)



Notes: Data are from the "Total credit to the non-financial sector" databases by the Bank for International Settlements.

growth. A positive γ would tell us that in those countries experiencing a larger credit boom, the effect of TFP growth on industry growth is higher. Conversely, a negative γ would work in the opposite direction: credit booms would be associated with a weaker relation between the productivity and the performance of an industry.

The measures of aggregate credit I use are from the BIS Statistics and include Credit to the Private Non-Financial Sector and Credit to Non-Financial Corporations (NFC).[11] All quantities are deflated by the CPI. To build the growth rate variables, I first took the year-by-year log-variation and multiplied by 100, and then computed the simple average from 2001 to 2007.[12] The results are reported in Figure 2.2.

As we can see, all countries went through a period of general credit growth with the sole exception of Germany, which reports a slight decrease in the Credit to the PNFS and to NFC during the examined period.[13] At the opposite extreme, Ireland and Spain, notably the two countries that suffered major banking crises, experienced an outstanding credit boom, as measured by both of the two quantities.

Data on industries are derived from the EU KLEMS Growth and Productivity Accounts.

[11] In this quantity the credit to households and non-profit institutions is excluded.

[12] I chose 2001 as the starting year for my analysis since it corresponds to the bottom of business cycle for most of Western countries. However, results are robust to small changes in the starting year.

[13] Notably, the credit boom similarly occurred in European countries, where bank loans are the main source of financing, as well as in the US where capital markets traditionally play a more relevant role.

The database contains industry-level measures of output, capital, employment, and TFP. Measurements and computations are based on the growth accounting methodology. Multifactor productivity growth is computed as the residual contribution to output growth, under the assumption of competitive markets, full input utilization and constant returns to scale. Indices for capital service flows, labor service flows and intermediate inputs are built as weighted sums of disaggregated components. In particular, in the labor index are weighted the hours worked by workers with different characteristics, such as educational attainment and age.[14]

Industrial classification is based on the NACE1, up to 32 industries. Since the focus here is on the allocation of factors to the Non-Financial Sector, I exclude from my sample the entire Finance sector. Measures of Capital and Value Added are in volume indices. The growth variables for my regressions are built in the same way as those for total credit.

Measures of input and output can tell us about the growth and allocation of productive factors across industries. In order to verify that the results are driven by the credit allocation channel, I integrated the data with a measure of financial leverage to be used as an additional dependent variable. Given that balance sheets data by industry are not available, I constructed a summary variable from Compustat Global and North America. For each company in the dataset I computed the average debt-to-equity ratio and its annual growth.[15] I then averaged across companies in each industry and country. Finally, I computed the average from 2001 to 2007. Note that the growth in leverage is only measured on the intensive margin without considering the entry and exit of firms in the dataset.[16]

The results for our main specification are reported in Table 2.1 and 2.2, respectively when we use the Credit to the Private Non-Financial Sector and the Credit to Non-Financial Corporations.[17] For every regression I include as a control the initial share in 2001 of the dependent variable in the total economy of the country. For the Debt-to-Equity ratio, the respective control is the initial level. I also show the results when controlling for the interaction with the initial level of credit, measured as the ratio to GDP. This is to avoid the results are driven by a convergence in levels of aggregate credit.[18] The growth in Debt-to-Equity ratio should help reveal those industries that increased their dependence on external finance. In order to avoid

[14]See O'Mahony and Timmer (2009) for a more detailed description of the dataset and the methodology.

[15]The ratio is computed as (Total Liabilities)/(Total Assets-Total Liabilities). Negative values and outlying values over 25 are dropped.

[16]This is a reasonable restriction given that the Compustat database is limited to the small sub-sample of publicly traded firms.

[17]Note that the different number of observations depends on the availability of data for the different industries in the different countries. In particular, data on capital are not available for Belgium, France and Ireland.

[18]For example high-growing credit countries may have started from lower levels of credit, which may imply a negative relation between the productivity and performance of the various industries.

the variation from a change in the value of assets, I also control for the average asset growth for the respective companies in the Compustat database.

The interaction between the TFP growth and credit growth is significantly negative in all cases except for the regressions with Capital Growth as dependent variable.[19] A likely explanation is that the sample is smaller since data on capital are not available for Belgium, France, and Ireland. In particular, all the remaining results are sensitive to the variation provided by Germany, Ireland, and Spain. When excluding one of these three countries from the sample, the negative effect is weakened or it disappears.[20]

Overall, these results play in favor of the hypothesis that credit booms are associated with a worse allocation of factors between the industries. In fact, those industries which experienced a bigger increase in productivity grew relatively less in countries which experienced a more rapid aggregate credit boomed. The effect is similar when we consider the increase in financial leverage of the Compustat companies. More productive industries showed a relative increase in their Debt-to-Equity ratio when the growth in aggregate credit was lower. This suggests that a misallocation of funds could be at the origin of the misallocation of factors.

A possible critique to the results above is that they could be driven by reverse causality: those countries having a worse allocation of factors may need a bigger increase in aggregate credit to reallocate resources between the industries. In particular, credit could be optimally allocated to low productive sectors to boost long-term development and promote convergence.[21] In order to offset the likelihood of reverse causality, I proxy the TFP growth of the industries in all countries with the TFP growth of the American industries, on the assumption that the growth in productivity of the American industries can be adopted as a measure of their technological advancement. Consistent with the chosen proxy variable, it is argued that all countries should optimally invest in those sectors showing the greatest progress. The model I estimate here is similar to the previous estimation, but the productivity measure is no longer country-specific, which means that the impact of the δ is now captured by the industry-fixed effects:

[19] At the same time, the overall effect of the TFP growth is (most of the time) significantly positive.

[20] Similarly, some industries have a bigger weight in driving the result. Construction and Real Estate are among these industries.

[21] Also note that there is an alternative hypothesis that the increase in credit to an industry reduces its productivity. This would still be in favor of the misallocation result, even though at the within-industry level.

$$Y\_growth_{k,j} = \alpha_k\,(industry_k) + \beta_j\,(country_j)$$

$$+\gamma\,(US\_TFP\_growth_k \times credit\_growth_j) + controls_{k,j} + \varepsilon_{k,j}.$$

The results are reported in Table 2.3 and 2.4, again for the Credit to the Private Non-Financial Sector and the Credit to Non-Financial Corporations. American industries are now excluded from the regressions. All the controls are similar to the previous specification. As we can see the effect of the interaction between the TFP growth in the American industries and the credit growth is significantly negative in almost all specifications.[22]

The evidence set out here contradicts the proposition of the productive efficiency role of bubbly credit advanced by recent literature on rational bubbles. However, in the following sections, I will show that the emergence of a bubble can be a natural way to admit the misallocation of factors during a credit boom.

2.3 A Model of Rational Bubbles with Capital Misallocation

In this section, I will introduce the theory supporting the main claim of the paper. The central purpose is to describe the mechanism by which a rational bubble can induce a misallocation of factors and provide the necessary conditions for the misallocation result.

I will first describe the framework and characterize the equilibrium without bubbles. Then I will introduce the possibility of bubbly credit and analyze the bubbly equilibria. Note that the setup is deterministic. I will focus only on the steady state equilibria, given that the model presents trivial dynamics. I will introduce unexpected shocks and examine the dynamics for the richer model proposed in Section 2.4.

2.3.1 The Bubble-Free Environment

The model is based on the classic Over-Lapping Generations framework set out by Diamond (1965) and Tirole (1985), with two-periods (young and old) lived agents.[23] In the framework, there are three different types of agents, each of measure one:[24] Workers, High-type investors

[22] Interestingly, in this model, the negative effect appears also when Capital Growth is the dependent variable. The estimate is not significant only in the case of Debt-to-Equity ratio when I use the credit to Non-Financial Corporations and control for the initial credit to GDP level.

[23] Note that qualitatively similar results could be obtained in an environment with infinitely-lived agents hit by uninsurable idiosyncratic shocks. Woodford (1990), for example, proposes an elegant way to reproduce Over-Lapping Generations behavior starting from infinitely-lived agents.

[24] Note that there is no population growth in the environment.

and Low-type investors. To make things more simple, I assume that all agents will only maximize their old-age consumption.

When young, workers receive a wage w.[25] While they may want to save their entire wage to consume when old, they have no technology to store it. Their only option is lending in the credit market to earn an income in the following period.

Investors, on the other hand, do not receive any wage. However, when they are born, they can install capital and rent in the following period to competitive firms owning production technologies of type H or L:[26]

$$A_j k_{j,t} \; for \; j \in \{H, L\} . \tag{2.1}$$

Capital is specific for the two types of technologies: once installed, a given type of capital cannot be intratemporally rented to a different technology. High-type and Low-type investors differ in the type of capital they can install and, then, on the technology they can access. We assume $A_H > A_L$. We will assume that capital fully depreciates in production.

Finally, young agents in this economy can meet in a competitive credit market. Specifically, young investors can get external financing by selling credit contracts. However, in keeping with the new literature on rational bubbles, a borrowing constraint limits the amount they can borrow:

$$R_{t+1} d_{j,t+1} \leq \phi MRK_{j,t+1} k_{j,t+1} \; for \; j \in \{H, L\} \tag{2.2}$$

with $\phi < 1$. On the left-hand side, R_{t+1} is the market interest rate, and $d_{j,t+1}$ is the debt issued by investor of type j. The promised repayment $R_{t+1} d_{j,t+1}$ cannot be higher than a fraction ϕ of the future capital income of the investor. Note that $MRK_{j,t+1}$ is the price of capital for the two types of production. This constraint is quite standard in the literature and can be interpreted as a limit on the pledgeable income of the borrower. In keeping with this literature, a binding borrowing constraint can push the interest rate below the growth rate of the economy and open the way for the existence of bubbles even if the economy is dynamically efficient.

Finally the budget constraint for an investor is:

$$k_{j,t+1} = d_{j,t+1} \; for \; j \in \{H, L\} . \tag{2.3}$$

[25]In Section 4 workers will earn their wage by supplying labor.

[26]In this chapter, the results of the model would not change if the investors also owned the production technologies. However, in the next chapter, I will assume that the firms are owned by the workers in order to introduce elastic labor supply and demand effects.

We can now define the equilibrium in this economy.

DEFINITION: *A competitive equilibrium is a list of consumption, debt, capital, labor, and prices such that:*

(i) Young workers maximize their old-age consumption by buying credit contracts in the value of w. Old workers consume $R_t w$

(ii) Young investors choose $k_{j,t+1}$ and $d_{j,t+1}$, given prices $(R_{t+1}, MRK_{j,t+1})$, maximizing future profits

$$MRK_{j,t+1}k_{j,t+1} - R_{t+1}d_{j,t+1} \; for \; j \in \{H, L\} \tag{2.4}$$

subject to budget constraints (2.3)*, borrowing constraints* (2.2) *and resource constraints*

$$d_{j,t+1} \geq 0.$$

Old investors consume their profits

(iii) Factors are paid at their marginal productivity:

$$MRK_{j,t} = A_j \; for \; j \in \{H, L\}\,. \tag{2.5}$$

(iv) All markets clear in every period. In particular, it must be:

$$d_{H,t+1} + d_{L,t+1} = w. \tag{2.6}$$

In this stylized economy with linear production technologies and borrowing constraints, High-type and Low-type investors can respectively offer rates ϕA_H and ϕA_L. In equilibrium, it must be $R^* = \phi A_H$ with only High-type investors obtaining funds in the credit market. Then, all capital is optimally allocated to the High-type production: $d^*_H = k^*_H = w$. Aggregate production and consumption are $Y^* = A_H w$ and $C = R^* w + (1-\phi)\,Y^* = Y^* = A_H w$.

In the next section, I will analyze how the emergence of a bubble distorts the allocation of capital in this economy. In order to introduce bubbles I will make the following assumption:

ASSUMPTION 1: $\phi < \frac{1}{A_H} \rightarrow R^* < 1$.

This is the traditional condition for the existence of bubbles: the interest rate must be lower than the growth rate of the economy. It is clear that R^* can be lower than 1, even if the economy is dynamically efficient, i.e., $A_H > 1$. In the following section, I will show how the effect of a bubble on the allocation of capital depends on the market interest rate and the returns on capital A_H and A_L.

2.3.2 Introducing Bubbly Debt

A bubble is an asset with no fundamental value, i.e., essentially a pyramid scheme. A young agent would buy a bubbly asset only with the purpose of reselling it in the following period. Usually, according to the literature on rational bubbles, the stock of bubbly assets is given and the analysis is focused on the exchange. Martin and Ventura (2012) introduced the possibility of issuing new bubbly assets or starting a new pyramid scheme. This aspect is relevant because the agent that introduces a new bubbly asset in the economy earns a windfall. As we will see, the privilege of being an issuer of bubbly assets is crucial for our misallocation result.

I will assume bubbles can be issued only by young investors.[27] A bubble can be interpreted as a credit note, apparently identical to the other credit notes secured by the future pledgeable income of the investors. The main difference is that the bubbly notes will not be repaid by borrowers, but will instead be repaid with the purchase by the future generation of workers.

Credit markets are still competitive. However, now an investor can issue two types of debt, secured and unsecured. For $j \in \{H, L\}$, these debt types are defined as:

$$d^S_{j,t+1} = \begin{cases} d_{j,t+1} & if \quad R_{t+1} d_{j,t+1} \leq \phi MRK_{j,t+1} k_{j,t+1} \\ \frac{\phi}{R_{t+1}} MRK_{j,t+1} k_{j,t+1} & if \quad R_{t+1} d_{j,t+1} > \phi MRK_{j,t+1} k_{j,t+1} \end{cases} \tag{2.7}$$

$$d^U_{j,t+1} = \begin{cases} 0 & if \quad R_{t+1} d_{j,t+1} \leq \phi MRK_{j,t+1} k_{j,t+1} \\ d_{j,t+1} - \frac{\phi}{R_{t+1}} MRK_{j,t+1} k_{j,t+1} & if \quad R_{t+1} d_{j,t+1} > \phi MRK_{j,t+1} k_{j,t+1}. \end{cases} \tag{2.8}$$

With the choice of secured funding, the investor will now face the following borrowing constraint:

$$R_{t+1} d^S_{j,t+1} \leq \phi MRK_{j,t+1} k_{j,t+1} \; for \; j \in \{H, L\}. \tag{2.9}$$

It is worthy to stress that both secured and unsecured notes must promise the same return R_{t+1} to be purchased in equilibrium.

[27] The assumption does not affect the qualitative results of my analysis. In the next section I will introduce a rationale for investors being the only possible issuers of bubbly debt.

When an investor can issue unsecured debt he earns a windfall, since he will not be responsible for its repayment. The budget constraint of an investor can now be rewritten as:

$$k_{j,t+1} + l^U_{j,t+1} = d^S_{j,t+1} + d^U_{j,t+1} \; for \; j \in \{H, L\} \tag{2.10}$$

where $l^U_{j,t+1}$ represents the purchase of unsecured notes by investors of type j. It is relevant to observe that the possibility of issuing unsecured debt depends on the beliefs of the agents in the economy. In our framework an investor cannot actively influence these beliefs. This means that he can choose $d^S_{j,t+1}$ but not $d^U_{j,t+1}$.

A bubbly scheme is sustainable if the future generations of agents have enough income to repurchase the unsecured notes issued in the market. The equilibrium interest rate is linked to the path of the unsecured debt by the following market clearing relation:

$$R_{t+1}\left(l^U_{H,t+1} + l^U_{L,t+1} + w_t - d^S_{H,t+1} - d^S_{L,t+1}\right) = w_{t+1} - \left(k_{H,t+2} + k_{L,t+2}\right). \tag{2.11}$$

The left-hand side represents the $t+1$-value of all unsecured notes issued before time $t+1$; the right-hand side represents the available income at time $t+1$ that young agents do not invest in capital.

We can define the competitive equilibrium when there is bubbly debt in the economy.

DEFINITION: *A competitive equilibrium with bubbly debt is a list of consumption, secured and unsecured debt, capital, labor, and prices such that:*

(i) Young workers maximize their old-age consumption by buying credit contracts in the value of w. Old workers consume $R_t w$.

(ii) Young investors choose $k_{j,t+1}$, $d^S_{j,t+1}$ and $l^U_{j,t+1}$, given $d^U_{j,t+1}$ and prices $(R_{t+1}, MRK_{j,t+1})$, maximizing future profits

$$MRK_{j,t+1}k_{j,t+1} - R_{t+1}\left(d^S_{j,t+1} - l^U_{j,t+1}\right) \; for \; j \in \{H, L\} \tag{2.12}$$

subject to budget constraints (2.10)*, borrowing constraints* (2.9) *and resource constraints*

$$d^S_{j,t+1} \geq -d^U_{j,t+1} \; and \; l^U_{j,t+1} \geq 0.$$

Old investors consume their profits

(iii) Factors are paid at their marginal productivity:

$$MRK_{j,t} = A_j \ for \ j \in \{H, L\} \, . \tag{2.13}$$

(iv) Agents hold consistent beliefs about the path of $d^U_{H,t+1}$ and $d^U_{L,t+1}$

(v) All markets clear in every period. In particular, it must be:

$$R_t \left(l^U_{H,t} + l^U_{L,t} + w_{t-1} - d^S_{H,t} - d^S_{L,t} \right) = w_t - (k_{H,t+1} + k_{L,t+1}) \, . \tag{2.14}$$

In this section, I will characterize the steady state equilibria with bubbly debt. An equilibrium with bubbly debt is supported by the beliefs of the agents, which in turn determine the equilibrium rate R^b. Furthermore, these beliefs also determine who can issue bubbly debt. This aspect is critical to understanding our misallocation result. The investors issuing unsecured debt earn a rent which they can use to increase their investment. Particularly, this issuing ability has nothing to do with the actual productivity of the issuer. In what follows I will assume that workers' beliefs are such that H-type and L-type investors always issue a fraction $(1 - \delta)$ and δ of the total value of new unsecured notes $d^U_H + d^U_L$.

In steady state, bubbly debt can exist only if the equilibrium rate R^b is higher than $R^* = \phi A_H$. In fact, if $R^b = R^*$, we know from the previous section that it must be $w = d^S_H$, i.e., the H-type investors would be able to secure the entire lending amount from the workers.

Investors choose to be borrowers or lenders in the credit market depending on whether their return A_j is higher or lower than the market interest rate. I will describe the equilibria with bubbly debt in the following three cases: $R^* < R^b \leq A_L$, $A_L < R^b \leq A_H$ and $A_H < R^b$.[28]

***CASE 1**: $R^* < R^b \leq A_L$*

If R^b is lower than A_L, both H-type and L-type investors want to be net borrowers in the credit market. This also implies $l^U_H = l^U_L = 0$, i.e., investors do not want to hold bubbly notes. The quantity $w - d^S_H - d^S_L$ then represents the aggregate value of bubbly debt in the economy, which is the value of newly and previously issued unsecured notes. Specifically, this quantity cannot be entirely transferred to the investors in the form of new unsecured debt - a part of it must be used to repurchase the existing unsecured debt. From market clearing condition (2.14)

[28] Note that the existence of the three intervals of equilibria is subject to $R^b \leq 1$.

we can solve for the total steady state value of new unsecured debt issued by the investors:

$$d_H^U + d_L^U = \left(1 - R^b\right)\left(w - d_H^S - d_L^S\right). \tag{2.15}$$

In this last equation, we find the traditional necessary condition for the sustainability of a bubbly equilibrium: $R^b \leq g = 1$. A higher R^b reduces the amount of new unsecured debt the investors can issue since the workers need to use a larger share of their income to buy existing credit notes. This is in keeping with the characteristic crowding-out effect of rational bubbles. In the extreme case of $R^b = 1$, there is no unsecured transfer from the workers to the investors in the steady state.

The equilibrium H-type and L-type capital are given by:

$$k_H = d_H^S + d_H^U = \frac{\phi}{R^b} A_H k_H + (1-\delta)\left(1 - R^b\right)\left[w - \frac{\phi}{R^b}\left(A_H k_H + A_L k_L\right)\right] \tag{2.16}$$

$$k_L = d_L^S + d_L^U = \frac{\phi}{R^b} A_L k_L + \delta\left(1 - R^b\right)\left[w - \frac{\phi}{R^b}\left(A_H k_H + A_L k_L\right)\right]. \tag{2.17}$$

With respect to the equilibrium without bubbles, now the Low-type investors can raise financing in the credit market as long as $\delta > 0$ and $R^b < 1$. Moreover, the Low-type investors will invest their rent in L-type capital given that the market rate R^b is lower than A_L. This, eventually, raises also the amount of secured debt issued by the Low-type investors. In the case of $R^b > A_L$, a Low-type investor who issues unsecured notes would use his rent to purchase credit contracts in the market instead.

