COMPUTING EFFECTS OF ELECTRIC AND MAGNETIC FIELDS NEAR OVERHEAD TRANSMISSION LINES

by

Muhammad Mubassir Noor Swapnil

B.A.Sc., The University of British Columbia, 2012

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF APPLIED SCIENCE

in

THE FACULTY OF GRADUATE AND POSTDOCTORAL STUDIES (ELECTRICAL AND COMPUTER ENGINEERING)

THE UNIVERSITY OF BRITISH COLUMBIA

(Vancouver)

September 2017

© Muhammad Mubassir Noor Swapnil, 2017
Abstract

Analysis of Electric and Magnetic Fields produced by overhead transmission lines plays a critical role prior to their construction by electrical utilities. They affect the width of the transmission corridor and terrain clearances from overhead conductors. There is substantial interest in learning about the health effects of short and long-term exposure to these fields, as induced currents and voltages from parallel lines pose a hazard to line crews working on a de-energized line.

A MATHCAD program has been developed to compute the strengths of Electric and Magnetic Fields and to determine any induction currents and/or voltages due to the coupling effect of these fields. MATHCAD is an engineering software that allows programming and development of engineering calculations using natural mathematical syntax and variables. An overview of the theory of electric and magnetic fields is also provided in this paper. This research tests a few scenarios in the MATHCAD program, and the test results are verified against EPRI (Electric Power Research Institute) and IEEE (Institute of Electrical and Electronic Engineers) guidelines to ensure consistency. The results are also compared with the safety limits published by ICNIRP (International Commission on Non-Ionizing Radiation Protection) and IEEE. We also discuss the practical use of these results, along with potential future updates to the MATHCAD program.
Lay Summary

This goal of this research work is to create a simple computer program to estimate the effects of Electric and Magnetic Fields from overhead transmission lines. High voltage on transmission lines generates electric fields and currents flowing through the conductors generate magnetic fields. These fields can induce current and/or voltage on nearby conducting objects depending on their strength, and there has been concern for a long time about any adverse health effects from prolonged exposure to these fields. A MATHCAD program has been developed to calculate the strength of these fields and the amount of induced current/voltage that could be produced by them. The program is suitable in visualizing the effects of these fields and conforms to the guidelines recommended by the EPRI and IEEE associations. The output obtained from this program can be easily compared with the recommended safety levels determined by IEEE and ICNIRP to alleviate any concerns.
This research is based principally on the published methods from IEEE in “IEEE-524 Std-2016: Guide for the Installation of Overhead Transmission Line Conductors” and from EPRI in “EPRI AC Transmission Line Reference Book-200kV and Above”. The idea of developing a MATHCAD program is based loosely on a fellow colleague’s personal software, experienced Transmission Line Engineer, Milind Khot (P. Eng). Mr. Khot provided his technical expertise in the initial stage of my work and discussed the benefits of this work with my research supervisor, Professor William Dunford. Professor Dunford has provided me with constant guidance, encouragement, and feedback during the course of the technical work and preparation of this thesis. I have been responsible for carrying out the initial literature review, creating the MATHCAD program, and analyzing the output and writing of this thesis.
### Table of Contents

Abstract .......................................................................................................................................... ii  
Lay Summary .................................................................................................................................. iii  
Preface ........................................................................................................................................... iv  
Table of Contents ........................................................................................................................... v  
List of Tables ................................................................................................................................... vii  
List of Figures ................................................................................................................................... viii  
List of Symbols ............................................................................................................................... x  
List of Abbreviations ..................................................................................................................... xi  
Acknowledgements ...................................................................................................................... xii  
Dedication ...................................................................................................................................... xiii  

**Chapter 1: Introduction** ................................................................................................................1  

1.1 Thesis Outline ......................................................................................................................... 1  
1.2 Electric and Magnetic Fields .................................................................................................... 2  
  1.2.1 Electric Fields from Overhead Transmission Lines .......................................................... 2  
  1.2.2 Magnetic Fields from Overhead Transmission Lines ....................................................... 7  
    1.2.2.1 Complex Depth ................................................................................................... 8  
1.3 Induction .................................................................................................................................... 10  
  1.3.1 Electric Field Induction .................................................................................................... 11  
  1.3.2 Magnetic Field Induction ................................................................................................. 12  
1.4 Assumptions of EMF Theory .................................................................................................... 15  
1.5 Shield Wires, Bundled Conductors and Sag ........................................................................... 16
Chapter 2: Computing EMF and Induction Effects in MATHCAD ........................................19

2.1  Program Layout ............................................................................................................. 19

2.2  Analysis of Program Output ........................................................................................ 25

  2.2.1  Case A – Electric Field of a Flat Configuration Structure ....................................... 25

  2.2.2  Case B - Magnetic Field of a Flat Configuration Structure ....................................... 28

  2.2.3  Case C - Magnetic Field due to Shield Wires ............................................................ 31

  2.2.4  Case D - Electric Field Induction ........................................................................... 33

  2.2.5  Case E - Magnetic Field Induction ........................................................................... 38

  2.2.6  Reduced Impedance Matrix .................................................................................... 44

2.3  Applications of Program Output .................................................................................. 45

Chapter 3: Conclusion ..............................................................................................................48

Bibliography ..........................................................................................................................50
List of Tables

Table 2-1 Case-A Comparing Electric Field Strengths between MATHCAD and EPRI .......... 26
Table 2-2 Case-B Comparing Magnetic Field Density without Image Current .................. 29
Table 2-3 Case-C Comparison of Magnetic Fields Accounting for Shield Wires ................. 32
Table 2-4 Case-D Double Circuit Parameters to Calculate Electric Field Induction ............ 34
Table 2-5 Comparison of Electric Field Induced Voltages and Currents .......................... 36
Table 2-6 Electric Field Induced Voltage and Current on Shield Wires ............................ 36
Table 2-7 Double Circuit Parameters to Calculate Magnetic Field Induction .................... 39
Table 2-8 Comparison of Magnetic Field Induced Voltages ............................................ 40
Table 2-9 Example of Impedance Variation ...................................................................... 40
Table 2-10 Difference in Induced Voltage due to Impedance Variation ............................. 41
Table 2-11 Comparison of Magnetically Induced Voltages using Reduced Impedance Matrix ... 42
Table 2-12 EMF Exposure Limits from ICNIRP and IEEE ................................................ 46
Table 2-13 Summary of EMF Values from Cases A-E ...................................................... 46
List of Figures

Figure 1-1 Distribution of Electric Field Lines from OHTL to Ground [2] ................................. 3
Figure 1-2 Mirror Image of Conductor Above Ground [2] ......................................................... 4
Figure 1-3 Conductors and their Images ..................................................................................... 5
Figure 1-4 Electric Field Generated by a Line Charge at an Observation Point [2] ..................... 6
Figure 1-5 Magnetic Field Intensity at an Observation Point [2] ................................................... 8
Figure 1-6 Image Conductors at Complex Depth ......................................................................... 9
Figure 1-7 Two Mutually Coupled Conductors and their Circuit Representation (© 2010 IEEE) [10] ........................................................................................................................................ 11
Figure 1-8 Sag of Conductor above Ground ................................................................................ 17
Figure 2-1 Input for Electric Field Calculation (Part-1) ............................................................... 20
Figure 2-2 Input for Electric Field Calculation (Part-2) ............................................................... 21
Figure 2-3 Location of Probes to Measure EMF (Distances are not to scale) ............................ 22
Figure 2-4 Input for Magnetic Field and Induction Calculations .................................................. 23
Figure 2-5 [P] and [C] Matrices, a Sample Output from MATHCAD ........................................... 25
Figure 2-6 Case-A 525kV Conductor Profile ............................................................................ 26
Figure 2-7 Case-A Lateral Profile of Variation of Electric Field Strength .................................... 27
Figure 2-8 Case-B Lateral Profile of Variation of Magnetic Field Density .................................... 29
Figure 2-9 Case-B Magnetic Field Density Variation (Linear) ....................................................... 30
Figure 2-10 Case-B Magnetic Field Density Profile Accounting for Ground Current ............... 30
Figure 2-11 Magnetic Field Profile with Shield Wires ................................................................. 31
Figure 2-12 Double Circuit Configuration-IEEE (© IEEE 2017)[6] .............................................. 35
viii
Figure 2-13 Double Circuit Configuration-MATHCAD .............................................................. 35
Figure 2-14 Electric Field Profiles of a 345kV Double Circuit: Only One Circuit Energized..... 37
Figure 2-15 Electric Field Profiles of a 345kV Double Circuit: Both Circuits Energized........ 37
Figure 2-16 Electric Field Profile of 345kV/525kV Double Circuit Line ................................. 38
Figure 2-17 Magnetic Field Profiles of Double Circuit Line with 1 Circuit Carrying Load (1kA) ................................................................................................................................................................................................. 43
Figure 2-18 Magnetic Field Profiles of Double Circuit Line-Both Carrying Load (1kA Same direction) ......................................................................................................................................................................................................................................................... 43
Figure 2-19 Magnetic Field Profiles of Double Circuit Line Carrying 1kA and 2kA Currents... 44
# List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>Volts</td>
</tr>
<tr>
<td>kV</td>
<td>Kilo Volts</td>
</tr>
<tr>
<td>A</td>
<td>Amperes</td>
</tr>
<tr>
<td>C</td>
<td>Coulombs</td>
</tr>
<tr>
<td>Ω</td>
<td>Ohms</td>
</tr>
<tr>
<td>mG</td>
<td>Milli Gauss</td>
</tr>
<tr>
<td>µT</td>
<td>Micro Tesla</td>
</tr>
<tr>
<td>δ</td>
<td>Skin Depth</td>
</tr>
<tr>
<td>ρ</td>
<td>Earth Resistivity</td>
</tr>
<tr>
<td>Ω</td>
<td>Radian frequency</td>
</tr>
<tr>
<td>ε₀</td>
<td>Dielectric Permittivity of Free-Space</td>
</tr>
<tr>
<td>µ₀</td>
<td>Magnetic Permeability of Free-Space</td>
</tr>
<tr>
<td>σ</td>
<td>Surface Charge Density</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>------------------------------</td>
</tr>
<tr>
<td>EMF</td>
<td>Electric and Magnetic Fields</td>
</tr>
<tr>
<td>OHTL</td>
<td>Overhead Transmission Line</td>
</tr>
<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>TEM</td>
<td>Transverse Electromagnetic</td>
</tr>
<tr>
<td>CI</td>
<td>Carson Integral</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
</tbody>
</table>
Acknowledgements

