Non-equilibrium Transport in Electron Solids

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in

The Faculty of Graduate and Postdoctoral Studies

(Physics)

THE UNIVERSITY OF BRITISH COLUMBIA
(Vancouver)

December 2016

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Abstract

Electron-electron interactions inside of two dimensional electron gases (2DEG) in out-of-plane magnetic field and at very low temperatures under certain conditions can lead to electron localization in Wigner crystals or even more complex periodic structures. These states are usually referred to as electron solid phases and result in Reentrant Integer Quantum Hall Effect (RIQHE) in transport measurements. However, their microscopic description remains unclear, as insulating phases with different microscopic structure demonstrate indistinguishable macroscopic transport properties. In this work the transport of the electron solids is investigated away from equilibrium conditions. This approach allows to break an insulating state by application of significant current bias to the 2DEG. As bias current increases, longitudinal and Hall resistivities measured for these states show multiple sharp breakdown transitions. Whereas the high bias breakdown of fractional quantum Hall states is consistent with simple heating, the nature of RIQH breakdown remains to be a subject of a considerable debate.

A comparison of RIQH breakdown characteristics at multiple voltage probes indicates that these signatures can be ascribed to a phase boundary between broken-down and unbroken regions, spreading chirally from source and drain contacts as a function of bias current and passing voltage probes one by one. It is shown, that the chiral sense of the spreading is not set by the chirality of the edge state itself, instead depending on electron- or hole-like character of the RIQH state. Although at high current bias the electron temperature is unmeasurable with standard techniques, the data shows that electron solid states appear to stay temperature sensitive even after the RIQH effect is destroyed. A comparison of temperature dependence and the spatial distribution of the Hall potential along the edge provides an evidence, that the bulk 2DEG remains insulating up to surprisingly high biases. Finally a metastable stripe phase around \( \nu = 9/2 \) is investigated under non-equilibrium conditions in the sample with electron density, which is close to the stripe reorientation critical point. The anisotropy of non-equilibrium stripe phase under high current biases shows a strong dependence of the natural orientation of stripes on exact filling factor.
Preface

This thesis describes work associated with the following publications:

1. **Low-temperature illumination and annealing of ultrahigh quality quantum wells**
   M. Samani, A. V. Rossokhaty, E. Sajadi, S. Luscher, J. A. Folk, J. D. Watson, G. C. Gardner, and M. J. Manfra
   Phys. Rev. B (Rapid) 90, 121405 (2014)

2. **Electron-Hole Asymmetric Chiral Breakdown of Reentrant Quantum Hall States**
   A. V. Rossokhaty, Y. Baum, J. A. Folk, J. D. Watson, G. C. Gardner, and M. J. Manfra

3. Versions of Ch. 5 and 6 are in preparation for publication

I helped to design and build the low temperature electrical filtering, wire cooling modules and electromagnetic radiation shielding. These changes to the refrigeration system significantly lowered its base temperature, which is crucial for experiments on reentrant states. I also built the fiber system in experimental setup, which allowed direct illumination of the sample and its temperature control during the cooldown process. This helped to develop sample preparation recipe, described in experiment [1], which was used in all other experiments. All experimental data from experiment [1] was acquired by M. Samani.

I identified initial direction of all experiments under supervision of Prof. J.A. Folk. I performed all of the data collection, data analyses and developed the interpretations/conclusions, shown in the main chapters of these thesis. The drafts were prepared by me, and these were edited together with the listed coauthors.

Semiconductor heterostructures and samples studied in this thesis were provided by the group of Prof. M.J. Manfra in Purdue University.

Numerical simulations were performed by Yuval Baum.
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- QHE ................................................. Quantum Hall Effect
- IQHE ................................. Integer Quantum Hall Effect
- RIQHE .......................... Reentrant Integer Quantum Hall Effect
- FQHE ................................. Fractional Quantum Hall Effect
- LL .......................................................... Landau Level
- FQHE ................................. Fractional Quantum Hall Effect
- 2DEG ......................................... Two Dimensional Electron Gas
- MBE ................................................ Molecular Beam Epitaxy
- MOSFET ............................... Metal Oxide Field Effect Transistor
- CDW ................................................ Charge Density Wave
- ELC .......................................................... Electron Liquid Crystal
- LED .......................................................... Light Emitting Diode
- RRR ................................................ Residual-resistance ratio
- WF .......................................................... Wiedemann-Franz
Chapter 1

Introduction

1.1 Overview

In 1879, eighteen years prior to the discovery of the electron and long before the discovery of its spin, Edwin Hall detected an effect, bearing his name nowadays. He measured a voltage, which appears across the sample due to the displacement of moving charge carriers in external electric and magnetic fields (Fig. 1.1). In the twentieth century, the Hall effect, well-known as a routine technique for semiconductor characterization, revealed a non-trivial quantum nature of two dimensional systems – quantum Hall effect (QHE).

Although the classical Hall effect has been discovered and studied more than a hundred years ago, QHE appeared to be absolutely unexpected. Even after the discovery of this phenomenon, no single even indirect prediction have been found in the literature. Two dimensional electron systems in a perpendicular magnetic field remain to have unforeseen and surprising behaviors even thirty years after the discovery of the QHE. Today this is still a field of an active research, since the solidity of this effect provides a vast of phenomena, connected to it [6–8].

The first experimental evidence of integer quantum Hall effect (QIHE) was discovered by Klaus von Klitzing in 1980 [2]. He detected appearance of quantized plateaux in Hall resistance traces as a function of the external out-of-plane magnetic field, measured in metal-oxide field effect transistor...
1.1. Overview

Figure 1.2: First experimental evidence of integer quantum Hall effect. Picture is adopted from [2].

(MOSFET) at low temperatures. Simultaneously with plateaux, longitudinal resistance demonstrated the deep minima, approaching zero. The values of those plateaux appear to be a universal combination of fundamental constants, independent of the specific sample:

\[ \rho_{xy} = \frac{V_{xy}}{I_{xx}} = \frac{h}{\nu e^2} , \]

(1.1)

where \( \nu \) is an integer number (Fig. 1.2) and the measurement is performed as shown on Fig. 1.1. This effect is called integer quantum Hall effect (IQHE).
Specifically, the filling factor

\[ \nu = \frac{h n_s}{eB} \]  

(1.2)
determines the electronic ground state of the system with 2D electron density \( n_s \) at a particular magnetic field \( B \). The physical meaning of this parameter is the ratio between areal densities of magnetic flux quanta \((eB/h)\) and conduction electrons \( (n_s)\). Filling factors 0 to 2 correspond to partial filling of the first spin-degenerate Landau level (LL) with orbital quantum number \( N=0 \); from \( \nu = 2 - 4 \) the first \((N=0)\) level is completely filled and the second \((N=1)\) is partially occupied, etc.

These two main features of the quantum Hall effect (existence of the plateaux and their universal values) are the consequences of fundamental quantum properties of 2D systems. The plateaux are usually explained by a single-particle localization on underlying impurities and result from discreteness of LLs: if the filling factor is slightly increased from integer \( \nu \), e.g., by lowering the magnetic field, electrons are promoted to the next LL. However they are localized, as the minima of the underlying impurities potential are populated first. Since localized electrons do not contribute to the electrical transport, Hall resistance stays constant in some range of magnetic fields.

The universal values of the plateaux are the result of a spectrum structure of a 2D system of charged particles. In a perpendicular magnetic field, charge carriers are confined to quantized cyclotron orbits. Each filled LL in a real space appears to be a dense packing of those orbits, representing coupling between a magnetic flux and charge carriers. Thus the charge density at each LL and, therefore, the value of \( \rho_{xy} \) depends on the internal structure of the system.

If the system is a two dimensional electron gas (2DEG), the density of states at spin resolved LL is \( n_L = B/\Phi_0 \), where \( \Phi_0 = h/e \) is the magnetic flux quantum. In this case, classical Hall conductivity of the 2DEG with electron density \( n_s \) could be rewritten in the following form:

\[ \sigma_{xy} = \frac{n_s e}{B} = \frac{n_L e}{B} \frac{n_s}{n_L} = \nu \frac{e^2}{h} \]  

(1.3)

This explains the “magic” value of Hall conductance in (1.1).

Interestingly, the same idea can be extended to a model of any non-interacting quasi particles. For instance, in high magnetic fields, i.e. low filling factors, the ground states of interacting electrons are described by a quantum liquid of Laughlin state at \( \nu = 1/(2s + 1) \) [9] or composite...
fermions at $\nu = p/(2ps + 1)$ [10], where $p, s$ are integers. Composite fermions experience a reduced magnetic field $(eB)^* = eB/(2ps \pm 1)$, and thus fill LLs with a filling factor $\nu^* = n_e h/(eB)^*$ as do electrons in case of the integer quantum Hall effect. These states account for a fractional quantum Hall effect (FQHE), realized experimentally as plateaux in $\rho_{xy}$ and vanishing resistances $\rho_{xx}$ at the appropriate fractional filling factors [7].

Further quality improvement of the samples and lowering the temperatures revealed a large variety of exotic collective states in 2DEG caused by many body interaction of electrons in high perpendicular magnetic fields, ranging from FQH liquids to charge-ordered electron solid states, such as Wigner crystals or nematic stripe phases [3, 11–14]. In some cases electron solid states show similar transport characteristics to their FQH cousins, though the underlying microscopic description of these phases is believed to be very different. When the electron solid is fully formed, $R_{xx}$ vanishes and $R_{xy}$ returns to the nearest integer value of $\nu$, so such states are often referred to as reentrant integer quantum Hall (RIQH) states. These transport characteristics are explained by crystallization of the electrons in the partially filled Landau level: when frozen they no longer contribute to transport.

However, completely different phases develop around the half filling of the uppermost Landau level. Corresponding to Landau level filling factors $\nu = 9/2, 11/2, 13/2$, and so on, these phases are characterized by a huge anisotropy in the longitudinal resistance of the 2DEG which develops rapidly on cooling below about 150 mK [11, 12]. This impressive signature has been widely interpreted as strong evidence for the unidirectional, or striped electron solid phases.

The nature of electron solid states can be understood if the following phenomenological interaction potential is considered [15]:

$$U(r) = \frac{1}{r} - Be^{-kr} + V_d. \quad (1.4)$$

The first term in (1.4) represents Coulomb repulsion, the second one is responsible for attraction, coming from electron screening and the last term represents the impact of random potential. Such kinds of interactions appear in many physical systems and were intensively studied theoretically. Similar to the normal crystal, under certain conditions, it becomes energetically favorable for particles to reorganize into periodic crystal structure or particle domains of higher particle density. Stripe and bubble phases have been shown to occur in such kinds of systems [15]. The onset of the different stripe and bubble phases can be controlled by directly varying the relative
strength of the repulsive and attractive interactions at a fixed density or by holding the interactions fixed while the density of the system is changed.

Figure 1.3: CDW patterns [3]. (a) Stripe pattern. (b) Bubble pattern. (c) WC. One cyclotron orbit is shown.

First numerical calculations showed, that under conditions of QHE electron solids can be formed at $N \geq 2$ by breaking 2DEG in alternating regions of neighboring integer filling factors. Depending on the occupation of last partially filled Landau level, these regions can have different patterns [14]. Close to the integer filling factor (i.e. at low electron density at the last Landau level) an ordinary Wigner crystal is formed. These states usually appear straight after conventional IQH plateaux and result in its widening in experimental measurements. If the electron density is increased (i.e. the filling factor increases under constant magnetic field), it becomes energetically favorable for the system to reorganise in more complex structure with two electrons per site. Although putting two electrons close to each other leads to energy loss, much higher gain is achieved as the lattice period doubled. Therefore, at high filling factors, electrons can form many types of, so called, super Wigner crystals or "bubble" phase with two, three, four, etc. electrons per lattice site. Eventually, around the half filling of the highest Landau level, the bubbles collapse, forming a unidirectional structure of periodic stripes.

Observation of RIQHE at N=1 LL [13, 16-18] caused more accurate calculation of the ground state at the second half filled LL [19, 20] and revealed extremely rich phase diagram of multiple FQH and electron solid states interspersed within a narrow range of magnetic field.
1.2 Motivation

The nature of electron solid states at intermediate filling factors, $2 < \nu < 4$, is much less clear. Here, the $N=1$ Landau level is partially filled and competition between Coulomb and magnetic energies intersperses multiple FQH and RIQH states within a narrow range of magnetic field [19] (Fig. 1.4a). One important puzzle is why four RIQH states appear between $\nu = 2$ and $\nu = 3$, or between $\nu = 3$ and $\nu = 4$, but only two such states appear at each higher filling factor. This might be explained by the stabilization of the $\nu = 7/3$ and $\nu = 8/3$ fractional states in the middle of reentrant states, splitting each in two parts. On the other hand, theory predicts existence of multiple bubble phases with a different number of electrons per bubble. Since all those phases are insulating, all corresponding reentrant states will demonstrate the same integer plateau in experiment. This fact makes different bubble phases indistinguishable in transport measurements. So the question of the internal structure of reentrant states at different Landau levels remains under discussion. Experimental input is even more important,

![Figure 1.4: Characteristic $R_{xy} \equiv dV_{xy}/dI$ and $R_{xx} \equiv dV_{xx}/dI$ traces, measured at base temperature (14 mK) in filling factors $\nu = 2 - 3$ ($I_{ac}=1$ nA, $I_{dc}=0$)](image_url)
as many theoretical tools used to understand RIQH states at high filling factor become less trustworthy below $\nu = 4$ [3, 21].

The microscopic structure of FQH and RIQH states may be explored via their thermodynamic characteristics, for example by measuring how robust they are to destabilization at finite temperature. Numerous gap measurements of FQH states in the literature fall into this category [22], as do measurements of the melting temperatures for electron solids [17, 18, 23]. For example, the melting temperatures for $\nu = 2 - 4$ RIQH states scale with Coulomb energy, indicating that they are stabilized by interactions (as expected for electron crystals) [23]. But large differences in melting temperature below and above $\nu = 4$ may indicate that the crystal structure changes with filling factor [24]. The problem with this approach is that it does not provide a tool for probing a microscopic structure at base temperatures. From a practical point of view RIQH states from $2 < \nu < 4$ are extremely fragile, limited to temperatures below 40 mK and the highest mobility samples [17, 23, 25, 26]. Even small temperature change can destroy low temperature electron crystal phases prior to their separation in the magnetic field.

Another way to destabilize RIQH states is to drive a large current bias through the sample, giving rise to an electric field in the 2DEG plane. Like other charge ordered systems across the field of strongly-correlated electronics, RIQH states are susceptible to destabilization by an electric field, which depins the crystal from an underlying disorder potential or reforms it with an altered long-range order [27–29]. Experimental measurements of the bias-driven sliding dynamics of charge density waves date back to work on NbSe$_3$ nearly four decades ago and remain an area of active research [30–32], as they provide a lot of information about the underlying microscopic structure of the electron system.

Transport signatures of depinning, whether for RIQH states or transition metal oxides, include sharp transitions out of the insulating state for increased bias, and excess resistance noise in the transition region or even narrow-band oscillations [27, 28]. At the same time, the temperature-induced transition out of insulating RIQH states was observed to be far more abrupt that would be expected for thermal activation in a gapped quantum Hall liquid. This led to an understanding that the onset of conduction in RIQH states at moderate temperatures is a collective effect associated with a phase change of the electronic system that is effectively a melting of the electron solid [23].

However, the unusual bias and temperature dependence of RIQH states must not be considered in isolation from each other. All previous interpre-
1.3. Structure of this thesis

Following this introduction, Chapter 2 starts with the theoretical basics of two dimensional physics of interacting electrons. After brief description of integer, fractional and reentrant quantum Hall effect theory, I focus on review of the main theoretical models of microscopic structure of reentrant states and induced anisotropic phases at higher Landau levels. The details of experimental procedures and setup comprise Chapter 3.

In the rest of this thesis I present the measurements results of properties of different electron solid states under non-equilibrium conditions of high bias.

Chapter 4 refers to the spatially-resolved measurements of RIQH breakdown, which indicate that a phase boundary between broken and unbroken regions spreads chirally from source and drain contacts as a function of bias current. As the phase boundary passes various contacts, its spreading generates multi-stage breakdown signatures like those observed elsewhere. The chiral sense of the spreading is not set by the chirality of the edge state itself, instead depending on electron- or hole-like character of the RIQH state.

The temperature measurements of high bias RIQH states breakdown at $\nu = 2 - 4$ are described in Chapter 5. The measurements of reentrant states presented here suggest that the electronic temperature of the bulk stays close to the cold GaAs lattice even when Joule heating is significant, in marked contrast to fractional states, which melt more easily with bias. Although the temperature range where the reentrants states are stable does not allow to accurately estimate electron temperature near the high bias breakdown,
presented thermal analysis show, that the observed effects can not result from a simple melting of the electron crystal by Joule heating. The possible explanation of the observed effects in reentrant states is given in terms of non-uniform break-down, discussed in previous chapter.

Chapter 6 describes experimental results of current induced anisotropy measurements of the stripe phases in the range of filling factors $\nu = 9/2 - 15/2$. The experimental findings are compared to the experiments from other groups. These data suggest an existence of multiple regions of magnetic fields around each half filling of LL with different alignment orientation of the stripe phase. The model of double-well potential is suggested to explain the experimental data, providing a new vision at the mechanism, responsible for alignment of anisotropic phases.

Conclusions and discussion of future possible experiments form Chapter 7.
Chapter 2

Literature review

2.1 Quantum Hall Effect

2.1.1 Low dimensional physics

Two dimensional electron gas is the most versatile playground for probing fundamental interactions in solid state physics. This term is usually referred to the systems where the movement of electrons is confined in one direction to the extent that the energy splitting between the levels of spatial quantization becomes significant. In real samples the confinement is usually achieved at the interfaces of materials with different band gaps, creating a potential barrier for electrons in one direction. The simplest realization of this approach is the interface of semiconductor and insulator. A metal gate on top of the insulator allows to apply the electric field perpendicular to the surface, providing a lot of control on the energy levels of the electrons at the interface.

First MOSFETs opened an access to the experimental probing of low dimensional physics. Further technology improvement of epitaxial growth allowed to create high quality heterostructures with pre-engineered complicated band structure of electrons. The most advanced are the MBE grown heterostructures based on the interface of GaAs/AlGaAs semiconductor. This technique provides a lot of control on the thickness of the thin films grown, as well as, “sandwiching” them in any desired manner. Implementation of the side δ-doping technology in 1975 [33] allowed to achieve extremely low disorder in crystal lattice and the record mobilities of electrons. Discovery of fractional quantum Hall effect in 1986 [7], made GaAs/AlGaAs heterostructures the main playground for probing low dimensional physics.

Consider the simplest case of a single quantum well when a 2DEG is confined between two infinite walls at the distance $a$ from each other. This model is a good approximation of a regular quantum well, since the difference of GaAs and AlGaAs band gaps is much larger compared to the energy splitting of spatially quantized levels. In this case, free motion of particles is limited to two degrees of freedom, while in the third one the momentum
2.1. Quantum Hall Effect

can have only certain discrete values. Similarly to pure 2D case, the density of state at each spatially quantized sub-band is constant and given by

\[ \frac{dn_s}{dE} = \frac{g_s m^*}{\pi \hbar^2}, \] (2.1)

where \( m^* \) and \( g_s \) are the effective mass and spin degeneracy of an electron respectively.

Following conventional notation, we set the \( z \) axis of coordinate system aligned along the confinement direction. The kinetic energy of non-interacting electrons is

\[ E_k(n_z) = \frac{\hbar k_x^2}{2m^*} + \frac{\hbar k_y^2}{2m^*} + E(n_z), \] (2.2)

where \( k_{x,y} \) are the wave vectors in the plane of 2DEG, and

\[ E(n_z) = \frac{(\pi \hbar n_z)^2}{2m^*a^2}. \] (2.3)

is the energy of spatially quantized sub-band \( n_z \).

If this system is cooled to temperatures below the energy splitting of spatial quantization \( \Delta E(n_z) = E(n_z + 1) - E(n_z) \), electrons condense to the lowest energy and occupy states at the first energy sub-band (unless electron density is higher, than the degeneracy of states at this level and few sub-bands are populated). This leads to the situation when electron momentum in the \( z \) direction is fixed and the system has only two degrees of freedom.

Application of the magnetic field \( \mathbf{B} \), perpendicular to the plane of the 2DEG, confines the electrons to cyclotron orbits. If an in-plane electric field \( \mathbf{E} \parallel \mathbf{x} \) is applied on top of the magnetic field, in addition to circular motion at cyclotron frequency \( \omega_c = eB/m^* \), electrons start to drift at speed \( v = E/B \) perpendicular to the electric and magnetic fields. Since average electron velocity has a component perpendicular to the magnetic field, the transport properties of 2DEG are described by the conductivity tensor \( \sigma \):

\[ j_x = \sigma_{xx} E_x + \sigma_{xy} E_y, \]
\[ j_y = \sigma_{yx} E_x + \sigma_{yy} E_y, \] (2.4)

where \( \sigma_{xy} = -\sigma_{yx} \) and \( \sigma_{xx} = \sigma_{yy} \) for an isotropic space.

