# FLOW CHARACTERISTICS OF SINGLE AND DOUBLE LIQUID JET IMPINGEMENT

by

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#### Abstract

In the past few decades, the increasing demands for superior cooling systems in various industries have shifted the focus onto impinging liquid jets as an efficient, powerful cooling technique. Although much has been done on the thermal aspects of jet impingement, the available knowledge still lacks an in-depth understanding of the fluid dynamics involved in the phenomena that dictate the associated transport mechanisms. The present thesis has been planned to analytically and experimentally study the fluid dynamics of the interaction between a liquid, free-surface impinging jet with a solid surface, and also with a neighboring jet.

The circular hydraulic jump as a key feature of a free-surface jet impingement was analyzed. The focus was given to the influence of the target plate on the behavior of the hydraulic jump. Two conditions for the target plate were examined: large plates with capillary limit at the edge, and also small target plates. It was experimentally and theoretically discussed that the circular jumps with these two conditions exhibit different behaviors from those presented in the literature. Furthermore, a systematic Froude number analysis on circular hydraulic jumps was carried out and the significant differences between circular jumps and the classical jumps (i.e. in open channels) were Highlighted. It was shown that due to the significant influence of the surface tension in circular jumps, the critical Froude number differs from that observed in the classical jumps and could be larger than unity.

Moreover, the interaction between the flow fields formed by two jets impinging on a solid surface was investigated in detail. Understanding of this interaction is of significant importance due to the promising potentials of multiple jets for high heat flux applications. Two different configurations were studied: two vertical jets, and two inclined jets. The fluid dynamics involved in the collision between two thin liquid films formed on the surface was theoretically analyzed. A systematic experimental study was also carried out to examine the effects of different parameters on the flow field interaction. The experimental results were then compared to the theoretical predictions to verify the presented models. Good agreements were observed between the presented theory and the experimental data.

### Preface

A version of Chapter 4, and some parts of Chapter 5 (Sections 5.4.2 - 5.4.4) have been published. Mohajer, B. and Li, R., 2015. Circular hydraulic jump on finite surfaces with capillary limit. Physics of Fluids, 27(11), 117102. I wrote the manuscript which was further revised by Dr. Li.

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## **Table of Contents**

Abstract	ii
Preface	
Table of C	ontentsiv
List of Tak	oles viii
List of Fig	uresix
List of Syn	nbolsxviii
Acknowled	lgementsxx
Dedication	xxi
Chapter 1	Introduction and Thesis Organization1
1.1	Introduction1
1.2	Jet Impingement
1.3	Multiple Jet Impingement
1.4	Research Objectives
1.4.1	Research Task 1 – Circular Hydraulic Jump Formed on Finite Surfaces with Capillary
	Limit
1.4.2	Research Task 2 – Circular Hydraulic Jump on Small Target Plates, and a Froude
	Number Analysis
1.4.3	Research Task 3 – Double Vertical Jet Impingement
1.4.4	Research Task 4 – Double Inclined Jet Impingement
Chapter 2	Literature Review 12
2.1	General
2.2	Single jet impingement
2.3	Froude number
2.4	Multiple Jet Impingement
2.5	Double Jet Impingement
Chapter 3	Experimental Setup and Methodology
3.1	Overview
3.2	Experimental Setup
3.2.1	Single Jet Setup
3.2.2	Double Jet Impingement
3.3	Working Fluid

3.4	Uncertainty	
Chapter 4	Hydraulic Jump with Capillary Limit at the Edge	
4.1	Overview	
4.2	Experimental Study	
4.2.1	Experimental methodology	
4.2.2	Experimental results	41
4.3	Theoretical Study	45
4.3.1	Force Analysis	45
4.3.	1.1 Surface tension force	
4.3.	1.2 Hydrostatic force	
4.3.	1.3 Viscous friction force	51
4.3.	1.4 Force from the pre-jump flow	
4.3.	1.5 Analysis of the 2 <sup>nd</sup> control volume	
4.3.2	Validation	55
4.4	Discussion	58
4.4.1	Gradual jump	58
4.4.2	Scale Analysis	61
4.5	Summary	
Chapter 5	Hydraulic Jump on Small Surfaces and a Froude Number Analys	is 64
5.1	Overview	64
5.2	Experimental methodology	65
5.3	Experimental Results	66
5.3.1	Hydraulic jump diameter	
5.3.2	Post-jump film depth	70
5.3.3	Post-jump region	73
5.4	Froude Number Analysis	75
5.4.1	Post-jump Froude number	75
5.4.	1.1 Constant post-jump Froude number	
5.4.2	Pre-jump Froude number	
5.4.3	Effect of surface tension	
5.4.4	Critical Froude number	
5.4.4	4.1 Theoretical Study	
5.4.4	4.2 Validation	
5.5	Summary	92
		v

Chapter 6	6 Double Vertical Jet Impingement	
6.1	Overview	94
6.2	Description of phenomenon	94
6.3	Experimental Study	97
6.3.1	Experimental methodology	97
6.3.2	Experimental observations	
6.3	3.2.1 Jump-jump interaction	
6.3.3	Rising sheet patterns and breakup regimes	
6.3	3.3.1 Regime I: Capillary instability	
(	6.3.3.1.1 Regime I-A Pre-sheet formation	104
(	6.3.3.1.2 Regime I-B Periodic drop	104
(	6.3.3.1.3 Regime I-C Closed rim	
(	6.3.3.1.4 Regime I-D Rim breakup	106
6.3	3.3.2 Regime II: Kelvin-Helmholtz (KH) instability	106
(	6.3.3.2.1 Regime II-A. Open-rim	107
(	6.3.3.2.2 Regime II-B. Transition into atomization	107
6.4	Theoretical Study	109
6.4.1	The rising sheet profile	109
6.4	4.1.1 Geometric relations	114
6.5	Results and Discussion	115
6.6	Unequal Double Jet Impingement	121
6.7	Summary	
Chapter 7	7 Double Inclined Jet Impingement	
7.1	Overview	124
7.2	Description of Phenomenon	
7.3	Experimental Methodology	127
7.4	Theoretical Analysis	
7.4.1	Single inclined jet impingement	
7.4	4.1.1 Impingement region	
7.4	4.1.2 Thin wall jet region	131
7.4.2	Double Inclined Jet Impingement	134
7.4	4.2.1 Geometric Relations	134
7.4	4.2.2 Stagnation Line – Theoretical Model	136
7.5	Experimental Results and Discussion	139
		vi

7.5.1	Single inclined jet	
7.5.2	Double inclined jet impingement	141
7.5.2	2.1 General	141
7.5.2	2.2 Stagnation line	
7.5.2	2.3 Influence of jet inclination angle	
7.5.2	2.4 Influence of flow rate	148
7.5.2	2.5 Influence of fluid viscosity	
7.5.2	2.6 Influence of jet-jet relative position	
7	5.2.6.1 Spacing defined by $S_{rel.}$ and $\psi_{rel.}$	154
7.6	Unequal Double Jet Impingement	
7.7	Summary	
Chapter 8	Summary, Conclusions, and Future Work	
8.1	Summary	
8.1.1	Hydraulic Jump with the Capillary Limit at the Edge	
8.1.2	Hydraulic Jump on Small Surfaces and a Froude Number Analysis	
8.1.3	Double Vertical Jet Impingement	
8.1.4	Double Inclined Jet Impingement	
8.2	Concluding Remarks	
8.3	Recommendations for the Future Work	
8.3.1	Double inclined jet impingement with varied configurations	
8.3.2	Thermal Analysis	
References	•••••••••••••••••••••••••••••••••••••••	
Appendice	s	
APPEND	IX A Detailed Derivation of the Surface Tension Force	
APPEND	IX B Detailed Derivation of the Viscous Friction Force	

## List of Tables

Table 3.1.	Physical properties of the working fluids.	35
Table 5.1.	Summary of the observations from Figure 5.15.	90

## List of Figures

Figure 1.1.	Jet impingement cooling applications: (a) hot metal rolling process [2], (b)
	cooling turbine blades [3], (c) Electronic cooling [4], (d) jet impingement
	packaging for electronic cooling in form of a cold plate [5]2
Figure 1.2.	(a) A water jet impingement, (b) Schematic of the flow field formed by a
	jet impingement
Figure 1.3.	Heat transfer coefficient distribution resulting from a single jet
	impingement on a heated surface with uniform heat flux
Figure 1.4.	Double water jet impingement, (a) identical vertical jets, (b) vertical jets
	with unequal flow rates, (c) inclined identical jets (bi-planar jets)
Figure 2.1.	Different types of jet impingement, (a) free-surface jet, (b) submerged
	jet, (c) confined submerged jet 12
Figure 2.2.	(a) A free-surface water jet impingement and the resulting circular
	hydraulic jump, (b) Schematic of the flow field on the surface 13
Figure 2.3.	Schematic of an array of free-surface jets impinging on a solid surface 23
Figure 3.1.	(a) Schematic of the experimental setup used for the single jet
	impingement experiments, (b) the nozzle holder
Figure 3.2.	A water jet impinging on a circular target plate. (a) Top view image of
	the jet impingement and the formed hydraulic jump on the target plate,
	(b) Side view image of the liquid film on the surface
Figure 3.3.	(a) Schematic of the experimental setup used for the double jet
	impingement experiments, (b) nozzle configurations
Figure 3.4.	Interaction between two impinging jets, (a) double vertical jet
	impingement, (b) double inclined jet impingement with $\phi$ =50° (jets are
	placed in two parallel planes), (1) underneath images, (2) side view
	images
Figure 3.5.	Flow meter calibration for viscous fluids, (a) water-solution wt. 50%, (b)
	water-solution wt. 65%
Figure 4.1.	(a) The hydraulic jump formed by a round water jet impinging on a
	circular disc, (b) Side view of the water jet impinging on a circular disc,

	where water flows off the disc only from one spot at the edge. (c)
	Schematic of jet impingement on a circular disc on which the post-jump
	film has a stable rim formed at the disc edge
Figure 4.2.	Experimental results of the hydraulic jump radius $R_j$ and the post-jump
	depth $H$ for varied flow rates, disc sizes, and working fluids. The slope of
	the linear fitting for each fluid is the post-jump Froude number defined by
	Eq. (4.1b). The inset graph shows the slope of the linear fitting ( $Fr_o$ ) for
	different disc sizes using water jets
Figure 4.3.	(a) The jump radius $R_j$ and the post-jump film depth H formed by water
	jets impinging on a circular disc of $R_s = 20$ mm. (b) The measured jump
	radius $R_j$ of all the tests conducted here in comparison with the scaling
	law $R_j \propto Q^{5/8}$ , and also with the critical radius $r_c$ for the boundary layer
	development in the pre-jump region
Figure 4.4.	(a) The simplified view of the first control volume with an angular
	dimension $\Delta \theta$ and radial dimension from the hydraulic jump $R_j$ to the
	edge of the disc $R_s$ ; (b) the central plane of the control volume with all
	forces projected on the plane
Figure 4.5.	(a) An arbitrary portion of Figure 4.4b shows the vectors on the free
	surface for the analysis of the surface tension force; (b) More geometric
	details of the 1 <sup>st</sup> control volume
Figure 4.6.	The 2 <sup>nd</sup> control volume is selected at the hydraulic jump location
Figure 4.7.	Contact angles formed by the free stable liquid film at the disc edge 56
Figure 4.8.	Comparison between the theory and the experimental data for varied flow
	rates, disc sizes, and working fluids. (a) Evaluation of Eqs. (4.15), (4.17),
	and (4.21) using the experimental data. The x-axis starts from
	$(r_c / a)^3 / \text{Re} = 0.043$ . (b) The theretical predictions of the jump radius versus
	the experimental measurements
Figure 4.9.	A water jet (Q=3.75 cm <sup>3</sup> /s) impinging on: (a) a large disc $R_S = 40$ mm;
	(b) a small disc $R_S = 17.5$ mm. (c) The height of free surface measured
	using a height gauge (the gauge needle is seen in the images). Quarter

circle fitting is applied to the free surface at the jump location. The inset
schematically shows a quarter circle jump shape
Comparison between different forces (theoretically calculated) in Eq. (4.2)
for a given condition of $R_s = 50mm$ , $a=0.375$ mm, $\beta_s = 100^\circ$ , and varied
flow rates of water jets
(a) Circular hydraulic jump formed by a liquid jet impinging on a solid
surface. (b) Schematic of a circular hydraulic jump formed by jet
impingement on a circular plate. The inset sketch shows more fluid
dynamics details
(a-f) A water jet with the diameter $d=0.75$ mm and the flow rate $Q=3.75$
cm <sup>3</sup> /s impinging on plates with different plate sizes $D_s$ , (g) The measured
jump diameters $D_j$ associated with images (a-f) versus the plate size $D_s 67$
(a-f) Water jets (d=0.75 mm) with varied flow rates impinging on a fixed
plate with $D_s = 30$ mm. (g) The measured jump diameters $D_j$ associated
with images (a-f) versus the flow rate $Q$
The hydraulic jump diameter versus the plate size for different flow rates
(water jets <i>d</i> =0.75 mm)
The hydraulic jump diameter versus the flow rate on different plates with
two jet diameters, (a) d=0.75 mm, (b) d=2 mm70
(a-f) Side view images of the same tests shown in Figure 5.2 with a water
jet of d=0.75 mm and a constant flow rate of $Q = 3.75 \text{ cm}^3/\text{s}$ on different
target plates, (g) the measured post-jump depths versus the plate size 71
(a-f) Side view images of the same tests shown in Figure 5.3 with water
jets of d=0.75 mm impinging on a plate of $D_s = 30$ mm and varied flow
rates, (g) the measured post-jump depths versus the flow rate72
The post-jump film depth versus the flow rate on varied plate sizes (water
jets with <i>d</i> =0.75 mm)73
The circular jump diameter $D_j$ is plotted as a non-dimensional post-jump
region $(D_s - D_j) / D_s$ versus the flow rate for different plate sizes. The same
results have been presented in Figure 5.574

- Figure 5.11. The data presented in Figure 5.5 are selected for several plate sizes to apply the relation presented by Duchesne et al. [23]:

$$Y = \frac{Q}{Fr_o} = \sqrt{g}\pi D_j \left[ H_{\infty}^4 + \frac{6}{\pi} \frac{\upsilon Q}{g} \ln \left( \frac{D_s}{D_j} \right) \right]^{3/8} \dots 78$$

Figure 5.12. The post- jump Froude number versus the flow rate for the jumps formed The pre-jump Froude number (calculated using Eq. (4.1a) with  $\lambda$ =1.12) Figure 5.13. Non-dimensional surface tension force Bo<sup>-1</sup> calculated for all the tests in Figure 5.14. comparison with the flow momentum force  $Fr_o^2$  and hydrostatic force 0.5.85 Figure 5.15. (a) Froude number analysis of circular hydraulic jumps with varied prejump velocity profiles and surface tension forces; (b) Enlargement of the region where  $F_{r} \sim 1$  to show critical Froude numbers and the coupling of Figure 5.16. (a) The post-jump Froude number versus the non-dimensional post-jump length. Gray solid symbols are used to highlight  $Fr_o > 1$ . The solid symbols are plotted in the inset graph comparing theoretical values of  $Fr_{o,max}$  with the actual supercritical values of  $Fr_o$ . (b) The  $Bo^{-1}$  versus the nondimensional post-jump length. .....91 Figure 6.1. (a) Schematic of two vertical, free-surface impinging jets showing major flow regions: (1) free jet, (2) jet stagnation point, (3) inner thin wall jet, (4) outer thin wall jet, (5) stagnation line (fountain formation region), (6) rising sheet. (b) Schematic of the flow fields formed on the surface (jump-Figure 6.2. Interaction between two vertical impinging water jets, (a) equal, distant 

Figure 6.3.	Interaction between two identical vertical jets with fixed flow rates of
	Q=300 cm <sup>3</sup> /min and varied spacings; (a) S=28 mm, (b) S=25 mm, (c)
	S=22.5 mm, (d) S=19 mm, (e) S=16 mm, (f) S=13.5 mm
Figure 6.4.	Interaction of two identical, vertical jets with constant spacing of S=19
	mm; (a) Q=200 cm <sup>3</sup> /min, (b) Q=250 cm <sup>3</sup> /min, (c) Q=300 cm <sup>3</sup> /min, (d)
	Q=350 cm <sup>3</sup> /min, (e) Q=400 cm <sup>3</sup> /min, (f) Q=500 cm <sup>3</sup> /min100
Figure 6.5.	Photographs showing droplet fall on the wall jets and destroying the
	steady jump profiles on the plate. (Sequence from left to right. The
	images have been chosen to highlight the droplet fall process) 101
Figure 6.6.	First set of impact waves due to the impingement of the liquid jet on the
	solid surface103
Figure 6.7.	Thee rising sheet formed by the impingement of two identical, vertical
	jets showing different subregimes of "Regime I" breakup mechanism.
	The tests were conducted by a fixed flow rate and varied jet-to-jet
	spacings (the spacing is decreasing from Regime I-A to Regime I-D).
	Images were chosen to highlight the characteristics of each breakup
	regimes
Figure 6.8.	The rising sheet formed by the impingement of two identical, vertical jets
	showing different subregimes of "Regime II" breakup mechanism. The
	tests were conducted by a fixed flow rate and varied jet-to-jet spacings (the
	spacing is decreasing from Regime II-A to Regime II-B). Images were
	chosen to highlight the characteristics of each breakup regimes 108
Figure 6.9.	Side view images of the rising sheet associated with different breakup
	regimes
Figure 6.10.	Sketch of a liquid sheet formed by impinging of two identical vertical jets
	on a horizontal plate. (a) Force balance at the edge of the sheet. The lower
	sketch is the rising sheet with the curved line representing the sheet edge;
	the upper sketch is the cross section of the sheet at the edge. (b) An
	arbitrary element selected on the rising sheet
Figure 6.11.	Schematic of the rising sheet edge profile on the y-z plane 114

Comparison between the experimental results and the theoretical
predictions. (a) Water jets with $Q=300 \text{ cm}^3/\text{min}$ , S=15.5 mm, (b) Water-
Glycerol solution (wt. 50%) with Q=370 cm <sup>3</sup> /min, S=11.6 mm, (c) Water-
Glycerol solution (wt. 65%) with Q=412 cm <sup>3</sup> /min, S=10.1 mm. (Solid lines
are the theoretical results)
Comparison between the theoretical and experimental values of the
maximum rising sheet height. The solid symbols represent the calculations
with $V = \sqrt{U_r^2 - 2gz}$ and the hollow symbol represent $V = U_r$
Experimental measurements of the edge profile of the rising sheet; (a)
Water with Q=300 cm <sup>3</sup> /min, (b) Water-Glycerol (wt 50%) with Q=450
$cm^3/min$ , (c) Water-Glycerol (wt 65%) with Q=450 cm <sup>3</sup> /min. The dashed
lines are plotted by applying a polynomial curve fitting to show the
approximate location of the rising sheet edge 120
Two unequal, vertical, water jets. (a, b) two different view angles to better
illustrate the phenomenon, $Q_1=250 \text{ cm}^3/\text{min}$ and $Q_2=500 \text{ cm}^3/\text{min}$ , (c)
$Q_1$ =150 cm <sup>3</sup> /min and $Q_2$ =600 cm <sup>3</sup> /min, (d) $Q_1$ =3000 cm <sup>3</sup> /min and $Q_2$ =600
cm <sup>3</sup> /min
Schematic of the interaction of two opposing inclined jets located on two
parallel planes. (a) 3D schematic view, (b) schematic of the hydraulic
jumps and the stagnation line formed on the surface
Interaction of two bi-planar, opposing, inclined jets. (a) $\phi = 70^{\circ}$ , (b) $\phi = 50^{\circ}$ ,
(c) $\phi = 30^{\circ}$
Schematic views of (a) an inclined jet impingement, (b) jet flow
streamlines in the stagnation zone, (c) non-circular hydraulic jump formed
on the surface, (d) the impingement region
Non-circular hydraulic jumps formed by inclined jets, (a) $\phi$ =50°, water-
glycerol wt. 50%, Q=300 cm <sup>3</sup> /min, (b) $\phi$ =40°, water-glycerol wt. 50%,
Q=300 cm <sup>3</sup> /min, (c) $\phi$ =50°, water-glycerol wt. 65%, Q=380 cm <sup>3</sup> /min, (d)
$\phi = 30^{\circ}$ , water, Q=350 cm <sup>3</sup> /min
Angular distributions of different flow properties at a constant radial
spacing of $r=5$ mm. Calculations have been done using a water jet with

	the flow rate of $Q=350$ cm <sup>3</sup> /min and the nozzle inclination angle of
	φ=50°
Figure 7.6.	Schematic of arbitrary streamlines colliding at the stagnation line, along
	with geometric parameters. (b) A schematic of the thin wall jets colliding
	at the impact zone
Figure 7.7.	Comparisons between the theory (Eq. 7.22) and the measured hydraulic
	jump profiles formed by single inclined impinging jets. Solid lines
	represent the theory. (a) $\phi=50^{\circ}$ , water, (b) $\phi=50^{\circ}$ , water-glycerol wt. 50%,
	(c) $\phi=50^{\circ}$ , water-glycerol wt. 65%, (d) $\phi=70^{\circ}$ , water-glycerol wt. 50%, (e)
	$\phi$ =70°, water-glycerol wt. 65%, (f) $\phi$ =40°, water-glycerol wt. 50% 140
Figure 7.8.	Interaction of two inclined jets on a horizontal plate. Top row images: the
	formed rising sheet. Bottom row images: underneath images of the flow
	field formed on the surface. (a) $\phi=70^{\circ}$ , (b) $\phi=50^{\circ}$ , (c) $\phi=30^{\circ}$ 142
Figure 7.9.	The normalized momentum distribution with the azimuth angle in the thin
	wall jet formed by an inclined impinging jet. Values are calculated using
	Eq. (7.13b) at a constant radial spacing of 5mm from the stagnation point.
	(a) varied nozzle inclination angles - water jets with $Q=350 \text{ cm}^3/\text{min}$ , (b)
	varied jet flow rates – water jets with , $\phi=50^{\circ}$ , (c) varied fluid kinematic
	viscosities - Q= 350 cm <sup>3</sup> /min, $\phi$ =50°144
Figure 7.10.	Flow field formed on the target surface due to the interaction between two
	inclined impinging jets. Photographs are from underneath the impingement
	plate. (a) $\phi=70^{\circ}$ , water, Q=300 cm <sup>3</sup> /min, S <sub>X</sub> =13.3 mm, S <sub>Y</sub> =4.2 mm, (b)
	$\phi$ =70°, water-glycerol wt. 65%, Q=410 cm <sup>3</sup> /min, S <sub>X</sub> =10.7 mm, S <sub>Y</sub> =4.7 mm,
	(c) $\phi=50^{\circ}$ , water, Q=300 cm <sup>3</sup> /min, S <sub>X</sub> =12.8 mm, S <sub>Y</sub> =3.5 mm, (d) $\phi=50^{\circ}$ ,
	water- glycerol wt. 50%, Q=370 cm <sup>3</sup> /min, $S_X$ =10.7 mm, $S_Y$ =4.7 mm, (e)
	$\phi$ =30°, water, Q=350 cm <sup>3</sup> /min, S <sub>X</sub> =21.1 mm, S <sub>Y</sub> =7.3 mm, (f) $\phi$ =30°, water-
	glycerol wt. 65%, Q=350 cm <sup>3</sup> /min, $S_X$ =8.2 mm, $S_Y$ =5 mm, (g) measured
	stagnation lines; inset demonstrates the coordinates used for the
	measurements 146
Figure 7.11.	Stagnation lines formed by water jets with the flow rate of $Q=350$ cm <sup>3</sup> /min,
	fixed jet relative positions and varied nozzle inclination angles. (a) $S_X=17.5$

	mm, $S_Y=9$ mm, (b) $S_X=13.5$ mm, $S_Y=7$ mm. The dashed lines represent the
	theoretical results
Figure 7.12.	(a) Water, $\phi = 50^{\circ}$ , S <sub>X</sub> =9 mm, S <sub>Y</sub> =3.5 mm, (b) water-glycerol wt.65%, $\phi =$
	70°, S <sub>X</sub> =11.4 mm, S <sub>Y</sub> =6.4 mm, (c) water-glycerol wt.50%, $\phi$ = 50°, S <sub>X</sub> =18.2
	mm, $S_Y=4.7$ mm, (d) water, $\phi=30^{\circ}$ , $S_X=21.2$ mm, $S_Y=7.3$ mm
Figure 7.13.	Stagnation lines formed by different working liquids, (a) $\phi=70^{\circ}$ , S <sub>X</sub> =12
	mm, and S <sub>Y</sub> =6 mm, (b) $\phi$ =50°, S <sub>X</sub> =11.5 mm, and S <sub>Y</sub> =4.5 mm, (b) $\phi$ =30°,
	$S_X$ =12.5 mm, and $S_Y$ =4.2 mm. Flow rates of Q=350, 3870, 380 cm3/min
	are used for water, water-glycerol wt. 50%, and water-glycerol wt. 65%,
	respectively151
Figure 7.14.	Stagnation lines formed with a fixed $S_Y$ and varied $S_X$ , (a) water-glycerol
	wt.65%, $\phi=70^{\circ}$ , Q=410 cm <sup>3</sup> /min, (b) water-glycerol wt.50%, $\phi=50^{\circ}$ ,
	Q=370 cm <sup>3</sup> /min, (b) water, $\phi$ =30°, Q=350 cm <sup>3</sup> /min
Figure 7.15.	Stagnation lines formed by a fixed $S_X$ and varied $S_Y$ , (a) water-glycerol
	wt.50%, $\phi=70^{\circ}$ , Q=370 cm <sup>3</sup> /min, (b) water-glycerol wt.50%, $\phi=50^{\circ}$ ,
	Q=370 cm <sup>3</sup> /min, (c) water, $\phi$ =30°, Q=350 cm <sup>3</sup> /min154
Figure 7.16.	Measured stagnation lines formed by fixed $\psi_{rel.}$ and varied $S_{rel.}$ . The
	working fluid is water with flow rate of Q=350 cm <sup>3</sup> /min (a) $\phi$ =70°, (b)
	$\phi = 50^{\circ}$ , (c) $\phi = 30^{\circ}$
Figure 7.17.	Measured stagnation lines formed by a fixed $S_{rel}=13$ mm and varied $\psi_{rel}$ .
	The working fluid is water with the flow rate of $Q=350 \text{ cm}^3/\text{min}$
	(a) $\phi = 70^{\circ}$ , (b) $\phi = 50^{\circ}$ , (c) $\phi = 30^{\circ}$
Figure 7.18.	Theoretical results of the locations of the stagnation lines with varied jets
	relative positions, (a) water, $\phi=30^{\circ}$ , Q=350 cm <sup>3</sup> /min, (b) water-glycerol
	wt.50%, $\phi$ =50°, Q=370 cm <sup>3</sup> /min, (c) water, $\phi$ =70°, Q=350 cm <sup>3</sup> /min, and
	$S_{rel.}$ =13 mm, (d) water, $\phi$ =70°, Q=350 cm <sup>3</sup> /min, (e) water, $\phi$ =50°, Q=350
	cm <sup>3</sup> /min
Figure 7.19.	Schematic of two arbitrary streamlines colliding at the stagnation line,
	along with the geometric parameters
Figure 7.20.	Comparison between the theoretical predictions and the experimental
	measurements of the stagnation lines formed due to the interaction

between two unequal jets. (a) S=14.5 mm , test $\#1: Q_1=200$ and $Q_2=500$
$cm^{3}/min$ , test #2: $Q_{1}$ =250 and $Q_{2}$ =500 $cm^{3}/min$ , test #3: $Q_{1}$ =300 and
Q <sub>2</sub> =500 cm <sup>3</sup> /min, (a) S=11 mm , test #1: Q <sub>1</sub> =200 and Q <sub>2</sub> =300 cm <sup>3</sup> /min,
test #2: $Q_1=200$ and $Q_2=400$ cm <sup>3</sup> /min, test #3: $Q_1=200$ and $Q_2=500$
cm <sup>3</sup> /min

## List of Symbols

a	Jet radius
$a_{ heta}$	Angular distribution of the jet radius
$Bo^{-l}$	Inverse Bond number
d	Jet diameter
$D_j$	Hydraulic jump diameter
$D_S$	Target plate diameter
$F_g$	Hydrostatic force
$F_{\mu}$	Viscous friction force
$F_{\sigma}$	Surface tension force
$F_{\sigma,j}$	Surface tension force at the jump
$F_{\sigma,s}$	Surface tension force at the edge of the plate
$f_i$	Momentum function for the pre-jump flow
$f_o$	Momentum function for the post-jump flow
fi,m	Minimum value of the pre-jump momentum function
$f_{o,m}$	Minimum value of the post-jump momentum function
Fr	Froude number
$Fr_o$	Post-jump Froude number
$Fr_i$	Pre-jump Froude number
Fr <sub>o,max</sub>	Maximum possible post-jump Froude number
$Fr_{i,min}$	Minimum possible pre-jump Froude number
$Fr_{o,m}$	Post-jump Froude number associated with $f_{o,m}$
$Fr_{i,m}$	Pre-jump Froude number associated with $f_{i,m}$
H	Post-jump liquid depth
h	Pre-jump liquid depth
$h_s$	Thickness of the rising sheet
Hjet	Spacing between the nozzle exit and the target surface
$H_{max}$	Maximum height of the rising sheet
Κ	Free surface curvature
М	Rate of flow momentum

М"	Momentum flux
Q	Volumetric flow rate
$Q_{ heta}$	Angular distribution of the flow rate
$r_c$	The radial location where the boundary layer reaches the free surface
$R_j$	Hydraulic jump radius
$R_{j-0}$	Hydraulic jump radius in the outer wall jet
R <sub>j-π</sub>	Hydraulic jump radius in the inner wall jet
R <sub>S</sub>	Target plate radius
Re	Reynolds number
S	Jet-to-jet spacing
$S_X$	Spacing between two stagnation points in x-direction
$S_Y$	Spacing between two stagnation points in y-direction
S <sub>rel.</sub>	Relative spacing between two stagnation points
и	Velocity profile in the thin wall jet
$U_j$	Jet velocity
x, y, z	Cartesian coordinate
r, θ, z	Cylindrical coordinate
$\phi$	Jet inclination angle
λ	Correction factor for the pre-jump Froude number
μ	Fluid dynamic viscosity
ρ	Fluid density
v	Fluid kinematic viscosity
σ	fluid surface tension
$\beta_S$	Contact angle at the edge of the target plate
ξ	Free surface profile
$\psi_{rel.}$	Relative angle between two stagnation points
γ	The angle between a stream line and the local normal of the stagnation line

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### **Chapter 1** Introduction and Thesis Organization

#### 1.1 Introduction

The increasing demands for efficient cooling systems in various industries have shifted the focus onto liquid cooling due to the superior thermal properties of liquids. Over the past few decades, jet impingement has been an attractive cooling technique in a number of industries such as the metal and drying industry, and also applications such as preventing overheating, cooling turbine blades, and processing materials [1].

Additionally, in electronic industry, the enlarged packaging densities and power levels of recent electronic components have resulted in high local heat fluxes, high device temperatures and appearance of sub-millimeter hot spots. Unless cooled properly below the designed operating temperature, the resulting high surface temperatures will lead to degradation in the performance and failure of the electronic component. The promising cooling capacity of liquid jet impingement have motivated researchers and engineers to take advantage of this powerful cooling technique for high heat flux electronic devices as well.

Jet impingement cooling is usually characterized by low pressure drops and favorable surface temperature uniformity (through the use of jet arrays). However, the most attractive characteristic of jet impingement is rather its relatively very high heat transfer coefficients. The impingement cooling approach also offers a compact hardware arrangement, which also makes it an appropriate cooling technique for electronic devices. In Figure 1, few examples of jet impingement cooling applications are illustrated. With such a broad application, the primary goal of the present research project is to carry out experimental and analytical studies on the flow characteristics of an impinging jet, its interaction with a solid surface, and also with a neighboring impinging jet.



**Figure 1.1.** Jet impingement cooling applications: (a) hot metal rolling process [2], (b) cooling turbine blades [3], (c) Electronic cooling [4], (d) jet impingement packaging for electronic cooling in form of a cold plate [5].

#### **1.2 Jet Impingement**

When a liquid jet impacts on a solid surface, the liquid spreads out radially on the surface, and forms a thin liquid film. Figure 1.1a shows a water jet impinging on a solid surface, and Figure 1.1b illustrates the flow field formed by a jet impingement. The small thicknesses of the liquid film lead to high flow velocities in the spreading flow. As a result, a very thin boundary layer that offers little resistance to the heat flow is formed on the surface, and consequently high heat transfer rates are attained in this area. These flow characteristics of jet impingement make it a promising cooling technique for high heat flux applications. As the liquid spreads on the surface,

it reaches a critical radius  $(R_j)$  where a steep increase in the fluid film depth arises and the liquid velocity drops dramatically (see Figure 1.1). The sudden increase of the fluid film is called a "circular hydraulic jump". A daily example of a circular hydraulic jump can be observed in kitchen sinks where tap water flow strikes the sink.



Figure 1.2. (a) A water jet impingement, (b) Schematic of the flow field formed by a jet impingement

A hydraulic jump is often observed in open channel flows such as rivers and spillways (i.e. also referred to as the classic hydraulic jump or 2D hydraulic jump). At the jump, the fast flowing liquid suddenly slows down and increases in height, converting some of its kinetic energy into an increase in the potential energy, with some energy irreversibly lost through turbulence to heat [6,7].

One of the primary dimensionless parameters regularly used to characterize free surface flows is Froude number. Froude number, Fr, is defined as the ratio between the flow inertia and gravitation. Froude number equal to unity, Fr = 1, is referred to as the critical Froude number, which is associated with the minimum value of the momentum function available for that particular flow (i.e. also accounts for the minimum energy that the flow can possess) [6,7]. The flow before the jump is called the "pre-jump" region and characterized as a super-critical flow with Froude numbers higher than unity ( $Fr_i > 1$ ). The pre-jump flow is also referred to as the "wall jet". The flow downstream of the jump is called the "post-jump" region and characterized as a sub-critical flow with Froude numbers lower than unity ( $Fr_o > 1$ ). A hydraulic jump occurs as a transition between a super-critical flow to a sub-critical flow.

Since a drastic drop in the fluid velocity occurs as the fluid flows through the jump, hydraulic jumps degrade the cooling performance of jet impingement and interrupt the uniformity of the heat transfer on the surface. Thus, the hydraulic jump is identified as an important feature of jet impingement performance, and therefore, estimating the location where the jump occurs has been an area of active research for the past few decades [8–15]. Parameters that play major roles in the behavior of jet impingement include the nozzle size and shape, nozzle angle with respect to the target plate, the flow rate, the working fluid properties, as well as the target plate characteristics. Regarding the target plate, the size of the target plate along with the flow condition at the edge of the plate are among the parameters that influence the behavior of the impinging jet and the hydraulic jump size formed on the surface.

With jet impingement, the heat transfer coefficient distribution on the surface is bell-shaped. Figure 1.2 illustrates an example of the heat transfer coefficient distribution on a heated surface with a uniform heat flux exposed to a water jet impingement. The maximum heat transfer rates are achieved at the impact point of the jet on the surface. This point is referred to as the "stagnation point". Although very high heat transfer coefficients can be achieved in the vicinity of the stagnation point (i.e. the impingement region), with a single impinging jet, the thermal performance degrades sharply as the fluid spreads out from the stagnation point. This is due to the boundary layer growth on the surface as well as the drop in the fluid velocity as a result of the radial spreading. Consequently, the heat transfer distribution is non-uniform and decreases with the distance from the stagnation point. It is perceived that jet impingement is a promising cooling technique for local cooling applications (i.e. hot-spot cooling or small area cooling application). If the intended area for cooling is large, a single jet impingement might be unable to provide the desired, uniform cooling results.



**Figure 1.3.** Heat transfer coefficient distribution resulting from a single jet impingement on a heated surface with uniform heat flux.

#### **1.3 Multiple Jet Impingement**

In many applications, the area that needs to be cooled is large, and the spatial variation of the heat transfer observed with a single jet is not ideal. To overcome this limitation, multiple jets are utilized in many applications to achieve, or approach, uniform cooling over the target surface [16,17]. Due to the outstanding thermal performance of jet arrays, they have long been an active area of research [18–22]. Different parameters have been reported to play significant roles in the flow and thermal behavior of an array of jets. Despite the parameters that influence the performance of a single jet, other parameters such as the arrangement of the jets, jet-to-jet

spacing, and the spacing between the jet/orifice plate to the target plate have been found to make great contributions to the performance of multiple jets.

Multiple jets are usually arranged in staggered or inline arrays. Heat transfer by each jet is influenced by the neighboring jets. It has been reported that the interaction between the flow fields formed by any two neighboring jets plays an important role in the thermal behavior of the array of jets. Thus, it seems necessary to have an in-depth understanding of the physics involved in the interaction between two jets impinging on a solid surface.