I offer my gratitude to the faculty, staff, and my friends from UBC, who have inspired me to work and continue my study in the field of Power Systems. I owe thanks to my supervisor, Professor Dunford, for continued support, guidance, and encouragement during the course of my Master’s Degree.

I thank my former colleague, Milind Khot, who helped me time and again with the technical expertise he has accumulated over the years as a Transmission Engineer.

I am grateful to Professor Dommel who lent me a copy of the EPRI Red Book to satisfy my curiosity when I told him about my interest in Electric and Magnetic Fields.

I am also thankful to my colleagues at AMEC Foster Wheeler in the Transmission Department who were supportive and patient with me while balancing my professional work and Master’s studies.

Lastly, my parents are my source of success, my greatest source of strength, and they have supported me unconditionally throughout my years of education. Special thanks to them for being on my side, and to my siblings and spouse who are my moral support.
To my parents and grandparents
Chapter 1: Introduction

Electric and Magnetic Fields (EMF), generated by overhead transmission lines (OHTL), are highly important elements that are thoroughly considered by power utilities during the lines’ design and maintenance. The primary effects of EMF are related to health concerns due to continuous or short-time exposure to transmission lines and causing a safety hazard for crews working near parallel lines. In turn, these effects result in the utilities directing the acquisition of the transmission corridor (or right-of-way), as well as the safety steps to be taken during maintenance work. This thesis will discuss the theories developed over the years to calculate the relative strengths of the EMFs away from the lines and their induction effects on parallel lines. The goal of this research is to coalesce these theoretical concepts and create a computer program that can, for any number of transmission lines, output the EMF strengths, impedance/admittance matrices, and induced currents/voltages on nearby lines – all in one place. To meet this goal, a MATHCAD program has been created and its output is compared against known scenarios for verification. The computations are completed using accepted engineering principles established by Electric Power Research Institute (EPRI) and Institute of Electrical and Electronics Engineers (IEEE).

1.1 Thesis Outline

This thesis is comprised of three chapters. Chapter 1 serves as a technical review of the Electric and Magnetic fields from the perspective of transmission line engineering. Chapter 2 elaborates on the workings of the MATHCAD software that computes the effects of EMF near transmission lines. The final chapter concludes by discussing the practical use of these calculations to electric
utilities, the limitations of the MATHCAD program, and the potential future projects to be built upon this work.

1.2 Electric and Magnetic Fields

Overhead transmission lines generate electric and magnetic fields at power frequency (50/60Hz). These fields are generated by the presence and/or movement of electrically charged particles. The term “electromagnetic field” refers to coupled electric and magnetic fields [1]. However, alternating current (AC) transmission lines typically operate at low frequencies (50Hz or 60Hz). 60 Hz frequency is common in North America and 50Hz in Europe, Australia and parts of Asia [2]. At these frequencies, the Electric and Magnetic fields (EMF) can be treated separately [1]. The fields are assumed “quasi-static” - that is, time varying, but at a sufficiently slow rate so that they are normally adequately approximated by a static field [3]. This section will review the relevant concepts of electric and magnetic fields as created by energized transmission lines.

1.2.1 Electric Fields from Overhead Transmission Lines

An electric field is created by any electric charge that is in motion or stationary. It is the electric force experienced by a unit positive charge within the field and mathematically defined as [4]:

\[ E = \lim_{q \to 0} \frac{F}{q} \]  

(1.1)

Of course, when the charge is in motion it is called “current”. The electric field is expressed in units of “V/m” or “N/C”. In energized transmission lines, AC voltages produce sinusoidal electric fields that drive capacitive current through the line [2]. The high voltage on the conductors creates a capacitance between the conductors and the ground, with the air acting as
the dielectric medium. When referring to capacitance or charges on overhead conductors, they are generally expressed as “capacitance per unit length (F/m)” or “charge per unit length (C/m)”, since different lines are built to different lengths. The charge on the conductor is assumed to be distributed along the centre line of the conductor located above the ground [1]. This line charge (denoted by $q$) induces a surface charge density (denoted by $\sigma$, C/m²) of opposite polarity below the ground plane, as shown in Figure 1-1.

![Figure 1-1 Distribution of Electric Field Lines from OHTL to Ground [2]](image)

The net electric field at any point above ground is thus the vector addition of individual electric fields created by $q$ and $\sigma$. In practice, the net electric field is calculated using the principle of “Method of Images” proposed by Reitz and Milford [5]. The method involves replacing the ground plane with a second line charge (-$q$) that is “mirror image” of $q$. This is depicted in Figure 1-2.
The electric field strength, $E$, of a line charge ($q$) above ground from a distance $x$ is given by the following Equation [1]:

$$E = \frac{q}{2\pi \epsilon x}$$

$\epsilon = \text{dielectric permittivity of medium}$

Note, in this case, that air is the medium between the conductor and ground, and hence, the permittivity is usually taken as that of free-space and written as $\epsilon_0$. The potential difference between two points A and B due to the charge on the conductor can be expressed as [6]:

$$V_{AB} = \int_{L_1}^{L_2} E \, dx = \int_{L_1}^{L_2} \frac{q}{2\pi \epsilon_0 x} \, dx = \frac{q}{2\pi \epsilon_0} \ln \frac{L_2}{L_1}$$

$L_1, L_2 = \text{distance of conductor from A and B}$

When there are multiple conductors above the ground, which is common in real life, Equation 1.3 can be expanded to find the relationship between the voltage and charge on each conductor. As an example, Figure 1-3 shows two conductors, A and B, with line charges, $Q_A$ and $Q_B$, above ground and their corresponding mirror images $A^*$ and $B^*$ with line charges, $-Q_A$ and $-Q_B$. 

Figure 1-2 Mirror Image of Conductor Above Ground [2]
If the external radius (i.e. radius to the surface) of the conductors are $r_A$ and $r_B$, then the voltages ($V_A$ and $V_B$) and charges ($Q_A$ and $Q_B$) on each conductor A and B are given by [6]:

$$
V_A = \frac{Q_A}{2\pi\epsilon_0} \ln \frac{2h_1}{r_A} + \frac{Q_A}{2\pi\epsilon_0} \ln \frac{D_{AB*}}{D_{AB}} \\
V_B = \frac{Q_B}{2\pi\epsilon_0} \ln \frac{2h_2}{r_B} + \frac{Q_B}{2\pi\epsilon_0} \ln \frac{D_{BA*}}{D_{BA}}
$$

This process can be extended for any number of conductors. Usually, the voltage is a known quantity along with the conductor coordinates. From there, the line charge for a given conductor can be calculated using the above relationship. Equation 1.4 can be rewritten in matrix form as:

$$
\begin{bmatrix}
V_A \\
V_B
\end{bmatrix} = [P] \begin{bmatrix}
Q_A \\
Q_B
\end{bmatrix}
$$

The matrix $[P]$ is called the Potential Coefficient, which in this case is given by [6]:
Capacitance is related to charge by \( Q = CV \). Hence, the inverse of the matrix \( [P] \): \( [P]^{-1} \) provides the capacitance matrix \( [C] \) in F/m. Once \( [Q] \) is found, it can be used to calculate the electric field strengths using Equation 1.2. For example, consider Figure 1-4 that shows a conductor above ground \((x_i, y_i)\), where the field strength is measured at an observation point \((X, Y)\):

\[
\begin{bmatrix}
P_{AA} & P_{AB} \\
P_{BA} & P_{BB}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{2\pi \varepsilon_0} \ln \frac{2H_1}{r_A} & \frac{1}{2\pi \varepsilon_0} \ln \frac{D_{AB}}{D_{BA}} \\
\frac{1}{2\pi \varepsilon_0} \ln \frac{D_{AB}}{D_{BA}} & \frac{1}{2\pi \varepsilon_0} \ln \frac{2H_2}{r_B}
\end{bmatrix}
\]

(1.6)

Figure 1-4 Electric Field Generated by a Line Charge at an Observation Point [2]

Based on Figure 1-4, Equation 1.2 can be written as:

\[
E = \frac{Q}{2\pi r} = \frac{Q}{2\pi \sqrt{(x_i - X)^2 + (y_i - Y)^2}}
\]

(1.7)

The electric field \( E \) can be further separated into its vertical \((y\text{-axis})\) and horizontal \((x\text{-axis})\) components by simple trigonometric manipulation that will yield the following equations [2]:

\[
E_x = \frac{Q(x_i - X)}{2\pi \left[(x_i - X)^2 + (y_i - Y)^2\right]}
\]

(1.8)
\[ E_y = \frac{Q(y_i - Y)}{2\pi[(x_i - X)^2 + (y_i - Y)^2]} \tag{1.9} \]

In the case of multiple conductors and their images, all of the individual contributions of charges from each conductor (or image) are summed together to obtain the net electric field strength at \( X, Y \).