Using the equation of classical free electron motion in an electromagnetic field in the limit of relaxation time \( \tau \):

\[ m \frac{d\mathbf{v}}{dt} = e\mathbf{v} \times \mathbf{B} + e\mathbf{E} - \frac{m^*\mathbf{v}}{\tau} \] (2.5)
2.1. Quantum Hall Effect

and Ohm’s law

\[ j = n_s e \nu, \]

(2.6)

the components of conductivity can be expressed in the following form:

\[ \sigma_{xx} = \frac{n_s e^2 \tau}{m^*} \frac{1}{1 + \omega_c^2 \tau^2}, \]

\[ \sigma_{xy} = \frac{n_s e^2 \tau}{m^*} \frac{\omega_c \tau}{1 + \omega_c^2 \tau^2}. \]

(2.7)

The quantity \( \mu = \frac{e \tau}{m^*} \) is usually referred to as the mobility of the 2DEG. It is proportional to the electron scattering rate and is the measure of the sample quality. In an ideal 2DEG, \( \tau \to \infty \) and (2.7) takes form:

\[ \sigma_{xx} = 0, \]

\[ \sigma_{xy} = \frac{n_s e}{B}. \]

(2.8)

In the quantum limit, perpendicular magnetic field splits the electron spectrum (2.2) into completely discrete Landau levels:

\[ E(N, n_z) = \hbar \omega_c (N + \frac{1}{2}) + \frac{(\pi \hbar n_z)^2}{2m^*a^2}. \]

(2.9)

Here \( \omega_c = \frac{eB}{m^*} \) is the cyclotron frequency and \( N=0,1,2... \) is the orbital quantum number of the Landau quantization. Degeneracy of each Landau level can be written in the form

\[ \gamma = \frac{S}{2\pi l_B^2}, \]

(2.10)

where \( S \) is the area of the 2DEG and \( l_B = (\frac{\hbar}{eB})^{1/2} \) is the magnetic length. Consider the density of electrons at fully filled Landau level, expressed in terms of magnetic flux quantum \( \Phi_0 = \frac{\hbar}{e} \),

\[ n_L = \frac{\gamma}{S} = (2\pi l_B^2)^{-1} = \frac{B}{\Phi_0}. \]

(2.11)

Equation (2.11) means, that in case of completely filled Landau level, each electron encircles one magnetic flux quantum. Therefore, one can define the filling factor

\[ \nu = \frac{n_s}{n_L} = \frac{n_s \hbar}{eB}, \]

(2.12)

as the number of filled Landau levels at zero temperature.
2.1. Quantum Hall Effect

Besides the orbital quantization, out-of-plane magnetic field removes spin degeneracy of electrons. Zeeman effect splits each Landau level in two spin polarized branches. Consequently, the state of the 2D electron in the magnetic field is set by three quantum numbers: orbital number \( N = 0, 1, \ldots \), spin \( s = \pm 1/2 \) and the number of spatial quantization sub-band \( n_z = 1, 2, \ldots \):

\[
E(N, s, n_z) = \hbar \omega_c (N + \frac{1}{2}) + \frac{(\pi \hbar n_z)^2}{2m^*a^2} + sg\mu_B B.
\]  

Here \( g \) is electron’s g-factor and \( \mu_B = e\hbar/2m_0 \) – Bohr’s magneton. Each Landau level corresponds to the cyclotron orbit of radius \( r_N \approx \sqrt{N}r_B \). Each orbit can be occupied by two electrons with opposite spins, corresponding to spin resolved branches. As the energy of the Zeeman splitting is much smaller than the cyclotron energy, filling factors 0 to 2 correspond to partial filling of the first spin-degenerate Landau levels with orbital quantum number \( N=0 \); from \( \nu = 2 – 4 \) the first (\( N=0 \)) level is completely filled and the second (\( N=1 \)) is partially occupied, etc.

Consider the sample in a fixed perpendicular magnetic field. As was mentioned above, at zero temperature electrons occupy the lowest possible energy levels. Therefore, if the electron density is increased, more and more states are being filled at the last Landau level, increasing the filling factor. Once this level is completely filled, the electrons start to occupy the upper one, etc. Since the occupation of each level is defined by the filling factor, this experiment is identical to magnetic field sweep at the fixed electron density in the sample. Formula (2.11) shows that the density of states at each Landau level depends on the magnetic field. As the magnetic field is lowered, the number of states at each level is decreased and electrons are prompted to occupy the higher energy states.

2.1.2 Integer Quantum Hall Effect

The important consequence of discrete energy levels is that at integer filling factors the finite energy, equal to the Landau level splitting \( \Delta E = \hbar \omega_c \), is needed to add the next electron to the system. In such a state the 2DEG does not screen an external electric field, i.e. demonstrates insulating behaviour. If, for instance, the density is varied by the voltage application on a metal top gate, the number of electrons in 2D channel at integer filling factor will stay constant in the range of voltages \( \Delta V_g = \hbar \omega_c / e \) until next electron can be added. According to (2.8) and (2.11) the conductivity of the 2DEG

\[
\sigma_{xy} = \frac{\nu n v_e}{B} = \nu \frac{e^2}{h}
\]  

is...
2.1. Quantum Hall Effect

will demonstrate the quantized plateau in this range.

In an ideal system, the application of voltage between the metal gate and electron system implies, that the voltage source forces electrons to move from the metal into the 2DEG. This charge difference on the plates creates a voltage between them. If no electron can enter the 2DEG (equivalent to the insulating state), the voltage in the range $\Delta V_g = \hbar \omega_c / e$ can not be applied by the source. However, in real systems the localized states are always present (at uncontrolled impurities, for example). When the intermediate voltage is applied, the extra electric field, not screened by the 2DEG, penetrates through the channel and is screened elsewhere in the sample.

Although this hypothetical experiment leads to the transverse conductivity quantization, it can not be considered as the explanation of IQHE. For example, it implies zero width of a plateaux in the magnetic field sweep ($\sigma_{xy}$ has quantized value exactly at one point, corresponding to integer filling factor), which is not the case in real samples.

Discreetness of electron spectra in perpendicular magnetic field, discussed above, is important, but not enough for explanation of Hall conductivity quantisation. Another important property, required for observation of the plateaux, is the presence of a random potential.

The “magic” conductance value at the plateau, coinciding with classical Hall resistance at completely filled Landau level, is rather a hint than a coincidence. According to (2.14), $\sigma_{xy}$ is defined by the filling factor and is independent on the magnetic field (providing the filling factor is the same). In other words the contribution to net Hall voltage, appearing in completely filled Landau level, is independent on the magnetic field, as both quantities, the transverse conductivity and electron density, depend linearly on $B$. Consequently, once the Landau level is filled, it contributes equally to the net conductivity of the 2DEG, at any value of the magnetic field. If the filling factor variation leads to population of localized states by extra electrons, the net conductance is independent on the total number of the electrons in the system, as it is defined only by the number of underlying filled Landau levels.

Random fluctuations of electron’s potential result from uncontrolled impurities and defects in a crystal structure, which are always present in real samples. The role of random potential is twofold. At first, it broadens Landau levels, which creates finite density of states in the some range of energies between Landau levels. This allows for gradual change of the Fermi level by sweeping the external parameters such as magnetic field or electron density. Secondly, random potential creates a number of localized states in certain range around integer filling factor. The former is important since only de-
localized electrons contribute to electronic transport. Therefore, to achieve a quantized conductance only small fraction of delocalized states has to be occupied \cite{34–36} (Fig. 2.1).

Figure 2.1: Structure of electron energy levels in the presence of random potential

When the filling factor of the 2DEG is changed from $\nu = i$ to $i + 1$ (by changing electron density, for instance), the Fermi level gradually moves from one Landau level to the next one. Underlying random potential leads to spatial variation of local filling factor of the 2DEG. First electrons occupy localized states at the bottom of the upper Landau level. Consequently, the Hall conductance remains unchanged, given by (2.14):

$$\sigma_{xy} = ie^2/h.$$  

(2.15)

Further filling leads to occupation of delocalized states. At this stage all extra electrons change the local filling factor at the parts of the sample, where the electron motion is not finite. Since only the density of delocalized electrons at the Fermi level is important for transport properties, at this point the transverse resistance deviates from quantized plateau. By the time, when all delocalized states are occupied, the density of “transport” electrons equals to $(i + 1)n_L$. Therefore, $\sigma_{xy}$ hits the next level of quantization.
2.1.3 Theory of the edge states

The importance of the edges in QHE was realized soon after its discovery. In [37] IQHE was explained in terms of a supercurrent due to the long-range phase rigidity of the wave functions around the closed loop. Then this picture was supplemented by discussion of the edge states which form at the boundary of the sample [4]. In 2DEG, placed in a strong perpendicular magnetic field, the effect of the edge can be described in terms of increasing energy of Landau levels due to additional confinement near the boundary (Fig. 2.2). In semiclassical limit this can be understood as the quantized energy levels lifting when the edge potential confines electron motion, preventing them from circling over cyclotron orbit.

![Figure 2.2: Structure of electron energy levels in the presence of the edges. After [4].](image)

Generalized theory of the edge states was given in [38]. The absence of backscattering results from the cyclotron motion of electrons: when scattered from the edge their trajectories are curved back by the magnetic field. The skipping orbits of such kind result in continuous unidirectional electron drift within a distance on the scale of the magnetic length from the edge. On
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the other hand, electrons feel electric field \( \frac{dU}{dy} \) due to bending of the bands close to the edge. This field is perpendicular to the boundary of the 2DEG, causing the centers of cyclotron orbits to drift along the edge. This property of the edges eliminates electron backscattering, turning edge channel into a perfect one dimensional conductor.

To describe this picture on the language of quantum mechanics, let’s consider an ideal two-dimensional strip of width \( w \). Let us set the \( Ox \) axis of coordinate system along the strip and \( Oy \) perpendicular to it. The spinless electron Hamiltonian of this system is

\[
H = \frac{1}{2m^*}(\left( p_x - eBy \right)^2 + p_y^2) + V(y),
\]

where we set the vector potential \( A = (-By, 0, 0) \) and edge electric potential \( V(y) \). The wavefunction is separable in the form \( \psi_{j,k} = e^{ikx} f_j(y) \), which leads to the eigenvalue problem:

\[
\left[ -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial y^2} + \frac{m^*}{2} \omega_c^2 (y_0 - y)^2 + V(y) \right] f = Ef, \tag{2.17}
\]

where parameter \( y_0 = -kl_B^2 \). In the bulk \( V(y) \equiv 0 \) and (2.17) turns into the problem of free electron in the magnetic field with quantized Landau levels \( E_{jk} = \hbar \omega_c (j + 1/2) \). Near the edges \( V(y) > 0 \) around \( y = 0 \) and \( y = w \) and Landau levels are dependent on \( y \). Edge effect lifts the degeneracy of the Landau quantization. At zero temperature all states are occupied below the Fermi level and vacant above it. As a result, electrons at the Fermi level are confined in lateral direction to certain distances from the edge, corresponding to the points where Landau level crosses the Fermi energy. This confinement creates \( N \) channels on each side of the sample, corresponding to \( N \) filled Landau levels below the Fermi level in the bulk.

Longitudinal electron velocity can be expressed in terms of \( y_0 \):

\[
v_{jk} = \hbar^{-1} \frac{dE_{jk}}{dk} = \hbar^{-1} \frac{dE_{jk}}{dy_0} \frac{dy_0}{dk}, \tag{2.18}
\]

where \( \frac{dE_{jk}}{dy_0} \) is proportional to the slope of Landau level. This slope has different signs on opposite sides of the sample, representing the chiral motion of edge electrons around the perimeter of the sample.

This natural absence of electron backscattering near sample’s edges in the magnetic field allows to describe quantum Hall system in terms of ideal conductors \cite{39}. Consider the strip connecting two electron reservoirs at chemical potentials \( \mu_1 \) and \( \mu_2 \), which serve as a source and a sink of carriers.
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and energy. Let us calculate the conductance of this conductor, assuming \( \mu_1 > \mu_2 \). Since backscattering is forbidden, reservoir emits carriers into current-carrying states up to its chemical potential. Once the carrier reached another reservoir, it is absorbed. Since below \( \mu_2 \) all the states, moving in both directions, are occupied, the net current is produced only by the electrons in energy interval between \( \mu_2 \) and \( \mu_1 \). The injected current in one channel is

\[
j = ev \frac{dn}{dE} \Delta \mu. \tag{2.19}
\]

Employing (2.18) and the density of states for 1D system \( dn/dk = 1/2\pi \), (2.19) turns into

\[
j = \frac{e}{\hbar} \frac{dE}{dk} \frac{dn}{dE} \Delta \mu = \frac{e}{\hbar} \Delta \mu. \tag{2.20}
\]

Thus, the current fed into an edge state by a reservoir is the same as the current fed into a quantum channel in a zero-field perfect conductor. The resulting two-terminal resistance for a single edge state:

\[
R = \frac{V}{I} = \frac{\Delta \mu}{e} \frac{h}{N e \Delta \mu} = \frac{h}{e^2}. \tag{2.21}
\]

Consequently, in ideal sample every Landau level below the Fermi energy creates a pair of edge channels, one on each side of the sample, which acts as a resistor with quantized conductance. At integer filling factor \( \nu = N \), every pair acts as a parallel resistor, i.e. the total 2 point resistance of the sample is

\[
R = \frac{h}{e^2 N}. \tag{2.22}
\]

This result has important consequences. First, it provides alternative explanation for the conductance quantization in quantum Hall regime: every Landau level below the Fermi level creates ideal one dimensional channel. Secondly, in order to change the direction of motion, electron has to move over macroscopic distance through the bulk to the opposite edge. Therefore, similar to an ideal 1D wire, the potential is constant along the edge channel, and one would measure vanishing longitudinal resistance between any contacts at one side of the sample.

It is important to mention, that described effect does not depend on the edge shape: the edge state is naturally formed everywhere, where the Landau level crosses the Fermi energy. This means, that (2.22) is independent on impurities at the edge: addition of an extra impurity changes only the shape of the edge state. This provides an explanation of the universality of the Hall effect: its total independence on the shape and size of the sample [40],
only topology of the measurement contacts with respect to the source/drain ones matters.

Interaction effects are also important in real samples. The detailed theory was developed in [41, 42] and led to introduction of compressible and incompressible stripes concept: competition between Coulomb interaction and electrostatic energy of electron in the effective edge electric field leads to 2DEG density redistribution. This becomes possible even at zero temperature due to a large number of unoccupied states in the vicinity of Landau and Fermi level crossing. Thus the sample breaks into compressible and incompressible strips, parallel to the edge. Although, the microscopic details of such systems are very different from the simple Landauer picture, described above, the accurate calculation shows, that (2.22) stays correct even in the interacting electrons case.

2.1.4 Non uniform samples

Previously, only samples with uniform density where considered. Here the effects of sample inhomogeneities are considered. Specifically, this paragraph is focused on macroscopic variations of a random potential, leading to different electron densities at different parts of the sample. For the purpose of this thesis, sharp density changes are considered, i.e. it is assumed that there is infinitely narrow finite density jump at the boundary between two uniform regions with different electron density. Since number of the edge states in the sample varies with the filling factor, some of them must be reflected at the boundary. The sharpness of the density jump allows to avoid the discussion of the question, how the edge states behave inside the boundary under the density gradients and simplify this problem to several independent edge channels, some of which are reflected by the density jump, while such boundary is transparent for the others. In a real world such conditions can be achieved by the voltage, applied to the macroscopic gate, covering part of the sample. Since in real structures the proximity effects at the edges are comparable to the depth of 2DEG, this assumption is feasible.

Scattering of the edge states has been a topic of intensive research for a long time [43]. For example, it was shown, that in a system with a narrow constriction, the electrical potential varies along the edge, and, longitudinal voltage deviates from zero. Therefore, diagonal measurements could probe different values at different contacts.

Here this theory is applied for the case of one reflected edge state at each side of the sample, modeling the sample with non-uniform density. Obtained results are important for interpretation of experimental findings
2.1. Quantum Hall Effect

Figure 2.3: Propagation of the edge states in inhomogeneous sample with two filling factors $\nu = N$ and $\nu = N + 1$. The outer thick arrow depicts first $N$ co-propagating edge states around the whole sample.

in the following chapters. The schematics of the considered system is shown on figure\[\text{2.3}\]. Source and drain ohmics are surrounded by the region of higher density and, consequently, have different number of propagating edge states in the “melted” (grey areas at Fig.[2.3]) and “frozen” regions (the meaning of such notation will be explained later). Assume, that the “frozen” bulk is at integer filling and, therefore, $N$ completely filled Landau levels create $N$ propagating edge states. In “melted” regions the density is higher, but the filling factor is still integer ($\nu = N + 1$). This adds one extra edge state at each “melted” area.

According to [38], the current flowing through the ohmic is given by the chemical potential and the number of incoming and outgoing edge states:

$$I_e = N_{out} \mu_{out} - N_{in} \mu_{in}. \quad (2.23)$$

Applying (2.23) to source, drain and four corner contacts, shown on Fig. [2.3].
2.1. Quantum Hall Effect

one gets the following system of equations:

\[ \begin{align*}
I e &= (N + 1)\mu_H - (N + 1)\mu_L, \\
I e &= (N + 1)\mu_H - (N + 1)\mu_L, \\
0 &= N\mu_H - N\mu_H^+, \\
0 &= N\mu_H^+ + \mu_L - (N + 1)\mu_H^-, \\
0 &= N\mu_L - N\mu_L^+, \\
0 &= N\mu_L^+ + \mu_H - (N + 1)\mu_L^-.
\end{align*} \tag{2.24} \]

Lets define the drain chemical potential \( \mu_L = 0 \), then \( \mu_H = V \). Then, (2.24) takes form:

\[ \begin{align*}
I e &= (N + 1)\left( V - \frac{V}{N + 1} \right) = NV, \\
\mu_H^- &= \frac{N\mu_H}{N + 1} = \frac{N}{N + 1}V, \\
\mu_L^- &= \frac{\mu_H}{N + 1} = \frac{V}{N + 1}, \\
\mu_H^+ &= \mu_H = V, \\
\mu_L^+ &= \mu_L = 0.
\end{align*} \tag{2.25} \]

If one defines diagonal voltages \( V^+ \) and \( V^- \) with as

\[ \begin{align*}
V^+ &= \mu_H^+ - \mu_L^+, \\
V^- &= \mu_H^- - \mu_L^-.
\end{align*} \tag{2.26} \]

and solve (2.26), the diagonal resistances could be defined as

\[ \begin{align*}
R^+ &= \frac{V^+}{I} = \frac{h}{e^2} \frac{1}{N}, \\
R^- &= \frac{V^-}{I} = \frac{h}{e^2} \frac{N - 1}{N(N + 1)},
\end{align*} \tag{2.27} \]

or, expressed with the resistance of the melted state:

\[ \begin{align*}
R^+ &= \frac{h}{e^2} \frac{1}{N + 1} \left( 1 + \frac{1}{N} \right), \\
R^- &= \frac{h}{e^2} \frac{1}{N + 1} \left( 1 - \frac{1}{N} \right).
\end{align*} \tag{2.28} \]
2.2. Coulomb interaction effects

The physical meaning of (2.28) becomes clear after it is expressed in terms of longitudinal resistance

\[ R_{xx} = \frac{\mu_H^+ - \mu_H^-}{I} = \frac{h}{e^2} \frac{1}{N + 1} \]  

(2.29)

and Hall resistance of the melted regions, adjacent to the source and drain contacts \( R_H = \frac{h}{e^2} \frac{1}{N+1} \):

\[ R^+ = R_H + R_{xx}, \]
\[ R^- = R_H - R_{xx}. \]  

(2.30)

In other words the diagonal voltages include not only the potential difference between the edges of the sample, but the potential drop along the edge.

2.2 Coulomb interaction effects

So far only the systems of non-interacting electrons were considered. Although single particle approximation can provide theoretical explanation for IQHE, it suggests only plateaux of \( \sigma_{xy} \) appearing at the integer multiples of \( e^2/h \). However, it was a great surprise, when the quantization of transverse conductivity was found at \( e^2/3h \) and \( 2e^2/3h \) [7, 44]. Another mystery was the width of the integer plateaux, which become wider in higher mobility samples (\( \mu > 10^6 \text{cm}^2/\text{V} \cdot \text{s} \)). These experiments could not be explained only with the impurity effects in non-interacting 2DEG. Consequently, it is evident, that under certain conditions, the Coulomb interaction becomes more important than random variation of underlying electron potential.

