Probing the Large-Scale Structure of the Universe with the Sunyaev-Zel’ dovich Effect

by

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Abstract

The Sunyaev-Zeldovich (SZ) effect is a spectral distortion in the Cosmic Microwave Background (CMB), due to up-scattering of CMB photons by high energy electrons in clusters of galaxies or any cosmic structure. The Planck satellite mission has measured the spectral distortion with great sensitivity and has produced a full-sky SZ ($y$) map, which can be used to trace the large-scale structure of the Universe.

In this dissertation, I construct the average SZ ($y$) profile of $\sim 65,000$ Luminous Red Galaxies (LRGs) from the Sloan Digital Sky Survey Data Release 7 (SDSS/DR7) using the Planck $y$ map and compare the measured profile with predictions from the cosmo-OWLS suite of cosmological hydrodynamical simulations. This comparison agrees well for models that include feedback from active galactic nuclei (AGN feedback).

In addition, I search for the SZ signal due to gas filaments between $\sim 260,000$ pairs of LRGs taken from the Sloan Digital Sky Survey Data Release 12 (SDSS/DR12), lying between 6-10 $h^{-1}$Mpc of each other in the tangential direction and within 6 $h^{-1}$Mpc in the radial direction. I find a statistically significant SZ signal between the LRG pairs. This is the first detection of gas plausibly located in filaments, expected to exist in the large-scale structure of the universe. I compare this result with the BAHAMAS suite of cosmological hydrodynamical simulations and find that it predicts a slightly lower, but marginally consistent result.

As an extension of my MSc. thesis work, I study CMB polarization. The B-mode component of CMB polarization is an important observable to test the theory of inflation in the early universe. However, foreground emissions in our own galaxy dominates the B-mode signal and therefore multi-frequency observations will be required to separate any CMB signal from the foreground emission. I assess the value of adding a new low-frequency channel at 10 GHz for the foreground removal problem by simulating realistic experimental data. I find that such a channel can greatly improve our determination of the synchrotron component which, in turn, significantly improves the reliability of the CMB separation.
Lay Summary

The evolution of structure in the Universe poses many challenges to our understanding. On the very largest scales, the evolution is relatively simple because it is dominated by gravity. But on smaller scales, the structure becomes more complicated because baryonic (gas and plasma) physics begins to become important. One of the key tracers of large scale structure is clusters of galaxies. These objects are the most massive bound systems in the Universe and they roughly mark the transition from the simple larger scales to the more complicated smaller scales. Observational probes of baryonic matter on these scales are difficult to come by. The primary focus of my PhD research has been to study the properties of baryonic gas in and between the clusters of galaxies using the Sunyaev-Zel’dovich(SZ) effect, which allows us to trace the gas distribution through high energy electron distributions in clusters of galaxies or any cosmic structure.
Preface

This dissertation is original, submitted or unpublished, mostly independent work by the author, Hideki Tanimura with the dedicated support from my supervisor, Gary Hinshaw. The images and plots from other materials are re-used based on the Creative Commons Attribution license (CC BY 4.0) in arXiv (https://arxiv.org/help/license).

In Chapters 2, 3, 4, and 5, I conducted all the data analysis including the observational and simulation data. The observational data are publicly available, obtained from the Sloan Digital Sky Survey and Planck satellite mission. The simulation data from the cosmo-OWLs and BAHAMAS suite of cosmological hydrodynamical simulations are kindly provided by Ian McCarthy at Liverpool John Moores University.

The work in Chapters 6 and 7 is an extension of my master’s thesis, “10GHz Sky Survey to probe Inflation with CMB Polarization” (Tanimura, 2013), which has been accepted by University of British Columbia. In this dissertation, the study is extended to the polarization mode, while a majority of the text in the chapters has been extracted from the manuscript in (Tanimura, 2013).
Table of Contents

Abstract .................................................. ii
Lay Summary ............................................ iii
Preface .................................................... iv
Table of Contents ........................................ v
List of Tables ............................................. ix
List of Figures ............................................ x
List of Acronyms ......................................... xiv
Acknowledgements ........................................ xvi

1 Introduction ............................................. 1
  1.1 Modern cosmology .................................... 1
  1.2 Expanding universe .................................. 2
     1.2.1 Friedmann equations ............................ 3
     1.2.2 The Hubble diagram ............................ 5
     1.2.3 Big Bang Nucleosynthesis ..................... 7
     1.2.4 Cosmic microwave background ................. 8
  1.3 Inflation ............................................. 11
     1.3.1 Horizon problem ................................ 13
     1.3.2 Solution to the horizon problem .............. 14
     1.3.3 Primordial power spectrum .................... 15
  1.4 Structure formation ................................ 16
     1.4.1 Linear structure evolution .................... 16
     1.4.2 Non-linear structure evolution ............... 19
     1.4.3 Evolution of baryon ............................ 20
     1.4.4 Clusters of galaxies ........................... 22
  1.5 Sunyaev-Zel’dovich effect ........................ 23
Table of Contents

1.5.1 Kompaneetz equation ........................................... 23
1.5.2 Sunyaev-Zeldovich effect ...................................... 25

2 Construction of Compton parameter $y$ map ..................... 30
  2.1 Introduction .................................................. 30
  2.2 Construction of $y$ map with internal linear combination tech-
      nique ......................................................... 30
  2.3 Comparison with Planck $y$ map ............................... 34
    2.3.1 Compton $y$ parameter profile of Luminous red galaxies 34
    2.3.2 Relation between integrated Compton $y$ parameter
          and stellar masses ........................................ 34
  2.4 Conclusion .................................................. 36

3 Probing hot gas in halos through the Sunyaev Zel’dovich
   effect .......................................................... 39
  3.1 Introduction .................................................. 39
  3.2 AGN feedback effects ......................................... 40
  3.3 Luminous red galaxies (LRGs) ................................ 42
    3.3.1 Luminous red galaxies ................................... 42
    3.3.2 LRG catalog ................................................ 43
  3.4 Compton $y$ parameter profile of the LRGs ................... 44
  3.5 Comparison to cosmo-OWLS hydrodynamic simulations to
      probe AGN feedback effect ................................. 44
    3.5.1 cosmo-OWLS hydrodynamic simulations .................... 44
    3.5.2 Comparison to cosmo-OWLS hydrodynamic simula-
          tions ...................................................... 47
  3.6 Comparison to prediction from halo model and universal pres-
      sure profile ................................................ 48
    3.6.1 The Stacked $y$ profile with cross-correlation of the tSZ
          and distribution of galaxy clusters ..................... 48
    3.6.2 Universal Pressure Profile ................................ 51
    3.6.3 Estimating halo masses of LRGs .......................... 52
    3.6.4 Comparison to the prediction ............................. 53
  3.7 Discussion .................................................. 54
  3.8 Conclusion .................................................. 56

4 Probing hot gas in the cosmic web through the Sunyaev
   Zel’dovich effect ............................................... 58
  4.1 Introduction .................................................. 58
  4.2 Missing baryon problem ....................................... 59
## Table of Contents

4.3 Pair stacking of LRG pairs ........................................ 61
   4.3.1 LRG pair catalog ........................................... 61
   4.3.2 Stacking on LRG pairs ...................................... 62
   4.3.3 Subtracting the halo contribution .......................... 63
   4.3.4 Null tests and error estimates ............................... 66
4.4 Interpretation of the detected tSZ signal between the LRG pairs ........................................ 71
4.5 Comparison to BAHAMAS hydrodynamic simulations ................. 74
   4.5.1 BAHAMAS hydrodynamic simulations ........................ 74
   4.5.2 Comparison with the hydrodynamic simulations .............. 75
4.6 Discussion .......................................................... 76
4.7 Conclusion .......................................................... 79

5 Probing hot gas in the cosmic web between galaxy groups and clusters ........................................ 80
   5.1 Introduction ...................................................... 80
   5.2 Pair stacking of galaxy group/clusters .......................... 80
      5.2.1 Galaxy groups and clusters for SDSS DR10 galaxies ........ 80
      5.2.2 SDSS DR10 group pair catalog .............................. 81
      5.2.3 Stacking on group pairs ..................................... 82
      5.2.4 Subtracting the halo contribution .......................... 82
      5.2.5 Null tests and error estimates ............................... 82
   5.3 Comparison to BAHAMAS hydrodynamic simulations .............. 87
   5.4 Interpretation of the detected tSZ signal between the group pairs ........................................ 88
   5.5 Discussion .......................................................... 88
   5.6 Conclusion .......................................................... 89

6 CMB polarization: A probe of the early universe ................. 91
   6.1 CMB polarization .................................................. 91
      6.1.1 E/B decomposition .......................................... 92
      6.1.2 Observable predictions and current observational constraints ........................................ 94
   6.2 Foreground emission in the microwave bands .................... 95
      6.2.1 Synchrotron emission ....................................... 95
      6.2.2 Free-Free emission .......................................... 97
      6.2.3 Thermal dust emission ....................................... 97
      6.2.4 Spinning dust emission ...................................... 98
7 A 10GHz polarization sky survey ........................................... 99
  7.1 Introduction .............................................................. 99
  7.2 Field of view ............................................................ 100
  7.3 Simulations of the observational data in polarization .......... 102
    7.3.1 Simulations of the polarized skies ......................... 102
    7.3.2 Noise estimate ..................................................... 106
  7.4 Markov chain monte carlo simulation ............................... 107
    7.4.1 Model function ..................................................... 107
    7.4.2 Improvement by the 10GHz data (without spinning dust) ......................................................... 108
    7.4.3 Improvement by the 10GHz data (with spinning dust) 109
  7.5 Conclusion ............................................................... 123

Bibliography ................................................................. 124
### List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>History of Modern Cosmology</td>
<td>2</td>
</tr>
<tr>
<td>1.2</td>
<td>ΛCDM cosmological parameters</td>
<td>13</td>
</tr>
<tr>
<td>2.1</td>
<td>Band data for the Planck $y$ maps (Van Waerbeke et al., 2014)</td>
<td>32</td>
</tr>
<tr>
<td>3.1</td>
<td>The baryon feedback models in the cosmo-OWLS simulation. Each model has been run in both Planck and WMAP7 cosmology (McCarthy et al., 2014).</td>
<td>47</td>
</tr>
<tr>
<td>7.1</td>
<td>Mean noises of stokes Q/U map at each frequency.</td>
<td>106</td>
</tr>
<tr>
<td>7.2</td>
<td>Uncertainty of the fit parameters by MCMC [uK] (without spinning dust).</td>
<td>109</td>
</tr>
<tr>
<td>7.3</td>
<td>Uncertainty of the fit parameters by MCMC [uK] (with spinning dust).</td>
<td>115</td>
</tr>
</tbody>
</table>
## List of Figures

1.1 A recent Hubble diagram of a large combined sample of galaxies using SNIa as standard candles ........................................... 7
1.2 Constraints on the baryon density from Big Bang Nucleosynthesis ................................................................. 9
1.3 CMB Spectrum measured by COBE/FIRAS .................. 10
1.4 CMB temperature fluctuation map from Planck satellite mission ................................................................. 12
1.5 Power spectrum of CMB temperature fluctuations from Planck satellite mission .................................................. 12
1.6 The linear matter power spectrum $P(k)$ versus wavenumber extrapolated to $z=0$ .............................................. 20
1.7 Structure formation of the universe at $z = 0$ in a N-body simulation box 100 Mpc/$h$ on side. ................................. 21
1.8 The mass function of dark matter halos .......................................................... 23
1.9 The CMB spectrum and the distorted spectrum by the SZ effect ................................................................. 25
1.10 Spectral distortion of the CMB radiation due to the Sunyaev-Zel’dovich effect ............................................. 28
1.11 Cleaned images of Abell 2256 at $z = 0.058$ observed by Planck at 100, 143, 217, 353, and 545 GHz ......................... 28
1.12 Planck all-sky Compton parameter maps .......................... 29

2.1 Maps of the Compton parameter, $y$, formed from linear combinations of the Planck HFI maps ......................... 33
2.2 The average $y$ profile of 63,398 LRGs using the Planck $y$ map (blue) is compared with the $y$ profile using our $y$ maps .... 35
2.3 Mean SZ signal vs. stellar mass for locally brightest galaxies derived with our $y$ map version D ................................. 37

3.1 The luminosity function of galaxies .......................................... 41
3.2 First measurement of the $M_{\text{BH}}$-$\sigma$ relation .............................. 42
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>Left: The stellar mass distribution of SDSS DR7 LRGs. Right: The redshift distribution of the LRGs.</td>
</tr>
<tr>
<td>3.4</td>
<td>The average Planck $y$ map stacked against 74,681 LRGs</td>
</tr>
<tr>
<td>3.5</td>
<td>Compton $y$ maps simulated by cosmo-OWLS hydrodynamic simulations</td>
</tr>
<tr>
<td>3.6</td>
<td>The average $y$ profile of LRGs is compared with the beam-convolved $y$ profile of the simulated central galaxies in Planck cosmology</td>
</tr>
<tr>
<td>3.7</td>
<td>The average $y$ profile of LRGs is compared with the $y$ profile of the simulated central galaxies in WMAP7 cosmology</td>
</tr>
<tr>
<td>3.8</td>
<td>Universal pressure profile</td>
</tr>
<tr>
<td>3.9</td>
<td>The relation between the stellar masses of central galaxies and halo masses</td>
</tr>
<tr>
<td>3.10</td>
<td>The average $y$-profile of LRGs is compared with the predictions based on the halo model and UPP</td>
</tr>
<tr>
<td>4.1</td>
<td>Evolution of the four cosmic baryon components in mass fraction</td>
</tr>
<tr>
<td>4.2</td>
<td>The baryon census in the local Universe</td>
</tr>
<tr>
<td>4.3</td>
<td>The distribution of redshift and tangential separations for the selected LRG pairs</td>
</tr>
<tr>
<td>4.4</td>
<td>The average Planck $y$ map stacked against 262,864 LRG pairs</td>
</tr>
<tr>
<td>4.5</td>
<td>The best-fit circular halo profiles fit to the map</td>
</tr>
<tr>
<td>4.6</td>
<td>The residual $y$-map after the best-fit radial halo signals are subtracted</td>
</tr>
<tr>
<td>4.7</td>
<td>A sample null map obtained by stacking the $y$ map against the LRG pairs that were rotated in galactic longitude by random amounts</td>
</tr>
<tr>
<td>4.8</td>
<td>An average $y$ map stacked against a catalog of LRG pseudo pairs</td>
</tr>
<tr>
<td>4.9</td>
<td>The result from 1000 rotated null stacks</td>
</tr>
<tr>
<td>4.10</td>
<td>The result from 1000 pseudo-pair null stacks</td>
</tr>
<tr>
<td>4.11</td>
<td>The single-halo model $y$ map is stacked against the same 262,864 LRG pairs</td>
</tr>
<tr>
<td>4.12</td>
<td>The stacked $y$ map of the central galaxy pairs from the BA-HAMAS simulations, at 10 arcsecond angular resolution (unsmoothed)</td>
</tr>
<tr>
<td>5.1</td>
<td>The distribution of redshift and tangential separations for the group pairs</td>
</tr>
</tbody>
</table>
List of Figures

5.2 The average Planck $y$ map stacked against 34,955 group pairs
5.3 The best-fit circular halo profiles fit to the map
5.4 The residual $y$-map after the best-fit radial halo signals are subtracted
5.5 The result from 1000 rotated null stacks
5.6 The result from 1000 pseudo-pair null stacks
6.1 $E$-mode and $B$-mode patterns of polarization
6.2 Angular power spectra of CMB with varying tensor-to-scalar ratio
6.3 Brightness temperature rms as a function of frequency and astrophysical component for temperature and polarization
7.1 The image of observation strategy at Penticton
7.2 Hit map of one-year of observations, in galactic coordinates ($N_{side} = 64$)
7.3 The simulated stokes Q maps
7.4 The simulated stokes U maps
7.5 Correlation maps among the fit parameters (without spinning dust)
7.6 Stokes Q spectra (without spinning dust)
7.7 Stokes U spectra (without spinning dust)
7.8 Input, output(best-fit) and residual maps of $Q_s$ estimated by MCMC, without spinning dust
7.9 Input, output(best-fit) and residual maps of $U_s$ estimated by MCMC, without spinning dust
7.10 Input, output(best-fit) and residual maps of $\beta_s$ estimated by MCMC, without spinning dust
7.11 Correlation maps among the fit parameters (with spinning dust)
7.12 Stokes Q spectra (with spinning dust)
7.13 Stokes U spectra (with spinning dust)
7.14 Input, output(best-fit) and residual maps of $Q_s$ estimated by MCMC, with spinning dust
7.15 Input, output(best-fit) and residual maps of $U_s$ estimated by MCMC, with spinning dust
7.16 Input, output(best-fit) and residual maps of $\beta_s$ estimated by MCMC, with spinning dust
7.17 Input, output(best-fit) and residual maps of $Q_{sp}$ estimated by MCMC, with spinning dust
7.18 Input, output (best-fit) and residual maps of $U_{sp}$ estimated by MCMC, with spinning dust. 

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## List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACT</td>
<td>Atacama Cosmology Telescope</td>
</tr>
<tr>
<td>AGN</td>
<td>Active Galactic Nuclei</td>
</tr>
<tr>
<td>BAO</td>
<td>Baryonic Acoustic Oscillation</td>
</tr>
<tr>
<td>BBN</td>
<td>Big Bang Nucleosynthesis</td>
</tr>
<tr>
<td>CMB</td>
<td>Cosmic Microwave Background</td>
</tr>
<tr>
<td>CDM</td>
<td>Cold Dark Matter</td>
</tr>
<tr>
<td>Dec</td>
<td>Declination</td>
</tr>
<tr>
<td>DR7</td>
<td>Data Release 7</td>
</tr>
<tr>
<td>DR12</td>
<td>Data Release 12</td>
</tr>
<tr>
<td>FOF</td>
<td>Friends-Of-Friends</td>
</tr>
<tr>
<td>FOV</td>
<td>Field Of View</td>
</tr>
<tr>
<td>FRW</td>
<td>Friedmann-Robertson-Walker</td>
</tr>
<tr>
<td>FWHM</td>
<td>Full Width Half Maximum</td>
</tr>
<tr>
<td>HMF</td>
<td>Halo Mass Function</td>
</tr>
<tr>
<td>IC</td>
<td>Inverse Compton</td>
</tr>
<tr>
<td>ICM</td>
<td>Intra Cluster Medium</td>
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<tr>
<td>IGM</td>
<td>Inter Galactic Medium</td>
</tr>
<tr>
<td>ILC</td>
<td>Internal Linear Combination</td>
</tr>
<tr>
<td>ISM</td>
<td>Inter Stellar Medium</td>
</tr>
<tr>
<td>LRG</td>
<td>Luminous Red Galaxies</td>
</tr>
<tr>
<td>LSS</td>
<td>Last Scattering Surface</td>
</tr>
<tr>
<td>PSF</td>
<td>Point Spread Function</td>
</tr>
<tr>
<td>RA</td>
<td>Right Ascension</td>
</tr>
<tr>
<td>SDSS</td>
<td>Sloan Digital Sky Survey</td>
</tr>
<tr>
<td>SHM</td>
<td>Stellar-to-Halo Mass (relation)</td>
</tr>
<tr>
<td>SMBH</td>
<td>Super Massive Black Hole</td>
</tr>
<tr>
<td>SNe</td>
<td>SuperNovae</td>
</tr>
<tr>
<td>SPT</td>
<td>South Pole Telescope</td>
</tr>
<tr>
<td>SZ</td>
<td>Sunyaev-Zel’ dovich (effect)</td>
</tr>
<tr>
<td>UPP</td>
<td>Universal Pressure Profile</td>
</tr>
<tr>
<td>WHIM</td>
<td>Warm Hot Intergalactic Medium</td>
</tr>
</tbody>
</table>
## List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMAP</td>
<td>Wilkinson Microwave Anisotropy Probe</td>
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<tr>
<td>ΛCDM</td>
<td>Lambda Cold Dark Matter</td>
</tr>
</tbody>
</table>
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Chapter 1

Introduction

1.1 Modern cosmology

Einstein’s theory of General Relativity, developed almost 100 years ago, has dramatically changed the view of our universe. The Einstein’s equations were applied to the universe and it was noticed that the universe is dynamical by Friedmann and LeMaître. At the time, it was believed that the universe was static, thus, a constant term called “cosmological constant” was added to enforce a static universe. However, after the Hubble’s discovery, the non-static universe was widely accepted and the cosmological constant was dropped. The Hubble’s observation implied that the universe is expanding, and was presumably much hotter and denser in the past. The idea of an expanding universe, called the Big Bang theory, is now supported by three important observational results: Hubble’s observed expansion, light element abundances implying primordial nucleosynthesis, and the cosmic microwave background (CMB) left over from 380,000 years after the creation of the universe, which are described in the next section.

Once the cosmological constant was dropped, however, recent observations strongly support a non-vanishing cosmological constant ($\Lambda$). The non-zero cosmological constant results in an acceleration of the expansion in the Friedman equations derived from the Einstein’s equations. The nature of this exotic component, called “dark energy”, has been studied to explain the acceleration of the universe discovered by measurements of distant type Ia supernovae. In addition, the existence of non-baryonic matter, called “(cold) dark matter” (CDM) is supported by observations such as rotation curve of our galaxy, gravitational lensing effects, CMB and so on. Including these mysterious dark components, currently, the standard model of cosmology is called “ΛCDM model”. The model is discussed further in this chapter. The main historical events supporting modern cosmology are summarized in Tab. 1.1.
1.2. Expanding universe

Modern cosmology is constructed based on the assumption that the universe is homogeneous and isotropic and no special place does not exist, which is called "Cosmological Principle". It is an important basis that the laws of physics are universal. Homogeneity means that the Universe looks the same anywhere, while isotropy states that the Universe looks the same in all directions. It seems contradictory to the fact that the distribution of planets, stars and galaxies in our nearby universe is highly inhomogeneous. However, the principle is consistent with the recent observations extended over a large scale. For isotropy, (i) radio galaxies are randomly distributed across the entire sky: NRAO VLA Sky Survey (NVSS) detected nearly $2 \times 10^6$ sources stronger than $S = 2.5$ mJy at 1.4 GHz. The distribution of the discrete sources on the sky is extremely isotropic (Condon et al., 1998), (ii) distant galaxies with the same distance (known by standard candles such as SNIa) are moving away from us with the same speed (redshift) in any direction of the sky, implying that the expansion of the universe is isotropic, (iii) the temperature of the cosmic microwave background radiation is same in all directions within a fractional precision better than $10^{-4}$, after subtracting the contribution from the CMB dipole component due to the proper motion of our solar system and also the contribution from point sources such as clusters of galaxies causing the distortion of the CMB spectra. For homogeneity, as a result of large galaxy surveys, it is found that their spatial distribution consists of a tangled cosmic web structure up to 400 Mpc (1 pc = 3.26 light year), but on scales larger than 400 Mpc, little structure is
found. Therefore, the cosmological principle is currently supported by the observational facts.

1.2. Expanding universe

1.2.1 Friedmann equations

Friedmann solved the Einstein’s field equations assuming the cosmological principle. A homogeneous and isotropic expanding universe is expressed by the Friedmann-Robertson-Walker (FRW) metric as \( c = 1 \) (c is the speed of light),

\[
ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right],
\]

where \( ds \) is a geodesics, \( dt \) is a time, \( a(t) \) is a time-dependent scale(expansion) factor, and \( K \) is a constant representing a curvature of the space (flat or positive/negative curvature space) in polar coordinates \( (r, \theta, \phi) \). The Einstein’s field equations describe the relation between the space-time geometry and energy-momentum,

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu},
\]

where \( R_{\mu\nu} \) is the Ricci tensor, which depends on the metric \( (g_{\mu\nu}) \) and its derivatives; \( R \) is the Ricci scalar \( (R \equiv g^{\mu\nu}R_{\mu\nu}) \); \( G \) is Newton’s constant; \( \Lambda \) is the cosmological constant; and \( T_{\mu\nu} \) is the energy-momentum tensor. By applying the FRW metric to the Einstein’s equations, the equations of motion called “Friedmann equations” can be derived, which determine the evolution of the scale factor,

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} + \frac{\Lambda}{3},
\]

\[
\ddot{a} = -\frac{4\pi G}{3} (\rho + 3P) + \frac{\Lambda}{3},
\]

where \( \rho \) is the energy density and \( P \) is the pressure of an isotropic fluid. In addition, the energy-momentum conservation law in an expanding universe, \( T^\mu_{\nu;\mu} = 0 \), gives

\[
\dot{\rho} + 3\frac{\dot{a}}{a} (\rho + P) = 0,
\]

which can be also acquired from the above two equations.

To solve for the scale factor as a function of cosmic time, an equation of state for the fluid, which is the relationship between the energy density and
1.2. Expanding universe

pressure, is necessary. It takes the form of $P = w \rho$, using a dimensionless number $w$. Eq. [1.5] can be solved for a constant $w$ as

$$ \rho \propto a^{-3(1+w)}. \quad (1.6) $$

The universe is filled with a mixture of different ingredients characterized by different equations of state. The essential components in the universe are radiation, matter and dark energy (cosmological constant). Since each component reacts to the expansion differently, $w$ should be also a function of time. However, one has a period when one component is dominant over the others, thus, the constant $w$ would be valid for most of the cosmic time.

The early universe was filled with radiation. In the radiation dominated era, $P = \frac{1}{3} \rho$, and the evolution of the scale factor is $\rho_r \propto a^{-4}$. The energy density of radiation decreased quickly and it was overcome by the one of matter. In the matter dominated era, the pressure is much smaller than the energy density, $|P| \ll \rho$, and $P \simeq 0$ gives $\rho_m \propto a^{-3}$, so that matter dilutes by following $1/V = 1/a^3$.

The energy density of matter also decreased with time and now the universe is dominated by dark energy (cosmological constant). Without the cosmological constant, Eq. [1.4] states that both the energy density and pressure cause a deceleration in the expanding universe. However, according to observations, the total matter density is only one third of the critical density ($\rho_{cr} = 3H_0^2/8\pi G$: characteristic density to make the geometry of the Universe flat) and $a(t)$ is accelerating, $\ddot{a} > 0$. Thus, the cosmological constant, which was once proposed by Einstein to achieve a stationary universe, has been revived to explain the acceleration of the universe. As in Eq. [1.4], the cosmological constant can cause an acceleration in the expansion of the universe. For the cosmological constant to be constant, $w = -1$ in Eq. [1.6]. In this case, the scale factor grows exponentially. Note that the expansion of the universe accelerates for any equation of state of $w < -1/3$.