### 1.2.2 Magnetic Fields from Overhead Transmission Lines

The movement of charged particles is the primary source of magnetic fields in transmission lines [4]. When the lines are carrying load (or fault) currents, they generate time-varying magnetic fields surrounding the conductors [6]. These field lines are concentric circles and the field vector is perpendicular to the radii of these circular field lines at any point [2]. Magnetic flux density (\( B \)) is expressed in units of Tesla (T). This is a rather large unit to measure fields generated by transmission lines, so the unit milliGauss (mG) or microTesla (\( \mu \)T) is used more often. 1mG is equal to 1x10^-7 T. The magnetic field intensity (\( H \)) is expressed in A/m and is related to flux density as follows:

\[ B = \mu H \tag{1.10} \]

\[ \mu = \text{magnetic permeability of medium} \]

Just as in the case of an electric field, air is the acting medium between conductors and the magnetic permeability is taken as \( \mu_0 \). The magnetic field can also be separated into vertical and horizontal components. The magnetic flux density (in mG) at a distance \( x \) due to the current \( I \) flowing through an infinitely long line is given by:

\[ B = \frac{\mu_0 I}{2\pi x} \tag{1.11} \]
Figure 1-5, which is similar to Figure 1-4, shows the breakdown of components of the magnetic field intensity ($\mathbf{H}$) observed at a point $(X, Y)$.

From Figure 1-5, the vector field $\mathbf{H}$ is perpendicular to the radius of the circular magnetic field line. Using Equation 1.11 and some trigonometric manipulation, it can be shown that the vertical and horizontal components of the fields can be expressed as:

$$H_y = \frac{I (X - x_i)}{2\pi [(x_i - X)^2 + (y_i - Y)^2]}$$  \hspace{1cm} (1.12)

$$H_x = \frac{-I (Y - y_i)}{2\pi [(x_i - X)^2 + (y_i - Y)^2]}$$  \hspace{1cm} (1.13)

Multiplying the above by $\mu_0$ will give the values of $B_x$ and $B_y$. In the case of multiple conductors and their images, all the individual contributions of currents from each conductor (or image) are summed together to obtain the net magnetic flux density at X, Y.

### 1.2.2.1 Complex Depth

Magnetic flux density from multiple conductors can be computed using the Method of Images discussed in Section 1.2.1. However, there is a slight variation to account for the conductivity of
the earth. The earth is not a perfect conductor and currents flow deeper into the earth causing a phase delay. The returning earth currents are accounted for by placing the image conductors at an additional “complex depth” as shown in Figure 1-6:

![Image Conductors at Complex Depth](image)

The complex depth, \( \delta \), in metres, is given by [7]:

\[
\delta = 2 \sqrt{\frac{\rho}{j\omega\mu}} \quad (1.14)
\]

The result without the factor 2 multiplication is also called the “skin depth”. \( \rho \) is the resistivity of the soil (usually taken as 100\( \Omega \cdot m \)) and \( \omega \) is the angular frequency (\( \approx 377 \) rad/s at 60Hz) of the flowing current. Equations 1.12 and 1.13 are valid for image conductors \( A^* \) and \( B^* \) by simply adding the value of \( \delta \) to the height of the conductors. In addition, a correction factor \( CF \) is multiplied with the values of magnetic field intensity obtained via Equations 1.12 and 1.13 [1]:

---

[Image 1]: #
[Figure 1-6]: #
\[
CF = \left( 1 + \frac{1}{3} \left( \frac{\delta}{D_{\text{image}}} \right)^4 \right) 
\]

\[
H_{\text{corrected}} = (CF) \times H_{x\text{ or } y}
\]

\[D_{\text{image}} = \text{Distance between image conductor and observation point}\]

Note, Equations 1.14 and 1.15 are still approximations of the proposed original solution by Carson [8]. The original formula involved derivation of ground return impedance using the following complex integral (CI) [9]:

\[
\begin{align*}
\text{Ground Impedance, } z &= \frac{j \omega \mu}{2\pi} \left[ \ln \frac{D_2}{D_1} + CI \right] \\
\text{Complex Integral, } CI &= \int_0^{\infty} \frac{2e^{-H\lambda}}{\lambda + \sqrt{\lambda^2 + j \omega \mu \sigma}} \cos(x\lambda) d\lambda 
\end{align*}
\]

These equations are based on a quasi-static approach that is valid for frequencies up to 10MHz.

The variables H, x, D1 and D2 vary depending on whether the objective is to find mutual impedance or to find the self-impedance of conductors [9]. Usually, the complex depth is very large compared to the height of a conductor above ground and can be neglected without loss of accuracy [1]. The MATHCAD program, developed as part of this thesis, provides the option to incorporate or neglect the effect of earth resistivity during magnetic field calculation.

1.3 Induction

Electric and magnetic fields generated by energized transmission lines have the capacity to induce current or voltage on a nearby conducting surface such as vehicle-roofs, steel objects, fences, other transmission lines, and so on. Further, voltages and currents induced into de-energized transmission lines pose a serious work hazard for line-crew personnel. Though voltage is considered to be the main cause of accidents involving de-energized line work, the induced voltage and subsequent current flowing through a worker’s body is the primary reason
for accidents [10]. Induction from transmission lines can be divided into two types: Electric Field Induction and Magnetic Field Induction.

### 1.3.1 Electric Field Induction

When transmission lines are parallel to each other, the energized lines induce voltage and currents on nearby conductors via electric field induction. The conductors, separated from each other by air acting as a dielectric medium, form a capacitance between themselves (called mutual capacitance) and the ground. Figure 1-7 shows two conductors parallel to each other and their equivalent circuit representation:

![Figure 1-7 Two Mutually Coupled Conductors and their Circuit Representation (© 2010 IEEE)](image)

Here, conductor \( a \) is energized to \( V_{ag} \), whereas conductor \( b \) is de-energized. Due to capacitive coupling via a dielectric medium (i.e. air), voltage \( V_{bg} \) is induced on conductor \( b \). \( C_{ag}, C_{bg} \) represent the capacitances between the conductor and the ground, while \( C_{ab} \) is the mutual capacitance between the conductors. During line maintenance, if node \( b \) of the de-energized conductor is grounded, a current will now flow through \( b \) to the ground. This is the “induced current” flowing through the line, which poses a hazard for line workers. This concept can be expanded to any number of conductors based on discussion of Section 1.2.1. The capacitance values are found by inverting the \([P]\) matrix in Equation 1.6 to give the capacitance matrix \([C]\). This matrix is then used to evaluate the induced voltage and current on de-energized line(s). The
current, \( I \), through a conductor is related to capacitance \( C \) and voltage \( V \) by the following equation:

\[
I = j \omega CV
\]  
(1.17)

The term “\( j \omega C \)” is also called the “Shunt Admittance”. In matrix representation, the voltage and current matrices could be partitioned into energized and de-energized conductors as follows [6]:

\[
\begin{bmatrix}
I_{\text{energized}} \\
I_{\text{de-energized}}
\end{bmatrix}
= j \omega [C]
\begin{bmatrix}
V_{\text{energized}} \\
V_{\text{de-energized}}
\end{bmatrix}
\]  
(1.18)

To determine induced voltage (unit of V) on de-energized lines, \([I_{\text{de-energized}}]\) is set to 0. To determine the induced current (unit A/m) when the de-energized lines are grounded, \([V_{\text{de-energized}}]\) is set to 0. Also, based on Equation 1.18, since the capacitance is expressed in F/m, the induced voltage is a function of the energizing voltage and is independent of the distance that the lines are parallel to each other. The induced current is dependent on the distance between the lines as well as how far they run in parallel. This is evident from the units of induced voltage (i.e. Volts) and induced current (i.e. Ampere) per metre.