First theoretical investigations demonstrated, that Coulomb interaction can cause stabilization of a large variety of qualitatively different states, revealing rich field of many-body physics. From mathematical point of view, the fundamental difference from the non-interacting picture, is that the variables of an interacting Hamiltonian can not be split. Therefore, the wave function of the system can not be described as a product of single electron wave functions. In order to find the ground state of 2D electron system, one has to solve the differential equation in \( 4N+1 \) dimensional space, where \( N \) is the number of electrons in a system. Different numerical techniques, such as Hartree-Fock approximation, are applied to address this problem, but in many cases the accuracy of those calculations is questioned, since its uncertainty often is comparable to the difference between energy levels of the system. Another problem was that neither of ground states, found by those calculations, could explain quantization of the transverse conductivity.
at fractional filling factors (what is now called FQHE). Therefore, this effect has to be explained by a fundamentally different ground state.

The purpose of this section is to provide a brief theoretical review of two main families of the interacting electron ground states in the quantum Hall regime and to give a qualitative explanation of their microscopic structures. As will be shown later, stabilization of each state is defined by the temperature and interaction energy. Similar to the atoms, depending on the ratio between temperature and interaction, the electrons can form a liquid – a structure with no long range order, and electron crystals (completely periodic phases, sometimes referred to as electron solids). Both of those types of states are interesting in the scope of this thesis, as they lead to quantization of Hall conductivity in some range of filling factors, however, the underlying physical mechanisms of these effects are completely different.

### 2.2.1 Fractional Quantum Hall Effect

Following the chronological order of realising the microscopic structure of many-body states in quantum Hall regime, first the experimental signatures of fractional Quantum Hall Effect (FQHE) are described. The detailed microscopic theory of fractional states is beyond the scope of this manuscript. Instead, a very general qualitative picture is provided, allowing to build a conceptual understanding of the formation mechanism of FQHE plateaux.

Since experimental evidence of FQHE appeared before the theoretical explanation, we first describe the basic experimental facts, which such a theory has to explain:

1. This effect takes place only at certain densities, defined by the strength of out-of-plane magnetic field.
2. Experimental manifestation of FQHE is very similar to IQHE, besides the plateaux correspond to a fractional level of quantization, i.e. $(1/3)e^2/h$
3. The effect appears only in high quality samples. It is very fragile and is affected by even small amount of impurities
4. Fractional quantization is observed only at low temperatures (below 1 K) and high magnetic fields
5. In addition to the simple $1/3$ and $2/3$ large family of quantized plateaux, corresponding to other fractions with odd denominators, such as $2/5$,
2.2. Coulomb interaction effects

2/7,..., has been observed. Those states are even more fragile and temperature sensitive.

The similarity between fractional and integer quantum Hall effects, hints on the similar underlying microscopic mechanisms (otherwise, one would need to find a completely different theory to explain quantized plateaux). It was shown, that Laughlin’s wave function of $\nu = 1/3$ fractional state $[9]$ 

$$\psi = \{ \prod_{j<k} f(z_j - z_k)^m \} \exp\left( -\frac{1}{4} \sum_l |z_l|^2 \right),$$  \hspace{1cm} (2.31)

where $z_j = x_j + iy_j$, was a good approximation to the ground state of a Coulomb interacting system. Therefore, its properties could be understood from those of Laughlin’s ground state. (2.31) describes a bound state with a number of zeros, corresponding to the number of flux quanta in the system. The fermionic nature of this system requires (2.31) to be an antisymmetric function, i.e. $m$ must be an odd integer. This means, that the ratio between the number of particles and magnetic flux (in units of the flux quantum), being an inverse of the filling factor, is an odd integer as well.

However, Laughlin’s wave function explains the ground state only at the filling factor, which is exactly $1/q$. Consequently, (2.31) itself can not account for an existence of finite width plateaux. In order to build a theory of fractional states, analogous to IQHE, one has to construct the system of non-interacting particles, localized on a random potential around the fractional filling factor $[45]$. When the area of the system is slightly changed, the number of flux quanta changes, and, therefore, Laughlin’s ground state has to be modified. It could be shown $[46]$, that the wave function changes by means of quasiparticles, which carry fractional charge $e^* = e/q$. At sufficiently low densities of quasiparticles, i.e. sufficiently close to $\nu = 1/q$, the interaction between particles becomes low and all the correlated states are destroyed by the potential fluctuations. In this case random potential localizes quasiparticles and their contribution to transport properties of the sample vanishes. This results in the quantized plateau around $\nu = 1/q$. Since the system is particle-hole symmetric, similar speculations could be applied to the $\nu = 1 - 1/q$ fractional states.

This approach can be generalized for the case of other fractions, such as $n/(2kn + 1)$. In $[47]$ the concept of composite fermions (CF) was introduced. This approach allows to combine weakly interacting quasiparticles by attaching to each electron $2k$ flux quanta in the direction opposite to the external magnetic field. As a result, every CF effectively feels smaller magnetic flux. For the Laughlin state $\nu = 1/q$, the residual flux $q - 2k = 1$, as
2.2. Coulomb interaction effects

$q$ is an odd number. In other words, formally $\nu = 1/q$ state corresponds to IQHE of CFs at $\nu^* = 1$. A quantized plateaux corresponds to every integer filling factor $n$ of CF with $1/n$ flux quanta per particle, i.e.

$$q - 2k = \frac{1}{n},$$

which gives the equation for FQHE plateaux sequence:

$$\nu = \frac{1}{q} = \frac{1}{1/n + 2k} = \frac{n}{2kn + 1},$$

where $k,n$ are positive integers.

Similarly, CFs could help to understand recently discovered FQHE at even denominators [48]. A two-dimensional electron system in an external magnetic field, with Landau-level filling factor $\nu = 1/2$, can be transformed to a system of CFs in zero effective magnetic field [49]. Nonetheless, the microscopic descriptions of some other even-denominator states are still under debate [50] and vary from exotic particles with non-Abelian statistics [51] to Bose-Einstein condensation of two-electron pairs with opposite spins [52]. Recently, the requirement of complete spin polarization in the ground state was also reconsidered. It was found that a partially filled lowest LL may contain Skyrmions [53]. But independently on the microscopic structure of the ground state, the general approach is always the same: an interacting electron system is described by means of complex weakly interacting quasiparticles in the vicinity of the fractional filling factor. If those new particles do not contribute to net current (localized on random potential, for example), the Hall resistance resides on the plateau, corresponding to the classical Hall value at the fractional filling factor.

2.2.2 Reentrant Quantum Hall States

The primary parameter, defining the state of interacting system, is the ratio between carrier potential and kinetic energies. In 2DEG the strength of interaction is measured by $r_s$, the ratio of Coulomb energy

$$E_C \sim \frac{e^2}{\epsilon a} \sim \sqrt{n_s}$$

(2.34)

to Fermi energy

$$E_F = \frac{\pi \hbar^2 n_s}{m^*} \sim n_s,$$

(2.35)
2.2. Coulomb interaction effects

Therefore,

\[ r_s \sim \frac{1}{\sqrt{n_s}} \]  \hspace{1cm} (2.36)

increases, when the carrier density is decreased.

Similar to atoms in a crystal lattice, the ground state of a 2DEG in a strong interaction limit (high \( r_s \)) in the absence of the disorder is expected to be a Wigner solid [54]. In such ideal system the critical density, above which the crystal melts even at \( T=0 \), was estimated to correspond to \( r_s \sim 37 \) [55]. However, in real systems disorder competes with Coulomb repulsion, resulting in even lower critical densities. Although the current level of disorder in the best quality samples does not allow to reach the critical values of \( r_s \), it is possible to create carrier densities where deviation of a strongly correlated system from Fermi liquid can be observed even at \( B = 0 \) [56, 57].

Under conditions of QHE, the perpendicular magnetic field quenches the kinetic energy of electrons to cyclotron motion, leading to higher values of \( r_s \) for the same densities. An electron Wigner crystal was predicted to be a fundamental state even in realistic samples in the limit of high magnetic fields [58]. Moreover, the main parameter in this regime, defining the ground state of the system, is not the electron density, but the filling factor. In real samples electron solids are believed to be pinned to the underlying disorder [59], producing an insulating state of the sample. Experimentally this state is very different from localization of free electrons or low interacting quasiparticles on random potential for the case of IQHE and FQHE. In contrast to quantum Hall states, where free carriers are delocalized near the edges, resulting in non-dissipative transport \( R_{xx} \to 0 \) as \( T \to 0 \), the resistivity of pinned electron solid is expected to demonstrate pure insulating behavior with diverging \( R_{xx} \to \infty \) at \( T \to 0 \).

Lots of attention has been drawn to reentrant insulating phases (RIP) in the chase of 2D electron Wigner crystal in high perpendicular magnetic fields. Experimentally RIPs were observed at the edges of fractional [60–62] and integer plateaux [63]. However, the role of the competition between random potential and electron-electron interaction, defining electrons ground state, is still under debate [64]. RIP’s transport characteristics [65, 66] are very similar to charge ordered structures in other systems and considered to be an experimental demonstration of the weak WC pinning [67, 68].

If the cyclotron motion is preserved and LL mixing is small, underlying, completely occupied LLs affect only the background screening of the system at high filling factors and can be excluded from the problem. Consequently, the ground state of such a system is defined only by few electrons left at the last partially filled LL. Namely, the partial filling factor of the last Landau
level $\nu^* = \nu - \lfloor \nu \rfloor$ at $\nu > 1$, plays the same role as real filling factor for the case of $\nu < 1$. This means, that the system behaviour in higher filling factors should be approximately periodic with $\nu$. For example, in the vicinity of integer fillings, effective charge carrier density in the problem is low and the system becomes highly interactive, creating conditions for electron solid formation, similar to WC in high magnetic fields. Consequently, the system in the vicinity of integer plateaux could turn into insulating state beyond the single electron localization on random potential. This effect is believed to be responsible for widening of IQH plateaux, observed in ultra-high mobility samples.

After discovery of RIQHE at the Landau levels with high ($N > 2$) orbital numbers [27], the theory of electron solids was reconsidered. First experiments demonstrated, that reentrant states (RS) and integer plateaux at elevated temperatures are interspersed by the free electron states. Furthermore, temperature measurements showed, that RSs collapse around partial filling factors $\nu^* = 1/4$ and $\nu^* = 3/4$, which can not be explained by low density Wigner crystallization. However, reenterance of Hall resistance to the values of integer quantization hinted on the pinning mechanism of electron localization at the last LL. Therefore, the electron crystallization theory was expanded to charge density waves (CDW) [3, 14]. The basic concept of CDW lies in the idea, that cyclotron motion alters electron-electron repulsive interaction and the features of a real single particle potential could at certain electron densities make energetically favourable to double the period of WC lattice for the price of putting two electrons per site [3]. Such approach extends a conventional single electron WC state to the sequence of the ordered electron structures with $n = 1, 2, \ldots$ electrons per site.

Qualitatively this result could be understood from analysis of a simple 1D toy model, adopted from [14]. Consider an electron gas, interacting in a boxlike potential

$$u(x) = u_0 \Theta(2R - |x|),$$

(2.37)

and compensated by a surrounding average uniform background potential. Let’s also assume, that the number of states “inside” each box is limited. In such a system the lowest energy state corresponds to the equidistant positions of boxes, minimizing repulsion between neighbours. Let’s now consider how the energy of this gas depends on the number of the particles per site. It costs zero energy for the particle to get “inside” the box, however, the repulsion energy decreases when the spacing between sites is increased. Therefore, such model system tends to maximize the lattice period and fill each box with the maximum possible number of electrons.
Turning back to the case of a reentrant quantum Hall system, the electrons appear to be broken in clusters with equal number of electrons, which play the role of the “box”. The number of electrons in the cluster is limited by the the local Landau level capacity. Thus, the $\nu^* = 1/4$ RS would correspond to Wigner-like lattice with electrons grouped in the clusters at maximum local density, corresponding to $\nu^* = 1$. For the particle-hole symmetric case at $\nu^* = 3/4$ the “bubbles” turn into $\nu^* = 0$ puddles in the sea of $\nu^* = 1$. Random potential in the sample, disturbs an ideal order in this structure, leading to formation of electron clusters or “bubbles”, as they were historically referred to. Consequently such microscopic structures are usually called the “bubble phases”.

Of course, electrons in 2DEG experience short range repulsive interaction. Thus, adding an extra electron to the bubble increases the energy of the system. However, in contrast to the case of vanishing magnetic field, where the repulsion between two electrons increases as $1/r$, the overlap of two electrons, orbiting around the cyclotron motion does not strongly depend on the distance between them, when the centers of the orbits are closer, than $2r_c$. Therefore, the model of the “box”-like potential physically results from electron cyclotron motion.

In order to explain RIQHE, one needs not only to construct a fundamental electron solid state away from integer filling factor, but also demonstrate the mechanism of its localization. Similar to ordinary Wigner crystal, the question of the CDW pinning was investigated theoretically [69, 70]. Fukuyama et.al. demonstrated, that competition between the random impurity potential and the elastic energy for phase fluctuations leads results in two limiting cases. In the strong pinning case the phase is pinned to each impurity site. However, in the weak pinning case the pinning mechanism is more subtle. The system is pinned by effectively breaking up into domains, suggesting a mechanism from intrinsic inhomogeneity of the sample.

### 2.2.3 Anisotropic states around LL half filling

Completely new anisotropic states were discovered next to the reentrant states [11, 12] around half filling of the Landau level. In contrast to all other experimental manifestations of quantum Hall effect, the bulk of the sample is not insulating in these states and, thus, transverse resistance does not quantize. Additionally, the longitudinal resistance experiences high anisotropy with respect to crystallographic axis of the sample.

Although the behavior of these new states was very different, the main candidate for their microscopic description is a correlated electron solid.
2.2. Coulomb interaction effects

Firstly, temperature measurements show sharp melting of such states with distinct onset temperature for the anisotropy. Above this critical point, the anisotropy vanishes and longitudinal resistance corresponds to conventional IQHE between integer plateaux. Secondly, the IV characteristic of those states are extremely nonlinear [12].

From a theoretical point of view, the unidirectional CDW states, which break rotational symmetry of the system, can be formed around half fillings of the Landau level. They can be considered as a limit of the isotropic bubble phase, which is collapsed into the periodic alternating stripes with residual filling factors $\nu^* = 1$ and $\nu^* = 0$. An edge state has to be formed at the boundary of each stripe due to the density jump, creating a natural set of Hall bars with low resistance along the edge and exponentially high resistance across the boundary. Therefore, along the stripe direction, one can consider this system, as a set of Hall bars, connected parallel to each other. Therefore, the resistance along this direction

$$R_{xx} = \frac{r_{xx}}{N_S} \to 0,$$

where $N_S \sim w/r_c$ is the number of stripes, formed at the edge of size $w$, and $r_{xx}$ is the longitudinal resistance of one Hall bar at current filling factor. On the opposite, for the direction, perpendicular to the stripe ordering, the stripes can be considered as a chain of Hall bars connected in series. In this case:

$$R_{xx} = r_{xx} N_S \to \infty.$$  

Despite an easy general explanation, the theory of the electron stripe phase at half filling of higher LLs appeared to be complicated in details. First intriguing question was: how do the stripes choose the orientation direction? From the first site, in isotropic sample the orientation of the stripe phase has to be spontaneously chosen, therefore, one would observe random anisotropy orientation not only in different samples, but in different cooldowns as well. However, in early experiments the stripe phase was always found to be oriented along one crystallographic direction. The result was confirmed by many experimental groups for different LLs in many samples and many multiple cooldowns. This observation led to a lot of research, trying to identify the rotational symmetry breaking parameter. Experimentalists tried to connect that to different factors, varying from tiny built-in potential, resulting from crystal imperfections due to the growth anisotropy of the sample [71], to the form factor of the cleaved sample.

It was found, that the stripe phase can be reoriented by many effects such as external parallel magnetic field [72, 73], mechanical deformation [74].
or density effects \cite{75}. However, the underlying microscopic mechanism of stripe alignment is still a topic of significant debates. Recent experiments revealed, that the orientation of stripes at the same LL can be different, depending on the specific filling factor, providing an evidence of complex microscopic structure of every macroscopic anisotropic state \cite{76}.

Another puzzle of the electron solids in general and stripe phases in particular is associated with the melting temperature. According to the theory, stripes at higher LLs melt at temperatures on the order of 1 K \cite{3,14}, which overestimates the real experimental values by about an order of magnitude \cite{11,12}. In order to deal with this problem, the effects of random potential were considered theoretically. It was shown, that potential variation disturbs CDW and lowers cohesive energy, but the amount of this effect is still an open question. Alternatively, electron phases without long range order, called electron liquid crystals (ELC), were considered \cite{77}. It was shown, that the depending on interaction strength and amount of disorder, introduced into the system, CDW can experience several phase transitions, first melting into the glassy state without long range order, and only after that, ELC melts into isotropic state, gaining the rotational symmetry \cite{78}.

Unfortunately, experimental investigation of these states is way behind the theory. Extreme fragility of electron solids limits the affordable experimental techniques and makes accessing the microscopic structure of such state a very challenging experimental task. In addition to transport anisotropy measurements as a response to an external disturbance by some parameters, a very few experiments, trying to directly measure microscopic features of the electron solids, have been performed. For instance, the high frequency resonance of stripe and bubble phases where investigated. Tsui et.al observed disappearance of cyclotron resonance, approaching integer quantum Hall plateaux, as well as in reentrant states at high Landau level and reentrant insulating states at the boundaries of fractional plateaux in high magnetic field limit. In \cite{79} Kukushkin et.al. used photoacoustic excitation of the stripe phases at $\nu = 9/2$ in order to measure the period of the stripes. Although they got a period of stripes around $2.9r_C$, which is pretty close to the theoretical estimate of $2.7r_C$, calculated by Koulakov et.al in \cite{3}, these techniques do not allow to distinguish different types of microscopic phases.

2.2.4 Correlated states at N=1 LL

In previous paragraphs two main types of interacting electron states were considered. They correspond to two different limits of magnetic field, i.e.
the limit of high and low magnetic confinement. In the former, the strong interaction is given by a large reduction of kinetic energy in high magnetic field. While in the latter case, the effective electron density is significantly increased due to the “condensation” of the majority of electrons under the cyclotron gap in completely filled Landau levels.

Filling factors $\nu = 2 - 4$, corresponding to the orbital number $N = 1$, represent the “grey” area, where $r_S$ is small and electron interaction could not be considered as the main part of electrical potential. On the other hand, it is not small enough to allow one to approximate this system with free electrons (this case corresponds to $r_S \ll 1$). Although, CDWs, which are believed to be responsible for the reentrant states at higher filling factors, were not predicted to exist at $N = 1$ LL [80], not only is RIQHE observed experimentally in the $\nu = 2 - 4$ range, but there exists four instead of conventional two reentrant states for each region between integer plateaux [16, 17]. This suggests a strong competition between many different electronic states, making the phase diagram of the system extremely complicated [81].

Electron ground state phase diagram was assessed by several different numerical methods [19–21]. In spite of theoretical predictions of different CDW phases with one and two electrons per bubble at the opposite sides of the fractional plateaux, no direct experimental observation of microscopic electronic structure under conditions of RIQHE was demonstrated so far. On the other hand, there is a growing experimental evidence, that particle-hole symmetry do not hold for the case of RIQHE at the second Landau level [23, 82], which creates even more doubts about the theoretical results. The widely used Hartree-Fock approach, for example, has many limitations. This technique is exact only in the limit of high Landau level occupation, which is not very accurate over the whole range of filling factors, where the RIQHE is observed. Secondly, the competition with the FQHE enhances fluctuations which might be critical for electron ordering. Eventually, this approximation does not take into account LL mixing, which was shown to significantly affect the energy gaps of FQH states [83]. Thus, despite more than a decade of intensive research, the microscopic structure of 2D electrons at the second Landau level is still far from being understood.
Chapter 3

Experimental setup

In this section technical details of the experimental setup are given. Although it was based on a commercial dilution refrigerator, the system has been modified in several ways in order to achieve lower electron temperatures and maximal sample quality. A detailed description of the heterostructures and sample preparation recipes is followed by the a description of the measurement scheme.

3.1 Sample details

Most measurements were performed on a square sample with Van-der-Pauw measurement geometry. Electrical contact to the 2DEG was achieved by diffusing indium contacts placed at the corners and midpoints of the \( \approx 5 \times 5 \) mm wafer. In few samples, however, a Ni/Ge/Au recipe was used to provide electrical contact with the 2DEG. This technique was typically used for lithographically defined smaller devices. In this case few more steps of electron lithography were made. The first one defined the etch trenches, which limit the mesa of the device. At the second step \( \approx 100 \times 100 \mu m \) square windows for contacts were defined. Subsequent evaporation and lift-off resulted in deposition of Ni/Ge/Au layers in appropriate places, which diffused inside the 2DEG during annealing at 430° C and formed ohmic contacts.