Considering the density evolution of the various cosmic components,

$$ \rho_m(t) = \rho_{m,0} a^{-3(t)}, \quad (1.7) $$

$$ \rho_r(t) = \rho_{r,0} a^{-4(t)}, \quad (1.8) $$

$$ \rho_\Lambda(t) = \rho_\Lambda = \Lambda/8\pi G = const, \quad (1.9) $$

where $\rho_{r,0}$ is the radiation energy density today, $\rho_{m,0}$ is the matter energy density today and $\rho_\Lambda$ is the dark energy density, and the dimensionless density parameters for matter, radiation, and dark energy can be defined as

$$ \Omega_m = \frac{\rho_{m,0}}{\rho_{cr}}; \quad \Omega_r = \frac{\rho_{r,0}}{\rho_{cr}}; \quad \Omega_\Lambda = \frac{\rho_{\Lambda,0}}{\rho_{cr}}. \quad (1.10) $$
1.2. Expanding universe

The density parameters influence the expansion of the universe. With the density parameters and \( \rho = \sum \rho_i \), the expansion equation (Hubble parameter) can be written as

\[
H^2(t) = H_0^2 [a(t)^{-4} \Omega_r + a(t)^{-3} \Omega_m + a(t)^{-2} \Omega_k + \Omega_\Lambda],
\]

(1.11)

for \( H(t_0) = H_0 \) and \( a(t_0) = 1 \), including the curvature density parameter \( \Omega_k = K/H_0^2 \).

The radiation energy density today is very small and can be neglected when compared to that of matter. However, since \( \rho_r \) evolves faster than \( \rho_m \), the radiation and matter had the same energy density at an epoch,

\[
\frac{\rho_r(t)}{\rho_m(t)} = \frac{\rho_{r,0}}{\rho_{m,0} a(t)} = \frac{\Omega_r}{\Omega_m a(t)} \simeq 1.
\]

(1.12)

From the radiation density today, \( a_{eq} \sim 4.2 \times 10^{-5} (\Omega_m h^2)^{-1} \) is an epoch when the radiation and matter had an equal value of energy density, and the radiation energy density was dominant before the time. These equations are the basis of the standard Big Bang cosmological model.

1.2.2 The Hubble diagram

The expansion of the universe causes galaxies to move away from each other. Therefore, we should see all the galaxies receding from us. However, we can not see the recession of distant galaxies directly. Otherwise, it is observed as a stretch of light wavelength, such as Doppler shift. This stretch factor is defined as redshift \( z \):

\[
1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}},
\]

(1.13)

where \( \lambda_{\text{emit}} \) is the wavelength of light at emission and \( \lambda_{\text{obs}} \) is the wavelength of light at observation. Hubble found that distant galaxies are in fact receding from us and, in addition, the velocity increases with distance. This is exactly expected result in a homogeneously expanding universe, which is called Hubble’s law:

\[
v = H_0 d,
\]

(1.14)

where \( H_0 \) is called “Hubble constant”, \( v \) is the receding velocity of an object from us and \( d \) is the distance to the object.

The redshift is actually caused by the expansion of the universe. The equation for a light wave in the isotropic universe is given from Eq.1.1,

\[
ds^2 = -dt^2 + \frac{a^2(t) dr^2}{1 - Kr^2},
\]

(1.15)
1.2. Expanding universe

and \( ds = 0 \) provides a straight line in a curved space-time for the light wave. Assuming a flat space \((K = 0)\), the total path is given by the integral in both space and time,
\[
\int \frac{dt}{a(t)} = \int dr.
\] (1.16)

Suppose a crest of the light wave was emitted at \( t = t_1 \) in the past and the subsequent crest was emitted at \( t = t_1 + \delta t_1 \), and for an observer, the first crest was observed at \( t = t_0 \) and the subsequent crest was observed at \( t = t_0 + \delta t_0 \). Since the both crests travel through the same (comoving) distance,
\[
\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_{t_1 + \delta t_1}^{t_0 + \delta t_0} \frac{dt}{a(t)}.
\] (1.17)

Note that the comoving distance is a distance that does not change in time due to the expansion and the physical(proper) distance can be obtained by multiplying it by the scale factor, \( a(t) \). Over the period of the light wave, the scale factor is essentially constant. This yields
\[
\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)}.
\] (1.18)

The wavelength at the source and observer are given by \( \lambda_1 = c \delta t_1 \) and \( \lambda_0 = c \delta t_0 \). Therefore, using Eq. 1.13 and \( a(t_0) = 1 \) (normalization of the scale factor), the redshift is obtained by
\[
1 + z = \frac{\lambda_0}{\lambda_1} = \frac{1}{a(t_1)}.
\] (1.19)

It is called “cosmological redshift”, which means that the wavelength emitted at the time with the scale factor, \( a(t) \), is stretched by \( 1/a(t) \) \( (a(t) \leq 1) \) due to the expansion.

The relation between the distance and redshift of distant objects is shown in the Hubble diagram. Redshifts are obtained by observations directly, but distances are difficult. One of the most useful techniques to measure distances is find a object with the same luminosity, called standard candle. Currently Type Ia supernovae (SNIa) is used to extend the Hubble diagram out to very large redshifts, \( z \sim 1.7 \). Fig. 1.1 is the recent Hubble diagram of a large combined sample of galaxies using SNIa as standard candles for distance measurement (Betoule et al., 2014). Using the dataset, they find \( \Omega_m = 0.295 \pm 0.034 \) for a flat \( \Lambda \)CDM cosmology. The result is consistent with the CMB measurements from the WMAP and Planck experiments. The best-fit \( \Lambda \)CDM cosmology for a fixed \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) is shown
in black line. The Hubble diagram is still the most direct evidence of the expanding universe.

![Graph](image)

**Figure 1.1:** *Top:* A recent Hubble diagram of a large combined sample of galaxies using SNIa as standard candles for distance measurement (Betoule et al., 2014) (arXiv:1401.4064). The graph presents distance (as distance modulus; proportional to log of distance) vs. redshift $z$. The different SNIa samples are denoted by different colors [low-z sample; Sloan SDSS sample; SN legacy survey, SNLS; and Hubble Space Telescope SNIa, HST]. The distance modulus redshift relation of the best-fit $\Lambda$CDM cosmology for a fixed $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ is shown in black line. *Bottom:* residuals from the best-fit $\Lambda$CDM cosmology as a function of redshift.

### 1.2.3 Big Bang Nucleosynthesis

In the middle of 20th century, it was considered that all the elements heavier than hydrogen were formed in stellar interiors by nuclear fusions or by supernova explosions. However, according to observations, it was found that $\sim$25% of (baryonic) matter in the universe comprises of helium in mass and it was much larger than the prediction of the stellar theory. A similar mystery was present also for deuterium. Even worse, it is destroyed inside stars rather than produced. To solve for the problem, George Gamow suggested a
1.2. Expanding universe

theory that the light elements should have been produced in the very early universe, which is now called Big Bang Nucleosynthesis (BBN).

The Big Bang model implies that the universe was much hotter and denser, and most of baryons existed as protons and neutrons at the temperature of $\geq 10^9$ [K]. At such a high temperature, atomic nuclei are quickly destroyed by high energy photons. Along with the expansion of the universe, the temperature cooled down less than binding energies of typical atomic nucleus and then nucleosynthesis began. In the nucleosynthesis, deuterium formed first, and subsequently, light elements such as helium and lithium followed. However, the atomic nuclei heavier than lithium were not produced because stable elements do not exist at the atomic number of 5 and 8, which would have been necessary to produce further heavier elements. The production of atomic nuclei stopped when the universe cooled down to the temperature of $\sim 4 \times 10^8$ [K]. Through the process, most of neutrons were captured in the atomic nuclei and the leftover decayed into protons. The BBN theory predicts the amount of the light elements produced in the early universe and it is consistent with observations. Thus, the abundance of light elements is the important basis to support the Big Bang model. Now it is recognized that light elements (deuterium, helium, and lithium) were mainly produced in the very early universe after the Big Bang, while heavy elements were produced inside of stars much later than that.

Fig.1.2 is the prediction from BBN for the light element abundances, which depend on the amount of baryons at the time of BBN. Therefore, observations of the light elements can constrain the baryon density in the universe. In particular, the primordial deuterium has a high sensitivity to the baryon density and the result is consistent with the outcome from the CMB measurements, though the primordial $^7$Li abundance derived from observations is lower than the expected. The derived baryon density indicates that the baryon contribution is only 5 % of the critical density. However, the total matter density is estimated to be $\sim 30$ %. This result supports the existence of non-baryonic matter, called “dark matter”.

1.2.4 Cosmic microwave background

The discovery of the cosmic microwave background (CMB) strongly supports the Big Bang theory. In the Big Bang model, the early universe was filled with a hot and dense plasma consisting of photons, electrons, baryons and dark matter. The expansion of the universe caused an adiabatic cooling of the plasma until the temperature allowing electrons to combine with protons and form neutral hydrogen atoms stably. This event is called “re-
1.2. Expanding universe

Figure 1.2: Constraints on the baryon density from Big Bang Nucleosynthesis (Fields et al., 2014) (arXiv:1412.1408). The abundances of $^4$He, D, $^3$He and $^7$Li as predicted by the standard model of the BBN. The bands show the 95% CL range. Boxes indicate the observed light element abundances. The narrow vertical band indicates the CMB measure of the cosmic baryon density by WMAP, while the wider band indicates the BBN concordance range (both at 95% CL).
1.2. Expanding universe

The recombination happened approximately 380,000 years after the Big Bang when the temperature went down to around 3,000 K. After the recombination, the universe became neutral and also transparent for photons since most of free electrons were captured in hydrogen atoms. This resulted in “decoupling” of matter and radiation, and the radiation began to travel freely through space. This ancient radiation is called “cosmic microwave background radiation” and reaches us today isotropically from a spherical surface called “last scattering surface”.

The Big Bang theory predicts a nearly blackbody spectrum of the CMB radiation with a lower temperature since it must have been emitted from a very optically thick plasma and the temperature should have been cooled down due to the expansion of the universe. It also predicts its isotropy since the radiation source should be very far and the last scattering surface is receding from us in the same way. The observations are completely consistent with these predictions and can not be explained by other sources. Currently the CMB radiation is measured very precisely (Fig. 1.3) and the temperature is found to be $2.725 \pm 0.002$ (Fixsen et al., 1996). This is one of the greatest successes of the Big Bang theory.

Figure 1.3: CMB Spectrum measured by COBE/FIRAS (Fixsen et al., 1996). The FIRAS instrument aboard COBE measured the spectrum of the CMB precisely, and found a blackbody spectrum with deviations limited to 50 ppm of the peak brightness, with a peak wavelength of 1.869 mm, corresponding to a temperature of $T = 2.725 \pm 0.002$ K.
1.3 Inflation

The Big Bang theory also predicts a temperature anisotropy in the CMB. The temperature anisotropy was first detected by COBE \cite{Smoot1992}. It is believed to be caused by inhomogeneities in the matter distribution at the epoch of recombination. The discovery provides an evidence that density fluctuations existed since a very early universe, perhaps caused by quantum fluctuations in the scalar field of inflation or by some other mechanisms. It is now widely accepted that the primordial density fluctuations grew with a gravitational collapse and seeded the formation of current cosmic structure such as galaxies and clusters of galaxies.

The CMB anisotropies have been measured more precisely by WMAP \cite{Hinshaw2013} and Planck \cite{Planck2016c} (Fig. 1.4). In particular, the Planck mission, with high sensitivity and small angular resolution, measured the CMB anisotropies up to smaller angular scales and several acoustic peaks were detected in the power spectrum of the CMB temperature fluctuations (Fig. 1.5). Note that the power spectrum characterizes the amplitude of fluctuations as a function of angular scale, \( l = \pi / \theta \).

The acoustic oscillations arise from a balance in the photon-baryon plasma. The pressure of photons erase the anisotropies, whereas the gravitational attraction of baryon and dark matter raises the anisotropies. These two effects compete and create acoustic oscillations in the CMB. The CMB photons decouple when a particular spatial wavelength is at its peak amplitude.

The \( \Lambda \)CDM theory predicts the acoustic peaks, depending on cosmological parameters. The angular scale of the first peak probes the curvature of the universe and the WMAP and Planck results demonstrate that the geometry of the Universe is flat, rather than curved. The ratio of the odd peaks to the even peaks determines the baryon density. The third peak can be used to get information about the dark matter density. The best-fit \( \Lambda \)CDM parameters are summarized in Tab. 1.2.

1.3 Inflation

The Big Bang model explains many observations such as the Hubble diagram, the abundance of the light elements, and CMB. However, this model raises new questions.
1.3. Inflation

Figure 1.4: CMB temperature fluctuation map by Planck satellite mission (Planck Collaboration 2016a) (arXiv:1502.01582).

Figure 1.5: Power spectrum of CMB temperature fluctuations from Planck satellite mission (Planck Collaboration 2016a) (arXiv:1502.01582).
1.3. Inflation

<table>
<thead>
<tr>
<th>Label</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>$\Omega_b$</td>
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<tr>
<td>$\Omega_{CDM}$</td>
<td>Dark matter fraction</td>
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<td>$\Omega_\Lambda$</td>
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<td>$\tau$</td>
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<td>$n_s$</td>
<td>Scalar index</td>
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<tr>
<td>$10^9 \Delta^2_R$</td>
<td>Scalar amplitude</td>
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<td>$H_0$ (km/s/Mpc)</td>
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<tr>
<td>$z_{reion}$</td>
<td>Reionization period</td>
<td>$8.5^{+1.0}_{-1.1}$</td>
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Table 1.2: ΛCDM cosmological parameters from Planck CMB power spectra, in combination with CMB lensing reconstruction and external data including BAO(6dFGRS, SDSS, BOSS and WiggleZ), SNe(JLA) and $H_0$(HST Cepheid+SNe) (Planck Collaboration, 2016c).

1.3.1 Horizon problem

In the theory of relativity, the speed of light is finite, meaning that we can only observe a limited part of the entire universe, which is called “horizon”. The horizon also existed at an earlier time. The horizon scale at the recombination ($z \sim 1, 100$) can be calculated as follows.

In a time interval $dt$, light can travel a distance of $c dt$. In a comoving distance, it corresponds to $d\chi = c dt / a(t)$. Therefore, the horizon at a time $t$ from the Big Bang is

$$\chi_{\text{hor}} = \int_0^t \frac{c dt}{a(t)}, \quad (1.20)$$

in comoving distance. This can be expressed with a scale parameter, $a(t)$, using $dt = da / \dot{a} = da / (aH)$,

$$\chi_{\text{hor}} = \int_0^{(1+z)^{-1}} \frac{c da}{a^2 H(a)}, \quad (1.21)$$

and $H(a)$ can be expressed with the density parameters,

$$H^2(a) = H_0^2[a^{-4}\Omega_r + a^{-3}\Omega_m + a^{-2}\Omega_k + \Omega_\Lambda]. \quad (1.22)$$

Since $z_{eq}(\sim 3, 400) \gg z_{rec}(\sim 1, 100)$ at the recombination and only a small fraction of time is in the radiation dominated era, it would be valid that
most of the time have been in the matter dominated era, which leads to $H(a) \approx H_0 \sqrt{\Omega_m a^{-3/2}}$. Substituting it to Eq. (1.21) gives

$$\chi_{\text{hor}} \approx 2 \frac{c}{H_0} \frac{1}{\sqrt{(1+\delta)\Omega_m}},$$

where $z_{eq} \sim 3400$ is the redshift (epoch) when radiation and matter had an equal value of energy density. Since the physical distance can be calculated by

$$x_{\text{hor}} \approx 2 \frac{c}{H_0} \Omega_{m}^{1/2}(1+z)^{-3/2},$$

the angular size of the horizon on the sky in case of $\Omega_\Lambda = 0$ can be simply given by

$$\theta_{\text{hor,rec}} = \frac{x_{\text{hor}}(z_{\text{rec}})}{D_A(z_{\text{rec}})} \approx \sqrt{\frac{\Omega_m}{z_{\text{rec}}}} \sim 1^\circ,$$

where $D_A$ is an angular diameter distance. It means that CMB radiations separated by more than about one degree were not causally connected before the recombination. However, due to the observations, the CMB temperatures are same from all the direction of the sky within only fluctuations of $\Delta T/T \sim 10^{-5}$, which is called “Horizon problem”.

The second problem is called “Flatness problem”. The WMAP and Planck observations suggest that the geometry of our universe is nearly flat, which means the energy density of the universe is nearly equal to the critical density and our universe is at a peculiar point between an eternal expansion and eventual collapse. Since the curvature of spacetime would grow with time, the probability of such a situation is very unlikely and it would requires an extreme fine-tuning in the past. However, the (traditional) Big Bang theory can not provide a solution for it.

### 1.3.2 Solution to the horizon problem

A. Guth came up with a solution for these questions (Guth, 1981). He postulates that our universe experienced a very rapid accelerated expansion, called “Inflation”, in a very early stage. It can be realized if the universe is dominated by a potential energy of slowly rolling scalar field.

In the Friedmann equation, the slow rolling scalar field takes the form of

$$H^2 = \frac{8\pi G}{3} \rho = \frac{8\pi G}{3} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right) \approx \frac{8\pi G}{3} V(\phi),$$

where $\dot{\phi}^2/2$ is the kinetic and $V(\phi)$ is the potential term of the scalar field, and the potential term is dominant over the kinetic term. For a constant
1.3. Inflation

\[ 8\pi GV(\phi)/3 = H^2 = \text{const} \] and the solution is simply \( a(t) \propto \exp(\text{H}t) \), resulting in an exponential expansion.

How much inflation is required to solve the horizon problem? For inflation to be valid, so-called “comoving Hubble radius” at the start of inflation had to be larger than the current comoving Hubble radius. Note that the comoving Hubble radius is the radius of the observable universe in comoving distance unit, which is \( 1/aH \). Inflation causes the decrease of comoving Hubble radius due to the increase of the scale factor.

Most inflationary models typically assume the energy scale at inflation to be order \( 10^{15} \) GeV or larger, thus,

\[
\frac{a_0 H_0}{a_e H_e} \simeq \frac{T_0}{10^{15}\text{GeV}} \simeq 10^{-28},
\]

where \( a_e \) and \( H_e \) are the scale factor and Hubble constant at the end of inflation, and \( T_0 \) is the CMB temperature today. Thus, inflation can solve the horizon problem if the universe expands exponentially for more than 60 e-folds \( \ln(10^{28}) \sim 64 \).

Inflation can also solve the flatness problem. Inflation and its subsequent expansion has essentially flattened the curvature of the universe, just as the world appears flat for a observer on the surface of the earth.

1.3.3 Primordial power spectrum

Inflation also explains the origin of structure in the universe. Before the inflation, the size of the universe observed today was microscopic and the quantum fluctuations in the microscopic scales expanded to astronomical scales during the inflation. The quantum fluctuations grow over a long period and build up a large-scale structure today such as stars, galaxies, and clusters of galaxies.

The slow-roll inflation produces a spectrum of curvature perturbations \( (P_R) \) that is almost scale-invariant since no characteristic length-scale existed then. The small deviation from the scale-invariance can be quantified by forming a spectral index \( n_s(k) \) as a function of wavenumber, \( k = 2\pi/L \),

\[
n_s - 1 \equiv \frac{d \ln P_R}{d \ln k},
\]

where \( n_s = 1 \) is a scale-free spectrum, called Harrison-Zel’dovich spectrum. A constant \( n_s \) is a power-law spectrum,

\[
P_R(k) = A_s(k_s) \left( \frac{k}{k_s} \right)^{n_s-1},
\]

15
where $k_*$ is a pivot scale.

The Planck observation finds $n_s = 0.9603 \pm 0.0073$ with a significance of more than 5 standard deviations (Planck Collaboration 2016a) and it is considered as a great success of the inflation theory. This scenario also explains several important observations about flatness, isotropy and homogeneity of the universe. Thus, the inflation model is the best theory to explain the earliest moment of the universe so far.

### 1.4 Structure formation

#### 1.4.1 Linear structure evolution

The formation and evolution of the large-scale structure induced by inhomogeneities. In the early universe, the inhomogeneities were subtle. The small inhomogeneities can be treated in a perturbation theory. In addition, Newtonian gravity is an adequate description for non-relativistic matter such as cold dark matter and baryons after recombination (Schneider 2006). Consider a non-relativistic fluid in an element of mass density $\rho$, pressure $P \ll \rho$ and velocity $\mathbf{u}$ with a position $\mathbf{r}$ and time $t$. For the fluid, the equations of motion are given by the continuity equation and Euler equation.

\[
\partial_t \rho = - \nabla \cdot (\rho \mathbf{u}), \tag{1.30}
\]

\[
\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{r} = - \frac{\nabla P}{\rho} - \nabla \Phi, \tag{1.31}
\]

where the gravitational potential, $\Phi$, is determined by the Poisson equation,

\[
\nabla^2 \Phi = 4\pi G \rho. \tag{1.32}
\]

In the expanding universe, physical coordinates, $\mathbf{r}$, can be replaced with comoving coordinates, $\mathbf{x}$ by $\mathbf{r}(t) = a(t)\mathbf{x}$. The velocity field is also replaced by $\mathbf{u}(t) = \dot{\mathbf{r}} = H\mathbf{r} + \mathbf{v}$ where $H\mathbf{r}$ is the Hubble flow and $\mathbf{v}$ is the proper velocity.

Introducing the density perturbation of

\[
\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}} \tag{1.33}
\]

in the continuity equation, Euler equation and Poisson equation, the first order equations result in

\[
\dot{\delta} = -\frac{1}{a} \nabla \cdot \mathbf{v}, \tag{1.34}
\]
\[ \dot{v} + Hv = -\frac{1}{a\dot{\rho}} \nabla \delta P - \frac{1}{a} \nabla \delta \Phi, \quad (1.35) \]

\[ \nabla^2 \delta \Phi = 4\pi G a^2 \dot{\rho} \delta, \quad (1.36) \]

where the overdot is the derivative with respect to time. Combining these equations finds

\[ \ddot{\delta} + 2H \dot{\delta} - \frac{c_s^2}{a^2} \nabla^2 \delta = 4\pi G \bar{\rho} \delta, \quad (1.37) \]

where \( \delta P = \frac{c_s^2}{a^2} \delta \rho \) and \( c_s \) is the sound speed of the fluid. These equations are valid in the linear regime \( (\delta \ll 1) \).

For a static space \( (H=0) \), the solution is \( \delta \propto e^{i(wt-k \cdot r)} \) with \( w^2 = c_s^2 k^2 - 4\pi G \bar{\rho} \). It has a critical wavenumber at \( k_J = \frac{4\pi G \bar{\rho}}{c_s} \) called "Jeans' length", where the frequency of the oscillation becomes zero. On small scales \( (k < k_J) \), the pressure dominates and the density fluctuations oscillate. On large scales \( (k > k_J) \), the gravity dominates and the density fluctuations grow exponentially.

In an expanding universe \( (H \neq 0) \), the equation includes a friction term, \( 2H \dot{\delta} \), and the growth of density fluctuations depends on time via \( \rho(t) \) and \( c_s(t) \). Below the Jeans length, the density fluctuations oscillate with decreasing amplitude (decaying mode) and, above it, the density fluctuations grow with a power-law (growing mode). The decaying mode does not contribute to the structure formation, and only the growing mode does. The solution of the growing mode is

\[ \delta(x,t) = D_+(t) \delta(x), \quad (1.38) \]

where

\[ D_+(t) \approx \frac{5}{2} a(t) \Omega_m \left[ \Omega_m^{4/7} - \Omega_\Lambda + \left( 1 + \frac{\Omega_m}{2} \right) \left( 1 + \frac{\Omega_\Lambda}{70} \right) \right]^{-1}. \quad (1.39) \]

The approximation in the equation is good to a few percent for plausible \( \Omega_m \) and \( \Omega_\Lambda \) (Lahav et al. 1991) (Carroll et al. 1992).

To describe the inhomogeneous universe quantitatively, the power spectrum \( P(k) \) can be used:

\[ \delta(x) = \int \frac{d^3k}{(2\pi)^3} \delta(k) e^{i k \cdot x}, \quad (1.40) \]

\[ \langle \delta(k) \delta(k') \rangle = (2\pi)^3 \delta_D^3(k-k') P(k), \quad (1.41) \]

where \( \delta_D \) denotes a delta function and \( \langle \rangle \) does an ensemble average for all the pairs of \( k \) and \( k' \). Note that the power spectrum \( P(k) \) describes the
1.4. Structure formation

level of structure as a function of the length-scale \( k = 2\pi/L \), where \( k \) is a comoving wavenumber. Using the power spectrum, the density fluctuations can be described by

\[
P(k, t) = D^2_+(t)P_0(k),
\]

where \( P_0(k) \) is a primordial power spectrum.

At early times, no natural length-scale existed in the universe as described in §1.3.3, hence, one should expect the primordial power spectrum to be a power-law form of

\[
P_0(k) \propto k^{n_s}.
\]

Using the primordial power spectrum, the power spectrum after the inflationary epoch can be written as

\[
P(k) = A \ k^{n_s}T^2(k),
\]

where \( T(k) \) is called “transfer function”. The transfer function provides the evolution of density perturbations through the epochs of horizon crossing and radiation/matter transition. The evolution of density perturbations freezes out after horizon crossing and remains approximately constant. After the end of inflation, the Hubble horizon grows and the macroscopic density perturbation from inflation re-enters the horizon from modes of small scales. However, if the density perturbation enters the horizon in the radiation-dominated era, the density fluctuation cannot grow due to the radiation pressure. On the other hand, if the density perturbation enters the horizon in the matter-dominated era, the density perturbations grow as \( \delta \propto D_+(t) \). According to the quantitative consideration of these effects, the transfer function can be computed numerically, or analytically for two limiting cases,

\[
T(k) \propto \begin{cases} 
1 & (k \ll k_{eq}) \\
\kappa^{-2} & (k \gg k_{eq}).
\end{cases}
\]

Therefore, the power spectrum for the cold dark matter with the Harrison-Zel’dovich spectrum \((n_s = 1)\) is derived to be

\[
P(k) \propto kT^2(k) \propto \begin{cases} 
k & (k \ll k_{eq}) \\
k^{-3} & (k \gg k_{eq}).
\end{cases}
\]

Fig. 1.6 shows the observed linear matter power spectrum by the CMB observation, extrapolated to \( z=0 \). The normalization of the power spectrum
1.4. Structure formation

cannot be determined from the theory but has to be determined by observations. For the normalization factor, a variance in a smoothed density field in $R = 8h^{-1}\text{Mpc}$ is often used,

$$\sigma_8^2(R) = \int \frac{dk}{k} \frac{k^3 P(k)}{2\pi^3} |W(kR)|^2,$$

where $W(kR)$ is a window function and if it is a tophat in real space, $W(kR) = [3/(kR)^3](\sin(kR) - (kR) \cos(kR))$. In the framework of $\Lambda$CDM model, $\sigma_8$ can be predictable from the CMB observation at scale of $k \simeq 10^{-3}$ and currently constrained to be $\sigma_8 = 0.8159 \pm 0.0086$ by the $(\text{Planck} + \text{ext})$ data.

In addition, the matter power spectrum can be determined from the galaxy distribution today. However, the formation and evolution of galaxies are not understood sufficiently to predict the relation between galaxies and dark matter. The connection between them is parametrized by so-called linear bias factor $b$,

$$\delta_g = \frac{\Delta n}{\bar{n}} = b \frac{\Delta \rho}{\bar{\rho}} = b \delta,$$

where $\bar{n}$ is the average number density of the galaxy population, and $\Delta n$ is the deviation from the average. The linear relation is not strictly justified from theories, however, it is plausible on scales where the density field is linear.