### 1.3.2 Magnetic Field Induction

When transmission lines are carrying load or fault currents, these currents produce magnetic field lines that link with nearby conductors via magnetic field induction. These fields are time-varying magnetic fields that can induce a voltage on de-energized lines if they are not grounded or are only grounded at one point [6]. The induced current travels in the opposite direction of the energizing line current [10].
In the case of magnetic coupling, it is necessary to develop a “Series Impedance” matrix, $[Z]$, similar to the “Shunt Admittance” matrix for capacitive coupling. This impedance matrix contains the self and mutual impedances of the conductors involved. A series impedance matrix of 3 conductors will have the following format:

$$ [Z] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} $$ (1.19)

$Z_{11}, Z_{22}, Z_{33}$ refers to the self-impedance of the conductors, whereas the off-diagonal terms define the mutual impedance between the conductors. For example, $Z_{13}$ is the mutual impedance between the 1st and 3rd conductor. These impedances are a function of geometric mean radius (GMR), physical location, and AC resistance of the conductors. GMR and AC resistance values are typically obtained from the cable manufacturer. Based on Figure 1-6 and Equation 1.16, the self-impedance of conductor A with AC resistance $R_{AC}$ is given by [1] [9]:

$$ Z_{AA} = R_{AC} + \frac{j \omega \mu}{2\pi} \left[ \ln \frac{2h_1}{GMR_A} + CI \right], \quad CI = \int_0^\infty \frac{2e^{-2h_1\lambda}}{\lambda + \sqrt{\lambda^2 + j\omega\mu\sigma}} d\lambda $$ (1.20)

The mutual impedance between conductors A and B are given by [1] [9]:

$$ Z_{AB} = \frac{j \omega \mu}{2\pi} \left[ \ln \frac{D_{AB^*}}{D_{AB}} + CI \right], \quad CI = \int_0^\infty \frac{2e^{-(h_1+h_2)\lambda}}{\lambda + \sqrt{\lambda^2 + j\omega\mu\sigma}} \cos(x\lambda) d\lambda $$ (1.21)

Due to the oscillating nature of Carson’s Integral published in 1926, there have been various approximations proposed by more recent authors [7, 9, 11, 12, 13] to make the computation easier. For example, a popular approximation of the Carson Integral has the following form [7]:

$$ Z_{AB} = \frac{j \omega \mu}{2\pi} \left[ \ln \frac{\sqrt{h_1 + h_2 + 2\delta + x^2}}{D_{AB}} \right] $$ (1.22)
A single logarithmic term was preferable to evaluating the original complex integral expression. Modern computer applications have obviously made it easier to evaluate oscillating integrals with sufficient accuracy and speed, as long as the user is aware of the functional limits. In 2015, a paper [14] was published that claims to obtain a closed-form expression of Carson’s Integral and provide the exact solution. This has not been applied in the MATHCAD software, since it has not been extensively verified by other researchers in the engineering community. Nevertheless, once sufficient verification is completed, past approximations will be rendered ineffective.

The series impedance of a conductor is related to its voltage and current by the following equation:

\[ \mathbf{V} = \mathbf{Z}\mathbf{I} \]  

(1.23)

In matrix representation, the voltage and current matrices can be partitioned as follows [6]:

\[
\begin{bmatrix}
\mathbf{V}_{\text{energized}} \\
\mathbf{V}_{\text{de-energized}}
\end{bmatrix} = \mathbf{Z} \begin{bmatrix}
\mathbf{I}_{\text{energized}} \\
\mathbf{I}_{\text{de-energized}}
\end{bmatrix}
\]

(1.24)

To determine the induced voltage (unit of V/m) on de-energized lines if they are ungrounded or grounded at one point, \(\mathbf{I}_{\text{de-energized}}\) is set to 0 and is then solved using simple matrix algebra [6]. To find the induced current (unit of A), \(\mathbf{V}_{\text{de-energized}}\) is instead set to 0. Note, in the case of magnetic field induction, the induced current is independent of the distance the lines are parallel to each other, but dependent on the distance between the lines, mutual impedance, and the current flowing in the energized lines. This is evident from the units of induced voltage (i.e. Volts) per metre and induced currents (i.e. Amperes).
Now that the basics of EMF strengths and Induction effects have been covered, the next two sections will summarize the assumptions made in the discussed methods and how the effects of other factors like shield wires, bundled conductors, and sags that are prevalent in real transmission lines can be incorporated.

1.4 Assumptions of EMF Theory

The methods discussed so far to calculate field strengths and induction effects are based on reasonable assumptions set out in EPRI text [1] and IEEE-524 [6] Standard. The formulae provided thus far are valid in 2-dimensions (2-D). In practice, fields exist in all 3 dimensions, and methods have been developed to compute field effects in 3-D. While 2-D methods work in most cases to obtain an initial impression, it is important in some cases to utilize 3-D methods, i.e. when determining fields around steel towers, suspension insulators, and substations. 3-D models are complex and generally require an extensive amount of computing time and resources. There are commercial applications that perform these tasks, such as CDEGS, WinIGS, and ASPEN OneLiner, among others, but they are quite expensive. Realistically, 2-D methods can provide a quicker and reasonable approximation before deeper analysis is required in transmission and substation projects.

As stated earlier, EMFs from transmission lines are assumed to be quasi-static TEM, i.e. the electric and magnetic fields are restricted only perpendicularly to the direction of propagation and can be treated separately at low frequencies like 50 or 60Hz, since the wavelengths are much larger than other considered dimensions [3]. In fact, Carson’s Integral is valid up to 10MHz [14].
Additional assumptions made in relation to electric fields are:

- Electric charges only reside on the surface of the conductor when there are no space charges around (e.g. during Corona in adverse weather).
- The transmission conductors are infinitely long and parallel to each other, and are located in a flat terrain.
- The earth is a perfect electrical conductor and the air is the only dielectric medium with a constant permittivity equal to that of free-space.

In relation to magnetic fields, the following assumptions are made:

- The transmission conductors are straight, infinitely long and parallel to each other.
- The earth is a poor conductor of magnetic fields. However, the presence of earth can be accounted for using the notion of Complex Depth (discussed in Section 1.2.2.1).

1.5 Shield Wires, Bundled Conductors and Sag

The examples mentioned so far have not considered the effect of shield wires, bundled conductors, and sags that are common in real-world transmission lines. Shield wires have a detectable, yet small effect on EMF [1], and there will be a mutual capacitance and impedance between transmission conductors and shield wires. Shield wires are incorporated into Capacitance and Impedance matrices easily by treating them as individual conductors (with relevant voltage or current depending on the analysis).

Bundled conductors are often used in transmission lines to increase load capacity and reduce corona losses. When bundled conductors are present, there are two ways to model their radius:
1. Treat each individual conductor in a bundle separately (this will increase matrix size).

2. Replace them with a single equivalent conductor.

The second method is preferable in terms of computation time. In the case of symmetrically bundled conductors, the formulae to find an equivalent radius is given by [15]:

\[ r_{\text{equivalent}} = \sqrt[n]{n r A^{n-1}} \]  

where, \( n = \) number of conductors in the bundle  
\( r = \) radius of individual conductor  
\( A = \) radius of the bundle

In the case of magnetic fields, \( r \) is replaced by GMR. GMR represents the radius of a hollow conductor that will have the same magnetic field pattern as that of a solid conductor with radius \( r \). GMR is usually provided by the cable manufacturers in the datasheet. The relationship between GMR and \( r \) of a solid cylindrical conductor with uniform current density is [1]:

\[ \text{GMR} = e^{-0.25r} \]  

In the real world, the conductors in transmission lines typically form a curved shape:

![Figure 1-8 Sag of Conductor above Ground](image)

This is caused by gravitational force pulling the conductor downward, making it “sag” and take a catenary shape as shown in Figure 1-8. The conductor is attached to a transmission structure at a
supporting point “$h$” metres above the ground, and then it sags so that its minimum point above
the ground is “$s$” metres. The conductor is assumed to be a perfectly straight line in EMF
calculations, so an average of conductor height is calculated based on its average sag and
attachment height above ground. This average height is calculated [16] as follows based on
Figure 1-8:

$$h_{average} = \frac{1}{3} h + \frac{2}{3} s$$  \hspace{1cm} (1.27)$$

So, $h_{average}$ represents the assumed height of the conductor above ground throughout the length of
the line. The MATHCAD program automatically calculates this average height based on sag and
attachment height data. It also provides a small tool to calculate the equivalent radius and
geometric mean radius of bundled conductors as an added benefit to the user.

We have now covered the theories pertinent to understanding the behaviour of EMF from
transmission lines. The next chapter focuses on the development of the MATHCAD program
that utilizes these theoretical concepts and the analysis of the obtained results.
Chapter 2: Computing EMF and Induction Effects in MATHCAD

A MATHCAD program has been developed as part of this research work to compute the EMF effects described in the previous chapter. This program is meant to serve as one simple tool to estimate field strengths and induction effects altogether. The MATHCAD software has been chosen because of its intuitive, user-friendly input and mathematics-friendly programming. MATHCAD also allows coding of Integrals, Summations, etc. in the same way that they are normally written in a mathematical notation. For example, to sum up all the electric field vectors from “n” conductors in the x-direction in Equation 1.8, the code in MATHCAD is simply:

\[
\sum_{i=1}^{n} E_x = \frac{Q_i (x_i - X)}{2\pi \left[ (x_i - X)^2 + (y_i - Y)^2 \right]}
\]

The tool does not require the use of explicit “for” loops as needed in C or MATLAB. Apart from this, MATHCAD code can be organized into Input, Process, and Output, providing a logical top to bottom flow for the reader. Overall, this is the preferable application for our work given its robust handling of engineering calculations compared to spreadsheets like Excel, and simplicity compared to programs like MATLAB. The next sections will outline the layout, processing, and output of this program and compare the results with examples from IEEE-524 [6] and EPRI [1].