3.1.1 Heterostructure design

The strength of the observed RIQHE states strongly depends on the heterostructure design of the wafer, of which the sample is made. Molecular beam epitaxy (MBE) is the primary technique for growth of the state of the art heterostructures; however, it takes a lot of technical ingenuity to achieve the record mobility and strength of RIQHE.

Historically, electron mobility was the main figure of merit for the quality of the 2DEG. In addition to the trend in the strength of FQH states in the first Landau level, measured in low mobility samples at the first stages of FQHE research, this belief was backed up by the simple reasoning, that
3.1. Sample details

mobility is limited by internal disorder, which prevents stabilization of cor-
related states. Although, the growing evidence of poor correlation between
the mobility and quality of FQH states (size of the 5/2 gap, for instance)
has been obtained in the last decade [84], electron mobility on a scale of
$10^7 \text{cm}^2/\text{Vs}$ is still a necessary, though insufficient, condition for observation
of RIQHE.

The key factor, defining the mobility of the sample, i.e. the random po-
tential, experienced by electrons, is the intrinsic disorder incorporated into
the sample’s crystal structure during MBE growth. In first place it is defined
by the number of uncontrolled impurities, which is directly connected to the
purity of the chamber and vacuum level during the growing process [5]. An-
other significant source of electron scattering centers and disorder comes
from the ionized doping centers, which lead to direct variation of the elec-
tron’s potential inside the channel. The quantity of dopants, defining the
density of 2D electrons, can not be decreased, but their effect can be sig-
ificantly lessened. Many improvements in this scope can be achieved by
adjusting the design of the heterostructure.

Although, the simplest heterostructure design to produce a high-mobility
2DEG is a single heterojunction at the interface of GaAs/Al$_{0.35}$Ga$_{0.65}$As,
the highest mobility 2DEGs grown today are all quantum well structures,
realized as a thin layer of GaAs between two AlGaAs barriers. An obvious
step to lower the amount of disorder, introduced by the dopants - is to move
them away from the channel. Introduction of structures with a $\delta$-doping
silicon layer result in a significant increase in mobility and is a common
practice nowadays. In the case of quantum well, the doping layers can be
placed on both sides of the electron channel, significantly increasing 2D
electron density. In reality, the highest mobility heterostructures, which are
also the ones typically used to study fragile quantum Hall states in the second
Landau level, are more complicated than what has just been described and
involve one of several variations of short-period superlattice doping [85].

Measurements were performed on a 300 Å symmetrically doped GaAs/AlGaAs
quantum wells with low temperature electron density $n_\text{s} \sim 3 \times 10^{11} \text{cm}^{-2}$
and mobility above $15 \times 10^6 \text{cm}^2/\text{Vs}$. The typical conduction band struc-
ture of our samples is shown in figure 3.1a. Silicon doping is incorporated
into the AlAs barriers on both sides of the quantum well (Fig. 3.1b). Al-
though AlAs acts as a potential barrier in the main $\Gamma$ valley of GaAs, the
features of the structures of other bands generate a significant amount of
charge to move from Si atoms into the AlAs layer. This has an important
consequence: the charge in these channels, although it does not appear as
a parallel channel in low bias conductance, can move under illumination at
3.1. Sample details

Figure 3.1: Conduction band edge and free charge density in the immediate vicinity of the doping well located 75 nm below the edge of the primary 30-nm GaAs quantum well. On the inset: the general structure of the conduction band edge and charge density profile for a whole quantum well. Adopted from [5].

intermediate temperatures and low fields, screening the potential of ionized donors from the 2DEG. This effect reveals another important step in sample preparation: illumination and cooldown protocol.

3.1.2 Cooldown protocol

As was noted above, in order to prepare a high quality 2DEG with strong reentrant states, it is important to freeze the charge in AlAs side channels in such a way that it provides the best screening of the electric field from ionized dopants. First this requires illumination of the sample at a temperature range below 1 K. This results in deionization of the majority of Si donors. At this point, electrons start to tunnel from shallow donor levels into the quantum well, surface states and localized states at the AlAs layer.
3.1. Sample details

Additionally, the elevated electron temperature provides the mechanism for charge equilibration by means of thermally activated conductivity in AlAs layers. On the other hand, thermal fluctuations prevent the charges from staying at equilibrium distribution. Thus the sample has to be warmed to a high enough temperatures to enable charges to completely redistribute and equilibrate donor potential, as well as, recover the electron population of the quantum well before the system is frozen. However, warming to very high temperatures makes the system sensitive to external electric noise and charge fluctuations. Therefore, the annealing procedure must be undergone under strict temperature control, and thorough parameter optimization is required.

Illumination of the sample was usually performed by a red LED, which shone on the sample during the natural cooldown of the system from nitrogen to base temperatures. In addition to the lack of control over light power density, this design has the drawback of uncontrolled sample temperature due to Joule heat dumped into the LED. To overcome this problem a multimode optical fibre was used to shine light onto the sample, coming from a red LED located outside the refrigerator. With a power of 140 mW applied to the LED (80 mA at 1.7 V), $6 \pm 2 \mu W$ of optical power reached the bottom of the fibre, as estimated from the 15 mK of the mixing chamber temperature rise from base temperature during illumination. Geometrical considerations suggest an optical intensity of $120 \pm 50 \text{nW/mm}^2$ at the sample location. Under continuous illumination, the sample temperature, monitored by local thermometer, was approximately 600 mK.

A detailed investigation of the effects of low temperature illumination and annealing on FQH characteristics of a GaAs/AlGaAs quantum well sample is presented in [1]. For the experiments, presented in this thesis, the following optimized protocol was used for sample preparation. From room temperature the sample was cooled down to roughly 30 mK and then illuminated for 30 minutes. Next, the light was turned off and, using a local heater the sample was annealed at a temperature between 2.2 K for 16 minutes. After annealing the sample was left in the dark with all contacts grounded to cool down to the system’s base temperature.

This approach has several advantages. Firstly it enables the separation of shining and heating stages, providing direct control over the ionization process. From experimental point of view, the external light source is stable, and the sample temperature is independent and has a controlled light power and wavelength. Secondly, annealing after the illumination has been turned off, eliminates extra uncontrolled sample heating due to the light absorbed in the sample. This allows for precise sample temperature control by a local
3.2. Low temperature system

3.2.1 Dilution refrigerator

Commercial $^3$He/$^4$He dilution refrigerator from Oxford Instruments with a base temperature of 13 mK was used to cool down the sample. Figure 3.2 demonstrates a cross-sectional view of the low temperature custom part below the mixing chamber plate. The design of this unit is the most crucial in refrigeration of the sample, since it provides thermal connection between the sample and the dilution unit, where the main cooling power originates.

The sample in a standard ceramic chip carrier is fixed inside the socket at the end of the cold finger, attached directly to the mixing chamber plate. The cold finger is covered by a hand made radiation shield thermally sunk into the mixing chamber. In addition to the sample screening from the background thermal radiation, this shield is made rigid, providing mechanical stability, and centering the sample in the magnet bore. Its structural strength comes from a paper-phenolic tube, but it is also covered by the vertical stripes of high RRR thin copper foil with insulating adhesive. These stripes are electrically connected to the mixing chamber and guarantee that the metal shield cools down close to the mixing chamber temperatures. Moreover, electrical resistance between different foil pieces prevents its heating due to Addy currents when the magnetic field is varied. The second radiation shield is a part of the Oxford commercial setup. It covers the handmade shield and the mixing chamber and is thermally connected to a 1K pot. The entire low-temperature unit below the mixing chamber has to fit inside the 3-inch superconducting solenoid bore therefore, space around the cold finger is quite limited. This requires extra efforts to ensure that all cylindrical shields are coaxial and no touches appear between them.

The chip carriers were custom manufactured in order to avoid the presence of any magnetic materials in the vicinity of the sample. Thus, magnetic field disturbance and extra heating due to magnetization of Ni nuclei in the regular industrial contact plating were minimized. Similarly, the socket Ni/Au-plated pins were replaced by the custom-made ones, lasercut from a
3.2. Low temperature system

Figure 3.2: Design of the dilution fridge low temperature side.
3.2. Low temperature system

Be/Cu plate.

Although the mixing chamber can reach temperatures below 20 mK, cooling the electron system is not straight-forward. The problem is that at this temperature range the electron and phonon systems are weakly coupled. On the other hand, electrons from wires diffuse into the 2DEG through low resistive electrical contacts. Thus, cooling electrical wires is essential for cooling two dimensional electron system. For the same reason induced electrical noise in the wires creates another source of electron heating in high resistive samples. This noise also has to be filtered as much as possible before the wires are cooled to the base temperature.

Few tricks were incorporated into the design of the cold finger in order to solve the problems described above. Electrical noise filtering was performed in several stages. First, the wires passed through a set of RC-filters mounted on PCB, screwed to the top of the mixing chamber plate. Although those filters were exposed to the external thermal radiation, they were warmer, but still close to the base temperature. This significantly decreased thermal fluctuations inside the filter resistors and made RF filtering much more effective. At the second stage of attenuation, the wires were cooled to the base temperatures inside an attenuator plate. This plate was made of low resistive wires, wrapped in copper foil and soldered to a copper plate pressed to the mixing chamber plate. Such design provided good thermal connection to the mixing chamber and realized effective cooling of the wires. At the last stage, the wires undergo the final step of cooling on a separate copper plate separately connected to the mixing chamber and eventually reached the chip carrier socket. More details on the cold finger design are given in [86].

Special attention was paid to grounding. In order to separate the system from external noise coming from electronics through the electrical ground, the sample circuit was made to be completely floating. All measurement and current source were separated from the world by high quality instrumentation amplifiers LT1167 (Fig. 3.3), powered by a battery. An extra reference contact at the output of preamp enabled the separation of measurement and instrumentation ground. Similarly, if connected in the opposite direction, such an amplifier could provide ground separation for the input current signal. Fig. 3.4 shows schematics for the voltage measurement and voltage source used in our measurement setup.

Because this experiment depends crucially on removing heat from the sample, the back of the chip was additionally soldered with indium to the gold surface of the chip carrier, which was thermally connected to the mixing chamber by wire-bonding the gold bottom to 16 of the cold finger’s electrical wires.
3.2. Low temperature system

Although dissipation due to high current biases in the bulk of the 2DEG was quite small in the described experiments—10’s of picowatts at most—the total Joule power dissipated in the sample was much larger (almost 2 nW at 400 nA) because the two-probe resistance between the source and drain (mostly $r_{xy}$) is much larger than $r_{xx}$. Here the question is addressed: how can 2 nW be dissipated at the sample without raising its temperature significantly above the mixing chamber temperature?

Almost all heat dissipation in the quantum Hall regime occurs in the source and drain contacts. With half of the power, 1 nW, dissipated in a single contact, the rise in contact temperature is set by the thermal resistance from the contact, through a bond wire and a chip carrier connector, to well-cooled measurement wires at the mixing chamber temperature. This thermal resistance can be estimated from the measured cooling power when heat was applied to a resistor mounted directly to the back of the chip carrier. Any heating applied to the chip carrier had to be carried away by the 16 leads bonded to the carrier backplane. 6 nW of power applied to the heater on
3.2. Low temperature system

Figure 3.4: Ground breaking schematics: a) voltage measurements; b) voltage source. Circles around the voltage probes depict cable shielding, connected to the measurement ground.

the back of the chip carrier was seen to warm the sample by 11 mK, giving a thermal resistance through each lead of 29 mK/nW. That is, 1 nW into the source or drain would warm it to a temperature around 40 mK. Is it possible to keep 2DEG cold if source and drain contacts are at 40 mK?

In the filling factor range $2 < \nu < 3$, source and drain ohmic contacts are separated from the 2DEG by $\sim 10 k\Omega$ contact resistance, and longitudinal conductivity per square through the 2DEG is $\ll 10^{-4} \Omega^{-1}$ ($\sigma_{xx} = \rho_{xx}/\rho_{xy}^2$). Together, these numbers give a Wiedemann-Franz (WF) thermal conductivity from the source or drain into the bulk of the 2DEG that is vanishingly small, $\ll 10^{-10} nW/mK$ or less at 15 mK. By comparison, the interior 3 mm$\times$3 mm region of the 2DEG is coupled to phonons with a thermal conductivity that is many orders of magnitude larger, between $10^{-6}$ and $10^{-2}$ nW/mK depending on 2DEG conductivity. This massive discrepancy between WF coupling and coupling to phonons ensures that electrons in the bulk of the 2DEG (though not the ones right next to the source and drain contacts) are locked to the phonon temperature of the chip rather than to the electron temperature of the leads.

With the source and drain at 40 mK, and the chip backplane at 15 mK, the phonon temperature will then be determined by the relative areas of source/drain contacts ($\sim 0.25 \, mm^2$ for each) compared to the area of contact to the backplane ($\sim 25 \, mm^2$). Even taking into account that phonon heat flux through the boundary at 40 mK is 10 times higher than at 15 mK, the phonon temperature will stay close to the (cold) backplane temperature.

Note that through these calculations we have assumed a homogeneous
3.2. Low temperature system

phonon temperature throughout the chip, despite the fact that heating is localized at source and drain contacts while cooling is spread across the backplane. This approximation can be justified by comparing lateral phonon thermal resistivity per square through the 0.3 mm-thick GaAs chip itself \((2 \times 10^8 \text{K/W, estimated from Ref. [89]})\), compared to the thermal boundary resistance \(3.5 \times 10^8 \text{mm}^2 \text{K/W estimated from Ref. [90]}\). Together these give a characteristic thermal “spreading” length of 1.3 mm from each contact, close to half of the chip dimension.

3.2.2 Temperature control

Sample temperature measurement was performed by several sensors to cover a wide temperature range. At higher temperatures \((\geq 50 \text{mK})\) a calibrated RuO\(_x\) resistor, bolted to the mixing chamber, was sensed. A second thermometer/heater pair, located on the back of the chip carrier, was calibrated using the mixing chamber thermometer/heater and enabled much faster heating and cool down times for temperature-dependent measurements.

The second thermometer was sanded down from a commercial resistor, suggested in [91], to roughly 0.3 mm thick film. After that, the sensor was glued to the back of a ceramic chip carrier using GE varnish, which also provided the sensor with electrical insulation. After that, electrical contact to the embedded metal contacts was made by bonding them to the contact pads of the chip carrier. It is important to mention that removal of oxide film from the metal contacts is needed to make a reliable bonding at low temperatures.

Another important detail, mentioned by Samkharadze et.al., was the need to insulate the carbon part from the air to prevent calibration drift during thermal cycling. In our setup the carbon sensor was not completely covered due to technical reasons. Therefore, it had to be recalibrated after every thermal cycling above liquid nitrogen boiling temperature.

Calibration of the second thermometer was done against RuO\(_x\) measurements above 50 mK and extrapolated down to the base temperature. However, at low temperatures, electrons are weakly thermally coupled to the crystal lattice, therefore, their temperature can differ significantly. The electronic temperature in the sample was monitored using temperature-dependent features in the magnetoresistance, which could be calibrated at higher temperatures where thermal equilibration is more reliable and then extrapolated to lower temperatures. In Ref. [23], for example, it was shown that the width of a reentrant state grows with temperature down to 6 mK.

In order to obtain the heater calibration, R2c temperature dependence
3.2. Low temperature system

was compared with its dependence on the heater power (Fig. 3.5). The former was measured by application of constant power to the mixing chamber. After the time, significant to stabilize the system (normally \(\sim 3\) hours for the lowest temperatures), there was no detectable difference between two magnetoresistance traces, taken one after the other. This allows us to assume, that the sample was thermally equilibrated with the mixing chamber and that its temperature was equal to the RuO\(_x\) calibrated resistor temperature at the mixing chamber. If the on-chip heater was used to warm the sample, there were no significant mixing chamber temperature changes observed within 10 minutes.

In the second step of calibration, constant power was applied to the on-chip heater and a few traces at different biases were taken one after another. The first and last ones were taken at zero bias. Their difference allowed to estimate chip carrier temperature change to be less than 1.5 mK after application of high biases. This difference can be also observed in figure 3.5 as zero heater power point corresponds to higher temperatures.
Consequently, the base temperature of the sample depends on the bias and introduces uncertainty of about 2 mK in our temperature measurements.

3.3 Electrical measurements

Electrical measurements of differential resistances \( R_{xx} \equiv dV_{xx}/dI \) and \( R_{xy} \equiv dV_{xy}/dI \) were performed using a low frequency lock-in technique with an AC current bias, \( I_{ac} \), between 1 and 5 nA. An additional DC bias current \( I_{dc} \) up to 5 uA was used to induce the in-plane electric field, \( E_y = V_{xy}/w \), in the plane of the sample and transverse to the current direction (\( w = 5 \text{ mm} \) is the sample width). The standard measurement scheme for driving DC and AC currents in the same direction is shown in figure 3.6a.

![Diagram of electrical measurements](image)

Figure 3.6: Schematics of electrical measurements: a). \( \text{AC} \parallel \text{DC} \); b). \( \text{AC} \perp \text{DC} \).

For the measurements, where AC and DC currents were applied in perpendicular directions, a scheme with floating AC current was used (Fig. 3.6b). In this case the AC ground was broken by means of the acoustic transformer with a high common mode rejection level. This enabled the construction of a floating AC circuit and the application of a small probing electric field perpendicular to the main Hall one generated by DC current.

In some cases, pure DC current was applied to the sample and appropriate DC voltage responses measured. In this case, the data were differentiated to obtain the values of differential resistances, and the size of the current step was an equivalent of the AC bias.
Chapter 4

Breakdown spatial resolution

4.1 Motivation

Reentrant states breakdown under high currents, driven through the sample, is a well known phenomenon, observed shortly after the discovery of the RIQHE [27]. These experiments are normally explained in terms of sliding dynamics of depinned charge density waves [31], or alignment of electron liquid crystal domains by the induced Hall electric field [29]. For example in [29] authors investigated the sample's state anisotropy with respect to the direction of high DC bias based on the macroscopic measurement of longitudinal resistance between corners of a square sample. They concluded, that the reentrant states become anisotropic in high bias, based on resistance difference, measured in the direction parallel or perpendicular to the DC bias component. Additionally, the scaling of the critical breakdown current with the size of the sample, led them to a conclusion, that the induced electric field is the primary parameter, defining the state of the 2DEG.

An important assumption in this interpretation of the experimental data is that the main effect of elevated DC bias current is to induce a homogeneous change in macroscopic samples, which may range from 100’s of microns to several mm’s depending on the specific realization. However, when current flow through the sample is strongly inhomogeneous (as is always the case in the quantum Hall regime) one’s ability to approximate bulk properties from non-local measurements is immediately suspect. For example, finite currents driven through a sample in the quantum Hall regime necessarily create temperature rise due to Joule heating, and that heating is in many cases strongly localized.

In this chapter it is shown, that large current biases, driven through a RIQH state, induce a sharply inhomogeneous breakdown. In other words, the electronic system becomes spatially fractured into macroscopic regions that are either melted (conducting) or frozen (insulating). This fracturing is monitored through a comparison of multiple voltages, probed simultaneously at different contact pairs. Notably, this phase-separated breakdown is entirely absent from quantum Hall liquid states, consistent with the distinc-
tion between collective phase transitions in RIQH electron solids vs activated transport in gaped FQH liquids. Considering numerous RIQH states, from \( \nu = 2 \) all the way up to \( \nu = 8 \), and multiple contacts arrangements, the RIQH breakdown is shown to propagate clockwise or counterclockwise from the source and drain contacts with a sense that depends on the electron- or hole-like character of the particular RIQH state. Supported by numerical simulations, the data are explained by a phase boundary between frozen and melted regions of an underlying electron solid that spreads around the chip, following the location of dissipation hot spots induced by local changes in \( R_{xy} \).

4.2 Experimental results

4.2.1 Measurement details

Measurements were performed on a 300 Å symmetrically doped GaAs/AlGaAs quantum well with low temperature electron density \( n_s = 3.1 \times 10^{11} \text{ cm}^{-2} \) and mobility \( 15 \times 10^6 \text{ cm}^2/\text{Vs} \). Electrical contact to the 2DEG was achieved by diffusing indium beads into the corners and sides of the 5×5 mm chip (Fig. 4.1a). Electronic temperature, \( T_e \), at low bias was monitored using temperature-dependent features in \( R_{xx} \), and confirmed to follow \( T_{\text{mix}} \) down to 13 mK. FQH characteristics were optimized following Ref. [1], in a dilution refrigerator with base temperature \( T_{\text{mix}} \sim 13 \text{ mK} \).