In Figure 1.3, the interaction between two water jets impinging on a horizontal plate is shown. It is seen that the thin pre-jump flows collide in the area between the jets and give rise to a liquid fountain/sheet, which spreads spatially from the impact region. The impact region where the two thin wall jets meet is referred to as the "stagnation line". The location and shape of the stagnation line depends on the relative strengths and orientations of the individual parent jets. This is clearly shown in Figure 1.3, where jets with varied configurations interact and generate different flow fields on the surface.



Figure 1.4. Double water jet impingement, (a) identical vertical jets, (b) vertical jets with unequal flow rates, (c) inclined identical jets (bi-planar jets).

#### **1.4 Research Objectives**

There is an extensive literature available regarding the heat transfer and thermal performance of jet impingement. However, most of these studies focused more on the thermal aspects of impinging jets and therefore lack an in-depth analysis of the fluid dynamics of the phenomenon that dictates the associated transport mechanisms. For instance, the circular hydraulic jump formed on finite size surfaces needs further investigation. Likewise, the interaction due to the impingement of two neighboring liquid jets has received very little attention.

The present thesis has been planned to analytically and experimentally study the fluid dynamics of single and double jet impingement. The thesis will focus on the gaps that exist in the literature in terms of the fluid dynamics involved in the phenomena. Circular hydraulic jump as a key feature of a jet impingement will be further analyzed. A systematic Froude number analysis for circular hydraulic jumps will be carried out, and the critical differences between the classical jumps and circular jumps will be pointed out. The interaction between two impinging jets on a solid surface will be investigated in detail, and the fluid dynamics involved in the collision of two pre-jump liquid films will be analytically and experimentally analyzed. Different jet configurations (vertical/ inclined jets) will be examined in the analysis.

Results of this research project will provide researchers/engineers with a more informative understanding of the flow characteristics of single and double jet impingement, and lead to improved designs for cooling of high heat flux sources. The overall objective of the present thesis is achieved through the following research tasks:

## 1.4.1 Research Task 1 – Circular Hydraulic Jump Formed on Finite Surfaces with Capillary Limit

In actual applications, impinging jets are employed on finite-size surfaces with free edges (i.e. no weir at the edge). In the case of free edges, when the flow rate is low or/and the surface size is relatively large (i.e. the jump is far from the plate edge), the capillary effect holds the liquid film along most of the plate edge, and the post-jump liquid only falls from individual points at the edge of the plate. This has also been reported by other researchers such as Duchesne et al. [23] and Craik et al. [24] using a working fluid with a surface tension close to that of water.

The above-mentioned, commonly seen condition is different from the previous theoretical models, for which a uniform flow at the edge of the plate was assumed, or the plate size was not taken into consideration by controlling the post-jump flow using a weir at the edge of the plate. Thus, the first objective of this research project is to find an analytical solution to predict the circular hydraulic jump location formed on surfaces with finite sizes and the capillary limit at the edge. The study will be presented in Chapter 4 of the present thesis.

## 1.4.2 Research Task 2 – Circular Hydraulic Jump on Small Target Plates, and a Froude Number Analysis

Although in a few previous studies the finite size of the plate was taken into account as a boundary condition, little attention has been paid to the plate size as a major test variable. In the preliminary experiments conducted in the present research project, an interesting phenomenon came to attention using *small* target plates. It was observed that with a fixed jet flow,

significantly larger hydraulic jumps were formed on smaller target plates. This observation motivated a systematic experimental study to characterize the circular hydraulic jumps formed on small target plates, the sizes of which are comparable to the sizes of the hydraulic jumps (i.e. jump arises close to the plate edge).

The experimental study will be followed by an analytical Froude number analysis. The classical 2D hydraulic jump in open channels is an established fluid mechanics problem with a rich body of information and analysis available in the literature. However, to the best of the author's knowledge, there is no Froude number analysis available focusing on the case of circular hydraulic jumps. The objective of this analysis is then to address a number of important questions. What are the differences between the classical jump and circular jump in terms of Froude number? What is the relation between the pre-jump and post-jump Froude numbers? What is the critical Froude number in the case of a circular hydraulic jump? Is it possible for the post-jump flow to exhibit Froude numbers higher than unity?

This research objective will be addressed in Chapter 5.

#### **1.4.3** Research Task 3 – Double Vertical Jet Impingement

Multiple jets have shown promising potential for providing strong, uniform cooling in high heat flux applications. To better understand the physics and performance of an array of jets, an indepth understanding of the fluid flow involved in the interaction between neighboring jets is essential. The next objective of the present thesis is to experimentally and analytically study the interaction between the flow fields formed on a horizontal plate due to the impingement of two vertical neighboring jets. The primary focus will be given to the fountain-like liquid sheet formed as a result of the flow field interaction. Important parameters involved in the phenomenon will be discussed. The breakup mechanisms that the rising sheet experiences will be characterized based on the experimental observations. Moreover, the shape of the rising sheet will be theoretically analyzed. This objective will be addressed in Chapter 6.

#### **1.4.4** Research Task 4 – Double Inclined Jet Impingement

Impinging jets are not always used with a vertical orientation relative to the target plate. Impinging jets with inclined angles have been employed in varied applications due to either limitations in the space or the nature of the application. Although there are a few previous studies available on the flow field produced by the impingement of a single inclined jet [25–32], little attention has been paid to the interaction between two inclined jets. This research task focuses on the flow field formed by the impingement of two inclined jets, when the jets are placed on two parallel planes (see Figure 1.3c). The two jets have the same nozzle inclination angles and the projected jet flow directions on the horizontal plate are in opposite directions. A systematic experimental study will be carried out to examine and characterize the formed flow field. Furthermore, a theoretical analysis will be developed to predict the shape and location of the generated stagnation line as a result of the interaction between the two flow fields on the surface. This study will be presented in Chapter 7 of the present thesis.

In addition to the foregoing research tasks, a literature review is performed on the flow characteristics of single and double jet impingement, which is presented in Chapter 2. Also, Chapter 3 elaborates on the experimental setup and methodology employed for this research

project. While a general description of the experimental rig and the measurement methods are presented in Chapter 3, more experimental details associated with each research task will be presented in its associated chapter. Finally, in Chapter 8, a summary of the thesis along with the concluding remarks will be presented, and related future work will be discussed.

### **Chapter 2** Literature Review

#### 2.1 General

An impinging liquid jet may be classified according to whether it is a free-surface or a submerged jet, and whether its cross section is circular or rectangular (planar). A free-surface jet is discharged into a fluid of different type before impinging upon the target plate [33], as schematically shown in Figure 2.1a (i.e. a liquid jet discharged into a gas ambient). Whereas, a submerged jet is discharged into a fluid of the same type, that is liquid into liquid or gas into gas (schematically illustrated in Figures 2.1b and c). The submerged jets can be described as being confined or unconfined. If the jet is unconfined, the nozzle plate (i.e. orifice plate) is far away from the target plate and has little or no effect on the flow structure of the flow field on the surface (see Figures 2.1a and b). However, if the jet is confined (see Figure 2.1c), the nozzle plate is close to the target plate and influences the flow structure by causing a recirculation zone [34]. Moreover, in the case of multiple jets, the flow is also influenced by the cross-flow effects associated with the interaction between neighboring jets [34].



Figure 2.1. Different types of jet impingement, (a) free-surface jet, (b) submerged jet, (c) confined submerged jet.

As a result of the formation of thin hydrodynamic and thermal boundary layers at the impingement surface, the achieved convection heat transfer coefficients are large, and therefore jet impingement is well-suited for cooling of high heat flux applications [33–35]. The present thesis will focus on the flow characteristics of free-surface, single and double jet impingement.



**Figure 2.2.** (a) A free-surface water jet impingement and the resulting circular hydraulic jump, (b) Schematic of the flow field on the surface.

Figure 2.2 shows the impingement of a free-surface, water jet on a solid surface. When a liquid jet impacts on the surface, the liquid spreads out radially and forms a thin fluid film on the surface exhibiting high velocities. As a result, a very thin boundary layer that offers little resistance to the heat flow is formed on the surface, and consequently high heat transfer rates are attained in this area [33,34,36–38]. Within the stagnation zone, hydrodynamic and thermal boundary layers are of uniform thickness [33]. Beyond the stagnation zone, the boundary layer begins to grow until it reaches the free surface of the liquid film (see the inset of Figure 2.2b). The liquid film thickness in this area is of order of tens/hundreds of microns. There are competing effects on the liquid film thickness in the pre-jump area, with the radial spreading

acting to decrease the liquid film thickness, while viscous effects act to thicken the film due to the deceleration in the developing boundary layers.

For Prandtl numbers associated with liquids (Pr >1), the hydrodynamic boundary layer reaches the free surface before the thermal boundary layer [39]. The radial spacing at which the boundary layer encompasses the entire liquid film depth is denoted by  $r_c$ . Once the hydrodynamic boundary layer covers the entire film depth (at  $r = r_c$ ), viscous effects extend throughout the liquid film thickness and the surface velocity begins to decrease with increasing the radial spacing (see Figure 2.2b). In this region, velocity profiles at different radial locations are selfsimilar [8]. The similarity region ends where the transition to turbulence begins.

Here, the literature summary is divided into two groups. First, a summary of the important studies and findings in the area of single, free-surface jet impingement will be introduced. This part will particularly focus on the flow characteristics of a circular hydraulic jump as a key feature related to the fluid flow of an impinging jet. The knowledge of the fluid dynamics of a single jet will then help to better understand the physics involved in the interaction of two neighboring jets impinging on a surface. Secondly, a description of the available literature in the area of double jet impingement will be presented.

#### 2.2 Single jet impingement

As mentioned before, an interesting fluid dynamics phenomenon that influences the performance of a free-surface jet is the circular hydraulic jump formed on the surface. The phenomenon is characterized as an abrupt increase in the film thickness and a corresponding reduction in the
film velocity, which is accompanied by a significant reduction in the heat transfer coefficients. This explains the importance of the hydraulic jump location for many cooling applications using jet impingement. Due to this significant role, prediction of the hydraulic jump radius/diameter has been the subject of many studies for the past few decades.

The first theoretical study was carried out by Rayleigh in 1914 [40]. An inviscid model was used along with the continuity and momentum equations to predict the jump location. A detailed inviscid theory for circular hydraulic jump can be found in Ref. [41]. However, due to the significance of viscous effects, the inviscid model is only of academic interest and known to be inadequate for real applications.

The pioneer work by Watson in 1964 was the first to consider the fluid viscosity [8]. He analytically analyzed the flow in the thin pre-jump film using the boundary layer approximation. The radial location where the boundary layer reaches the free surface  $(r_c)$  was considered as a separation point. Karman-Pohlhausen method and a similarity method were employed for the flow before and after  $r = r_c$ , respectively. Karman-Pohlhousen method is referred to the general momentum integral solution using a fourth-order polynomial velocity profile [42]. Employing a force-momentum balance at the jump location, a solution was obtained to show the relation of the jump radius with the jet flow and the post-jump film depth. In Watson's theory, the target surface size was not included in the solution. Also, the post-jump film depth was treated as a known parameter. They justified this assumption by utilizing a weir at the edge of the plate to control the post-jump film depth.

Watson's study was a great step forward. However, the theory and experimental results were not always in a satisfactory agreement. This can be found in many previous studies including those in Refs. [12,24,43–47]. The general conclusion of all these studies was that Watson's theory is in a good agreement with the experimental data for large jump radii. However, the theory predicts poorly for small jump radii.

In Watson's theory, the surface tension effect was neglected and not considered in the analysis. The influence of the surface tension on a circular hydraulic jump was explored in studies such as those in Refs. [12,24,48–51]. Particularly, Bush & Aristoff in 2003 included the surface tension effect in Watson's theory and presented a revised expression for the jump diameter [12]. They took into account the surface tension effect as a radial force that originates from the curvature of the circular hydraulic jump. They reported that the surface tension force becomes significant for small jump radii. This explains the poor agreement of Watson's theory with experimental data for small jump radii. Although Bush & Aristoff's work considerably enhanced the accuracy of Watson's theory, the post-jump film thickness was still treated as a known parameter. Also, the target plate size was not taken into account either.

Another group of studies explored circular hydraulic jumps on plates with a free edge, i.e. no weir/barrier at the edge [9,11,13–15,52–54]. It was assumed that the post-jump fluid film falls freely around the entire plate edge. Therefore, the hydraulic jump adapts itself to the imposed boundary condition at the edge governed by the effect of inertia and surface tension [13,55]. In this approach, the plate size is taken into account by imposing a boundary condition at the edge of the plate. Using simplified equations of the shallow-water type with viscosity included, Bohr

et al. [9] provided a scaling relation for the jump radius (i.e. the shallow-water equations are derived by integrating the Navier–Stokes equations over the depth of the liquid film, when the stream-wise length scale is much greater than the length scales in other directions [56]). By connecting the pre-jump flow and the post-jump flow through a shock, the scaling relation resulted in  $R_j \propto Q^{5/8}$ , where Q is the jet flow rate. The scaling law was shown to agree with previous and recent experimental data [57–59]. Higuera [11] numerically studied the hydraulic jump of viscous laminar flow using the boundary layer approximation for the flow around the jump. The position and structure of the jump were determined numerically by imposing a boundary condition at the edge of the plate where the liquid turns around and falls under the action of gravity. Further developments on these theories can be found in a few later studies [14,15,60].

Other important studies assuming a uniform flow at the edge of the plate edge including those presented by Kasimov [61] and Rojas et al. [52,53]. Kasimov [61] proposed another shallow-water model by including a plate-slope function into the simplified radial momentum equation. Mathematically describing the bottom topography of the flow, the function allows the model to have a smooth connection between the post-jump flow and the flow over the plate edge. With careful treatment of the edge condition, the model improves on previous work by Bohr et al. [9]. Rojas et al. [52,53] introduced the inertial lubrication theory, in which the circular hydraulic jump is a singularity line connecting the inertial lubrication theory, Rojas et al. [53] presented

a corrected scaling law for the jump radius, which depends on the post-jump film depth, density, and surface tension.

As mentioned before, although in this group of studies, the finite size of the target plate was taken into account, the primary assumption was considering a target plate with a uniform edge flow. However, in actual applications, liquid does not always flow off uniformly at the edge of the plate. In the case of low flow rates or/and relatively large surface sizes, the surface tension effect holds the liquid film along most of the plate edge and the liquid only falls from individual points at the edge. This has also been observed by other researchers such as Duchesne et al. [23] and Craik et al. [24] using working fluids with surface tension close to that of water. They observed de-wetting occurring on the lateral edges of the target plate, with liquid flowing off through a few rivulets forming on the plate perimeter.

Following the forgoing review of the available literature, an analytical solution seems necessary to predict the circular hydraulic jump location formed on the target plates with free edges and also the capillary limit at the edge. This will be addressed in "Research Task 1" of this thesis, which will be presented in Chapter 4.

Moreover, the target plate sizes considered in most of the above-mentioned studies were of orders of magnitudes larger than the size of the hydraulic jumps formed on the surface. One may expect to see a different behavior of the hydraulic jump when the jump gets close to the edge of the plate. Higuera [11,15] and Rao & Arakeri [13,55] reported that the subcritical flow (i.e. post-jump flow) approaching the plate edge accelerates and becomes locally critical due to the gravitational acceleration prevailing at the edge of the plate. Likewise, Bohr et al. [9] and

Higuera [11] explained that, in the post-jump region, the inertia terms cannot be neglected in a short distance from the edge of the plate. Consequently, if the hydraulic jump occurs close to the edge, the assumption that the flow velocity after the jump is small is not valid anymore. Thus, Bohr et al. [9] and Higuera [11] stated that, when the spacing between the jump and the plate edge is small, their theories are not able to predict the jump dimeter.

Therefore, the influence of the target plate size, especially when the plate size is comparable to the size of the hydraulic jump still needs more in-depth understanding. This will be addressed in "Research Task 2" of this research project, which will be presented in Chapter 5.

### 2.3 Froude number

In hydrodynamics, "hydraulic jump" is known as a sudden transition from a high-speed, supercritical flow to a subcritical one, with a sudden jump of the fluid depth [6,7,62–66]. This phenomenon is commonly seen at very different length scales: in dam spillways [65], in tidal bores on rivers [66], or in kitchen sinks when the tap water hits the sink surface. The dimensionless number used to describe this phenomenon is the Froude number, Fr, defined as the ratio of the flow inertia to the external field (i.e. in many applications simply due to gravity) [6,7]. Froude number is generally expresses as  $Fr = U/\sqrt{gL_0}$ , where U is the flow velocity, g is

the acceleration of gravity, and  $L_0$  is the characteristics length, which is usually represented by the flow depth.

Generally, the hydraulic jump characteristics mostly depend on the Froude number value, and the comparison between the flow Froude number and the critical Froude number. The critical Froude number is associated with the minimum value of the momentum function available for that particular flow (i.e. also accounts for the minimum energy that the flow can possess). Classically, the critical Froude number is known to be equal to unity ( $Fr_{cr} = 1$ ) [6,7]. Upstream of the jump (i.e. pre-jump region), the flow is characterized as a supercritical flow with Froude numbers higher than unity ( $Fr_i > 1$ ). Through the hydraulic jump, some of the flow kinetic energy is converted into an increase in the potential energy, with some energy irreversibly lost through turbulence to heat. Downstream of the jump (post-jump), the flow is a subcritical flow with  $Fr_o < 1$ . The two values of the flow Froude numbers upstream and downstream of a jump are dependent on each other via the mass and momentum conservations expressed at the jump location [6,7,64].

Most of the available studies on circular hydraulic jumps have focused rather on the prediction of the jump location, for which two key theories are available [8,9], among a rich literature aiming to improve these ones or to propose alternatives [11,12,53,54,61,67]. However, very little attention has been paid to the Froude number as the primary dimensionless number characterizing the flow involved in the occurrence of a hydraulic jump.

One of the first studies examining the Froude number of a circular hydraulic jump was carried out by Ishigai *et al.* [44]. They claimed to distinguish four categories on the shape of the liquid surface at the jump controlled by the pre-jump Froude number,  $Fr_i = U(gh)^{-1/2}$ , where U and h are the mean velocity and the liquid depth just upstream of the jump. The four categories are (a) a stable smooth surface with a few standing waves ( $Fr_i < 2$ ), (b) a smooth gradual rise in the film thickness (2 <  $Fr_i$  < 7), (c) a narrower more rounded jump (7 <  $Fr_i$  < 15) and (d) an unstable jump at which air bubbles are constantly entrained ( $Fr_i$  > 15).

There was a shortage of quantitative data to support such detailed categorization. However, there is no doubt that the jump structure does go through considerable changes as the flow parameters alter. A number of previous studies have focused on the structure of the jump, the boundary layer separation, and the flow recirculation at the jump location [10,24,44,45,48,57,68]. Craik et al. carried out experiments on the structure of the jump as well as the associated Froude numbers [24]. They reported that the categorization presented by Ishigai *et al.* [44] appears to be improper. They also stated that although the flow structure inside the jump is complex and varies significantly with the flow conditions, the mass conservation and momentum balance across the jump must still apply.

Another study that explored the flow Froude number at a circular hydraulic jump was conducted by Liu & Linehard [48]. In their experimental work, they pointed out several differences between the open-channel hydraulic jump and the circular hydraulic jump. They explained that the classical hydraulic jump of undergraduate textbooks is usually expressed as a roiling turbulent free-surface linking the supercritical and subcritical flows. However, circular hydraulic jump exhibits a liquid surface that is smooth (unless the flow undergoes severe turbulent regimes), with a relatively sharp and steady jump (see Figure 2.2a). In addition, the pre-jump liquid film in circular jumps is very thin (in contrast with open channel jumps), with thicknesses of about tens/hundreds microns. Despite these obvious differences in the flow field, circular jumps differ from the classical jumps by the range of the pre-jump Froude numbers as well. With impinging jets, the pre-jump Froude numbers can be as high as several hundreds, whereas typical openchannel flows exhibit supercritical Froude numbers of usually no more than 20 or 30. Each of these aspects may contribute to interesting deviations from the standard theory of the hydraulic jump. For instance, Liu & Linehard stated that as the pre-jump Froude number increases, the classical jump theory applied to the circular jumps (i.e. the force analysis of Watson's theory [8]) fails to predict the jump size [48].

Despite the above-mentioned discussion on the pre-jump Froude number, recently, Duchesne et al. in 2014 carried out an experimental study on the post-jump Froude number and witnessed an interesting phenomenon [23]. They experimentally observed that the post-jump Froude number based on the measured post-jump depth is locked on a constant value ( $Fr_o \sim 0.33$ ). Interestingly, they also reported that the constant value of the Froude number is independent of the flow rate, kinematic viscosity, and surface tension. However, they observed a slight dependence on the nozzle diameter, ~10% for a variation of the nozzle diameter from 1.1 to 7mm.

Although jet impingement and the resulting circular hydraulic jump have been a subject to a wide range of studies, very little attention has been paid to the differences between the classical, 2-D hydraulic jump and the circular one. Moreover, Froude number as a primary dimensionless parameter determining the state of the flows involved in a hydraulic jump needs to be further analyzed. "Research Task 2" of the present research project will address the mentioned gap in the literature, which will be presented in Chapter 5.

### 2.4 Multiple Jet Impingement

Employing multiple jets to cover a target surface leads to significant improvements in the efficiency and uniformity of the heat transfer rates on the surface. For a typical single jet impingement, the heat transfer coefficient values can decay by a factor of 4 or 5 from r/d = 0 to 9 (i.e. *r* is the radial position and *d* is the nozzle diameter). Employing multiple jets can reduce this variation to a factor of 2 [69].

Different parameters have been reported to play major roles in the thermal performance of an array of jets [70–75]. Figure 2.3 illustrates a schematic of a jet array impinging on a surface. Parameters including the jet diameter (d), jet-to-jet spacing (S), the spacing between the nozzle exit to the target surface (H), etc. have been reported to make great contributions to the magnitude and uniformity of the heat transfer rate on the surface.



Figure 2.3. Schematic of an array of free-surface jets impinging on a solid surface.

The jet-to-jet positioning in an array (also referred to as the pitch, S) determines the degree of the flow interaction [69]. At the target surface, the wall jets (i.e. thin liquid films) of two adjacent jets may collide and form another local stagnation zone accompanied with boundary layer separations. The wall flows are then transformed into a "fountain" shape flow (see Figure 2.3).

The formation of the fountain can alter the transfer rates in the collision region. Depending on the nozzle to plate spacing (H) and also the jet-to-jet spacing (S), the fountain effect may enhance the thermal performance by generating turbulence, mixing, and interrupting the boundary layer growth [76,77]. The interaction may also develop undesirable flow patterns. It may disturb the wall flow and reduce the average heat transfer rate. At small jet-to-jet spacings (S), the fountain may interfere with the falling free jets. If the fountain exchanges momentum with the free jet, the heat transfer rates attained at the target surface are found to decrease [76,78].

As briefly explained above, different parameters influence the thermal performance of a set of multiple jets. Several researchers have tried to characterize and quantify these parameters and their associated influences on the heat transfer rates achieved by multiple jets. For instance, most studies have reported that the jet-to-jet spacing (S) is of considerable significance, however their recommendations differ (e.g. S/d = 20 in Refs. [79,80], versus S/d ~ 6 in Ref. [81]). Whereas, Jiji & Dagan [19] reported no noticeable effect of this parameter.

Unfortunately, most of the available studies often employed systems with large numbers of jets, which makes it difficult to distinguish the influence of individual parameters. For example, San & Lai reported that the jet interaction is significant for S/d < 4 [82]. They also stated that the jet interaction plays a minor role for S/d > 8 and H/d > 2, and the undesired effect will intensify as S/d and H/d decrease from these values. However, in another study, Behnia et al. [83] observed that the fountain effect degrades the thermal performance of a jet array for

0.25 < H/d < 1, and have a minimal influence on the heat transfer rates for H/d > 1. Characterization of the effects of these parameters can be also found in Refs. [70–75].

It can be concluded from the above review that the fluid dynamics involved in the performance of jet arrays is greatly complex. All the above-mentioned parameters are relative and dependent on each other. In other words, characterizing the influence of one parameter is impossible without taking the other parameters into consideration. In addition, it adds to the complexity of the phenomenon, when the flow rate and the fluid properties are taken into consideration. Thus, this complexity of the phenomenon highlights the necessity of an in-depth analysis exploring the detailed fluid dynamics involved in the interaction between two neighboring jets.

## 2.5 Double Jet Impingement

The impingement of two neighboring jets on a solid plate (double jet impingement) has been the focus of a number of studies. The available literature can be divided into two groups. The first group studied double "air" jet impingement [84–87]. The primary application of this group of studies was focused on VTOL (vertical take-off and landing aircrafts). They studied the flow field produced by the interaction of two air jets impinging on the ground. The influences of the interaction and the generated upwash fountain on the lift forces and the performance of the aircraft during take-off and landing were examined [88–94]. Although valuable findings and discussions have been presented in these studies, very little attention was paid to the cooling applications of the phenomenon and the influence of the generated flow field on the transfer rates. More importantly, "air" jet impingement is categorized as a submerged jet (i.e. a gas jet is

discharged into a gas environment), which exhibits different physics compared to free surface jets (i.e. a liquid jet is discharged into a gas environment) [34].

The formed fountain/upwash flow is one of the important differences between the interaction between two air jets and the interaction between two free-surface liquid jets. There are two key reasons for this difference. First, with submerged air jets, there is a significant entrainment from the ambient medium both into the thin film flow and into the upwash flow. Such a strong entrainment does not occur when free-surface liquid jets are impinging on a solid surface. The other difference is regarding the formation of hydraulic jumps on the surface and the interaction between the formed jumps in the case of free-surface liquid jets, which is absent in air jet flows. The interaction between the formed hydraulic jumps gives rise to a series of complex fluid dynamic phenomena and creates interesting jump–jump interaction patterns [95].

Relatively much has been done to study the interaction between two air jet impingement. However, very little has been done on the interaction between two liquid free-surface jets and the influence of the generated flow field on the cooling applications. The first study on double liquid jets was carried out by Ishigai et al. [44]. They reported that the local heat transfer is not significantly altered at the interaction zone if the interacting wall flows have already gone through the transition to turbulence. However, they observed noticeable enhancement in the thermal performance if the interaction occurs prior to the transition within the wall flows.

The next studies on double free-surface jet impingement were carried out by Slayzak et al. in 1993 and 1994 [76,78]. They conducted two experimental works, one with two adjacent planar liquid jets (i.e. 2D, slot jets) [78], and the other one with two rows of circular, free-surface jets

[76]. They investigated the influences of the interaction between the two wall flows on the heat transfer rates at the surface. They reported that the heat transfer coefficients recorded beneath the interaction zone were comparable to those associated with the stagnation points of the parent jets. They also utilized two jet guards to protect the free jets from interacting with the formed fountain (rising sheet). They observed that if either of the free jets and the rising fountain interact (i.e. exchange momentum), the heat transfer rates attained at the surface were degraded appreciably.

The other noteworthy study on double, free-surface jet impingement was done by Kate et al. in 2007 [95] who examined the interaction between two hydraulic jumps formed by two vertical water jets. Their work experimentally explained the jump–jump interactions formed by different jet-to-jet spacings and different strengths (flow rate) of the individual parent jets. They also described the formed fountain flow as an arch-shaped liquid sheet surrounded by a thick rim. However, they did not present any quantitative analysis of the shape and height of the formed rising sheet.

To the best of the author's knowledge, in addition to the above four important studies related to the interaction between two free-surface liquid jets, there are only a few other works in this area including Refs. [77,96–99]. However, all these studies, except the ones presented by Kate et al. [95] and Haustain et al. [77], only examined the thermal behavior of double jet impingement without any in-depth analysis of the fluid dynamics of the phenomenon. The complexity involved in the interaction between two adjacent, impinging, liquid jets requires a comprehensive understanding of the fluid flow and the resulting rising sheet and interaction zones formed on the

surface. In Research Tasks 3 and 4 of this research project, which will be presented in Chapters 6 and 7, the interaction of two free-surface liquid jets with vertical and inclined orientations will be experimentally investigated. Furthermore, analytical models will be developed to predict the locations and shapes of the formed stagnation line and rising sheet.

# **Chapter 3** Experimental Setup and Methodology

## 3.1 Overview

The overall objective of this chapter is to present a general description of the experimental rig and methodology employed to carry out the experimental part of the present thesis. In addition to the information presented here, more experimental details will be provided in the following chapters, which will be more specific to the set goals of each chapter.

## **3.2 Experimental Setup**

In order to address the set objectives of this research project, the experimental setup should meet the following requirements:

- Smooth flow at the nozzle outlet.
- Accurate flow measurements.
- Flexibility in using different working liquids.
- Flexibility in the impingement plate configurations: plate size and edge condition.
- Flexibility in the jet configurations: nozzle diameter, nozzle inclination angle, nozzle spatial position, etc.
- Accurate measurements of the jet impingement features such as the hydraulic jump radius, the post-jump liquid depth, etc.
- Capability to accommodate double impinging jets with varied configurations (vertical and inclined jets).

• Accurate measurements of the interaction features in the double jet impingement experiments such as the formed stagnation line, jump-jump interaction profiles, the rising sheet, etc.

Two arrangements for the experimental setup are designed in the view of the set objectives. One setup is designed to accommodate the single jet impingement study, which is schematically shown in Figure 3.1. Another design is employed to carry out the double jet impingement experiments, which is schematically depicted in Figure 3.3. Both setups take advantage of the same flow supply system, which is comprised of a pressurized water tank and an air compressor. The pressurized water tank is utilized in order to deliver a smooth flow at the nozzle outlet and minimize the flow fluctuations caused by a pump. The air compressor is capable to increase the pressure inside the tank up to 140 psi, which results in flow rates of up to 3 lit/min with the present tubing configuration.



Figure 3.1. (a) Schematic of the experimental setup used for the single jet impingement experiments, (b) the nozzle holder.

## 3.2.1 Single Jet Setup

For the single jet impingement, the flow rate is measured using a FLR1010ST, OMEGA flow meter which gives an accuracy of 1%. With the nozzle holder designed and fabricated for the

present experimental setup, the spatial position of the nozzle with respect to the target plate can be easily adjusted using two translation stages (see Figure 3.1b). Using the nozzle holder shown in Figure 3.1b, the nozzle inclination angle can be changed from 0 to 90 degree with steps of 10 degrees. However, since the single jet experiments are only done with the vertical orientation (relative to the target plate), the nozzle angle is fixed on 90 degrees.

The jet nozzles are made of stainless steel tubes with inner diameters of d = 0.75, 1, 2, and 4.2 mm. The lengths of the nozzles are carefully picked in a way that the fully developed condition is reached before the nozzle exit. Since several previous studies reported that the nozzle to plate spacing has negligible effects on the behavior of free-surface jet impingement [36,100], a fixed spacing of 5d is set in most of the experiments conducted here. The target plates examined for the single jet impingement experiments are cut out of an aluminum plate using a water-jet machine. Circular aluminum discs with diameters ranging from  $D_S = 20$  to 100 mm are tested. All plates are sanded around the edges using a fine sandpaper in order to smoothen the rough edges. However, no systematic care is given to the plates' edges in the microscopic level.

A camera (Nikon D800) is used to take images of the experiments. For each test, top-view and side-view images are taken. The top view images are used to measure the jump radius/diameter. The side view images are used for measuring the post-jump liquid film depth *H*. Figure 3.2 shows a top view image and a side view image of a water jet impinging on a solid surface. Using a calibrated length scale, the image processing software "Image J" is employed to measure the hydraulic jump radius/diameter and also the post-jump liquid depth. For each test, five top view images and five side-view images are taken, and the average values are used for the analysis.

Due to the steady nature of circular hydraulic jumps, the five values measured for each test are very close to each other, therefore, taking more images would not change the average values that are used in the analysis. A height gauge is also used to measure the free liquid surface profile around the hydraulic jump.



**Figure 3.2.** A water jet impinging on a circular target plate. (a) Top view image of the jet impingement and the formed hydraulic jump on the target plate, (b) Side view image of the liquid film on the surface.

## **3.2.2** Double Jet Impingement

In order to study the interaction between two neighboring jets, the experimental setup is modified. A schematic of the designed experimental setup is presented in Figure 3.3. To have separate control over the flow rate of each jet, two sets of valves and flow meters (rotameters) are installed in parallel.

In all the experiments carried out in the present thesis with double impinging jets, both jets obtain the same orientation (i.e. nozzle inclination angle). The nozzle holder shown in Figure 3.1b is used to set the first nozzle. The second nozzle is installed using a clamp. Once the second nozzle is placed with the desired orientation (i.e. either vertical or inclined), it is clamped and remains fixed for all the tests conducted with the set orientation. Once this jet is fixed, the

relative spacing between the jets is adjusted by the first jet using the translation stages mounted on the nozzle holder. In the present research project, four different nozzle inclination angles are investigated ( $\phi = 90, 70, 50, 30$  degrees) with varied relative positions.



**Figure 3.3.** (a) Schematic of the experimental setup used for the double jet impingement experiments, (b) nozzle configurations.

As explained before, the interaction between the flow fields on the surface gives rise to a liquid fountain. The formed fountain makes it difficult to take top view images and capture the flow field on the surface. To resolve this problem, a transparent plate is used as the target plate accompanied by a camera underneath the plate (see Figure 3.3a). A light source is used above the test station, which illuminates the surface and enables the camera to capture the flow field on the surface. Figures 3.4a and b illustrate sample images taken using the described test setup with vertical and inclined configurations, respectively. The flow fields formed on the surface due to the impingement of two neighboring jets are presented in Figures 3.4a1 and b1 (photography from underneath). It is seen that photography from underneath the transparent plate provides a

clear picture of the jump profiles and stagnation lines formed on the surface. Additionally, a high speed camera (PHANTOM – MIRO M310) is used to record the size and shape of the generated rising sheet. Figures 3.4a2 and b2 show the captured rising sheets.



**Figure 3.4.** Interaction between two impinging jets, (a) double vertical jet impingement, (b) double inclined jet impingement with  $\phi = 50^{\circ}$  (jets are placed in two parallel planes), (1) underneath images, (2) side view images.

### 3.3 Working Fluid

In order to examine the influence of the fluid properties on the flow characteristics of single and double jet impingement, different liquids are employed as the working fluid. Throughout the present thesis, five different working fluids are examined: distilled water, a water-surfactant solution, and three different water-glycerol solutions with wt. (mass fraction) 50%, 60%, and 65%. The water-surfactant solution is made by adding Pluronic L92 (BASF Corporation) into distilled water with a concentration of 2.5 g per liter. The surface tension of the resulting fluid is  $\sigma = 0.037$  N/m. Negligible influences are assumed on the density and viscosity of the fluid (i.e. considered to be the same as those of water).

Viscosity is known to play a significant role in the fluid dynamics of a flow. To study the influence of viscosity on the fluid flow of single and double jet impingement, glycerol is mixed with water with three different mass fractions, 50%, 60%, and 65%. The resulting fluids exhibit dynamic viscosities that are 5.11, 9, and 12.45 times higher than that of water, respectively. The physical properties of the working fluids used in this research project are listed in Table 3.1.

Fluid type	Density	Dynamic viscosity	Kinematic viscosity	Surface tension
	$(kg/m^3)$	(mpa s)	(mm <sup>2</sup> /s)	( <b>mN/m</b> )
Distilled water	998	1	1	72
Water-surfactant	998	1	1	37
Water-glycerol wt. 50%	1130.5	5.11	4.52	68.1
Water-glycerol wt. 60%	1156.6	9	7.778	67.3
Water-glycerol wt. 65%	1169.6	12.45	10.64	66.9

Table 3.1. Physical properties of the working fluids.

Generally speaking, flow meters are sensitive to the fluid viscosity. In other words, the flow rate that a flow meter measures depends on the viscosity of the working fluids. All flow meters are normally calibrated by the manufacturing company for the use of a particular working fluid. The flow meters used in the present work have been calibrated for water as the working fluid. Therefore, a change in the viscosity of the working fluid results in wrong flow rate measurements. To avoid this considerable source of error, before running any tests, the flow meters must be calibrated for each working fluid with a viscosity different from that of water (i.e. water-glycerol solutions). For this purpose, flows with different flow rates are pumped through the flow meter. The actual flow rate is measured using a stopwatch and is then plotted versus the flow rate that the flow meter reads. Employing a curve fitting, a relation can be found to correlate the actual flow rates with those measured by the flow meter. In Figure 3.5, two sample flow rate calibrations are presented for the working fluids of water-glycerol wt. 50% and 65%. Only the ranges of the flow rates that are examined for each viscous fluid are calibrated and plotted in Figure 3.5.



Figure 3.5. Flow meter calibration for viscous fluids, (a) water-solution wt. 50%, (b) water-solution wt. 65%.

### 3.4 Uncertainty

The principal measurement errors in the present experimental study are those in the flow measurement, jet diameter, post-jump fluid depth, and jump diameter. The uncertainty related to identifying the radial position of the hydraulic jump is  $\pm 5$  pixels, which is associated with the average width of the dark region seen as the hydraulic jump. Similarly, the uncertainty regarding the post-jump film depth is  $\pm 2$  pixels. The pixel measurements are then mapped to physical dimensions by calibrating the images using a reference length scale. In the experiments, the reference length scale is usually the outer diameter of the nozzle. As for the photography from

underneath, the reference length scale is determined by an image taken before conducting the experiments (normally using a ruler as the reference length). The camera is then fixed in place and will not move until the experiments are done. Consequently, the resulting nominal uncertainty of each instantaneous radial measurement is  $\pm 1\%$  and post-jump film depth is  $\pm 0.5\%$ . Moreover, the flow meters give an accuracy of  $\pm 1\%$  and the uncertainty in the jet radius for the smallest nozzle is  $\pm 5\%$ .