2.1 Program Layout

The first section of the MATHCAD program is user input. It asks for relevant inputs in the form of numbers and tables similar to a spreadsheet, and the inputs are categorized as:

A) Electric Field Input (for calculating and generating Electric Field Strength and its profile)

B) Magnetic Field Input (for calculating and generating Magnetic Field Strength and its profile)
C) Induction Effects (on shield wires and/or de-energized lines).

Screenshot in Figure 2-1 captures the first part of the Electric Field Input from the program:

![Figure 2-1 Input for Electric Field Calculation (Part-1)](image)

The texts and table highlighted in Cyan are the only data that the user needs to fill-in. For fields in table format, if the user has a spreadsheet of data in Excel (which is typical for utilities), it can be copied directly into the MATHCAD cells without typing it in manually. Most of the input is self-explanatory and few sections require special attention. For example, physical data for shield wires must be entered “last” and the user must assign numbers sequentially to the de-energized lines, i.e. 4, 5, 6 instead of 6, 4, 5. The program asks for both attachment heights and sags to
calculate average height. Depending on the utility, some prefer to provide the maximum sag of the conductor from attachment point while some prefer the height of the conductor above ground at maximum sag. If the latter is known, as in ‘s’ in Figure 1-8, SAG_FLAG is set to 0 and the “sag” table can remain empty. Otherwise, SAG_FLAG is set to 1 and the user will proceed to fill out the “sag” table and leave “h_s” empty. MATHCAD uses either of this information in conjunction with attachment height to find the average height. Figure 2-2 captures the second part of the Electrical Field Input:

<table>
<thead>
<tr>
<th>Number of subconductors in a bundle (single integer)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In case of bundled conductors, use this mini-section on the right to compute equivalent radius of each and plug the value back in r or r-gmr:</td>
</tr>
<tr>
<td>spacing = spacing between subconductors in cm</td>
</tr>
<tr>
<td>r_s = external radius/GMR (see section 5) of each subconductor in cm</td>
</tr>
<tr>
<td>b = calculated bundle radius/GMR in cm</td>
</tr>
<tr>
<td>r sequel = calculated equivalent bundle radius/GMR in meters</td>
</tr>
<tr>
<td>Insert r sequel calculated on the right to the table for external radius ext.</td>
</tr>
</tbody>
</table>

| External radii of conductors, in meters (vector with 'K' entries) |
| In case of bundled conductors, provide equivalent radius from above |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 14.95 | 24.95 | 24.95 | |

Table: | x | y |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
</tr>
</tbody>
</table>

Table: | x_min | x_max |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2525</td>
</tr>
<tr>
<td>2</td>
<td>2525</td>
</tr>
<tr>
<td>3</td>
<td>2525</td>
</tr>
<tr>
<td>4</td>
<td>2525</td>
</tr>
</tbody>
</table>

Table: | y_p | y_p_min | y_p_max |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-120</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2-2 Input for Electric Field Calculation (Part-2)

To begin with, the orange highlighted portion shown above is a simple mini-tool to compute the equivalent radius of bundled conductors as stated previously. The user can skip this if these values are already known and/or calculated separately. Note, the inputs (bundle spacing, bundle
radius) to find equivalent radius are in “cm” as this is often the case with manufacturer datasheets. This prevents the user from making any errors while doing menial unit conversions and instead allows the user to focus on actual parameters. Once the external radius is found, this is plugged into the table after the mini-tool, along with values for voltage, phase angles and horizontal coordinates.

The program requires “associated” line-line voltage for each conductor, not the actual phase voltage. This implies, if there is a 3-phase line with a line-to-line voltage of 100kV, that for each of the three conductors the user will insert 100kV. The program will internally calculate the actual phase-to-ground voltage by dividing the line voltage by √3. The reason for adopting this convention in this program is because when utilities talk about voltage, it refers to the RMS line-to-line voltage in practice, unless stated otherwise. This way, the user is relieved from remembering to do the √3 conversions.

![Figure 2-3 Location of Probes to Measure EMF (Distances are not to scale)](image-url)
The final input is for “probes”. This is basically a series of observation points at which the user wants to measure the EMF strength, as shown in Figure 2-3 above. For example, if the user wants to measure the field strength of a 3-phase line laterally up to 10m from the “edge” of left and right conductors at 1m interval and 2m above ground, then the value of $y_p$ is 2, $probe_{-}spacing$ is 1, $range_{-}min$ is -10, and $range_{-}max$ is +30. The range values are in the context of the conductors’ x-coordinates, as is the case in Figure 2-2, where the left conductor starts from a horizontal position of x=0. Guidelines from the International Commission on Non-Ionizing Radiation Protection (ICNIRP) [17] and IEEE C95.6 [18] specify the height above ground for field measurements and different utilities may prefer to use their internal values.

Once the Electric Field Input is provided, the user then proceeds to supply additional data for magnetic field and induction calculations as shown in Figure 2-4:
Magnetic field calculations are based on the physical locations of the conductors and probes set up in “Electrical Field Input”. If the user is only interested in magnetic fields, other inputs (like voltage, phase angle, etc.) in the EF can be disregarded. The first line of the Magnetic Field Input requires the value of uniform soil resistivity. This can be set to 0 if the user chooses to ignore ground return currents, as discussed in Section 1.2.2.1. The next two fields take in conductor currents (i.e. phase current for each conductor) and their corresponding phase angles, followed by the orange colored mini-tool (shown in Figure 2-4) to determine the GMR of a conductor. The height of the probe above ground to measure magnetic field density is given by the variable \( y_{p\_mag} \). The APPROX_IND_FLAG value of 1 forces MATHCAD to use logarithmic approximation (Equation 1.22) to calculate series impedance. This is followed by a table of AC resistance values of the conductor (at 25°C or another relevant operating temperature).

The final two lines are related to the calculation of induction effects. They are basically Boolean variables that are set to 1 if the user is interested in calculating shield wire and/or de-energized line induction values; otherwise they are set to 0. Once all the relevant input is keyed in, MATHCAD computes the field strengths, induction voltages, and currents based on the theories discussed in Chapter 1. The user can scroll down to see the various output. The program highlights the output variable in blue, as in Figure 2-5, shows any values/matrices the user might be interested in, and plots the graphs showing conductor positions and field profiles in 2-D.
2.2 Analysis of Program Output

This section compares the output of the program against some sample cases provided in EPRI and IEEE-524 standards. This analysis is to measure the accuracy of the program relative to software based on the same standards. The numerical values in each case are stated in the same number of significant figures as given by EPRI (or IEEE) for an even comparison. Note, EPRI uses a Java-based Applet (2002 version) and IEEE uses a “C” program (1992 version) to perform these calculations.

2.2.1 Case A – Electric Field of a Flat Configuration Structure

This case [1] considers a three-phase 525-kV line in a flat configuration, 10m between phases, 10.6m height above ground, and bundles of three conductors 3.3cm in diameter with 45cm spacing. The electric field is calculated at a point 20m from centreline and 2m above ground.
Based on this description, Figure 2-6 shows the location of conductors above ground:

![Graph: Conductor Coordinates](image)

**Figure 2-6 Case-A 525kV Conductor Profile**

MATHCAD determines the corresponding potential coefficient \([P]\) and capacitance matrices \([C]\) for the line, and calculates the field strength based on those values. Table 2-1 compares the capacitance and electric field strength values obtained from MATHCAD and the EPRI book:

<table>
<thead>
<tr>
<th></th>
<th>EPRI</th>
<th>MATHCAD</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_{11} (F/m)</td>
<td>11.6E-12</td>
<td>11.6E-12</td>
<td>0.0%</td>
</tr>
<tr>
<td>C_{12} (F/m)</td>
<td>-1.9E-12</td>
<td>-1.9E-12</td>
<td>0.0%</td>
</tr>
<tr>
<td>C_{13} (F/m)</td>
<td>-560.0E-15</td>
<td>-555.0E-15</td>
<td>-0.9%</td>
</tr>
<tr>
<td>C_{21} (F/m)</td>
<td>-1.9E-12</td>
<td>-1.9E-12</td>
<td>0.0%</td>
</tr>
<tr>
<td>C_{22} (F/m)</td>
<td>11.9E-12</td>
<td>12.0E-12</td>
<td>0.8%</td>
</tr>
<tr>
<td>C_{23} (F/m)</td>
<td>-1.9E-12</td>
<td>-1.9E-12</td>
<td>0.0%</td>
</tr>
<tr>
<td>C_{31} (F/m)</td>
<td>-560.0E-15</td>
<td>-555.0E-15</td>
<td>-0.9%</td>
</tr>
<tr>
<td>C_{32} (F/m)</td>
<td>-1.9E-12</td>
<td>-1.9E-12</td>
<td>0.0%</td>
</tr>
<tr>
<td>C_{33} (F/m)</td>
<td>11.6E-12</td>
<td>11.6E-12</td>
<td>0.0%</td>
</tr>
<tr>
<td>E_x (V/m)</td>
<td>-381-939j</td>
<td>-381-939j</td>
<td>0.0%</td>
</tr>
<tr>
<td>E_y (V/m)</td>
<td>1750+4438j</td>
<td>1750+4438j</td>
<td>0.0%</td>
</tr>
<tr>
<td>E_{rms} (V/m)</td>
<td>4877</td>
<td>4877</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 2-1 Case-A Comparing Electric Field Strengths between MATHCAD and EPRI
Comparing the results between EPRI and MATHCAD, they are virtually identical with a maximum difference of 1%. This could have arisen due to a number of reasons, such as the difference in the way matrix inversions and floating digits are handled in Java versus MATHCAD. Source code for the Java applet from EPRI is not available, so it is possible that some of the constants (like $\pi$, $\varepsilon_0$) are hardcoded to specific precision. All or some of these can play a role to prevent a 100% match in all cases. The same principle could be applied when using the C-program from IEEE in Cases D and E. Regardless, the goal is to keep the differences to a minimum and presumably not beyond 5% without reasonable justification.