We can solve further for k_H and $k_L$to obtain:

$$k_H = \frac{(1-\delta)\frac{1-R^b}{R^b}\frac{R^b-\phi A_L}{1-\phi A_L}}{1 - \frac{\phi A_H}{R^b} + (1-\delta)\frac{1-R^b}{R^b}\frac{\phi A_H - \phi A_L}{1-\phi A_L}} w \tag{2.18}$$

$$k_L = \frac{\delta\frac{1-R^b}{R^b}\frac{R^b-\phi A_H}{1-\phi A_H}}{1 - \frac{\phi A_L}{R^b} + \delta\frac{1-R^b}{R^b}\frac{\phi A_L - \phi A_H}{1-\phi A_H}} w. \tag{2.19}$$

In the bubble-free environment, the higher productivity was driving the allocation of financing and the installment of capital in the High-type sector. Here, secured debt still depends on A_H and A_L. However, the relative allocation of unsecured funding has no relation to the productivity of the borrower - it is completely driven by the agents' beliefs about δ. In particular, an increase in δ expands the relative allocation of capital in favor of the Low-type sector.

CASE 2: $A_L < R^b \leq A_H$

When R^b is higher than A_L but lower than A_H, only the High-type investors will be net borrowers in the credit market. Young Low-type investors will sell their unsecured notes to purchase credit contracts in the market, or they will simply keep their unsecured notes and sell them when old. In steady state, the market clearing condition (2.14) becomes:

$$d_H^U + R^b d_L^U = \left(1 - R^b\right)\left(w - d_H^S\right). \tag{2.20}$$

Given $d_L^U = \frac{\delta}{1-\delta} d_H^U$, the value of new unsecured debt issued by the High-type investors is:

$$d_H^U = \frac{1-\delta}{1-\delta\left(1-R^b\right)}\left(1 - R^b\right)\left(w - \frac{\phi}{R^b} A_H k_H\right). \tag{2.21}$$

Aggregate High-type capital is:

$$k_H = d_H^S + d_H^U = \frac{1-\delta\left(1-R^b\right) - R^b}{1-\delta\left(1-R^b\right) - \phi A_H} w. \tag{2.22}$$

In this second case, there is no capital accumulated in the Low-type sector. A higher δ reduces the amount of High-type capital as it increases the rent consumed by the Low-type investors.

CASE 3: $A_H < R^b$

When the interest rate is higher than A_H, both types of young investors want to be net lenders in the market. In this scenario it must be $d_H^S = d_L^S = k_H = k_L = 0$. From the market clearing condition (2.14), all resources are employed to purchase existing unsecured notes in every time:

$$R^b\left(d_H^U + d_L^U\right) = \left(1 - R^b\right) w. \tag{2.23}$$

We can now summarize our results:

$$k_H = \begin{cases} w & if\ R^b = R^* \\ \dfrac{(1-\delta)\frac{1-R^b}{R^b}\frac{R^b-\phi A_L}{1-\phi A_L}}{1-\frac{\phi}{R^b}A_H+(1-\delta)\frac{1-R^b}{R^b}\frac{\phi A_H-\phi A_L}{1-\phi A_L}} w & if\ R^* < R^b \leq A_L \\ \dfrac{1-\delta\left(1-R^b\right)-R^b}{1-\delta(1-R^b)-\phi A_H} w & if\ A_L < R^b \leq A_H \\ 0 & if\ R^b > A_H \end{cases}, \tag{2.24}$$

$$k_L = \begin{cases} 0 & if\ R^b = R^* \\ \frac{\delta \frac{1-R^b}{R^b} \frac{R^b - \phi A_H}{1-\phi H}}{1 - \frac{\phi}{R^b} A_L + (1-\delta) \frac{1-R^b}{R^b} \frac{\phi A_L - \phi A_H}{1-\phi A_H}} w & if\ R^* < R^b \leq A_L \\ 0 & if\ A_L < R^b \leq A_H \\ 0 & if\ R^b > A_H \end{cases} . \tag{2.25}$$

Figures 2.3, 2.5 and 2.7 plot the allocation of High-type and Low-type capital against R^b if $1 \leq A_L < A_H$, $A_L < 1 \leq A_H$ and $A_L < A_H < 1$, given $\delta = 0.5$. The figures confirm that the emergence of a bubble misallocates capital only if $R^b < A_L$. We can state the following Proposition.

PROPOSITION 1: *A necessary condition for bubbles inducing a misallocation of factors is $R^b < A_L$.*

We also want to examine how the bubble affects the aggregate accumulation of capital, the output and the welfare of the economy.

PROPOSITION 2: *The emergence of a bubble always reduces aggregate output and capital. The effect on aggregate consumption can be positive only if $A_H < 1$.*

The proof of Proposition 2 is in Appendix 1. Figures 2.4, 2.6 and 2.8 plot the steady state values of total output and consumption against R^b, given $\delta = 0.5$. In this model, a bubble is always contractionary. This result does not only derive from the typical crowding-out effect. The model adds an additional contractionary effect associated with the misallocation of factors. However, a bubble may still increase aggregate consumption, but only when the economy is dynamically inefficient.[29]

Finally, we can analyze how the identity of the investor that issues bubbly debt influences the aggregate economy.

PROPOSITION 3: *An increase in δ always reduces aggregate output and capital. The effect on aggregate consumption can be positive only if $A_H < 1$.*

Proposition 3 is proved in Appendix 2. Intuitively, when Low productivity investors issue a larger share of unsecured notes, factors are misallocated and output is lower. In addition, since Low-type investors earn lower returns, a larger share of the workers' future endowment must be allocated to the repayment of bubbly debt. This, eventually, reduces the total stock of capital.

[29] This is in line with the original theory by Tirole.

To conclude, the model described here adds a new dimension to existing theories of rational bubbles. Bubbles do not only affect the aggregate accumulation of capital, they also have a re-allocation effect. Productive factors can be crowded out from specific sectors to be re-allocated to others. For a given interest rate, the effect of a bubble depends on this re-allocation of factors. In particular, the cost of a bubbly episode may be higher if it involves a large misallocation of capital towards low productive sectors.

This model, however, still does not tell us which investors would have an advantage in the issuing of bubbly debt. In addition, the contraction in output is always associated with a reduction in the stock of capital. In the next section, I will introduce some risk in the activity of the investors - which will affect their life expectancy on the market and, thereafter, their ability to maintain a bubbly scheme and accumulate capital over time. I will show that a bubble can boost aggregate capital accumulation even if that induces a misallocation of factors and a decrease in total production.

2.4 Credit Bubbles and Misallocation in a Model with Risky Investments

In this section I will extend the previous model by introducing a mechanism which predicts the misallocation equilibrium in a unique way. Importantly, the same mechanism will also open the doors for capital accumulation even if the bubble is contractionary. Here, investors will live for more than two periods but they will face some risk in their investment activity which will affect their life expectancy on the market and, thereafter, their ability to maintain a bubbly scheme and accumulate capital over time. In addition, workers will now supply labor and make an intertemporal consumption choice when young.[30] All agents are assumed to be risk neutral.

I will describe the problem faced by workers and investors in the following subsection. Note that, for simplicity, agents behave as if bubbles were deterministic. In looking at the dynamics, I will assume that the shocks to the system are unexpected.

2.4.1 OLG Workers

Workers live for two periods as in the previous version of the model. However, they now choose their total labor supply when young and their consumption in both young and old

[30] This aspect will be relevant when introducing nominal rigidities in the second chapter.

periods, by maximizing the following utility:

$$\log\left(c_{Y,t}\right) - \varphi h_t + \log\left(c_{O,t+1}\right) \tag{2.26}$$

subject to $c_{Y,t} = w_t h_t - l_{t+1}$ and $c_{O,t+1} = R_{t+1} l_{t+1}$, where l_{t+1} denotes lending in the credit market. For simplicity I assume that the disutility from working is linear. The solution to the problem gives the aggregate supply of labor and lending:

$$h_t = \frac{2}{\varphi} \tag{2.27}$$

$$l_{t+1} = \frac{1}{2} w_t h_t = \frac{w_t}{\varphi}. \tag{2.28}$$

2.4.2 Risky Investments

Investors are still grouped into two categories of mass one - High-type and Low-type - but they now live for more than two periods. Specifically, each investor has an i.i.d. probability σ of surviving in each period t. Then, in each period a mass $(1-\sigma)$ of old investors leave the market and the same number of new investors enter the market with endowment e.[31] Similarly to the previous section, the investors want to maximize their consumption in their last period of life.[32] Then, in all the previous periods, they will always reinvest and continue to accumulate capital.

Again, the investors have a storing technology that will allow the installation of a specific kind of capital to rent in the following period to High-type or Low-type production. Unlike the activities described in the previous section, here the storing activity is risky. In particular, with respective probabilities $(1-\varepsilon_H)$ and $(1-\varepsilon_L)$, the storing can fail and the investor can end up with no capital in the following period. I make the assumption that these shocks are idiosyncratic and not insurable.

Production functions are now Cobb-Douglas combining capital and labor:

$$A_j k_{j,t}^{\alpha} h_{j,t}^{1-\alpha} \; for \; j \in \{H, L\}. \tag{2.29}$$

[31] Borrowing banks are modeled in a similar fashion in the model of bank runs described by Gertler and Kiyotaki (2015).

[32] Note that the same decision would derive if investors maximized a linear utility over consumption in different periods, $\sum_{t=0}^{\infty} c_{m,t}$, and the return from borrowing and investing was always higher than 1.

I make the following assumptions:

ASSUMPTION 2: $\varepsilon_H < \varepsilon_L$.

ASSUMPTION 3: $\varepsilon_H^\alpha A_H > \varepsilon_L^\alpha A_L$.

Assumption 2 states that the probability of failing is higher for an H-type investor. Nonetheless, Assumption 3 confirms that the overall H-type productivity is still higher. These premises describe an environment in which higher productivity sectors are also riskier. Conversely, low productive sectors offer more stability over time. Then, the two types of investment offer a different combination in the risk-return spectrum.

An investor m, of type H or L, raises external funding in the credit market and faces a similar borrowing constraint:

$$R_{t+1}d_{m,t+1}^S \leq \phi MRK_{j,t+1}\varepsilon_j i_{m,t+1} \; for \; j \in \{H, L\}\,, \tag{2.30}$$

where $d_{m,t+1}^S$ and $i_{m,t+1}$ are secured debt and investment. That is to say, an investor of type j can secure his borrowing up to a fraction ϕ of his expected capital income. The investors can also expand their borrowing by issuing bubbly debt. At this point, a further restriction is imposed:

ASSUMPTION 4: *A debt contract can be exchanged as long as the issuer has positive equity.*

Assumption 4 comes with an important implication: when an investor fails or dies, all the bubbly notes that he has issued will burst.[33] This is a more accurate description of what happens in the real world where tradable securities fail automatically with their issuers' failure, or where financial institutions issue short-term notes which are rolled over under the same roof. Assumption 4 introduces a gap in the expected duration of H-type and L-type activities. It is worth pointing out that, given that H-type investors experience a shorter life expectancy on the market, they have a lower probability of rolling over a bubbly scheme.

2.4.3 Equilibrium and Steady State Solutions

The equilibrium in the economy is now defined as follows:

DEFINITION: *A competitive equilibrium is a list of consumption, lending, secured and unsecured debt, capital, labor, and prices such that:*

[33]Note that an investor whose storage activity fails will also end up with zero consumption.

(i) Young workers maximize their utility (2.26) *by choosing* h_t *and* l_{t+1}. *Old workers consume* $R_t l_t$

(ii) An investor m *of type* j *who is still active in the market in period* t *chooses* $i_{m,t+1}$, $l^U_{m,t+1}$ *and* $d^S_{m,t+1}$, *given* $d^U_{m,t+1}$ *and prices* $(R_{t+1}, MRK_{j,t+1})$, *maximizing profits in the last period of his life*

$$\sum_{q=1}^{\infty} (1-\sigma)\,\sigma^{q-1}\varepsilon_j \left[MRK_{j,t+q} i_{m,t+q} - R_{t+q}\left(d^S_{m,t+q} - l^U_{m,t+1}\right)\right] \; for \; j \in \{H, L\} \tag{2.31}$$

subject to budget constraint

$$c_{m,t} + i_{m,t+1} + l^U_{m,t+1} = MRK_{j,t} i_{m,t} - R_t d^S_{m,t} + d^S_{m,t+1} + d^U_{m,t+1}, \tag{2.32}$$

borrowing constraint (2.30) *and resource constraints*

$$d^S_{m,t+1} \geq -\left(MRK_{j,t} i_{m,t} - R_t d^S_{m,t} + d^U_{m,t+1}\right) \; and \; l^U_{m,t+1} \geq 0.$$

An investor who dies in period t, *consumes his final income* $c_{m,t} = MRK_{j,t} i_{m,t} - R_t\left(d^S_{m,t} - l^U_{m,t}\right)$, *while an investor who fails leaves the market with no final consumption*

(iii) Factors are paid at their marginal productivity:

$$w_t = (1-\alpha)\,A_H \left(\frac{k_{H,t}}{h_{H,t}}\right)^{\alpha} = (1-\alpha)\,A_L \left(\frac{k_{L,t}}{h_{L,t}}\right)^{\alpha} \tag{2.33}$$

$$MRK_{j,t} = \alpha A_j \left(\frac{h_{j,t}}{k_{j,t}}\right)^{1-\alpha} \; for \; j \in \{H, L\} \tag{2.34}$$

with $k_{j,t} = \varepsilon_j \int_{m \in j} i_{m,t} \; for \; j \in \{H, L\}$

(iv) Agents hold consistent beliefs about the path of $d^U_{j,t+1}$ $for \; j \in \{H, L\}$

(v) All markets clear in every period.

As described in the previous section, the necessary condition to have bubbles misallocating resources is that both borrowing constraints are binding. Therefore, I make the following assumption.

ASSUMPTION 5: $R_{t+1} < \varepsilon_L MRK_{L,t+1} \; \forall \; t.$ [34]

[34] The condition is on variables endogenously determined in the model. Therefore, it implicitly sets restrictions on parameters so that all the equilibria we will characterize (with or without bubbles) respect the inequality.

Note that Assumption 5 implies $R_{t+1} < \varepsilon_H MRK_{H,t+1}$ a fortiori. All investors will try to borrow until their constraints bind and only the workers will lend in the credit market.

I begin by characterizing the steady state equilibria of the economy. Without bubbles in the economy, the steady state interest rate is:[35]

$$R^* = 2\phi \frac{\alpha}{1-\alpha}. \tag{2.36}$$

The previous section described bubbly debt equilibria as possible if R^b was lower than the growth rate of the economy. This was possible because a debt security could also be exchanged after the death of the issuer. Here a bubbly scheme will burst if the issuer dies or fails, which means that in a steady state with bubbles, High-type and Low-type investors cannot promise a return higher than $\sigma\varepsilon_H$ and $\sigma\varepsilon_L$.[36] From now on, I will make the following assumption:

ASSUMPTION 6: $\sigma\varepsilon_H \leq R^* < \sigma\varepsilon_L$.

Assumption 6 implies that only L-type investors can run a bubbly scheme in steady state. By backward induction, only L-type investors can credibly initiate a bubbly scheme because their survival rate in the market is higher given a lower probability of failure. A rational bubbly scheme relies on the expectation that the agents will continue to buy in the long run. Borrowers with riskier projects have a lower probability of survival and cannot sustain a long term pyramid scheme. In this course of event, a bubble will necessarily prompt the misallocation of resources from higher to lower productive borrowers. The interest rate R^b in this bubbly equilibria will be such that $R^* \leq R^b \leq \sigma\varepsilon_L < 1$.

The dynamics of aggregate capital can now be set out in both sectors:

$$k_{H,t+1} = \varepsilon_H \left\{ (1-\sigma)\, e + \sigma\,(1-\phi)\, \frac{\alpha}{1-\alpha} w_t h_{H,t} + \frac{\phi}{R_{t+1}} \frac{\alpha}{1-\alpha} w_{t+1} h_{H,t+1} \right\} \tag{2.37}$$

$$\begin{aligned} k_{L,t+1} = \varepsilon_L &\left\{ (1-\sigma)\, e + \sigma\,(1-\phi)\, \frac{\alpha}{1-\alpha} w_t h_{L,t} + \frac{\phi}{R_{t+1}} \frac{\alpha}{1-\alpha} w_{t+1} h_{L,t+1} \right\} \\ &+ \varepsilon_L \left\{ l_{t+1} - \frac{\phi}{R_{t+1}} \frac{\alpha}{1-\alpha} w_{t+1} h_{t+1} - R_t \left(l_t - \frac{\phi}{R_t} \frac{\alpha}{1-\alpha} w_t h_t \right) \right\}. \end{aligned} \tag{2.38}$$

[35] We can solve by plugging (33) in

$$R^* \frac{w}{\varphi} = \phi\alpha \left(A_H k_H^{\alpha} h_H^{1-\alpha} + A_L k_L^{\alpha} h_L^{1-\alpha} \right). \tag{2.35}$$

[36] Note that the implicit assumption is that unsecured funds are randomly allocated inside the mass of H-type and L-type investors who are in the market at time t.

The curly braces refer to the H-type and L-type aggregate investments in time t. Newly-arrived investors of both types invest their endowment e. Pre-existing investors who remain in the market in period t, on aggregate reinvest their income: $(1-\phi) MRK_{j,t}k_{j,t} = (1-\phi) \frac{\alpha}{1-\alpha} w_t h_{j,t}$, $for\ j \in \{H, L\}$. All investors in the market will also invest all external funding they are able to raise in the credit market: $\frac{\phi}{R_{t+1}} MRK_{j,t+1}k_{j,t+1} = \frac{\phi}{R_{t+1}} \frac{\alpha}{1-\alpha} w_{t+1}h_{j,t+1}$, $for\ j \in \{H, L\}$. In addition, L-type investors can invest the rent they obtain from issuing unsecured debts. In the last line we can see that the rent is given by the portion of current unsecured debt that is not allocated to the repayment of past unsecured debt. In an equilibrium with no bubbles, the rent is equal to 0. Finally, both types of aggregate investments are fractioned by the respective storage survival rate.

In steady state the two equations can be simplified:

$$k_H = \varepsilon_H \left\{ (1-\sigma) e + \left[\sigma (1-\phi) + \frac{\phi}{R^b} \right] \frac{\alpha}{1-\alpha} w h_H \right\} \tag{2.39}$$

$$k_L = \varepsilon_L \left\{ (1-\sigma) e + \left[\sigma (1-\phi) + \frac{\phi}{R^b} \right] \frac{\alpha}{1-\alpha} w h_L + \left(1-R^b\right) \frac{R^b - R^*}{R^b} \frac{w}{\varphi} \right\}. \tag{2.40}$$

Substituting into (2.33), the steady state labor allocation is finally obtained as a function of w:

$$h_H = \frac{(1-\sigma) e}{\left(\frac{w}{(1-\alpha)\varepsilon_H^\alpha A_H} \right)^{\frac{1}{\alpha}} - \left[\sigma (1-\phi) + \frac{\phi}{R^b} \right] \frac{\alpha}{1-\alpha} w} \tag{2.41}$$

$$h_L = \frac{(1-\sigma) e + \left(1-R^b\right) \frac{R^b - R^*}{R^b} \frac{w}{\varphi}}{\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L} \right)^{\frac{1}{\alpha}} - \left[\sigma (1-\phi) + \frac{\phi}{R^b} \right] \frac{\alpha}{1-\alpha} w}. \tag{2.42}$$

It is easy to see that in the bubble-free equilibrium, i.e. when $R^b = R^*$, the allocation of capital and labor is driven by the aggregate productivities $\varepsilon_H^\alpha A_H$ and $\varepsilon_L^\alpha A_L$. Since the latter is smaller, High-type investors receive more capital and labor. The rise of a bubble misallocates factors in favor of the Low-type investors.

The following Proposition can now be stated.

PROPOSITION 4: *A bubble always reduces total output.*

A formal proof is provided in the Appendix. Intuitively, it would seem that a bigger R^b increases the amount of unsecured debt in the economy, which would raise both the misallocation and the crowding-out of capital. As expected, bubbles in this section are always contractionary. Nevertheless, this does not necessarily imply a reduction in the aggregate stock of capital as it did in the previous section.

PROPOSITION 5: *There exist steady state equilibria with bubbles in which aggregate capital increases.*

The proposition is proved in Appendix 4. The bubbly episodes preceding a financial crisis are typically characterized by a fast accumulation in capital. In particular, in the years prior to 2008 we saw a boom in housing and mortgage loans. The original theory of rational bubbles could not explain this phenomenon. In Tirole's framework, a bubble would reduce capital when the economy is dynamically inefficient. The addition of credit constraints in the new literature on rational bubbles, has introduced a new class of bubbly equilibria: by improving the intratemporal allocation of funding, bubbles can boost output and capital. In this section I introduced a further type of bubbly episode. This bubble reduces output and increases capital by misallocating resources towards low productive sectors which, nonetheless, have a higher propensity to accumulate. Our result is driven by the assumption that low productive sectors have a lower fundamental risk. Interestingly, the model predicts the emergence of non-fundamental risk in sectors that are fundamentally more stable.