The best fit $\Lambda$CDM model is shown in Fig. [1.6]. As one can see, the fit is quite good. The fact that the CMB and LSS data agree over a substantial region of overlap gives a confidence in the correctness of the concordance model.

1.4.2 Non-linear structure evolution

The linear perturbation theory is applicable on large scales, but the evolution of structures like clusters of galaxies can not be treated within the framework of linear perturbation theory. Instead, the power spectrum, $P(k)$, turns non-linear around $k \sim [0.1, 1.0]h/\text{Mpc}$. Higher-order perturbation theory can be used to follow slightly larger values of the density fluctuations, but the achievements do not justify the large mathematical effort in general. Additionally, the fluid approximation is no longer valid if gravitationally bound systems form.

For this reason, numerical simulations have become a main tool to predict the structure formation. Since the matter distribution in the universe is dominated by dark matter, it is often sufficient to compute the behavior of
1.4. Structure formation

Figure 1.6: Linear matter power spectrum $P(k)$ versus wavenumber extrapolated to $z=0$, from various measurements of cosmological structure (Tegmark et al., 2004) (arXiv:astro-ph/0310725). The best fit ΛCDM model is shown in solid line.

the dark matter and thus to consider only gravitational interactions. The results of the simulations have contributed very substantially to establish the standard model of cosmology.

In recent years, the computational power has increased and baryons can be treated with their hydrodynamic processes such as radiative transfer and other baryonic feedback processes. Using the hydrodynamical simulations, one can predict the distribution of both baryon and dark matter. The simulations can be compared directly with most of the observables and the influence of the heating and cooling of the baryonic effects can be examined, however, it is still challenging on cosmological scales.

1.4.3 Evolution of baryon

The matter distribution is not directly observable since it is mostly dark matter. What we see is mainly baryon through photon. However, it would have grown in a different manner from dark matter. Before recombination,
1.4. Structure formation

baryons, electrons and photons existed as one plasma fluid. The density perturbations in the baryon-electron-photon plasma did not grow due to the high pressure from photons. Instead, the perturbations caused sound waves propagating in the plasma with time-independent amplitudes, which is called “Baryon Acoustic Oscillations” (BAO). Hence, the density perturbations of baryons only began to grow after recombination when baryons decoupled from photons.

In the matter dominated universe, the gravitational instability grows with \((\delta \rho/\rho)(t) \propto a(t)\). Hence, in a universe without dark matter, the growth factor of baryon density perturbations would have been at most \(\delta \rho/\rho \sim 10^{-3}\) since \(z_{\text{rec}} \sim 1,100 (1+z = 1/a)\). However, the amplitude of baryon density perturbations at recombination is known to be \(\delta \rho/\rho \sim 10^{-5}\) from the CMB anisotropy measurements.

After recombination, the universe became neutral and the baryonic matter was no longer influenced by photons. It induced the baryons to behave like dark matter, only subject to gravitational forces. However, since dark matter decoupled from the plasma much earlier than baryons, the perturbations of the dark matter had begun to grow much earlier. The baryons fell
into the potential wells already formed by the dark matter, then the perturbations of both dark matter and baryon have developed together soon after recombination. It induced the baryon perturbations to grow faster than expected and galaxies formed in the regions where dark matter was overdense originally.

1.4.4 Clusters of galaxies

The large-scale structure of the universe has been developed by the gravitational instability originated from the primordial density fluctuations. Small sub-clumps form first, then undergo a merging process to form larger structures, up to clusters of galaxies.

The galaxy clusters trace fields of dark matter density fluctuations, and their number density as a function of mass and redshift, so-called “halo mass function” (HMF) \( n(M, z) \), has a dependance on cosmological parameters. Therefore, by comparing the theoretical one with observations, one can constrain \( \Omega_m \) especially, however, degenerate with \( \sigma_8 \). The degeneracy can be solved by studying the redshift evolution of the mass function (e.g., (Borgani, 2008)).

In theory, the HMF can be described by the “Press-Schechter model” (Press and Schechter, 1974), under the assumption of initial Gaussian perturbations. It provides the mass function of

\[
\frac{dn(M, z)}{dM} = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M^2 \sigma_M(z)} \left| \frac{d\ln \sigma_M(z)}{d\ln M} \right| \exp \left( -\frac{\delta_c^2}{2\sigma_M(z)^2} \right), \tag{1.51}
\]

where \( \bar{\rho} = M/V_M \), \( \sigma_M(z) \) is the variance at the mass scale \( M \) at redshift \( z \), and \( \delta_c \) is the critical overdensity given by \( \delta_c \approx 1.69 \) under the assumption of spherical collapse in the Einstein-de Sitter universe. It assumes that the fraction of matter ending up in objects of a given mass \( M \) can be found by looking at the portion of the initial density field, smoothed on the mass-scale \( M \), lying at an overdensity exceeding a given critical threshold value, \( \delta_c \).

With the advent of N-body simulations such as Millennium simulation (Springel et al., 2005), significant deviations from the Press-Schechter model have been found as seen in Figure. [L8]. Thus, analytical descriptions have been improved along with simulations. More realistic ellipsoidal collapse model has been investigated, and in addition, universal analytic fitting functions are proposed. These advanced models are found to be in very good agreement with numerical simulations as demonstrated in Figure. [L8] providing a good description of \( n(M, z) \).
However, the accuracy of the cosmological parameter measurements is currently limited by uncertainties to trace the masses, in the relation between the cluster masses and observables such as luminosity or temperature. In simulations, the mass and radius of galaxy clusters are estimated with a spherical overdensity (SO) algorithm. The virial radius is defined such that within a sphere of radius $r_{\text{vir}}$, the average mass density of the cluster is about 200 times the critical density $\rho_{\text{cr}}$ of the universe. The mass within $r_{\text{vir}}$ is called the virial mass $M_{\text{vir}}$ ($\simeq M_{200}$). However, it is still challenging for the “real” galaxy clusters.

![Figure 1.8: The mass function of dark matter halos (pants with errorbars) identified at different redshifts in the Millennium simulation (Springel et al., 2005) (arXiv:astro-ph/0504097). Solid lines are predictions from an analytic fitting function. The dashed lines give the Press-Schechter model.](image)

### 1.5 Sunyaev-Zel’dovich effect

#### 1.5.1 Kompaneetz equation

After recombination, CMB photons free-stream, but they are influenced by free electrons in the middle of the path, if baryons become ionized in their evolution. For example, through X-ray observations, it is found that galaxy
clusters are filled with large clouds of hot ionized gas. It changes the phase space density of the CMB photons, $n(\nu)$. The evolution of the photon phase space density $n(\nu, t)$ in the presence of electron gas can be described by the Boltzmann equation,

$$\frac{\partial n(\nu, t)}{\partial t} = \int d^3p \int d\Omega \frac{d\sigma}{d\Omega} [f_e(p_1)n(\nu_1, t)(1+n(\nu, t)) - f_e(p)n(\nu, t)(1+n(\nu_1, t))],$$

(1.52)

where $f_e(p)$ is an electron phase space density with momentum $p$, $\sigma$ is a cross section and $\Omega$ is a solid angle. This equation describes an energy transfer via scattering events $p + \nu \leftrightarrow p_1 + \nu_1$. The first term describes the population of $\nu$ state by incoming photons with $\nu_1$, while the second term describes the de-population. $(1 + n)$ factors are due to the stimulated absorption.

For thermal non-relativistic electrons,

$$f_e(E) = n_e(2\pi m_e T_e)^{-3/2}e^{-E/T_e} \quad \text{with} \quad E = \frac{p^2}{2m_e},$$

(1.53)

where $n_e$ is the number density of electrons, $m_e$ is the mass of electron and $T_e$ is the temperature of electrons. Assuming the energy transfer is small compared to the electron kinetic energy, it holds

$$\Delta \equiv \nu_1 - \frac{\nu}{T_e} \ll 1. \quad (1.54)$$

The energy conservation holds $E_1 = E - T_e \Delta$, therefore

$$f_e(E_1) \approx \left(1 + \Delta + \frac{\Delta^2}{2}\right) f_e(E). \quad (1.55)$$

For photons scattered by electrons,

$$n(\nu_1) \approx n(\nu) + T_e \Delta \partial_\nu n(\nu) + \frac{1}{2}(T_e \Delta)^2 \partial^2_\nu n(\nu). \quad (1.56)$$

By defining

$$\tilde{x} = \frac{\nu}{T_e}, \quad (1.57)$$

so that $T_e \partial_\nu n = \partial_{\tilde{x}} n \equiv n'$, the Boltzmann equation can be simplified to

$$\partial_y n(\tilde{x}, y) = \tilde{x}^{-2}\partial_{\tilde{x}}[\tilde{x}^4(n' + n + n^2)] \quad (1.58)$$

with time replaced by so-called “Compton y parameter” ($y = \int dt \frac{n_e \sigma_T T_e}{m_e}$). This equation is called “Kompaneetz equation” (Kompaneetz, 1956).
1.5. Sunyaev-Zel’dovich effect

1.5.2 Sunyaev-Zeldovich effect

The Sunyaev-Zeldovich (SZ) effect is inverse Compton scattering of the CMB photons off electrons in clusters of galaxies or any cosmic structure with unbound electrons. X-ray observations show that galaxy clusters are filled with large clouds of hot gas with the temperature of \( T_e \sim 10^7 - 10^8 \) [K]. The high energy electrons in the hot gas inject energy into the CMB photons through the inverse Compton scattering and the photons are pumped to occupy higher energy states.

![Figure 1.9: The CMB spectrum (dashed line) and the distorted spectrum by the SZ effect (solid line) (Carlstrom et al., 2002).](image)

The Kompaneetz equation (Eq. 1.58) provides the distortion of the CMB spectrum due to the SZ effect. Since the CMB spectrum is initially Planck function with temperature \( T_{\text{CMB}} \ll T_e \), the term proportional to the radiation phase space density of \( n(n+1) \) can be neglected,

\[
\partial_y n(x, y) = x^{-2} \partial_x [x^4 \partial_x n(x, y)], \quad (1.59)
\]

where \( x \) is the dimensionless photon frequency of \( x = \frac{h \nu}{k_B T_{\text{CMB}}} \), \( h \) is the Planck constant and \( k_B \) is the Boltzmann constant. (Note that \( \tilde{x} = \frac{h \nu}{k_B T_e} \) is replaced with \( x = \frac{h \nu}{k_B T_{\text{CMB}}} \).) Substituting the Planck function of \( n =
(e^x - 1)^{-1}$ gives
\[
\frac{\partial n}{\partial y} = \frac{xe^x}{(e^x - 1)^2} \left( x \coth \frac{x}{2} - 4 \right),
\]
and
\[
\frac{\Delta n}{n_0} = \frac{\Delta I_\nu}{I_{\nu, \text{CMB}}} = \frac{xe^x}{e^x - 1} \left( x \coth \frac{x}{2} - 4 \right) y,
\]
or equivalently,
\[
\frac{\Delta T_{\text{CMB}}}{T_{\text{CMB}}} = \left( x \coth \frac{x}{2} - 4 \right) y,
\]
in thermodynamic temperature unit. This is called (thermal) “Sunyaev-Zeldovich” (tSZ) effect (Sunyaev, 1980; Sunyaev and Zeldovich, 1970, 1972; Zeldovich and Sunyaev, 1969) and the amplitude of the distortion is defined by the Compton $y$ parameter.

It is amongst the major sources of secondary anisotropies of the CMB on sub-degree angular scales. When the CMB photons pass through a cloud of free electrons in equilibrium with number density $n_e$, they are subject to scattering with a probability characterized by the optical depth,
\[
\tau_e = \int \sigma_T n_e dl \sim 2 \times 10^{-3} \left( \frac{n_e}{10^{-3}\text{cm}^{-3}} \right) \left( \frac{l}{\text{Mpc}} \right),
\]
where $\sigma_T$ is the Thomson scattering constant and
\[
y = \int \sigma_T n_e \frac{k_B T_e}{m_e c^2} dl \sim 4 \times 10^{-5} \left( \frac{n_e}{10^{-3}\text{cm}^{-3}} \right) \left( \frac{T_e}{10^8\text{K}} \right) \left( \frac{l}{\text{Mpc}} \right),
\]
which is essentially the dimensionless electron pressure integrated along the line of sight and only a tiny effect even for a massive galaxy cluster.

The thermal SZ effect is the spectral distortion in order of $(v_e/c)^2$ ($v_e$ is electron velocity). To the lowest order in $v_e/c$, a small change in $T_{\text{CMB}}$ is caused by the Doppler effect, which is called kinetic SZ (kSZ) effect,
\[
\frac{\Delta T_{\text{CMB}}}{T_{\text{CMB}}} = \int \sigma_T n_e \frac{v_\parallel}{c} dl \sim 7 \times 10^{-6} \left( \frac{n_e}{10^{-3}\text{cm}^{-3}} \right) \left( \frac{v_\parallel}{10^3\text{km s}^{-1}} \right) \left( \frac{l}{\text{Mpc}} \right),
\]
where $v_\parallel$ is the line-of-sight component of $v_e$. Note that random velocities cancel out, so the coherent motion with respect to the CMB is responsible for the kSZ effect. In the non-relativistic limit, the spectral shape of the kinetic SZE is still described by a Planck spectrum, but at a slightly different temperature, lower (higher) for positive (negative) peculiar velocities. Even though the tSZ effect is in order of $(v_e/c)^2$, it is dominant over the kSZ
1.5. Sunyaev-Zel’dovich effect

effect, typically by one order of magnitude for galaxy clusters, because the thermal velocity of electrons is much larger than the bulk motions of $\lesssim 10^3$ km s$^{-1}$.

In the non-relativistic limit, the spectral shapes of the kSZ and tSZ effects depend only on the frequency. Corrections due to higher-order terms are non-negligible once electrons become relativistic (e.g., Challinor and Lasenby (1998); Itoh et al. (1998); Nozawa et al. (1998); Sazonov and Sunyaev (1998)). In the relativistic case, the spectral shape of the tSZ effect begins to depend on $T_e$ and that of the kSZ effect on both $T_e$ and the bulk velocity. However, for a massive cluster with $k_B T_e \sim 10$ keV ($M \sim 10^{15} M_\odot$), the relativistic correction to the SZ effect is of order a few percent in the Rayleigh-Jeans (RJ) portion of the spectrum. In any case, the observed amplitude and spectral shape of the SZ effect are both independent of $z$, because $T$ and $I_\nu$ are redshifted in exactly the same way as $T_CMB$ and $I_\nu, CMB$.

The characteristic spectral shape helps to separate it from other components such as emissions from radio galaxies and primary CMB anisotropies. Recent developments of wide-field surveys by the South Pole Telescope (SPT) (Reichardt et al., 2013; Staniszewski et al., 2009; Vanderlinde et al., 2010; Williamson et al., 2011), the Atacama Cosmology Telescope (ACT) (Hasselfield et al., 2013; Hincks et al., 2010; Marriage et al., 2011), and the Planck satellite (Planck Collaboration, 2014b, 2016f) have boosted the number of galaxy clusters observed through the SZ effect by more than an order of magnitude over the last decade.

Especially, the Planck team constructed all-sky $y$-maps [1] (Planck Collaboration, 2016d) as one of the dataset from the Planck 2015 data release and provided in HEALpix [2] format with a resolution of $N_{\text{side}} = 2048$. Two types of $y$-maps are publicly available: MILCA and NILC $y$-map, both of which are obtained by the Internal Linear Combination (ILC) approach, but different algorithm (Fig. 1.12).

The $y$ maps can be used for a variety of cosmological and astrophysical purposes. The Planck team have performed a blind search for objects and identified about 1,600 galaxy clusters such as Fig. 1.11 (Planck Collaboration, 2011). They also shows that the sensitivity of the $y$ map is sufficient to detect faint and diffuse structures such as bridges between merging clusters. Moreover, they have proved via a stacking analysis that even low signal-to-noise regions in the $y$ map preserve the tSZ signal for small galaxy groups (with tens of galaxies) (Planck Collaboration, 2016d).

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1 Planck $y$-maps, http://pla.esac.esa.int/pla/#results
2 http://healpix.sourceforge.net/
1.5. Sunyaev-Zel’dovich effect

Figure 1.10: Spectral distortion of the CMB radiation due to the Sunyaev-Zel’dovich effect. The thick solid line is the thermal SZE and the dashed line is the kinetic SZE. For reference the 2.7 K thermal spectrum for the CMB intensity scaled by 0.0005 is shown by the dotted line. The cluster properties used to calculate the spectra are an electron temperature of 10 keV, a Compton y parameter of $10^{-4}$, and a peculiar velocity of 500 km s$^{-1}$ (Carlstrom et al., 2002).

Figure 1.11: Cleaned images of Abell 2256 at $z = 0.058$ observed by Planck at 100, 143, 217, 353, and 545 GHz (left to right) over a size of 1 square degree. Blue, green, and red colors indicate negative, null, and positive intensities with respect to the CMB, respectively (Planck Collaboration, 2011) (arXiv:1101.2024).
1.5. Sunyaev-Zel’dovich effect

Figure 1.12: Planck all-sky Compton parameter maps for NILC (top) and MILCA (bottom) in orthographic projections (Planck Collaboration 2016d) (arXiv:1502.01596).
Chapter 2

Construction of Compton parameter $y$ map

2.1 Introduction

In this chapter, all-sky Compton parameter ($y$) maps are constructed from the individual Planck frequency maps. The Compton $y$ parameter is the amplitude of the thermal Sunyaev-Zeldovich (tSZ) effect, produced by the inverse Compton scattering of cosmic microwave background (CMB) photons by high energy electrons along the line of sight. It provides a direct measurement of thermal pressure due to electrons, allowing to study the baryonic (gaseous) physics of clusters of galaxies as well as structure formation in the universe. The Planck satellite mission has made sensitive measurements at 9 frequency bands with all-sky coverage, which provides a unique data set to produce all-sky Compton parameter ($y$) maps.

“Internal Linear Combination” (ILC) technique, described in the next section, is applied to make $y$ maps that extract only the thermal SZ signal and remove other emissions. Amongst the various emissions, the $y$ map is primarily focusing on removing dust emission. The dust emission is the most dominant emissions in the band maps, thus, its contamination in the $y$ map is carefully checked by using a variety of dust models with varying spectral index.

2.2 Construction of $y$ map with internal linear combination technique

The ILC method (e.g., (Eriksen et al., 2004; Remazeilles et al., 2011)) assumes little about the properties of the data. It simply assumes that observed temperature maps $T(\nu)$ at frequency $\nu$ can be written in a combination of different components. The ILC provides coefficients to extract a component of interest by forming the linear combination of the observed maps, so that it has a unit response to the target based on “known” spectral
2.2. Construction of $y$ map with internal linear combination technique

shape from theories or observations.

In the frequency range of Planck observations, the dominant emission components are the CMB, thermal Sunyaev-Zel’dovich distortion and thermal dust emission from interstellar dust grains in our galaxy and from dusty external galaxies including the Cosmic Infrared Background (CIB). Therefore, the band maps can be expanded to

$$T(\nu) = S_{SZ}(\nu) T'_{SZ} + S_{CMB}(\nu) T'_{CMB} + S_{dust}(\nu) T'_{dust}, \quad (2.1)$$

where $S_i$ is the spectral shape of $i$ component and $\nu'$ is a pivot frequency, which can be different for each component. Here, we assume that the extragalactic dust emission consist of the same spectral shape with the galactic dust emission.

The spectra of the CMB and tSZ are well known. The CMB has a constant frequency spectrum in thermodynamic unit and the tSZ distortions have a frequency dependence of $\Delta T_{CMB}(\nu)/T_{CMB} = y S_{SZ}(x)$, where $T_{CMB} = 2.725 \text{ K}$ is the monopole temperature, $y$ is the Compton parameter along the line-of-sight, and $S_{SZ}(x) = x \coth(x/2) - 4$, with $x \equiv h\nu/k_B T_{CMB}$, as described in §1.5.

However, the spectra of the dust emission is less known. It varies from pixel to pixel in the maps. The Planck team finds that it spans a range of spectra in the vicinity of the nominal spectra index of $\beta_d \approx 1.6$ based on the greybody spectrum ($s_d \propto \nu^{\beta_d} B(\nu)$) where $B(\nu)$ is the blackbody spectrum (Planck Collaboration, 2016g). Here, we simply assume that the dust spectra is a power-law with a constant index at all the pixels ($s_d \propto \nu^{\beta_d}$). Though the power-law model would be valid for given frequencies we use, thus, the $y$ maps we construct will inevitably have residual contamination from the dust emission and the level of contamination varies among the different estimates we form. We check the contamination by assigning different spectral indices for the dust emission. Given the band coefficients used in our estimates, we can predict the fractional foreground residual as a function of the “true” spectral index, as discussed below. In practice, we find a very weak dependence on the constructed $y$ maps from varying $\beta_d$.

We construct $y$ map by forming linear combinations of four lowest-frequency HFI maps: 100 GHz, 143 GHz, 217 GHz, and 353 GHz. The band maps are provided in thermodynamic CMB temperature unit from the Planck release 2 products. We smooth the band maps to a common Gaussian FWHM beam of 10 arcmin since the beams at different band maps are different. The band coefficients are chosen to extract the tSZ distortion, but to project out the CMB and thermal dust emission.
2.2. Construction of $y$ map with internal linear combination technique

We form a given $y$ map, $y$, from a linear combination of the band maps with the common beam,

$$y = \sum_\nu b_\nu T(\nu)/T_{\text{CMB}},$$

(2.2)

where the sum is over frequency band, $T(\nu)$ is the map at frequency $\nu$, and $b_\nu$ is the coefficients for $y$ map. The coefficients for various $\beta_d$ values are given in Table 2.1 and the resulting maps are shown in Figure 2.1. In all cases, these coefficients satisfy the following constraints, 1) $\sum_\nu b_\nu S_{\text{SZ}}(\nu) = 1$ to produce a map in unit of the Compton $y$ parameter $y$; 2) $\sum_\nu b_\nu S_{\text{CMB}}(\nu) = 0$ to null the primary CMB; 3) $\sum_\nu b_\nu S_{\text{dust}}(= c_\nu (\nu/\nu_0)^{\beta_d}) = 0$ to null the dust emission with a spectral index $\beta_d$ (the factor $c_\nu$ given in Table 2.1 converts an antenna temperature to thermodynamic temperature). One last freedom is used to minimize the noise in the $y$ map, $\partial \sigma_y^2/\partial b_\nu = 0$, where $\sigma_y^2 = \sum b_\nu^2 \sigma(\nu)^2$ and $\sigma(\nu)$ is the mean noise of $T(\nu)$.

The last column in Table 2.1 gives an indication of how much dust signal might remain in a given $y$ map. The quoted value is the factor by which a dust signal with true index $\beta_d = 2.0$ would be suppressed (or amplified) relative to its 100 GHz amplitude, in the given $y$ map. For example, in map B where we assume $\beta_d = 1.4$, a dust signal with $\beta_d = 2.0$ would survive with an amplitude of -2 times its 100 GHz amplitude. Similar results are readily tabulated for other dust indices.

Table 2.1: Band data for the Planck $y$ maps (Van Waerbeke et al., 2014)

<table>
<thead>
<tr>
<th>Map</th>
<th>$\beta_d$</th>
<th>$b_{100}$</th>
<th>$b_{143}$</th>
<th>$b_{217}$</th>
<th>$b_{353}$</th>
<th>$r_{2.0}$</th>
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<tbody>
<tr>
<td>A</td>
<td>...</td>
<td>-0.1707</td>
<td>-0.1148</td>
<td>0.0085</td>
<td>0.2770</td>
<td>44.42</td>
</tr>
<tr>
<td>B</td>
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<td>-0.7089</td>
<td>-0.1372</td>
<td>0.9388</td>
<td>-0.0927</td>
<td>-1.99</td>
</tr>
<tr>
<td>C</td>
<td>1.6</td>
<td>-0.6952</td>
<td>-0.1378</td>
<td>0.9169</td>
<td>-0.0839</td>
<td>-3.13</td>
</tr>
<tr>
<td>D</td>
<td>1.8</td>
<td>-0.6826</td>
<td>-0.1385</td>
<td>0.8969</td>
<td>-0.0758</td>
<td>-0.95</td>
</tr>
<tr>
<td>E</td>
<td>2.0</td>
<td>-0.6710</td>
<td>-0.1393</td>
<td>0.8787</td>
<td>-0.0684</td>
<td>0.00</td>
</tr>
<tr>
<td>F</td>
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<td>-0.7235</td>
<td>-0.1367</td>
<td>0.9624</td>
<td>-0.1022</td>
<td>-4.4</td>
</tr>
<tr>
<td>G</td>
<td>1.0</td>
<td>-0.7389</td>
<td>-0.1364</td>
<td>0.9876</td>
<td>-0.1124</td>
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</tr>
<tr>
<td>$c_\nu$</td>
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<td>1.657</td>
<td>3.003</td>
<td>13.012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{\text{SZ}}(x)$</td>
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<td>-1.037</td>
<td>-0.001</td>
<td>2.253</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

32
2.2. Construction of $y$ map with internal linear combination technique

Figure 2.1: Maps of the Compton parameter, $y$, formed from linear combinations of the Planck HFI maps, shown on a scale $0 < y < 1 \times 10^{-4}$. The residual contamination from foreground signals, primarily thermal dust emission, varies widely among the different maps. In all cases, the primary CMB fluctuations have been projected out by enforcing the condition $\sum_\nu b_\nu \cdot 1 = 0$. Very top: A (no dust rejection), Top left: $y$ map version B (reject $\beta_d = 1.4$), top right: version C (reject $\beta_d = 1.6$), middle left: version D (reject $\beta_d = 1.8$), middle right: version E (reject $\beta_d = 2.0$), bottom left: version F (reject $\beta_d = 1.2$), and bottom right: version G (reject $\beta_d = 1.0$).
2.3 Comparison with Planck $y$ map

2.3.1 Compton $y$ parameter profile of Luminous red galaxies

To compare our $y$ maps with the one constructed by the Planck team, we use Compton $y$ parameter profile of luminous red galaxies (LRGs), studied in Chapter 3. The detail is described in Chapter 3, so we describe the summary here.

Luminous red galaxies (LRGs) are early-type and massive galaxies, consist mainly of old stars with little ongoing star formation, and typically reside in the centers of galaxy groups and clusters. We construct the mean tSZ Compton $y$ profile of $\sim 65,000$ LRGs from the Sloan Digital Sky Survey Data Release 7 using the Planck $y$ map. The $y$ profile shows the pressure profile in galaxy groups and clusters, and can be used to probe the baryonic effects (effect of gas and plasma) inside them.

In Figure 2.2, the $y$ profile of the LRGs using the Planck $y$ map is compared to the ones using our $y$ maps assuming different dust spectral indices. The $y$ profile with Planck $y$ map is completely consistent with the $y$ profiles with our $y$ maps. In addition, the $y$ profiles using our different $y$ maps show little difference even for widely varying dust spectral indices. It suggests that the ILC technique works well to extract the tSZ signal and the contamination due to the galactic dust emission should not be significant.