# **Chapter 4** Hydraulic Jump with Capillary Limit at the Edge

### 4.1 Overview

As an interesting fluid dynamics phenomenon and also an important feature of jet impingement, circular hydraulic jumps have been an active area of research for the past few decades. In actual applications of jet impingement, jets impact on surfaces with finite sizes and free edges (i.e. no weir at the edge). In the case of plates with free edges, the liquid film on the surface does not always flow off uniformly along the entire perimeter of the plate. Instead, in the case of low flow rates or/and relatively large surface sizes, the capillary effect holds the liquid film along most of the plate edge, and the liquid only falls from one (or multiple) spot at the edge (see Figure 4.1b). As discussed in the literature review, this has also been observed by other researchers such as Duchesne et al. [23] and Craik et al. [24] using working fluids with surface tension close to that of water. In this chapter, an analytical study is conducted to develop a model for predicting the location of the circular hydraulic jump on plates with finite sizes and capillary limit at the edge. Surface tension force, gravitational force, viscous friction force, and rate of flow momentum are taken into account. Based on the force-momentum analysis of two control volumes, a system of equations are derived, which can be used to solve for the jump radius and the post-jump film depth. This is different from previous theoretical models for which the post-jump film thickness was treated as a known parameter, or a uniform flow at the edge of the plate was assumed. Experimental tests using water and a water-surfactant solution are carried out to verify the presented model. Both the theory and experiments demonstrate the significance of surface

tension for this type of hydraulic jumps. Moreover, the shape of the free surface at the jump is analyzed to evaluate the assumption of *steep jump* employed for the theoretical model.

#### 4.2 Experimental Study

When a vertical liquid jet impacts on a horizontal plate, it changes direction and spreads out radially on the surface in a very thin liquid layer. The liquid flows radially with high velocities until an abrupt increase in the liquid film thickness arises due to the occurrence of a circular hydraulic jump. Downstream of the jump, the liquid velocity drops significantly. Figure 4.1 illustrates a hydraulic jump formed by a water jet on a surface. The detailed flow characteristics of jet impingement were discussed in Chapters 1 and 2, and schematically shown in Figure 2.2b.

## 4.2.1 Experimental methodology

A series of experimental tests with round liquid jets impinging on circular discs have been conducted. As schematically shown in Figure 4.1, a vertical liquid jet with radius *a* and flow rate *Q* impacts on a circular disc with radius  $R_s$ . The spreading of the jet consists of a circular thin area (i.e. pre-jump region) surrounded by a thick film (i.e. post-jump region). The hydraulic jump appears at a radial location  $r = R_j$  where the film depth changes from *h* to *H*. The postjump thick film is pinned at the edge of the disc and maintains a stable rim around the edge of the circular disc except one spot where the fluid flows off the disc (see Figure 4.1b).

In this study it is assumed that the liquid flows off only from one point at the edge. The one-spot edge flow at the disc edge is controlled by attaching a small piece of absorbent paper to the disc edge and flush with the top surface of the disc. The same method was used by Craik et al. [24] for achieving a uniform edge flow, for which absorbent paper was attached all around the disc edge. During our experiments, the flow rate was changed with care so that the post-jump liquid film always had a stable rim with one-spot edge flow. Despite the one-spot edge flow, it was observed that all the hydraulic jumps remain circular (axisymmetric). For each disc, the maximum tested flow rate was when the one-spot edge flow was about to disappear. If the flow rate was further increased, the fluid would flow off at multiple spots and then all over the disc edge.



**Figure 4.1.** (a) The hydraulic jump formed by a round water jet impinging on a circular disc, (b) Side view of the water jet impinging on a circular disc, where water flows off the disc only from one spot at the edge. (c) Schematic of jet impingement on a circular disc on which the post-jump film has a stable rim formed at the disc edge.

Two working fluids are tested: distilled water and a water-surfactant solution, whose physical properties are presented in Table 3.1. The jet nozzle used for this set of experiments has an inner radius of a=0.375 mm and a length to radius ratio equal to 68. Circular aluminum discs with eight varied radii,  $R_s$  ranging from 20 to 50 mm ( $R_s/a$  ranges from 53 to 133), are tested. The

nozzle to surface spacing is 10*a* and remains fixed for all the experiments. The side view images and top view images of the flow field on the surface are used to measure the jump radius and the post-jump film thickness, respectively. Additionally, the side view images are used to measure the contact angle  $\beta_s$  at the edge as shown in Figure 4.1c. Employing a height gage, the free surface profile at the jump location is also captured.

### 4.2.2 Experimental results

For a jet impinging on a surface, Froude number can be used to compare the flow inertia to the gravitation. If a steep jump is assumed (i.e. at  $r = R_j$  the film depth abruptly changes from *h* to *H*), two Froude numbers can be defined as

$$Fr_{i} = \lambda Q / \left( 2\pi R_{j} \sqrt{gh^{3}} \right)$$
(4.1a)

$$Fr_o = Q / \left( 2\pi R_j \sqrt{gH^3} \right) \tag{4.1b}$$

Here  $Fr_i$  is the pre-jump Froude number based on the pre-jump film depth, while  $Fr_o$  is the postjump Froude number based on the post-jump film depth. The assumption here is a uniform velocity profile across the depth. This assumption is suitable for the post-jump film, as the inertial force is insignificant as compared to the hydrostatic force. However, the velocity profile is important for the pre-jump region, where the inertial force is dominant. A correction factor  $\lambda$  is presented to take into account the non-uniformity of the velocity profile in the thin pre-jump film. The correction factor  $\lambda$  will be derived in Chapter 5 along with a detailed discussion and analysis of the circular hydraulic jump Froude number. The experimentally measured data of the jump radius  $R_i$  and the post-jump film thickness H are

plotted in Figure 4.2. The y-axis is the experimental data in form of  $\frac{Q}{\pi a^2} / \sqrt{g \frac{a^2}{2R_j}}$  (i.e. defined as the pre-jump Froude number for the impingement of an *inviscid* jet, which has the pre-jump

film depth h equal to  $a^2/2R_j$  and a uniform velocity equal to the jet velocity  $Q/\pi a^2$ ). The x-axis

is the experimental data in form of  $\left(H/\frac{a^2}{2R_j}\right)^{3/2}$  (i.e. defined as the post-jump film depth

normalized by the inviscid pre-jump film depth). The results are presented as two groups of tests using two different working fluids. Linear fitting with intercept set at the origin is carried out. Each group shows a linear slope, which is the post-jump Froude number  $Fr_o$  expressed as Eq. (4.1b). This indicates that in each group of tests, the post-jump Froude number remains constant. The tests using the water-surfactant solution show a constant post-jump Froude number of  $Fr_o = 0.15$ , while the tests using water show  $Fr_o = 0.10$ . The Froude numbers here show dependence on the surface tension of the fluid (contrary to what reported by Duchesne et al. [23]). Also, both values are smaller than the constant Froude number reported by Duchesne et al. [23] (i.e. ~0.32). The major reason might be related to the edge flow condition as the other study mainly focused on hydraulic jumps formed with uniform edge flow around the disc. An extensive Froude number analysis is carried out in Chapter 5 and the results will be compared to those of Duchesne et al. [23].



**Figure 4.2.** Experimental results of the hydraulic jump radius  $R_j$  and the post-jump depth *H* for varied flow rates, disc sizes, and working fluids. The slope of the linear fitting for each fluid is the post-jump Froude number defined by Eq. (4.1b). The inset graph shows the slope of the linear fitting ( $Fr_a$ ) for different disc sizes using water jets.

The data points in Figure 4.2 appear in clusters. Each cluster is for a specific flow rate, and each data point in the cluster represents a specific disc size. The inset of Figure 4.2 shows the effect of the disc size on the post-jump Froude number. The data is associated with water as the working fluid. For this purpose, linear fitting is carried out to the data points associated with each disc size (with varied flow rates). The slope of each line is  $Fr_o$  corresponding to a certain disc size, and is then plotted versus the disc radius in the inset of Figure 4.2. Although not very clear, the post-jump Froude number seems to decrease with increasing the disc size. However, the change is in a quite narrow range from ~0.098 to ~0.103.

From the experiments, it was observed that the post-jump depth *H* formed on a disc shows negligible dependence on the flow rate *Q*. As an example, Figure 4.3*a* shows the hydraulic jump formed on a disc with  $R_s$ =20 mm. The post-jump depth, H~3.92±0.07 mm, remains almost constant with varied flow rates. Thus, one would expect if both  $Fr_o$  and *H* are constant, according to Eq. (4.1*b*), the jump radius  $R_j$  should change linearly with the flow rate *Q*. This linear trend can be seen in Figure 4.3a, which shows  $R_j$  linearly increasing with *Q*. The error bars are eliminated from the figure because the ranges of the measured errors are smaller than the size of the symbols plotted here, and thus the error bars would not be visible in the figure.



Figure 4.3. (a) The jump radius  $R_j$  and the post-jump film depth H formed by water jets impinging on a circular disc of  $R_s = 20$  mm. (b) The measured jump radius  $R_j$  of all the tests conducted here in comparison with the scaling law  $R_j \propto Q^{5/8}$ , and also with the critical radius  $r_c$  for the boundary layer development in the pre-jump region.

Figure 4.3b shows  $R_j$  versus Q for the two groups of tests presented in Figure 4.2. Each group shows a linear trend of  $R_j$  increasing with Q. The scaling law presented by Bohr et al. [9] states that the jump radius changes with  $Q^{5/8}$ . To compare with the scaling law, a relation  $R_j \propto Q^{5/8}$  is plotted by matching with the data point of the smallest  $R_j$ . Clearly, the present work with the edge condition of stable rim is different from the scaling law presented for the circular hydraulic jumps with uniform edge flow.

Here, a theoretical study is carried out to develop a model to predict the hydraulic jump radius with capillary limit at the edge.

### 4.3 Theoretical Study

### 4.3.1 Force Analysis

From the present experimental tests, it is observed that when the flow rate is low or/and the plate is relatively large, the post-jump film forms a stable rim around the circular disc and the liquid flows off the disc only from one point at the edge (see Figure 4.1*b*). As the flow rate increases, the jump radius grows toward the edge of the disc, and the post-jump fluid flows off at multiple edge spots, and eventually all over the edge. In the present study, the focus is on the jet impingement with stable rim at the edge.

Since a stable rim outlines the plate edge, the flow momentum in the radial direction at the disc edge is zero. The flow field on the surface is assumed to be axisymmetric for the following three reasons. First, the hydraulic jumps in the present work are always observed to remain circular despite the one-spot edge flow. Second, although as a result of the stable rim, the radial flow needs to turn into an azimuthal flow when getting close to the plate edge (the flow eventually converges to the one-spot flow at the edge), this azimuthal flow is weak and negligible. This can be justified by evaluating a local Froude number close to the disc edge, which can be calculated

by replacing  $R_j$  in Eq. (4.1b) with  $R_s$ . This local Froude number must be even smaller than the post-jump Froude numbers that have been shown in Figure 4.2. Third, neglecting the azimuthal flow can also be justified from the observation of the contact angle at the disc edge, which shows relatively constant contact angles around the disc perimeter. The measurement of the contact angle will be discussed in Section 4.3.2.

In the present theoretical study, two control volumes will be introduced and analyzed. The forcemomentum balance will be carried out on these two control volumes to obtain two relations between the jump radius ( $R_i$ ) and the post-jump film thickness (H).



Figure 4.4. (a) The simplified view of the first control volume with an angular dimension  $\Delta \theta$  and radial dimension from the hydraulic jump  $R_j$  to the edge of the disc  $R_s$ ; (b) the central plane of the control volume with all forces projected on the plane.

The first control volume is a small angular section  $\Delta\theta$ , with *r* ranging from the jump location  $R_j$  to the edge of the disc  $R_s$ . A 3D schematic of the control volume is shown in Figure 4.4*a*. For easy presentation, the free liquid surface is not shown as a smooth curved surface, but instead is composed of three facets (2*a*, 2*b*, 2*c*). The angle  $\alpha$  is used to indicate the azimuth angle within

the control volume, which ranges from  $-\Delta\theta/2$  to  $\Delta\theta/2$  with its center located at  $\alpha = 0$ . The control volume consists of a radial flow input from the pre-jump liquid film as well as the surface tension force on the free surfaces (2*a*, 2*b*, 2*c*), the hydrostatic pressure symmetrically on facets 1*a* and 1*b*, and the shear stress on the bottom of the control volume (facet 3) due to the interaction with the disc surface. Based on the foregoing discussion on the azimuthal flow, the azimuthal flows entering and exiting through the facets 1*a* and 1*b* are neglected.

In this study, the force balance in the radial direction of the control volume will be analyzed. All the forces are projected to the center radial plane ( $\alpha = 0$ ) of the control volume (see Figure 4.4*b*), which is orthogonal to the free surface of the liquid film. The free surface is expressed by  $z = \xi(r)$ , and z = 0 is the liquid-solid interface. Due to the pressures and stress mentioned above, there are surface tension force ( $F_{\sigma}$ ), hydrostatic force ( $F_{g}$ ), viscous friction force from the disc surface ( $F_{\mu}$ ), and force from the pre-jump region (*F*). In Figure 4.4*b*, for clearer comprehension, force vectors are shown in the presumed directions, and they are positive when acting in the positive radial direction. Applying a force balance to the control volume (Figure 4.4*b*) gives

$$F + F_{\sigma} + F_{\mu} + F_{\varrho} = 0 \tag{4.2}$$

#### 4.3.1.1 Surface tension force

An arbitrary segment of the central plane (see Figure 4.4*b*) is presented in Figure 4.5. Here  $\hat{n}$  is the unit vector of the local surface normal,  $\hat{r}$  is the unit radial vector, and *dS* is an element of the length of the free surface. To obtain the surface tension force shown in Figure 4.4*b*, surface

integral should be applied to the entire free surface of the control volume, and the local force should be projected to the radial direction at  $\alpha$ =0. Hence, the surface tension force is given by

$$F_{\sigma} = \int_{-\Delta\theta/2}^{\Delta\theta/2} \int_{S} \sigma \left( K_{1} + K_{2} \right) \left( \hat{n} \cdot \hat{r} \right) r dS \cos \alpha d\alpha$$
(4.3)

where  $K_1$  and  $K_2$  are the two principal curvatures, and *S* is the length of the free surface from  $R_j$  to  $R_s$ . For a small angle  $\Delta\theta$ ,  $2\sin(\Delta\theta/2)\sim\Delta\theta$ , and this approximation is used throughout the present study. Applying the approximation, Eq. (4.3) reduces to



**Figure 4.5.** (a) An arbitrary portion of Figure 4.4b shows the vectors on the free surface for the analysis of the surface tension force; (b) More geometric details of the 1<sup>st</sup> control volume.

Detailed analysis of Eq. (4.4) is provided in Appendix A , which results in Eq. (A13). Eq. (A13) shows two components for  $F_{\sigma}$ , which can be written as

$$F_{\sigma} = F_{\sigma,j} + F_{\sigma,s} \tag{4.5}$$

Where  $F_{\sigma,j}$  is the surface tension force (in the radial direction) at the jump location and  $F_{\sigma,s}$  is the surface tension force (in the radial direction) at the plate edge. The two components are differentiated based on the geometry shown in Figure 4.5b. Here, a jump region is defined ( $R_j \leq$  $r \leq R_{j,c}$ ), where the free surface rises from *h* to *H* and has a length  $S_j$ . There is also a region close to the surface edge ( $R_{s,c} \leq r \leq R_s$ ), where the free surface descends from *H* to zero and has a length  $S_s$ . There is another region in between ( $R_{j,c} \leq r \leq R_{s,c}$ ) where the film depth *H* remains relatively constant. On small surfaces, it is possible that  $R_{j,c} = R_{s,c}$ .

According to Eq. (A13) and Eq. (4.5), the surface tension force at the jump is expressed by

$$F_{\sigma,j} = -\sigma \left[ S_j - \left( R_{j,c} - R_j \right) \right] \Delta \theta$$
(4.6)

Here,  $\beta_j$  in Eq. (A13) (i.e. the local slope angle of the film surface at  $r = R_j$ , see Figure 4.5*b*) has been taken to be zero, as the entire free surface from the pre-jump region to the post-jump region must be differentiable. Simplification can be made by assuming a steep jump, i.e.  $(R_{j,c} - R_j) \rightarrow$ 0. As a result,  $S_j \rightarrow H(1 - h/H)$ . Since  $h/H \ll 1$ , the approximation  $S_j \sim H$  can be used. Thus, Eq. (4.6) reduces to

$$F_{\sigma,j} = -\sigma H \Delta \theta \tag{4.7}$$

which was also obtained by Bush & Aristoff [12].

According to Eq. (A13) and Eq. (4.5), the surface tension force at the disc edge is expressed by  $F_{\sigma,s} = -\sigma \left[ S_s - \Delta R_s + R_s \left( 1 - \cos \beta_s \right) \right] \Delta \theta$ (4.8) where  $\Delta R_s = (R_s - R_{s,c})$ . If we assume a uniform curvature  $K_1$  for the free surface at  $R_{s,c} \le r \le R_s$  (see Figure 4.5),  $\Delta R_s$  and  $S_s$  can be determined using the geometric relations given by

$$\Delta R_{s} = \frac{H \sin \beta_{s}}{1 - \cos \beta_{s}}$$

$$S_{s} = \frac{H \beta_{s}}{1 - \cos \beta_{s}}$$
(4.9)

Plugging Eq. (4.9) into Eq. (4.8), and substituting Eqs. (4.7) and (4.8) into Eq. (4.5), the total surface tension force associated with the free surface is

$$F_{\sigma} = -\sigma H \left[ 1 + \frac{\beta_s - \sin \beta_s}{1 - \cos \beta_s} + \frac{R_s}{H} (1 - \cos \beta_s) \right] \Delta \theta$$
(4.10)

### 4.3.1.2 Hydrostatic force

There is hydrostatic pressure on the side surfaces of the defined control volume (facets 1*a* and 1*b* in Figure 4.4*a*). The angle between the inward-pointing surface normal and the radial vector  $\hat{r}(\alpha = 0)$  is  $(\pi/2 - \Delta\theta/2)$ . Hence the resulted force projected to the center radial direction can be determined by

$$F_g = 2\sin\left(\frac{\Delta\theta}{2}\right) \int_{R_j}^{R_s} \int_{0}^{\xi} \rho gz dz dr$$
(4.11)

For the jet impingement with relatively low flow rates on relatively large discs, a major portion of the post-jump liquid film shows a relatively uniform depth *H*. For simplicity, a uniform depth from  $R_j$  to  $R_s$  is assumed in the analysis, i.e.  $\xi(r) \sim H$ . Integrating from  $R_j$  to  $R_s$  results in

$$F_g = \frac{1}{2}\rho g H^2 \left(R_s - R_j\right) \Delta\theta \tag{4.12}$$
The error associated with this assumption could be significant for small discs, on which the shape of the post-jump film is curved rather than flat. This will be shown and discussed in Section 4.4.1.

#### 4.3.1.3 Viscous friction force

Downstream of the jump, the fluid film continuously flows on the disc. There is a viscous friction force on the bottom of the control volume (facet 3 in Figure 4.4a), which is related to the velocity gradient at the solid-fluid interface. If the post-jump film is sufficiently large as compared to the pre-jump region, the complex flow around the jump location as a result of the boundary layer separation can be assumed to already end at  $r = R_{j,c}$ , which have been reported by previous studies [11,14,15,52]. Therefore, the flow after the jump can be assumed radially unidirectional, and the velocity is given by

$$U_0 = \frac{Q}{2\pi H R_{j,c}} \tag{4.13}$$

The analysis of the viscous friction force in the post jump region is provided in Appendix B. The viscous force projected to the central radial direction of the control volume is

$$F_{\mu} = -0.383 \left(\rho\mu\right)^{1/2} \left(\frac{Q}{2\pi H R_{j,c}}\right)^{3/2} \left(R_{s,c} - R_{j,c}\right)^{1/2} \left(R_{s,c} + 2R_{j,c}\right) \Delta\theta$$
(4.14)

The discussion in Section 4.4.2 will show that the viscous friction force is much smaller than the other forces in the present force analysis. Therefore, the error associated with the unidirectional flow assumption is insignificant for the entire force analysis.

All the forces required for Eq. (4.2) have been obtained. A steep hydraulic jump is assumed such that  $R_{j,c} = R_j$ . For simplicity,  $R_{s,c}$  in Eq. (4.14) is replaced with  $R_s$ . Plugging Eqs. (4.10), (4.12), and (4.14) into Eq. (4.2), and dividing through by  $\Delta\theta$ , the force analysis of the control volume shown in Figure 4.4 results in

$$F' = \sigma H \left[ 1 + \frac{\beta_s - \sin \beta_s}{1 - \cos \beta_s} + \frac{R_s}{H} (1 - \cos \beta_s) \right] - \frac{1}{2} \rho g H^2 (R_s - R_j) +$$

$$0.383 (\rho \mu)^{1/2} \left( \frac{Q}{2\pi H R_j} \right)^{3/2} (R_s - R_j)^{1/2} (R_s + 2R_j)$$
(4.15)

Here the unit of F' is force per radian. On the right hand side, the first, second, and third terms are associated with the surface tension force, the hydrostatic force, and the viscous friction force, respectively.

# 4.3.1.4 Force from the pre-jump flow

For the defined control volume, the force F shown in Eq. (4.2) comes from the film flow in the pre-jump region, which includes the rate of the flow momentum and the hydrostatic force. Projected to the central radial direction, the force from the pre-jump film can be written as

$$F = \left(\rho \int_{0}^{h(R_j)} u^2 \Big|_{r=R_j} dz + \frac{1}{2}\rho g h^2\right) R_j \Delta\theta$$
(4.16)

Since the flow prior to the jump is characterized as a supercritical flow with high Froude numbers (for the present work,  $Fr_i>10$ ), in Eq. (4.16), the inertia term is dominant, and the hydrostatic force term can be neglected. Dropping the hydrostatic term, and applying Watson's solution for the flow in the pre-jump thin region [8], Eq. (4.16) can be expressed as

$$F' = \frac{\rho}{2} \left(\frac{Q}{\pi a}\right)^2 \left[1 - 1.0213 \left(\frac{R_j}{a \operatorname{Re}}\right)^{1/2} \left(\frac{R_j}{a}\right)\right], \text{ for } R_j < r_c$$
(4.17a)

$$F' = \frac{\rho}{2} \left(\frac{Q}{\pi a}\right)^2 \left[ 3.8486 \frac{1}{\text{Re}} \left(\frac{R_j}{a}\right)^3 + 1.1039 \right]^{-1}, \text{ for } R_j > r_c$$
(4.17b)

where Re is the jet Reynolds number based on the jet diameter defined as  $Re = 2Q/\pi av$ . The two Eqs. (4.17a) and (4.17b) are associated with the conditions for which the hydraulic jump occurs in a radial position before and after where the boundary layer in the pre-jump region reaches the surface (i.e.  $r = r_c$ ), respectively. Figure 3b has shown that for all the tests in the present work, the jump takes place after the boundary layer absorbs the entire film depth (i.e.  $R_j > r_c$ ). Thus, Eq. (4.17b) will be used for calculations.

Equations (4.15) and (4.17) provide the force-momentum balance relation for the control volume shown in Figure 4.4. In Eq. (4.15), the surface tension term depends on the contact angle  $\beta_s$ . This contact angle could be related to the fluid surface tension, the wettability of the disc, the local shape and structure of the disc edge, the disc size, and the flow rate. Ideally, if these relations were known,  $\beta_s$  could be determined separately by another correlation equation. In the present study,  $\beta_s$  is an input from the experimental observations. As a result, there are two unknown parameters in the present force-momentum balance relation,  $R_j$  and H, and therefore another relation is needed.

# 4.3.1.5 Analysis of the 2<sup>nd</sup> control volume

To obtain another relation between  $R_j$  and H, a portion of the first control volume from  $R_j$  to  $R_{j,c}$  (see Figure 4.5*b*) is chosen as the 2<sup>nd</sup> control volume. The projection of the control volume to its central radial direction is shown in Figure 4.6. The force-momentum balance of the control volume requires

$$F - M_{out} + F_{g,j} + F_{\sigma,j} = 0 \tag{4.18}$$

where  $F_{g,j}$  represents the hydrostatic force, and  $\dot{M}_{out}$  is the rate of the flow momentum at the outlet. The viscous friction force is neglected because the radial interval  $(R_{j,c} - R_j)$  is considered to be small. The surface tension force  $(F_{\sigma,j})$  has been given by Eq. (4.6) and in a simplified form by Eq. (4.7).



Figure 4.6. The 2<sup>nd</sup> control volume is selected at the hydraulic jump location.

The velocity is still assumed to be unidirectional right after the jump, and the velocity is already given by Eq. (4.13). The error associated with this assumption should be small for the entire force analysis, as the flow inertial force in the post-jump region is insignificant as compared to the hydrostatic force (i.e the post-jump region flow is characterized as a subcritical flow with low Froude numbers). Projected to the central radial direction of the control volume, the rate of the output flow momentum on the post-jump side is

$$\dot{M}_{out} = \rho \left(\frac{Q^2}{4\pi^2 H R_{j,c}}\right) \Delta \theta \tag{4.19}$$

The hydrostatic force for the second control volume can be expressed by

$$F_{g,j} = -\frac{1}{2}\rho g H^2 R_{j,c} \Delta \theta + \frac{1}{2}\rho g \left( \int_{R_j}^{R_{j,c}} \xi^2 dr \right) \Delta \theta$$
(4.20)

The second term in Eq. (4.20) is the hydrostatic force on the sides of the control volume (i.e. similar to Eq. (4.11) for the 1<sup>st</sup> control volume).

Equations (4.7), (4.19) and (4.20) are then plugged into Eq. (4.18). By assuming a steep jump,  $R_{j,c}$  can be replaced by  $R_j$  in all the equations. After dividing through by  $\Delta\theta$ , Eq. (4.18) becomes

$$F' = \frac{\rho Q^2}{4\pi^2 H R_j} + \frac{1}{2} \rho g H^2 R_j + \sigma H$$
(4.21)

The above analysis of the two control volumes results in a system of three equations: Eqs. (4.15), (4.17), and (4.21), which contain three unknowns: F',  $R_j$ , and H, which can be solved for the jump radius and the post-jump thickness.

# 4.3.2 Validation

To apply the force-momentum balance equations, the contact angle  $\beta_s$  needs to be known. Employing an image processing software (ImageJ), the contact angle is measured using the five side view images taken at different azimuth positions away from the one-spot edge flow. Averages are taken with standard deviations generally within  $\pm 5^{\circ}$ , which could be due to the inconsistency of the disc edge condition. For each test, the measurement does not show any trend that relates the contact angle to the relative location from the one-spot edge flow. Therefore, the observation of the relatively constant contact angle around the disc edge indicates negligible effect of the non-uniform azimuthal flow. The data for the two fluids are presented in Figure 4.7. Overall, the two groups of tests show different ranges:  $\beta_s < 90^\circ$  for the test using the water-surfactant solution, and  $\beta_s > 90^\circ$  for the tests using water. Figure 4.7 shows that  $\beta_s$  does not remain constant as the flow condition and the disc size change. For example, for a specific flow rate of water, by changing the disc size, the contact angle varies within almost 20°. However, the measurements do not show any clear trends of changing with the flow rate and the disc size. In addition to the flow rate and disc size, the contact angle could also be affected by the contact angle hysteresis and the inconsistency of the surface edge condition.



Figure 4.7. Contact angles formed by the free stable liquid film at the disc edge.

It is worth-noting that as an input parameter to the theoretical model,  $\beta_s$  carries the geometric information of the rim at the edge (see Eq. 4.9) rather than the local wetting at the disc edge. This is true as the model does not include any wetting equilibrium relation such as the Young's equation. In other words,  $\beta_s$  does not have to be an equilibrium wetting angle. Therefore, the measurement deviation of  $\pm 5^{\circ}$  is the only major uncertainty associated with  $\beta_s$  that affects the accuracy of the theoretical model.



Figure 4.8. Comparison between the theory and the experimental data for varied flow rates, disc sizes, and working fluids. (a) Evaluation of Eqs. (4.15), (4.17), and (4.21) using the experimental data. The x-axis starts from  $(r_c / a)^3 / \text{Re} = 0.043$ . (b) The theretical predictions of the jump radius versus the experimental measurements.

Two methods are used to validate the theoretical model (Eqs. 4.15, 4.17, and 4.21), which examine the agreement of the presented equations with the experimental data. The presented equations are plotted in Figure 4.8a, where the force F' normalized by  $0.5\rho Q^2/(\pi a)^2$  is plotted versus  $(R_j/a)^3/Re$ . First, Eq. (4.17) is plotted as a continuous line, which represents the force calculated based on the flow analysis of the pre-jump liquid film. Second, the experimental conditions ( $Q, R_s, \sigma$ , and a) and the measured data ( $R_j, \beta_s$ , and H) are plugged into Eqs. (4.15) and (4.21), which are then plotted as scatter data points. Calculations using Eq. (4.15) estimates the force (F') based on the force analysis of the 1<sup>st</sup> control volume from the jump to the disc edge, while Eq. (4.21) estimates the force based on the force analysis at the jump location (i.e.  $2^{nd}$  control volume). A better agreement between Eq. (4.21) and Eq. (4.17) is visible. The deviation of Eq. (4.15) from Eq. (4.17) could be due to the uncertainties associated with the contact angle measurement, the assumption of a flat post-jump film, and the assumption of a uniform curvature at the edge.

Figure 4.8a also shows that for all the tests  $(R_j/a)^3/Re > (0.366)^3$ , which indicates  $R_j > r_c$ . This is due to the force limitation of the stable rim at the surface edge. For a hydraulic jump to appear where  $R_j < r_c$ , the normalized force from the post-jump film must be higher than ~0.8 as shown in Figure 4.8a.

The second method to verify the proposed theory is to solve Eqs. (4.15), (4.17), and (4.21) for  $R_j$  using the measured  $\beta_s$ . Presented in a normalized form,  $R_j/R_s$ , the theoretical predictions are plotted versus the experimental measurements in Figure 4.8b. If the theory estimates the accurate values of the jump radii, all the data points will fall on the line y=x. Figure 4.8b shows a fairly good agreement between the theory and experiments.

# 4.4 Discussion

# 4.4.1 Gradual jump

In the presented theory, a steep jump was assumed for the analysis. As mentioned before, a steep jump means a sudden increase in the film thickness from *h* to *H*, at the location of the jump  $r = R_j$ . In other words, the maximum depth of the post-jump film, *H*, was assumed as the local film depth at  $r = R_j$ . However, in reality, the film free surface "jumps" gradually rather than steeply. Experimental and numerical observations of gradual jumps have been reported in a few previous studies [11,14,24,48,101]. In this section, our experimental observation of the gradual jump will

be discussed to evaluate the effect of the steep jump assumption employed in the proposed theory.

As shown in Figure 4.9, a height gauge is used to measure the local height of the film free surface at varied radial locations downstream from the hydraulic jump  $r = R_j$ . The jet impingement produces a relatively flat post-jump film on the large disc (see Figure 4.9a), and a curved film on the small disc (see Figure 4.9b). The measured data are plotted in Figure 4.9c, which shows a gradual increase of the film depth around the jump locations. As schematically shown by the inset of Figure 4.9c, the jump occurs within a radial range from  $R_j$  to  $R_{j,c} = R_j + \Delta R_j$ . For both cases, the radial distance  $\Delta R_j$  is close to H, and comparable to  $R_j$ .



**Figure 4.9.** A water jet (Q=3.75 cm<sup>3</sup>/s) impinging on: (a) a large disc  $R_S$  = 40 mm; (b) a small disc  $R_S$  = 17.5 mm. (c) The height of free surface measured using a height gauge (the gauge needle is seen in the images). Quarter circle fitting is applied to the free surface at the jump location. The inset schematically shows a quarter circle jump shape.

To analyze the gradual jump, the force equation for the 2<sup>nd</sup> control volume considering a steep jump (Eq. 4.21) is rewritten taking into account the gradual change in the free surface at the

jump location. In this light, neglecting the viscous friction force on the disc surface (from  $R_j$  to  $R_{j,c}$ ), from Eqs. (4.6), (4.19) and (4.20), the force per unit radian can be written as

$$F'_{\Delta} = \frac{Q^2}{4\pi^2 \left(R_j + \Delta R_j\right) H} +$$

$$\frac{1}{2} g H^2 \left(R_j + \Delta R_j\right) - \frac{1}{2} g \left(\int_{R_j}^{R_j + \Delta R_j} \xi^2 dr\right) + \frac{\sigma}{\rho} \left(S_j - \Delta R_j\right)$$

$$(4.22)$$

Here  $F'_{\Delta}$  denotes the force for the gradual jump (replacing F' associated with a steep jump).

To continue the analysis of Eq. (4.22), the surface shape of the gradual jump is needed. The height gauge measurement shows  $\Delta R_j \sim H$ , which can also be perceived from the previous studies as well [14,24,60,101]. Hence, as an approximation, a quarter circle fitting is applied to the data points in the jump region. Fairly good fitting is visible for both cases as shown in Figure 4.9c. The radius of the circle is  $\Delta R_j$ , and  $H = \Delta R_j + h \approx \Delta R_j$ . Hence,  $S_j = \Delta R_j \pi/2$ , and  $\xi^2 = 2\Delta R_j (r - R_j) - (r - R_j)^2$ . Applying these conditions to Eq. (4.22) and dividing it through by  $\rho [(Q/2\pi)^4 g/R_j]^{1/3}$  gives

$$f_{o,\Delta} = Fr_o^{-4/3} \left[ Fr_o^2 \left( 1 + \frac{H}{R_j} \right)^{-1} + 0.5 \left( 1 + \frac{H}{3R_j} \right) + Bo^{-1} \left( \frac{\pi}{2} - 1 \right) \right]$$
(4.23)

where  $\Delta R_j/R_j$  has been replaced with  $H/R_j$ . Here,  $f_{o,\Delta}$  is the non-dimensional form of  $F'_{\Delta}$  for the post-jump region of a gradual jump. Whereas for a steep jump,  $f_o$  is defined as the non-dimensional form of F', which can be derived by dividing Eq. (4.21) through by  $\rho [(Q/2\pi)^4 g/R_j]^{1/3}$  as follows

$$f_o = Fr_o^{-4/3} \left( Fr_o^2 + 0.5 + Bo^{-1} \right)$$
(4.24)

where  $Bo^{-1} = \sigma / \rho g R_j H$  is the inverse Bond number, which compares the surface tension force and the gravitational force. The effect of the gradual jump can be evaluated by comparing the difference between Eq. (4.23) and Eq. (4.24), which is

$$\frac{f_{o,\Delta} - f_o}{f_o} = \left[\frac{1}{6}\frac{H}{R_j} - Fr_o^2\frac{H}{R_j} + Bo^{-1}\left(\frac{\pi}{2} - 2\right)\right] \left(Fr_o^2 + 0.5 + Bo^{-1}\right)^{-1}$$
(4.25)

For obtaining Eq. (4.25), the approximation  $(1 + H/R_j)^{-1} \sim (1 - H/R_j)$  for  $H/R_j < 1$  has been used for Eq. (4.23). In Eq. (4.25), the first term in the brackets is the increase of the hydrostatic force, the second term is the decrease of the inertia force, and the third term is the decrease of the surface tension force. The gradual jump could increase  $(f_{o,\Delta} > f_o)$  or decrease  $(f_{o,\Delta} < f_o)$  the total force depending on  $Fr_o$ ,  $H/R_j$ , and  $Bo^{-1}$ . For the two cases in Figure 4.9,  $Fr_o \sim 0.1$ ,  $H/R_j \sim 0.6$ ,  $Bo^{-1} \sim 0.25$ , which result in  $(f_{o,\Delta} - f_o)/f_o \sim -2\%$ . However, for the 2% reduction of total force, there is a 13% increase of the hydrostatic force, a 1% reduction in the inertia force, and a 14% reduction of the surface tension force. The individual effects on the hydrostatic and surface tension forces are not negligible. However, due to the counterbalance of the individual effects, the overall effect of the gradual jump is small. Therefore, *steep jump* is a reasonable assumption in present theory.