2.4.4 The Dynamics of the Model

This section will set out the simulated dynamics of the model when the system is hit by unexpected shocks to the interest rate R_t. A summary of the three experiments proposed in this section is presented in Table 2.5.

I start by analyzing the transition dynamics between the bubble-free steady state, characterized by R^*, and the bubbly steady state with $R^b = \sigma\varepsilon_L$. The model is solved numerically. The share of capital is in line with data from developed countries: $\alpha = 0.35$. The selection of the remaining parameters respects the assumptions set out in the previous section. Specifically, I set $A_H = 1.9$, $A_L = 1.1$, $\varepsilon_H = 0.13$, $\varepsilon_L = 0.6$, $\sigma = 0.75$, $\phi = 0.4$, $e = 0.001$, $\varphi = 1$. ϕ is low enough so that Assumption 5 is respected. Similarly, the choice for σ, ε_H and ε_L is made to meet Assumption 6. In particular, to confirm Proposition 5, ε_H is set sufficiently small relatively to ε_L that a reallocation of funding towards L-type investors would boost capital accumulation. In this simulation the economy starts from a bubble-free steady state: in period 11 the interest rate rises from R^* to $R^b = \sigma\varepsilon_L$; in period 71 the bubble bursts, the return drops and converges to R^*.

Figure 2.9 and 2.10 report the path for the allocation of capital and labor for the first experiment. While the reallocation in the labor market is symmetrical, given a fixed total labor supply, we can see how the increase in the amount of L-type capital overtakes the reduction in H-type capital when the bubble appears. This can also be observed in the path for aggregate capital presented in Figure 2.11. However, the rise in the aggregate stock of capital is not reflected in a long run expansion in output. Total production gradually decreases at the emergence of the bubble and only returns to its initial steady state level when the bubble bursts (Figure 2.12).

In the two remaining exercises, I analyze the system in the case in which Assumption 6 is not respected and both types of investors can potentially issue unsecured notes. With respect to before, I assume $\varepsilon_H = \varepsilon_L = 0.13$ and $\phi = 1 \times 10^{-100}$ and I study the transition between the bubble-free steady state and a bubbly steady state characterized by $R^b = R^* + 1 \times 10^{-5}$. Note that R^* and R^b are now set extremely small in order to minimize the crowding-out effect of bubbles and reproduce the positive growth result proposed by the recent literature on rational bubbles. In Figure 2.13, I report the path for H-type capital, L-type capital and aggregate output when assuming that only H-type investors issue unsecured notes. As we can see, when the bubble emerges, the high productivity sector increases its capital while the low productivity one reduces it. In particular, the crowding-out in investment does not offset this positive reallocation of capital and the aggregate output increases. Finally, in Figure 2.14, I replicate the same exercise in the case in which only the L-type investors issue unsecured notes. As in the initial simulation, the bubble induces a recession given that the negative reallocation of capital is summed to the crowd-out effect. However, aggregate capital can not increase given that the two sectors now face the same risk.

An apparent drawback of this model is the timing of the expansion and the recession. Bubbly times are generally expansionary, at least in the short run; recessions typically start at the burst of the bubble. In the second chapter of the dissertation, I will add nominal rigidities to our environment and show how a rise and drop in nominal returns can induce both a short-run demand effect and a long-run reallocation of factors.

2.5 Conclusions

Financial crises are typically preceded by a credit boom. According to a widespread view, the cost of a crisis originates in the sudden freezing of the credit markets. Recent contributions to the literature on rational bubbles associate these fluctuations in credit to bubbly episodes. In these papers, bubbles expand output and capital by improving the allocation of funding when productive agents are financially constrained. The burst of the bubble would then lead to a recession.

The evidence, however, shows that a rapid growth in credit promotes a misallocation of resources towards low productive industries. For example, housing and real estate sectors are the usual recipients of an increased share of capital in a credit boom. This chapter shows how this phenomenon can be explained in the rational bubble framework. Here, investors with different productivities can borrow by pledging their future income as collateral or by pledging the repurchase of debt by future lenders. The key intuition for the misallocation result is that borrowing through unsecured debt does not require high productivity. Instead, a credit bubble favors those borrowers who have a low probability of exiting the market in the future and can maintain a long-lived scheme.

An important result of the theory is that bubbles can promote capital accumulation even if they are contractionary. Funding would be reallocated towards lower productive sectors which have a higher propensity for accumulation. This explains both the investment misallocation and the growth in capital stock that can be observed during a credit boom.

Table 2.1: The Effect of the Growth of Credit to the PNFS on Factors' Allocation when Industry's TFP Growth is Country Specific

	Value Added Growth		Capital Growth		Employment Growth		Hours worked growth		Debt-to-Equity Ratio Growth	
	(1)	**(2)**	**(1)**	**(2)**	**(1)**	**(2)**	**(1)**	**(2)**	**(1)**	**(2)**
TFP Growth	0.794***	2.312***	0.004	-0.187	0.076**	0.039	0.046	0.170	0.538***	1.175**
	(0.051)	(0.119)	(0.044)	(0.146)	(0.032)	(0.102)	(0.032)	(0.133)	(0.137)	(0.567)
Interaction (TFP Growth X Credit Growth PNFS)	-0.072***	-0.051***	-0.003	-0.008	-0.014***	-0.014***	-0.015***	-0.014***	-0.074***	-0.073***
	(0.007)	(0.006)	(0.006)	(0.007)	(0.004)	(0.004)	(0.004)	(0.004)	(0.021)	(0.021)
Interaction (TFP Growth X Credit PNFS to GDP in 2001)		-0.010***		0.001		0.000		-0.001		-0.005
		(0.001)		(0.001)		(0.001)		(0.001)		(0.004)
Industry's share of total Value Added in 2001	0.132	0.260*								
	(0.178)	(0.143)								
Industry's share of total Capital in 2001			-0.043*	-0.044*						
			(0.026)	(0.026)						
Industry's share of total Employment in 2001					-0.026	-0.026				
					(0.055)	(0.055)				
Industry's share of total Hours Worked in 2001							-0.026	-0.027		
							(0.060)	(0.060)		
Debt-to-Equity Ratio in 2001									-0.325	-0.258
									(0.598)	(0.601)
Average Asset Growth of Compustat Companies									-0.109***	-0.110***
									(0.032)	(0.032)
Number of observations	384	384	284	284	379	379	378	378	312	312
R2	0.639	0.766	0.629	0.631	0.765	0.765	0.749	0.750	0.272	0.275

Notes: Data on credit are from the "Total credit to the non-financial sector" database by the Bank for International Settlements. Data on industry growth and productivity are from the "EU KLEMS" database by the Groningen Growth and Development Center. Debt-to-Equity ratios are average across companies computed from Compustat Global and North America.

Table 2.2: The Effect of the Growth of Credit to NFC on Factors' Allocation when Industry's TFP Growth is Country Specific

	Value Added Growth		Capital Growth		Employment Growth		Hours worked growth		Debt-to-Equity Ratio Growth	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
TFP Growth	0.819***	1.610***	-0.030	0.045	0.059*	0.057	0.034	0.110	0.534***	0.508*
	(0.047)	(0.088)	(0.044)	(0.117)	(0.031)	(0.064)	(0.031)	(0.070)	(0.135)	(0.282)
Interaction (TFP Growth X Credit Growth NF Corp.)	-0.078***	-0.100***	0.002	0.004	-0.011***	-0.011***	-0.013***	-0.017***	-0.081***	-0.080***
	(0.006)	(0.006)	(0.006)	(0.006)	(0.004)	(0.004)	(0.004)	(0.005)	(0.022)	(0.024)
Interaction (TFP Growth X Credit NF Corp. to GDP in 2001)		-0.008***		-0.001		0.000		-0.001		0.000
		(0.001)		(0.001)		(0.001)		(0.001)		(0.003)
Industry's share of total Value Added in 2001	0.077	0.415***								
	(0.170)	(0.153)								
Industry's share of total Capital in 2001			-0.043*	-0.043						
			(0.026)	(0.026)						
Industry's share of total Employment in 2001					-0.025	-0.025				
					(0.056)	(0.056)				
Industry's share of total Hours Worked in 2001							-0.025	-0.025		
							(0.060)	(0.060)		
Debt-to-Equity Ratio in 2001									-0.347	-0.350
									(0.597)	(0.598)
Average Asset Growth of Compustat Companies									-0.108***	-0.108***
									(0.032)	(0.032)
Number of observations	384	384	284	284	379	379	378	378	312	312
R2	0.668	0.745	0.628	0.629	0.763	0.763	0.748	0.749	0.273	0.273

Notes: Data on credit are from the "Total credit to the non-financial sector" database by the Bank for International Settlements. Data on industry growth and productivity are from the "EU KLEMS" database by the Groningen Growth and Development Center. Debt-to-Equity ratios are average across companies computed from Compustat Global and North America.

Table 2.3: The Effect of the Growth of Credit to the PNFS on Factors' Allocation when the US Industry's TFP Growth is used as Proxy

	Value Added Growth		Capital Growth		Employment Growth		Hours worked growth		Debt-to-Equity Ratio Growth	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
Interaction (TFP Growth in the US X Credit Growth PNFS)	-0.033**	-0.037***	-0.026***	-0.027***	-0.032***	-0.034***	-0.030***	-0.032***	-0.056**	-0.055**
	(0.013)	(0.013)	(0.009)	(0.009)	(0.006)	(0.006)	(0.006)	(0.006)	(0.026)	(0.027)
Interaction (TFP Growth in the US X Credit PNFS to GDP in 2001)		-0.004***		-0.001		-0.002***		-0.002***		0.000
		(0.002)		(0.001)		(0.001)		(0.001)		(0.003)
Industry's share of total Value Added in 2001	0.190	0.225								
	(0.254)	(0.252)								
Industry's share of total Capital in 2001			-0.046*	-0.047*						
			(0.026)	(0.026)						
Industry's share of total Employment in 2001					-0.062	-0.074				
					(0.057)	(0.056)				
Industry's share of total Hours Worked in 2001							-0.053	-0.065		
							(0.063)	(0.062)		
Debt-to-Equity Ratio in 2001									-0.450	-0.449
									(0.665)	(0.666)
Average Asset Growth of Compustat Companies									-0.114***	-0.115***
									(0.035)	(0.035)
Number of observations	348	348	255	255	346	346	345	345	284	284
R2	0.331	0.345	0.640	0.641	0.768	0.776	0.749	0.756	0.244	0.244

Notes: Data on credit are from the "Total credit to the non-financial sector" database by the Bank for International Settlements. Data on industry growth and productivity are from the "EU KLEMS" database by the Groningen Growth and Development Center. Debt-to-Equity ratios are average across companies computed from Compustat Global and North America.

Table 2.4: The Effect of the Growth of Credit to NFC on Factors' Allocation when the US Industry's TFP Growth is used as Proxy

	Value Added Growth		Capital Growth		Employment Growth		Hours worked growth		Debt-to-Equity Ratio Growth	
	(1)	**(2)**	**(1)**	**(2)**	**(1)**	**(2)**	**(1)**	**(2)**	**(1)**	**(2)**
Interaction (TFP Growth in the US X Credit Growth NF Corp.)	-0.025*	-0.062***	-0.021**	-0.022**	-0.027***	-0.041***	-0.026***	-0.037***	-0.052**	-0.046
	(0.013)	(0.015)	(0.009)	(0.009)	(0.006)	(0.007)	(0.006)	(0.007)	(0.026)	(0.030)
Interaction (TFP Growth in the US X Credit NF Corp. to GDP in 2001)		-0.008***		-0.002		-0.003***		-0.002***		0.001
		(0.002)		(0.001)		(0.001)		(0.001)		(0.004)
Industry's share of total Value Added in 2001	0.178	0.277								
	(0.255)	(0.250)								
Industry's share of total Capital in 2001			-0.046*	-0.046*						
			(0.026)	(0.026)						
Industry's share of total Employment in 2001					-0.057	-0.072				
					(0.058)	(0.057)				
Industry's share of total Hours Worked in 2001							-0.050	-0.063		
							(0.063)	(0.063)		
Debt-to-Equity Ratio in 2001									-0.468	-0.473
									(0.665)	(0.666)
Average Asset Growth of Compustat Companies									-0.114***	-0.115***
									(0.035)	(0.035)
Number of observations	348	348	255	255	346	346	345	345	284	284
R2	0.326	0.363	0.634	0.637	0.763	0.772	0.746	0.752	0.243	0.244

Notes: Data on credit are from the "Total credit to the non-financial sector" database by the Bank for International Settlements. Data on industry growth and productivity are from the "EU KLEMS" database by the Groningen Growth and Development Center. Debt-to-Equity ratios are average across companies computed from Compustat Global and North America.

Figure 2.3: Steady state values of k_H and k_L as a function of R^b: $\delta = 0.5$ and $1 \leq A_L < A_H$

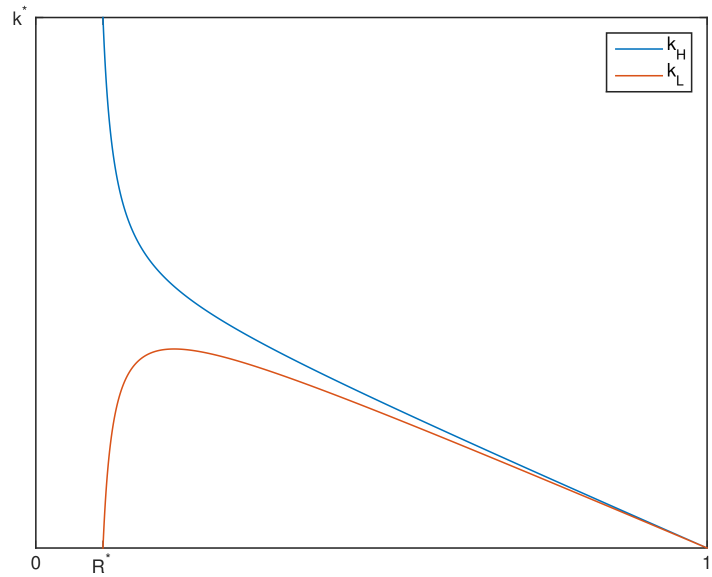


Figure 2.4: Steady state values of aggregate output and consumption as a function of R^b: $\delta = 0.5$ and $1 \leq A_L < A_H$

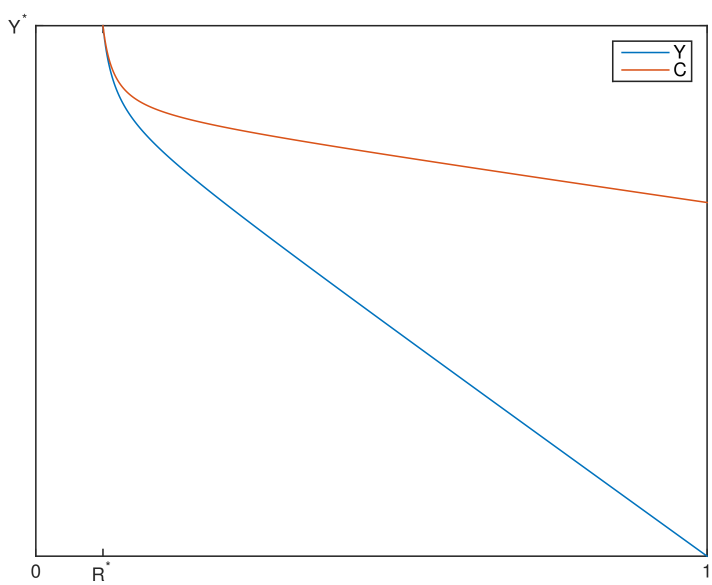


Figure 2.5: Steady state values of k_H and $k_L$as a function of R^b: $\delta = 0.5$ and $A_L < 1 \leq A_H$

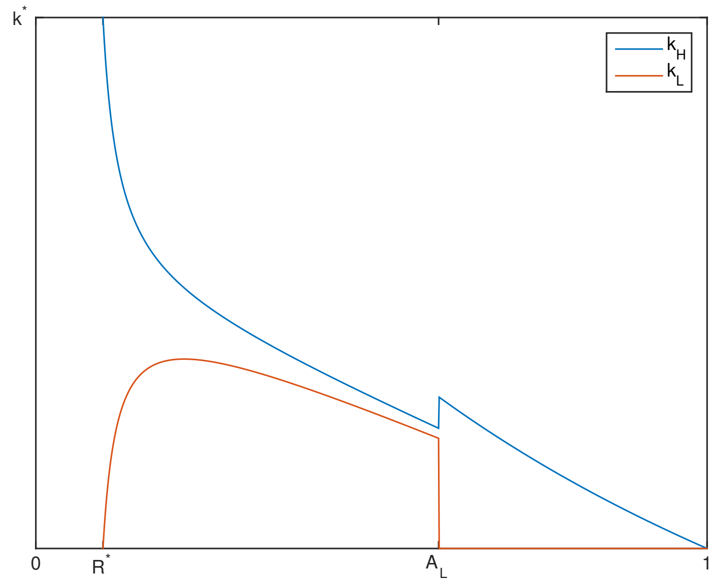


Figure 2.6: Steady state values of aggregate output and consumption as a function of R^b: $\delta = 0.5$ and $A_L < 1 \leq A_H$

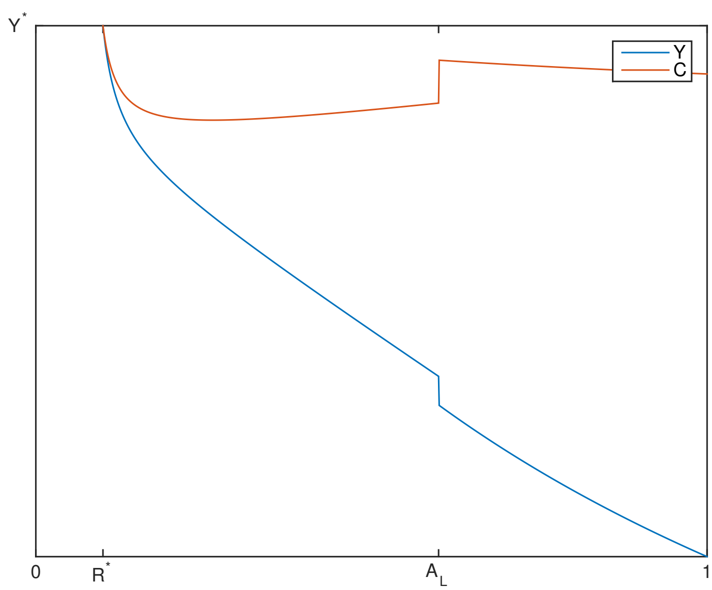


Figure 2.7: Steady state values of k_H and k_L as a function of R^b: $\delta = 0.5$ and $A_L < A_H < 1$

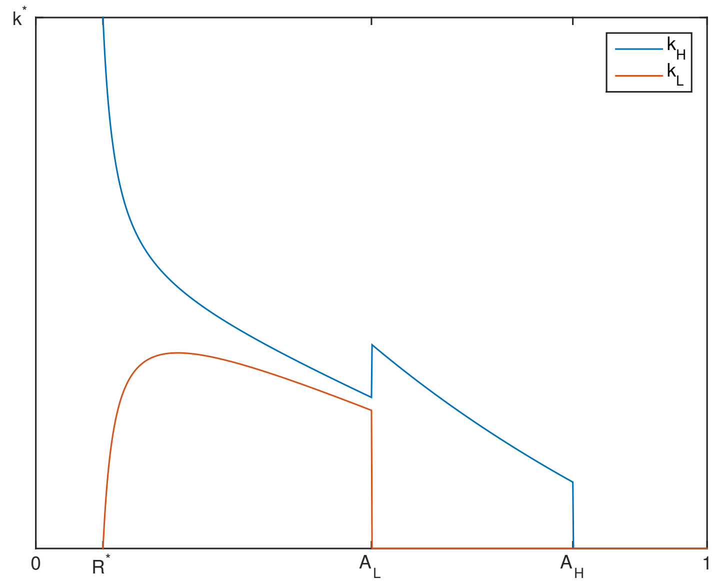


Figure 2.8: Steady state values of aggregate output and consumption as a function of R^b: $\delta = 0.5$ and $A_L < A_H < 1$