Differential resistances \( \tilde{R} \equiv dV/dI_b \) for various contact pairs were measured by lockin amplifier with an AC current bias, \( I_{AC} = 5 \text{ nA} \), at 71 Hz. An additional DC current bias \( I_{DC} \) was added to the AC current in many cases, providing 2D maps of various resistances as both bias voltage and filling factor are changed (see e.g. Fig. 4.1b).

At zero DC bias, characteristic \( \tilde{R}_{xx} \) and \( \tilde{R}_{xy} \) traces over the region \( 2 < \nu < 3 \) show an example of fractional quantum Hall states at filling factors \( \nu = 2 + 1/5 \), \( 2+1/3 \), \( 2+1/2 \), \( 2+2/3 \) and \( 2+4/5 \), with four RIQH states labelled R2a-R2d following conventional notation (dashed lines in Fig. 4.1c). At high current bias (solid lines in Fig. 4.1c) the reentrant states disappear, with \( \tilde{R}_{xy} \) moving close to the classical Hall resistance, while most fractional states remain well-resolved: quantized in \( \tilde{R}_{xy} \) with vanishing \( \tilde{R}_{xx} \).

The breakdown process is most clearly seen in 2D maps of \( I_{DC} \) and magnetic field, typically showing multiple regions with sharply delineated boundaries. Fig. 4.1 presents several such maps for the R2c reentrant state. Considering first the measurement of \( \tilde{R}_{xx} \) (Figs. 4.1b), the breakdown transitions follow a pattern that is similar to those observed by other groups [29, 82];
4.2. Experimental results

Figure 4.1: a) Schematic of a measurement combining AC (wiggly arrow) and DC (solid arrow) current bias through contacts 1 and 5 (see numbering). $\tilde{R}_{xx} = dV_{xx}/dI$, $\tilde{R}_D^+ = dV_{26}/dI$, and $\tilde{R}_D^- = dV_{84}/dI$. Curved arrows indicate edge state chirality. b) Evolution of $\tilde{R}_{xx}$ with DC bias for the R2c reentrant and $\nu = 5/2$ FQH state, showing breakdown regions ‘A’, ‘B’, and ‘C’ ($I_{AC}=5\,\text{nA}$). c) $\tilde{R}_{xx}$ and $\tilde{R}_{xy}$ (contacts 3 and 7) for filling factors $\nu = 2 - 3$, showing the breakdown at very high DC bias ($I_{AC}=5\,\text{nA}$). (d,e) Simultaneous measurements of $\tilde{R}_D^+$ (d) and $\tilde{R}_D^-$ (e), taken together with $\tilde{R}_{xx}$ in b). Dashed lines indicate identical parameters in each panel.
4.2. Experimental results

these transitions naturally divide the map into three distinct subregions, labelled ‘A’, ‘B’, and ‘C’.

Region A is characterized by very low $\tilde{R}_{xx}$: here the electron solid state is presumably pinned and completely insulating. The sharp transition to region B corresponds to a sudden rise in $R_{xx}$, while for higher bias (region C) the differential resistance drops again to a very small value. Unlike the flat zero of region A, however, $\tilde{R}_{xx}$ in region C fluctuates up and down as a function of $I_{DC}$ and $B$, giving an undulating pattern of ripples in the 2D data.

It is worth noting that the sharp transitions in the RIQH state breakdown are entirely absent from the neighbouring $\nu = 5/2$ state, a distinction seen for all RIQH states compared to all fractional states. Data for different cooldowns and different RIQH states were slightly different in the details, but qualitative characteristics were consistent for every well-developed RIQH state in every cooldown.

4.2.2 Measurements at different contacts

The observation of sharp delineations in the resistance of a bulk sample, measured between voltage probes separated by 5 mm, at first glance would seem to imply that the entire sample must suddenly change its electronic state for certain values of bias current and field. If this were true, one would expect simultaneous jumps in resistance monitored at any pair of voltage probes, although at a quantitative level, of course, the resistance jumps might be by differing amounts. Considering the two ‘diagonal’ pairs of voltage probes, marked $\tilde{R}_+^D$ and $\tilde{R}_-^D$ in Fig. 4.1a, one sees immediately that this is not the case. $\tilde{R}_+^D$ (Fig. 4.1d) exhibits the same transitions as $\tilde{R}_{xx}$ (Fig. 4.1b), but for $\tilde{R}_-^D$ (Fig. 4.1c) the A-B transition is entirely missing.

Qualitative differences between diagonal measurements $\tilde{R}_+^D$ and $\tilde{R}_-^D$ are expected when samples are inhomogeneous, especially in the quantum Hall regime[43]. $\tilde{R}_+^D$ and $\tilde{R}_-^D$ contacts are distinguished by the chirality of quantum Hall edge states: moving from source or drain contacts following the edge state chirality, one first comes to the $\tilde{R}_+^D$ contacts, then to $\tilde{R}_{xy}$ contacts in the middle of the sample, and finally to the $\tilde{R}_-^D$ contacts. This distinction is often used to isolate the conductance (via $\tilde{R}_+^D$) of a mesoscopic gated region in the middle of a quantum Hall sample, whereas $\tilde{R}_-^D$ would include combinations of gated and bulk filling factors[38].

Given the stark difference in breakdown characteristics for $\tilde{R}_+^D$ and $\tilde{R}_-^D$ maps, one could ask what happens halfway in between, that is, for the middle
4.2. Experimental results

Figure 4.2: (a) Simultaneous measurements showing the evolution of $\tilde{R}_D^+$, $\tilde{R}_{xy}$, and $\tilde{R}_D^-$, with DC bias, in the middle of the R2c reentrant state ($I_{AC}=5\,\text{nA}$); Evolution of (b) $\tilde{R}_D^-$, (c) $\tilde{R}_{xy}$ and (d) $\tilde{R}_D^+$ of R7a reentrant state with DC bias.

set of contacts, $\tilde{R}_{xy}$. As shown in Fig. 4.2a, the A-B transition (signified by a sharp rise resistance) simply moves to higher bias following the edge state chirality, from $\tilde{R}_D^+$ to $\tilde{R}_{xy}$ to $\tilde{R}_D^-$. An analogous progression was seen for every reentrant state, even up to much higher filling factor. Figs. 4.2b-d show the breakdown progression for R7a, albeit extending up to much higher bias currents.

4.2.3 Electron- vs hole-like reentrant states

Although the sequence for R7a is at first glance qualitatively similar to that seen for R2c, the direction of the sequence is opposite: whereas $\tilde{R}_D^+$ breaks
down before $\tilde{R}_D^-$ for R2c (Figs. 4.1d,e and Fig. 4.2a), $\tilde{R}_D^+$ breaks down after $\tilde{R}_D^-$ for R7a (Figs. 4.2b-d). This distinction was found to be linked to the electron- or hole-like (e/h-like) character of the reentrant state, considering every reentrant state observed in this experiment, independent of the number of occupied Landau levels. In other words, all reentrant states corresponding to a less-than-half-filled Landau level [$R_{2ab}, R_{3ab}, R_{4a}, R_{5a}$, etc.] broke down at lower bias for $\tilde{R}_D^-$, higher bias for $\tilde{R}_D^+$, whereas all states corresponding to a more-than-half-filled Landau level [$R_{2cd}, R_{3cd}, R_{4b}, R_{5b}$, etc.] broke down at lower bias for $\tilde{R}_D^+$, higher bias for $\tilde{R}_D^-$. Although slight differences in quantum Hall traces for various contact pairs are common, the qualitative distinctions between $\tilde{R}_D^+$ and $\tilde{R}_D^-$ did not depend on the specific contact used in the measurement, but only on the chiral distance of voltage probes with respect to source/drain current leads. For example, rotating the contact configuration by $90^\circ$ with respect to the chip axes leaves the $\tilde{R}_D^+/\tilde{R}_D^-$ distinction intact, shown in Fig. 4.3 for R3 reentrant states. As before, $\tilde{R}_D^+$ and $\tilde{R}_D^-$ are defined by the chirality of edge state transport away from the source or drain, together with the e/h character of the reentrant state.
Figure 4.3: Diagonal measurement with the current flowing in two perpendicular orientations with respect to the sample. It is clearly seen, that the strong/weak reentrant state is defined by the current contacts and any kind of measurement can be obtained on the same voltage probes.
4.3 Discussion

4.3.1 Model

The fact that $\tilde{R}_D^+$ and $\tilde{R}_D^-$ switch roles for electron- versus hole-like reentrant states provides an important hint as to the origin of this effect. Edge state chirality is fixed by the magnetic field direction, and would not be expected to suddenly reverse when crossing half-filling for each Landau level. Instead, the proposed explanation is based on localized dissipation in the quantum Hall regime, a phenomenon that is known to give rise to “hotspots” any time a significant bias is applied to a quantum Hall sample.

Consider current injected into a sample in the integer quantum Hall (IQH) regime, where $\rho_{xx}$ is exponentially close to zero but $R_{xy}$ is large. Driving a current $I_{bias}$ through such a sample requires a potential difference $R_{xy}I_{bias}$ between source and drain, and this potential drops entirely at the source and drain contacts (no voltage drop can occur within the sample since $\rho_{xx} \to 0$). Specifically, the voltage drops where the current carried along a few-channel edge state is dumped into the metallic source/drain contact – a region of effectively infinite filling factor.

In the language of chemical potentials, current-carrying edge states must be filled to a chemical potential well above equilibrium due to their limited density of states, but the chemical potential in the drain contact can remain very close to equilibrium. The voltage drop always occurs where current moves from a region of lower filling factor (smaller density of states, larger chemical potential) to a region of higher filling factor.

Moving now to a sample in the reentrant IQH regime ($\rho_{xx} \to 0$ as before), hotspots again appear at any location where current flows from a region of higher to lower $R_{xy}$. But now the local value of $R_{xy}$ is strongly temperature dependent, with a sharp melting transition in both longitudinal and transverse resistances as previously observed in Ref. [23]. The R2a reentrant state, for example, has $R_{xy} = 0.5(h/e^2)$ in the low temperature, low bias limit [Fig. 1b], but at higher temperature or bias the state melts to $R_{xy} \simeq 0.425(h/e^2)$. In the limit of very low current bias applied to a sample in the R2a reentrant state, the entire sample is effectively at $\nu = 2$ and only the two hotspots associated with IQH are observed. As the bias increases, however, the regions around those two hotspots melt to $R_{xy} \simeq 0.425(h/e^2)$, and an extra two hotspots will appear, where the current passes from the bulk (at $R_{xy} \simeq 0.5(h/e^2)$) into the melted region (at $R_{xy} \simeq 0.425(h/e^2)$) near the contact.

The localized heat dissipation, however, is not enough for existence of the
4.3. Discussion

A localized area with elevated temperature, i.e., high temperature gradients. Another requirement is low heat conductance, which prevents the dissipated energy from spreading over the large areas. The temperature distribution inside the 2DEG is set by the Wiedemann-Franz law, defining the lateral heat conductivity, and electron-phonon coupling, which provides physical mechanism for a heat flow through the crystal lattice, perpendicular to the surface of the chip. Competition of those two heat flows defines the size of the hot spot. Although the precise calculation of the heat dissipation requires complicated simulations, the order-of-magnitude estimate of the spot size could be given found from the comparison of Wiedeman-Franz and electron-phonon heat conductance. The first one could be directly calculated from the 2DEG conductivity

\[ \kappa_{el} = L \sigma_{xx} T, \]  

(4.1)

where \( \sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2) \) and \( L = 2.44 \times 10^{-8} \text{W} \Omega K^{-2} \) is known as Lorentz constant. After the breakdown of the reentrant state, measured longitudinal resistance of the sample was on the scale of a 100\( \Omega \), leading to \( \sigma_{xx} = 10^{-6} \Omega^{-1} \) for R2c reentrant state \( (R_{xy} \sim 10k\Omega) \).

Electron-phonon heat conductance could be estimated following [92]. The power, dissipated into phonon system, is given as following:

\[ dP = C dT, \]  

(4.2)

where \( 1/\tau = \alpha T^3 \) is the phonon scattering time and \( C = \beta T \) is the heat capacity of electrons. For the free electron case \( \alpha \) was experimentally measured to be \( 2.9 \times 10^9 s^{-1} K^{-3} \) [88] and \( \beta = \frac{\pi k_B^4 m^* S}{3 \hbar^2} \), where \( k_B, m^*, S \) are the Boltzmann constant, the effective electron mass in GaAs and the hotspot area respectively. Therefore, in the general case,

\[ \kappa_{ph} = \frac{dP}{dT} = \alpha \beta T^4. \]  

(4.3)

Equilibration of lateral and perpendicular thermal conductivities provides an estimate for the hotspot area:

\[ S = \frac{3\hbar^2 L \sigma_{xx}}{\pi \alpha k_B^2 m^* T^3}, \]  

(4.4)

At 20 mK this formula gives the area of heat dissipation localization about \((30 \mu m)^2\), justifying the concept of the hotspots.

Therefore, a conceptual picture that explains the data at a qualitative level is following:
4.3. Discussion

1. Starting from low bias, conventional quantum Hall hotspots near the source and drain contacts locally melt the reentrant state, giving rise to two extra hotspots at the boundary between melted and unmelted reentrant phases, either upstream or downstream from the source and drain depending on electron- or hole-like character of the reentrant state.

2. As bias is increased, the dissipation at each hotspot increases also. The melted region (that is, the area where the local temperature is above the melting temperature) spreads, shifting the location of the reentrant hotspots upstream or downstream away from the contacts. The location of the reentrant hotspots stabilizes approximately when the dissipation per unit area drops to the point that the local temperature in the melted region is just above the melting temperature. Here we assume that the cooling rate from the electron system to the phonons (assumed to be cold) scales with area.

3. The transition between regions A and B in 2D maps such as Figs. 4.1b, 4.1d and 4.1f—that is, the transition from low $R_{xx}$ and integer-quantized $R_{xy,D}$ to high $R_{xx}$ with associated jump up or down in $R_{xy,D}$—occurs when the boundary between melted and frozen regions passes the relevant voltage probe.

4.3.2 Numerical simulation

Although the chemical potential analysis is appealing, it is not necessary to understand the location of quantum Hall hotspots. Even a classical electrostatics simulation of current flow between regions with different $R_{xy}$ (and $R_{xx}$ close to zero) is sufficient to indicate the same result: dissipation always occurs at the locations where current passes from regions of high $R_{xy}$ to lower $R_{xy}$.

To backup this argument with qualitative theoretical justification, broken (A) and unbroken (B) regions are approximated as the ones with different transverse conductivity $\sigma_{xy}$. Within this approximation, the classical solution of Kirchhoff equations is considered in a two dimensional domain $\Omega$ (Fig. 4.1). For any point $(x, y) \in \Omega$, Kirchhoff's laws dictate:

$$\nabla \cdot j = 0, \nabla \times E = 0,$$

where $j$ and $E$ are the electric current density and field respectively. Introducing the electric potential, $E = -\nabla \phi$, the equation for $E$ is trivially fulfilled. Finally, assuming the local relation $j = \sigma E$, we get:
4.3. Discussion

Figure 4.4: Geometry of the domain Ω, where simulation is performed. Regions A and B denote the areas with different densities, corresponding to different $\sigma_{xy}$’s in the simulation.

$$\nabla \cdot (\sigma \nabla \phi) = 0,$$

(4.6)

where $\sigma$ has the general form (see eq. 2.4):

$$\sigma = \begin{bmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{xx} \end{bmatrix}$$

Next, the boundary conditions have to be defined. In presented simulation two cases, corresponding to electron-like and hole-like reentrant state, are considered:

1. $\sigma_{xy}^A = 2.5e^2/h$, $\sigma_{xy}^B = 2e^2/h$
2. $\sigma_{xy}^A = 2.5e^2/h$, $\sigma_{xy}^B = 3e^2/h$

In both cases $\sigma_{xx} = 0.02e^2/h$ for any $(x, y) \in \Omega$. A current I is injected uniformly through the red boundary (contact) and it is collected from the blue one.

Two extra hotspots are clearly seen in the dissipation map in Fig. 4.5a, based on a numerical solution of the problem, described above.\[1\] Current

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\[1\] Numerical simulations in this paragraph were performed by Yuval Baum, Weizmann Institute of Science
4.3. Discussion

Figure 4.5: Classical simulation of dissipation due to current flow in a sample divided into three regions: semicircles corresponding to the melted state near each contact (hollow hatched) with $R_{xy} = \hbar/(2.5e^2)$, and the bulk (dark blue) reentrant state with $R_{xy} = \hbar/(2e^2)$ (a) or $R_{xy} = \hbar/(3e^2)$ (b). Hotspots appear at different corners of the melted region in a) and b).
flows predominantly along the edges due to the inequality $R_{xy} \gg R_{xx}$, so hotspots appear at intersections of the boundary between $R_{xy}$ regions and the sample edge. Note that only one corner of each melted region has a hotspot, but whether that corner is where the edge state enters or leaves the melted region depends on the relative value of $R_{xy}$ as compared to the bulk [compare Fig. 4.5a to 4.5b].

The difference between Figs. 4.5a and 4.5b justifies at a qualitative level why the boundary between melted and frozen reentrant phases should propagate in opposite directions for electron-like and hole-like reentrant states. In order to make quantitative connection between precise hotspot location and the spreading direction for the melted region, one would need to understand the detailed feedback process by which the boundary of melted phase changes as dissipated power increases—a thermal analysis is beyond the scope of the present chapter and require future theoretical research.

## 4.3.3 Comparison with experiment

Now let’s compare experimental details against the theoretical conclusions, which could be derived from suggested model, in order to test it for the consistency.

### Breakdown propagation along the edge

As further confirmation that a melted pool of free carriers around source/drain contacts spreads into the bulk of the sample at a frozen reentrant state along the edge, the spreading of potential jumps around the sample are mapped out in detail for the R2c reentrant state (Fig. 4.6). This is the hole-like reentrant state with $\tilde{R}_{xy}$ corresponding to $i=3$ ($\tilde{R}_{xy} = h/3e^2 = 8.6 \text{k}\Omega$) in the frozen state, and $\nu=2.56$ ($\tilde{R}_{xy} = h/2.56e^2 = 10 \text{k}\Omega$) in the melted state. Consequently, for R2c reentrant state the potential jump at the hot spot

$$\Delta = I \frac{h}{e^2} \left( \frac{1}{\nu} - \frac{1}{3} \right).$$

(4.7)

Similar to previous notation sample contacts are enumerated clockwise with the current sourced into contact 1 and drained from contact 5, assumed to be the ground potential.

Fig. 4.6a and b denote three plausible extents of the melted region with their associated hotspots, as well as, the edge state potentials expected for one of this situations. Figure 4.6c demonstrates the potential difference between multiple contact pairs all starting from contact 8 on the low-potential
4.3. Discussion

Figure 4.6: a) Hole-like reentrant state breakdown propagation in the sample; hexagonal, diamond and round marks depict the position of the hot spots for three different sizes of broken regions (dark grey, light grey and hatched), corresponding to different bias currents; b) electrical potential distribution in the sample with half-way broken reentrant state; comparison of breakdown propagation, measured on different ohmic contacts along one side with common contact in c) frozen and d) melted region. The marks depict current value, corresponding to the hot spot locations, shown in a)
side of the sample, compared to contacts 2, 3, and 4 on the high-potential side. The relevant data for contact pairs 6-2, 6-3, and 6-4 are shown in Fig. 4.6d.

For the first situation (S1), corresponding to the dark grey melted region with the hexagonal hotspots (Fig. 4.6a), the potential drops along the upper edge state occurred before (following edge state chirality) contacts 2, 3, 4, and the potential drop along the lower edge state occurred before contacts 6 or 8, so none of the voltage contact pairs 8-2, 8-3, 8-4, 6-2, 6-3, or 6-4 have recorded a resistance jump relative to the zero bias measurement. For situation 2, corresponding to the light grey melted region and diamond hotspot, the potential drop in the upper edge state occurs after contact 2, but before contacts 3 or 4; along the lower edge the potential drop occurs in between contacts 6 and 8. Considering pairs 8-2, 8-3 and 8-4 in case of S2, we, therefore, expect a resistance jump only in 8-2, whereas potential jumps have occurred for all three pairs 6-2, 6-3, and 6-4. The magnitude of the jump in 6-2 is twice what it is in 6-3 or 6-4, because that pair includes jumps along both the upper and lower edge states (see Fig. 7b). However, in S3, when the hot spot passes contact 3, putting both contacts in the pair 6-3 in the melted state, the 6-3 deviates from the 6-4 trace and jumps to the twice higher level, similar to the measurement of the 6-2 pair.