#### 4.4.2 Scale Analysis

Here, we aim to theoretically evaluate the contribution that each force makes in Eq. (4.2). In Figure 4.10, the individual forces of Eq. (4.2) are non-dimensionalized and plotted versus the jet Reynolds number in a logarithmic scale. In this regard, the system of Eqs. (4.15), (4.17), and (4.21) is first solved for water jets with varied flow rates impinging on a disc of  $R_s$ =50 mm. As for the contact angle  $\beta_s$ , a constant angle of  $\beta_c = 100^\circ$  is employed, which is approximately the average of all the measured contact angles presented in Figure 4.7 for water. The calculated  $R_j$  and H for each condition are then plugged into Eqs. (4.10), (4.12), (4.14), and (4.17) to calculate the individual forces. The forces are also non-dimensionalized by dividing them by  $\rho[(Q/2\pi)^4 g/R_j]^{1/3}$ . In Figure 4.10,  $f_\sigma$ ,  $f_g$ ,  $f_\mu$ , and  $f_i$  represent the non-dimensional surface tension force  $F_\sigma$ , non-dimensional hydrostatic force  $F_g$ , non-dimensional force from the pre-jump region F, respectively.



Figure 4.10. Comparison between different forces (theoretically calculated) in Eq. (4.2) for a given condition of  $R_s = 50mm$ , a=0.375 mm,  $\beta_s = 100^\circ$ , and varied flow rates of water jets.

It is seen that all the non-dimensional forces decrease with increasing Re. This is because the jump radius grows and moves further downstream from the impingement zone as Re increases. Likewise, the post-jump region shrinks. Both reasons contribute to the decreasing trend of the forces observed in Figure 4.10. The viscous force is observed to play a small role and can be

neglected from the calculations. The surface tension force has two major components, one at the jump location and the other one at the rim of the post-jump film. It is seen that the surface tension plays a major role in the circular hydraulic jumps formed on finite surfaces, for which there is a stable rim on the edge. The surface tension force balances mainly against the gravitational force of the post-jump film and the force originating from the pre-jump region.

# 4.5 Summary

The circular hydraulic jump formed by a liquid jet impinging on finite-size circular discs with free edge was studied, where the post-jump film is bounded by a stable rim. Jet impingement with capillary limit at the plate edge is commonly observed for low flow rates. Surface tension was observed to play a major role for the post-jump region from the jump to the disc edge. The jump radius in case of capillary limit at the edge was experimentally observed to increase linearly with the flow rate. This trend is different from the scaling law presented for the uniform edge flow. Detailed force analysis was conducted for the two control volumes, one including the entire post-jump film and one at the jump location. The theoretical model developed based on the force-momentum analyses of the two control volumes were shown to agree with the experimental results using water and water-surfactant solution.

Steep jump has been the major assumption for the present theoretical analysis. Based on the experimentally measured jump profiles, the gradual jump was found to reduce the surface tension force and increase the hydrostatic force. However, the overall effect is small, and steep jump is a reasonable assumption in the present theory.

# Chapter 5 Hydraulic Jump on Small Surfaces and a Froude Number Analysis

#### 5.1 Overview

Although in a number of the previous studies the finite size of the target plate was taken into account as a boundary condition, little attention has been paid to the plate size as a major test variable. As a result, there is limited information available regarding the influence of the plate size on the hydraulic jump, especially when the plate size is small. In this chapter, an experimental study is carried out to examine the circular hydraulic jumps formed on small target plates, i.e. the sizes of these plates are comparable to the sizes of the hydraulic jumps formed on them. It will be shown that the hydraulic jump is significantly dependent on the size of the plate when the plate size is small or/and the jet flow rate is high. With a fixed jet flow rate, an increase of up to 68% was observed in the diameter of the jump only by changing the target plate size. It will be discussed that the change observed in the jump diameter is related to the change of the post-jump film depth.

All the foregoing observations are shown to be attributed to the reduction of the post-jump region on the plate, which causes the flow inertia in the post-jump flow to become significant. This will be explained by examining the post-jump Froude number. The relation between the pre-jump and post-jump Froude number will be theoretically analyzed. The analysis reveals the significant importance of the surface tension as one of the primary differences between classical hydraulic jumps and circular jumps. Finally, the possibility of obtaining a post-jump flow with a

Froude number higher than unity (the critical Froude number in classical jumps) will be examined.

## 5.2 Experimental methodology

The experimental setup employed for this work has been described in Chapter 3. The discharged liquid from the nozzle impacts on the horizontal target plate and forms a circular hydraulic jump on the surface (see Figure 5.1a). Three different nozzles with inner diameters d = 0.75, 2, and 4.2 mm are used. Circular aluminum plates with free edges and varied diameters  $D_s$  ranging from 20 to 100 mm are tested. Without any weir attached around the edge of the plate, the flow is free to adjust itself when flowing off the plate edge, i.e. the liquid film could form a stable rim and flow off the plate only from one spot, or it could flow off from several spots, or it could freely flow off around the entire edge. Three different working fluids are used: distilled water, water-glycerol solution wt. 60%, and water-surfactant solution. The physical properties of these fluids are reported in Table 3.1.



**Figure 5.1.** (a) Circular hydraulic jump formed by a liquid jet impinging on a solid surface. (b) Schematic of a circular hydraulic jump formed by jet impingement on a circular plate. The inset sketch shows more fluid dynamics details.

## 5.3 Experimental Results

#### 5.3.1 Hydraulic jump diameter

For circular hydraulic jumps, the jet flow condition, especially the flow rate, is usually considered as the major factor determining the diameter of the circular jump. In Figure 5.2 the flow condition is maintained constant (a water jet with d=0.75 mm and Q = 3.75 cm<sup>3</sup>/s), while the diameter of the target plate is changed from  $D_s=45$  mm to  $D_s=20$  mm. Clearly it is observed that despite the same impinging jet, the circular jumps formed on the large plates are smaller than those formed on the small plates.

The jump diameters associated with Figures 5.2a-f are measured and plotted versus the plate size in Figure 5.2g. It is seen that the jump diameter increases as the plate size decreases. As a result, the hydraulic jump becomes closer to the plate edge. This can be seen in Figure 5.2g, where the data points approach the line  $D_j = D_s$  as  $D_s$  decreases. This influences the post-jump region, which can be quantified by  $(D_s - D_j)$ . Figure 5.2 reveals that the effect of the plate size on the jump diameter is significant. The jump diameter is  $D_j = 14$  mm on the plate with  $D_s = 45$  mm, while  $D_j = 21$  mm on the plate of  $D_s = 25$  mm, showing an increase of 50% due to the change of the plate size (with constant flow condition). When the plate size decreases to  $D_s = 20$  mm (see Figure 5.2), the plate is entirely covered by the thin liquid film without appearance of a hydraulic jump. This is a promising condition for cooling applications due to the elimination of the hydraulic jump.



**Figure 5.2.** (a-f) A water jet with the diameter d=0.75 mm and the flow rate Q=3.75 cm<sup>3</sup>/s impinging on plates with different plate sizes  $D_s$ , (g) The measured jump diameters  $D_i$  associated with images (a-f) versus the plate size  $D_s$ .

Tests are also conducted by keeping the plate size  $D_s$  constant and varying the flow rate Q. In Figure 5.3, water jets with flow rates increasing from 2.5 to 4.58 cm<sup>3</sup>/s impact on a plate with  $D_s = 30$  mm. As expected, the hydraulic jump grows as the flow rate increases. As a result, the post-jump region,  $D_s - D_j$ , reduces. The trend can also be observed from Figure 5.3g, which shows the measured jump diameters versus the flow rate. It is interesting to see that the trend of  $D_j$  changing with Q shows a steeper rise from test (c) to test (d). It can be perceived that  $D_j$ becomes more sensitive to the increase of the flow rate as  $D_j$  approaches  $D_s$ .

To further investigate the foregoing observations, an extensive set of tests are conducted using water jets with varied flow rates and plate sizes, and the results are presented in Figures 5.4 and 5.5. As mentioned before, for each test, five images of the jump diameter and another five images of the post-jump film depth are taken, and the averages are plotted and used for the analysis. Due to the steadiness of the phenomenon and consequently very small deviations of the

recorded data, error bars are not shown throughout this chapter (i.e. error bars are smaller or approximately the same size of the plotted symbols).



Figure 5.3. (a-f) Water jets (d=0.75 mm) with varied flow rates impinging on a fixed plate with  $D_s$  =30 mm. (g) The measured jump diameters  $D_i$  associated with images (a-f) versus the flow rate Q

Figure 5.4 plots the jump diameter versus the plate size for varied flow rates. Each data line represents a constant flow rate. Each line has a horizontal portion which shows the independence of the jump diameter from the plate size. However, each line also exhibits a rising portion, where the jump diameter  $(D_j)$  increases with decreasing the plate size  $(D_s)$ . This rise indicates that on these small surfaces, the hydraulic jump is not only a function of the flow rate, but also a function of the target plate size. The increase of the jump diameter on small plates is significant. Taking the flow rate of 2.5 cm<sup>3</sup>/s as an example, reducing the plate size from  $D_s = 25$  mm to  $D_s = 20$  mm, the jump diameter increases by 68%. As shown in Figure 5.4, the influence of the plate size on the hydraulic jump diameter tends to appear when the data points are close to the

line  $D_j = D_s$ . In other words, the rise in  $D_j$  occurs when the hydraulic jump arises close to the plate edge, i.e. the post-jump region  $D_s - D_j$  shrinks.



Figure 5.4. The hydraulic jump diameter versus the plate size for different flow rates (water jets d=0.75 mm).

The trend shown in Figure 5.4 can also be presented by plotting  $D_j$  versus Q for varied  $D_s$ . The experimental results of water jets with d = 0.75 mm (shown in Figure 5.4) are plotted in Figure 5.5a. Additionally, the test results of a larger nozzle, d = 2 mm, are plotted in Figure 5.5b. The data associated with the nozzle d = 2 mm are not shown in Figure 5.4, because the tests using the large nozzle focus more on the variation of the flow rate but less on the variation of the plate size. In Figure 5.5, each data line represents a specific plate size. On large plates or/and with relatively low flow rates, all data lines almost coincide, showing a common trend of  $D_j$  increasing with Q. This observation indicates the independence of the hydraulic jump from the plate size. However, most lines in Figure 5.5 show a shift-up. The shift-ups evidence a change in the jump growth trend with the flow rate. This deviation indicates a steeper increase in the jump

diameter with the flow rate. It is seen that the flow rate at which the shift-up occurs depends on the plate size. As the plate size increases, the shift-up occurs at higher flow rates.

In summary, the data points shown in Figure 5.5 can be divided into two groups: 1) those prior to the shift-ups are independent of the plate size, and are shown using solid symbols; 2) those after the shift-ups are affected by the plate size, and are shown as the hollow symbols. Throughout this chapter, we will be consistent with this definition of the solid and hollow symbols.



**Figure 5.5.** The hydraulic jump diameter versus the flow rate on different plates with two jet diameters, (a) d=0.75 mm, (b) d=2 mm.

# 5.3.2 Post-jump film depth

To better understand the trends of the jump diameter changing with the flow rate and plate size, the depth of the post-jump fluid film, H, is analyzed based on the side view images of the fluid film formed on the plate (see Figure 5.6). The images presented in Figures 5.6a-f are the side view images of the same tests presented in Figure 5.2, for which the plate size  $D_s$  decreases while the flow rate Q remains constant. The post-jump film thicknesses are measured and plotted in Figure 5.6g. It is seen that on large plates the post-jump film depth remains approximately constant. However, as the plate size further decreases, the post-jump film starts thinning. Eventually, the hydraulic jump disappears for  $D_s=20$  mm, and H = 0 is seen in Figure 5.6g. Comparing Figure 5.6 with Figure 5.2 reveals that the substantial change observed in the jump diameter  $D_j$  is associated with the change of the post-jump film thickness.



Figure 5.6. (a-f) Side view images of the same tests shown in Figure 5.2 with a water jet of d=0.75 mm and a constant flow rate of Q = 3.75 cm<sup>3</sup>/s on different target plates, (g) the measured post-jump depths versus the plate size.

Similarly, the side view images of the tests presented in Figure 5.3 are shown in Figures 5.7a-f, for which the flow rate increases while the plate size is kept fixed. The measured values of the post-jump thickness, H, are plotted in Figure 5.7g. This figure shows that the post-jump film depth remains approximately constant with the flow rate. However, as the flow rate increases further, the post-jump depth drops. The steep decrease of H shown in Figure 5.7g is associated with the steep rise of the jump diameter in Figure 5.3.



Figure 5.7. (a-f) Side view images of the same tests shown in Figure 5.3 with water jets of d=0.75 mm impinging on a plate of  $D_s$  =30 mm and varied flow rates, (g) the measured post-jump depths versus the flow rate.

The post-jump liquid film depths associated with the experiments using water jets of d=0.75 mm are measured and shown in Figure 5.8. The hydraulic jump diameters of the same tests have been shown in Figure 5.5a. The results are plotted versus the flow rate for only a few plate sizes. Consistent with Figure 5.5, the same symbols are used in Figure 5.8, i.e. solid symbols for the jump unaffected by the plate size, and hollow symbols for the jumps affected by the plate size. It is seen that the post-jump depth remains approximately constant (independent of both Q and  $D_s$ ) if the plate is large or the flow rate is low. All the data points with the constant H trend are solid indicating no change in the jump growth trend. By further increasing the flow rate or reducing the plate size, the depth is observed to decrease with increasing the flow rate. It is clear that the decrease of H in Figure 5.8 coincide with the shift-ups in Figure 5.5 (hollow symbols). It is known from the literature review (both 2D and circular jumps) that the post-jump depth directly influences the position of the hydraulic jump [8,24] (i.e. with equal conditions, a thinner H results in a larger  $D_j$ , and a thicker H results in a smaller  $D_j$ ). Thus, the decrease observed in the post-jump depth explains the steep growth in the jump diameter seen in Figure 5.5.



Figure 5.8. The post-jump film depth versus the flow rate on varied plate sizes (water jets with d=0.75 mm).

# 5.3.3 Post-jump region

Flowing through a hydraulic jump, the fluid velocity drops dramatically. As a result, the postjump flow is dominated by pressure and viscous forces, while the flow inertia could be negligible [9,52,53]. This is valid if the post-jump region,  $D_s - D_j$ , is large. This situation can be achieved by using large plates or/and low flow rates. In this case, although increasing the flow rate causes the hydraulic jump to expand, the change in the post-jump depth is very small. Higuera [11] and Rao & Arakeri [13,55] reported that the subcritical post-jump flow approaching the edge of the plate speeds up and becomes locally critical (in a small area close to the edge) due to the gravitational acceleration prevailing at the edge of the plate. In other words, at the edge, the flow inertia could become significant due to the local gravitational acceleration. If the post-jump region ( $D_s - D_j$ ) is small and limited to a small area close to the edge, the flow inertia becomes significant in the entire post-jump flow. This has been reported by Bohr et al. [9], who pointed out that if the jump occurs close to the edge, the inertia terms cannot be neglected for the postjump flow. In this case, increasing the flow rate or reducing the plate size would increase the velocity of the post-jump flow. A higher speed of the post-jump flow results in a thinner post-jump liquid depth [11], which allows the circular jump to further expand.



**Figure 5.9.** The circular jump diameter  $D_j$  is plotted as a non-dimensional post-jump region  $(D_s - D_j)/D_s$  versus the flow rate for different plate sizes. The same results have been presented in Figure 5.5.

The discussion above indicates that the adjacency of the hydraulic jump to the plate edge plays a significant role on the behavior of the jump. In order to quantify this parameter, a nondimensional post-jump length is defined as  $(D_s - D_j)/D_s$ . Using this new parameter to replace  $D_j$  in Figure 5.5, Figure 5.5 is replotted in Figure 5.9 using the same symbols (solid and hollow). It is seen that most of the hollow symbols, which represent the experiments in which the hydraulic jump is affected by the plate size, fall in the region where  $(D_s - D_j)/D_s < 0.5$ . In other words, Figure 5.9 shows that when the spacing between the jump and the plate edge becomes smaller than half of the plate radius, the jump gets influenced by the plate size. When the post-jump region is large,  $(D_s - D_j)/D_s > 0.5$ , the effect of the plate size is negligible.

#### 5.4 Froude Number Analysis

The results above have shown that when the jump appears close to the plate edge, the flow inertia in the post-jump flow becomes significant. At the same time, the post-jump depth decreases, resulting in a decrease in the gravitational force accompanied by the increase of the inertia force in the post-jump flow. This must affect the Froude number of the post-jump flow, which is the ratio of the two forces. In this section, the post-jump Froude number of circular hydraulic jumps with varied conditions is studied.

# 5.4.1 Post-jump Froude number

In Chapter 4, Eqs. (4.1a) and (4.1b) were introduced to calculate the pre-jump Froude number  $Fr_i$  and the post-jump Froude number  $Fr_o$ , respectively. Employing Eq. (4.1b), the post-jump Froude numbers are calculated for the tests shown in Figure 5.8. The data are associated with water jets with d=0.75 mm and different flow rates and plate sizes. The results are plotted in Figure 5.10. Consistent with Figures 5.8 and 5.5a, the hollow symbols represent the tests in which the hydraulic jumps are affected by the plate size. From Figure 5.10, three important trends come to attention:

First, for large plates or low flow rates, the post-jump Froude number remains constant with the flow rate,  $Fr_o \approx 0.1$ . These data points are solid symbols, indicating that the jump diameter  $D_j$  and the post-jump depth *H* are independent of the target plate size  $D_s$  (see Figures 5.8 and 5.5a).

Second, if the effects of the plate size on  $D_j$  and H are significant, represented as hollow symbols,  $Fr_o$  increases with the flow rate. Increase in the post-jump Fr number indicates an increase in the flow inertia compared to the gravitational force. Comparing different target plates shows that  $Fr_o$  also increases with decreasing the plate size.



Figure 5.10. The post-jump Froude number is calculated for the hydraulic jumps formed by the water jets (d=0.75 mm) with varied flow rates impinging on plates with varied sizes. The same tests have been shown in Figures 5.8 and 5.5a.

Third, although the post-jump flow is commonly considered as a subcritical flow (Froude number lower than unity), it is observed that in some cases,  $Fr_o > 1$ .

Following the above observations, the discussion here continues on, first, the constant post-jump Froude number, which is observed to be independent of the jet flow rate, and secondly, the supercritical values of the Froude number for the post-jump flow, i.e.  $Fr_o > 1$ .

#### 5.4.1.1 Constant post-jump Froude number

Figure 5.10 has shown a constant value of  $Fr_o = 0.1$  for the hydraulic jumps formed on large plates or with low flow rates. Very recently, Duchesne et al. [23] reported that, based on their

experiments, the post-jump Froude number was locked on a constant value,  $Fr_o = 0.32$ . They stated that the constant value is independent of the flow rate, kinematic viscosity, and surface tension of the fluid. They, however, observed a slight dependence on the nozzle diameter, ~10% for a variation of the nozzle diameter from 1.1 to 7mm.

For low-viscosity fluids, Duchesne et al. [23] provides a relation as  $Q = Fr_o Y$ , where  $Y = \sqrt{g\pi D_j} \left[H_{\infty}^4 + \frac{6}{\pi} \frac{vQ}{g} \ln\left(\frac{D_s}{D_j}\right)\right]^{3/8}$  and  $H_{\infty}$  is the liquid film thickness at the edge of the plate. Plotting Q versus Y, they observed that all their experimental data points fell on a straight line whose slope represents the post-jump Froude number. In this section, the data of water jets presented in Figure 5.5 are selected for several plate sizes and are used to calculate Y, which is then plotted versus Q in Figure 5.11. The solid and hollow symbols are used consistent with Figure 5.5. Since the liquid film thickness at the plate edge,  $H_{\infty}$ , was not recorded in the present study, the characteristic surface tension length  $\sqrt{\frac{\sigma}{\rho g}}$  is used to approximate the film depth at the plate edge. For both jet diameters (d = 0.75, and 2 mm), the solid data points show a clear linear trend. As for the tests where the plate size influences the jump diameter (i.e. hollow symbols), the data points deviate from the linear trend-line. Following the solution of Duchesne et al. [23], the slope of the linear trend represents the value of  $Fr_o$  (i.e.  $Fr_o=0.18$  for d = 0.75 mm and  $Fr_o=0.27$  for d = 2 mm). From Figure 5.11, the following should be noted:



**Figure 5.11.** The data presented in Figure 5.5 are selected for several plate sizes to apply the relation presented by Duchesne et al. [23]:  $Y = \frac{Q}{Fr_o} = \sqrt{g} \pi D_j \left[ H_{\infty}^4 + \frac{6}{\pi} \frac{\upsilon Q}{g} \ln \left( \frac{D_s}{D_i} \right) \right]^{3/8}$ .

Similar to Duchesne et al. [23], the linear lay-out of the solid symbols indicates that  $Fr_o$  remains constant with the flow rate as long as the plate size does not influence the jump behavior.

The calculated slopes of the lines ( $Fr_o=0.18$  and 0.27) are different from the constant value ( $Fr_o = 0.32$ ) reported by Duchesne et al. [23].

Contrary to Duchesne et al. [23] who stated that the jet diameter does not have a major impact on the constant value of  $Fr_o$ , it seems that there is a clear dependence of  $Fr_o$  on the jet diameter.

The slope of the linear trend, e.g.  $Fr_o=0.18$  for d=0.75 mm, differs from the value calculated from the measured post-jump film depth H ( $Fr_o = 0.1$  shown in Figure 5.10). The discrepancy could be due to the uncertainty on the value of  $H_{\infty}$ , which was approximated with the characteristic capillary length in the calculation. The author examined more realistic values for  $H_{\infty}$  and the results tend to approach the experimental values. In view of the above discrepancy in  $Fr_o$  compared to Duchesne et al. [23], more tests are conducted to investigate the constant value of the post-jump Froude number. Varied nozzle diameters (d = 0.75, 2, and 4.2 mm), flow rates, and working fluids (distilled water, waterglycerol solution, and water-surfactant solution wt. 60%) are tested. The objective here is to identify the influential parameters that could affect the constant value of  $Fr_o$ . Due to the focus on the constant value of  $Fr_o$ , the largest plate,  $D_s = 100 \text{ mm}$ , is used. It is worth-noting that for this set of experiments, in order to be consistent with the test conditions carried out by Duchesne et al. [23], an absorbent paper is attached around the entire plate edge, flush with the top surface, to generate a uniform flow at the edge. With a given condition specified by the nozzle diameter and the working fluid, the flow rate is changed in a range such that the jump remains far from the edge (i.e.  $(D_s - D_j)/D_s > 0.5$ ) and unaffected by the plate size.

The post-jump Froude number is calculated using Eq. (4.1b) based on the measured post-jump film depth (*H*) and the jump diameter ( $D_j$ ). The results are then plotted versus the flow rate in Figure 5.12. Each group of data is under the same test conditions specified by the jet diameter and the working fluid, while the flow rate is changed. Figure 5.12 shows that for each test condition, the post-jump Froude number maintains a relatively constant value independent of the flow rate. This can be related to the constant post-jump depth shown in Figure 5.8, which occurs when the post-jump region is large.

Figure 5.12 also reveals that the constant value of  $Fr_o$  depends on both the jet diameter and the working fluid. First, it is seen that  $Fr_o$  increases as the jet diameter increases. For instance, with water as the working fluid, the post-jump Froude number increases from  $Fr_o = 0.11$  for d = 0.75

mm to  $Fr_o = 0.22$  for d = 4.2 mm. This observation is different from that of Duchesne et al. [23] that reported only 10% change in  $Fr_o$  as a result of jet diameter change.



Figure 5.12. The post- jump Froude number versus the flow rate for the jumps formed on a plate with  $D_s = 100$  mm.

The effect of the surface tension is clear by comparing the results of the tests using water as the working liquid and those with the water-surfactant solution. Take the jet diameter d = 2 mm as an example, the reduction in the surface tension leads the post-jump Froude number to increase from  $Fr_o = 0.15$  with water to  $Fr_o = 0.32$  with the water-surfactant solution. The effect of viscosity can also be shown by comparing the results of the water-glycerol solution with those of water. The water-glycerol solution is around nine times more viscous than water. Figure 5.12 illustrates that increasing the viscosity of the working liquid causes the post-jump Froude number to increase. In the case of d = 0.75 mm, the post-jump Froude number is  $Fr_o \sim 0.11$  for water, while  $Fr_o \sim 0.2$  for the water-glycerol solution.

Form the above discussion, it is concluded that when the hydraulic jump is far from the plate edge and unaffected by the plate size, the post-jump Froude number remains constant with the flow rate. However, the constant value is a function of the nozzle diameter, and the fluid surface tension and viscosity.

## 5.4.2 Pre-jump Froude number

As for the pre-jump flow, Eq. (4.1a) was introduced to calculate the pre-jump Froude number  $Fr_i$ . It was discussed that the velocity profile plays an important role in the pre-jump region, and a correction factor  $\lambda$  is introduced to take into account the non-uniformity of the velocity profile in the thin pre-jump liquid film. As discussed in Chapter 4, in all the tests carried out for this work, the hydraulic jump always arises at a radial position downstream of the position where the boundary layer covers the entire film depth, i.e.  $R_j > r_c$ . Thus, this observation is considered in all the analysis conducted here.

In Eq. (4.1a), the depth h that is the pre-jump liquid thickness at the location of the jump, is given by [8]

$$h = 3.798 \frac{\upsilon R_j^3 + 0.182a^2 Q}{QR_j}$$
(5.1)

To calculate  $Fr_i$  using Eq. (4.1a), the correction factor  $\lambda$  needs to be determined. In addition to Eq. (4.1a), the pre-jump Froude number can also be defined in a general form as

$$Fr_{i} = \left(\frac{1}{h} \int_{0}^{h} u^{2} dz\right)^{1/2} / (gh)^{1/2}$$
(5.2)

Here *z* is the vertical position relative to the plate surface (solid-fluid interface), and *u* is the velocity profile in the pre-jump region at  $r = R_j$ . Introducing a similarity function *f* such that  $f = u/u_{z=h}$  [102], and applying  $Q = 2\pi R_j \int_0^h u dz$ , Eq. (5.2) can be rewritten as

$$Fr_{i} = \left(\sqrt{\int_{0}^{1} f^{2} d\delta} / \int_{0}^{1} f d\delta\right) \frac{Q}{2\pi R_{j} \sqrt{gh^{3}}}$$
(5.3)

The non-dimensional location  $\delta = z/h = 0$  is the solid-fluid interface, and  $\delta = 1$  is the freesurface of the pre-jump film. Watson [8] provided a solution of the similarity function, which is  $c\delta = \int_0^f (1-x^3)^{-0.5} dx$  with c = 1.402 and the boundary conditions of f(1) = 1 and f(0) = 0. Thus, the correction factor in Eq. (4.1a) is

$$\lambda = \sqrt{\int_{0}^{1} f^{2} d\delta} / \int_{0}^{1} f d\delta = \frac{3c^{3/2}}{\sqrt{2\pi}} = 1.12$$
(5.4)

It should be noted that for the classic hydraulic jump in large open channels  $\lambda \sim 1$  as the pre-jump fluid depth is usually much larger than the boundary layer thickness.

Employing Eq. (4.1a), the pre-jump Froude number  $Fr_i$  is calculated for the tests in which the jump is unaffected by the target plate size, i.e. the post-jump Froude number  $Fr_o$  remains constant with flow rate. The results are presented in Figure 5.13 versus the flow rate Q for two working fluids, water and water-surfactant solution. Clearly, despite the constant post-jump Froude number  $Fr_o$  with the flow rate, for both fluids, the pre-jump Froude number changes with the flow rate. This indicates a significant difference from the classic hydraulic jump in rectangular channel flows, for which the pre- and post-jump Froude numbers must change

together to maintain the force/momentum balance at the jump. The discussion in this section focuses on a Froude number-based relation between the pre- and post-jump flows. The effect of surface tension and the coupling between the pre- and post-jump Froude numbers will be analyzed.



Figure 5.13. The pre-jump Froude number (calculated using Eq. (4.1a) with  $\lambda$ =1.12) versus the flow rate. d=0.75 mm.

The force-momentum balance equations derived in Chapter 4 can be expressed in terms of Froude numbers. Here, we consider the second control volume presented in Chapter 4, located at the location of the jump. Dividing Eq. (4.21) through by  $\rho [(Q/2\pi)^4 g/R_i]^{1/3}$  gives

$$f_o = Fr_o^{-4/3} \left( Fr_o^2 + 0.5 + Bo^{-1} \right)$$
(5.5)

Here,  $f_o$  is the non-dimensional force from the post-jump region, and  $Bo^{-1} = \sigma / \rho g R_j H$  is the inverse Bond number representing the surface tension force at the jump.

As for the force from the pre-jump region, Eq. (4.16) is chosen instead of Eq. (4.17). Dividing Eq. (4.16) through by  $\rho [(Q/2\pi)^4 g/R_j]^{1/3} \Delta \theta$  gives

$$f_i = \left(\frac{Fr_i}{\lambda}\right)^{-4/3} \left(Fr_i^2 + 0.5\right) \tag{5.6}$$

where  $f_i$  is the non-dimensional force from the pre-jump region. Equations (5.5) and (5.6) can also be referred to as the "momentum functions" of the post- and pre-jump flows, respectively. The conservation of energy at the jump requires  $f_o = f_i$ . The following discussion will focus on the relation between  $f_o$  and  $f_i$ .

#### 5.4.3 Effect of surface tension

It was observed that for the tests where the hydraulic jump is unaffected by the target plate size, the post-jump Froude number is constant while the pre-jump Froude number is not (see Figure 5.13). From Eqs. (5.5) and (5.6), it is perceived that the surface tension force must play an important role in maintaining the force balance/conservation of momentum at the jump location. In Eq. (5.5), the first term in the bracket (i.e.  $Fr_o^2$ ) represents the inertia force, the second term (i.e. 0.5) represents the hydrostatic force, and the third term (i.e.  $Bo^{-1}$ ) represents the surface tension force. To evaluate the significance of surface tension,  $Bo^{-1}$  is calculated and plotted in Figure 5.14 in comparison with the two other terms in Eq. (5.5): 0.5 and  $Fr_o^2$ . Generally, as seen in Figure 5.14, the surface tension force is larger than the inertia force but less than the hydrostatic force, i.e.  $Fr_o^2 < Bo^{-1} < 0.5$ . Also, it is seen that the surface tension force is more significant for the tests with low flow rates but less for the high flow rates. This is due to the increase of the jump radius with increasing the flow rate. The tests using water exhibit more

significance of the surface tension than the tests using the water-surfactant solution. This is attributed to the properties of the two fluids, i.e. water-surfactant solution has lower surface tension than water. The trend of  $Bo^{-1}$  changing with the flow rate presented in Figure 5.14 is similar to the trend of  $Fr_i$  shown in Figure 5.13.



Figure 5.14. Non-dimensional surface tension force  $Bo^{-1}$  calculated for all the tests in comparison with the flow momentum force  $Fr_o^2$  and hydrostatic force 0.5.

# 5.4.4 Critical Froude number

# 5.4.4.1 Theoretical Study

Figure 5.10 has shown that the post-jump Froude number could be larger than unity. In this section, the theoretical possibility of observing a post-jump flow with  $Fr_o$  higher than unity is discussed. Additionally, further data analysis will be carried out to validate the theoretical discussion.

Hydraulic jump is a transition from a super-critical flow (high Froude number) to sub-critical flow (low Froude number). For each flow, plotting the non-dimensional momentum function ( $f_o$  or  $f_i$  in Eqs. 5.5 and 5.6) versus the Froude number would show a minimum point, which is referred to as the critical point where the minimum momentum/energy exists. The minimum point can be calculated by

$$\left(\frac{\partial f}{\partial Fr}\right)_{Fr=Fr_m} = 0 \tag{5.7}$$

Here the subscript "m" represents the condition at which the momentum function has a minimum value. In the classical hydraulic jump theory, the minimum point in the momentum function is referred to as the critical point and the corresponding Froude number is referred to as the critical Froude number. From Eqs. (5.5) and (5.6), the minimum values of the pre- and post-jump momentum functions and the corresponding Froude numbers are

$$f_{i,\min} = 1.5\lambda^{4/3}, \ Fr_{i,m} = 1$$
 (5.8a)

$$f_{o,\min} = 1.5 (1 + 2Bo^{-1})^{1/3}, \quad Fr_{o,m} = \sqrt{1 + 2Bo^{-1}}$$
 (5.8b)

Equation (5.8b) shows that if  $Bo^{-1} > 0$ , as a result  $Fr_{o,m} > 1$ . Also,  $Fr_{o,m}$  increases as  $Bo^{-1}$  increases. This points out the theoretical possibility of having supercritical post-jump flows. However, since the momentum is conserved at the jump location (i.e.  $f_o = f_i$ ), the actual range of the post-jump Froude number is determined by the conjugation between the pre-jump and post-jump flows. The conjugation of the two flows is shown in Figure 5.15. In this figure, the pre-jump momentum function (i.e.  $f_i - \text{Eq. 5.6}$ ) is plotted for  $Fr \ge Fr_{i,m}$  with two values of  $\lambda=1$  and 1.12. Also, the post-jump momentum function (i.e.  $f_o - \text{Eq. 5.5}$ ) is plotted for  $Fr \le Fr_{o,m}$
with four different values of  $Bo^{-1} = 0, 0.2, 0.3, and 0.45$ . Different values of  $Bo^{-1}$  are examined to introduce different possible scenarios. To have a better picture, the enlargement of Figure 5.15a close to Fr ~1 is illustrated in Figure 5.15b. Four scenarios can be realized from Figure 5.15 and are discussed below.

Scenario I is represented by the curves  $f_i(\lambda = 1)$  and  $f_o(Bo^{-1} = 0)$ , which show the conventional theory of the classic hydraulic jump with uniform velocity profiles and no surface tension. These two curves intersect at Fr = 1 (see points  $A_o$  and  $A_i$ ), which is known as the critical Froude number.



Figure 5.15. (a) Froude number analysis of circular hydraulic jumps with varied pre-jump velocity profiles and surface tension forces; (b) Enlargement of the region where  $Fr \sim 1$  to show critical Froude numbers and the coupling of the pre- and post-jump Froude numbers.

Scenario II is represented by the curve  $f_i(\lambda = 1.12)$  and curve  $f_o(Bo^{-1} = 0.2)$ , which do not intersect. In this case,  $f_o|_{F_{r_o}=F_{r_{i,m}}} < f_{i,\min}$ , which requires

$$Bo^{-1} < 1.5(\lambda^{4/3} - 1) \tag{5.9}$$

The ranges of the possible pre-jump and post-jump Froude numbers are determined by the coupling of the two curves (see points  $B_o$  and  $B_i$  in Figure 5.15b). The point  $B_o$  represents the maximum possible post-jump Froude number, denoted by  $Fr_{o,max}$ . This means that the post-jump Froude number cannot be higher than the value of  $Fr_{o,max}$ . The value of  $Fr_{o,max}$  can be obtained from solving the equation  $f_o = f_{i,min}$ . The point  $B_i$  shows the minimum possible pre-jump Froude number, which is  $Fr_{i,min} = 1$ .  $Fr_{i,min}$  is defined as the smallest Froude number that the pre-jump flow can attain. In this scenario, although  $Fr_{o,max} < 1$  (see point  $B_o$  in Figure 5.15b).

Scenario III is represented by the curves  $f_i(\lambda = 1.12)$  and  $f_o(Bo^{-1} = 0.3)$ , which intersect in the region of Fr > 1. In this case,  $f_o|_{Fr_o = Fr_{i,m}} > f_{i,\min}$  and also  $f_{o,\min} < f_i|_{Fr_i = Fr_{o,m}}$ , which requires

$$1.5(\lambda^{4/3} - 1) < Bo^{-1} < \frac{1.5(\lambda^{4/3} - 1)}{2(1.5 - \lambda^{4/3})}$$
(5.10)

Following the above condition, both  $Fr_{o,max}$  and  $Fr_{i,min}$  are equal to the Froude number at the intersection point (see points  $C_o$  and  $C_i$  in Figure 5.15b). Consequently, the maximum possible post-jump Froude number is higher that unity, i.e. $Fr_{o,max} > 1$ . The value of  $Fr_{o,max}$  can be determined from  $f_i = f_o$ , which gives

$$Fr_{o,\max} = \sqrt{\frac{Bo^{-1}}{\lambda^{4/3} - 1} - 0.5}$$
(5.11)

Scenario IV is represented by the curves  $f_i(\lambda = 1.12)$  and  $f_o(Bo^{-1} = 0.45)$ , which do not intersect. In this case,  $f_{o,\min} > f_i|_{Fr_i = Fr_{o,m}}$ , which requires

$$Bo^{-1} > \frac{1.5(\lambda^{4/3} - 1)}{2(1.5 - \lambda^{4/3})}$$
(5.12)

With the condition of this scenario, the maximum possible post-jump Froude number is larger than unity, i.e.  $Fr_{o,max} > 1$ , and the value of  $Fr_{o,max}$  is associated with the minimum point of the post-jump momentum function (see point  $D_o$  in Figure 5.15b). Hence,

$$Fr_{o,\max} = Fr_{o,m} = \sqrt{1 + 2Bo^{-1}}$$
(5.13)

In Scenario IV,  $Fr_{i,min}$  can be calculated from  $f_i = f_{o,min}$  (see point  $D_i$  in Figure 5.15b).