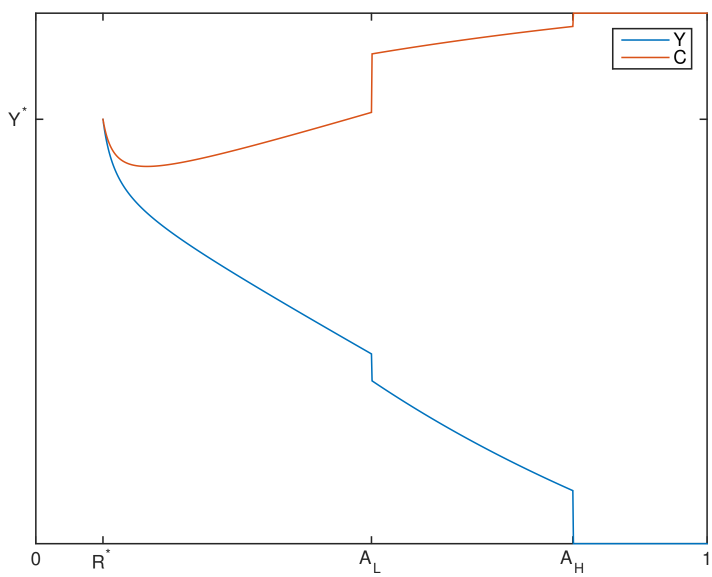
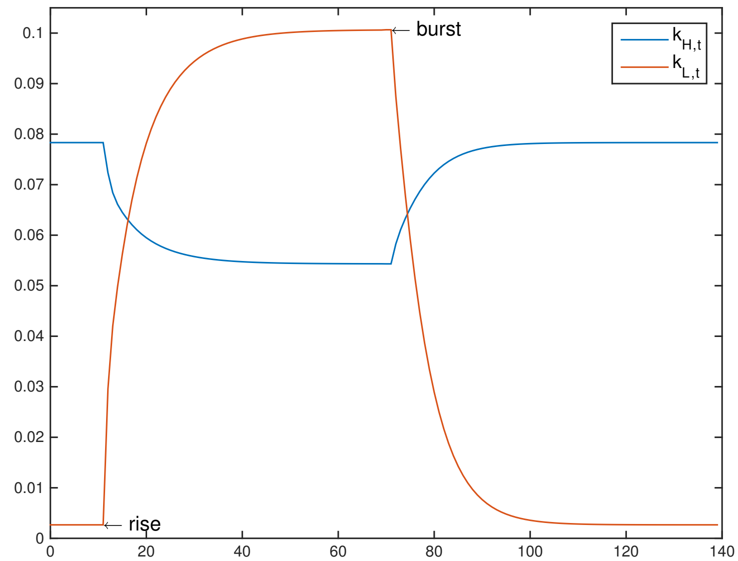


Table 2.5: Summary of simulated dynamics experiments

	Permanent positive shock to R_t *at* $t = 11$	*Permanent negative shock to* R_t *at* $t = 71$	ε_H	ε_L	ϕ	*Who issues unsecured notes*
1)	From $R^* = 0.43$ to $R^b = 0.45$	From $R^b = 0.45$ to $R^* = 0.43$	0.13	0.6	0.4	L-type, given $\sigma\varepsilon_H \leq R^* < \sigma\varepsilon_L$
2)	From $R^* \approx 0$ to $R^b = R^* + 1 \times 10^{-5}$	From $R^* + 1 \times 10^{-5}$ to $R^* \approx 0$	0.13	0.13	1×10^{-100}	H-type, by assumption
3)	From $R^* \approx 0$ to $R^b = R^* + 1 \times 10^{-5}$	From $R^* + 1 \times 10^{-5}$ to $R^* \approx 0$	0.13	0.13	1×10^{-100}	L-type, by assumption

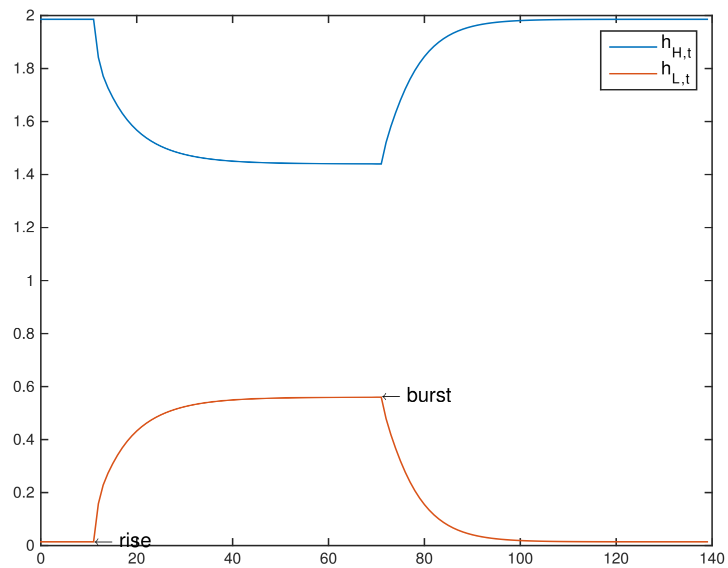
Notes: All remaining parameters are the same in the three experiments: $\alpha = 0.35$, $A_H = 1.9$, $A_L = 1.1$, $\sigma = 0.75$, $e = 0.001$ and $\varphi = 1$.

Figure 2.9: Simulation of the dynamics for $k_{H,t}$ and $k_{L,t}$ (Experiment 1)



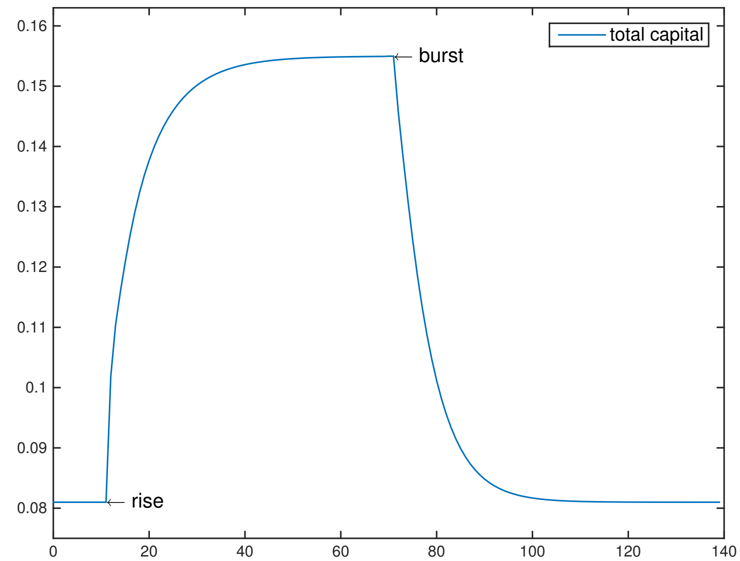
The dynamics is initiated by an unexpected positive shock to R_t at time 11 and a negative shock at time 71.

Figure 2.10: Simulation of the dynamics for $h_{H,t}$ and $h_{L,t}$ (Experiment 1)



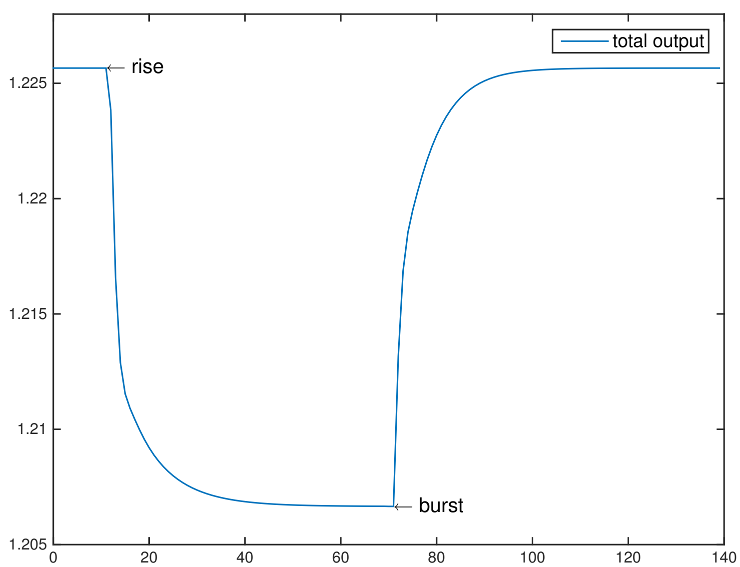
The dynamics is initiated by an unexpected positive shock to R_t at time 11 and a negative shock at time 71.

Figure 2.11: Simulation of the dynamics for the total capital $k_{H,t} + k_{L,t}$ (Experiment 1)



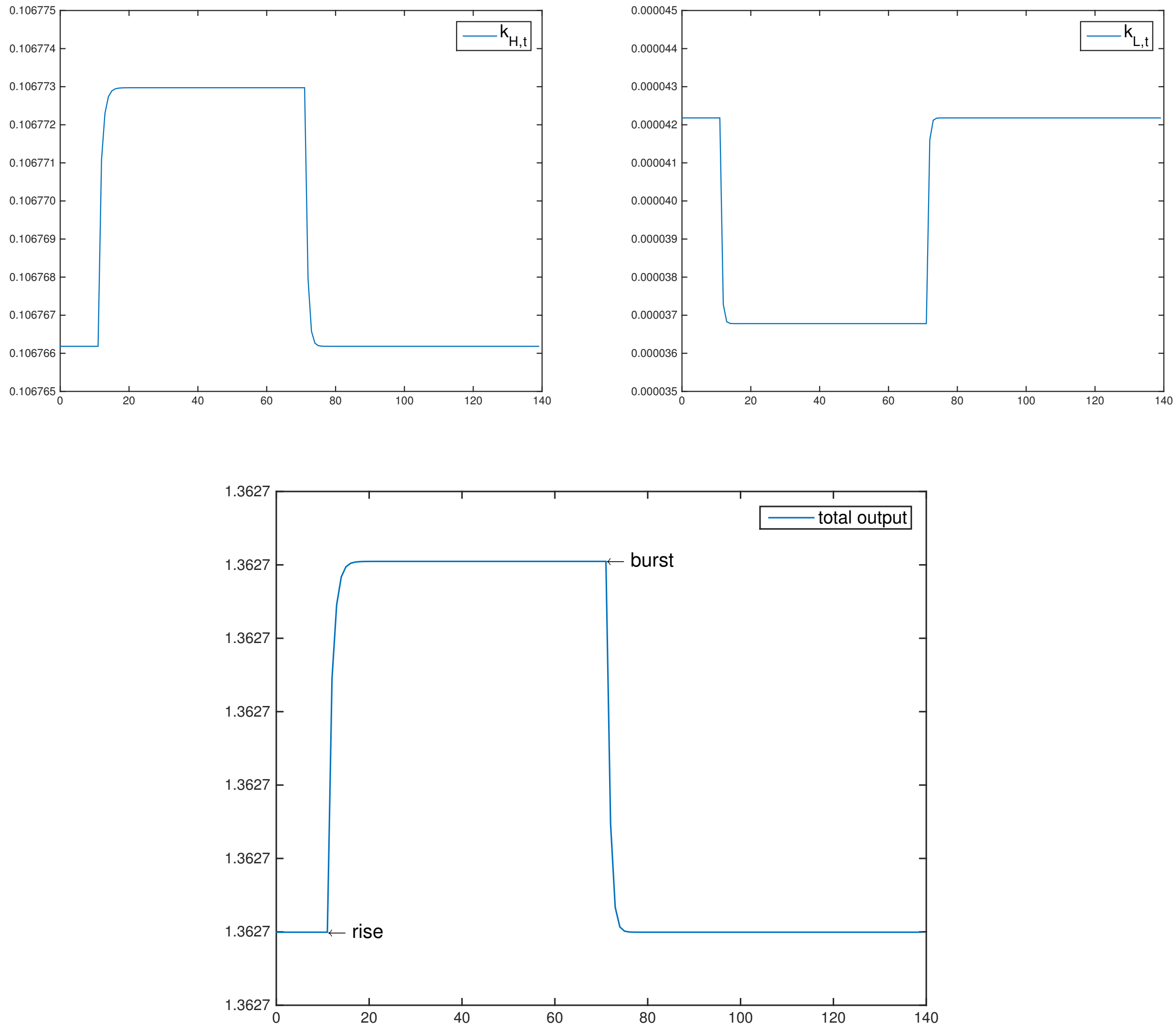
The dynamics is initiated by an unexpected positive shock to R_t at time 11 and a negative shock at time 71.

Figure 2.12: Simulation of the dynamics for the total output $Y_t = Y_{H,t} + Y_{L,t}$ (Experiment 1)



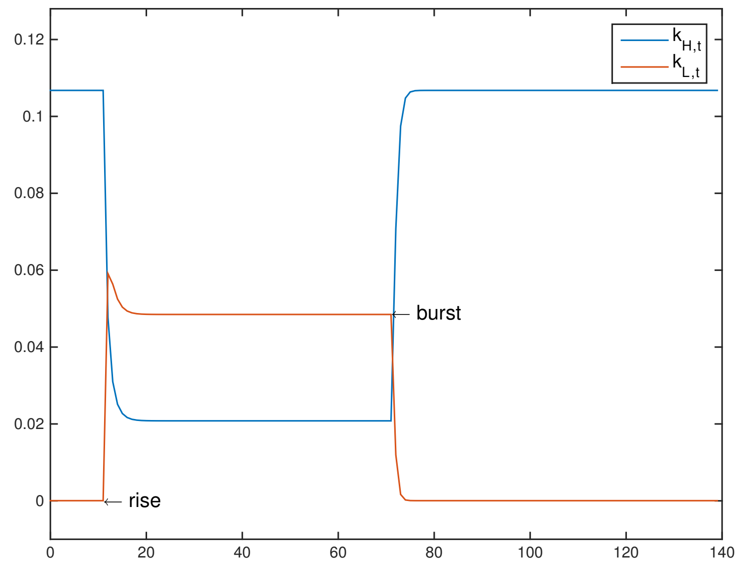
The dynamics is initiated by an unexpected positive shock to R_t at time 11 and a negative shock at time 71.

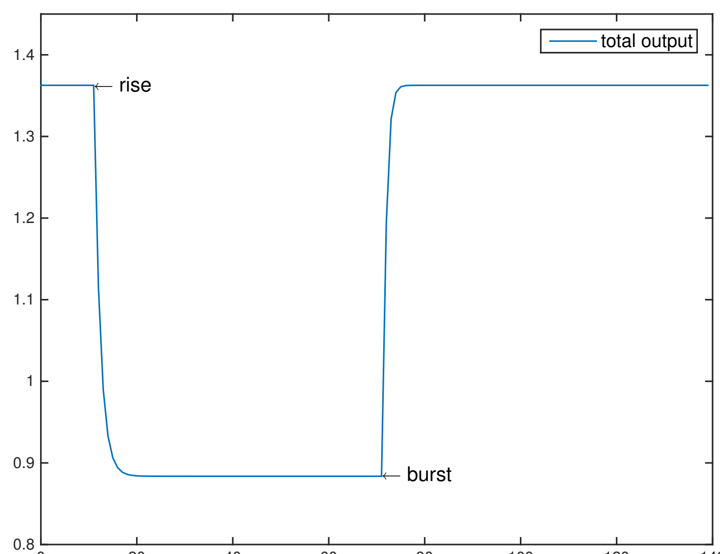
Figure 2.13: Simulation of the dynamics for $k_{H,t}$, $k_{L,t}$ and total output: $\varepsilon_H = \varepsilon_L$ and only the H-type investors issue unsecured notes (Experiment 2)



The dynamics is initiated by an unexpected positive shock to R_t at time 11 and a negative shock at time 71. The bubble induces a positive reallocation of capital that boosts output. Note that the scale in the first two graphs is the same.

Figure 2.14: Simulation of the dynamics for $k_{H,t}$, $k_{L,t}$ and total output: $\varepsilon_H = \varepsilon_L$ and only the L-type investors issue unsecured notes (Experiment 3)





The dynamics is initiated by an unexpected positive shock to R_t at time 11 and a negative shock at time 71.

Chapter 3

Credit Bubbles in a Model with Nominal Rigidities

3.1 Introduction

Financial bubbles typically have a bad reputation among the general public. However, boom-and-bust cycles in credit and housing prices are correlated with output and investments. For this reason, recent papers in the literature of rational bubbles have suggested that bubbles themselves are beneficial to the economy; the problem is that they burst. In the first chapter, I challenged the productive efficiency role of bubbles and proposed a theory of rational bubbles with misallocation of factors. In this chapter, I will propose a different channel by which the increase in asset prices potentially produce a positive effect on output.

Adding to the previous chapter, here the main contribution is the introduction of a demand effect associated to nominal rigidities. Specifically, this demand effect will be consistent with short-run variations in the real return R_t. The main intuition is that a boom in prices can be sustained by two different forces: a higher demand and the emergence of a bubble. A combination of the two effects can explain both the positive correlation with output and the negative correlation with TFP.

In Section 3.2 I describe the mechanism driving the demand effect. A nominal rise in the credit market return, given price rigidities, implies a higher demand and an optimal increase in the labor supply. This, ultimately, boosts the returns on capital and confirms the original rise in the credit market rate. In Section 3.3 I characterize the equilibrium of the model, while in Section 3.4 I show how the return can fluctuate because of a monetary shock or a change in a bubble scheme. The simulations of the model report the opposite effect on output induced by a monetary shock and the rise of a bubble. In particular, given a zero inflation path, an initial increase supported by aggregate demand can turn into a bubble. Then, the output is boosted in the short run but falls toward a lower steady state in the long run.

Finally, in Section 3.5, I analyze the optimal policy prescriptions. In my environment, stabilizing the output gap is not enough to prevent the emergence of a bubble. A social planner should limit the issuing of unsecured credit notes by the private sector and keep the monopoly power to create bubbly notes.

The role of nominal prices in an environment with asset bubbles has been already studied in Galí (2014) and Asriyan, Fornaro, Martin, and Ventura (2016). However, my framework departs from both papers. In Galí (2014) the presence of nominal rigidities allows a central bank to manage the short-run real interest rate, and through this, to influence the short-run fluctuations of an asset bubble. However, similarly to my model, the monetary policy does not affect the long run conditions for the existence of bubbles. In Asriyan, Fornaro, Martin, and Ventura (2016) there are no price rigidities. The authors focus on the role of inflation when expectations on bubble returns are set in nominal terms. In their framework money is an additional asset with no fundamental value; then, agents are exogenously constrained to hold it. By controlling the money supply the monetary authority determine the inflation rate and, ultimately, it influences the growth rate of bubbles. In my model, inflation may still influence the real price of bubbly notes, but the monetary authority cannot generate it by printing money. Importantly, the demand effect introduced in my model is not present in any of the previous works.

The chapter is also related to the literature on credit booms and financial crises. In a number of papers, the cost of a financial crisis has been associated with the freezing of credit markets because of adverse selection. For example, in Gorton and Ordoñez (2014), the booming period is characterized by an increasing opacity regarding the quality of collateral; the crisis bursts when the lenders have the incentive to collect information about the true quality of the investments. In Boissay, Collard, and Smets (2016), credit supply collapses after an expansion in which less efficient banks self-select on the borrowing side of the interbank market. Differently from these papers, in my model, the deep recession that follows the credit boom is rather explained by the misallocation of factors resulting from a credit bubble, combined with a negative demand effect.

The chapter is organized as follows. Section 3.2 describes the additions to the model presented in Chapter 2. In Section 3.3 I characterize the equilibrium. Section 3.4 describes how the return on secured notes is related to both the existence of bubbles and the aggregate demand and reports the simulations of the model under three different scenarios: shocks to a bubble scheme, demand shocks and a mix of the two. Section 3.5 analyzes the policy prescriptions. Section 3.6 concludes.

3.2 A Model of Rational Bubbles with Nominal Rigidities

The model described in this section is an extension of the one exposed in Section 2.4 of the previous chapter. The economy is still populated by OLG workers, High-type investors and Low-type investors. Workers supply labor, while investors produce capital goods. Labor and capital are combined by firms of type H and L to produce consumption goods. To introduce demand effects associated to nominal rigidities I will now assume that firms produce differentiated goods and compete in a monopolistic fashion. Specifically, agents in the economy consume two perfectly substitutable composite goods produced by firms of type H and L:

$$Y_t = Y_{H,t} + Y_{L,t} = \left[\int_{n\in H} y_{n,t}^{\frac{\eta-1}{\eta}} dn\right]^{\frac{\eta}{\eta-1}} + \left[\int_{n\in L} y^{\frac{\eta-1}{\eta}} dn\right]^{\frac{\eta}{\eta-1}} \tag{3.1}$$

for $\eta \geq 1$, where $y_{n,t}$ is the output of a single firm, while $Y_{H,t}$ and $Y_{L,t}$ are the aggregate outputs of High-type and Low-type firms. I assume that the firms in each of two sectors compose a continuum of mass one. The implied composite prices are:

$$P_t = P_{H,t} = P_{L,t} = \left[\int_{n\in H} p_{n,t}^{1-\eta} dn\right]^{\frac{1}{1-\eta}} = \left[\int_{n\in L} p_{n,t}^{1-\eta} dn\right]^{\frac{1}{1-\eta}}. \tag{3.2}$$

In equilibrium, the two composite prices must be equal, given that the two goods are perfect substitutes.