The observed resistances corresponding to various contact pairs (Figs. 4.6c and 4.6d) follow precisely the sequence, described above. Bias currents corresponding to each situation are marked with the corresponding hotspot symbol in the graphs, that is, bias current ∼50nA corresponds to the hexagonal hotspot symbol marking to situation 1.

Figures 4.6c and d also include a trace (dashed line) for the two-wire resistance of the sample, that is, the potential measured across source and drain contacts. A resistance of 2.6 kOhms is subtracted from the trace to take into account wire resistance in the cryostat as well as imperfect ohmic contacts. At zero DC bias, when the entire sample is in the frozen reentrant state corresponding to $R_{xy}(i = 3) = \hbar/3e^2$, the two wire resistance must be $R = 8.6k\Omega$ (after subtracting wire and ohmic contact resistances). But once a finite melted region has spread away from source and drain contacts, resulting in e.g. situation 1 with the hexagonal hotspots in Fig. 4.6a, the two-wire resistance increases to include the two additional potential drops at the hotspots. Thus the two-wire resistance records the first appearance of a melted region spreading beyond source and drain contacts, whereas the voltage probes 2 and 6 only register the movement of the hotspot past these two voltage probes.

On the other hand, the drop of two-wire resistance to classical Hall
value, indicates that two melted regions become electrically connected, i.e. this is the indication of the breakdown of the insulating bulk. Notably, this transition coincides with the breakdown transition, measured at the strongest 8-4 pair. This fact indicates, that the insulating bulk state breaks down before the hotspots reach contacts 4 and 8 and region C in figures 4.1d,e corresponds to the conductive bulk state. Detailed investigation of region C is a subject of the following chapter.

Anisotropic measurement

Although the following chapter focuses on anisotropic measurements, here experimental data is compared to the findings from other groups. Specifically, Ref. [29] compared $R_{xx}$, measured parallel or perpendicular to a large DC current bias, by rotating the $R_{xx}$ voltage probes and AC current bias contacts by 90° with respect to the DC bias contacts [Fig. 4.7a,b]. It was observed that the low-$R_{xx}$ region A extended to much higher bias for the ($AC \perp DC$) orientation, compared to the conventional ($AC \parallel DC$) orientation. While Ref. [29] focused on R4 states exclusively, we found analogous behaviour for all reentrant states measured, from filling factor 2 through 8 (see e.g. R3a in Figs. 4.7c,d).

However, instead of complicated picture, involving reorientation of electron liquid crystal domains, such behaviour can be simply explained by the hotspot-movement mechanism outlined above. Figs. 4.7a and 4.7b schematics include dashed lines to show the melted-frozen boundary at an intermediate bias, with associated hotspots marked with ⋆’s; the boundary propagates from the DC (not AC) current contacts, because the measurement is done in the limit of vanishing AC bias. The local potential along the edge of the sample drops sharply when passing the DC source and drain (the conventional IQH hotspots), but a second smaller potential drop occurs at each ⋆. For the bias-induced distribution of melted and frozen phases indicated in Fig. 4.7, the small potential drop occurs between the $R_{xx}$ voltage probes in Fig. 4.7a, but not in Fig. 4.7b, so large $R_{xx}$ in region B (e.g. 50 nA) is recorded only in the $AC \parallel DC$ configuration. For the small DC biases, when the hot spot is in between voltage probes, the potential drop is not detected by the AC measurement, as the probes are connected only to the edge states, associated with the underlying completely filled Landau levels.
4.3. Discussion

a) AC||DC

Rxx
e
B (T) B (T)
IDC (nA)
3.88 3.84 3.80 3.76
250
200
150
100
50
0
-50
3.88 3.84 3.80 3.76
c) d)

Rxx (kW)
0.8
0.4
0.0
Rxx
e
b) AC   DC

Figure 4.7: Comparison of two possible measurement geometries: a) AC||DC and b) AC ⊥ DC; arrows label the source and drain contacts for DC (straight) and AC (wiggly) bias, and chirality of the edge states (curved). Hatched areas denotes partially-extended melted regions for a hole-like reentrant state at intermediate DC bias, corresponding to $I_{DC} \sim 50$nA in panels (c,d). Hotspots at the melted/frozen boundary indicated by ⋆. Yellow line indicates potential along the edge. (c,d) $R_{xx}$ maps in $(I_{DC}, B)$ plane for c) AC||DC and d) AC ⊥ DC measurement.

4.3.4 RIQHE vs FQHE breakdown

As was pointed out earlier, described inhomogeneity of the breakdown was observed only for the reentrant states, while in fractional quantum Hall regime, the sample demonstrates similar activated transport behaviour at any location of the sample. This result is of the great importance, since it provides a direct observation of the differences between local thermodynamic properties of the 2D electron system in different correlated states. On the mundane level this can be explained by very sharp resistance temperature dependence of the RIQH around critical temperatures [23]. As heat conductance is proportional to the resistance, the more $\sigma_{xx}$ drops with temperature, the higher heat localization is.

60
4.4 Conclusions

However, such a dramatic local temperature dependence clearly implies a difference in the thermodynamical properties of the RIQH and FQH ground states and creates a probe of microscopic structure of those states at the hot spot. The most popular theoretical picture of the bubble phases has no direct experimental confirmation yet, as it requires an investigation of the correlated state local order. However, all experimental techniques, currently used, allow to study reentrant state microscopic structure only in indirect way, driving a conclusion about microscopic properties from macroscopic measurements, averaged over the whole area of the sample (for instance, microwave techniques). Therefore, the effect, which provides technique to probe the local microscopic properties of the electron system under conditions of RIQHE is important finding.

In conclusion, it was demonstrated that bias-induced breakdown of the RIQH effect is strongly inhomogeneous across macroscopic (mm-scale) samples, at least for large ranges of intermediate DC biases. As bias increases, the RIQH breakdown propagates away from source and drain contacts with a chiral sense that depends on the electron- or hole-like character of the reentrant state, leading to different critical breakdown biases for different pairs of voltage probes.

This phenomenon may result from opposite hotspot locations for the two types of reentrant states, giving rise to melted (no longer reentrant) regions near source and drain contacts that spread in opposite directions as bias increases. Suggested model was qualitatively confirmed by the numerical simulation and comparison with the data. It was experimentally shown, that hotspots propagate chirally along the edges of the sample, sequentially passing contacts, following edge state chirality. The voltage, probed at different contacts is in agreement with the potential differences, derived from the electrical potential distribution. This effect creates a nice tool for distinguishing different conducting states of the 2DEG, surrounding the ohmic contacts.

Experimental data was also compared with similar results from other groups [29]. Despite the general agreement, theoretical explanations suggested before fail to explain new experimental results. This experiment highlights an example of counterintuitive phenomena that can appear in transport measurements of correlated electronic states. It also shows the danger in interpreting macroscopic measurements at a microscopic level, in
samples, where electronic phase transitions are sharp and phase segregation may occur.

The observed macroscopic phase separation was demonstrated to be a distinct signature of correlated reentrant states, as it was not observed in activated transport during the breakdown of fractional quantum Hall phases. This difference most likely results from a sharp temperature dependence of the resistance around breakdown of RIQHE, observed before. The properties of the breakdown are directly connected to the thermodynamics of the microscopic electron state, and bias-breakdown measurements therefore offer a valuable insight into the dynamics of RIQH states in general.
Chapter 5

High bias reentrant states

5.1 Motivation

One of the main candidates for microscopic description of high bias RIQHE breakdown is a destabilization of the electron solid by Hall electric field, which is thought to depin the crystal from an underlying disorder potential or reform it with altered long-range order \[27,29\]. These hypotheses are based on the analogy with other charge ordered systems across the field of strongly-correlated electronics. Experimental measurements of the bias-driven sliding dynamics of charge density waves date back to work on NbSe\(_3\) nearly four decades ago, and remain an area of active research \[30–32\].

Transport signatures of high bias RIQHE breakdown, in contrast to smooth free electron traces at higher temperatures, include sharp transitions out of the insulating state, and excess resistance noise in the transition region \[27,28\]. A simple free-electron picture fails to explain this experimental fact. On the other hand RIQH states, especially in high magnetic fields \((2 < \nu < 4)\), are extremely fragile, limited to temperatures below 40 mK and the highest mobility samples \[17,23,25,26\]. However, high currents under conditions of QHE (high perpendicular magnetic field and ultra-low temperatures) unambiguously dissipate a lot of heat in the electronic system, questioning the existence of the high bias electron solid state under such conditions.

Furthermore, spatially resolved measurements of the RIQHE breakdown demonstrated a macroscopic phase separation within the sample at intermediate biases (see Ch. \[4\]). This led to a conclusion, that the RS breakdown results from partial electron solid melting by localized hot spots rather than from the sliding dynamics. However, at intermediate currents, some parts of the fractured sample demonstrate reentrant behaviour, implying that electrons there stay below crystal melting temperature. What is the microscopic structure of the intermediate bias states (region C) at low temperatures immediately after breakdown from insulating reentrant state?

In this chapter, the measurements of high bias breakdown process, probed simultaneously at various contact pairs, are analyzed. This allows to under-
stand details of electrical potential distribution along the edge of the sample, providing an evidence for the insulating bulk state breakdown, while some regions of the 2DEG stay frozen. Additionally, the first temperature measurements of high bias RIQH states in the second (N=1) Landau level presented at the second part of this chapter offer new insights into the nature of electron solids under non-equilibrium conditions. Experiment is focused on finely-featured phase transitions, which appear as a function of bias current, before the state melts completely at very high bias. It is found that these transport signatures retain extremely fine bias and temperature sensitivity even down to 15 mK, under conditions where simple estimations of self-heating by bias current would imply electron temperatures at the hot spots well above 100 mK.

Together these data hint on the correlated electron nature of non-insulating reentrant states under intermediate biases. Presumably they stay cold, as highly localized dissipation allows to draw most of Joule power away from the 2DEG through the bottom of the sample. Besides, it is demonstrated, that the bulk breakdown mechanism is different from a simple melting, since the heat dissipation is likely localized near the edges of the 2DEG. Finally, alternative conducting bulk states, suggested in literature, and their possible experimental manifestations are reviewed at the end of the chapter.

5.2 Transport measurements

Longitudinal and transverse voltages, probed at the edges of the sample in quantum Hall regime, depend on whether its bulk is insulating or not. At the same time those parameters of electronic transport are independent on the specific location of measurement contacts, i.e. matters only their topological position with respect to the bulk. This immediately leads to a conclusion, that the change of the bulk state has identical effect on the transport measurements at any contacts of the sample. Contrary, if the sample is inhomogeneous, as was shown in the previous chapter, completely different voltages could be simultaneously sensed at various contact pairs of the same sample. This provides a criteria for distinguishing transport features caused by bulk from those, which are just a result of a local potential change around the specific ohmics. In the following paragraph I focus on investigation of the RIQHE bias breakdown features, measured at different contacts in order to drive a conclusion about state of the 2DEG in the bulk of the sample.

The measurements where done at the same chip, as in Ch.4. A sample
5.2. Transport measurements

Figure 5.1: (a) Characteristic $\tilde{R}_{xy} \equiv dV_{xy}/dI$ and $\tilde{R}_{xx} \equiv dV_{xx}/dI$ traces, measured at base temperature (13 mK) in filling factors $\nu = 3−4$ ($I^{ac}_b=5$ nA, $I^{dc}_b=0$); (b) evolution of $\tilde{R}_D^+$ ($I^{ac}_b=5$ nA) at the R3a reentrant state with dc bias current $I^{dc}_b$. Characteristic regions denoted ‘A’, ‘B’, and ‘C’ in the R2c breakdown; (c) schematic of the measurement; (d) traces of $\tilde{R}_D^+$ and $\tilde{R}_D^-$ breakdown with bias. Inset demonstrates $\tilde{R}_D^+$ measurement 2D map in region C with correlated pattern of ripples.
low bias measurement of the RIQHE for filling factors $\nu = 3 - 4$ is shown on the figure 5.1a. Fig. 5.1b demonstrates the characteristic 2D map of $R_D^+$ measurement of R3a reentrant state in the measurement geometry, shown at Fig. 5.1c. Figure 5.1d demonstrates a characteristic $R_D^+$, $R_D^-$ traces at the center of the R3a reentrant state. In addition to the chiral behaviour of the break-down described in previous chapter both traces simultaneously transfer into the region C with Hall resistance showing ripples around the classical Hall value. The 2D map of that region shown on the inset to the graph ensures that the ripples are consistent and totally reproducible in bias and magnetic field. Since two traces coincide in region C, this behaviour suggests, that transition to region C is caused by the change of the electronic bulk state, rather than local potential redistribution due to redistribution of various regions in inhomogeneous samples.

### 5.2.1 Electrical potential spatial distribution

First let’s consider electrical potential distribution in the sample. An electron experiences a potential jump (the hotspot) at every boundary between melted and frozen regions, where the local filling factor increases along the electron flow. Figures 5.2a and 5.2b denote a propagation of the melted regions and appropriate hotspots with bias current for electron-like and hole-like reentrant states. Following the previous labeling, the sample contacts are marked clockwise, with current sourced into contact 1 and drained from contact 5, assumed to be a ground potential.

In general case of the RIQHE at filling factor $\nu$, reentering to the integer plateau $i$, the electrical potential jump

$$\Delta = I \frac{\hbar}{e^2} \left| \frac{1}{i} - \frac{1}{\nu} \right|$$  \hspace{1cm} (5.1)

where $\nu, i$ are the filling factors, corresponding to the current and nearest to it integer value of filling factor and $I$ is the bias current through the sample.

To reconstruct the value of electrical potential in all regions of the sample for the case of electron-like reentrant state (Fig. 5.2a) let’s start from the contact 6. Its potential equals to the potential of the ground, which is set to be equal zero, as it is electrically connected to the drain contact by the edge state. In this case contact 8, which is “after” the hot spot in the direction, opposite to the propagation of the edge state, has potential $\Delta$. On the other hand, contact 2 has the potential $Ih/ie^2$, corresponding to the integer quantized resistance of the unbroken reentrant state. Hence, the potential of the last contact 4 is lowered by $\Delta$ due to the potential jump at the hot spot.
5.2. Transport measurements

Figure 5.2: Breakdown propagation in the sample in a) electron and b) hole case, diamond and round marks depict the position of the hotspots for two different sizes of broken regions (grey and hatched), corresponding to different bias currents; comparison of two diagonal measurements for the c) electron (R3a) and d) hole-like (R3c) reentrant states (data taken from Fig. 4.3a)

Following similar logic, one can reconstruct electrical potential distribution for the case of the hole-like reentrant state. Specific values of the electric potential at all 4 corner contacts for the case of hole-like R3c reentrant state are shown in Fig. 5.2b.

Now, consider Hall voltages, which could be possibly measured between different pairs of contacts. Assume, that the potential difference within every microscopic region is small. Therefore, the ohmic state can be approximated by “melted” or “frozen”, defined by the state of the 2DEG around it. Hence, transverse resistance between two given ohmic contacts changes only when the 2DEG around at least one of them changes its state. In other words, there are only three possible voltages, which could be probed at any Hall contact pair for each reentrant state. The corresponding values of the transverse resistances, calculated from the potential differences at different
5.2. Transport measurements

contact states with use of (5.1), are summarised in the following table (F/M labels frozen or melted state of the corresponding ohmic):

<table>
<thead>
<tr>
<th>High/low potential</th>
<th>F</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>$\frac{n_{\theta}}{ue^2}$</td>
<td>$\frac{n_{\theta}}{ue^2}$</td>
</tr>
<tr>
<td>M</td>
<td>$\frac{n_{\theta}}{ue^2} \pm 2\Delta I$</td>
<td>$\frac{n_{\theta}}{ue^2}$</td>
</tr>
</tbody>
</table>

Although the detailed discussion of the transport signatures of the RIQHE breakdown in high bias is the subject of following paragraphs, such a simple speculation allows to explain, for instance, such a non-obvious fact, as transverse resistance below classical Hall value, measured for electron-like reentrant states at intermediate biases (Fig. 5.2c). In homogeneous sample it would mean, that the number of electrons, contributing to the transport is higher, than one would expect from the filling factor of the system. However, for the case of partially broken reentrant state such ”undershoot” of resistance results from two potential jumps at two hot spots, one at each side of the sample. Similar behaviour, although reversed, is observed for the hole-like reentrant states (Fig.5.2d). Transverse resistance trace overshoots the free electron Hall one due to reversed direction of the breakdown propagation, resulting in different potential distribution (compare figures 5.2a and 5.2b).

5.2.2 Comparison with experiment

In order to support the theory, described above, with more qualitative explanations, the specific values of the features in experimental traces are considered in details. Figures5.3c,d contrast the breakdown of the R7a reentrant state, measured at different contacts. The dashed vertical lines depict current thresholds, corresponding to boundaries between regions A, B and C, defined earlier. In region A the hotspots are located ”before” all contacts (Fig.5.3a), so every voltage pair probes integrally quantized value. This area corresponds to non-broken reentrant states and was previously labeled as region A.

Region B was defined as the intermediate state, when the sample is fractured into several macroscopic broken and non-broken regions. As the bias is increased, the transverse resistance, measured at different probes deviates from the integer plateau and gets to the classical Hall value or even lower, depending on the state of the voltage probes being measured. For the measurement geometry shown on Fig.5.2a, the voltage probes are symmetric with respect to the propagation of the hot spot. Thus, in ideal
5.2. Transport measurements

Figure 5.3: (a) schematics of the hot spot propagation with current bias; (b) schematics of the melted regions and current flow through the bulk in region C; (c)-(d) breakdown traces of electron-like reentrant state R7a with bias; (e) comparison of the breakdown of hole-like reentrant state R2c, measured at different voltage probes, and 2-point measurement.
5.2. Transport measurements

sample, one should see a direct transfer from integer plateau to the value twice lower, then the classical Hall resistance. However, in real samples, the hotspots do not simultaneously reach the contacts on each side, therefore, the trace demonstrates the step in the middle (light grey area on figures 5.3c-d). Since, the conductance of the melted area is high, contact potential is independent on the position of the hotspot, thus, measured Hall resistance is constant in bias independently to the exact position of hotspot. This can be easily seen for the case, where the contact asymmetry is large enough to create a significant difference between the chiral position of two hot spots. Fig. 5.3c depicts $R_D - D$ trace with the step at current biases between 50 and 100 nA. It is clearly seen, that measured resistance is constant in this range currents. This situation is schematically shown on Fig. 5.3a for contacts 3 and 7 and the hotspot position, marked by the diamond. Similar behaviour is observed for the hole-like reentrant states, shown on figure 5.3e. At the region, corresponding to two measurement contacts surrounded with the melted reentrant state, the transverse resistance jumps twice as high, as the difference between integer plateau and classical Hall value in complete agreement with (5.1).

Another interesting observation is that under certain conditions the sample could still be in the inhomogeneous state, corresponding to region B, while all the measurement contact pairs are situated in the melted regions and transverse resistance is not quantized anywhere within the sample. An example of such situation is marked by dark grey area on Fig. 5.3c-d and shown schematically on Fig. 5.3a (the hot spot position is marked by star). Despite the sample does not demonstrate integrally quantized Hall resistances in transport measurements, the boundary between regions B and C appears to be at higher biases. It can be identified, as the bias current, where all measurements simultaneously experience a jump to the classical Hall resistance level (for example, jump around 290 nA on Fig. 5.3c-d). The fact that this transition happens at exactly the same bias for any contact pair (including source/drain ones, Fig. 5.3e) means, that the Hall resistance change in region C is caused not by the local movement of the hotspot (which has already passed all contacts), but by the transition of the bulk from insulating to conducting state. The sample in such state is shown schematically in Fig. 5.3b. In contrast to the 4-wire voltage probes, the 2-point measurement is set by the potential difference between opposite edges of the sample. When the hot spots are formed, source and drain contacts automatically appear in the melted region, which means, that their breakdown to classical Hall value could be caused only by shorting two opposite edges through the bulk of the sample.
5.2. Transport measurements

5.2.3 Longitudinal voltage

Finally, I want to draw attention to another interesting experimental fact, shown at Fig. 5.4. For hole-like reentrant state R2c longitudinal DC voltage $V_{xx}(I_{dc})$ remains large into region C, even though the differential resistance $\widetilde{R}_{xx}$ (measured simultaneously) drops to a very small value.

![Figure 5.4](image)

Figure 5.4: Evolution of R2c with DC current bias: a) longitudinal voltage $V_{xx}$ and b) longitudinal differential resistance $\widetilde{R}_{xx}$ ($I_{ac} = 5nA$). Note the very different color scale in (b) compared to previous figures. c). Characteristic traces of $V_{xx}$ and $\widetilde{R}_{xx}$ at $\nu = 2.569$.