Figure 2-7 shows how the electric field strength varies horizontally as the probe is moved away from the line:

![Electric Field Profile](image)

**Figure 2-7 Case-A Lateral Profile of Variation of Electric Field Strength**

As evident from the figure above, the electric field strength is highest (9.36kV/m) near the edge of the line and gradually decreases from there. The field profile is symmetrical about the centre.
conductor (at x=10) since the flat configuration itself is symmetric. Different configurations like triangular, angled, double-circuit, etc., will affect the shape of the profile.

Changing the values of sag, conductor radius, voltage, and heights inside the MATHCAD software will provide an indication of how these parameters individually affect field strength. In general, the following conclusions can be drawn [1]:

1. Electric Field strength increases with increased voltage.
2. Increasing the line height decreases the maximum field strength at ground level.
3. Effect of sag is negligible if the lowest conductor point (maximum sag) is chosen. The field strength accuracy drops near suspension insulators and a 3-D method is suitable to account for the geometry of the suspension insulators as well as the tower’s shielding effect.
4. Increased conductor size equals increase surface radius, which increases field strength.
5. The smaller separation between conductors produces lower field strengths as the fields tend to cancel each other.

Now that it is known how the electric field behaves for the flat 525kV line, the next section discusses how the magnetic field strength varies for the same line when it is carrying a load current.

2.2.2 Case B - Magnetic Field of a Flat Configuration Structure

Using the same physical dimensions as Case-A, the case is now altered slightly by injecting balanced and symmetric currents of 1000A into all three phases. Table 2-2 shows the magnetic
flux density values obtained from MATHCAD and EPRI without accounting for ground return current:

<table>
<thead>
<tr>
<th>Distance from Centre Line</th>
<th>EPRI Field (mG)</th>
<th>MATHCAD Field (mG)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>210</td>
<td>210</td>
<td>0.0%</td>
</tr>
<tr>
<td>100</td>
<td>3.5</td>
<td>3.5</td>
<td>0.0%</td>
</tr>
<tr>
<td>200</td>
<td>0.9</td>
<td>0.9</td>
<td>0.0%</td>
</tr>
<tr>
<td>500</td>
<td>0.14</td>
<td>0.14</td>
<td>0.0%</td>
</tr>
<tr>
<td>1000</td>
<td>0.035</td>
<td>0.035</td>
<td>0.0%</td>
</tr>
<tr>
<td>2000</td>
<td>0.009</td>
<td>0.009</td>
<td>0.0%</td>
</tr>
<tr>
<td>5000</td>
<td>0.0014</td>
<td>0.0014</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 2-2 Case-B Comparing Magnetic Field Density without Image Current

As seen in Table 2-2, MATHCAD and EPRI field values are in agreement. Magnetic Field Profiles can also be drawn similar to that of Electric Field Profiles. The vertical axis is usually logarithmic since the field values of interest lie outside the right-of-way and are very small.

Figure 2-8 shows the field profile based on Table 2-2:

![Magnetic Flux Density Profile](image)

**Figure 2-8 Case-B Lateral Profile of Variation of Magnetic Field Density**

The field densities are separated into Elliptical and Phasor flux values. Phasor Flux simply implies a net field based on the current’s RMS value, which is the value that affects the
transmission corridor. In three-phase lines, the sinusoidal current values vary with time and are typically not in phase with each other. Hence, they generate an elliptical vector field away from the line [1]. Elliptical Flux, shown by MATHCAD for informational purposes, is the “maximum” of the major and minor field components of such an ellipse formed by the magnetic fields. In this figure, the maximum resultant field density is 210mG. Figure 2-9 shows the same field profile but on both sides of the line (±100m) on a linear vertical axis:

![Magnetic Flux Density Profile](image1)

**Figure 2-9 Case-B Magnetic Field Density Variation (Linear)**

If ground return current is accounted for by setting the soil resistivity to 100Ωm, then the field density takes the following shape:

![Magnetic Flux Density Profile](image2)

**Figure 2-10 Case-B Magnetic Field Density Profile Accounting for Ground Current**
It is clear by comparing the above two figures that the ground current is too small to affect the field density. The difference can only be realized if more trailing digits after the decimal place are accounted for, and this would only give a minute difference of 0.02mG. Such a small magnitude is impractical to consider given the precision of physical measurements in practice. Therefore, ground current is often ignored in calculations involving magnetic flux density.

2.2.3 Case C - Magnetic Field due to Shield Wires

The addition of shield wires to a line has a detectable effect on the magnetic field. Using the previous case, now two shield wires are added at ± 6.5 m from the centre conductor, 15.6m above ground, resistance = 6.7 Ω/mile (1.1185Ω/m), reactance = 1.8 Ω/mile @ 1foot spacing (implies a GMR of 1.1006×10⁻⁷m), and ground resistivity 100Ωm. Shield wire currents are 16.3 A @ 174° (from centre phase current) and 15.4 A @ 27° [1].

Figure 2-11 depicts the behaviour of magnetic field density due to the addition of shield wires.

![Magnetic Field Profile with Shield Wires](image-url)
As can be seen, the currents in the shield wire change the maximum flux density slightly which is now 210.4mG. If the shield wires are included but the currents are ignored (i.e. set to 0), then the maximum flux density is 210.5mG. Table 2-3 compares the values obtained from EPRI when shield wires are present and are carrying currents:

<table>
<thead>
<tr>
<th>Distance from Centre Line</th>
<th>EPRI Field (mG)</th>
<th>MATHCAD Field (mG)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-200</td>
<td>0.78</td>
<td>0.79</td>
<td>1.3%</td>
</tr>
<tr>
<td>-100</td>
<td>3.31</td>
<td>3.33</td>
<td>0.6%</td>
</tr>
<tr>
<td>0</td>
<td>210.4</td>
<td>210.4</td>
<td>0.0%</td>
</tr>
<tr>
<td>100</td>
<td>3.65</td>
<td>3.64</td>
<td>0.3%</td>
</tr>
<tr>
<td>200</td>
<td>0.96</td>
<td>0.95</td>
<td>1.0%</td>
</tr>
</tbody>
</table>

Table 2-3 Case-C Comparison of Magnetic Fields Accounting for Shield Wires

The values between MATHCAD and EPRI are in close agreement. Notice, in both cases the magnetic flux density decreases slightly on one side of the centreline and increases on the other. Just like in Electric Fields, changing the values of sag, conductor radius, current, and height in MATHCAD will provide an indication of how these parameters affect the field density. In general, the following conclusions can be drawn [1]:

1. Magnetic flux density decreases with increasing conductor height and distance away from the line, but at a slower rate than the electric field.
2. Flux density is unaffected by conductor diameter, unlike the Electric Field.
3. Magnetic field increases with increased current. For this reason, it is typical to find the field level specified at maximum expected load current and corresponding operating temperature.
4. Earth return currents have a negligible effect, even up to few hundred metres from the line.
5. Shield wires have a minor but noticeable effect on flux density.

This covers the example of a magnetic field profile. The next sections will focus on comparing cases of EMF inductions.

2.2.4 Case D - Electric Field Induction

This case considers Electric Field Induction in a double-circuit line with shield wires as provided in the test case in IEEE-524 Standard [6]. One circuit is energized at 345kV and the other is de-energized. The induced voltage (in Volts) and currents per metre are to be determined on the de-energized line. The circuit configuration presented in Table 2-4 on the next page, which summarizes the parameters provided in the IEEE-524 test case. The original dimensions in the IEEE document are in FPS (feet-pound-force), and have been converted to SI units in Table 2-4. The “C” source code provided by IEEE to calculate induction effects has also been modified as part of this research work to make it compatible with SI units and easier to compare. The IEEE program includes some additional assumptions and error corrections inside the code. These are left unchanged and the output is compared to MATHCAD to see how closely they match.
<table>
<thead>
<tr>
<th>No. of Energized Lines</th>
<th>3</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of De-energized Lines</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Shield Wires</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tower Attachment Point, ( h_i(m) )</td>
<td>30.5</td>
<td>22.9</td>
<td>22.9</td>
<td>30.5</td>
<td>22.9</td>
<td>22.9</td>
<td>35.4</td>
<td>35.4</td>
</tr>
<tr>
<td>Mid-Span Sag, ( h_s(m) )</td>
<td>20.4</td>
<td>12.8</td>
<td>12.8</td>
<td>20.4</td>
<td>12.8</td>
<td>12.8</td>
<td>27.7</td>
<td>27.7</td>
</tr>
<tr>
<td>Line Voltage (kV)</td>
<td>345</td>
<td>345</td>
<td>345</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Voltage Phases (degrees)</td>
<td>0</td>
<td>120</td>
<td>240</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equivalent radius, ( r_{eq}(m) )</td>
<td>9.350E+00</td>
<td>9.350E+00</td>
<td>9.350E+00</td>
<td>9.350E+00</td>
<td>9.350E+00</td>
<td>9.350E+00</td>
<td>5.49E-01</td>
<td>5.49E-01</td>
</tr>
<tr>
<td>X-coordinate(m)</td>
<td>5.94</td>
<td>0.0</td>
<td>6.1</td>
<td>16.3</td>
<td>16.2</td>
<td>22.3</td>
<td>4.27</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Table 2-4 Case-D Double Circuit Parameters to Calculate Electric Field Induction
The figures below show the relative positions of the conductors based on Table 2-4. Figure 2-12 is taken from IEEE-524 (Imperial units) and Figure 2-13 from the MATHCAD program (SI units).