I will describe the decisions of the workers, investors, and firms in separate subsections. For simplicity, I still assume that agents behave as if there were no shocks. In what follows, all variables in nominal terms will have a superscript N.

3.2.1 Workers with Elastic Labor Supply

Workers live for two periods and choose their total labor supply and consumption by maximizing the utility

$$\log\left(c_{Y,t}\right)-\varphi h_t+\log\left(c_{O,t+1}\right) \tag{3.3}$$

subject to $c_{Y,t}=\frac{w_t^N}{P_t}h_t+\frac{\Pi_t^N}{P_t}-l_{t+1}$ and $c_{O,t+1}=\left(R_{t+1}^N\frac{P_t}{P_{t+1}}\right)l_{t+1}$. l_{t+1} denotes the real amount of lending in the credit market. $w_t^N$and Π_t^N respectively denote the nominal wage and profits. R_{t+1}^N is the nominal return. I am assuming that young workers own the monopolistic firms and earn their profits. Then, the optimal supply of labor and lending are:

$$h_t=\frac{w_t^N h_t}{\Pi_t^N+w_t^N h_t}\frac{2}{\varphi} \tag{3.4}$$

$$l_{t+1}=\frac{1}{2}\left(\frac{w_t^N}{P_t}h+\frac{\Pi_t^N}{P_t}\right)=\frac{w_t}{\varphi}. \tag{3.5}$$

It is worthy to note that the labor supply increases with the relative share of labor income to profits. This is crucial to generating demand effects in the economy.

3.2.2 Investors

Investors of type H and L behave as in Subsection 2.4.2 of the previous chapter. Every time they invest to obtain a specific type of capital to rent in the following period. I keep Assumption 2, 3 and 4 to reproduce the correlation between bubbles and misallocation.

Finally, the borrowing constraint can be rewritten in nominal terms:

$$R_{t+1}^N\frac{P_t}{P_{t+1}}d_{m,t+1}^S\leq\phi\frac{Q_{j,t+1}^N}{P_{t+1}}\varepsilon_j i_{m,t+1}\ for\ j\in\{H,L\}\,. \tag{3.6}$$

3.2.3 Firms

Each firm n maximizes its profits

$$p_{n,t}A_j k_{n,t}^{\alpha}h_{n,t}^{1-\alpha}-Q_{j,t}^N k_{n,t}-w_t^N h_{n,t}\ for\ j\in\{H,L\} \tag{3.7}$$

given prices $Q_{j,t}^N$ and w_t^N, and demand constraint $y_{n,t}=\left(\frac{p_{n,t}}{P_t}\right)^{-\eta}Y_{j,t}$. Note that the price of capital $Q_{j,t}$ can now be different from the marginal return on capital $MRK_{j,t}$, given the presence of monopolistic rents. I assume that the price of a good is set one period in advance: as long as no shock hits the economy, a firm will set the price at a constant markup $\frac{\eta}{\eta-1}$ over his marginal cost. Optimal capital and labor demand will be such that

$$\lambda_{n,t} p_{n,t} \alpha A_j \left(\frac{h_{n,t}}{k_{n,t}}\right)^{1-\alpha} = Q_{j,t}^N \ for \ j \in \{H, L\} \tag{3.8}$$

$$\lambda_{n,t} p_{n,t} \left(1-\alpha\right) A_j \left(\frac{k_{n,t}}{h_{n,t}}\right)^{\alpha} = w_t^N \ for \ j \in \{H, L\}, \tag{3.9}$$

where $\lambda_{n,t}$ is the portion of revenues allocated to the payment of factors. Note that, when a firm can optimally set his price, it must be $\lambda_{n,t} = \frac{\eta-1}{\eta} \ \forall \ n$[1]. The aggregate λ_t in period t can be defined as

$$\lambda_t = \lambda_{H,t} \frac{Y_{H,t}}{Y_t} + \lambda_{L,t} \frac{Y_{L,t}}{Y_t}, \tag{3.10}$$

where $\lambda_{H,t}$ and $\lambda_{L,t}$ are the respective shares of High and Low type firms. Fluctuations in λ_t will be associated to demand effects. The incomes and profits in the economy can be rewritten as a function of λ_t and aggregate output Y_t^N:

$$Q_{H,t}^N k_{H,t} + Q_{L,t}^N k_{L,t} = \lambda_t \alpha Y_t^N, \tag{3.11}$$

$$w_t^N h_t = \lambda_t \left(1-\alpha\right) Y_t^N, \tag{3.12}$$

$$\Pi_t^N = \left(1-\lambda_t\right) Y_t^N. \tag{3.13}$$

Then, from (3.4), I can express the labor supply as an increasing function of λ_t:

$$h_t = \frac{\left(1-\alpha\right)\lambda_t}{\left(1-\lambda_t\right) + \left(1-\alpha\right)\lambda_t} \frac{2}{\varphi}. \tag{3.14}$$

The workers are willing to increase their labor supply when the share of revenues allocated to the payment of factors is larger.

3.3 Equilibrium and Steady State

The equilibrium in the economy is now defined as follows:

DEFINITION: *A competitive equilibrium is a list of consumption, lending, secured and unsecured debt, capital, labor, and prices such that:*

(i) Young workers maximize their utility (3.3) *by choosing* h_t *and* l_{t+1}. *Old workers consume* $R_t l_t$

[1] $\lambda_{n,t}$ is indeed the inverse of the firm's markup.

(ii) An investor m of type j who is still active in the market in period t chooses $i_{m,t+1}$, $l^U_{m,t+1}$ and $d^S_{m,t+1}$, given $d^U_{m,t+1}$ and prices $\left(R^N_t, Q^N_{j,t}, P_t\right)$, maximizing profits in the last period of his life

$$\sum_{q=1}^{\infty}(1-\sigma)\,\sigma^{q-1}\varepsilon_j\left[\frac{Q^N_{j,t+q}}{P_{t+q}}i_{m,t+q}-R^N_{t+q}\frac{P_{t+q-1}}{P_{t+q}}\left(d^S_{m,t+q}-l^U_{m,t+1}\right)\right] \tag{3.15}$$

for $j \in \{H, L\}$, subject to budget constraint

$$c_{m,t}+i_{m,t+1}+l^U_{m,t+1}=\frac{Q^N_{j,t}}{P_t}i_{m,t}-R^N_t\frac{P_{t-1}}{P_t}d^S_{m,t}+d^S_{m,t+1}+d^U_{m,t+1}, \tag{3.16}$$

borrowing constraint (3.6) *and resource constraints*

$$d^S_{m,t+1}\geq-\left(\frac{Q^N_{j,t}}{P_t}i_{m,t}-R^N_t\frac{P_{t-1}}{P_t}d^S_{m,t}+d^U_{m,t+1}\right)\ \text{and}\ l^U_{m,t+1}\geq 0.$$

An investor who dies in period t, consumes $c_{m,t}=\frac{Q^N_{j,t}}{P_t}i_{m,t}-R^N_t\frac{P_{t-1}}{P_t}\left(d^S_{m,t}-l^U_{m,t}\right)$, while an investor who fails leaves the market with no final consumption

(iii) Each firm n chooses $k_{n,t}$, $h_{n,t}$ at time t and $p_{n,t}$ at time $t-1$, maximizing (3.7)

(iv) Agents hold consistent beliefs about the path of $d^U_{j,t+1}$ for $j \in \{H, L\}$

(v) All markets clear in every period.

As in the previous chapter, I keep the assumption that the credit market return is lower than the marginal return of both types of investors. In addition, I still assume that $\sigma\varepsilon_H \leq R^* < \sigma\varepsilon_L$, so that only L-type investors can issue unsecured debt.

In a deterministic world, nominal rigidities cannot play any role. If firms optimally set their price, it must be:

$$h_t=\frac{(\eta-1)(1-\alpha)}{1+(\eta-1)(1-\alpha)}\frac{2}{\varphi}. \tag{3.17}$$

Given (3.17), the bubble-free steady state equilibrium interest rate is pinned down from the binding borrowing constraint:

$$R^*=2\phi\frac{\alpha}{1-\alpha}\frac{(\eta-1)(1-\alpha)}{1+(\eta-1)(1-\alpha)}. \tag{3.18}$$

The dynamics of capital is the same as in Subsection 2.4.3:

$$k_{H,t+1} = \varepsilon_H \left\{ (1-\sigma)\, e + \sigma\,(1-\phi)\, \frac{\alpha}{1-\alpha} w_t h_{H,t} + \frac{\phi}{R_{t+1}} \frac{\alpha}{1-\alpha} w_{t+1} h_{H,t+1} \right\} \tag{3.19}$$

$$\begin{aligned} k_{L,t+1} &= \varepsilon_L \left\{ (1-\sigma)\, e + \sigma\,(1-\phi)\, \frac{\alpha}{1-\alpha} w_t h_{L,t} + \frac{\phi}{R_{t+1}} \frac{\alpha}{1-\alpha} w_{t+1} h_{L,t+1} \right\} \\ &+ \varepsilon_L \left\{ l_{t+1} - \frac{\phi}{R_{t+1}} \frac{\alpha}{1-\alpha} w_{t+1} h_{t+1} - R_t \left(l_t - \frac{\phi}{R_t} \frac{\alpha}{1-\alpha} w_t h_t \right) \right\}. \end{aligned} \tag{3.20}$$

The steady state capital allocation is then:

$$k_H = \varepsilon_H \left\{ (1-\sigma)\, e + \left[\sigma\,(1-\phi) + \frac{\phi}{R} \right] \frac{\alpha}{1-\alpha} w h_H \right\} \tag{3.21}$$

$$k_L = \varepsilon_L \left\{ (1-\sigma)\, e + \left[\sigma\,(1-\phi) + \frac{\phi}{R} \right] \frac{\alpha}{1-\alpha} w h_L + (1-R) \left[\frac{1}{R^*} - \frac{1}{R} \right] \phi \frac{\alpha}{1-\alpha} w h \right\}. \tag{3.22}$$

From the last equations we solve for the steady state labor allocation as a function of w and aggregate labor supply h:

$$h_H = \frac{(1-\sigma)\, e}{\left(\frac{\frac{\eta}{\eta-1} w}{(1-\alpha)\varepsilon_H^\alpha A_H} \right)^{\frac{1}{\alpha}} - \left[\sigma\,(1-\phi) + \frac{\phi}{R} \right] \frac{\alpha}{1-\alpha} w} \tag{3.23}$$

$$h_L = \frac{(1-\sigma)\, e + (1-R) \left[\frac{1}{R^*} - \frac{1}{R} \right] \phi \frac{\alpha}{1-\alpha} w h}{\left(\frac{\frac{\eta}{\eta-1} w}{(1-\alpha)\varepsilon_L^\alpha A_L} \right)^{\frac{1}{\alpha}} - \left[\sigma\,(1-\phi) + \frac{\phi}{R} \right] \frac{\alpha}{1-\alpha} w}. \tag{3.24}$$

Finally, the steady state wage w can be pinned down from the labor market clearing:

$$h_H + h_L = \frac{(\eta-1)(1-\alpha)}{1+(\eta-1)(1-\alpha)} \frac{2}{\varphi}. \tag{3.25}$$

In the next section, I will study how the system behaves when hit by unexpected shocks to the credit market return. I will show that, in this version of the model, the return is subject to two shocks of different nature. First, as in the previous chapter, the interest rate can change because of the emergence and burst of a bubble. Second, a monetary shock can induce a demand effect because of the presence of nominal rigidities.

3.4 Simulated Dynamics after Demand and Bubbly Shocks

In a deterministic environment, there is no difference between the expected and realized returns. This also implies that both secured and unsecured notes would be associated with the same interest rate, both ex-ante and ex-post. In order to properly analyze the effect of unexpected shocks in the model, we need to formally introduce a distinction between these rates. In what follows I denote the ex-ante interest rate with $R_{EA,t}$. In equilibrium, this return must be the same across secured and unsecured notes. The realized returns from secured and bubbly notes are denoted instead with R_t and $R_{B,t}$. Obviously, the two returns can diverge. For example, in the time in which a bubble bursts, $R_{B,t}$ collapses, while R_t does not.

The relation between R_t and fluctuations to the aggregate demand can be derived from the binding borrowing constraint:

$$R_t^N = \bar{R}_t \left(\frac{\lambda_t Y_t}{\frac{\eta-1}{\eta} \bar{Y}_t} \right) \left(\frac{P_t}{P_{t-1}} \right). \tag{3.26}$$

In Appendix 5 I describe how to obtain the formula above. The realized nominal return is a function of a natural rate $\bar{R}_t$, a gap in the repayment of factors $\left(\frac{\lambda_t Y_t}{\frac{\eta-1}{\eta} \bar{Y}_t} \right)$ and inflation. $\bar{R}_t$ is the return that prevails without any demand shock. Then, it must be $\bar{R}_t = R_{EA,t}$. For our simulations, I will set:

$$\left(\frac{\lambda_t Y_t}{\frac{\eta-1}{\eta} \bar{Y}_t} \right) \left(\frac{P_t}{P_{t-1}} \right) = \nu_t. \tag{3.27}$$

An unexpected change in ν_t can be interpreted as a monetary or demand shock. A positive shock will induce an increase in $\left(\frac{\lambda_t Y_t}{\frac{\eta-1}{\eta} \bar{Y}_t} \right)$ and, ultimately, in R_t. Intuitively, a higher supply of labor given a same amount of capital would raise $Q_{j,t}$ and, by a relaxation of the borrowing constraint, R_t. In normal times, on a path with zero inflation, it must be $\nu_t = 1$.

The emergence and burst of a bubble influences R_t through $\bar{R}_t$. We can write:

$$\bar{R}_t = R_{EA,t} = 2\phi \frac{\alpha}{1-\alpha} \frac{(\eta-1)(1-\alpha)}{1+(\eta-1)(1-\alpha)} \frac{\bar{w}_t}{w_{t-1}} + \xi_t. \tag{3.28}$$

The first element in the sum is the return we pin down from the binding borrowing constraint in the absence of monetary shocks and bubbles. The additional component $\xi_t \geq 0$ appears during a bubbly episode. In normal times, it must be $\xi_t = 0$. Therefore, an increase in the real return R_t can be associated with an economy contraction when driven by a change in $\bar{R}_t$, and an economy expansion when driven by a positive monetary shock. In this section, I will analyze the effect of both types of shock.

I will simulate the dynamics of the model under three different scenarios. First, I will

Table 3.1: Summary of simulated dynamics experiments

	Permanent rise in R_t^N *at* $t = 11$	*Permanent falls in* R_t^N *at* $t = 51$
1)	Driven by permanent positive shock to ξ_t (bubble rise)	Driven by permanent negative shock to ξ_t (bubble burst)
2)	Driven by permanent positive shock to ν_t (positive demand effect)	Driven by permanent negative shock to ν_t (negative demand effect)
3)	Driven by temporary positive shock to ν_t and permanent positive shock to ξ_t	Driven by temporary negative shock to ν_t and permanent negative shock to ξ_t

describe how the system reacts in response to a change in ξ_t. This is the scenario described in Chapter 2 where a change in the real return is associated with the emergence and the burst of a bubble. In the second simulation, I will analyze instead the effect of a permanent shock to ν_t. In this scenario, the shocks will only produce temporary deviations from the same steady state. Finally, I will propose a simulation in which a temporary monetary shock turns into a long run change in $\bar{R}_t$. A summary of three experiments is presented in Table 3.1.

The model is solved numerically. η is set equal to 11, in order to have a 10% markup. The remaining parameters are similar to the ones chosen in Chapter 2: $\alpha = 0.35$, $A_H = 1.9$, $A_L = 1.1$, $\varepsilon_H = 0.13$, $\varepsilon_L = 0.6$, $\sigma = 0.75$, $\phi = 0.45$, $e = 0.001$, $\varphi = 1$. In all three simulations I analyze the effects of a positive and a negative shock to R_t^N. In particular, I assume that in period 11 the changes in ξ_t and ν_t induce a permanent rise in the nominal rate R_t^N, from $R^* = 0.42$ to 0.43; then, from period 51 the rate R^N falls back to 0.42. Specifically, the starting and final values are always $\xi_t = 0$ and $\nu_t = 1$. It is worthy to note that R_t will be different from $R_{EA,t}$ only in periods 11 and 51.

In Figure 3.1, I plotted the path for R_t^N, R_t, h_t, inflation $\pi_t = \frac{P_t}{P_{t-1}}$ and total output Y_t for the first simulation. Here the change in the nominal return is associated with an identical change in the real return: prices are constant along the entire interval. Moreover, nominal rigidities do not play any role since there is no demand shock: the labor supply is constant. This scenario is similar to the one described in Chapter 2. The emergence of the bubble in period 11 induces a misallocation of factors reducing the total output. The burst of the bubble brings the economy back to the higher original steady state.

Figure 3.2 reports the path for the same variables when the shocks to R_t^N is driven by a permanent change in ν_t. In this case, there are no bubbles appearing in the economy. The sudden increase in R_t^N triggers a demand effect in the time of the shocks, given that prices are rigid. The real return reacts in an identical way only in the time of the unexpected shocks. Starting from the periods that follow the positive (negative) shock, the inflation rises (drops) and the real interest rate returns to the bubble-free steady state level. The monetary shocks produce a temporary real effect. In the period of the increase (fall) in R_t^N, the labor supply reacts in a positive (negative) way. The total output is boosted in period 11 because of the higher labor supply. Then it gradually goes back to its original steady state. The opposite dynamics is triggered from period 51 on. Note that R_t and π_t do not follow a smooth path after the shocks. For example, R_t falls below the steady state level after the initial positive shock. The reason can be seen in equation (3.28): a positive monetary shock that boosted the output at time $t-1$, induces a reduction in $\bar{R}_t$ given an increase in the total lending.

Finally, in Figure 3.3 I present the results of the third simulation. In this last experiment, the initial rise (drop) in R_t^N is driven by a positive (negative) change in ν_t and induces an immediate demand effect as in the previous case. However, in the following periods, I assume that prices do not adjust, ν_t falls (jumps) back to 1, and the higher (lower) real return is supported by a change in ξ_t, i.e., by the emergence (burst) of a bubble. Such a dynamics may be justified by a coordination on zero-inflation equilibria.[2] Therefore, inflation keeps stationary as in the first scenario; the total labor reacts in period 11 and 51 as in the second scenario. This experiment allows me to reproduce a situation in which the initial boom in market returns induces an immediate positive demand effect which gradually vanishes and gets replaced by a long run misallocation of factors. The output boom in period 11 turns into a recession in the following periods. Similarly, the output drop in period 51 is followed by an expansion that brings the system back to the initial steady state.

The last exercise proposed in this section provides a potential interpretation for the dynamics of output and TFP that we observed in the recent times of low inflation. While an initial boom is associated with a positive demand effect triggered by higher market returns, the reduction in TFP is driven by the emergence of a bubble and a misallocation of factors. In the next section, I will study the policy implications suggested by the model.

3.5 Policy Prescriptions

In this section, I will discuss how a social planner can restore an optimal equilibrium in the economy. A monetary authority, controlling the nominal value of the secured credit contracts, can effectively close the gap between the output Y_t and its natural level $\bar{Y}_t$. However, this has

[2] For example, a central bank may play the role of coordination device in supporting this equilibrium selection.

no effect on $\bar{R}_t$.

In order to influence the real return $\bar{R}_t$ and the allocation of factors, other instruments are needed. Given the simple structure of the framework set out here, an optimal allocation of factors is one in which all credit and labor are assigned to high productive investors.[3] In this context, a planner would promote a reallocation by discouraging L-type activity. A simple route to this would be to tax the capital income of the L-type investors. In particular, for a proportional tax $\tau_{L,t}$, L-type investors would prefer to lend to H-type investors if

$$(1-\tau_{L,t})\,\varepsilon_L Q_{L,t} < R_t; \tag{3.29}$$

i.e., if the equilibrium interest rate was higher than the expected return from the L-type investment.[4] In the following I assume $\tau_{L,t}$ is large enough to allow the condition to be respected.