2.3.2 Relation between integrated Compton $y$ parameter and stellar masses

Additionally, to compare our $y$ maps with the one constructed by the Planck team, we follow the analysis in (Planck Collaboration 2013c)(PCXI), but using our $y$ map (version D) instead, and compare the results. First, we select “locally brightest galaxies” (LBG) as central galaxies in their dark matter halos using the same criterion used in Planck Collaboration (2013c) from the SDSS DR7 galaxy catalog (Abazajian et al., 2009). The SDSS DR7 galaxy catalog provides the positions, magnitudes, spectroscopic redshifts and stellar masses in the New York University Value-Added Catalog (NYU-VAGC) (Blanton et al., 2005). The stellar masses are estimated with the K-correct software of Blanton and Roweis (2007) by fitting the five-band SDSS photometry to more than 400 spectral templates,
2.3. Comparison with Planck $y$ map

Figure 2.2: The average $y$ profile of 63,398 LRGs using the Planck $y$ map (blue) is compared with the $y$ profiles using our $y$ maps constructed with different dust spectral index models. Green: $y$ map version B ($\beta_d = 1.4$), red: version C ($\beta_d = 1.6$), yellow: version D ($\beta_d = 1.8$) and cyan: version E ($\beta_d = 2.0$). The 1 $\sigma$ statistical uncertainties are complemented for the $y$ profiles from the Planck and $y(D)$ maps as their width.

most of which are based on stellar evolution synthesis models of Bruzual and Charlot (2003) assuming the stellar initial mass function of Chabrier (2003).

From the SDSS DR7 galaxy catalog, we reject a galaxy if a brighter galaxy in $r$-band resides within a tangential distance of 1.0 Mpc and within a radial velocity difference of $|c\Delta z| < 1000$ km s$^{-1}$. we also use SDSS photometry to eliminate all objects with a companion that is close and bright enough that it might violate the above criteria using the “photometric redshift 2” catalogue (photoz2 (Cunha et al., 2009)) from the SDSS DR7 website to search for additional companions. We then eliminate any candidate with a companion in this catalog of equal or brighter $r$-magnitude and projected within 1.0 Mpc, unless the photometric redshift distribution of the “companion” is inconsistent with the spectroscopic redshift of the candidate (inconsistent means that the total probability for the companion to have a redshift equal to or less than that of the candidate is less than 0.1). This procedure leaves us with a sample of 248,643 locally brightest galaxies.
2.4 Conclusion

For each LBG, we extract from our $y$ map, a circular aperture of angular radius $\theta_c \equiv 5 \times \theta_{500}$ and an annular aperture of inner radius $\theta_c$ and outer radius $\theta_c + \text{FWHM}$, where FWHM is the Planck beam of 10 arcmin. We define the observed signal to be the sum of all pixels inside the circular aperture minus the mean pixel value inside the annular aperture ($Y_{cyl}^{cyl}$). This subtraction is meant to remove large-scale foreground contamination, assuming it is roughly constant over the extracted aperture. Then $Y_{cyl}^{cyl}$ is converted to $Y_{500}^{sph}$ (the Comptonization parameter integrated over a sphere of radius $R_{500}$) using the knowledge of the electron pressure profile in dark matter halos. We use the Universal Pressure Profile (UPP) from (Arnaud et al., 2010).

Finally, we scale it by $E^{-2/3}(z) \times (d_A(z)/500\text{Mpc})^2 (E(z) = (\Omega_m(1+z)^3+\Omega_d)^{1/2})$ to account for the effects of comparing similar objects at different redshifts. They are stacked in 10 stellar mass bins between $10^{11} - 10^{12} M_\odot$ to estimate the binned averages and uncertainties. Our result is compared with the Planck result (PCXI) in Figure 2.3. The results are consistent with each other, though the uncertainties are relatively larger in our result especially in lower-mass range. It also appears in the fact that the Planck result is stable in the power-law relation, though our result slightly fluctuates around the relation.

The Planck (MILCA) $y$ map is constructed using more Planck band maps including 545 GHz and 857 GHz, and ILC coefficients are allowed to vary as a function of multipole $\ell$ and are computed independently on different sky regions (a maximum of 3072 regions at high resolution map) to incorporate spatially varying foreground emissions such as the spectral index of galactic dust emission and dust temperature. It can reduce the uncertainties in the Planck $y$ map and have a better sensitivity even for low-mass objects.

2.4 Conclusion

We construct all-sky Compton parameter ($y$) map from the individual Planck frequency maps with the Internal Linear Combination (ILC) technique, used to make the $y$ map explicitly to remove contamination from the CMB and preserve the thermal SZ signal. Amongst the various other sources of contamination, our $y$ maps are primarily focused on removing contamination due to galactic dust emission by assuming several dust models with varying spectral index, $\beta_d$. Our $y$ maps are compared with the $y$ map constructed by the Planck team.
2.4. Conclusion

Figure 2.3: Mean SZ signal vs. stellar mass for locally brightest galaxies derived with our $y$ map version D, compared to the original Planck result in (Planck Collaboration, 2013c)(PCXI). Error bars show 1 σ statistical uncertainty in each mass bin of 0.1 dex.

- The $y$ profile of the LRGs are constructed using the Planck $y$ map and our $y$ maps. They are completely consistent with each other, and moreover the $y$ profiles using our different $y$ maps show little difference even for widely varying dust spectral indices. It suggests that the ILC technique works well to extract the tSZ signal and the contamination due to the galactic dust emission should not be significant.

- We follow the analysis in (Planck Collaboration, 2013c)(PCXI) and compare the relation between the integrated $y$ signals of central galaxies from SDSS DR7 galaxies and their stellar masses. The results are consistent with each other, though the uncertainties are relatively larger in our results especially in lower-mass range. It also appears in the fact that the Planck result is stable in the power-law relation, though our result slightly fluctuates around the relation. These imply that our $y$ maps might have more uncertainties than the Planck $y$ map.

The comparison of our $y$ maps with Planck $y$ map provides the consistent results, which allows to use either of them. However, in the study of this
2.4. Conclusion

chapter, the Planck $y$ map should have a better sensitivity with less uncertainties, therefore we use the Planck $y$ map in the studies in the following chapters.
Chapter 3

Probing hot gas in halos through the Sunyaev Zel’dovich effect

3.1 Introduction

The prominent advances in observational cosmology have provided six key cosmological parameters within a few percent, where in the current ΛCDM cosmology, most of the energy density of the universe is due to the dark energy and dark matter and the contribution of the baryonic matter is only \( \sim 4.6\% \) according to the CMB observation by WMAP and Planck satellite \(^{[2]}\) Hinshaw et al. \( 2013 \) Planck Collaboration \( 2016\).c.

However, the evolution of the Universe from the very primitive initial state to the current state is not well understood such as the evolution of galaxies and large scale structure of their distribution. One of the important tracers of them is clusters of galaxies. They are the most massive bound structures and therefore mark the most prominent density peaks of the large scale structure in the universe. Their cosmological evolution is therefore directly related to the growth of cosmic structures.

X-ray observations have discovered that galaxy clusters are intense X-ray sources which are emitted by hot gas \( (T \sim 10^7 \text{[K]}) \) located between galaxies. This intergalactic gas (intraccluster medium, ICM) contains more baryons than the stars seen in the member of galaxies. It indicates that the evolution of the ICM is more complex and regulated by the radiative cooling and also non-gravitational heating sources such as the active galactic nucleus (AGN).

AGN feedback has a wide range of impacts on galaxies and galaxy clusters: the observed relation between the black hole mass and bulge velocity dispersion, the regulation of cool cores and, the suppression of producing massive galaxies predicted by N-body simulations \(^{[3]}\) Gitti et al. \( 2012 \) Schneider \( 2006 \). Thus, the interplay of hot gas with the relativistic plasma
3.2. AGN feedback effects

The AGN feedback effects are key for understanding the growth and evolution of galaxies and the formation of large-scale structures. It has thus become clear that AGN feedback effects on the ICM must be incorporated in any model of galaxy evolution (e.g., Battaglia et al., 2010; McCarthy et al., 2014; Schaye et al., 2010; Sijacki et al., 2007; Steinborn et al., 2015; Vogelsberger et al., 2013). However, the non-gravitational processes beyond simple gravity, gas dynamics, and radiative cooling are not well understood. To study the effect of non-gravitational processes, particularly, galaxy groups and low-mass clusters are ideal laboratories since their relatively shallow potentials compared to massive clusters would have a noticeable impact of non-gravitational effects on their formation and evolution (e.g., Battaglia et al., 2012; Dong et al., 2010; Giodini et al., 2010; Johnson et al., 2009; Le Brun et al., 2014).

In this chapter, we probe the hot gas in the luminous red galaxies (LRG’s) halos from the Sloan Digital Sky Survey seventh data release (Abazajian et al., 2009) (SDSS DR7 LRG (Kazin et al., 2010)) using the Planck Compton parameter ($y$) map from the 2015 Planck data release (Planck Collaboration, 2016c).

3.2 AGN feedback effects

One of the main problems of the current cosmological model is why so few baryons have formed stars (Cole, 1991; White and Frenk, 1991). Only 10% have been observed in the form of stars (e.g., Balogh et al., 2001), which is much less than predicted by numerical simulations of cosmological structure formation including the hydrodynamics of baryons. Especially, simulations including only gravitational heating predict an excessive cooling of baryons and result in a population of too massive and bright galaxies with respect to the ones observed, thus fail to reproduce the truncation of the high-luminosity end of the galaxy luminosity function (Benson et al., 2003; Sijacki and Springel, 2006).

This problem can be solved by the non-gravitational heating supplied by supernovae (SNe). According to simulations, the heating due to SNe suppresses the low-luminosity side of galaxy luminosity function, however, it is not enough to quench cooling in massive galaxies (Borgani et al., 2002). Energetically, AGN heating appears to be the most likely mechanism to explain the quenching of massive galaxies (Voit, 2005; Voit and Donahue, 2005) in Figure 3.1.

AGNs are powered by accretion of material onto supermassive black
3.2. AGN feedback effects

Figure 3.1: The luminosity function of galaxies from observations is compared schematically to the one predicted from the CDM-motivated theory.

Holes (SMBH), which are located at the centers of galaxies. For a SMBH with \( M \sim 10^{9}M_{\odot} \), the energy release during the formation and growth is of the order of \( E_{\text{BH}} \sim 2 \times 10^{62} \text{ erg s}^{-1} \). Even a tiny fraction (\( \lesssim 1\% \)) of the released energy could heat and blow away the gas in the bulge, thus preventing the star formation in the region. In particular, the correlation between the mass of central black hole (\( M_{\text{BH}} \)) and the stellar velocity dispersion (\( \sigma \)) in the galaxy’s bulge is found as in Figure 3.2. The data shows that the formation of SMBHs and bulges are closely linked (Ferrarese and Merritt, 2000; Gebhardt et al., 2000; Magorrian et al., 1998), and the formation and evolution of galaxies are largely influenced by the SMBHs. The physical process regulating these phenomena is called “feedback”, and the understanding of its origin and influence in detail is still an important open question in extragalactic astrophysics.
3.3 Luminous red galaxies (LRGs)

3.3.1 Luminous red galaxies

It has long been known that the most luminous galaxies in clusters are a very homogeneous population (e.g., (Postman and Lauer, 1995)). For example, Hogg et al. (2005) tests the homogeneity with the luminous red galaxy (LRG) spectroscopic sample of the Sloan Digital Sky Survey. The survey sky area is divided into 10 disjoint regions of each \( \sim 2 \times 10^7 h^{-3} \text{Mpc}^3 \), and the fractional rms number density variations of the LRG sample in the redshift range \( 0.2 < z < 0.35 \) among the regions is found to be 7% of the mean density.

Because these objects are intrinsically very luminous, they can be observed to great distance and used as powerful tracers of large-scale structure of the universe. In particular, luminous red galaxies are early-type and massive galaxies, consist mainly of old stars with little ongoing star formation, and typically reside in the centers of galaxy groups and clusters. Therefore, LRGs have been used to detect and characterize the remnants of baryon acoustic oscillations (BAO) at low to intermediate redshift (Anderson et al., 2014; Eisenstein et al., 2005; Kazin et al., 2010).

In the Sloan Digital Sky Survey, LRGs are selected using a variant of
the photometric redshift technique and are meant to comprise a uniform, approximately volume-limited sample of objects with the reddest colors in the rest frame. The sample is selected via cuts in the \((g-r, r-i, r)\) color-color-magnitude cube. The resulting LRG sample has nearly constant comoving number density at low to intermediate redshift with a passively-evolved luminosity threshold that is close to constant \(^5\).

3.3.2 LRG catalog

The SDSS DR7 LRG catalog (N=105,831) by \(^6\) provides the positions, magnitudes and spectroscopic redshifts. The stellar masses of the LRGs are provided in the New York University Value-Added Catalog (NYU-VAGC) \(^7\) (Blanton et al., 2005). They are estimated with the K-correct software \(^8\) of Blanton and Roweis (2007) by fitting the five-band SDSS photometry to more than 400 spectral templates, most of which are based on stellar evolution synthesis models of Bruzual and Charlot (2003) assuming the stellar initial mass function of Chabrier (2003).

Not all LRG’s are central galaxies in massive halos. \(^{9}\) estimates that 89 % of LRG’s are central based on the shape of the two-point correlation function of LRGs, but \(^{9}\) find that, at a halo mass of \(10^{14.5} M_\odot\), only 73% of LRG’s are central based on comparing LRG positions to cluster centers, as measured by X-rays in the redMaPPer cluster catalog, however, this is a relatively small sample compared to the full LRG catalog. To minimize the fraction of satellite LRG’s in our sample (which could bias our results in chapter 3 and 4), we add an additional constraint and select the locally most-massive LRG’s (based on stellar mass) using a criterion that is analogous to that used in Planck Collaboration (2013c): we reject a galaxy if a more massive galaxy resides within a tangential distance of 1.0 Mpc and within a radial velocity difference of \(|c\Delta z|<1000 \text{ km s}^{-1}\), which leaves \(\sim 101,000\) LRGs. Using an analogous selection based on \(r\)-band magnitude in the DR7 galaxy catalog, Planck Collaboration (2013c) assesses that the fraction of the central galaxies is at least \(\sim 85 \%\) for \(M_\star > 10^{11.3} M_\odot\) and reaches up to \(\sim 90 \%\) in higher stellar masses based on their simulation studies. It suggests that the LRGs we use are mostly centrals. The distributions of redshift and stellar masses of the LRGs is shown in Figure 3.3.

\(^5\)http://classic.sdss.org/
\(^6\)http://cosmo.nyu.edu/ eak306/SDSS-LRG.html
\(^7\)http://sdss.physics.nyu.edu/vagc/
\(^8\)http://howdy.physics.nyu.edu/index.php/Kcorrect
3.4 Compton $y$ parameter profile of the LRGs

In this section, we describe our procedure for stacking a $y$ map against the LRGs and construct the mean $y$ profile. For each LRG in the catalog, we place one LRG at the center in a 2-dimensional angular coordinate system of $-40' < \Delta l < 40'$ and $-40' < \Delta b < 40'$ divided in $80 \times 80$ bins. The mean $tSZ$ signal in the annular region between 30 and 40 arcmin is subtracted as an estimate of the local background signal.

The left panel in Figure 3.4 shows the average $y$ map stacked against the 74,681 LRGs in which the $tSZ$ signal at each pixel is smoothed by taking the average of $5 \times 5$ square pixels nearby. The right panel in Figure 3.4 is the average $y$ profile of the LRGs (blue) and the width of the blue line represents $1\sigma$ statistical uncertainty of the $y$ profile. The Planck beam of 10 arcmin in FWHM is shown as the black dashed line for comparison, the amplitude of which is normalized to the central peak of the data $y$ profile. The average $y$ profile has a central peak of $y \sim 1.8 \times 10^{-7}$ and we detect the $tSZ$ emission out to $\sim 30'$ well beyond the Planck beam.

3.5 Comparison to cosmo-OWLS hydrodynamic simulations to probe AGN feedback effect

3.5.1 cosmo-OWLS hydrodynamic simulations

The cosmo-OWLS suite is an extension of the OverWhelmingly Large Simulations project (Schaye et al. 2010) designed with cluster cosmology and
3.5. Comparison to cosmo-OWLS hydrodynamic simulations to probe AGN feedback effect

Figure 3.4: Left: The average Planck $y$ map stacked against 74,681 LRGs in an angular coordinate system of $-40' < \Delta l < 40'$ and $-40' < \Delta b < 40'$ divided in $80 \times 80$ bins. The tSZ signal at each pixel is smoothed by taking the average of $5 \times 5$ square pixels nearby. Right: The average $y$ profile of the LRGs (blue). The $1\sigma$ statistical uncertainty is represented in a width of the blue line. The Planck beam of 10 arcmin in FWHM is supplemented in black dashed line for comparison, the peak of which is normalized to the center of the LRGs’ $y$ profile.

large scale-structure surveys in mind (Le Brun et al., 2014; McCarthy et al., 2014; van Daalen et al., 2014). The cosmo-OWLS suite consists of box-periodic hydrodynamical simulations, the largest of which have volumes of $(400h^{-1}\text{Mpc})^3$ and contain $1024^3$ each of baryonic and dark matter particles. The suite employs two different cosmological models: Planck release 1 cosmology (Planck Collaboration, 2014a) with $\{\Omega_m, \Omega_b, \Omega_{\Lambda}, \sigma_8, n_s, h\} = \{0.3175, 0.0490, 0.6825, 0.834, 0.9624, 0.6711\}$ and WMAP7 cosmology (Komatsu et al., 2011) with $\{\Omega_m, \Omega_b, \Omega_{\Lambda}, \sigma_8, n_s, h\} = \{0.272, 0.0455, 0.728, 0.81, 0.967, 0.704\}$ respectively. Each simulation is run with 5 different models of baryon sub-grid physics: “NOCOOL”, “REF”, “AGN 8.0”, “AGN 8.5” and “AGN 8.7”, which is summarized in Table 3.1 Earlier studies demonstrate that the AGN 8.0 model reproduces a variety of observed gas features in local groups and clusters of galaxies by optical and X-ray data. For each model of simulation, 10 almost-independent mock galaxy catalogues are generated on a light cone and 10 corresponding $y$ maps are generated from the hot gas (McCarthy et al., 2014). Each of these light cones contain about one million galaxies out to $z \sim 3$, and each spans a $5^{\circ} \times 5^{\circ}$ patch of sky. To compare with data, we convolve the simulated $y$ maps with a Gaussian kernel of 10 arcmin, FWHM, corresponding to the Planck beam.
3.5. Comparison to cosmo-OWLs hydrodynamic simulations to probe AGN feedback effect

Figure 3.5: Compton $y$ maps simulated by cosmo-OWLS hydrodynamic simulations compared to different AGN feedback models (McCarthy et al., 2014). (Top: AGN 8.0, middle left: NOCOOL, middle right: REF, bottom left: AGN 8.5, and bottom right: AGN 8.7)
Table 3.1: The baryon feedback models in the cosmo-OWLS simulation. Each model has been run in both Planck and WMAP7 cosmology (McCarthy et al., 2014).

<table>
<thead>
<tr>
<th>Simulation</th>
<th>UV/X-ray background</th>
<th>Cooling</th>
<th>Star formation</th>
<th>SN feedback</th>
<th>AGN feedback</th>
<th>∆T_{heat}</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOCOOL</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>...</td>
</tr>
<tr>
<td>REF</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>...</td>
</tr>
<tr>
<td>AGN 8.0</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>$10^{8.0}$ K</td>
</tr>
<tr>
<td>AGN 8.5</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>$10^{8.5}$ K</td>
</tr>
<tr>
<td>AGN 8.7</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>$10^{8.7}$ K</td>
</tr>
</tbody>
</table>

3.5.2 Comparison to cosmo-OWLS hydrodynamic simulations

We compare the average $y$ profile to simulations. To do so, we analyze light cones from the cosmo-OWLS suite of simulations (§3.5.1) as we did the real data. As LRGs, we select simulated “central” galaxies with the same stellar mass and redshift distribution as the real data. To match the distributions, the average $y$ profile in each stellar mass and redshift bin is constructed in the simulations and they are summed up using the number weight of the LRGs,

$$y(\theta)_{\text{sim}} = \sum_{M_s, z} [\bar{y}(\theta, M_s, z)_{\text{sim}} \times w(M_s, z)_{\text{LRG}}],$$

where $\bar{y}(\theta, M_s, z)_{\text{sim}}$ is the average $y$ profile of simulated central galaxies in a stellar mass and redshift bin and $w(M_s, z)_{\text{LRG}}$ is a normalized number of actual LRGs in the same stellar mass and redshift bin. However, since the field of view of each light cone (25 [deg$^2$]) is much smaller than the overlapping region of SDSS-Planck survey (~8000 [deg$^2$]), massive central galaxies are scarce in the simulations. Therefore, we restrict the maximum stellar mass of LRGs to $10^{11.7} M_\odot$ in order to match the stellar mass distribution and also avoid a Poisson noise due to a few massive objects. As a result, the number of LRGs is reduced to 63,398, but this would be an appropriate procedure considering that we probe the baryonic effects, which would be more evident in low-mass group/clusters.

The average $y$ profile of 63,398 LRGs is compared to the simulations with different AGN feedback models in Figure. 3.6, where the gray lines show the average $y$ profiles of the simulations. In the comparison, the clear difference between the data and REF model can be seen. In general, the energy release around the center of halos heats the gas and prevents cooling,
3.6. Comparison to prediction from halo model and universal pressure profile

thus the star formation around the center. Therefore, in the same system (the same total mass of halos), the stellar mass of central galaxy is less with a powerful energy release from AGN. Since we select the central galaxies based on the stellar mass, lower-mass halos are selected in the REF model compared to the models including AGN feedback. This appears in the lower central peak of the $y$ profile in the REF model than others.

This effect is not significant among the AGN feedback models in the simulations. However, a visible trend is seen that the higher the power of feedback, the lower the peaks of $y$ profile. This is due to the fact that the AGN feedback ejects the gas around the center of halos outward and the gas density is lowered.

As a result of this comparison, the REF model is strongly rejected. This suggests that the model without AGN feedback can not account for the $y$ profile of the LRGs. Furthermore, the AGN 8.0 and AGN 8.5 models agree well with the real data, but AGN 8.7 does not, which constrains the AGN feedback models. In our study, a similar result is obtained using the simulations of WMAP7 cosmology. Our result is consistent with the other studies that the AGN 8.0 model reproduces a variety of observed gas features in local groups and clusters of galaxies, using optical and X-ray data.

3.6 Comparison to prediction from halo model and universal pressure profile

3.6.1 The Stacked $y$ profile with cross-correlation of the tSZ and distribution of galaxy clusters

For the calculation of the stacked $y$ profile, we follow the method in Fang et al. (2012) and work in the flat-sky and Limber approximation (Limber, 1954).

The cross power spectrum for the tSZ signal and the distribution of galaxy clusters is given by the sum of the one-halo term (contribution from own halos) and two-halo term (contribution from the correlated environment surrounding the halos):

$$C_{\ell}^{yh} = C_{\ell}^{yh,1h} + C_{\ell}^{yh,2h}. \quad (3.2)$$

The one-halo term is calculated by

$$C_{\ell}^{yh,1h} = \frac{f(x)}{n^2 V} \int dz \frac{d^2V}{dzd\Omega} \int dM \frac{dn}{dM}(M,z) \times S(M,z) \tilde{y}_{\ell}(M,z), \quad (3.3)$$
3.6. Comparison to prediction from halo model and universal pressure profile

Figure 3.6: The average $y$ profile of LRGs (blue) is compared with the beam-convolved $y$ profile of the simulated central galaxies in *Planck* cosmology (grey), with the same stellar mass and redshift distributions in different AGN feedback models respectively. *Top left:* AGN 8.0 model, *Top right:* AGN 8.5 model, *Bottom left:* AGN 8.7 model and *Bottom right:* REF model.

Figure 3.7: The average $y$ profile of LRGs (blue) is compared with the $y$ profile of the simulated central galaxies in *WMAP7* cosmology (grey).
where $d^2V/dzd\Omega$ is the comoving volume element and $dn/dM$ is the halo mass function. We adopt the halo mass function of Tinker et al. (2010), calculated using a code called ‘HMFcalc’ (Murray et al., 2013). In a cluster survey, galaxy clusters are selected depending on the observational strategy. The selection function can be defined by their observed redshift and estimated mass, which is given by

$$S(M, z) = \Theta(z - z_{\text{min}})\Theta(z_{\text{max}} - z) \times \Theta(M - M_{\text{min}})\Theta(M_{\text{max}} - M),$$

(3.4)

where $\Theta$ stands for the Heaviside step function. The average 2-dimensional angular number density of the selected galaxy clusters is calculated by

$$\bar{n}^{2D} = \int dz d^2V dzd\Omega \int dM \frac{dn}{dM}(M, z)S(M, z).$$

(3.5)

$\tilde{y}_\ell(M, z)$ is 2D Fourier transform of $y$ profile for a cluster with a pressure profile $P_e(x, M, z)$, which is calculated by

$$\tilde{y}_\ell(M, z) = \frac{\sigma_T}{m_e c^2} \frac{4\pi r_s}{\ell_s^2} \int dx x^2 \sin(\ell x/\ell_s) P_e(x, M, z),$$

(3.6)

where

$$x = \frac{r}{r_s}, \quad \ell_s = \frac{d_A}{r_s},$$

(3.7)

where $r_s$ is a characteristic scale radius of ICM pressure profile where $x = r/r_s$ is defined to be a dimensionless radial scale, and $d_A$ is an angular diameter distance where $\ell_s = d_A/r_s$ is defined to be an associated multipole moment.

The two-halo term is calculated by

$$C_{\ell}^{yh,2h} = f(x) \int dz d^2V dzd\Omega P^L_m(k = \frac{\ell}{\chi}, z)W_h(z)W^y_\ell(z),$$

(3.8)

where $P^L_m(k, z)$ is a linear matter power spectrum. The function $W_h(z)$ is defined to be

$$W_h(z) = \frac{1}{\bar{n}^{2D}} \int dM \frac{dn}{dM}(M, z)S(M, z)b(M, z),$$

(3.9)

and $W^y_\ell(z)$ is

$$W^y_\ell(z) = \int dM \frac{dn}{dM}(M, z)b(M, z)\tilde{y}_\ell(M, z),$$

(3.10)
3.6. Comparison to prediction from halo model and universal pressure profile

where $b(M, z)$ is a halo bias. We use the halo bias from Tinker et al. (2010).

By putting them together, a Fourier-transform of the stacked $y$ profile, $C_{y}^{bh}$, is calculated. In an angular space, the stacked $y$ profile is given by

$$\bar{y}(\theta) = \frac{1}{f(x)} \int \frac{\ell \, d\ell}{2\pi} J_0(\ell \theta) C_{y}^{bh},$$

(3.11)

where $J_0$ is the zeroth order Bessel function. Finally, we convolve it with a point spread function of an observation,

$$\bar{y}(\theta)_{\text{obs}} = \frac{1}{f(x)} \int \frac{\ell \, d\ell}{2\pi} J_0(\ell \theta) C_{y}^{bh} B_l,$$

(3.12)

where $B_l = \exp[(l + 1)\sigma^2/2]$ and $\sigma = \theta_{\text{FWHM}}/\sqrt{8\ln(2)}$ with $\theta_{\text{FWHM}} = 10$ arcmin, corresponding to the Planck beam.