Figure 2-12 Double Circuit Configuration-IEEE (© IEEE 2017)[6]

Figure 2-13 Double Circuit Configuration-MATHCAD
The IEEE program calculates the induced voltages and currents on the de-energized lines only.

Table 2-5 compares these values with the ones obtained from MATHCAD:

<table>
<thead>
<tr>
<th>Conductor</th>
<th>Induced Voltage (V)</th>
<th>Induced Current (A/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IEEE</td>
<td>MATHCAD</td>
</tr>
<tr>
<td>4</td>
<td>1.740E+04</td>
<td>1.740E+04</td>
</tr>
<tr>
<td>5</td>
<td>1.540E+04</td>
<td>1.540E+04</td>
</tr>
<tr>
<td>6</td>
<td>9.127E+03</td>
<td>9.127E+03</td>
</tr>
</tbody>
</table>

Table 2-5 Comparison of Electric Field Induced Voltages and Currents

As shown above, the values from the IEEE and MATHCAD programs are in close agreement.

Notice, the induced voltages are in the order of 1000 Volts, and if the de-energized lines are
grounded within 200m, a current in the order of magnitude 12mA can flow through Conductor 4.
This is beyond the let-go current of 10mA stated earlier. Increasing the voltage of the energized
gle will further increase the values of the induced voltage and current. MATHCAD also
calculates the magnitude of induced voltages and currents on the Shield Wires, which can be just
as high:

<table>
<thead>
<tr>
<th>Conductor</th>
<th>Induced Voltage (V)</th>
<th>Induced Current (A/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>3.535E+04</td>
<td>8.611E-05</td>
</tr>
<tr>
<td>8</td>
<td>1.534E+04</td>
<td>3.157E-05</td>
</tr>
</tbody>
</table>

Table 2-6 Electric Field Induced Voltage and Current on Shield Wires

Section 2.2.1 showed the Electric Field Profile of a flat configuration line. This case gives the
opportunity to see the Electric Field Profile from a typical double circuit line with shield wires
when one side is energized, both sides are energized, and the voltage is increased. Figures 2-14
and 2-15 on the next page capture these cases using MATHCAD. The physical locations of the
conductors are kept unchanged.
Comparing the figures above, the peak field strength is 2.10kV/m when only one of the 3-phase line circuits is energized. This value increases to 2.42kV/m when both circuits are energized to 345kV. The maximum field strength has not changed significantly when both lines are energized, and in both cases, the field is strongest at the edge of the line. This supports the
previous conclusion in Section 1.2.1, that compact lines with smaller separation produce lower electric fields due to cancellation effects [1]. When the voltage of one of the circuits is increased to 525kV the profile shape changes:

![Electric Field Profile](image)

**Figure 2-16 Electric Field Profile of 345kV/525kV Double Circuit Line**

The maximum field strength is now increased to 3.45kV/m. As expected, the voltage has a directly proportional effect on electric field strength.

---

### 2.2.5 Case E - Magnetic Field Induction

Using the same double circuit line, it is now possible to determine the order of induction from magnetic fields when one circuit is carrying a load current and the other remains de-energized. In this case, a balanced positive sequence of 1000A currents are injected into a load-carrying line, where the other line is grounded at both ends. Additional parameters like GMR and AC resistance of the cables are now required to find the induced voltage (V/m) and current (A). All these parameters, from the IEEE-524 test case, are consolidated in Table 2-7.
<table>
<thead>
<tr>
<th>No. of Energized Lines</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of De-energized Lines</td>
<td>3</td>
</tr>
<tr>
<td>No. of Shield Wires</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Energized Lines</th>
<th>De-Energized Lines</th>
<th>Shield Wires</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line Number</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Tower attachment height, ( h(m) )</td>
<td>30.5</td>
<td>22.9</td>
</tr>
<tr>
<td>Mid-Span Sag, ( h_s(m) )</td>
<td>20.4</td>
<td>12.8</td>
</tr>
<tr>
<td>Line Current (A)</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Current Phases (degree)</td>
<td>0</td>
<td>120</td>
</tr>
<tr>
<td>AC Resistance @ 50°C (( \Omega/m ))</td>
<td>2.107E-05</td>
<td>2.107E-05</td>
</tr>
<tr>
<td>X-coordinate(m)</td>
<td>5.94</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 2-7 Double Circuit Parameters to Calculate Magnetic Field Induction
IEEE has a separate C-program for magnetic fields to output induced voltages (but not current) on de-energized lines. Table 2-8 now compares these values with those obtained from MATHCAD:

<table>
<thead>
<tr>
<th>Conductor</th>
<th>Induced Voltage (V/m)</th>
<th>Induced Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IEEE</td>
<td>MATHCAD</td>
</tr>
<tr>
<td>4</td>
<td>3.180E-02</td>
<td>3.655E-02</td>
</tr>
<tr>
<td>5</td>
<td>3.831E-02</td>
<td>3.044E-02</td>
</tr>
<tr>
<td>6</td>
<td>2.707E-02</td>
<td>2.089E-02</td>
</tr>
</tbody>
</table>

Table 2-8 Comparison of Magnetic Field Induced Voltages

At first glance, the voltage values from the two programs seem significantly different based on the highest variance of 22.8%. Note, all the values are in the same order of magnitude. There are two possible explanations for this, the first of which is based on the difference that stems from the way the series impedance matrix is calculated. To understand this, consider the impedance values in Table 2-9:

<table>
<thead>
<tr>
<th></th>
<th>Z (4,1)</th>
<th>Z (4,2)</th>
<th>Z (4,3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference (%)</td>
<td>4.9</td>
<td>3.6</td>
<td>1.9</td>
</tr>
<tr>
<td>Phase Currents</td>
<td>1000\angle0°</td>
<td>1000\angle120°</td>
<td>1000\angle240°</td>
</tr>
</tbody>
</table>

Table 2-9 Example of Impedance Variation

In Table 2-9, Z represents the mutual impedances between conductor 4 and conductors 1, 2 and 3. Suppose, the top row values are calculated using the Carson Integral and the second-row values deviate slightly from the top. The differences between these impedances are reasonable, with the maximum being 4.9%. To obtain induced voltage, each phase current is now multiplied
by the corresponding impedance column and then added together. Table 2-10 shows the obtained voltage values using these two sets of impedance values, which are seemingly close:

<table>
<thead>
<tr>
<th>Voltage using Method 1 Impedance</th>
<th>Voltage using Method 2 Impedance</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0221+j0.0291</td>
<td>0.0192+j0.0379i</td>
<td>25.3</td>
</tr>
</tbody>
</table>

*Table 2-10 Difference in Induced Voltage due to Impedance Variation*

When complex current is multiplied by the complex impedance the difference between induced voltages goes as high as 25%, even though the maximum difference between impedances was about 5%. The IEEE program evaluates Carson’s Integral in terms of an infinite series expansion and uses a modified form of the expansion terms. Meanwhile, MATHCAD can evaluate Carson’s Integral with its internal optimization function or use the logarithmic approximation given in Equation 1.22. The difference in magnitude between impedance matrices using MATHCAD’s “integral function” and “logarithmic approximation” itself can vary between 0% and 2%. Although this difference is acceptable given the variation in the computation methods, they can add up during the calculation of induced voltage. So, a 23% difference in value against the IEEE program that uses a modified version of Carson’s equations is not surprising.

However, upon further investigation of the source code of the IEEE program, a second and more plausible explanation surfaced. The IEEE program computes the induced voltage by using a Reduced Impedance Matrix ‘by default’ that accounts for shield wire impedances. The Reduced Impedance Matrix is discussed in Section 2.2.6 in detail. Essentially, this is a technique to absorb shield wire impedances into actual line impedances. Table 2-11 compares the induced voltage
from MATHCAD and IEEE program again, but this time the MATHCAD result is based on the Reduced Impedance Matrix:

<table>
<thead>
<tr>
<th>Conductor</th>
<th>Induced Voltage (V/m)</th>
<th>Induced Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IEEE</td>
<td>MATHCAD</td>
</tr>
<tr>
<td>4</td>
<td>3.180E-02</td>
<td>3.180E-02</td>
</tr>
<tr>
<td>5</td>
<td>3.831E-02</td>
<td>3.832E-02</td>
</tr>
<tr>
<td>6</td>
<td>2.707E-02</td>
<td>2.708E-02</td>
</tr>
</tbody>
</table>

Table 2-11 Comparison of Magnetically Induced Voltages using Reduced Impedance Matrix

As evident from the table, the voltage values are now virtually the same. So, it is justifiable to conclude that the second argument is the correct version. As stated earlier, the IEEE source code was not modified to change its algorithm to compute these induced voltages/currents, so the program calculated the series impedance matrix differently when shield wires were involved.