Bubbles are still possible even if high productive borrowers obtain the entire funds. In a long run steady state, the social planner would like to target the return $\bar{R}^g$ that maximizes the aggregate welfare of the economy:

$$\begin{aligned}\log\left[\frac{1}{2}\left(1-\alpha\frac{\eta-1}{\eta}\right)Y_H\left(\bar{R}^g\right)\right]+\log\left[\frac{1}{2}\bar{R}^g\left(1-\alpha\frac{\eta-1}{\eta}\right)Y_H\left(\bar{R}^g\right)\right]\\ +(1-\phi)\,\alpha\frac{\eta-1}{\eta}Y_H\left(\bar{R}^g\right)+\bar{R}^g\frac{1-\sigma}{1-\sigma\bar{R}^g}e.\end{aligned} \tag{3.30}$$

The first line refers to the utility of the workers; the second line reports the utility of the H-type investors and L-type investors. Given an optimal allocation of factors, a bubbly scheme is certainly contractionary - the only effect is to crowd-out H-type capital. However, by transferring resources from younger to older agents, the consumption of the latter may increase if the economy is dynamically inefficient.

A social planner can target an optimal rate $\bar{R}^g_{t+1}$ by imposing its monopoly on the creation of bubbly notes. The planner would set a cap on the debt creation by the private sector: it must be $d_{H,t+1} \leq \frac{\phi}{\bar{R}^g_{t+1}}\alpha Y_{H,t+1}\left(\bar{R}^g_{t+1}\right)\ \forall\, t$. In addition he can directly introduce bubbly notes when $\bar{R}^g_{t+1}$ is larger than the bubble-free rate R^*_{t+1} and the workers have extra resources to lend. An optimal amount of unsecured notes $d^U_{t+1}\left(\bar{R}^g_{t+1}\right)$ can be issued by a government in

[3] Since $\varepsilon_H^\alpha A_H > \varepsilon_L^\alpha A_L$, it is always $\varepsilon_H MRK_{H,t} > \varepsilon_L MRK_{L,t}$.

[4] Note that a policy which reallocates resources towards productive borrowers is also desirable in the absence of a bubble in the economy, as long as $e > 0$.

the form of government bonds,[5] or by a central bank in the form of bank notes. Clearly, the fraction that the planner earns as a rent would be transferred to subsidize H-type investment.

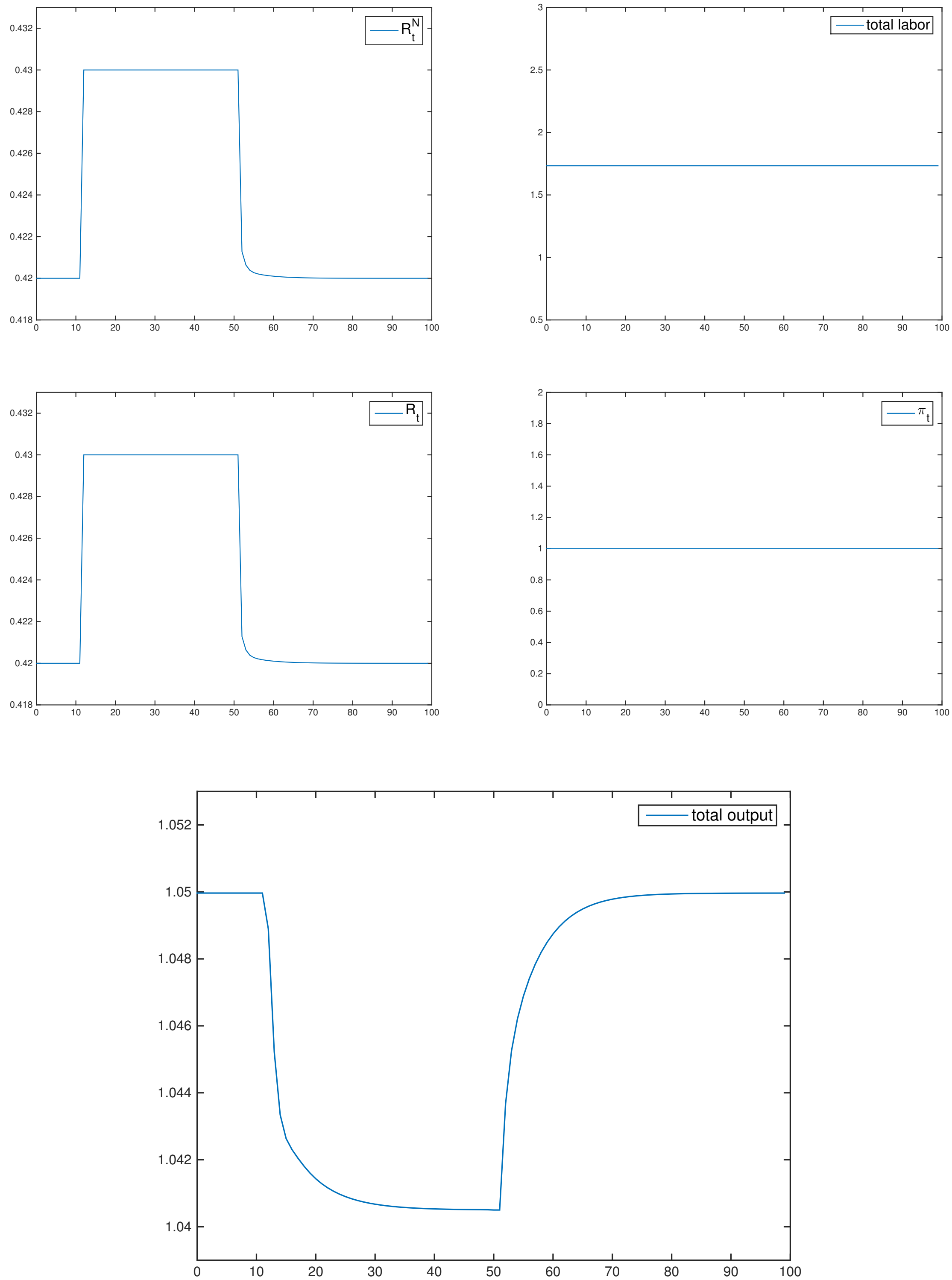
3.6 Conclusions

Boom-and-bust cycles in credit and asset prices can be associated with different effects. While we usually label these events as bubbly episodes, they may also be supported by a change in the aggregate demand. In this chapter, I showed how the actual emergence (burst) of a bubble can be anticipated by an initial increase (fall) in the aggregate demand. Such a dynamics can explain why, during the recent cycles, output and credit were positively correlated, while TFP and credit were not.

From a policy perspective, targeting the output gap is not enough to prevent the emergence of a bubble in the private sector. In order to control bubbles, a monetary or fiscal authority needs to retain the monopoly on the creation of bubbly assets. Caps on the debt creation by private entities is a possible instrument to reach this goal.

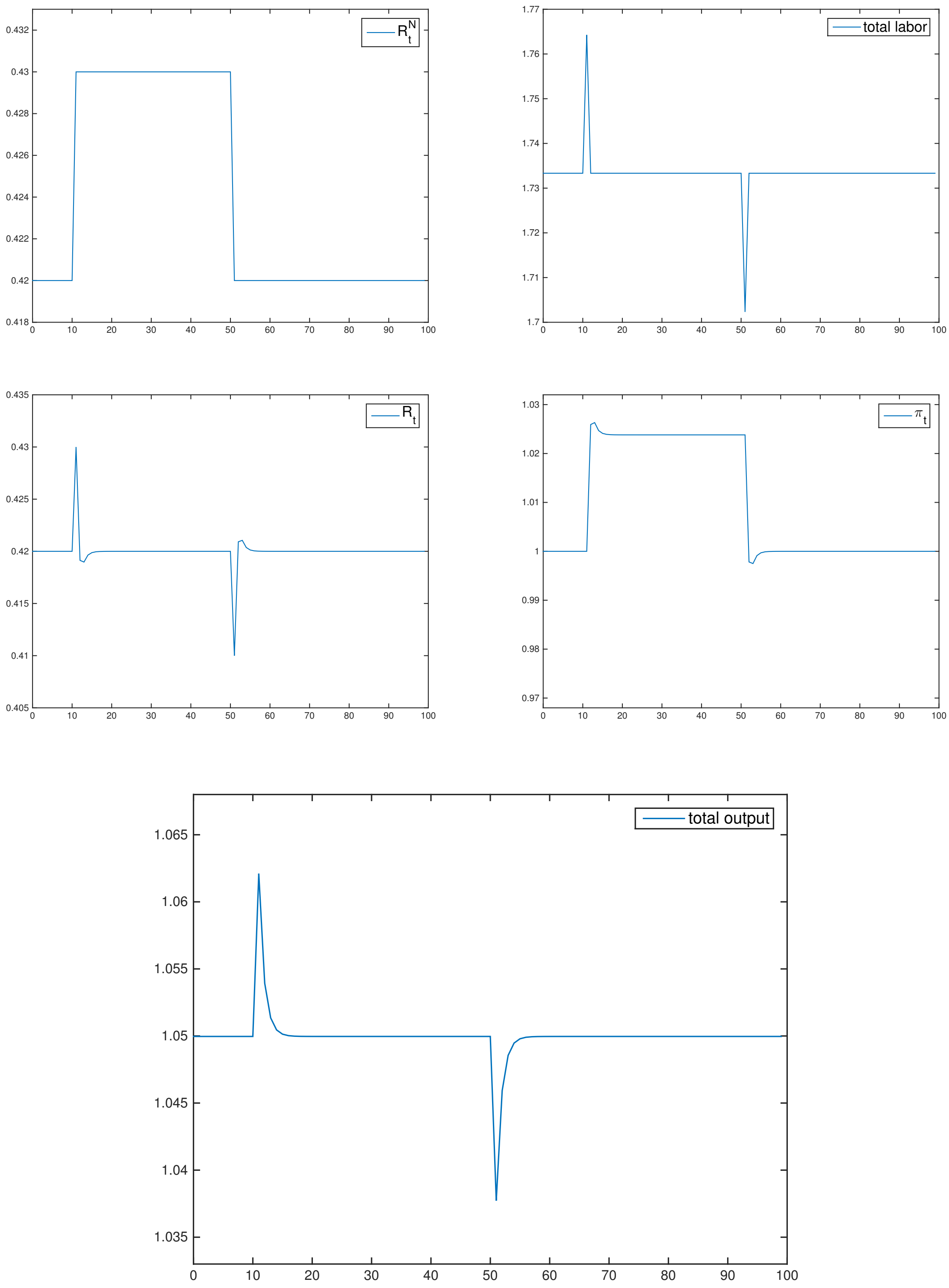
[5] A similar policy is suggested by Woodford (1990).

Figure 3.1: Simulation of the dynamics for R_t^N, h_t, R_t, π_t and Y_t: shocks to the value of bubbly debt (Experiment 1)



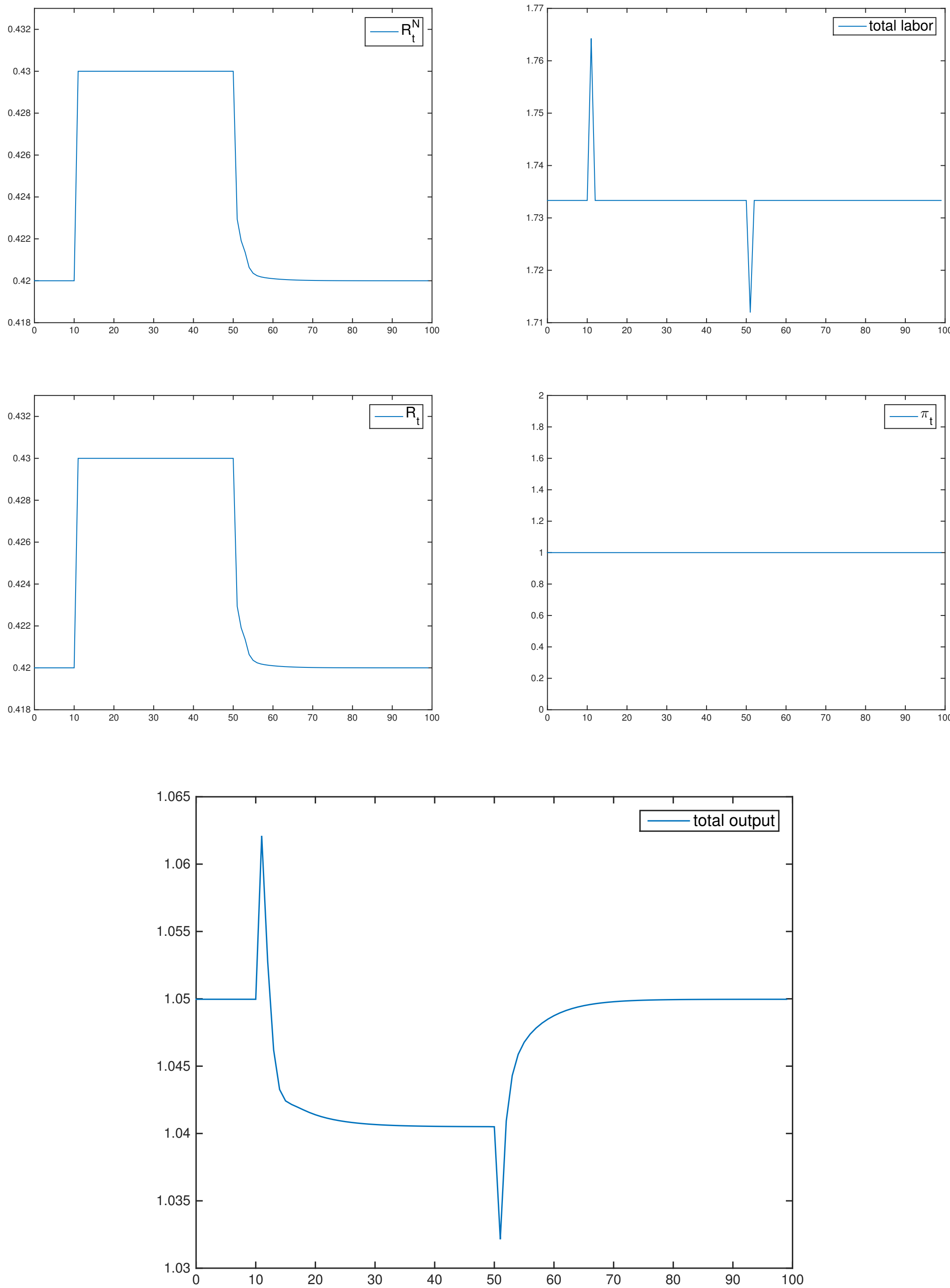
The dynamics is initiated by an unexpected positive shock to $\bar{R}_t$ at time 11 and a negative shock at time 51. Nominal rigidities play no role.

Figure 3.2: Simulation of the dynamics for R_t^N, h_t, R_t, π_t and Y_t: demand shocks (Experiment 2)



The dynamics is initiated by an unexpected positive shock to R_t^N at time 11 and a negative shock at time 51. Nominal rigidities play a role only in the periods of the shocks.

Figure 3.3: Simulation of the dynamics for R_t^N, h_t, R_t, π_t and Y_t: demand and bubbly shocks (Experiment 3)



The dynamics is initiated by an unexpected positive shock to R_t^N at time 11 and a negative shock at time 51. Nominal rigidities play a role only in the periods of the initial shocks.

Chapter 4

Liquidity Mismatch or Bubbly Mismatch?

4.1 Introduction

The 2008 financial crisis was triggered by a systemic run on unconventional short-term debt securities such as commercial papers and repurchase agreements. This has raised new concerns about the risk of liquidity mismatch between assets and liabilities of financial institutions. The last Basel Accord has introduced explicit liquidity ratios for banks; recent papers by Brunnermeier, Gorton, and Krishnamurthy (2012) and Bai, Krishnamurthy, and Weymuller (2016) have proposed a new measure to assess the liquidity risk of an institution. According to the traditional theories of liquidity mismatch, banks issue short-term debt to finance long-term investments. This process is beneficial to the economy, but it exposes financial institutions to the risk of runs. However, a run on short-term debt can also be explained by the collapse of a bubbly scheme on bank debt. The appearance of such a bubble would still raise a mismatch between assets and liabilities, but not to support the financing of long-term projects.

In this chapter, I investigate the balance sheets of American Bank Holding Companies to test theories of liquidity transformation versus theories of bubbles. In particular, I want to find out if periods of rising mismatch between assets and liabilities are associated with the transformation of illiquid assets into liquid debt, or rather with the emergence of a bubble on liabilities. Answering this question is particularly relevant, given that the two hypothesis imply different policy prescriptions. While traditional deposits are, to a large extent, insured by the government, new types of liquid debt contracts are not. An extension of government insurance may be justified if that supports a process of liquidity transformation. On the contrary, a government should typically prevent the appearance of a bubble, as I discussed in the previous chapters.

In Section 4.2, I will rely on stylized theories of liquidity creation to obtain testable predictions. I compare two scenarios: a first one in which the bank can temporarily roll-over its debt to finance projects that are currently illiquid, but that will pay out in a future time; a second one in which the bank can roll-over its debt forever, by effectively issuing bubbles. I will show that in both scenarios, a mismatch arises between the cash from assets that the bank earns and the repayment of debt. However, this mismatch is supported by a different underlying motivation and produces different predictions. The bank investing in illiquid projects can issue new debt to pay the old one, but not to increase its current dividends. The bank issuing bubbles has no such a constraint and will increase its dividend payouts. Although oversimplified, the framework captures some relevant features of the traditional models with rolling-over of debt. In particular, in models of liquidity transformation and bank runs such as Diamond and Dybvig (1983), Gertler and Kiyotaki (2015), and Dang, Gorton, Hölmstrom, and Ordoñez (2016), banks do not roll-over forever, and they eventually use their funds to repay the debt.

In Section 4.3, I present the results of my empirical investigation over the balance sheets of American Bank Holding Companies from 1986 to today. First, I analyze the balance sheet complementarities of financial institutions. I show that new types of short-term debt contract are more likely to be backed by securities rather than loans, while traditional deposits are the usual complement of illiquid loans. Second, I test the previous theories by looking at the correlation between the cash-flow mismatch, measured as the ratio between the repayment of short-term debt and the cash from assets, and the payment of dividends. While the correlation is typically negative, it is flattened when focusing on the years after 2001 and controlling for time and bank fixed effect. In particular, I show that the issuing of non-traditional short-term debt securities is correlated with higher dividends.

To the best knowledge of the author, the analysis proposed in this chapter is novel. Most empirical papers try to measure the liquidity mismatch of financial institutions by looking at the stock values from balance sheet data (generally combined with market price data). A typical problem with liquidity ratios based on stock information is that they cannot report the actual liquidity mismatch in case of emergency asset liquidation.[1] In this paper, I do not attempt to provide a correct measure of the unobservable liquidity mismatch of a bank. Instead, I look at the actual cash flow from assets and its allocation to infer variations in the transformation of illiquid assets or the build-up of a bubbly scheme.

The chapter is organized as follows. In Section 4.2 I describe the theoretical motivation and derive the testable predictions. In Section 4.3 I report the empirical results. Section 4.4 concludes.

4.2 Theoretical Predictions

In this section, I will describe the decision of an infinitely lived bank and derive the predictions to be tested on the data. The bank's optimal decision is restrained by a pledgeability constraint, similar to the one described in the previous chapters. The constraint sets a limit to the repayment of debt that the bank can credibly promise, based on the cash flow from assets that it will earn. I will describe two different scenarios in which this constraint is violated and a mismatch between assets and liabilities arises. In the first scenario, the bank can invest in an illiquid project, which is an asset that pays in $T > 1$ periods and it is not pledgeable. However, the constraint is temporarily relaxed and the bank can effectively roll-over its debt until the project pays off. In this scenario, I want to depict a process of liquidity transformation conducted by a financial institution. In the second scenario, the bank is allowed to violate the pledgeability constraint by issuing bubbly debt. In this scenario, the bank is not responsible for the repayment of its debt in excess.

The bank maximizes its utility:

$$\sum_{t=0}^{\infty} \beta^t \frac{\chi_t^{1-\sigma}}{1-\sigma}, \tag{4.1}$$

where χ_t are the dividends at time t. Every time, the bank can invest in a liquid asset a_{t+1}, and obtain Qa_{t+1} in the following period. In addition, it can raise external debt d_{t+1} at a fixed interest rate R. When not investing in illiquid assets nor issuing bubbly debt, the decision is subject to constraint:

$$Rd_t \leq \phi Q a_t. \tag{4.2}$$

[1] Bai, Krishnamurthy, and Weymuller (2016) try to do that by inferring liquidation prices from the market. However, this remains an imperfect way given that market prices in case of a systemic bank run are not really observable.

The constraint imposes that the total repayment on debt must be lower than a fraction ϕ of the revenues. It is worthy to note that such a constraint prevent the bank from engaging in Ponzi-schemes. However, the condition is stricter than a usual no-Ponzi game one. In particular, in this environment, the bank cannot be punished in case of default. Then, $\phi Q a_t$ should be interpreted as the fraction of future income that the bank can credibly pledge.