Both scenarios III and IV show that it is theoretically possible for the post-jump flow to be supercritical, i.e.  $Fr_{o,max} > 1$ . From Eqs. (5.10) and (5.12), it is clear that these two cases require  $Bo^{-1} > 1.5(\lambda^{4/3} - 1)$  (5.14)

In summary, depending on  $Bo^{-1}$  and  $\lambda$ , the maximum possible post-jump Froude number  $(Fr_{o,max})$  could be smaller, equal, or larger than unity. In particular, if the relation between  $Bo^{-1}$  and  $\lambda$  given by Eq. (5.14) is satisfied, theoretically it is possible for the post-jump flow to obtain a Froude number higher than one, i.e.  $Fr_o > 1$ . To be clearer on the definition of  $Fr_{o,max}$ , it is worth noting that  $Fr_{o,max}$  is a theoretical value determining the upper limit of the Froude number range that the post-jump flow can possibly obtain. If the value of  $Fr_{o,max}$  for a flow is higher than unity, the actual post-jump Froude number of the flow  $(Fr_o)$  is not necessarily higher than unit (i.e.  $Fr_o$  could still be smaller than one). However, it is physically possible for the flow to obtain a post-jump Froude number in the range of  $1 < Fr_o < Fr_{o,max}$ .

If  $\lambda = 1$  and  $Bo^{-1} = 0$ , it becomes the classic hydraulic jump theory without the surface tension, for which  $Fr_{o,max} = 1$ . The discussion is also summarized in Table 5.1.

Scenario	Bo <sup>-1</sup> & λ	Fr <sub>o,max</sub>	Fr <sub>i,min</sub>
Ι	$Bo^{-1} = 0; \lambda = 1$	1	1
Ш	$0 < Bo^{-1} < 1.5 \left(\lambda^{4/3} - 1\right)$	$Fr_{o,max} < 1$ evaluated form $f_o = f_{i,min}$ Eqs. (5.5) and (5.8a)	1
III	$1.5(\lambda^{4/3} - 1) < Bo^{-1} < \frac{1.5(\lambda^{4/3} - 1)}{2(1.5 - \lambda^{4/3})}$	$Fr_{o,max} = Fr_{i,min} > 1$ $Fr_{o,max} = Fr_{i,min} = \sqrt{\frac{Bo^{-1}}{\lambda^{4/3} - 1} - 0.5}$	
IV	$Bo^{-1} > \frac{1.5(\lambda^{4/3} - 1)}{2(1.5 - \lambda^{4/3})}$	$Fr_{o,max} = Fr_{o,m} > 1$ $Fr_{o,max} = \sqrt{1 + 2Bo^{-1}}$	$Fr_{i,min} > 1$ evaluated form $f_i = f_{o,min}$ Eqs. (5.6) and (5.8b)

Table 5.1. Summary of the observations from Figure 5.15.

# 5.4.4.2 Validation

The experimental data are further analyzed to find supercritical post-jump values in the tests. Using the measured jump diameter and the post-jump depth,  $Fr_o$  and  $Bo^{-1}$  are calculated for all the tests of water jets with d = 0.75 and 2 mm. The data of  $Fr_o$  are plotted versus  $(D_s - D_j)/D_s$ in Figure 5.16a. When the post-jump region is large, i.e.  $(D_s - D_j)/D_s > 0.5$ ,  $Fr_o$  remains relatively constant as the data points fall into a small range of  $Fr_o$ . However, as the post-jump region shrinks, i.e.  $(D_s - D_j)/D_s < 0.5$ ,  $Fr_o$  increases, and the data points spread. In Figure 5.16a, twelve data points exhibit  $Fr_o > 1$ , which are highlighted by grey solid symbols.



**Figure 5.16.** (a) The post-jump Froude number versus the non-dimensional post-jump length. Gray solid symbols are used to highlight  $Fr_o > 1$ . The solid symbols are plotted in the inset graph comparing theoretical values of  $Fr_{o,max}$  with the actual supercritical values of  $Fr_o$ . (b) The  $Bo^{-1}$  versus the non-dimensional post-jump length.

The results of  $Bo^{-1}$  are also plotted in Figure 5.16b versus the non-dimensional post-jump length. Generally, the effect of the surface tension becomes more significant as  $(D_s - D_j)/D_s$ decreases. In Figure 5.16b,  $Bo^{-1} = 1.5(\lambda^{4/3} - 1)$  and  $Bo^{-1} = \frac{1.5(\lambda^{4/3} - 1)}{2(1.5 - \lambda^{4/3})}$  are plotted as two horizontal lines, for which the correction factor  $\lambda = 1.12$  is used. It can be seen that a number of data points satisfy the condition presented by Eq. (5.14) (i.e.  $Bo^{-1} > 1.5(\lambda^{4/3} - 1)$ ) when  $(D_s - D_j)/D_s$  is small. Within these data points are the twelve supercritical points with  $Fr_o > 1$ .

Since all the twelve points satisfy Eq. (5.12) as well (i.e.  $Bo^{-1} > \frac{1.5(\lambda^{4/3} - 1)}{2(1.5 - \lambda^{4/3})}$ ), Eq. (5.13) is used

to calculate the theoretical maximum possible Froude number,  $Fr_{o,max}$ . The results are plotted in the inset graph of Figure 5.16a, showing that all the supercritical values of  $Fr_o$  are smaller than their theoretical possible maxima.

### 5.5 Summary

In the present chapter, the circular hydraulic jumps formed on small impingement plates were studied, when the sizes of the target plates are comparable to those of the hydraulic jumps. Focus was put on the effects of the plate size on the jump diameter, post-jump film depth, and the postjump Froude number.

It was observed that with relatively low flow rates and large plates (i.e. the jump is far from the plate edge), the hydraulic jump is independent of the plate size. In this case, the hydraulic jump is only a function of the flow condition. However, as the plate size reduces and/or flow rate increases (i.e. the hydraulic jump approaches the plate edge), the effect of the plate size appears, showing two interesting trends. One trend is that the jump diameter increases with reducing the plate size. The other trend is that, for small plates, the jump diameter becomes more sensitive to the change of the flow rate (i.e. the jump changes more sharply with the flow rate). Both trends were observed to be related to the post-jump film depth, which decreases with increasing the flow rate or decreasing the plate size.

All the above observations can be attributed to the reduction of the post-jump region on the plate, i.e.  $(D_s - D_j)$ . It was found that the influence of the plate size on the hydraulic jump becomes significant when the non-dimensional post-jump length, i.e.  $(D_s - D_j)/D_s$ , is less than ~0.5. It was explained that when the jump approaches the plate edge (i.e. the post-jump length reduces), the flow inertia becomes significant in the post-jump region. This discussion was proved by the analysis of the post-jump Froude number, which demonstrated two trends with respect to the flow rate and plate size. First, for large plates or low flow rates, the Froude number maintains a constant value independent of the flow rate. However, contrary to what Duchesne et al. [23] reported, the constant value depends on the fluid properties and the jet diameter. Secondly, for small plates or high flow rates, the value of the Froude number increases with increasing the flow rate and decreasing the plate size. This increase in the post-jump Froude number indicates that the flow inertia becomes important in the post-jump region.

Contrary to the classical hydraulic jump, it was observed that despite the post-jump Froude numbers being constant and independent of the flow rate, the pre-jump Froude number changes with the flow rate. This was explained to be a result of the surface tension force at the jump location. A theoretical study was conducted by coupling the pre-jump and post-jump momentum functions through the conservation of momentum at the jump location. The maximum possible post-jump Froude number and the minimum possible pre-jump Froude number were theoretically derived. It was pointed out that depending on the pre-jump velocity profile ( $\lambda$ ) and the surface tension force at the jump ( $Bo^{-1}$ ), the maximum possible post-jump Froude number could be larger than unity. This indicates the possibility of obtaining a post-jump flow with supercritical Froude numbers ( $Fr_o > 1$ ). The post-jump Froude number was calculated for all the tests carried out for this study, and few tests have exhibited post-jump Froude number higher than one.

# **Chapter 6 Double Vertical Jet Impingement**

# 6.1 Overview

In this chapter, the interaction between the flow fields formed on a horizontal surface due to the impingement of two identical, vertical, free-surface jets will be studied. To have a better picture of the phenomenon, a detailed description of double jet impingement will be presented. The jump-jump interaction patterns and the associated influential parameters will be generally characterized. The primary focus of the present chapter will be given to the generated rising sheet (i.e. fountain) as a result of the flow field interaction. The breakup mechanism of the rising sheet will be categorized into two main regimes and six subregimes based on the experimental observations. Moreover, an analytical model will be developed to predict the shape of the rising sheet (i.e. the edge profile of the rising sheet). To validate the presented theory, the results of the model will be compared to the experimental measurements obtained using different working fluids. At the end, a general description of the interaction between two unequal, vertical jets will be presented. The double, unequal jet impingement will be followed up and further analyzed in Chapter 7.

# 6.2 Description of phenomenon

The flow field caused by two free-surface, liquid jets impacting on a surface can be generally characterized as six different regions as illustrated in Figure 6.1.



Figure 6.1. (a) Schematic of two vertical, free-surface impinging jets showing major flow regions: (1) free jet, (2) jet stagnation point, (3) inner thin wall jet, (4) outer thin wall jet, (5) stagnation line (fountain formation region), (6) rising sheet. (b) Schematic of the flow fields formed on the surface (jump-jump interaction).

The free jet and the stagnation point regions are similar to the case of a single jet impingement. After a liquid jet impacts on a surface at the stagnation point, the liquid spreads out radially. The outer wall jet remains unaffected from any adjacent flow fields and behaves similar to a single jet impingement, i.e. spreads out until a sudden increase in the film thickness due to the hydraulic jump occurs. The inner thin wall jets, on the other hand, collide midway between the jets and give rise to a liquid sheet, which spreads spatially from the impact region (see Figure 6.2b). The impact region where the two inner wall jets meet is called the "stagnation line". The location and the shape of the stagnation line depend on the relative strengths of the individual parent jets. If the parent jets are two identical, vertical jets impinging on a horizontal plate, the stagnation line will be a straight line exactly midway between the two stagnation points of the individual jets (i.e. a straight line perpendicular to the line connecting the two stagnation points). On the other hand, if the jets have unequal strengths (e.g. unequal jet flow rates or nozzle diameters), the stagnation line will show a curvature (see Figure 6.2c). In this case, the stagnation line will not be located midway between the jets, and will be shifted towards the weaker jet [95].



**Figure 6.2.** Interaction between two vertical impinging water jets, (a) equal, distant jets, (b) equal, adjacent jets, (c) unequal jets.

The schematic of the flow field formed on a surface due to the impingement of two identical, vertical jets is illustrated in Figure 6.1b. When two inner wall jets interact, the collision results in local high pressures, which deviate the spreading fluid from its original radial direction. As a result, the radial flow takes a turn and leave the interaction area, either vertically upward (i.e. form the rising sheet), or towards outside of the interaction zone almost perpendicular to the line connecting the two stagnation points. The streamlines outside of the interaction zone (i.e. outer wall jet) remain unaffected, and spread outward radially.

At the collision zone, the opposing thin wall jets join and give rise to a fan-shaped liquid fountain. The rising sheet flows upward and spreads spatially. The formed liquid sheet has the shape of an arch or an approximate segment of a circle. Though the rising sheet is a thin liquid film, its edge is characterized by a relatively thick rim or crown with a nearly circular crosssection. The maximum height of the rising sheet falls above the intersection point of the stagnation line and the line connecting the two stagnation points of the parent jets. The strength and direction of the rising sheet depend on the strengths and flow characteristics of the individual parent jets. At any point along the impact zone, the rising flow exhibits three-dimensional flow structures [95]. As the rising sheet grows, the rim surrounding the standing sheet disappears and the rising sheet goes through different breakup regimes. If the parent jets are of equal strengths, the formed fountain will rise vertically up along a straight stagnation line. Though, if the jets have unequal strengths, the rising sheet will be curved and inclined towards the weaker jet (see Figure 6.2c).

# 6.3 Experimental Study

## 6.3.1 Experimental methodology

The experimental setup employed for the present work was described in Chapter 3. Few images of double vertical jet impingement and the interaction between the formed flow fields are shown in Figure 6.2. The nozzle size used for the double jet impingement experiments is d = 1 mm and is kept fixed for all the experiments. In most part of this chapter, two identical jets are examined, meaning both jets have the same flow rates, nozzle sizes and working fluid. Three different working fluids are used: distilled water, water-glycerol solutions wt. 50% and 65%. The physical properties of these fluids are reported in Table 3.1. A large, transparent target plate is utilized, and photography from underneath is carried out to capture the jump-jump interaction on the surface. To study the formed rising sheet a high speed camera is employed as schematically illustrated in Figure 3.3.

# 6.3.2 Experimental observations

#### 6.3.2.1 Jump-jump interaction

From our flow visualization experiments, it has been observed that the flow field caused by the impingement of two neighboring jets depends on the jet flow rates, nozzle sizes, fluid properties, jet inclination angles, and the relative positions of the two jets. As for two vertical, identical jets, the influential parameters are the jet flow rates, fluid properties and the spacing between the jets.

It was observed that with a fixed flow rate, the flow field mainly depends on the spacing between the jets. The images in Figure 6.3 are associated with two water jets with fixed flow rates of  $Q=300 \text{ cm}^3/\text{min}$  impinging on a horizontal plate with varied jet-to-jet spacings. When two jets are placed with a large distance, the radial symmetry of each individual hydraulic jump remains uninfluenced by the neighboring jet. This indicates no noticeable interaction between the hydraulic jumps, as can be seen in Figure 6.3a. This condition is referred to as "far-distant jets".

With reducing the spacing, the hydraulic jumps of the two individual jets start interacting, which results in disturbing the radial symmetry of the individual jump profiles. As a result of the interaction, jump radius in the inner wall jet region reduces. Although, the hydraulic jumps are affected by each other, a rising fountain is not yet formed (see Figures 6.3b and c). The streamlines immediately after the jump in the interaction area are deflected from their radial path and leave the region between the two hydraulic jumps (also referred to as the "bridge" between the two jumps). This condition is called "distant jets".

At smaller spacings, the two thin, inner wall jets with supercritical velocities collide. At the collision region, a stagnation line is formed and the thin wall jets take a sudden 90 degree turn upward and form a vertical liquid sheet (see Figures 6.3d-f). This condition is referred to as "adjacent jets". Depending on the strengths of the wall jets at the stagnation line, the generated liquid sheet rises higher. As the jet-to-jet spacing decreases, due to the reduction of the radial spacing between the stagnation points and the stagnation line, the thin wall jets at the stagnation line become stronger (i.e. they obtain higher velocities with thinner film thicknesses). Therefore, as seen in Figures 6.3d-f, the formed liquid sheets become wider and rise higher.



**Figure 6.3.** Interaction between two identical vertical jets with fixed flow rates of Q=300 cm<sup>3</sup>/min and varied spacings; (a) S=28 mm, (b) S=25 mm, (c) S=22.5 mm, (d) S=19 mm, (e) S=16 mm, (f) S=13.5 mm.

As discussed before, flow characteristics of the formed rising sheet depend on the strengths of the opposing thin wall jets at the stagnation line. Apart from the jet-to-jet spacing, the strengths of the thin wall jets are also a function of the flow rate of the parent jets. It is known that as the flow rate of an impinging jet increases, at a constant radial spacing from the stagnation point, stronger wall jets (with higher momentum) are formed (i.e. this has been discussed in the literature review, and also Chapters 4 and 5 regarding single jet impingement).



Figure 6.4. Interaction of two identical, vertical jets with constant spacing of S=19 mm; (a) Q=200 cm<sup>3</sup>/min, (b)  $Q=250 \text{ cm}^3/\text{min}$ , (c) Q=300 cm<sup>3</sup>/min, (d) Q=350 cm<sup>3</sup>/min, (e) Q=400 cm<sup>3</sup>/min, (f) Q=500 cm<sup>3</sup>/min.

Experiments have been also conducted in which the jets are set at a fixed spacing of S=19 mm and the flow rate of the impinging jets vary from 200 to 500 cm<sup>3</sup>/min. The images are shown in Figure 6.4. It is seen that at the low flow rates, the hydraulic jumps are unaffected by the neighboring jet (see Figure 6.4a). As the flow rate increases, the hydraulic jumps of individual jets grow larger. Therefore, the area between the two hydraulic jumps becomes smaller, and eventually the two jumps start interacting (see Figure 6.4b). When the flow rate is as high as  $Q=300 \text{ cm}^3/\text{min}$  (see Figure 6.4c), the radii of the individual hydraulic jumps are extended to an extent that the two inner thin wall jets collide and a vertical rising sheet is formed. With further increasing the flow rate, the thin wall jets at the stagnation line become stronger, and the formed rising sheet grows wider and higher (see Figure 6.4c-f).

In the images shown in Figures 6.3 and 6.4, it is seen that the individual hydraulic jump profiles are not always perfectly circular and some disturbances are observed along the jump boarders. The reason of the observed unstable, shaky profiles is that the disintegrated droplets from the rising sheet fall on the plate, disturb the wall jets, and destroy the steady nature of the hydraulic jumps formed on the surface. In order to avoid the messy look of the hydraulic jumps, several images of each test had to be taken in order to capture an approximate steady, circular jump. In Figure 6.5, consecutive images are shown where droplets, which have been detached from the rising sheet, fall on the wall jets and disturb the jump profiles. When a drop falls, it disturbs the flow field on the surface. However, due to the high velocities of the wall jets, the flow field recovers quickly and retains its steady look until the next drop disturbs it again. As the rising sheet becomes stronger, the sheet goes through severe breakup regimes, larger numbers of droplets are generated, and therefore makes it less likely for the flow field on the surface to retain its steady behavior.



**Figure 6.5.** Photographs showing droplet fall on the wall jets and destroying the steady jump profiles on the plate. (Sequence from left to right. The images have been chosen to highlight the droplet fall process).

# 6.3.3 Rising sheet patterns and breakup regimes

Several types of rising sheet patterns and breakup regimes are observed in the experiments and illustrated in Figures 6.7 and 6.8. All the rising sheets formed by two identical, vertical jets are flat and in a plane perpendicular to the horizontal plate. It has been reported by previous studies that the sheet breakup occurs through two principal mechanisms [103–105]. First, a fairly unruffled sheet may be produced, which eventually breaks down through the superposition of the aerodynamic waves. Secondly, the sheet breaks down due to the impact waves that are produced at the impact zone. The latter type of waves were even found in the vacuum environment [104,106].

It is worth noting that all the studies available regarding liquid sheet patterns and breakups focused on the liquid sheets that are formed by the direct collision of two liquid jets (i.e. the jets directly collide without impinging on a surface). It was concluded from these studies that the impact waves are not triggered by aerodynamic reactions, but resulted from the collision of the two parent jets at the impact point.

In the present study, the liquid sheet is not formed by the direct collision of two liquid jets. Here, the liquid jets first impact on a solid surface and then the wall jets collide and form a rising sheet. In this case, there are two impact zones. First, the impact of the liquid jet on the solid surface, and then the collision of the two wall jets. Thus, the first set of impact waves is produced at the stagnation points of the individual jets and propagates radially outward. These waves are easily seen in Figure 6.6, which was taken with a high shutter speed. These waves travel along the wall jets and eventually migrate to the formed rising sheet. As a result, even with low wall jet

velocities, the liquid sheet formed here is ruffled. It can be clearly seen in the images of Figure 6.7. This is one of the differences between the present study and those reported in the literature with the direct collision of the liquid jets.



Figure 6.6. First set of impact waves due to the impingement of the liquid jet on the solid surface.

Here two major breakup regimes are defined according to the type of the instability that causes the breakup.

# 6.3.3.1 Regime I: Capillary instability

In this regime, a fairly ruffled sheet exists surrounded by a relatively thick liquid rim. The liquid rim is due to the balance between the momentum forces and the surface tension force at the edge of the rising sheet. Due to small disturbances, bead-like shapes are generated on the rim. At low sheet velocities, the liquid rim and the formed beads grow larger but since they do not have sufficient momentum to detach, they fall down under the effect of the gravitational force. Thus, the standing sheet is destroyed, though rises again very quickly. At higher velocities, the beads are detached from the edge. These beads result in the formation of drops and ligaments, which may further break down and generate smaller droplets. In this case, droplet formation is due to the capillary instability at the rim. The size of the rising sheet becomes larger when the jet velocities increase. This regime is further divided into four subregimes as follows, and is illustrated in Figure 6.7.

### 6.3.3.1.1 Regime I-A Pre-sheet formation

This subregime occurs when the parent jet flow rates are low or/and the spacing between the two jets is large. The hydraulic jumps on the horizontal plate are unaffected by the adjacent jet. As the spacing decreases and/or the flow rate increases, the bridge between the two jumps becomes narrower and thinner. Eventually a thick rim starts to form, though it has not risen up yet. No obvious growth of beads is observed. This subregime is shown in Figure 6.7 from I-A1 to I-A3.

# 6.3.3.1.2 Regime I-B Periodic drop

As the wall jet velocity increases, the sheet starts rising up, and a thick rim surrounds the entire sheet. There exists equilibrium between the capillary and momentum forces at the edge of the sheet. The upflow of the liquid in the rising sheet is characterized by radial spreading from the stagnation line, while the downward flow is only through the peripheral rim surrounding the rising sheet. At these low sheet velocities, the liquid reaching the edge slows down and forms a thick rim. The thick rim grows and becomes larger and heavier, and eventually falls down under the effect of the gravitational force (see Figure 6.7 I-B3). This results in destroying the sheet. The sheet rises up again and the same process repeats. This rise-and-fall trend continues periodically, which results in an unstable-looking sheet. Since the sheet velocity is low, nearly no droplet detachment is observed. Images associated with this subregime are shown in Figure 6.7 from I-B1 to I-B3.



**Figure 6.7.** The rising sheet formed by the impingement of two identical, vertical jets showing different subregimes of "Regime I" breakup mechanism. The tests were conducted by a fixed flow rate and varied jet-to-jet spacings (the spacing is decreasing from Regime I-A to Regime I-D). Images were chosen to highlight the characteristics of each breakup regimes.

# 6.3.3.1.3 Regime I-C Closed rim

At higher sheet velocities, small disturbances at the edge cause the local momentum forces become greater than the local surface tension force. As a result of this local imbalance, some bead-like shapes are generated at the rim, which keep growing. After enlarging to some extent, the beads lead to the formation of large ligaments, which may break up from the rim and produce large droplets. The sheet may still exhibit some periodic drop behavior. Images in Figure 6.7 from I-C1 to I-C3 represent this subregime.

## 6.3.3.1.4 Regime I-D Rim breakup

The disturbances initiating from the impact zone, propagate throughout the entire sheet. At the same time, as the sheet progresses higher, its thickness reduces as a result of the spreading. Eventually at some heights, these disturbances cause the rim formed around the sheet to disintegrate from the upper edge of the sheet. This is due to the local increase in the momentum forces. The detached rim further breaks down and form smaller drops. Capillary instability still dominates in producing droplets at the edge of the sheet. This subregime is illustrated in Figure 6.7 from I-D1 to I-D3.

# 6.3.3.2 Regime II: Kelvin-Helmholtz (KH) instability

When the wall jet velocity is increased further, the rising sheet becomes distinctly unstable. The principal source of the instability is the interaction of the sheet with the ambient air. Disintegration arises when the wave amplitude extends to a critical value, and the rising sheet is fragmented. The fragments subsequently break down into smaller drops. Generally, either decreasing the jet-to-jet spacing and/or increasing the parent jet flow rates results in the formation of wider rising sheets. However, the experimental observations show that, contrary to the capillary regime, in this regime, increasing the wall jet velocities does not lead to an increase in the height of the rising sheet. As compared to regime I, the drop size in this regime is mostly

smaller, and lies in a narrower range. This regime can be divided into two subregimes as follows, and is shown in Figure 6.8.

## 6.3.3.2.1 Regime II-A. Open-rim

By further increasing the sheet velocity, there does not exist a closed thick rim around the sheet anymore. The top part of the rim is broken up and two separate thick rims exist on the sides of the sheet. Large ligaments and drops are detached from the side rims, while smaller drops are detached from the top part of the sheet. Most part of the sheet is still undestroyed, though the sheet is more ruffled due to the amplification of the waves propagating along the sheet. Although, there still exists the balance between the capillary and momentum forces at the edge of the sheet, the amplified disturbances frequently make the local momentum forces to overcome the capillary force. With further increasing the sheet velocity, the sheet becomes wider. The sheet loses its arch shape and a nearly rectangular shape sheet is formed, which is confined by two thick edges on the sides. Images associated with this subregime are shown in Figure 6.8 from II-A1 to II-A3.

# 6.3.3.2.2 Regime II-B. Transition into atomization

With further increasing the sheet velocity, the sheet keeps widening to an extent that the thick edges on the sides disappear, and small drops are being sprayed vigorously. In this subregime, the breakup becomes much more violent than the foregoing subregimes. The upstream part of the rising sheet does not remain stable anymore. Disintegration initiates from a small breakup downstream of the impact line, which propagates throughout the entire sheet. Eventually the sheet is fragmented. The fragments break up into ligaments, following with ligaments breakup into small drops. As the sheet velocity increases, smaller drops with higher velocities are generated. This subregime is illustrated in Figure 6.8 from II-B1 to II-B6.



**Figure 6.8.** The rising sheet formed by the impingement of two identical, vertical jets showing different subregimes of "Regime II" breakup mechanism. The tests were conducted by a fixed flow rate and varied jet-to-jet spacings (the spacing is decreasing from Regime II-A to Regime II-B). Images were chosen to highlight the characteristics of each breakup regimes.

Images of the foregoing breakup regimes are shown in Figure 6.9 from another angle. In this figure, in order to observe the different breakup regimes, the spacing between the nozzles is changed resulting in different rising sheet velocities. Each image represents one of the abovementioned breakup regimes. The side view images of the rising sheet mostly show the range of the drop sizes disintegrating from the rising sheet. It is clear that at low sheet velocities, there is a standing liquid sheet with almost no drops detaching from the rim (see Figure 6.9 I-B). As the sheet velocity increases as a result of reducing the jet-to-jet spacing, few large drops are detached from the sheet (see Figures 6.11 I-C and I-D). At higher velocities, the number of drops being sprayed increases drastically, and the drop sizes lie in a smaller range (see Figures 6.11 II-A and II-B).



Figure 6.9. Side view images of the rising sheet associated with different breakup regimes.

# 6.4 Theoretical Study

# 6.4.1 The rising sheet profile

As mentioned before, at the edge of the rising sheet there is a balance between the momentum and capillary forces. Ideally the liquid sheet continues growing up to a point where this balance is satisfied. In this section, an analytical model is developed to predict the height and shape of the rising sheet. The height of the rising sheet can be important in determining the spacing between the orifice plate and the target plate. If the formed rising sheet interacts with the orifice plate, the liquid drops on the spreading thin film on the target plate and may result in undesired effects. This is achieved by determining the edge profile of the rising sheet. However, in reality, as mentioned before, there are several forms of disturbances and instabilities that locally disrupt the force-momentum balance. As a result of these disturbances, the liquid sheet does not always reach its nominal/theoretical height (i.e. the balance could break up at some point with lower heights than those predicted theoretically).

To find a general solution for the rising liquid sheet, a force balance analysis at the edge of the sheet is carried out.



**Figure 6.10.** Sketch of a liquid sheet formed by impinging of two identical vertical jets on a horizontal plate. (a) Force balance at the edge of the sheet. The lower sketch is the rising sheet with the curved line representing the sheet edge; the upper sketch is the cross section of the sheet at the edge. (b) An arbitrary element selected on the rising sheet.

Figure 6.10a illustrates a schematic of the rising sheet and a part of its cross section close to the rim. The rim that is represented by a half circle in the upper sketch is picked as the control

volume. Carrying out a 2D analysis, the rate of change of the momentum (force per unit length of the rim),  $F_m$ , is

$$F_m = \dot{m}V\cos\varphi \tag{6.1}$$

where V is the local fluid velocity at the edge, and  $\varphi$  is the angle between the velocity vector and the local normal to the sheet edge. The mass flow rate,  $\dot{m}$ , is expressed by

$$\dot{m} = \rho V \cos \varphi h_s \tag{6.2}$$

where  $h_s$  is the local thickness of the rising sheet at the edge. Substituting Eq. (6.2) into Eq. (6.1) gives an expression for the rate of change of momentum per unit length as

$$F_m = \rho (V \cos \varphi)^2 h_s \tag{6.3}$$

The surface tension force at the edge of the sheet per unit length is expressed by

$$F_{\sigma} = \sigma(\frac{2}{h_s} + \frac{1}{R_c})h_s \tag{6.4}$$

where  $R_c$  is the local radius of the rim. Since in most cases studied here  $R_c$  is much larger than  $h_s$ ,  $(h_s << R_c)$ , Eq. (6.4) can be rewritten as

$$F_{\sigma} = 2\sigma \tag{6.5}$$

In order to have a stable rim, the rate of change of the fluid momentum must be balanced by the force caused by the surface tension at the rim location. Thus,

$$F_m = F_\sigma \tag{6.6}$$

Substituting Eqs. (6.3) and (6.5) into Eq. (6.6) results in an expression for the sheet thickness right at the edge.

$$h_s = \frac{2\sigma}{\rho(V\cos\varphi)^2} \tag{6.7}$$

Furthermore, conservation of mass must be satisfied along the sheet. It means that the sum of the volume flow rates coming from the two thin wall films at the stagnation line must be equal to the volume flow rate of the rising sheet. Therefore, an arbitrary segment of the thin wall jets is chosen followed by an element of the rising sheet as shown in Figure 6.10b. Conservation of mass for the shaded element on the rising sheet can be written as

$$2(U_r h_r r d\gamma) = h_s V dS \tag{6.8}$$

where  $r = \frac{S/2}{\cos\theta}$  is the radial spacing between the jet stagnation points and the bottom of the element on the rising sheet,  $dS = rd\gamma + Rd\gamma = (r+R)d\gamma$ , and  $\theta$  is an arbitrary angle determining the selected element as shown in Figure 6.10. Also,  $h_r$  is the thickness of the thin wall jets on the plate immediately before the stagnation line, which can be calculated using Watson's solution as

$$h_r = 3.799 \frac{\upsilon (r^3 + 0.2864 a^3 \text{ Re})}{Qr}$$
 [8]. In addition,  $U_r$  is the average wall jet velocity of the selected

element right at the stagnation line, which can be calculated as  $U_r = \frac{Q}{2\pi h_r r}$ . Equation (6.8) then

becomes

$$h_s = \frac{2U_r h_r r}{(r+R)V} \tag{6.9}$$

Combining Eqs. (6.7) and (6.9) gives an expression for the radius of the rim as

$$R = r(\frac{\rho U_r h_r V \cos^2 \varphi}{\sigma} - 1) \tag{6.10}$$

In the above equation, the sheet velocity at the edge, V, and also the angle between the velocity vector and the local normal to the sheet edge,  $\varphi$ , are still to be determined. The angle  $\varphi$  will be 112

discussed later along with introducing the geometric relations. As for the sheet velocity V, in most of the previous studies and theories, an inviscid model has been used for the liquid sheet formed by the interaction of two jets [103,107–111]. They assumed a uniform fluid velocity across the liquid sheet, which is equal to the jet velocities at the impact point/line, i.e.  $V = U_r$ . In the present work, the same assumption is employed with one difference. Since the liquid sheet rises vertically upward, the fluid flow is influenced by the gravitational force. As a result, part of the kinetic energy is converted to the potential energy, which must be considered in determining the sheet velocity, V. Conservation of energy from the bottom of the sheet to the edge results in

$$V = \sqrt{U_r^2 - 2gz} \tag{6.11}$$

where z is the vertical elevation of the rising sheet.

Equation (6.10) leads to a solution for the radial spacing between the collision point of the streamlines at the stagnation line to the edge of the rising sheet. In order to predict the actual location of the edge, in addition to R, the angle  $\psi$ , as shown in Figure 6.10a, needs to be determined.

The stagnation line location and also the flow direction in the rising sheet can be explained by the momentum components of each wall jet flow element entering the rising sheet. Since negligible energy loss due to turning is assumed [90], the total momentum in an upflow element of the rising sheet is taken to be the sum of the momentum of the individual wall jet flow elements. It is assumed that the momentum components normal to the stagnation line cause the upward direction of the fluid flow of the rising sheet. Also, the tangential components of the momentum result in the outward motion of the rising liquid following the direction of the stagnation line. Since the focus of the present chapter is on two identical, vertical jets, for which the stagnation line is a straight line located exactly midway between the two jets (i.e. symmetry line), it is realized that at any point along the stagnation line  $\psi = \theta$ . Similar discussion can be found in a general form in Ref. [90]. Consequently, by calculating *R* using Eq. (6.10), along with the angle  $\psi$ , the actual location of the edge of the rising sheet can be estimated.



Figure 6.11. Schematic of the rising sheet edge profile on the y-z plane.

# 6.4.1.1 Geometric relations

The intersection of the stagnation line and the line connecting the two stagnation points is considered as the origin in y-z plane as shown schematically in Figure 6.11. The position of the edge of the rising sheet is defined as f(y,z) = 0. Employing the Cartesian coordinate system (y, z), the following geometric relations are obtained

$$R = \sqrt{z^2 + (y - \frac{S}{2} \tan \theta)^2}$$

$$r = \frac{S/2}{\cos \theta}$$
(6.12)

$$\theta = \tan^{-1} \left( \frac{y}{z + \frac{S}{2}} \right) \tag{6.14}$$

And eventually, the angle between the velocity vector and the local normal to the sheet edge,  $\varphi$ , can be expressed as

$$\varphi = \pi - \tan^{-1} z' - \theta \tag{6.15}$$

From the experimental observations, it is seen that the maximum height of the rising sheet is at y=0. At this point, the velocity vector is normal to the edge of the sheet, meaning  $\varphi=0$ . Thus, the maximum height of the rising sheet can be calculated combining Eqs. (6.10) and (6.11) with  $\varphi=0$  and r=S/2 as follows

$$z\big|_{y=0} = H_{\max} = \frac{\rho Q \sqrt{U_r^2 - 2gH_{\max}}}{2\pi\sigma} - \frac{S}{2}$$
(6.16)

Combing Eqs. (6.10), (6.11), and (6.15) results in an ordinary differential equation that can be solved using a boundary condition set at the maximum height of the rising sheet,  $H_{max}$ , to determine f(y, z) = 0 as the edge profile of the rising sheet. A numerical solution was developed in MATLAB to solve this differential equation. The solution starts from the maximum height of the edge as the boundary condition (using Eq. 6.16), and proceeds with

$$z' = \frac{z_2 - z_1}{y_2 - y_1} \tag{6.17}$$

#### 6.5 Results and Discussion

Figure 6.12 compares the results of the theoretical model with the experimental measurements of the edge profiles. Three different working liquids are presented in this figure with varied viscosities and slight differences in the surface tension values (physical properties of theses fluids are available in Table 3.1). Figure 6.12a is associated with a rising sheet formed by the impingement of two vertical water jets with equal flow rates of Q=300 cm<sup>3</sup>/min and a jet-to-jet spacing of S=15.5 mm. Figure 6.12b shows a rising sheet formed by two vertical jets with water-glycerol solution wt. 50% as the working fluid, flow rates of Q=370 cm<sup>3</sup>/min, and a spacing of S=11.6 mm. Figure 6.12c is associated with two water-glycerol wt. 65% jets with flow rates of Q=412 cm<sup>3</sup>/min and a spacing of S=10.1 mm. In the bottom graphs of Figure 6.12, comparisons are illustrated between the measured location of the rising sheet edge and the presented theoretical solution. The theoretical model could be significantly simplified by assuming the angle  $\varphi$  is equal to zero. The results plotted in Figure 6.12 show reasonable agreements between the theory and the experimental measurements.



**Figure 6.12.** Comparison between the experimental results and the theoretical predictions. (a) Water jets with Q=300 cm<sup>3</sup>/min, S=15.5 mm, (b) Water-Glycerol solution (wt. 50%) with Q=370 cm<sup>3</sup>/min, S=11.6 mm, (c) Water-Glycerol solution (wt. 65%) with Q=412 cm<sup>3</sup>/min, S=10.1 mm. (Solid lines are the theoretical results)

It should be mentioned that the presented theory is only applicable to the rising sheets that maintain relatively stable rim and are not experiencing severe breakup modes. Thus, the tests that exhibit a fairly stable rim around the rising sheet is considered for the comparison. In these tests, droplet detachments only occur at the edge of the sheet, i.e. breakup regime "T" as categorized in section 6.3.3.1.

In regime I, the rising sheet usually exhibits a periodic fall-and-rise behavior. This behavior could be due to the gravitational force, which causes the enlarged rim to fall after reaching a certain size, and re-rise again. It also could be a result of the droplets falling on the wall jets and disturbing the rising sheet (as shown in Figure 6.5). Therefore, in order to capture the location of the edge, several images were recorded using a high speed camera. Measurements were then done at different instants of time for each test condition (i.e. when the sheet reaches its maximum height) and the recorded results are presented here.

It can be deduced from the above discussion that the presented theory determines the size of the rising sheet in an ideal condition (i.e. without any types of disturbances). However, it has been reported previously that, even in a vacuum environment, the impacts waves still exist [104,106]. Hence, as a result of the existing disturbances, the rising sheet breaks up before reaching its maximum theoretical height. Thus, the experimental results usually show values smaller than the values predicted by the theory. Another source of uncertainty in the theory could be originated from the approximation used for the radius of the rim surrounding the rising sheet. In the theory, the rim cross-section was modeled as a half circle with a diameter equal to the sheet thickness.