3.6.2 Universal Pressure Profile

For the pressure profile, we adopt the universal pressure profile (UPP). The UPP has an analytical formulation given by Nagai et al. (2007) for the generalised Navarro-Frenk-White (GNFW) profile (Navarro et al., 1997),

$$P(x) = \frac{P_0}{(c_{500}x)\gamma[1 + (c_{500}x)^\alpha/(\beta - \alpha)]/\gamma},$$

(3.13)

where $x = r/R_{500}$, and the model is defined by the following parameters: $P_0$, normalization; $c_{500}$, concentration parameter defined at a characteristic radius $R_{500}$; and the slopes in the central ($x \ll 1/c_{500}$), intermediate ($x \sim 1/c_{500}$) and outer regions ($x \gg 1/c_{500}$), given by $\gamma$, $\alpha$ and $\beta$, respectively.

The scaled pressure profile for a halo with $M_{500}$ and $z$ is represented by

$$\frac{P(r)}{P_{500}} = P(x),$$

(3.14)

with

$$P_{500} = 1.65 \times 10^{-3} h(z)^{8/3} \times \left[ \frac{M_{500}}{3 \times 10^{14} h^{-1}_{70} M_\odot} \right]^{2/3} h_{70}^2 \text{ keV cm}^{-3},$$

(3.15)

using a characteristic pressure, $P_{500}$, reflecting the mass variation expected in the standard self-similar model, purely based on gravitation (Arnaud et al., 2010). The deviation from the standard self-similar scaling appears
as a variation of the scaled pressure profile. As in Arnaud et al. (2010), this variation can be expressed as a function of total mass,

\[
P(r) = P(\frac{M_{500}}{3 \times 10^{14}h_{70}^{-1}M_\odot})^{\alpha_p},
\]

where \( \alpha_p \) can be approximated by \( \alpha_p = 0.12 \). For the parameters in the GNFW, we adopt the best-fit values of \( [P_0, c_{500}, \gamma, \alpha, \beta] = [6.41, 1.81, 0.31, 1.33, 4.13] \), estimated using Planck tSZ and XMM-Newton X-ray data in Planck Collaboration (2013a), which is shown in Figure 3.8.

Figure 3.8: Universal pressure profile derived from (Planck Collaboration, 2013a) (red line) compared to the one in (Arnaud et al., 2010) (blue line).

### 3.6.3 Estimating halo masses of LRGs

In order to construct the y profile of the LRGs using the halo model and UPP, we estimate the halo masses of the LRG halos using the SHM relation from Planck Collaboration (2013c) (P13-SHM), Coupon et al. (2015) (C15-SHM) and Wang et al. (2016) (W16-SHM). In P13-SHM, the halo masses are estimated using a semi-analytic N-body simulation of galaxy population evolution from Guo et al. (2013). This simulation is tuned to reproduce the observed stellar mass function of the SDSS galaxies and the SHM relation is obtained by the abundance-matching method. In W16-SHM, the P13-SHM
3.6. **Comparison to prediction from halo model and universal pressure profile**

The relation is recalibrated with the gravitational lensing measurements from Reyes et al. (2012). In C15-SHM, it is estimated in the CFHTLenS/VIPERS field by combining deep observations from the near-UV to the near-IR, supplemented by ~70 000 secure spectroscopic redshifts, and analyzing galaxy clustering, galaxy-galaxy lensing and the stellar mass function. The three SHM relations are shown in Figure. 3.9 on top of the SHM relations between central galaxies and corresponding halos in the AGN 8.0 simulation: P13-SHM in cyan, C15-SHM in magenta and W16-SHM in yellow. In the stellar mass range of the LRGs, the halo mass estimates from C15-SHM and W16-SHM are consistent and the one from P13-SHM is estimated higher by ~0.1 dex than the others.

![Figure 3.9](image)

**Figure 3.9:** Black points show the relation between the stellar masses of central galaxies and halo masses in 0.16 < z < 0.47 from the AGN 8.0 simulation and its mean relation is shown in red. The three other SHM relations are shown on top of it: P13-SHM (Planck Collaboration, 2013c) in cyan, C15-SHM (Coupon et al., 2015) in magenta and W16-SHM (Wang et al., 2016) in yellow.

### 3.6.4 Comparison to the prediction

Using the estimated halo masses, we can calculate the average y profile of the LRG halos, using the halo model and UPP, via the procedure described in §3.6.1. To match the mass and redshift distributions, we apply the same
method described in §3.5.2 the average $y$ profile in each mass and redshift bin is calculated, and convolved with the Planck beam, and then they are summed up using the number weight of the LRGs. The constructed $y$ profiles with different halo mass estimates are shown in Figure.3.10 as well as the $y$ profile of the LRGs and the one from the AGN 8.0 simulation. Note that the AGN 8.0 simulation is customized to a larger field of view of $10^\circ \times 10^\circ$ [deg$^2$] but a limited redshift of $z < 1$ in this figure to increase the number of objects. It shows a better agreement with the $y$ profile of the LRGs.

The predictions from C15-SHM + UPP (magenta) and W16-SHM + UPP (yellow) agree well with the $y$ profile of the LRGs. Assuming the C15-SHM or W16-SHM relation is correct, it implies that the UPP, estimated for galaxy clusters in the mass range of $10^{14} - 10^{15} M_\odot$, is applicable even for low-mass halos, as suggested by the scaling relation of $Y_{500} - M_h$ ($Y_{500}$: the Comptonization parameter integrated over a sphere of radius $R_{500}$) reported in Planck Collaboration (2013c)(P13). However, we adopt the “average” SHM relation for all the LRGs and the scatter is not included. For example, applying the Gaussian scatter of halo mass in the C15-SHM relation from Moster et al. (2010) raises the peak of the $y$ profile by $\sim 25\%$. In addition, the distribution of the scatter is not guaranteed to be symmetric and it would also affect the model prediction of the $y$ profile. Furthermore, according to Le Brun et al. (2015), the AGN 8.0 simulation, which also agrees with the $y$ profile of the LRGs, predicts lower and more extended pressure profiles in low-mass halos than the UPP. Therefore, it is still hard to conclude a definitive statement about pressure profiles in low-mass halos because of the degeneracy between pressure profiles and SHM relations in the model prediction. For the conclusion, we need a better understanding of the SHM relation including its scatter, which is not well understood so far.

3.7 Discussion

This study is partially motivated by the contradictory result between (Planck Collaboration 2013c)(P13) and (Anderson et al. 2015)(A15) on the state of hot gas in galaxy group/clusters through the scaling relation. The self-similar scaling relation is valid under the assumption that the process is dominated by gravity and the deviation from the relation points to the presence of more complex processes such as baryonic effects. Using the Locally Brightest Galaxies (LBGs) in SDSS DR7, P13 finds self-similar scaling in $Y - M_h$, implying the hot gas represents roughly the mean cosmic fraction of the mass even in low-mass systems. On the other hand, A15 finds a steeper
3.7. Discussion

Figure 3.10: The average $y$ profile of LRGs (blue) is compared to the predictions using the halo model (Tinker et al., 2010) and UPP (Planck Collaboration, 2013a) when the halo masses of the LRGs are estimated using the SHM relation of P13-SHM (cyan), C15-SHM (magenta) and W16-SHM (yellow). The $y$ profile of the simulated central galaxies in the AGN 8.0 simulation is shown in grey. Note that the AGN 8.0 simulation is customized to a larger field of view of $10^\circ \times 10^\circ$ [deg$^2$] but a limited redshift of $z < 1$ in this figure to improve the number of objects. As a comparison, we also apply the UPP to the halo mass distribution of the AGN 8.0 simulation, which is shown in grey dashed line.

scaling than the self-similar relation in $L_X-M_h$, suggesting the importance of non-gravitational heating such as AGN feedback, as numerous X-ray studies of galaxy groups find a deficit of baryons (e.g., (Gastaldello et al., 2007; Gonzalez et al., 2013; Pratt et al., 2009; Sun et al., 2009)). These results can be reconciled by the idea that the low-mass halos may contain the cosmic fraction of baryons, just like galaxy clusters, but the density profile of the gas in the halos may be less concentrated.

We find that the measured $y$ profile of the LRGs agree well with the one from AGN 8.0 simulation, but does not without the AGN feedback. The result is consistent with other studies in which the AGN 8.0 model reproduces a variety of observed gas features in local groups and clusters of galaxies by optical and X-ray data. The AGN 8.0 model predicts lower
3.8 Conclusion

In this chapter, using the the *Planck* $y$ map, we present a stacking analysis of the SDSS DR7 LRGs, which are considered to be mostly central galaxies in dark matter halos. We construct the average $y$ profile centered on the LRGs and study the thermodynamic state of the gas in groups and low-mass clusters. The major results of our analysis are summarized as follows:

- The central tSZ signal is $y \sim 1.8 \times 10^{-7}$ and we detect tSZ emission out to $\sim 30$ arcmins well beyond the extent the *Planck* beam of the 10 arcmin.

- We compare the average $y$ profile of LRGs with the predictions from the cosmo-OWLS suite of cosmological hydrodynamical simulations. This comparison agrees well with the models including the AGN feedback (AGN 8.0 and AGN 8.5), but not with the model not including it (REF). This result suggests that an additional heating mechanism is required over and above SNe feedback and star formation, which is consistent with other studies showing that the AGN 8.0 model reproduces a variety of observed gas features in optical and X-ray data.

On the other hand, we also demonstrate that the measured $y$ profile of the LRGs agree well with the predictions using the UPP with the SHM relation from C15-SHM (Coupon et al., 2015) or W16-SHM (Wang et al., 2016), estimated by gravitational lensing measurements. This implies that the UPP, estimated for galaxy clusters in the mass range of $10^{14}$-$10^{15} M_\odot$, can be applied to low-mass systems down to $M_h \sim 2 \times 10^{13} M_\odot$ such as LRG halos, and thus little deviation from the self-similar scaling relation in $Y-M_h$.

However, for the model prediction, the degeneracy between pressure profiles and SHM relations remains. Especially, the scatter in the SHM is not well understood and has a negligible impact on the model prediction of $y$ profile. Because of the degeneracy, it is still hard to conclude a definitive statement about pressure profiles in low-mass halos, in turn, a deviation from self-similar relation due to non-gravitational heating such as AGN feedback. For the conclusion, a better understanding of the SHM relation is important in our analysis.
3.8. Conclusion

- The average $y$ profile of LRGs is also compared with the predictions using the halo model (Tinker et al., 2010) and UPP (Planck Collaboration, 2013a). The predicted $y$ profile is consistent with the data, if one accounts for the two-halo clustering term in the model, and assuming the halo mass distribution by the C15-SHM and W16-SHM estimated with gravitational lensing measurements. This may imply that the UPP, estimated for galaxy clusters in the mass range of $10^{14} - 10^{15} M_\odot$, is applicable even in low-mass halos down to $M_h \sim 2 \times 10^{13} M_\odot$ as implied in P13. However, for a definitive statement, a better understanding of the SHM relation is needed.

In our analysis, the dominance of the two-halo term in the low-mass systems is partially due to the coarse Planck beam. We emphasize that more precise measurements with a better angular resolution and sensitivity such as ACTPol (Niemack et al., 2010) and SPTpol (Austermann et al., 2012) will shed light further light on the issue and help to clarify the impact of AGN feedback on the formation and evolution of galaxies. Moreover, as an extension of our studies, the newer data release of the SDSS DR13 LRG catalog will help to probe the evolution of the gas inside halos to higher redshift, however, a better understanding of the SHM relation is needed.
Chapter 4

Probing hot gas in the cosmic web through the Sunyaev Zel’dovich effect

4.1 Introduction

In the distant Universe ($z \gtrsim 2$), most of the expected baryons are found in the Ly$\alpha$ absorption forest: the diffuse, photo-ionized intergalactic medium (IGM) with a temperature of $10^4$-$10^5$ K (e.g., (Rauch et al., 1997; Weinberg et al., 1997)). However, in the local Universe ($z \lesssim 2$), the observed baryons in stars, the cold interstellar medium, residual Ly$\alpha$ forest gas, OVI and BLA absorbers, and hot gas in clusters of galaxies account for only $\sim 50\%$ of the expected baryons (e.g., (Fukugita and Peebles, 2004; Nicastro et al., 2008; Shull et al., 2012)). Hydrodynamical simulations suggest that 40-50$\%$ of baryons could be in the form of shock-heated gas in a cosmic web between clusters of galaxies. This so-called Warm Hot Intergalactic Medium (WHIM) has a temperature range of $10^5$-$10^7$ K (Cen and Ostriker, 2006). The WHIM is difficult to observe due to its low density: several detections in the far-UV and X-ray have been reported, but few are considered definitive (Yao et al., 2012). It appears that $\sim 30\%$ of the baryons are still unaccounted for in the local universe (Shull et al., 2012).

Recently, (Clampitt et al., 2016) searched for evidence of massive filaments between proximate pairs of LRG’s taken from the Sloan Digital Sky Survey seventh data release (SDSS DR7). Using weak gravitational lensing signal, stacked on 135,000 pairs of LRG’s, they find evidence for filament mass at $\sim 4.5\sigma$ confidence. Similarly, Epps and Hudson (2017) detects the weak lensing signal using the Canada France Hawaii Telescope Lensing Survey (CFHTLenS) mass map from stacked filaments between SDSS-III/BOSS LRG’s at $5\sigma$ confidence.

The tSZ signal provides an excellent tool for probing baryonic gas at low and intermediate redshifts. Atrio-Barandela and Mückel (2006) and
4.2 Missing baryon problem

Atrio-Barandela et al. (2008) suggest that electron pressure in the WHIM would be sufficient to generate potentially observable tSZ signals. The measurement is challenging due to the morphology of the source and the relative weakness of the signal, however, the Planck team reports a significant tSZ signal in the inter-cluster region between the merging clusters of A399–A401 (Planck Collaboration, 2013b). In conjunction with ROSAT X-ray data, they estimate the temperature and density of the inter-cluster gas to be \( kT = 7.1 \pm 0.9 \text{ keV} \) with a baryon density of \((3.7 \pm 0.2) \times 10^{-4} \text{ cm}^{-3}\).

Van Waerbeke et al. (2014), Ma et al. (2015) and Hojjati et al. (2015) report correlations between gravitational lensing and tSZ signals in the field using the CFHTLenS mass map and Planck tSZ map. Similarly, Hill and Spergel (2014) reports a statistically significant correlation between the Planck CMB lensing potential and the Planck tSZ map. These results show clear evidence for hot gas tracing dark matter. Further, in the context of a halo model, there is clear evidence for contributions from both the one- and two-halo terms, but there is no statistically significant evidence for contributions from diffuse, unbound gas not associated with (correlated) collapsed halos.

In this chapter, we search the Planck data for a tSZ signal due to gas filaments between pairs of Luminous Red Galaxies taken from the Sloan Digital Sky Survey Data Release 12 (SDSS/DR12) (Alam et al., 2015; Prakash et al., 2016).

4.2 Missing baryon problem

The energy density of baryons in the Universe, \( \Omega_b \), can be measured by the CMB power spectrum, especially through the second peak. In addition, an independent measurement can be obtained from the observed abundances of light elements (D, He, Li), which constrain the baryon fraction through the Big Bang Nucleosynthesis model. The result from the BBN is currently \( \Omega_b h^2 = 0.021 - 0.025 \) (e.g. the review by (Fields et al., 2014)) and consistent with the WMAP \((0.02264 \pm 0.00050)\) (Hinshaw et al., 2013) and Planck \((0.02225 \pm 0.00016)\) (Planck Collaboration, 2016c) CMB measurements.

After reionization, most of the baryons in the Universe are ionized. Even though the remaining neutral fraction is tiny \((\sim 10^{-4}-10^{-5})\), it produces the Ly-\( \alpha \) forest. For example, a continuum spectra from a quasar emitted at \( \lambda < 1215.7 \text{ Å} \) is absorbed by intervening neutral gas when it is redshifted to the energy of Lyman-\( \alpha \). Therefore, the absorption lines trace the column density of the neutral gas along the time of the universe. At high redshift
4.2. Missing baryon problem

(z \gtrsim 2), most of the expected baryons are found in the Lyα absorption forest (e.g., (Rauch et al., 1997; Weinberg et al., 1997)).

However, at low redshift (z \lesssim 2), the observed baryons in stars, the cold interstellar medium, residual Lyα forest gas, OVI and BLA absorbers, and hot gas in clusters of galaxies account for only \sim 50\% of the expected baryons – the remainder has yet to be identified (e.g., (Fukugita and Peebles, 2004; Nicastro et al., 2008; Shull et al., 2012)).

Where is the remainder of the baryons in the local Universe? Along with the structure formation, accretion shocks heat the intergalactic medium (IGM), roughly to the virial temperature of the structures onto which the material is accreting. This changes the ionization equilibrium of the IGM and causes the neutral fraction to drop quickly with the temperature increase. In addition, feedback from star formation and AGN activity in galaxies heats the IGM as well. With these processes, a large amount of IGM has been converted into so-called warm hot intergalactic medium (WHIM). The WHIM no longer shows up in the Lyman-α forest. Hydrodynamical simulations suggest that 40-50\% of baryons could be in the form of WHIM located in a cosmic web between clusters of galaxies with a temperature range of \sim 10^5-10^7 K as seen in Fig. 4.1 (Cen and Ostriker, 2006).

The effort to detect the WHIM has been focused on metal absorption lines such as OVII and OVI corresponding to a series of soft X-ray lines around 0.6-0.8 keV. A few detections of WHIM filaments have been claimed by XMM-Newton and Chandra observations, but they have little sensitivity to the WHIM and most have not been confirmed significantly. The WHIM also produces the diffuse X-ray emission, however, the emissivity is proportional to the square of the gas density and drops significantly for the low-density plasma. Fig. 4.2 from Shull et al., (2012) shows the current census of baryons including the detected WHIM. The sum of the known baryons falls short of the expected and \sim 30\% of the baryons are still missing.

On the other hand, the tSZ effect has a linear response to the gas density and higher sensitivity to low density plasma. The optical depth of the SZ effect is subtle and the detection is challenging. For example, in a filament of length 1 Mpc and electron density \sim 10^{-6} \text{ cm}^{-6} (\delta \sim 10 \text{ at } z \sim 0), the optical depth is only \tau \sim 4 \times 10^{-5}, however this is potentially observable through a stacking method or cross-correlation with other tracers of structure.
4.3 Pair stacking of LRG pairs

4.3.1 LRG pair catalog

SDSS Data Release 12 (DR12) is the final release of data from SDSS-III. The catalog provides the position, spectroscopic redshift, and classification type for each object. We extract objects identified as `sourcetype=LRG`. The stellar masses of these objects have been estimated by three different groups. We use the estimate based on a principal component analysis method by (Chen et al., 2012), which uses stellar evolution synthesis models from (Bruzual and Charlot, 2003), and a stellar initial mass function from (Kroupa, 2001). For the stacking analysis, we select LRG’s with

\[\text{http://www.sdss.org/dr12/spectro/galaxy}\]
4.3. Pair stacking of LRG pairs

Figure 4.2: The baryon census in the local Universe (Shull et al., 2012).

$M_* > 10^{11.3} M_\odot$. According to the scaling relation of $Y_{500} - M_*$ ($Y_{500}$: the Comptonization parameter integrated over a sphere of radius $R_{500}$) reported in (Planck Collaboration, 2013c), these LRG’s should have a central tSZ signal-to-noise ratio of order unity. Since our analysis requires us to estimate and subtract the tSZ signal associated with the halos of the individual LRG’s, this cut enhances the reliability of that estimation, via the procedure described in §4.3.3.

As we discussed in §3.3, not all LRG’s are central galaxies in massive halos. We reject a given galaxy if a more massive galaxy resides within a tangential distance of $1.0 \, h^{-1}$ Mpc and within a radial velocity difference of $|c \Delta z| < 1000 \, \text{km} \, \text{s}^{-1}$. We construct the LRG pair catalog from this central LRG catalog by finding all neighbouring LRG’s within a tangential distance of 6-10 $h^{-1}$ Mpc and within a proper radial distance of ± 6$h^{-1}$ Mpc. The resulting catalog has 262,864 LRG pairs to redshifts $z \sim 0.4$. Their redshift and separation distributions are shown in Figure 4.3.

4.3.2 Stacking on LRG pairs

The angular separation between LRG pairs in our catalog ranges between 27 and 203 arcmin. For each pair in the catalog, we follow Clampitt et al. (2016)
4.3. Pair stacking of LRG pairs

Figure 4.3: Left: The redshift distribution of LRG pairs peaks at $z \sim 0.35$. Top right: The distribution of tangential separations between LRG pairs. Bottom right: The distribution of radial separations between LRG pairs.

and form a normalized 2-dimensional image coordinate system, $(X, Y)$, with one LRG placed at $(-1, 0)$ and the other placed at $(+1, 0)$. The corresponding transformation from sky coordinates to image coordinates is also applied to the $y$ map and the average is taken over all members in the catalog. The mean tSZ signal in the annular region $9 < r < 10$ ($r^2 \equiv X^2 + Y^2$) is subtracted as an estimate of the local background signal.

The top panel of Figure 4.4 shows the average $y$ map stacked against 262,864 LRG pairs over the domain $-3 < X, Y < +3$, and the lower panel shows a slice of this map at $Y = 0$. Not surprisingly, the average signal is dominated by the halo gas associated with the individual LRGs in each pair. The peak amplitude of this signal is $\Delta y \sim 1.4 \times 10^{-7}$, consistent with the study in §3.4 in the previous chapter.

4.3.3 Subtracting the halo contribution

We estimate the average contribution from single LRG halos as follows. Since we have selected central LRGs for our pair catalog, we assume that the average single-halo contribution is circularly symmetric about each LRG in a pair. To determine the radial profile of each single halo, we fit the map (indexed by pixel $p$) to a model of the form

$$y_h(p) = y_{L,i}(p) + y_{R,j}(p),$$  \hspace{1cm} (4.1)

where $y_{L,i}$ is the single-halo signal in the $i$th radial bin centred on the “left” LRG at $(-1, 0)$, $y_{R,j}$ is the single-halo signal in the $j$th radial bin centred
4.3. Pair stacking of LRG pairs

Figure 4.4: Top: The average Planck $y$ map stacked against 262,864 LRG pairs in a coordinate system where one LRG is located at $(X,Y) = (-1,0)$ and the other is at $(X,Y) = (+1,0)$. The square region, $-3 < X,Y < +3$, comprises $151 \times 151$ pixels. Bottom: The corresponding $y$ signal along the X axis.
4.3. Pair stacking of LRG pairs

on the “right” LRG at (+1, 0), and \( p (=p(X,Y)) \) is a pixel on the map. When performing the fit, we choose radial bins of size \( \Delta r = 0.02 \), and we weight the map pixels uniformly. To avoid biasing the profiles with non-circular filament signal, we exclude the central region \(-1 < X < +1 \) and \(-2 < Y < +2 \) from the fit. Figure 4.5 shows the resulting best-fit profiles for the LRG halos.

Figure 4.5: Top: The best-fit circular halo profiles fit to the map in Figure 4.4. Bottom: The best-fit radial profile of the left and right halos shown above.

Figure 4.6 shows the residual \( y \) map after subtracting the best-fit circular profiles shown in Figure 4.5 (note the change in color scale from Figures 4.4 and 4.5). The bright halo signals appear to be cleanly subtracted, while a residual signal between the LRGs is clearly visible. The lower panels of Figure 4.6 show the residual signal in horizontal \((Y = 0)\) and vertical \((X = 0)\) slices through the map. The shape of the signal is consistent
4.3.1 Pair stacking of LRG pairs

with an elongated filamentary structure connecting average pairs of central LRGs. The mean residual signal in the central region, \(-0.8 < X < +0.8\) and \(-0.2 < Y < +0.2\), is found to be \(\Delta y = 1.31 \times 10^{-8}\).

\[
\Delta y = 1.31 \times 10^{-8}
\]

4.3.4 Null tests and error estimates

To assess the reality of the residual signal and estimate its uncertainty, we perform two types of Monte Carlo-based null tests. In the first test, we rotate the center of each LRG pair by a random angle in galactic longitude (while..
4.3. Pair stacking of LRG pairs

keeping the galactic latitude fixed, in case there is a systematic galactic background signal). We then stack the \( y \) map against the set of rotated LRG pairs, and we repeat this stacking of the full catalog 1000 times to determine the \( \text{rms} \) fluctuations in the background (and foreground) sky. Figure [4.7] shows one of the 1000 rotated, stacked \( y \) maps: the map has no discernible structure. We can use this ensemble of maps to estimate the uncertainty of the filament signal quoted above. Taking the same region used before \((-0.8 < X < +0.8 \text{ and } -0.2 < Y < +0.2)\), we find that the ensemble of null maps has a mean and standard deviation of \( \Delta y = (-0.03 \pm 0.24) \times 10^{-8} \) in Figure [4.10]. Since the average signal in this null test is consistent with zero, we cautiously infer that our estimator is unbiased, however, we present another test in the following.

The second null test is to stack the \( y \) map against “pseudo-pairs” of LRGs: that is, pairs of objects that satisfy the transverse separation criterion, but which have a large separation along the radial direction. Such pairs are not expected to be connected by filamentary gas. We generate a pseudo-pair catalog as follows: for each pair in the original catalog, we pick one of the two members at random, then pick a new partner LRG that is located within 6-10 \( h^{-1}\text{Mpc} \) of it in the transverse direction, but which is more than 30 \( h^{-1}\text{Mpc} \) away in the radial direction. We select the same number of pairs meeting this criterion as in default LRG pair catalog, so that the stacked image is of approximately the same depth. We repeat this selection 1000 times to generate an ensemble of pseudo-pair catalogs.

The top panel of Figure [4.8] shows an average \( y \) maps stacked against one of the pseudo pair catalog realizations. This map is similar to the genuine pair-stacked map shown in Figure [4.4], but with less apparent signal between the LRGs. We perform the same single-halo model fit described above and subtract it from the map with the result is shown in the middle panel of Figure [4.8]. As with the rotated null test above, this map shows no discernible structure. To generate statistics, we repeat this test 1000 times: we find that the ensemble of null maps has a mean and standard deviation of \( \Delta y = (0.00 \pm 0.25) \times 10^{-8} \) in Figure [4.10], virtually the same as with the rotated ensemble. We adopt this standard deviation as the final uncertainty of the mean filament signal due to instrument noise, sky noise (i.e., cosmic variance and foreground rejection errors), and halo subtraction errors.
4.3. Pair stacking of LRG pairs

Figure 4.7: *Top:* A sample null map obtained by stacking the $y$ map against the LRG pairs that were rotated in galactic longitude by random amounts. *Bottom:* The tSZ signal along the X and Y axes of the $y$ map shown above.
4.3. Pair stacking of LRG pairs

Figure 4.8: Top: An average $y$ map stacked against a catalog of LRG pseudo pairs (see text for a definition). Middle: The residual $y$ map after subtracting the best-fit circular halos from the above map, using the same procedure that was applied to the genuine pair stack. Bottom: The tSZ signal along the X and Y axes of the residual map shown in the middle panel.
4.3. Pair stacking of LRG pairs

Figure 4.9: The result from 1000 rotated null stacks. The data $y$ value ($\Delta y = 1.31 \times 10^{-8}$) is expressed in red-dash line.