Assuming $Q > R$ and $\phi Q < R$, the constraint must be binding. This means that, without investment in illiquid assets and bubbly debt, the mismatch ratio must be

$$\frac{Rd_t}{Qa_t} = \phi. \tag{4.3}$$

In the two cases that I will describe below, the pledgeability constraint is violated and the mismatch ratio increases for two different reasons. In the first case, at time 0, the bank can also invest in an illiquid asset z_T that pays at time T. Between 0 and T, the constraint is relaxed and the bank can temporarily raise more debt. In the second case, between 0 and T, the bank can increase the mismatch by issuing bubbly debt. Then, at time T, the bubble bursts and the mismatch goes back to ϕ. The goal is to derive how the bank optimally changes its path of dividends when having the option of creating a liquidity or a bubbly mismatch.

LIQUIDITY MISMATCH:

At time 0, I assume that the bank can invest z_T in an illiquid asset paying in T periods. The asset is illiquid not just because it does not pay off immediately; it also cannot be efficiently traded by the lenders.[2] Therefore, the bank cannot use it as a pledge until the time in which it pays off. However, I will assume that banks own a technology to relax their pledgeability constraint when investing in the illiquid asset. Effectively, the bank can temporarily roll-over its debt. The scheme ends when the illiquid asset finally pays off.

In what follows, I will assume that the illiquid asset pays $Q_z z_T$ at time T. The bank chooses a_{t+1}, z_T and d_{t+1}. The problem can be expressed in the following way:

$$\max \sum_{t=0}^{\infty} \beta^t \frac{\left(Qa_t + \mathbf{1}_{\{t=T\}} Q_z z - Rd_t + d_{t+1} - a_{t+1} - \mathbf{1}_{\{t=0\}} z_T\right)^{1-\sigma}}{1-\sigma} \tag{4.4}$$

[2] For example, in Dang, Gorton, Hölmstrom, and Ordoñez (2016), the consumers cannot efficiently hide the information regarding the investment outcome. In Gertler and Kiyotaki (2015), the households must pay a management cost if directly holding capital; however, in this last paper, the asset is not per-se illiquid, since it pays out in every period.

subject to constraint

$$Rd_t \leq \phi Q a_t + \mathbf{1}_{\{0<t\leq T\}} \frac{1}{R^{T-t}} \phi Q_z z_T. \tag{4.5}$$

The bank can effectively pledge the repayment from the illiquid asset only at time T. However, from 0 to $T-1$, the constraint is relaxed and the bank is allowed to issue more debt by holding the illiquid asset. A simple interpretation is that the bank can pledge a part of its future debt to repay the current one. Note that, even if the asset cannot be pledged, I am assuming that, for any $0 < t < T$, the debt in excess is a discounted amount of the final pledgeable repayment. Therefore, debt can be rolled-over as long as it helps the bank to invest in z_T.

The problem is solved in Appendix 6. From 0 to $T-1$ and from T onwards, the bank must hold some liquid assets. In fact, they are the only instruments available to smooth consumption across these periods. Given that $Q > R$ and $\phi Q < R$, the constraint is binding and the optimal dividends path is such that:

$$\frac{\chi_{t+1}}{\chi_t} = \left(\beta Q \frac{1-\phi}{1-\phi\frac{Q}{R}}\right)^{\frac{1}{\sigma}} \forall\, 0 \leq t < T-1 \; and \; t \geq T. \tag{4.6}$$

Dividends would grow at a constant rate, which depends on the prices Q and R, and the tightness of the borrowing constraint.

Between $T-1$ and T, it must be:

$$\frac{\chi_T}{\chi_{T-1}} = \max\left\{\left(\beta Q \frac{1-\phi}{1-\phi\frac{Q}{R}}\right)^{\frac{1}{\sigma}} ; \left(\beta \frac{Q_z}{\rho\left(Q, Q_z, R, T\right)} \frac{\beta^{T-1}-\phi}{\left(\beta Q \frac{1-\phi}{1-\phi\frac{Q}{R}}\right)^{T-1} - \phi \frac{Q_z}{\rho(Q,Q_z,R,T)R}}\right)^{\frac{1}{\sigma}}\right\}, \tag{4.7}$$

where $\rho\left(Q, Q_z, R, T\right) < 1$ is a function of the prices and the number of periods T. The bank does not invest in the illiquid asset if the second term in the curly brackets is smaller. This is the case in which the illiquid asset is not profitable enough. If instead the second term is larger, it must be $z_T > 0$. In this case, the bank optimally reduces its stock of liquid asset between 0 and $T-1$, expecting for time T in which the illiquid asset is finally liquidated.

When the bank invests in the illiquid asset a mismatch between debt repayment and income from assets arises. Specifically, it is

$$\frac{Rd_t}{Qa_t} = \frac{\phi Q a_t + \frac{1}{R^{T-t}} \phi Q_z z_T}{Qa_t} > \phi \; \forall\, 0 < t \leq T-1. \tag{4.8}$$

In addition, the dividends jump to a higher level at time T, when the mismatch ratio goes back to ϕ. By investing in the illiquid asset, the bank keeps its dividends at a lower level in the initial periods to raise them when the project pays off. Therefore, the analysis predicts a negative correlation between a liquidity mismatch and the level of dividends.

BUBBLY MISMATCH:

In the second scenario, I assume that the bank is allowed to violate the borrowing constraint by issuing bubbly notes as in the framework described in the previous chapters. However, at time T, I assume that the bubble bursts unexpectedly and the bank stops issuing bubbly notes. The bank now chooses both secured d_{t+1}^S and unsecured debt d_{t+1}^U, solving the problem:

$$\max \sum_{t=0}^{\infty} \beta^t \frac{\left(Qa_t - Rd_t^S + d_{t+1}^S + d_{t+1}^U - \omega_t - a_{t+1}\right)^{1-\sigma}}{1-\sigma} \tag{4.9}$$

subject to constraints

$$Rd_t^S \leq \phi Q a_t \tag{4.10}$$

and

$$d_t^U \leq \bar{d}_t^U . \tag{4.11}$$

The borrowing constraint is still binding for secured debt. However, by issuing d_{t+1}^U, the bank can raise its financing beyond the pledgeability limit without being responsible for its repayment. Every time, the bank would maximize its unsecured debt until the supply limit $\bar{d}_{t+1}^U$. The quantity ω_t is taken as exogenous and must be equal to $R\bar{d}_t^U$.[3] From 0 to $T-1$, it is $\bar{d}_{t+1}^U \geq R\bar{d}_t^U > 0$; from T onwards it is $\bar{d}_{t+1}^U = 0$ and $\omega_t = 0$. Therefore, the bank earns positive rents until $T-1$; from T onwards, the rents unexpectedly drop to 0.

By violating the pledgeability constraint, the mismatch ratio must be

$$\frac{Rd_t}{Qa_t} = \frac{R\left(d_t^S + d_t^U\right)}{Qa_t} = \frac{\phi Q a_t + R\bar{d}_t^U}{Qa_t} > \phi \; \forall \; 0 < t \leq T-1, \tag{4.12}$$

as in the liquidity case. At time T it falls back to ϕ. In addition, the optimal path for dividends is still:

$$\frac{\chi_{t+1}}{\chi_t} = \left(\beta Q \frac{1-\phi}{1-\phi\frac{Q}{R}}\right)^{\frac{1}{\sigma}} \; \forall \; 0 \leq t < T-1 \; and \; t \geq T \; . \tag{4.13}$$

[3]Note that, with respect to the previous chapters, in this specification, d_{t+1}^U includes the resources necessary to repurchase the existing unsecured notes.

Figure 4.1: Balance Sheet Summary Statistics

ASSETS		LIABILITIES	
Loans to Banks	5.3% (0.04)	80% (0.1)	Deposits
Reverse Repos	1.8% (0.04)	2.8% (0.05)	Other Short Term Debt (Commercial Papers and Repos)
Loans	62.5% (0.14)	0.1% (0.01)	Tradable Liabilities
Non Government Securities	11% (0.09)	6.3% (0.07)	Other Borrowing
Government Securities	10.3% (0.09)	2.5% (0.05)	Other Liabilities
Other Assets	5.8% (0.04)	9% (0.04)	Equity

Notes: Data are from the FRY-9C Consolidated Report of Condition and Income, completed on a quarterly basis by Bank Holding Companies.

However, when the bubble bursts at time T, if the bank expected positive rents $\bar{d}^U_{t+1} - R\bar{d}^U_t > 0$ for $t \geq T$, the level of dividends must be reduced:

$$\frac{\chi_T}{\chi_{T-1}} < \left(\beta Q \frac{1-\phi}{1-\phi\frac{Q}{R}}\right). \tag{4.14}$$

Since the bank obtains a rent when issuing bubbly debt, a bubbly mismatch must be positively correlated to dividends.

In the next section, I will verify if the correlation between the mismatch ratio and the dividends is positive or negative, by analyzing the balance sheet of the American Bank Holding Companies over the last thirty years. Given the theoretical predictions of this section, a positive correlation would likely be associated with the appearance of a bubble rather than with the transformation of illiquid assets into liquid debt.

4.3 Empirical Test

In this section, I will report the results of my empirical test. Data are from the FRY-9C report completed by American Bank Holding Companies. Observations are quarterly and available from 1986 for almost 3000 banks. Figure 4.1 shows the structure of the balance sheet.

For each item, I report the average share across all banks over time (with relative standard deviations). Deposits represent by far the largest component. Repos and Commercial Papers represent less than 3% of the total assets.[4] In Table 4.1 and 4.2 I report the correlation between the two sides of the balance sheet. The variables are measured respectively as fractions of total assets and levels. I regress the total Deposits and Other Short-Term Debt on the Loans to Banks, Reverse Repos, Loans, Government Securities, Non-Government Securities and Other Assets. Regressions include time and bank fixed effects. The results show that while traditional deposits are mainly backed by loans, other short-term liabilities require a relatively larger share of securities.[5] The expansion in liquid debt contracts that preceded the 2008 crisis was primarily driven by a surge in Repos and Commercial Papers. A preliminary analysis reveals that these contracts are not typically backed by illiquid loans as it happens instead for traditional deposits.

In order to test the theories presented in the previous section I start by estimating the following model:

$$\log \chi_{b,t} = bank_b + quarter_t + \gamma \left(\frac{R_t D_{b,t}}{Q_{b,t} A_{b,t}} \right) + \varepsilon_{b,t}. \tag{4.15}$$

The goal is to evaluate the correlation between the cash-flow mismatch and the ratio of dividends to assets. The variable $\chi_{b,t}$ is given by the total dividends from preferred and common shares. The repayment on debt $R_t D_{b,t}$ is given by the total amount of short-term debt (Deposits and Other Short Term Debt) multiplied by the quarterly interest rate on T-bills. Finally $Q_{b,t} A_{b,t}$ is equal to the total income from assets. In Table 4.3 I present the results of my estimation.[6] The coefficient γ is always significantly negative. However, when we include both time and bank fixed effect the magnitude of the effect is much smaller. The main reason is the large size heterogeneity of the banks included in the sample.[7]

In the following tables I investigate if the relation between the two ratios has changed in

[4]Unfortunately, our data do not include the large amount of short-term securities issued by investment banks through their off-balance sheet conduits. Gorton and Metrick (2012) provide an analysis of the unregulated repo market relying on survey data. They estimate that in 2004 the size of the total bilateral repo market, which was dominated by unregulated institutions, was $3.857 trillion. In a 2008 speech, Timothy Geithner, at the time President of the FED New York, assessed the combined size of the asset-backed commercial paper conduits in the beginning of 2007 at $2.2 trillion.

[5]The coefficients for Government Securities appear to flip between the table with shares of total assets and the one with levels. A possible explanation comes from the weak correlation between Government Securities and Loans, the largest asset component, where the remaining asset items are highly correlated with it. Therefore, an increase in the Government Securities to Total Asset ratio may also be induced by an independent reduction in the level of Loans. This would explain the relatively higher effect on Other Short-Term Debt and the lower effect on Deposits when moving from the levels to the shares analysis.

[6]Outlier observations with $\left(\frac{R_t D_{b,t}}{Q_{b,t} A_{b,t}} \right) > 100$ are dropped from the analysis.

[7]Indeed, the magnitude of the coefficient with no fixed effects can be halved by excluding the largest 10% of Total Assets observations.

Table 4.1: Balance Sheet Complementarities: shares of total assets

	Deposits on Total Assets	Other STD on Total Assets
Loans to Banks on TA	0.555***	-0.055***
	(0.010)	(0.005)
Reverse Repos on TA	0.584***	0.053***
	(0.011)	(0.006)
Loans on TA	0.499***	-0.012**
	(0.009)	(0.005)
Government Securities on TA	0.418***	0.078***
	(0.009)	(0.005)
Non Government Securities on TA	0.400***	0.058***
	(0.009)	(0.005)
Bank Fixed Effects	Yes	Yes
Quarter Fixed Effects	Yes	Yes
Number of observations	70,379	74,115
R2	0.160	0.085

Notes: Data are from the FRY-9C Consolidated Report of Condition and Income.

the years of the boom-and-bust cycle associated with the 2008 crisis. In Table 4.4 I replicate the previous exercise including a dummy identifying the quarters starting after 2001 and its interaction with $\left(\frac{R_t D_{b,t}}{Q_{b,t} A_{b,t}}\right)$:

$$\log \chi_{b,t} = bank_b + quarter_t + dummy\_2001_p + \gamma_p \left[dummy\_2001_p \times \left(\frac{R_t D_{b,t}}{Q_{b,t} A_{b,t}} \right) \right] + \varepsilon_{b,t}. \quad (4.16)$$

The choice of the 2001 threshold is justified by the appearance of Repos and Commercial Papers in the balance sheets of the companies. In all cases, the coefficient is significantly less negative when considering the period after 2001. In Table 4.5 I estimate the model in (4.16) excluding the quarters between 2007 and 2009 in which many banks suffered a run. This is in order to exclude the possibility that our results are driven by the banking crisis. The results are similar to before. In particular, the total effect disappears in the period after 2001 when including both bank and quarter fixed effects.

The years preceding the 2008 financial crisis were characterized by a boom in non conventional deposits. In the last exercise, I want to isolate the effects of a variation in traditional Deposits and Other Short Term Debt. In Figure 4.2 and 4.3 I plotted the aggregate path of $\left(\frac{R_t d_t}{Q_t A_t}\right)$ and $\left(\frac{R_t o_t}{Q_t A_t}\right)$, where d_t and o_t are respectively the aggregate amount of Deposits and

Other Short Term Debt, with $D_t = d_t + o_t$. In both graphs I also included the path of the log aggregate dividends. As we can see, on aggregate the mismatch ratio $\left(\frac{R_t o_t}{Q_t A_t}\right)$ is more positively correlated than $\left(\frac{R_t d_t}{Q_t A_t}\right)$. In the model we now include both mismatch ratios as explanatory variables:

$$\log \chi_{b,t} = bank_b + quarter_t + \gamma_d \left(\frac{R_t d_{b,t}}{Q_{b,t} A_{b,t}}\right) + \gamma_o \left(\frac{R_t o_{b,t}}{Q_{b,t} A_{b,t}}\right) + \varepsilon_{b,t}. \tag{4.17}$$

Note that the data on Other Short Term Debt start only from 2002; then, the regressions cover only this period. The results are presented in Table 4.6 and 4.7, including or not the 2007-2009 period. The tables reveal that non-conventional deposits have a relevant impact for the near-zero correlation of the after-2001 period.[8] While traditional Deposits still induce a negative effect, $\left(\frac{R_t o_{b,t}}{Q_{b,t} A_{b,t}}\right)$ tends to be positively correlated to dividends.

Overall, the empirical investigation shows that the relation between the mismatch ratio and dividends have become more positive during the years of the recent credit cycle. Those banks that increased their cash-flow mismatch did not reduce the payment of dividends. These results support the claim that the increased leverage and mismatch were not entirely justified by the transformation of illiquid assets into liquid instruments but could also be explained by the emergence of a debt bubble.

4.4 Conclusions

The recent financial crisis was anticipated by an increase in the mismatch between assets and liabilities of banks. The transformation of illiquid loans into liquid debt instruments is one of the main roles of the financial sector. For this reason, most countries in the world have introduced government insurance schemes to prevent the risk of a bank run. However, in this chapter, I showed that an increasing cash-flow mismatch could also be associated with the emergence of a bubble on bank debt. From a stylized model, I derived a simple condition to test theories of liquidity transformation versus theories of bubbles in bank's balance sheet. Specifically, I showed that the correlation between dividends and cash-flow mismatch is negative when a bank is running a liquidity mismatch and positive when it is running a bubbly mismatch. Importantly, the condition does not require any analysis of the assets' composition.

I find evidence that the boom in short-term debt that preceded the 2008 crisis could have been fed by the emergence of a bubble. Specifically, those banks with a higher mismatch were

[8] Observations with $\left(\frac{R_t o_{b,t}}{Q_{b,t} A_{b,t}}\right) > 0.15$ are dropped from the analysis.

not reducing their dividends payout.

The results depend on the simple definition of mismatch that my theory provides. Future research efforts should aim to relax the definition of mismatch taking into account the heterogeneous degree of pledgeability of different types of asset.

From a normative point of view, a regulatory authority should identify when an expansion in short-term debt is matched by illiquid long-term investment or if instead it is supported by the build-up of a bubbly scheme. In this second case, government insurance may not be the optimal policy. A stricter regulation controlling the issuing of short-term debt by financial institutions would be desirable instead.

Table 4.2: Balance Sheet Complementarities: levels

	Deposits	Other Short Term Debt
Loans to Banks	0.874***	0.030***
	(0.004)	(0.001)
Reverse Repos	0.255***	0.028***
	(0.005)	(0.001)
Loans	0.814***	0.013***
	(0.002)	(0.000)
Government Securities	1.164***	-0.221***
	(0.010)	(0.002)
Non Government Securities	0.666***	0.039***
	(0.005)	(0.001)
Other Assets	-0.245***	-0.010***
	(0.004)	(0.001)
Bank Fixed Effects	Yes	Yes
Quarter Fixed Effects	Yes	Yes
Number of observations	70,377	74,114
R2	0.960	0.205

Notes: Data are from the FRY-9C Consolidated Report of Condition and Income

Table 4.3: Effect of Mismatch on Dividends: entire sample

	log Dividends			
Mismatch Ratio	-0.029***	-0.024***	-0.048***	-0.001***
	(0.000)	(0.000)	(0.001)	(0.000)
Bank Fixed Effects	No	Yes	No	Yes
Quarter Fixed Effects	No	No	Yes	Yes
Number of observations	84,425	84,425	84,425	84,425
R2	0.050	0.138	0.110	0.505

Notes: Data are from the FRY-9C Consolidated Report of Condition and Income.

Table 4.4: Effect of Mismatch on Dividends: before and after 2001

	log Dividends					
Mismatch Ratio	-0.051***	-0.045***	-0.052***	-0.037***	-0.200***	-0.005***
	(0.001)	(0.001)	(0.000)	(0.000)	(0.002)	(0.001)
Mismatch Ratio X Dummy_after2001	0.025***	0.018***	0.034***	0.012***	0.170***	0.004***
	(0.001)	(0.001)	(0.000)	(0.000)	(0.003)	(0.001)
Dummy_after2001		0.224***		0.759***		
		(0.025)		(0.011)		
Bank Fixed Effects	No	No	Yes	Yes	No	Yes
Quarter Fixed Effects	No	No	No	No	Yes	Yes
Number of observations	84,425	84,425	84,425	84,425	84,425	84,425
R2	0.074	0.075	0.285	0.324	0.153	0.505

Notes: Data are from the FRY-9C Consolidated Report of Condition and Income.

Table 4.5: Effect of Mismatch on Dividends: before and after 2001, excluding 2007-2009

	log Dividends					
Mismatch Ratio	-0.048***	-0.045***	-0.050***	-0.037***	-0.200***	-0.005***
	(0.001)	(0.001)	(0.000)	(0.000)	(0.002)	(0.001)
Mismatch Ratio X Dummy_after2001	0.023***	0.020***	0.033***	0.013***	0.173***	0.005***
	(0.001)	(0.001)	(0.000)	(0.000)	(0.003)	(0.001)
Dummy_after2001		0.112***		0.701***		
		(0.026)		(0.011)		
Bank Fixed Effects	No	No	Yes	Yes	No	Yes
Quarter Fixed Effects	No	No	No	No	Yes	Yes
Number of observations	76,765	76,765	76,765	76,765	76,765	76,765
R2	0.069	0.069	0.281	0.315	0.150	0.510

Notes: Data are from the FRY-9C Consolidated Report of Condition and Income.