However, from the experimental observations it can be inferred that the rim is usually thicker than the sheet thickness.

The results of the maximum height ( $H_{\text{max}} = z|_{y=0}$ ) predicted by the theory are plotted in Figure 6.13 versus the measured maximum heights from the experimental tests. The results represent three different liquids: water and two different concentrations of water-glycerol solutions (wt 50% and 65%). Additionally, the results of the maximum heights ( $H_{\text{max}}$ ) calculated without considering the gravitational effects (i.e. using  $V=U_r$  instead of Eq. 6.11) are presented as hollow symbols. From Figure 6.13, the following points are deduced.

First, despite the uncertainties in the height of the rising sheet due to the existing disturbances, there is a fairly good agreement between the theory and the experimental measurements.

Second, as the height increases the discrepancy between the theory and the experimental measurements increases. The reason is that as the sheet becomes stronger and spreads larger, the disturbances become more substantial, thus, the sheet is more likely to break up prior to reaching its theoretical height.

Third, majority of the data points fall above the line y=x indicating that the theoretical values are higher than the experimental measurements. This has been explained earlier that the disturbances propagating along the sheet are responsible for this discrepancy between the theory and experiments. In addition, it is seen that the influence of the gravitational effects (i.e. Eq. 6.11) in determining the rising sheet velocity can be pronounced. Taking into account the conversion of the kinetic energy into the potential energy along the sheet reduces the predicted height by up to 20%. It is seen in Figure 6.13 that as the sheet rises higher, the differences between the hollow and solid symbols are increased, which indicates the significance of the potential energy at higher elevations.



Figure 6.13. Comparison between the theoretical and experimental values of the maximum rising sheet height. The solid symbols represent the calculations with  $V = \sqrt{U_r^2 - 2gz}$  and the hollow symbol represent  $V = U_r$ 

With a fixed jet flow rate, the wall jets colliding at the stagnation line become stronger as the spacing between the jets decreases. As a result, the liquid sheet rises higher. Figure 6.14 illustrates the edge profile of the rising sheet with fixed flow rates and varied jet-to-jet spacings. Figure 6.14a represents tests with water as the working fluid and a fixed flow rate of Q=300 cm<sup>3</sup>/min. Figure 6.14b and c are associated with water-glycerol solutions with mass fractions of 50% and 65%, and fixed flow rates of Q=450 and 412 cm<sup>3</sup>/min, respectively.



**Figure 6.14.** Experimental measurements of the edge profile of the rising sheet; (a) Water with  $Q=300 \text{ cm}^3/\text{min}$ , (b) Water-Glycerol (wt 50%) with  $Q=450 \text{ cm}^3/\text{min}$ , (c) Water-Glycerol (wt 65%) with  $Q=450 \text{ cm}^3/\text{min}$ . The dashed lines are plotted by applying a polynomial curve fitting to show the approximate location of the rising sheet edge.

A close look at Figure 6.14a reveals that as the spacing reduces, the liquid sheet rises higher. However, at small spacings, the height of the rising sheet does not increase proportionately. The data points of small spacings lie close to each other. The reason is that as the strength of the rising sheet increases with reducing the jet-to-jet spacing, the sheet goes through more severe breakup regimes. As described in Section 6.3.3.2 (i.e. breakup regime II), increasing the rising sheet velocity does *not* increase the height of the rising sheet anymore. This is attributed to the amplified disturbances that break down the sheet before reaching its nominal height. This trend is similar for the two other more viscous fluids as well (see Figures 6.17b and c). However, looking at all the tests carried out in this study, it is inferred that as the viscosity of the working fluid increases, the rising sheet becomes more resistant towards the disturbances. As a result, even with small spacings, the sheet can grow higher and closer to its nominal height.

# 6.6 Unequal Double Jet Impingement

After conducting an extensive set of experiments with two identical jets, experiments are also carried out with two unequal, vertical jets. The jets employed in the present work are of equal diameters, though with different flow rates. The resulting jump-jump interaction patterns are observed to be similar to those produced by two identical neighboring jets, i.e. far distant jets, distant jets, and adjacent jets.



Figure 6.15. Two unequal, vertical, water jets. (a, b) two different view angles to better illustrate the phenomenon,  $Q_1=250 \text{ cm}^3/\text{min}$  and  $Q_2=500 \text{ cm}^3/\text{min}$ , (c)  $Q_1=150 \text{ cm}^3/\text{min}$  and  $Q_2=600 \text{ cm}^3/\text{min}$ , (d)  $Q_1=300 \text{ cm}^3/\text{min}$  and  $Q_2=600 \text{ cm}^3/\text{min}$ 

A few images of the flow field interaction due to the impingement of two unequal jets are shown in Figure 6.15. It is observed that contrary to two identical jets, the stagnation line formed with two unequal jets is curved. Also, the stagnation line is no longer located midway between the two stagnation points, instead, it is shifted towards the weaker jet. Additionally, the formed rising sheet, as seen in Figure 6.15, is inclined towards the weaker jet. All these observations are attributed to the collision of dissimilar streamlines at the collision zone. Contrary to the interaction of two identical jets, with unequal jets, at any point along the stagnation line, the colliding streamlines exhibit different strengths (i.e. momentum). As a result, the formed stagnation line is moved towards the weaker jet, and the formed rising sheet is inclined. This will be thoroughly discussed in a theoretical discussion in Chapter 7.

From the experimental observations, it was also observed that as the difference between the two flow rates increases, the stagnation line moves closer to the weaker jet, and its curvature increases (see Figure 6.15c and d). In addition, the rising sheet inclination angle increases as well (i.e. the angle between the rising sheet and a vertical plane), which in some cases, makes the rising sheet to interact with the weaker free jet (see Figure 6.15c).

In Chapter 7, a theoretical model will be proposed to predict the location and shape of the formed stagnation line due to the interaction of two unequal jets. The comparison between the theoretical predictions and the experimental measurements will be also presented in Chapter 7.
# 6.7 Summary

The flow field formed by the impingement of two vertical, free-surface jets on a horizontal plate was studied. The jump-jump interaction was experimentally explored and characterized as three types: far-distant jumps, distant jumps, and adjacent jumps. The primary focus of this chapter was given to the formed rising sheet as a result of the interaction between two adjacent jumps. The breakup mechanism of the rising sheet was studied based on the experimental observations. The sheet patterns and breakup mechanisms were categorized into two main regimes and six subregimes.

In addition, an analytical model was developed to predict the size and shape of the rising sheet. This was achieved by determining the edge profile of the sheet. It was explained that at the edge of the liquid sheet the capillary force balances the momentum forces. Thus, a force balance analysis at the edge of the sheet was carried out. The analytical model has led to a differential equation that can be solved for the edge profile of the rising sheet. The results of the presented theory were compared to the experimental measurements of the edge profile of the rising sheets obtained by three different working fluids: water, water-glycerol solutions wt. 50% and 65%. It was discussed that the presented theory is applicable to the rising sheets that are not experiencing severe breakup modes. Due to the propagation of the disturbances along the sheet, the local balance at the edge frequently breaks down and the liquid sheet gets disturbed before reaching its theoretical height. Despite these uncertainties in the height of the rising sheet due to the existing disturbances, a fairly good agreement was observed between the theory and the experimental measurements.

# **Chapter 7 Double Inclined Jet Impingement**

# 7.1 Overview

Contrary to a single vertical jet impingement, the flow field formed on a plate by the impingement of an inclined jet is no longer axisymmetric. With an inclined jet, the formed hydraulic jump is elliptic, and all the flow properties on the surface exhibit a dependence on the azimuthal position. In this chapter, the interaction between two flow fields formed by the impingement of two inclined jets on a surface is experimentally and theoretically studied. The jets considered for the present study are placed on two parallel planes, and their projected flow directions on the horizontal target plate are in opposite directions (see Figure 7.1). It will be observed that the jump-jump interaction in this case exhibits a curved stagnation line and forms a tilted rising sheet.

First, the generated flow field on the surface along with the formed stagnation line and the rising sheet will be generally described. A single inclined jet will be analytically studied to highlight the differences between the flow characteristics of inclined and vertical jet impingement. Next, a theory will be developed using a "momentum flux" balance to predict the location and shape of the formed stagnation line as a result of the impingement of two inclined jets. A systematic experimental study will be carried out to characterize the influential parameters on the generated flow field. The influences of different parameters on the formed stagnation line, such as the jet inclination angle, flow rate, working fluid, and the relative position of the jets, will be

experimentally examined. Moreover, the results of the presented theory will be compared to the measured stagnation line locations to verify the correctness of the model.

#### 7.2 Description of Phenomenon

In Chapter 6, the flow field produced by the impingement of two identical, vertical jets was discussed. With two identical, vertical jets, regardless of the flow rate and jet-to-jet spacing, the stagnation line is always a straight line perpendicular to the line connecting the two individual stagnation points exactly midway between the two jets. Even if the two jets are inclined with opposing jet flow directions, as long as the jets are placed on the same plane, the stagnation line behaves similar to the case of two vertical jets, i.e. a straight line normal to the plane that the jets are laid on. In both cases, i.e. uniplanar, inclined and vertical double jet impingement, the generated liquid sheet rises straight up vertically. The rising sheet will be on a plane perpendicular to the jet plane located equidistant from both jets (See Figure 6.2b).

In the present chapter, the flow field formed by the impingement of two inclined jets is studied, in which the jets are laid on two parallel planes as schematically illustrated in Figure 7.1a. Two jets have the same nozzle inclination angles,  $\phi$ , which is defined as the angle between the nozzle and the horizontal plane. The projected jet flow directions on the horizontal plate are parallel though in opposite directions, as shown with the arrows in Figure 7.1. The relative position of the jets can be identified in two ways: ( $S_X$ ,  $S_Y$ ) defined as the spacing between the two stagnation points in x and y directions, respectively (see Figure 7.1b). Second, ( $S_{rel.}$ ,  $\psi_{rel.}$ ) defined as the relative angle and the relative spacing between the two stagnation points, (see Figure 7.1b).



**Figure 7.1.** Schematic of the interaction of two opposing inclined jets located on two parallel planes. (a) 3D schematic view, (b) schematic of the hydraulic jumps and the stagnation line formed on the surface.

Contrary to a single vertical jet impingement, the flow formed on a plate by the impingement of an inclined jet is no longer axisymmetric, and exhibits a three-dimensional nature. With an inclined jet, the formed hydraulic jump is elliptic [25,29,58]. A schematic of the jump-jump interaction of two inclined, bi-planar jets (i.e. jets are placed on two parallel planes) are shown in Figure 7.1b.

When two identical, uniplanar jets interact (either vertical or inclined), the streamlines that meet and collide at the impact zone are symmetric with respect to the stagnation line. In other words, the colliding streamlines are similar with the same angle and strength originating from the parent stagnation points. However, in case of bi-planar inclined jets, the streamlines that collide are no longer symmetric, which results in the formation of a curved stagnation line (see Figures 7.1 and 7.2). Additionally, the experimental observations show that the generated rising sheet is no longer a vertical sheet. The formed liquid sheet will be tilted in two opposite directions towards the closer stagnation point, which will be described further in this chapter. The flow characteristics of the interaction between two bi-planar, inclined jets depend on the jet inclination angles ( $\phi$ ), relative position of the jets (i.e. ( $S_X$ ,  $S_Y$ ) or ( $S_{rel.}$ ,  $\psi_{rel}$ )), flow rates (Q), and the fluid properties. The present work only focuses on the interaction of two identical, inclined jets (i.e. equal flow rates, jet inclination angles, and fluid properties) with jets flowing in opposing directions.



**Figure 7.2.** Interaction of two bi-planar, opposing, inclined jets. (a)  $\phi = 70^{\circ}$ , (b)  $\phi = 50^{\circ}$ , (c)  $\phi = 30^{\circ}$ .

# 7.3 Experimental Methodology

The experimental facility employed for this work has been described in Chapter 3. The relative spacing of the jets, as discussed, is set by keeping one jet fixed and changing the position of the other jet using the translation stages. Since the formed rising sheet sprays droplets around at relatively high rates, photography from underneath of the impingement plate is carried out. Three different working fluids are used: distilled water, and two water-glycerol solutions wt. 50% and 65%. The physical properties of these fluids are available in Table 3.1. The nozzle diameter of d=1 mm is used to conduct the experiments presented in the current chapter.

In addition to the interaction between two inclined impinging jets, the flow field formed by a single inclined jet is also studied. The purpose of the latter is to have a better understanding of the flow characteristics of an inclined jet beyond the impingement, which will be used for the

analysis of the interaction between the two flow fields. Moreover, the experimental results of a single inclined jet impingement (i.e. the location of the measured hydraulic jump) will be compared to the theory presented by Kate & Chakraborty [25] in order to verify the correctness of the measured parameters. The results are presented in Section 7.5.1.



**Figure 7.3.** Schematic views of (a) an inclined jet impingement, (b) jet flow streamlines in the stagnation zone, (c) non-circular hydraulic jump formed on the surface, (d) the impingement region.

# 7.4 Theoretical Analysis

# 7.4.1 Single inclined jet impingement

A single inclined impinging jet is schematically illustrated in Figure 7.3. In the case of an inclined jet impingement, contrary to vertical impinging jets, the axial symmetry only exists in the free jet region. The flow exhibits three-dimensional nature elsewhere (i.e. downstream of the stagnation point) [58]. The impingement zone, which is known to be the cross-section of the free jet in vertically impinging jets, changes in shape and size with the jet inclination (see Figure

7.3d). The stagnation point in the impingement area is no longer coincident with the geometrical center of the jet. It has been reported by previous researchers that with an inclined impinging jet, the stagnation point is shifted towards the upstream side of the geometrical center of the non-circular impingement zone [25,27–29,58] (see Figures 7.3b and d).

From the impingement zone outwards, the flow no longer remains axisymmetric. The radial flow spreading out from the stagnation point results in non-circular hydraulic jump profiles (see Figure 7.3c). Figure 7.4 shows a few non-circular hydraulic jumps formed on the surface due to the impingement of inclined jets. As observed in Figure 7.4 and schematically plotted in Figure 7.3c, the profile of the hydraulic jump is an elliptic shape. The change in the shape of the jump profiles from circular with vertical impinging jets, to elliptic shapes in the case of an inclined jet impingement, is attributed to the changes in the location of the stagnation point and also the shape of the impingement region as a result of the jet inclination.



Figure 7.4. Non-circular hydraulic jumps formed by inclined jets, (a)  $\phi = 50^{\circ}$ , water-glycerol wt. 50%, Q=300 cm<sup>3</sup>/min, (b)  $\phi = 40^{\circ}$ , water-glycerol wt. 50%, Q=300 cm<sup>3</sup>/min, (c)  $\phi = 50^{\circ}$ , water-glycerol wt. 65%, Q=380 cm<sup>3</sup>/min, (d)  $\phi = 30^{\circ}$ , water, Q=350 cm<sup>3</sup>/min.

## 7.4.1.1 Impingement region

A jet with radius *a* and flow rate *Q* obliquely impinges on a flat, horizontal surface with an angle  $\phi$  (see Figure 7.3a). The geometric intersection of the jet and the surface is an elliptical region, which is referred to as the impingement region (see Figure 7.3c and d). The ellipse can be described by  $r_e$  and  $\theta$ , as shown schematically in Figure 7.3d, and the geometric relation is

$$\left(\frac{r_e}{a}\sin\theta\right)^2 + \left(\frac{r_e}{a}\cos\theta - \frac{S}{a}\right)^2\sin^2\phi = 1$$
(7.1)

where *S* is the stagnation point shift (i.e. the distance between the stagnation point and the geometrical center of the jet, *O*, see Figure 7.3d). The stagnation point, as shown in Figure 7.3b, is referred to as the intersection point of the separation streamline and the target surface [58]. The flow rate per radian at the angular location  $\theta$ ,  $Q'_{\theta}$  satisfies

$$Q_{\theta}' d\theta = r_e^2 \frac{d\theta}{2} U_j \sin\phi \tag{7.2}$$

where  $U_j$  is the jet velocity. Equation (7.2) can be reduced to

$$Q'_{\theta} = \frac{r_e^2}{2} U_j \sin\phi \tag{7.3}$$

Conservation of momentum between the impinging jet and the flow in the radial direction parallel to the surface requires

$$\int_{0}^{2\pi} Q'_{\theta} V_{r} \cos \theta d\theta = Q U_{j} \cos \phi$$
(7.4)

where  $V_r$  is the radial flow velocity parallel to the surface. Assuming no energy loss due to turning, mechanical energy balance results in  $V_r$  to be equal to  $U_j$ . Applying Eq. (7.3) to Eq. (7.4) along with  $Q = \pi a^2 U_j$  and  $V_r = U_j$  gives

$$\int_0^{\pi} \left(\frac{r_e}{a}\right)^2 \cos\theta d\theta = \pi \cot\phi$$
(7.5)

From Eqs. (7.1) and (7.5), we can solve for both S and  $r_e$ , which are

$$S = a \cot \phi \tag{7.6}$$

$$r_e = a \left( \frac{\sin \phi}{1 - \cos \phi \cos \theta} \right) \tag{7.7}$$

If it is assumed that the flow at any angular position  $\theta$  is produced by a vertical jet impinging on the surface, the flow rate of the vertical jet is  $Q_{\theta} = 2\pi Q'_{\theta}$ . From Eqs. (7.3) and (7.7), the flow rate can be expressed as

$$Q_{\theta} = Q \frac{\sin^3 \phi}{\left(1 - \cos \theta \cos \phi\right)^2} \tag{7.8}$$

The jet velocity is still  $U_j$ . The jet radius of the equivalent vertical jet corresponding to the azimuthal position  $\theta$  can be found by dividing Eq. (7.8) by  $\pi U_j$ .

$$a_{\theta}^{2} = a^{2} \frac{\sin^{3} \phi}{\left(1 - \cos \theta \cos \phi\right)^{2}}$$

$$(7.9)$$

Hence, the impingement region of an inclined circular jet can be modelled by that of a vertical jet impinging on a horizontal surface whose radius and flow rate are  $a_{\theta}$  and  $Q_{\theta}$ , respectively.

# 7.4.1.2 Thin wall jet region

After the impingement region, the free jet flow changes direction and spreads out radially in a very thin layer on the surface, i.e. thin wall jet region. The conservation of mass in the wall jet region gives

$$\frac{Q_{\theta}}{2\pi r} = \int_0^h u dz \tag{7.10}$$

where *u* is the velocity profile in the thin film and *h* is the thickness of the thin wall jet. The flow momentum in the thin wall jet per unit of circumferential length at the azimuthal position  $\theta$  is expressed as

$$\dot{M}' = \rho \int_0^h u^2 dz \tag{7.11}$$

If an inviscid jet flow is assumed, the conservation of energy requires  $u=U_j$ . Thus, from Eq. (7.10) the thickness of the thin film can be expressed as  $h = \frac{Q_{\theta}}{2\pi r U_j}$ . Substituting *u* and *h* into Eq.

# (7.11) gives

$$\dot{M}' = \rho U_j^2 h = \rho U_j \frac{Q_\theta}{2\pi r}$$
(7.12)

As for viscous jets, a boundary layer flow is considered for the flow outside the impingement region. Watson's solution is used to solve for the flow momentum as discussed in Chapters 2 and 4 [8]. Prior to the hydraulic jump, the thin wall jet is divided into two regions: before and after the radius where the boundary layer reaches the free surface of the thin wall jet, i.e.

 $r_c = 0.1974 \frac{Q_{\theta} a_{\theta}^2}{\upsilon}$  (see Figure 2.2b). Therefore, the flow momentum per unit of circumferential

length can be expressed as

$$\dot{M}' = \rho U_j^2 \left[ \frac{a_\theta^2}{2r} - 0.64 \sqrt{\frac{\nu r a_\theta^2}{Q_\theta}} \right], \text{ for } r < r_c$$
(7.13a)

$$\dot{M}' = 0.00838 \frac{\rho Q_{\theta}^3}{\upsilon r \left(r^3 + 0.183 \frac{Q_{\theta} a_{\theta}^2}{\upsilon}\right)}, \text{ for } r > r_c$$
(7.13b)

Also, following Watson's theory, the thin wall jet thickness as a function of the radial and angular positions can be expressed as

$$h = 3.799 \frac{\upsilon}{Q_{\theta} r} \left( r^{3} + 0.183 \frac{Q_{\theta} a_{\theta}^{2}}{\upsilon} \right)$$
(7.14)

**Figure 7.5.** Angular distributions of different flow properties at a constant radial spacing of r=5 mm. Calculations have been done using a water jet with the flow rate of Q=350 cm<sup>3</sup>/min and the nozzle inclination angle of  $\phi=50^{\circ}$ .

To have a better picture of the angular distributions of the flow properties in the thin wall jet, in Figure 7.5, the flow momentum  $\dot{M}'$  and the film thickness h are plotted versus the angular position  $\theta$ . Additionally, the angular distributions of the flow rate  $Q_{\theta}$  and the jet radius  $a_{\theta}$  are presented as well. Equations (7.8), (7.9), (7.13), and (7.14) are employed, at a constant radial spacing of r=5 mm, to calculate the flow rate, the jet radius, the flow momentum, and the thin wall jet thickness, respectively. A water jet with the flow rate of Q=350 cm<sup>3</sup>/min and the nozzle

inclination angle of  $\phi=50^{\circ}$  is considered for the calculations. All the calculated parameters are then normalized by their maximum values, i.e. the maximum values of  $Q_{\theta}$ ,  $a_{\theta}$ , and  $\dot{M}'$  are at  $\theta=0$ , and the maximum value of h is at  $\theta=\pi$ . The normalized data is plotted versus  $\theta$  in Figure 7.5. The angular distributions of the flow parameters will be used later on this chapter to explain the phenomena observed with the interaction of two inclined impinging jets.

# 7.4.2 Double Inclined Jet Impingement

#### 7.4.2.1 Geometric Relations

Two inclined jets impinging on a solid plate create two flow fields on the surface, which could interact and generate a rising liquid sheet. The jets are placed on two parallel planes, which are perpendicular to the impingement plate. Projected onto the surface, the two jet flows are in opposite directions, as shown by the gray arrows in Figure 7.6a.



Figure 7.6. Schematic of arbitrary streamlines colliding at the stagnation line, along with geometric parameters. (b) A schematic of the thin wall jets colliding at the impact zone.

Taking the midpoint of the line connecting the two impairment points as the origin, their impingement points are then located at  $\left(-\frac{S_x}{2}, -\frac{S_y}{2}\right)$  and  $\left(\frac{S_x}{2}, \frac{S_y}{2}\right)$ . A rising liquid sheet is

formed, and the location of the rising sheet on the horizontal plate (also referred to as the stagnation line) is f(x, y) = 0. Figure 7.6a illustrates the schematic of two arbitrary streamlines colliding at an arbitrary point (*x*,*y*) on the stagnation line. The geometric parameters shown in Figure 7.6a can be expressed in terms of *x*, *y*, S<sub>X</sub>, and S<sub>Y</sub> using the following relations. In all the equations presented here, the subscript "1" and "2" are used to refer to the parameters associated with jet #1 and jet #2, respectively.

$$r_{1} = \sqrt{\left(x + \frac{S_{x}}{2}\right)^{2} + \left(y + \frac{S_{y}}{2}\right)^{2}}$$
(7.15a)

$$r_2 = \sqrt{\left(x - \frac{S_x}{2}\right)^2 + \left(y - \frac{S_y}{2}\right)^2}$$
 (7.15b)

$$\cos \theta_1 = \frac{x + \frac{S_x}{2}}{r_1}$$
 (7.16a)

$$\cos\theta_2 = \frac{\frac{S_x}{2} - x}{r_2} \tag{7.16b}$$

The angle that the streamlines make with the local normal to the stagnation line is denoted by  $\gamma$  and can be expressed as

$$\theta_1 + \gamma_1 = \frac{\pi}{2} - \left(\pi - \tan^{-1} y'\right)$$
(7.17a)

$$\theta_2 + \gamma_2 = \frac{\pi}{2} - \left(\pi - \tan^{-1} y'\right)$$
(7.17b)

where y' = dy / dx and the streamline angles  $\theta_1$  and  $\theta_2$  can be simply defined by

$$\theta_1 = \tan^{-1} \frac{y + (S_Y / 2)}{x + (S_X / 2)}$$
 and  $\theta_2 = \tan^{-1} \frac{(S_Y / 2) - y}{(S_X / 2) - x}$ . Thus, Eq. (7.17) is rewritten as

$$\gamma_{1} = \tan^{-1} y' - \frac{\pi}{2} - \tan^{-1} \frac{y + (S_{Y}/2)}{x + (S_{X}/2)}$$
(7.18a)

$$\gamma_2 = \tan^{-1} y' - \frac{\pi}{2} - \tan^{-1} \frac{(S_Y + 2) - y}{(S_X / 2) - x}$$
(7.18b)

# 7.4.2.2 Stagnation Line – Theoretical Model

One may develop an analytical model to predict the location of the stagnation line by employing the azimuthal distribution of the radial flow momentum presented in Eq. (7.13). This model would establish the location of the stagnation line by balancing the rates of the flow momentum in the opposing wall jets (per unit circumferential length) in a direction locally normal to the stagnation line. If the wall jet flow momentum are balanced in the direction normal to the stagnation line, the rising sheet does not acquire any momentum normal to the stagnation line. As a result of the mentioned momentum balance, the rising sheet has to rise vertically (i.e. in a plane perpendicular to the impingement plate, although a velocity component tangential to the stagnation line is allowed). In other words, the mentioned momentum balance would fix the stagnation line location such that a rising sheet normal to the impingement plate is established. However, as observed in the experiments, the rising sheet is indeed inclined relative to the impingement plate.

Contrary to the interaction between two identical vertical jets, with two bi-planar, inclined jets, the stagnation line is not a symmetry line for the flow field formed on the surface anymore. As seen in Figure 7.6a, the streamlines colliding at the stagnation line leave their parent stagnation points with different angles (i.e.  $\theta_1$  and  $\theta_2$ ). Therefore, due to the jet inclination and consequently

the angular distribution of the flow properties (see Figure 7.5), the two thin wall jets colliding at any point along the stagnation line exhibit unequal flow properties. For instance, the colliding wall jets exhibit different film thicknesses at the interaction region (schematically shown in Figure 7.6b). The mentioned dissimilarity of the opposing film thicknesses requires the balance to be applied on an average over the film thicknesses of the wall jets. In other words, the location of the stagnation line can be predicted by balancing the wall jet flow momentum per unit area of the wall jets in a direction normal to the stagnation line. Here, the momentum per unit area of the wall jet is called "momentum flux", however, in some other references, the term "momentum density" has been used for the momentum per unit area [88,90]. The momentum flux at the azimuthal position  $\theta$  can be defined as

$$\dot{M}'' = \rho \frac{1}{h} \int_0^h u^2 dz$$
(7.19)

Thus, the momentum flux balance in a direction locally normal to the stagnation line can be expressed as

$$\dot{M}_{1}''\cos^{2}\gamma_{1} = \dot{M}_{2}''\cos^{2}\gamma_{2}$$
(7.20)

The implementation of this criterion results in an imbalance of the total wall jet flow momentum at the stagnation line in a direction normal to the stagnation line. Consequently, the remaining momentum appears in the rising sheet in a direction normal to the stagnation line. As a result, the rising sheet is allowed to be inclined relative to the impingement surface (rises in a non-vertical trajectory). The momentum flux (Eq. 7.19) can be estimated by considering a boundary layer flow for the wall jet region. The Watson's solution for viscous jets is employed and the momentum flux is derived and expressed as

$$\dot{M}'' = \rho U_j^2 \left( \frac{a_\theta^2}{2r} - 0.64 \sqrt{\frac{\nu r a_\theta^2}{Q_\theta}} \right) / \left( \frac{a_\theta^2}{2r} + 1.76 \sqrt{\frac{\nu r a_\theta^2}{Q_\theta}} \right), \text{ for } r < r_c$$
(7.21a)

$$\dot{M}'' = 0.00221 \frac{\rho Q_{\theta}^{4}}{\upsilon^{2} \left(r^{3} + 0.183 \frac{Q_{\theta} a_{\theta}^{2}}{\upsilon}\right)^{2}}, \text{ for } r > r_{c}$$
(7.21b)

With viscous jet flows, depending on the radial position r (whether  $r < r_c$  or  $r > r_c$ ), either Eq. (7.21a) or (7.21b) is employed to determine  $\dot{M}''$ . The azimuthal distributions of the flow rate  $Q_{\theta}$ , and the jet radius  $a_{\theta}$ , are evaluated from Eqs. (7.8) and (7.9), respectively. Consequently, a solution for the stagnation line location, f(x, y) = 0, can be obtained by substituting Eq. (7.21) along with the geometric relations of Eqs. (7.15), (7.16), and (7.18) into the momentum flux balance of Eq. (7.20).

Derivation of the presented model results in an ordinary differential equation that needs to be solved to find a relation f(x, y) = 0 as the stagnation line location. The streamlines that are laid on the line connecting the two stagnation points are the only pair of the colliding streamlines with the same flow properties (i.e. they leave their parent stagnation points with the same angle  $\theta_1 = \theta_2 = \psi_{rel.}$ ). As a result, they collide exactly midway between the two stagnation points. This point is picked as the origin for the theoretical analysis in Figure 7.6a. Hence, the origin (0, 0) is chosen as the boundary condition. A numerical solution is developed in MATLAB to solve the

obtained differential equation. The solution starts from the boundary condition set at the origin and proceeds from this point with  $y' = \frac{y_2 - y_1}{x_2 - x_1}$ .

# 7.5 Experimental Results and Discussion

### 7.5.1 Single inclined jet

As mentioned in the experimental methodology, to verify the correctness of the measured parameters, experiments are carried out using a single inclined jet. It was explained that the impingement region of an inclined, circular, impinging jet can be modelled as the impingement region of an equivalent vertical elliptic jet, with the jet radius of  $a_{\theta}$  and the jet flow rate of  $Q_{\theta}$ (both functions of azimuthal position  $\theta$ ). Kate & Chakraborty in 2007 [58] employed the scaling technique introduced by Bohr et al. [9] to predict the jump radius with respect to the azimuthal position,  $R_i(\theta)$ , as follows

$$R_{j}(\phi,\theta) = C \left[ \frac{a^{2}}{2} \frac{\sin^{3}\phi}{(1-\cos\phi\cos\theta)^{2}} U_{j} \right]^{5/8} v^{-3/8} g^{-1/8}$$
(7.22)

where v is the fluid kinematic viscosity, and the constant *C* depends on the chosen velocity profile for the thin wall jet. For a parabolic profile, for instance, *C* turns to be approximately 0.73. Values of *C* for higher-order velocity profiles are available in Ref. [58].

In Figure 7.7, three jet inclination angles with varied flow rates are tested using three different working fluids (water, water-glycerol solution wt. 50%, and water-glycerol solution wt. 65%). To examine the results, the experimental measurements of the hydraulic jump profiles are

 $Q=290 \text{ cm}^3/\text{min}$  $Q=200 \text{ cm}^3/\text{min}$  $Q=300 \text{ cm}^3/\text{min}$ (a) (b) (c) 10  $Q=350 \text{ cm}^3/\text{min}$  $Q=250 \text{ cm}^3/\text{min}$ 0 0  $Q=370 \text{ cm}^3/\text{min}$  $Q=300 \text{ cm}^3/\text{min}$  $Q=410 \text{ cm}^3/\text{min}$  $Q=410 \text{ cm}^3/\text{min}$ X(mm) X(mm) X(mm 28 -10 -15  $Q=300 \text{ cm}^3/\text{min}$  $Q=290 \text{ cm}^3/\text{min}$ (d)  $Q=300 \text{ cm}^3/\text{min}$ (e) (f)  $Q=370 \text{ cm}^3/\text{min}$ 0  $Q=350 \text{ cm}^3/\text{min}$ 0  $Q=370 \ cm^3/min$ 0  $Q=410 \text{ cm}^3/\text{min}$  $Q=410 \text{ cm}^3/\text{min}$  $Q=410 \text{ cm}^3/\text{min}$ X(mm) X(mm)

compared to the results of Eq. (7.22). The theoretical results are shown with the solid lines. A reasonable agreement is observed between the experimental results and the theory.

**Figure 7.7.** Comparisons between the theory (Eq. 7.22) and the measured hydraulic jump profiles formed by single inclined impinging jets. Solid lines represent the theory. (a)  $\phi=50^{\circ}$ , water, (b)  $\phi=50^{\circ}$ , water-glycerol wt. 50%, (c)  $\phi=50^{\circ}$ , water-glycerol wt. 65%, (d)  $\phi=70^{\circ}$ , water-glycerol wt. 50%, (e)  $\phi=70^{\circ}$ , water-glycerol wt. 65%, (f)  $\phi=40^{\circ}$ , water-glycerol wt. 50%.

As mentioned in the literature review, with vertical, free-surface jets, the spacing between the nozzle exit and the target plate ( $H_{jet}$  - see Figure 7.3a) has been reported to have negligible influence on the flow field formed on the surface [100]. However, with inclined jets, one would expect that the gravitational effects may deflect the jet (i.e. change the angle  $\phi$ ), and consequently influence the flow field formed on the surface. Employing a kinematics analysis, the following relation determines the jet deflection as a function of the jet velocity ( $U_j$ ), jet height ( $H_{jet}$ ), and the nozzle inclination angle with the horizontal plane ( $\phi$ ):

$$\phi_{s} = \tan^{-1} \left( \frac{\sqrt{U_{j}^{2} \sin^{2} \phi + 2gH_{jet}}}{U_{j} \cos \phi} \right)$$
(7.23)

Here,  $\phi_s$  is the new jet angle at the impact point on the surface (after deflection). From Eq. (7.23), it is inferred that the deflection increases as the drop height ( $H_{jet}$ ) increases and/or the jet velocity ( $U_j$ ) decreases. With the ranges of the flow rates and jet heights used in the present work, the deflection from the original nozzle angle (i.e.  $\frac{\phi_s - \phi}{\phi} \times 100$ ) has been found to be less than 1% and assumed to be negligible. Thus, the original nozzle angle  $\phi$  is used for calculations. In applications with high jet drops and/or low flow rates, the jet deflection must be taken into consideration.

# 7.5.2 Double inclined jet impingement

#### 7.5.2.1 General

The flow field formed by the interaction between two inclined jets impinging on a surface generally depends on the jet inclination angles ( $\phi$ ), jet flow rates (Q), relative position of the jets, and the fluid properties. As seen in Figures 7.1 and 7.2, the generated stagnation line is not a straight line anymore, instead, exhibits a curvature. Also, the rising sheet formed at the stagnation zone is no longer a vertical sheet. The rising sheet in this case is tilted towards the closer stagnation point.

Figures 7.8a, b, and c show the rising sheet formed due to the interaction of two bi-planar inclined jets with the jet inclination angles of  $\phi$ =70, 50, and 30 degrees, respectively.

Additionally, the images taken from underneath the impingement plate are presented, which show the stagnation line and the jump profiles formed on the surface. As mentioned in the experimental methodology, to capture the underneath images, a light source was used right above the impingement plate. The gray shade observed around the stagnation lines in the underneath images of Figure 7.8 are the shadow of the tilted rising sheet on the surface. It is clearly perceived that the rising sheet is tilted in two opposite directions.



**Figure 7.8.** Interaction of two inclined jets on a horizontal plate. Top row images: the formed rising sheet. Bottom row images: underneath images of the flow field formed on the surface. (a)  $\phi = 70^{\circ}$ , (b)  $\phi = 50^{\circ}$ , (c)  $\phi = 30^{\circ}$ .

Any point along the stagnation line is associated with the collision of two streamlines originating from the parent stagnation points (see Figure 7.6a). These streamlines leave their parent stagnation points with different angles ( $\theta_1$  and  $\theta_2$  in Figure 7.6a). As a result, due to the angular distribution of the flow momentum (Eq. 7.13), at any point along the stagnation line (except the center point – origin), the colliding streamlines carry unequal momentum. This results in a momentum imbalance, as discussed in the theoretical analysis. The remaining momentum causes the inclination of the rising sheet. From Eq. (7.13), and as seen in Figure 7.5, the streamline that leaves its parent stagnation point with a smaller angle obtains a larger flow momentum. As a result, the rising sheet requires to be tilted towards the other wall jet, i.e. closer stagnation point.

At the origin (i.e. intersection of the stagnation line and the line connecting the two stagnation points), since the colliding streamlines make equal angles (i.e.  $\theta_1 = \theta_2 = \psi_{rel.}$ ), the formed liquid sheet at the origin rises normal to the plate. Further away from the origin along the stagnation line, the rising sheet inclination angle increases. Also, it is observed that as the jet inclination angle ( $\phi$ ) decreases, the tilt of the rising sheet increases. Other parameters such as the relative position of the jets and the flow rate also make significant contributions to the tilt observed in the rising sheet.