Figure 4.10: The result from 1000 pseudo-pair null stacks. The data $y$ value ($\Delta y = 1.31 \times 10^{-8}$) is expressed in red-dash line.
4.4 Interpretation of the detected tSZ signal between the LRG pairs

We can estimate the physical conditions of the gas we detect by considering a simple, isothermal, cylindrical filament model of electron over-density with a density profile proportional to $r_c/r$, at redshift $z$. The Compton $y$ parameter produced by the tSZ effect is given by

$$y = \frac{\sigma_T k_B}{m_e c^2} \int n_e T_e \, dl. \quad (4.2)$$

In general, the electron density at position $\mathbf{x}$ may be expressed as

$$n_e(\mathbf{x}, z) = \bar{n}_e(z)(1 + \delta(\mathbf{x})), \quad (4.3)$$

where $\delta(\mathbf{x})$ is the density contrast, and $\bar{n}_e(z)$ is the mean electron density in the universe at redshift $z$,

$$\bar{n}_e(z) = \frac{\rho_b(z)}{\mu_e m_p}, \quad (4.4)$$

where $\rho_b(z) = \rho_c \Omega_b (1 + z)^3$ is the baryon density at redshift $z$, $\rho_c$ is the present value of critical density in the universe, $\Omega_b$ is the baryon density in units of the critical density, $\mu_e = \frac{2}{1 + \chi} \simeq 1.14$ is the mean molecular weight per free electron for a cosmic hydrogen abundance of $\chi = 0.76$, and $m_p$ is the mass of the proton.

We can express the profile in the Compton parameter as a geometrical projection of a density profile with $n_e(r, z)$:

$$y(r) = \frac{\sigma_T k_B T_e}{m_e c^2} \int_{r_\perp}^{R} \frac{2r n_e(r, z)}{\sqrt{r^2 - r_\perp^2}} \, dr, \quad (4.5)$$

where $r_\perp$ is the tangential distance from the filament axis on the map and $R$ is the cut-off radius of the filaments. Assuming negligible evolution of the filaments and constant over-density, $\delta_c$, at the core since $z = 0.4$,

$$n_e(r = 0, z) = \frac{n_e(r = 0, z)}{\bar{n}_e(z)} \bar{n}_e(z) = \delta_c \bar{n}_e(0) (1 + z)^3. \quad (4.6)$$

We consider three density profiles,

$$n_e(r) = \begin{cases} \text{constant} & (r < r_c), \\ \frac{n_e(0)}{\sqrt{1 + (r/r_c)^2}} & (r < 5r_c), \\ \frac{n_e(0)}{1 + (r/r_c)^2} & (r < 5r_c). \end{cases} \quad (4.7)$$

$$n_e(r) = \begin{cases} \text{constant} & (r < r_c), \\ \frac{n_e(0)}{\sqrt{1 + (r/r_c)^2}} & (r < 5r_c), \\ \frac{n_e(0)}{1 + (r/r_c)^2} & (r < 5r_c). \end{cases} \quad (4.8)$$

$$n_e(r) = \begin{cases} \text{constant} & (r < r_c), \\ \frac{n_e(0)}{\sqrt{1 + (r/r_c)^2}} & (r < 5r_c), \\ \frac{n_e(0)}{1 + (r/r_c)^2} & (r < 5r_c). \end{cases} \quad (4.9)$$
where $r_c$ is the core radius. To regularize the profiles, we adopt a cutoff radius of $5r_c$ for the second and third profile. Applying the profiles to the simulations described in §4.5, we find the best-fit density profile to follow $(r_c/r)$.

For this model, the predicted tSZ signal in the region $(-0.8 < X < +0.8, -0.2 < Y < +0.2)$ for the 262,864 filaments can be written as

$$\Delta \tilde{y} = 4.9 \times 10^{-8} \times \left( \frac{\delta_c}{10} \right) \left( \frac{T_e}{10^7 \text{ K}} \right) \left( \frac{r_c}{0.5h^{-1} \text{ Mpc}} \right).$$  \hspace{1cm} (4.10)

Applying the observational constraint on mean $\Delta y$, we have,

$$\delta_c \left( \frac{T_e}{10^7 \text{ K}} \right) \left( \frac{r_c}{0.5h^{-1} \text{ Mpc}} \right) = 2.7 \pm 0.5.$$  \hspace{1cm} (4.11)

Assuming the same temperature and morphology estimates from the simulations in §4.5 apply to the observational data, the mean filament over-density between LRG pairs is $\delta \sim 3.2 \pm 0.7$. This implies that the gas in the filaments between the LRGs can be widely spread over the large scale with very low density.

Is the signal we detect due to unbound diffuse gas outside of halos or bound gas in halos between the LRG pairs? To investigate this, we simulate a model $y$ map using only bound gas in the SDSS DR12 LRGs with $10^{10} M_\odot < M_* < 10^{12} M_\odot$ and $0. < z < 0.8$ and compare the result with Planck $y$ map. To make the single-halo model $y$ map, we select “central” LRGs described in §4.3.1, which leaves $\sim 1,100,000$ LRGs, and estimate the halo masses of the LRGs with the stellar-to-halo mass relation of Coupon et al. (2015) (§3.6.3). Then we locate $y$ profiles within the virial radius ($r < r_{200}$) of the LRG halos on the map using the universal pressure profile (UPP) (§3.6.2). For the model $y$ map, we perform the same analysis. The peak $y$ value of the LRG halos is dimmer than the $y$ map in Figure 4.4 since we only include the contribution within the virial radius of the LRG halos and in addition, no sub-halos are included. After the circular halo subtraction, we obtain $\Delta y = 0.29 \times 10^{-8}$ from the bridge region. Moreover, we simulate a model $y$ map including $y$ profiles within $r < 3 \times r_{200}$ of the LRG halos and the result is $\Delta y = 0.36 \times 10^{-8}$. These results suggest that most of the $y$ signal we detect between the LRGs should originate in unbound diffuse gas, although the contributions from other types of galaxies, less massive galaxies and galaxies in higher redshift should be present at some level.

Some systematic effects that might enhance or diminish the tSZ signal in the filaments may exist. For example, in our analysis, although we assume
4.4. Interpretation of the detected tSZ signal between the LRG pairs

Figure 4.11: Left: The single-halo model $y$ map described in the text is stacked against the same 262,864 LRG pairs as in the data analysis. Right: The residual model $y$ map after subtracting the best-fit circular halos from the map at left, using the same procedure that was applied to the genuine pair stack.

that the average single-halo contribution is circularly symmetric about each LRG, one might speculate that the signal we detect between nearby LRG pairs is a result of tidal effects, in which single-object halos are elongated along the line joining the two objects. However, considering that the $\sim 260,000$ LRG pairs are made from $\sim 220,000$ LRGs and each LRG has roughly 2 pairs (One is aligned between our LRG pairs and the other is not aligned), the direction of the elongation is not obvious. While tidal effects must be present at some level, if they were the dominant explanation for the residual signal we see, we would expect the elongated halo structure to extend in both directions along the line joining the objects. The fact that we see no significant excess signal outside the average pair suggests that tidal effects are not significant.

On the other hand, there are possible systematic effects that might lower the tSZ signal in the filaments. For example, some of the LRG pairs are not connected by filaments, or some of the filaments may not straight but curved. However, in a study of $N$-body simulations, Colberg et al. (2005) found that cluster pairs with separations $< 5h^{-1} \text{Mpc}$ are always connected by dark matter filaments, mostly straight filaments. Further, they found filaments connecting $\sim 85\%$ of pairs separated by $5-10h^{-1} \text{Mpc}$ and $\sim 70\%$ of pairs separated by $10-15h^{-1} \text{Mpc}$. This effect would lower the average $y$-value in the stacked filament and/or broaden the shape at some level, but the simulation study implies that most LRG pairs should be connected by
dark matter (and presumably gas) filaments, and that dilution is unlikely
to be significant.

In addition, there may be effects from asymmetric feature outside halos,
due to the filaments extending out to different direction other than be-
tween our LRG pairs. We estimate the effect assuming circularly symmetric
distribution of unaligned filaments around the halos. The aligned-filaments
between the LRG’s occupy roughly 10% with \( \Delta y \sim 1.0 \times 10^{-8} \) region around
a circular halo, so one unaligned filament makes only a few % extra excess
on top of the circular halo profile and the effect should be negligible.

Finally, there might be an effect due to the Planck beam. We study the
beam effect for the filaments using the BAHAMAS simulations. Using the
smoothed \( y \) maps, the result is \( \Delta y = 0.84 \times 10^{-8} \) and it is \( \Delta y = 1.00 \times 10^{-8} \)
with the unsmoothed \( y \) maps. With the study, we find that the beam dilutes
the amplitude of \( y \)-value by \( \sim 15\% \), although the mean separation angle
between the LRGs is \( \sim 0.7 \) deg, therefore, the beam effect should not be
significant in our study.

4.5 Comparison to BAHAMAS hydrodynamic simulations

4.5.1 BAHAMAS hydrodynamic simulations

To compare our results with theory, we analyze the BAHAMAS suite of cos-
ological smoothed particle hydrodynamics (SPH) simulations (McCarthy
et al., 2017) in the same manner as the data. The BAHAMAS suite is
a direct descendant of the OWLS (Schaye et al., 2010) and cosmo-OWLS
projects (Le Brun et al., 2014; McCarthy et al., 2014; van Daalen et al.,
2014). The simulations reproduce a variety of observed gas features in groups
and clusters of galaxies in the optical and X-ray bands. The BAHAMAS
suite consists of box-periodic hydrodynamical simulations, the largest of
which have volumes of \((400h^{-1}\text{Mpc})^3\) and contain 1024\(^3\) each of baryonic and
dark matter particles. The suite employs two different cosmological mod-
els: WMAP9 cosmology (Hinshaw et al., 2013) with \( \{ \Omega_m, \Omega_b, \Omega_\Lambda, \sigma_8, n_s, h \} =\)
\( \{ 0.2793, 0.0463, 0.7207, 0.821, 0.972, 0.700 \} \), and Planck 2013 cosmology
(Planck Collaboration, 2014a) with \( \{ \Omega_m, \Omega_b, \Omega_\Lambda, \sigma_8, n_s, h \} =\)
\( \{ 0.3175, 0.0490, 0.6825, 0.834, 0.9624, 0.6711 \} \). We have four realizations with
the WMAP9 cosmology and one with the Planck 2013 cosmology.

From each realization, 10 almost-independent mock galaxy catalogues
are generated on a light cone and 10 corresponding \( y \) maps are generated
from the hot gas (McCarthy et al., 2014). Each of these light cones contain about one million galaxies out to $z \sim 1$, and each spans a $10^\circ \times 10^\circ$ patch of sky.

4.5.2 Comparison with the hydrodynamic simulations

We compare our results to these simulations. To do so, we analyze light cones from the BAHAMAS suite of simulations (§4.5.1) as we did the real data. For each cosmology, we construct simulated LRG pairs by selecting central galaxies with the same separation criteria as the real data. We invoke a stellar mass threshold such that the mean stellar mass of the sample matches the mean of the data. The resulting catalog has 242,669 pairs. Prior to stacking, we also convolve the simulated $y$ map in each light cone with a 10 arcmin FWHM Gaussian kernel to match the Planck map. After stacking and radial halo subtraction, we find the residual tSZ signal between central galaxy pairs to be $\Delta y = (0.84 \pm 0.24) \times 10^{-8}$ with the WMAP9 cosmology. The uncertainty is estimated by drawing 1000 bootstraps samples from among the 40 light cones. We have also analyzed the simulations based on the Planck 2013 cosmology and find $\Delta y = (1.14 \pm 0.33) \times 10^{-8}$. However, this model only has one realization of the initial conditions, instead of four, so it has a larger uncertainty than the WMAP9 estimate.

The comparison of the simulations to the data is not entirely straightforward because of possible selection effects. In particular, the methods for estimating stellar mass in these two systems are different. The data estimates we use are based on the principal component method in Chen et al. (2012), which are, on average, $\sim 0.2$ dex higher than those based on spectro-photometric model fitting (Maraston et al., 2013). The simulation estimates we use are based on directly counting the baryonic mass within 30 kpc of a given central galaxy. As noted in §4.3.1, we adopt a stellar mass threshold of $10^{11.3} M_\odot$ for the data. In order to match the mean stellar mass of the simulation population, we must adopt a stellar mass threshold of $10^{11.2} M_\odot$. This procedure produces the same peak $y$ values at the center of each mean halo: data and simulation. We believe this selection should produce comparable filament amplitudes.

In addition, we examine four independent realizations of the WMAP9 cosmology and find that the mean residual tSZ signal between central galaxy pairs varies from $\Delta y = 0.26 \times 10^{-8}$ to $1.37 \times 10^{-8}$ by factor of $\sim 5$. This suggests the cosmic variance has a large impact on the simulations considering the limited volumes in the simulations compared to the data covering almost one quarter of the sky.
With the caveats noted above, we can use the simulations to further probe the physical conditions in the observed filaments using the “unsmoothed” maps. In Figure 4.12 we separately examine the electron over-density and temperature in the stacked simulation data. For each light cone in our simulation box, we form optical depth and electron temperature maps,

$$
\tau = \sigma_T \int n_e \, dl,
$$

(4.12)

$$
\langle T_e \rangle = \langle y \rangle \langle \tau \rangle \times \frac{k_B}{m_e c^2},
$$

(4.13)

where $\sigma_T$ is the Thomson scattering cross section, $k_B$ is the Boltzmann constant, $m_e$ is the electron mass, $c$ is the speed of light, $n_e$ is the electron number density, $T_e$ is the electron temperature, and the integral is taken along the radial direction. Using the mean over-density map, we fit a variety of density profile models (§4.4) to the stacked $\tau$ map, and we find the best-fit profile to follow ($r_c/r$, where $r$ is the perpendicular distance from the cylinder axis, and $r_c = 0.5h^{-1}\text{Mpc}$, where $r_c$ is the core radius of the density profile. Assuming this profile, the best-fit central over-density is $\delta = 1.5 \pm 0.4$. From the $\langle T_e \rangle$ map, the mean (electron density-weighted) temperature of the electron gas in the filament region ($-0.8 < X < +0.8$, $-0.2 < Y < +0.2$) is found to be $(0.82 \pm 0.06) \times 10^7 \text{ K}$.

4.6 Discussion

Other groups have studied filamentary gas in the large scale structure. We compare and contrast those results to ours as follows.

The Planck Team (Planck Collaboration 2013b) studied the gas between the merging Abell clusters A399 and A401, which have a tangential separation of $3h^{-1}\text{Mpc}$. Using a joint analysis of Planck tSZ data and ROSAT X-ray data, they estimate a gas temperature of $kT = 7.1 \pm 0.9 \text{ keV}$ ($T \sim 8.2 \times 10^7 \text{ K}$), and a central electron density of $n_e = (3.72 \pm 0.17) \times 10^{-4} \text{ cm}^{-3}$ ($\delta \sim 1500$). Assuming a filament diameter of 1.0 Mpc with a cylindrical shape, it corresponds to $y \sim 10^{-5}$. This high density and temperature may be because the filaments in merger systems have been shock-heated and compressed more than normal.

Using XMM-Newton observations, Werner et al. (2008) study the gas properties in a filament connecting the massive Abell clusters A222 and A223, at redshift $z \sim 0.21$. Assuming a separation of 15 Mpc, they find $kT = 0.91 \pm 0.25 \text{ keV}$ ($T \sim 1.1 \times 10^7 \text{ K}$) and $n_e = (3.4 \pm 1.3) \times 10^{-5} \text{ cm}^{-3}$ ($\delta \sim$...
4.6. Discussion

Figure 4.12: Top left: The stacked \( y \) map of the central galaxy pairs from the BAHAMAS simulations, at 10 arcsecond angular resolution (unsmoothed). Top right: The same \( y \) map after the best-fit circular halos are subtracted. Middle left: The stacked \( \tau \) map for the same pair sample as above. Middle right: The same \( \tau \) map after circular halo subtraction. Bottom: The electron density-weighted temperature (\( T_e \)) map on a \( \log_{10} \) scale.
4.6. Discussion

In addition, Eckert et al. (2015) find filamentary structures around the galaxy cluster Abell 2744, at \( z \sim 0.3 \), and estimate a gas temperature of \( T \sim 10^7 \) K, and an over-density of \( \delta \sim 200 \) on scales of 8 Mpc. Assuming a filament diameter of 1.0 Mpc with a cylindrical shape, it corresponds to \( y \sim 10^{-7} \), which is one order of magnitude higher than our result.

Interestingly, as a study with similar targets, but using the CFHTLenS mass map, Epps and Hudson (2017) studied the weak lensing signal of filaments between SDSS-III/BOSS LRG’s and find a mass of \((1.6 \pm 0.3) \times 10^{13} M_\odot\) for a stacked filament region of 7.1 \( h^{-1}\)Mpc long and 2.5 \( h^{-1}\)Mpc wide. Assuming a uniform-density cylinder, they estimate \( \delta \sim 4 \) in the filaments, consistent with our result.

Considering the limited number of X-ray detections, it could be that our result of \( y \sim 10^{-8} \) is typical of filaments between lower-mass galaxy groups and clusters, but systematic effects may also contribute to the difference as we discussed.

A similar study using simulations is presented in Colberg et al. (2005), wherein 228 filaments between pairs of galaxy clusters are studied using the \( \Lambda \)CDM N-body simulation of Kauffmann et al. (1999). They identify straight mass filaments longer than 5\( h^{-1}\) Mpc, normalize the length of each filament to unity, and find the average density of matter contained within 2\( h^{-1}\) Mpc of the (normalized) filament axis to be \( \delta \sim 7 \). This is somewhat higher than our estimate of \( \delta = 3.2 \pm 0.7 \), however, the following factors may compromise this comparison.

1) They select halos with masses larger than \( 10^{14} M_\odot \), whereas we select LRGs with the stellar masses larger than \( 10^{11.3} M_\odot \). According to the SHM relation used in Planck Collaboration (2013c) (see also Wang et al. (2016)), this corresponds to halo masses with \( M_{200} \sim (5 - 7) \times 10^{13} M_\odot \), and may include lower mass systems, given the scatter in the SHM relation. The mass of filaments can be correlated with the mass of the halos associated with the filaments as West et al. (1995) suggests the cluster formation along filaments. This selection of low-mass halo pairs can result in the lower tSZ signal from the filaments between the LRGs.

2) The \( \Lambda \)CDM simulation tracks dark matter, whereas we analyze hydrodynamic simulations which include baryonic effects such as radiative cooling, star formation, SN feedback and AGN feedback. The baryonic gas in the filaments may not trace the dark matter faithfully.

3) They examine filaments between cluster pairs separated by 5\( h^{-1}\) to 25\( h^{-1}\) Mpc, whereas we study smaller separations of 6-10\( h^{-1}\) Mpc.
4.7 Conclusion

In this chapter, using the Planck Sunyaev-Zeldovich (tSZ) map and the SDSS DR12 catalog of Luminous Red Galaxies (LRGs), we search for warm/hot filamentary gas between pairs of LRGs by stacking the $y$ map on a grid aligned with the pairs. We detect a strong signal associated with the LRG host halos and subtract that using a best-fit, circularly symmetric model. We detect a statistically significant residual signal and draw the following conclusions.

- The residual tSZ signal in the region between LRG pairs is $\Delta y = (1.31 \pm 0.25) \times 10^{-8}$, with a 5.3$\sigma$ significance. Assuming a simple, isothermal, cylindrical filament model of electron over-density with a radial density profile proportional to $r_c/r$ (as determined from simulations), we constrain the physical parameters of the gas in the filaments to be

$$\delta_c \left(\frac{T_e}{10^7 \text{ K}}\right) \left(\frac{r_c}{0.5h^{-1}\text{Mpc}}\right) = 2.7 \pm 0.5. \quad (4.14)$$

- We apply the same analysis to the BAHAMAS suite of cosmological hydrodynamic simulations (McCarthy et al., 2017). The results are marginally consistent, but the simulations predict a slightly lower mean tSZ signal of $\Delta y = (0.84 \pm 0.24) \times 10^{-8}$.

- Our result is comparable with the result in Epps and Hudson (2017) for the overdensity in the filaments. They study the weak lensing signal of filaments between SDSS-III/BOSS LRG’s and estimate $\delta \sim 4$ assuming a uniform-density cylinder of filaments. We also compare our results to complementary X-ray results, such as Werner et al. (2008) and Eckert et al. (2015). Our result is lower by a factor of $\sim 10$, but our systems are much different.

Our investigation can be extended with larger spectroscopic surveys such as extended BOSS (eBOSS) in SDSS-IV and the Dark Energy Spectroscopic Instrument (DESI). Their larger samples would improve the signal-to-noise and allow for a more detailed study of the physical conditions as a function of LRG properties, such as stellar mass and redshift. Large-area experiments with higher tSZ angular resolution, such as the Atacama Cosmology Telescope and the South Pole Telescope, would also help to ascertain the state of filament gas.
Chapter 5

Probing hot gas in the cosmic web between galaxy groups and clusters

5.1 Introduction

In the previous chapter, we detect the tSZ signal due to gas filaments between the \( \sim 260,000 \) LRG pairs using the Planck \( y \) map. In this chapter, we search for the tSZ signal of the gas filaments between galaxy group and clusters, identified by (Tempel et al., 2014) based on the spectroscopic sample of the galaxies of SDSS data release 10 (DR10) by following the same analysis procedure in chapter 4.

The SDSS DR10 groups and clusters catalog is constructed from SDSS DR10 galaxies by friends-of-friends (FoF) algorithm, which has been the most frequently applied method of identifying groups and clusters in galaxy redshift data. The advantage to use the group catalog is that they are clearly identified as galaxy group/clusters in their dark matter halos. The drawback is that the number of available systems is limited to massive ones in the local Universe so far since the galaxy survey has to be complete in a redshift range, otherwise the identification can be biased.

5.2 Pair stacking of galaxy group/clusters

5.2.1 Galaxy groups and clusters for SDSS DR10 galaxies

Galaxies tend to gather in groups of several members, or even larger companions since gravitationally bound galaxy systems are linked by an underlying dark matter halo. However, there is no straightforward way to identify them. The friends-of-friends (FoF) algorithm has been the most frequently applied to identify groups and clusters in galaxy redshift data. The FoF method uses galaxy distances as the main basis of grouping, and thus it is relatively
simple and straightforward. (Tempel et al., 2014) uses the FOF method for the SDSS DR10 galaxies (Ahn et al., 2014) with a variable linking length in the transverse and radial directions to identify as many realistic groups as possible and constructed a flux-limited FoF group and cluster catalog, consisting of 82,458 galaxy groups.

Dynamical masses of the galaxy group are estimated based on radial velocity dispersions and group extent in the sky to the extracted groups. For the stacking analysis, we select the groups with \( M_{\text{dyn}} > 10^{13} M_\odot \). According to the scaling relation of \( Y_{500} - M_* \) (\( Y_{500} \): the Comptonization parameter integrated over a sphere of radius \( R_{500} \)) reported in (Planck Collaboration, 2013c), these groups should have a central tSZ signal-to-noise ratio of order unity. Since our analysis requires us to estimate and subtract the tSZ signal associated with the halos of the individual groups, this cut enhances the reliability of that estimation.

### 5.2.2 SDSS DR10 group pair catalog

We construct the pair catalog from the SDSS DR10 group catalog by finding all neighboring groups within a tangential distance of 6-10 \( h^{-1} \)Mpc and within a proper radial distance of \( \pm 6 h^{-1} \)Mpc. The resulting catalog has 34,955 group pairs to redshifts \( z \sim 0.2 \). Their redshift and separation distributions are shown in Figure 5.1.

![Figure 5.1](image)

Figure 5.1: Left: The redshift distribution of group pairs peaks at \( z \sim 0.08 \). Top right: The distribution of tangential separations between group pairs. Bottom right: The distribution of radial separations between group pairs.
5.2.3 Stacking on group pairs

We follow the same analysis in the previous chapter and form a normalized 2-dimensional image coordinate system, \((X, Y)\), with one galaxy group placed at \((-1, 0)\) and the other placed at \((+1, 0)\). The corresponding transformation from sky coordinates to image coordinates is also applied to the \(y\) map and the average is taken over all members in the catalog. The mean tSZ signal in the annular region \(6 < r < 8\) \((r^2 \equiv X^2 + Y^2)\) is subtracted as an estimate of the local background signal.

The top panel of Figure 5.2 shows the average \(y\) map stacked against 34,955 group pairs over the domain \(-3 < X, Y < +3\), and the lower panel shows a slice of this map at \(Y = 0\). The average signal is dominated by the halo gas associated with the individual groups in each pair. The peak amplitude of this signal is \(\Delta y \sim 2.5 \times 10^{-7}\), which is higher than \(\Delta y \sim 1.4 \times 10^{-7}\) of the LRG pairs. It suggests that more massive halos are selected for this study than the LRG halos in the previous chapter.

5.2.4 Subtracting the halo contribution

We estimate the average contribution from single group halos by assuming that the average single-halo contribution is circularly symmetric about each group in a pair. When determining the radial profile of each single halo and performing the fit, we mask the central region \(-1 < X < +1\) and \(-0.5 < Y < +0.5\) from the fit. Figure 5.3 shows the resulting best-fit profiles for the group halos.

Figure 5.4 shows the residual \(y\) map after subtracting the best-fit circular profiles shown in Figure 5.3. The bright halo signals appear to be cleanly subtracted, while a residual signal between the groups is clearly visible. The lower panels of Figure 5.4 show the residual signal in horizontal \((Y = 0)\) and vertical \((X = 0)\) slices through the map. The shape of the signal is consistent with an elongated filamentary structure connecting average pairs of central LRGs. The mean residual signal in the central region, \(-0.8 < X < +0.8\) and \(-0.2 < Y < +0.2\), is found to be \(\Delta y = 2.28 \times 10^{-8}\).

5.2.5 Null tests and error estimates

To assess the reality of the residual signal and estimate its uncertainty, we perform two types of Monte Carlo-based null tests: rotated null stacks and pseudo-pair null stacks as we did in §4.3.4. We find that the ensemble of null maps has a mean and standard deviation of \(\Delta y = (0.01 \pm 0.33) \times 10^{-8}\) from
5.2. Pair stacking of galaxy group/clusters

Figure 5.2: Top: The average Planck $y$ map stacked against 34,955 group pairs in a coordinate system where one LRG is located at $(X, Y) = (-1, 0)$ and the other is at $(X, Y) = (+1, 0)$. The square region, $-3 < X, Y < +3$, comprises $151 \times 151$ pixels. Bottom: The corresponding $y$ signal along the $X$ axis.
5.2. Pair stacking of galaxy group/clusters

Figure 5.3: Top: The best-fit circular halo profiles fit to the map in Figure 5.2. Bottom: The best-fit radial profile of the left and right halos shown above.