Table 4.6: Effect of Mismatch on Dividends: Deposits and Other Short Term Debt

	log Dividends			
Mismatch Ratio (Deposits)	-0.027***	-0.027***	-0.063***	-0.007***
	(0.000)	(0.000)	(0.001)	(0.001)
Mismatch Ratio (Other Short Term Debt)	42.286***	-0.811	38.727***	2.081***
	(0.888)	(0.546)	(0.850)	(0.492)
Bank Fixed Effects	No	Yes	No	Yes
Quarter Fixed Effects	No	No	Yes	Yes
Number of observations	44,794	44,794	44,794	44,794
R2	0.116	0.299	0.206	0.435

Notes: Data are from the FRY-9C Consolidated Report of Condition and Income. Observations for Other Short Term Deposits are available only from 2002.

Table 4.7: Effect of Mismatch on Dividends: Deposits and Other Short Term Debt, excluding 2007-2009

	log Dividends			
Mismatch Ratio (Deposits)	-0.025***	-0.026***	-0.061***	-0.006***
	(-0.001)	(0.000)	(-0.001)	(-0.001)
Mismatch Ratio (Other Short Term Debt)	44.079***	-0.705	40.377***	2.787***
	(-1.005)	(-0.626)	(-0.959)	(-0.554)
Bank Fixed Effects	No	Yes	No	Yes
Quarter Fixed Effects	No	No	Yes	Yes
Number of observations	37,328	37,328	37,328	37,328
R2	0.109	0.295	0.205	0.452

Notes: Data are from the FRY-9C Consolidated Report of Condition and Income. Observations for Other Short Term Deposits are available only from 2002.

Figure 4.2: Path of the Mismatch Ratio of Deposits and log Dividends for the aggregate economy

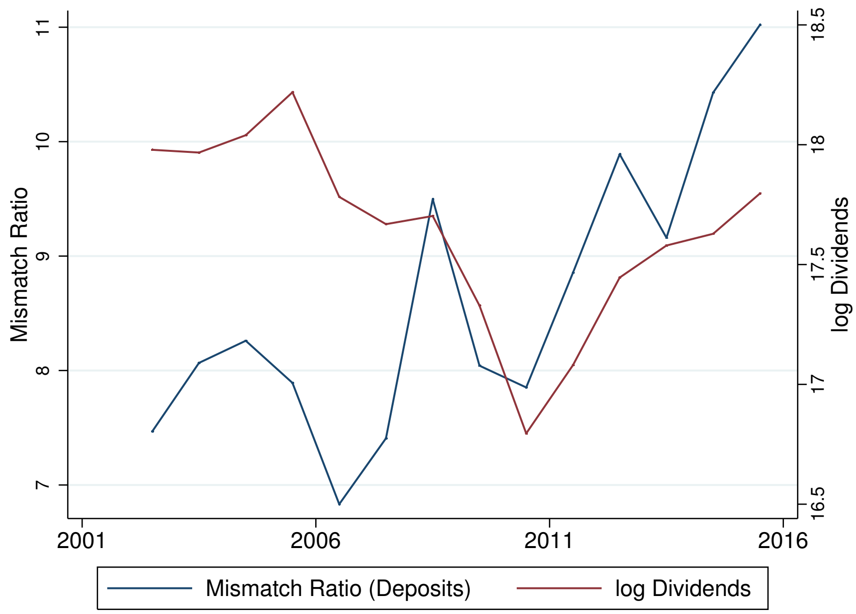
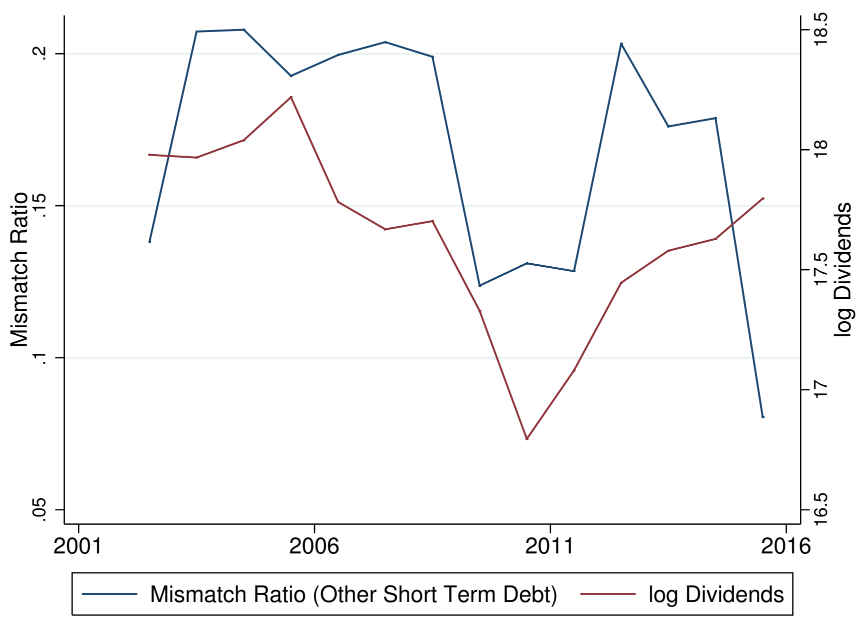


Figure 4.3: Path of the Mismatch Ratio of Other Short Term Debt and log Dividends for the aggregate economy



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Appendix 1: Proof of Proposition 2

We will prove that aggregate capital and output is always lower if $R^b > R^*$. When it is $R^b \leq A_L$, the aggregate capital is

$$K = \frac{\chi}{\Upsilon} w =$$

$$\frac{\left(1 - \frac{\phi}{R^b} A_L + \delta q^b \phi \frac{A_L - A_H}{1-\phi A_H}\right)(1-\delta)\, q^b \frac{R^b - \phi A_L}{1-\phi A_L} + \left(1 - \frac{\phi}{R^b} + (1-\delta)\, q^b \phi \frac{A_H - A_L}{1-\phi A_L}\right) \delta q^b \frac{R^b - \phi A_H}{1-\phi A_H}}{\left(1 - \frac{\phi}{R^b} A_L + \delta q^b \phi \frac{A_L - A_H}{1-\phi A_H}\right)\left(1 - \frac{\phi}{R^b} A_H + (1-\delta)\, q^b \phi \frac{A_H - A_L}{1-\phi A_L}\right)} w. \tag{4.18}$$

with $q^b = \frac{1-R^b}{R^b}$. The denominator is bigger than the numerator if $R^b > \phi A_H = R^*$. In fact, it is

$$\Upsilon = \chi + \left(\frac{R^b - \phi A_H}{R^b} \frac{R^b - \phi A_L}{R^b}\right) \left\{1 - \left(1 - R^b\right) \frac{1 - \phi \left[(1-\delta) A_H + \delta A_L\right]}{(1-\phi A_H)(1-\phi A_L)}\right\}, \tag{4.19}$$

where both the quantities in the round and curly brackets are positive. Then it must be $K < w = K^*$. When it is $A_L < R^b \leq A_H$, the aggregate capital is

$$K = k_H = \frac{1 - \delta\left(1 - R^b\right) - R^b}{1 - \delta\,(1 - R^b) - \phi A_H} w. \tag{4.20}$$

Also in this case the ratio is lower than one as long as $R^b > R^*$, and it must be $K < K^*$.

The result trivially follows for the aggregate output, given that $Y = A_H k_H + A_L k_L \leq A_H\,(k_H + k_L) < A_H w = Y^*$.

Appendix 2: Proof of Proposition 3

We will start showing that $\frac{\partial K}{\partial \delta}$ and $\frac{\partial Y}{\partial \delta}$ are negative for both cases with $R^* < R^b \leq A_L$ and $A_L < R^b \leq A_H$. In the first case the aggregate capital K can be expressed as a function of k_H or k_L:

$$K = \frac{\left(1 - R^b\right) w + \phi \left(A_L - A_H\right) k_L}{1 - \phi A_H} = \frac{\left(1 - R^b\right) w + \phi \left(A_H - A_L\right) k_H}{1 - \phi A_L}. \tag{4.21}$$

Given that

$$\frac{\partial k_L}{\partial \delta} = \frac{\frac{1-R^b}{R^b} \frac{\left(R^b - \phi A_H\right)\left(R^b - \phi A_L\right)}{R^b (1 - \phi A_H)}}{\left(1 - \frac{\phi}{R^b} A_L + \delta \frac{1-R^b}{R^b} \phi \frac{A_L - A_H}{1 - \phi A_H}\right)^2} w > 0, \tag{4.22}$$

it must be $\frac{\partial K}{\partial \delta} < 0$. The result follows for the aggregate output, since it is $Y = \frac{K - \left(1 - R^b\right) w}{\phi}$.

In the case of $A_L < R^b \leq A_H$ we can derive

$$\frac{\partial k_H}{\partial \delta} = \frac{\left(1 - R^b\right) \left(\phi A_H - R^b\right)}{\left[1 - \delta \left(1 - R^b\right) - \phi A_H\right]^2} w < 0. \tag{4.23}$$

Then, it must be $\frac{\partial K}{\partial \delta} < 0$ and $\frac{\partial Y}{\partial \delta} < 0$, since $Y = A_H K = A_H k_H$.

The effect of an increase in δ on the aggregate consumption varies in the two cases with $R^* < R^b \leq A_L$ and $A_L < R^b \leq A_H$. Aggregate consumption in steady state is given by:

$$C = \begin{cases} A_H w & if\ R^b = R^* \\ R^b w + (1 - \phi) Y & if\ R^* < R^b \leq A_L \\ R^b w + (1 - \phi) Y + R^b d_L^U & if\ A_L < R^b \leq A_H \\ w & if\ R^b > A_H \end{cases}. \tag{4.24}$$

When $R^* < R^b \leq A_L$, it is $\frac{\partial C}{\partial \delta} = (1 - \phi) \frac{\partial Y}{\partial \delta} < 0$. In the case of $A_L < R^b \leq A_H$ we can simplify the derivative:

$$\frac{\partial C}{\partial \delta} = \frac{1 - R^b}{\left[1 - \delta\left(1 - R^b\right) - \phi A_H\right]^2} \left(R^b - \phi A_H\right)\left(1 - A_H\right) w. \tag{4.25}$$

A higher δ increases the aggregate consumption only if $A_H < 1$.

Appendix 3: Proof of Proposition 4

We will prove that an increase in R^b always reduces the steady state wage. From labor market clearing it is:

$$\frac{(1-\sigma)\,e}{\left(\frac{w}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}}-\left[\sigma\left(1-\phi\right)+\frac{\phi}{R^b}\right]\frac{\alpha}{1-\alpha}w}+\frac{(1-\sigma)\,e+\left(1-R^b\right)\frac{R^b-R^*}{R^b}\frac{w}{\varphi}}{\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L}\right)^{\frac{1}{\alpha}}-\left[\sigma\left(1-\phi\right)+\frac{\phi}{R^b}\right]\frac{\alpha}{1-\alpha}w}=\frac{2}{\varphi}. \tag{4.26}$$

Taking derivatives with respect to R^b, we get:

$$-\left[\frac{\frac{R^*}{2(R^b)^2}w}{\left(\frac{w}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}}-p\left(R^b,w\right)}-\frac{\frac{R^*}{2(R^b)^2}w}{\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L}\right)^{\frac{1}{\alpha}}-p\left(R^b,w\right)}\right]h_H-\frac{\frac{w}{\varphi}}{\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L}\right)^{\frac{1}{\alpha}}-p\left(R^b,w\right)}$$

$$=\frac{1}{w}\frac{\partial w}{\partial R^b}$$

$$\left\{\frac{\left[\frac{1}{\alpha}\left(\frac{w}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}}-p\left(R^b,w\right)\right]h_H}{\left(\frac{w}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}}-p\left(R^b,w\right)}+\frac{\left[\frac{1}{\alpha}\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L}\right)^{\frac{1}{\alpha}}-p\left(R^b,w\right)\right]h_L-q\left(R^*,R^b,w\right)}{\left(\frac{w}{(1-\alpha)\varepsilon_L^\alpha A_L}\right)^{\frac{1}{\alpha}}-p\left(R^b,w\right)}\right\} \tag{4.27}$$

with $q\left(R^*,R^b,w\right)=\left(1-R^b\right)\frac{R^b-R^*}{R^b}\frac{w}{\varphi}$ and $p\left(R^b,w\right)=\left[\sigma\left(1-\phi\right)+\frac{\phi}{R^b}\right]\frac{\alpha}{1-\alpha}w$. The equation always implies $\frac{\partial w}{\partial R^b}<0$. In fact, the left hand side is negative for sure, given $\varepsilon_H^\alpha A_H>\varepsilon_L^\alpha A_L$.

Appendix 4: Proof of Proposition 5

Aggregate capital in steady state is given by:

$$K = (\varepsilon_H + \varepsilon_L)(1-\sigma)e + (\varepsilon_H h_H + \varepsilon_L h_L)\left[\sigma(1-\phi) + \frac{\phi}{R^b}\right]\frac{\alpha}{1-\alpha}w + \varepsilon_L\left(1-R^b\right)\frac{R^b - R^*}{R^b}\frac{w}{\varphi}. \tag{4.28}$$

Taking derivatives with respect to R^b at $R^b = R^*$ we obtain:

$$\frac{\partial K}{\partial R^b} = \varepsilon_L\left\{\left[\sigma(1-\phi) + \frac{\phi}{R^*}\right]\frac{\alpha}{1-\alpha}\left[h_L^*\frac{\partial w}{\partial R^b} - w^*\frac{\partial h_H}{\partial R^b}\right] + \left[\frac{1-R^*}{\varphi R^*} - \frac{1}{2R^*}h_L^*\right]w^*\right\}$$

$$+\varepsilon_H\left\{\left[\sigma(1-\phi) + \frac{\phi}{R^*}\right]\frac{\alpha}{1-\alpha}\left[h_H^*\frac{\partial w}{\partial R^b} + w^*\frac{\partial h_H}{\partial R^b}\right] - \frac{1}{2R^*}h_H^* w^*\right\}. \tag{4.29}$$

It is easy to see that $\frac{\partial K}{\partial R^b}$ would certainly be negative if $\varepsilon_H = \varepsilon_L$.

In what follows we will show that $\frac{\partial K}{\partial R^b}$ would be positive for small ϕ and ε_H. For simplicity, we will prove the claim for the case $\varepsilon_H^\alpha A_H = \varepsilon_L^\alpha A_L$. We start by taking the limit of $\frac{\partial K}{\partial R^b}$ for ε_H approaching 0 given a constant $\rho = \varepsilon_H^\alpha A_H = \varepsilon_L^\alpha A_L$, i.e. by assuming $A_H = A_H(\varepsilon_H) = \frac{\rho}{\varepsilon_H^\alpha}$. Note that, as soon as $\varepsilon_H^\alpha A_H$ and $\varepsilon_L^\alpha A_L$ are constant, h_H^*, h_L^*, w^*, $\frac{\partial w}{\partial R^b}$ and $\frac{\partial h_H}{\partial R^b}$ do not change as ε_H goes to 0. Then it will be:

$$\lim_{\varepsilon_H \to 0}\frac{\partial K}{\partial R^b} = \varepsilon_L\left\{\left[\sigma(1-\phi) + \frac{1-\alpha}{\alpha}\right]\frac{\alpha}{1-\alpha}\left[\frac{1}{\varphi}\frac{\partial w}{\partial R^b} - w^*\frac{\partial h_H}{\partial R^b}\right] + \left[\frac{1-2R^*}{2\varphi R^*}\right]w^*\right\}. \tag{4.30}$$

$\left[\frac{1-2R^*}{2\varphi R^*}\right]$ would be positive for a small R^*. Moreover, we can rewrite $\left[\frac{1}{\varphi}\frac{\partial w}{\partial R^b} - w^*\frac{\partial h_H}{\partial R^b}\right]$ as:

$$\frac{(w^*)^2}{2\varphi}$$

$$\left[\frac{\frac{1+R^*}{R^*}}{\left(\frac{w^*}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}} - \left[\sigma\left(1-\phi\right) + \frac{\phi}{R^*}\right]\frac{\alpha}{1-\alpha}w^*} - \frac{1}{\frac{1}{\alpha}\left(\frac{w^*}{(1-\alpha)\varepsilon_H^\alpha A_H}\right)^{\frac{1}{\alpha}} - \left[\sigma\left(1-\phi\right) + \frac{\phi}{R^*}\right]\frac{\alpha}{1-\alpha}w^*}\right], \tag{4.31}$$

where the quantity in the square brackets is always positive.

Appendix 5

From the binding borrowing constraint we can find the real return at time t:

$$R_t = \frac{2}{\varphi}\phi \frac{\alpha\lambda_t}{(1-\lambda_t) + (1-\alpha)\lambda_t} \frac{w_t}{d_t^S}; \tag{4.32}$$

with flexible prices it must be:

$$\bar{R}_t = \frac{2}{\varphi}\phi \frac{\alpha(\eta-1)}{1 + (1-\alpha)(\eta-1)} \frac{\bar{w}_t}{d_t^S}. \tag{4.33}$$

Then, we can rewrite:

$$R_t = \bar{R}_t \frac{\lambda_t w_t}{(1-\lambda_t) + (1-\alpha)\lambda_t} \frac{1 + (1-\alpha)(\eta-1)}{(\eta-1)\bar{w}_t}. \tag{4.34}$$

From the equilibrium in the labor market it is:

$$w_t = \frac{\varphi}{2}\left[(1-\lambda_t) + (1-\alpha)\lambda_t\right] Y_t; \tag{4.35}$$

then, we finally obtain

$$R_t^N = R_t \left(\frac{P_t}{P_{t-1}}\right) = \bar{R}_t \left(\frac{\lambda_t Y_t}{\frac{\eta-1}{\eta}\bar{Y}_t}\right)\left(\frac{P_t}{P_{t-1}}\right). \tag{4.36}$$

Appendix 6

The first order conditions of the problem with respect to a_{t+1}, z_T and d_{t+1} are:

$$\chi_t^{-\sigma} = \beta Q \chi_{t+1}^{-\sigma} + \lambda_{t+1}\phi Q \tag{4.37}$$

$$\chi_0^{-\sigma} = \beta^T Q_z \chi_T^{-\sigma} + \phi Q_z \sum_{i=1}^{T} \frac{\lambda_i}{R^{T-1}} \tag{4.38}$$

$$\chi_t^{-\sigma} = \beta R \chi_{t+1}^{-\sigma} + \lambda_{t+1} R. \tag{4.39}$$

For $0 < t \leq T-1$ and $t > T$ the bank must invest in a_{t+1}. Combining the first and third equation we obtain:

$$\lambda_t = \beta \frac{Q-R}{R-\phi Q}\chi_t^{-\sigma}. \tag{4.40}$$

In addition, it must be

$$\frac{\chi_{t+1}}{\chi_t} = \left(\beta Q \frac{1-\phi}{1-\phi\frac{Q}{R}}\right)^{\frac{1}{\sigma}} \forall\, 0 \leq t < T-1 \; and \; t \geq T. \tag{4.41}$$

We can solve recursively in (35):

$$\chi_0^{-\sigma} = \beta^T \frac{Q_z}{\rho\,(Q, Q_z, R, T)}\chi_T^{-\sigma} + \phi \frac{Q_z}{\rho\,(Q, Q_z, R, T)}\lambda_T, \tag{4.42}$$

with

$$\rho\,(Q, Q_z, R, T) = 1 - \phi \frac{Q_z}{R^T}\beta \frac{Q-R}{R-\phi Q}\sum_{i=1}^{T-1}\left(\beta Q \frac{1-\phi}{1-\phi\frac{Q}{R}} R\right)^i. \tag{4.43}$$

Combining (36), (38) and (39), finally we obtain:

$$\chi_0^{-\sigma} = \zeta\left(Q, Q_z, R, T\right)\chi_T^{-\sigma} = \beta^T \frac{Q_z}{\rho\left(Q, Q_z, R, T\right)} \frac{\left(\beta^{T-1} - \phi\right)\left(Q\frac{1-\phi}{1-\phi\frac{Q}{R}}\right)^{T-1}}{\left(\beta Q\frac{1-\phi}{1-\phi\frac{Q}{R}}\right)^{T-1} - \phi\frac{Q_z}{\rho(Q,Q_z,R,T)R}} \chi_T^{-\sigma}. \quad (4.44)$$

If $\zeta\left(Q, Q_z, R, T\right)$ is larger than $\left(\beta Q\frac{1-\phi}{1-\phi\frac{Q}{R}}\right)^T$, then it must be:

$$\frac{\chi_T}{\chi_{T-1}} = \left(\beta\frac{Q_z}{\rho\left(Q, Q_z, R, T\right)}\frac{\beta^{T-1} - \phi}{\left(\beta Q\frac{1-\phi}{1-\phi\frac{Q}{R}}\right)^{T-1} - \phi\frac{Q_z}{\rho(Q,Q_z,R,T)R}}\right)^{\frac{1}{\sigma}} > \left(\beta Q\frac{1-\phi}{1-\phi\frac{Q}{R}}\right)^{\frac{1}{\sigma}}. \quad (4.45)$$