The discussion above reveals the importance of the angular distribution of the flow momentum. It can be inferred that a change in the angular momentum distribution of the individual wall jets could influence the location of the stagnation line. Here, the influences of different parameters on the angular distribution of the flow momentum produced by a single, inclined impinging jet are investigated. Parameters such as the jet inclination angle ( $\phi$ ), the flow rate (Q), and the fluid kinematic viscosity (v) are examined and illustrated in Figure 7.9. To have a better comparison, for each case, the calculated momentum distributions (at a constant radial position) are normalized by their maximum values (at  $\theta$ =0).

First, the nozzle inclination angle  $\phi$  is examined. Figure 7.9a shows the normalized momentum distributions for three different nozzle inclination angles,  $\phi$ =70, 50, and 30 degrees. Other

parameters such as the flow rate and fluid properties are considered to be fixed. It is seen that the change in the nozzle inclination angle impacts the momentum distribution, i.e. as  $\phi$  decreases, a narrower area in front of the jet obtains high momentum, and the momentum distribution becomes sharper at small angular positions ( $\theta$ ). On the other hand, at larger nozzle angles, the momentum distribution is flatter over a larger a range of the angular positions.

Figure 7.9b demonstrates the momentum distribution for varied flow rates. The nozzle inclination angle and the fluid properties are kept fixed. Compared to the case of the change in the nozzle inclination angle, it is observed that the momentum distribution does not change significantly with changing the flow rate from Q=200 cm<sup>3</sup>/min to Q=500 cm<sup>3</sup>/min (the range of the flow rate used in the present work).



Figure 7.9. The normalized momentum distribution with the azimuth angle in the thin wall jet formed by an inclined impinging jet. Values are calculated using Eq. (7.13b) at a constant radial spacing of 5mm from the stagnation point. (a) varied nozzle inclination angles - water jets with Q=350 cm<sup>3</sup>/min, (b) varied jet flow rates – water jets with ,  $\phi=50^{\circ}$ , (c) varied fluid kinematic viscosities - Q= 350 cm<sup>3</sup>/min,  $\phi=50^{\circ}$ .

The normalized momentum is also plotted versus the angular position  $\theta$  for varied fluid kinematic viscosities in Figure 7.9c. The jet flow rate and the jet inclination angle are kept fixed while changing the viscosity. It is observed that as the fluid viscosity increases, the high

momentum area is concentrated in a narrower area in front of the jet, i.e. the momentum distribution becomes sharper at small angular positions. This trend of change in the momentum distribution is similar to that with different nozzle angles, i.e. increasing the viscosity alters the momentum distribution in the same way as decreasing the nozzle inclination angle does. The foregoing discussion will later be employed to analyze the location of the formed stagnation line.

## 7.5.2.2 Stagnation line

Figure 7.10 shows images of the flow field formed on the plate due to the interaction between two inclined impinging jets. The images represent tests with three different jet inclination angles ( $\phi$ =30, 50, and 70 degrees), three different working liquids (water, and two water-glycerol solutions wt. 50% and 65%), and varied flow rates and relative jet positions. The detailed test conditions are explained in the caption of the figure. The stagnation line associated with each image is measured and plotted in Figure 7.10g. For each test, the midpoint of the line connecting the two stagnation points is picked as the origin, as illustrated on the inset of Figure 7.10g.

It is seen that, all the measured stagnation lines are symmetric with respect to the origin. In other words, the intersection of the stagnation line and the line connecting the two stagnation points is the point of symmetry. A close look at all the experimental data and images carried out in this study reveals that the origin is not only the symmetry point for the stagnation line, but is also the point of symmetry for the entire flow field formed on the surface. Hence, throughout this dissertation, only the positive values on x-axis will be shown for the measured stagnation lines.



Figure 7.10. Flow field formed on the target surface due to the interaction between two inclined impinging jets.
Photographs are from underneath the impingement plate. (a) φ=70°, water, Q=300 cm<sup>3</sup>/min, S<sub>X</sub>=13.3 mm, S<sub>Y</sub>=4.2 mm, (b) φ=70°, water-glycerol wt. 65%, Q=410 cm<sup>3</sup>/min, S<sub>X</sub>=10.7 mm, S<sub>Y</sub>=4.7 mm, (c) φ=50°, water, Q=300 cm<sup>3</sup>/min, S<sub>X</sub>=12.8 mm, S<sub>Y</sub>=3.5 mm, (d) φ=50°, water- glycerol wt. 50%, Q=370 cm<sup>3</sup>/min, S<sub>X</sub>=10.7 mm, S<sub>Y</sub>=4.7 mm, (e) φ=30°, water, Q=350 cm<sup>3</sup>/min, S<sub>X</sub>=21.1 mm, S<sub>Y</sub>=7.3 mm, (f) φ=30°, water-glycerol wt. 65%, Q=350 cm<sup>3</sup>/min, S<sub>X</sub>=8.2 mm, S<sub>Y</sub>=5 mm, (g) measured stagnation lines; inset demonstrates the coordinates used for the measurements.

#### 7.5.2.3 Influence of jet inclination angle

Next, the influence of the jet inclination angle ( $\phi$ ) on the position of the stagnation line is studied. For this purpose, all the test conditions were kept fixed and different nozzle inclination angles were examined. Figure 7.11 plots the stagnation lines measured with water as the working fluid and a constant flow rate of Q=350 cm<sup>3</sup>/min. Experiments were carried out with three different nozzle inclination angles of  $\phi$ =70, 50, and 30 degrees, while the relative position of the stagnation points were kept fixed. Figures 7.11a and b represent experimental results of two different relative positions. It is observed that with fixed test conditions, as the nozzle inclination angle decreases, the formed stagnation line makes a smaller angle with the horizontal axis. In other words, the stagnation line rotates counter-clockwise around the origin and becomes closer to the horizontal axis. The change observed in the location of the stagnation line with the jet inclination angle ( $\phi$ ) is attributed to the influence of  $\phi$  on the angular distribution of the flow momentum in the thin wall jets (see Figure 7.9b).



Figure 7.11. Stagnation lines formed by water jets with the flow rate of Q=350 cm<sup>3</sup>/min, fixed jet relative positions and varied nozzle inclination angles. (a)  $S_X=17.5$  mm,  $S_Y=9$  mm, (b)  $S_X=13.5$  mm,  $S_Y=7$  mm. The dashed lines represent the theoretical results.

Along with the experimental data, the theoretical results using the presented model (Eq. 7.20) are plotted as dashed lines in Figure 7.11a. It is seen that the theory predicts the stagnation line corresponding to  $\phi=70^{\circ}$  with a relatively good accuracy. As the nozzle inclination angle

decreases, the discrepancy between the theoretical predictions and the experimental results increases. Although the theory does not estimate the accurate location of the stagnation line associated with small inclination angles, it predicts the trend of change observed in the location of the stagnation line with the nozzle inclination angle.



**Figure 7.12.** (a) Water,  $\phi = 50^{\circ}$ ,  $S_X = 9 \text{ mm}$ ,  $S_Y = 3.5 \text{ mm}$ , (b) water-glycerol wt.65%,  $\phi = 70^{\circ}$ ,  $S_X = 11.4 \text{ mm}$ ,  $S_Y = 6.4 \text{ mm}$ , (c) water-glycerol wt.50%,  $\phi = 50^{\circ}$ ,  $S_X = 18.2 \text{ mm}$ ,  $S_Y = 4.7 \text{ mm}$ , (d) water,  $\phi = 30^{\circ}$ ,  $S_X = 21.2 \text{ mm}$ ,  $S_Y = 7.3 \text{ mm}$ .

#### 7.5.2.4 Influence of flow rate

Here, the influence of the flow rate on the location of the stagnation line is investigated. Varied test conditions (specified by the nozzle inclination angle, jet relative positions, and working

fluid) have been chosen and presented in Figure 7.12. For each test condition, different flow rates were examined and the positions of the stagnation lines were measured and compared.

In Figure 7.12a, images of the flow field formed on the surface with varied flow rates are shown. The images are associated with water jets with the nozzle inclination angle of  $\phi$ =50° and a fixed relative position of S<sub>X</sub>=9 mm, S<sub>Y</sub>=3.5 mm. The flow rate increases from Q=200 cm<sup>3</sup>/min in Figure 7.12a1 to Q=400 cm<sup>3</sup>/min in Figure 7.12a5. The measured locations of these stagnation lines are depicted in Figure 7.12a6. It is observed that all the stagnation lines fall on each other. As the flow rate increases, the stagnation lines extend longer, which is due to the formation of larger hydraulic jumps on the surface.

Figures 7.12b, c, and d each represents stagnation lines related to a different jet inclination angle ( $\phi$ =70, 50, 30 degrees), different working fluids (water, and two water-glycerol solutions wt. 50% and 65%), and different relative positions. The detailed test conditions are described in the caption. In all the tests presented here, the stagnation lines exhibit an approximate independence from the flow rate. Although, some discrepancy is observed at the distant points from the origin, the data points associated with the positions where the two thin wall jets meet and collide fall on each other. The stagnation line extends further as the flow rate increases. The discrepancy observed in the data lines at distant points from the origin is associated with the area outside of the collision zone of the two thin wall jets (i.e. the thin wall jet meets the thick liquid film on the surface downstream of the neighbor hydraulic jump). Technically speaking, those data points are not part of the stagnation line since the stagnation line is defined as the line formed by the collision of two thin wall jets. Therefore, it is concluded that the stagnation line formed by the

interaction of two inclined jets impinging on a surface is independent of the flow rate. As seen in Figure 7.9b, the normalized angular distribution of the flow momentum is relatively independent of the jet flow rate. This is responsible for the independence of the stagnation line location from the flow rate observed in Figure 7.12.

In Figure 7.12c, the theoretical predictions determined using Eq. (7.20) are plotted along with the experimental measurements. The working fluid is water-glycerol solution wt. 50% with varied flow rates. The nozzle inclination angle is  $\phi$ =50° for this set of experiments. Although a discrepancy is observed between the experimental and theoretical results, the theory indeed predicts the independence of the stagnation line from the flow rate as well.

## 7.5.2.5 Influence of fluid viscosity

To have a better understanding of the phenomenon, three different working fluids have been examined, i.e. water, and two different concentrations of water-glycerol solutions (i.e. wt. 50% and 65%). The major difference between the properties of these three liquids is the viscosity (they have approximately similar surface tension and density). Figures 7.13a, b, and c illustrate the experimental results obtained by three different nozzle inclination angles  $\phi$ =70, 50, and 30 degrees, respectively. In order to study the influence of the fluid viscosity on the location of the stagnation line, for each nozzle inclination angle, the relative position of the jets was kept fixed, while different liquids were pumped through the test setup. Although it has been concluded in a previous discussion that the stagnation line is independent of the flow rate (see Figure 7.12), relatively similar flow rates were chosen using different fluids.



Figure 7.13. Stagnation lines formed by different working liquids, (a)  $\phi = 70^{\circ}$ ,  $S_X = 12$  mm, and  $S_Y = 6$  mm, (b)  $\phi = 50^{\circ}$ ,  $S_X = 11.5$  mm, and  $S_Y = 4.5$  mm, (b)  $\phi = 30^{\circ}$ ,  $S_X = 12.5$  mm, and  $S_Y = 4.2$  mm. Flow rates of Q=350, 3870, 380 cm3/min are used for water, water-glycerol wt. 50%, and water-glycerol wt. 65%, respectively.

Two trends are observed in Figures 7.13a, b, and c. First, as the fluid viscosity increases the stagnation line tends to make a smaller angle with the horizontal axis, i.e. rotates counterclockwise around the origin. This is attributed to the change in the angular momentum distribution due to a change in the fluids viscosity. As discussed before, increasing the kinematic viscosity influences the angular momentum distribution in the same way as decreasing the nozzle inclination angle ( $\phi$ ) does (see Figures 7.9Figure 7.9a and c). Consequently, similarly to decreasing the nozzle inclination angle (see Figure 7.11), as the viscosity increases, the stagnation line rotates counter-clockwise around the origin. Therefore, it can be generally concluded that if a parameter changes the angular distribution of the flow momentum in the wall jet formed by a single inclined jet, it will influence the stagnation line formed by the interaction of two bi-planar inclined jets.

The other trend observed in Figure 7.13 is that as the viscosity increases the stagnation line does not extend as far (even though the experiments with more viscous fluids are conducted with slightly higher flow rates). This could be attributed to the negative power of kinematic viscosity

in Eq. (7.22). This indicates that an increase in the fluid viscosity leads to a decrease in the jump radii. Consequently, shorter stagnation lines are formed with the interaction of more viscous liquids.

The theoretical predictions (dashed lines) are also plotted along with the experimental results in Figure 7.13a. Although a small discrepancy is observed between the predictions and the experimental measurements, the theory successfully determines the influence of the fluid viscosity on the stagnation line location.

# 7.5.2.6 Influence of jet-jet relative position

Generally, two groups of parameters are responsible for the flow characteristics formed on the surface and eventually the location of the stagnation line. First, the parameters that influence the angular distribution of the flow momentum in the thin wall jets (as discussed above). The other group is associated with the geometric placement of the two jets, in other words, the relative position of the jets. Jets relative position determines where and how the wall jets interact. In this section, the influence of the relative position of the jets on the location of the stagnation line is explored. First, the experimental observations are presented. In order to avoid messy looks of the figures, the theoretical predictions are presented separately in Figure 7.18.

Since the jets are on two parallel planes, the relative position of the jets can be determined by  $S_X$  and  $S_Y$  defined as the spacing between the individual stagnation points in x and y directions, respectively (as depicted in Figure 7.1b). First, experimental tests are conducted with different working fluids and nozzle inclination angles for which  $S_Y$  is kept fixed while  $S_X$  is changed. The experimental results of the shape and location of the stagnation lines are presented in Figure 152

7.14. It is observed that in all the tests with varied test conditions, as the spacing between the jets in x-direction (S<sub>X</sub>) decreases, the stagnation line rotates counter-clockwise around the point of symmetry (origin). Thus, the stagnation line becomes closer to the horizontal axis. As mentioned before, two uniplanar jets generate a stagnation line perpendicular to the line connecting the two stagnation points (i.e. the stagnation line falls on the vertical axis). As the ratio  $S_X/S_Y$  increases, the stagnation lines approach the case of uniplanar jets.



**Figure 7.14.** Stagnation lines formed with a fixed  $S_Y$  and varied  $S_X$ , (a) water-glycerol wt.65%,  $\phi=70^\circ$ , Q=410 cm<sup>3</sup>/min, (b) water-glycerol wt.50%,  $\phi=50^\circ$ , Q=370 cm<sup>3</sup>/min, (b) water,  $\phi=30^\circ$ , Q=350 cm<sup>3</sup>/min.

Experiments are also carried out where  $S_X$  is kept fixed and  $S_Y$  is changed. The results of these tests (plotted in Figure 7.15) reveal the significance of the spacing between the two parallel planes where the individual jets are placed on. Figure 7.15 shows that as  $S_Y$  increases, the angle between the stagnation line and the horizontal axis reduces (i.e. stagnation line rotates counter-clockwise around the origin). In other words, a decrease in  $S_Y$  increases the ratio  $S_X/S_Y$ , which causes the stagnation line to approach the condition of the uniplanar jets (i.e. straight vertical stagnation line). On the other hand, an increase in  $S_Y$  reduces the ratio  $S_X/S_Y$ , and the stagnation line to become closer to the horizontal axis.



Figure 7.15. Stagnation lines formed by a fixed  $S_X$  and varied  $S_Y$ , (a) water-glycerol wt.50%,  $\phi=70^\circ$ , Q=370 cm<sup>3</sup>/min, (b) water-glycerol wt.50%,  $\phi=50^\circ$ , Q=370 cm<sup>3</sup>/min, (c) water,  $\phi=30^\circ$ , Q=350 cm<sup>3</sup>/min.

# **7.5.2.6.1** Spacing defined by $S_{rel.}$ and $\psi_{rel.}$

The relative position of the impinging jets can also be expressed by the relative angle ( $\psi_{rel.}$ ) and

the relative spacing  $(S_{rel.})$  between the two stagnation points defined as  $\psi_{rel.} = \tan^{-1}\left(\frac{S_{\gamma}}{S_{X}}\right)$  and

 $S_{rel.} = \sqrt{S_x^2 + S_y^2}$  (see Figure 7.1b). Here, the effects of  $\psi_{rel.}$  and  $S_{rel.}$  on the stagnation lines formed by two inclined jets are separately studied. Figure 7.16 shows the experimental data of the stagnation lines obtained by a fixed relative angle ( $\psi_{rel.}$ ), while changing the relative spacing ( $S_{rel.}$ ). The working fluid is water and the flow rate is 350 cm<sup>3</sup>/min in all the tests. Figures 7.16a, b, and c represent the experimental data collected by the nozzle inclination angles of 70, 50, and 30 degrees, respectively.



**Figure 7.16.** Measured stagnation lines formed by fixed  $\psi_{rel.}$  and varied  $S_{rel.}$ . The working fluid is water with flow rate of Q=350 cm<sup>3</sup>/min (a)  $\phi$ =70°, (b)  $\phi$ =50°, (c)  $\phi$ =30°.

In order to carry out the experiments with a constant  $\psi_{rel.}$ , the position of one jet is kept fixed, while the other jet is moved along a straight line radially spreading out from the stagnation point of the fixed jet. The angle that the radial line makes with the horizontal axis is then the fixed relative angle ( $\psi_{rel.}$ ). For each nozzle inclination angle ( $\phi$ ), two different relative angles ( $\psi_{rel.}$ ) are tested. The top row graphs in Figure 7.16 (denoted by "i") represent a small relative angle, and the bottom row graphs (denoted by "ii") demonstrate a larger relative angle.

Figure 7.16a-i, which represents the  $\phi=70^{\circ}$  and  $\psi_{rel.}=9^{\circ}$ , shows that all the stagnation lines fall on each other. It means that the location of the stagnation line does not change as the relative spacing changes. However, with the same nozzle inclination angle but a larger relative angle of  $\psi_{rel.} = 28^{\circ}$  (see Figure 7.16a-ii), the stagnation lines slightly deviate. It is seen that as the relative spacing increases, the stagnation line tends to make a smaller angle with the horizontal axis. This change in the location of the stagnation line with the relative spacing is more pronounced as the nozzle inclination angle decreases. It is seen in Figure 7.16c-i that even with a small relative angle  $\psi_{rel.}=10^{\circ}$ , the stagnation line location moves towards the horizontal axis as the relative spacing increases. The change observed in the location of the stagnation line with  $S_{rel.}$  is clearly substantial with larger relative angles  $\psi_{rel}$  as seen in Figure 7.16c-ii.

It can be concluded from Figure 7.16 that the dependency of the stagnation line location on the relative spacing between the jets increases as the nozzle inclination angle ( $\phi$ ) decreases. At the same time, the stagnation line is more sensitive to  $S_{rel}$ , when the relative angle  $\psi_{rel}$  is larger.

Next step, experiments are conducted in which the relative spacing ( $S_{rel.}$ ) was kept fixed while changing the relative angle between the jets  $\psi_{rel.}$ . To implement this objective, one jet is kept fixed, while the other jet is moved on the circumference of a circle whose center is set on the stagnation point of the fixed jet. As a result, a constant spacing equal to the radius of the circle is achieved with varied relative angles. Figures 7.17a, b, and c illustrate the experimental results obtained with three different nozzle inclination angles,  $\phi$ =70, 50, and 30 degrees, respectively. The working fluid is water with a constant flow rate of Q=350 cm<sup>3</sup>/min.



**Figure 7.17.** Measured stagnation lines formed by a fixed  $S_{rel.}=13$  mm and varied  $\psi_{rel.}$ . The working fluid is water with the flow rate of Q=350 cm<sup>3</sup>/min (a)  $\phi$ =70°, (b)  $\phi$ =50°, (c)  $\phi$ =30°.

It is seen in all three graphs of Figure 7.17 that as the relative angle increases, with a fixed  $S_{rel}$ , the stagnation line rotates counter-clockwise around the origin and becomes closer to the horizontal axis. An increase in  $\psi_{rel}$  with a constant  $S_{rel}$  is equivalent to a decrease in  $S_X$  while increasing  $S_Y$ . As observed in Figures 7.14 and 7.15, both cause the stagnation line to move towards the horizontal axis. It is worth-noting that the relative angle of  $\psi_{rel} = 0$  represents the uniplanar impinging jets and makes a straight stagnation line, which falls on the vertical axis. Comparing Figures 7.17a, b, and c, it is seen that as the nozzle inclination angle decreases, with approximately similar  $\psi_{rel}$ , the formed stagnation line makes a larger angle with the vertical axis. Moreover, the relative spacing used in Figure 7.13 is  $S_{rel}=13$  mm. As concluded from Figure 7.16 and also observed in the experiments, setting a larger relative spacing causes the stagnation lines to make a larger angle with the vertical axis (with the same relative angles used in Figure 7.17).

As mentioned before, the theoretical results regarding the influence of the jets relative position are plotted in Figure 7.18. Since comparisons of the theory and the experimental measurements

for varied test conditions have been made several times throughout this chapter, in order to prevent a messy look of the figures, the experimental and theoretical results are presented separately. Figures 7.18a, b, c, d, and e are associated with the test conditions employed in Figures 7.14c, 7.15b, 7.17a, 7.16a-i, and 7.16b-ii, respectively. It is seen that the presented theory successfully predicts the influence of the relative position of the jets on the location of the stagnation line.



**Figure 7.18.** Theoretical results of the locations of the stagnation lines with varied jets relative positions, (a) water,  $\phi=30^{\circ}$ , Q=350 cm<sup>3</sup>/min, (b) water-glycerol wt.50%,  $\phi=50^{\circ}$ , Q=370 cm<sup>3</sup>/min, (c) water,  $\phi=70^{\circ}$ , Q=350 cm<sup>3</sup>/min, and  $S_{rel}=13$  mm, (d) water,  $\phi=70^{\circ}$ , Q=350 cm<sup>3</sup>/min, (e) water,  $\phi=50^{\circ}$ , Q=350 cm<sup>3</sup>/min.

# 7.6 Unequal Double Jet Impingement

The general description of the interaction between two unequal, vertical jets impinging on a horizontal surface has been presented in Chapter 6. It was explained that the formed stagnation
line is shifted closer to the weaker jet and exhibits a curvature. Additionally, the generated rising sheet is curved and inclined towards the weaker jet. Due to the unequal strength of the parent jets, the stream lines colliding at any point along the stagnation line exhibit different properties (e.g. flow momentum, film thickness, velocity, etc.). Therefore, the momentum flux balance can be applied to the colliding wall jets in order to predict the location and shape of the stagnation line. Here, the geometric relations associated with the interaction of two unequal impinging jets are presented.

Taking the stagnation point of the weaker jet as the origin, the impingement points of the individual jets are then located at (0,0) and (S,0). A rising sheet is formed, and the location of the stagnation line on the horizontal plate is f(x, y) = 0. The geometric parameters shown in Figure 7.19 can be expressed in terms of x, y, and S using the following relations. In all the equations presented here, the subscripts "1" and "2" are used to refer to the parameters associated with the weaker jet (jet #1) and the stronger jet (jet #2), respectively.



Figure 7.19. Schematic of two arbitrary streamlines colliding at the stagnation line, along with the geometric parameters

$$r_1 = \sqrt{x^2 + y^2}$$
(7.24a)

$$r_2 = \sqrt{(S-x)^2 + y^2}$$
(7.24b)

$$\theta_1 = \tan^{-1} \left( \frac{y}{x} \right) \tag{7.25a}$$

$$\theta_2 = \tan^{-1} \left( \frac{y}{S - x} \right) \tag{7.25b}$$

The angle that the streamlines make with the local normal to the stagnation line is denoted by  $\gamma$  and can be expressed as

$$\gamma_1 = \theta_1 + \frac{\pi}{2} - \tan^{-1} y' \tag{7.26a}$$

$$\gamma_2 = \theta_2 - \frac{\pi}{2} + \tan^{-1} y' \tag{7.26b}$$

where y' = dy/dx. A solution for the stagnation line location, f(x, y) = 0, can be obtained by substituting Eq. (7.21) along with the geometric relations of Eqs. (7.24), (7.25), and (7.26) into the momentum flux balance of Eq. (7.20). In Eq. (7.21), for vertical jets, the azimuthal distributions of the flow rate  $Q_{\theta}$ , and the jet radius  $a_{\theta}$  are simply substituted with the jet flow rate Q and radius a, respectively.

As explained for the case of two inclined jets, derivation of the presented model results in a differential equation that needs to be solved to find a relation f(x, y) = 0 as the stagnation line location. The intersection of the stagnation line and the line connecting the two stagnation points (see  $(X_1, 0)$  in Figure 7.19) is picked as the boundary condition to solve the differential equation,

and the solution proceeds from this point with  $y' = \frac{y_2 - y_1}{x_2 - x_1}$ . Since the line connecting the two stagnation points is the symmetry line for the formed flow field, the streamlines that fall on this line are locally normal to the stagnation line (i.e.  $\gamma_1 = \gamma_2 = 0$ ). Thus,  $X_1$  can be determined in a general form using the following relation

$$\frac{\left[X_{1}^{3}+0.183\frac{Q_{1}a_{1}^{2}}{\upsilon_{1}}\right]}{\left[\left(S-X_{1}\right)^{3}+0.183\frac{Q_{2}a_{2}^{2}}{\upsilon_{2}}\right]} = \sqrt{\frac{\rho_{1}}{\rho_{2}}}\frac{Q_{1}^{2}\upsilon_{2}}{Q_{2}^{2}\upsilon_{1}}$$
(7.27)

Applying the present experimental conditions (i.e. jets with the same fluids, and nozzle sizes), the Eq. (7.27) is reduced to

$$\frac{\upsilon X_1^3 + 0.183Q_1 a^2}{\upsilon \left(S - X_1\right)^3 + 0.183Q_2 a^2} = \left(\frac{Q_1}{Q_2}\right)^2$$
(7.28)

A set of experiments with two unequal, vertical jets are carried out and the results of the stagnation line locations are compared to the present theoretical predictions. The two jets employed for the present set of experiments are similar in every aspects expect the flow rate, i.e. they have the same nozzle size (d = 1 mm) and working fluid (water), with different flow rates. Few images of the experiments have been shown in Figure 6.15. Figure 7.20a illustrates the experimental results along with the theoretical predictions for a jet-to-jet spacing of S = 14.5 mm, where the larger flow rate remains fixed at Q<sub>2</sub>=500 cm<sup>3</sup>/min and the smaller flow rate changes from Q<sub>1</sub>=200 to 300 cm<sup>3</sup>/min. Similarly, Figure 7.20b represents the data with S = 11 mm and a fixed smaller flow rate of Q<sub>1</sub>=200 cm<sup>3</sup>/min and varied larger flow rate from Q<sub>2</sub> = 300

to  $500 \text{ cm}^3/\text{min}$ . Since the line connecting the two stagnation points is the symmetry line for the entire flow field, only the positive values on the *y*-axis is shown. It is seen that the presented theory predicts the locations of the formed stagnation lines with a fairly good agreement.



Figure 7.20. Comparison between the theoretical predictions and the experimental measurements of the stagnation lines formed due to the interaction between two unequal jets. (a) S=14.5 mm , test #1:  $Q_1=200$  and  $Q_2=500$  cm<sup>3</sup>/min, test #2:  $Q_1=250$  and  $Q_2=500$  cm<sup>3</sup>/min, test #3:  $Q_1=300$  and  $Q_2=500$  cm<sup>3</sup>/min, (a) S=11 mm , test #1:  $Q_1=200$  and  $Q_2=300$  cm<sup>3</sup>/min, test #2:  $Q_1=200$  and  $Q_2=400$  cm<sup>3</sup>/min, test #3:  $Q_1=200$  and  $Q_2=500$  cm<sup>3</sup>/min.

### 7.7 Summary

In the present chapter, the flow field formed by the interaction between two inclined jets was experimentally and analytically studied. The study focused on two inclined jets that are placed on two parallel planes, and their projected flow directions on the horizontal plate are in opposite directions. The experimental observations revealed that the formed stagnation line in this case is curved and the rising sheet is tilted in two opposite directions towards the closer jet. Also, the intersection of the stagnation line and the line connecting the parent stagnation points is the point of symmetry for the flow field formed on the target surface.

The theoretical study started with the analysis of a single inclined jet. It was discussed that the impingement region of an inclined jet can be modeled by the impingement region of a vertical jet whose radius and flow rate are both functions of the azimuthal angle expressed by Eqs. (7.8) and (7.9). The flow field formed on the surface with an inclined impinging jet is no longer axisymmetric, and the hydraulic jump is elliptic. Experiments with single inclined jets were conducted, and the measured hydraulic jump profiles were compared to the results of the theory presented by Kate & Chakraborty [58].

In a theoretical discussion, it was explained that the location of the stagnation line can be estimated by solving a "momentum flux" balance in the opposing wall jets in a direction locally normal to the stagnation line. Momentum flux is a term used to refer to the rate of the wall jet flow momentum per unit area. The theory leads to a differential equation that needs to be numerically solved.

An extensive experimental study was carried out to examine the influence of different parameters on the location of the formed stagnation line. Parameters such as the jet inclination angle, flow rate, fluid viscosity, and relative position of the jets were systematically examined. It was discussed that two groups of parameters are mostly responsible for the flow characteristics formed on the surface and eventually the location of the stagnation line. First, the parameters that influence the angular distribution of the flow momentum in the thin wall jets, including the jet inclination angle and the fluid viscosity. The other group is associated with the geometric placement of the two jets, in other words, the relative position of the jets.

It was observed that increasing the fluid viscosity impacts the angular distribution of the flow momentum in the same way as decreasing the nozzle inclination angle does. As a result, they both influence the location of the formed stagnation line in the same way. It was also observed that the location of the stagnation line is independent of the jet flow rates. This was attributed to the independence of the angular momentum distribution in the wall jet from the jet flow rate.

The presented theory was solved and the results were compared to the experimental measurements. It was observed that the theory predicts the stagnation line location with fairly good agreement when the jet inclination angle is relatively large. However, the theoretical predictions deviate from the experimental measurements as the jet inclination angle decreases. Although discrepancies were observed between the theoretical predictions and the experimental measurements, the theory successfully determined the influences of different parameters on the location of the formed stagnation line.

## **Chapter 8** Summary, Conclusions, and Future Work

## 8.1 Summary

Over the past few decades, jet impingement has been an attractive cooling technique in a number of industries such as the metal and drying industry, and also applications such as preventing overheating, cooling turbine blades, and processing materials. Although jet impingement cooling is characterized by low pressure drops, and also uniform thermal coverage over large surfaces (through the use of jet arrays), the most attractive characteristic of jet impingement is rather its relatively very high heat transfer coefficients. The impingement cooling approach also offers a compact hardware arrangement, which also makes it an appropriate cooling technique for electronic devices.

With such a broad application, the primary goal of the present thesis was to carry out an indepth study on the flow characteristics of an impinging jet, its interaction with a solid surface, and also with a neighboring impinging jet. The circular hydraulic jump as a key feature of jet impingement was further analyzed. A systematic Froude number analysis was conducted, and the critical differences between circular jumps and open-channel jumps were highlighted. Moreover, the interaction between two free-surface jets impinging on a target surface was studied in detail. The fluid flow involved in the collision between two pre-jump liquid films was analytically and experimentally analyzed. Different jet configurations (vertical/inclined jets) were examined in the analysis.

#### 8.1.1 Hydraulic Jump with the Capillary Limit at the Edge

In Chapter 4, circular hydraulic jumps formed on target plates with free edges and the capillary limit at the edge were studied. This is a commonly seen condition especially with low flow rates or/and large plates. It was explained that in the case of plates with free edges, liquid does not always flow off uniformly along the entire perimeter of the plate. Instead, if the flow rate is low or/and the surface size is relatively large, the surface tension holds the liquid film along most of the plate edge, and the liquid only falls from individual points at the edge. In this case, the jump radius was experimentally observed to increase linearly with the flow rate. This trend is different from the scaling law presented for the uniform edge flow that states  $R_j \propto Q^{5/8}$  [9].

An analytical model was proposed to predict the location of the circular hydraulic jump with the above-mentioned capillary limit condition. Based on the force-momentum analysis of two control volumes, a system of equations was derived, which can be solved for the jump radius and the post-jump film depth. The present study was different from the previous theoretical models in which the post-jump film thickness was treated as a known parameter, or a uniform flow at the edge of the plate was assumed. Experimental tests using water and a water-surfactant solution were carried out to verify the proposed model. A good agreement was observed between the theoretical predictions and the measured values of the jump radius. Both the theory and experiments demonstrate the significance of surface tension for this type of hydraulic jumps (i.e. circular jumps). Moreover, the assumption of considering a steep jump in the proposed theory was further analyzed using the experimentally measured jump profiles. Although the gradual jump was found to individually reduce the surface tension force and increase the hydrostatic

force, the overall effect was shown to be small, and "steep jump" was a reasonable assumption in the present theory.

#### 8.1.2 Hydraulic Jump on Small Surfaces and a Froude Number Analysis

In most of the previous studies, the target plates considered for the analysis were mostly much larger than the formed hydraulic jumps. In Chapter 5, a systematic experimental study was conducted to examine the circular hydraulic jumps formed on small target plates, when the sizes of the target plates are comparable to the sizes of the hydraulic jumps formed on them. It was observed that with relatively low flow rates or/and large plates (i.e. the jump is far from the plate edge), the hydraulic jump is only a function of the flow condition and independent of the plate size.

However, on small plates or/and with high flow rates (i.e. the hydraulic jump approaches the plate edge), the target plate influences the jump, showing two interesting trends. One trend is that the jump diameter grows with reducing the plate size. The other trend is that, on small plates, the hydraulic jump becomes more sensitive to the change of the flow rate (i.e. the jump grows more sharply with the flow rate). Both trends were observed to be related to the post-jump film depth, which decreases with increasing the flow rate or/and decreasing the plate size. All these observations were attributed to the reduction of the post-jump length on the target plate defied as  $(D_s - D_j)$ . It was experimentally observed that the influence of the plate size on the hydraulic jump becomes significant when the non-dimensional post-jump length (i.e.  $(D_s - D_j)/D_s$ ) is less than ~0.5.

It was also explained that when the jump approaches the plate edge, the flow inertia becomes significant in the post-jump region. This discussion was examined by analyzing the post-jump Froude number, which demonstrated two trends with respect to the flow rate and the plate size. First, when the jump is independent form the plate size (i.e large plate/low flow rates), the Froude number maintains a constant value independent of the flow rate. However, the experimental study revealed that, contrary to what Duchesne et al. [23] reported, the constant value depends on the fluid properties and the jet diameter. Secondly, for small plates or high flow rates, the post-jump Froude number increases with increasing the flow rate or/and decreasing the plate size. The increase in the post-jump Froude number indicates that the flow inertia becomes important in the post-jump region.

Moreover, contrary to the classical hydraulic jump, it was observed that despite the post-jump Froude numbers being constant and independent of the flow rate, the pre-jump Froude number changes with the flow rate. This was explained to be a result of the surface tension force at the jump location. Through coupling the pre-jump and post-jump momentum functions, the maximum possible post-jump Froude number and the minimum possible pre-jump Froude number were theoretically derived. It was pointed out that depending on the pre-jump velocity profile ( $\lambda$ ) and the surface tension force at the jump ( $Bo^{-1}$ ), the maximum possible post-jump Froude number could be larger than unity. This indicates the possibility of obtaining a post-jump flow with supercritical Froude numbers ( $Fr_o > 1$ ). The post-jump Froude numbers were calculated for all the tests carried out for the present study, and a few tests exhibited the postjump Froude numbers higher than one, which verified the presented theory.

### 8.1.3 Double Vertical Jet Impingement

Multiple jets have shown promising potentials for providing a strong, uniform cooling in high heat flux applications. To better understand the physics and performance of an array of jets, an in-depth understanding of the fluid dynamics involved in the interaction between the neighboring jets is essential. Thus, the next objective of the present thesis was to experimentally and analytically study the interaction between the flow fields formed on a horizontal plate due to impingement of two neighboring jets.

In Chapter 6, the interaction between two identical, vertical impinging jets was studied. The jump-jump interaction was experimentally explored and characterized as three types: far-distant jumps, distant jumps, and adjacent jumps. The formed rising sheet as a result of the interaction between two adjacent jumps was further studied. The breakup mechanism of the rising sheet was investigated based on the experimental observations. The sheet patterns and breakup mechanisms were categorized into two main regimes and six subregimes. The sheet breakup was observed to occur through two principal mechanisms: aerodynamic waves and impact waves. It was explained that with two jets impinging on a solid surface, there are two sources for the impact waves; first, the waves that originate at the impact point of the free jet on the plate, and secondly, the waves that originate due to the collision of the two wall jets at the stagnation line. It was stated that due to these two sources of impact waves, the formed rising sheet is always ruffled, even at low velocities.

An analytical model was also developed to predict the size and shape of the rising sheet. This was achieved by determining the edge profile of the rising sheet through a balance between the

capillary and momentum forces at the edge of the rising sheet. The results of the presented theory were compared to the experimental measurements of the edge profiles obtained by three different working fluids: water, water-glycerol solutions wt. 50% and 65%. It was discussed that the presented theory is applicable to the rising sheets that are not experiencing severe breakup modes. Due to the propagation of the disturbances along the sheet, the local balance at the edge frequently breaks down and the liquid sheet breaks down before reaching its theoretical height. Despite these uncertainties in the height of the rising sheet due to the existing disturbances, a fairly good agreement was observed between the theory and the experimental measurements.