Now, just like electric fields, it is worthwhile to observe the variation of magnetic fields in the double circuit line in the following situations: one circuit carrying the load current, both circuits carrying load currents, and when the current value is increased. The figures on the next two pages capture these cases.
In the above figures, the maximum flux density is 60.4mG when one line is carrying load and 64.0mG when both are carrying load currents in the same direction. Of course, the flux density will significantly increase (about 84mG) when the load is in opposite directions. The in-between fields repel each other as they have the same vector direction. Figure 2-19 shows the effect of increasing the load current to 2000A on one circuit:
Increasing the current by twice the amount has increased the magnetic flux density to 120.3 mG; by about the same margin. This is consistent with the previous conclusion that flux density is directly proportional to increased current.

### 2.2.6 Reduced Impedance Matrix

In addition to providing the field strength and inductive current/voltage values, the MATHCAD program also allows the user to obtain the reduced impedance matrix by eliminating shield wires. This method is very useful in reducing the size of the matrix for hand calculations and even when using computers dealing with large systems. A 3-phase line with phases \( a, b \) and \( c \) with one shield wire, \( s \), has the following voltage, current, and impedance relationship in matrix form:

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
V_s \\
\end{bmatrix} =
\begin{bmatrix}
Z_{aa} & Z_{ab} & Z_{ac} & Z_{as} \\
Z_{ba} & Z_{bb} & Z_{bc} & Z_{bs} \\
Z_{ca} & Z_{cb} & Z_{cc} & Z_{cs} \\
Z_{sa} & Z_{sb} & Z_{sc} & Z_{ss} \\
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c \\
I_s \\
\end{bmatrix}
\]

Figure 2-19 Magnetic Field Profiles of Double Circuit Line Carrying 1kA and 2kA Currents
Based on this matrix, the shield wire current, $I_s$, can also be written as:

$$I_s = \frac{-1}{Z_{ss}}(Z_{sa}I_a + Z_{sb}I_b + Z_{sc}I_c)$$

Replacing $I_s$ in terms of phase currents and impedances will lead to the following derivation for the reduced impedance matrix [19]:

$$[V_{phase}] = [Z_{reduced}] \ast [I_{phase}]$$

$$[Z_{reduced}] = [Z_{phase}] - [Z_s][Z_{ss}]^{-1}[Z_s]^T \quad (1.28)$$

where,

$$[Z_s] = \begin{bmatrix} Z_{as} \\ Z_{bs} \\ Z_{cs} \end{bmatrix}$$

Using this principle, the 8x8 impedance matrix in the double circuit line case can be reduced to a 6x6 matrix for faster calculation, while trading off the ability to see individual effects on shield wires.

### 2.3 Applications of Program Output

Now that the MATHCAD program has been discussed in detail, it is worth considering the practical applications of these obtained results in the real world. There are no regulations setting safety standards for exposure to EMFs in Canada [20]. However, professional associations like ICNIRP, IEEE and the ACGIH (American Conference of Governmental Industrial Hygienists) do indeed set limits on EMF exposure to workers and public. These limits dictate the right-of-way that utilities typically need to acquire before construction of new transmission lines. Land purchase can be expensive and electrical utilities often end up sharing the right-of-way with pipelines, water, and gas supply utilities [21]. So, it is important to figure out the amount of EMF
that is generated within and at a reasonable distance beyond the right-of-way of a transmission line.

ICNIRP and IEEE have set the following recommended limits of EMF exposure in their guidelines [17] and [18]:

<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>ICNIRP</th>
<th>IEEE</th>
<th>ICNIRP</th>
<th>IEEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worker</td>
<td>8.3</td>
<td>20</td>
<td>1000</td>
<td>2710</td>
</tr>
<tr>
<td>Public</td>
<td>4.2</td>
<td>5, Maximum 10 under High-Voltage Lines</td>
<td>200</td>
<td>904</td>
</tr>
</tbody>
</table>

*Table 2-12 EMF Exposure Limits from ICNIRP and IEEE*

These limits are set to prevent any adverse health effects on human bodies due to short or long-term exposure to EMF. This is a good opportunity to compare these restricted values with the results obtained from the flat 345kV and the 525kV double circuit configuration lines in previous sections of this study. These values are summarized below:

<table>
<thead>
<tr>
<th>Line Configuration</th>
<th>Maximum Electric Field Strength (kV/m)</th>
<th>Maximum Magnetic Flux Density (μT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat Configuration: 345kV and 1000A Load</td>
<td>9.36</td>
<td>21</td>
</tr>
<tr>
<td>Double Circuit Configuration: 525kV and 1000A Load</td>
<td>3.45</td>
<td>6.04</td>
</tr>
<tr>
<td>Double Circuit Configuration: 345kV and 2000A Load</td>
<td>2.1</td>
<td>12.0</td>
</tr>
</tbody>
</table>

*Table 2-13 Summary of EMF Values from Cases A-E*

The magnetic flux density values are well below the IEEE and ICNIRP thresholds. The electric field strengths are closer to the 5kV/m limit. The flat configuration line exceeds 5kV/m but is still under 10kV/m for high voltage lines. Electrical utilities do set their own exposure limits based on the guidelines. For example, BC Hydro sets the maximum limit of Electric Field
Strength to 10kV/m and a few US states limit the magnetic flux density at 150 or 200mG at the edge of the right-of-way[22].

Apart from the field values, induced voltages and currents are sources of hazards for utilities during line maintenance or short circuits. Induced currents from electric field induction alone can be above the “let-go” threshold of 10mA. Temporary protective grounds are typically installed to mitigate induced currents from EMF, but line workers face a hazard both at the beginning of ground installation and at the removal of those grounds. In the first case, it is the electric and magnetic fields that can contribute to the current flowing through a worker if the person finds themselves in series with the current path. In the latter case, there is the hazard of arcing during attempts to interrupt the capacitive or inductive current [10]. So, having knowledge of the order of these induced currents and voltages can help utilities to take appropriate safety steps for line crews.

The MATHCAD program can accept a different combination of current and voltage angles, so it is possible to determine induced voltage and/or current due to asymmetrical faults as long as proper magnitude and angles are provided. This feature is lacking in the IEEE program because it only accepts positive sequence currents or voltages. This has been addressed in another literature review [23], where it was proposed to extract the impedance matrices from the IEEE program in a text file and then manually use them with pre-computed asymmetric fault currents. This is exactly what the MATHCAD application does internally, and further adds to its usefulness in this study.
Chapter 3: Conclusion

This paper has discussed the theoretical concepts that lay the foundation for the calculation of Electric and Magnetic Fields and their corresponding Induction Voltages and Currents. The MATHCAD application built as part of this work has been verified against different cases from the EPRI book and the IEEE-524 Standard. The results from MATHCAD are within 0-2% of the values provided by EPRI and IEEE references, with the exclusion of magnetically-induced voltages. As discussed, this variation could easily be explained in terms of the distinct ways the programs utilize the series impedance matrix.

Comparing the IEEE and MATHCAD applications, the latter is more user-friendly, can accommodate any theoretical number of lines, and has better visual representation. The IEEE application is limited to one set of 3-phase energized lines, one set of 3-phase de-energized lines, and shield wires. It only computes the induction currents and voltages and uses a DOS Window prompt to take user input (heights, resistances, etc.) sequentially. If an error is made halfway through, the user is forced to restart the process, whereas MATHCAD dynamically updates its results when any of the input tables are modified.

The MATHCAD program is quite handy for obtaining a quick and initial estimate of both electric and magnetic field effects for same sets of lines. The results obtained from the program can be seen as a first step before undertaking detailed analysis in any expensive commercial applications. In the real-world, this kind of initial estimation is necessary during the definition phase or feasibility study of a transmission project, when all parties are only interested in approximate values and do not want to spend resources performing a detailed 3D analysis.
Given the current stage of the MATHCAD program, a few additional features and improvements could be made to the program as part of future work:

1. Implement the recently proposed algorithm [14] to determine the series impedance matrix by computing an exact solution of Carson Integral.

2. Adding an option to perform 3-D Calculations of EMF strengths as per EPRI guidelines. This will take line angles of transmission lines into account.


4. Determine Corona effects based on various weather conditions.

5. Calculate induction effects on underground pipelines.

6. Estimate induced current flowing through a worker using typical human resistance.

Implementing all of these together will convert the program into a complete all-in-one package for estimating various effects from transmission lines at the initial stage of any project.

The effects of Electric and Magnetic Fields have financial and safety implications for electrical utilities and the general public. There are misconceptions about the adverse health impacts of EMF (especially magnetic fields) among the general population, and several research projects have been carried out over the years – a list of some of them is maintained by utilities like BC Hydro [24] to allay public fears. So far, it seems that a direct causal link has not been established between human health and magnetic field exposure from transmission lines [17]. However, in light of the above study, it is important to continue to be investigative about EMF effects and to have an objective idea about their magnitude, which led to the development of the MATHCAD program. It is sincerely hoped that this program will act as a starting point toward satisfying that interest from any member of the public or researcher from any scientific field.
Bibliography