1000 rotated null stacks and $\Delta y = (0.07 \pm 0.36) \times 10^{-8}$ from 1000 pseudo-pair null stacks, which are shown in Figure 5.6. The average signals in these null tests are consistent with zero and the standard deviations are consistent with each other. We adopt the standard deviation from the pseudo-pair null stacks as the final uncertainty of the mean filament signal due to instrument noise, sky noise (i.e., cosmic variance and foreground rejection errors), and halo subtraction errors.
5.2. Pair stacking of galaxy group/clusters

Figure 5.4: Top: The residual y-map after the best-fit radial halo signals are subtracted. Bottom: The residual tSZ signal along the X and Y axes.
5.2. Pair stacking of galaxy group/clusters

Figure 5.5: The result from 1000 rotated null stacks. The data $y$ value ($\Delta y = 2.28 \times 10^{-8}$) is expressed in red-dash line.

Figure 5.6: The result from 1000 pseudo-pair null stacks. The data $y$ value ($\Delta y = 2.28 \times 10^{-8}$) is expressed in red-dash line.
5.3 Comparison to BAHAMAS hydrodynamic simulations

We compare our results to simulations. To do so, we analyze light cones from the BAHAMAS suite of simulations. However, the number of groups in \( z < 0.2 \) is limited in the original light cones, therefore the simulations are customized for this study to span a \( 25^\circ \times 25^\circ \) patch of sky for each light cone by limiting the redshift up to \( z \sim 0.13 \). To hold the independence among the generated light cones, only 3 \( y \) maps are generated for each realization of four, instead.

For WMAP9 cosmology, we construct simulated group pairs by selecting group pairs with the same separation criteria as the real data. We invoke a mass threshold such that the mean mass of the sample matches the mean of the data. For the simulations, we use \( M_{200m} \) (mass enclosed within a sphere of radius \( R_{200} \) such that the enclosed density is 200 times the mean “matter” density). The resulting catalog has 175,844 pairs.

Prior to stacking, we also convolve the simulated \( y \) map in each light cone with a 10 arcmin FWHM Gaussian kernel to match the Planck map. After stacking and radial halo subtraction, we find the residual tSZ signal between central galaxy pairs to be \( \Delta y = (0.85 \pm 0.37) \times 10^{-8} \) with the WMAP9 cosmology. The uncertainty is estimated by drawing 1000 bootstraps samples from among the 12 light cones.

The result from the BAHAMAS simulations is not consistent with the data. However, the comparison of the simulations to the data is not entirely straightforward because of possible selection effects. In particular, the methods for estimating mass in these two systems are different. The data estimates we use are based on radial velocity dispersions, which is only 1-dimensional information for 3-dimensional velocity. On the other hand, the simulation estimates we use are based on directly counting the particle mass within \( R_{200} \) of a given galaxy group. As noted, we adopt a mass threshold of \( 10^{13} \, M_\odot \) for the data. We check that the mean mass as well as luminosity and velocity dispersions are consistent between the data and simulations. We believe this selection should produce comparable filament amplitudes.

In addition, the galaxy groups are identified by the FOF algorithm for both, but the group catalog in the data is identified by a varying linking length, on the other hand, the simulations use one typical value of linking length, which might be causing the difference in the identification of galaxy groups, thus physical parameters such as mass of the groups.

Moreover, we examine four independent realizations of the WMAP9 cos-
5.4 Interpretation of the detected tSZ signal between the group pairs

We can estimate the physical conditions of the gas we detect by considering the same model used in \(\S 4.4\) with a density profile proportional to \(r_c/r\) at redshift \(z\). For the 34,955 group pairs, the predicted tSZ signal in the region \((-0.8 < X < +0.8, -0.2 < Y < +0.2)\) can be written as

\[
\Delta \bar{y} = 3.4 \times 10^{-8} \times \left(\frac{\delta_c}{10}\right) \left(\frac{T_e}{10^7 \text{ K}}\right) \left(\frac{r_c}{0.5 h^{-1} \text{ Mpc}}\right). \tag{5.1}
\]

Applying the observational constraint on mean \(\Delta y\), we have,

\[
\delta_c \left(\frac{T_e}{10^7 \text{ K}}\right) \left(\frac{r_c}{0.5 h^{-1} \text{ Mpc}}\right) = 6.7 \pm 1.1. \tag{5.2}
\]

Since the simulations are not consistent with the data unlike the case of the LRG pairs, we cannot assume the temperature or density from the simulations. As we discussed, the comparison between the data and simulations is not straightforward. Since we are limited to access the entire simulations so far, it is probably fair to say that the issue is still under consideration.

5.5 Discussion

Our result from the group pairs is \(y \sim 2.28 \times 10^{-8}\), which is a slightly higher than the result from the LRG pairs \((y \sim 1.31 \times 10^{-8})\), however, it is still lower than the results from the Planck team (Planck Collaboration, 2013b) \((y \sim 10^{-5})\) and X-ray observations \((y \sim 10^{-7})\) such as (Eckert et al., 2015; Werner et al., 2008). Therefore, the same discussion in \(\S 4.6\) applies to the group pairs. Here, we focus on the comparison in our results (LRG pairs and group pairs) as follows.

For the difference, one of the possible reasons is the correlation between the mass of filaments and the mass of halos associated with the filaments. The mean mass of halos consisting of the group pairs \((y \sim 2.5 \times 10^{-7})\) is
5.6. Conclusion

clearly higher than the mass of the LRG halos \((y \sim 1.4 \times 10^{-7})\) according to the peak \(y\) signal. Along with the higher SZ signal from the filaments between the group pairs, this might imply the correlation between the mass of filaments and the mass of halos associated with the filaments as [West et al. (1995)] suggests the cluster formation along filaments.

Another possibility is that the evolution of the filaments. The mean redshift of the LRGs is \(z \sim 0.3\), on the other hand, the mean of the groups is \(z \sim 0.1\). During the time, the overdensity in the filaments grow and it may cause the difference. The time evolution of the mean density for filaments is studied in [Cautun et al. (2014)] using cosmological simulations and they find that it is only a little in \(0.3 < z < 0.1\). However, to study these hypothesis using the data, we run short of statistics for filaments (or sensitivity of the SZ effect) so far.

On the other hand, it could be explained by some systematic effects. The tidal effect due to the group pairs enhances the tSZ signal in the region between the pairs and it would have more impact for more massive pairs. In addition, the unconnected pairs by filaments lower the average tSZ signal between the pairs and it would have more impact for the LRG pairs since they are not clearly identified as “central galaxies” in galaxy groups. These effects would mitigate the difference, although we believe that these effects are not significant from our studies in \[4.6\].

5.6 Conclusion

In this chapter, using the Planck Sunyaev-Zeldovich (tSZ) map and the SDSS DR10 groups and clusters catalog constructed by [Tempel et al. (2014)], we search for warm/hot gas filamentary gas between pairs of groups by stacking the \(y\) map on a grid aligned with the pairs. We detect a strong signal associated with the group halos and subtract that using a best-fit, circularly symmetric model. We detect a statistically significant residual signal and draw the following conclusions.

- The residual tSZ signal in the region between group pairs is \(\Delta y = (2.28 \pm 0.36) \times 10^{-8}\), with a 6.1\(\sigma\) significance. Assuming a simple, isothermal, cylindrical filament model of electron over-density with a radial density profile proportional to \(r_c/r\) (same model used for the filamentary gas between the LRG pairs in the previous chapter[4]), we constrain the physical parameters of the gas in the filaments to be

\[
\delta_c \left( \frac{T_e}{10^7 \text{K}} \right) \left( \frac{r_c}{0.5h^{-1}\text{Mpc}} \right) = 6.7 \pm 1.1.
\]
5.6. Conclusion

- We apply the same analysis to the BAHAMAS suite of cosmological hydrodynamic simulations (McCarthy et al., 2017). The simulations predict a lower mean tSZ signal of $\Delta y = (0.85 \pm 0.37) \times 10^{-8}$ and it is not consistent with our result. However, there are some factors making the comparison hard, especially mass.

- Our estimate of $\delta = 6.7 \pm 1.1$ from the group pairs we use assuming the temperature of $T \sim 10^7$ [K] is somehow higher than another our estimate of $\delta = 3.2 \pm 0.7$ from the LRG pairs we use in the previous chapter. The mean mass of halos associated with the group pairs ($y \sim 2.5 \times 10^{-7}$) is clearly higher than the mass of the LRG halos ($y \sim 1.4 \times 10^{-7}$) according to the peak $y$ signal. It might imply the correlation between the mass of filaments and the mass of the halos associated with the filaments, but some of the systematics has to be taken into account.

Our investigation can be extended with larger spectroscopic surveys such as extended BOSS (eBOSS) in SDSS-IV and the Dark Energy Spectroscopic Instrument (DESI). Their larger samples would improve the signal-to-noise and allow for a more detailed study of the physical conditions as a function of group properties, such as mass and redshift. Large-area experiments with higher tSZ angular resolution, such as the Atacama Cosmology Telescope and the South Pole Telescope, would also help to ascertain the state of filament gas.
6.1 CMB polarization

In the very early universe, a tiny second after the Big Bang, it is hypothesized that the universe experienced a huge expansion of the space, called “Inflation”. The theory explains several important observations such as flatness, isotropy and homogeneity of the universe. Currently, it is the best theory to explain what happened at the earliest moment of the universe, however, no direct evidence for inflation has been observed yet.

One of the key observables is the so-called “$B$-mode polarization” in the CMB. The accelerated expansion of space during inflation would have created ripples of gravitational waves in the space, and these gravitational waves could have left a distinctive polarization pattern in the CMB. Therefore, CMB polarization is one of the most important tools to probe inflation.

A linear polarization field is specified with two types of CMB polarization called “$E$-mode” and “$B$-mode”. They are analogous to the $E$ and $B$ vector fields of electromagnetism because they have similar behavior under parity inversion. The $E$-mode polarization has an even-parity and it is characterized by a curl-free mode with polarization vectors that are radial around cold spots ($E < 0$) and tangential around hot spots ($E > 0$) on the sky. The Planck stack more than 11,000 cold and 10,000 hot spots in the CMB and measure the $E$-mode polarization patterns to high precision. They actually oscillates from the radial pattern to tangential pattern (or vice versa) around the cold(hot) spots as a function of scale. On the other hand, the $B$-mode polarization has an odd-parity and it is characterized by a divergence-free (curl) mode.

Both $E$-mode and $B$-mode are invariant under rotations, but when reflected with a line through the center, the $E$-patterns remain unchanged, while $B$-patterns change the sign (Figure 6.1). The $E$-mode polarization is produced by scalar perturbations (density fluctuations) and tensor perturba-
6.1. CMB polarization

CMB polarization (primordial gravitational waves). However, the $B$-mode polarization is only produced by tensor perturbations, therefore, the $B$-mode polarization can be used to probe inflation.

A key cosmological parameter is characterized by the tensor-to-scalar ratio, $r$, which determines the energy scale of inflation. Some inflationary models predict the energy scale to be near the GUT scale (the energy scale above which, it is believed, the electromagnetic force, weak force, and strong force are unified to one force) of $\sim 10^{15}$ GeV. Such high energies cannot be generated in laboratory, so the early universe provides an environment to test theories at these energy scales. Therefore, many experiments are underway to search for $B$-mode polarization in the CMB.

Figure 6.1: $E$-mode and $B$-mode patterns of polarization. Note that $E$-mode patterns are symmetric, while the $B$-mode patterns are anti-symmetric under parity (Hu and White 1997).

6.1.1 E/B decomposition

The mathematical description of CMB temperature and polarization anisotropies are summarized as follows. The linear polarization can be described by the Stokes parameters $Q$ and $U$, and the magnitude and angle are $P = \sqrt{Q^2 + U^2}$ and $\alpha = \frac{1}{2} \tan^{-1} (U/Q)$. The CMB temperature anisotropy is
expanded in terms of scalar (spin-0) spherical harmonics
\[ T(\hat{n}) = \sum_{l,m} a_{lm}^T Y_{lm}(\hat{n}), \quad (6.1) \]
where \( \hat{n} \) denotes the direction on the sky. The quantity \( T \) is invariant under a rotation in the plane perpendicular to \( \hat{n} \). On the other hand, the quantities \( Q \) and \( U \) transform under rotation by an angle \( \psi \) as a spin-2 field \( (Q \pm iU)(\hat{n}) \rightarrow e^{\pm 2i\psi}(Q \pm iU)(\hat{n}) \). Therefore, the harmonic analysis of \( Q \pm iU \) requires expansion on the sphere in terms of tensor (spin-2) spherical harmonics
\[ (Q \pm iU)(\hat{n}) = \sum_{l,m} a_{lm}^{(\pm 2)} Y_{lm}(\hat{n}). \quad (6.2) \]
The linear combination of multipole coefficient \( a_{lm}^{(\pm 2)} \) can be defined,
\[ a_{lm}^E \equiv -\frac{1}{2} (a_{lm}^{(+2)} + a_{lm}^{-2}), \quad a_{lm}^B \equiv -\frac{1}{2i} (a_{lm}^{(+2)} - a_{lm}^{-2}). \quad (6.3) \]
Then one can define two scalar (spin-0) fields instead of the spin-2 quantities \( Q \) and \( U \),
\[ E(\hat{n}) = \sum_{l,m} a_{lm}^E Y_{lm}(\hat{n}), \quad B(\hat{n}) = \sum_{l,m} a_{lm}^B Y_{lm}(\hat{n}), \quad (6.4) \]
to specify the linear polarization field of \( E \) and \( B \) modes (Baumann et al., 2009). Usually, angular power spectra of the \( E \)-mode and \( B \)-mode fields are used to evaluate their amplitude,
\[ C_{l}^{EE} = \langle a_{lm}^E a_{lm}^{E*} \rangle, \quad C_{l}^{BB} = \langle a_{lm}^B a_{lm}^{B*} \rangle. \quad (6.5) \]
Cross-correlations among the \( T \)-mode, \( E \)-mode and \( B \)-mode of harmonic coefficients can be also evaluated mathematically, but only \( C_{l}^{TT}, C_{l}^{TE}, C_{l}^{EE}, \) and \( C_{l}^{BB} \) are non-zero because the \( T \)-mode and \( E \)-mode have a positive parity and the \( B \)-mode has a negative parity (Baumann et al., 2009).

Fig. 6.2 shows the CMB angular power spectra of \( T \)-mode, \( E \)-mode and \( B \)-mode with varying values of the tensor-to-scalar ratio \( r \) based on the Planck cosmology. The \( B \)-mode polarization in the CMB is generated only by the primordial gravitational waves. However, the CMB is distorted by the cosmic structure between us and the last scattering surface, due to the lensing effect (CMB lensing), and it produces “apparent” \( B \)-mode. The intrinsic CMB \( B \)-mode polarization is dominant over the \( B \)-mode due to CMB lensing for \( \ell < 100 \), but if \( r < 0.001 \), it would be less dominant even for the scales.
6.1. CMB polarization

Figure 6.2: Angular power spectra of CMB with varying tensor-to-scalar ratio. The angular power spectra shows total intensity, $E$ modes, primordial $B$ modes and lensing $B$ modes.

6.1.2 Observable predictions and current observational constraints

In 2014, the BICEP2 experiment announced the detection of a B-mode polarization signal with an amplitude of $r = 0.2$ (BICEP2 Collaboration, 2014). This result requires an inflationary energy scale of about $E_{\text{inf}} = V^{1/4} \approx 2 \times 10^{16}$ GeV. However, as will be described in §6.2, foreground components from our Galaxy also produce $E$- and $B$-mode polarization signals and they are much stronger than the CMB polarization. After the result was published, the Planck team together with BICEP and Keck teams reanalyzed the data using the detailed galactic dust §6.2.3 measurements made possible with the large frequency coverage of Planck’s since the BICEP2 data come from only one frequency. The conclusion was that the BICEP findings are compatible with a pure dust signal, yielding an upper limit for the tensor to scalar ratio of $r < 0.11$ (BICEP2/Keck Collaboration, 2015).

The current constraint on $r$ mainly driven by the measurements of the Planck satellite mission:

$$r < 0.09 \quad (95\% \text{CL with TT, TE, EE + lowP}),$$

where lowP denotes the polarization power spectrum in $\ell < 30$ and the
smaller scales of the foreground dominated region are not used. The Planck measurement of the B-mode polarization contributes little to the limit on $r$ so far. The current limit on $r$ is constrained by $T$-mode, and it is somewhat degenerate with $n_s$. Thus, better limits on $n_s$ provide better limits on $r$.

6.2 Foreground emission in the microwave bands

In the microwave range, four foreground emission components from our Galaxy are known besides the CMB signal: synchrotron, free-free (unpolarized), thermal dust, and spinning dust emission (Fig. 6.3). The BB polarization power spectra due to foregrounds (dust and synchrotron) is much higher than the CMB polarization spectrum. The spinning dust emission is very uncertain and hard to distinguish with the synchrotron emission. Therefore, new data at lower frequencies will provide important information on the galactic emissions and can be used to subtract the polarized foreground emissions from the CMB. In the next chapter, we simulate new data at 10 GHz and evaluate its ability for the foreground removal. Before that, in this chapter, we summarize emission mechanisms of the foreground sources (Rybicki and Lightman, 1979) and their current constraints (Dickinson, 2016).

6.2.1 Synchrotron emission

Synchrotron radiation is emitted by relativistic cosmic ray electrons accelerated by the Galactic magnetic field. In the magnetic field, the electrons spiral around the field lines and emit radiation as traversing the circular path. The emission reflects the number and energy spectrum of the CR electrons and also the strength of the magnetic field. Therefore, it can vary across the sky, but the spectrum is well-approximated by power-law.

At GHz frequencies, the typical spectral indexes are $\beta \approx -2.7$ with variations $\Delta \beta \approx \pm 0.2$ (Platania et al., 1998, 2003). At higher frequencies, the spectrum appears to steepen further, presumably due to radiative losses, with $\beta \approx -3.0$ around WMAP and Planck frequencies (Davies et al., 2006). The polarization of synchrotron is less known. Below a few GHz, it is affected by the Faraday Rotation (Wolleben et al., 2006) and difficult to probe the original state. It can be mapped at frequencies above a few GHz. WMAP and Planck have polarization measurements at low frequencies, however, the measurements are limited in S/N ratio and the accurate spectral indexes are not derived yet. It is considered that the synchrotron emission can be polarized at as high as 75% in a uniform and regular magnetic field and
6.2. Foreground emission in the microwave bands

Figure 6.3: Brightness temperature rms as a function of frequency and astrophysical component for temperature (top) and polarization (bottom) (Planck Collaboration 2016b) (arXiv:1502.01588). For temperature, each component is smoothed to an angular resolution of 1° FWHM, and the lower and upper edges of each line are defined by masks covering 81 and 93 % of the sky, respectively. For polarization, the corresponding smoothing scale is 40′, and the sky fractions are 73 and 93 %. 

96
seems to be polarized at a level of 10 - 40 % \cite{Planck2016e,Vidal2015}.

### 6.2.2 Free-Free emission

Free-free (bremsstrahlung) emission is the radiation by free electrons accelerated due to the interaction with ions in an ionized gas (usually protons). Its spectrum is well understood. At frequencies above a few GHz, it has a spectral index of $\beta = -2.1$ with very little variations with electron temperatures. At higher frequencies around 100 GHz, the spectral index is steepened slightly to $\beta = -2.13 \cite{Planck2014d}$.

Free-free emission is intrinsically unpolarized. Coulombic interactions are by their nature random and no significant alignment with the magnetic field. Residual polarization can be generated on sharp edges due to Thomson scattering, however, the polarization is expected to be very low ($\ll 1\%$) with current upper limit at $< 3\%$ for diffuse emission and $\ll 1\%$ for compact HII regions \cite{Macellari2011}. Therefore, the free-free emission would not be a major foreground for CMB polarization measurements.

### 6.2.3 Thermal dust emission

Thermal dust emission is produced by dust grains, absorbing ultra-violet photons from the exciting radiation field and re-radiating thermally. The majority of the dust emission is radiated from star forming regions, where the dust is heated by nearby young stars. Dust grains are made of silicate and graphite, or coated with ices in cold regions. The distribution of the grain size shows more small grains and fewer large grains with an average size of $\sim 0.1$ mm. Dust is usually located with $\text{H}_2$ in molecular clouds, with the mass ratio of $M(\text{dust})/M(\text{H}_2) \sim 0.01$. Thermal dust emission is common in the Milky Way as well as other galaxies.

Thermal dust emission is represented by a modified black body function of the form in antenna temperature:

$$T_A(\nu) \propto \nu^\beta_d B(\nu, T_d),$$

where $\beta_d$ is the dust spectral index and $T_d$ is the equilibrium temperature of dust grains. According to the \textit{Planck} measurements combined with \textit{IRAS}/\textit{COBE} data, the thermal dust emission is well modeled by a single modified blackbody with mean temperature $T_d \approx 19$ K and index $\beta_d \approx 1.6 \cite{Planck2016g}$. Thermal dust emission can be significantly polarized. Elongated dust grains emit preferentially along their shortest
6.2. Foreground emission in the microwave bands

axes while large dust grains can align efficiently by the Galactic magnetic field, causing a net polarization. The Planck measurements find the mean polarization fraction of $\approx 10\%$ at high latitude and it is polarized up to $\approx 20\%$ (Planck Collaboration 2015, 2016h).

6.2.4 Spinning dust emission

In our own galaxy and other galaxies, there is recent evidence of so-called “anomalous radio emission”, plausibly from spinning dust grains. Draine and Lazarian (1998) proposed that very small grains containing 10 - 100 carbon atoms could be spun up to tens of GHz frequencies in the ISM (Inter Stellar Medium). Electric dipole moments from the spinning dust grains likely produce radio emission at these frequencies with the spectrum depending on their local conditions. Thus, in addition to free-free, thermal and synchrotron emission, spinning dust emission is a fourth component that is present in radio emission from other external galaxies and our own Milky Way.

According to Ali-Hamoud et al. (2009) and Silsbee et al. (2011), current spinning dust models can predict the spectral shape including a variety of physical processes such as collisions with neutral and ionized gas, plasma drag, absorption of photons etc. The peak frequency of the spinning dust spectrum is determined by the size of the smallest grains, and the total power follows the same dependence, the power emitted by the smallest grains. In a few Galactic clouds, observations show a good fit to the model spectrum with a typical peak frequency of $\approx 30$ GHz. However, at high Galactic latitudes, the spectrum of spinning dust has not been measured clearly due to the difficulty of component separation. The theoretical work also suggests that the spinning dust emission is not highly polarized at frequencies above a few GHz. Measurements show that the level of polarization is only a few percent for an upper limit (Dickinson et al., 2011; Rubiñó Martín et al., 2012) and thus, the spinning dust emission should not be a major foreground for CMB polarization measurements.
Chapter 7

A 10GHz polarization sky survey

7.1 Introduction

$B$-mode polarization in the CMB is a direct tracer of tensor perturbations caused by gravitational waves in the inflationary period of the early universe. However, galactic foreground sources also produce $E$- and $B$-mode polarization, and are much stronger than the CMB polarization. The dominant source of the polarized emission is galactic synchrotron and dust emission, but especially synchrotron emission is currently characterized with low signal/noise. We are developing a project called the Canadian Galactic Emission Mapper (CGEM) to produce half-sky maps of total intensity and linear polarization at 10 GHz to constrain synchrotron emission as well as spinning dust emission.

In this chapter, we simulate polarized skies using version 1.0 of Python Sky Model (PySM) software package (Thorne et al., 2016). The software is used to generate full-sky simulations of Galactic foregrounds in intensity and polarization at microwave frequencies. The components simulated are thermal dust, synchrotron, AME (Spinning dust) and CMB based on publicly available data from the WMAP and Planck satellite missions. Small-scale realizations of these components at resolutions greater than current all-sky measurements are also added.

Using this software, we simulate the 10 GHz polarized sky and then observations with estimated noise appropriate to one-year observation from Penticton BC in Canada. The simulated 10 GHz data is combined with currently available data from WMAP + Planck and data from the future satellite mission of LiteBIRD (Matsumura et al., 2014) (Their estimated instrumental noises are also simulated) and then the improvement of foreground estimates with/without the new data is evaluated. We use a Markov chain Monte Carlo (MCMC) algorithm to derive the probability distributions of parameters for the foreground emissions and CMB such as their
amplitudes and spectral indexes.

### 7.2 Field of view

Here, we will verify the field of view of the telescope in galactic coordinates. In CGEM, the telescope is proposed to rotate with an opening angle 40° from the zenith at Penticton at a rotation rate of 1 rpm, see in Fig. [7.1](#).

To simulate this, we transform the line of sight from telescope coordinates to Penticton coordinates. With the Euler angles, the coordinates are transformed by

$$
\begin{pmatrix}
x_t \\
y_t \\
z_t
\end{pmatrix}
= R_z(\psi)R_x(\theta)R_z(\phi)
\begin{pmatrix}
x_p \\
y_p \\
z_p
\end{pmatrix}
\tag{7.1}
$$

where \((x_t, y_t, z_t) = (0, 0, 1)\) is in telescope coordinates and \((x_p, y_p, z_p)\) is in Penticton coordinates, and where a precession opening angle is \(\psi = 40^\circ\), a precession angle is \(\theta = 360^\circ \times t(\sec)/60(\sec)\) and a spin angle is \(\phi = 0^\circ\).

![Figure 7.1: The image of observation strategy at Penticton. The red arrow is the direction of observation, which rotates once a minute with an opening angle 40° and blue arrow is the direction to NCP.](#)
Next, we transform the line of sight from Penticton coordinates to Earth coordinates. Since Penticton is located at the latitude, 49°29′ N, the direction of the telescope in Earth coordinates is given by

\[
\begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} = R_y(90° - \theta) \begin{pmatrix} x_e \\ y_e \\ z_e \end{pmatrix}
\]

(7.3)

\[
= \begin{pmatrix} \cos (90° - \theta) & 0 & -\sin (90° - \theta) \\ 0 & 1 & 0 \\ \sin (90° - \theta) & 0 & \cos (90° - \theta) \end{pmatrix} \begin{pmatrix} x_e \\ y_e \\ z_e \end{pmatrix},
\]

(7.4)

where \((x_e,y_e,z_e)\) is in Earth coordinates and \(\theta = 49.5°\).