Previously regions A, B and C were shown to correspond to a chiral movement of the boundary between regions of melted and frozen reentrant states in the sample (see Ch. 4). Turning to $\widetilde{R}_{xx}$ measurement, region A corresponds to both ohmic contacts appearing in the frozen insulating area. In this case, no current is leaking out of the edge states between the voltage probes, therefore, the measured longitudinal voltage is zero. On the contrary, region B corresponds to the case, where the melted boundary is
between two $\tilde{R}_{xx}$ probes. In this case the measurement represents the potential drop at the boundary, showing high signal. Once the bulk starts to conduct, the voltage, measured between $\tilde{R}_{xx}$ probes experiences a drop (at around 200 nA, Fig. 5.4c), remaining, however, at high level, demonstrating nearly current independent trace.

Such a diode-like I-V curve is rather surprising. As was mentioned in previous section, the bulk breakdown happens before the hot spots leave the region between $\tilde{R}_{xx}$ voltage probes. The constant longitudinal voltage means, that the potential jump at the boundary of two regions remains the same, which requires constant current to flow through the hotspot at the edge. Thus, when the bias is increased in region C, all extra current flows nearly dissipationlessly through the bulk, maintaining constant potential difference at the density jump near the edge. This means, that relatively large current could flow through the bulk of the sample in region C, which questions an existence of correlated electron state. In order to bring some light to this puzzle, I performed temperature measurements, which are the topic of the next paragraph.

5.3 Temperature measurements

5.3.1 Experimental technique

Temperature measurements were performed on the same sample and in the same cooldown. Chip temperature was controlled using the mixing chamber heater or a thermometer/heater pair on the back of the chip carrier for faster response. Differential resistances $\tilde{R}_{xx} \equiv dV_{xx}/dI_b$ and $\tilde{R}_{xy} \equiv dV_{xy}/dI_b$ were measured by lockin amplifier with an AC current bias, $I_{ac}^b = 5$ nA, at 71 Hz.

Characteristic $\tilde{R}_{xx}$ and $\tilde{R}_{xy}$ traces at $I_{dc}^b = 0$ show well-developed fractional quantum Hall states at $\nu = 2 + 1/5$, $7/3$, $5/2 (\Delta_{5/2} \sim 0.45 \text{ K})$, $8/3$ and $2+4/5$ (Fig. 5.5a). The diamond of $\tilde{R}_{xx}$ features surrounding the center of the reentrant state, as before, demonstrates three distinct subregions, labelled ‘A’, ‘B’, and ‘C’ in Fig. 5.5b. The differential resistance is very small for higher bias, in region C, where $\tilde{R}_{xx}$ fluctuates up and down as a function of $I_{dc}^b$ and $B$. Resulting pattern of ripples in the 2D data becomes sharper as temperature is reduced. These features are completely reproducible for consecutive $I_{dc}^b$ sweeps, and are, therefore, not oscillations or noise of the type reported in [28]. Images with features as sharp as those seen in Figs. 5.5b and 5.6 were obtained only below 15 mK and only for very well developed reentrant states, especially R2c and R2a but not R2b in Fig. 5.5a (in fact,
5.3. Temperature measurements

Figure 5.5: (a) Characteristic $\tilde{R}_{\text{xy}} \equiv dV_{\text{xy}}/dI$ and $\tilde{R}_{\text{xx}} \equiv dV_{\text{xx}}/dI$ traces, measured at base temperature (13 mK) in filling factors $\nu = 2-3$ ($I_{b\text{ac}}^c=1 \text{nA}$, $I_{b\text{dc}}^c=0$); (b) evolution of $\tilde{R}_{\text{xx}}$ ($I_{b\text{ac}}^c=5 \text{nA}$) at the R2c reentrant state and $\nu = 5/2$ FQH state with dc bias current $I_{b\text{dc}}^c$. Characteristic regions denoted ‘A’, ‘B’, and ‘C’ in the R2c breakdown. Color scale is saturated in region B. Inset demonstrates cross section of the graph, marked by dash-dotted line.
5.3. Temperature measurements

R2b was never observed to be well-developed. Similar data were also seen for every well-developed RIQH state between \( \nu = 2 \) and \( \nu = 4 \), taking into account many samples and many cooldowns, though high bias features were often less sharp and/or intricate than those shown here.

The electronic temperature in the sample was monitored using temperature-dependent features in the magnetoresistance, which could be calibrated at higher temperatures where thermal equilibration is more reliable and then extrapolated to lower temperatures. In Ref. [23], for example, it was shown that the width of a reentrant state grows with temperature down to 6 mK. Figs. 5.6a and 5.6b show temperature dependence of the R2c reentrant state. Following a similar approach to [23] we confirm that, in the absence of significant current bias, our sample temperature follows that of the mixing chamber down to its base, 13 mK (Fig. 5.6c).

When DC bias is added, the shape of the reentrant feature changes dramatically, and one must be careful to separate the direct effects of the induced electric field from those of Joule heating. Figure 5.6 contrasts \( R_{xx} \) for the reentrant state at \( I_{dc} = 0 \) (Fig. 5.6b) and \( I_{dc} = 300 \) nA (Fig. 5.6d), over a range of temperatures. One clue that the electric field is responsible for most of the reentrant state breakdown in region C comes from the existence of extremely sharp features in the magneto-fingerprint for low lattice temperatures. When the temperature is high enough to melt the reentrant state, on the other hand, the magnetoresistance traces even at high bias are essentially featureless.

5.3.2 Experimental results

The remainder of this chapter focuses on region C (Fig. 5.5b). The 2D data shows numerous fine features that shift in bias and magnetic field. The rippled pattern around \( \nu = 2.58 \) and above 150 nA, for example, may correspond to pinning/depinning transitions of an electron crystal state. Narrow-band noise previously reported in a high-bias phase analogous to region C, but for \( \nu > 4 \) [28], is sometimes attributed to a bubble phase sliding over a pinning potential. One difficulty with this explanation, however, is that the frequency associated with the bubble lattice spacing is many orders of magnitude away from what was observed.

Because region C appears only at high bias, after passing the highly-dissipative region B, it may be tempting to associate it with a RIQH state that has been destroyed by bias-induced heating. It is clear, that C does not represent the remains of a completely molten RIQH electron crystal. Such a melting transition is observed for even higher biases (dashed bound-
5.3. Temperature measurements

Figure 5.6: Evolution of a) $\tilde{R}_{xy}$ and b) $\tilde{R}_{xx}$ at the R2c reentrant state, with temperature between 15 and 48 mK, $I_{ac}=1\,\text{nA}$, $I_{dc}=0$; c) temperature dependence of R2c width at $I_{dc}=0$; d) $R_{xx}$ evolution at $I_{ac}=1\,\text{nA}$, $I_{dc}=300\,\text{nA}$.
5.3. Temperature measurements

Figure 5.7: Region C remains highly temperature dependent, despite significant Joule heating. (a) the full breakdown diamond of R2c, analogous to Fig. 1b but after a refrigerator cycle to 4K. Sharp features in region C serve as effective thermometers, because they are extremely temperature sensitive. Interior box denotes area in (b,c), with hashed region at higher bias considered to be beyond breakdown. (b) Zoom in to box from (a). Sharp features are qualitatively similar in low (200-300nA) and high (400-500nA) bias regions. (c) Analogous data after warming the mixing chamber to 15.6 mK. Sharp features are completely washed out, at both low and high bias.
5.3. Temperature measurements

Figure 5.8: Field sweeps over the spike on the boundary of insulating area near $\nu = 2.55$, see Fig. 1b. Horizontal axis represents estimated lattice temperature controlled by chip carrier resistor, taking into account additional dissipation in sample's contacts due to bias current ($I_{ac}=5$ nA, $I_{dc}=300$ nA) in Fig. 5.5b), where $R_{xx}$ becomes again nearly flat with temperature-independent resistance.

In general, Joule heating of electrons is a severe problem at ultra-low temperatures because the coupling between electrons and the crystal lattice falls off as $T^5$. This is due to a collapse in electronic heat capacity and phonon density of states [88, 92, 93]. On the contrary, region C appears to be anomalously cold (Fig. 5.6d), significantly colder than might be expected by balancing the measured dissipation in the sample with electron-phonon cooling of the 2DEG into the lattice [87].

Figure 5.7 illustrates an extreme sensitivity of high bias data to small increases in lattice temperature. $R_{xx}$ data at $T_{mix} = 14$ mK exhibits numerous sharp features that shift smoothly in bias and magnetic field (Figs. 5.7a,b). Increasing $T_{mix}$ from 14 to 15.6mK (Fig. 5.7c) melts all of these features.
within region C, though a small amount of rippling remains and a sharp transition is still observed at the high-bias boundary (transition to hatched region in the figure).

To study this more quantitatively, I focus on a sharp peak in $R_{xx}$ at $I_{dc}=300$ nA that appears as the reentrant state breaks down (refer to inset in Fig. 5.5b). Fig. 5.8 shows that this peak moves with temperature, just like the edge of the reentrant state at zero bias. This strong temperature dependence indicates that the electronic system remains tightly coupled to $T_{mix}$ even at extremely low temperature, and even with hundreds of nA of bias current applied.

Since described high bias features exist in a very narrow region of temperatures below 30 mK, conventional methods of electron temperature measurement, such as Arrhenius plots of resistivity, are not relevant in region C where the insulating state is already broken. However, an order-of-magnitude estimate of electron overheating by dissipation in region C can be extracted from the observation that qualitative characteristics (sharp features inside region C) appear similar in the low and high bias regions in Fig. 5.7b (14 mK), whereas the character at both low and high bias changes dramatically after only a 1.6 mK increase in mixing chamber temperature (Fig. 5.7c). In other words, raising the bias from 200 to 400 nA apparently affects the electron gas less than raising the mixing chamber from 14 to 15.6 mK.

5.4 Discussion

A dramatic contrast between differential (AC) and static (DC) resistances at high bias, as well as, the analysis of the breakdown, measured at different contacts indicate a nearly-frictionless sliding mechanism that is activated above a critical electric field in the bulk of the sample. On the other hand, low electron temperature immediately suggests electron solid as a microscopic description of the bulk state.

Correlated electron systems under conditions of RIQHE are predicted to have many different phases, especially Wigner-like solids (bubble phases) and nematic/smectic liquid electron crystals (ELC) [78]. As was already mentioned in previous chapters, such phases are susceptible to the external Hall electric field. For example, under certain conditions electron crystal may simply depin from the underlying potential, resulting in a sliding of the entire electron crystal structure instead of a single electron transport [31]. In this case the charge is transferred between the opposite edges of the sample,
However, cold electrons form correlated state. The assumption of cold bulk electrons is possible, as the power is dissipated locally at the boundary between melted and frozen regions, where the potential jump takes place.

Although in high mobility systems the number of scattering centers is extremely low, in the case of reentrant state their pinning might be much more effective due to the fact that they are coupled to the electron crystal. In high biases, when the insulating state is broken, under certain conditions a “plastic flow” can occur [15]. In this case transport is given by few random electrons occasionally hopping between pinned clusters. This creates another mechanism for the current flowing through the bulk of the sample, while the majority of electrons is localized within pinned correlated electron state. However, it has to be mentioned, that in this case delocalized electrons dissipate the energy at every jump, i.e. straight into the bulk. Thus, it is unlikely, that such state could stay cold under high currents.

Alternatively, after the insulating state is broken down, the bulk current distribution can be non uniform. In the most extreme case the current flow is localized to one narrow channel. While specific shape and conductance of this current is set by surrounding insulating cold bulk, the Joule heat, dissipated locally, heats few electrons inside the hot current flow. Even though no mechanism is known, allowing for such strong non-uniformity, the question of current distribution in the bulk after breakdown remains open.

### 5.5 Conclusions

The breakdown of the reentrant states under high current bias was previously explained by the partial melting of the correlated electron solid state by Joule heat, dissipated into electron system. In this chapter I investigated the structure of high bias states, which are stabilized after RIQHE is completely destroyed. Compared with the temperature measurements, the spatial distribution of the Hall potential along the edge, measured at various contact pairs, allows to extract an information about electron properties in the bulk. Although Joule heating is the primary effect, causing RIQH breakdown, the 2DEG in the bulk of the sample turns into conducting state before the correlated electron state is destroyed. It was shown, that after the insulating bulk state is broken, the system retains extremely high temperature sensitivity down to 15 mK.

These data suggest the depinning of electron crystal phase from the underlying lattice as the main candidate for microscopic description of the
5.5. Conclusions

non-equilibrium RIQHE states. Such scenario is qualitatively consistent with existing predictions for sliding dynamics[15], however, a more quantitative analysis of this mechanism awaits further theoretical consideration.
Chapter 6

Bias induced anisotropy at half-filled Landau level

6.1 Motivation

Anisotropic transport around half filling of the higher Landau levels ($N > 1$), usually referred to as a stripe phase, was one of the first experimental evidences of the stabilization of a correlated electron solid states under conditions of quantum Hall effect [11, 12]. On a microscopic level transport anisotropy is believed to result from a unidirectional charge density wave (CDW). Similar to its isotropic sibling, in the anisotropic state, electrons form alternating stripes at neighbouring integer filling factors [3, 14].

Broken rotational symmetry is a fundamental property of a stripe phase, which implies that the electron system must have a preferred direction for orientation of the stripes. Over the last few decades, strong experimental evidence has been collected, showing that the alignment of stripes is connected to the crystallographic orientation. The stabilization of the stripe phase in GaAs samples results, in most cases, in a low $R_{xx}$ probed along "easy" ⟨110⟩ crystal orientation and high $R_{xx}$ along the orthogonal "hard" axis. Despite the large amount of experimental data, the origin of the particular favored stripe orientation remains poorly understood. The theory "by default" does not predict the existence of a preferred direction for CDW. Different groups tried to connect the alignment of the stripes to anisotropic roughness of MBE growth [71] or to the asymmetry of the electron potential in the direction perpendicular to the 2DEG [94]. However, experimentally, no complete correlation between those features and the stripe orientation was found [95, 96]. Furthermore, theoretical studies suggest, that under periodic potential modulation, unidirectional CDW may align both parallel or perpendicular to the modulation direction, depending on a potential strength [97].

In the search of the native underlying potential responsible for such alignment, stripe phases were shown to rotate by 90° under external directional
perturbations. For example, transport anisotropy can be switched by in-plane magnetic field [72, 73], current bias [29], surface acoustic waves [79], or by application of an external strain to the chip [74]. In all these experiments, the directional perturbation is believed to create a symmetry breaking potential which is competing with the intrinsic one. Above the critical values of the external parameter, the native potential becomes weaker and the stripe orientation is set by the direction of the external influence [98].

Most of early experiments on the anisotropic CDW states were limited to the electron densities below $3 \times 10^{11} \text{cm}^{-2}$. However, different transport anisotropy not only was demonstrated at the samples with higher density, but the rotation of the stripes was detected, while the density of the 2DEG was varied within the same sample [75]. Additionally, weakly anisotropic metastable states were observed around half filling of higher Landau levels [99]. All these experimental facts led to the conclusion that the intrinsic alignment potential is more complex than the unidirectional rotational symmetry breaking potential. It was suggested that the alignment of the stripes is actually set by two competing orthogonal components, tending to align CDW along $\langle 110 \rangle$ or $\langle 1\bar{1}0 \rangle$. The relative strength of these potentials changes with electron density, leading to the stripe reorientation at certain density. Around the critical point, the two potentials have similar effects, allowing for metastability and anisotropy suppression.

However, it was recently demonstrated that in high mobility samples transport anisotropy can depend on the filling factor [76], revealing an even more complicated structure of the CDW states around half filling of Landau level. By application of in-plane magnetic field, the authors induced a state with opposite anisotropies away from the half filling and in the center of the stripe phase. Such switching of the anisotropy axis within a single Landau level hints to a strong dependence of the native symmetry breaking potential on the filling factor.

In this experiment a metastable stripe phase around $\nu = 9/2$ is investigated under non-equilibrium conditions in the sample with electron density $n_s \sim 3 \times 10^{11} \text{cm}^{-2}$, which is close to the stripe reorientation density. Despite the metastability, the anisotropy of the non-equilibrium stripe phase under high current biases allows to restore the orientation of the fundamental alignment potential in the sample. These experiments suggest the existence of multiple microscopic phases within each anisotropic state and provide a new vision at the CDW alignment mechanism.
6.2 Experimental results

6.2.1 Measurement details

Most of the data, demonstrated in this chapter, was taken at the same square sample with Wan-der-Pauw contacts and in the same cooldown as in previous experiments. However, the main details of the experimental results were reproduced in multiple cooldowns as well as in an another sample, although the quantitative values, such as anisotropy factor could vary. The temperature of the sample was estimated by the carbon thermometer, glued to the back of the chip carrier and confirmed to be below 20 mK all time during the experiment. All resistances in this chapter were probed using standard lock-in technique with 4 nA AC at 81 Hz.

The GaAs sample was cleaved along $\langle 110 \rangle$ and $\langle 1\overline{1}0 \rangle$ crystallographic planes, defining naturally the longitudinal resistance orientation relative to the crystal lattice. Independently on the direction of bias current applied, the $\tilde{R}_{xx}$ and $\tilde{R}_{yy}$ resistances denote differential resistance measured along corresponding edge of the sample (refer to 3.1.3 for the electrical measurement setup details).

Characteristic longitudinal and Hall resistance traces are shown on figure 6.1. The integer quantum Hall plateaux are well developed showing sharp steps between quantized values. Longitudinal resistances $R_{xx}$ and $R_{yy}$ measured along two orthogonal edges of the square chip demonstrate weak anisotropy for all peaks up to $\nu = 11/2$. This allows to assign easy and hard orientations to the chip (refer to the schematics at Fig. 6.1). The last anisotropic state around $\nu = 9/2$ demonstrates much stronger anisotropy, albeit easy/hard directions are swapped compared to all higher filling factors.

The screening from underlying completely filled Landau levels can vary the strength of inherent potential. Hence the anisotropy switching of the strongest stripe phase in the sample with electron density close to the critical is not that surprising. Nonetheless, before turning to the discussion of this effect, which is the main subject of the current chapter, I want to demonstrate that at half fillings of higher Landau levels ($\nu \geq 11/2$), the ordinary stripe phases are formed despite the fact that anisotropy is strongly suppressed.

In the samples with lower densities with strong anisotropic state around $\nu = 9/2$, the high current biases were shown to stabilize or destabilize the stripe phases depending on their direction with respect to the orientation of the stripes [29]. Specifically, if the current is driven along the stripes,
6.2. Experimental results

Figure 6.1: Characteristic $\tilde{R}_{xx}$, $\tilde{R}_{yy}$ and $\tilde{R}_{xy}$ (for current driven along Y axis) for filling factors $\nu = 4 - 14$, showing the weak transport anisotropy ($I_{AC}=4 \text{ nA}$). Schematics show the preferred orientation of the stripes at high filling factors.

i.e. in the easy direction, the stripe phase is enhanced and its anisotropy remains unchanged up to high currents, until the melting of the entire state due to the Joule heating. On the opposite, if the current bias is driven perpendicular to the stripes in the hard direction, the anisotropy disappears at much lower biases. This effect is usually explained by destruction of the long range stripe order, leading to vanishing transport anisotropy, while the melting of the correlated state by bias is roughly independent on the current direction.

Similar behavior was confirmed in our sample for the anisotropic states in the range of $\nu = 11/2 - 15/2$. Figure 6.2 shows longitudinal resistance measurements for all four possible AC/DC orientations, relative to the crystal axis. Small schematic on the side of each graph demonstrates the relative orientation of AC/DC currents and alignment of stripes with respect to the sample. At zero DC bias $\tilde{R}_{yy}$ measures high resistance state independently
Figure 6.2: Evolution of $\tilde{R}_{yy}$ with a) $DC||Oy$, d) $DC||Ox$ and $\tilde{R}_{xx}$ with b) $DC||Oy$, c) $DC||Ox$. Dashed lines show approximate melting boundary of the stripe phase. Schematics depict stripe alignment and measurement orientation relative to DC bias, driven in a-b) hard and c-d) easy direction. Short (long) wiggly lines depict orientation of stripes suppressed (enhanced) by bias.
6.2. Experimental results

on the orientation of DC component (Fig. 6.2a,d). Similarly \( \tilde{R}_{xx} \) probes low signal in accordance with easy measurement orientation along the stripes (Fig. 6.2b,c). However, in this sample remains of a metastable state could be observed even at high filling factors at very low biases (\( I_{DC} < 100\ nA \)). It is represented by weak zero bias anomalies on figure 6.2 and is very likely to be responsible for the low \( \tilde{R}_{xx}/\tilde{R}_{yy} \) anisotropy at field sweep traces (Fig. 6.1).