#### 8.1.4 Double Inclined Jet Impingement

Impinging jets are not always used with a vertical orientation relative to the target plate. Impinging jets with an inclined angle have been presented in varied applications due to either limitations in the space or the nature of the application. Contrary to a single vertical jet impingement, the flow field formed on a plate by the impingement of an inclined jet is no longer axisymmetric, and thus, the formed hydraulic jump is elliptic.

In Chapter 7, the interaction between two inclined impinging jets was experimentally and theoretically studied. The jets considered for this study were placed on two parallel planes, and the projections of their flow directions on the target plate were in opposite directions. A detailed description of the phenomenon was presented based on the experimental observations and the theoretical analysis of a single inclined jet. In the case of inclined double jet impingement, it was observed that the formed stagnation line due to the jump-jump interaction is curved, and the rising sheet is tilted in two opposite directions towards the closer jet.

In a theoretical discussion, it was explained that the location of the stagnation line can be estimated by solving a "momentum flux" balance in the opposing wall jets in a direction locally normal to stagnation line. As a result, an imbalance exists in the total flow momentum of the wall jets, which causes the inclination of the rising sheet.

Through a systematic experimental study, it was discussed that two groups of parameters are mostly responsible for the flow characteristics formed on the target surface, and consequently, the location of the stagnation line. First, the parameters that alter the angular distribution of the flow momentum in the thin wall jet of an individual jet, including the jet inclination angle and the fluid viscosity. The other group is associated with the geometric placement of the two jets, in other words, the relative position of the jets.

It was observed that increasing the fluid viscosity alters the angular distribution of flow momentum in the same way as decreasing the nozzle inclination angle does. As a result, they both influence the location of the formed stagnation line in the same way. It was also observed that the location of the stagnation line is independent of the jet flow rates. This was attributed to the independence of the angular momentum distribution in the wall jet from the jet flow rate.

The experimental measurements were also compared to the theoretical predictions. It was observed that the proposed theory accurately predicts the stagnation line location when the jet inclination angle is relatively large. However, as the jet inclination angle decreases, the theoretical predictions deviate from the experimental measurements. Although discrepancies were observed between the theoretical predictions and the experimental measurements, the theory successfully predicted the effects of different parameters on the location of the formed stagnation line. In addition, it was shown that the presented theory is also able to predict the location of the stagnation line formed by the interaction between two unequal, vertical jets.

## 8.2 Concluding Remarks

- A theory has been presented to predict the location of the circular hydraulic jump formed on finite size surfaces with capillary limit at the edge.
- On small target plates, the circular hydraulic jump is not only a function of the jet flow condition, but also a function of the target plate size.
- On large plates, in which the jump is independent of the target plate size, the post-jump Froude number remains constant with the jet flow rate. However, the constant value depends on the nozzle size and fluid viscosity and surface tension.
- Circular hydraulic jump differs from the classical theory of hydraulic jumps, due the existence of surface tension effect.
- It has been analytically and experimentally showed that the post-jump Froude number, in the case of circular hydraulic jumps, could be higher than unity.
- The rising sheet formed as a result of the interaction between two vertical jets was categorized based on its breakup mechanism and patterns.
- An analytical model has been developed to predict the rising sheet edge profile.
- A theory has been presented to predict the location of the stagnation line formed as a result of the interaction between two inclined jets, and also two unequal, vertical jets.
- Angular distribution of the flow momentum in the spreading thin film and also the relative position of the jets are responsible for the location of the stagnation line.

## 8.3 Recommendations for the Future Work

### **8.3.1** Double inclined jet impingement with varied configurations

The present work focused on the interaction between two inclined impinging jets, in which the two jets are placed on two parallel planes with opposite flow directions. Two inclined jets can exhibit different configurations. They could be on two planes that intersect with varied angles. Also, the projected jet flow directions on the target plate could be in opposite direction or the same directions. Each configuration will result in the observations of new, interesting flow fields. As a recommendation for the future work, I would suggest to examine the interaction between the flow fields formed by the impingement of two inclined jets with varied configurations. The study can be carried out experimentally and theoretically. The author believes that the proposed theory in the present work can be applied to any form of double inclined jet configuration, only with using the modified forms of the geometric relations.

## 8.3.2 Thermal Analysis

As a promising cooling technique, the thermal analysis of jet impingement is an interesting topic of research. Although there is an extensive literature available regarding the heat transfer and thermal performance of jet impingement, inclined impinging jets have received little attention. More importantly, to the best of the author's knowledge, there is not any study available on the thermal performance of double inclined impinging jets. Generally, as experimentally observed, the interaction between two inclined impinging jets results in interesting fluid dynamics phenomena. Thus, one may expect to also observe interesting thermal performances due to the impingement of two inclined jets with varied configurations.

## References

- [1] H. Cho, K. Kim, and J. Song, "Applications of impingement jet cooling systems," *Syst. Energy, Eng. Appl. first Ed. Nov. Publ. New York*, 2011.
- [2] Z. Malinowski, T. Telejko, and B. Hadala, "Implementation of the axially symmetrical and three dimensional finite element models to the determination of the heat transfer coefficient distribution on the hot plate," *Key Eng.*, 2012.
- [3] A. Terzis, P. Ott, J. von Wolfersdorf, B. Weigand, and M. Cochet, "Detailed Heat Transfer Distributions of Narrow Impingement Channels for Cast-In Turbine Airfoils," *J. Turbomach.*, vol. 136, no. 9, p. 91011, 2014.
- [4] M. E. Jr and R. Simons, "High powered chip cooling—air and beyond," *Electron. Cool.*, 2005.
- [5] S. V. Garimella and D. A. West, "Two-Phase Liquid Jet Impingement Cooling. Cooling Technologies Research Center, Purdue University."
- [6] Y. A. Çengel and J. M. Cimbala, *Fluid mechanics: fundamentals and applications*. 2006.
- [7] F. White, "Fluid Mechanics," *McGraw-Hill, New York*, p. 862, 2010.
- [8] E. J. Watson, "The radial spread of a liquid jet over a horizontal plane," *J. Fluid Mech.*, vol. 20, p. 481, 1964.
- [9] T. Bohr, P. Dimon, and V. Putkaradze, "Shallow-water approach to the circular hydraulic jump," *J. Fluid Mech.*, vol. 254, no. 1, p. 635, 1993.
- [10] T. Bohr *et al.*, "Separation and pattern formation in hydraulic jumps," *Phys. A Stat. Mech. its Appl.*, vol. 249, no. 1–4, pp. 111–117, 1998.
- [11] F. J. Higuera, "The hydraulic jump in a viscous laminar flow," *J. Fluid Mech.*, vol. 274, no. 1, p. 69, 1994.
- [12] J. W. M. Bush and J. M. Aristoff, "The influence of surface tension on the circular hydraulic jump," *J. Fluid Mech.*, vol. 489, pp. 229–238, 2003.
- [13] A. Rao and J. H. Arakeri, "Integral analysis applied to radial film flows," *Int. J. Heat Mass Transf.*, vol. 41, no. 18, pp. 2757–2767, 1998.
- [14] T. Bohr, V. Putkaradze, and S. Watanabe, "Averaging Theory for the Structure of Hydraulic Jumps and Separation in Laminar Free-Surface Flows," *Phys. Rev. Lett.*, vol. 79, no. 6, pp. 1038–1041, 1997.
- [15] F. J. Higuera, "The circular hydraulic jump," *Phys. Fluids*, vol. 9, no. 5, p. 1476, 1997.
- [16] M. Hamed and M. Akmal, "Determination of heat transfer rates in an industry-like spray quench system using multiple impinging water jets," *Int. J. Mater.*, 2005.
- [17] L. Thielen, H. Jonker, and K. Hanjalić, "Symmetry breaking of flow and heat transfer in multiple impinging jets," *Int. J. heat fluid*, 2003.

- [18] B. P. Whelan and A. J. Robinson, "Nozzle geometry effects in liquid jet array impingement," *Appl. Therm. Eng.*, vol. 29, no. 11–12, pp. 2211–2221, 2009.
- [19] L. M. Jiji and Z. Dagan, "Experimental Investigation of Single Phase Multi-jet Impingement Cooling of an Array of Microelectronic Heat Sources," *Cool. Technol. Electron. Equip.*, pp. 333–351, 1988.
- [20] E. N. Wang et al., "Micromachined jets for liquid impingement cooling of VLSI chips," J. Microelectromechanical Syst., vol. 13, no. 5, pp. 833–842, 2004.
- [21] Y. Xing, S. Spring, and B. Weigand, "Experimental and Numerical Investigation of Heat Transfer Characteristics of Inline and Staggered Arrays of Impinging Jets," *J. Heat Transfer*, vol. 132, no. 9, p. 92201, 2010.
- [22] B. Weigand and S. Spring, "Multiple Jet Impingement A Review," *Heat Transf. Res.*, vol. 42, no. 2, pp. 101–142, 2011.
- [23] A. Duchesne, L. Lebon, and L. Limat, "Constant Froude number in a circular hydraulic jump and its implication on the jump radius selection," *EPL (Europhysics Lett.*, vol. 107, no. 5, p. 54002, 2014.
- [24] A. D. D. Craik, R. C. Latham, M. J. Fawkes, and P. W. F. Gribbon, "The circular hydraulic jump," *J. Fluid Mech.*, vol. 112, no. 1, p. 347, 1981.
- [25] R. P. Kate, P. K. Das, and S. Chakraborty, "Investigation on non-circular hydraulic jumps formed due to obliquely impinging circular liquid jets," *Exp. Therm. Fluid Sci.*, vol. 32, no. 8, pp. 1429–1439, 2008.
- [26] R. P. Kate, P. K. Das, and S. Chakraborty, "Effects of Jet Obliquity on Hydraulic Jumps Formed by Impinging Circular Liquid Jets on a Moving Horizontal Plate," J. Fluids Eng., vol. 131, no. 3, p. 34502, 2009.
- [27] R. P. Kate, P. K. Das, and S. Chakraborty, "Effects of Jet Obliquity on Hydraulic Jumps Formed by Impinging Circular Liquid Jets on a Moving Horizontal Plate," J. Fluids Eng., vol. 131, no. 3, p. 34502, 2009.
- [28] R. P. Kate, P. K. Das, and S. Chakraborty, "Hydraulic jumps with corners due to obliquely inclined circular liquid jets," *Phys. Rev. E - Stat. Nonlinear, Soft Matter Phys.*, vol. 75, no. 5, pp. 1–6, 2007.
- [29] C. F. Ma, Q. Zheng, H. Sun, K. Wu, T. Gomi, and B. W. Webb, "Local characteristics of impingement heat transfer with oblique round free-surface jets of large Prandtl number liquid," *Int. J. Heat Mass Transf.*, vol. 40, no. 10, pp. 2249–2259, 1997.
- [30] J. Stevens and B. W. Webb, "The effect of inclination on local heat transfer under an axisymmetric, free liquid jet," *Int. J. Heat Mass Transf.*, vol. 34, no. 4–5, pp. 1227–1236, 1991.
- [31] A. Y. Tong, "On the impingement heat transfer of an oblique free surface plane jet," *Int. J. Heat Mass Transf.*, vol. 46, no. 11, pp. 2077–2085, 2003.
- [32] K. Ibuki, T. Umeda, H. Fujimoto, and H. Takuda, "Heat transfer characteristics of a planar

water jet impinging normally or obliquely on a flat surface at relatively low Reynolds numbers," *Exp. Therm. Fluid Sci.*, vol. 33, no. 8, pp. 1226–1234, 2009.

- [33] J. H. Lienhard, "Heat Transfer by Impingement of Circular Free-Surface Liquid Jets," 2005.
- [34] F. P. Incropera and S. Ramadhyani, "Single-Phase, Liquid Jet Impingement Cooling of High-Performance Chips," in *Cooling of Electronic Systems*, Dordrecht: Springer Netherlands, 1994, pp. 457–506.
- [35] B. Agostini, M. Fabbri, J. E. Park, L. Wojtan, J. R. Thome, and B. Michel, "State of the Art of High Heat Flux Cooling Technologies," *Heat Transf. Eng.*, vol. 28, no. 4, pp. 258– 281, 2007.
- [36] J. Stevens and B. W. Webb, "Local Heat Transfer Coefficients Under an Axisymmetric, Single-Phase Liquid Jet," *J. Heat Transfer*, vol. 113, no. February, p. 71, 1991.
- [37] K. Jambunathan, E. Lai, M. A. Moss, and B. L. Button, "A review of heat transfer data for single circular jet impingement," *Int. J. Heat Fluid Flow*, vol. 13, no. 2, pp. 106–115, 1992.
- [38] X. Liu and J. H. L. V, "Liquid Jet impingement heat transfer on a uniform heat flux surface." pp. 1–8, 1989.
- [39] F. P. Incropera, D. P. DeWitt, T. L. Bergman, and A. S. Lavine, *Fundamentals of Heat and Mass Transfer*, vol. 6th. 2007.
- [40] L. Rayleigh, "On the theory of long waves and bores," Proc. R. Soc. London. Ser. A, 1914.
- [41] G. Birkhoff and E. Zarantonello, "Jets, Wakes, and Cavities Academic," New York, 1957.
- [42] I. G. Currie, "Fundamental Mechanics of Fluids," *Mech. Eng.*, vol. 1, p. xiv, 525 p, 2003.
- [43] R. Olsson and E. Turkdogan, "Radial spread of a liquid stream on a horizontal plate," *Nature*, 1966.
- [44] S. Ishigai, S. Nakanishi, M. Mizuno, and T. Imamura, "Heat transfer of the impinging round water jet in the interference zone of film flow along the wall," *Bull. JSME*, 1977.
- [45] V. Nakoryakov, B. Pokusaev, and E. Troyan, "Impingement of an axisymmetric liquid jet on a barrier," *Int. J. Heat*, 1978.
- [46] M. Errico, "A Study of the Interaction of Liquid Jets with Solid Surfaces (Impingement, Cleaning)," *ProQuest Dissertations and Theses.* 1986.
- [47] V. Vasista, "Experimental study of the hydrodynamics of an impinging liquid jet," 1989.
- [48] X. Liu and J. H. V Lienhard, "Exl riments in Fluids The hydraulic jump in circular jet impingement and in other thin liquid films," *Exp. Fluids*, vol. 116, pp. 108–116, 1993.
- [49] K. Yokoi and F. Xiao, "Mechanism of structure formation in circular hydraulic jumps: Numerical studies of strongly deformed free-surface shallow flows," *Phys. D Nonlinear Phenom.*, vol. 161, no. 3–4, pp. 202–219, 2002.
- [50] K. Yokoi and F. Xiao, "Relationships between a roller and a dynamic pressure distribution

in circular hydraulic jumps," Phys. Rev. E, vol. 61, no. 2, p. 10, 2000.

- [51] J. W. M. Bush, J. M. Aristoff, and a. E. Hosoi, "An experimental investigation of the stability of the circular hydraulic jump," *J. Fluid Mech.*, vol. 558, p. 33, 2006.
- [52] N. O. Rojas, M. Argentina, E. Cerda, and E. Tirapegui, "Inertial lubrication theory," *Phys. Rev. Lett.*, vol. 104, no. 18, pp. 1–4, 2010.
- [53] N. Rojas, M. Argentina, and E. Tirapegui, "A progressive correction to the circular hydraulic jump scaling," *Phys. Fluids*, vol. 25, no. 4, 2013.
- [54] R. Dasgupta and R. Govindarajan, "Nonsimilar solutions of the viscous shallow water equations governing weak hydraulic jumps," *Phys. Fluids*, vol. 22, no. 11, 2010.
- [55] A. Rao and J. H. Arakeri, "Wave structure in the radial film flow with a circular hydraulic jump," vol. 31, pp. 542–549, 2001.
- [56] D. Randall, "The shallow water equations," Dep. Atmos. Sci. Color. State, 2006.
- [57] I. Tani, "Water jump in the boundary layer," J. Phys. Soc. Japan, 1949.
- [58] R. P. Kate, P. K. Das, and S. Chakraborty, "Hydraulic jumps due to oblique impingement of circular liquid jets on a flat horizontal surface," *J. Fluid Mech.*, vol. 573, pp. 247–263, 2007.
- [59] Y. Brechet, "On the circular hydraulic jump," Am. J. Phys., vol. 67, no. 8, p. 723, 1999.
- [60] S. Watanabe and V. Putkaradze, "Integral methods for shallow free-surface flows with separation," *J. Fluid*, 2003.
- [61] A. R. Kasimov, "A stationary circular hydraulic jump, the limits of its existence and its gasdynamic analogue," *J. Fluid Mech.*, vol. 601, pp. 189–198, 2008.
- [62] W. H. Hager, *Energy dissipators and hydraulic jump*, vol. 8. 1992.
- [63] H. Chanson, *The Hydraulics of Open Channel Flow: An Introduction*. 2004.
- [64] H. Chanson, "Current knowledge in hydraulic jumps and related phenomena. A survey of experimental results," *European Journal of Mechanics, B/Fluids*, vol. 28, no. 2. pp. 191– 210, 2009.
- [65] R. M. Khatsuria, *Hydraulics of spillways and energy dissipators*. Marcel Dekker, 2005.
- [66] H. Chanson, *Tidal Bores, Aegir, Eagre, Mascaret, Pororoca*. WORLD SCIENTIFIC, 2011.
- [67] B. Mohajer and R. Li, "Circular hydraulic jump on finite surfaces with capillary limit," *Phys. Fluids*, vol. 27, no. 11, p. 117102, Nov. 2015.
- [68] C. Ellegaard, A. Hansen, A. Haaning, and T. Bohr, "Experimental results on flow separation and transitions in the circular hydraulic jump," *Phys. Scr.*, 1996.
- [69] N. Zuckerman and N. Lior, "Jet impingement heat transfer: Physics, correlations, and numerical modeling," *Adv. Heat Transf.*, vol. 39, no. C, pp. 565–631, 2006.
- [70] M. Attalla and E. Specht, "Heat transfer characteristics from in-line arrays of free

impinging jets," Heat mass Transf., 2009.

- [71] A. Huber and R. Viskanta, "Convective heat transfer to a confined impinging array of air jets with spent air exits," *J. Heat Transfer*, 1994.
- [72] L. W. Florschuetz, D. E. Metzger, and C. C. Su, "Heat Transfer Characteristics for Jet Array Impingement With Initial Crossflow," in *Volume 4: Heat Transfer; Electric Power*, 1983, p. V004T09A001.
- [73] E. A. Browne, G. J. Michna, M. K. Jensen, and Y. Peles, "Experimental Investigation of Single-Phase Microjet Array Heat Transfer," *J. Heat Transfer*, vol. 132, no. 4, p. 41013, 2010.
- [74] M. Fabbri and V. K. Dhir, "Optimized Heat Transfer for High Power Electronic Cooling Using Arrays of Microjets," J. Heat Transfer, vol. 127, no. 7, p. 760, 2005.
- [75] A. J. Robinson and E. Schnitzler, "An experimental investigation of free and submerged miniature liquid jet array impingement heat transfer," *Exp. Therm. Fluid Sci.*, vol. 32, no. 1, pp. 1–13, 2007.
- [76] S. J. Slayzak, R. Viskanta, and F. P. Incropera, "Effects of Interactions Between Adjoining Rows of Circular, Free-Surface Jets on Local Heat Transfer From the Impingement Surface," *J. Heat Transfer*, vol. 116, no. February, pp. 88–95, 1994.
- [77] H. D. Haustein, J. Joerg, W. Rohlfs, and R. Kneer, "Influence of micro-scale aspects and jet-to-jet interaction on free-surface liquid jet impingement for micro-jet array cooling," *Thermomechanical Phenom. Electron. Syst. -Proceedings Intersoc. Conf.*, pp. 904–911, 2014.
- [78] S. J. Slayzak, R. Viskanta, and F. P. Incropera, "Effects of Interaction Between Adjacent Free Surface Planar Jets on Local Heat Transfer From the Impingement Surface," *J. Heat Transfer*, vol. 37, no. February, pp. 269–282, 1994.
- [79] M. Fabbri, S. Jiang, and V. K. Dhir, "A Comparative Study of Cooling of High Power Density Electronics Using Sprays and Microjets," *J. Heat Transfer*, vol. 127, no. 1, p. 38, 2005.
- [80] A. Bhunia and C. L. Chen, "Pressure Drop in Generating Free-Surface Liquid Microjet Array From Short Cylindrical Orifices," *J. Fluids Eng.*, vol. 133, no. 6, p. 61103, 2011.
- [81] G. J. Michna, E. A. Browne, Y. Peles, and M. K. Jensen, "The effect of area ratio on microjet array heat transfer," *Int. J. Heat Mass Transf.*, vol. 54, no. 9, pp. 1782–1790, 2011.
- [82] J.-Y. San and M.-D. Lai, "Optimum jet-to-jet spacing of heat transfer for staggered arrays of impinging air jets," *Int. J. Heat Mass Transf.*, vol. 44, no. 21, pp. 3997–4007, 2001.
- [83] M. Behnia, S. Parneix, Y. Shabany, and P. A. Durbin, "Numerical study of turbulent heat transfer in confined and unconfined impinging jets," *Int. J. Heat Fluid Flow*, vol. 20, no. 1, pp. 1–9, 1999.
- [84] M. J. Siclari, D. Migdal, T. W. Luzzi Jr., J. Barche, and J. . L. Palcza, "Development of

Theoretical Models for Jet-Induced Effects on V/STOL Aircraft," J. Aircr., vol. 13, no. 12, pp. 938–944, Dec. 1976.

- [85] W. G. Hill and R. C. Jenkins, "Effect of Nozzle Spacing on Ground Interference Forces for a Two-Jet V/STOL Aircraft," J. Aircr., vol. 17, no. 9, pp. 684–689, Sep. 1980.
- [86] W. G. HILL, "Effects of a central fence on upwash flows," *J. Aircr.*, vol. 22, no. 9, pp. 771–775, Sep. 1985.
- [87] P. M. CABRITA, A. J. SADDINGTON, and K. KNOWLES, "PIV measurements in a twin-jet STOVL fountain flow," *Aeronaut. J.*, vol. 109, no. 1100, pp. 439–449.
- [88] D. R. Kotansky, "Multiple Jet Impingement Flowfields," in *Recent Advances in Aerodynamics*, New York, NY: Springer New York, 1986, pp. 435–469.
- [89] D. KOTANSKY and L. GLAZE, "The effects of ground wall-jet characteristics on fountain upwash flow formation and development," in 14th Fluid and Plasma Dynamics Conference, 1981.
- [90] M. J. Siclari, W. G. Hill Jr., and R. C. Jenkins, "Stagnation Line and Upwash Formation of Two Impinging Jets," *AIAA J.*, vol. 19, no. 10, pp. 1286–1293, Oct. 1981.
- [91] L. W. Glaze, D. R. Bristow, and D. R. Kotansky, "V/STOL Fountain Force Coefficient." 1983.
- [92] D. R. Kotansky and L. W. Glaze, "Investigation of impingement region and wall jets formed by the interaction of high aspect ratio lift jets and a ground plane," 1978.
- [93] D. R. Kotansky, N. A. Durando, D. R. Bristow, and P. W. Saunders, "Multi Jet Induced Forces and Moments on VTOL Aircraft Hovering in and Out of Ground Effect." 1977.
- [94] R. C. Jenkins and W. G., J. Hill, "Investigation of VTOL Upwash Flows Formed by two Impinging Jets,." 1977.
- [95] R. P. KATE, P. K. DAS, and S. CHAKRABORTY, "An experimental investigation on the interaction of hydraulic jumps formed by two normal impinging circular liquid jets," J. Fluid Mech., vol. 590, pp. 355–380, 2007.
- [96] M. A. Teamah and M. M. Khairat, "Heat transfer due to impinging double free circular jets," *Alexandria Eng. J.*, vol. 54, no. 3, pp. 281–293, 2015.
- [97] R. Kneer, H. D. Haustein, C. Ehrenpreis, and W. Rohlfs, "Flow Structures and Heat Transfer in Submerged and Free Laminar Jets," no. February 2016, pp. 1–21, 2014.
- [98] M. M. Seraj, E. Mahdi, and M. S. Gadala, "Numerical Assessments of Impingement Flow over Flat Surface due to Single and Twin Circular Long Water Jets," *Trans. Control Mech. Syst.*, vol. 1, no. 7, pp. 290–299, 2012.
- [99] M. L. Hosain, R. Bel Fdhila, and A. Daneryd, "Heat transfer by liquid jets impinging on a hot flat surface," *Appl. Energy*, vol. 164, pp. 934–943, 2016.
- [100] J. H. Lienhard, "Impingement cooling with free-surface liquid jets," 18th Natl. 7th ISHMT-ASME Heat Mass Transf. Conf., 2006.

- [101] C. Ellegaard *et al.*, "Creating corners in kitchen sinks," *Nature*, vol. 392, no. 6678, pp. 767–768, Apr. 1998.
- [102] H. Schlichting and K. Gersten, Boundary-layer theory. 2003.
- [103] R. Li and N. Ashgriz, "Characteristics of liquid sheets formed by two impinging jets," *Phys. Fluids*, vol. 18, no. 8, pp. 1–13, 2006.
- [104] N. D. Dombrowski and P. C. Hooper, "A study of the sprays formed by impinging jets in laminar and turbulent flow," *J. Fluid Mech.*, vol. 18, no. 3, p. 392, Mar. 1964.
- [105] M. F. Heidmann, R. J. Priem, and J. C. Humphrey, "A study of sprays formed by two impinging jets," *NACA Technical Note*. p. 33 pages, 1957.
- [106] K. R. Stehling, "Injector Spray and Hydraulic Factors in Rocket Motor Analysis," J. Am. Rocket Soc., vol. 22, no. 3, pp. 132–138, May 1952.
- [107] K. D. Miller, "Distribution of Spray From Impinging Liquid Jets," J. Appl. Phys., vol. 31, no. 6, p. 1132, 1960.
- [108] D. Hasson and R. E. Peck, "Thickness distribution in a sheet formed by impinging jets," *AIChE J.*, vol. 10, no. 5, pp. 752–754, Sep. 1964.
- [109] E. A. Ibrahim and A. J. Przekwas, "Impinging jets atomization," *Phys. Fluids A Fluid Dyn.*, vol. 3, no. 12, p. 2981, 1991.
- [110] J. Naber and R. D. Reitz, "Modeling Engine Spray/Wall Impingement," 1988.
- [111] W. E. Ranz, "Some Experiments on the Dynamics of Liquid Films," J. Appl. Phys., vol. 30, no. 12, p. 1950, 1959.

# Appendices

## **APPENDIX A Detailed Derivation of the Surface Tension Force**

To obtain the surface tension force  $F_{\sigma}$  (see Figure 4.4b), geometric details of the free surface have been shown in Figure 4.5. The expression of the surface tension force will be derived here based on Eq. (4.4), which is

$$F_{\sigma} = \Delta \theta \int_{S} \sigma \left( K_{1} + K_{2} \right) \left( \hat{n} \cdot \hat{r} \right) r dS$$
(A1)

To determine the surface normal  $\hat{n}$ , we define the free surface as

$$J(r,z) = z - \xi(r) \tag{A2}$$

which comes from the curve line  $z = \xi(r)$ . The gradient of the curve in Cartesian coordinates is

$$\nabla J = \frac{\partial J}{\partial z} \hat{z} + \frac{\partial J}{\partial r} \hat{r} = \hat{z} - \xi' \hat{r}$$
(A3)

The unit vector of the surface normal then can be expressed as

$$\hat{n} = \frac{\nabla J}{|\nabla J|} = \frac{1}{\sqrt{1 + {\xi'}^2}} \,\hat{z} - \frac{\xi'}{\sqrt{1 + {\xi'}^2}} \,\hat{r} \tag{A4}$$

where  $\xi' = d\xi/dr$ . Hence

$$\hat{n} \cdot \hat{r} = -\frac{\xi'}{\sqrt{1 + {\xi'}^2}} \tag{A5}$$

The first principal curvature can be obtained by taking the divergence of the surface normal in the Cartesian coordinates system, which is

$$K_{1} = -\left(\hat{z}\frac{\partial}{\partial z} + \hat{r}\frac{\partial}{\partial r}\right) \cdot \hat{n} = \frac{\partial}{\partial r} \left(\frac{\xi'}{\sqrt{1 + {\xi'}^{2}}}\right)$$
(A6)

For the other curvature  $K_2$  that is orthogonal to  $K_1$ , the radius is  $r/\cos \gamma$ , and  $\cos \gamma = -\hat{n} \cdot \hat{r}$  (see Fig. 5a). Thus,

$$K_2 = -\frac{1}{r}\hat{n}\cdot\hat{r} = \frac{1}{r}\frac{\xi'}{\sqrt{1+{\xi'}^2}}$$
(A7)

where Eq. (A5) has been used.

Substituting Eqs. (A5), (A6), and (A7) into Eq. (A1) results in

$$F_{\sigma} = -\Delta\theta\sigma \int_{S} \left[ \frac{\xi'^2}{1+\xi'^2} + \frac{r}{2} \frac{d}{dr} \left( \frac{\xi'^2}{1+\xi'^2} \right) \right] dS$$
(A8)

The total curvature can also be obtained by simply replacing the Cartesian divergence in Eq. (A6) with the cylindrical divergence.

Replacing  $\xi'^2$  in the numerators with  $(1 + \xi'^2 - 1)$ , we change Eq. (A8) to

$$F_{\sigma} = -\Delta\theta\sigma \int_{S} \left[ 1 - \frac{1}{1 + {\xi'}^2} - \frac{r}{2} \frac{d}{dr} \left( \frac{1}{1 + {\xi'}^2} \right) \right] dS$$

which can be re-organized as

$$F_{\sigma} = -\Delta\theta\sigma \left[ S - \int_{S} \frac{1}{2r} \frac{d}{dr} \left( \frac{r^2}{1 + {\xi'}^2} \right) dS \right]$$
(A9)

The differential length dS can be expressed as  $dS = \sqrt{(dr)^2 + (d\xi)^2}$ . We define  $r' = dr/d\xi$ . As shown by Figure 4.5b, the outer film shape can be divided into three sections, which, depending on how  $\xi$  changes with r, can be expressed by

$$\frac{dS}{dr} = \frac{\sqrt{r'^2 + 1}}{r'} \quad \text{for } R_j \le r \le R_{j,c}$$

$$\frac{dS}{dr} = 1 \quad \text{for } R_{j,c} \le r \le R_{s,c}$$

$$\frac{dS}{dr} = -\frac{\sqrt{r'^2 + 1}}{r'} \quad \text{for } R_{s,c} \le r \le R_s$$
(A10)

Applying Eq. (A10) to the integral term in Eq. (A9) gives

$$\begin{split} &\int_{S} \frac{1}{2r} \frac{d}{dr} \left( \frac{r^{2}}{1 + \xi'^{2}} \right) dS \\ &= \int_{R_{j}}^{R_{j,o}} \frac{1}{2} \frac{\sqrt{r'^{2} + 1}}{rr'} d\left( \frac{r^{2}r'^{2}}{r'^{2} + 1} \right) + \int_{R_{j,o}}^{R_{s,i}} \frac{1}{2r} d\left( \frac{r^{2}}{1 + \xi'^{2}} \right) - \int_{R_{s,i}}^{R_{s}} \frac{1}{2} \frac{\sqrt{r'^{2} + 1}}{rr'} d\left( \frac{r^{2}r'^{2}}{r'^{2} + 1} \right) \\ &= \frac{rr'}{\sqrt{r'^{2} + 1}} \bigg|_{R_{j}}^{R_{j,c}} + r \bigg|_{R_{j,c}}^{R_{s,c}} - \frac{rr'}{\sqrt{r'^{2} + 1}} \bigg|_{R_{s,c}}^{R_{s}} \end{split}$$
(A11)

where  $\xi' = 0$  for  $R_{j,c} \le r \le R_{s,c}$  has been used.

Based on Figure 4.5b, the boundary conditions required for continuing Eq. (A11) are

$$r'(r = R_{j}) = \cot \beta_{j}; \ r'(r = R_{j,c}) = +\infty; \ r'(r = R_{s,c}) = -\infty; \ r'(r = R_{s}) = -\cot \beta_{s}$$

Here  $\beta_j$  is the slope angle at  $r = R_j$ , and  $\beta_s$  is the contact angle at the disc edge. Applying the above conditions to Eq. (A11), and then substituting Eq. (A11) into Eq. (A9), results in

$$F_{\sigma} = -\sigma \left[ S - \left( R_s \cos \beta_s - R_j \cos \beta_j \right) \right] \Delta \theta$$
(A12)

If there is no jump,  $S = R_s - R_j$ , and  $\beta_j = \beta_s = 0$ . As a result, the surface tension force vanishes. If we consider  $S = S_j + (R_{s,c} - R_{j,c}) + S_s$  as shown by Figure 4.3b, Eq. (A12) can be re-organized as

$$F_{\sigma} = -\sigma \Big[ S_j - \left( R_{j,c} - R_j \cos \beta_j \right) + S_s - \left( R_s \cos \beta_s - R_{s,c} \right) \Big] \Delta \theta$$
(A13)

Eq. (A13) shows two components, one is the surface tension force at the jump, and the other component is the surface tension force at the disc edge.

#### **APPENDIX B** Detailed Derivation of the Viscous Friction Force

The analysis here focuses on the viscous force imposed by the disc on the post-jump film flow. This analysis will result in Eq. (4.14). To be consistent with the assumption of axisymmetric flow for the control volume analysis, here the flow in the post-jump region is assumed to be in radial direction without considering the azimuthal flow. The post-jump film has been assumed as a unidirectional flow with zero boundary layer right after jump at  $r = R_{j,c}$ . From  $R_{j,c}$  to  $R_{s,c}$ , the velocity boundary layer grows. The free stream velocity,  $U_0$ , has been given by Eq. (4.13). For  $R_{s,c} \gg R_{j,c}$ , the governing equations in cylindrical coordinates with boundary layer approximations are

$$\frac{\partial (ru)}{\partial r} + \frac{\partial (rw)}{\partial z} = 0 \tag{B1-a}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = v\frac{\partial^2 u}{\partial z^2}$$
(B1-b)

where u and w are the velocity components in r and z directions, respectively, and v is the kinematic viscosity of the fluid. The pressure term in Eq. (B1-b) has been dropped, which is a valid approximation for large flat post-jump film with stable rim at the edge.

From  $R_{j,c}$  to  $R_{s,c}$ , the velocity boundary layer is assumed to never reach the free surface. The film can be considered as semi-infinite with boundary conditions given by

$$\begin{aligned} u\big|_{z=0} &= w\big|_{z=0} = 0\\ u\big|_{z\to\infty} &= U_0 \end{aligned} \tag{B2}$$

This is a Blasius boundary layer problem for radial flow, and the boundary layer depth is  $\sim \left[v(r-R_{j,c})/U_0\right]^{1/2}$ . The problem can be solved by introducing a stream function given by

$$\psi = \left[\frac{2}{3}\upsilon\left(r - R_{j,c}\right)U_0\right]^{1/2} f\left(\eta\right)$$
(B3)

where  $f(\eta)$  is the similarity function with the similarity variable given by

$$\eta = \left[\frac{3U_0}{2\nu(r-R_{j,c})}\right]^{1/2} z \tag{B4}$$

Substituting  $u = \partial \psi / \partial z$  and  $w = -\partial \psi / \partial r$  into Eq. (B1-b) gives

$$f''' + ff'' = 0$$
 (B5)

According to Eq. (B2), the boundary conditions for Eq. (B5) are

$$f(0) = f'(0) = 1$$
  

$$f'(\infty) = 1$$
(B6)

According to Schlichting & Gersten [102], it follows that f''(0) = 0.4696, which is

$$\left[\frac{2\upsilon(r-R_{j,c})}{3U_0}\right]^{1/2} \frac{1}{U_0} \frac{\partial u}{\partial z}\Big|_{z=0} = 0.4696$$
(B7)

Hence, the velocity gradient at the solid-fluid interface is

$$\left. \frac{\partial u}{\partial z} \right|_{z=0} = 0.575 U_0 \left[ \frac{U_0}{\upsilon \left( r - R_{j,c} \right)} \right]^{1/2} \tag{B8}$$

The viscous friction force projected to the central radial direction of the control volume is

$$F_{\mu} = -\mu \int_{R_{j,c}}^{R_{s}} \frac{\partial u}{\partial z} \bigg|_{z=0} r dr \Delta \theta$$
(B9)

where  $\mu$  is the dynamic viscosity of the fluid. Inserting Eq. (4.13) into Eq. (B8), and then substituting Eq. (B8) into Eq. (B9), we have

$$F_{\mu} = -0.383 \left(\rho\mu\right)^{1/2} \left(\frac{Q}{2\pi H R_{j,c}}\right)^{3/2} \left(R_{s,c} - R_{j,c}\right)^{1/2} \left(R_{s,c} + 2R_{j,c}\right) \Delta\theta$$
(B10)