The third step is to transform the line of sight from Earth coordinates to celestial coordinates. The earth spins around its axis once a day based on the sidereal time, which is the time it takes the earth to make one rotation relative to the vernal equinox. A mean sidereal day is 23 hours, 56 minutes, 4.0916 seconds, a little shorter than a solar day.

\[
\begin{pmatrix} x_e \\ y_e \\ z_e \end{pmatrix} = R_z(LST) \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix}
\]

(7.5)

\[
= \begin{pmatrix} \cos (LST) & \sin (LST) & 0 \\ -\sin (LST) & \cos (LST) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix},
\]

(7.6)

where \((x_c,y_c,z_c)\) is in celestial coordinates, and \(LST\), a local sidereal time, is given by

\[
LST(\text{deg}) = 360° \times \frac{0.671262 + 1.00273790935 \times (\text{MJD} - 40000) + \frac{\text{longitude}}{360°}}{7.7}
\]

where longitude is \(-119.6°\) at Penticton, MJD stands for Modified Julian Date, and "frac" represents the decimal part of the result.

The final step is to transform the line of sight from celestial coordinates to galactic coordinates. The right ascension and declination of the galactic center is 266.40500 deg and -28.93617 deg, so the rotation along the z-axis by 266.40500° and along the y-axis by 28.93617° points to the galactic center. In addition, the rotation along the x-axis by 58.59866° determines the angle.
7.3 Simulations of the observational data in polarization

of the galactic place, so

\[
\begin{pmatrix}
    x_g \\
    y_g \\
    z_g
\end{pmatrix} = R_x(\psi)R_y(\theta)R_z(\phi)
\begin{pmatrix}
    x_c \\
    y_c \\
    z_c
\end{pmatrix}
\] (7.8)

\[
= \begin{pmatrix}
    1 & 0 & 0 \\
    0 & \cos \psi & \sin \psi \\
    0 & -\sin \psi & \cos \psi
\end{pmatrix}
\begin{pmatrix}
    \cos \theta & 0 & -\sin \theta \\
    0 & 1 & 0 \\
    \sin \theta & 0 & \cos \theta
\end{pmatrix}
\begin{pmatrix}
    \cos \phi & \sin \phi & 0 \\
    -\sin \phi & \cos \phi & 0 \\
    0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    x_c \\
    y_c \\
    z_c
\end{pmatrix}
\] (7.9)

where \((x_g, y_g, z_g)\) is in galactic coordinates, and where \(\psi = 58.59866^\circ, \theta = 28.93617^\circ\) and \(\phi = 266.40500^\circ\).

With this sequence of transformations, we can obtain the direction of the telescope in galactic coordinates at any time. Fig. 7.2 show the hit maps of one-year observation in galactic coordinates. 42% of all the sky is covered with this observing strategy.

![Figure 7.2: Hit map of one-year of observations, in galactic coordinates (N\(_{\text{side}}\) = 64).](image)

7.3 Simulations of the observational data in polarization

7.3.1 Simulations of the polarized skies

The PySM software (Thorne et al., 2016) has template maps and spectral index maps of four known components in the microwave range: synchrotron,
7.3. Simulations of the observational data in polarization

thermal dust, spinning dust and CMB. The CMB map is simulated from an input power spectrum defined by a set of cosmological parameters ($\Omega_b = 0.0486$, $\Omega_m = 0.3075$, $\Omega_k = 0.0$, $H_0 = 67.74$ km s$^{-1}$ Mpc$^{-1}$, $A_s = 2.14 \times 10^{-9}$, $n_s = 0.9667$, $\tau = 0.066$) derived by Planck+BAO+JLA+$H_0$. We use $r = 0.05$ for the tensor-to-scalar ratio, just below the current upper limits of $r < 0.09$ (BICEP2/Keck Collaboration 2015). Note that we use a pivot scale of $k_0 = 0.05$ Mpc$^{-1}$ but the results are very weakly dependent on this because we have no tilt in the tensor spectrum ($n_T = 0.0$). The input theoretical power spectrum is calculated using the CAMB software (Lewis et al. 2000). We do not include the effects of gravitational lensing, which would contribute significant power at scales $\ell > 100$.

Using these maps, the polarized sky at 10 GHz and other frequency ranges can be simulated with

$$Q_{\text{model}}(\nu, \hat{n}) = Q_s(\hat{n}) \times \left(\frac{\nu}{23}\right)^{\beta_s(\hat{n})} + Q_d(\hat{n}) \times \left(\frac{\nu}{353}\right)^{\beta_d(\hat{n})} B_\nu(T_d(\hat{n})) + Q_{\text{CMB}}(\hat{n}),$$

$$U_{\text{model}}(\nu, \hat{n}) = U_s(\hat{n}) \times \left(\frac{\nu}{23}\right)^{\beta_s(\hat{n})} + U_d(\hat{n}) \times \left(\frac{\nu}{353}\right)^{\beta_d(\hat{n})} B_\nu(T_d(\hat{n})) + U_{\text{CMB}}(\hat{n}),$$

where the subscripts, $s, d, sp$ and $CMB$, represent the synchrotron, thermal dust, spinning dust and CMB respectively at a direction, $\hat{n}$, and $\epsilon$ is the emissivity function of spinning dust emission calculated using Spdust2 (Ali-Hamoud et al. 2009; Silsbee et al. 2011), evaluated for a cold neutral medium, where $\nu_{\text{peak}}$ is the peak frequency of the emission varying spatially. We set $T_d(\hat{n}) = 18$ [K] and $\nu_{\text{peak}}(\hat{n}) = 30$ [GHz] in our simulation as Remazeilles et al. (2016) did.

For the frequencies, $\nu$, we simulate the sky at 10 GHz, at the frequencies of LiteBIRD (60, 78, 100, 140, 195, 280 [GHz]) and at lower frequencies currently available from WMAP and Planck (23, 30, 33, 41, 44 [GHz]), which would be profitable to reconstruct the low frequency components such as synchrotron and spinning dust emissions, which have not been well understood. The simulated polarization Q and U maps are shown in Fig.7.3 and Fig.7.4.
Figure 7.3: The simulated stokes Q maps at 10,23,30,33,41,44,60,78,100,140,195,280 GHz in the galactic coordinates (N_{side} = 128).
7.3. Simulations of the observational data in polarization

Figure 7.4: The simulated stokes U maps at 10, 23, 30, 33, 41, 44, 60, 78, 100, 140, 195, 280 GHz in the galactic coordinates (N\textsubscript{side} = 128).
7.3. Simulations of the observational data in polarization

7.3.2 Noise estimate

We estimate the noise of CGEM’s 10 GHz observations assuming $T_{\text{sys}}$ (system temperature) = 50 (K), $\Delta \nu$ (band width) = $2 \times 10^9$ (Hz) and $\tau$ (sampling interval) = 0.05 (sec),

$$\sigma_0(@10\,GHz) = \frac{T_{\text{sys}}}{\sqrt{\Delta \nu \tau}} = \frac{50}{\sqrt{2 \times 10^9 \times 0.05}} = 5\,\text{mK}. \quad (7.12)$$

This gives a noise per observation of 5 mK. The noise for the map is calculated by $\sigma(\nu, \hat{n}) = \sigma_0(\nu)/\sqrt{N_{\text{obs}}(\nu, \hat{n})}$, where $\sigma_0(\nu)$ is the noise per observation at each frequency and $N_{\text{obs}}(\nu, \hat{n})$ is the number of observations at each pixel and frequency. $N_{\text{obs}}(\nu, \hat{n})$ of the 10 GHz data is obtained from the hit map of one-year observation (Fig. 7.2). With the given pixel noise, $\sigma(\nu, \hat{n})$, we can simulate random noise at each pixel using a Gaussian random number generator, $N(0,1)$, followed by $Q(U)_{\text{noise}}(\nu, \hat{n}) = N(0,1) \times \sigma(\nu, \hat{n})$.

Here, we simply estimate the mean noise at each frequency, $\bar{\sigma}(\nu) = \sigma_0(\nu)/\sqrt{\bar{N}_{\text{obs}}(\nu)}$, where $\bar{N}_{\text{obs}}(\nu)$ is the mean number of observations at a pixel, estimated by $\bar{N}_{\text{obs}}(\nu) = \sum \bar{N}_{\text{obs}}(\nu, \hat{n})/N_{\text{pix}}$. We distribute the random noise to each pixel by $Q(U)_{\text{noise}}(\nu, \hat{n}) = N(0,1) \times \bar{\sigma}(\nu)$ and simulate the observational data,

$$Q(U)_{\text{obs}}(\nu, \hat{n}) = Q(U)_{\text{model}}(\nu, \hat{n}) + Q(U)_{\text{noise}}(\nu, \hat{n}). \quad (7.13)$$

The data now include three foreground emissions, CMB and noise.

For the WMAP data, the noise per observation at each frequency is described in [Jarosik et al., 2011] and the hit maps are provided in [10]. For the Planck data, the covariance matrices of $I$, $Q$ and $U$ maps, and hit maps are provided in [11]. The mean noise at each frequency for this study is summarized in Table 7.1.

<table>
<thead>
<tr>
<th>Frequency [GHz]</th>
<th>10</th>
<th>23</th>
<th>30</th>
<th>33</th>
<th>41</th>
<th>44</th>
<th>60</th>
<th>78</th>
<th>100</th>
<th>140</th>
<th>195</th>
<th>280</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean noise [uK]</td>
<td>15</td>
<td>15</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>12</td>
<td>0.375</td>
<td>0.236</td>
<td>0.170</td>
<td>0.135</td>
<td>0.113</td>
<td>0.138</td>
</tr>
</tbody>
</table>

The mean noise of stokes Q/U map at each frequency is given in CMB thermodynamic unit at $N_{\text{side}} = 128$.

---

*10* https://lambda.gsfc.nasa.gov/product/map/dr5/m_products.cfm

*11* http://pla.esac.esa.int/pla/#results
7.4 Markov chain monte carlo simulation

In this section, we assess the utility of 10 GHz data to separate galactic foreground emissions and CMB from the simulated observational maps. These emission components are expressed with 12 parameters, \{ \( Q_s, Q_d, Q_{sp}, Q_{CMB}, U_s, U_d, U_{sp}, U_{CMB} \) \} (the amplitude of each emission component at a reference frequency) and \{ \( \beta_s, \beta_d, T_d, \nu_{peak} \) \} (the spectral index and shape parameter of the relevant emission component). A Markov Chain Monte Carlo (MCMC) method is used to acquire a probability distribution of each parameter. We show that 10 GHz data helps to determine these parameters more precisely by comparing the result with/without the 10 GHz data.

For this analysis, we adopt "emcee" software (Foreman-Mackey et al., 2013), the affine invariant ensemble sampler proposed by Goodman and Weare (2010). The software is provided as open source and it has been used for many astrophysics projects.

7.4.1 Model function

MCMC can be used to find a set of parameters that best fits data for a proposed model function. For our model, we use the same function used to simulate data (Eq. 7.10, 7.11).

\[
y_1(\nu, \hat{n}) = p_0(\hat{n}) \times \left( \frac{\nu}{23} \right)^{p_2(\hat{n})} \times + p_3(\hat{n}) \times \left( \frac{\nu}{353} \right)^{p_5(\hat{n})} B_{\nu}(p_6(\hat{n})) \times + p_7(\hat{n}) \\
+ p_9(\hat{n}) \times \epsilon(\nu, p_{11}(\hat{n})),
\]

\[
y_2(\nu, \hat{n}) = p_1(\hat{n}) \times \left( \frac{\nu}{23} \right)^{p_2(\hat{n})} \times + p_4(\hat{n}) \times \left( \frac{\nu}{353} \right)^{p_5(\hat{n})} B_{\nu}(p_6(\hat{n})) \times + p_8(\hat{n}) \\
+ p_{10}(\hat{n}) \times \epsilon(\nu, p_{11}(\hat{n})),
\]

where \( y_{1,2} \) is a function for each Q/U component.

The \( \chi^2 \) of our fit is

\[
\chi^2(\hat{n}) = \sum_{\nu} \left\{ \frac{(Q(\nu, \hat{n})_{obs} - y_1(\nu, \hat{n}))^2}{\sigma(\nu)^2} + \frac{(U(\nu, \hat{n})_{obs} - y_2(\nu, \hat{n}))^2}{\sigma(\nu)^2} \right\},
\]

and the likelihood function is calculated by assuming that the probability follows Gaussian distribution,

\[
Probability(\hat{n}) = \exp(-\chi^2/2).
\]
7.4. Markov chain monte carlo simulation

\[ \text{Likelihood}(\hat{n}) = -0.5 \times \chi. \quad (7.17) \]

The likelihood is maximal when the sum of square errors is minimal. We find that the dust temperature can not be well constrained with the chosen frequency range, therefore, we adopt a Gaussian prior for the dust temperature \( T_d = 18 \pm 0.05 \) K. With this prior, we confirm that the input parameter values can be recovered successfully by the MCMC fitting, before including instrumental noises.

7.4.2 Improvement by the 10GHz data (without spinning dust)

Next we simulate the observational data, \( Q(U)_{\text{obs}}(\nu, \hat{n}) \), including instrumental noises. For the noises, we consider two cases at 10 GHz, 62 uK and also 15 uK, which can be reached by recent polarimeters. (In this paper, the main results are shown with the noise of 15 uK.) First, we consider a simple case for the foregrounds, assuming that the spinning dust emission is negligible. Later, we include the spinning dust emission in the foregrounds both for the input data and model function assuming it is polarized at the 2% level.

With the emcee package, the Markov chain provides a probability distribution of each parameter in the model function, \( y(\nu, \hat{n}) \). Fig. 7.5 shows the correlation matrix of the fit parameters from the MCMC fitting at a sample pixel. The red lines are the result with the 10 GHz data (w/10GHz) and blue lines are the one without the 10 GHz data (w/o10GHz). The histograms on the diagonal show the marginalized distribution for each parameter and the green lines show the input ("true") values. The off-diagonal figures represent the correlation between any two components and the three circles show 1, 2, 3 \( \sigma \) confidence level. The figure shows that the parameters for the synchrotron emissions are constrained better with the 10 GHz data as expected, however, the improvement is marginal for the other parameters.

With the best-fit ("output") values, the spectra of each emission component and the total emission in the target range of frequency can be reconstructed with the 10 GHz data and without it. One example, at a sample pixel, is shown in Fig. 7.6 and Fig. 7.7. The solid lines are the input spectra and the dash lines are the output spectra from the MCMC fitting. As seen in the figures, the input synchrotron spectra (green solid line) can be recovered with the MCMC fitting well when including the 10 GHz data, however it can not without it.

In other pixels, since each emission component contributes to the total with a different amplitude, the improvement should be dependent on it.
7.4. Markov chain monte carlo simulation

Therefore, we run MCMC for all the pixels to generate output parameter maps that can be compared to the input parameter maps. We also produce the residual maps, which are made by subtracting the output parameter maps from the input parameter maps. Fig. 7.8, Fig. 7.9 and Fig. 7.10 show the input, output, residual maps for $Q_s$, $U_s$, $\beta_s$. In the figures, the w/10GHz case and the w/o10GHz case are also compared. In the w/o10GHz case, large residuals are seen, but they are not clear in the w/10GHz case.

Finally, we show the uncertainties of the fit parameters in Table 7.2. The uncertainty is defined by one sigma standard deviation as a result of Gaussian fit to the probability distribution for each parameter. The uncertainty of the synchrotron amplitude is improved by a factor of $\sim 2.7$ and $\sim 3.1$ for the spectral index. However, it is $\sim 1.3$ for the CMB and little for the dust emission because the CMB and dust emission are already relatively well constrained by the sensitive LiteBIRD data at higher frequencies.

According to current observations, the $B$-mode polarization in the CMB is expected to be extremely faint and many “sensitive” experiments are under way to search for it. Therefore, we need to remove all the contaminating signals accurately, otherwise it biases the view of the early universe. Our result shows that new 10 GHz data will allow the contamination to be subtracted more accurately by combining with proposed sensitive high-frequency measurements such as LiteBIRD, and play an essential role in questing for the tiny signal in the CMB and understanding the origin of our universe.

Table 7.2: Uncertainty of the fit parameters by MCMC [uK] (without spinning dust).

<table>
<thead>
<tr>
<th></th>
<th>$Q_s$</th>
<th>$U_s$</th>
<th>$\beta_s$</th>
<th>$Q_d$</th>
<th>$U_d$</th>
<th>$\beta_d$</th>
<th>$Q_{cmb}$</th>
<th>$U_{cmb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>w/o 10GHz</td>
<td>3.6</td>
<td>2.9</td>
<td>0.58</td>
<td>0.069</td>
<td>0.064</td>
<td>0.26</td>
<td>0.16</td>
<td>0.14</td>
</tr>
<tr>
<td>w/ 10GHz(62uK)</td>
<td>2.1</td>
<td>1.8</td>
<td>0.30</td>
<td>0.065</td>
<td>0.060</td>
<td>0.22</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>w/ 10GHz(15uK)</td>
<td>1.3</td>
<td>1.1</td>
<td>0.19</td>
<td>0.060</td>
<td>0.055</td>
<td>0.19</td>
<td>0.12</td>
<td>0.11</td>
</tr>
</tbody>
</table>

7.4.3 Improvement by the 10GHz data (with spinning dust)

Next we include the spinning dust emission both in the input data and model function, and repeat the study.

Fig. 7.11 shows the correlation matrix of the fit parameters from the MCMC fitting at a sample pixel. The figure tells that the fit parameters for synchrotron and spinning dust emission are constrained better with the 10
Figure 7.5: Correlation maps among the fit parameters (without spinning dust) obtained by the MCMC fitting at a sample pixel. Red lines are the result with the 10 GHz data and blue lines are the result without the 10 GHz data. The histograms at the diagonal are projections of the samples by MCMC to each parameter axis and the green lines show the input values. The figures at the off-diagonal represent the correlation between any two components and the three circles show 1,2,3 $\sigma$ confidence level. It is clearly seen that the synchrotron parameters are well constrained with the 10 GHz data. The CMB is better constrained modestly along with it.
7.4. Markov chain monte carlo simulation

Figure 7.6: (Left): Stokes $Q$ spectra (without spinning dust) estimated by the MCMC fitting with the 12 band data including 10 GHz in a sample pixel. The solid lines are the input spectra and the dash lines are the output spectra by the MCMC fitting. (Right): Spectra obtained by the MCMC fitting with the 11 band data, not including 10 GHz, in the same pixel.

Figure 7.7: (Left): Stokes $U$ spectra (without spinning dust) estimated by the MCMC fitting with the 12 band data including 10 GHz in a sample pixel. The solid lines are the input spectra and the dash lines are the output spectra by the MCMC fitting. (Right): Spectra obtained by the fitting with the 11 band data, not including 10 GHz, in the same pixel.
Figure 7.8: Input, output (best-fit) and residual maps of $Q_s$ estimated by MCMC, without spinning dust. *Top:* Input $Q_s$ map at 23 GHz, *middle left:* Output map with 10 GHz data, *middle right:* Residual map with 10 GHz data, *bottom left:* Output map without 10 GHz data, and *bottom right:* Residual map without 10 GHz data.
7.4. Markov chain monte carlo simulation

Figure 7.9: Input, output(best-fit) and residual maps of $U_s$ estimated by MCMC, without spinning dust. **Top:** Input $U_s$ map at 23 GHz, **middle left:** Output map with 10 GHz data, **middle right:** Residual map with 10 GHz data, **bottom left:** Output map without 10 GHz data, and **bottom right:** Residual map without 10 GHz data.
Figure 7.10: Input, output(best-fit) and residual maps of $\beta_s$ estimated by MCMC, without spinning dust. *Top:* Input $\beta_s$ map, *middle left:* Output map with 10 GHz data, *middle right:* Residual map with 10 GHz data, *bottom left:* Output map without 10 GHz data, and *bottom right:* Residual map without 10 GHz data.)
GHz data as expected, however, the improvement is marginal for the other parameters.

Fig. 7.12 and Fig. 7.13 show the spectra of each emission component and the total emission reconstructed with the output values at a sample pixel. As seen in the figures, the input synchrotron spectra (green solid line) and spinning dust spectra (cyan solid line) can be reconstructed by the MCMC fitting well with the 10 GHz data, however it can not without it.

We run MCMC for all the pixels to estimate the average improvement. Fig. 7.14, Fig. 7.15 and Fig. 7.16 show the input, output, residual maps for $Q_s$, $U_s$, $\beta_s$. In the figures, the w/10GHz case and w/o10GHz case are also compared. In the w/o10GHz case, large residuals are seen, but that is not clear in the w/10GHz case. We also show the input, output, residual maps for $Q_{sp}$ and $U_{sp}$ in Fig. 7.17 and Fig. 7.18. Unlike the correlation matrix in a sample pixel, the spinning dust emission is not improved very much with the 10 GHz data. The reason is because spinning dust emission is negligibly weak in any frequency for most of the pixels. Such a weak signal is by no means recovered well, however the effect to the CMB would be also limited.

Finally, we show the uncertainties of the fit parameters in Table 7.3. The uncertainty of the synchrotron amplitude is improved by a factor of $\sim 2.4$ and $\sim 2.7$ for the spectral index. This is a little worse than the case without spinning dust. This suggests that some of the contribution of the 10 GHz data are used to improve the spinning dust emission. The improvement factor is $\sim 1.3$ for the CMB and $\sim 1.5$ for the spinning dust emission, and little for the dust emission. The marginal average improvement of spinning dust emission is due to the same reason as mentioned above.

Table 7.3: Uncertainty of the fit parameters by MCMC [uK] (with spinning dust).

<table>
<thead>
<tr>
<th></th>
<th>$Q_s$</th>
<th>$U_s$</th>
<th>$\beta_s$</th>
<th>$Q_d$</th>
<th>$U_d$</th>
<th>$\beta_d$</th>
<th>$Q_{cmb}$</th>
<th>$U_{cmb}$</th>
<th>$Q_{sp}$</th>
<th>$U_{sp}$</th>
<th>$\nu_{sp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>w/o 10GHz</td>
<td>3.4</td>
<td>2.7</td>
<td>0.54</td>
<td>0.074</td>
<td>0.068</td>
<td>0.24</td>
<td>0.16</td>
<td>0.14</td>
<td>0.21</td>
<td>0.17</td>
<td>8.5</td>
</tr>
<tr>
<td>w/ 10GHz</td>
<td>2.1</td>
<td>1.8</td>
<td>0.31</td>
<td>0.070</td>
<td>0.064</td>
<td>0.21</td>
<td>0.14</td>
<td>0.13</td>
<td>0.18</td>
<td>0.14</td>
<td>7.2</td>
</tr>
<tr>
<td>w/ 10GHz</td>
<td>1.4</td>
<td>1.1</td>
<td>0.20</td>
<td>0.065</td>
<td>0.060</td>
<td>0.19</td>
<td>0.12</td>
<td>0.11</td>
<td>0.14</td>
<td>0.12</td>
<td>6.2</td>
</tr>
</tbody>
</table>
7.4. Markov chain monte carlo simulation

Figure 7.11: Correlation maps among the fit parameters (with spinning dust) obtained by the MCMC fitting at a sample pixel. Red lines are the result with the 10 GHz data and blue lines are the result without the 10 GHz data. The histograms at the diagonal are projections of the samples by MCMC to each parameter axis and the green lines show the input values. The figures at the off-diagonal represent the correlation between any two components and the three circles show 1,2,3 σ confidence level. It is clearly seen that the synchrotron parameters are well constrained with the 10 GHz data. The spinning dust emission and CMB are better constrained along with it.
7.4. Markov chain monte carlo simulation

Figure 7.12: (Left) Stokes $Q$ spectra (with spinning dust) estimated by the MCMC fitting with the 12 band data including 10 GHz, in a sample pixel. The solid lines are the input spectra and the dash lines are the output spectra by the MCMC fitting. (Right) Spectra obtained by the MCMC fitting with the 11 band data, not including 10 GHz, in the same pixel.

Figure 7.13: (Left) Stokes $U$ spectra (with spinning dust) estimated by the MCMC fitting with the 12 band data including 10 GHz, in a sample pixel. The solid lines are the input spectra and the dash lines are the output spectra by the MCMC fitting. (Right) Spectra obtained by the fitting with the 11 band data, not including 10 GHz, in the same pixel.
7.4. Markov chain monte carlo simulation

Figure 7.14: Input, output(best-fit) and residual maps of $Q_s$ estimated by MCMC, with spinning dust. Top: Input $Q_s$ map at 23 GHz, middle left: Output map with 10 GHz data, middle right: Residual map with 10 GHz data, bottom left: Output map without 10 GHz data, and bottom right: Residual map without 10 GHz data.
7.4. Markov chain monte carlo simulation

Figure 7.15: Input, output(best-fit) and residual maps of $U_s$ estimated by MCMC, with spinning dust. *Top:* Input $U_s$ map at 23 GHz, *middle left:* Output map with 10 GHz data, *middle right:* Residual map with 10 GHz data, *bottom left:* Output map without 10 GHz data, and *bottom right:* Residual map without 10 GHz data.
Figure 7.16: Input, output (best-fit) and residual maps of $\beta_s$ estimated by MCMC, with spinning dust. *Top:* Input $\beta_s$ map, *middle left:* Output map with 10 GHz data, *middle right:* Residual map without 10 GHz data, *bottom left:* Output map with 10 GHz data, and *bottom right:* Residual map without 10 GHz data.
7.4. Markov chain monte carlo simulation

Figure 7.17: Input, output(best-fit) and residual maps of $Q_{sp}$ estimated by MCMC, with spinning dust. Top: Input $Q_{sp}$ map at 22.8 GHz, middle left: Output map with 10 GHz data, middle right: Residual map with 10 GHz data, bottom left: Output map without 10 GHz data, and bottom right: Residual map without 10 GHz data.
7.4. Markov chain monte carlo simulation

Figure 7.18: Input, output(best-fit) and residual maps of $U_{sp}$ estimated by MCMC, with spinning dust. Top: Input $U_{sp}$ map at 22.8 GHz, middle left: Output map with 10 GHz data, middle right: Residual map with 10 GHz data, bottom left: Output map without 10 GHz data, and bottom right: Residual map without 10 GHz data.
7.5 Conclusion

Using version 1.0 of the Python Sky Model (PySM) software package (Thorne et al., 2016), we simulate the 10 GHz polarized maps and observations with estimated noise appropriate to one-year observation from Penticton BC with the CGEM telescope. The simulated 10 GHz data is combined with currently available data from WMAP + Planck and simulated data from the proposed satellite mission LiteBIRD (Matsumura et al., 2014). The improvement of foreground reconstruction with/without the new data is evaluated using the Markov chain Monte Carlo (MCMC) algorithm.

- Assuming the spinning dust emission is negligible, the uncertainty of the synchrotron amplitude is improved by a factor of ≈2.7 and the spectral index is by a factor of ≈3.1, on average.

- By including spinning dust emission in the simulation, the uncertainty of the synchrotron amplitude is improved by a factor of ≈2.4 and the spectral index is by a factor of ≈2.7, on average.

- The improvement in the recovery of the spinning dust, thermal dust, and CMB signals are modest (less than a factor of 2) over most of the sky. In the case of the CMB and thermal dust components, the data are rather well constrained by the more-sensitive higher frequency data, especially from the proposed LiteBIRD mission. However, the improvement in the synchrotron recovery, and in the spinning dust recovery in regions where this component is bright, will greatly improve our confidence that our foreground models are sensible and correct. This, in turn, will greatly improve our confidence in any future detection of B-mode polarization in the CMB.
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