Once even a small DC bias is applied, the metastable state is destroyed and high transport anisotropy revives. If the bias current is driven along the stripes (Fig. 6.2c-d) the anisotropic state survives up to the biases above 3 uA. This is opposed to the case with current driven in the perpendicular direction (Fig. 6.2a-b), where no significant anisotropy is detected above approximately 1 uA DC.

This behavior is in complete agreement with the measurements, reported by other groups [29]. Therefore, \( Ox \) axis can be unambiguously identified as the preferred stripe orientation in a low electron density limit. Following the conventional notation, hereafter the direction showing high longitudinal resistance is referred to as “hard”, while low resistive direction is called “easy” orientation.

6.2.2 Bias measurements at \( \nu = 9/2 \)

Now I turn to the description of measurement results at \( \nu = 9/2 \). Figure 6.3a depicts schematically the stripe orientation preferred by 9/2 state in this sample. The metastable state is thought to have two alignment potentials of approximately equal strength, therefore, the stripes are randomly oriented along \( Ox \) or \( Oy \) axis (Fig. 6.3a top). However, the electron system can relax into anisotropic state, for example as a result of annealing [99], aligning all stripes along one orientation. This case is depicted at the bottom of Fig. 6.3a. Note, that at 9/2 state transport anisotropy is opposite to the one measured at half filling of higher Landau levels. Therefore stripes are shown to be oriented along \( Oy \) axis on the bottom schematics of Fig 6.3a.

Figure 6.3b shows zero DC bias traces along two perpendicular crystallographic directions. The longitudinal resistance is clearly anisotropic, although anisotropy is weak. Another sign of nearly metastable state comes from hysteretic behavior between up and down field sweeps [99]. However, zero DC bias traces undergo a dramatic change after a 3 uA DC bias current is applied for a short period of time (\( \sim 10\ s \)) at each point (Fig. 6.3c,d). After the application of the bias, transport properties of 9/2 state become strongly dependent on the exact filling factor. In the vicinity of half filling, the anisotropy disappears and the longitudinal resistance does not depend
6.2. Experimental results

Figure 6.3: a) Schematic of a measurement with at metastable (top) and anisotropic (bottom) $\nu = 9/2$ stripe phase; b) comparison of $\tilde{R}_{xx}$ and $\tilde{R}_{yy}$ field scans at zero DC bias, solid (dashed) arrow mark field sweep down (up); 0 nA DC $\tilde{R}_{xx}$ and $\tilde{R}_{yy}$ field scans annealed at 3 uA DC along c) $O_y$ and d) $O_x$ axis at every filling factor.

(\text{within the uncertainty of few hundred Ohms}) neither on measurement direction, nor on the orientation of the DC bias applied at each point. Away from $\nu = 9/2$, on the contrary, anisotropy is strongly enhanced in comparison to field sweeps on Fig 6.3b.

Since this effect does not depend on the direction of current bias, it suggests that the annealing due to Joule heating is the primary effect causing the switching of the state. Interestingly, the new anisotropic state is perpendicular to the conventional orientation of stripes previously identified for higher Landau levels. This means that the ground state of the stripe phase away from 9/2 is orthogonal to the conventional alignment of the stripes at
lower electron densities or higher filling factors (Fig 6.2).

These experimental findings are consistent with the metastability measurements of the stripe state at high density samples, reported in literature [99]. Two different anisotropic directions at \( \nu = 9/2 \) state were recently reported in by other group [76]. However, the main experimental finding of this work is that the critical in-plane magnetic field, which re-aligns the CDW phase, strongly depends on exact filling factor within one anisotropic state. Although the fact that the stripe phase behavior depends dramatically on filling factor is intriguing, it does not provide any information about the intrinsic ground state of the CDW. For example, these data could be explained by transition between Coulomb dominant and modulation dominant (modulation is given by in-plane field in this case) regimes, suggested theoretically [97].

Anisotropic states around \( \nu = 9/2 \) were previously investigated in bias at significantly lower electron densities [29]. This resulted in the stabilization of the conventional anisotropic state and no metastability was reported. However, as was already confirmed at the beginning of this chapter for other stripe states at higher Landau levels, such measurement allows to put the system away from the metastable state and directly probe intrinsic alignment potential. In contrast to effects of the in-plane magnetic field, which just creates the primary modulation potential above the critical point, driving the current affects the pinning of the stripes in an additive way. In other words, if the current is driven parallel to natural orientation of the stripes, the anisotropy of the state is strongly enhanced under high biases. This allows to investigate the initial state of the system by its response to the external perturbation instead of investigation of the system affected by an external field. In order to study the fundamental stripe state around \( \nu = 9/2 \) in our sample, the transport anisotropy around was probed away from equilibrium at different filling factors.

Figures 6.4a,d demonstrate the direction of DC bias with respect to the sample as well as \( O_x \) and \( O_y \) directions of AC current used during the \( \bar{R}_{xx} \) and \( \bar{R}_{yy} \) measurements correspondingly. The 2D maps on Fig.6.4b,c show \( \bar{R}_{xx} \) and \( \bar{R}_{yy} \) under DC current applied along the \( O_y \) axis. Similar data for the case of \( DC \parallel O_x \) is shown on figure 6.4e,f. One can easily identify three anisotropic high bias regions on Fig.6.4 in the middle of the stripe state \((\nu \sim 4.45 - 4.6)\) and two regions, one on each side \((\nu \sim 4.375 - 4.45\) and \( \nu \sim 4.6 - 4.675 \).
6.2. Experimental results

Figure 6.4: Schematic description of stripe orientation in the vicinity of $\nu = 9/2$ relative to DC bias, driven in a) hard and d) easy direction. Short (long) wiggly lines depict orientation of stripes destabilized (stabilized) by bias. Evolution of $\tilde{R}_{yy}$ with b) $DC||Oy$ and f) $DC||Ox$. Evolution of $\tilde{R}_{xx}$ with c) $DC||Oy$ and e) $DC||Ox$. Dashed lines show approximate melting boundary of the stripe phase.
6.2. Experimental results

For the purpose of this chapter, I investigate the anisotropy of high bias (above 100 nA) states. Consider first the side regions at $\nu \sim 4.375 - 4.45$ and $\nu \sim 4.6 - 4.675$. If the DC bias is applied along $Oy$ axis (Fig. 6.4a-c), the transport demonstrates strong anisotropy ($\tilde{R}_{xx} \gg \tilde{R}_{yy}$) even at $I_{DC} > 1$ uA. At the same time high bias anisotropy already vanishes at 500 nA of DC bias applied in perpendicular direction along $Ox$ (Fig. 6.4a-c). In order to make this distinction clearer $\tilde{R}_{xx}$ and $\tilde{R}_{yy}$ bias traces at $\nu = 4.42$ (Fig. 6.5a,b) and $\nu = 4.62$ (Fig. 6.5c,d) are contrasted for two perpendicular orientations of DC component. These filling factors correspond to the center of the new anisotropic states stabilized by annealing (Fig. 6.3) and coincide with the filling factor of inverse anisotropy states under parallel magnetic field in [76].

Fig. 6.5a,c compares $\tilde{R}_{xx}/\tilde{R}_{yy}$ bias scans at different sides from the half filling when DC drive enhances the stripes. At both filling factors $\tilde{R}_{xx}$ probes high resistance, corresponding to the “hard” measurement orientation. Notably, significant anisotropy survives up to 2 uA of bias in both cases as well. If the bias is applied along $Ox$ axis (Fig. 6.4b,d), the “hard” and “easy” orientation remain the same, however no anisotropic transport is detected above approximately 400 nA. These results are completely consistent with previous findings: the bias driven perpendicular to the stripes destabilizes the stripe phase, killing the long range order. This leads to an anisotropy vanish at the currents well below the critical melting current. Presumable orientation of stripes is shown on the insets on figure 6.5. As before the short (long) wiggles mark suppression (enhancement) of the stripe by bias driven perpendicular to (along) the direction of their initial alignment potential.

Notice that anisotropy of a low bias state ($< 100$ nA) could be significantly different from the one above $\sim 200$ nA. This effect is most clear on Fig. 6.4b,c and could be attributed to a bias reorientation threshold. At low currents, the effects of current alignment are negligible and the sample state is mostly independent on bias. However, above $\sim 200$ nA the zero bias state is destroyed and the anisotropy demonstrates a sharp change.

Turning back now to Fig. 6.4 let’s focus on the center of the state $\nu = 4.45 - 4.6$. In this range stripe phase is enhanced by DC current flowing along $Ox$ axis (Fig. 6.4d-f). Longitudinal resistance anisotropy is clearly seen at high biases, albeit the isotropic state is observed around zero DC currents, which is consistent with the measurements on Fig. 6.3. Nonetheless, there is no measurable transport anisotropy under the bias above 200 nA when DC current is driven along $Oy$ (Fig. 6.4a-c). This effect might result from a very weak pinning potential. Therefore the stripe phase is immedi-
6.3 Discussion

Figure 6.5: Bias scans of anisotropic states, stabilized by annealing at a,b) \( \nu = 4.42 \); c,d) \( \nu = 4.62 \). The direction of the DC bias as well as the expected orientation and strength of the stripes is shown on the schematics.

...ately destructed by bias beyond the threshold and anisotropic state is never stabilized under such conditions.

Experimental data show, that the orientation of natural alignment potential changes with the filling factor: it tends to align the stripes along \( O_x \) axis in the center region and in perpendicular direction on the sides.

6.3 Discussion

The schematic phase diagram on figure 6.6 summarizes experimental data. Initial orientation of the stripe phase, as well as the direction of AC and DC bias components, are schematically shown on the left side of each graph. The anisotropic phase is divided into three regions with different anisotropy in the center and on the sides. The dark (light) background of each state depicts high (low) longitudinal resistance under high bias, corresponding to
Figure 6.6: Schematic phase diagram of the stripe phase in samples with electron density around the critical point.
the measurement along hard (easy) axis. The icon of the stripes demonstrates the direction of the alignment, while the wiggle length shows the enhanced by bias, regular or suppressed stripe phase correspondingly. Horizontal (vertical) orientation of stripes symbol depicts alignment along $Ox$ ($Oy$).

The data shows that for the center state the bias stabilizes regular CDW orientation (along $Ox$), which coincides with the self-organized orientation of the stripe phase at higher Landau levels. This results in the existence of anisotropy up to high biases (strong state) if the DC current is driven along $Ox$ (Fig. 6.6c,d), while the same anisotropy survives to much lower biases (weak state) if the DC orientation is switched to $Oy$ axis (Fig. 6.6a,b). It is important to mention, that initial orientation of stripe phase around $\nu = 9/2$, stabilized during the field sweeps at zero bias, was identified along $Oy$ orientation. However, the fact that it vanishes at zero bias after annealing, as well as high bias measurements, hint at the isotropic state results from application of bias, while the inherent alignment potential is oriented along the $Ox$ axis. Analogous behavior is observed for the high bias states on the sides of $9/2$. These states seem to align along $Oy$ axis in bias, retaining low bias orientation of the anisotropy, which appears after annealing. Although the stripe orientation can be varied at zero bias, the experimental data suggests that the natural potential is strongly dependent on the filling factor.

I suggest the following model explaining experimental results. The CDW pinning potential possesses two important features. First, it consists of two competing components, tending to align the stripes along two orthogonal orientations. Second, those components are strongly dependent on the filling factor. Figure 6.7a shows qualitative profile of two potentials and their relative strength for the case of conventional low density sample. The anisotropic region is marked by color background. In this case, the $Ox$ alignment potential is much lower than $Oy$, therefore the stripes are always aligned in that orientation. However, the increase of density leads to decrease of difference between $Ox$ and $Oy$ potentials, making them overlap in a much narrower region (Fig. 6.7). This leads to a much smaller energy difference between the two states with different orthogonal orientations, which allows the existence of the metastable state around $9/2$. Conversely, it is the anisotropic state with the opposite $Oy$ anisotropy that gets stabilized on the sides. The system with relative pinning potential distribution shown on the figure 6.7b has two different ground states with opposite alignment in the middle and on the sides of the stripe phase. Although the close energy difference between two states is small, this anisotropy does not realign by itself. However, the ground state affects the properties of the 2DEG in non-equilibrium state under high bias currents.
6.4 Conclusions

In conclusion, the bias induced anisotropy of the stripe phase around 9/2 was investigated in the high electron density sample. This puts the electron system under conditions of strong competition between different potentials, tending to align the state along two perpendicular orientations. Although the origin of those states is still unclear, it was demonstrated that the change of the transport anisotropy is most likely caused by a change of the structure of ground pinning potential, rather than by an electron phase change. The orientation of the stripes was observed to be extremely sensitive to the filling factor, leading to a different high bias transport anisotropies on the sides and in the center of the state. The data are consistent with recent reports from other groups, providing another experimental evidence for the strong dependence of pinning potential on the electron filling factor [76].

An alternative explanation to the model of competing pinning potentials could be the stripe realignment due to the change of electron-electron interaction strength, suggested in [97]. Such interpretation suggests that there is a single pinning potential, however the electron-electron interaction changes when the density or filling factor is varied. Around the critical point, the system experiences transition between Coulomb dominant and modulation dominant regimes which makes the stripes to switch the alignment direction between orthogonal and parallel to the external modulation.

Figure 6.7: Schematic description of the competition of two pinning stripe potentials in a) low electron density; b) close to the critical point. Wiggly lines in the middle depict the anisotropy of the ground state.
Chapter 7

Summary and future experiments

In this thesis, I have studied the transport properties of the electron solid state, which are set away from equilibrium by application of high bias. Although this technique requires driving high currents through the sample at low temperatures, it was demonstrated that under certain conditions cold correlated electron states can survive. This allows one to probe the cold electron solid states away from equilibrium, which is very important in this particular case. Since correlated electron solid states are normally insulating at zero bias, different microscopic phases are indistinguishable from each other in transport. On the other hand, electron solids form at temperatures below 100 mK (in the most extreme cases of N=1 Landau levels, the melting temperature is even lower, less than 50 mK in the best quality samples). Such fragile properties of correlated electron solid states leaves very few experimental techniques available for their probing. Electrical transport is the most suitable one.

Although probing of an electron system away from equilibrium provides a lot of information about its state, the interpretation of experimental results is difficult. This is especially true in the case of the transport measurements in macroscopic sample, since various inhomogeneities in the 2DEG lead to stabilization of different local states which are averaged in the resulting signal.

In the first part (Ch. 4) of this work I focus on the investigation of high bias states at filling factors, corresponding to the conditions of RIQHE. Specifically, the mechanism of insulating state breakdown was investigated in details. It was shown, that the sample is strongly inhomogeneous during breakdown However, investigation of spatial distribution of the electrical potential enables the probing of some microscopic properties of the system. For example, one could distinguish electron- and hole-like reentrant states. The breakdown was found to propagate chirally from source and drain contacts with a sense that depends on the electron- or hole-like character of the reentrant state. This phenomenon appears to result from a thermal
Chapter 7. Summary and future experiments

runaway effect due to phase segregation and dissipation hotspots; it was observed only in (correlated) RIQH states, not fractional or integer states, pointing to their qualitatively distinct thermodynamic properties. This experiment shows the danger in interpreting macroscopic measurements at a microscopic level, especially where electronic phase transitions are sharp. On the other hand, it demonstrates the power of using breakdown characteristics as a probe into the thermal properties of correlated electron states. Looking ahead, it would be particularly interesting to investigate similar breakdown phenomenology in combined Corbino/hall bar geometries, where hotspot-induced breakdown could be included or avoided as desired. Similarly, adding multiple small contacts within the interior of a sample could enable melted and frozen phases to be probed separately and the nature of the transition region to be measured directly.

Surprisingly, the electron temperature under high bias was shown to be unexpectedly low after RIQHE breakdown. This means that high currents could be driven through the sample without significant heating, suggesting the formation of a correlated non-insulating state after breakdown. The second part of this thesis (Ch. 5) was devoted to the investigation of conducting high bias states, arising after breakdown of the normal insulating reentrant states. By analyzing the distribution of electrical potential along the sample’s edge, it was experimentally demonstrated that non-insulating correlated bulk electron states exist even after the conventional RIQHE is completely destroyed in the entire sample. This shows that investigation of the non-equilibrium states might help not only to access the thermodynamic properties of the system, but to probe properties of the underlying pinning potential. Furthermore, the theory predicts a wide variety of dynamic phases formed in electron crystals under external force. This makes phase transitions between different correlated dynamic electron states a good alternative for microscopic explanation of the cold non-insulating states under high bias. Although there are several reports from literature about observation of electrical noise after the RIQHE breakdown, the data from Chapter 5 demonstrate a lot of small reproducible features in the electrical signal in this region. The signal results in correlated pattern of ripples in DC bias/magnetic field coordinates. Since this state is cold, it is hard to connect such behaviour with a simple depinning of the electronic crystal. Therefore, driving high bias current through the sample might be an efficient instrument for probing dynamic electron solid states.

Finally, the ground pinning potential of the anisotropic phases was investigated by means of anisotropic stripe phase enhancement by current bias (Ch. 6). This effect was used as a probe for the direction of the intrinsic
alignment potential at different filling factors. The data suggest that at high density samples the strong dependence of transport anisotropy on the filling factor is set by the change of stripe orientation, rather than the phase transition between different CDW states with orthogonal anisotropy. Investigation of the high bias state anisotropy around filling factor $\nu = 9/2$, supported the conclusion that the fundamental aligning potential in the center of that state does not depend on the number of the Landau level, even though the state demonstrates orthogonal anisotropy in zero bias. This experimental fact suggests, that the Coulomb interaction plays a crucial role in the stripe alignment mechanism. For instance, instead of rotation of alignment potential, stripe reorientation could be caused by the change of optimal direction from parallel to orthogonal orientation relative to the modulation potential.

Eventually, I would like to suggest several experiments as a possible direction for future research. First of all, I would like to draw the reader’s attention to the zero bias anomaly appearing at very low current biases in many non-integer filling factors all over the range of $\nu = 2-8$. It was demonstrated in Ch. 6 that around the half filling of the Landau level this anomaly is very likely to result from metastability of the stripe phase. However, similar data were observed at the edges of the reentrant states at $N=1$ Landau levels. Figure 7.1 shows an example of the zero bias peak at the edges of an R2c reentrant state at several temperatures. Since those states are believed to be isotropic and no reliable experimental evidence of anisotropic transport in RIQHE has been reported so far, it is hard to connect these effects to the metastability of a unidirectional CDW phase all over the entire region of filling factors. On the other hand, theory predicts instability of the CDW state with respect to the melting into smectic and nematic phases [77]. In [29] the authors hypothesize, that the isotropy of reentrant states in macroscopic samples is given by the formation of the electron liquid crystal phase without long range order, rather than the bubble phase. In this case local alignment of the stripe domains along the Hall electric field could lead to enhancement of the insulating state and, consequently, zero bias peak. The key feature of metastability is hysteretic behavior with respect to the direction of the magnetic field sweep. Investigation of temperature dependence and hysteresis in the magnetic field could, therefore, help to favor one theoretical explanation over the other.

Another interesting future experiment might be related to the breakdown of reentrant state at higher Landau levels. Figure 7.2 shows a 2D map of $R_{xy}$ in the range of $\nu = 4 - 8$. Reentrant states at $N=2$ Landau level demonstrate ordinary diamond-like collapse in bias (Fig. 7.2a). However, at $N=3$ (Fig. 7.2b) the diamonds show splitting. In other words, reentrant
Figure 7.1: $\tilde{R}_{xx}$ bias dependence of R2c at different temperatures a) 24.8 mK b) 22.9 mK c) 21.2 mK. Color scale is saturates at 500 Ω.
states R6a and R7a, for instance, at a bias current of around 2.5 uA split into two quantized parts, while in the middle ($\nu \sim 6.8$ and $\nu \sim 7.8$ respectively) $R_{xy}$ deviates from the quantized integer plateau. Such splitting was always observed only for N=3 Landau level in several samples and cooldowns and never for N=2. However, the diamond splitting was not reproducible in every cooldown of the same sample. Although, such non-reproducibility could be attributed to the absence of complete control on the sample preparation process, more samples need to be tested in several cooldowns to obtain reliable experimental evidence for such splitting. If this effect is real and not caused by inhomogeneities in the sample, such splitting is very interesting. Effectively, this means that reentrant states corresponding to N=3 Landau levels, in contrast to N=2 ones, are less stable at the center than on the sides. The microscopic description of reentrant states at higher Landau levels predicts a sequence of bubble phases with a different number of electrons per bubble within one reentrant state. Numerical calculations [19, 21] confirmed the stabilization of a different number of bubble phases within R4,5 and R6,7 reentrant states. Therefore, such splitting could be the first direct experimental observation of the different microscopic bubble phases.
Figure 7.2: $R_{xy}$ bias scans in the range a) $\nu = 4 - 6$ and b) $\nu = 6 - 8$. 
Bibliography


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